Three Essays on Macroeconomics of Informality
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General Introduction

The informal sector plays a significant role in the global labor market. According to the International Labor Organization (2018), over 60 percent of the world’s employed population works in the informal sector. Informality characterizes a wide variety of economic activities, including self-employed individuals (which accounts for more than half of informal employment), small enterprises, and informally hired employees working for formal firms. The term informal economy “refers to all economic activities by workers and economic units that are – in law or in practice – not covered or insufficiently covered by formal arrangements and does not cover illicit activities” (ILO, 2015, p. 23).

Informal employment is found in countries at all levels of socioeconomic development. However, it is the main feature of labor markets in developing countries, where the informal sector has persisted at high rates over the last couple of decades. According to the ILO (2018), informal employment accounts for more than half of non-agricultural employment in most developing countries: around 72 percent in Africa, 63 percent in Asia and the Pacific, 64 percent in the Arab States, 50 percent in Latin America, and 30 percent in Europe and Central Asia. In the case of developed countries, only 17 percent of the urban labor force is employed in informal activities.

Informality is a multidimensional concept, differing in nature across workers and countries (Perry et al., 2007). It is, in some cases, the result of agents that find it optimal to avoid taxation and regulation despite not having the protection and advantages that the law and the state can provide. But in most cases, it is the outcome of agents who cannot find a job in the formal sector and depend on informal jobs as a means of subsistence. In developed countries, informality usually involves tax evasion and undeclared labor rather than a significant share of unregistered businesses. While in developing countries the exclusion factor, coming from costs of formality and the lack of formal jobs, is still quite prevalent (Oviedo et al. 2009).

While offering the advantage of employment and de facto flexibility, a large informal sector is associated with adverse macroeconomic and development outcomes. On average, economies with larger informal sectors tend to have lower productivity, slower physical and human capital accumulation, higher poverty and inequality, and smaller fiscal resources (World Bank, 2019). Since the informal sector is labor-intensive, tends to employ low-skilled workers, has credit constraints, and lacks economies of scale, it has on average lower productivity than the formal sector (Loayza, 2018). Informality
can also hinder growth by distorting firm decisions (e.g., staying small to stay hidden) and allowing unproductive firms to survive, further distorting the allocation of resources (Ulyssea, 2018). Employment in the informal sector can provide a safety net by keeping or creating employment during periods when the formal sector is shedding jobs (Loayza and Rigolini, 2011). However, workers in the informal economy are largely excluded from the social security system and less protected against negative shocks than workers in the formal sector. A large informal sector also reduces tax revenues that limit the ability of the governments to provide public goods and social transfers. According to Besley and Persson (2014), low-income countries typically collect taxes of between 10 to 20 percent of Gross Domestic Product (GDP), while the average for high-income countries is more about 40 percent. This difference is not necessarily explained by a choice of low tax rates but rather by the challenges associated with tax collection: these include informality and misreporting (Besley and Persson 2009).

Because of the wide prevalence of the informal economy and their adverse outcomes for development, understanding how the informal sector emerges and affects the economy and evaluating the aggregate impacts of policies toward informality are central questions in economic development. The incidence and implications of informality in developing economies have attracted the attention of researchers and policy-makers over the last decades. A considerable amount of economic research has focused on empirical evidence, particularly on measuring and determining the causes of informality (Schneider 2005, Perry et al. 2007, Rei and Battacharya 2008, ILO 2002, ILO 2018, Bosch and Maloney 2008 ). Other researchers use a theoretical framework to understand how the informal sector emerges and its implications on the economy (see Rauch, 1991, and Loayza, 1996). In particular, some researchers study how the informal sector generates both positive and negative externalities for the formal economy (see e.g. Fugazza and Jacques, 2003 and Zenou, 2008), and how labor market policies affect the size of the informal sector (see e.g. Bosch and Esteban-Pretel, 2012, Albrecht et al. 2008 and Satchi and Temple, 2009, and Ulyssea, 2018).

Early modeling in the economics of informality started from the classical Harris-Todaro (1970) framework. In these models, informality is captured by building a model of two distinct markets that are segmented and in which two different wage equilibrium prevail (wage duality), where wages in the formal sector can turn out to be higher than the market-clearing wages. Brueckner and Zenou (1999) add a land market to the standard Harris-Todaro framework where wages are endogenously determined. The
idea of identifying the informal labor market with the disadvantaged sector of a market segmented by rigidities in the formal sector dates back to Lewis (1954). Most recent literature has developed more sophisticated models to represent formal, informal, and integrated labor markets (see Boeri and Garibaldi 2005, Fugazza and Jacques 2003, and Badaoui et al. 2006). Most of these models incorporate the search and matching model of Mortensen-Pissarides into Harris and Todaro’s model. Under this framework, it is possible to determine the rules governing the flows between the formal and informal sectors on the one hand and to and for unemployment on the other hand.

The previously cited literature focuses on the intersectoral margin for workers and firms. Other papers like Albrecht et al. (2008), Zenou (2008), and Satchi and Temple (2009) focus on the intersectoral margin for workers. Albrecht et al. (2008) develop a search and matching model with endogenous job destruction. They assume that workers have the same productivity in the informal sector (unregulated self-employment) while they have different productivity when they work in the formal sector. In this way, the relative productivity in the two sectors is an important factor in the workers’ choice. In this model, unemployment is the residual state in the sense that workers whose employment in either an informal-sector or a formal-sector job ends flow back into unemployment.

In general, these papers aim to study how informal jobs in the labor market are created, and the effect of fiscal policy and labor market institutions (such as employment protection legislation, tax wedge, unemployment benefits, unemployment benefit duration, and union density) on informal economic activity. These studies focus on the real economy and do not analyze the interaction between the informal sector and monetary policy. Surprisingly, few papers have been devoted to the monetary policy analysis when the economy displays a large informal sector. Notable exceptions are Castillo and Montoro (2010), Batini et al. (2011), and Alberola and Urrutia (2020).

Given the importance of the labor market structure in determining output, inflation, and the response of the economy to aggregate shocks, it is of great importance to analyze the implications of informality for monetary policy in developing countries. Alberola and Urrutia (2020), for instance, find that that monetary policy is less effective in stabilizing the economy in the presence of an informal sector. They state that “[...] the buffering effect of informality implies that the contraction in labor demand associated to an interest rate hike can be absorbed by a rapid decline in informal workers together with a smaller fall in real wages. This labor channel flattens the Phillips curve, making disinflation harder to achieve. On the other hand, interest rates have a direct impact on
unit labor costs through the financial channel, also making it more difficult for monetary policy to reduce inflation”.

In addition to decreasing the monetary policy’s effectiveness, what are the implications of informality for inflation stabilization and optimal monetary policy design? Aruoba (2010), using a data set covering 118 countries for the period 1996-2004, finds a positive correlation between inflation and the size of the informal sector (with a correlation coefficient of 0.28). I use data for 50 developing countries, for which the information about informal employment is available in the ILOSTAT database. I analyze the correlation between inflation and the size of the informal sector for those developing and emerging countries with an inflation target policy and a flexible exchange rate regime. I find that the correlation between the level of inflation and the size of the informal sector is positive (see Figure 1), with a correlation coefficient of 0.24. Additionally, I find that inflation volatility and the size of the informal sector are positively correlated, with a correlation coefficient of 0.20 (see Figure 1).

![Figure 1 Inflation and Informality](image)

Inflation rate vs. Informality  
Inflation Volatility vs. Informality

**Source:** Own calculation using data of informal employment from the ILOSTAT database, and of inflation from the World Development Indicators for the period 2000-2017.

**Note:** Informality is measured without the agricultural sector. I exclude from the sample the dollarized economies and countries without an inflation target policy. I also take the average inflation and informality for Benin, Côte d’Ivoire, Mali, Niger, and Senegal because they all have the same currency.

These correlations naturally cannot be interpreted as implying causation without a structural model. Castillo and Montoro (2010) is the first paper that analyses the

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I exclude countries with interrupted series, such as Zimbabwe, Maldives, and Mozambique where the data for inflation were not available for some periods. I compute inflation volatility using the standard deviation of the cyclical component of the Hodrick-Prescott (HP) filter applied to the natural logarithm of inflation. In order to avoid overweight observations very close to zero (and to miss observations where inflation is negative).
effect of informal labor markets on monetary policy. They model a dual labor market economy with formal and informal labor contracts within a New Keynesian (NK) model with labor market frictions. They find that during periods of high aggregate demand the informal sector expands due to lower hiring costs. In particular, the authors show that “informal workers act as a buffer stock of labor that allows firms to expand output without putting pressure on wages.” Batini et al. (2011) adopt a similar approach by modeling the presence of a classical labor market together with a wage norm to summarize frictions in the formal sector. They construct a two-sector, formal-informal New Keynesian closed-economy. The informal sector is more labor-intensive, is untaxed, has a classical labor market, faces high credit constraints in financing investment, and is less visible in terms of observed output. They find that the importance of commitment increases in economies characterized by a large informal sector. Simple implementable optimized rules that respond only to observed aggregate inflation and formal-sector output can be significantly worse in welfare terms than their optimal counterpart but are still far better than discretion.

In the same line, Alberola and Urrutia (2020) develop a simple general equilibrium closed economy model with nominal rigidities, labor, and financial frictions to analyze the transmission of shocks and monetary policy. In the model, the informal sector provides a flexible margin of adjustment to the labor market at the cost of lower productivity. In addition, only formal firms have access to financing, which is instrumental in their production process. In a quantitative version of the model, they find that informality dampens the impact of demand and financial shocks, as well as of technology shocks specific to the formal sector, on wages and inflation, but heightens the inflationary impact of aggregate technology shocks.

The first two chapters of this thesis contribute to this literature by analyzing optimal monetary policy with informality. Different from Castillo and Montoro (2010), Batini et al. (2011), and Alberola and Urrutia (2020), the first and second chapters of this thesis provide a theoretical framework to analyze optimal monetary policy with informality.

The first chapter analyzes the main principles of monetary policy according to the relative size of the informal sector. We propose a canonical model in the NK framework in order to derive these principles analytically. We choose to focus on tax avoidance as the key feature of the informal sector. Our model is an NK two-sector economy with taxation only in the formal sector. In this simple model, it is possible to derive the optimal policy recommendations from an approximated quadratic welfare function.
and then characterize the role of the informality size for monetary policy. We find that the presence of variable taxes in the formal sector generates an inflation bias under a discretionary policy, which increases with the size of the informal sector. Secondly, we find that only the formal sector is responsible for the cost-push shocks which are amplified in a more informal economy. The trade-off between inflation and the formal output gap is then dependent on the elasticity of the former variable with respect to the formal output gap. However, the optimal management of inflation also depends on the elasticity of the informal output gap with respect to the formal output gap. Since this elasticity decreases with the size of the informal sector, whether inflation volatility (in terms of the aggregate output gap) is lower or higher in a more informal economy is ambiguous. By simulation, we show that economies with a larger informal sector should stabilize more inflation relative to the two sectoral output gaps and less relative to the total output gap.

In the second chapter, I also analyze optimal monetary policy design using a more sophisticated framework. I develop a closed economy model with dual labor markets, formal and informal, that integrates labor market search into a New Keynesian model with nominal price and wage rigidities. Following Thomas (2008) and Gertler and Trigari (2009), I introduce staggered nominal wage bargaining under which firms and workers in the formal sector bargain over wages in a setting with search and matching frictions. Under this framework, it is possible to analyze the implications of informality for optimal monetary policy design in a scenario where there is a trade-off between inflation and unemployment. I find that the contribution of wage inflation volatility to the welfare loss, relative to the contribution of price inflation volatility, is lower for the case with informality. This result is explained by the fact that in the presence of an informal sector, the proportion of firms in the economy facing wage rigidities is lower. As a result, the optimal policy will result in lower price inflation volatility for a given level of wage inflation volatility.

Additionally, I find that, in the presence of an informal sector, the inefficient fluctuations on the labor market variables such as employment, labor market tightness, and formal hiring rate are higher. This is because, in response to an aggregate productivity shock, only a fraction of firms in the formal sector can adjust their nominal wages. This wage rigidity generates a gap between the actual and the natural formal wage (the target wage), which translates into wage dispersion and inefficient job creation in the formal sector. In the presence of informality, the response of the target wage to productivity
shocks is higher. The target wage in the formal sector depends on the informal wage (the outside option) and the formal labor market tightness. After a negative productivity shock, the decrease in both variables is higher than in the case without informality. As a result, in response to aggregate productivity shocks, the Central Bank should use price inflation to avoid excessive wage inflation volatility that causes excessive unemployment volatility and excessive dispersion in the formal hiring rate. By controlling the inflation rate, the Central Bank can affect the real value of nominal wages and then bring real formal wages closer to their flexible-wage levels. The presence of an informal sector requires a higher adjustment of price inflation to reduce this gap.

To illustrate the implications of the trade-off faced by the Central Bank, I analyze the behavior of a decentralized economy when the monetary authority implements a policy of full inflation stabilization. I find that the welfare loss under a zero price inflation policy is about 1.26 times as large as under the optimal policy. For the case without informality, the welfare loss under a zero price inflation policy is about 0.015 times larger than the regime under the optimal policy. These results show that a policy designed to minimize inflation volatility can generate significant welfare losses in the presence of formal wage rigidities and informality, as is the case for most emerging countries.

