UNIVERSITÉ DE REIMS CHAMPAGNE-ARDENNE

ÉCOLE DOCTORALE SCIENCES DU NUMERIQUE ET DE L'INGENIEUR n°620

THÈSE

Pour obtenir le grade de

DOCTEUR DE L'UNIVERSITÉ DE REIMS CHAMPAGNE-ARDENNE

Discipline : **Automatique, signal, productique, robotique**

Spécialité **:Automatique**

Présentée et soutenue publiquement par

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Le 14 janvier 2021

Cooperative Control of Multi-Agent Systems in the Clustered Network

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JURY

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Acknowledgements

Obtaining a Ph.D. degree in Electrical and Computer Engineering at URCA (University of Reims Champagne-Ardenne) is my ultimate goal during the last four years. Along my journey, I am proud to be a member of CReSTIC, where I have been very fortunate to interact with many great people. My Ph.D. advisor, Prof. Noureddine MANAMANNI, and my Ph.D. co-advisor, Prof. Nadhir MESSAI obviously play a significant role in my success at CReSTIC. During my time working under their supervision, Noureddine and Nadhir showed me their constant guidance and support for my studies and career. They taught me how to conduct research, how to write a paper, and above all how to shape my career. Whenever I seek advice, they have always been there with the best solution. I am thankful and indebted for their great care and concern about my career and my family.

My deep thanks to all my friends and colleagues, who gave me unforgettable and precious friendship, crossed the barriers of language and culture and who were always present in good times as well as in the most difficult. I thank Prof. Dinh Hoa Nguyen and Prof. Thinh Thanh Doan for our collaborations during my last semester here.

I would also like to extend my sincere thanks to all the members of the jury for agreeing to review my presentation, despite their busy schedule. My sincere appreciation to Prof. Lydie NOUVELIERE and Prof. Michel BASSET, who do me the honor of examining this work. I also express my deep thanks to Prof. Frederic KRATZ and Prof. Seddik M. DJOUADI, for agreeing to evaluate this work.

I thank the VIED (Vietnamese International Education Department) of MOET (Ministry of Education and Training) for their financial support through my assistantship and fellowship during my Ph.D. studies.

Most importantly, I thank my family for their unconditional love and unlimited support for my career success. I extremely appreciate my mother-in-law for being here in 6 months to take care of my kids, so that I would have more time for my studies. I also thank my kids, Gia Binh, and Nhat Nam (Kévin) for the time taken from them. They are always a big motivation to push me forward. I wish to thank my wife, Thanh Quynh, for being with me and showing her constant support for my career. She has also been very patient with all of my frustrations during my studies.

I wish to show my gratitude and love to my parents for their unlimited support of my career choice. They have worked very hard and given up many things to make my success possible. This thesis is dedicated to them.

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List of Notations

$\mathbb{C}$	Set of complex numbers
$\mathbb{R}$	Set of real numbers
$\mathbb{Z}^+$	Set of positive integer numbers
$\mathbb{F}^{m\times n}$	Set of matrices with m rows, n columns, and entries in $\mathbb{R}$ or $\mathbb{C}$
$\mathbb{R}^{m\times n}$	Set of matrices with m rows, n columns, and entries in $\mathbb{R}$
$x \in \mathbb{R}^{n_x}$	Real-valued column vector of dimension n_x
x^*	The conjugate transpose of x
t	Continuous-tiem instant, real-valued
$x(t)$	Continuous time-variant vector variable
$x^i(t)$	Continuous time-variant vector variable of agent i
$\|x\|_p$	The p-norm of vector x, for $p \geq 1$
$\|\mathcal{N}^i\|$	Cardinality of the set $\mathcal{N}^i$
a^{ij}	The entry at the position of ith row and jth column of matrix A
A^T	Transpose of matrix A
A^H	Hermitian conjugate of matrix $A \in \mathbb{C}^{n\times m}$
A^*	Hermitian transpose or conjugate transpose of matrix A
$A > 0$	Positive definite matrix A
$A \geq 0$	Positive semi-definite matrix A
$A < 0$	Negative definite matrix A
$A \leq 0$	Negative semi-definite matrix A
$det(A)$	Determination of matrix A
$rank(A)$	Rank of matrix A
$ker(A)$	Kernel of matrix A
$A \otimes B$	Kronecker product of matrices A and B
I	Identity matrix with appropriate dimensions
0	Zero matrix with appropriate dimensions
$\triangleq$	Equals by definition
$\bar{z}$	Conjugate of $z \in \mathbb{C}$
$diag(A_1, \dots, A_n)$	Block-diagonal matrix $\begin{bmatrix} A_1 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & A_n \end{bmatrix}$, where $A_i \in \mathbb{F}^{n_i \times m_i}$

List of Abbreviations

MASs	**M**ulti **A**gent **S**ystems
UAVs	**U**nmaned **A**tutonous **V**ehilces
LMIs	**L**inear **M**atrix **I**nequalities
CPS	**C**yber **P**hysical **S**ystems
NCS	**N**etwork **C**ontrol **S**ystems
LQR	**L**inear **Q**uadratic **R**eguarator

Introduction Générale

Au cours des dernières décennies, les progrès rapides de la science et des sciences et des technologies de l'information et de la communication ont rendu possible le déploiement d'un grand nombre d'agents autonomes qui travaillent en coopération pour assurer des missions civiles et militaires. Comparativement à un seul agent complexe, cela peut améliorer considérablement l'efficacité opérationnelle, réduire les coûts et fournir des degrés supplémentaires de redondance. Le fait plusieurs agents autonomes pour travailler ensemble efficacement afin d'obtenir des comportements collectifs de groupe est généralement appelé contrôle coopératif des systèmes multi-agents (MASs). Le contrôle coopératif des MASs a fait l'objet d'une attention particulière de la part de diverses communautés scientifiques, en particulier de la communauté des systèmes et du contrôle.

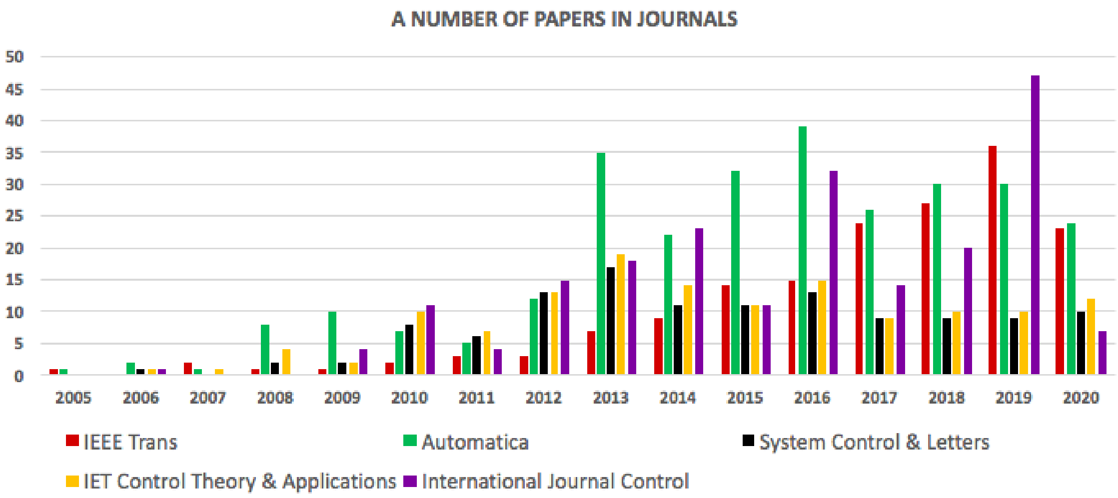

FIGURE 1: Statistiques d'un certain nombre d'articles relatifs au *"consensus, coopérative, MASs"* dans les revues entre 2005 – Juillet, 2020

Afin de confirmer ce constat, nous avons analysé les données des articles publiés dans certaines revues de qualité Q1 au cours de la période 2005 - juillet 2020 avec les mots clés *"consensus, cooperative, MASs"*, le résultat de ce travail est illustré par la Fig.1, qui montre que le sujet du "contrôle coopératif" est de plus en plus étudié. Par exemple, le nombre d'articles dans "Automatica" en 2015 augmente de 30 fois par rapport à 2005, tandis que les publications "IEEE Trans" augmentent d'environ 10 fois. En outre, de plus en plus d'applications potentielles dans divers domaines tels que le vol en formation, l'informatique distribuée, la robotique, la surveillance, les systèmes de reconnaissance, les systèmes d'alimentation électrique, l'attaque coopérative de plusieurs missiles et les systèmes de transport intelligents sont analysées. En particulier, 31% des articles publiés dans Automatica représentant une proportion importante (voir Fig.2). Ensuite, 23%, 18%, 15%, et 13% sont respectivement le pourcentage d'articles publiés dans "International Journal Control", "IEEE Trans", "IET Control Theory & Applications", et "System Control &

Letters". De plus, à partir des histogrammes, nous constatons qu'il y a beaucoup de groupes de recherche qui se penchent sur ce sujet, ce qui entraîne une concurrence considérablement accrue entre les groupes et exige également des contributions considérables en termes de théorie et d'applications dans les publications. Par conséquent, le contrôle coopératif est non seulement un sujet intéressant mais aussi un sujet plus stimulant.

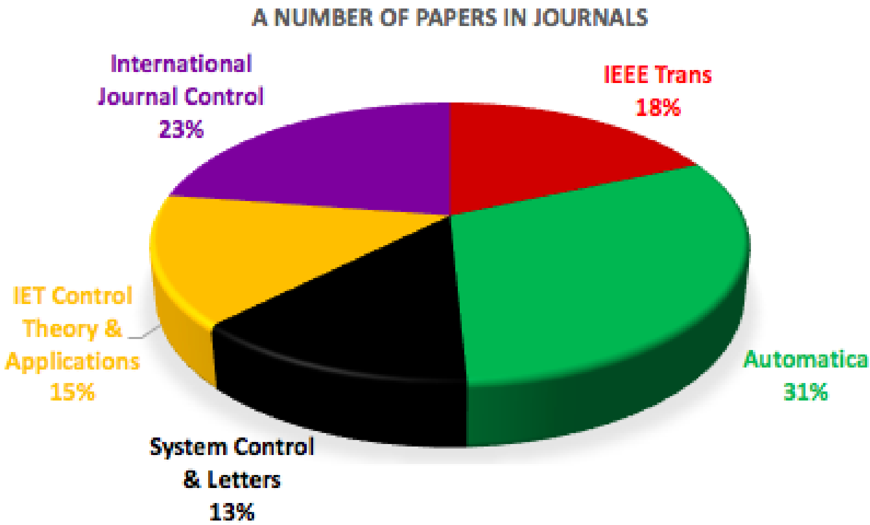

FIGURE 2: Statistiques du pourcentage d'articles liés textit "consensus, coopérative, MASs" dans les revues au cours de 2005 – Juillet, 2020

Pour le problème du contrôle coopératif, la tâche principale est de concevoir des protocoles de contrôle pertinents pour atteindre l'objectif de coordination souhaité. Ce concept est né d'un article de Vicseck et al, 1995 dans Physical Review Letters (voir Fig.3), où l'émergence d'un mouvement auto-ordonné dans les systèmes biologiques de particules, appelé *modèle de Vicseck*, était étudiée par les règles du plus proche voisin. Cependant, ce travail montre des résultats de simulation sans explications théoriques claires. Ce problème a été résolu dans Jadbabaie and Morse, 2003, où, sur la base de la théorie des graphes, le *modèle de Viceck* a été représenté comme un système linéaire commuté dont le signal de commutation prend des valeurs dans l'ensemble des indices qui paramètrent la famille des graphes. En outre, en raison du grand nombre d'agents, de la répartition spatiale des actionneurs, de la capacité de détection limitée des capteurs et des courtes portées de communication sans fil, il est considéré comme trop coûteux, voire impossible en pratique, de mettre en œuvre des contrôleurs centralisés. Ainsi, le contrôle distribué (l'une des premières mentions dans Olfati-Saber and Murray, 2004), dépendant uniquement des informations locales des agents et de leurs voisins, semble être un outil prometteur pour le traitement des MASs. Dans Olfati-Saber and Murray, 2004, le problème du consensus a été particulièrement analysé avec des topologies fixes et de commutation qui ont été considérées comme des graphiques non orientés ou fortement connectés. Les résultats ont été étendus dans Ren and Beard, 2005, où il a été démontré qu'il

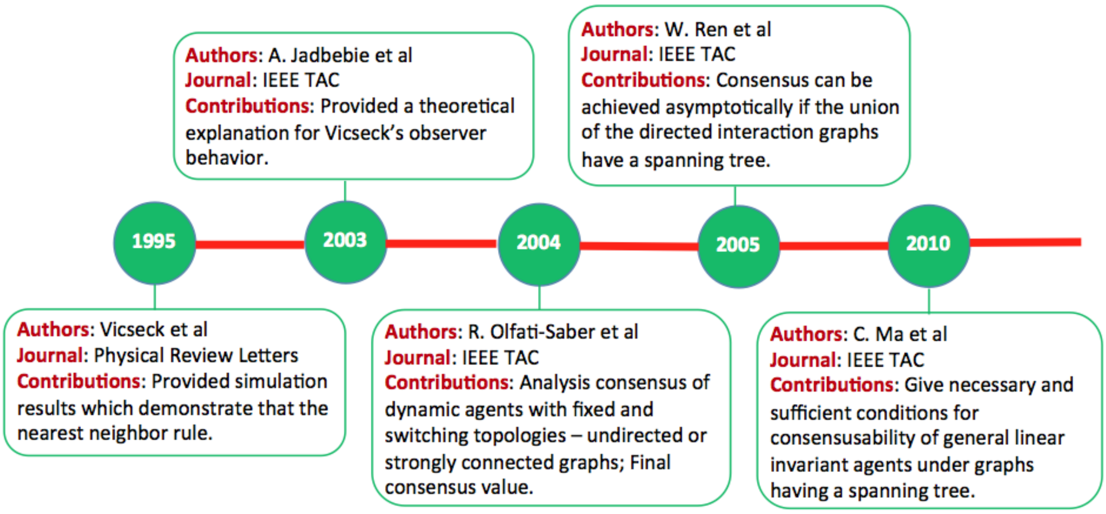

FIGURE 3: Chronologie du contrôle coopératif majeur de MAS

est suffisant d'avoir un graph de communication avec seulement un arbre de couverture pour obtenir un consensus. Outre l'importance de la topologie du réseau, la dynamique des MAS joue un rôle important dans l'étude et la conception des algorithmes coopératifs, ainsi que dans les valeurs finales de consensus. Les conditions nécessaires et suffisantes pour un consensus des MASs linéaires générales sont données dans Ma and Zhang, 2010. Suivant cette ligne de recherche, il existe une série de résultats axés sur l'analyse et la conception d'algorithmes coopératifs avec des MASs homogènes et hétérogènes. En outre, le contrôle coopératif sous des contraintes de temps, d'entrée, d'état, ainsi que la quantification et le contrôle coopératif déclenché par un événement sont également étudiés (voir dans le chapitre 1 pour plus de détails).

En raison du fait que les agents dans les MASs sont généralement limités en ressources, comme les limies de communication sans fil (pour échanger des informations entre les agents), les capteurs (pour mesurer les informations relatives entre les agents voisins) et les actionneurs (pour piloter les agents), ainsi que les contraintes énergétiques liées aux interactions de longue durée, un ingénieur doit parfois diviser un grand réseau en grappes. Des exemples de ces réseaux, appelés réseaux en grappes [1] comprennent à la fois les communications en ondes millimétriques 5G dans le réseau sans fil[2] (voir Fig.4), le consensus de l'avis dans les réseaux sociaux[3] (voir Fig.5).

Certains résultats concernant le problème de consensus dans les réseaux d'agents dynamiques de type intégrateur ont été présentés dans Bragagnolo et al., 2016, où une stratégie de réinitialisation de l'état des leaders est conçue, ce qui permet de parvenir à un consensus dans le réseau clusterisé. Cependant, plusieurs problèmes

[1]Dans cette thèse, nous utilisons "réseau en grappes" pour représenter le réseau, qui s'est divisé/composé en/des grappes qui sont presque toujours isolées les unes des autres.

[2]Long Zhang, et al, A Survey on 5G millimeter Wave Communications for UAV-Assisted Wireless Networks, *IEEE Access*, July 2019

[3]https://softwarerecs.stackexchange.com

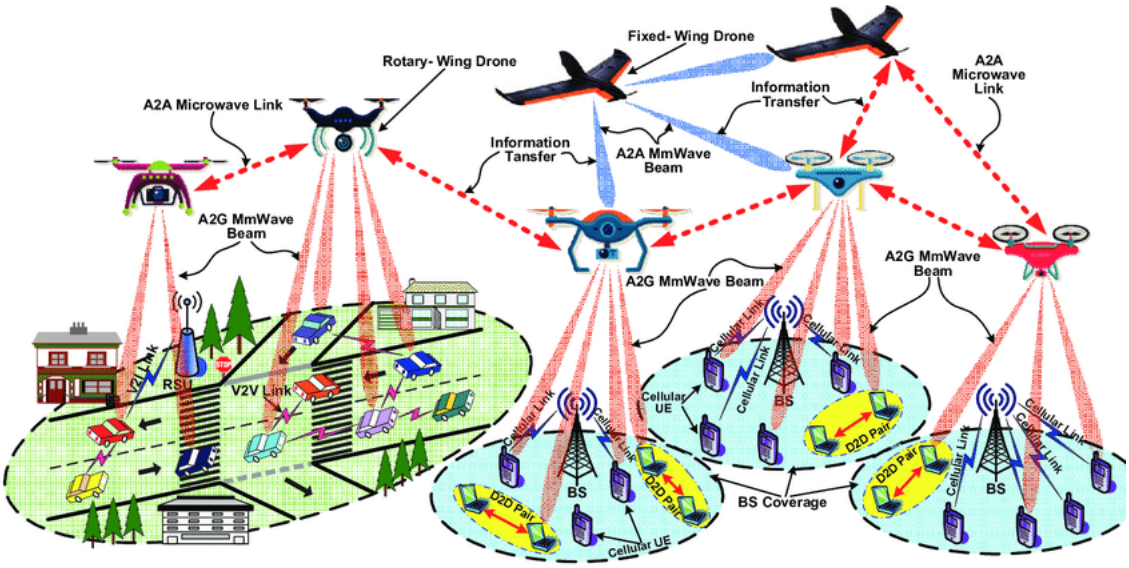

FIGURE 4: Communications en ondes millimétriques 5G pour les réseaux sans fil assistés par des drones[2]

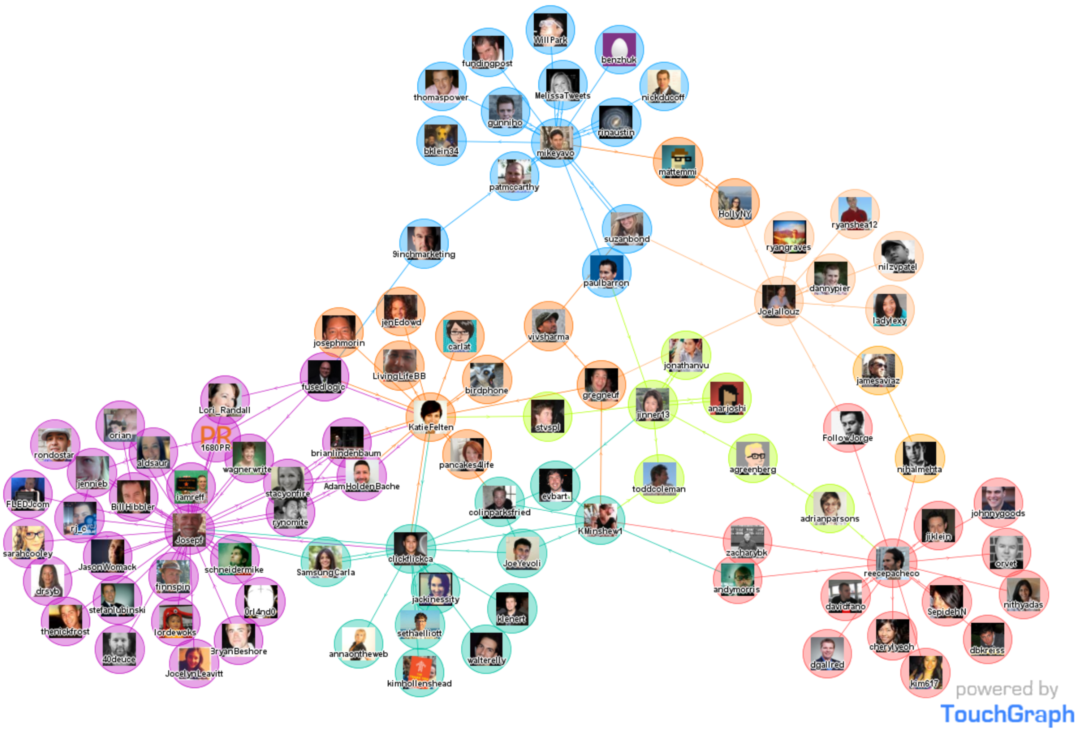

FIGURE 5: Le réseau social [3]

ouverts liés au contrôle coopératif dans le réseau de clusters restent à étudier. La conception de contrôleurs distribués pour les MASs à dynamique complexe (homogène et hétérogène), ou d'algorithmes de contrôle de formation pour le réseau groupé de multi-UAV sous état, les contraintes d'entrée sont quelques exemples de problèmes ouverts intéressants et stimulants. Par conséquent, dans cette thèse, nous étendons ces directions de recherche en suivant la motivation ci-dessus.

Contributions et Résumés

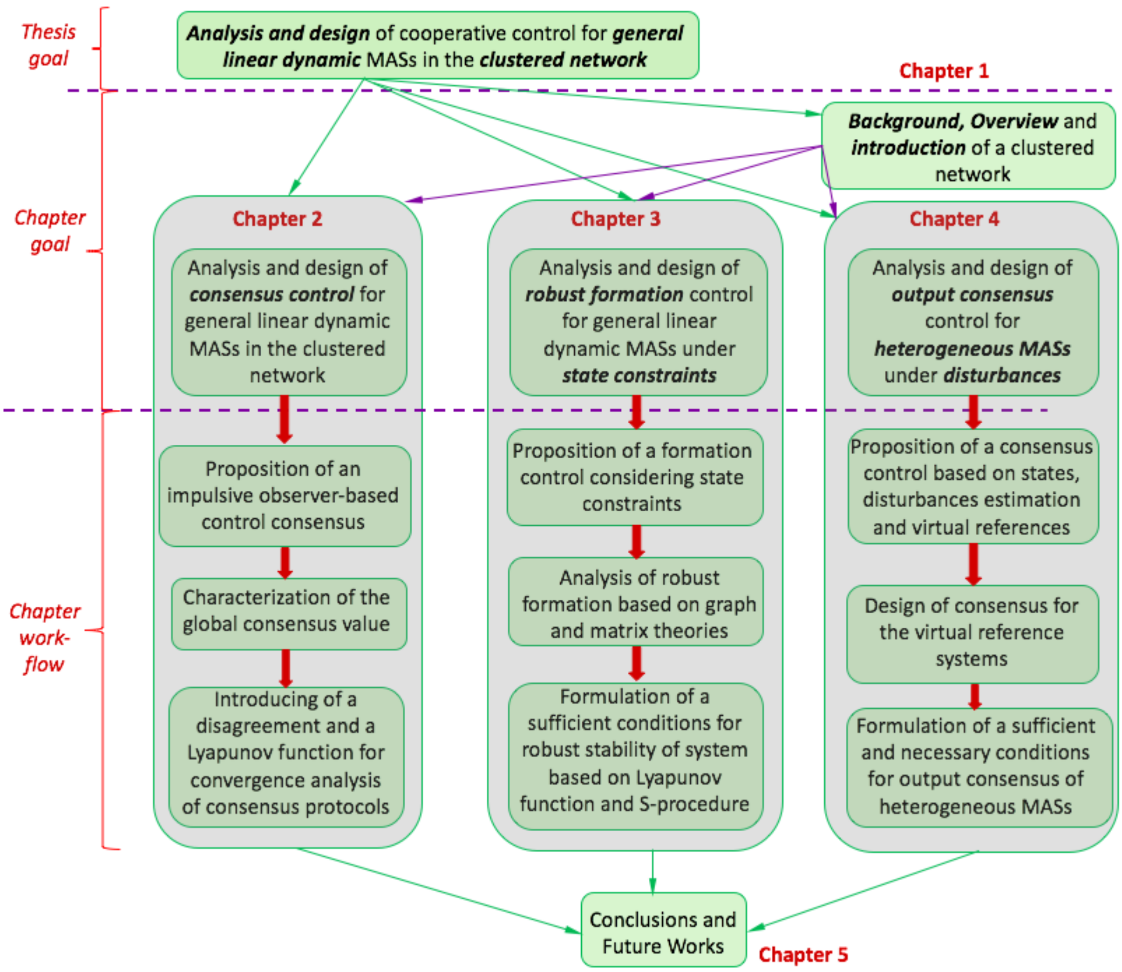

FIGURE 6: Aperçu général de la thèse et des principales contributions

Dans ce qui suit, nous présentons les grandes lignes et résumons les principales contributions de la thèse. La structure de cette thèse est illustrée dans la Fig.6. Les principales contributions de chaque chapitre sont les suivantes :

Le chapitre 1 présente une vue d'ensemble de l'état de la technique en matière de contrôle coopératif. Nous étudions ce problème en nous basant sur quatre facteurs principaux : la topologie du réseau, la dynamique des MASs, les contraintes des MASs et les méthodologies. Nous montrons que la topologie du réseau et la dynamique des MASs sont deux éléments importants dans l'analyse et la conception des algorithmes de contrôle coopératif. Nous poursuivons avec des contributions dans le domaine du contrôle coopératif sous contraintes, telles que les contraintes de communication et les contraintes d'agent. Ensuite, nous présentons les méthodologies de la littérature en matière d'analyse et de résolution du contrôle coopératif des MASs dans les réseaux. Enfin, nous discutons de la littérature présentée et des questions de recherche ouvertes.

Le chapitre 2 aborde le problème du consensus dans le réseau, où chaque nœud du graphe du réseau représente un agent à dynamique linéaire. Le comportement coopératif des MASs linéaires avec la dynamique générale du système dans le réseau en clusters est défini non seulement par les protocoles de contrôle dynamique concernant les clusters isolés, mais aussi par les interactions discrètes entre les leaders. Cela rend le problème de consensus dans le réseau en clusters avec des agents linéaires beaucoup plus difficile que celui du cas de l'intégrateur. Ainsi, un contrôle impulsif basé sur l'observation est proposé pour traiter le problème de consensus. Les principales contributions de ce chapitre sont au nombre de trois.

- L'analyse et la caractérisation de la valeur du consensus dans le cadre considéré est analysée. Nous montrons que la valeur du consensus global ne dépend que de la dynamique de chaque agent, des graphiques des clusters, de l'interaction entre les leaders et des conditions initiales.

- Un contrôle impulsif basé sur l'observation, qui utilise uniquement les informations de sortie relatives locales, et l'interaction discrète entre les groupes de leaders, est conçu. Ensuite, nous montrons que la conception consensuelle des réseaux en clusters peut être indirectement résolue en considérant la stabilité d'un système équivalent. Pour étudier la stabilité de ce système équivalent, nous proposons un algorithme permettant de choisir de manière appropriée les matrices de rétroaction, gain des observateurs et les poids de couplage sous la forme de certaines LMI.

- L'interaction de conception entre les grappes de leaders et le contrôleur impulsif basé sur l'observateur, assurant que les agents dans les réseaux en grappe renferment une cible prescrite.

Le chapitre 3 nous étudions le problème de la formation dans les systèmes composés d'agents linéaires qui sont soumis à des contraintes d'état. La structure de communication en temps continu dans chaque grappe est représentée par un clusters fixe et non orienté. Les principales contributions de ce chapitre sont résumées comme suit :

- Un protocole de formation robuste, qui traite de la communication en temps continu à l'intérieur des clusters et de l'échange d'informations en temps discret entre les clusters, est introduit. Il est ensuite montré que le problème de contrôle de la formation robuste considéré peut être indirectement résolu en étudiant la stabilité robuste d'un système équivalent basé sur la théorie des matrices et la théorie des graphes algébriques. De plus, il montre le rôle important de la communication entre les leaders à certains moments discrets spécifiques, représentés par la matrice stochastique.

- Une condition suffisante est obtenue en termes d'inégalités matricielles linéaires (LMI) pour la formation distribuée robuste de réseaux composés d'agents linéaires génériques sous contraintes d'état et de communications hybrides. De plus, nous montrons que les LMI obtenues peuvent être solutionnées de manière entièrement distribuée, c'est-à-dire que chaque agent peut calculer la matrice de gain par lui-même et mettre en œuvre le protocole de formation robuste en utilisant uniquement des informations locales (ses informations et celles de ses voisins).

Le chapitre 4 examine le problème du consensus de sortie dans les réseaux composés de MASs hétérogènes qui sont soumis à différentes perturbations. Chaque cluster est représentée par un graphe fixe et dirigé. Un contrôle de consensus de sortie est proposé pour gérer le consensus dans le réseau considéré. Les principales contributions de ce chapitre peuvent être résumées comme suit.

- Investigation du problème du consensus dans un réseau de MASs en grappes dirigées, où les agents ont une dynamique linéaire distincte et générique sous différentes perturbations.

- Un modèle de référence interne dynamique pour chaque agent est introduit, qui prend en compte les communications en temps continu entre les modèles de référence internes dans les clusters virtuels et les échanges d'informations discrètes entre ces clusters virtuels. Par conséquent, le consensus de sortie des agents hétérogènes est indirectement résolu par le consensus des références virtuelles. Pour y parvenir, un protocole de contrôle de consensus hybride est proposé pour le réseau en grappe virtuel. Grâce aux résultats de la théorie des matrices et de la théorie des graphes algébriques, le consensus du réseau en grappes virtuel est résolu.

- Une condition suffisante et nécessaire est obtenue pour le consensus de sortie des agents hétérogènes linéaires sous différentes perturbations dans le réseau en grappe.

Finalement, **le chapitre 5** présente un bref résumé des principaux résultats et contributions de cette thèse, et indique quelques sujets/directions possibles pour la recherche future.

Publications

La plupart des résultats de cette thèse ont été publiés dans des revues internationales à comité de lecture ou lors de conférences internationales. Certains de ces résultats ont été obtenus en collaboration avec d'autres chercheurs. Les publications résultant de certaines parties de ma recherche de thèse sont énumérées ci-dessous :

Journal Papers

1. **Van Thiem Pham**, Nadhir Messai, and Noureddine Manamanni. Impulsive Observer-Based Control in Clustered Networks of Linear Multi- Agent Systems. *IEEE Transactions on Network Science and Engineering*, 2019 (Early Acess), **IF: 5.213** according to Journal Citation Reports (JCR) 2020.

2. **Van Thiem Pham**, Nadhir Messai, Dinh Hoa Nguyen, and Noureddine Manamanni. Robust Formation Control Under State Constraints of Multi-Agent Systems in Clustered Networks. *Systems & Control Letters*, 2019 (accepted), **5-Year IF: 3.048** according to Journal Citation Reports (JCR) 2020.

3. **Van Thiem Pham**, Nadhir Messai, and Noureddine Manamanni. Output Consensus Design in Clustered Networks of Heterogeneous Linear Multi-Agent Systems under Disturbances. *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, 2020 **(revision)**, **IF: 9.309** according to Journal Citation Reports (JCR) 2020.

Communications

4. **Van Thiem Pham**, Nadhir Messai, and Noureddine Manamanni. Consensus of Multi-Agent Systems in Clustered Networks. *2019 18th European Control Conference (ECC)*, pages 1085–1090, 2019.

5. **Van Thiem Pham**, Nadhir Messai,D Hoa Nguyen, and Noureddine Manamanni. Adaptive Output Consensus Design in Clustered Networks of Heterogeneous Linear Multi-Agent Systems. *58th Conference on Decision and Control (CDC).* Nice, France, pages 5426-5431, 2019.

6. **Van Thiem Pham**, Quynh T T Nguyen, Nadhir Messai, and Noureddine Manamanni. Fault-Tolerant Tracking Control for Heterogeneous Multi-Agent Systems. *59th Conference on Decision and Control (CDC)*, 2020. (accepted)

General Introduction

Over the past decades, rapid advances of science and technology in miniaturizing of computing, communication, sensing, and actuation have made it feasible to deploy a large number of autonomous agents to work cooperatively to fulfill civilian and military missions. This, compared to a single complex agent, can significantly improve the operational effectiveness, reduce the costs, and provide additional degrees of redundancy. Having multiple autonomous agents to work together efficiently to achieve collective group behaviors is usually referred to as cooperative control of multi-agent systems (MASs). Cooperative control of MASs has received compelling attention from various scientific communities, especially the systems and control community.

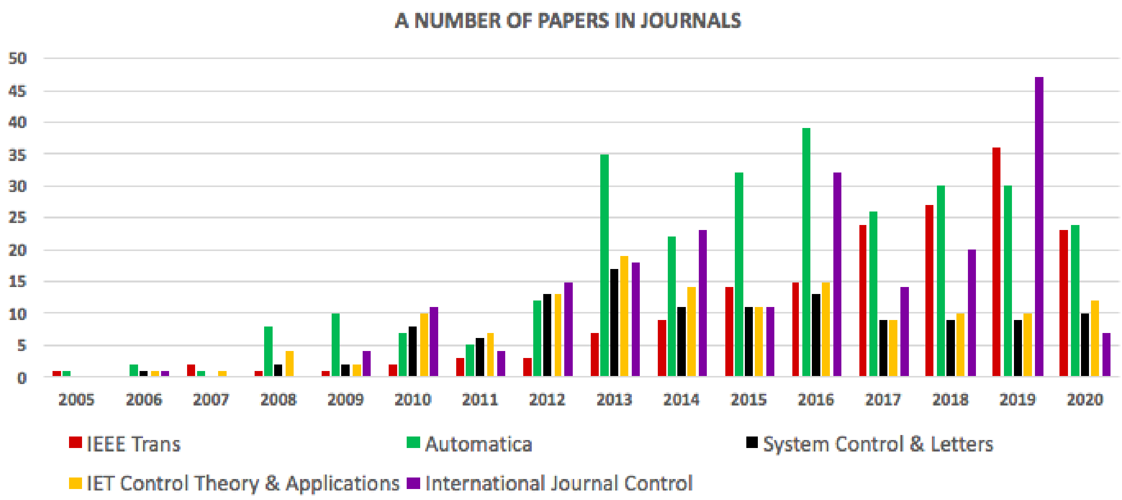

FIGURE 7: Statistics of a number of papers related *"consensus, cooperative, MASs"* in Journals during 2005 – July, 2020

In order to demonstrate the above observation, we record the data of papers in some top journals during 2005 – July 2020 with keywords *"consensus, cooperative, MASs"*, which is depicted in Fig.7. It shows that the topic "cooperative control" is increasingly concerned, for example, a number of papers in "Automatica" in 2015 increase 30 times compared to those in 2005, while those of papers in "IEEE Trans" move upward approximately 10 times. Moreover, more and more potential applications in various areas such as satellite formation flying, distributed computing, robotics, surveillance, reconnaissance systems, electric power systems, cooperative attack of multiple missiles, and intelligent transportation systems are analyzed and discovered in duration from 2005 to 2020. Especially, there are 31% papers published in Automatica, which are a significant proportion (see Fig.8). Next, 23%, 18%, 15%, and 13% are respectively the percentage of published papers in "International Journal Control", "IEEE Trans", "IET Control Theory & Applications", and "System Control & Letters". Furthermore, according to bar charts, we recognize that there is a lot of research groups considering this topic, which leads to significantly increased competition among groups and also demands considerable contributions in terms of theory and applications in publications. Therefore, cooperative control is not only an interesting topic but also a more challenging topic.

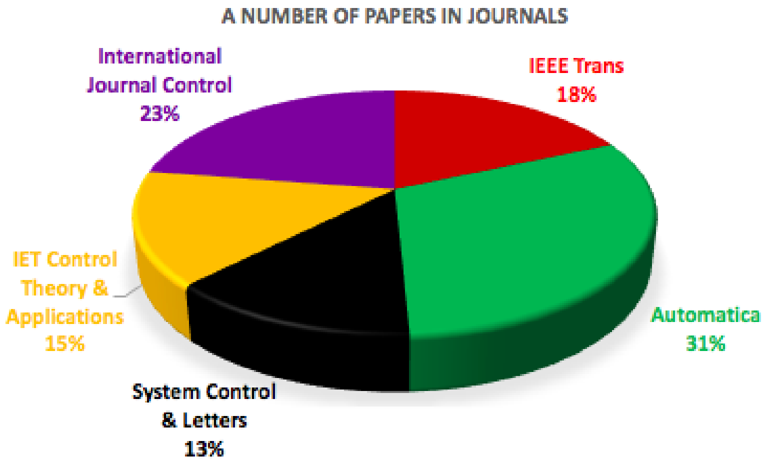

FIGURE 8: Statistics of percentage of papers related *"consensus, co-operative, MASs"* in Journals during 2005 – July, 2020

For the cooperative control problem, the main task is to design relevant control protocols to achieve the desired coordination objective. This concept started from a paper of Vicseck et al, 1995 in Physical Review Letters (see Fig.9), where the emergence of self-ordered motion in biological systems of particles, called *Vicseck's model*, was investigated by the nearest neighbor rules. However, it was only simulation results that had not a clear theoretical explanation. This was overcome in Jadbabaie and Morse, 2003, where based on graph theory, the *Vicseck's model* was represented as an n-dimensional switched linear system whose switching signal takes values in the set of indices which parameterize the family of graphs. Moreover, due to a large number of agents, the spatial distribution of actuators, limited sensing capability of sensors, and short wireless communication ranges, it is considered too expensive or even infeasible in practice to implement centralized controllers. Thus, distributed control (one of the first mentions in Olfati-Saber and Murray, 2004), depending only on local information of the agents and their neighbors, appears to be a promising tool for handling MASs. In Olfati-Saber and Murray, 2004, the consensus problem was particularly analyzed with fixed and switching topologies that were considered as undirected or strongly connected graphs. The results were further extended in Ren and Beard, 2005, where it has been shown that it is sufficient to have a communication graph with only a spanning tree in order to achieve consensus. Besides the importance of the network topology, the complex dynamics of MASs play an important role in the investigation and design of the cooperative algorithms, as well as the final consensus values. The necessary and sufficient conditions for consensus of the general linear MASs are given in Ma and Zhang, 2010. Following this line of research, there is a series of results-focused on the analysis and design of cooperative algorithms with homogeneous and heterogeneous MASs.

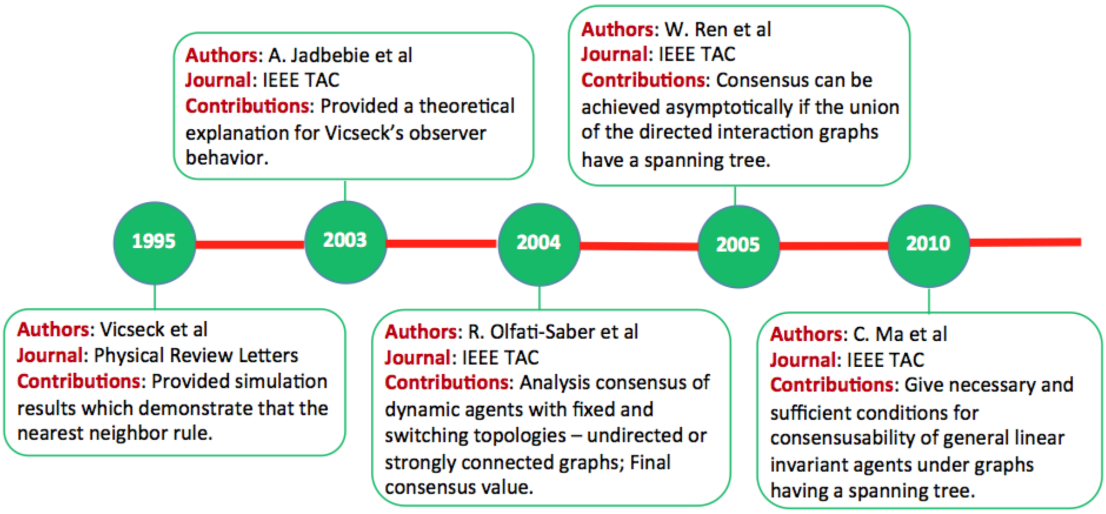

FIGURE 9: Timeline of the major cooperative control of MAS

Moreover, the cooperative control under time-delays, input, state constraints, as well as quantization and event-triggered cooperative control are also studied (see in Chapter 1 for more detail).

An arise from the fact that agents in MASs are usually resource-limited, such as limited ranges of wireless communication (for exchanging information among agents), sensors (for measuring relative information between neighboring agents) and actuators (for driving the agents), as well as energy constraints related to long time interactions, an engineer should sometimes partition a large network into clusters. Examples of these networks, called clustered networks [4] include both of the 5G Millimeter-wave communications in the wireless network[5] (see Fig.10), the consensus of the opinion in the social networks[6] (see Fig.11).

Some results of consensus problem in cluster networks of *integrator dynamic agents* was presented in Bragagnolo et al., 2016, where a reset strategy of the leaders' state is designed, which allows to reach consensus in the clustered network. However, several open problems related to cooperative control in the clustered network continuously rise. Designing appropriately distributed consensus controllers for MASs with complex dynamics (homogeneous and heterogeneous general linear dynamics), or formation control algorithms for the clustered network of multi-UAVs under state, input constraints are some examples of open interesting and challenging problems. Therefore, in this thesis, we extend these research directions by following the above motivation.

[4]In this thesis, we use "clustered network" to represent the network, which divided/composed into/of clusters that are almost all the time isolated one from another.

[5]Long Zhang, et al, A Survey on 5G Millimeter Wave Communications for UAV-Assisted Wireless Networks, *IEEE Access*, July 2019

[6]https://softwarerecs.stackexchange.com

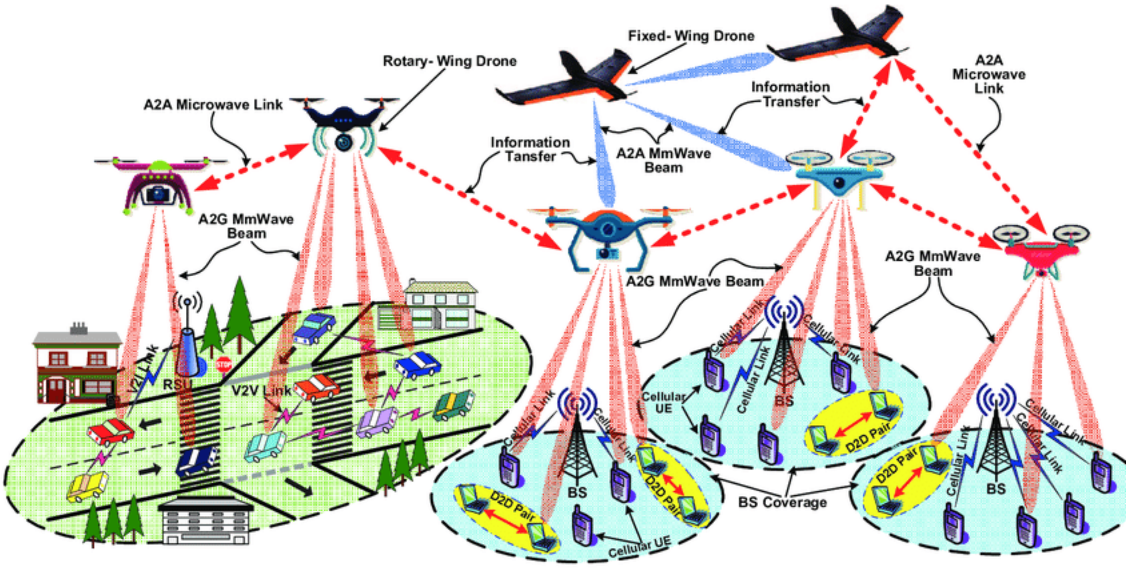

FIGURE 10: 5G mmWave communications for UAV-assisted wireless networks[5]

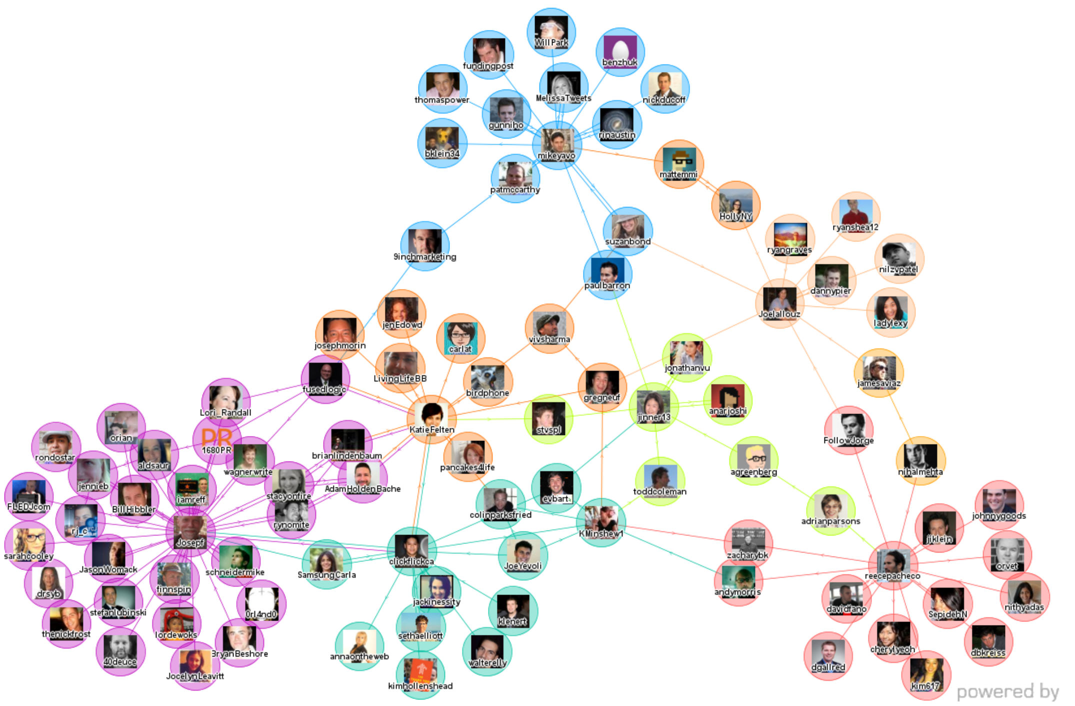

FIGURE 11: The social network[6]

Contributions and Outline

In this following, we present the outline and summarize the main contributions, of the thesis. The structure of this thesis is illustrated in Fig.12. The main contributions of the individual chapter are as follows:

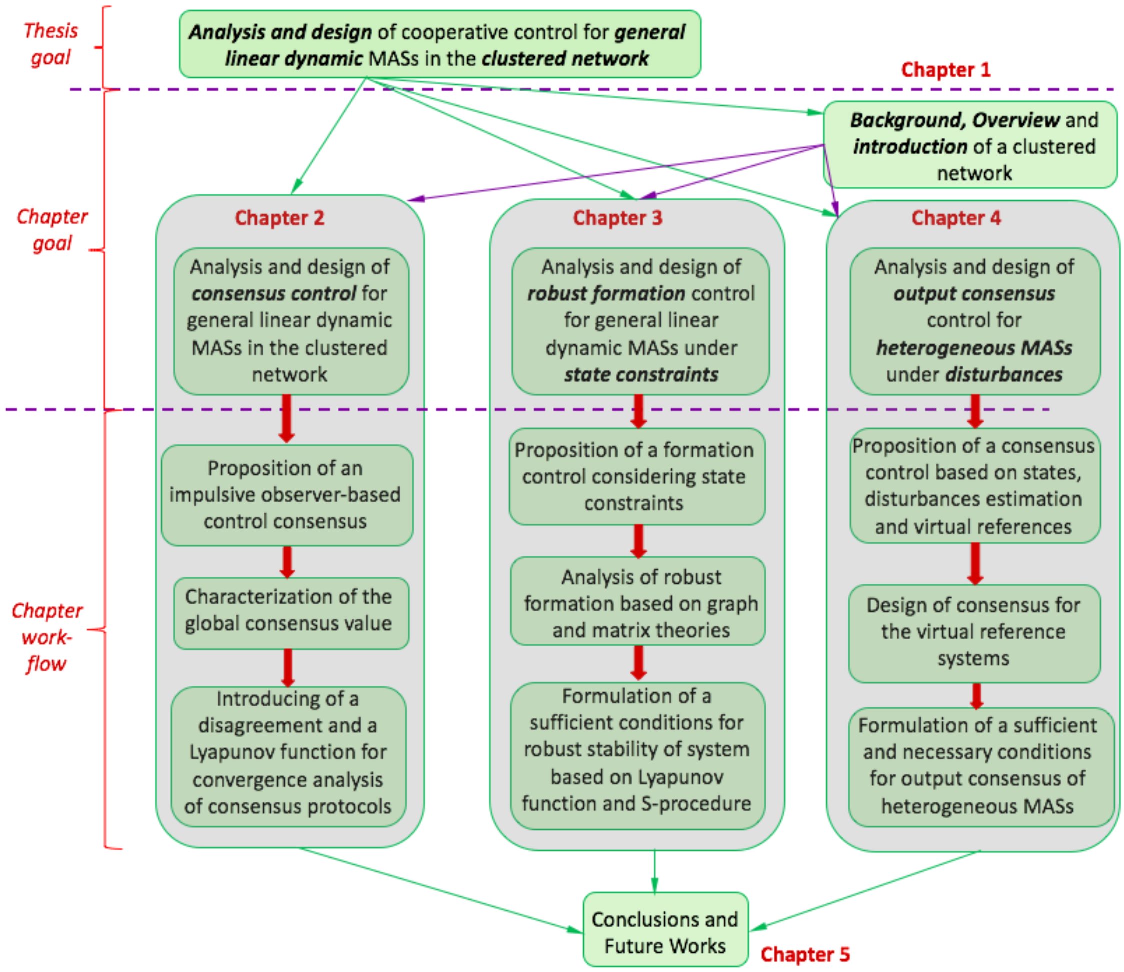

Figure 12: General overview of the thesis and main contributions

Chapter 1 provides an overall view of the state of art in the cooperative control problem. We study this problem based on four main factors: network topology, dynamics of MASs, constraints of MASs, and methodologies. We show that the network topology and dynamics of MASs are two important elements in analyzing and designing the cooperative control algorithms. We continue with contributions in the field of cooperative control under constraints, such as communication constraints and agent constraints. Afterward, we present the methodologies of the literature in analyzing and solving the cooperative control of MASs in networks. Finally, we discuss the presented literature and open research questions.
Chapter 2 addresses the problem of consensus in the clustered network, where each node of the network graph represents an agent with linear dynamics. The cooperative behavior of linear MASs with general system dynamics in the clustered network is defined by not only the dynamical control protocols concerning the isolated clusters but also the discrete interactions among the leaders. This makes a consensus problem in the clustered network with general linear agents much more challenging

than that of the integrator case. Thus, an impulsive observer-based control is proposed to handle the consensus problem. The main contributions of this chapter are threefold.