The third chapter deviates from the analysis of monetary policy and focuses on analyzing the incidence of job polarization in economies with a large informal sector. Most of the literature on the polarization of employment (see Autor et al. 2003, Autor et al. 2006, Autor and Dorn 2013, Goos et al. 2014, Michaels et al. 2014, Feng and Graetz 2015) focuses on analyzing this phenomenon in developed countries. Recently, there have been some studies that find evidence of job polarization in several low and middle-income countries. The World Development Report (2016) finds that the average decline in the share of routine employment has been 7.8 percentage points for the period 1995-2012. Reijnders and de Vries (2018) also find evidence of an increase in the share of non-routine jobs in total employment for a group of advanced and major emerging countries during the period 1999-2007. They find that for all these countries, technological change was the main force behind employment changes.

In developing and emerging countries, in contrast with developed countries, a substantial fraction of the urban labor force is self-employed in the informal sector performing labor-intensive activities. How then does the job polarization process in developing countries affect the size of the informal sector and vice-versa? In chapter 3, I aim to answer this question by analyzing the incidence of job polarization in developing coun-
tries and its implications for informality. I focus on analyzing how technological change, which is one of the leading explanations for job polarization, could affect the structure of employment and wages in emerging countries. I develop a general equilibrium model with informality and endogenous occupational choice, based on Autor and Dorn (2013). I consider a labor market in which some workers are low-skill while others are high-skill. I assume three sectors in this economy: the goods sector, the formal service sector, and the informal service sector. Workers in the informal service sector can avoid taxation but are less productive.

The analytical solution of the model shows that job polarization, driven by a Routine-Biased Technological Change (RBTC), could lead to an increase in the share of employment in the informal sector and a reduction in the wage inequality at the bottom of the skill distribution. I find that employment and wages in both the formal and the informal service sectors increase due to the increased demand for services. The allocation of labor in the service sector depends on the level of labor income taxes, the degree of substitution between the two types of services, and the level of efficiency in the formal service sector. I find that the share of informal employment in the service sector decreases with technological progress.

**Introduction Générale**

Le secteur informel joue un rôle important sur le marché mondial du travail. Selon l’Organisation internationale du travail (2018), plus de 60% de la population active mondiale, occupe un emploi informel. L’informalité touche des activités économiques multiples, elle concerne notamment les travailleurs indépendants (qui représentent plus de la moitié de l’emploi informel), les petites entreprises et les employés embauchés de manière informelle travaillant pour des entreprises formelles. Le terme d’économie informelle "désigne l’ensemble des activités économiques des travailleurs et des unités économiques qui sont- en droit ou en pratique - non couvertes ou insuffisamment couvertes par des dispositifs formels et ne couvre pas les activités illicites" (ILO, 2015, p. 23).

L’emploi informel est présent dans des pays ayant attend des niveaux de développement socio-économique très variés. Cependant, il est la principale caractéristique des marchés du travail des pays en développement, où le secteur informel a continué de représenter
une part importante l’économie nationale au cours des deux dernières décennies. Selon l’ILO (2018), le secteur informel représente plus de la moitié des emplois non agricoles dans la plupart des pays en développement : environ 72% en Afrique, 63% en Asie et dans le Pacific, 64% dans les États arabes, 50% en Amérique latine et 30% en Europe et en Asie centrale. Dans les pays développés, seule 7 pour cent de la main-d’œuvre urbaine est employée dans des activités informelles.

L’informativité est un concept multidimensionnel, dont la nature diffère selon les travailleurs et les pays concernés (Perry et al. 2007). Certains agents économiques choisissent le travail informel dans un calcul coût-bénéfice conscient, considérant que l’absence de cadre fiscal et réglementaire revaut bien le sacrifice de la protection et des divers avantages offerts par la loi et l’État. Mais la plupart des agents économiques dépendent plutôt du secteur informel comme moyen de subsistance, dans la mesure où ils ont échoué à trouver un emploi dans le secteur formel. Dans les pays développés, l’informativité implique l’évasion fiscale et le travail non déclaré plutôt qu’une part importante d’entreprises non enregistrées. Dans les pays émergents au contraire, le facteur d’exclusion, provenant des coûts de la formalité et du manque d’emplois formels, est encore assez répandu (Oviedo et al. 2009).

Tout en offrant l’avantage de de permettre plus d’emplois et d’offrir une flexibilité naturelle, un secteur informel important est associé à des conséquences macroéconomiques défavorables, y compris pour le développement. En moyenne, les économies dotées d’un secteur informel plus important ont tendance à avoir une productivité plus faible, une accumulation de capital physique et humain plus lente, une pauvreté et des inégalités plus élevées, et des ressources fiscales plus faibles (World Bank, 2019). Étant donné que le secteur informel est à forte intensité de main-d’œuvre, qu’il a tendance à employer des travailleurs peu qualifiés, qu’il est soumis à des contraintes de crédit et qu’il ne bénéficie pas d’économies d’échelle, il montre en moyenne une productivité et une accumulation de capital physique et humain inférieures à celles du secteur formel (Loayza, 2018). L’informativité peut également entraver la croissance en faussant les décisions des entreprises (par exemple, une entreprise non déclarée, peut ainsi décider de rester de petite taille pour éviter de se faire repérer, quand bien même la décision optimale d’un point de vue économique serait de croître) et en permettant aux entreprises peu productives de survivre, faussant alors davantage l’allocation des ressources (Ulyssea, 2018). Certes, L’emploi dans le secteur informel peut constituer un filet de sécurité en maintenant ou en créant des emplois pendant les périodes où le secteur formel supprime
des emplois (Loayza et Rigolini, 2011). Cependant, les travailleurs de l’économie informelle sont largement exclus du système de sécurité sociale et moins protégés contre les crises économiques que les travailleurs du secteur formel. Un secteur informel important réduit également les recettes fiscales limitant ainsi la capacité du gouvernement à fournir des biens publics et à opérer des transferts sociaux.


déterminer des règles régissant les flux entre les deux secteurs, ainsi que vers et depuis le groupe des chômeurs. La plupart de ces modèles incorporent le modèle de recherche et d’appariement de Mortensen-Pissarides au modèle de Harris et Todaro.


Étant donné l’importance de la structure du marché du travail dans la détermination de la production, de l’inflation et de la réponse de l’économie aux chocs de productivité agrégés, il est essentiel d’analyser les implications de l’informalité pour la politique monétaire. C’est seulement en s’appuyant sur ce genre d’analyse, que les banques centrales pourront espérer formuler des politiques monétaires bien adaptées aux économies en développement et émergentes. Alberola et Urrutia (2020) constatent que la politique monétaire est moins efficace pour stabiliser l’économie en présence d’un secteur informel. Ils affirment que l’effet tampon de l’informalité implique que la contraction de la demande de travail associée à une hausse du taux d’intérêt peut être absorbée par une baisse rapide du nombre de travailleurs informels, ainsi que par une baisse plus faible des salaires réels. Ce canal du travail aplatit la courbe de Phillips, rendant la désinflation plus difficile à atteindre. D’autre part, les taux d’intérêt ont un impact
direct sur les coûts unitaires de main-d’œuvre par le biais du canal financier, ce qui rend également plus difficile la réduction de l’inflation par la politique monétaire.

En plus de diminuer l’efficacité de la politique monétaire, quelles sont les implications de l’informalité pour la stabilisation de l’inflation et la conception d’une politique monétaire optimale ? Aruoba (2010), utilisant un ensemble de données couvrant 118 pays pour la période 1996-2004, trouve une corrélation positive entre l’inflation et la taille du secteur informel (avec un coefficient de corrélation de 0,28). Pour ma part, j’utilise des données pour 50 pays en développement, pour lesquelles les informations sur l’emploi informel sont disponibles dans la base de données ILOSTAT. J’analyse la corrélation entre l’inflation et la taille du secteur informel pour les pays en développement et les pays émergents ayant une politique d’objectif d’inflation et un régime de taux de change flexible. Je constate que la corrélation entre le niveau d’inflation et la taille du secteur informel est positive (voir figure 1), avec un coefficient de corrélation de 0,24. En outre, je constate que la volatilité de l’inflation et la taille du secteur informel sont positivement corrélées, avec un coefficient de corrélation de 0,20 (voir la figure 1).

Ces corrélations ne peuvent naturellement pas être interprétées comme impliquant une causalité sans un modèle structurel. Très peu de recherches ont été consacrées à l’analyse des implications de l’informalité sur la politique monétaire. Castillo et Montoro (2010) est le premier article qui s’intéresse à l’effet des marchés du travail informels sur la politique monétaire. Ils modélisent une économie dont le marché du travail est doublé (formel et informel) dans un modèle d’équilibre général Néo Keynésien (NK) incluant des frictions sur le marché du travail. Ils constatent qu’en période de forte demande agrégée, le secteur informel se développe en raison des coûts d’embauche plus faibles. En particulier, les auteurs montrent que "les travailleurs informels agissent comme un stock tampon de main-d’œuvre qui permet aux entreprises d’accroître la production sans agir sur les salaires." Batini et al. (2011) adoptent une approche similaire en modélisant un marché du travail classique associé à une norme salariale qui intègre toutes les frictions du secteur formel. Ils construisent une économie fermée néo keynésienne à deux secteurs, formel et informel. Le secteur informel est plus intensif en main-d’œuvre, ne paye pas d’impôt, a un marché du travail classique, fait face à des contraintes de crédit élevées pour financer l’investissement et est moins visible en termes de production observée. Ils constatent que l’importance de la politique monétaire sous engagement augmente dans les économies caractérisées par un secteur informel impor-
tant. Les règles optimales simples qui ne répondent qu’à l’inflation globale observée et à la production du secteur formel peuvent être nettement moins bonnes en termes de bien-être que leur contrepartie optimale, mais restent bien meilleures que la discrétion.


Le premier chapitre, co-écrit avec Jean-Olivier Hairault, étudie les grands principes de la politique monétaire en fonction de la taille relative du secteur informel. Nous proposons un modèle canonique dans le cadre NK afin de dériver analytiquement ces principes. Nous choisissons de nous concentrer sur l’évasion fiscale comme caractéristique principale du secteur informel. Notre modèle est une économie NK à deux secteurs où seul le secteur formel est soumis à l’impôt. Dans ce modèle simple, il est possible de caractériser la politique monétaire optimale à partir d’une fonction de bien-être quadratique approximée, puis de calculer analytiquement l’impact de la taille du secteur informel sur la politique monétaire. Nous trouvons que la présence de taxes variables dans le secteur formel génère un biais d’inflation sous une politique discrétionnaire, qui augmente avec la taille du secteur informel. Deuxièmement, nous constatons que seul le secteur formel est responsable des chocs de coûts qui sont amplifiés dans une économie plus informelle. L’arbitrage entre l’inflation et l’écart de production du secteur formel dépend alors de l’élasticité de l’inflation par rapport à l’écart de production du secteur formel. Cependant, la gestion optimale de l’inflation dépend également de l’élasticité de l’écart de production du secteur informel par rapport à l’écart de production du secteur informel. Comme cette élasticité est décroissante avec la taille du secteur informel,
l’effet de l’informalité sur la volatilité de l’inflation (en termes d’écart de production global) est ambigu. Par simulation, nous montrons que les économies ayant un secteur informel plus important devraient stabiliser moins d’inflation par rapport à l’écart de production total.

Dans le deuxième chapitre, j’analyse également la conception optimale de la politique monétaire en utilisant un cadre plus sophistiqué. Je développe un modèle d’économie fermée avec deux marchés du travail, formel et informel, qui intègre la recherche sur le marché du travail dans un nouveau modèle keynésien avec des rigidités nominales sur les prix et les salaires. À la suite de Thomas (2008) et de Gertler et Trigari (2009), j’introduis des négociations salariales nominales échelonnées dans lesquelles les entreprises et les travailleurs du secteur formel négocient les salaires dans un contexte de frictions de recherche et d’appariement. Dans ce cadre, il est possible d’analyser les implications de l’informalité pour la conception de la politique monétaire optimale dans un scénario de compromis entre inflation et chômage. Je constate que la contribution de la volatilité de l’inflation des salaires à la perte de bien-être, par rapport à la contribution de la volatilité de l’inflation des prix, est plus faible dans le cas de l’informalité. Ce résultat s’explique par le fait qu’en présence d’un secteur informel, la proportion d’entreprises de l’économie confrontées à des rigidités salariales est plus faible. Par conséquent, en présence d’un secteur informel, la politique optimale se traduira par une plus faible volatilité de l’inflation des prix pour un niveau donné de volatilité de l’inflation des salaires.

De plus, je trouve qu’en présence d’un secteur informel, les fluctuations inefficaces sur les variables du marché du travail telles que l’emploi, la tension sur ce marché et le taux d’embauche dans le secteur formel sont plus élevées. Ce résultat s’explique par le fait que, en réponse à un choc de productivité globale, seule une fraction des entreprises du secteur formel peut ajuster ses salaires nominaux. Cette rigidité des salaires génère un écart entre le salaire réel et le salaire naturel dans le secteur formel (le salaire cible) qui se traduit par une dispersion des salaires et une création d’emplois inefficace dans le secteur formel. En présence d’informalité, la réponse du salaire cible aux chocs de productivité est plus élevée. Le salaire cible dans le secteur formel dépend du salaire informel (l’option extérieure) et de l’étanchéité du marché du travail formel. Après un choc de productivité négatif, la diminution des deux variables est plus élevée que dans le cas sans informalité. Par conséquent, en réponse aux chocs de productivité globale, la banque centrale devrait utiliser l’inflation des prix pour éviter une volatilité excessive de
l’inflation des salaires qui entraîne une volatilité excessive du chômage et une dispersion excessive du taux d’embauche formel. En contrôlant le taux d’inflation, la banque centrale peut affecter la valeur réelle des salaires nominaux et ensuite rapprocher les salaires formels réels de leurs niveaux de salaires flexibles. La présence d’un secteur informel nécessite un ajustement plus important de l’inflation des prix pour réduire cet écart.

Pour illustrer les implications de l’arbitrage auquel est confrontée la banque centrale, j’analyse le comportement d’une économie décentralisée lorsque l’autorité monétaire met en œuvre une politique de stabilisation totale de l’inflation. J’ai constaté que la perte de bien-être dans le cadre de la politique d’inflation zéro est environ 1.26 fois plus importante que dans le cadre de la politique optimale. Dans le cas sans informalité, cette perte de bien-être est seulement 0.015 fois plus importante qu’avec la politique optimale. Ces résultats montrent qu’une politique conçue pour minimiser la volatilité de l’inflation peut générer des pertes de bien-être significatives en présence de rigidités salariales formelles et d’informalité, comme c’est le cas pour la plupart des pays émergents.


Dans les pays en développement et émergents, à la différence des pays développés, une fraction importante de la main-d’œuvre urbaine est dans le secteur informel et exerce des activités à forte intensité de main-d’œuvre. Comment le processus de polarisation des emplois dans les pays en développement affecte-t-il la taille du secteur

La solution analytique du modèle montre que la polarisation de l’emploi, induite par un changement technologique axé sur la routine (RBTC), pourrait conduire à une augmentation de la taille du secteur informel et à une réduction de l’inégalité des salaires au bas de la distribution des compétences. Je constate que l’emploi et les salaires dans les secteurs de services formels et informels augmentent en raison de la demande accrue de services. L’allocation de la main-d’œuvre dans le secteur des services dépend du niveau d’imposition des revenus du travail, du degré de substitution entre les deux types de services et du niveau d’efficacité du secteur formel des services. Je constate aussi que la part de l’emploi informel dans le secteur des services diminue avec le progrès technologique.
References


1 Optimal Monetary Policy with Informality: A First Pass

Abstract

Our paper aims to unveil how much monetary policy shall deviate from the flexible-price allocation in an economy with a large informal sector. First of all, the presence of variable taxes in the formal sector generates an inflation bias under a discretionary policy, which increases with the size of the informal sector. Secondly, we find that only the formal sector is responsible for the cost-push shocks that are amplified in a more informal economy. The trade-off between inflation and the formal output gap is then dependent on the elasticity of the former variable with respect to the formal output gap. However, the optimal management of inflation also depends on the elasticity of the informal output gap with respect to the formal output gap. As this elasticity is decreasing with the size of the informal sector, whether inflation volatility (in terms of the aggregate output gap) is lower or higher in a more informal economy is ambiguous. By simulation, we show that economies with a larger informal sector should stabilize less inflation relative to the total output gap.

Keywords: Informality, optimal monetary policy, New-Keynesian macroeconomics, tax distortion.

JEL classification: E26, E52, E12, H21

1.1 Introduction

Surprisingly enough, few papers have been devoted to the analysis of the monetary policy when the economy displays an informal sector, where value-added activities avoid taxation. A lot of countries share these features, especially emerging countries where the rule of law and tax compliance are not well established\(^3\). Generally, informal labor markets are the result of agents who want to avoid taxation and regulation, despite the

\(^2\)This chapter is the product of joint work with Jean-Olivier Hairault

\(^3\)Labor markets in developing countries are particularly affected by the existence of a large informal sector. According to the ILO (2018), informal employment accounts for more than half of non-agricultural employment in most developing countries: around 72 percent in Africa, 63 percent in Asia and the Pacific, 64 percent in the Arab States, 50 percent in Latin America, and 30 percent in Europe and Central Asia. In the case of developed countries, only 17 percent of the urban labor force is employed in informal activities.
protection and advantages that the state can provide in the formal sector; it is also the
outcome of agents who cannot find a job in the formal sector and depend on informal
jobs as a means of subsistence.

Given the importance of the labor market structure in determining output, inflation,
and the response of the economy to aggregate shocks, it is of great importance to an-
alyze the implications of informality for monetary policy in developing countries. What
is then the effect of the informality scale on the optimal trade-off between inflation and
output or the inflation bias? From the seminal work of Clarida et al. (1999), monetary
policy has been built on very firm theoretical foundations. The New Keynesian (NK)
framework offers clear guidelines for central banks of developed countries. It remains
to propose a canonical model encompassing what is the major issue for emerging coun-
tries: the existence of a large informal sector. Our paper aims at deriving the first
principles of monetary policy according to the relative size of the informal sector. We
propose a canonical model in the NK framework in order to derive analytically these
principles. We choose to focus on tax avoidance as the key feature of the informal
sector: our model is an NK two-sector economy with taxation only in the formal sector.
In this simple model, it is possible to derive the optimal policy recommendations from
an approximated quadratic welfare function, and then to characterize the role of the
informality size for monetary policy analytically.

We show that the presence of distortive taxes generates an inflation bias under the
discretionary policy as a result of the central bank’s incentive to boost production above
its natural level. This inflationary bias increases with the size of the informal sector,
even though only the formal sector displays a distorted steady state. Additionally, we
show that only the formal sector due to tax distortion fluctuations is responsible for
cost-push shocks: in this sector, the gap between the natural rate and the first best
allocation varies due to fluctuations in taxes. On the other hand, cost-push shocks are
amplified in a more informal economy. The trade-off between inflation and the formal
output gap is then dependent on the elasticity of the former variable with respect to the
latter one (the inverse of the sacrifice ratio in terms of formal output). Unambiguously,
this elasticity is lower in a more informal sector, which would lead to higher (relative)
volatility of inflation in such an economy. However, this is only one dimension of the
optimal management of inflation, as the formal output gap is only one dimension of the
policy-relevant aggregate output gap, which must also take into account the elasticity of
the informal output gap with respect to the formal output gap (the sectoral integration).
As this elasticity is decreasing with the size of the informal sector, whether inflation volatility (in terms of the aggregate output gap) is lower or higher in a more informal economy is ambiguous. By simulation, we show that economies with a larger informal sector should stabilize less inflation relative to the welfare based output gap.

Our paper is the first one to investigate the implication of informality for monetary policy in the standard NK framework. Castillo and Montoro (2010) and Batini et al. (2011) propose too complex theoretical frameworks to derive analytical results. Our simple model can deliver very clear insights about inflation dynamics: emerging economies with a large informal sector should display higher mean and inflation volatility (relative to the policy relevant output gap and the total output gap).

A key assumption in our model is the absence of public debt, and then the absence of tax smoothing allowed by debt management over the business cycle. It is certainly an extremely simplifying assumption, but it allows us to unveil some basic properties. It also reflects the fact that in emerging countries the public debt management is more constrained than in developed countries. Overall, our framework certainly overemphasizes tax variations over the business cycles, but we do believe that it delivers the key differences of the optimal monetary policy across developed and emerging countries.

The rest of the chapter is organized as follows. In Section 1.2 we develop a NK model with two sectors, and endogenous labor income taxes in the formal sector. In Section 1.3, we show how the presence of varying taxes generates a trade-off between stabilizing inflation and stabilizing the policy-relevant output gap. In Section 1.4 we characterize optimal monetary policy. In section 1.5 we realize a quantitative analysis of the model. Section 1.6 concludes.

### 1.2 A New Keynesian model with informality

We propose a New-Keynesian closed economy framework with a formal sector (F) and an informal one (I). Only workers in the formal sector have to pay a wage income tax, which is a source of distortions for working hours. We consider an economy populated by infinitely-lived households whose utility depends on leisure and the consumption of

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4 According to Besley and Persson (2014), low-income countries typically collect taxes of between 10 to 20 percent of GDP, while the average for high-income countries is more like 40 percent. This difference is not necessarily explained by a choice of low tax rates but by the challenges associated with tax collection: these include informality and misreporting (Besley and Persson 2009).
market goods produced by a continuum of monopolistically-competitive firms. Formal and informal firms hire labor from households in order to produce a wholesale good that is sold in a competitive market. Retail firms use wholesale goods as an input and transform them into differentiated final goods.

1.2.1 Households

An exogenous fraction $N_S$ of individuals works in sector $S$, $S \in (F, I)$, and maximize the following expected discounted utility function:

$$U_{S,t} = E_0 \sum_{t=0}^{\infty} \beta^t \left[ \frac{c_{S,t}^{1-\sigma} - 1}{1 - \sigma} - \frac{h_{S,t}^{1+\eta}}{1 + \eta} \right].$$

Households in the formal sector have to pay a labor income tax $\tau^w_t$. They maximize their utility subject to the following budget constraint:

$$c_{F,t} + b_{F,t} \leq (1 - \tau^w_t) w_{F,t} N_F h_{F,t} + \frac{(1 + i_{t-1})}{1 + \pi_t} b_{F,t-1} + T_F.$$  

On the other side, households in the informal sector consume their current income and do not pay taxes. Their budget constraint writes:

$$c_{I,t} \leq N_I w_{I,t} h_{F,t},$$

where $w_S$ is the real wage in sector $S$, $c_S$ is the aggregate CES basket of $i$ goods consumed by the $S$ sector, $b_F$ is a one-period bond and $h_S$ is the working hours supplied by individuals in sector $S$, and $\pi_t = \left( \frac{P_t}{P_{t-1}} - 1 \right)$ is the inflation rate between $t - 1$ and $t$ (where $P_t$ is the level of prices in $t$). $\beta = \frac{1}{1+\rho}$, with $\rho > 0$, is the subjective discount rate. $\sigma$ and $\eta$ are positive parameters which define the curvature of leisure and consumption preferences. $\tau^w$ is the tax rate paid by workers in the formal sector. The choice of working hours in the formal sector is then distorted. $T_F$ is a lump-sum transfer to households in the formal sector.

The first-order conditions of the intertemporal program for households working in the formal and the informal sectors are given by:

$$\left( \frac{c_{F,t+1}}{c_{F,t}} \right)^\sigma = \beta E_t \left( \frac{1 + i_t}{1 + \pi_{t+1}} \right),$$

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\[ h_{F,t}^\eta = (c_{F,t})^{-\sigma} (1 - \tau_t^w) w_{F,t}, \]  
(5)

\[ h_{I,t}^\eta = (c_{I,t})^{-\sigma} w_{I,t}. \]  
(6)

Each period, households choose optimally the quantity of each variety \( j \):

\[ c_{S,j,t} = \left( \frac{p_{jt}}{P_t} \right)^{-\theta} c_{S,t}, \]  
(7)

where \( \theta \) measures the elasticity of substitution between varieties.

### 1.2.2 Firms

**Wholesale firms**  Formal and informal firms in the wholesale sector hire \( N_F h_{F,t} \) and \( N_I h_{I,t} \) labor hours from households in order to produce \( Y_{F,t} \) and \( Y_{I,t} \) units, respectively, of the intermediate good, using the following technology:

\[ Y_{S,t} = A_{S,t} N_S h_{S,t}. \]  
(8)

where \( A_{S,t} \) is the stochastic sectoral productivity common to all firms in sector \( S \). We assume that \( \ln(A_{S,t}) \) follows a first-order stationary auto-regressive process with auto-regressive coefficient \( \rho_S \).

In a competitive environment, the maximization of profits implies that the wholesale real price \( P_Y \) equals the real marginal cost \( \phi_{S,t} \) in each sector:

\[ P_t^Y = \frac{w_{S,t}}{A_{S,t}} \equiv \phi_{S,t}, \quad \forall S = F, I. \]  
(9)

**Retail firms**  Retail firms owned by households working in the formal sector purchase wholesale output at price \( P_Y \) and transform it without labor or capital into differentiated final goods \( j \). Following Calvo (1983), each monopolistic retailer is assumed to reset its price with probability \( (1 - \omega) \) in any given period, independent of the time elapsed since their last adjustment. A firm that can adjust its price in period \( t \) chooses
\( P_t^* \) in order to maximize its intertemporal flows of profits:

\[
\max_{P_t^*} \mathbb{E}_t \sum_{j=0}^{\infty} \Gamma_{t,t+j} \omega^j \left[ P_t^* Y_{t+j/t} - P_{t+j}(1 - \tau_m) P_{t+j} Y_{t+j/t} \right],
\]

subject to the sequence of demand constraints:

\[
Y_{t+j/t} = \left( \frac{P_t^*}{P_{t+j}} \right)^{-\theta} Y_{t+j},
\]

where \( \Gamma_{t,t+j} = \beta j \left( \frac{e_{F,t+j}}{e_{F,t}} \right)^{-\sigma} \left( \frac{P_t}{P_{t+j}} \right) \) is the stochastic discount factor for nominal payoffs, and \( Y_{t+j/t} \) denotes the output in period \( t + j \) for a firm whose last reset of prices was in period \( t \). In order to focus on the distortions imposed by taxation in the formal sector, a subsidy \( \tau_m = \frac{1}{\theta} \) financed by lump-sum taxes is assumed to offset the monopolistic distortion at the steady state.