- The analysis and the charcterization of the global consensus value in the considered framework is analyzed. We show that the value of global consensus depends on the dynamics of each agent, the graphs of clusters, the interaction between leaders, and the initial conditions.

- An impulsive observer-based control, which uses only the local relative output information, and discrete interaction between leaders' clusters, is designed. Then, we show that the consensus design for clustered networks can be indirectly solved by considering the stability of an equivalent system. To study the stability of this equivalent system, we propose an algorithm to suitably choose the feedback and observer gain matrices and coupling weights in the form of some LMIs.

- The design interaction among leaders' clusters and impulsive observer-based controller, ensuring agents in clustered networks enclose a prescribed target.

Chapter 3 studies the formation control problem in clustered network systems of linear agents that are subjected to state constraints. The continuous-time communication structure in each cluster is represented by a fixed and undirected graph. The main contributions of this chapter are summarised as follows:

- A robust formation protocol, which deals with the continuous-time communication inside clusters and discrete-time information exchange between clusters, is introduced. It is then shown that the considered robust formation control problem can be indirectly solved by studying the robust stability of an equivalent system based on matrix theory and algebraic graph theory. Moreover, it shows the important role of communication between leaders at some specific discrete instants, represented by the stochastic matrix.

- A sufficient condition is derived in terms of linear matrix inequalities (LMIs) for the robust distributed formation of clustered networks of generic linear agents under state constraints and hybrid communications. Moreover, we show that obtained LMIs can be saved in fully distributed fashion i.e., each agent can compute the gain matrix by itself and implement the robust formation protocol using only local information (its information and its neighbors' information).

Chapter 4 discusses the output consensus problem in the clustered networks composed of heterogeneous MASs that are subjected to different disturbances. Each cluster is represented by a fixed and directed graph. An output consensus control is proposed to handle the consensus in the considered network. The main contributions of this chapter can be summarized as follows.

- Investigation of the consensus problem in a directed clustered network of MASs, where agents have distinct and generic linear dynamics under different disturbances.

- A dynamic internal reference model for each agent is introduced, which takes into account the continuous-time communications among internal reference models in virtual clusters and discrete information exchanges between those virtual clusters. Therefore, the output consensus of heterogeneous agents is indirectly solved through the consensus of the virtual references. To achieve that, a hybrid consensus control protocol is proposed for the virtual clustered network. Thanks to results from matrix theory and algebraic graph theory, the consensus of the virtual clustered network is solved.

- A sufficient and necessary condition is derived for the output consensus of linear heterogeneous agents under different disturbances in the clustered network.

Chapter 5 presents a short summary of the main results and contributions of this thesis, and indicates some possible topics/directions for future research.

Finally, in Appendix A and B we provide few tools for understanding the main results of the thesis such as the definitions of stochastic matrices, symmetric matrices, and the concept of LMIs and graph theory.

Publications

Most of the results in this thesis have been published at refereed international journals or international conferences. Some of these results have been achieved in collaboration with other researchers. Publications as a result of parts of my thesis research are listed as below:

Journal Papers

1. **Van Thiem Pham**, Nadhir Messai, and Noureddine Manamanni. Impulsive Observer-Based Control in Clustered Networks of Linear Multi- Agent Systems. *IEEE Transactions on Network Science and Engineering*, 2019 (Early Acess), **IF: 5.213** according to Journal Citation Reports (JCR) 2020.

2. **Van Thiem Pham**, Nadhir Messai, Dinh Hoa Nguyen, and Noureddine Manamanni. Robust Formation Control Under State Constraints of Multi-Agent Systems in Clustered Networks. *Systems & Control Letters*, 2019 (accepted), **5-Year IF: 3.048** according to Journal Citation Reports (JCR) 2020.

3. **Van Thiem Pham**, Nadhir Messai, and Noureddine Manamanni. Output Consensus Design in Clustered Networks of Heterogeneous Linear Multi-Agent Systems under Disturbances. *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, 2019 **(revision)**, **IF: 9.309** according to Journal Citation Reports (JCR) 2020.

Conference Papers

4. **Van Thiem Pham**, Nadhir Messai, and Noureddine Manamanni. Consensus of Multi-Agent Systems in Clustered Networks. *2019 18th European Control Conference (ECC)*, pages 1085–1090, 2019.

5. **Van Thiem Pham**, Nadhir Messai,D Hoa Nguyen, and Noureddine Manamanni. Adaptive Output Consensus Design in Clustered Networks of Heterogeneous Linear Multi-Agent Systems. *58th Conference on Decision and Control (CDC).* Nice, France, pages 5426-5431, 2019.

6. **Van Thiem Pham**, Quynh T T Nguyen, Nadhir Messai, and Noureddine Manamanni. Fault-Tolerant Tracking Control for Heterogeneous Multi-Agent Systems. *59th Conference on Decision and Control (CDC)*, 2020. (accepted)

Chapter 1

An Overview and Open Research Questions

Contents

Résumé du Chapitre 1

Ce chapitre donne une vue d'ensemble de l'état de l'art en matière de contrôle coopératif des MAS. Nous commençons le chapitre par une introduction au problème des MAS et du contrôle coopératif (problème de consensus et de formation). Pour étudier le contrôle coopératif des MAS, nous étudions ce problème en nous basant sur quatre facteurs principaux : la topologie du réseau, la dynamique des MAS, les contraintes des MAS et les méthodologies. Nous montrons que la topologie du réseau et la dynamique des MAS sont deux éléments importants dans l'analyse et la conception des algorithmes de contrôle coopératif. Nous poursuivons le chapitre avec des contributions dans le domaine du contrôle coopératif sous contraintes, telles que les contraintes de communication et les contraintes d'agent. Ensuite, nous présentons les méthodologies de la littérature en matière d'analyse et de résolution du contrôle coopératif des MAS dans les réseaux. Le chapitre se termine par une discussion de la littérature présentée et des questions de recherche ouvertes.

This chapter provides an overall view of the state of art in cooperative control of MASs. We begin the chapter by providing an introduction to the MASs and cooperative control problem (consensus and formation problem). To investigate the cooperative control of MASs, we study this problem based on four main factors: network topology, dynamics of MASs, constraints of MASs, and methodologies. We show that the network topology and dynamics of MASs are two important elements in analysing and designing the cooperative control algorithms. We continue the chapter with contributions in the field of cooperative control under constraints, such as communication constraints and agent constraints. Afterward, we present the methodologies of the literature in analyzing and solving the cooperative control of MASs in networks. The chapter ends with the discussion of the literature presented and open research questions.

1.1 An Overview on Cooperative Control of MASs

The MASs is generally referred to as a system composed of a set of dynamical agents that interact through a communication network to reach a coordinated behavior or operation. Especially, the cooperative control problems of MASs have been extensively investigated in the past two decades because there are many practical applications such as unmanned aerial vehicles (Dong et al., 2014), wireless sensor networks (Halgamuge, Guru, and Jennings, 2003), autonomous underwater vehicles (Nguyen, Messai, and Manamanni, 2017), transportation networks (Salvi, Santini, and Valente, 2017), etc.

For the cooperative control problem, the main task is to design relevant control protocols to achieve the desired coordination objective. There are two approaches in the cooperative control design, namely, the centralized approach and the distributed approach. The centralized approach assumes that at least a central (which lies on one of the agents, called central agent) is available and has the capability to receive information from and send control signals to all the other agents. The distributed approach is based on local interactions only, i.e., each agent exchanges information with its neighbors. Due to a large number of agents, the spatial distribution of actuators, limited sensing capability of sensors, and short wireless communication ranges, it is considered too costly or even infeasible in practice to implement centralized controllers. Thus, distributed control, depending only on local information of the agents and their neighbors, appears to be a promising resolution for MASs. Furthermore, there are two fundamental tasks in the study of the cooperative control of MASs: consensus and formation control.

- **Consensus/synchronization**: Consensus refers to the group behavior that all of the agents asymptotically reach a certain common agreement through a locally distributed protocol. The idea behind consensus serves as a fundamental principle for the design of distributed multi-agent coordination algorithms. Therefore, investigating consensus has been the main research direction in the study of distributed multi-agent coordination. To bridge the gap between the study of consensus algorithms and many physical properties inherited in practical systems, it is necessary and meaningful to study consensus by considering many practical factors, such as control algorithm, communication, constraints, and agent dynamics, which characterize some important features of practical systems. This is the main motivation to study consensus (Cao et al., 2013).

- **Formation control**: Distributed formation refers to the group behavior that all of the agents form a predesigned geometrical configuration through local interactions with or without a common reference. Compared with the consensus problem where the final states of all agents typically reach a singleton, the final states of all agents can be more diversified under the formation control scenario. Indeed, formation control is more desirable in many practical applications such as formation flying, cooperative transportation, sensor networks, and reconnaissance. For its broad applications and advantages, formation control has been a very active research subject in the control systems community, where a certain geometric pattern is aimed to form with or without a group reference. More precisely, the main objective of formation control is to coordinate a group of agents such that they can achieve some desired formation (Cao et al., 2013).

The research framework of cooperative control of MASs is depicted in Fig. 1.1. The network topology is a significant element to investigate the consensus, where it is considered over many types, such as fixed, time-varying, switching, leader-follower, and clustered network topology. Moreover, dynamics of MASs, including homogeneous, heterogeneous, and nonlinear MASs, play an important in determining the final consensus state and in designing the corresponding consensus protocols. Another direction on the cooperative control of MASs usually concerns with constraints. To fulfill the gap between studying cooperative algorithms and some inherent properties or constraints in practical models, it is necessary to investigate consensus problems with some practical factors, such as communication constraints (time delays, samplings, hybrid communication), quantization, and input, state saturations, which can be regarded as key constrained features in practical models. Afterward, in order to deal with these cooperative problems under the above considerations, three methods of analysis and design are widely used in the literature, such as the Lyapunov approach, frequency domain, and matrix approach. Finally, our research framework considering the dynamics of MASs (homogeneous and heterogeneous) with constraints (hybrid communication and state constraints) in the clustered network is shown in Fig. 1.1 (violet color). Because of the mentioned dynamics, constraints, and clustered network, the system performance might be degraded. Even worse, the stability could be destroyed. Therefore, it is of great importance and challenge to design controllers to guarantee the desired performance under these above considerations. This is one of the motivations in this research thesis.

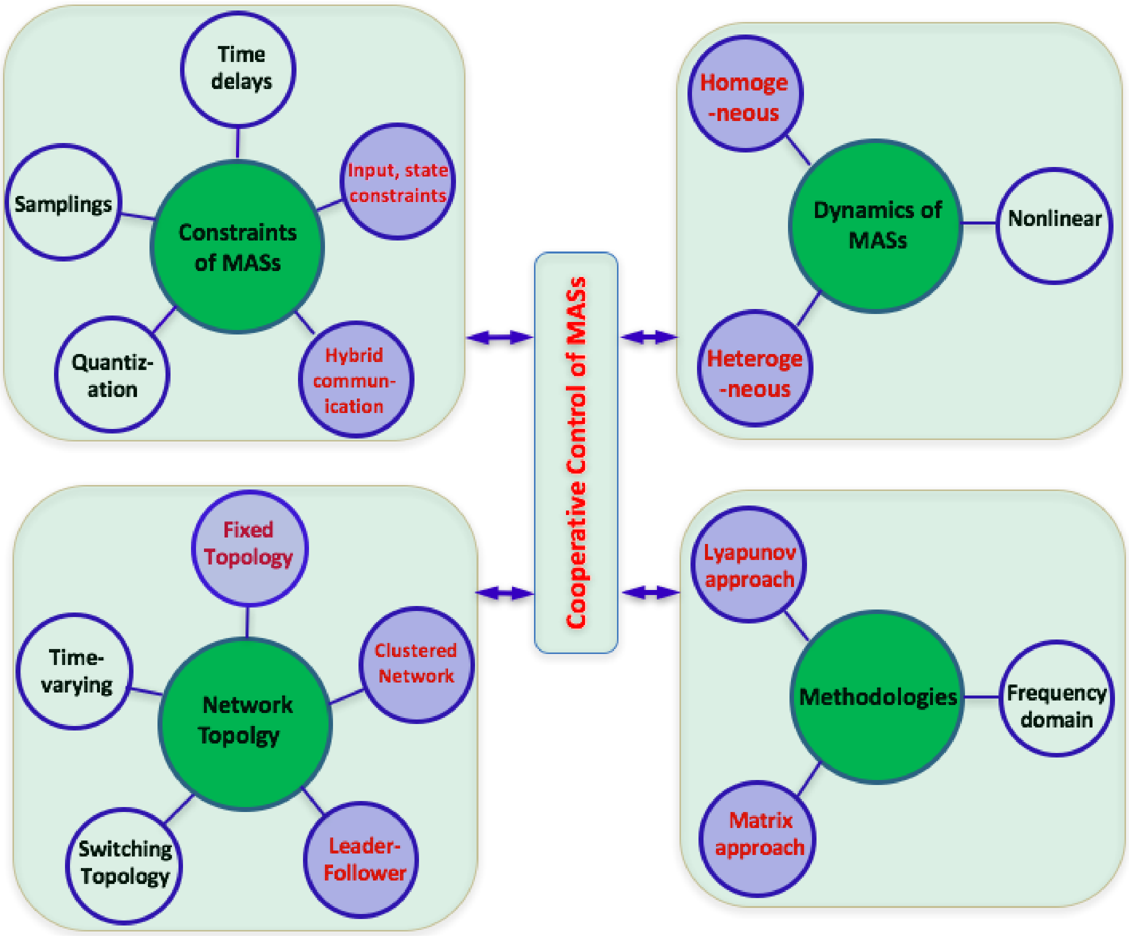

FIGURE 1.1: The research framework of cooperative control of MASs.

1.1.1 Dynamics of MASs

Since consensus and formation problems are concerned with the behavior of a group of agents, it is natural to consider the system dynamics for practical agents in the study of the cooperative problem. Although the study of cooperative control under various system dynamics is due to the existence of complex dynamics in practical systems, it is also interesting to observe that system dynamics play an important role in determining the final consensus and formation state, and in designing various types of consensus and formation algorithms. Generally, dynamics of MASs normally refers to homogeneous (dynamics of MASs are same), heterogeneous (dynamics of MASs are different), and nonlinear, as depicted in Fig. 1.2. In the following, the effect of dynamics of MASs on the cooperative control is investigated.

A. Homogeneous MASs

In the simplest, homogeneous case, a group of N identical agents is considered in a continuous-time setup with agent having a special single-integrator form

$$\dot{x}_i(t) = u_i(t), \forall i = 1, 2, \cdots, N. \tag{1.1}$$

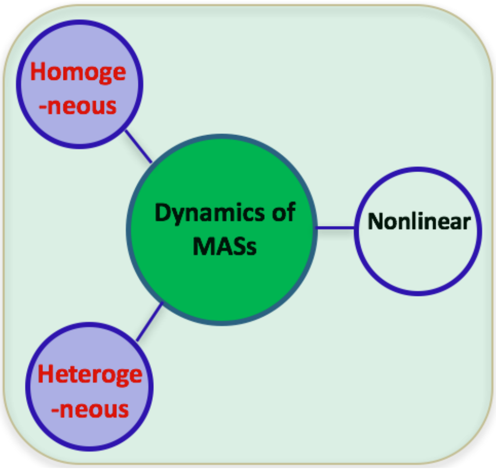

FIGURE 1.2: The research framework of dynamics of MASs.

The controller typically takes the form

$$u_i(t) = \sum_{i=1}^{N} a_{ij}(x_j(t) - x_i(t)) \tag{1.2}$$

where $x_i \in \mathbb{R}$ is state of agent i and a_{ij} is $(i,j)-th$ entry of the corresponding adjacency matrix $\mathcal{A}$, which describes the connection graph among the agents. According to Section B.0.2, a Laplacian matrix $\mathcal{L}$ can be defined based on $\mathcal{A}$ such that

$$\dot{x}(t) = -\mathcal{L}x(t), x(t) = [x_1(t), x_2(t), \cdots, x_N(t)]^T. \tag{1.3}$$

As shown in Olfati-Saber and Murray, 2004, the motivation to use (1.2) is that each agent converges towards the weighted average of the states of its neighbors, called average-consensus problem. It shows that after reaching a consensus, the group decision value is

$$x^\star = \frac{\sum_{i=1}^{N} r_i x_i(0)}{\sum_{i=1}^{N} r_i} \tag{1.4}$$

i.e., the decision value belongs to the convex hull of the initial values, where $r = [r_1, r_2, \cdots, r_N]^T$ is a left eigenvector of $\mathcal{L}$ (see in Section B.0.2). In particular, it is not concerned with the performance evolution in the network dimension as the length of the network dimension is finite, denoted by N.

In the same time, agents with integrator form are also considered in case of discrete-time. As shown in original papers (Jadbabaie and Morse, 2003; Olfati-Saber and Murray, 2004; Ren and Beard, 2005), the discrete dynamics of the consensus protocol has a strong relationship with the theory of Markov chains. Therefore, the consensus protocol for discrete cases can be represented as a stochastic matrix P. This means that the ij^{th} entry of the matrix P, denoted P_{ij}, is the probability of a random variable x having a state i at time k, and state j at time $k+1$. Therefore, the overall dynamics of the discrete network can define as

$$x(k+1) = Px(k), \tag{1.5}$$

where P is a stochastic matrix, which can be defined as in Appendix A.

Consider the (1.3), we can discretize its continuous dynamics by assuming a constant sampling interval δ. This leads to the following equation

$$x(k+1) = e^{-\mathcal{L}\delta}x(k), \tag{1.6}$$

According to Proposition 3.18 of Mesbahi and Egerstedt, 2010, it shows that all digraphs and sampling intervals $\delta > 0$, one has $e^{-\mathcal{L}\delta}\mathbf{1} = \mathbf{1}$ and $e^{-\mathcal{L}\delta} \geq 0$. That is, for all digraphs and $\delta > 0$, $e^{-\mathcal{L}\delta}$ is a stochastic matrix. Therefore, thanks to this Proposition, the connection between (1.3) and (1.5) is given.

Extensions of consensus algorithms to double-integrator dynamics were also investigated in Moreau, 2005; Wei Ren and Atkins, 2007 for instance. Moreover, the consensus problem for agents with general linear dynamics, which has form in (1.7), is also investigate.

$$\begin{aligned} \dot{x}_i(t) &= Ax(t) + Bu_i(t), \\ y_i(t) &= Cx_i(t) \end{aligned} \tag{1.7}$$

where A, B, C are constant matrices of appropriate dimension. A range of references focused on consensus protocols such as

$$u_i(t) = cK\sum_{j=1}^{N} a_{ij}(x_j(t) - x_i(t)), \tag{1.8}$$

and the objective is to find feedback control laws K and coupling gain c such that consensus can be achieved. Particularly, the controller gains were obtained using LQR optimal control method (for example in Zhongkui Li et al., 2010; Zhang, Lewis, and Das, 2011; Nguyen, 2017; Qian, Liu, and Feng, 2018a), robust consensus protocols with obtained controller gains by solving LMIs (for example in Hongyong Yang and Zhang, 2011; Mo and Lin, 2018; Nguyen, Narikiyo, and Kawanishi, 2018), finite-time consensus protocols (for example in Li, Du, and Lin, 2011; Liu and Geng, 2015; Liu, Cao, and Xie, 2019; Liu et al., 2016), and event-trigger consensus controls (for instance in Hu et al., 2016; Yi et al., 2019; Jiang et al., 2018).

B. Heterogeneous MASs

When considering heterogeneous MASs dynamics, it means that the dynamics of MASs i^{th} is described by the local state space representation A_i, B_i, and C_i (instead of a general A, B, and C in (1.7)). This class of problem is usually referred to as a "synchronization problem". In the literature, there are two basic research approaches: cooperative output regulation problem and virtual exosystem approach, which are shown below

- *Cooperative output regulation problem*: The dynamics of MASs subjected to exogenous signal $v(t)$ is modeled as

$$\begin{aligned} \dot{x}_i(t) &= A_i x_i(t) + B_i u_i(t) + E_i v_i(t), \\ y_i(t) &= C_i x_i(t) \end{aligned} \tag{1.9}$$

where $v_i(t)$ represents the reference input to be tracked and it is generated by an exosystem $\dot{v}_i(t) = Sv_i(t), w_i(t) = Rv_i(t)$ (Su and Huang, 2012; Huang and Ye, 2014; Xiang, Li, and Hill, 2017; Adib Yaghmaie, Lewis, and Su, 2016). The exosystem plays a role as the leader, which generates the reference signal to all outputs of heterogeneous MASs. To do this, a subset of the N heterogeneous MASs are able to access to the exogenous signal $v_i(t)$ by a feedback control. Therefore, the objective is to design a distributed controller to guarantee the local tracking error $e_i(t) = y_i(t) - w_i(t)$ approaches zero for all $t \to \infty$. The cooperative output regulation problem of linear multiagent system has been studied using feedforward control in Su and Huang, 2012; Xiang, Li, and Hill, 2017. Extensions of this direction to heterogeneous MASs with uncertainties were investigated in Huang and Ye, 2014, where the robust output regulation was proposed. Moreover, a novel concept of output regulation region for heterogeneous MASs using state feedback was proposed in Adib Yaghmaie, Lewis, and Su, 2016 such that selecting the controller coupling gain from this region guarantees that the sufficient condition is always satisfied.

- *Virtual exosystem approach*: The dynamics of heterogeneous MASs has form

$$\begin{aligned} \dot{x}_i(t) &= A_i x_i(t) + B_i u_i(t), \\ y_i(t) &= C_i x_i(t) \end{aligned} \tag{1.10}$$

It was shown in references (Wieland, Sepulchre, and Allgöwer, 2011; Li et al., 2015; Yang, Huang, and Wang, 2016; Hu, Liu, and Feng, 2017; Kiumarsi and Lewis, 2017; Qian, Liu, and Feng, 2018b) that each agent has an internal reference model embedded in a controller, which is generated from a cyber command center. Those internal reference models have identical dynamics and can be viewed as virtual systems that generate virtual reference inputs for agents. They also interact through a network called virtual networks which have the same structure as the physical network of heterogeneous agents. Therefore, the distributed consensus control has a form

$$\begin{aligned} u_i(t) &= K_i \xi_i(t) + E_i v_i(t) \\ \dot{\xi}_i(t) &= G_i x_i(t) + M_i y_i(t) + O_i v_i(t), \end{aligned} \tag{1.11}$$

where $v_i(t)$ represents the reference input to be tracked and it is generated by an exosystem

$$\begin{aligned} \dot{v}_i(t) &= Sv_i(t) + L_i \sum_{j=1}^{N} a_{ij}(w_j(t) - w_i(t)) \\ w_i(t) &= Rv_i(t) \end{aligned} \tag{1.12}$$

The objective of (1.11) is to find a necessary or/and sufficient conditions, which guarantee the synchronizability of the network. Moreover, the output consensus problem of heterogeneous discrete-time MASs was studied in Li et al., 2015; Kiumarsi and Lewis, 2017. Another direction proposes a distributed even-triggered control protocol (Hu, Liu, and Feng, 2017; Qian, Liu, and Feng, 2018b). It is shown that all agents achieve asymptotically output consensus with intermittent communication among agents in a network.

Up to now, the cooperative control problem of homogeneous and heterogeneous MASs is still promised problem in the control community.

C. Nonlinear MASs

The synchronization/consensus problem studied for nonlinear MASs is different from that for homogeneous and heterogeneous MASs although there is no strict division. Specifically, the former is more concerned with control design methodologies. For instance, it aims to seek a controller $u_i(t)$ for a MASs of time-varying nonlinear dynamics

$$\dot{x}_i(t) = \phi_i(x_i(t), t) + B_i u_i(t), \tag{1.13}$$

A number of results has been studied under a class of globally Lipschitz condition

$$\|\phi_i(x_{i1}(t), t) - \phi_i(x_{i2}(t), t)\| \leq c\|x_{i1}(t) - x_{i2}(t)\|, \forall x_{i1}(t), x_{i2}(t) \tag{1.14}$$

It was shown in Yu et al., 2010 that a simple control for MASs with second-order nonlinear dynamics, which relies only on information from direct neighbors, still guarantees the consensus of MASs. The result also holds for general systems Li et al., 2013; Wen et al., 2014a; Yu et al., 2018. Moreover, when the nonlinearity does not have to satisfy the global Lipschitz-like condition which excludes some benchmark nonlinear systems such as van der Pol systems, Duffing systems, and so on. Then, a distributed adaptive state feedback control law, which integrates the conventional adaptive control technique with the adaptive distributed observer, was given in Liu and Huang, 2017. The synchronization problem was also considered for the nonlinear heterogeneous systems described by Euler-Lagrange equations. For example, Mei, Ren, and Ma, 2011 developed suitable estimation and control strategies to track a dynamic leader. The controllers proposed in Wang, 2013 guarantee synchronization or flocking of Euler-Lagrange systems with uncertain kinematics and dynamics or uncertain parameters.

1.1.2 Network Topology

In MASs, the network topology among all agents plays a crucial role in solving cooperative issues. The objective here is to explicitly identify necessary and/or sufficient conditions on the network topology such that cooperative issues can be achieved under properly designed algorithms. Especially, the network topology is considered such as fixed topology, time-varying topology, switching topology, leader-follower, and clustered network depicted in Fig. 1.3.

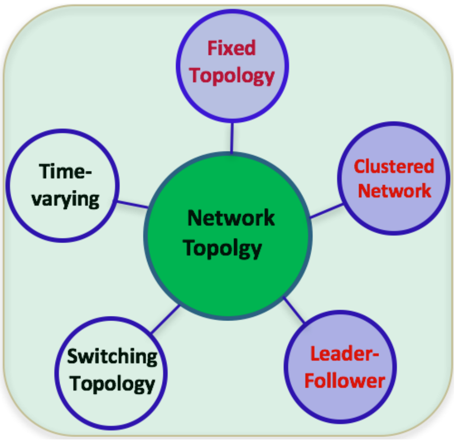

FIGURE 1.3: The research framework of network topology of MASs.

A. Fixed topology

Many verifiable consensus algorithms have been developed based on fixed topology in the past decade, where the close relations between the network connectivity and consensus behavior of MASs are constructed, for example in Jadbabaie and Morse, 2003; Olfati-Saber and Murray, 2004; Ren and Beard, 2005; Ren, 2007a; Wei Ren and Atkins, 2007. It was shown in Jadbabaie and Morse, 2003 that, for an undirected network of double-integrators, velocity consensus can be achieved if the fixed graph is connected. In Olfati-Saber and Murray, 2004, the result about the average consensus of continuous-time MASs with fixed topology was investigated, which has shown that the strongly connected and balanced directed graphs play a key role is solved average consensus problem. Moreover, the problem was also studied in Ren and Beard, 2005 with a fixed topology and constant weighting factors. It is proved that the consensus can be achieved asymptotically if the union of the directed interaction graphs contains a spanning tree frequently enough. The result was extended to a directed network of second-order dynamics in Ren, 2007a; Wei Ren and Atkins, 2007 and general dynamics in Hu et al., 2018; Pham et al., 2019; Guo, Liang, and Lu, 2019, for example.

B. Switching topology

In a network of distributed agents, some of the existing communication links can fail simply due to the existence of an obstacle between two agents, as well as the creation of new links between nearby agents. This leads to non-fixed topologies. In terms of the network topology, this means that a certain number of edges are added or removed from the graph. We note here that in the area of non-fixed topologies, the term “switching topologies” describes the case where the topology changes over time but only switches between a finite, known set of distinct communication graphs. The paper Olfati-Saber and Murray, 2004 studied directed networks with switching topologies. It shows that network of simple-integrators with switching topologies taken from the collection of strongly connected and balanced digraphs can asymptotically achieve average consensus for any switching signal. A weaker condition was proposed in Ren and Beard, 2005 showing that consensus can be achieved asymptotically if the union of the collection of interaction graphs across some time intervals has a spanning tree frequently enough. The result was extended to a network of double-integrators with an undirected graph in Ren, 2007b; Wen et al., 2014b; Dong et al., 2016; Hua et al., 2019, as well as the nonlinear dynamics Zhai and Yang, 2014, the general dynamics Wen et al., 2013; Zhu and Yuan, 2014.

In the case of randomly switching topologies, You, Li, and Xie, 2013 presented the switching such as a time-homogeneous Markov process, whose state corresponds to a possible interaction topology among agents. It is shown that the effect of switching topologies on consensus is determined by the union of topologies associated with the positive recurrent states of the Markov process.

C. Time-varying topology

Different from the "switching topologies", the “time-varying topologies” include all networks where an infinite set of arbitrary graph structures is considered. The time-varying topologies are the so-called “nearest neighbor rule”. Each agent interacts with all and only all agents within its limited communication radius. Such networks always have an undirected neighborhood graph. In this sense, these results were weaker than those for directed networks with switching topologies discussed above. Jadbabaie and Morse, 2003 used a simple but compelling discrete-time model of autonomous agents using nearest neighbor rule (called a Vicsek model). It is shown that if there exists an infinite sequence of continuous, nonempty, and bounded time intervals such that the union of the collection of time-varying undirected graphs across each time interval is connected (called joint connectivity condition), all agents converge to a common steady-state provided the agents, which are all “linked together” via their neighbors with sufficient frequency as the system evolves. The problem was further studied in Zhang, Zhai, and Chen, 2011 considering a weaker joint connectivity condition. In Cao, Zheng, and Zhou, 2011, a necessary and sufficient condition for achieving the average consensus of continuous-time agents in undirected networks is investigated, where the joint connectivity condition defined by the integral of adjacency matrix over a certain time interval. It is emphasized that the infinite integral connectivity is not equivalent to the piecewise joint connectivity or the piecewise integral connectivity, but equivalent to the piecewise integral K-connectivity. Other than the joint connectivity condition, it was also proved in Hendrickx and Tsitsiklis, 2013 that convergence (not necessarily to consensus) can be guaranteed if the time-varying topologies are cut-balanced (if a group of agents influences the remaining ones, the former group is also influenced by the remaining ones by at least a proportional amount). The results have further been extended for the time-varying nonlinear networks in some works such as Manfredi and Angeli, 2017; Manfredi and Angeli, 2018.

D. Clustered network

In cooperative control, for cooperative control strategies to be effective, agents need to reach consensus on shared data. A group of agents must be able to respond to unanticipated situations or any changes when a cooperative task is carried out. This might result in that the agreements are different from the changes in environments, situations, cooperative tasks, or even time. To do this, a network is usually divided into multiple sub-networks (called clusters), where information exchange exists not only among agents in clusters but also different clusters. This is more suitable for complex practical applications, such as the energy optimization in the wireless sensor network (Halgamuge, Guru, and Jennings, 2003; Chen and Wen, 2013), the consensus of the opinion in the social networks (Morarescu, Martin, and Girard, 2014; Morarescu et al., 2016), and the problem of formation of multiple unmanned aerial vehicles (Pham, Messai, and Manamanni, 2019a; Pham et al., 2019, Pham et al., 2020b; Pham, Messai, and Manamanni, 2019b; Pham et al., 2020a; Pham, Doan, and Nguyen, 2020). The consensus of this kind of network is called by ***cluster consensus or group consensus*** problem.

One notes here that the difference between group consensus and cluster consensus lies in their task. A cluster consensus is meant that for any initial states of the agents, all the agents clustered network finally reach complete consensus, while there may or may not be a consensus within the clusters. If for any initial states of the nodes, not only all the nodes within the same cluster reach complete consensus, but also there is no consensus between any two different clusters, then group consensus is said to be achieved. Group consensus implies cluster consensus.

In Xiao and Wang, 2008a; Wu, Zhou, and Chen, 2009 authors shown that to achieve the group consensus, the couplings among agents from different clusters which may be negatively weighted (which play a role are as an inhibitory mechanism to desynchronize the motions of agents from different clusters). Meanwhile, agents within the same cluster have the positively weighted couplings, which are used for synchronizing the agents in clusters. However, these negatively weighted couplings may also cause some negative effects such as making the state trajectories of the agents within the same cluster oscillating or even divergent. This can be considered as one of the main reasons why positively weighted couplings among need to be strong enough (see Yu and Wang, 2010; Xia and Cao, 2011). However, in some realistic physical systems, the coupling strength among agents may be weak and they are not allowed to be arbitrarily large. To handle this disadvantage, Qin and Yu, 2013; Qin, Yu, and Anderson, 2016; Qin et al., 2017 indicated that a sufficient condition to achieve group consensus is a directed acyclic interaction topology.

Different from the group consensus problem considered above. The works in Bragagnolo et al., 2014; Rejeb, Morarescu, and Daafouz, 2015; Bragagnolo et al., 2016; Morarescu et al., 2016 considered the cluster consensus problem, where the interaction among agents inside each cluster happens in continuous-time, and interaction in inter-cluster is cooperative. In each cluster, there exists an agent called a leader who can exchange information outside of its cluster at some specific discrete-time. The further results on cluster cosensus problem are investigated in Pham, Messai, and Manamanni, 2019a; Pham, Messai, and Manamanni, 2019b; Pham et al., 2020c.

D. Leader-Follower topology

The leader-following scenario can be seen as a special case in the directed networks, which are discusssed above. In case there exists one agent in the network without any incoming links, this agent can be regarded as the leader of the network and is labeled by "0". In the cooperative control, due to the existence of a leader, a distributed tracking control of MASs is proposed, where all agents from 1 to N track to the reference trajectory, which is set by an active leader. The authors in Hu and Hong, 2007 designed a suitable neighbor-based local controller together with a neighbor-based state estimator to track an active leader whose velocity is unknown to the agents. In Ni and Cheng, 2010, a local information is used to design and analysis of the leader-following consensus is presented for both fixed and switching interaction topologies. Other researchers proposed suitable distributed tracking control laws for networks of second-order nonlinear MASs (Haibo Du, Yingying Cheng and Jia, 2015; Liu and Huang, 2016; Han et al., 2017b), high-order MASs (Qin, Yu, and Anderson, 2016; Wang and Song, 2018), and general dynamics MASs (Li and Jaimoukha, 2009; Yaghmaie, Lewis, and Su, 2016; Tan, Cao, and Li, 2018).

1.1.3 Constraints in MASs

As shown in Section 1.1.1, consensus always focuses on the behaviors of MASs. Therefore, it is natural to understand that system dynamics of practical models should be considered when studying the consensus problem in MASs. To fully fill the gap between studying consensus algorithms and some inherent properties or constraints in practical models, it is necessary to investigate consensus problems with some practical factors, such as communication constraints (time delays, samplings, hybrid communication), quantization, and input, state saturations, which can be regarded as key constrained features in practical models (see Fig. 1.4).

A. Time delays

In general, time delay reflects an important property, which is inherited in practical systems due to actuation, control, communication, and computation. Particularly, it can be generated by several reasons, such as limited communication speed when information transmission exists, the extra time required by the sensor to get the measurement information, the computation time required for generating the control inputs, and the execution time required for the inputs being acted. In the literature, there are two types of time delays, which have been considered such as, communication delay, and input delay. More precisely, if it takes time T_{ij} for agent i to receive information from agent j, then the cooperative control (1.1) becomes

$$u_i(t) = \sum_{j=1}^{N} a_{ij}(x_j(t - T_{ij}) - x_i(t)), \tag{1.15}$$

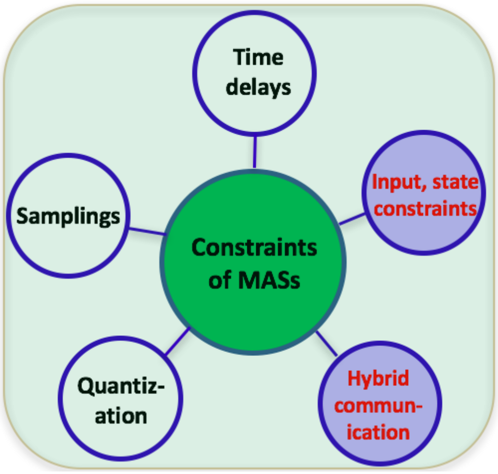

FIGURE 1.4: The research framework of contraints of MASs.

an interpretation of (1.15) is that at time t, agent i receives information from agent j and uses data $x_j(t - T_{ij})$ instead of $x_j(t)$ due to the time delay. Moreover, if the input delay for agent i is given by T_i^p, then the cooperative control (1.1) becomes $u_i(t - T_i^p)$. Then closed-loop form of system (1.1) is given by

$$\dot{x}_i = \sum_{j=1}^{N} a_{ij}(x_j(t - T_i^p) - x_i(t - T_i^p)), \tag{1.16}$$

Because time delay might affect the system stability, it is necessary to study under what conditions consensus can still be guaranteed even if time delay exists. In other words, can one find conditions on the time delay such that consensus can be achieved?. To do this, the effect of time delay on the consensuality of the network was investigated. For example, a sufficient condition on the time delay to guarantee consensus under a fixed undirected interaction graph is presented in Olfati-Saber and Murray, 2004. Specifically, an upper bound for the time delay is derived under which consensus can be achieved. Next, the robustness of consensus in discrete-time single-integrator multi-agent systems to arbitrarily large delays was discussed in Xiao and Wang, 2008b. It is shown that if there exists an arbitrary upper bound and the union of graphs has a spanning tree, then consensus is achieved. In Münz, Papachristodoulou, and Allgöwer, 2010; Münz, Papachristodoulou, and Allgower, 2011, linear MAS models with different feedback delays, e.g., affecting only the neighbor's output, or affecting both the agent's own and its neighbors' output has been considered. The work in Wang, 2014 investigated the consensus problem of networked uncertain mechanical systems subjected to nonuniform communication delays, where interaction among agents is a directed graph containing a spanning tree. Based on Lyapunov-like analysis and frequency-domain input-output analysis, it is shown that the proposed unified consensus control scheme ensures agents achieving a scaled weighted average consensus. These results are extended for the case of consensus of nonlinear multi-agent systems with self and communication time delays (Ma et al., 2015), and group consensus in networked mechanical systems with communication delays (Yu et al., 2017a). And a stochastic consensus of MASs with both time-delays and measurement noises is also studied in Zong, Li, and Zhang, 2019.

B. Input and state constraints

Other significant and realistic issues have been encountered such as the constraints on the agent's inputs, states, or relative states because of the physical limitations of agents. This includes, for example, the formation of vehicles with limited speeds and limited working space, smart buildings energy control with constraints on temperature and humidity in specific ranges and etc. Therefore, the cooperative control for constrained MASs is not only theoretically challenging but also practically important. When considering the constraint in cooperative control, can the distributed algorithms reported before still be effective with the saturation constraint. This problem is pretty significant since the answer to it determines whether we should design new coordinated algorithms when the saturation constraint exists in MASs. Motivated by these observations, the cooperative control in constrained multi-agent coordination were widely studied. Recently, some studies have considered the cooperative control of MASs under the constraints on agent's inputs, and states. In Nedi, Ozdaglar, and Parrilo, 2010, a constrained consensus algorithm and distributed optimization problems were proposed, where agents state constraints are investigated and they are required to lie in individual closed convex sets. In another work, Wei, Xiang, and Li, 2011 studied a consensus problem of simple integrator MASs under input constraints. Following this research line, a distributed consensus of second-order MASs with nonconvex input constraints was addressed in Mo and Lin, 2018. It is shown that the input constrained consensus is achieved if the graph has a directed spanning tree. And the global consensus problem for discrete-time MASs with input saturation constraints under fixed undirected topologies was studied in Yang et al., 2014. Another direction to deal with input and state constraints, discarded consensus algorithms are employed (Zhou and Wang, 2018). Next, in order to achieve the global consensus in the presence of agents' inputs, states, or relative state constraints (Nguyen, Narikiyo, and Kawanishi, 2017; Nguyen, Narikiyo, and Kawanishi, 2018), the MASs is reformulated in form of a network of Lure systems. Moreover, a distributed consensus of high-order continuous-time MASs with nonconvex input constraints, switching topologies, and delays was als studied in Wang et al., 2019.

C. Hybrid communication

It has been noticed that interaction among agents in the Section 1.1.1 and Section 1.1.2 is either continuous-time or discrete-time. Nevertheless, in several practical applications, e.g., cooperative intelligent transportation systems, robots fleet cooperation, consensus control on a social network, etc, due to either energy constraints occurring in long-time interactions or communication constraints, agents can only impulsively exchange information with their neighbors or be subjected to abrupt changes at specific instants. This leads to a hybrid interaction that combines both continuous and discrete interactions among agents. In Guan, Wu, and Feng, 2012, a sufficient result has been derived for the impulsive consensus of first-order MASs, where the graphs of continuous-time and impulsive-time topologies contain a spanning tree. Following this research line, there are several types of research Guan et al., 2012a; Hu et al., 2013, which have dealt with the consensus problem of the second-order MASs under an impulsive control strategy. Moreover, inspired by the results in Jadbabaie and Morse, 2003; Ren and Beard, 2005, the necessary condition of consensus on graph connections among agents may require. This is investigated in Liu, Zhang, and Xie, 2017, where the first-order MASs with hybrid delay consensus protocols are described in the form of impulsive systems. However, there still exist many challenges in investigating the cooperative problem of MASs in hybrid communication. For example, analysis and design cooperative controls in fixed/swithching network of agents with general dynamics system under time delays is up to now still open questions.

D. Samplings

In MASs sampled data are often only sent to the neighbors periodically at discrete time instances. A framework for studying the consensus problem of multi-agent systems via sampled control was introduced in Xie et al., 2009b and Xie et al., 2009a for a fixed topology and switching topologies, respectively. Two sampled data based discrete-time coordination algorithms were studied in Polyakov, Efimov, and Perruquetti, 2015 and Cao and Ren, 2010 which gave necessary and sufficient conditions on the interaction graph, the damping gain and the sampling period to guarantee coordination. In Guan et al., 2012b, the distributed consensus problem for second-order continuous-time multi-agent networks with sampled-data communication was investigated. Necessary and sufficient conditions based on the stability theory of impulsive systems and properties of the Laplacian matrix are obtained to ensure the consensus of the controlled networks. Next, a novel distributed event-triggered sampled-data transmission strategy, which allows the event-triggering condition to be intermittently examined at constant sampling instants was studied in Guo, Ding, and Han, 2014. Independent and asynchronous sampling times in a directed network of continuous-time second-order agents were considered in Yu et al., 2013. In the case of generally linear multiagent systems with aperiodic sampling intervals, Zhang, Shi, and Yu, 2018 proposed a new consensus control subject to sampling interval changing from a finite set. By using the properties of Laplacian matrix and the newly developed protocol, the containment control problem is transformed into the stability problem of a discrete-time switched linear system.

E. Quantization

The data transferred in networks is usually rounded off and represented with finite bits. Because of this, there exits a difference between the real data and transmitted data, which may effect the system in terms of performance and stability. Quantized control approach, as an effective control strategy, can deal with those limitations in network systems, where uniform quantizer has been popular with researchers. The authors of Kashyap, Başar, and Srikant, 2007 proposed a quantized gossip algorithm, that forces the network to converge to a set of quantized consensus distributions for an arbitrary initial vector and arbitrary connected graph. The term gossip algorithm describes a control protocol where at each time instant exactly one agent updates its state based on the information transmitted from only one of its neighbors. Follow this research, the average consensus problem on a network of digital links based on pairwise "gossip" communications and updates was investigated Carli et al., 2010. Moreover, uniform quantizer has been popular with researchers. For example, Nedić et al., 2009 adopted uniform quantizer to address the consensus of single-integrator discrete-time MASs. The work in Yu and Antsaklis, 2012 first uses quantized absolute state measurements to design event-triggered consensus algorithms for undirected networks with single integrator agent dynamics. The authors prove that the quantized states of agents achieve consensus asymptotically. In Garcia et al., 2013, the authors consider a uniform quantizer and prove that all agents will converge to a certain ball whose radius is related to the quantization error. Moreover, Li et al., 2014 discussed the quantized consensus problem for a group of agents over directed networks with switching topologies. For general linear systems, Yu et al., 2017b; Zhang et al., 2017 adopted the Lyapunov method to deal with the quantized consensus of MASs with event-triggered strategy, where quantized relative state measurements are considered. Next, the quantized control and the event-triggered control of MASs with external disturbance on the basis of an undirected graph was studied in Wu et al., 2018. It is shown that “Zeno behavior” phenomenon can be excluded under the event-triggered quantized control mechanisms, and the boundness of the relative state error can be adjusted by selecting the different parameters.

1.1.4 Methodologies

In order to analyze and design the consensus and formation in network topology (investigated in Section 1.1.2) of MASs with dynamics (investigated in Section 1.1.1) with/without constraints (investigated in Section 1.1.3), the following methods, including matrix theory approach, frequency-domain method, and Lyapunov theory approach, are widely deployed (see Fig. 1.5).

A. Matrix theory approach

Due to the nature of MASs, matrix theory has been frequently used in the stability analysis of their distributed coordination. When applying this kind of methods, the state-transition matrices of MASs are usually transformed into stochastic matrices (see Jadbabaie and Morse, 2003; Ren and Beard, 2005). For example, given $\mathcal{T}$as a sampling period, the closed-loop form of system (1.1) is given by

$$x_i(k+1) = \frac{1}{\sum_{j=1}^{N} a_{ij}[k]G_{ij}[k]} \sum_{j=1}^{N} a_{ij}[k]G_{ij}[k]x_j(k), i = 1, 2.\cdots, N. \tag{1.17}$$

where $k \in \{1, 2, \cdots\}$ is the discrete-time index, $a_{ij}[k] > 0$ are positive constants, and $G_{ij}[k]$ is 1 if information flows from agent j to agent i, and 0 otherwise.

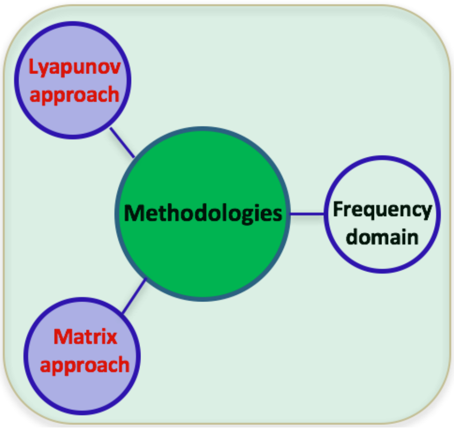

FIGURE 1.5: The research framework of cooperative control of MASs.

Equation (1.17) can be written in matrix form as $x(k+1) = D[k]x(k)$, where $x = [x_1, \cdots, x_N]^T, D = [d_{ij}]$ with $d_{ij} = a_{ij}[k]G_{ij}[k] / \sum_{j=1}^{N} a_{ij}[k]G_{ij}[k]$, and $D[k]$ is called state-transition matrices. Based on the production convergence property on an infinite sequence of stochastic matrix, consensus can be ensured. In addition, the continuous-time cosensus scheme was also proposed in Ren and Beard, 2005. These resutls was extended for formation tracking with a time-varying reference in Ren, 2007b; Ren, 2008. Moreovere, the matrix theory approach can also deal with the consensus problem with hybrid communication in fixed and switching topologies (Guan, Wu, and Feng, 2012; Liu, Zhang, and Xie, 2017).

B. Frequency domain method

Frequency domain methods are also powerful tool to solve the consensus problems and analyze the consensusability. By some proper transformations, consensus problems can be bridged to the stabilization problems of error dynamics. And the frequency domain methods like Nyquist criterion (Olfati-Saber and Murray, 2004; Tian and Liu, 2008; Tian and Liu, 2009), stability margin optimization (Gu, Marinovici, and Lewis, 2012; Qi, Qiu, and Chen, 2013; Chen and Shi, 2020), and pole analysis (You and Xie, 2011; Chen and Shi, 2017; Hengster-Movric and Lewis, 2013; Wang, 2014; Ma et al., 2015), can be employed. More precisely, in Tian and Liu, 2008, the first-order multi-agent system with input and communication delays has been studied based on the frequency-domain analysis. Based on the symmetry of system structure allowing one to obtain much less conservative consensus condition. However, a very small perturbation may destroy the symmetry. Thus, the robustness of multi-agent systems becomes a very important issue, which has been addressed in Tian and Liu, 2009. In this paper, the robust leader-following consensus algorithm for second-order multi-agent systems with diverse input delays was investigated. Based on some early results for the congestion control system with diverse communication delays, decentralized conditions with some preconditions are obtained for the multi-agent system with symmetric coupling weights.