### 1.2.3 Government

The government runs a balanced budget. The distortive taxes to formal workers are used to finance constant public transfers to households in the formal sector \( T_F \). Therefore, government’s budget constraint regarding these transfers writes as follows:

\[
\tau_w N_F w_{F,t} h_{F,t} = T_F. \tag{10}
\]

The tax rate \( \tau_w \) varies over the business cycle in order to balance the fluctuating tax base.

### 1.3 Phillips, Sectoral Integration and IS curves

In what follows, the variables with hat represent the log-deviation of the variable from their steady-state value.

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\(^5\)On the other hand, the sum of all subsidies given to retailers at the steady state are fully financed by lump-sum taxes.
1.3.1 Sectoral integration curve

Using Equations (9), (5) and (6), we derive, after log-linearizing around the steady-state, the following expressions for the marginal cost in the informal and formal sectors (see Appendix A1 for the derivation):

\[ \hat{\phi}_{I,t} = (\eta + \sigma) X_{I,t}, \]  
\[ \hat{\phi}_{F,t} = (\eta + \sigma) X_{F,t} + \frac{\tau_w}{1 - \tau_w} \hat{\tau}_w, \]

where \( X_{S,t} = \hat{Y}_{S,t} - \hat{Y}^e_{S,t} \) is the welfare relevant output gap in sector \( S \), i.e. the deviation between the actual output \( \hat{Y}_{S,t} \) and its efficient level \( \hat{Y}^e_{S,t} \). The marginal cost in the formal sector is proportional to the (welfare-relevant) formal output gap, and it also depends on tax variations in the formal sector.

From Equation (9), \( \hat{\phi}_{F,t} = \hat{\phi}_{I,t} \). This sectoral integration condition implies a relationship between the sectoral output gaps:

\[ (\eta + \sigma) (X_{F,t} - X_{I,t}) = -\frac{\tau_w}{1 - \tau_w} \hat{\tau}_w. \]

A change in labor tax impacts the marginal cost in the formal sector and implies working hours differential across sectors over the business cycle. Cyclical variations in taxes disconnect the formal output gap from the informal one, especially in an economy with a large informal sector. In an otherwise homogeneous economy across sectors, the two sectoral output gaps would perfectly co-move. Taking into account the expression of the tax variation:

\[ \hat{\tau}_w = -\hat{\phi}_{F,t} - A_{F,t} - \hat{h}_{F,t}, \]

we get:

\[ X_{I,t} = \Omega_x X_{F,t} - \frac{\tau_w}{\eta + \sigma} \hat{Y}^e_{F,t}, \]

where \( \Omega_x = \frac{(\eta + \sigma)(1 - \tau_w) - \tau_w}{\eta + \sigma} \).

Let us note that there is a perfect sectoral integration, \( X_{I,t} = X_{F,t} \), when taxes are zero or constant. With variable distortionary taxes in the formal sector, we have \( \Omega_x \neq 1 \), and it can be either positive or negative. When \( X_{F,t} \) increases, it then pushes upward \( X_{I,t} \) because the marginal cost increases. But this decreases taxes, which in turn decreases
the marginal cost all the more that \( \eta \) and \( \sigma \) are low, so the increase in \( X_{I,t} \) is lower than the increase in \( X_{F,t} \). The higher is the size of the informal sector \( (N_I) \) the lower is \( \Omega_x \), and therefore the weaker is the positive co-movement across sectoral output gaps. More precisely, the informal sector is less volatile than the formal one, due to the counter-cyclical behavior of distortive taxes in the formal sector.

### 1.3.2 The Phillips Curve

From the optimal pricing program, it is possible to derive the following traditional expression for the dynamics of inflation (see Appendix A2 for the derivation):

\[
\hat{\pi}_t = \Upsilon \hat{\phi}_t + \beta E_t \hat{\pi}_{t+1},
\]

(15)

where \( \Upsilon = \frac{(1-\omega \beta)(1-\omega)}{\omega} \) and \( \hat{\phi}_t \) is the real marginal cost equal to \( \hat{P}_t^Y = \hat{\phi}_{F,t} = \hat{\phi}_{I,t} \). The parameter \( \Upsilon \) is decreasing in the degree of price rigidity, \( \omega \). Thus the higher is the price rigidity, the less sensitive is inflation to changes from the marginal cost.

The expression of the marginal cost can be obtained either from (11) and (14) or from (12) and (13):

\[
\hat{\phi}_t = \kappa X_{F,t} - cp_t,
\]

(16)

where

\[
\kappa = (\eta + \sigma) (1 - \tau^w) - \tau^w > 0, \quad \frac{\partial \kappa}{\partial N_I} < 0.
\]

\[
 cp_t = \frac{\tau^w \hat{Y}_{F,t}}{\eta + \sigma}, \quad \frac{\partial cp_t}{\partial N_I} > 0.
\]

This expression of the marginal cost unveils a crucial feature of our economy with informality. It is only in the formal sector, through the tax variation, that there are cost-push shocks, denoted as \( cp_t \). Following a negative productivity shock in the formal sector, for instance, there is an increase in the marginal cost due to a higher tax rate; this increase is all the more intense when the economy is more informal \( \left( \frac{\partial \tau^w}{\partial N_I} > 0 \right) \).

The parameter \( \kappa \) represents the elasticity of the real marginal cost with respect to the formal output gap. As it is traditional in a NK framework, this elasticity is small when there are large convexities on preferences (small \( \sigma \) and \( \eta \)). But in the formal sector, as the marginal cost depends on the marginal tax, the coefficient \( \kappa \) also depends on the informality size. It is reduced by the informality size through the feedback effect of tax variation on the marginal cost \( \left( \frac{\partial \kappa}{\partial N_I} < 0 \right) \).
Combining Equations (15) and (16), the Phillips curve can then be written as follows:

$$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \Upsilon \kappa X_{F,t} - \Upsilon c p_t. \quad (17)$$

From the previous discussions, it is straightforward that an economy with larger informality faces higher cost-push shocks on inflation. The Central Bank must decide whether to accept a higher level of inflation volatility or instead stabilizing inflation at the expense of allowing for more fluctuations in the welfare-relevant formal output gap. In this economy, the sacrifice ratio in terms of the formal output gap is particularly high. It implies that stabilizing inflation through adjustments in the formal output gap is particularly costly in such economies.

### 1.3.3 IS Curve

By log-linearizing the Euler equation (4), we obtain:

$$\hat{Y}_{F,t} = E_t \left[ \hat{Y}_{F,t+1} \right] - \frac{1}{\sigma} \hat{p}_t + \frac{1}{\sigma} E_t \left[ \hat{\pi}_{t+1} \right].$$

Expressing this function in terms of the aggregate welfare output gap ($\hat{Y}_t - \hat{Y}^e_t = X_t$), we get the aggregate IS curve (see Appendix A3 for the derivation):

$$X_t = E_t \left[ X_{t+1} \right] - \frac{\theta^m}{\sigma} \left[ \hat{p}_t - \hat{r}^n_t - E_t \left[ \hat{\pi}_{t+1} \right] \right] + (\theta^e - \theta^m) \frac{1}{\sigma} \hat{r}^n_t, \quad (18)$$

where $\hat{r}^n_t$ is the natural interest rate, $\theta^m = \frac{Y_F}{Y}$ and $\theta^e = \frac{Y_F}{Y^e}$.

The primary impact of the interest rate on the aggregate demand depends on the formal output share, as households only in the formal sector have access to the financial market. Equation 18 shows that an increase in the size of the informal sector (i.e. a decrease in the formal output share $\theta^m$) decreases the elasticity of aggregate demand to real interest rate, making the monetary policy less effective in containing demand. As the share of the informal sector tends to 1, $\theta^m$ decreases towards zero and $\theta^e \simeq \theta^m$. Under this scenario, monetary policy is ineffective when nobody has access to the financial market.
1.4 Optimal monetary policy

1.4.1 The welfare loss function

The second-order Taylor approximation of the aggregate utility function around the steady-state \((c_F, c_I, h_F, h_I)\) yields to the following welfare loss function \(\mathcal{W}\) (see Appendix A4 for the derivation):

\[
\mathcal{W} = E_0 \sum_{t=0}^{\infty} \beta^t \left( \frac{1}{2} \left[ \frac{\theta}{\eta} \pi_t^2 + (\eta + \sigma) \left( N_F X^2_{F,t} + (1 - N_F) X^2_{I,t} \right) \right] - N_F \tau^w X_{F,t} \right).
\] (19)

Welfare losses are expressed in terms of welfare-equivalent permanent loss in consumption, measured as a fraction of steady-state consumption. As it is traditional in a NK framework, the weight of inflation volatility is increasing in \(\theta\), the elasticity of substitution among goods, and decreasing in the degree of price stickiness \(\omega\). An increase in \(\omega\) will increase the degree of price dispersion resulting from a deviation from zero inflation. Additionally, the effect of any given price dispersion in the welfare losses will increase with the elasticity of substitution across goods \(\theta\).

The weight associated with the two sectoral welfare relevant output gap volatilities in the loss function is increasing with \(\eta\) and \(\sigma\), which determine the curvature of the utility function. The higher those parameters, the higher the change in the marginal rate of substitution between consumption and hours compared with the change in the marginal cost.

The presence of the linear term \(X_{F,t}\) in the welfare loss function (19) implies that any increase in the formal output gap decreases the welfare losses because in the (flexible-price) equilibrium formal output is below its efficient level. This term measures the steady-state wedge between the marginal rate of substitution between consumption and leisure and the marginal product of labor in the formal sector, and hence the inefficiency of the formal sector steady-state generated by the tax distortion. Note that this term is weighted by the level of taxes and the share of the formal sector. The presence of this term will give rise to the traditional inflation bias when the monetary policy is discretionary.
1.4.2 Optimal discretionary policy

In this section, we characterize the optimal monetary policy under discretion. The Central Bank chooses \( \pi_t, X_{F,t}, \) and \( X_{I,t} \) for a sequence of given expected inflation levels in order to minimize the welfare function subject to the Phillips curve (17) and to the sectoral integration curve (14). The first-order conditions of this optimization problem are as follows:

\[
\begin{align*}
\left[ \hat{\pi}_t \right] \quad \frac{\theta}{Y} \pi_t &= \lambda_1, \\
\left[ X_{F,t} \right] \quad (\eta + \sigma) \frac{Y_F}{Y} X_{F,t} - \frac{Y_F}{Y} \tau^w + \lambda_1 \kappa + \lambda_2 \Omega_x &= 0, \\
\left[ X_{I,t} \right] \quad (\eta + \sigma) \frac{Y_I}{Y} X_{I,t} - \lambda_2 &= 0,
\end{align*}
\]

where \( \lambda_1 \) and \( \lambda_2 \) are the Lagrangian multipliers associated to (14) and (17) respectively. Reorganizing these equations, we get:

\[
(\eta + \sigma) (N_F X_{F,t} + \Omega_x (1 - N_F) X_{I,t}) - N_F \tau^w + \theta \kappa \pi_t = 0. \tag{20}
\]

First of all, as already noted, there is a positive bias in the inflation level under a discretionary regime. Note that this term is weighted by the level of \( \tau^w \) and the share of the formal sector. The higher the distortion created by the tax level (when informality is widespread), the further the formal sector from its efficient level. However, the larger the formal sector, the higher the impact on total welfare loss. More informality creates a higher tax level but reduces the scope of this distortion specific to the formal sector. By combining Equations (10), (5) and (6) at the steady-state we obtain

\[
\tau^w N_F = \frac{T_F}{A_F h_F}.
\]

From this expression, it is straightforward that the inflation bias increases with the size of the informal sector.

Moreover, Equation (20) summarizes the optimal trade-off between inflation and the policy-relevant aggregate output gap defined as \( N_F X_{F,t} + \Omega_x (1 - N_F) X_{I,t} \). As in the traditional NK model, the lower \( \theta \) is, and the higher the curvature coefficients (\( \eta \) and \( \sigma \)) are, the higher the relative volatility of inflation should be. This effect is independent of the informality size.

The first insight about how informality matters for the volatility trade-off, comes from the analysis conducted above about the Phillips curve. The larger the size of the informal sector, the lower \( \kappa \) will be. Therefore, the less the monetary authorities should intervene to stabilize inflation at the expense of the formal output gap volatility. In
order to evaluate all the welfare losses due to output gap volatilities, the destabilizing effect on the informal output gap must be accounted for through the integration condition. The parameter $\Omega_x$ determines how the informal sector is destabilized by the central bank’s intervention. It defines the policy-relevant aggregate output gap $(N_F X_{F,t} + \Omega_x (1 - N_F) X_{I,t})$, which ultimately matters in the inflation-activity trade-off, as it is the measure of the welfare loss due to the sectoral output gap variations. This measure is weighted by the relative size of the two sectors, as expected, but also by the sectoral integration parameter $\Omega_x$. The larger the informal sector, the lower $\Omega_x$, and the lower the policy-relevant aggregate output volatility for a given level of output volatility in the formal sector, which turns out to be ultimately less costly to stabilize inflation. When $\Omega_x$ is close to 0, only the volatility in the formal sector is a source of welfare loss.

Therefore, an increase in the informality size has two opposite effects on inflation volatility: an increase through the sacrifice ratio and a decrease through a lower sectoral integration. The aggregate effect on inflation volatility relative to output volatility would depend on which effect dominates.