Moreover, some gain margins-based frequency domain consensus results are developed only considering undirected communication topologies. Qi, Qiu, and Chen, 2013 further explored the case of systems with directed topologies based on the result in Gu, Marinovici, and Lewis, 2012 by considering the gain and phase margins optimization problem. However, the problem of determining the stability margins and regions, especially for directed communication topologies, still requires further investigation. Therefore, by bridging the Laplacian spectra-based stability region to gain and phase margins, a stability margin-based results for agents of single-input dynamics was established (Chen and Shi, 2020). The result is further extended to a condition directly depending on the unstable poles of the agents' dynamics, in which the conservativeness is reduced by introducing an appropriate tuning parameter.

Considering the pole analysis, a new input-output property of the strictly proper transfer function was introduced in Wang, 2014. Moreover, via a constructive approach in the frequency domain, the limit value of the proper transfer function was obtained. Accordingly, thanks to Lyapunov-like analysis and frequency-domain input-output analysis, it is shown that a unified consensus control framework ensures consensus of multiple mechanical systems with communication delays on directed graphs containing a spanning tree. Following this research line, Ma et al., 2015 solved the consensus problem of nonlinear second-order MASs with parametric uncertainties on a directed graph containing a spanning tree. Next, the agent with general linear systems under time-varying delays in undirected and fixed topology was studied in Chen and Shi, 2017. By analyzing the delay-dependent gains, and in light of the small gain theorem, sufficient frequency domain consensus criteria for both continuous and discrete-time systems are established.

C. Lyapunov theory approach

Although matrix theory is a relatively simple approach for stability analysis of the formation and consensus problem, it is not applicable in many consensus and formation producing scenarios, especially with general dynamics and nonlinear systems. It is then natural to consider the Lyapunov function approach. The basic idea of Lyapunov stability theory-based methods is to transform the original multi-agent dynamics to associate error dynamics. Then the consensus condition can be evaluated by the properly constructed Lyapunov function. More specially, with a final consensus value (1.4) in Section 1.1.1 the disagreement vector δ,

$$\delta = x - x^{\star}\mathbf{1}, \tag{1.18}$$

was employed to convert the consensus problem into stability systems (Olfati-Saber and Murray, 2004). These results was also further extended for finite-time consensus (Wang and Xiao, 2010; Chen et al., 2011), event-trigger consensus control (Meng and Chen, 2013; Yi et al., 2019). For general dynamics, Zhongkui Li et al., 2010 also used the disagreement vector, related to the left and right eigenvector of the Laplacian matrix corresponding to the network graph. It is shown that the consensus problem can be indirectly solved by considering the stability systems. This kind of methods not only can be applied to LTI systems or linear systems but also can be used to deal with time-varying systems (Dong et al., 2015; Dong et al., 2016; Dong et al., 2018) or nonlinear systems (Ji and HaiBo, 2017; Yu et al., 2013; Huang and Ye, 2015).

In addition, Lyapunov stability theory-based methods are compatible to many advanced control schemes, like adaptive control (Li, Chen, and Ding, 2016; Liu and Huang, 2017; Mao, Akyol, and Zhang, 2017), event-triggered consensus (Almeida, Silvestre, and Pascoal, 2017; Hu, Liu, and Feng, 2017; Qian, Liu, and Feng, 2018a). Moreover, many researchers working in the field of consensus of multi-agent systems with delays use sums of Lyapunov-Krasovskii functionals or Lyapunov-Razumikhin functions. The former has been applied to investigate single integrator networks in Hu and Hong, 2007; Qin, Gao, and Zheng, 2011; Di Bernardo, Salvi, and Santini, 2015; Han et al., 2017a. The main disadvantage of using Lyapunov-Krasovskii functions is that the underlying graph needs to be undirected or weight-balanced. Lyapunov- Razumikhin functions are used to obtain results for more general multi-agent systems of single integrators and directed, uniformly quasi-strongly connected graphs in Salvi, Santini, and Valente, 2017; Santini et al., 2017.

1.2 Research Motivations

According to the four aspects discussed in this chapter, the research on cooperative control of MASs is a broad area. In this thesis, our research framework foccus on the dynamics of MASs (homogeneous and heterogeneous - presented in Section 1.1.1) with constraints (hybrid communication and state constraints - presented in Section 1.1.3) in the clustered network (see in Section 1.1.2) shown in Fig. 1.1 (violet color). Because of the mentioned dynamics, constraints, and clustered network, the system performance might be degraded. Even worse, the stability could be destroyed. Therefore, it is of great importance and challenge to design controllers to guarantee the desired performance under these above considerations.

In the subsection, we introduce the clustered nerwork (i.e., networks divided into subnetworks, also called clusters, where each node of the network graph represents an agent with linear dynamics. Each subnetwork is represented by a directed graph. Moreover, the agents in each cluster cannot communicate with agents from other clusters, except one single agent of each subnetwork, which is called a leader. These leaders interact at instant times via fixed and strongly connected directed graph). Then, some open research questions are given.

1.2.1 Cluster networks and MAS

In the sequel, we consider that the network $\mathcal{G}$ is subdivided into m directed (undirected) subnetworks $\mathcal{C}_\tau, \forall \tau \in \{1, \cdots, m\}$ represented by the graphs $\mathcal{G}_1, \cdots, \mathcal{G}_m$ such that $\mathcal{G}_1 = (\mathcal{V}_1, \mathcal{E}_1), \cdots, \mathcal{G}_m = (\mathcal{V}_m, \mathcal{E}_m)$, where $\mathcal{V} = \mathcal{V}_1 \cup \mathcal{V}_2 \cup \ldots \cup \mathcal{V}_m$ and $V_\tau \cap V_g = \oslash$ for all $\tau, g = 1, \cdots m, \tau \neq g$ and $\mathcal{E} = \mathcal{E}_1 \cup \mathcal{E}_2 \cdots \cup \mathcal{E}_m$. The communication graph of each subnetwork $\mathcal{G}_\tau$ is represented by a Laplacian matrix $\mathcal{L}_\tau$. Each cluster has a specific agent called the leader, and denoted in the following by $l_\tau \in \mathcal{V}_\tau, \forall \tau \in \{1, \cdots, m\}$. The remaining agents are called followers and are denoted by f_h. The set of leaders will be denoted by $\mathcal{I} = \{l_1, \cdots, l_m\}$. At particular time instant $t_k, k \in \mathbb{N}, t_k \geq 0$ of a time sequence $\{t_k\}$ that satisfies $t_1 < t_2 < \cdots, lim_{t_k \to \infty} t_k = \infty$, the leaders interact following a predefined interaction

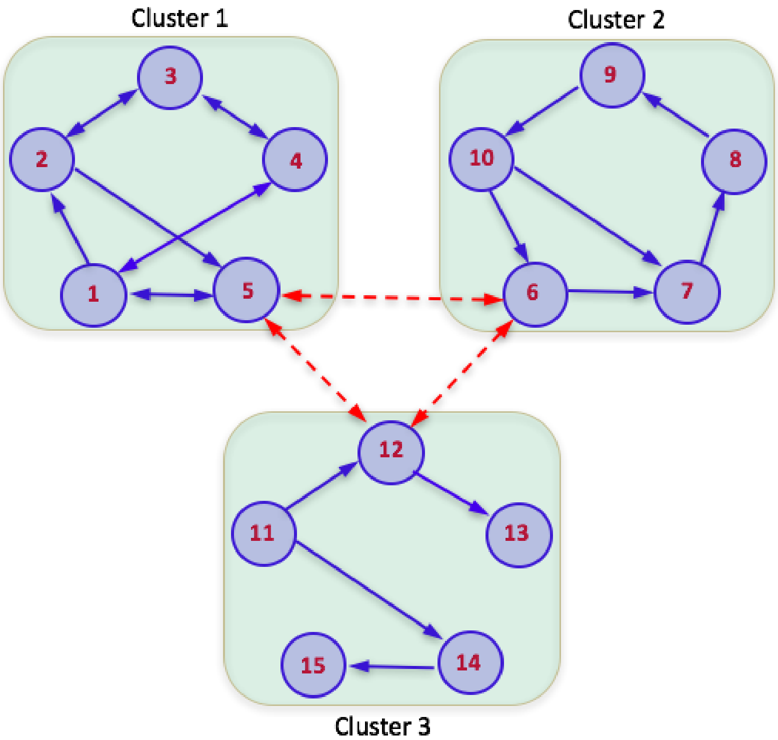

FIGURE 1.6: The clustered network of 15 agents

map $\mathcal{E}_l \subset \mathcal{I} \times \mathcal{I}$. The leaders communication graph $\mathcal{G}_l = (\mathcal{I}, \mathcal{E}_l)$ is also supposed to be a undirected graph. Finally, without loss of generality, each leader is considered as the first agent of its cluster

$$\mathcal{C}_\tau = \{l_\tau, f_{m_{\tau-1}+2}, \cdots, f_{m_\tau}\}, \quad \forall \tau \in \{1, \cdots, m\}, \tag{1.19}$$

where $m_0 = 0, m_m = N$ and the cardinality of $\mathcal{C}_\tau$ is given by $\|\mathcal{C}_\tau\| = n_\tau = m_\tau - m_{\tau-1}, \forall \tau \geq 1$.

Moreover, $P_l \in \mathbb{R}^{m \times m}$ is a row stochastic matrix associated to the graph $\mathcal{G}_l$ is defined as

$$\begin{cases} P_{l(i,j)} = 0, & if\ (i,j) \notin \mathcal{E}_l \\ P_{l(i,j)} > 0, & if\ (i,j) \in \mathcal{E}_l; i \neq j \\ \displaystyle\sum_{j=1}^{m} P_{l(i,j)} = 1, & \forall i = 1, \cdots, m \end{cases} \tag{1.20}$$

and $\mathcal{L}$ has the following block diagonal structure

$$\mathcal{L} = \begin{bmatrix} \mathcal{L}_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \mathcal{L}_m \end{bmatrix}, \mathcal{L}_\tau \in \mathbb{R}^{N_\tau}. \tag{1.21}$$

Example 2.2: To illustrate the notation (1.19), we consider a simple network of 15 agents partitioned in 3 clusters having 5 elements depicted in Fig. 1.6. Then

$$\mathcal{C}_1 = \{l_1, f_1, f_2, f_3, f_4\}, \quad \mathcal{C}_2 = \{l_2, f_7, f_8, f_9, f_{10}\} \text{ and } \mathcal{C}_3 = \{l_3, f_{11}, f_{13}, f_{14}, f_{15}\}$$

and the set of leaders will be denoted by $\mathcal{I} = \{l_1, l_2, l_3\}$. The leaders interact following a predefined interaction map $\mathcal{E}_l \subset \mathcal{I} \times \mathcal{I}$. Thus, the leaders communication graph is defined by $\mathcal{G}_l = (\mathcal{I}, \mathcal{E}_l)$.

Moreover, P_l is a row stochastic matrix associated to the graph $\mathcal{G}_l$ is given by

$$P_l = \begin{bmatrix} 0.4 & 0.2 & 0.4 \\ 0.2 & 0.3 & 0.5 \\ 0.6 & 0.1 & 0.3 \end{bmatrix} \in \mathbb{R}^{3\times3} \tag{1.22}$$

and $\mathcal{L}$ has the following block diagonal structure

$$\mathcal{L} = \begin{bmatrix} \mathcal{L}_1 & 0 & 0 \\ 0 & \mathcal{L}_1 & 0 \\ 0 & 0 & \mathcal{L}_3 \end{bmatrix} \in \mathbb{R}^{15\times15}, \tag{1.23}$$

where $\mathcal{L}_1 = \mathcal{L}_c, \mathcal{L}_2 = \mathcal{L}_b, \mathcal{L}_3 = \mathcal{L}_d$, are determined in **Example B.1**.

1.2.2 Open Research Questions

In the subsection, we would like to propose the following open research questions.

1) The first question for considering the analysis consensus in a clustered network arises from problems that would benefit from a division of a large network into subnetworks (called clusters). They are almost all the time isolated one from another, such as the energy optimization in the wireless sensor network (see Halgamuge, Guru, and Jennings, 2003; Chen and Wen, 2013), or the consensus of the opinion in the social networks (Jadbabaie and Morse, 2003). In each cluster, there exists an agent called a leader who can exchange information outside of its cluster at some specific discrete-time, while the interaction among agents inside each cluster happens in continuous-time. The main distinctness of this question compared to the analysis consensus in Jadbabaie and Morse, 2003; Olfati-Saber and Murray, 2004; Ren and Beard, 2005 lies on two main aspects: 1) The communication among agents in the network either continuous-time or discrete-time, meanwhile the communication in the consideration network is hybrid (see Section 1.1.3); 2) The network is composed/ divided of/into by several clusters (see Section 1.1.2). This leads to the problem that although each cluster can achieve consensus, the consensus of the overall network is not guaranteed. Thus, thanks to the results from the matrix theory approach in Section 1.1.4 and algebraic graph theory, this thesis provides a solution to this question.

2) In some literature on consensus, for instance in Olfati-Saber and Murray, 2004; Wei Ren and Atkins, 2007; Zhongkui Li et al., 2010, one of the interesting

research problems is a final consensus value of the network. It relates to the dynamics of agents, the communication topology, and the initial conditions. For the clustered network, due to the impulsive effects on the communication of leaders as well as the change of structure of the Laplacian matrix corresponding to the network, the final consensus value problem is more complex. Therefore, the second question is to state what the global consensus value is ?. This thesis provides a approach to address a such issue.

3) The cooperative behavior of linear MASs with general system dynamics, related Section 1.1.1, in the clustered network is defined by not only the dynamical control protocols concerning the isolated clusters but also the discrete interactions among the leaders. This evidence makes a consensus problem in the clustered network with general linear agents much more challenging than that of the integrator case. Thus, the next question is to state under which conditions the considered network of general linear agents can achieve and maintain consensus behavior. Another challenge we face is how to rebuild the full state information of each agent by using only the local relative output information, and discrete interaction between leaders' clusters. Based on the Lyapunov theory approach in Section 1.1.4, we provide a novel approach to address such issues.

4) The significant and realistic issues have been encountered such as the constraints on the agent's inputs or states because of the physical limitations of agents (investigated in Section 1.1.3). Moreover, the formation problem is also an active research subject in cooperative control. Thus, the next questions are how to design a robust formation control and which conditions such that the considered network of general linear agents under state constraints can reach the desired formation. Thanks to results from the matrix theory and Lyapunov theory approaches given in Section 1.1.4, these problem are addressed in this thesis.

5) How to achieve synchronization in the clustered network of different agents dynamics (heterogeneous MASs mentioned in Section 1.1.1) under different disturbances using output is an open research question. This thesis provides a novel approach to address such issues based on Lyapunov theory approach (see Section 1.1.4).

In this thesis, our objective is to address the above identified gaps in the literature.

Chapter 2

Impulsive Observer-Based Control in Clustered Network

Contents

Résumé du Chapitre 2

Ce chapitre aborde le problème du consensus dans les réseaux divisés en sous-réseaux (également appelés clusters), où chaque nœud du graphe de réseau représente un agent à dynamique linéaire. Un protocole de contrôle basé sur l'observation impulsive est proposé (correspondant à la troisième question de la section 1.2). Sur la base de ce protocole proposé, la dynamique collective des réseaux de MAS est décrite sous le terme de systèmes hybrides. La caractérisation de la valeur consensuelle globale de ce type de réseaux est analysée (correspondant à la deuxième question de la section 1.2). Ensuite, nous montrons que le problème de consensus peut être indirectement résolu en considérant la stabilité d'un système équivalent. Esnuite, une condition suffisante pour la stabilité asymptotique de ce système équivalent est proposée. Nous développons un algorithme qui calcule les paramètres d'interaction entre, le gain de rétroaction, les matrices de gain des observateurs et les poids de couplage. Des résultats de simulation sont donnés pour démontrer l'efficacité des résultats théoriques.

This chapter addresses the problem of consensus in networks divided into subnetworks (also called clusters), where each node of the network graph represents an agent with linear dynamics. An impulsive observer-based control protocol is proposed (corresponding to the third question in Section 1.2). Based on this proposed protocol, the collective network dynamics of MASs is described in the term of hybrid systems. The characterization of the global consensus value of this kind of networks is analyzed (corresponding to the second question in Section 1.2). Secondly, we show that the problem of consensus design for clustered networks can be indirectly solved by considering the stability of an equivalent system. Then, a sufficient condition for the asymptotical stability of this equivalent system is proposed. Finally, an algorithm properly selects the interaction network of the leaders, feedback gain, observer gain matrices, and coupling weights. This allows agents in the clustered network to enclose a prior fixed target. Simulation results are given to demonstrate the effectiveness of the theoretical results.

2.1 Related Work and Contributions

In the last decade, the problem of consensus for networked MASs has attracted increasing consideration in a variety of fields such as engineering, biology, sociology and so on (Nguyen, Messai, and Manamanni, 2017, Bragagnolo, Messai, and Manamanni, 2019 for example). The primary objective in the consensus problems is to design relevant protocols and algorithms such that agents reach an agreement. For instance, the consensus problem for first-order and second-order dynamics systems based on continuous-time models, such as Olfati-Saber and Murray, 2004; Qin, Gao, and Zheng, 2011; Wang et al., 2019, or discrete-time models, such as Ren and Beard, 2005; Jadbabaie and Morse, 2003; Zhou and Wang, 2009; Moreau, 2005, are discussed in different frameworks: directed or undirected networks, fixed or switching topology, as well as communication with delays. Moreover, the consensus problem for agents with general linear dynamics is also investigated in Zhongkui Li et al., 2010; Zhang, Lewis, and Das, 2011; Nguyen, 2017; Qian, Liu, and Feng, 2018a; Liu and Yang, 2017; Su et al., 2019. In these works the distributed consensus protocols Zhongkui Li et al., 2010; Zhang, Lewis, and Das, 2011 were designed by utilizing relative information of the fixed communication graph, and the controller gains were obtained using LQR optimal control method. However, Zhongkui Li et al., 2010; Zhang, Lewis, and Das, 2011 considered only a local LQR performance index. Thus, an approach using a global LQR performance index was introduced in Nguyen, 2017. Next, the event-triggered control in Qian, Liu, and Feng, 2018a was introduced to solve the problem of the energy resourse constraints in fixed network of linear MASs with external disturbances. Alternatively, the consensus problem in switching topologies was addressed in several works Liu and Yang, 2017; Su et al., 2019. Accordingly, Su et al., 2019 studied the semi-global output consensus problem for multiagent systems depicted by discrete-time dynamics subject to external disturbances and input saturation. Among the aforementioned works, the exchanging information among agents in the network is considered as continuous or discrete.

In several practical applications of MASs, the communication network is partitioned into several groups or clusters. This includes for example the traffic control by consensus on the speed, vehicles platooning in the intelligent transportation systems (Jia and Ngoduy, 2016; Li et al., 2018), or the consensus control on fixed topology social network (i.e., no new person leave or join a group) (Yaghmaie et al., 2017), where individuals can change their opinions by interacting with individuals outside of their community. Moreover, another issue could concern the communication constraints, where agents can be either subject to abrupt changes at specific instants (Lu, Ho, and Cao, 2010; Guan, Wu, and Feng, 2012; Liu, Zhang, and Xie, 2017) or can only exchange information in an impulsive way with their neighbors. For those scenarios, the typical results of the works mentioned above cannot be applied directly to achieve consensus.

Motivated by both theoretical and practical issues mentioned above, this research focuses on clustered networks of agents with continuous intra-cluster and discrete inter-cluster communications. Two basic questions can be raised: The first question is to state under which conditions the network of agents can achieve and maintain consensus behavior ?, and the second question is to state what the global consensus value is ?. In order to find the answer to the first question, in Bragagnolo et al., 2016, the authors proposed a quasi-periodically reset strategy and provide some LMI conditions that guarantee the global uniform exponential stability of the consensus of systems with subnetwork represented by directed and strongly connected graphs. When the reset instants are event-triggered, i.e., defined by the occurrence of specific events, the sufficient conditions for consensus were proposed in Morarescu et al., 2016. The second question was investigated in Bragagnolo et al., 2016 for agents with a simple integrator. However, the final global consensus value of agents with general linear dynamics is still an open problem.

Moreover, most of the literature considers the consensus of cluster-divided MASs only with integrator dynamics rather than general system dynamics. Therefore, instead of investigating the consensus problem of the clusters-divided network for agents with integrator dynamics, we focus on a more general model. Because there are only interactions among clusters at the specific instants t_k, the cooperative behavior of a linear multi-agent system in the clustered network is defined by not only the dynamical control protocols concerning the isolated clusters but also the discrete interactions among the leaders. This evidence makes a consensus problem in the clustered network with general linear agents much more challenging than that of the integrator case, which is also indicated in Pham, Messai, and Manamanni, 2019c; Pham et al., 2020b. Another challenge we face is how to rebuild the full state information of each agent by using only the local relative output information, and discrete interaction between leaders' clusters. Motivated by the observer design approach for a single system (Zhongkui Li et al., 2010 and Li et al., 2011a; Wen et al., 2017; Li et al., 2011b; Wan et al., 2017), an impulsive distributed observer, which takes into account continuous intra-cluster and discrete inter-cluster communications, is designed for each agent. This impulsive observer online estimates its full state information. Since the separation principle can not be satisfied for controller and observer design in this hybrid system, a Lyapunov function-based design approach is approved. Furthermore, the target enclosing problem (Zheng, Liu, and Sun, 2015; Dong et al., 2018) is also considered by designing the interaction among leaders' clusters.

The above-mentioned problems and limitations mainly motivate this work, whose main contributions are threefold.

- The characterization of the global consensus value in the considered framework is analyzed. We show that the value of global consensus depends only on the dynamics of each agent, the graphs of clusters, the interaction between leaders, and the initial conditions.

- An impulsive observer-based control, which uses only the local relative output information, and discrete interaction between leaders' clusters, is designed. Then, we show that the consensus design for clustered networks can be indirectly solved by considering the stability of an equivalent system. To study the stability of this equivalent system, we propose an algorithm to suitably choose the feedback and observer gain matrices and coupling weights in the form of some LMIs.

- The interaction among leaders' clusters, ensuring agents in clustered networks enclose a prescribed target, is designed.

2.2 Problem Formulation

2.2.1 Impulsive Observer-based Control

Suppose that there is a group of N linear identical agents that interact in m clusters. The dynamics of each agent i is described by

$$\dot{x}_i = Ax_i + Bu_i, \quad y_i = Cx_i, \tag{2.1}$$

where $x_i = [x_{i,1}, \cdots, x_{i,n}]^\mathsf{T} \in \mathbb{R}^n$ is the state, $u_i \in \mathbb{R}^p$ is the control input, and $y_i \in \mathbb{R}^q$ is the measured output.

According to (1.19), we concede that each agent has a vector state denoted by $x_{l_\tau} = [x_{l_\tau,1}, \cdots, x_{l_\tau,n}]^\mathsf{T} \in \mathbb{R}^n$ for the leaders l_τ and $x_{f_h} = [x_{f_h,1}, \cdots, x_{f_h,n}]^\mathsf{T} \in \mathbb{R}^n$ for the followers $f_h, \forall \tau \neq h = 1, \cdots, N$.

As mentioned above, all agents in a cluster are connected, and the clusters interact together via their leaders at some instant times t_k. The design of controller and observer will consider the information exchanged among agents. Differing from Zhongkui Li et al., 2010; Li et al., 2011b; Li et al., 2011a, an impulsive observer, which deals with continuous intra-cluster and discrete inter-cluster communications, is introduced. This observer ensures not only consensus of states, but also convergences of observation errors. The following impulsive observer for the i^{th} agent is constructed:

$$\begin{aligned} \dot{\hat{x}}_i &= A\hat{x}_i + Bu_i + H(\hat{y}_i - y_i) + qL\xi_i, , \quad t \in (t_{k-1}, t_k), \\ \hat{y}_i &= C\hat{x}_i, \end{aligned} \tag{2.2}$$

and the interaction between the leader's states of the observer after the reset time can be described by

$$\hat{x}_{l_i}(t_k) = \sum_{j=1}^{m} (P_{l(i,j)} \otimes I_n)\hat{x}_{l_j}(t_k^-), \quad t = t_k, \tag{2.3}$$

where $P_l \in \mathbb{R}^{m\times m}$ is a row stochastic matrix associated to the graph $\mathcal{G}_l$, represented in (1.20), and $\hat{x}_i = [\hat{x}_{i,1}, \cdots, \hat{x}_{i,n}]^\top \in \mathbb{R}^n$ is the state of observer, $H, L \in \mathbb{R}^{n\times q}$ denote the observer gain matrices, and $q > 0$ is called a coupling gains. ξ_i is the relative output estimation output of i^{th} agent, and is defined as follows:

$$\xi_i = \sum_{j=1}^{N} a_{ij}[(\hat{y}_j - y_j) - (\hat{y}_i - y_i)], \tag{2.4}$$

where a_{ij} are entries of the adjacency matrix. Moreover, due to ξ_i is based on information exchanges from neighboring nodes, this impulsive observer (2.2)–(2.4) is different from the centralized architecture.

Next, an impulsive observer-based control protocol for a clustered network is designed, where only the leaders of these clusters can communicate together at the reset time t_k,

$$x_{l_i}(t_k) = \sum_{j=1}^{m} (P_{l(i,j)} \otimes I_n)x_{l_j}(t_k^-), \quad t = t_k. \tag{2.5}$$

The protocol herein designed considers that each agent has access to the relative estimated state measurement of its neighbors, and is given by

$$u_i = pK\sum_{j=1}^{N} a_{ij}(\hat{x}_j - \hat{x}_i), \quad t \in (t_{k-1}, t_k), \tag{2.6}$$

where $a_{ii} = 0$ and $a_{ij} > 0$ if agent i can receive information from agent j and 0 otherwise. $p > 0 \in \mathbb{R}$ is called a coupling gains or weights, which can be regarded as a scaling factor on the communication graph $\mathcal{G}$, and $K \in \mathbb{R}^{p\times n}$ denote the feedback gain matrices.

For the i^{th} agent, the observation error vector is defined $e_i = x_i - \hat{x}_i$, thus it follows

$$\begin{aligned}
\dot{e}_i &= (A + HC)e_i + qLC\sum_{j=1}^{N} a_{ij}(e_j - e_i), \quad t \in (t_{k-1}, t_k), \\
e_{l_i}(t_k) &= \sum_{j=1}^{m} (P_{l(i,j)} \otimes I_n)e_{l_j}(t_k^-), \quad t = t_k.
\end{aligned} \tag{2.7}$$

Then, by using (2.6), system (2.1) can be rewritten as

$$\dot{x}_i = Ax_i + pBK\left[\sum_{j=1}^{N} a_{ij}(x_j - x_i) - \sum_{j=1}^{N} a_{ij}(e_j - e_i)\right], \ t \in (t_{k-1}, t_k). \tag{2.8}$$

Finally, the hybrid dynamics of system (2.1) under the impulsive observer-based control protocol (2.6) and the interaction between leaders (2.5) can be rewritten as

$$\begin{cases} \dot{z}_i = \mathcal{A}z_i - \sum_{j=1}^{N} l_{ij}\mathcal{H}z_j, & t \in (t_{k-1}, t_k), \\ z_{l_i}(t_k) = \sum_{j=1}^{m} (P_{l(i,j)} \otimes I_{2n}) z_{l_j}(t_k^-), & t = t_k \end{cases} \tag{2.9}$$

where $z_i = \begin{bmatrix} x_i \\ e_i \end{bmatrix} \in \mathbb{R}^{2n}$, $\mathcal{A} = \begin{bmatrix} A & 0 \\ 0 & A + HC \end{bmatrix} \in \mathbb{R}^{2n\times 2n}$, and $\mathcal{H} = \begin{bmatrix} pBK & -pBK \\ 0 & qLC \end{bmatrix} \in \mathbb{R}^{2n\times 2n}$.

It can be seen that the evolution of the dynamic system described by (2.9) is influenced by the events that happen at the reset moments. Thus, the evolution of the whole system can be viewed as a hybrid system that evolves as follows: during interval time (t_{k-1}, t_k), the interaction among nodes in each cluster $\mathcal{C}_\tau$ is related only to the graph $\mathcal{G}_\tau$. However, at each instant time t_k, the leaders update their states instantaneously according to the topology of $\mathcal{G}_l$. The objective of the above collaboration is to achieve a specific global objective namely consensus defined as follows.

Definition 2.2.1 *The consensus of system (2.9), is said to be reached if there exists an impulsive observer-based control protocol (2.6), such that*

$$\lim_{t\to\infty} \|x_i - x_j\| = 0, \quad \forall i, j = 1, \cdots, N \tag{2.10}$$

$$\lim_{t\to\infty} \|x_i - \hat{x}_i\| = 0, \quad \forall i, j = 1, \cdots, N, \tag{2.11}$$

for all initial conditions.

Assumption 2.2.1 *Graph $G_\tau, \forall\tau \in \{1, \cdots, m\}$ is strongly connected.*

2.2.2 Useful Lemmas

In the sequel, the following lemma are considered.

Lemma 2.2.1 *(Olfati-Saber, Fax, and Murray, 2007) Let P_l be a row stochastic nonnegative matrix with left and right eigenvectors w and $\mathbf{1}_m$ respectively, satisfying $P_l\mathbf{1}_m = \mathbf{1}_m, w^\top P_l = w^\top$, and $\mathbf{1}_m^\top w = 1$.*

Lemma 2.2.2 *(Yu et al., 2010) Suppose Assumption 2.2.1 holds. Then, there exists a positive vector $\theta_\tau = [\theta_1, \cdots, \theta_{N_\tau}]^\mathsf{T} \in \mathbb{R}^{N_\tau}$ such that*

$$\Theta_\tau \mathcal{L}_\tau + \mathcal{L}_\tau^\mathsf{T} \Theta_\tau \geq 0, \forall \tau \in \{1, \cdots, m\}, \tag{2.12}$$

where $\Theta_\tau = diag\{\theta_1, \cdots, \theta_{N_\tau}\} \in \mathbb{R}^{N_\tau \times N_\tau}$. Moreover, the general algebraic connectivity is defined by

$$a_\tau(\mathcal{L}_\tau) = \min_{\theta_\tau^\mathsf{T} x = 0, x \neq 0} \frac{x^\mathsf{T}(\Theta_\tau \mathcal{L}_\tau + \mathcal{L}_\tau^\mathsf{T} \Theta_\tau) x}{2 x^\mathsf{T} \Theta_\tau x} \tag{2.13}$$

where $\theta_\tau^\mathsf{T} \mathcal{L}_\tau = 0_{1 \times N_\tau}, \sum_{i=1}^{N_\tau} \theta_i = 1$.

Remark 2.2.1 *Assumption 2.2.1 is needed to guarantee that the Laplacian matrix $\mathcal{L}_\tau$ of $\mathcal{G}_\tau, \tau \in \{1, \cdots, m\}$ satisfies the following proprieties $\mathcal{L}_\tau \mathbf{1}_{N_\tau} = 0, r_\tau^\mathsf{T} \mathcal{L}_\tau = 0$ and $r_\tau^\mathsf{T} \mathbf{1}_{N_\tau} = 1$, where $\mathbf{1}_{N_\tau}$, and $r_\tau^\mathsf{T} = \frac{1}{N_\tau} \mathbf{1}_{N_\tau}$ are the right and left eigenvectors of $\mathcal{L}_\tau$ associated with zero eigenvalue, respectively.*

The reminder of this chapter will deal with the following problems: Consider a group of N agents with their auxiliary systems defined in (2.9), and suppose that Assumption 2.2.1 holds. We now are interested in the solution of the following problems.

Problem 1 *Characterization of the global consensus value z^*.*

Problem 2 *Design of an impulsive observer-based control protocol (2.6) such that the hybrid systems (2.9) satisfies Definition 2.2.1.*

Problem 3 *Design of a row stochastic matrix $P_l \in \mathbb{R}^{m \times m}$ and an impulsive observer-based control protocol (2.6) such that the agents of clustered networks enclose a prescribed target z^*.*

2.3 Agreement Behavior Analysis

This section will focus on **Problem 1**. We firstly show that under the impulsive observer-based control protocol (2.9) each cluster, has a local agreement, which is piecewise constant. In the following, our consideration focuses on analyzing the changing of the local agreement value at instant times t_k. The global consensus value $z^\star$ is determined based on the set of local agreements of clusters and interaction between the leaders of these clusters, respectively. Particularly, we show that it depends on the system dynamics, the initial conditions and the interaction between the leaders.

2.3.1 Local Agreement Behavior

If one considers the consensus problem of each cluster, without taking into account the interaction between the clusters at the instant time t_k, a sufficient condition that guarantees the local agreements (i.e., the consensus in each cluster) is given in the following Lemma 2.3.1, which is formulated in the same way as in Zhongkui Li et al., 2010.

Lemma 2.3.1 *Consider the $\mathcal{C}_\tau$ represented by a graph $\mathcal{G}_\tau$ satisfying the Assumption 2.2.1, the protocol (2.9) solves the consensus problem for the cluster $\mathcal{C}_\tau$ if and only if the matrices $A-\lambda_{\tau,i}pBK$ and $A+HC-\lambda_{\tau,i}qLC$ are Hurwitz for all the non-zero eigenvalues $\lambda_{\tau,i}$ of the Laplacian $\mathcal{L}_\tau$ of $\mathcal{G}_\tau$. Moreover, the local consensus value of the cluster $\mathcal{C}_\tau$ is*

$$z^\star_\tau(t) = (r_\tau^\top \otimes e^{\mathcal{A}t})z_\tau(0), \forall \tau \in \{1, \cdots, m\}. \tag{2.14}$$

where $r_\tau \in \mathbb{R}^{N_\tau}$ is such that $r_\tau^\top \mathcal{L}_\tau = \boldsymbol{0}_{1\times N_\tau}$ and $r_\tau^\top \mathbf{1}_{N_\tau} = 1$.

Proof 2.3.1 *We consider the local agreement of cluster $\mathcal{C}_\tau, \tau = 1, \cdots, m$. Using the Eq. (2.9) with $i = 1, \cdots, N_\tau$ and the global model of cluster $\mathcal{C}_\tau$ can be expressed as the following*

$$\dot{z}_\tau = (I_{N_\tau} \otimes \mathcal{A} - \mathcal{L}_\tau \otimes \mathcal{H})z_\tau \tag{2.15}$$

the disagreement vector is also used $\xi_\tau = [(I_{N_\tau} - \mathbf{1}_{N_\tau} r_\tau^\top) \otimes I_n]z_\tau$, (2.15) becomes

$$\dot{\xi}_\tau = (I_{N_\tau} \otimes \mathcal{A} - \mathcal{L}_\tau \otimes \mathcal{H})\xi_\tau \tag{2.16}$$

Since $\mathbf{1}_{N_\tau}$ and $r_\tau^\top$ are the right and left eigenvectors of $\mathcal{L}_\tau, v = 1, \cdots, m$ associated with the zero eigenvalue, respectively. Therefore, there exists a matrix $V_\tau \in \mathbb{R}^{N_\tau \times N_\tau}$ such that $V_\tau^{-1}\mathcal{L}_\tau V_\tau = \Lambda_\tau$ is diagonal. Then we also introduce a new variable $\psi_\tau = (V_\tau^{-1} \otimes I_n)\xi_\tau$, the Eq. (2.16) is equivalent to

$$\begin{aligned} \dot{\psi}_\tau &= (V_\tau^{-1} \otimes I_n)(I_{N_\tau} \otimes \mathcal{A})\xi_\tau - (V_\tau^{-1} \otimes I_n)(\mathcal{L}_\tau \otimes \mathcal{H})\xi_\tau \\ &= (I_{N_\tau} \otimes \mathcal{A})(V_\tau^{-1} \otimes I_n)\xi_\tau - (\Lambda_\tau \otimes \mathcal{H})(V_\tau^{-1} \otimes I_n)\xi_\tau \\ &= (I_{N_\tau} \otimes \mathcal{A} - \Lambda_\tau \otimes \mathcal{H})\psi_\tau \end{aligned} \tag{2.17}$$

Choosing $V_\tau = [\mathbf{1}_{N_\tau} \quad Y_\tau]; V_\tau^{-1} = \begin{bmatrix} r_\tau^\top \\ W_\tau \end{bmatrix}$, It is easy to recognize that $\psi_{v,1} = (r_\tau^\top \otimes I_n)\xi_\tau = 0$ and the elements of the state matrix of (2.17) are block diagonal. Hence, $\psi_{v,i};\ i = 2, \cdots, N_\tau$ converge asymptotically to zero if and only if the following subsystems

$$\dot{\psi}_{\tau,i} = (\mathcal{A} - \lambda_{\tau,i} \otimes \mathcal{H})\psi_{\tau,i}$$

are asymptotically stable. By using H in (2.9), the matrices $\mathcal{A} - \lambda_{\tau,i} \otimes \mathcal{H}$ have the following form

$$\begin{bmatrix} A - p\lambda_{\tau,i}BK & pBK \\ 0 & A + HC - q\lambda_{\tau,i}LC \end{bmatrix}; i = 2, \cdots, N_\tau$$

Therefore, the system (2.16) is asymptotically stable iff $A - p\lambda_{v,i}BK$ and $A + HC - q\lambda_{\tau,i}LC, i = 2, \cdots, N_\tau$ are Hurwitz. Then (2.15) will be reached consensus.

Next, we show that the local agreement of cluster $\mathcal{C}_\tau$, the solution of Eq. (2.15) can be obtained as

$$\begin{aligned}
&z_\tau(t) = e^{(I_{N_\tau}\otimes\mathcal{A}-\mathcal{L}_\tau\otimes\mathcal{H})t}z_\tau(0)\\
&= [\mathbf{1}_{N_\tau}\otimes I_{2n}\ \ Y_\tau\otimes I_{2n}]e^{(I_{N_\tau}\otimes\mathcal{A}-\Lambda_\tau\otimes\mathcal{H})t}\begin{bmatrix} r_\tau^{\mathsf{T}}\otimes I_{2n}\\ W_\tau\otimes I_{2n}\end{bmatrix}z_\tau(0)\\
&= [\mathbf{1}_{N_\tau}\otimes I_{2n}\ \ Y_\tau\otimes I_{2n}]\begin{bmatrix} e^{\mathcal{A}t} & 0\\ 0 & e^{(I_{N_\tau-1}\otimes\mathcal{A}-\Lambda_{N_\tau-1}\otimes\mathcal{H})t}\end{bmatrix}\begin{bmatrix} r_\tau^{\mathsf{T}}\otimes I_{2n}\\ W_\tau\otimes I_{2n}\end{bmatrix}z_\tau(0)\\
&= (\mathbf{1}_{N_\tau}r_\tau^{\mathsf{T}})\otimes e^{\mathcal{A}t}z_\tau(0) + (Y_\tau\otimes I_{2n})e^{(I_{N_\tau-1}\otimes\mathcal{A}-\Lambda_{N_\tau-1}\otimes\mathcal{H})t}(W_\tau\otimes I_{2n})z_\tau(0)
\end{aligned} \tag{2.18}$$

Because $A - p\lambda_{\tau,i}BK$ and $A + HC - q\lambda_{\tau,i}LC$; $i = 2, \cdots, N_\tau$ are Hurwitz corresponding to $(I_{N_\tau-1}\otimes\mathcal{A} - \Lambda_{N_\tau-1}\otimes\mathcal{H})$ is Hurwitz, thus $e^{(I_{N_\tau-1}\otimes\mathcal{A}-\Lambda_{N_\tau-1}\otimes\mathcal{H})t} \to \infty$ when $t\to\infty$. Then, one has

$$z_\tau(t) \to (\mathbf{1}_{N_\tau}r_\tau^{\mathsf{T}})\otimes e^{\mathcal{A}t}z(0) \ \ \text{as}\ \ t\to\infty$$

Remark 2.3.1 *According to the dynamics of system (2.9), if this system achieves consensus and the corresponding local consensus value is $z^\star$, then $z^\star\mathbf{1}_m$ belongs to the same subspace. In addition, we see that even if a local agreement of the clusters is achieved, the value of the local agreement will change at the reset time t_k.*

$$(r_\tau^{\mathsf{T}}\otimes e^{\mathcal{A}t})z_\tau(t_k) \neq (r_\tau^{\mathsf{T}}\otimes e^{\mathcal{A}t})z_\tau(t_{k-1}), \forall\tau\in\{1,\cdots,m\}.$$

Therefore, the global consensus can be achieved only if the local agreements converge.

2.3.2 Global Agreement Behavior

Before the characterization of the global consensus value, let us introduce the vectors $z^\star\in\mathbb{R}^{2mn}$ and $w\in\mathbb{R}^m$ as following

$$\begin{aligned} z^\star &= [z_1^\star, z_2^\star, \cdots, z_m^\star]^{\mathsf{T}}\in\mathbb{R}^{2mn},\\ w &= [w_1, w_2, \cdots, w_m]^{\mathsf{T}}\in\mathbb{R}^m, \end{aligned} \tag{2.19}$$

where $z_\tau^\star, \tau\in\{1,\cdots,m\}$ represents local agreement of cluster $\mathcal{C}_\tau$ and w is the left eigenvector of P_l associated with the eigenvalue 1 such that $w^{\mathsf{T}}\mathbf{1}_m = 1$.

Let us also introduce a matrix corresponding to the left eigenvectors of the clusters

$$\mathcal{Q} = \begin{bmatrix} r_1^{\mathsf{T}}\otimes e^{\mathcal{A}t} & \cdots & 0\\ \vdots & \ddots & \vdots\\ 0 & \cdots & r_m^{\mathsf{T}}\otimes e^{\mathcal{A}t}\end{bmatrix}\in\mathbb{R}^{2mn\times 2Nn}. \tag{2.20}$$

In addition, according to the expressed structure of (1.19), we assume that each vector r_τ, satisfying $r_\tau^{\mathsf{T}}\mathbf{1}_{N_\tau} = 1$, can be decomposed in its component $r_{\tau,l}$ (representing the leader of cluster $\mathcal{C}_\tau$) and the remaining of its components grouped in the vector $r_{\tau,f}$ (representing the followers of cluster $\mathcal{C}_\tau$). Particularly, $r_\tau^{\mathsf{T}} = [r_{\tau,l}^{\mathsf{T}},\ r_{\tau,f}^{\mathsf{T}}]$; $r_\tau^{\mathsf{T}}\otimes e^{\mathcal{A}t} = [r_{\tau,l}^{\mathsf{T}}\otimes e^{\mathcal{A}t},\ r_{\tau,f}^{\mathsf{T}}\otimes e^{\mathcal{A}t}]$.

We also introduce the following vector

$$\mu^{\mathsf{T}} = \phi^{\mathsf{T}} \otimes (e^{\mathcal{A}t})^{-1} \in \mathbb{R}^{2n\times 2mn}, \tag{2.21}$$

where $\phi^{\mathsf{T}} = [\phi_1, \cdots, \phi_\tau, \cdots, \phi_m] \in \mathbb{R}^{1\times m}$ and $\phi_\tau = w_\tau / r_{\tau,l}$; $\forall \tau \in \{1, \cdots, m\}$.

Based on the above definitions, we are now able to give our first result concerning the global consensus value.

Theorem 2.3.1 *Consider a network of clusters $\mathcal{C}_\tau$ represented by the graphs $\mathcal{G}_\tau, \forall \tau \in \{1, \cdots, m\}$ satisfying Assumption 2.2.1. If the network represented by system (2.9) achieves a consensus, then the global consensus value is:*

$$z^\star = \frac{e^{\mathcal{A}t}\mu^{\mathsf{T}}\mathcal{Q}z(0)}{\sum_{\tau=1}^{m}\phi_\tau}. \tag{2.22}$$

Proof 2.3.2 *Let us define a permutation matrix $\mathcal{M}$ such that $\mathcal{Q}(\mathcal{M} \otimes I_{2n})^{\mathsf{T}} = [\mathcal{Q}_1, \mathcal{Q}_2]$ where*

$$\mathcal{Q}_1 = Q_1 \otimes e^{\mathcal{A}t} \in \mathbb{R}^{2mn\times 2mn}, \mathcal{Q}_2 = Q_2 \otimes e^{\mathcal{A}t} \in \mathbb{R}^{2mn\times(2Nn-2mn)}, \tag{2.23}$$

and Q_1 is a block diagonal matrix with respect to the leaders

$$Q_1 = \begin{bmatrix} r_{1,l}^{\mathsf{T}} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & r_{m,l}^{\mathsf{T}} \end{bmatrix} \in \mathbb{R}^{m\times m}.$$

Q_2 is a block diagonal matrix with respect to the followers

$$Q_2 = \begin{bmatrix} r_{1,f}^{\mathsf{T}} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & r_{m,f}^{\mathsf{T}} \end{bmatrix} \in \mathbb{R}^{m\times(N-m)}.$$

Moreover, according to Lemma 2.3.1, the local consensus values before the reset instants can be written as:

$$z^\star = \mathcal{Q}z(0) = \mathcal{Q}(\mathcal{M} \otimes I_{2n})^{\mathsf{T}}(z_l(0), z_f(0)). \tag{2.24}$$

The states of the leader's agent are updated at the reset times t_k, so that (2.24) can be represented as:

$$z^\star(t_k) = \mathcal{Q}(\mathcal{M} \otimes I_{2n})^{\mathsf{T}}(z_l(t_k), z_f(t_k)), \tag{2.25}$$

after, by considering the fact that $z_l(t_k) = (P_l \otimes I_{2n})z_l(t_k^-)$ *and* $z_f(t_k) = z_f(t_k^-)$ *at each reset times* t_k*, this yields*

$$\begin{aligned} z^\star(t_k) - z^\star(t_k^-) &= \mathcal{Q}(\mathcal{M} \otimes I_{2n})^\top(z_l(t_k) - z_l(t_k^-), z_f(t_k) - z_f(t_k^-)) \\ &= \mathcal{Q}_1(z_l(t_k) - z_l(t_k^-)) + \mathcal{Q}_2(z_f(t_k) - z_f(t_k^-)) \\ &= \mathcal{Q}_1(z_l(t_k) - z_l(t_k^-)) + 0 \\ &= \mathcal{Q}_1(P_l \otimes I_{2n} - I_{2mn})z_l(t_k^-). \end{aligned} \tag{2.26}$$

By multiplying both sides of (2.26) by $\mu^\top$*, one has*

$$\begin{aligned} \mu^\top z^\star(t_k) =& \mu^\top z^\star(t_k^-) + \\ &+ (w^\top P_l \otimes I_{2n} - (w^\top \otimes I_{2n})I_{2mn})z_l(t_k^-). \end{aligned} \tag{2.27}$$

where $\mu^\top \mathcal{Q}_1 = [\phi^\top \otimes (e^{At})^{-1}](Q_1 \otimes e^{At}) = w^\top \otimes I_{2n}$.

Then, by using the property $w^\top P_l = w^\top$ *of Lemma 1, we obtain*

$$\mu^\top z^\star(t_k) = \mu^\top z^\star(t_k^-). \tag{2.28}$$

When the hybrid dynamics of system (2.9) is considered, Lemma 2.3.1 implies that the agents belonging to the same cluster try to approach a local agreement. It means z *remains a constant in the interval time* $t \in (t_{k-1}\ t_k)$*, and leads to*

$$\mu^\top z^\star = \mu^\top z^\star(0). \tag{2.29}$$

Finally, by considering that system (2.9) can achieve consensus, which means that $z^\star \to z^\star \mathbf{1}_N$ *as* $t \to \infty$*, Eq. (2.29) becomes*

$$\mu^\top z^\star \mathbf{1}_N = \mu^\top \mathcal{Q} z(0). \tag{2.30}$$

this equation leads to Eq. (2.22). ■

Remark 2.3.2 *According to Theorem 2.3.1, ones notes that:*

(i) By considering the particular case of simple integrator $\dot{x}_i = u_i$*, the obtained global consensus value is*

$$x^\star = \frac{\phi^\top Q x(0)}{\sum_{\tau=1}^{m} \phi_\tau}, Q = diag\{r_1^\top, r_2^\top \cdots, r_m^\top\}.$$

(ii) The global consensus value $x^\star$ *and observation error* $e^\star$ *can be deduced from (2.22) such as*

$$x^\star = \frac{e^{At}\mu^\top \mathcal{Q} x(0)}{\sum_{\tau=1}^{m} \phi_\tau}, \quad e^\star = \frac{e^{(A+HC)t}\mu^\top \mathcal{Q} e(0)}{\sum_{\tau=1}^{m} \phi_\tau}. \tag{2.31}$$

(iii) There are four factors affecting the value of global consensus: the dynamics of agents, the graph of each cluster, the interaction between leaders, and the

initial conditions. Moreover, the global consensus value does not depend on the reset sequence.

2.4 Impulsive Observer-Based Consensus Controller

In this next section, we will at first show that the problem of consensus design for clustered networks can be indirectly solved by considering the stability of an equivalent system. Then, **Problem 2** will be solved by giving some sufficient conditions in a LMI form. Moreover, based on the analysis given in Section 2.3, the obtained LMIs are adapted to compute the reset matrix in order to solve **Problem 3**.

2.4.1 Design Equivalence

Let us also introduce vectors

$$\begin{aligned} x &= [x_{l_1}^\top, x_{f_2}^\top, \cdots, x_{f_{m_1}}^\top, \cdots, x_{l_m}^\top \cdots, x_{f_{m_m}}^\top = x_{f_N}^\top]^\top \in \mathbb{R}^{Nn}, \\ x_l &= [x_{l_1}^\top, \cdots, x_{l_m}^\top]^\top \in \mathbb{R}^{mn}, \end{aligned}$$

containing states of the agents and leader's states, respectively. We are ready now to define variables

$$\begin{aligned} e &= [e_{l_1}^\top, e_{f_2}^\top, \cdots, e_{f_{m_1}}^\top, \cdots, e_{l_m}^\top, \cdots, e_{f_{m_m}}^\top = e_{f_N}^\top]^\top \in \mathbb{R}^{Nn} \\ e_l &= [e_{l_1}^\top, \cdots, e_{l_m}^\top]^\top \in \mathbb{R}^{mn}, \end{aligned}$$

which collect observation errors and leader's observation errors, respectively.