### 1.5 Quantitative Analysis

This section quantifies the effect of a productivity shock, given numerical values for the model’s parameters. In the baseline calibration of the model a period $t$ corresponds to a quarter, the discount rate is assumed $\beta = 0.988$, which implies a real interest rate of about 5 percent. We also assume log utility function ($\sigma = 1$), a unitary Frisch elasticity of labor supply ($\eta = 1$), and the elasticity of substitution between varieties $\theta = 6$. These are values commonly found in the business cycle literature (Galí, 2008). In addition, following the empirical evidence found in Galí, Gertler, and López-Salido (2001), Sbordone (2002), and Galí (2008) we set the probability for a firm of not changing prices equal to $\omega = 2/3$, which implies an average price duration of three quarters. $N_I$ is assumed to be equal to 0.5, since, according to the ILO (2018), informal employment accounts for more than half of non-agricultural employment in most developing countries. Transfers to households in the formal sector are calibrated in order to obtain tax revenue of 15 percent of the Gross Domestic Product (GDP) at the steady-state. According to Besley and Persson (2014), low-income countries typically collect taxes of
between 10 to 20 percent of GDP, while the average for high-income countries is more like 40 percent. Finally, we assume an auto-regressive coefficient of the productivity shock equal to $\rho = 0.9$. Table 1.1 summarizes the parameter values of the model.

<table>
<thead>
<tr>
<th>Description</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discount rate</td>
<td>$\beta$</td>
<td>0.988</td>
</tr>
<tr>
<td>Inverse of the intertemporal elasticity of substitution</td>
<td>$\sigma$</td>
<td>1</td>
</tr>
<tr>
<td>Elasticity of labor supply</td>
<td>$\eta$</td>
<td>1</td>
</tr>
<tr>
<td>Probability for a firm of not changing prices</td>
<td>$\omega$</td>
<td>$\frac{2}{3}$</td>
</tr>
<tr>
<td>Elasticity of substitution between varieties</td>
<td>$\theta$</td>
<td>6</td>
</tr>
<tr>
<td>Size of the informal sector</td>
<td>$N_I$</td>
<td>0.5</td>
</tr>
<tr>
<td>Transfers to households in the formal sector</td>
<td>$T_F$</td>
<td>0.136</td>
</tr>
<tr>
<td>Auto-regressive coefficient</td>
<td>$\rho$</td>
<td>0.9</td>
</tr>
</tbody>
</table>

Figure 1.1 shows the optimal response of inflation, formal, informal, and total welfare-relevant output gaps, to a 1% standard deviation negative productivity shock. We consider different rates of informality, which are indicated as follows: a size of the informal sector equal to 0.5, $N_I = 0.5$ (solid line), $N_I = 0.3$ (dashed line), and $N_I = 0.1$ (dotted line). In all cases, inflation and informal output gap increase after an aggregate negative productivity shock, while formal output gap and total output gap decrease. In particular, note that when the size of the informal sector is large, $N_I = 0.5$, the optimal policy implies a larger increase in inflation, compared with the case when the size of the informal sector is lower $N_I = 0.3$ and $N_I = 0.1$. Additionally, it is worth noticing from Figure 1.1 that the higher the size of the informal sector, the lower the decrease of the total welfare-based output gap after a negative productivity shock. These results suggest that an increase in the size of the informal increases inflation volatility. The fact that an increase in the informal sector increases the sacrifice ratio, implies that the monetary authorities should intervene less to stabilize inflation at the expense of the formal output gap volatility.
Figure 1.1. Optimal responses to a negative productivity shock

Figure 1.2 shows the ratio of inflation and total welfare-relevant output gap volatility for different sizes of the informal sector under the optimal policy. Note that this ratio increases with the size of the informal sector. Similarly, Figure 1.3 represents the optimal ratio of inflation and formal welfare-relevant output gap volatility for different values of $N_I$, which initially decreases with the size of the informal sector and then increases when the size of the informal sector is bigger than 0.5. This result suggests that economies with a larger informal sector should stabilize less inflation relative to the total welfare-based output gap.
1.6 Conclusions

The focus of this paper is the study of the implications for the Central Bank and the optimal design of monetary policy in economies with a large informal sector. Our results can be summarized as follows. First of all, we find that informality amplifies cost-push shocks on inflation. Secondly, the sacrifice ratio increases with the weight of the informal sector, which would lead to recommending less inflation stability in economies with higher levels of informality. Thirdly, however, in this type of economy, the policy-relevant output gap, which takes into account the degree of the sectoral linkages is less volatile for any level of inflation volatility. Finally, the inflation bias increases with the size of the informal sector.
Therefore, we find that an increase in the informality size has two opposite effects on inflation volatility: an increase through the sacrifice ratio and a decrease through a lower sectoral integration. The aggregate effect on inflation volatility relative to output volatility would depend on which effect dominates. By simulation, we show that economies with a larger informal sector should stabilize less inflation relative to the total output gap.

If these results may be specific to the simple framework considered, we think that our paper is quite general about the key factors at stake in the implications of informality for optimal monetary policy. First of all, our results emphasize the impact of informality on steady-state structural inefficiencies (the gap between the natural and efficient output levels), which depends on both the sectoral location of the inefficiencies and how they are linked to the informality size. Secondly, they unveil the impact of informality on the sacrifice ratio in terms of the policy-relevant output gap, which brings together the Phillips curve sacrifice ratio and the sector integration degree. We think that any more complex frameworks could be analyzed in the same vein by identifying the sector where the structural inefficiencies are, how they are affected by the size of the informal sector, and what the strength of sectoral links in the business cycle is.

Let us examine the case of a model which introduces search frictions only in the formal sector. This adds inefficiencies in the formal sector. If the size of the informal sector increases, there is less employment in the formal sector, which reduces the cost of search frictions in the steady-state. In this model, a more informal economy could display a lower inflationary bias. On the other hand, the formal sector remains the only source of cost-push, and only a more in-depth analysis of the model would make it possible to understand the impact of an increase in informality on the magnitude of cost-push shocks and on the sacrifice ratio in the formal sector. Intuitively, a smaller formal sector decreases the elasticity of the vacancy costs and then increases the sacrifice ratio. In addition, a lower sectoral correlation can be expected when the hiring elasticity to a vacancy is greater in the formal sector.

Another important insight brought about by our analysis is that monetary policy should not target only the formal sector. Only considerations related to informational issues in the measurement of the informal sector could lead to recommend to favor the formal output gap in the monetary policy rule. But the estimated sacrifice ratio of the Phillips curve in the formal sector would be a wrong indicator of the optimal trade-off between inflation and output gap, which would lead to failing to stabilize the inflation volatility.
1.7 Appendix

Appendix A1: Sectoral integration curve

The law of one price implies $\phi_{f,t} = \phi_{I,t}$. This implies a relationship between the sectoral outputs as a sectoral integration condition.

Let us define the marginal cost:

$$\phi_{S,t} = \frac{W_{S,t}}{A_{S,t}P_{t}} \quad S = F, I$$

Let us reconsider the expression of the marginal cost in terms of the welfare based output gap.

From (5) and (12)

$$h_{F,t}^\eta = (c_{Ft})^{-\sigma} (1 - \tau^w) A_{F,t} \phi_{F,t}$$

$$h_{I,t}^\eta = (c_{It})^{-\sigma} A_{I,t} \phi_{I,t}$$

We obtain, after log-linearizing the previous two equations, the following equations:

$$\hat{\phi}_{I,t} = (\eta + \sigma) \left( \hat{\Psi}_{I,t} \right) - (\eta + 1) \hat{A}_{I,t}$$

and

$$\hat{\phi}_{F,t} = (\eta + \sigma) \left( \hat{\Psi}_{f,t} \right) + \frac{\tau^w}{1 - \tau^w} \hat{r}^w_t - (\eta + 1) \hat{A}_{f,t}$$

It is also possible to write the same equations for the natural rate equilibrium and the first-best equilibrium:

$$0 = (\eta + \sigma) \left( \hat{\Psi}_{I,t}^{e} \right) - (\eta + 1) \hat{A}_{I,t}$$

$$0 = (\eta + \sigma) \left( \hat{\Psi}_{f,t}^{e} \right) - (\eta + 1) \hat{A}_{f,t}$$
\[(\eta + \sigma)\left(\hat{Y}^e_{I,t}\right) = (\eta + 1)\hat{A}_{I,t}\]

\[(\eta + \sigma)\left(\hat{Y}^e_{f,t}\right) = (\eta + 1)\hat{A}_{f,t}\]

therefore

\[\hat{\phi}_{I,t} = (\eta + \sigma)\left(\hat{X}_{I,t}\right)\]

and

\[\hat{\phi}_{F,t} = (\eta + \sigma)\left(\hat{X}_{F,t}\right) + \frac{\tau^w}{1 - \tau^w}\hat{\tau}^w_t\]

where \(\hat{X}_{S,t} = \hat{Y}_{S,t} - \hat{Y}^e_{S,t}\) is the welfare-relevant output gap in sector \(S\), i.e. the deviation between the actual output \(\hat{Y}_{S,t}\) and its efficient level \(\hat{Y}^e_{S,t}\).

From Equation (9), \(\hat{\phi}_{F,t} = \hat{\phi}_{I,t}\). This sectoral integration condition implies a relationship between the sectoral output gaps:

\[(\eta + \sigma)\left(\hat{X}_{F,t} - \hat{X}_{I,t}\right) = -\frac{\tau^w}{1 - \tau^w}\hat{\tau}^w_t\]

Replacing the expression of the tax variation \(\hat{\tau}^w_t = -\hat{\phi}_{F,t} - \hat{A}_{F,t} - \hat{h}_{F,t}\), into the previous equation we obtain:

\[\hat{\phi}_{F,t} = (\eta + \sigma)\hat{X}_{F,t} - \frac{\tau^w}{1 - \tau^w}\left(\hat{\phi}_{F,t} + \hat{A}_{F,t} + \hat{h}_{F,t}\right)\]

reorganizing

\[\hat{\phi}_{F,t} = (\eta + \sigma)\hat{X}_{F,t} - \frac{\tau^w}{1 - \tau^w}\left(\hat{\phi}_{F,t} + \hat{Y}_{F,t}\right)\]

\[\frac{(1-\tau^w)}{\tau^w} (1 + \frac{\tau^w}{1 - \tau^w}) \hat{\phi}_{F,t} = \frac{(1-\tau^w)(\eta + \sigma)}{\tau^w} \hat{X}_{F,t} - \left(\hat{Y}_{F,t}\right)\]

\[\frac{(1-\tau^w)}{\tau^w} (1 + \frac{\tau^w}{1 - \tau^w}) \hat{\phi}_{F,t} = \frac{(1-\tau^w)(\eta + \sigma)}{\tau^w} \hat{X}_{F,t} - \left(\hat{Y}_{F,t} - \hat{Y}^e_{F,t} + \hat{Y}^e_{F,t}\right)\]

\[\frac{1}{\tau^w} \hat{\phi}_{F,t} = \frac{(1-\tau^w)(\eta + \sigma)}{\tau^w} \hat{X}_{F,t} - \hat{X}_{F,t} + \hat{Y}^e_{F,t}\]

\[\frac{1}{\tau^w} \hat{\phi}_{F,t} = \left(\frac{(1-\tau^w)(\eta + \sigma)}{\tau^w} - 1\right) \hat{X}_{F,t} - \hat{Y}^e_{F,t}\]

\[\hat{\phi}_{F,t} = ((\eta + \sigma)(1 - \tau^w) - \tau^w) \hat{X}_{F,t} - \tau^w \hat{Y}^e_{F,t}\]
Therefore, the marginal cost in both sector can be written as follows

\[
\hat{\phi}_{F,t} = ((\eta + \sigma) (1 - \tau^w) - \tau^w) \hat{X}_{F,t} - \tau^w \hat{Y}_e^{e}, \quad (21)
\]

\[
\hat{\phi}_{I,t} = (\eta + \sigma) \hat{X}_{I,t} \quad (22)
\]

Finally, with \(\hat{\phi}_{F,t} = \hat{\phi}_{I,t}\), the sectoral integration condition takes the form:

\[
\hat{X}_{I,t} = \Omega_x \hat{X}_{F,t} - \frac{\tau^w \eta + \sigma \hat{Y}_e^{e}}{\eta + \sigma} \quad (23)
\]

where \(\Omega_x = \frac{(\eta + \sigma)(1 - \tau^w) - \tau^w}{\eta + \sigma}\)

Appendix A2: The Phillips Curve

From the first-order condition associated with the optimal pricing program we obtain:

\[
\frac{P^*_t}{P_t} = \frac{E_t \sum_{j=0}^{\infty} \beta^j c_t^{1-\sigma} \omega^j \left[ \phi_{t+j} \left( \frac{P_t}{P_{t+j}} \right)^{-\theta} \right]}{E_t \sum_{j=0}^{\infty} \beta^j c_t^{1-\sigma} \omega^j \left[ \left( \frac{P_t}{P_{t+j}} \right)^{1-\theta} \right]} \quad (24)
\]

log-linearizing (24) around the steady state, and denoting \(P_t^E = \frac{P^*_t}{P_t}\) we have that

\[
P_t^E E_t \sum_{j=0}^{\infty} \beta^j c_t^{1-\sigma} \omega^j \left[ \phi_{t+j} \left( \frac{P_t}{P_{t+j}} \right)^{-\theta} \right] = E_t \sum_{j=0}^{\infty} \beta^j c_t^{1-\sigma} \omega^j \left[ \phi_{t+j} \left( \frac{P_t}{P_{t+j}} \right)^{-\theta} \right]
\]

\[
\hat{P}_t^E \left( \frac{c_t^{1-\sigma}}{1-\omega^j} \right) + (\theta - 1) c_t^{1-\sigma} E_t \sum_{j=0}^{\infty} (\omega^j)^j \left( \hat{P}_{t+j} - \hat{P}_t \right) + (1 - \sigma) c_t^{1-\sigma} E_t \sum_{j=0}^{\infty} (\omega^j)^j \hat{\phi}_{t+j} + \theta c_t^{1-\sigma} E_t \sum_{j=0}^{\infty} \beta^j \omega^j \left[ \hat{\phi}_{t+j} \right] + \theta c_t^{1-\sigma} E_t \sum_{j=0}^{\infty} \beta^j \omega^j \left[ \hat{P}_{t+j} - \hat{P}_t \right]
\]

reorganizing previous equation

\[
\left( \frac{\hat{P}_t^E}{1 - \omega^j} \right) = E_t \sum_{j=0}^{\infty} \beta^j \omega^j \left[ \hat{\phi}_{t+j} \right] + E_t \sum_{j=0}^{\infty} \beta^j \omega^j \left[ \hat{P}_{t+j} - \hat{P}_t \right]
\]