Then, system (2.9) can be written in the following system, which describes the overall network dynamics

$$\begin{cases} \dot{z} = (I_N \otimes \mathcal{A} - \mathcal{L} \otimes \mathcal{H})z, & t \in (t_{k-1}, t_k), \\ z_l(t_k) = (P_l \otimes I_{2n}) z_l(t_k^-), & t = t_k, \end{cases} \tag{2.32}$$

where z, z_l are augmented vectors defined as

$$\begin{aligned} z &= [z_{l_1}^\top, z_{f_2}^\top, \cdots, z_{f_{m_1}}^\top, \cdots, z_{l_m}^\top, \cdots, z_{f_{m_m}}^\top]^\top \in \mathbb{R}^{2Nn}, \\ z_l &= [z_{l_1}^\top, \cdots, z_{l_m}^\top]^\top \in \mathbb{R}^{2mn}, \\ z_{l_i} &= [x_{l_i}\ e_{l_i}]^\top \in \mathbb{R}^{2n}, z_{f_i} = [x_{f_i}\ e_{fi}]^\top \in \mathbb{R}^{2n}, \end{aligned} \tag{2.33}$$

and $\mathcal{L}$ is Laplacian matrix associated to the graph $\mathcal{G}$ represented in (1.21).

Next, let us present some algebraic properties of $\mathcal{L}$ in the following Proposition.

Proposition 2.4.1 *Let us consider a network of m clusters satisfying Assumption 2.2.1, with the Laplacian $\mathcal{L} \in \mathbb{R}^{N\times N}$, then rank$(\mathcal{L}) = N - m$ and $\mathcal{L}$ has m eigenvalues at zero and all the other $N - m$ eigenvalues of the Laplacian $\mathcal{L} \in \mathbb{R}^{N\times N}$ have non-negative real parts.*

Proof 2.4.1 *Based on the Assumption 2.2.1, $rank(\mathcal{L}_\tau) = N_\tau - 1$ and $\mathcal{L}_\tau$ has a simple eigenvalue at zero (Olfati-Saber and Murray, 2004), where N_τ is a number of agents in cluster $\mathcal{C}_\tau$. Therefore, $rank(\mathcal{L}) = rank(\mathcal{L}_1) + \cdots + rank(\mathcal{L}_\tau) + \cdots + rank(\mathcal{L}_m) = \sum_{\tau=1}^{m} N_\tau - m = N - m$. Moreover, taking into account the particular form of $\mathcal{L}$, it has m zero eigenvalues, all the other $N - m$ eigenvalues have non-negative real parts.*

Now, let $r_\tau^\mathsf{T} = [r_1, \cdots, r_{N_\tau}] \in \mathbb{R}^{1\times N_\tau}$ be the left eigenvector of $\mathcal{L}_\tau$ associated with zero eigenvalue, satisfying $r_\tau^\mathsf{T}\mathcal{L}_\tau = 0_{1\times N_\tau}$ and $r_\tau^\mathsf{T}\mathbf{1}_{N_\tau} = 1$, $\forall \tau \in \{1, \cdots, m\}$. Since Proposition 2.4.1 and $\mathcal{L}\mathbf{1}_N = \mathbf{0}_N$, $\mathbf{r}^\mathsf{T}\mathcal{L} = \mathbf{0}_{1\times N}$, where $\mathbf{r}^\mathsf{T} = [r_1^\mathsf{T}, \cdots, r_m^\mathsf{T}] \in \mathbb{R}^{1\times N}$, vectors $\mathbf{1}_N$ and $\mathbf{r}^\mathsf{T}$ are respectively the right and left eigenvectors of the Laplacian of $\mathcal{G}$ associates with m zero eigenvalues. Moreover, a straightforward calculation shows that $\mathbf{r}^\mathsf{T}\mathbf{1}_N = m$.

Let us also introduce the extended stochastic matrix P_e as follows:

$$P_e = \mathcal{M}^\mathsf{T} \begin{bmatrix} P_l & 0 \\ 0 & I_{N-m} \end{bmatrix} \mathcal{M} \in \mathbb{R}^{N\times N}, \tag{2.34}$$

where $\mathcal{M}$ is a permutation matrix used in the Section 2.3. Thus, the second equation in (2.32) can be expressed by

$$z(t_k) = (P_e \otimes I_{2n})z(t_k^-), \quad t = t_k.$$

Finally, let us introduce a new variable

$$\psi = z - \frac{1}{m}(\mathbf{1}_N\mathbf{r}^\mathsf{T} \otimes I_{2n})z, \tag{2.35}$$

where $\mathbf{r}^\mathsf{T} = [r_1^\mathsf{T}, \cdots, r_m^\mathsf{T}] \in \mathbb{R}^{1\times N}$ is the left eigenvector of $\mathcal{L}$ satisfying $\mathbf{r}^\mathsf{T}\mathbf{1}_N = m$ and $\mathbf{r}^\mathsf{T}\mathcal{L} = 0_{1\times N}$. According to the new variable (2.35), we are now ready to formulate our statement.

Proposition 2.4.2 *By considering the new variable (2.32), the overall network dynamics system (2.35) becomes*

$$\begin{cases} \dot{\psi} = (I_N \otimes \mathcal{A} - \mathcal{L} \otimes \mathcal{H})\psi, & t \in (t_{k-1}, t_k), \\ \psi(t_k) = (P_\psi \otimes I_{2n})\psi(t_k^-), & t = t_k, \end{cases} \tag{2.36}$$

where $P_\psi = MP_e, M = I_N - \frac{1}{m}\mathbf{1}_N\mathbf{r}^\mathsf{T}$.

Proof 2.4.2 *By using the property $\mathcal{L}\mathbf{1}_N = \mathbf{0}_N$, one can show*

$$\begin{aligned} (\mathcal{L} \otimes \mathcal{H})z &= (\mathcal{L} \otimes \mathcal{H})[(I_N - \frac{1}{m}\mathbf{1}_N\mathbf{r}^\mathsf{T}) \otimes I_{2n}]z \\ &= (\mathcal{L} \otimes \mathcal{H})\psi. \end{aligned} \tag{2.37}$$

Thus, the first equation in (2.32) is rewritten as

$$\dot{z} = (I_N \otimes \mathcal{A})z - (\mathcal{L} \otimes \mathcal{H})\psi. \tag{2.38}$$

Taking the derivative of both sides of Eq. (2.35), using (2.38) and the property $\mathbf{r}^{\mathsf{T}}\mathcal{L} = 0_{1\times N}$*, this yields*

$$\begin{aligned}\dot{\psi} =& [(I_N - \frac{1}{m}\mathbf{1}_N\mathbf{r}^{\mathsf{T}}) \otimes I_{2n}](I_N \otimes \mathcal{A})z- \qquad (2.39)\\ &- [(I_N - \frac{1}{m}\mathbf{1}_N\mathbf{r}^{\mathsf{T}}) \otimes I_{2n}](\mathcal{L} \otimes \mathcal{H})\psi \qquad (2.40)\\ =& (I_N \otimes \mathcal{A})[(I_N - \frac{1}{m}\mathbf{1}_N\mathbf{r}^{\mathsf{T}}) \otimes I_{2n}]z - (\mathcal{L} \otimes \mathcal{H})\psi.\end{aligned}$$

According to Eq. (2.39), we can indicate that the first equation in (2.36) is equivalent to the first equation in (2.32).

Moreover, Eq. (2.35) allows to

$$\psi(t_k^-) = z(t_k^-) - (\frac{1}{m}\mathbf{1}_N\mathbf{r}^{\mathsf{T}} \otimes I_{2n})z(t_k^-). \qquad (2.41)$$

It is noteworthy that $P_e\mathbf{1}_N = \mathbf{1}_N$*. Then, by multiplying both sides of Eq. (2.41) by* $(P_e \otimes I_{2n})$*, one obtains*

$$z(t_k) = (P_e \otimes I_{2n})\psi(t_k^-) + (\frac{1}{m}\mathbf{1}_N\mathbf{r}^{\mathsf{T}} \otimes I_{2n})z(t_k^-). \qquad (2.42)$$

Moreover, at each reset time t_k*, one has* $\psi(t_k) = [(I_N - \frac{1}{m}\mathbf{1}_N\mathbf{r}^{\mathsf{T}}) \otimes I_{2n}]z(t_k)$*. Then, using (2.42), one has*

$$\begin{aligned}\psi(t_k) =& [(I_N - \frac{1}{m}\mathbf{1}_N\mathbf{r}^{\mathsf{T}}) \otimes I_{2n}](P_e \otimes I_{2n})\psi(t_k^-)+\\ &+ [(I_N - \frac{1}{m}\mathbf{1}_N\mathbf{r}^{\mathsf{T}}) \otimes I_{2n}](\frac{1}{m}\mathbf{1}_N\mathbf{r}^{\mathsf{T}} \otimes I_{2n})z(t_k^-)\\ &= [(I_N - \frac{1}{m}\mathbf{1}_N\mathbf{r}^{\mathsf{T}}) \otimes I_{2n}](P_e \otimes I_{2n})\psi(t_k^-)+\\ &+ (I_N\frac{1}{m}\mathbf{1}_N\mathbf{r}^{\mathsf{T}} - \frac{1}{m^2}\mathbf{1}_N\mathbf{r}^{\mathsf{T}}\mathbf{1}_N\mathbf{r}^{\mathsf{T}}) \otimes I_{2n}z(t_k^-). \qquad (2.43)\end{aligned}$$

Finally, the proof is completed by using the property $\mathbf{r}^{\mathsf{T}}\mathbf{1}_N = m$*, one shows that the second equation in (2.36) corresponds to the second equation in (2.32).* ■

Remark 2.4.1 *One remarks that consensus of system (2.32) is equivalent to the stability of (2.36). In fact, (2.35) can be rewritten by* $\psi = (M \otimes I_{2n})z$*, where*

$$M = \begin{bmatrix} 1-\frac{r_1}{m} & -\frac{r_2}{m} & \cdots & -\frac{r_{N_\tau}}{m} & \cdots & -\frac{r_N}{m} \\ -\frac{r_1}{m} & 1-\frac{r_2}{m} & \cdots & -\frac{r_{N_\tau}}{m} & \cdots & -\frac{r_N}{m} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ -\frac{r_1}{m} & -\frac{r_2}{m} & \cdots & 1-\frac{r_{N_\tau}}{m} & \cdots & -\frac{r_N}{m} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ -\frac{r_1}{m} & -\frac{r_2}{m} & \cdots & -\frac{r_{N_\tau}}{m} & \cdots & 1-\frac{r_N}{m} \end{bmatrix}.$$

According to definition of $\mathbf{r}^\mathsf{T}$*, it is not difficult to recognize that zero is a simple eigenvalue of matrix* M*. Moreover, by taking agent* i *as an example, one has*

$$\psi_i = z_i - \frac{1}{m}(r_1 z_1 + \cdots + r_{N_\tau} z_{N_\tau} + \cdots + r_N z_N).$$

It is easy to see that $\psi_i = 0, i = 1, \dots N$*, if and only if* $z_1 = \cdots = z_N$*. Therefore, instead of ensuring the condition (2.10)–(2.11) in Definition 2.2.1, we need only to prove that system (2.36) is stable at the equilibrium point.*

2.4.2 Impulsive Observer-Based Control Design

In next subsection, we deal with **Problem 2**. According to analysis above in Subsection 2.4.1, **Problem 2** is indirectly solved by considering the stability of the equivalent system (2.36) at the equilibrium point $\psi = 0$. Moreover, by using Lemma 2.2.2 and taking into account the particular form of $\mathcal{L}$ in (1.21), one has $\Theta_N \mathcal{L} + \mathcal{L}^\mathsf{T} \Theta_N > 0$, where

$$\Theta_N = \begin{bmatrix} \Theta_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \Theta_m \end{bmatrix}, \Theta_N = \Theta_N^\mathsf{T} > 0, \tag{2.44}$$

and the general algebraic connectivity of $\mathcal{L}$ is defined by

$$a(\mathcal{L}) = \min_{\tau=1,\cdots,m} a_\tau(\mathcal{L}_\tau). \tag{2.45}$$

By considering the subgraphs $\mathcal{G}_\tau$, there exists a unique a positive left eigenvector r_τ of $\mathcal{L}_\tau$ associated with the zero eigenvalue (Yu et al., 2010). Therefore, for the sake of simplicity, we will refer to the θ_τ as the r_τ.

Now, the objective of **Problem 2** is to design the matrices $K \in \mathbb{R}^{p\times n}, H, L \in \mathbb{R}^{n\times q}$, and the coupling weights $p, q > 0$ such that the consensus of system (2.32) can be achieved. The following algorithm is presented to select the corresponding parameters.

Now, let us introduce the next result of this section.

Theorem 2.4.1 *Consider that the network dynamics system (2.32) satisfies the Assumption 2.2.1. If there exists solutions to LMIs (2.46)–(2.48). Then, Problem 2 is solvable by Algorithm 1. Specifically,*

$$\begin{aligned} x_i &\to \frac{e^{At}\mu^\mathsf{T} Q x(0)}{\sum_{\tau=1}^m \phi_\tau}, \\ e_i &\to 0, i = 1, \cdots N, \ \text{as } t \to \infty. \end{aligned} \tag{2.49}$$

Proof 2.4.3 *According to Remark 2.4.1, the consensus problem of system (2.32) can be indirectly solved by considering the stability of system (2.36). Therefore, let*

Algorithm 1 Determing the control gain

1: **procedure** CALCULATE MATRICES $K \in \mathbb{R}^{p\times n}, H, L \in \mathbb{R}^{n\times q}$
2: Choose matrix $H \in \mathbb{R}^{n\times q}$ such that $A + HC$ is Hurwitz.
3: Choose c_1, c_2 and solve the following LMIs for variables P, γ_1, γ_2

$$P = P^{\mathsf{T}} > 0, \gamma_1 > 0, \gamma_2 > 0 \tag{2.46}$$

$$AP_1 + P_1A^{\mathsf{T}} - \gamma_1 BB^{\mathsf{T}} + c_1P_1 < 0 \tag{2.47}$$

$$A_H^{\mathsf{T}}P_2 + P_2A_H - \gamma_2 C^{\mathsf{T}}C + c_2P_2 < 0 \tag{2.48}$$

4: Calculate matrices $K = B^{\mathsf{T}}P_1^{-1},\ L = P_2^{-1}C^{\mathsf{T}}$
5: **procedure** CHOOSE p
6: Calculate λ_{min} which is real part of nonzero eigenvalues of $\Theta\mathcal{L} + \mathcal{L}^{\mathsf{T}}\Theta$
7: Calcuate the $\theta_{max} = max\{r_i\}, i = 1, \cdots, N$
8: Choose any coupling gain $p > \frac{\gamma_1\theta_{max}}{2a(\mathcal{L})}, q > \frac{\theta_{max}\gamma_2}{2a(\mathcal{L})}$

us consider the candidate Lyapunov function

$$\begin{aligned} V &= \begin{bmatrix}\psi_1^{\mathsf{T}} & \psi_2^{\mathsf{T}}\end{bmatrix} \begin{bmatrix}\Theta_N \otimes P_1^{-1} & 0 \\ 0 & \Theta_N \otimes P_2\end{bmatrix} \begin{bmatrix}\psi_1 \\ \psi_2\end{bmatrix} \\ &= \underbrace{\psi_1^{\mathsf{T}}(\Theta_N \otimes P_1^{-1})\psi_1}_{V_1} + \underbrace{\psi_2^{\mathsf{T}}(\Theta_N \otimes P_2)\psi_2}_{V_2} \end{aligned} \tag{2.50}$$

For $t \in (t_{k-1},\ t_k)$, one has $\dot{V} = \dot{V}_1 + \dot{V}_2$, where

$$\begin{aligned} \dot{V}_1 =& \psi_1^{\mathsf{T}}[\Theta_N \otimes (A^{\mathsf{T}}P_1^{-1} + P_1^{-1}A) - p(\Theta_N\mathcal{L} + \mathcal{L}^{\mathsf{T}}\Theta_N) \otimes P_1^{-1}BK]\psi_1 + \\ &+ p\psi_1^{\mathsf{T}}[(\Theta_N\mathcal{L} + \mathcal{L}^{\mathsf{T}}\Theta_N) \otimes P_1^{-1}BK]\psi_2. \end{aligned} \tag{2.51}$$

then according to Lemma 2.2.2, one has

$$\begin{aligned} -\psi_1^{\mathsf{T}}(\Theta_N\mathcal{L} + \mathcal{L}^{\mathsf{T}}\Theta_N)\psi_1 &\le -2a(\mathcal{L})\psi_1^{\mathsf{T}}\Theta_N\psi_1^{\mathsf{T}} \\ &\le -\frac{2a(\mathcal{L})}{\theta_{max}}\psi_1^{\mathsf{T}}\Theta_N\psi_1, \end{aligned} \tag{2.52}$$

by subtitute Eq. (2.52) into Eq. (2.51), we obtain

$$\begin{aligned} \dot{V}_1 \le& \psi_1^{\mathsf{T}}[\Theta_N \otimes (A^{\mathsf{T}}P_1^{-1} + P_1^{-1}A - 2p\frac{a(\mathcal{L})}{\theta_{max}}P_1^{-1}BK)]\psi_1 + \\ &+ p\psi_1^{\mathsf{T}}[(\Theta_N\mathcal{L} + \mathcal{L}^{\mathsf{T}}\Theta_N) \otimes P_1^{-1}BK]\psi_2. \end{aligned} \tag{2.53}$$

Then according to the LMI (2.47), we get

$$A^{\mathsf{T}}P_1^{-1} + P_1^{-1}A - \gamma_1 P_1^{-1}BB^{\mathsf{T}}P_1^{-1} < -c_1P_1^{-1},$$

and employing $K = B^\mathsf{T} P_1^{-1}$, $p > \frac{\gamma_1 \theta_{max}}{2a(\mathcal{L})}$ *in Algorithm 1, Eq. (2.53) becomes*

$$\dot{V}_1 \leq -c_1 \psi_1^\mathsf{T} (\Theta_N \otimes P_1^{-1}) \psi_1 + p \psi_1^\mathsf{T} [(\Theta_N \mathcal{L} + \mathcal{L}^\mathsf{T} \Theta_N) \otimes P_1^{-1} BK] \psi_2. \tag{2.54}$$

Moreover, there exists a constant $\bar{c}_1$*, satisfying* $0 < \bar{c}_1 \ll c_1$*, such that*

$$\begin{aligned} \dot{V}_1 \leq & -(\bar{c}_1 + c_1) \psi_1^\mathsf{T} (\Theta_N \otimes P_1^{-1}) \psi_1 + \\ & + p \psi_1^\mathsf{T} [(\Theta_N \mathcal{L} + \mathcal{L}^\mathsf{T} \Theta_N) \otimes P_1^{-1} BK] \psi_2. \end{aligned} \tag{2.55}$$

Similarly, the derivative of V_2 *also is given as*

$$\begin{aligned} \dot{V}_2 = & \psi_2^\mathsf{T} [\Theta_N \otimes (A_H^\mathsf{T} P_2 + P_2 A_H) - \\ & - q (\Theta_N \mathcal{L} + \mathcal{L}^\mathsf{T} \Theta_N) \otimes P_2 LC] \psi_2, \end{aligned} \tag{2.56}$$

by using (2.52)*, Eq. (2.56) becomes*

$$\dot{V}_2 \leq \psi_2^\mathsf{T} [\Theta_N \otimes (A_H^\mathsf{T} P_2 + P_2 A_H - q \frac{2a(\mathcal{L})}{\theta_{max}} P_2 LC)] \psi_2. \tag{2.57}$$

Then according to the LMI (2.48), one gets

$$A_H^\mathsf{T} P_2 + P_2 A_H - \gamma_2 C^\mathsf{T} C < -c_2 P_2, \tag{2.58}$$

and employing $q > \frac{\gamma_2 \theta_{max}}{2a(\mathcal{L})}$, $L = P_2^{-1} C^\mathsf{T}$ *in Algorithm 1, Eq. (2.57) becomes*

$$\dot{V}_2 \leq -c_2 \psi_2^\mathsf{T} (\Theta_N \otimes P_2) \psi_2 < 0. \tag{2.59}$$

Moreover, there exists a constant $\bar{c}_2 > 0$*, such that*

$$\dot{V}_2 \leq -(c_2 + \bar{c}_2) \psi_2^\mathsf{T} (\Theta_N \otimes P_2) \psi_2 < 0. \tag{2.60}$$

Finally, the derivative of the Lyapunov function V *is given by Eq. (2.55) and Eq. (2.60)*

$$\begin{aligned} \dot{V} \leq & - c_1 \psi_1^\mathsf{T} (\Theta_N \otimes P_1^{-1}) \psi_1 - c_2 \psi_2^\mathsf{T} (\Theta_N \otimes P_2) \psi_2 - \\ & - \bar{c}_1 \psi_1^\mathsf{T} (\Theta_N \otimes P_1^{-1}) \psi_1 - \bar{c}_2 \psi_2^\mathsf{T} (\Theta_N \otimes P_2) \psi_2 + \\ & + p \psi_1^\mathsf{T} [(\Theta_N \mathcal{L} + \mathcal{L}^\mathsf{T} \Theta_N) \otimes P_1^{-1} B B^\mathsf{T} P_1^{-1}] \psi_2. \end{aligned} \tag{2.61}$$

Eq. (2.61) *is equivalent to*

$$V \leq -c(V_1 + V_2) + \begin{bmatrix} \psi_1^\mathsf{T} & \psi_2^\mathsf{T} \end{bmatrix} \Phi \begin{bmatrix} \psi_1 \\ \psi_2 \end{bmatrix}, \forall t \in (t_{k-1}\ t_k) \tag{2.62}$$

where $c = \min\{c_1, c_2\}$*, and*

$$\Phi = \left[\begin{array}{cc} -\bar{c}_1 (\Theta_N \otimes P_1^{-1}) & * \\ p \Theta_N \mathcal{L} \otimes P_1^{-1} B B^\mathsf{T} P_1^{-1} & -\bar{c}_2 (\Theta_N \otimes P_2) \end{array} \right].$$

Since $-\bar{c}_2(\Theta_N \otimes P_2) < 0$*, one has* $\Phi < 0$ *if and only if* $\bar{c}_1$ *and* $\bar{c}_2$ *satisfy*

$$\bar{c}_1(I_N \otimes P_1^{-1}) - p^2\Phi_{21}^{\mathsf{T}}[\bar{c}_2(I_N \otimes P_2)]^{-1}\Phi_{21} > 0. \tag{2.63}$$

where $\Phi_{21} = I_N\mathcal{L} \otimes P_1^{-1}BB^{\mathsf{T}}P_1^{-1}$.

Therefore, one obtains

$$\dot{V} < -cV, \quad \forall t \in (t_{k-1}\ t_k). \tag{2.64}$$

On the other hand, at the reset time $t = t_k$*, one has*

$$\begin{aligned} V(\psi(t_k)) &= \psi^{\mathsf{T}}(t_k^-)[(P_\psi \otimes I_{2n})^{\mathsf{T}}\mathbb{P}(P_\psi \otimes I_{2n})]\psi(t_k^-) \\ &= \psi^{\mathsf{T}}(t_k^-)[(P_\psi^{\mathsf{T}}\Theta_N P_\psi - \alpha\Theta_N) \otimes P + \alpha(\Theta_N \otimes P)]\psi(t_k^-), \end{aligned} \tag{2.65}$$

where

$$\mathbb{P} = \begin{bmatrix} \Theta_N \otimes P_1^{-1} & 0 \\ 0 & \Theta_N \otimes P_2 \end{bmatrix} = \Theta_N \otimes P \in \mathbb{R}^{2Nn \times 2Nn}.$$

According to definition of P_ψ *in* (2.36)*, we alway choose* $0 < \alpha < 1$ *such that* $P_\psi^{\mathsf{T}}\Theta_N P_\psi - \alpha\Theta_N \leq 0$*. Then, we get*

$$V(t_k) - \alpha V(t_k^-) \leq 0. \tag{2.66}$$

In general, one has

$$\dot{V} \leq \alpha^k e^{-c(t-t_0^+)}V(t_0^+), \quad t \in (t_{k-1}, t_k). \tag{2.67}$$

Then, from Theorem 2.3.1 and step 2 of the Algorithm 1, one has

$$x_i = x_j \to \frac{e^{At}\mu^{\mathsf{T}}\mathcal{Q}x(0)}{\sum_{\tau=1}^m \phi_\tau}, e_i = e_j \to \frac{e^{(A+HC)t}\mu^{\mathsf{T}}\mathcal{Q}e(0)}{\sum_{\tau=1}^m \phi_\tau} = 0, i, = 1, \cdots, N. \qquad \blacksquare$$

Remark 2.4.2 *The condition (2.63) is satisfied if and only if all eigenvalues of matrix* $\bar{c}_1(\Theta_N \otimes P_1^{-1}) - p^2\Phi_{21}^{\mathsf{T}}[\bar{c}_2(\Theta_N \otimes P_2)]^{-1}\Phi_{21}$ *are greater than zero, where* P_1^{-1}, P_2, p *are the solutions of LMIs (2.46)-(2.48), and* $0 < \bar{c}_1 \ll c_1, \bar{c}_2 > 0$.

Remark 2.4.3 *One remarks form Theorem 2.4.1 that the convergence of proposed algorithm depends on the choosen parameters* c_1 *and* c_2 *as well as the coupling weights* p, q *that are directly propotional to the scalar* γ_1 *and* γ_2*. Thus, if one would like to optimise these parameters, the second step of Algorithm 1 can be rewritten as*

$$\begin{aligned} &\textit{minimize}\ \ \beta\gamma_1 + (1-\beta)\gamma_2 \\ &\textit{subject to}\ \ \gamma_1 > 0, \gamma_2 > 0 \text{ and } (2.46) - (2.48) \end{aligned}$$

2.4.3 Target Enclosing Problem

We are now searching a reset matrix $P_l \in \mathbb{R}^{m\times m}$, as defined in (1.20), and the parameters $K \in \mathbb{R}^{p\times n}$, $H, L \in \mathbb{R}^{n\times q}$ and $p, q > 0$, that allow agents in the clustered network to enclose a prescribed fixed target. Note here that the position of the target is not moving and the objective is to guarantee that the agents of the different clusters collaborate together in order to achieve and keep a formation that encloses the known target.

By the way, an impulsive observer-based formation control protocol for the clustered network, where only the leaders can communicate together at the reset time t_k, is given by

$$u_i = pK \sum_{j=1}^{N} a_{ij}[(\hat{x}_j - r_j) - (\hat{x}_i - r_i), \quad t \in (t_{k-1}, t_k), \tag{2.68}$$

where $r_i \in \mathbb{R}^n$ denotes formation of agent i. Let us denote $\delta_i = x_i - r_i \in \mathbb{R}^n$ be state formation of agent i. Then, the interaction among leaders is described by

$$\delta_{l_i}(t_k) = \sum_{j=1}^{m} (P_{l(i,j)} \otimes I_n)\delta_{l_j}(t_k^-), \quad t = t_k. \tag{2.69}$$

Next, by using (2.69), system (2.8) can be rewritten as

$$\dot{\delta}_i = A\delta_i + pBK\left[\sum_{j=1}^{N} a_{ij}(\delta_j - \delta_i) - \sum_{j=1}^{N} a_{ij}(e_j - e_i)\right] + Ar_i, \quad t \in (t_{k-1}, t_k). \tag{2.70}$$

Then, one obtains the hybrid system (2.9) by using $z_i = [\delta_i \quad e_i]^\mathsf{T} \in \mathbb{R}^{2n}$ and $Ar_i = 0$. As mentioned above in Theorem 2.4.1 and Theorem 2.3.1, the agents in the clustered network can achieve consensus, meaning

$$\lim_{t\to\infty} \|\delta_i - \delta_j\| = 0, \lim_{t\to\infty} \|x_i - \hat{x}_i\| = 0.$$

It is equivalent to

$$\lim_{t\to\infty} \|x_i - r_i - \delta^\star\| = 0, \quad \forall i = 1, \cdots, N, \tag{2.71}$$

and the global consensus value $\delta^\star$, which deduce directly from Theorem 2.3.1, is given by

$$\delta^\star = \frac{e^{At}\mu^\mathsf{T}\mathcal{Q}}{\sum_{\tau=1}^{m}\phi_\tau}[x(0) + R], \tag{2.76}$$

where $R = [r_1, \cdots, r_i, \cdots, r_N] \in \mathbb{R}^{Nn}$ is the desired state formation of N agents. If $\sum_{i=1}^{N} r_i = 0$, it can be obtained from (2.71) that

$$\lim_{t\to\infty}\left[\frac{\sum_{i=1}^{N} x_i}{N} - \delta^\star\right] = 0, \quad \forall i = 1, \cdots, N.$$

Algorithm 2 Determing the control gain

procedure CALCULATE MATRICES $K \in \mathbb{R}^{p\times n}, H, L \in \mathbb{R}^{n\times q}$ AND $P_l \in \mathbb{R}^{m\times m}$

Check the condition $Ar_i = 0$ and $\sum_{i=1}^{N} r_i = 0$.

Choose matrix $H \in \mathbb{R}^{n\times q}$ such that $A + HC$ is Hurwitz.

Choose c_1, c_2 and solve the following LMIs for variables P, γ_1, γ_2

$$(2.46) - (2.48)$$

$$P_\psi^\top + P_\psi - \Theta_N^{-1} - \Theta_N \le 0, \tag{2.72}$$

$$0 < \sum_{j=1}^{m} P_{l(i,j)} - 1 \le \epsilon, \tag{2.73}$$

$$0 < w^\top P_l - w^\top \le \epsilon, \tag{2.74}$$

where ϵ is a small enough positive constant, and

$$P_\psi = (I_N - \frac{1}{m}\mathbf{1}_N \mathbf{r}^\top)\mathcal{M}^\top \begin{bmatrix} P_l & 0 \\ 0 & I_{N-m} \end{bmatrix} \mathcal{M}. \tag{2.75}$$

Calculate matrices $K = B^\top P_1^{-1},\ L = P_2^{-1} C^\top$

procedure CHOOSE p

Calculate λ_{min} which is real part of nonzero eigenvalues of $\Theta\mathcal{L} + \mathcal{L}^\top\Theta$

Calcuate the $\theta_{max} = max\{r_i\}, i = 1, \cdots, N$

Choose any coupling gain $p > \frac{\gamma_1\theta_{max}}{2a(\mathcal{L})}, q > \frac{\theta_{max}\gamma_2}{2a(\mathcal{L})}$

It means that the target enclosing problem (Zheng, Liu, and Sun, 2015; Dong et al., 2018) is solved.

Moreover, it is worth noting that each cluster is considered fixed and known. Under this consideration, the matrices $\mathcal{Q} \in \mathbb{R}^{2mn\times 2Nn}$ and $A \in \mathbb{R}^{n\times n}$ in (2.76) are fixed and given. Therefore, a value of target $\delta^\star$ is imposed by a certain choice of w such that $w^\top \mathbf{1}_m = 1$, where $w^\top$ is left eigenvector of P_l associated with the eigenvalue 1.

In order to find a reset matrix $P_l \in \mathbb{R}^{,\times m}$, firstly notice that

$$(P_\psi^\top \Theta_N - I_N)\Theta_N^{-1}(\Theta_N P_\psi - I_N) \geq 0$$

leads to

$$P_\psi^\top \Theta_N P_\psi \geq P_\psi^\top + P_\psi - \Theta_N^{-1}. \tag{2.77}$$

Thus, one solution to

$$\begin{aligned} &V(\psi(t_k)) - V(\psi(t_k^-)) = \psi^\top(t_k^-)[(P_\psi^\top \Theta_N P_\psi - \Theta_N)\otimes P]\psi(t_k^-) \leq 0, \\ &\Leftrightarrow P_\psi^\top \Theta_N P_\psi - \Theta_N \leq 0, \end{aligned} \tag{2.78}$$

It is equivalent to Eq. (2.64), and using (2.77), one obtains

$$P_\psi^\top + P_\psi - \Theta_N^{-1} - \Theta_N \leq 0.$$

Secondly, a row stochastic matrix must be satisfied Eq. (1.20) and the constraint $w^\top P_l = w^\top$. However, to ensure the solvability of the LMIs, we modified these above conditions corresponding to LMIs (2.73) and (2.74), respectively.

Thus, the Algorithm 1 is modified to select the reset matrix $P_l \in \mathbb{R}^{m\times m}$ and corresponding parameters in **Problem 3**.

2.5 Illustrative Examples

In this section, we will in a first time motivate our approach by an example in which we show that the combination of classical consensus algorithms with discrete resets between the leaders can not always guarantee the consensus of the clustered networks. Then, we provide three simulations results to illustrate the contributions of the chapter. In the first case, a numerical example is presented to describe the characterization of the global consensus value, stated in **Problem 1**. The second case calculates the parameters $K \in \mathbb{R}^{p\times n}, H, L \in \mathbb{R}^{n\times q}$ and $p, q > 0$ mentioned in **Problem 2**, which ensures the stability of the system. Finally, the target enclosing problem of multiquadrotor unmanned aerial vehicle (UAV) is considered by using the solution in **Problem 3**. Particularly, by considering that the position of a target is known and corresponds to a global consensus value, vehicles in the clustered network are controlled to protect this target.

In the following, let us consider a network of 10 agents partitioned into 3 clusters as depicted in Fig. 2.1. The dynamics of each agent, mentioned in Yaghmaie et al.,

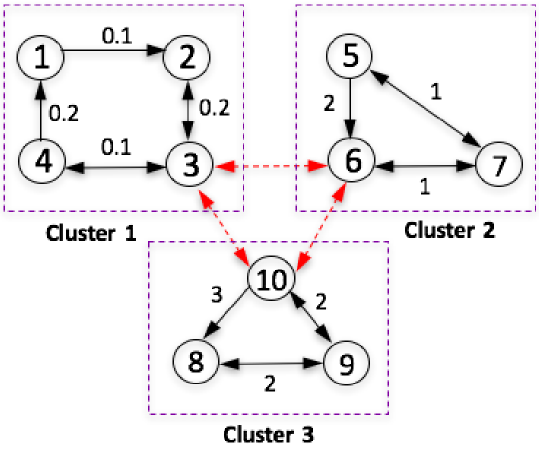

FIGURE 2.1: The communication of the network.

2017, and the interaction between the leaders are characterised by

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & \beta \\ 0 & -\lambda & -\theta \end{bmatrix}, C = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix},$$
$$B = \begin{bmatrix} 0 \\ 0 \\ \sigma \end{bmatrix}, P_l = \begin{bmatrix} 0.3 & 0.3 & 0.4 \\ 0.5 & 0.4 & 0.1 \\ 0.3 & 0.5 & 0.2 \end{bmatrix}, \tag{2.79}$$

and supposing that $x(0) = [-16 - 12 - 13 - 13 - 12 - 14 - 12 - 13 - 12 - 16 - 11 - 15\ 10\ 12\ 17\ 21\ 31\ 17\ 26\ 30\ 14\ -1 - 2 - 0.2 - 1.3 - .3 - 3.4 - 5 - 9 - 2]^{\mathsf{T}}$, and $\hat{x}(0) = 0.5 * x(0)$.

In this example, we suppose that agents 3, 6 and 10 have more communication capability than other agents. Thus, we choose them as leaders of clusters 1, 2 and 3, respectively.

In order to illustrate the advantage of the proposed approach, let us in the first time choose $\beta = 1, \lambda = 1, \theta = 2, \sigma = 1$. Moreover, in order to simplify the presentation and without loss of generality let us consider that the controller is calculated using Lemma 2.3.1 . This leads to:

$$K = [0.1\ 0.2\ 0.15], p_1 > 5, p_2 > \frac{1}{3}, p_3 > \frac{1}{6}. \tag{2.80}$$

Then, by choosing $p_1 = 6, p_2 = 1.3, p_3 = 1.2$ the states of agents in each cluster are depicted in Fig. 2.2 (left) for a value of the reset sequence $\Delta = 1$. Let us now carry out other simulations by varying the interval of the reset of the leaders. Moreover, define the combinational error as

$$E_i = \sum_{j=1}^{N} |x_i - x_j|, i = 1, \cdots, N. \tag{2.81}$$

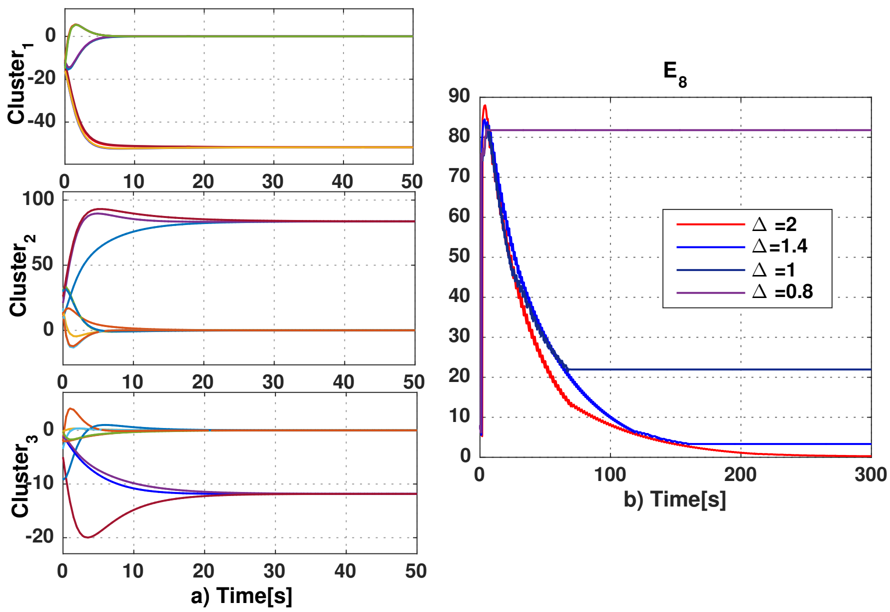

FIGURE 2.2: a) The state of agents in each cluster; b) Combinational error E_8 with different impulsive interval.

It is obvious that $E_i \to 0$ if and only if the clustered network reaches consensus, and otherwise. Fig. 2.2 (right) shows the combinational error E_8, and it indicates clearly that the consensus cannot be achieved for all the reset sequence Δ. This can be explained by the fact that the reset of the agents will have more influence than the consensus algorithm, which leads to a consensus between the leaders but not all the agents. Thus, other approaches are needed to handle this problem, particularly for small reset time.

After this motivation of our approach, the remainder of this section will illustrate our contributions.

2.5.1 The Global Consensus Value

To determine the global consensus value in Theorem 2.3.1, the eigenvectors $w^\mathsf{T}, r_1^\mathsf{T}, r_2^\mathsf{T}, r_3^\mathsf{T}$ are firstly determined as:

$$w^\mathsf{T} = [0.3772\ 0.386\ 0.2368], r_1^\mathsf{T} = [0.25\ 0.25\ 0.25\ 0.25],$$
$$r_2^\mathsf{T} = [0.33\ 0.33\ 0.33], r_3^\mathsf{T} = [0.33\ 0.33\ 0.33].$$

Then, according to (2.20) and (2.21), the matrices $\mathcal{Q}_1$, $\mathcal{Q}$ and μ^T are determined as:

$$\begin{aligned}
\mathcal{Q}_1 &= \begin{bmatrix} \frac{1}{4} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{3} \end{bmatrix} \otimes e^{At} \in \mathbb{R}^{9\times 9}, \\
\mathcal{Q} &= \begin{bmatrix} r_1^\mathsf{T} \otimes e^{At} & 0 & 0 \\ 0 & r_2^\mathsf{T} \otimes e^{At} & 0 \\ 0 & 0 & r_3^\mathsf{T} \otimes e^{At} \end{bmatrix} \in \mathbb{R}^{9\times 30}, \\
\mu^\mathsf{T} &= \begin{bmatrix} 0.0943 & 0.1287 & 0.0789 \end{bmatrix} \otimes [e^{At}]^{-1} \in \mathbb{R}^{3\times 9}.
\end{aligned}$$

Applying (2.22) of Theorem 2.3.1, the following global consensus value is calculated as:

$$\begin{aligned}
\hat{x}^\star = x^\star = \begin{bmatrix} \frac{25998}{8443} - \frac{158613t-145328}{42215} e^{-3t} \\ \frac{52871t+30819}{42215} e^{-3t} \\ -\frac{158613t+39586}{42215} e^{-3t} \end{bmatrix} = \begin{bmatrix} 3.0792 \\ 0 \\ 0 \end{bmatrix} \\
when\ t \to \infty.
\end{aligned} \tag{2.82}$$

Notice that the global consensus value x^* depends on the topology of each cluster $\mathcal{G}_\tau$, communication between leaders $\mathcal{G}_l$, and the dynamics of agent. If matrix A has eigenvalues with positive real parts, the final consensus value x^* will tend to infinity exponentially. Moreover, if matrix A is Hurwitz, then the final consensus value is zero.

2.5.2 Illustrative Example of Problem 2

In this scenario, we again show that the clustered network reaches consensus at x^* in (2.82), based on calculating parameters of consensus controller (2.6). By the way, the coupling weights q, p, observer gain $H, L \in \mathbb{R}^{n\times q}$, and control input gain $K \in \mathbb{R}^{p\times n}$ are determined according to Algorithm 1.

Firstly, choosing $H = [4\ \ 2 - 2]^\mathsf{T}$, and setting $c_1 = 1, c_2 = 1$. Then, solving the LMIs (2.46), (2.48) yields

$$\begin{aligned}
P_1 &= \begin{bmatrix} 12.2173 & -8.0215 & 2.2299 \\ -8.0215 & 8.1368 & -5.5423 \\ 2.2299 & -5.5423 & 6.8913 \end{bmatrix}, \\
P_2 &= \begin{bmatrix} 3.7886 & -3.1834 & 1.5852 \\ -3.1834 & 13.8289 & 6.8753 \\ 1.5852 & 6.8753 & 9.5114 \end{bmatrix},
\end{aligned}$$

and $\gamma_1 = 12.0702, \gamma_2 = 24.8656$. Therefore, one can select feedback matrices L and K as

$$\begin{aligned}
K &= [1.0535\ 1.9967\ 1.4033], p > 30.1755, \\
L &= [0.7098\ 0.3469\ \ -0.3690]^\mathsf{T}, q > 62.1640.
\end{aligned}$$

and choosing $p = 30.2, q = 62.2$.

Convergence of the new varible ψ in (2.35) is depicted in Fig. 2.3, as well as the Lyapunov function V is displayed in Fig. 2.7 with reset sequence $\Delta = 1$. It can show that the hybrid system (2.9) is stable, and the state of agents x_i, as well as observation error e_i in the clustered network reach consensus. This results are shown in Fig. 2.4, Fig. 2.5 and Fig. 2.6, respectively.

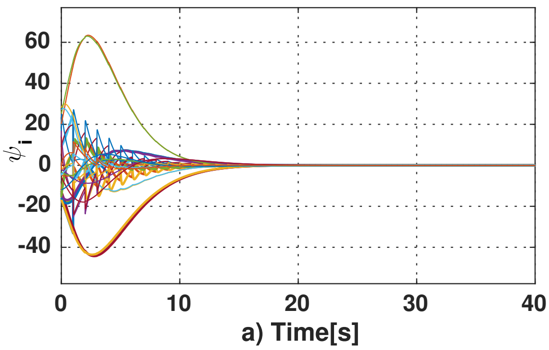

FIGURE 2.3: Convergence of the new variable ψ_i

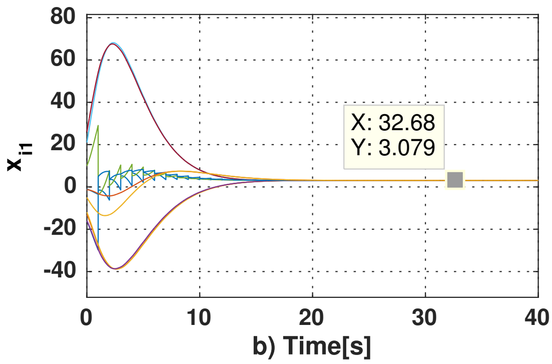

FIGURE 2.4: The first state curve of agents x_{i1} in the clustered network.

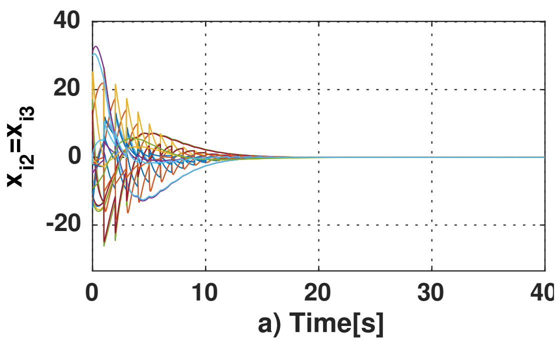

FIGURE 2.5: The state curve of agents x_{i2}, x_{i3} in the clustered network

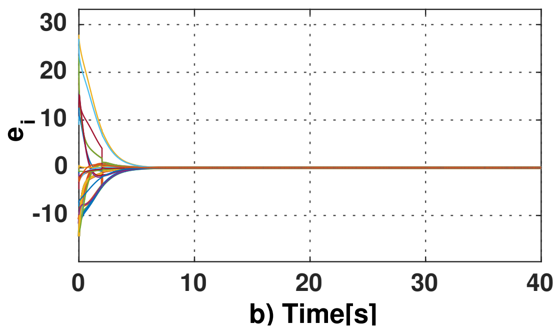

FIGURE 2.6: Convergence of observation error e_i of agents in the clustered network.

Moreover, it notices that by applying Algorithm 1, the clustered network can achieve consensus and the global consensus value $x_{i1} = 3.079, x_{i2} = x_{i3} = 0, \forall i = 1, \cdots, N$ as depicted in Fig. 2.3 and Fig. 2.4, which is the same results in (2.82) of Subsection 2.5.1.

The influence of interval Δ between two successive resets on the convergence of system (2.9) will be illustrated in Fig.2.7 (right). Particularly, the combinational error E_8 as defined above is depicted in Fig. Fig.2.7 (right). It shows that the consensus of clustered network can be reached by employing Algorithm 1.

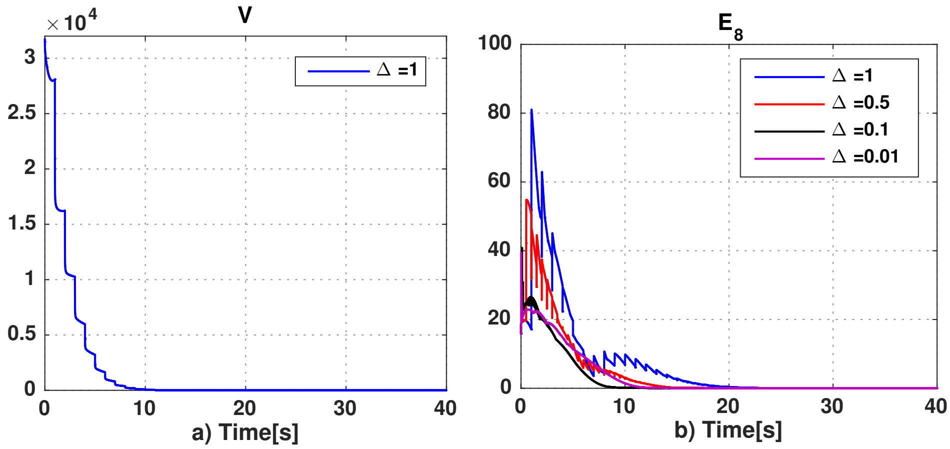

FIGURE 2.7: a) The Lyapunov function V in (2.54), b) Combinational error E_8 with different Δ.

2.5.3 Application to Target Enclosing of UAVs

According to Dong et al., 2018, the dynamics of UAV is represented by

$$\begin{aligned} \dot{p}_i &= v_i, \\ \dot{v}_i &= \alpha_p p_i + \alpha_v v_i + u_i, \end{aligned} \tag{2.83}$$

where $p_i \in \mathbb{R}^n, v_i \in \mathbb{R}^n$, and $u_i \in \mathbb{R}^n$ represent the position, velocity, and control input vectors, respectiely.

Letting $x_i = [p_{iX}\ v_{iX}\ p_{iY}\ v_{iY}]^\mathsf{T}$ and $u_i = [u_{iX}\ u_{iY}]^\mathsf{T}$, the dynamics of agent is depicted as (2.1), where

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \alpha_p & \alpha_v & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & \alpha_p & \alpha_v \end{bmatrix}, C^\mathsf{T} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}.$$

Next, the network is depicted in Fig. 2.1, and the formation specified by R is given by

$$\begin{aligned} &r_1 = [0\ 4\sqrt{3}]^\mathsf{T}, r_2 = [2\ 2\sqrt{3}]^\mathsf{T}, r_3 = [4\sqrt{3}\ 2\sqrt{3}]^\mathsf{T}, \\ &r_4 = [2\sqrt{3}\ 0]^\mathsf{T}, r_5 = [4\ \ -4\sqrt{3}]^\mathsf{T}, r_6 = [0\ \ -4\sqrt{3}]^\mathsf{T}, \\ &r_7 = [-4\ \ -4\sqrt{3}]^\mathsf{T}, r_8 = [-2\sqrt{3}\ 0]^\mathsf{T}, r_9 = [-4\sqrt{3}\ 2\sqrt{3}]^\mathsf{T}, \\ &r_{10} = [-2\ \ 2\sqrt{3}]^\mathsf{T}. \end{aligned}$$

and $R = [r_1\ r_2\ r_3\ r_4\ r_5\ r_6\ r_7\ r_8\ r_9\ r_{10}]^\mathsf{T} \otimes [1\ 0]^\mathsf{T}$. Each cluster is color coded, where the first cluster is red, second is blue and third is black.