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\[ \hat{p}_t^E = (1 - \omega \beta) E_t \sum_{j=0}^{\infty} \beta^j \omega^j [\hat{\phi}_{t+j} + \hat{P}_{t+j} - \hat{P}_t] \]

then, with \( \hat{p}_t^E = \hat{P}_t^* - \hat{P}_t \)

\[ \hat{P}_t^* = (1 - \omega \beta) E_t \sum_{j=0}^{\infty} \beta^j \omega^j [\hat{\phi}_{t+j} + \hat{P}_{t+j}] \]

Recursively

\[ \hat{P}_t^* = \hat{p}_t^E + \hat{P}_t = (1 - \omega \beta) \left[ \hat{\phi}_t + \hat{P}_t \right] \left( 1 - \omega \beta \right) \omega \beta E_t \sum_{j=0}^{\infty} \beta^j \omega^j \left[ \hat{\phi}_{t+j+1} + \hat{P}_{t+j+1} \right] \]

\[ \hat{p}_t^E = (1 - \omega \beta) \hat{\phi}_t + \omega \beta \left( E_t \hat{P}_t^E + E_t \hat{\pi}_{t+1} \right) \quad (25) \]

Log-linearizing the price Index we obtain:

\[ 0 = \omega \left( \hat{P}_{t-1} - \hat{P}_t \right) + (1 - \omega) \left( \hat{p}_t^E \right) \]

\[ \hat{p}_t^E = \frac{\omega}{(1 - \omega)} (\hat{\pi}_t) \quad (26) \]

replacing (26) into (25)

\[ \frac{\omega}{(1 - \omega)} \hat{\pi}_t = (1 - \omega \beta) \hat{\phi}_t + \omega \beta \left( E_t \frac{\omega}{(1 - \omega)} \hat{\pi}_{t+1} + E_t \hat{\pi}_{t+1} \right) \]

\[ \hat{\pi}_t = \frac{(1 - \omega \beta) (1 - \omega)}{\omega} \hat{\phi}_t + \beta E_t \hat{\pi}_{t+1} \]
\[ \hat{\pi}_t = \Upsilon \hat{\phi}_t + \beta E_t \hat{\pi}_{t+1} \]

where \( \Upsilon = \frac{(1 - \omega \beta)(1 - \omega)}{\omega} \) and \( \hat{\phi}_t \) is the real marginal cost equal to \( \hat{P}_t^Y = \hat{\phi}_{F,t} = \hat{\phi}_{I,t} = \hat{\phi}_t \).

The expression of the marginal cost can be obtained either from (22) and (14) or from (21).

\[ \hat{\phi}_t = \kappa X_{F,t} + c p_t \]

where

\[ \Omega_x = \frac{(\eta + \sigma)(1 - \tau^w) - \tau^w}{\eta + \sigma} \]
\[ \kappa = (\eta + \sigma) (1 - \tau^w) - \tau^w > 0, \quad \frac{\partial \kappa}{\partial N_F} > 0 \]
\[ c p_t = -\tau^w \hat{Y}_{e,F,t}, \quad \frac{\partial c p_t}{\partial N_F} > 0 \]

The Phillips curve can then be written as follows:

\[ \hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \Upsilon \kappa X_{F,t} + \Upsilon c p_t \quad (27) \]

### Appendix A3: IS Curve

Now log-linearizing the Euler equation (4) we obtain:

\[ \hat{Y}_{F,t} = E_t \left[ \hat{Y}_{F,t+1} \right] - \frac{1}{\sigma} \hat{i}_t + \frac{1}{\sigma} E_t \left[ \hat{\pi}_{t+1} \right] \quad (28) \]

on the other side, the social planner Euler equation can be represented as follows

\[ \hat{Y}_{e,F,t} = E_t \left[ \hat{Y}_{e,F,t+1} \right] - \frac{1}{\sigma} \hat{r}^n \]

where \( \hat{r}^n \) is the natural interest rate.

From the last two equations we can find ans expression for \( \hat{Y}_{F,t} - \hat{Y}_{e,F,t} \), this is:

\[ \hat{Y}_{F,t} - \hat{Y}_{e,F,t} = E_t \left[ \hat{Y}_{F,t+1} - \hat{Y}_{e,F,t+1} \right] - \frac{1}{\sigma} \left[ \hat{i}_t - \hat{r}^n - E_t \left[ \hat{\pi}_{t+1} \right] \right] \]
\[ X_{F,t} = E_t [X_{F,t+1}] - \frac{1}{\sigma} \left[ \hat{i}_t - \hat{r}_t^n - E_t [\hat{\pi}_{t+1}] \right] \]

Multiplying (28) by \( \frac{Y}{Y} \)

\[ \frac{Y}{Y} \hat{Y}_{F,t} = E_t \left[ \frac{Y}{Y} \hat{Y}_{F,t+1} \right] - \frac{1}{\sigma} \frac{Y}{Y} \hat{i}_t + \frac{1}{\sigma} \frac{Y}{Y} E_t [\hat{\pi}_{t+1}] \]

adding \( \frac{Y}{Y} \hat{Y}_{I,t} \) to both sides

\[ \frac{Y}{Y} \hat{Y}_{F,t} + \frac{Y}{Y} \hat{Y}_{I,t} = E_t \left[ \frac{Y}{Y} \hat{Y}_{F,t+1} + \frac{Y}{Y} \hat{Y}_{I,t+1} \right] - \frac{1}{\sigma} \frac{Y}{Y} \hat{i}_t + \frac{1}{\sigma} \frac{Y}{Y} E_t [\hat{\pi}_{t+1}] \]

given that we have that for the informal sector \( E_t \left[ \hat{Y}_{I,t+1} \right] = \hat{Y}_{I,t+1} \), previous equations becomes

\[ \hat{Y}_t = E_t \left[ \hat{Y}_{t+1} \right] - \frac{\hat{Y}_t}{\sigma} \left( \hat{i}_t - E_t [\hat{\pi}_{t+1}] \right) \]

Expressing this function in terms of the aggregate welfare output gap \( \hat{Y}_t - \hat{Y}_e = X_t \), we get the aggregate IS curve:

\[ \hat{Y}_t - \hat{Y}_e = \left( E_t \left[ \hat{Y}_{t+1} \right] - \frac{Y}{Y} \frac{1}{\sigma} \left( \hat{i}_t - E_t [\hat{\pi}_{t+1}] \right) \right) - \left( \frac{\hat{Y}_e}{\hat{Y}_e} - \frac{1}{\sigma} \right) \frac{\hat{r}_t}{\hat{r}_t} \]

\[ \hat{Y}_t - \hat{Y}_e = E_t \left[ \hat{Y}_{t+1} - \hat{Y}_{t+1} \right] - \frac{Y}{Y} \frac{1}{\sigma} \left( \hat{i}_t - \hat{r}_t^n - E_t [\hat{\pi}_{t+1}] \right) + \left( \frac{\hat{Y}_e}{\hat{Y}_e} - \frac{Y}{Y} \right) \frac{1}{\sigma} \hat{r}_t^n \]

we get the aggregate IS curve:

\[ X_t = E_t [X_{t+1}] - \frac{Y}{Y} \frac{1}{\sigma} \left( \hat{i}_t - \hat{r}_t^n - E_t [\hat{\pi}_{t+1}] \right) + \left( \frac{\hat{Y}_e}{\hat{Y}_e} - \frac{Y}{Y} \right) \frac{1}{\sigma} \hat{r}_t^n \]

\[ X_t = E_t [X_{t+1}] - \frac{\theta^m}{\sigma} \left( \hat{i}_t - \hat{r}_t^n - E_t [\hat{\pi}_{t+1}] \right) + \left( \theta^e - \theta^m \right) \frac{1}{\sigma} \hat{r}_t^n \]

where \( \theta^m = \frac{Y}{Y} \) and \( \theta^e = \frac{\hat{Y}_e}{\hat{Y}_e} \)
Appendix A4: The welfare loss function

The second order approximation of the utility function in the formal and in the informal sector around an steady state yields, for each sector

\[ U_{F,t} - U_F \simeq U_c C_F \left( \frac{c_{F,t} - c_F}{c_F} \right) + U_{h_F} h_F \left( \frac{h_{F,t} - h_F}{h_F} \right) + \frac{1}{2} U_{c_F c_F} C_F^2 \left( \frac{c_{F,t} - c_F}{c_F} \right)^2 + \frac{1}{2} U_{h_F h_F} h_F^2 \left( \frac{h_{F,t} - h_F}{h_F} \right)^2 \]

\[ U_{I,t} - U_I \simeq U_c C_I \left( \frac{c_{I,t} - c_I}{c_I} \right) + U_{h_I} h_I \left( \frac{h_{I,t} - h_I}{h_I} \right) + \frac{1}{2} U_{c_I c_I} C_I^2 \left( \frac{c_{I,t} - c_I}{c_I} \right)^2 + \frac{1}{2} U_{h_I h_I} h_I^2 \left( \frac{h_{I,t} - h_I}{h_I} \right)^2 \]

We use the following second-order approximation of relative deviations in terms of log deviations

\[ \frac{z_t - z}{z} \simeq \hat{z}_t + \frac{1}{2} \hat{z}_t^2, \text{ where } \hat{z}_t = z_t - z \]

\[ U_{F,t} - U_F \simeq U_c C_F \left( \hat{c}_{F,t} + \frac{1}{2} \hat{c}_{F,t}^2 \right) + U_{h_F} h_F \left( \hat{h}_{F,t} + \frac{1}{2} \hat{h}_{F,t}^2 \right) + \frac{1}{2} U_{c_F c_F} C_F^2 \left( \hat{c}_{F,t} + \frac{1}{2} \hat{c}_{F,t}^2 \right)^2 + \frac{1}{2} U_{h_F h_F} h_F^2 \left( \hat{h}_{F,t} \right)^2 \]

\[ U_{I,t} - U_I \simeq U_c C_I \left( \hat{c}_{I,t} + \frac{1}{2} \hat{c}_{I,t}^2 \right) + U_{h_I} h_I \left( \hat{h}_{I,t} + \frac{1}{2} \hat{h}_{I,t}^2 \right) + \frac{1}{2} U_{c_I c_I} C_I^2 \left( \hat{c}_{I,t} + \frac{1}{2} \hat{c}_{I,t}^2 \right)^2 + \frac{1}{2} U_{h_I h_I} h_I^2 \left( \hat{h}_{I,t} \right)^2 \]

then

\[ U_{F,t} - U_F \simeq U_c C_F \left[ \hat{c}_{F,t} + \frac{1 - \sigma}{2} \hat{c}_{F,t}^2 \right] + U_{h_F} h_F \left( \hat{h}_{F,t} + \frac{1 + \eta}{2} \hat{h}_{F,t}^2 \right) \]

\[ U_{I,t} - U_I \simeq U_c C_I \left[ \hat{c}_{I,t} + \frac{1 - \sigma}{2} \hat{c}_{I,t}^2 \right] + U_{h_I} h_I \left( \hat{h}_{I,t} + \frac{1 + \eta}{2} \hat{h}_{I,t}^2 \right) \]

with \( \hat{y}_S = \hat{c}_S \) and \( \hat{y}_{s,t} = \hat{y}_{s,t} - \hat{A}_{s,t} + d_t \) where \( d_t = \log \int_0^1 \left( \frac{p_t(i)}{p_t} \right)^{-\theta} d_t \). Galí (2008, p87) shows that \( d_t \) is proportional to the cross-sectional variance of relative prices. Therefore, \( d_t \simeq \frac{\theta}{2} \text{var}_i \{ p_t(i) \} \)
Now the period $t$ utility can be writing as follows:

$$U_{F,t} - U_F \simeq U_c C_F \left[ \hat{y}_{F,t} + \frac{1-\sigma}{2} \hat{y}_{F,t}^2 \right] + U_{h_F} h_F \left( \hat{y}_{F,t} + \frac{\eta}{2} \text{var}_i \{p_t(i)\} + \frac{1}{2} \left( \hat{y}_{F,t} - \hat{A}_{F,t} \right)^2 \right) + t.i.p$$

$$U_{I,t} - U_I \simeq U_c C_I \left[ \hat{y}_{I,t} + \frac{1-\sigma}{2} \hat{y}_{I,t}^2 \right] + U_{h_I} h_I \left( \hat{y}_{I,t} + \frac{\eta}{2} \text{var}_i \{p_t(i)\} + \frac{1}{2} \left( \hat{y}_{I,t} - \hat{A}_{I,t} \right)^2 \right) + t.i.p$$

where $t.i.p$ represents all terms independent of policy.