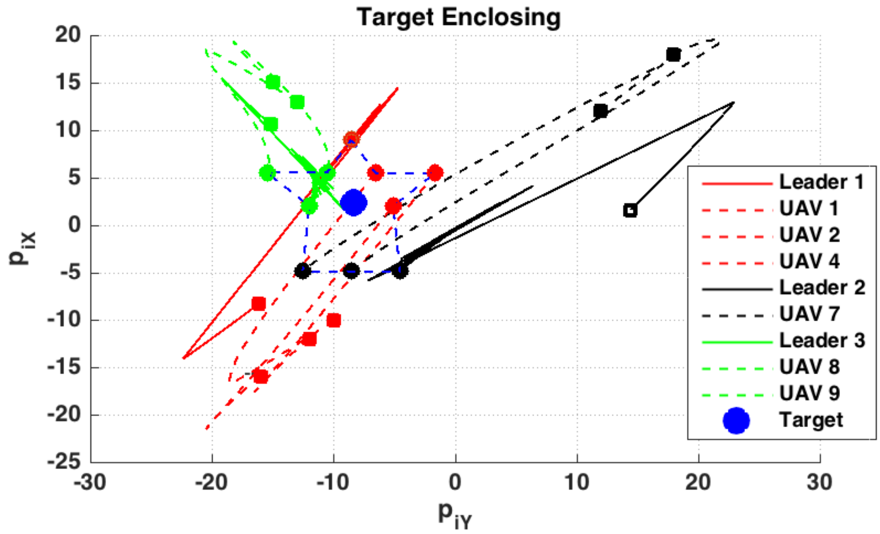

FIGURE 2.8: Formation of ten UAVs'positions encloses the target.

Moreover, initial conditions of three clusters are given as:

$$\begin{aligned}x_0^\mathsf{T} = [&-16-12-16-12-10-12-10-12-12-13\\&-12-13-16-17-16-17\ 10\ 12\ 10\ 12\ 18\ 15\ 18\ 15\ 12\\&13.5\ 12\ 13.5\ 11\ 12-11-12\ 13\ 13\ \ -13-13\ 15\ 9-15-9]\end{aligned}$$

Then, with a certain left eigenvector of P_l is $w^\mathsf{T} = [0.4\ \ 0.35\ 0.35]$, and $\alpha_p = 0, \alpha_v = -1.98$, the position, velocity of target $\delta^\star = [p_{iX}, v_{iX}, p_{iY}, v_{iY}]^\mathsf{T}$ are calculated by

$$\begin{bmatrix} p_{iX} \\ v_{iX} \end{bmatrix} = \begin{bmatrix} 2.4134 - 0.6108e^{-5.94t} \\ 1.2095e^{-5.94t} \end{bmatrix},$$
$$\begin{bmatrix} p_{iY} \\ v_{iY} \end{bmatrix} = \begin{bmatrix} -8.2939 + 2.6379e^{-5.94t} \\ -5.223e^{-5.94t} \end{bmatrix}.$$

After that, the reset matrix $P_l \in \mathbb{R}^{m\times m}, K \in \mathbb{R}^{p\times n}$, $H, L \in \mathbb{R}^{n\times q}$, and the coupling weights p, q can be designed by Algorithm 2. Firstly, checking the condition $Ar_i = 0, \sum_{i=1}^{N} r_i = 0$, and choosing $H = [1.0200 - 0.0196]^\mathsf{T}$, and setting $c_1 = 1, c_2 = 1$. Then, solving the LMIs (2.46)-(2.48), (2.72)-(2.74), yields

$$P_l = \begin{bmatrix} 0.2232 & 0.3884 & 0.3884 \\ 0.4439 & 0.1678 & 0.3883 \\ 0.4439 & 0.3883 & 0.1678 \end{bmatrix}, L = I_2 \otimes \begin{bmatrix} 0.2766 \\ -0.0233 \end{bmatrix},$$
$$K = I_2 \otimes \begin{bmatrix} 1.0520 & 1.0857 \end{bmatrix}, p = 32.3934, q = 51.0816.$$

The simulation result is shown in Fig. 2.8. We can observer that agents in the clustered network reach formation, while enclosing the target $p_Y = -8.29, p_X = 2.41$.

2.6 Chapter summary

In this chapter, a distributed observer-based control problem for consensus on complex dynamical networks is studied. Networks of linear agents are partitioned in several clusters and disconnected one of each other. On the other hand, we have analyzed its stability, and we have given the consensus value in general case. In order to reach a global consensus value, the interconnection network between the leaders is designed. Methods of selecting the feedback gain matrices as well as the coupling strengths for both controllers and observers are given with a designed algorithm. Two academic examples illustrate the main results.

Chapter 3

Robust Formation Control Under State Constraints

Contents

Résumé du Chapitre 3

Ce chapitre étudie le problème du contrôle de la formation dans les systèmes de réseaux composés d'agents linéaires qui sont soumis à des contraintes d'état (correspondant à la première des quatre questions de la section 1.2). Un protocole de formation robuste est proposé pour traiter la communication hybride décrite ci-dessus et les contraintes d'état des agents. Il est ensuite montré que le concept hybride de contrôle de formation robuste pour les réseaux multi-agents en cluster peut être indirectement résolu par le concept de stabilisation robuste d'un système équivalent obtenu par la théorie des matrices et la théorie des graphes algébriques. Ensuite, un contrôleur robuste est conçu pour le système initial en termes d'inégalités matricielles linéaires. Enfin, un problème de formation pour les véhicules aériens sans pilote est simulé pour illustrer l'efficacité de la méthode de conception de contrôle de formation hybride proposée.

This chapter studies the formation control problem in clustered network systems of linear agents that are subjected to state constraints (corresponding to the first the fourth questions in Section 1.2). A robust formation control protocol is proposed to deal with the hybrid communication described above and the constraints on states of agents. It is next shown that the hybrid robust formation control design for clustered multi-agent networks can be indirectly solved through the robust stabilization design of an equivalent system obtained by matrix theory and algebraic graph theory. Then, a robust controller is designed for the initial clustered network system in terms of linear matrix inequalities. Finally, a formation design for unmanned aerial vehicles is carried out and simulated to illustrate the effectiveness of the proposed hybrid formation control design method.

3.1 Related Work and Contributions

Currently, consensus-based control approaches are widely employed in solving the formation problem of MASs. The continuous-time first-order MASs was studied in Ren, 2008, where it was proved that they could achieve formation if the directed interaction topology contains a directed spanning tree. Extensions of consensus-based formation algorithms to second-order with linear or nonlinear dynamics under the fixed or the time-varying interaction graphs were also investigated in Wei Ren and Atkins, 2007. In Oh and Ahn, 2014, a formation control law based on local measurement of relative-positions was proposed for first-order MASs. In another research (Dong et al., 2015), based on consensus approaches, necessary and sufficient conditions were derived to deal with the time-varying formations for second-order UAV swarm systems. Another direction is to study the formation of second-order MASs with time delays (Liu and Tian, 2009). Accordingly, the sufficient conditions were proposed for MASs to achieve desired stationary and moving formations. Another direction considers the formation tracking control problems (Han et al., 2017a), which uses neighboring relative state, and position information for second-order MASs with time-varying delays. Moreover, the formation tracking problems were also investigated in Yang et al., 2018, where the agents' local coordinate systems are applied such that the centroid of the controlled formation tracks a given trajectory.

Other significant and realistic issues have been encountered such as the constraints on the agent's inputs, states, or relative states because of the physical limitations of agents. This includes, for example, the formation of vehicles with limited speeds and limited working space, smart buildings energy control with constraints on temperature and humidity in specific ranges and so on. Recently, some studies have considered the cooperative control of MASs under the constraints on agent's inputs, states, or relative states (Nedi, Ozdaglar, and Parrilo, 2010; Nguyen, Narikiyo, and Kawanishi, 2017; Nguyen, Narikiyo, and Kawanishi, 2018; Zhou and Wang, 2018; Mo and Lin, 2018; Wei, Xiang, and Li, 2011). In Nedi, Ozdaglar, and Parrilo, 2010, a constrained consensus algorithm and distributed optimization problems were proposed, where agents state constraints are investigated and they are required to lie in individual closed convex sets. In another work, Wei, Xiang, and Li, 2011 studied a consensus problem of simple integrator MASs under input constraints. Following this research line, a distributed consensus of second-order MASs with nonconvex input constraints was addressed in Mo and Lin, 2018. It is shown that the input constrained consensus achieved if the graph has a directed spanning tree. Another direction to deal with input and state constraints, discarded consensus algorithms are employed by Zhou and Wang, 2018. Moreover, in order to achieve the global consensus in the presence of agents' inputs, states, or relative state constraints (Nguyen, Narikiyo, and Kawanishi, 2017; Nguyen, Narikiyo, and Kawanishi, 2018), the MASs is reformulated in form of a network of Lure systems.

On the other hand, it has been noticed that interaction among agents in the aforementioned networks is either continuous-time or discrete-time. However, due to either energy constraints occurring in long-time interactions or communication constraints, agents can only impulsively exchange information with their neighbors or be subjected to abrupt changes at specific instants (Guan, Wu, and Feng, 2012; Guan et al., 2012a; Hu et al., 2013; Liu, Zhang, and Xie, 2017). This leads to a hybrid interaction that combines both continuous and discrete interactions among agents. In Guan, Wu, and Feng, 2012, a sufficient result has been derived for the impulsive consensus of first-order MASs, where the graphs of continuous-time and impulsive-time topologies contain a spanning tree. Following this research line, there are several types of research (Guan et al., 2012a; Hu et al., 2013), which have dealt with the consensus problem of the second-order MASs under an impulsive control strategy. Moreover, inspired by the results in Jadbabaie and Morse, 2003; Ren and Beard, 2005, the necessary condition of consensus on graph connections among agents may require. This is investigated in Liu, Zhang, and Xie, 2017, where the first-order MASs with hybrid delay consensus protocols are described in the form of impulsive systems. In other directions on consensus problem under hybrid communication, the network is partitioned into several groups or clusters, where continuous intra-cluster and discrete inter-cluster communications (Bragagnolo et al., 2014; Rejeb, Morarescu, and Daafouz, 2015; Bragagnolo et al., 2016; Morarescu et al., 2016). The works in Bragagnolo et al., 2016 proposed a quasi-periodically reset strategy and provided some LMI conditions to guarantee the globally uniformly exponential consensus where intra-cluster communication structures are represented by directed and strongly connected graphs. The researches in Morarescu et al., 2016; Rejeb, Morarescu, and Daafouz, 2015 investigated the sufficient conditions for event-triggered consensus. However, in most of the above studies on clustered MASs, constraints on the states of agents are not considered, and the dynamics of agents correspond to a simple integrator.

Motivated by both theoretical and practical issues mentioned above, this chapter investigates the state formation control problem under state constraints in clustered MASs where agents have generic linear dynamics. Our approach covers broader systems and scenarios than those in the existing studies (Bragagnolo et al., 2016; Morarescu et al., 2016). Next, a robust formation protocol, which deals with the continuous-time communication inside clusters and discrete-time information exchange between clusters, is introduced. Compared with the previous results (Pham, Messai, and Manamanni, 2019c; Pham et al., 2019), the protocol is more practical and complicated. It is then shown that the considered robust formation control problem can be indirectly solved by studying the robust stability of an equivalent system by matrix theory and algebraic graph theory. In comparison with the one in Bragagnolo et al., 2016; Pham, Messai, and Manamanni, 2019c; Pham et al., 2019, our approach shows the important role of communication between leaders at some specific discrete instants, represented by the stochastic matrix. Accordingly, a sufficient condition will be derived in terms of LMIs for the robust distributed formation of clustered networks of generic linear agents under state constraints and hybrid communications

3.2 Problem Formulation

3.2.1 Robust Formation Control

We consider a group of N linear identical agents that interact in m clusters. The dynamics of each agent i is described by

$$\dot{x}_i = Ax_i + Bu_i, \tag{3.1}$$

where $x_i = [x_{i,1}, \cdots, x_{i,n}]^\intercal \in \mathbb{R}^n$ is the state, $u_i \in \mathbb{R}^p$ is the control input; $A \in \mathbb{R}^{n\times n}, B \in \mathbb{R}^{n\times p}$.

According to (1.19), we concede that each agent has a vector state denoted by $x_{l_\tau} = [x_{l_\tau,1}, \cdots, x_{l_\tau,n}]^\intercal \in \mathbb{R}^n$ for the leaders l_τ and $x_{f_h} = [x_{f_{h,1}}, \cdots, x_{f_{h,n}}]^\intercal \in \mathbb{R}^n$ for the followers $f_h, \forall \tau \neq h = 1, \cdots, N$. Moreover, the formation of MASs refers to a geometric shape that satisfies some prescribed inter-agent geometric constraints achieved and preserved by a group of agents. The formation may represent a variety of physical quantities of an agent, such as position, velocity and altitude (Dong et al., 2014). The desired state formation is denoted by $R = [r_1, r_2, \cdots, r_N] \in \mathbb{R}^{Nn}$, where $r_i \in \mathbb{R}^n, \forall i \in \mathcal{V}$ is a formation variable of agent i. Therefore, the desired state formation R is meaning that agents in a network need to achieve a reference coordinate frame corresponding to their positions and velocities.

However, in practice, the measurement part may have bounded nonlinearities or saturation constraints due to sensor limitations as well as physical limitations. Moreover, different agents inside each cluster may have different interactive capabilities. Therefore, the upper and lower bounds of saturation constraints may also be different. This leads to the heterogeneous asymmetric saturation levels.

Because only the leaders of these clusters can communicate together at some reset times t_k, a formation protocol for a clustered network with state constraints based on local information is given by

$$u_i = K\sum_{j=1}^{N} a_{(ij)} \left(\phi_j(z_j(t)) - \phi_i(z_i(t))\right) + Qr_i, t \in (t_k, t_{k+1}) \tag{3.2}$$

where $z_i(t) = x_i(t) - r_i$ denote the state formation variables of agent i, and $K, Q \in$

$\mathbb{R}^{p\times n}$ denote the protocol gain matrices, respectively. The general continous functions $\phi_i(z_i) : \mathbb{R}^n \to \mathbb{R}^n, i = 1, \cdots, N$ satisfying the following sector-bounded conditions

$$(\phi_i(z_i(t)) - \Upsilon_1 z_i(t)) \circ (\phi_i(z_i(t)) - \Upsilon_2 z_i(t)) \leq 0 \; \forall z_i(t) \in \mathbb{R}^n, \tag{3.3}$$

where

$$\Upsilon_1 = diag\{\upsilon_{k,1}\}_{k=1,\cdots,n} \in \mathbb{R}^n,$$
$$\Upsilon_2 = diag\{\upsilon_{k,2}\}_{k=1,\cdots,n} \in \mathbb{R}^n$$

are matrices composed of known sector slopes $\upsilon_{k,1} < \upsilon_{k,2}$. An example of ϕ_i is the saturation function $s_i(\bullet)$ defined by

$$s_i(z_i(t)) \triangleq \begin{cases} \underline{s}_i : z_i(t) \leq \underline{s}_i, \\ z_i(t) : \underline{s}_i < z_i(t) < \overline{s}_i, \\ \overline{s}_i : z_i(t) \geq \overline{s}_i \end{cases} \tag{3.4}$$

where $\underline{s}_i \leq 0 \leq \overline{s}_i, \underline{s}_i < \overline{s}_i$ are known constants which are called saturation levels.

The exchanged information between leader's states at the reset time t_k can be described by

$$\Delta z_{l_i}(t_k) = \sum_{j=1}^{m} a_{l(ij)}(z_{l_j}(t_k) - z_{l_i}(t_k)), \;\; t = t_k, \tag{3.5}$$

where $\Delta z_{l_i}(t_k) = z_{l_i}(t_k^+) - z_{l_i}(t_k^-)$, where $z_{l_i}(t_k^+)$ and $z_{l_i}(t_k^-)$ represent the right and left limit of z_{l_i} at t_k, respectively. Without loss of generality, we assume that $z_{l_i}(t_k^-) = z_{l_i}(t_k)$. Furthermore, $a_{l(i,j)}$ is the (i,j)th entry of the weighted adjacent matrix $\mathcal{A}_l = [a_{l(ij)}]$, and m denotes the set of leaders in graph $\mathcal{G}_l$. The Laplacian matrix $\mathcal{L}_l = [L_{l(ij)}] \in \mathbb{R}^{m\times m}$ is defined as $L_{l(ii)} = \sum_{j\neq i=1}^{m} a_{l(ij)}; L_{l(ij)} = -a_{l(ij)}$.

Then the collective dynamics of system (3.1) under the consensus protocol (3.2) and the interaction between leaders (3.5) can be rewritten as

$$\begin{cases} \dot{z}_i(t) = Az_i(t) - BK\sum_{j=1}^{N} L_{(ij)} w_j(t) + (A + BQ)r_i, \;\; t \in (t_k, t_{k+1}), \\ \Delta z_{l_i}(t_k) = -\sum_{j=1}^{m} L_{l(ij)} z_{l_j}(t_k), \;\; t = t_k, \\ w_i(t) = \phi_i(z_i(t)). \end{cases} \tag{3.6}$$

It can be seen that the evolution of the dynamic system described by (3.6) is influenced by the events that happen at the reset moments t_k. The objective of the above collaboration is to achieve a specific global objective namely state formation defined as follows.

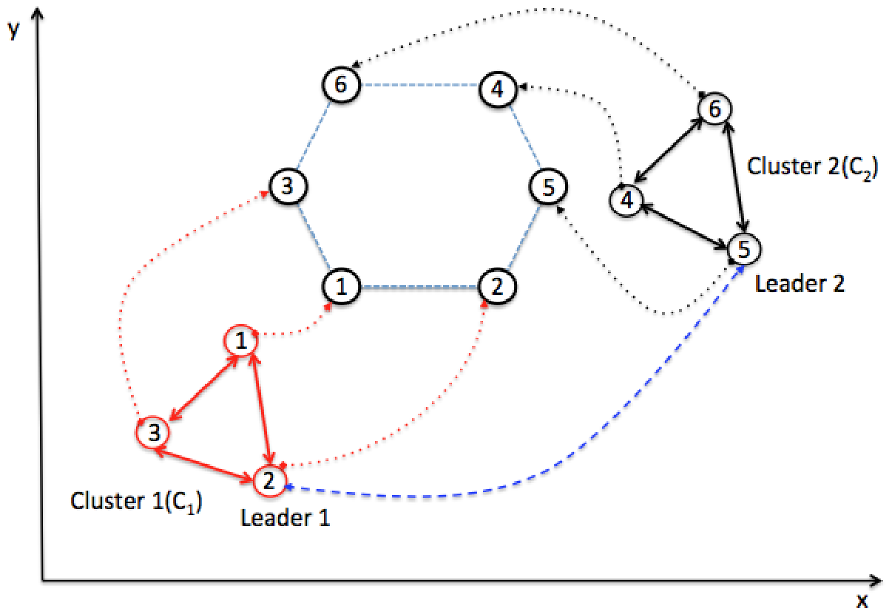

FIGURE 3.1: The scheme of formation problem

Definition 3.2.1 *The multi-agent system* (3.1) *is said to be achieve state formation anticipated by* R *if there is vector* $h(t) \in \mathbb{R}^n$ *such that*

$$\lim_{t\to\infty}(x_i(t) - r_i - h(t)) = 0, \tag{3.7}$$

for any given bounded initial condition.

For the sake of clarity, an example, as depicted in Fig. 3.1 is given to illustrate this kind of problem. There are 6 agents divided into two clusters, red and black, that have to realize a hexagon formation. Each agent receives only the state information of its neighbors in the same subnetwork (*the same color*). If there is no communication between subnetworks (*for example, agents 2 and 5 or agents 3 and 6*), then the 6-agent network cannot achieve the desired formation. Therefore, in order to ensure the task of 6 agents, at some discrete-time instants, a communication between one red and one black agent (*called leader 1 and leader 2, respectively*) is activated. Next, the following assumptions are utilized.

3.2.2 Useful Assumptions and Lemmas

Assumption 3.2.1 *The graphs* $\mathcal{G}_\tau$ *and* $\mathcal{G}_l$ *are undirected and connected.*

Assumption 3.2.2 $a_{l(ij)} > 0, \sum_{j\neq i=1}^{m} a_{l(ij)} < 1.$

Assumption 3.2.3 *The matrix pair* (A, B) *is stabilizable.*

Remark 3.2.1 *Assumption 3.2.1 is needed to guarantee that the Laplacian matrix* $\mathcal{L}_\tau$ *of* $\mathcal{G}_\tau, \forall \tau \in \{1, \cdots, m\}$ *satisfies the following proprieties* $\mathcal{L}_\tau \mathbf{1}_{N_\tau} = 0, r_\tau^{\mathsf{T}} \mathcal{L}_\tau = 0$

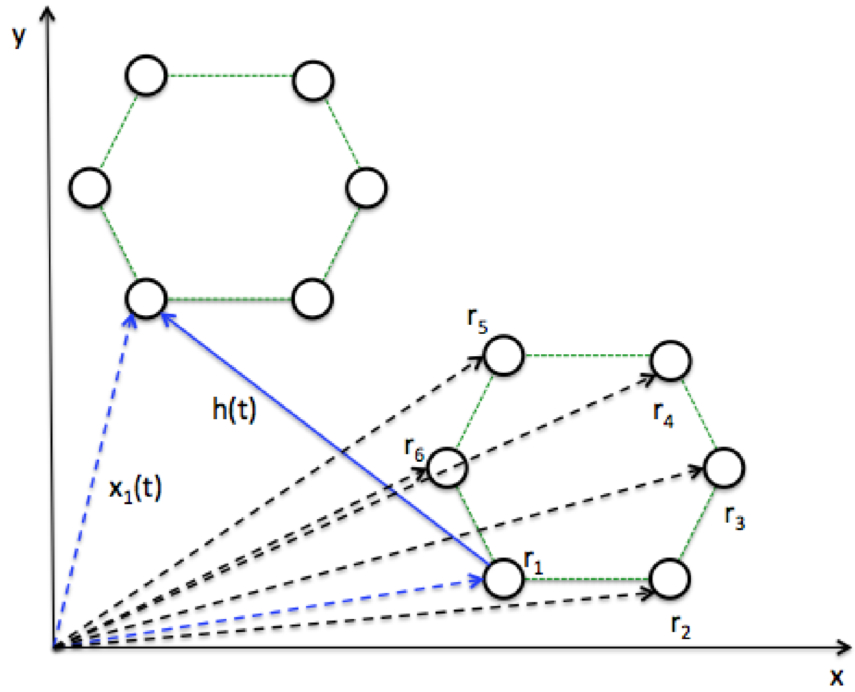

FIGURE 3.2: State formation of six agents moving in the XY plane

and $r_v^\top \mathbf{1}_{N_\tau} = 1$, *where* $\mathbf{1}_{N_\tau}$, *and* $r_\tau^\top = \frac{1}{N_\tau}\mathbf{1}_{N_\tau}$ *are the right and left eigenvectors of* $\mathcal{L}_\tau$ *associated with zero eigenvalue, respectively. Assumption 3.2.2 ensures the matrix* $P_l = I - \mathcal{L}_l$ *is a stochastic matrix with positive diagonal elements. Moreover, Assumption 3.2.3 is for the existence of a controller.*

Lemma 3.2.1 *(see Ren and Beard, 2005) Let* Γ *be a compact set consisting of* $n \times n$ *SIA matrices with the property that for any nonnegative integer* k *and any* $B_1, \cdots, B_k \in \Gamma$, *the matrix product* $\prod_{i=1}^k B_i$ *is SIA. Then, for given any infinite sequence* $B_1, B_2, \cdots$, *there exits a column vector* $c^\top$ *such that* $\lim_{k\to\infty}\prod_{i=1}^k B_i = \mathbf{1}c^\top$.

Lemma 3.2.2 *(see Jadbabaie and Morse, 2003) If* $B = [b_{ij}]_{n\times n}$ *is a stochastic matrix with positive diagonal elements, and the graph associated with* B *has a spanning tree, then* B *is SIA.*

Lemma 3.2.3 *(see Jadbabaie and Morse, 2003) Let* $m \geq 2$ *be a positive integer and let* $D_1, \cdots, D_m$ *be nonnegative* $n \times n$ *matrices with positive diagonal elements, then*

$$D_1 D_2 \cdots D_m \geq \gamma(D_1 + D_2 + \cdots + D_m),$$

where $\gamma > 0$ *can be specified from* $D_i, i = 1, \cdots, m$

Hereafter, the time index t is omitted in expressions of x_i, h and other variables just for conciseness of mathematical representations. Now, the considered problem in the current research is stated as follows.

3.3 Robust Distributed Formation Design

In this section, we propose a solution for the above hybrid robust formation control problem with fixed topologies $\mathcal{G}_\tau$ and $\mathcal{G}_l$ satisfying Assumption 3.2.1. The proposed design is composed of two steps. First, by employing results from matrix theory and algebraic graph theory, we show that the considered problem can be indirectly solved by the robust stability of an equivalent system. Then, the robust stability design of the equivalent system is derived in terms of LMIs.

3.3.1 Prerequisites

From the Geršgorin theorem (Olfati-Saber and Murray, 2004), we know that $\lambda_m \leq 2d_{max}(\mathcal{G}_l)$, where λ_m is the largest eigenvalue of the Lalapcian of the graph $\mathcal{G}_l$, and $d_{max}(\mathcal{G}_l)$ is the maximum out-degree of the nodes of $\mathcal{G}_l$, where

$$\deg_{out}(v_{li}) = \sum_{i=1}^{m} a_{l(ij)}$$

Therefore, we can get $0 < \lambda_m < 2$. Let us introduce

$$\begin{cases} P_{l(ij)} = -L_{l(ij)} = a_{l(ij)} > 0 \\ P_{l(ii)} = 1 - L_{l(ii)} = 1 - \sum\limits_{j \neq i = 1}^{m} a_{l(ij)} > 0, \end{cases} \tag{3.8}$$

then $\sum_{j=1}^{m} P_{l(ij)} = 1$, and $P_l = I - \mathcal{L}_l$ is a row stochastic matrix with positive diagonal elements, and according to Assumptions 3.2.1 and 3.2.2, it has an eigenvalue $\lambda_1 = 1$ with algebraic multiplicity equal to one, and all the other eigenvalues satisfy $0 < |\lambda_i| < 1, i = 2, \cdots, m$.

Moreover, as mentioned above the network is subdivided into m undirected sub-networks. Then, $\mathcal{L} \in \mathbb{R}^{N \times N}$ stands for the Laplacian matrix associated with the graph $\mathcal{G}$ represented in (1.21).

Some algebraic properties of $\mathcal{L}$ are presented in the following *Proposition*.

Proposition 3.3.1 *Let us consider a network of m clusters satisfying Assumption 3.2.1, with the Laplacian $\mathcal{L} \in \mathbb{R}^{N \times N}$, then $rank(\mathcal{L}) = N - m$ and $\mathcal{L}$ has m eigenvalues at zero and all the other $N - m$ eigenvalues of the Laplacian $\mathcal{L} \in \mathbb{R}^{N \times N}$ are positive.*

Next, it follows (3.8), the system (3.6) can be written in a form of the overall network dynamics

$$\begin{cases} \dot{z} = (I_N \otimes A)z - (\mathcal{L} \otimes BK)w + (I_N \otimes \mathbb{A})R, \; t \in (t_k, t_{k+1}) \\ z_l(t_k^+) = (P_l \otimes I_n) z_l(t_k), \quad t = t_k \\ w = \Phi(z), \end{cases} \tag{3.9}$$

where $P_l = I - \mathcal{L}_l, \mathbb{A} = A + BQ, \Phi(z) = [\phi_1^\top(z_1), \cdots, \phi_N^\top(z_N)]^\top$ and

$$\begin{aligned} z &= [z_{l_1}^\top, z_{f_2}^\top, \cdots, z_{f_{m_1}}^\top, \cdots, z_{l_m}^\top, \cdots, z_{f_{mm}}^\top = z_{f_N}^\top]^\top \in \mathbb{R}^{Nn}, \\ z_l &= [z_{l_1}^\top, \cdots, z_{l_m}^\top]^\top \in \mathbb{R}^{mn}. \end{aligned}$$

containing respectively the states of agents and leader's states.

Let us introduce the extended stochastic matrix P_e as follows

$$P_e = \mathcal{M}^\top \begin{bmatrix} P_l & 0 \\ 0 & I_{N-m} \end{bmatrix} \mathcal{M} \in \mathbb{R}^{N\times N}, \tag{3.10}$$

where $\mathcal{M}$ is a permutation matrix.

Then, the second equation in (3.9) can be expressed by

$$z(t_k^+) = (P_e \otimes I_n) z(t_k), \quad t = t_k.$$

In the following, let $U \in \mathbb{R}^{N\times N}$ be an orthogonal matrix, and employing Proposition 3.3.1, we obtain

$$\begin{aligned} U^{-1}\mathcal{L}U &= \begin{bmatrix} 0_m & 0 \\ 0 & \Gamma \end{bmatrix} = \Lambda \in \mathbb{R}^{(N)\times(N)}, \\ \Gamma &= diag\{\gamma_{m+1}, \cdots, \gamma_N\} \in \mathbb{R}^{(N-m)\times(N-m)}. \end{aligned} \tag{3.11}$$

Finally, let us also introduce the new variable

$$\psi = (U^{-1} \otimes I_n) z. \tag{3.12}$$

It follows the variable ψ in (3.12), we now formulate our statement as the following

$$\begin{cases} \dot{\psi} = (I_N \otimes A)\psi - (\Lambda U^{-1} \otimes BK) w + \mathcal{H}R, t \in (t_k, t_{k+1}) \\ \psi(t_k^+) = (P_\psi \otimes I_n)\psi(t_k), \quad t = t_k, \\ z = (U \otimes I_n)\psi, \\ w = \Phi(z), \end{cases} \tag{3.13}$$

where $P_\psi = U^{-1} P_e U$ and $\mathcal{H} = (U^{-1} \otimes \mathbb{A})$.

In the next part of this chapter, thanks to results from matrix theory and algebraic graph theory, we show that the robust formation control problem of MASs in clustered network (3.6) is indirectly solved by considering the robust stability of the system (3.13)

3.3.2 Formation Analysis in Clustered Network

In order to simplify the presentation of the next results let us partition the matrices U^{-1}, U into

$$U^{-1} = [U_3^T \;\; U_4^T]^T, U = [U_1 \;\; U_2], \tag{3.14}$$

where $U_3^T \in \mathbb{R}^{m\times N}, U_4^T \in \mathbb{R}^{(N-m)\times N}$ and $U_1 \in \mathbb{R}^{N\times m}$, $U_2 \in \mathbb{R}^{N\times(N-m)}$.

$$U_1 = \begin{bmatrix} \mathbf{1}_{N_1} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \mathbf{1}_{N_m} \end{bmatrix}, U_3^T = \begin{bmatrix} r_1^T & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & r_m^T \end{bmatrix}, \tag{3.15}$$

which satisfies $\mathcal{L}U_1 = 0_{N\times n}, U_3^T\mathcal{L} = 0_{m\times N}$. This allows to decompose (3.12) into two parts :

$$\psi_1 = (U_3^T \otimes I_n)z, \quad \psi_2 = (U_4^T \otimes I_n)z, \tag{3.16}$$

where $\psi_1 \in \mathbb{R}^{mn}$ and $\psi_2 \in \mathbb{R}^{Nn-mn}$. Now we are able to introduce the first main results of this chapter.

Theorem 3.3.1 *Consider the overall network dynamics system* (3.21)*, satisfying Assumptions 3.2.1–3.2.3, the hybrid robust formation control problem is solved if the following formation feasibility condition holds*

$$(A + BQ)(r_i - r_j) = 0, \quad \forall i, j = 1, \cdots, N. \tag{3.17}$$

and

$$\lim_{t\to\infty} \psi_2 \to 0 \tag{3.18}$$

for any given bounded initial conditions.

Proof 3.3.1 *If the condition (3.17) holds, then one has that*

$$[\mathcal{L} \otimes (A + BQ)]R = 0. \tag{3.19}$$

Pre-multiplying both sides of (3.19) *with* $(U^{-1} \otimes I_n)$ *yields*

$$[\Lambda U^{-1} \otimes (A + BQ)]R = 0. \tag{3.20}$$

Then pre-multiplying both the sides of (3.20) *with*

$$\begin{bmatrix} 0_m & 0 \\ 0 & \Gamma^{-1} \end{bmatrix} \otimes I_n$$

gives us $[U^{-1} \otimes (A + BQ)]R = 0$, *which is equivalent to* $\mathcal{H}R = 0$. *Therefore, the system* (3.13) *leads to the following system*

$$\begin{aligned} \dot{\psi} &= (I_N \otimes A)\psi - (\Lambda U^{-1} \otimes BK)w, t \in (t_k, t_{k+1}) \\ \psi(t_k^+) &= (P_\psi \otimes I_n)\psi(t_k), \quad t = t_k, \\ z &= (U \otimes I_n)\psi, \\ w &= \Phi(z). \end{aligned} \tag{3.21}$$

The proof is given in two steps. First, we show that ψ_1 *reaches a constant value, which depends on the dynamics of agents, the graph of each cluster, the interaction between leaders, and the initial conditions. Second, based on the analysis of the first step, the hybrid robust formation control problem, satisfying Definition 3.2.1, is solved.*

First, by employing (3.21), (3.14) *and* (3.16), *the dynamics of* ψ_1 *can be represented as:*

$$\begin{cases} \dot{\psi}_1 = (I_m \otimes A)\psi_1, \\ \psi_1(t_k^+) = (U_3^\top P_e U_1 \otimes I_n)\psi_1(t_k) + (U_3^\top P_e U_2 \otimes I_n)\psi_2(t_k) \end{cases} \tag{3.22}$$

Then, the solution of (3.22) *with initial condition* $\psi_1(t_0) = \psi_{10}$ *can be obtained by*

$$\psi_1 = e^{(I_m \otimes A)(t-t_k)}\psi_1(t_k^+) = (I_m \otimes e^{A(t-t_k)})\psi_1(t_k^+), \tag{3.23}$$

and if $\lim_{t\to\infty} \psi_2 \to 0$, *then* $\psi_1(t_k^+)$ *can be expressed as:*

$$\begin{aligned} \psi_1(t_k^+) &= \lim_{k\to\infty} \prod_{i=1}^{k} (U_3^\top P_e U_1 \otimes I_n) e^{(I_m \otimes A)(t_i - t_{i-1})}\psi_1(t_0) \\ &= \lim_{k\to\infty} (U_3^\top P_e U_1)^k \otimes e^{A(t_k - t_0)}\psi_1(t_0). \end{aligned} \tag{3.24}$$

In the following, by using results from Lemma 3.2.1 and Lemma 3.2.2, we prove that matrix $U_3^\top P_e U_1 \in \mathbb{R}^{m\times m}$ *is a row stochastic matrix with positive diagonal elements.*

According to (3.8), *the extended stochastic matrix* P_e *in* (3.10) *can be re-expressed as follows*

$$P_e = \begin{bmatrix} P_{l_{11}} & 0 & \cdots & P_{l_{1m}} & 0 \\ 0 & I_{N_1-1} & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ P_{l_{m1}} & 0 & \cdots & P_{l_{mm}} & 0 \\ 0 & 0 & \cdots & 0 & I_{N_m-1} \end{bmatrix} \in \mathbb{R}^{N\times N}, \tag{3.25}$$

where

$$P_l = \begin{bmatrix} P_{l_{11}} & \cdots & P_{l_{1m}} \\ \vdots & \vdots & \vdots \\ P_{l_{m1}} & \cdots & P_{l_{mm}} \end{bmatrix} = \begin{bmatrix} P_{l_1} \\ \vdots \\ P_{l_m} \end{bmatrix} \in \mathbb{R}^{m\times m}, \tag{3.26}$$

and the matrix $U_1 \in \mathbb{R}^{N\times m}$ *and* $U_3^{\mathsf{T}} \in \mathbb{R}^{m\times N}$ *are given in* (3.14)*. Now, the matrix* P_eU_1 *is calculated as*

$$P_eU_1 = \begin{bmatrix} P_{l_{11}} \cdots & P_{l_{1m}} \\ \mathbf{1}_{N_1-1} & 0_{(N_1-1)\times(N_1-1)} \\ \vdots & \vdots \\ P_{l_{m1}} \cdots & P_{l_{mm}} \\ \mathbf{1}_{N_m-1} & 0_{(N_m-1)\times(N_m-1)} \end{bmatrix} = \begin{bmatrix} E_1 \\ \vdots \\ E_m \end{bmatrix}, \tag{3.27}$$

where $\forall \tau \in \{1, \cdots, m\}$*, and*

$$E_\tau = \begin{bmatrix} P_{l_{\tau 1}} \cdots & P_{l_{\tau m}} \\ \mathbf{1}_{N_\tau-1} & 0_{(N_\tau-1)\times(N_\tau-1)} \end{bmatrix} \in \mathbb{R}^{N_\tau\times m}. \tag{3.28}$$

Then, the matrix $U_3^{\mathsf{T}}P_eU_1$ *is determined as follows*

$$U_3^{\mathsf{T}}P_eU_1 = \begin{bmatrix} r_1^{\mathsf{T}} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & r_m^{\mathsf{T}} \end{bmatrix} \begin{bmatrix} E_1 \\ \vdots \\ E_m \end{bmatrix} = \begin{bmatrix} r_1^{\mathsf{T}}E_1 \\ \vdots \\ r_m^{\mathsf{T}}E_m \end{bmatrix}, \tag{3.29}$$

where $r_\tau^{\mathsf{T}} = [r_{\tau_1}, \cdots, r_{\tau_{N_\tau}}] \in \mathbb{R}^{1\times N_\tau}$*,* $\forall \tau \in \{1, \cdots, m\}$*, and*

$$r_\tau^{\mathsf{T}}E_\tau = [r_{\tau_1}P_{l_{\tau 1}} + r_{\tau_2} + \cdots + r_{\tau_{N_\tau}}, r_{\tau_1}P_{l_{\tau 2}}, \cdots, r_{\tau_1}P_{l_{\tau m}}].$$

The sum of the row matrix $r_\tau^{\mathsf{T}}E_\tau$ *is calculated by*

$$\sum_{k=1}^{N_\tau} r_\tau^{\mathsf{T}}E_\tau = r_{\tau_1}P_{l_{\tau 1}} + \sum_{k=2}^{N_\tau} r_{\tau_k} + r_{\tau_1}\sum_{k=2}^{N_\tau} P_{l_{\tau k}}. \tag{3.30}$$

According to Assumptions 3.2.1, 3.2.2 and (3.8)*,* $P_{l_{\tau 1}} = 1 - \sum_{k=2}^{m} P_{l_{\tau k}}$*, then*

$$\sum_{k=1}^{N_\tau} r_\tau^{\mathsf{T}}E_\tau = \sum_{k=1}^{N_\tau} r_{\tau_k} = 1, \tag{3.31}$$

and $P_{l_{\tau k}} > 0$*, then*

$$\begin{aligned} r_{\tau_1}P_{l_{\tau 1}} + r_{\tau_2} + \cdots + r_{\tau_{N_\tau}} > 0, \\ r_{\tau_1}P_{l_{\tau 2}} > 0, \cdots, \\ r_{\tau_1}P_{l_{\tau m}} > 0. \end{aligned} \tag{3.32}$$

Subsequently, by employing (3.31) *and* (3.32)*, and according to Definition A.0.1, we see that the matrix* $U_3^{\mathsf{T}}P_eU_1 \in \mathbb{R}^{m\times m}$ *is a row stochastic matrix with positive diagonal elements.*

Furthermore, by employing (3.8)*, and* $\forall i, \tau \in \{1, \cdots, m\}$ *Eq.* (3.32) *becomes*

$$\begin{aligned} 1 + r_{\tau_1}(P_{l_{\tau 1}} - 1) = 1 - r_{\tau_1} \sum_{i \neq j = 1}^{m} a_{l(ij)}, \\ r_{\tau_1} P_{l_{\tau 2}} = r_{\tau_1} a_{l_{i2}}, \\ \cdots \\ r_{\tau_1} P_{l_{\tau m}} = r_{\tau_1} a_{l_{im}}. \end{aligned} \tag{3.33}$$

then the $(i,j)^{th}$ *entry of* $U_3^\mathsf{T} P_e U_1$ *is* $r_{\tau_1} a_{l(ij)}$*, which implies that the graph* $\mathcal{G}_l$ *and the graph of* $U_3^\mathsf{T} P_e U_1$ *have the same edge set. Thus, the graph of the matrix* $U_3^\mathsf{T} P_e U_1$ *is undirected and connected. It means that the graphs of* $U_3^\mathsf{T} P_e U_1$ *has at least one spanning tree.*

Based on the above analysis, we showed that the matrix $U_3^\mathsf{T} P_e U_1$ *is a row stochastic matrix with positive diagonal elements and its graph has at least one spanning tree. Then, according to Lemma 3.2.1, the matrix* $U_3^\mathsf{T} P_e U_1$ *is SIA.*

Therefore, from Lemma 3.2.2, there exits a column vector c^T *such that*

$$\lim_{k \to \infty} (U_3^\mathsf{T} P_e U_1)^k = \mathbf{1}_m c^\mathsf{T}. \tag{3.34}$$

Then, by substituing (3.34) *and* (3.24) *into* (3.22)*, one has*

$$\psi_1 = \mathbf{1}_m c^\mathsf{T} \otimes e^{A(t-t_0)} \psi_{10}. \tag{3.35}$$

Second, by introducing the variables

$$\mu_1 = (U \otimes I_n) \begin{bmatrix} \psi_1 \\ 0 \end{bmatrix}, \mu_2 = (U \otimes I_n) \begin{bmatrix} 0 \\ \psi_2 \end{bmatrix}, \tag{3.36}$$

one has $z = \mu_1 + \mu_2$*. Then, according to* (3.14) *the variable* μ_1 *is written such as*

$$\mu_1 = [U_1 \otimes I_n \; U_2 \otimes I_n] \begin{bmatrix} \psi_1 \\ 0 \end{bmatrix} = (U_1 \otimes I_n)\psi_1. \tag{3.37}$$

It follows that $\mu_2 = z - \mu_1$*, where* $\mu_2 = (U_2 \otimes I_n)\psi_2$*. And, by using* (3.15) *and* (3.35)*, we obtain*

$$\begin{aligned} \mu_1 &= U_1 \mathbf{1}_m c^\mathsf{T} \otimes e^{A(t-t_0)} \psi_{10}, \\ &= \mathbf{1}_N c^\mathsf{T} \otimes e^{A(t-t_0)} \psi_{10}. \end{aligned} \tag{3.38}$$

If it follows $\lim_{t\to\infty}\psi_2 \to 0$ *and recalling that* $(U \otimes I_n)$ *is nonsingular, then follows from* (3.36) *that* $\lim_{t\to\infty}\mu_2 \to 0$. *Finally, it follows from* $z = \mu_1 + \mu_2$ *that*

$$\begin{aligned}
\lim_{t\to\infty}\mu_2 &= \lim_{t\to\infty}(z - \mu_1) \\
&= \lim_{t\to\infty}(x - R - \boldsymbol{1}_N \underbrace{c^T \otimes e^{A(t-t_0)}\psi_{10}}_{h(t)}) \to 0, \\
&= \lim_{t\to\infty}(x_i - r_i - \underbrace{c^T \otimes e^{A(t-t_0)}\psi_{10}}_{h(t)}) \to 0,
\end{aligned} \tag{3.39}$$

which implies that the system (3.6) *can achieve state formation anticipated by* R, *meaning the hybrid robust formation control problem was solved. This completes the proof.*

Remark 3.3.1 $h(t)$ *in* (3.39) *generally can be used to guide a group of agents to achieve an anticipated formation specified by* R *as shown in Fig.3.2 and it is considered as the formation position function. Moreover, the formation function* $h(t)$ *in considered clustered network is described as* (3.39)*, which depend on agents' initial states and formation vector, agent's dynamics, communication networks' cluster and leaders. In addition, according from Definition 3.2.1, one see that when* $r_i = r_j, \forall i,j \in \mathcal{V}$, *then* $\lim_{t\to\infty}(x_i(t) - x_j(t)) = 0$ *or* $\lim_{t\to\infty}(x_i(t) - h(t)) = 0$. *In this case, the formation centre function* $h(t)$ *is equivalent to the global final consensus as shown in Pham, Messai, and Manamanni, 2019c.*

Remark 3.3.2 *According to Thorem 3.3.1, one sees that to ensure the state formation* R, *not only the communication topology is required to be connected and the Laplacian matrix is a symmetric matrix, but also the formation vector should satisfy the constraint* (3.17)*. Therefore, Theorem 3.3.1 establishes the relationship between the formability and the communication topology, the agents' dynamics and the formation vector.*

3.3.3 Robust Stabilization Controller Design

Based on the above analysis in Subsection 3.3.2, the objective now is to design the matrix $K \in \mathbb{R}^{p\times n}$, such that the system (3.21) is robustly stable, i.e., $\lim_{t\to\infty}\psi_2 \to 0$. The design of such robust stabilization controller gain K is given in the following theorem.

Theorem 3.3.2 *Consider the system* (3.21) *satisfying Assumptions 3.2.1–3.2.3 and condition* (3.17)*. It is robustly stable if there exist positive-definite and diagonal* *matrices* $P, \Pi, Z \in \mathbb{R}^{n\times n}$ *and* $X \in \mathbb{R}^{p\times n}$ *such that the following LMIs are feasible,*

$$\begin{bmatrix} \Xi_1 & \gamma_2 BX + \frac{(\Upsilon_1-\Upsilon_2)}{2}Z \\ * & -Z \end{bmatrix} \le 0, \tag{3.40}$$

$$\begin{bmatrix} \Xi_2 & -\gamma_N BX + \frac{(\Upsilon_1-\Upsilon_2)}{2}Z \\ * & -Z \end{bmatrix} \le 0, \tag{3.41}$$

$$\begin{bmatrix} Z & P \\ P & \Pi^{-1} \end{bmatrix} \ge 0, \tag{3.42}$$

where

$$\Xi_1 = sym(AP + \gamma_2 BX\Upsilon_2) + \alpha P,\ \gamma_2 = min\{\Gamma\},$$
$$\Xi_2 = sym(AP + \gamma_N BX\Upsilon_2) + \alpha P,\ \gamma_N = max\{\Gamma\}.$$

Furthermore, $K = XP^{-1}$.

Proof 3.3.2 *Firstly, we define*

$$V = V(\psi) = \psi^{\mathsf{T}}(\Theta \otimes P^{-1})\psi, \tag{3.43}$$

where

$$P = P^{\mathsf{T}} > 0, \Theta = \begin{bmatrix} 0_m & 0 \\ 0 & I_{N-m} \end{bmatrix}.$$

Obviously, V *is positive semi-definite. If we can prove that between impulses* t_k *and* t_{k+1}*, the function* V *is decreasing*

$$\dot{V} < 0, \quad \forall t \in (t_k, t_{k+1}), \tag{3.44}$$

where

$$\dot{V} = \psi^{\mathsf{T}}[\Theta \otimes (A^{\mathsf{T}}P^{-1} + P^{-1}A)]\psi - \psi^{\mathsf{T}}[(\Theta\Lambda U^{-1} + U\Lambda\Theta) \otimes P^{-1}BK]w. \tag{3.45}$$

and at reset time t_k

$$V(t_k) \ge V(t_k^+). \tag{3.46}$$

Then according to the Lasalle's invariance principle, $\psi(t)$ *globally exponentially converges to the largest invariance set contained in* $\{\psi \in \mathbb{R}^{Nn} | \dot{V}(\psi) = 0\}$ *for any initial conditions. It can be seen from (3.45) and definition of matrix* Θ *in (3.43) that* $\dot{V}(\psi) = 0$ *if and only if* $\lim_{t\to\infty} \psi_2 \to 0$*, where* $\psi_2 = [\psi_{m+1}, \cdots, \psi_N]^{\mathsf{T}} \in \mathbb{R}^{Nn-mn}$.