Equilibrium of the steady stated implies $\frac{U_{F,t}}{U_F} = MPH_F = (1-\tau^w) \frac{Y_F}{K_F}$, $\frac{U_{I,t}}{U_I} = MPH_I = \frac{Y_I}{K_I}$, and $Y_s = C_s$.

$$\frac{U_{F,t} - U_F}{U_c C_F} \simeq \left[ \hat{y}_{F,t} + \frac{1-\sigma}{2} \hat{y}_{F,t}^2 \right] - (1-\tau^w) \left( \hat{y}_{F,t} + \frac{\eta}{2} \text{var}_i \{p_t(i)\} + \frac{1}{2} \left( \hat{y}_{F,t} - \hat{A}_{F,t} \right)^2 \right) + t.i.p$$

$$\frac{U_{I,t} - U_I}{U_c C_I} \simeq \left[ \hat{y}_{I,t} + \frac{1-\sigma}{2} \hat{y}_{I,t}^2 \right] - \left( \hat{y}_{I,t} + \frac{\eta}{2} \text{var}_i \{p_t(i)\} + \frac{1}{2} \left( \hat{y}_{I,t} - \hat{A}_{I,t} \right)^2 \right) + t.i.p$$

under the small distortion assumption (so that the product of $\tau^w$ with a second order term can be ignored as negligible),

$$\frac{U_{F,t} - U_F}{U_c C_F} \simeq \left[ \hat{y}_{F,t} + \frac{1-\sigma}{2} \hat{y}_{F,t}^2 \right] - (1-\tau^w) \left( \hat{y}_{F,t} + \frac{\eta}{2} \text{var}_i \{p_t(i)\} + \frac{1}{2} \left( \hat{y}_{F,t} - \hat{A}_{F,t} \right)^2 \right) + t.i.p$$

$$\frac{U_{I,t} - U_I}{U_c C_I} \simeq \left[ \hat{y}_{I,t} + \frac{1-\sigma}{2} \hat{y}_{I,t}^2 \right] - \left( \hat{y}_{I,t} + \frac{\eta}{2} \text{var}_i \{p_t(i)\} + \frac{1}{2} \left( \hat{y}_{I,t} - \hat{A}_{I,t} \right)^2 \right) + t.i.p$$

then

$$\frac{U_{F,t} - U_F}{U_c C_F} \simeq -\frac{1}{2} \left( -2\tau^w \hat{y}_{F,t} + \theta \text{var}_i \{p_t(i)\} + (\eta + \sigma) \hat{y}_{F,t}^2 - 2 (1 + \eta) \left( \hat{y}_{F,t} \hat{A}_{F,t} \right) \right) + t.i.p$$

$$\frac{U_{I,t} - U_I}{U_c C_I} \simeq -\frac{1}{2} \left( \theta \text{var}_i \{p_t(i)\} + (\eta + \sigma) \hat{y}_{I,t}^2 - 2 (1 + \eta) \left( \hat{y}_{I,t} \hat{A}_{I,t} \right) \right) + t.i.p$$
\[ Y^e_{I,t} = \frac{1 + \eta}{\eta + \sigma} \hat{A}_{I,t} \]
\[ Y^e_{F,t} = \frac{1 + \eta}{\eta + \sigma} \hat{A}_{F,t} \]

we obtain

\[ \frac{U_{F,t} - U_F}{U_c C_F} \simeq -\frac{1}{2} \left( -2\tau w X_{F,t} + \theta \text{var}_i \{ p_t(i) \} + (\eta + \sigma) (X_{F,t})^2 \right) + t.i.p \]
\[ \frac{U_{I,t} - U_I}{U_c C_I} \simeq -\frac{1}{2} \left( \theta \text{var}_i \{ p_t(i) \} + (\eta + \sigma) (X_{I,t})^2 \right) + t.i.p \]

Accordingly, a second-order approximation can be written to the consumer’s welfare losses (up to additive terms independent of policy), and expressed as a fraction of steady state consumption as

\[
\mathcal{W} = -E_0 \sum_{t=0}^{\infty} \beta^t \left[ N_F \frac{U_{F,t} - U_F}{U_c C_F} + (1 - N_F) \frac{U_{I,t} - U_I}{U_c C_I} \right]
\]

\[
\mathcal{W} = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{1}{2} \left[ \theta \text{var}_i \{ p_t(i) \} + (\eta + \sigma) \left( N_F (X_{F,t})^2 + (1 - N_F) (X_{I,t})^2 \right) - N_F \tau w X_{F,t} \right] \right\}
\]

In Woodford (2003, chapter 6) is proved that \( \sum_{t=0}^{\infty} \beta^t \text{var}_i \{ p_t(i) \} = \frac{\omega}{(1-\beta \omega)(1-\omega)} \sum_{t=0}^{\infty} \beta^t \pi_t^2 \)

with \( \Upsilon = \frac{(1-\beta \omega)(1-\omega)}{\omega} \) we have \( \sum_{t=0}^{\infty} \beta^t \text{var}_i \{ p_t(i) \} = \frac{1}{4} \sum_{t=0}^{\infty} \beta^t \pi_t^2 \)

Therefore, the second-order Taylor approximation of the aggregate utility function around the steady state \( (c_F, c_I, h_F, h_I) \) yields to the following welfare loss function \( \mathcal{W} \):

\[
\mathcal{W} = E_0 \sum_{t=0}^{\infty} \beta^t \left( \frac{1}{2} \left[ \frac{\theta}{T} \pi_t^2 + (\eta + \sigma) \left( N_F X_{F,t}^2 + (1 - N_F) X_{I,t}^2 \right) - N_F \tau w X_{F,t} \right] \right)
\]
1.8 References


2 Optimal monetary policy in developing countries: the role of informality

Abstract

In this paper, I analyze optimal monetary policy in developing countries whose labor markets are characterized by the presence of a large informal sector. I develop a closed economy model with nominal price and wage rigidities, search and matching frictions, and a dual labor market. A formal one characterized by matching frictions and nominal wage rigidities, and an informal one where wages are fully flexible. Under this framework, a trade-off between price and wage inflation emerges. I find that informality increases the response of price and wage inflation to aggregate productivity shocks. As a result, the presence of an informal sector increases the inefficient fluctuations of the labor market variables, such as unemployment, labor market tightness, and formal hiring rate. I find that optimal policy with informality features significant deviations from price stability in response to aggregate productivity shocks.

JEL classification: E26, E52, E12, E61.

Keywords: Informality, Monetary policy, Nominal wage and price rigidities, Inflation targeting.

2.1 Introduction

In developing and emerging countries, the informal sector often accounts for a substantial fraction of the urban labor force. According to the International Labor Office (2018), informal employment accounts for more than half of non-agricultural employment in most developing countries: around 72 percent in Africa, 63 percent in Asia and the Pacific, 64 percent in the Arab States and 50 percent in Latin America. Most of the workers in this sector are self-employed. Their income comes from operating small unincorporated enterprises\(^6\) that are hidden from regulatory and monetary authorities, and are hardly registered by official statistics. While offering the advantage

\(^6\)These include activities such as trading on the streets or in markets; sales of cooked food from kiosks; the transport of people or goods by pedal-power or motorbikes; repairing clothes, shoes, or motor scooters; dwelling construction or adding extensions to them; scavenging for reusable waste; or providing a range of personal services like hairdressing, fortune-telling, shoe cleaning, street theater, house cleaning, and the like (Blades et., al. 2011).
of employment flexibility in some economies, a large informal sector is associated with low productivity, reduced tax revenues, poverty, and income inequality (World Bank, 2019).

While offering the advantage of employment and de facto flexibility, a large informal sector is associated with adverse macroeconomic and development outcomes. On average, economies with larger informal sectors tend to have lower productivity, slower physical and human capital accumulation, higher poverty and inequality, and smaller fiscal resources (World Bank, 2019). Since the informal sector is labor-intensive, tends to employ low-skilled workers, has credit constraints, and lacks economies of scale, it has on average lower productivity and physical and capital accumulation than the formal sector (Loayza, 2018). Informality can also hinder growth by distorting firm decisions (e.g., staying small to stay hidden) and allowing unproductive firms to survive, then distorting the allocation of resources further (Ulyssea, 2018). Employment in the informal sector can provide a safety net by keeping or creating employment during periods when the formal sector is shedding jobs (Loayza and Rigolini, 2011). However, workers in the informal economy are largely excluded from the social security system and less protected against negative shocks than workers in the formal sector. A large informal sector also reduces tax revenues that limit the ability of the government to provide public goods and social transfers. According to Besley and Persson (2014), low-income countries typically collect taxes of between 10 to 20 percent of Gross Domestic Product (GDP), while the average for high-income countries is more like 40 percent. This difference is not necessarily explained by a choice of low tax rates but by the challenges associated with tax collection: these include informality and misreporting (Besley and Persson 2009).

The implications of informality have drawn considerable attention in the literature. Most of the research on this topic aims to study how informal jobs in the labor market are generated and analyze the effect of fiscal and labor market policies on informal economic activity. The existing literature focuses mainly on the real economy, and not many papers have been devoted to monetary policy analysis in the presence of a large informal sector. The main objective of this paper is to contribute to this literature by studying the design of optimal monetary policy in economies with informality.

I develop a closed economy model with dual labor markets, formal and informal, that integrates labor market search into a New Keynesian model with nominal price and wage rigidities. Following Thomas (2008) and Gertler and Trigari (2009), I introduce
staggered nominal wage bargaining under which firms and workers in the formal sector bargain over wages in a setting with search and matching frictions. Motivated by the fact that the informal sector is mainly characterized by self-employed workers, I assume that wages in the informal sector are flexible.

I obtain the approximated quadratic welfare loss function and then characterize optimal monetary policy under commitment. I find that welfare decreases as inflation and wage volatility increases. Inflation causes inefficient dispersion on prices across retail firms, and similarly, wage inflation generates an inefficient dispersion on wages across formal firms. Welfare also decreases with output and labor market tightness volatility. This is because the composition of total production between formal and informal goods is distorted when the formal labor market tightness differs from its efficient value. Finally, the inefficient fluctuations in employment are an additional source of welfare losses. To my knowledge, this is the first paper to introduce both nominal price and wage rigidities in a model with informality and to characterize optimal monetary policy in these types of economy.

I show that for the case in which only price rigidities are present, wages in the formal sector are Nash bargained every period, and the steady-state is efficient, a zero inflation policy is optimal. I also show that when the negotiation of formal wages is staggered, a trade-off between price inflation and unemployment stabilization emerges. In the presence of price and formal wage rigidities, complete price-level stabilization is no longer optimal. As a consequence, the Central Bank should consider both price and formal wage stability, since fluctuations in price and wage inflation generate inefficient fluctuations in the allocation of resources in the economy.

To better understand the implications of informality for optimal monetary policy, I compare the predictions of the model against a case in which there is no informal sector in the economy. I find that the contribution of wage inflation volatility to the welfare loss, relative to the contribution of price inflation volatility, is lower for the case with informality. This result is explained by the fact that in the presence of an informal sector, the proportion of firms in the economy facing wage rigidities is lower. Therefore, in the presence of an informal sector, the optimal policy will result in a lower price inflation volatility for a given level of wage inflation volatility.

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7 Under this setting, formal wages are going to affect employment at an extensive margin. They influence the rate at which firms in the formal sector add new workers to their respective labor forces. As emphasized by Hall (2005), in this kind of setting the Barro’s critique does not apply (Gertler and Trigari, 2009).

8 For a discussion of the reasons why the informal sector should have less frictions see Zenou (2008).
Additionally, I find that, in the presence of an informal sector, the inefficient fluctuations on the labor market variables such as employment, labor market tightness, and formal hiring rate are higher. This result is explained by the fact that, in response to an aggregate productivity shock, only a fraction of firms in the formal sector can adjust their nominal wages. This wage rigidity generates a gap between the actual and the natural formal wage (the target wage) that translates into wage dispersion and inefficient job creation in the formal sector. In the presence of informality, the response of the target wage to productivity shocks is higher. The target wage in the formal sector depends on the informal wage (the outside option) and the formal labor market tightness. After a negative productivity shock, the decrease in both variables is higher than in the case without informality: on the one hand, the outside option decreases with an adverse productivity shock; and on the other hand, the informal sector works as a buffer that absorbs workers in bad times, and vice-versa. Consequently, after a negative productivity shock, the increase in unemployment is lower in the presence of an informal sector. Hence, the probability to fill a formal vacancy is also lower, pinning down the formal firm’s surplus and their incentive to hire.

As a result, in response to aggregate productivity shocks, the Central Bank should use price inflation to avoid excessive wage inflation volatility that causes excessive unemployment volatility and excessive dispersion in the formal hiring rate. By controlling the inflation rate, the central bank can affect the real value of nominal wages and then bring real formal wages closer to their flexible-wage levels. The presence of an informal sector requires a higher adjustment of price inflation to reduce this gap.