In the following, the condition (3.44) is equivalent to $\dot{V} + \alpha V \le 0$*, where* $\alpha > 0$*. Thus, Eq. (3.45) becomes*

$$\begin{aligned} \dot{V} + \alpha V = & \psi^{\mathsf{T}}[\Theta \otimes (A^{\mathsf{T}}P^{-1} + P^{-1}A + \alpha P^{-1})]\psi - \\ & - \psi^{\mathsf{T}}[(\Theta\Lambda U^{-1} + U\Lambda\Theta) \otimes P^{-1}BK]w \le 0. \end{aligned} \tag{3.47}$$

Then, using the S-procedure and the sector-bounded conditions (3.3), there exits a diagonal matrix $\Pi \in \mathbb{R}^{n\times n}, \Pi \geq 0$, *such that*

$$
\begin{aligned}
&\dot{V} + \alpha V - \sum_{j=1}^{N} (w_j - \Upsilon_1 z_j)^\mathsf{T} \Pi (w_j - \Upsilon_2 z_j) \leq 0. \\
\Leftrightarrow &\dot{V} + \alpha V - w^\mathsf{T}(I_N \otimes \Pi) w - z^\mathsf{T}(I_N \otimes \Pi\Upsilon_1\Upsilon_2) z + \\
&+ w^\mathsf{T}[I_N \otimes \Pi(\Upsilon_1 + \Upsilon_2)] z \leq 0.
\end{aligned}
\tag{3.48}
$$

Then, with $z = (U \otimes I_n)\psi, U^\mathsf{T} = U^{-1}$ (3.48) *becomes*

$$
\begin{aligned}
&\dot{V} + \alpha V - w^\mathsf{T}(I_N \otimes \Pi) w - \psi^\mathsf{T}(I_N \otimes \Pi\Upsilon_1\Upsilon_2)\psi + w^\mathsf{T}[U \otimes \Pi(\Upsilon_1 + \Upsilon_2)]\psi \leq 0. \\
\Leftrightarrow &\begin{bmatrix} \psi \\ w \end{bmatrix}^\mathsf{T} \begin{bmatrix} \Psi_1 & \Psi_2 \\ \Psi_3 & \Psi_4 \end{bmatrix} \begin{bmatrix} \psi \\ w \end{bmatrix} \leq 0. \\
\Leftrightarrow &\begin{bmatrix} \Psi_1 & \Psi_2 \\ \Psi_3 & \Psi_4 \end{bmatrix} \leq 0.
\end{aligned}
\tag{3.49}
$$

where

$$
\begin{aligned}
\Psi_1 &= \Theta \otimes (A^\mathsf{T} P^{-1} + P^{-1} A + \alpha P^{-1}) - I_N \otimes \Pi\Upsilon_1\Upsilon_2, \\
\Psi_2 &= -\Theta\Lambda U^{-1} \otimes P^{-1} B K + \frac{1}{2} U^\mathsf{T} \otimes \Pi(\Upsilon_1 + \Upsilon_2), \\
\Psi_3 &= -U\Lambda\Theta \otimes K^\mathsf{T} B^\mathsf{T} P^{-1} + \frac{1}{2} U \otimes \Pi(\Upsilon_1 + \Upsilon_2), \\
\Psi_4 &= -I_N \otimes \Pi.
\end{aligned}
$$

Subsequently, taking the Schur complement to (3.49) *results in* $\Psi_1 - \Psi_2\Psi_4^{-1}\Psi_3 \leq 0$, *in which*

$$
\begin{aligned}
&-\Psi_2\Psi_4^{-1}\Psi_3 = \Theta\Lambda^2\Theta \otimes P^{-1} B K \Pi^{-1} K^\mathsf{T} B^\mathsf{T} P^{-1} + \\
&+ \frac{1}{4} I_N \otimes \Pi(\Upsilon_1 + \Upsilon_2)^2 + \frac{1}{2} \Lambda\Theta \otimes (\Upsilon_1 + \Upsilon_2) K^\mathsf{T} B^\mathsf{T} P^{-1} + \\
&+ \frac{1}{2} \Theta\Lambda \otimes P^{-1} B K (\Upsilon_1 + \Upsilon_2).
\end{aligned}
$$

Next, by considering $\Psi_1 - \Psi_2\Psi_4^{-1}\Psi_3 \leq 0$, *and since* Λ, *and* Γ *in* (3.11) *are diagonal, one obtains*

$$
\begin{aligned}
&(A^\mathsf{T} P^{-1} + P^{-1} A + \alpha P^{-1}) - \Pi\Upsilon_1\Upsilon_2 + \\
&+ \gamma_k^2 P^{-1} B K \Pi^{-1} K^\mathsf{T} B^\mathsf{T} P^{-1} + \frac{1}{4} \Pi(\Upsilon_1 + \Upsilon_2)^2 + \\
&+ \frac{1}{2} \gamma_k (\Upsilon_1 + \Upsilon_2) K^\mathsf{T} B^\mathsf{T} P^{-1} + \frac{1}{2} \gamma_k P^{-1} B K (\Upsilon_1 + \Upsilon_2) \leq 0,
\end{aligned}
\tag{3.50}
$$

where $\gamma_k, k = m+1, \cdots, N$. *After, multiplying both sides* (3.50) *with* P, *we get*

$$\begin{aligned}
&PA^\mathsf{T} + AP + \alpha P + \frac{1}{4}P\Pi(\Upsilon_1 - \Upsilon_2)^2 P + \\
&+ \gamma_k^2 BK\Pi^{-1}K^\mathsf{T}B^\mathsf{T} + \frac{1}{2}\gamma_k P(\Upsilon_1 + \Upsilon_2)K^\mathsf{T}B^\mathsf{T} + \\
&+ \frac{1}{2}\gamma_k BK(\Upsilon_1 + \Upsilon_2)P \leq 0.
\end{aligned} \tag{3.51}$$

Since $P, \Upsilon_1, \Upsilon_2$ *are diagonal matrices,* $P\Upsilon_1 = \Upsilon_1 P, P\Upsilon_2 = \Upsilon_2 P$, *and* (3.51) *is equivalent to*

$$\begin{aligned}
&PA^\mathsf{T} + AP + \alpha P + \gamma_k \Upsilon_2 PK^\mathsf{T}B^\mathsf{T} + \gamma_k BKP\Upsilon_2 + \\
&+ \frac{1}{4}P\Pi(\Upsilon_1 - \Upsilon_2)^2 P + \gamma_k^2 BK\Pi^{-1}K^\mathsf{T}B^\mathsf{T} + \\
&+ \frac{1}{2}\gamma_k P(\Upsilon_1 - \Upsilon_2)K^\mathsf{T}B^\mathsf{T} + \frac{1}{2}\gamma_k BK(\Upsilon_1 - \Upsilon_2)P \leq 0.
\end{aligned} \tag{3.52}$$

It leads to

$$\begin{aligned}
&sym(AP + \gamma_k BKP\Upsilon_2) + \alpha P + \\
&+ [\gamma_k BKP + \frac{1}{2}(\Upsilon_1 - \Upsilon_2)\Pi P^2] \times \\
&\times (\Pi P^2)^{-1}[\gamma_k BKP + \frac{1}{2}(\Upsilon_1 - \Upsilon_2)\Pi P^2]^\mathsf{T} \leq 0.
\end{aligned} \tag{3.53}$$

Taking $Z \succeq \Pi P^2, Z \succ 0$ *and* $K = XP^{-1}$. *Then, using the Schur complement again with* (3.53) *leads to*

$$\begin{bmatrix} \Xi & \gamma_k BX + \frac{(\Upsilon_1 - \Upsilon_2)}{2}Z \\ * & -Z \end{bmatrix} \leq 0, \tag{3.54}$$

where $\Xi = sym(AP + \gamma_k BX\Upsilon_2) + \alpha P$ *and* $\gamma_k, k = m+1, \cdots, N$ *are eigenvalues of Laplacian matrix* $\mathcal{L}$.

Since $\gamma_2 = min\{\Gamma\}, \gamma_N = max\{\Gamma\}$ *and* $\gamma_2 \leq \gamma_{m+1} \leq \cdots \leq \lambda_N$, *we can represent* $\gamma_p, p = m+2, \cdots, N-1$ *as convex combination of* γ_2 *and* γ_N. *Thus, we derive* (3.40) *and* (3.41). *The LMI* (3.42) *is obtained straigthforward from* $Z \succeq \Pi P^2, Z \succ 0$.

On the other hand, at the reset time $t = t_k$ *one has*

$$\begin{aligned}
&V(\psi(t_k^+)) - V(\psi(t_k)) \\
&= \psi(t_k)^\mathsf{T}[(P_\psi \otimes I_n)^\mathsf{T}(\Theta \otimes P^{-1}(P_\psi \otimes I_n) - (\Theta \otimes P^{-1})]\psi(t_k).
\end{aligned}$$

Then, to guarantee the second condition (3.46), *one needs*

$$(P_\psi \otimes I_n)^\mathsf{T}(\Theta \otimes P^{-1})(P_\psi \otimes I_n) - (\Theta \otimes P^{-1}) \leq 0. \tag{3.55}$$

by multiplying both sides (3.55) *by* $(I_N \otimes P) > 0$*, we obtain*

$$(P_\psi \otimes I_n)^\top (\Theta \otimes P)(P_\psi \otimes I_n) - (\Theta \otimes P) \leq 0. \tag{3.56}$$

by employing $P_\psi = U^{-1} P_e U, U^{-1} = U^\top$ *and using* (3.8)*, and* (3.10)*, it is easy to verify that* $P_e^\top \Theta P_e - \Theta \leq 0$*. Thus, the condition* (3.56) *is always true.* ■

Remark 3.3.3 *In case of homogeneous constraints, the upper and lower sectors and bounds for state constraints of all agents are the same, then* Υ_1 *and* Υ_2 *are multiple of identity matrices, i.e.,* $\Upsilon_1 = \upsilon_1 I_n$ *and* $\Upsilon_2 = \upsilon_2 I_n$*. Then, the variable* $P \in \mathbb{R}^{n\times n}$ *in Theorem 3.3.2 is not required to be diagonal. Thus, the associated LMI problem is less conservative and its feasibility would be improved.*

Remark 3.3.4 *According to the LMIs* (3.40)*–*(3.42)*, one sees that the dimension of variables* $P \in \mathbb{R}^{n\times n}, X \in \mathbb{R}^{p\times n}$ *are just equal to that of the matrix* $A \in \mathbb{R}^{n\times n}$ *of each agent. Thus, the complexity of those LMI problems is low. If* γ_2, γ_N *are computed by a given Laplacian matrix* $\mathcal{L}$ *with respect to graph* $\mathcal{G}$*, then we can solve LMIs* (3.40)*–*(3.42) *in fully distributed fashion i.e., each agent can compute the gain matrix* K *by itself and implement the formation protocol* (3.2) *using only local information (its information and its neighbors' information).*

3.4 Application to Formation of UAVs

In this section, we consider a group of N UAV's motion in $d-$dimensional Euclidean space, which is modeled as the second order dynamics in Lafferriere et al., 2005; Wang and Xin, 2013, where the state variable consists of the configuration states (position-p_i^x, p_i^y) and their derivatives (velocity-v_{x_i}, v_{y_i}), the control input $u_i \in \mathbb{R}^p$ denotes the acceleration commands. Finally, the system matrices are given such as

$$A = I_d \otimes \begin{bmatrix} 0 & 1 \\ a_{21}^1 & a_{22}^1 \end{bmatrix}, B = I_d \otimes \begin{bmatrix} 0 \\ 1 \end{bmatrix}. \tag{3.57}$$

In order to illustrate that the proposed approaches are implemented in the complex network, we consider the following network which has ten UAVs, and the network is divided into three clusters in Fig. 3.3

The dynamics of the agents and the Laplacian matrix of leader network $\mathcal{G}_l$ are given by

$$A = I_2 \otimes \begin{bmatrix} 0 & 1 \\ 0 & -1 \end{bmatrix}, B = I_2 \otimes \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$

$$\mathcal{L}_l = \begin{bmatrix} 0.9 & -0.4 & -0.5 \\ -0.4 & 0.7 & -0.3 \\ -0.5 & -0.3 & 0.8 \end{bmatrix} \Rightarrow P_l = \begin{bmatrix} 0.1 & 0.4 & 0.5 \\ 0.4 & 0.3 & 0.3 \\ 0.5 & 0.3 & 0.2 \end{bmatrix}.$$

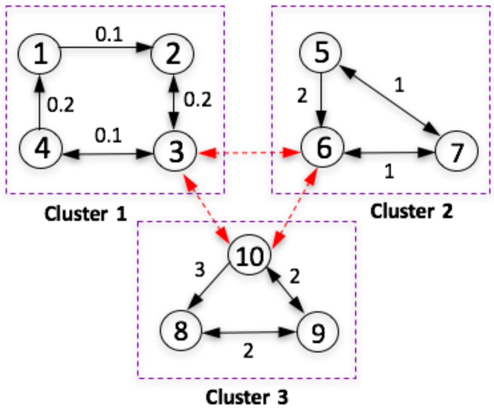

FIGURE 3.3: The communication of the network

In this example, we suppose that agents 3, 6 and 10 have more communication capability than other agents. Thus, we choose them as leaders of clusters 1, 2 and 3, respectively.

The formation considered this time is pentacle. Then, the formation specified by $R(m)$ are given by

$$
\begin{aligned}
&r_1 = [0\ 4\sqrt{3}], r_2 = [2\ 2\sqrt{3}], r_3 = [4\sqrt{3}\ 2\sqrt{3}], r_4 = [2\sqrt{3}\ 0],\\
&r_5 = [4\ \ -4\sqrt{3}], r_6 = [0\ \ -2\sqrt{3}], r_7 = [-4\ \ -4\sqrt{3}],\\
&r_8 = [-2\sqrt{3}\ 0], r_9 = [-4\sqrt{3}\ 2\sqrt{3}], r_{10} = [-2\ \ 2\sqrt{3}].
\end{aligned}
$$

and $R = [r_1\ r_2\ r_3\ r_4\ r_5\ r_6\ r_7\ r_8\ r_9\ r_{10}]^\intercal \otimes [1\ 0]^\intercal$. It is clear from R that

$$
\left[\begin{bmatrix} r_1 \\ 0 \end{bmatrix}, \dots, \begin{bmatrix} r_{10} \\ 0 \end{bmatrix}\right]^\intercal = \left[\begin{bmatrix} r_1^x & r_1^y \\ 0 & 0 \end{bmatrix}, \dots, \begin{bmatrix} r_{10}^x & r_{10}^y \\ 0 & 0 \end{bmatrix}\right]^\intercal.
$$

It means that two scenarios related to UAV's positions and velocities are taken into account: ten UAVs will be controlled to reach a regular pentacle formation in the 2D plane corresponding to their positions, and all of ten UAV's velocites will be achieve a common value such as

$$
\begin{aligned}
&\lim_{t\to\infty} \|(p_j - p_i) - (r_j - r_i)\| = 0,\\
&\lim_{t\to\infty} \|(v_j - v_i)\| = 0.
\end{aligned}
$$

Each cluster is color coded, where the first cluster is red, second is blue and third is black.

3.4.1 Heterogeneous Constraints

Due to a limited range of sensor, and the limited velocity of each UAV, the states of connected agents are bounded i.e., the state constraints are the saturation function

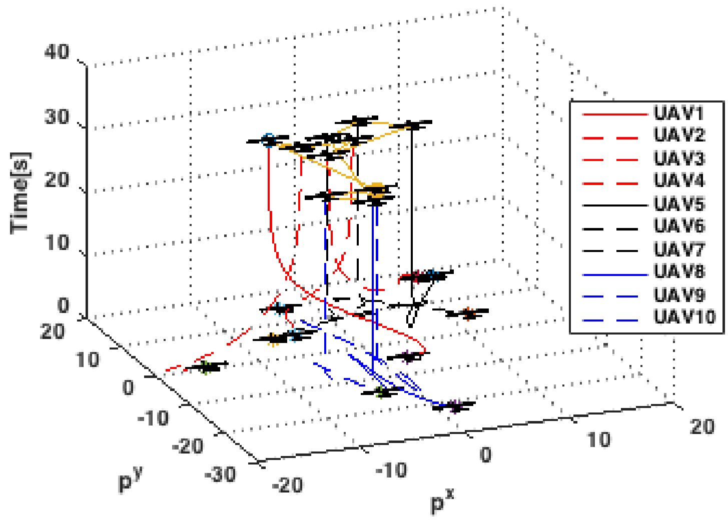

FIGURE 3.4: Pentacle formation of ten UAVs'positions without communication network of leaders.

(3.4) with heterogeneous constraints such as Table 3.1, and the initial conditions of three clusters are randomized.

TABLE 3.1: The sate constraints of ten agents

1st state	$\underline{s}_j(m)$	$\overline{s}_j$	2nd state	$\underline{s}_j$	$\overline{s}_j(m/s)$
p_1^x, p_1^y	-5	15	v_{x1}, v_{y1}	-2	0
p_2^x, p_2^y	-10	0	v_{x2}, v_{y2}	-1	3
p_3^x, p_3^y	-19	15	v_{x3}, v_{y3}	-1.3	3.1
p_4^x, p_4^y	-12	5	v_{x4}, v_{y4}	-1.8	2
p_5^x, p_5^y	-17	12	v_{x5}, v_{y5}	0	3
p_6^x, p_6^y	-20	10	v_{x6}, v_{y6}	-4	5
p_7^x, p_7^y	-18	7	v_{x7}, v_{y7}	-3.7	1.2
p_8^x, p_8^y	-19	11	v_{x8}, v_{y8}	-4	3
p_9^x, p_9^y	-22	15	v_{x9}, v_{y9}	-1.5	1.3
p_{10}^x, p_{10}^y	-10	15	v_{x10}, v_{y10}	-3	3

In $t \in (t_k,\ t_{k+1})$, the control protocol now is

$$u_i = K \sum_{j=1}^{N} a_{(ij)} \begin{bmatrix} \phi_j(z_j^{p_x}) - \phi_i(z_i^{p_x}) & \phi_j(z_j^{p_y}) - \phi_i(z_i^{p_y}) \\ \phi_j(v_j^x) - \phi_i(v_i^x) & \phi_j(v_j^y) - \phi_i(v_i^y) \end{bmatrix} + Q \begin{bmatrix} r_i^x & r_i^y \\ 0 & 0 \end{bmatrix}, \tag{3.58}$$

where $z_i^{p_x} = p_i^x - r_i^x; z_i^{p_y} = p_i^y - r_i^y$. Then, employing (3.58), the dynamics of agent i can be reformulated as

$$\begin{bmatrix} \dot{p}_i^x & \dot{p}_i^y \\ \dot{v}_i^x & \dot{v}_i^y \end{bmatrix} = \begin{bmatrix} v_i^x & v_i^y \\ -v_i^x & -v_i^y \end{bmatrix} + Q \begin{bmatrix} r_i^x & r_i^y \\ 0 & 0 \end{bmatrix} +$$
$$+ BK \sum_{j=1}^{N} a_{(ij)} \begin{bmatrix} \phi_j(z_j^{p_x}) - \phi_i(z_i^{p_x}) & \phi_j(z_j^{p_y}) - \phi_i(z_i^{p_y}) \\ \phi_j(v_j^x) - \phi_i(v_i^x) & \phi_j(v_j^y) - \phi_i(v_i^y) \end{bmatrix}.$$

At $t = t_k$, the interaction of leaders can expressed as

$$x_l(t_k^+) = (P_l \otimes I_2)x_l(t_k) + r_l - (P_l \otimes I_2)r_l,$$

where $l = 3, 6, 10$, and

$$x_{l_i} = \begin{bmatrix} p_{l_i}^x & p_{l_i}^y \\ v_{l_i}^x & v_{l_i}^y \end{bmatrix}, x_l = \begin{bmatrix} x_{l_1} \\ x_{l_2} \\ x_{l_3} \end{bmatrix}, \quad \begin{matrix} x_{l_1} = x_3, \\ x_{l_2} = x_6, \\ x_{l_3} = x_{10}. \end{matrix}$$

Choosing $Q = [0\ 1]$ to satisfy Theorem 3.3.1, and $\alpha = 0.1, \Upsilon_1 = diag\{0, 0.1\}, \Upsilon_2 = diag\{0.1, 0.2\}$. Then, solving LMIs in Theorem 3.3.2, one has the feedback matrix $K = [-5.6625\ \ -6.0109]$.

The interaction among leaders occurs at some instant times. These are defined based on some events or particularly demand of systems, which is decided by an operator. In our simulation, we assume that the reset time of the leader's communication is periodic and it happens at each second as depicted in Fig. 3.5 (upper). The evolution of leaders' states is also depicted in Fig. 3.5. It is clear that leaders' values are updated at the reset time t_k through the communication graph $\mathcal{G}_l$.

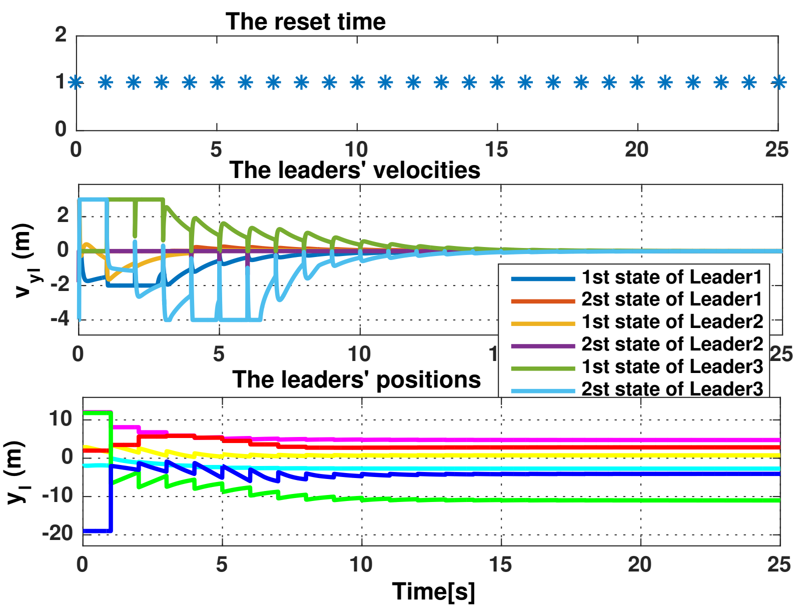

FIGURE 3.5: A reset signal and leaders' states of UAVs

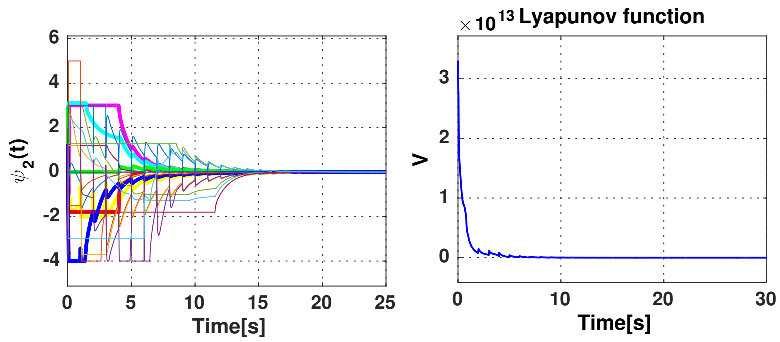

FIGURE 3.6: Convergence of the variable ψ_2 and the Lyapunov function.

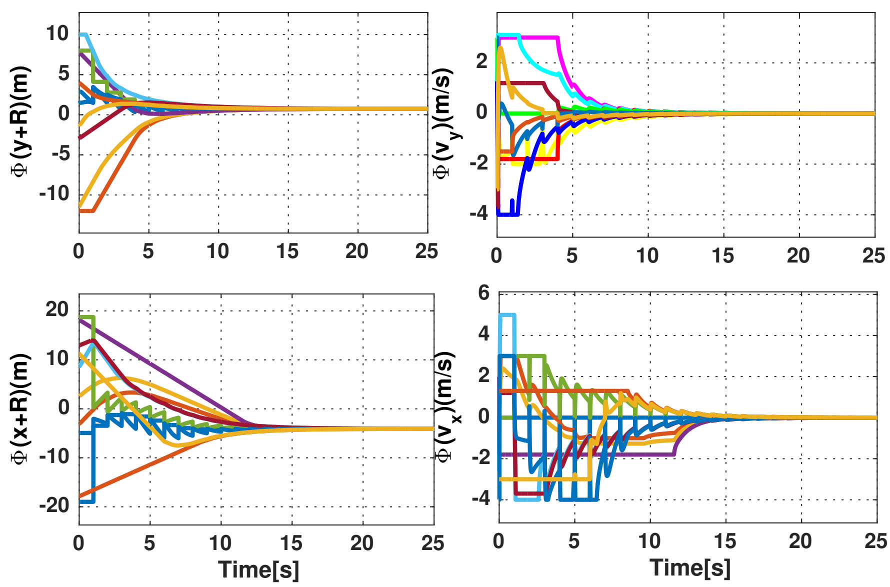

FIGURE 3.7: Ten-UAVs'positions (x_i, y_j) constraints (left) and ten-UAVs'velocities (v_{x_i}, v_{y_i}) constraints (right).

Convergence of both the variable ψ_2 in (3.16) and the Lyapunov function (3.43) is depicted in Fig. 3.6. It again verifies that the clustered network achieves formation under robust formation control protocol (3.17) if $\lim_{t\to\infty} \psi_2 \to 0$, shown in Theorem 3.3.1, and the Lyapunov function satisfies the condtions (3.44) and (3.46). Moreover, according to the simulation results depicted respectively in Fig. 3.7, and Fig. 3.8, we see that all positions and velocities of UAVs in clustered network reach the pentacle formation and consensus under the state constraints.

3.4.2 Homogeneous Constraints

Finally, in oder to investigate the influence of state constraints on the formation performance, we have carried out two simulations. In the first simulation, we consider constraints on positions that belong to $[-15\ 15](m)$ and we suppose that the speed

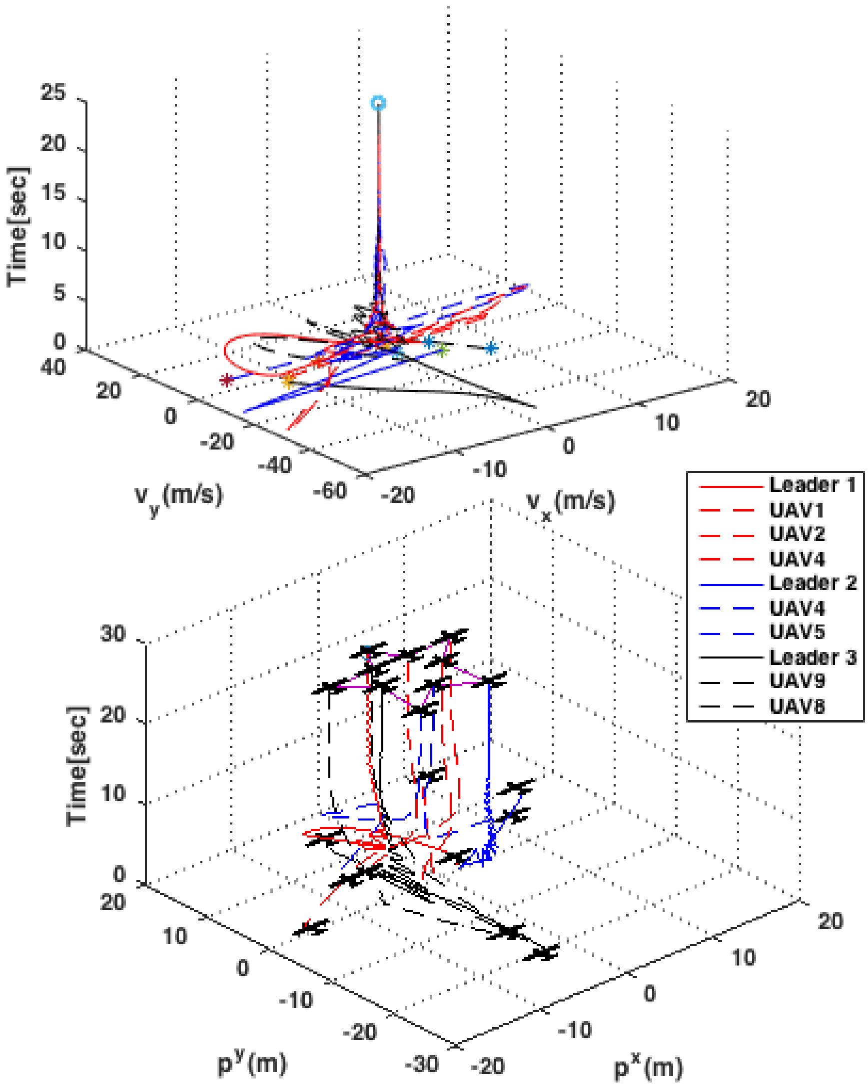

FIGURE 3.8: Pentacle formation of ten UAVs'positions (x_i, y_i) (lower) and consensus of ten UAVs'velocities (v_{x_i}, v_{y_i}) under state constraints (upper).

shoud be in the interval $[-15\ 8](m/s)$. The second simuation suppose that the constraints on positions belong to $[-3\ 3](m)$ and that speed should be in $[-3\ 3](m/s)$. The obtained simulation results are depicted in Fig. 3.9 and Fig. 3.10. In both of cases, one can see that agents achieve and keep the desired formation and that the values of state variables are in the defined region. Moreover, one can remark that when the constraints on the speed are stricter (the second case) the achievement of formation takes more time.

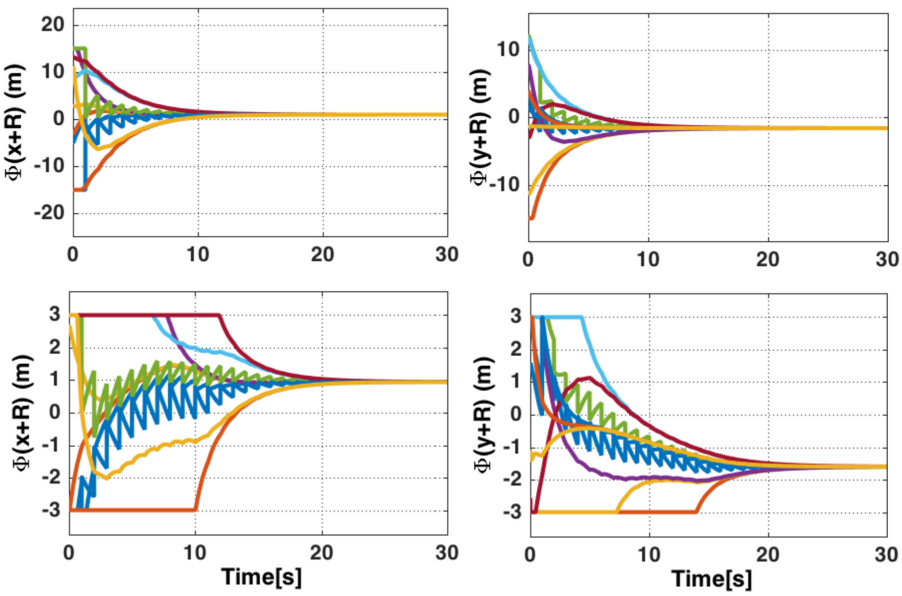

FIGURE 3.9: Ten-UAVs' positions (x_i, y_j) under state constraints belonging to $[-15\ 15](m)$ (upper); Ten-UAVs' positions (x_i, y_j) under state constraints belonging to $[-3\ 3](m)$ (lower).

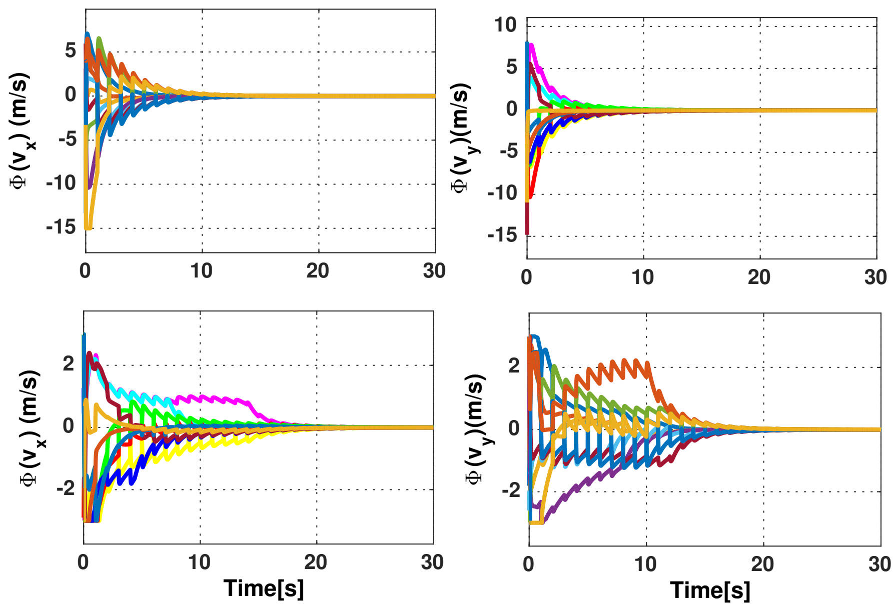

FIGURE 3.10: Ten-UAVs' velocities (v_{x_i}, v_{y_i}) under state constraints belonging to $[-15\ 8](m/s)$ (upper); Ten-UAVs' velocities (v_{x_i}, v_{y_i}) under state constraints belonging to $[-3\ 3](m/s)$ (lower).

3.5 Chapter summary

In this chapter, a novel approach has been proposed to design distributed robust formation controllers for general linear MASs under state constraints with the following features. First, the considered networks are partitioned into clusters, where the communication between agents inside each cluster is continuous, but the cluster leaders interact at some reset times. Second, it is shown that the robust formation design with state constraints can be indirectly solved by considering the stability of an equivalent system. Third, sufficient conditions for the robust stability of this equivalent system were derived from solutions of local convex LMI problems, which can be solved in a distributed manner. A possible application of our proposed approaches to the UAVs formation flying was illustrated.

Chapter 4

Output Consensus of Heterogeneous MASs under Disturbances

Contents

Résumé du Chapitre 4

Ce chapitre se concentre sur le problème du consensus de sortie dans les réseaux divisés en grappes composées d'agents hétérogènes qui sont soumis à différentes perturbations (correspondant à la cinquième question de la section 1.2). En introduisant un modèle de référence interne pour chaque agent, qui prend en compte les communications en temps continu entre les modèles de référence internes dans les grappes virtuelles et les échanges d'informations discrètes entre les clusters virtuelles, un protocole de contrôle par consensus distribué est proposé. Il est montré que le problème du consensus de sortie peut être indirectement résolu par le consensus des références virtuelles. Ensuite, en utilisant les résultats de la théorie des matrices et de la théorie des graphes algébriques, une condition suffisante pour le consensus dans des clusters virtuelles est proposée. Enfin, une condition suffisante et nécessaire est obtenue pour le consensus de sortie des agents hétérogènes linéaires sous différentes perturbations dans le réseau en grappes considéré. Enfin, un exemple illustratif est donné pour montrer l'efficacité des résultats théoriques proposés.

This chapter focuses on the output consensus problem in networks divided into clusters composed of heterogeneous agents that are subjected to different disturbances (corresponding to the fifth question in Section 1.2). By introducing a dynamic internal reference model for each agent, that it takes into account the continuous-time communications among internal reference models in virtual clusters and discrete information exchanges between virtual clusters, a distributed consensus control protocol is proposed. It is shown that the output consensus problem is indirectly solved through the consensus of the virtual references. Then, by using results from matrix theory and algebraic graph theory, a sufficient condition for the consensus in virtual clusters is proposed. Next, a sufficient and necessary condition is derived for the output consensus of linear heterogeneous agents under different disturbances in the considered clustered network. Finally, an illustrative example is given to show the effectiveness of the proposed theoretical results.

4.1 Related Work and Contributions

In many fields of applications, the dynamics of agents could be different, and state consensus is no longer valid. Hence, output consensus should be studied instead (Wieland, Sepulchre, and Allgöwer, 2011; Kim, Shim, and Seo, 2011; Huang and Ye, 2014; Li et al., 2015; Adib Yaghmaie, Lewis, and Su, 2016; Hu, Liu, and Feng, 2017; Kiumarsi and Lewis, 2017; Qian, Liu, and Feng, 2018b). The output consensus problem of heterogeneous linear MASs was studied in Wieland, Sepulchre, and Allgöwer, 2011. It is shown that an internal model principle is necessary and sufficient for the synchronizability of the network. Following this research line, a cooperative output regulation problem of linear MASs subjected to disturbances were investigated in Kim, Shim, and Seo, 2011; Huang and Ye, 2014. These works, which represent a special case of output consensus of heterogeneous MASs are based on the output regulation theory. The proposed approaches allow the output of heterogeneous MASs tracking an internal reference input while rejecting external disturbances. In another work, Adib Yaghmaie, Lewis, and Su, 2016 addressed the output regulation problem of linear heterogeneous MASs in directed and fixed communication graph, where agents have an arbitrary number of inputs and outputs. Moreover, the output consensus problem of heterogeneous discrete-time MASs was studied in Li et al., 2015; Kiumarsi and Lewis, 2017. Another direction proposes a distributed even-triggered control protocol (Hu, Liu, and Feng, 2017; Qian, Liu, and Feng, 2018b). It is shown that all agents achieve asymptotically output consensus with intermittent communication among agents in a network.

It has been noticed that interaction among agents in the aforementioned networks is either continuous-time or discrete-time. Nevertheless, in several practical applications, e.g., cooperative intelligent transportation systems, robots fleet cooperation, consensus control on a social network, etc, agents can be subject to abrupt changes at specific instants. Also, because of some constraints related to the energy or the range of communication, agents should impulsively exchange information with their neighbors at some discrete times (Guan et al., 2012a; Hu et al., 2013; Liu, Zhang, and

Xie, 2017). This leads to a hybrid communication that combines both continuous-time and discrete-time interaction among agents. In Guan et al., 2012a, a sufficient and necessary result has been derived for the impulsive consensus of second-order MASs with sampled-data communication, where the graphs of continuous-time and impulsive-time topologies contain a spanning tree. In another work, Hu et al., 2013 dealt with the time-delayed impulsive consensus problem of second-order MASs with switching topologies. Following this research line, the necessary condition of impulsive consensus on graph connections among agents is investigated in Liu, Zhang, and Xie, 2017. Moreover, sometimes the network should be divided into several groups or clusters (Bragagnolo et al., 2014; Rejeb, Morarescu, and Daafouz, 2015; Bragagnolo et al., 2016; Morarescu et al., 2016; Pham, Messai, and Manamanni, 2019a; Pham et al., 2019) with continuous intra-cluster and discrete inter-cluster communications. This is different from the consensus problem considered in Xia and Cao, 2011; Qin and Yu, 2013; Qin, Yu, and Anderson, 2016, where agents within a cluster are cooperative but are competitive with those in other clusters. The works in Bragagnolo et al., 2016 proposed a quasi-periodically reset strategy and provided some LMI conditions to guarantee the global uniform exponential consensus where the communication structures in clusters are represented by directed and strongly connected graphs. The research in Morarescu et al., 2016 investigated the sufficient conditions for event-triggered consensus. In other works (see Pham, Messai, and Manamanni, 2019a; Pham et al., 2019), the hybrid consensus control protocol was proposed to deal with the consensus problem of homogeneous MASs in the clustered network. Nevertheless, most of the above studies on clustered MASs consider the homogeneous agent with simply the integrator and identical.

Motivated by both above-mentioned limitations and practical issues mentioned above, this research focuses on clustered networks of MASs where each cluster is represented by a fixed directed graph and in each cluster there exists an agent called a leader who can instantly communicate with other leaders in other clusters. The main contributions of this chapter can be summarized as follows.

This chapter investigates a general setting of the consensus problem in directed clustered network of MASs, where agents have distinct and generic linear dynamics under different disturbances. A dynamic internal reference model for each agent is introduced, which takes into account the continuous-time communications among internal reference models in virtual clusters and discrete information exchanges between those virtual clusters. Therefore, the output consensus of heterogeneous agents is indirectly solved through the consensus of the virtual references. To achieve that, a hybrid consensus control protocol is proposed for the virtual clustered network. Thanks to results from matrix theory and algebraic graph theory, the consensus of the virtual clustered network are solved which forms the second contribution of this work. A sufficient and necessary condition is derived for the output consensus of linear heterogeneous agents under different disturbances in the clustered network.

4.2 Problem Formulation

4.2.1 Output Consensus Control

Consider N heterogeneous agents which interact in a network partitioned into m clusters. Each agent has the following linear generic dynamics

$$\begin{aligned} \dot{x}_i(t) &= A_i x_i(t) + B_i u_i(t) + E_{di} d_i(t), \\ y_i(t) &= C_i x_i(t). \end{aligned} \tag{4.1}$$

where $x_i(t) \in \mathbb{R}^{n_i}$, $u_i(t) \in \mathbb{R}^{p_i}$, and $y_i(t) \in \mathbb{R}^q$ are respectively the state, input, and output vector. $d_i(t) \in \mathbb{R}^{h_i}$ is the immeasurable disturbance, which has the following dynamics

$$\dot{d}_i(t) = A_{di} d_i(t). \tag{4.2}$$

Then, system (4.1)-(4.2) can be rewritten by the following augmented form:

$$\begin{bmatrix} \dot{x}_i(t) \\ \dot{d}_i(t) \end{bmatrix} = \underbrace{\begin{bmatrix} A_i & E_{di} \\ 0 & A_{di} \end{bmatrix}}_{A_i^d} \begin{bmatrix} x_i(t) \\ d_i(t) \end{bmatrix} + \begin{bmatrix} B_i \\ 0 \end{bmatrix} u_i(t),$$

$$y_i(t) = \underbrace{\begin{bmatrix} C_i & 0 \end{bmatrix}}_{C_i^d} \begin{bmatrix} x_i(t) \\ d_i(t) \end{bmatrix}. \tag{4.3}$$

Definition 4.2.1 *(Output Consensus Problem) The MAS (4.1) is said to achieve output consensus for all initial conditions if the following condition is satisfied*

$$\lim_{t \to \infty} \| y_i(t) - y_j(t) \| = 0, \ \forall i, j = 1, \dots, N. \tag{4.4}$$

In order to deal with the output consensus problem in the clustered network, we propose that each agent has an internal reference model embedded in a controller, which is generated from a cyber command centre (see Fig. 4.1). Those internal reference models have identical dynamics and can be viewed as virtual exosystems which generate virtual reference inputs for agents. They also interact through a network called virtual clustered networks which have the same structure as the physical clustered network of heterogeneous agents.

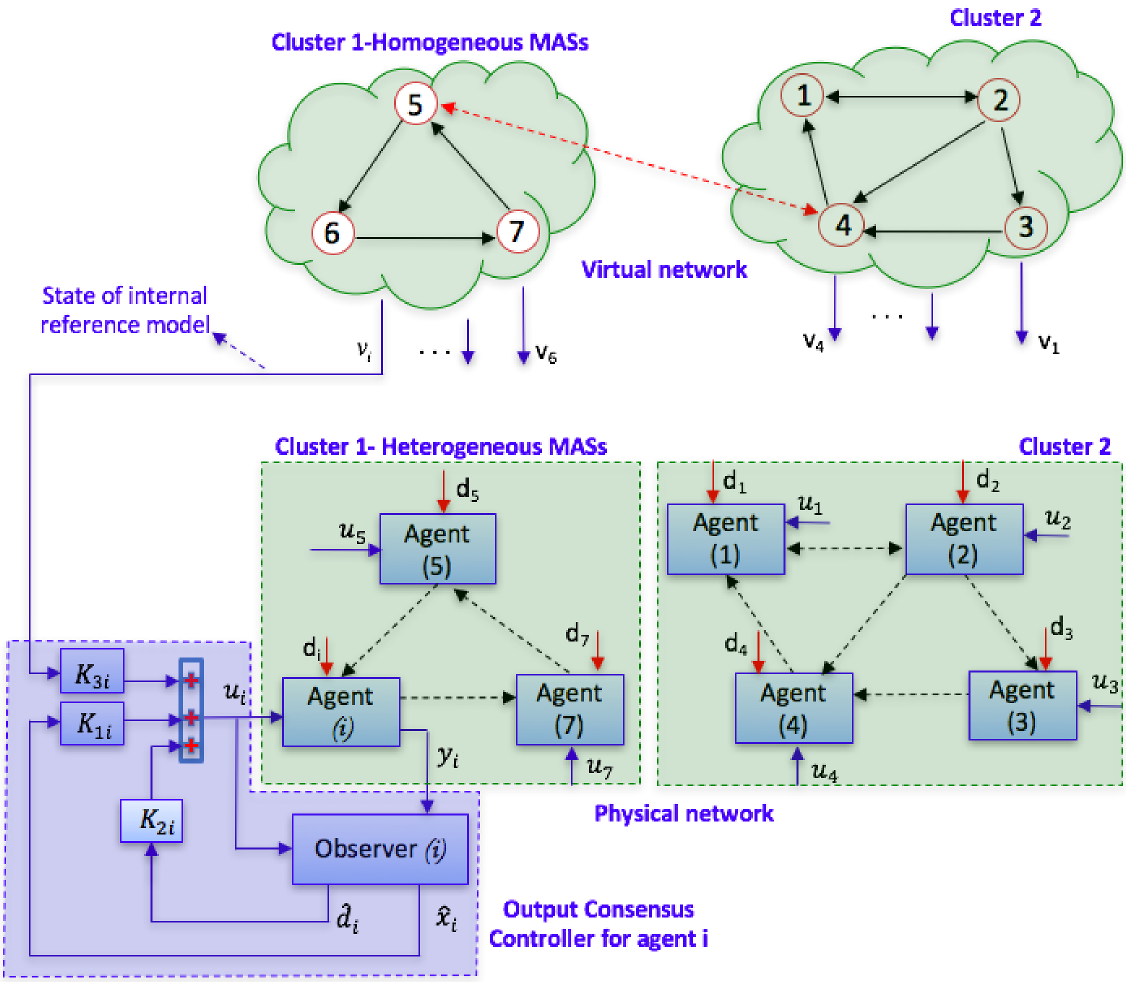

FIGURE 4.1: The heterogeneous MASs are in the physical space while internal reference models are generated from a cyber command center through the virtual clustered network.

In the following, we propose a distributed consensus control law for the considering clustered network of heterogeneous MASs (4.1) and their virtual references,

$$u_i(t) = K_{1i}\hat{x}_i(t) + K_{2i}\hat{d}_i(t) + K_{3i}v_i(t), \tag{4.5}$$

$$\begin{bmatrix} \dot{\hat{x}}_i(t) \\ \dot{\hat{d}}_i(t) \end{bmatrix} = \begin{bmatrix} A_i & E_{di} \\ 0 & A_{di} \end{bmatrix} \begin{bmatrix} \hat{x}_i(t) \\ \hat{d}_i(t) \end{bmatrix} + \begin{bmatrix} B_i \\ 0 \end{bmatrix} u_i + \underbrace{\begin{bmatrix} F_i^x \\ F_i^d \end{bmatrix}}_{F_i} (C_i\hat{x}_i(t) - y_i(t)), \tag{4.6}$$

$$\dot{v}_i(t) = A_v v_i(t) + cK_v \sum_{j=1}^{N} a_{(ij)} C_v (v_j(t) - v_i(t)), \tag{4.7}$$

$\forall t \in (t_k, t_{k+1})$, where $\hat{x}_i(t) \in \mathbb{R}^{n_i}$ is the observer state, $\hat{d}_i(t) \in \mathbb{R}^{h_i}$ is the estimation disturbance, $v_i(t) \in \mathbb{R}^{v}$ is the internal reference model state, $c \in \mathbb{R}$ is an coupling gain, and $K_v \in \mathbb{R}^{v\times q}$, $K_{1i} \in \mathbb{R}^{p_i\times n_i}$, $K_{2i} \in \mathbb{R}^{p_i\times h_i}$, $K_{3i} \in \mathbb{R}^{p_i\times v}$, $F_i^x \in \mathbb{R}^{q\times n_i}$, $F_i^d \in \mathbb{R}^{h_i\times n_i}$, $i = 1, \cdots, N$ are determined later.

Moreover, the discrete communication occurring between the leaders of clusters of heterogeneous MASs needs to assume that their states have the same dimensions. However, this assumption is challenging in some applications. Thus, to avoid this limitation, this will be implemented on the leaders of virtual communication clusters. Thus, exchange information among leader's states of virtual clusters at the reset time t_k can be described by

$$\Delta v_{l_i}(t_k) = \sum_{j=1}^{m} a_{l(ij)}(v_{l_j}(t_k) - v_{l_i}(t_k)), \quad t = t_k. \tag{4.8}$$

where $\Delta v_{l_i}(t_k) = v_{l_i}(t_k^+) - v_{l_i}(t_k^-)$, where $v_{l_i}(t_k^+)$ and $v_{l_i}(t_k^-)$ represent the right and left limit of v_{l_i} at t_k, respectively. Without loss of generality, we assume that $v_{l_i}(t_k^-) = v_{l_i}(t_k)$. Moreover, $a_{l(i,j)}$ is the (i,j)th entry of the weighted adjacent matrix $\mathcal{A}_l = [a_{l(ij)}]$, and m denotes the set of leader in graph $\mathcal{G}_l$. The Laplacian matrix $\mathcal{L}_l = [L_{l(ij)}] \in \mathbb{R}^{m\times m}$ is defined as $L_{l(ii)} = \sum_{j\neq i=1}^{m} a_{l(ij)}, L_{l(ij)} = -a_{l(ij)}$.

According to (4.7) and (4.8), the evolution of the whole system can be viewed as a hybrid system that evolves as follows: during interval time (t_{k-1}, t_k), the interaction among nodes in each cluster $\mathcal{C}_\tau$ is related only to the graph $\mathcal{G}_\tau$. Moreover, at each instant time t_k, the leaders update their states instantaneously according to the topology of $\mathcal{G}_l$.

4.2.2 Useful Assumptions

Hereafter, the time index t is ommited in expressions of $x_i, \hat{x}_i, u_i, y_i, v_i, d_i, \hat{d}_i, u_i$ and other variables just for conciseness of mathematical representations. Next, the following assumptions, defintion, and lemmas are used.

Assumption 4.2.1 *The graphs $\mathcal{G}_\tau$ has a spanning tree.*

Assumption 4.2.2 $a_{l(ij)} \succ 0, \sum_{j\neq i=1}^{m} a_{l(ij)} \prec 1.$

Assumption 4.2.3 *A_{di} has no eigenvalues with negative real parts.*

Assumption 4.2.4 *Matrices pairs (A_i, B_i), and $\left(\begin{bmatrix} C_i & 0 \end{bmatrix}, \begin{bmatrix} A_i & E_{di} \\ 0 & A_{di} \end{bmatrix}\right)$ are controllable and detectable, respectively.*

Lemma 4.2.1 *(Roger A. Horn, 2013) For any positive-definite matrix $\Phi_1 \in \mathbb{R}^{n\times n}$ and symmetric matrix $\Phi_2 \in \mathbb{R}^{n\times n}$, it can be verified that*

$$x^\mathsf{T}\Phi_2 x \leq \lambda_{max}(\Phi_1^{-1}\Phi_2)x^\mathsf{T}\Phi_1 x.$$

Remark 4.2.1 *Assumption 4.2.1 is needed to guarantee that the Laplacian matrix $\mathcal{L}_\tau$ of $\mathcal{G}_\tau, \forall\tau \in \{1, \cdots, m\}$ satisfies the following proprieties $\mathcal{L}_\tau \mathbf{1}_{N_\tau} = 0, r_\tau^\mathsf{T}\mathcal{L}_\tau = 0$, where $\mathbf{1}_{N_\tau}$, and $r_\tau^\mathsf{T} = \frac{1}{N_\tau}\mathbf{1}_{N_\tau}$ are the right and left eigenvectors of $\mathcal{L}_\tau$ associated with zero eigenvalue, respectively. Assumption 4.2.2 ensures the matrix $P_l = I - \mathcal{L}_l$ is a stochastic matrix with positive diagonal elements. Moreover, Assumptions 4.2.3 and 4.2.4 is for the existence of a controller and an observer.*

Remark 4.2.2 *It is noticed that the internal reference models (4.7) are constructed based on the relative output measurement, where A_v is the state matrix with appropriate dimensions, and output matrix C_v determines the dimensions of transmitted information between agents. The objective of internal reference models (4.7) generates the consensus trajectories of heterogeneous MASs (4.1).*

According to above our approach, the proposed design composes of two steps. The problem consensus of homogeneous MASs, described in (4.7)–(4.8), in virtual clustered network will be first given. By the way, we first show that the problem of consensus design for virtual clustered networks can be indirectly solved by considering the stability of an equivalent system. Then, the stability of this equivalent system will be given by means of LMI condition. In the second step, a sufficient and necessary condition is derived for the output consensus of linear heterogeneous agents in the considered clustered network.

4.3 Consensus of the Virtual Reference Systems

4.3.1 Prerequisites

From the Gershgorin theorem (Olfati-Saber and Murray, 2004), we know that $\lambda_m \preceq 2d_{max}(\mathcal{G}_l)$, where λ_m is the largest eigenvalue of the Laplacian of the graph $\mathcal{G}_l$, and $d_{max}(\mathcal{G}_l)$ is the maximum out-degree of the nodes of $\mathcal{G}_l$, where $\deg_{out}(v_{li}) = \sum_{i=1}^{m} a_{l(ij)}$. Therefore, we can get $0 \prec \lambda_m \prec 2$. Let us introduce

$$\begin{cases} P_{l(ij)} = -L_{l(ij)} = a_{l(ij)} \succ 0, \\ P_{l(ii)} = 1 - L_{l(ii)} = 1 - \sum_{j \neq i = 1}^{m} a_{l(ij)} \succ 0. \end{cases} \tag{4.9}$$

Then $\sum_{j=1}^{m} P_{l(ij)} = 1$, and $P_l = I - \mathcal{L}_l$ is a row stochastic matrix with positive diagonal elements, and according to Assumption 4.2.2, it has an eigenvalue $\lambda_1 = 1$ with algebraic multiplicity equal to one, and all the other eigenvalues satisfy $0 \prec |\lambda_i| \prec 1, i = 2, \cdots, m$.

By employing (4.9), and denoting $z_i = e^{\alpha t} v_i, \alpha \succ 0$, the internal reference models (4.7)–(4.8) can be rewritten as

$$\begin{cases} \dot{z}_i = (A_v + \alpha I_v) z_i + cK_v \sum_{j=1}^{N} a_{(ij)} C_v (z_j - z_i), t \in (t_k, t_{k+1}), \\ z_{l_i}(t_k^+) = \sum_{j=1}^{m} P_{l(ij)} z_{l_j}(t_k), \quad t = t_k. \end{cases} \tag{4.10}$$

Then, system (4.10) can be written in the following, which describes the overall network dynamics

$$\begin{cases} \dot{z} = (I_N \otimes (A_v + \alpha I_v) - c\mathcal{L} \otimes K_v C_v)z, \ t \in (t_k, t_{k+1}) \\ z_l(t_k^+) = (P_l \otimes I_n) z_l(t_k), \quad t = t_k, \end{cases} \tag{4.11}$$

where

$$z = [z_{l_1}^\top, z_{f_2}^\top, \cdots, z_{f_{m_1}}^\top, \cdots, z_{l_m}^\top, \cdots, z_{f_{m_m}}^\top = z_{f_N}^\top]^\top \in \mathbb{R}^{Nn},$$
$$z_l = [z_{l_1}^\top, \cdots, z_{l_m}^\top]^\top \in \mathbb{R}^{mn}.$$

containing respectively the states of agents and leader's states, and $P_l = I - \mathcal{L}_l$, and $\mathcal{L} \in \mathbb{R}^{N \times N}$ is the Laplacian matrix associated with the graph $\mathcal{G}$ represented in (1.21).

Now, some algebraic properties of $\mathcal{L}$ are presented in the following Proposition.

Proposition 4.3.1 *Let us consider a network of m clusters satisfying Assumption 4.2.1, with the Laplacian $\mathcal{L} \in \mathbb{R}^{N \times N}$, then rank$(\mathcal{L}) = N - m$ and $\mathcal{L}$ has m eigenvalues at zero and all the other $N - m$ eigenvalues of the Laplacian $\mathcal{L} \in \mathbb{R}^{N \times N}$ are positive real parts.*

Let us introduce the extended stochastic matrix P_e as follows

$$P_e = \mathcal{M}^\top \begin{bmatrix} P_l & 0 \\ 0 & I_{N-m} \end{bmatrix} \mathcal{M} \in \mathbb{R}^{N \times N}, \tag{4.12}$$

where $\mathcal{M}$ is a permutation matrix.

Then, the second equation in (4.11) can be expressed by

$$z(t_k^+) = (P_e \otimes I_n) z(t_k), \quad t = t_k. \tag{4.13}$$

In the following, let $U \in \mathbb{R}^{N \times N}$ be an invertible matrix, and employing Proposition 4.3.1, we obtain

$$U^{-1}\mathcal{L}U = \begin{bmatrix} 0_m & 0 \\ 0 & \Gamma \end{bmatrix} = \Lambda \in \mathbb{R}^{(N)\times(N)}, \tag{4.14}$$
$$\Gamma = diag\{\gamma_{m+1}, \cdots, \gamma_N\} \in \mathbb{R}^{(N-m)\times(N-m)}.$$

Next, let us also introduce the new variable

$$\psi = (U^{-1} \otimes I_n) z. \tag{4.15}$$

According to the variable ψ, we are now ready to formulate our statement as the following

$$\begin{cases} \dot{\psi} = (I_N \otimes (A_v + \alpha I_v) - c\Lambda \otimes BK)\psi, t \in (t_k, t_{k+1}), \\ \psi(t_k^+) = (P_\psi \otimes I_n)\psi(t_k), \ \ t = t_k, \end{cases} \tag{4.16}$$

where $P_\psi = U^{-1}P_e U$.

Finally, in order to simplify the presentation of the next results let us partition the matrices U^{-1}, U into

$$U^{-1} = [U_3^\top \ \ U_4^\top]^\top, U = [U_1 \ \ U_2], \tag{4.17}$$

where $U_3 \in \mathbb{R}^{m\times N}, U_4 \in \mathbb{R}^{(N-m)\times N}$ and $U_1 \in \mathbb{R}^{N\times m}$, $U_2 \in \mathbb{R}^{N\times(N-m)}$.

$$U_1 = \begin{bmatrix} \mathbf{1}_{N_1} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \mathbf{1}_{N_m} \end{bmatrix}, U_3 = \begin{bmatrix} r_1^\top & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & r_m^\top \end{bmatrix}, \tag{4.18}$$

where $\mathcal{L}U_1 = 0_{N\times m}, U_3^\top \mathcal{L} = 0_{m\times N}$. Then, (4.15) becomes

$$\psi_1 = (U_3 \otimes I_n)z, \quad \psi_2 = (U_4 \otimes I_n)z, \tag{4.19}$$

where $\psi_1 \in \mathbb{R}^{mn}$ and $\psi_2 \in \mathbb{R}^{Nn-mn}$.

4.3.2 Consensus Analysis in Clustered Network

According to results from Lemma 3.2.1 and Lemma 3.2.2, the consensus of homogeneous virtual reference systems (4.7)–(4.8) is analyzed and shown that it is indirectly solved by considering the stability of the system (4.17). The result is shown in the following theorem.

Theorem 4.3.1 *Consider the overall network dynamics system* (4.17)*, and suppose that assumptions 4.2.1–4.2.2, the consensus problem is solved if* $\lim_{t\to\infty} \psi_2 \to 0$ *for any given bounded initial conditions.*

Proof 4.3.1 *The proof is given in two steps. First, we show that ψ_1 reaches a constant value, which depends on the dynamics of agents, the graph of each cluster, the interaction between leaders, and the initial conditions. Second, based on the analysis of the first step, the hybrid rconsensus control problem is solved.*

Firstly, according to (4.14), (4.17) *and* (4.19)*, the dynamics of ψ_1 can be represented as the following*

$$\begin{cases} \dot{\psi}_1 = (I_m \otimes (A_v + \alpha I_v))\psi_1, \\ \psi_1(t_k^+) = (U_3 P_e U_1 \otimes I_n)\psi_1(t_k) + (U_3 P_e U_2 \otimes I_n)\psi_2(t_k) \end{cases} \tag{4.20}$$

Then, the solution of (4.20) *with initial condition $\psi_1(t_0) = \psi_{10}$ can be obtained by*

$$\begin{aligned} \psi_1 &= e^{(I_m\otimes(A_v+\alpha I_v))(t-t_k)}\psi_1(t_k^+) \\ &= (I_m \otimes e^{(A_v+\alpha I_v)(t-t_k)})\psi_1(t_k^+), \end{aligned} \tag{4.21}$$

and if $\lim_{t\to\infty}\psi_2 \to 0$, *then* $\psi_1(t_k^+)$ *can be expressed as*

$$\begin{aligned}\psi_1(t_k^+) &= \lim_{k\to\infty}\prod_{i=1}^{k}(U_3P_eU_1\otimes I_n)e^{(I_m\otimes A_v^\alpha)(t_i-t_{i-1})}\psi_1(t_0)\\ &= \lim_{k\to\infty}(U_3P_eU_1)^k\otimes e^{A_v^\alpha(t_k-t_0)}\psi_1(t_0).\end{aligned} \tag{4.22}$$

where $A_v^\alpha = A_v + \alpha I_v$.

In the following, by using results from Lemma 3.2.1 and Lemma 3.2.2, we prove that matrix $U_3P_eU_1 \in \mathbb{R}^{m\times m}$ *is a row stochastic matrix with positive diagonal elements.*

The extended stochastic matrix P_e *in (4.12) can be expressed as follows*

$$P_e = \begin{bmatrix} P_{l_{11}} & 0 & \cdots & P_{l_{1m}} & 0\\ 0 & I_{N_1-1} & \cdots & 0 & 0\\ \vdots & \vdots & \ddots & \vdots & \vdots\\ P_{l_{m1}} & 0 & \cdots & P_{l_{mm}} & 0\\ 0 & 0 & \cdots & 0 & I_{N_m-1}\end{bmatrix} \in \mathbb{R}^{N\times N}, \tag{4.23}$$

where

$$P_l = \begin{bmatrix} P_{l_{11}} & \cdots & P_{l_{1m}}\\ \vdots & \vdots & \vdots\\ P_{l_{m1}} & \cdots & P_{l_{mm}}\end{bmatrix} = \begin{bmatrix} P_{l_1}\\ \vdots\\ P_{l_m}\end{bmatrix} \in \mathbb{R}^{m\times m}, \tag{4.24}$$

and the matrix $U_1 \in \mathbb{R}^{N\times m}$ *and* $U_3 \in \mathbb{R}^{m\times N}$ *are in (4.18). Now, the matrix* P_eU_1 *is calculated as*

$$P_eU_1 = \begin{bmatrix} P_{l_{11}}\cdots & P_{l_{1m}}\\ \mathbf{1}_{N_1-1} & 0_{(N_1-1)\times(N_1-1)}\\ \vdots & \vdots\\ P_{l_{m1}}\cdots & P_{l_{mm}}\\ \mathbf{1}_{N_m-1} & 0_{(N_m-1)\times(N_m-1)}\end{bmatrix} = \begin{bmatrix} E_1\\ \vdots\\ E_m\end{bmatrix}, \tag{4.25}$$

where $\forall\tau \in \{1,\cdots,m\}$, *and*

$$E_\tau = \begin{bmatrix} P_{l_{\tau1}}\cdots & P_{l_{\tau m}}\\ \mathbf{1}_{N_\tau-1} & 0_{(N_\tau-1)\times(N_\tau-1)}\end{bmatrix} \in \mathbb{R}^{N_\tau\times m}. \tag{4.26}$$

Then, the matrix $U_3P_eU_1$ *is determined as follows*

$$U_3P_eU_1 = \begin{bmatrix} r_1^\mathsf{T} & \cdots & 0\\ \vdots & \ddots & \vdots\\ 0 & \cdots & r_m^\mathsf{T}\end{bmatrix}\begin{bmatrix} E_1\\ \vdots\\ E_m\end{bmatrix} = \begin{bmatrix} r_1^\mathsf{T}E_1\\ \vdots\\ r_m^\mathsf{T}E_m\end{bmatrix}, \tag{4.27}$$

where $r_\tau^\mathsf{T} = [r_{\tau_1},\cdots,r_{\tau_{N_\tau}}] \in \mathbb{R}^{1\times N_\tau}$, $\forall\tau \in \{1,\cdots,m\}$, *and*

$$r_\tau^\mathsf{T}E_\tau = [r_{\tau_1}P_{l_{\tau1}} + r_{\tau_2} + \cdots + r_{\tau_{N_\tau}}, r_{\tau_1}P_{l_{\tau2}},\cdots,r_{\tau_1}P_{l_{\tau m}}].$$

The sum of the row matrix $r_\tau^\mathsf{T} E_\tau$ is calculated by

$$\sum_{k=1}^{N_\tau} r_\tau^\mathsf{T} E_\tau = r_{\tau_1} P_{l_{\tau 1}} + \sum_{k=2}^{N_\tau} r_{\tau_k} + r_{\tau_1} \sum_{k=2}^{m} P_{l_{\tau k}}. \tag{4.28}$$

According to Assumptions 4.2.1 and 4.2.2 , $P_{l_{\tau 1}} = 1 - \sum_{k=2}^{m} P_{l_{\tau k}}$, then

$$\sum_{k=1}^{N_\tau} r_\tau^\mathsf{T} E_\tau = \sum_{k=1}^{N_\tau} r_{\tau_k} = 1, \tag{4.29}$$

and $P_{l_{\tau k}} > 0$, then

$$\begin{aligned} r_{\tau_1} P_{l_{\tau 1}} + r_{\tau_2} + \cdots + r_{\tau_{N_\tau}} &\succ 0, \\ r_{\tau_1} P_{l_{\tau 2}} &\succ 0, \\ &\vdots \\ r_{\tau_1} P_{l_{\tau m}} &\succ 0. \end{aligned} \tag{4.30}$$

Subsequently, by employing (4.29) and (4.30), and according to Definition A.0.1, we see that the matrix $U_3 P_e U_1 \in \mathbb{R}^{m\times m}$ is a row stochastic matrix with postive diagonal elements.

Furthermore, by employing (4.9), and $\forall i, \tau \in \{1, \cdots, m\}$ Eq. (4.30) becomes

$$\begin{aligned} 1 + r_{\tau_1}(P_{l_{\tau 1}} - 1) = 1 - r_{\tau_1} &\sum_{i\neq j=1}^{m} a_{l(ij)}, \\ r_{\tau_1} P_{l_{\tau 2}} &= r_{\tau_1} a_{l_{i2}}, \\ &\vdots \\ r_{\tau_1} P_{l_{\tau m}} &= r_{\tau_1} a_{l_{im}}. \end{aligned} \tag{4.31}$$

then the $(i,j)^{th}$ entry of $U_3^\mathsf{T} P_e U_1$ is $r_{\tau_1} a_{l(ij)}$, which implies that the graph $\mathcal{G}_l$ and the graph of $U_3^\mathsf{T} P_e U_1$ have same the edge set. Thus, the graph of the matrix $U_3^\mathsf{T} P_e U_1$ has a spanning tree.

Based on the above analysis, we showed that the matrix $U_3 P_e U_1$ is a row stochastic matrix with positive diagonal elements and the graph of it has a spanning tree. Then, according to Lemma 3.2.2, the matrix $U_3 P_e U_1$ is SIA.

Therefore, from Lemma 3.2.1, there exits a column vector c^T such that

$$\lim_{k\to\infty} (U_3 P_e U_1)^k = \mathbf{1}_m c^\mathsf{T}. \tag{4.32}$$

Then, by subtitute (4.32) and (4.22) into (4.21), one has

$$\psi_1 = \mathbf{1}_m c^\mathsf{T} \otimes e^{A_v^\alpha (t-t_0)} \psi_{10}. \tag{4.33}$$

Secondly, by introducing the variables

$$\mu_1 = (U \otimes I_n) \begin{bmatrix} \psi_1 \\ 0 \end{bmatrix}, \mu_2 = (U \otimes I_n) \begin{bmatrix} 0 \\ \psi_2 \end{bmatrix}, \tag{4.34}$$

one has $z = \mu_1 + \mu_2$. *Then, according to (4.34) the variable* μ_1 *is written such as*

$$\mu_1 = [U_1 \otimes I_n \; U_2 \otimes I_n] \begin{bmatrix} \psi_1 \\ 0 \end{bmatrix} = (U_1 \otimes I_n)\psi_1. \tag{4.35}$$

It follows that $\mu_2 = z - \mu_1$, *where* $\mu_2 = (U_2 \otimes I_n)\psi_2$. *And, by using (4.35), it obtain*

$$\begin{aligned} \mu_1 &= U_1 \mathbf{1}_m c^{\mathsf{T}} \otimes e^{A_v^\alpha (t-t_0)} \psi_{10}, \\ &= \mathbf{1}_N c^{\mathsf{T}} \otimes e^{A_v^\alpha (t-t_0)} \psi_{10}. \end{aligned} \tag{4.36}$$

Then, if $\lim_{t\to\infty} \psi_2 \to 0$ *and recalling that* $(U \otimes I_n)$ *is nonsingular, we obtain*

$$\lim_{t\to\infty} \mu_2 = \lim_{t\to\infty} (z - \mu_1) = 0. \tag{4.37}$$

Finally, according to (4.37) and (4.36), *one has*

$$\lim_{t\to\infty} (z - \mathbf{1}_N c^{\mathsf{T}} \otimes e^{A_v^\alpha (t-t_0)} \psi_{10}) \to 0. \tag{4.38}$$

which implies that the system (4.11) can achieve consensus, meaning: $z_i = z_j = \mathbf{1}_N c^{\mathsf{T}} \otimes e^{A_v^\alpha (t-t_0)} \psi_{10}), \forall i, j = 1, \cdots N$.

4.3.3 Stabilization Controller Design

Based on the above analysis, the objective now is to design the matrix $K_v \in \mathbb{R}^{v \times q}$, such that the system (4.16) is stable, i.e., $\lim_{t\to\infty} \psi_2 \to 0$. The design of such stabilization controller gain K_v and c are given in the following theorem.

Theorem 4.3.2 *Consider the homogeneous virtual reference systems (4.16) with Assumptions 4.2.1–4.2.2. If there exists a positive constant* $\beta \succ 0$ *and symmetric, positive definite matrix* $P \in \mathbb{R}^{v \times v}$ *such that the following LMI is feasible,*

$$A_v^{\mathsf{T}} P + P A_v - \beta C_v^{\mathsf{T}} C_v + 2\alpha P \prec 0. \tag{4.39}$$

Then, $\lim_{t\to\infty} \psi_2 \to 0$. *Moreover, the stabilization control gain* K_v *and coupling gain* c *are given by* $K_v = P^{-1} C_v^{\mathsf{T}}, c \succeq \frac{\beta}{2 \max \gamma_i}, \forall i = m+1, \cdots, N$.

Proof 4.3.2 *We consider the stability of the system* (4.16) *by introducing the Lyapunov function such as*

$$V = \sum_{i=m+1}^{N} \psi_i^{\mathsf{T}} P^{-1} \psi_i = \psi^{\mathsf{T}} \left(\underbrace{\begin{bmatrix} 0_m & 0 \\ 0 & I_{N-m} \end{bmatrix}}_{\Phi} \otimes P^{-1} \right) \psi. \tag{4.40}$$

Then, V is positive semidefinite, and it is sufficient to prove that between impulses t_k and t_{k+1} the function V is a decreasing

$$\dot{V} \preceq 0, \quad \forall t \in (t_k, t_{k+1}) \tag{4.41}$$

and with $\mu > 0$ at reset time t_k

$$V(t_k^+) \preceq \mu V(t_k) \tag{4.42}$$

For $t \in (t_k, t_{k+1})$, the derivative of V with respect to (4.16) is

$$\begin{aligned}\dot{V} =& \psi^\top[\Phi \otimes (A_v^\top P^{-1} + P^{-1}A_v + 2\alpha P^{-1}) - \\ & - c((\Phi\Lambda + \Lambda^\top\Phi) \otimes K_v C_v P^{-1})]\psi \\ =& \sum_{i=m+1}^{N} \psi_i^\top (A_v^\top P^{-1} + P^{-1}A_v + 2\alpha P^{-1} - 2c\gamma_i K_v C_v P^{-1})\psi_i.\end{aligned} \tag{4.43}$$

By choosing c such that $c \succeq \frac{\beta}{2\max\gamma_i}, \forall i = m+1, \cdots, N$

$$\begin{aligned}& A_v^\top P^{-1} + P^{-1}A_v + 2\alpha P^{-1} - 2c\gamma_i K_v C_v P^{-1} \\ & \preceq A_v^\top P^{-1} + P^{-1}A_v + 2\alpha P^{-1} - \beta K_v C_v P^{-1} \prec 0\end{aligned} \tag{4.44}$$

Therefore, $\dot{V} \leq 0$, and the LMI (4.39) is obatained by multiplying two sides of (4.44) by P, and letting $K_v = P^{-1}C_v^\top$.

On the other hand, at the reset time $t = t_k$ one has

$$\begin{aligned}& V(\psi(t_k^+)) - \mu V(\psi(t_k)) \\ & = \psi^\top(t_k)[(P_\psi \otimes I_n)^\top(\Phi \otimes P^{-1})(P_\psi \otimes I_n) - \\ & \qquad - \mu(\Phi \otimes P^{-1})]\psi(t_k) \\ & = \psi^\top(t_k)[(P_\psi^\top \Phi P_\psi - \mu\Phi) \otimes P^{-1}]\psi(t_k)\end{aligned} \tag{4.45}$$

by employing $P_\psi = U^{-1}P_eU$, (4.17) and (4.40), one has

$$P_\psi^\top \Phi P_\psi = \begin{bmatrix} (U_4P_eU_1)^\top(U_4P_eU_1) & (U_4P_eU_1)^\top(U_4P_eU_2) \\ (U_4P_eU_2)^\top(U_4P_eU_1) & (U_4P_eU_2)^\top(U_4P_eU_2) \end{bmatrix}$$

then, by using (4.33), we obtain

$$\begin{aligned}(U_4P_eU_1 \otimes P^{-1})\psi_1(t_k) &= U_4P_eU_1\mathbf{1}_m c^\top \otimes e^{A_v^\alpha(t-t_0)}\psi_{10} \\ &= U_4P_e\mathbf{1}_N c^\top \otimes e^{A_v^\alpha(t-t_0)}\psi_{10}.\end{aligned}$$

Moreover, according to (4.17) *and* (4.18)*, one has* $U_4 U_1 = 0_{(N-m)\times m}$*, where*

$$U_4 = \begin{bmatrix} U_{41}^\top & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & U_{4m}^\top \end{bmatrix}, U_1 = \begin{bmatrix} \mathbf{1}_{N_1} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \mathbf{1}_{N_m} \end{bmatrix}$$

Now, since P_e *in* (4.16) *is the stochastic matrix, one has* $P_e \mathbf{1}_N = \mathbf{1}_N$*. It becomes obvious that*

$$(U_4 P_e U_1 \otimes P^{-1})\psi_1(t_k) = 0_{(N-m)\times m}.$$

and

$$\begin{bmatrix} \psi_1^\top(t_k) \\ \psi_2^\top(t_k) \end{bmatrix}^\top P_\psi^\top \Phi P_\psi \begin{bmatrix} \psi_1(t_k) \\ \psi_2(t_k) \end{bmatrix} = \psi_2^\top(t_k) P_{\psi_1}^\top P_{\psi_1} \psi_2(t_k).$$

where $P_{\psi_1} = U_4 P_e U_2$*. Then, from* (4.45)*, one has*

$$V(\psi(t_k^+)) - \mu V(\psi(t_k)) = \psi_2^\top(t_k)[(P_{\psi_1}^\top P_{\psi_1} - \mu I_{N-m}) \otimes P^{-1}]\psi_2(t_k) \tag{4.46}$$

Then, according to Lemma 4.2.1, with $P_\psi^\top P_\psi$ *is a symmetric matrix and* $\lambda_{max}(P_\psi^\top P_\psi) > 0$*, we can conclude from* (4.46) *that*

$$V(\psi(t_k^+)) - \mu V(\psi(t_k)) \leq 0 \ \ , \mu = \lambda_{max}(P_\psi^\top P_\psi). \tag{4.47}$$

Thus, according to the Lasalle's invariance principle, ψ *globally exponentially converges to the largest invariance set contained in* $\{\psi \in \mathbb{R}^{Nn} | \dot{V}(\psi) = 0\}$ *for any initial conditions. It can be seen from* (4.43) *and definition of matrix* Φ *that* $\dot{V}(\psi) = 0$ *if and only if* $\lim_{t\to\infty} \psi_2 \to 0$*, where* $\psi_2 = [\psi_{m+1}, \cdots, \psi_N]^\top \in \mathbb{R}^{Nn-mn}$.

Remark 4.3.1 *By using Theorem 4.3.2, one can conclude that* $\lim_{t\to\infty} z \to 0$*. On the other hand, since* $z_i = e^{\alpha t} v_i$*, it becomes that* $\lim_{t\to\infty}(v_i - v_j) \to 0$ *exponentially converges, that is, state consensus can be exponentially achieved for all agents with rate of* $e^{-\alpha t}$.

4.4 Output Consensus of the Heterogeneous Agents

Having the consensus of internal reference models (4.7)–(4.8), the output consensus of heterogeneous clustered MASs under the distributed controller (4.5) can now be solved. The result is shown in the following theorem.

Theorem 4.4.1 *Suppose that Assumptions 4.2.1– 4.2.4 hold. Moreover, the LMI* (4.39) *is satisfied and matrices* $K_v \in \mathbb{R}^{v\times q}$ *is designed as in Theorem* **??***. For each agent* i*, let* $[K_{2i} \ \ K_{3i}] = M_i - K_{1i} N_i$ *and choose the matrices* $K_{1i} \in \mathbb{R}^{p_i \times n_i}$ *and* $F_i \in \mathbb{R}^{(q+h_i)\times n_i}$ *such that* $A_i + B_i K_{1i}$ *and* $A_i^d + F_i C_i$ *are Hurwitz. Then, under the distributed consensus control law* (4.5)–(4.8)*, the heterogeneous clustered MASs*

(4.1) achieve asymptotically the output consensus, if and only if the following linear matrix equations

$$N_i A_v^d = A_i N_i + B_i M_i + E_i, \tag{4.48}$$
$$R_v^0 = C_i N_i. \tag{4.49}$$

have solutions $N_i \in \mathbb{R}^{n_i \times (h_i+v)}$ and $M_i \in \mathbb{R}^{p_i \times (v+h_i)}$, where

$$A_v^d = \begin{bmatrix} A_{di} & 0 \\ 0 & A_v \end{bmatrix}, E_i = \begin{bmatrix} E_{di} & 0 \end{bmatrix}, R_v^0 = \begin{bmatrix} 0 & R_v \end{bmatrix}.$$

Proof 4.4.1 *Let $x_c = [x_{c1}^\top, x_{c2}^\top, \cdots, x_{cN}^\top]^\top \in \mathbb{R}^\mu$, with $x_{ci} = [x_i^\top, \hat{x}_i^\top, \hat{d}_i^\top, v_i^\top]^\top$ and $\mu = \sum_{i=1}^N (2n_i + h_i + v)$. By employing (4.3)–(4.8), the closed-loop system can be rewritten as*

$$\dot{x}_c = A_c x_c + B_c v_c^d, \quad t \in (t_k, t_{k+1}), \tag{4.50}$$
$$y = C_c x_c, \quad t \in (t_k, t_{k+1}), \tag{4.51}$$
$$x_c(t_k^+) = P_c x_c(t_k), \quad t = t_k. \tag{4.52}$$

where

$$\begin{aligned}
&e_d = [e_{d1}, \cdots, e_{dN}]^\top, e_{di} = d_i - \hat{d}_i, \forall i = 1, \cdots, N, \\
&v_c^d = [e_d^\top \; v_c^\top]^\top, v_c = (\mathcal{L} \otimes I_v) v, \\
&y = [y_1^\top, \cdots, y_N^\top]^\top, A_c = diag\{A_{c1}, \cdots, A_{cN}\}, \\
&B_c = diag\{B_{c1}, \cdots, B_{cN}\}, C_c = diag\{C_{c1}, \cdots, C_{cN}\},
\end{aligned}$$

$$P_c = \begin{bmatrix} I_n & 0 & 0 & 0 \\ 0 & I_n & 0 & 0 \\ 0 & 0 & I_h & 0 \\ 0 & 0 & 0 & P_e \otimes I_v \end{bmatrix}, n = \sum_{i=1}^N n_i, h = \sum_{i=1}^N h_i.$$

It can be shown that the ith diagonal elements of A_c, B_c and C_c have the following forms, respectively

$$A_{ci} = \begin{bmatrix} A_i & B_i K_{1i} & B_i K_{2i} + E_{di} & B_i K_{3i} \\ -F_i^x C_i & A_i + F_i^x C_i + B_i K_{1i} & B_i K_{2i} + E_{di} & B_i K_{3i} \\ -F_i^d C_i & F_i^d C_i & A_{di} & 0 \\ 0 & 0 & 0 & A_v \end{bmatrix},$$
$$B_{ci} = \begin{bmatrix} E_{di} & 0 & 0 & 0 \\ 0 & 0 & 0 & K_v C_v \end{bmatrix}^\top, \tag{4.53}$$
$$C_{ci} = \begin{bmatrix} C_i & 0 & 0 & 0 \end{bmatrix}. \tag{4.54}$$

Sufficiency

Denote $e_i = x_i - N_i \begin{bmatrix} d_i \\ v_i \end{bmatrix}$, $e_{xi} = x_i - \hat{x}_i$, $e_{di} = d_i - \hat{d}_i$ *the disagreement vector, state observation error of agent* i, *and disturbance estimation error, respectively. Then, by* (4.3), (4.5)–(4.8), *and* (4.48), *where* $[K_{2i}\ K_{3i}] = M_i - K_{1i}N_i$, *the error dynamics can be rewritten as*

$$\begin{bmatrix} \dot{e}_{xi} \\ \dot{e}_{di} \end{bmatrix} = (A_i^d + F_i C_i^d) \begin{bmatrix} e_{xi} \\ e_{di} \end{bmatrix}, \tag{4.55}$$

and

$$\begin{aligned} \dot{e}_i =& A_i x_i + B_i K_{1i} \hat{x}_i + B_i [K_{2i}\ K_{3i}] \begin{bmatrix} d_i \\ v_i \end{bmatrix} - B_i K_{2i} e_{di} + \\ &+ [E_{di}\ 0] \begin{bmatrix} d_i \\ v_i \end{bmatrix} - N_i \begin{bmatrix} A_{di} & 0 \\ 0 & A_v \end{bmatrix} \begin{bmatrix} d_i \\ v_i \end{bmatrix} - \\ &- N_i \begin{bmatrix} 0 \\ K_v C_v \sum_{j=1}^N a_{(ij)}(v_i - v_j) \end{bmatrix} \end{aligned} \tag{4.56}$$

$$\begin{aligned} \dot{e}_i =& A_i x_i + B_i K_{1i} x_i - B_i K_{1i} N_i \begin{bmatrix} d_i \\ v_i \end{bmatrix} - B_i K_{1i} e_{xi} + \\ &+ \underbrace{\left(B_i M_i - N_i \begin{bmatrix} A_{di} & 0 \\ 0 & A_v \end{bmatrix} + [E_{di}\ 0] \right)}_{-A_i N_i} \begin{bmatrix} d_i \\ v_i \end{bmatrix} - \\ &- B_i K_{2i} e_{di} - N_i \begin{bmatrix} 0 \\ K_v C_v \sum_{j=1}^N a_{(ij)}(v_i - v_j) \end{bmatrix} \end{aligned} \tag{4.57}$$

$$\dot{e}_i = (A_i + B_i K_{1i}) e_i - B_i K_{1i} e_{xi} - B_i K_{2i} e_{di} - N_i \begin{bmatrix} 0 \\ K_v C_v \sum_{j=1}^N a_{(ij)}(v_i - v_j) \end{bmatrix}. \tag{4.58}$$

According to Theorem **??**, *one has* $\lim_{t\to\infty}(v_i - v_j) = 0$. *Moreover, under Asumptions* 4.2.3 *and* 4.2.4, *we can choose a proper* F_i, K_{1i} *such that* $A_i^d + F_i C_i^d, A_i + B_i K_{1i}, i = 1, \ldots, N$ *are Hurwitz. It follows from* (4.55) *that* $\lim_{t\to\infty} e_{xi} = 0$ *and* $\lim_{t\to\infty} e_{di} = 0$. *Therefore, we can concluded from equation* (4.58) *that* $\lim_{t\to\infty} e_i = 0$, *which leads to*

$$\lim_{t\to\infty} x_i = \lim_{t\to\infty} \left(e_i + N_i \begin{bmatrix} d_i \\ v_i \end{bmatrix} \right) = \lim_{t\to\infty} N_i \begin{bmatrix} d_i \\ v_i \end{bmatrix}. \tag{4.59}$$

Then, by employing (4.49) *and* (4.59), *one has*

$$\begin{aligned} \lim_{t\to\infty} (y_i - y_j) &= \lim_{t\to\infty} \left(\begin{bmatrix} 0 & R_v \end{bmatrix} \begin{bmatrix} d_i - d_j \\ v_i - v_j \end{bmatrix} \right) \\ &= \lim_{t\to\infty} R_v (v_i - v_j) = 0. \end{aligned} \tag{4.60}$$

that is, the output consensus problem of heterogeneous multi-agent system (4.1) in clustered network is solved.

Necessity

*According to Theorem **??**, one further has $v_c = (\mathcal{L}_c \otimes I_v)v \to 0$ as $t \to \infty$. Moreover, the closed-loop system (4.50)–(4.52) has $y_i = y_j$ and $v_i = v_j$ for $i, j = 1, \cdots, N$. Therefore, it can be rewritten by*

$$\dot{x}_c = A_c x_c, \quad t \in (t_k, t_{k+1}) \tag{4.61}$$
$$y = C_c x_c, \quad t \in (t_k, t_{k+1}), \tag{4.62}$$
$$x_c(t_k^+) = P_c x_c(t_k), \quad t = t_k \tag{4.63}$$

Since $A_{ci} \in \mathbb{R}^{(2n_i+h_i+v)\times(2n_i+h_i+v)}$ and $A_c \in \mathbb{R}^{\mu\times\mu}$, there exits an invertible matrix $\mathcal{V}$ such that

$$\mathcal{V}^{-1} A_c \mathcal{V} = \begin{bmatrix} S & 0 \\ 0 & \star \end{bmatrix}. \tag{4.64}$$

for a matrix $S \in \mathbb{R}^{(v+h_i)\times(v+h_i)}$, and the notation "$\star$" stands for a matrix irrelevant to the problem. Let $\mathcal{V} = [\Upsilon \;\; \Xi]$, then

$$[\Upsilon \;\; \Xi]^{-1} A_c [\Upsilon \;\; \Xi] = \begin{bmatrix} S & 0 \\ 0 & \star \end{bmatrix} \Rightarrow A_c \Upsilon = \Upsilon S, \tag{4.65}$$
$$C_c [\Upsilon \;\; \Xi] = [\Psi \;\; \star]. \tag{4.66}$$

Note that $y_i = y_j \neq 0$, so $\lim_{t\to\infty}(\mathcal{L} \otimes I_q) C_c x_c = 0$. It means that the kernel of the Laplacian matrix is span$\{\boldsymbol{1}\}$. Thus, there exits an $R_v^0 = [0 \; R_v] \in \mathbb{R}^{p\times m}$ such that $\Psi = \boldsymbol{1}_N \otimes R^0, R_v \neq 0$, and $\lim_{t\to\infty}(\mathcal{L} \otimes I_q)(\boldsymbol{1}_N \otimes R_v^0) = 0$. It leads to

$$C_c \Upsilon = \boldsymbol{1}_N \otimes R_v^0. \tag{4.67}$$

Partition $\Upsilon = [\Upsilon_1^\mathsf{T}, \cdots, \Upsilon_N^\mathsf{T}]^\mathsf{T}, \Upsilon_i \in \mathbb{R}^{(2n_i+h_i+v)\times(h_i+v)}$. It follows from (4.65) and (4.67) that

$$A_{ci} \Upsilon_i = \Upsilon_i S, \tag{4.68}$$
$$C_{ci} \Upsilon_i = R_v^0. \tag{4.69}$$

and Υ_i can be rewritten as $\Upsilon_i = [N_i^\mathsf{T} \; Q_i^\mathsf{T} \; Y_{1i}^\mathsf{T} \; Y_{2i}^\mathsf{T}]^\mathsf{T}$, where $N_i \in \mathbb{R}^{n_i\times(h_i+v)}, Q_i \in \mathbb{R}^{n_i\times(h_i+v)}, Y_{1i} \in \mathbb{R}^{h_i\times(h_i+v)}, Y_{1i} \in \mathbb{R}^{v\times(h_i+v)}$.

It follows from (4.68), and (4.53) that

$$A_iN_i + B_iK_{1i}Q_i + (E_{di} + B_iK_{2i})Y_{1i} + B_iK_{3i}Y_{2i} = N_iS, \tag{4.70}$$
$$-F_i^xC_iN_i + (A_i + F_i^xC_i + B_iK_{1i})Q_i + (E_{di} + B_iK_{2i})Y_{1i} + B_iK_{3i}Y_{2i} = Q_iS, \tag{4.71}$$
$$-F_i^dC_iN_i + F_i^dC_iQ_i + A_{di}Y_{1i} = Y_{1i}S, \tag{4.72}$$
$$A_vY_{2i} = Y_{2i}S. \tag{4.73}$$

It is equivalent to

$$B_i\left(K_{1i}Q_i + \begin{bmatrix}K_{2i} & K_{3i}\end{bmatrix}\begin{bmatrix}Y_{1i}\\Y_{2i}\end{bmatrix}\right) + \begin{bmatrix}E_{di} & 0\end{bmatrix}\begin{bmatrix}Y_{1i}\\Y_{2i}\end{bmatrix} + A_iN_i = N_iS, \tag{4.74}$$
$$B_i\left(K_{1i}Q_i + \begin{bmatrix}K_{2i} & K_{3i}\end{bmatrix}\begin{bmatrix}Y_{1i}\\Y_{2i}\end{bmatrix}\right) + \begin{bmatrix}E_{di} & 0\end{bmatrix}\begin{bmatrix}Y_{1i}\\Y_{2i}\end{bmatrix} + + F_i^xC_i(Q_i - N_i) + A_iQ_i = Q_iS, \tag{4.75}$$
$$\begin{bmatrix}F_i^dC_i(Q_i - N_i)\\0\end{bmatrix} + \begin{bmatrix}A_{di} & 0\\0 & A_v\end{bmatrix}\begin{bmatrix}Y_{1i}\\Y_{2i}\end{bmatrix} = \begin{bmatrix}Y_{1i}\\Y_{2i}\end{bmatrix}S. \tag{4.76}$$

the solution of (4.74)–(4.76) is $Q_i = N_i$, $\begin{bmatrix}Y_{1i}\\Y_{2i}\end{bmatrix} = I$, $S = \begin{bmatrix}A_{di} & 0\\0 & A_v\end{bmatrix}$, *and* (**??**) *is obtained with* $M_i = K_{1i}N_i + \begin{bmatrix}K_{2i} & K_{3i}\end{bmatrix}$.

Finally, (4.49) can be obtained directly from (4.69).

Remark 4.4.1 *The solvability of the output consensus of heterogeneous clustered MASs depends on the solvability of the linear matrix equation (4.48)–(4.49), which can be checked by the following condition*

$$rank\begin{bmatrix}A_i - \lambda_lI_{n_i} & B_i\\C_i & 0\end{bmatrix} = n_i + q\ \forall i \in N, \tag{4.77}$$

for each $\lambda_l, l = 1, \cdots, v$, *which is an eigenvalue of* A_v.

Remark 4.4.2 *The control gains of each heterogeneous agent is designed independently based on the linear matrix equation (4.48)–(4.49). Morever, the proposed output consensus protocol (4.5)–(4.8) in considered clustered network is only only dependent on the local information of agents. It means that this protocol is distributed.*

4.5 Illustrative example

Let's us consider a network of 7 agents respectively partitioned into 2 clusters having 4 and 3 elements as depicted in Fig. 4.1. The dynamics of each agent i is characterized by

- Agent 1 and 5

$$A_i = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & a_i \\ 0 & -b_i & c_i \end{bmatrix}, B_i = \begin{bmatrix} 0 \\ 0 \\ d_i \end{bmatrix}, C_i = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix},$$
$$Ed_i = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}, Ad_i = \begin{bmatrix} 0 & w_i \\ -w_i & 0 \end{bmatrix}.$$

The parameters $\{a_i,\ b_i,\ c_i,\ d_i,\ w_i\}, i = 1,\ 5$ are set as $\{2,\ 3,\ 4\ ,1,\ 1.4\}$ and $\{1,\ 4,\ 2,\ 1,\ 1.2\}$, respectively.

- Agent 2 and 6

$$A_i = \begin{bmatrix} 0 & 1 \\ 0 & -a_i \end{bmatrix}, B_i = \begin{bmatrix} 0 \\ b_i \end{bmatrix}, C_i = \begin{bmatrix} 1 & 0 \end{bmatrix},$$
$$Ed_i = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, Ad_i = \begin{bmatrix} 0 & w_i \\ -w_i & 0 \end{bmatrix}.$$

The parameters $\{a_i,\ b_i\}, i = 2,\ 6$ are set as $\{2,\ 1,\ 1.6\}$, and $\{0.5,\ 0.5,\ 1.7\}$, respectively.

- Agent 3, 4 and 7

$$A_i = \begin{bmatrix} 1 & a_i \\ 0 & 1 \end{bmatrix}, B_i = \begin{bmatrix} a_i^2 \\ a_i \end{bmatrix}, C_i = \begin{bmatrix} 1 & 0 \end{bmatrix},$$
$$Ed_i = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, Ad_i = \begin{bmatrix} 0 & w_i \\ -w_i & 0 \end{bmatrix}.$$

The parameters $\{a_i,\ w_i\}, i = 3, 4, 7$ are set as $\{4,\ 1.9\}$, $\{2,\ 1.5\}$ and $\{6,\ 1.1\}$, respectively.

Moreover, suppose that agent 3 in cluster 1 and agent 5 in cluster 2 are the leaders of those clusters, respectively. We consider the internal reference models, where

$$A_v = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}, C_v = \begin{bmatrix} 1 & 0 \end{bmatrix}.$$

and the Lalacian matrix of leader network $\mathcal{G}_l$ is given by

$$\mathcal{L}_l = \begin{bmatrix} 0.6 & -0.6 \\ -0.5 & 0.5 \end{bmatrix} \Rightarrow P_l = \begin{bmatrix} 0.4 & 0.6 \\ 0.5 & 0.5 \end{bmatrix}$$

By choosing the parameter $\alpha = 0.4$ and $R_v = [1\ \ 0]$, consensus matrix gains for the virtual references are determined by solving the LMIs (4.39) in *Theorem* **??** as

$$K_v = \begin{bmatrix} 0.2038 & 0.0598 \end{bmatrix}^{\mathsf{T}}, c = 6.3.$$

Next, the solution of (4.48) and (4.49) with respect to each agent i is

$$N_{i=2,6} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 1 \end{bmatrix}, M_i = \begin{bmatrix} \frac{a_i}{b_i} & -\frac{1}{b_i} & -\frac{1}{b_i} & \frac{a_i}{b_i} \end{bmatrix},$$

$$N_{i=1,5} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 1 \\ -0 & -\frac{2}{a_i} & -\frac{1}{a_i} & 0 \end{bmatrix}, M_i = \begin{bmatrix} \frac{2w_i - a_i b_i}{a_i d_i} & \frac{2c_i}{a_i d_i} & \frac{c_i}{a_i d_i} & \frac{a_i b_i - 1}{a_i d_i} \end{bmatrix},$$

$$N_{i=3,4,7} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ \frac{1}{w_i} & \frac{1}{a_i w_i} & \frac{1}{a_i} & \frac{1}{a_i} \end{bmatrix}, M_i = \begin{bmatrix} -\frac{a_i + w_i}{a_i^2 w_i} & -\frac{1}{a_i^2 w_i} & -\frac{2}{a_i^2} & 0 \end{bmatrix},$$

and the other consensus gains K_{1i}, K_{2i}, K_{3i} and F_i can be obtained based on *Theorem 4.4.1*

$$\begin{aligned}
&K_{11} = -\begin{bmatrix} 70 & 38.5 & 20 \end{bmatrix}, K_{21} = -\begin{bmatrix} 40 & 16 \end{bmatrix}, K_{31} = \begin{bmatrix} 62 & 41 \end{bmatrix}, \\
&F_1 = \begin{bmatrix} -19 & -168.2961 & -365.8197 & 15.2561 & 15.7831 \end{bmatrix}^{\mathsf{T}} \\
&K_{12} = \begin{bmatrix} -14 & -7 \end{bmatrix}, K_{22} = \begin{bmatrix} -5 & -1 \end{bmatrix}, K_{32} = \begin{bmatrix} 13 & 9 \end{bmatrix}, \\
&F_2 = \begin{bmatrix} -12 & -14.139 & -30.3 & -22.138 \end{bmatrix}^{\mathsf{T}}, \\
&K_{13} = \begin{bmatrix} -1.5 & 3.25 \end{bmatrix}, K_{23} = \begin{bmatrix} -1.9046 & -0.4605 \end{bmatrix}, \\
&K_{33} = \begin{bmatrix} 0.5625 & -0.8125 \end{bmatrix}, K_{34} = \begin{bmatrix} 0.25 & -0.3611 \end{bmatrix}, \\
&F_3 = \begin{bmatrix} -16.0000 & -18.6473 & -7.8009 & -18.8578 \end{bmatrix}^{\mathsf{T}},
\end{aligned}$$

$$\begin{aligned}
&K_{14} = \begin{bmatrix} -0.6667 & 2.1667 \end{bmatrix}, K_{24} = \begin{bmatrix} -2.1490 & -0.3535 \end{bmatrix}, \\
&F_4 = \begin{bmatrix} -16.0000 & -15.3200 & 7.1298 & -18.3000 \end{bmatrix}^{\mathsf{T}} \\
&K_{15} = -\begin{bmatrix} 24 & 22 & 11 \end{bmatrix}, K_{25} = -\begin{bmatrix} 23.6 & 18 \end{bmatrix}, K_{35} = \begin{bmatrix} 15 & 25 \end{bmatrix}, \\
&F_5 = \begin{bmatrix} -17 & -133.596 & -452.981 & 20.036 & 41.118 \end{bmatrix}^{\mathsf{T}}, \\
&K_{16} = \begin{bmatrix} -28 & -17 \end{bmatrix}, K_{26} = \begin{bmatrix} -16 & -2 \end{bmatrix}, K_{36} = \begin{bmatrix} 26 & 18 \end{bmatrix}, \\
&F_6 = \begin{bmatrix} -13.5000 & -37.6456 & -23.7144 & -37.6603 \end{bmatrix}^{\mathsf{T}}, \\
&K_{17} = \begin{bmatrix} -6 & 6.5 \end{bmatrix}, K_{27} = -\begin{bmatrix} 4.92 & 2.33 \end{bmatrix}, K_{37} = \begin{bmatrix} 2.25 & -3.25 \end{bmatrix}, \\
&F_7 = \begin{bmatrix} -16.0000 & -43.4434 & 3.1368 & -34.1038 \end{bmatrix}^{\mathsf{T}}.
\end{aligned}$$

The initial conditions $x_i(0), \hat{x}_i(0), \hat{d}_i(0)$ are randomly chosen within $[-10\ 10]$. Then, the states of internal reference models and variable states ψ_2 are indicated in Fig. 4.2 It is clear that the variables ψ_2 comes to zero, which means that v_i reaches consensus. Moreover, the states of leader 1 and 2 are updated based on the leader's communication $\mathcal{G}_l$, which are shown in Fig.2 (red and blue color).

Then, α is varied in $[0.04, 0.4]$ and the LMIs (4.39) is solved corresponding to those values of α. Afterward, it observes that the consensus of internal references is also achieved in this circumstance. Particularly, Fig. 4.3 indicated the consensus

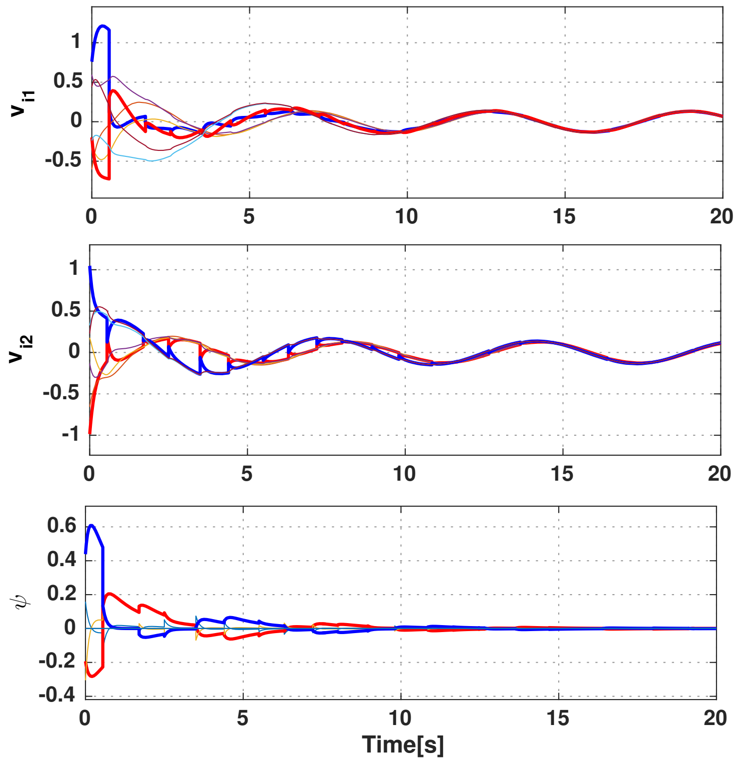

FIGURE 4.2: The states of internal reference models v_i, the new variable ψ_2.

responses with different values of α, as well as the evolution of $E = \psi_2^T \psi_2$ with different α is depicted in Fig. 4.4. It shows that the consensus depends on α, i.e., it is faster as α is bigger.

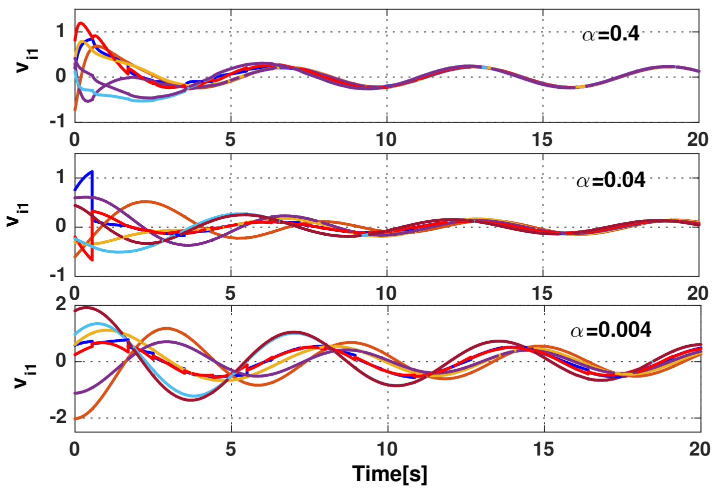

FIGURE 4.3: The states of of internal reference model v_{i1} with different α.

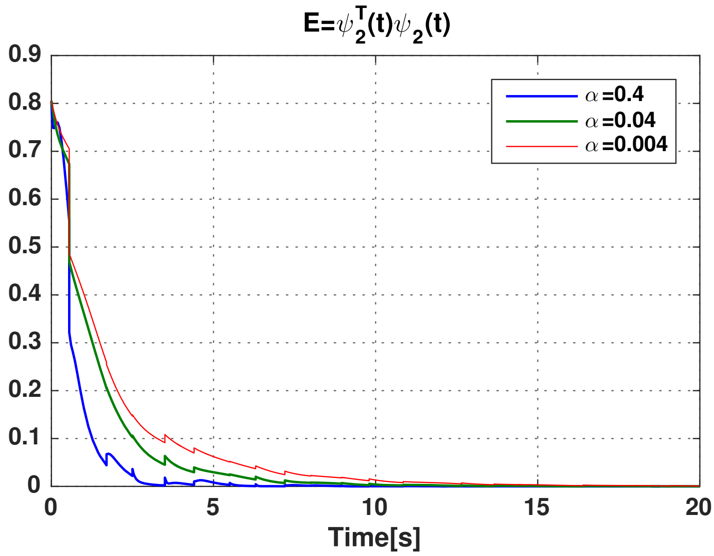

FIGURE 4.4: $E = \psi_2^T \psi_2$ with different α.

The estimation of disturbances $\hat{d}_1$ and estimated states of agents $\hat{x}_2$ in (4.6) are displayed in Fig. 4.6. Moreover, estimation disturbance errors e_{di}, observation errors $e_{xi}, i = 1, 2, \cdots, N$ are also depicted in Fig. 4.5. This clearly shows that by choosing appropriate gains $K_{1i} \in \mathbb{R}^{p_i \times n_i}$ and $F_i \in \mathbb{R}^{(q+h_i)\times n_i}$ such that $A_i + B_i K_{1i}$ and $A_i^d + F_i C_i$ are Hurwitz, the disturbances and states of agents can be estimated well. Furthermore, Fig. 4.6 illustrates that all agents achieve output consensus and track the internal reference states v_i (corresponding to disagreement vector e_i goes to zeros), which demonstrates the effectiveness of the proposed control law for output consensus.

4.6 Chapter summary

In this chapter, a distributed control consensus protocol, based on internal reference models, for heterogeneous MASs with different disturbances on complex networks with fixed and directed topology has been proposed. We indicated that the consensus of internal reference models in virtual clustered network can be indirectly solved by considering the stability of an equivalent system. Then, a sufficient and necessary condition were derived for the output consensus of linear heterogeneous MASs. Finally, an illustrative example was given to show the effectiveness of the proposed theoretical results.

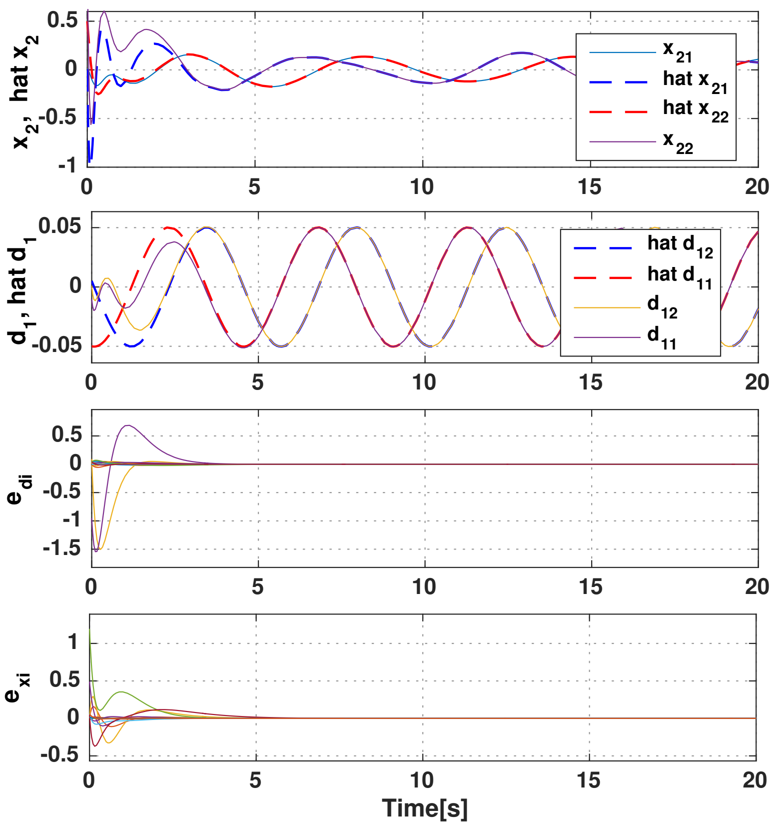

FIGURE 4.5: Estimated states $\hat{x}_2$, estimated disturbance $\hat{d}_1$, estimated disturbance error e_{di}, and observation error e_{xi}.

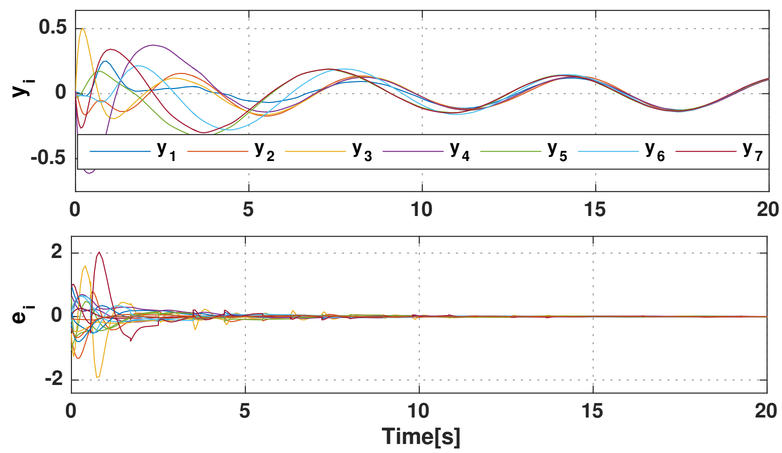

FIGURE 4.6: The output of agent y_i and the disagreement vector e_i.

Chapter 5

Conclusion and Future Research Directions

Contents

Conclusions

Nous avons étudié le contrôle et l'analyse des réseaux en grappes. Nous pouvons résumer cette thèse dans les points suivants :

- L'état de l'art relatif aux problèmes de contrôle coopératif a été présenté au chapitre 1. Nous avons présenté les résultats de la littérature sur le contrôle coopératif des MASs en nous basant sur quatre facteurs: la topologie du réseau, la dynamique des MASS, les contraintes et les méthodologies. Nous avons également présenté nos motivations et les questions de recherche ouvertes pour les sujets de recherche.

- Le chapitre 2 aborde le problème du consensus dans le réseau en grappe, où chaque nœud du graphe de réseau représente un agent à dynamique linéaire. Le comportement coopératif des MASs linéaires avec la dynamique générale du système dans le réseau en grappe est défini non seulement par les protocoles de contrôle dynamique concernant les grappes isolées, mais aussi par les interactions discrètes entre les leaders. Cette évidence rend le problème du consensus dans le réseau en grappe avec des agents linéaires généraux beaucoup plus difficile que celui du cas de l'intégrateur. En outre, un autre défi auquel nous sommes confrontés est de savoir comment reconstruire l'ensemble des informations sur l'état de chaque agent en utilisant uniquement les informations de sortie relatives locales et les interactions discrètes entre les grappes de leaders. Par conséquent, un contrôle impulsif basé sur l'observation est proposé pour traiter le problème du consensus. Ensuite, nous avons fourni un moyen de calculer la valeur du consensus en ne tenant compte que des conditions initiales du système et de la topologie du réseau.

- Le chapitre 3 étudie le problème du contrôle de la formation dans les systèmes de réseaux en grappes composés d'agents linéaires qui sont soumis à des contraintes d'état. La structure de communication en temps continu dans chaque grappe est représentée par un graphe fixe et non dirigé. Un protocole de formation robuste, qui traite de la communication en temps continu à l'intérieur des grappes et de l'échange d'informations en temps discret entre les grappes, est introduit. Il est ensuite montré que le problème de contrôle de la formation robuste considéré peut être indirectement résolu en étudiant la stabilité robuste d'un système équivalent basé sur la théorie des matrices et la théorie des graphes algébriques. La stabilité du système est également démontrée.

- Le chapitre 4 traite du problème du consensus de sortie dans les réseaux groupés composés de MASs hétérogènes qui sont soumis à différentes perturbations. Un contrôle de consensus de sortie est proposé pour gérer le consensus dans le réseau considéré. Nous proposons que chaque agent dispose d'un modèle de référence interne intégré dans un contrôleur, qui est généré à partir d'un centre de commande cybernétique. Ces modèles de référence internes ont une dynamique identique et peuvent être considérés comme des écosystèmes virtuels qui génèrent des entrées de référence virtuelles pour les agents. Ils interagissent également par le biais d'un réseau appelé réseau virtuel en grappe qui a la même structure que le réseau physique en grappe d'agents hétérogènes.

Contributions

Les contributions de la thèse sont les suivantes.

- Le regroupement en réseaux est souvent considéré uniquement par les agents ayant un seul type de dynamique intégratrice (soit continue, soit discrète). Un consensus hybride a été étudié, mais selon une approche très différente de celle proposée ici. L'ajout d'une connexion en dehors des clusters qui réinitialise la valeur d'un seul agent permet de diminuer la taille d'un réseau tout en le maintenant connecté. On peut voir cela appliqué aux réseaux qui souffrent de communications longue distance ou de transmissions défectueuses.

- La caractérisation de la valeur du consensus global dans le cadre considéré est analysée. Nous montrons que la valeur du consensus global ne dépend que de la dynamique de chaque agent, des graphiques des clusters, de l'interaction entre les leaders, et des conditions initiales. En outre, un contrôle impulsif basé sur l'observation, qui n'utilise que les informations de sortie relatives locales et l'interaction discrète entre les groupes de leaders, est conçu. Ensuite, nous montrons que la conception consensuelle des réseaux en grappes peut être indirectement résolue en considérant la stabilité d'un système équivalent. Pour étudier la stabilité de ce système équivalent, nous proposons l'algorithme 1 pour choisir de manière appropriée les matrices de rétroaction et de gain des observateurs et les poids de couplage sous la forme de quelques LMIs. Nous avons également conçu l'interaction entre les grappes de leaders, en veillant à ce que les agents des réseaux en grappes renferment une cible prescrite (voir l'algorithme 2).

- Nous avons proposé le problème du contrôle de la formation des états sous contraintes d'état dans les MASs en grappes où les agents ont une dynamique linéaire générique. Notre approche couvre des systèmes et des scénarios plus larges que ceux des études existantes d'un réseau groupé. Ensuite, un protocole de formation robuste, qui traite de la communication en temps continu à l'intérieur des grappes et de l'échange d'informations en temps discret entre les grappes, est introduit. Par rapport aux résultats précédents du réseau en grappes, le protocole est plus pratique et plus compliqué. Il est alors montré que le problème de contrôle de formation robuste considéré peut être indirectement résolu en étudiant la stabilité robuste d'un système équivalent par la théorie des matrices et la théorie des graphes algébriques. En comparaison avec celle du réseau en grappe, notre approche montre le rôle important de la communication entre les leaders à certains instants discrets spécifiques, représentés par la matrice stochastique. En conséquence, une condition suffisante sera dérivée en termes LMIs pour la formation distribuée robuste de réseaux groupés d'agents linéaires génériques sous contraintes d'état et de communications hybrides.

- Nous avons étudié un cadre général du problème de consensus dans les réseaux groupés dirigés de MASs, où les agents ont des dynamiques linéaires distinctes et génériques sous différentes perturbations. Un modèle de référence interne dynamique pour chaque agent a été introduit, qui prend en compte les communications en temps continu entre les modèles de référence internes dans les grappes virtuelles et les échanges d'informations discrètes entre ces grappes virtuelles. Nous avons fourni une condition suffisante et nécessaire dérivée pour le consensus de sortie des agents linéaires hétérogènes sous différentes perturbations dans le réseau en grappe.

5.1 Conclusions

We have investigated the control and analysis of cluster-patterned networks. We can summarize this dissertation in the following points:

- The state of the art related to cooperative control problems was presented in Chapter 1. We presented the results of the literature for cooperative control of MASs based on four factors: network topology, dynamics of MASs, constraints, and methodologies. We also presented our motivation and open research questions for the research topics.

- Chapter 2 addresses the problem of consensus in the clustered network, where each node of the network graph represents an agent with linear dynamics. The cooperative behavior of linear MASs with general system dynamics in the clustered network is defined by not only the dynamical control protocols concerning the isolated clusters but also the discrete interactions among the leaders. This evidence makes a consensus problem in the clustered network with general linear agents much more challenging than that of the integrator case. Moreover, another challenge we face is how to rebuild the full state information of each agent by using only the local relative output information, and discrete interaction between leaders' clusters. Therefore, an impulsive observer-based control is proposed to handle the consensus problem. Afterward, we provided a way to compute the consensus value given only the initial conditions of the system and the topology of the network.

- Chapter 3 studies the formation control problem in clustered network systems composing of linear agents that are subjected to state constraints. The continuous-time communication structure in each cluster is represented by a fixed and undirected graph. A robust formation protocol, which deals with the continuous-time communication inside clusters and discrete-time information exchange between clusters, is introduced. It is then shown that the considered robust formation control problem can be indirectly solved by studying the robust stability of an equivalent system based on matrix theory and algebraic graph theory. The stability of the system is also proven.

- Chapter 4 discusses the output consensus problem in the clustered networks composed of heterogeneous MASs that are subjected to different disturbances. An output consensus control is proposed to handle the consensus in the considered network. We propose that each agent has an internal reference model embedded in a controller, which is generated from a cyber command center. Those internal reference models have identical dynamics and can be viewed as virtual ecosystems that generate virtual reference inputs for agents. They also interact through a network called virtual clustered networks which have the same structure as the physical clustered network of heterogeneous agents.

5.2 Contributions

The contributions of the thesis are the following.

- Clustering in networks is often considered solely by agents with one type only of integrator dynamics (either continuous or discrete). Hybrid consensus has been studied but in a very different approach than the approach herein proposed. Adding a connection outside of clusters that resets the value of only one agent allows to diminish the size of a network while keeping it connected. We can see this applied to networks that suffer from long distance communications or faulty transmissions.

- The characterization of the global consensus value in the considered framework is analyzed. We show that the value of global consensus depends only on the dynamics of each agent, the graphs of clusters, the interaction between leaders, and the initial conditions. In addition, an impulsive observer-based control, which uses only the local relative output information, and discrete interaction between leaders' clusters, is designed. Then, we show that the consensus design for clustered networks can be indirectly solved by considering the stability of an equivalent system. To study the stability of this equivalent system, we propose Algorithm 1 to suitably choose the feedback and observer gain matrices and coupling weights in the form of some LMIs. We also designed the interaction among leaders' clusters, ensuring agents in clustered networks enclose a prescribed target (see Algorithm 2).

- We proposed the state formation control problem under state constraints in clustered MASs where agents have generic linear dynamics. Our approach covers broader systems and scenarios than those in the existing studies of a clustered network. Next, a robust formation protocol, which deals with the continuous-time communication inside clusters and discrete-time information exchange between clusters, is introduced. Compared with the previous results of the clustered network, the protocol is more practical and complicated. It is then shown that the considered robust formation control problem can be indirectly solved by studying the robust stability of an equivalent system by matrix theory and algebraic graph theory. In comparison with the one in the clustered network, our approach shows the important role of communication between leaders at some specific discrete instants, represented by the stochastic matrix. Accordingly, a sufficient condition will be derived in terms of LMIs for the robust distributed formation of clustered networks of generic linear agents under state constraints and hybrid communications.

- We investigated a general setting of the consensus problem in directed clustered network of MASs, where agents have distinct and generic linear dynamics under different disturbances. A dynamic internal reference model for each agent was introduced, which takes into account the continuous-time communications among internal reference models in virtual clusters and discrete information exchanges between those virtual clusters. We provided a sufficient and necessary condition derived for the output consensus of linear heterogeneous agents under different disturbances in the clustered network.

5.3 Suggested Future Research

We think that the obtained results in this thesis can be further extended in several directions. We suggest some of these directions below.

- As shown in Chapter 2, the reset sequence Δ effects on the convergence rate of the system. Moreover, the former is a specified value. However, when the clustered network reaches consensus, the exchanging information between leaders is not necessary. The latter even leads to redundancy of information at the moment. In order to overcome this problem, we will use the event-trigger communication technique, which is first introduced to MASs in Dimarogonas and Johansson, 2009.

- In Chapter 3, according to the LMIs (3.40)–(3.42), one sees that the dimension of variables $P \in \mathbb{R}^{n\times n}, X \in \mathbb{R}^{p\times n}$ are just equal to that of the matrix $A \in \mathbb{R}^{n\times n}$ of each agent. Thus, we can solve LMIs (3.40)–(3.42) in fully distributed fashion i.e., each agent can compute the gain matrix K by itself and implement the formation protocol (3.2) using only local information (its information and its neighbors' information). However, to obtain LMIs (3.40)–(3.42), the Laplacian matrix is required as a symmetric matrix (i.e., the graph of clusters is undirected and connected). The latter is a quite strong assumption. Therefore, based on the gain scheduling techniques, we will investigate the robust formation in the clustered network with a spanning tree topology in each cluster.

- Considering the output consensus of heterogeneous MASs in Chapter 4, designing output controller gains K_{1i}, K_{2i} and K_{3i} in Theorem 4.4.1 are obtained by solving the linear matrix equations (4.48)-(4.49). However, the general solution to this kind of linear matrix equations is not easy to find. It depends on not only the dynamics of each agent A_i, B_i, but also the dynamics of the internal reference model A_v. Thus, to overcome this disadvantage, we suggest to apply reinforcement learning (such as in Modares, Lewis, and Jiang, 2015) in order to determine controller gains because the method does not require knowledge of the system dynamics.

- The input and communication delay existing in the MASs leads to extra integral terms in the transformed systems, and the analysis of the integral terms makes the derived conditions more conservative. We suggest that a problem worth investigating in the consensus of clustered network with input and communication delay is to characterize the consensus value, with synchronous and asynchronous resets.

- Most existing works assume that every agent is autonomous. The coupling among agents is only introduced with the designed cooperative control. However, in many real scenarios, direct physical coupling exists among agents. In a power network, each bus is coupled to neighboring buses through the so-called tie lines. Therefore, the group behavior is influenced by both the physical coupling and the designed cooperative connection. In the literature, the research on physical coupling of MASs is rare. The research on this topic is closely related to that on large-scale systems, but the research may have more focuses on network behaviors.

Appendix A

Mathematical Preliminaries

we present some definitions of certain kinds of matrices such as stochastic matrix, positive definite and negative definite matrices and so on.

A.0.1 Stochastic matrix

A stochastic matrix (also named Markov matrix) is a matrix used to describe the transitions of probabilities, with each of its entries as a nonnegative real number.

Definition A.0.1 *(Jadbabaie and Morse, 2003) Let F_p a nonnegative and square matrix whose row sums are all equal 1 (i.e., $F_p\boldsymbol{1} = \boldsymbol{1}$). Then, matrix F_p is called row stochastic.*

A.0.2 Positive definite and negative definite matrices

A positive definite matrix is a special case of symmetric matrix[1].

Definition A.0.2 *(Bernstein, 2009) A symmetric $n \times n$ real matrix A is said to be positive (semi)definite if $z^T Az$ is positive (non-negative) for every non-zero column vector of n real numbers. Similarly, a negative definite matrix is defined by the same expression, but in this case $z^T Az$ must be always negative.*

A.0.3 Irreducible matrix

Definition A.0.3 *(Roger A. Horn, 2013) Let A be non-negative $n \times n$. Then A is irreducible if and only if $(I + A)^{n-1} > 0$.*

In a special case related to graph theory, by replacing non-zero entries in the matrix by one, and viewing the matrix as the adjacency matrix of a directed graph, the matrix is irreducible if and only if such directed graph is strongly connected.

A.0.4 SIA matrix

Definition A.0.4 *(Seneta, 1981) A matrix P is said to be a SIA matrix (i.e., stochastic, irreducible, and aperiodic) if it is stochastic and $Q = \lim_{n\to\infty} P^n$ exits and all rows of Q are the same.*

[1]A symmetric matrix is a square matrix that is equal to its transpose. In other words, considering a matrix A, A is symmetric if $A = A^T$

A.0.5 Kronecker product

Definition A.0.5 *(Bernstein, 2009) The Knonecker product of matrices $A \in \mathbb{R}^{m\times n}$ and $B \in \mathbb{R}^{p\times q}$ with $a_{ij} = [A]_{ij}$ and $b_{ij} = [B]_{ij}$, and $B \in \mathbb{R}^{p\times q}$ is the $mp \times nq$ matrix defined as*

$$A \otimes B = \begin{bmatrix} a_{11}B & \dots & a_{1n}B \\ \vdots & \ddots & \vdots \\ a_{m1}B & \dots & a_{mn}B \end{bmatrix}$$

which satisfies the following properties:

$$\begin{aligned} &(A \otimes B)(C \otimes D) = (AC) \otimes (BD) \\ &(A \otimes B)^T = A^T \otimes B^T \\ &A \otimes B + A \otimes C = A \otimes (B + C) \end{aligned}$$

$$\begin{aligned} &(A + B) \otimes C = A \otimes C + B \otimes C \\ &(A + \mu B) \otimes C = A \otimes C + \mu B \otimes C \\ &(A \otimes B)^{-1} = A^{-1} \otimes B^{-1} \end{aligned}$$

A.0.6 Gershgorin circle theorem

Definition A.0.6 *(Roger A. Horn, 2013) Let A be a complex $n \times n$ matrix, with entries a_{ij}. For $i \in \{1, 2, \cdots, n\}$ let $R_i = \sum_{i\neq j} |a_{ij}|$ be the sum of the absolute values of the non-diagonal entries in the i^{th} row. Let $D(a_{ii}, R_i)$ be the closed disc centered at a_{ii} with radius R_i. Such a disc is called a Gershgorin disc.*

A.0.7 Linear Matrix Inequality (LMI)

A LMI has the following form (see Boyd et al., 1994):

$$F(x) \triangleq F_0 + \sum_{i=1}^{m} x_i F_i > 0, \tag{A.1}$$

where $x \in \mathbb{R}^m$ is the variable symmetric matrices $F_i = F_i^T \in \mathbb{R}^{n\times n}, i = 0, \dots, m$ are given. We also encounter the nonstrict form of (A.1) as:

$$F(x) \geq 0 \tag{A.2}$$

The strict LMI (A.1) and the nonstrict LMI (A.2) are closely related.

The Schur complement converts a class of convex nonlinear inequalities that appears regularly in control problems to an LMI. The convex nonlinear inequlities are

$$\begin{cases} R(x) > 0 \\ Q(x) - S(x)R(x)^{-1}S(x)^T > 0 \end{cases} \tag{A.3}$$

where $Q(x) = Q(x)^T$, $R(x) = R(x)^T$, and $S(x)$ depend affinely[2] on x. The Schur complememtß converts this set of convex nonlinear inequalities into the equivalent LMI

$$\begin{bmatrix} Q(x) & S(x) \\ S(x)^T & R(x) \end{bmatrix} > 0, \tag{A.4}$$

A.0.8 Sector nonlinearities and S-procedure

Stability analysis and design for systems with saturation nonlinearities has received tremendous attention in recent years. To easily be used for controller synthesis, the conditions can be expressed as LMIs in terms of all the varying parameters. One of useful tools is S-procedure, which is frequently used in system theory (Han, 2005).

Sector nonlinearities

A function $\phi : \mathbb{R} \to \mathbb{R}$ is said to be in sector $[l, u]$ if for all $q \in \mathbb{R}, p = \phi(q)$ lies between lq and uq,

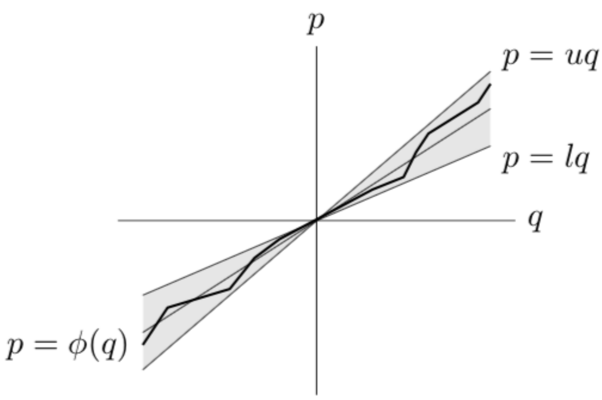

FIGURE A.1: Sector nonlinearities

which is alo can be expressed as quadratic inequality

$$(p - uq)(p - lq) \leq 0 \quad \forall q, p = \phi(q) \tag{A.5}$$

There are some equivalent statements related to sector $[l, u]$

- ϕ is in sector $[l, u]$ if and only if for all q,

$$\left| \phi(q) - \frac{u + l}{2} q \right| \leq \frac{u - l}{2} |q| \tag{A.6}$$

- ϕ is in sector $[l, u]$ if and only if for each q there is $\theta(q) \in [l, u]$ with $\phi(q) = \theta(q)q$.

[2] We say a function $f : \mathbb{R}^m \to \mathbb{R}^n$ is affine if there is a linear function $l : \mathbb{R}^m \to \mathbb{R}^n$ and a vector b in $\mathbb{R}^n$ such that $f(x) = l(x) + b$ for all x in $\mathbb{R}^m$.

Note that a very common type of sector nonlinearity is the standard saturation function $p = sat(q)$ and saturation like functions such as $p = tan^{-1}(q)$, which is used to represents the states constraints in Chapter 3.

S-procedure

The following S-procedure is used for controller synthesis in Chapter 3. Let $\alpha_0, \cdots, \alpha_m$ be quadratic scalar functions of $x \in \mathbb{R}^n$

$$\alpha_i(x) = x^T T_i x + 2u_i^T x + \beta_i, i = 0, \cdots, m; T_i = T_i^T \tag{A.7}$$

The existence of $\tau_i \geq 0, \cdots, \tau_m \geq 0$ such that

$$\alpha_0(x) - \sum_{i=1}^{m} \tau_i \alpha_i(x) \geq 0, \forall x, \tag{A.8}$$

implies that

$$\alpha_0(x) \geq 0, \quad \forall x \text{ such that } \alpha_i(x) \geq 0, \quad i = 1, \cdots, m \tag{A.9}$$

Note that (A.8) is equivalent to

$$\begin{bmatrix} T_0 & u_0 \\ u_0^T & \beta_0 \end{bmatrix} - \sum_{i=1}^{m} \tau_i \begin{bmatrix} T_i & u_i \\ u_i^T & \beta_i \end{bmatrix} \geq 0 \tag{A.10}$$

Appendix B

Basic Algebraic Graph Theory

B.0.1 Basic concept

In the MASs, a team of agents interacts with each other via communication or sensing networks to achieve collective objectives. It is convenient to model the information exchanges by using graph theory. Thus, some definition and notation related to both directed or undirected graphs (Mesbahi and Egerstedt, 2010) are given in this section

Definition B.0.1 *A directed graph $\mathcal{G}$ is a pair $(\mathcal{V}, \mathcal{E})$, where $\mathcal{V} = \{v_1, v_2, \cdots, v_N\}$ is a nonempty finite node set and $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ is an edge set of ordered pairs of nodes, called edges.*

Note here that the edge (v_i, v_j) in the edge set $\mathcal{E}$ denotes that agent v_j can obtain information from agent v_i, but not necessarily vice versa. For an edge (v_i, v_j), node v_i is called the parent node, v_j is the child node, and v_i is a neighbor of v_j. The in-degree of v_i is the number of edges having v_i as a head. The out-degree of a node v_i is the number of edges having v_i as a tail. The set of neighbors of node v_i is denoted as N_i, whose cardinality is called the in-degree of node v_i.

Definition B.0.2 *A graph is said to be undirected if $a_{ij} = a_{ji}, \forall i, j$ that is, if it is bidirectional* [1] *and the weights of edges (v_i, v_j) and (v_j, v_i) are the same.*

If the in-degree equals the out-degree for all nodes $v_i \in \mathcal{V}$ the graph is said to be balanced. Associated with each edge $(v_i, v_j) \in \mathcal{E}$ is a weight a_{ij}. We assume in this chapter that the nonzero weights are strictly positive. Moreover, a directed path from node v_{i1} to node v_{il} is a sequence of ordered edges of the form $(v_{ik}, v_{ik+1}), k = 1, \cdots, l-1$.

Definition B.0.3 *A directed graph is strongly connected if there is a directed path from every node to every other node.*

A (rooted) *directed tree* is a directed graph in which every node has exactly one parent except for one node, called the root, which has no parent and has directed paths to all other nodes.

Definition B.0.4 *A directed tree is defined as spanning when it connects all the nodes in the graph.*

Therefore, a strongly connected graph contains at least one directed spanning.

---

[1] If $(v_i, v_j) \in \mathcal{E}, \forall i, j$ the graph is said to be *bidirectional*

B.0.2 Graph-related matrices

As we have seen before, graphs are constructed to represent a relation between a finite number of agents. Graphs not only admit a graphical representation but can also be associated to certain matrices. In this subsection we will present some important matrices related to graphs (Mesbahi and Egerstedt, 2010).

Adjacency matrix

The *adjacency matrix* is defined as $\mathcal{A} = [a_{(ij)}]$ associated with the directed graph $\mathcal{G}$. It is defined such that $a_{(ij)} > 0$ if $(v_j, v_i) \in \mathcal{E}$, and $a_{ij} = 0$ otherwise. The adjacency matrix $\mathcal{A}$ of an undirected graph is symmetric, $\mathcal{A} = \mathcal{A}^T$.

Define the weighted in-degree of node v_i as the i^{th} row sum of $\mathcal{A}$

$$d_i^{in} = \sum_{i=1}^{N} a_{ij} \tag{B.1}$$

and the weighted out-degree of node $v_i$as the i^{th} column sum of $\mathcal{A}$

$$d_i^{o} = \sum_{i=1}^{N} a_{ji} \tag{B.2}$$

The in-degree and out-degree are local properties of the graph. A graph is said to be *weight balanced* if the weighted in-degree equals the weighted out-degree for all i. If all the nonzero edge weights are equal to 1, this is the same as the definition of balanced graph. An undirected graph is weight balanced, since if $\mathcal{A} = \mathcal{A}^T$ then the i^{th} row sum equals the i^{th} column sum. The adjacency matrix of an undirected graph is defined analogously except that $a_{ij} = a_{ji}$ for all $i \neq j$.

Laplacian matrix

The *Laplacian matrix* $\mathcal{L} = [L_{ij}] \in \mathbb{R}^{N\times N}$ is defined as

$$\begin{cases} L_{ij} = 0, \text{ if } (i,j) \notin \mathcal{E}, \\ L_{ij} = -a_{ij} < 0, \text{ if } (i,j) \in \mathcal{E}, \\ L_{ii} = \sum_{j\neq i} a_{ij}, \forall i = 1, \cdots, N \end{cases} \tag{B.3}$$

$L_{ii} = \sum_{j\neq i} a_{ij}; L_{ij} = -a_{ij}$. The Laplacian matrix can be written into a compact form as $\mathcal{L} = \mathcal{D} - \mathcal{A}$, where $\mathcal{D} = diag\{d_1, d_2, \cdots, d_N\}$ is the degree matrix with d_i as the in-degree of the i^{th} node.

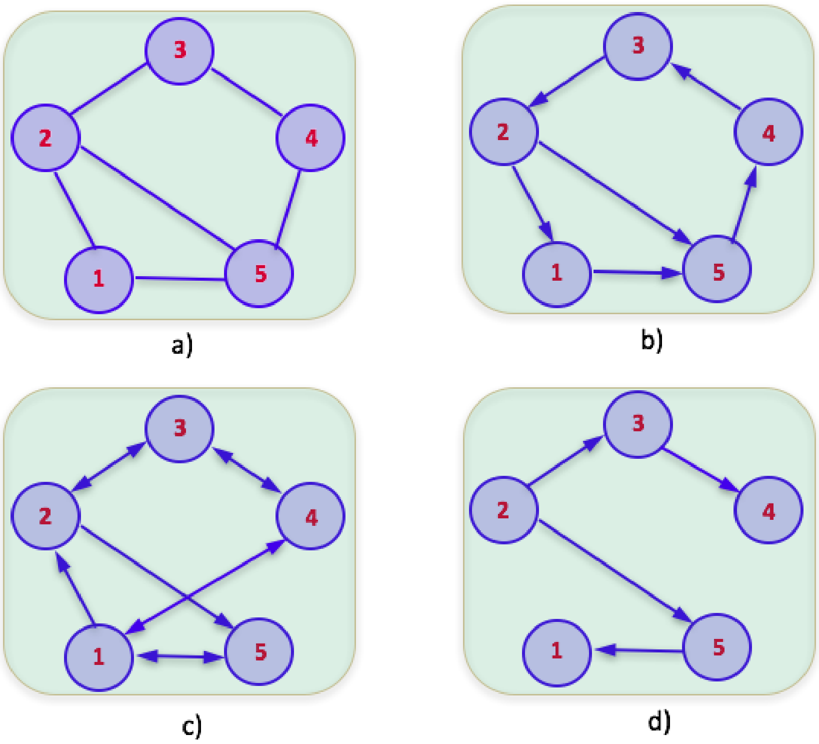

FIGURE B.1: Different types of graphs of five nodes: a) an undirected connected graph, b) a strongly connected graph, c) a balanced and strongly connected graph, d) a directed spanning tree.

Example B.1: For the graphs depicted in Fig B.1, we compute the corresponding Laplacian matrices as

$$\mathcal{L}_a = \begin{bmatrix} 2 & -1 & 0 & 0 & -1 \\ -1 & 3 & -1 & 0 & -1 \\ 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & -1 & 2 & -1 \\ -1 & -1 & 0 & -1 & 3 \end{bmatrix}, \mathcal{L}_b = \begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 & -1 \\ -1 & -1 & 0 & 0 & 2 \end{bmatrix},$$

$$\mathcal{L}_c = \begin{bmatrix} 2 & 0 & 0 & -1 & -1 \\ -1 & 2 & -1 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 \\ -1 & 0 & -1 & 2 & 0 \\ -1 & -1 & 0 & 0 & 2 \end{bmatrix}, \mathcal{L}_d = \begin{bmatrix} 1 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 \\ -1 & 0 & 0 & 0 & 1 \end{bmatrix},$$

For an undirected graph, it is known that $\mathcal{L} \geq 0$, i.e., $y^T \mathcal{L} y \geq 0, \forall y \in \mathbb{R}^N$. In this case, it is not difficult to verify that $\mathcal{L}$ satisfies the following sum-of-squares (SOS) property:

$$y^T \mathcal{L} y = \frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} a_{ij} (y_i - y_j)^2, \tag{B.4}$$

where y_i denotes the i^{th} item of y. For strongly connected and balanced directed graphs, $y^T\mathcal{L}y \geq 0, \forall y \in \mathbb{R}^N$, still holds, due to the property that $\frac{1}{2}(\mathcal{L}+\mathcal{L}^T)$ represents the Laplacian matrix of the undirected graph obtained from the original directed graph by replacing the directed edges with undirected ones (Olfati-Saber, Fax, and Murray, 2007). However, for general directed graphs which are not balanced, the Laplacian matrix $\mathcal{L}$ is not symmetric and $y^T\mathcal{L}y$ can be sign-indefinite.

The Laplacian matrix is very useful in the study of consensus of continuous time MASs, which is shown in the following Lemmas.

Lemma B.0.1 *(Zhongkui Li et al., 2010) The Laplacian matrix $\mathcal{L}$ of a directed graph $\mathcal{G}$ has at least one zero eigenvalue with $\mathbf{1}$ as a corresponding right eigenvector and all nonzero eigenvalues have positive real parts. Furthermore, zero is a simple eigenvalue of $\mathcal{L}$ if and only if $\mathcal{G}$ has a directed spanning tree. In addition, there exists a nonnegative left eigenvector r of $\mathcal{L}$ associated with the zero eigenvalue, satisfying $r^T\mathcal{L} = 0$ and $r^T\mathbf{1} = 1$. Moreover, r is unique if $\mathcal{G}$ has a directed spanning tree.*

Lemma B.0.2 *(Olfati-Saber and Murray, 2004) For an undirected graph, zero is a simple eigenvalue of $\mathcal{L}$ if and only if the graph is connected. The smallest nonzero eigenvalue λ_2 of $\mathcal{L}$ satisfies $\lambda_2 = \min_{x\neq 0, \mathbf{1}^T x=0} \frac{x^T\mathcal{L}x}{x^Tx}$*

Lemma B.0.3 *(Olfati-Saber and Murray, 2004) Suppose that $x = [x_1, x_2, \cdots, x_N]^T$ with $x_i \in \mathbb{R}$. Let $\mathcal{A}, \mathcal{L} \in \mathbb{R}^{N\times N}$ be, respectively, the adjacency matrix and the Laplacian matrix associated with the directed graph $\mathcal{G}$. Then, consensus is reached in the sense of $\lim_{t\to\infty} \|x_i(t) - x_j(t)\| = 0, \forall i, j = 1, 2, \cdots, N$, for the closed-loop system $\dot{x} = -\mathcal{L}z$ or equivalently $\dot{x}_i = -\sum_{j=1}^{N} a_{ij}(x_i - x_j)$, where a_{ij} denotes the $(i, j)^{th}$ entry of $\mathcal{A}$, if and only if $\mathcal{G}$ has a directed spanning tree. Furthermore, the final consensus value is given by $r^T x(0)$, where r is the normalized left eigenvector of $\mathcal{L}$ associated with the zero eigenvalue.*

B.0.3 Eigenstructure of Laplacian matrix

We shall see that the eigenstructure of the graph Laplacian matrix $\mathcal{L}$ plays a key role in the analysis of dynamical systems on Graphs (Lewis et al., 2014). Define the Jordan normal form of the Laplacian matrix by

$$\mathcal{L} = UJU^{-1} \tag{B.5}$$

with the Jordan form matrix and transformation matrix given as

$$J = \begin{bmatrix} \lambda_1 & & & \\ & \lambda_2 & & \\ & & \ddots & \\ & & & \lambda_N \end{bmatrix}, U = \begin{bmatrix} p_1, p_2, \cdots, p_N \end{bmatrix} \tag{B.6}$$

where the eigenvalues λ_i and right eigenvectors u_i satisfy

$$(\lambda_i I - \mathcal{L})p_i = 0 \tag{B.7}$$

with I being the identity matrix. In general, the λ_i in (B.6) are not scalars but are Jordan blocks of the form

$$\begin{bmatrix} \lambda_i & 1 & & \\ & \lambda_i & \ddots & \\ & & \ddots & 1 \\ & & & \lambda_N \end{bmatrix} \tag{B.8}$$

The inverse of the transformation matrix U is given as

$$U^{-1} = \begin{bmatrix} w_1^T & w_2^T & \cdots & w_N^T \end{bmatrix} \tag{B.9}$$

where the left eigenvectors w_i satisfy

$$w_i^T(\lambda_i I - \mathcal{L}) = 0 \tag{B.10}$$

and are normalized so that $w_i^T p_i = 1$.

We assume the eigenvalues are ordered so that $|\lambda_1| \leq |\lambda_1| \leq \cdots \leq |\lambda_N|$. Any undirected graph has $\mathcal{L} = \mathcal{L}^T$ so all its eigenvalues are real and one can order them as $\lambda_1 \leq \lambda_1 \leq \cdots \leq \lambda_N$. Since $\mathcal{L}$ has all row sums zero, one has

$$\mathcal{L}\mathbf{1}c = 0 \tag{B.11}$$

and $\lambda_1 = 0$ is an eigenvalue with a right eigenvector of $\mathbf{1}c$. That is, $\mathbf{1}c \in N(\mathcal{L})$ the null-space of $\mathcal{L}$. If the dimension of the null-space of $\mathcal{L}$ is equal to one, i.e., the rank of $\mathcal{L}$ is $N-1$, then $\lambda_1 = 0$ is nonrepeated and $\mathbf{1}c$ is the only vector in $N(\mathcal{L})$.

B.0.4 Geršgorin circle criterion

All eigenvalues of matrix $E = [e_{ij}] \in \mathbb{R}^{N \times N}$ are located within the union of N disks.

$$\bigcup_{i=1}^{N} \left\{ z \in \mathbb{C} : |z - e_{ij}| \leq \sum_{i \neq j} |e_{ij}| \right\} \tag{B.12}$$

The i^{th} disk in the Geršgorin circle criterion (Olfati-Saber and Murray, 2004) is drawn with a center at the diagonal element e_{ii} and with a radius equal to the i^{th} absolute row sum with the diagonal element deleted, $\sum_{i \neq j} |e_{ij}|$. Thus, the Geršgorin disks for the graph Laplacian matrix $\mathcal{L} = \mathcal{D} - \mathcal{A}$ are centered at the in-degrees d_i and have radius equal to d_i. Let d_{max} be the maximum in-degree of $\mathcal{G}$. Then, the largest Geršgorin disk of the Laplacian matrix $\mathcal{L}$ is given by a circle centered at d_{max} and having radius of d_{max}. This circle contains all the eigenvalues of $\mathcal{L}$ (see Fig. B.2).

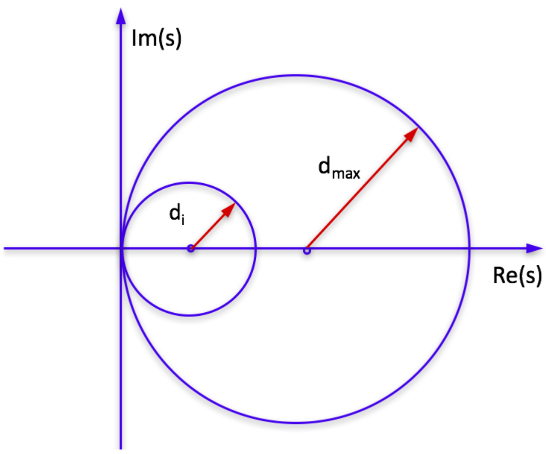

FIGURE B.2: Geršgorin disks of $\mathcal{L}$ in the complex plane

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Contrôle coopératif des systèmes multi-agents dans un réseau en cluster

Le résumé Le fait d'avoir plusieurs agents autonomes pour travailler ensemble efficacement afin d'obtenir des comportements collectifs de groupe est généralement appelé contrôle coopératif des systèmes multi-agents (MAS). En raison du fait que les agents des SMA sont généralement limités en termes de ressources, telles que la portée limitée des communications sans fil (pour échanger des informations entre les agents), des capteurs (pour mesurer les informations relatives entre agents voisins) et des actionneurs (pour piloter les agents), ainsi que des contraintes énergétiques liées aux interactions de longue durée, un ingénieur doit parfois diviser un grand réseau en grappes. Nous abordons d'abord le problème du consensus dans le réseau en grappes, où chaque nœud du graphe du réseau représente un agent à dynamique linéaire. Le comportement coopératif des MAS linéaires avec la dynamique générale du système dans le réseau en grappe est défini non seulement par les protocoles de contrôle dynamique concernant les grappes isolées, mais aussi par les interactions discrètes entre les leaders. Cela rend un problème de consensus dans le réseau en grappe avec des agents linéaires généraux beaucoup plus difficile que celui du cas de l'intégrateur. Ainsi, un contrôle impulsif basé sur l'observation est proposé pour traiter le problème de consensus. Ensuite, nous étudions le problème du contrôle de la formation dans les systèmes de réseaux groupés d'agents linéaires qui sont soumis à des contraintes d'état. La structure de communication en temps continu dans chaque grappe est représentée par un graphique fixe et non dirigé. Pour ce faire, un protocole de formation robuste, qui traite de la communication en temps continu à l'intérieur des grappes et de l'échange d'informations en temps discret entre les grappes, est introduit. Il est ensuite montré que le problème de contrôle de la formation robuste considéré peut être indirectement résolu en étudiant la stabilité robuste d'un système équivalent basé sur la théorie des matrices et la théorie des graphes algébriques. De plus, il montre le rôle important de la communication entre les leaders à certains moments discrets spécifiques, représentés par la matrice stochastique. Enfin, nous discutons du problème du consensus de sortie dans les réseaux groupés composés de MAS hétérogènes qui sont soumis à différentes perturbations. Chaque grappe est représentée par un graphique fixe et dirigé. Un modèle de référence interne dynamique pour chaque agent est introduit, qui prend en compte les communications en temps continu entre les modèles de référence internes dans les grappes virtuelles et les échanges d'informations discrètes entre ces grappes virtuelles. Par conséquent, le consensus de sortie des agents hétérogènes est indirectement résolu par le consensus des références virtuelles. Pour y parvenir, un protocole de contrôle de consensus hybride est proposé pour le réseau en grappe virtuel. Grâce aux résultats de la théorie des matrices et de la théorie des graphes algébriques, le consensus du réseau en grappes virtuel est résolu. Une condition suffisante et nécessaire est dérivée pour le consensus de sortie des agents hétérogènes linéaires sous différentes perturbations dans le réseau en grappe.

Estimation coopérative, systèmes multirobots, réseau en cluster

Cooperative Control of Multi-Agent Systems in the Clustered Network

Having multiple autonomous agents to work together efficiently to achieve collective group behaviors is usually referred to as cooperative control of multi-agent systems (MASs). An arise from the fact that agents in MASs are usually resource-limited, such as limited ranges of wireless communication (for exchanging information among agents), sensors (for measuring relative information between neighboring agents) and actuators (for driving the agents), as well as energy constraints related to long time interactions, an engineer should sometimes partition a large network into clusters. We first address the problem of consensus in the clustered network, where each node of the network graph represents an agent with linear dynamics. The cooperative behavior of linear MASs with general system dynamics in the clustered network is defined by not only the dynamical control protocols concerning the isolated clusters but also the discrete interactions among the leaders. This makes a consensus problem in the clustered network with general linear agents much more challenging than that of the integrator case. Thus, an impulsive observer-based control is proposed to handle the consensus problem. Next, we study the formation control problem in clustered network systems of linear agents that are subjected to state constraints. The continuous-time communication structure in each cluster is represented by a fixed and undirected graph. To do this, a robust formation protocol, which deals with the continuous-time communication inside clusters and discrete-time information exchange between clusters, is introduced. It is then shown that the considered robust formation control problem can be indirectly solved by studying the robust stability of an equivalent system based on matrix theory and algebraic graph theory. Moreover, it shows the important role of communication between leaders at some specific discrete instants, represented by the stochastic matrix. Finally, we discuss the output consensus problem in the clustered networks composed of heterogeneous MASs that are subjected to different disturbances. Each cluster is represented by a fixed and directed graph. A dynamic internal reference model for each agent is introduced, which takes into account the continuous-time communications among internal reference models in virtual clusters and discrete information exchanges between those virtual clusters. Therefore, the output consensus of heterogeneous agents is indirectly solved through the consensus of the virtual references. To achieve that, a hybrid consensus control protocol is proposed for the virtual clustered network. Thanks to results from matrix theory and algebraic graph theory, the consensus of the virtual clustered network is solved. A sufficient and necessary condition is derived for the output consensus of linear heterogeneous agents under different disturbances in the clustered network.

Cooperative control, multi-robot systems, clustered network

Discipline : AUTOMATIQUE, SIGNAL, PRODUCTIQUE, ROBOTIQUE

Spécialité : Automatique

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