In summary, the existence of a large informal sector has two opposite implications for optimal monetary policy. On the one side, given that in the presence of informality the proportion of firms in the economy facing wage rigidities is lower, the optimal policy results in a lower price inflation volatility for a given level of wage inflation volatility. On the other side, in the presence of a large informal sector, wage inflation and unemployment volatility are higher. As a result, the Central Bank should use price inflation to avoid excessive unemployment volatility and excessive dispersion in the formal hiring rate. The aggregate effect on price inflation volatility would depend on which effect dominates. For a standard calibration of the model for an economy with a large informal sector, I find that under the optimal monetary policy, the volatility of inflation is relatively higher in the presence of informality.

Finally, to illustrate the implications of the trade-off faced by the Central Bank, I
analyze the behavior of the decentralized economy when the monetary authority implements a policy of zero price inflation. I find that the welfare loss under a zero price inflation policy is about 1.26 times as large as under the optimal policy. For the case without informality, the welfare loss under a zero price inflation policy is about 0.015 times larger than the regime under the optimal policy. These results show that a policy designed to minimize price inflation volatility can generate significant welfare losses in the presence of nominal wage rigidities and informality, which might be the case for most emerging countries.

2.1.1 Related Literature

Early modeling in the area of informal economics started from the classical Harris-Todaro (1970) framework. In these models, informality is captured by building a model of two distinct markets that are segmented and in which two different wage equilibrium prevail (wage duality), where wages in the formal sector can turn out to be higher than the market-clearing wages. Brueckner and Zenou (1999) add a land market to the standard Harris-Todaro framework where wages are endogenously fixed. The idea of identifying the informal labor market with the disadvantaged sector of a market segmented by rigidities in the formal sector dates back to Lewis (1954). Most recent literature has developed more sophisticated models to represent formal, informal, and integrated labor markets (see Boeri and Garibaldi 2005, Fugazza and Jacques 2003, and Badaoui et al. 2006). In these models, trading frictions in the formal and the informal sectors are important, and it is possible to determine rules governing the flows between the two sectors, as well as to and from the pool of the unemployed. Most of these models incorporate the search and matching model of Mortensen-Pissarides into Harris and Todaro’s model.

The previous literature focuses on the intersectoral margin for workers and firms. Other papers like Albrecht et al. (2008), Zenou (2008), and Satchi and Temple (2009) focus on the intersectoral margin for workers. Albrecht et al. (2008) develop a search and matching model with endogenous job destruction. Workers have the same productivity in the informal sector (unregulated self-employment) while they have different productivity if they decide to enter the formal sector. In this way, the relative productivity in the two sectors is an important factor in the workers’ choice. In this model, unemployment is the residual state in the sense that workers whose employment in either an informal-sector or a formal-sector job ends flow back into unemployment.
In general, these papers aim to study how informal jobs in the labor market are created, and the effect of fiscal policy and labor market institutions (such as employment protection legislation, tax wedge, unemployment benefits, unemployment benefit duration, and union density) on informal economic activity. These studies focus on the real economy and do not analyze the interaction between the informal sector and monetary policy. Surprisingly, few papers have been devoted to the monetary policy analysis when the economy displays a large informal sector. An exception is Castillo and Montoro (2010), Batini et al. (2011), and Alberola and Urrutia (2020).

Castillo and Montoro (2010) is the first paper that analyses the effect of informal labor markets on monetary policy. They extend Blanchard and Gali (2010) by modeling a dual labor market economy with formal and informal labor contracts within a New Keynesian model with labor market frictions. In this framework, informality is a result of hiring costs, which are a function of the ratio of vacancies to unemployment. They find that informal workers act as a buffer on employment that allows firms to increase output without generating pressure on wages. Batini et al. (2011) study how informality affects the conduct of monetary policy. They develop a two-sector, formal and informal, New Keynesian model. The informal sector is more labor-intensive, can avoid taxation, has a classical labor market, faces high credit constraints in financing investment, and is less visible in terms of observed output. They find that the importance of commitment increases in economies characterized by a large informal sector and that optimal simple rules that respond only to observed aggregate inflation and formal output can be significantly worse in welfare terms than their optimal counterpart. In the same line, Arberola and Urrutia (2020) analyze the effect of informality on monetary policy. They develop a general equilibrium closed economy model with labor and financial frictions and nominal price rigidities. They find that informality has a buffering effect on the propagation of demand and supply shocks to prices. As a result, informality dampens the impact of demand and financial shocks on wages and inflation but amplifies the impact of technology shocks. Informality also increases the sacrifice ratio of monetary policy.

In contrast to Castillo and Montoro (2010), Alberola and Urrutia (2020), and Batini et al. (2011), I consider both price and wage rigidities and characterize optimal monetary policy under commitment. Under this framework, it is possible to analyze optimal monetary policy with informality in a scenario where there is a trade-off between inflation and unemployment.
The rest of the chapter is organized as follows: Section 2.2 presents the model. In section 2.3, I consider both the equilibrium of the model with flexible price and wages, and the Social Planner Solution. In section 2.4, I derive a log-linear approximation of the rational expectations equilibrium around the efficient steady-state under staggered wage bargaining in the formal sector. Section 2.5 analyzes the optimal monetary policy under commitment and the role of informality on the optimal monetary policy design. Section 2.6 concludes.

2.2 Model

The analysis builds on a New-Keynesian framework with dual labor markets. The model consists of households whose utility depends on the consumption of market goods and whose members are either employed in the formal or the informal sector, or are unemployed. Wholesale formal firms employ formal labor to produce a wholesale formal good that is sold in a competitive market. The labor market in this sector is characterized by search and matching frictions and nominal wage rigidities. Wholesale informal firms employ informal labor to produce a wholesale informal good sold in a competitive market. The labor market in this sector is not subject to search and matching frictions and wages are flexible. Retails firms aggregate the two wholesale goods and transform them into differentiated final goods that are sold to households in an environment of monopolistic competition.

2.2.1 Wholesale Firms

I assume two types of firms in the wholesale sector: formal and informal. Wholesale formal firms produce a homogeneous formal intermediate good that is sold to retailers at a competitive price $p^{f}_{t}$. The labor market in this sector is characterized by the presence of search and matching frictions and staggered wage bargaining. On the other side, wholesale informal firms produce a homogeneous informal intermediate good that is sold to retailers at a competitive price $p^{i}_{t}$. The labor market in this sector is not subject to search and matching frictions and wages are flexible.
**Wholesale informal firms**

Every period, each worker in the informal sector produces $y_{it}$ units of output under a production technology linear in labor $l_t^i$.

The aggregate output of the informal sector is given by:

$$y_t^i = z_t z^l l_t^i,$$

where $z_t$ is an aggregate productivity shock, and $z^l$ is a parameter denoting the productivity associated with workers in the informal sector. $ln(z_t)$ follows a first-order auto-regressive process, $ln(z_t) = \rho z ln(z_{t-1}) + \varepsilon_t^z$, where $\varepsilon_t^z$ is an independent and identically distributed shock.

Workers in this sector are self-employed, and wages equal the marginal productivity of labor:

$$w_t^i = p_t^i z_t z^l.$$

**Wholesale formal firms**

**The matching function**

Wholesale formal firms produce a homogeneous formal intermediate good $y_t^f$. In this sector, the number of hires is determined by a search and matching process. Each period, the number of successful matches between firms that post vacancies $v_t$ and unemployed workers looking for a job in the formal sector $l_t^u$ is determined by the matching function:

$$m(v_t, l_t^u) = N(l_t^u)\mu (v_t)^{1-\mu},$$

where $N$ is a scale parameter that reflects the efficiency of the matching process, and $(1 - \mu) \in (0, 1)$ measures the elasticity of the matching function with respect to vacancies. The probability to fill a vacancy, $q(\theta_t)$, is equal to:

$$q(\theta_t) = \frac{m(v_t, l_t^u)}{v_t} = N(\theta_t)^{-\mu},$$

where $\theta_t = \frac{n_t}{l_t}$ is the labor market tightness in the formal sector.

Similarly, the probability that an unemployed worker find a job in the formal sector, $p(\theta_t)$, is equal to:
\[ p(\theta_t) = \frac{m(v_t, l_t^u)}{l_t^u} = N(\theta_t)^{1-\mu}, \quad (33) \]

Equation (33) implies that an increase in the number of vacancies relative to the number of unemployed that search for a job in the formal sector increases the probability for an unemployed of finding a job in this sector. On the other side, an increase in \( \theta_t \) decreases the probability to fill a vacancy. Both firms and workers take \( q(\theta_t) \) and \( p(\theta_t) \) as given.

Assuming that firms in this sector are sufficiently large, \( q(\theta_t) \) represents the fraction of vacancies filled in period \( t \). New hires do not become productive until the next period, given the time involved in recruiting and training these new workers.

Therefore, the aggregate of employed workers in the formal sector at time \( t + 1 \) can be represented as follows:

\[ l_{it+1}^f = (1 - \rho) l_{it}^f + q(\theta_t) v_t, \quad (34) \]

where \( \rho \) is the exogenous destruction rate of formal employment.

**Formal production**

In the formal sector, firms are indexed by \( i \). Each firm employs \( l_{it}^f \) workers in period \( t \) and also posts \( v_{it} \) vacancies to attract new workers for the next period of operation. The total number of vacancies and employed workers in the formal sector are \( v_t = \sum_i v_{it}di \) and \( l_t^f = \sum_i l_{it}^f di \), respectively. If the search process is successful, the firm in the formal sector operates with the following technology:

\[ y_{it}^f = z_t z^f l_{it}^f, \]

where \( z^f \) is a parameter that represents the labor productivity specific to the formal sector, with \( z^f > z^t \). \( y_{it}^f \) is sold to retailers at a price \( p_t^f \).

The hiring rate \( \mathcal{F}_{it} \), is defined as the ratio between the number of vacancies \( v_{it} \), and the number of hired workers \( l_{it}^f \):

\[ \mathcal{F}_{it} = \frac{v_{it}}{l_{it}^f}. \]

Because of the staggered wage bargaining process, there will be wage dispersion across firms. As in Thomas (2008) and Gertler and Trigari (2009), I assume convexity in
vacancy-posting cost to ensure an equilibrium where all formal firms post vacancies in the presence of wage dispersion\(^9\). This cost is measured in terms of utility for the firm’s management, and is given by:

\[
\frac{\kappa}{2} F_{it}^2 l_{it}.
\]

Conditional on the current wage and employment, the present value of the flow of benefits \(Q_{it}^o\) for each firm in the formal sector, can be expressed as:

\[
Q_{it}^o = \max_{v_{it}} \left\{ p_{it} y_{it} - w_{it} l_{it} - \frac{\kappa}{2} F_{it}^2 l_{it} + E_t \Gamma_t, t+1 Q_{it+1}^o \right\},
\]

subject to the law of motion of employment in firm \(i\):

\[
l_{it+1} = (1 - \rho) l_{it} + q(\theta_t) v_{it}.
\]

\(\Gamma_{t,t+s} = \beta s \frac{u'(c_{t+s})}{u'(c_t)}\) is the stochastic discount factor between periods \(t\) and \(t + s\). Firms choose the hiring rate by setting the number of vacancies in period \(t\). They maximize the present value of the flow of benefits, taken as given the probability of filling a vacancy and the current path of expected wages. In case the firm \(i\) can renegotiate wages, it bargains with its workforce over a new contract. Otherwise, the firm sets the wage at the previous period’s level. The first-order condition with respect to vacancies is given by:

\[
\frac{\kappa F_{it}}{u'(c_t)} = q(\theta_t) E_t \Gamma_{t,t+1} \frac{\partial Q_{it+1}^o}{\partial l_{it+1}}.
\]

The value of the marginal worker for the firm is given by:

\[
J_{it}^f = \frac{\partial Q_{it}^o}{\partial l_{it}} = p_{it} m_l l_{it} - w_{it} + \frac{\kappa F_{it}^2}{2u'(c_t)} + (1 - \rho) E_t \Gamma_{t,t+1} \frac{\partial Q_{it+1}^o}{\partial l_{it+1}}.
\]

Therefore, the value for a formal firm of having an occupied job at time \(t\) is equal to the marginal product of a worker \(mpl_{it}^f\), minus the real wage \(w_{it}^f\), plus the saving on adjustment cost, plus the discounted value of having a match in the following period. Combining equations (36) and (37) yields the following condition for the hiring rate:

\(^9\)Wage dispersion creates a dispersion in the marginal benefit of posting vacancies. When the cost of a vacancy is linear, then the marginal cost of posting vacancies would be the same for all firms, and only the firm with the lower wage would post vacancies.
\[
\frac{nF_{it}}{q(\theta_t)} = \beta E_t \left[ u'(c_{t+1}) \left( p_{t+1}^f m p_{t+1}^f - w_{t+1}^f + \frac{\kappa F_{it+1}^2}{2u'(c_{t+1})} \right) + (1 - \rho) \frac{\kappa F_{it+1}}{q(\theta_{t+1})} \right]. \tag{38}
\]

Equation (38) equates the cost of hiring a worker, discounted by the probability of filling a vacancy, to the expected value of a match. The hiring rate depends on the discounted streams of benefits from having a filled job, plus the savings on adjustment cost. Note that the wage \( w_{it}^f \) set by the firm \( i \), is the only firm-specific variable that affects the hiring rate \( F_{it} \). Consequently, all firms with the same wage \( w_{it}^f \) are going to choose the same hiring rate, independent of their respective employment size.

The dividends that the household receive from formal firms are equal to:

\[
div_{it}^f = p_{it}^f y_{it}^f - w_{it}^f l_{it}^f.
\]

Given the constant returns to scale in production, it is possible to express aggregate output of the formal intermediate good as follows:

\[
y_{it}^f = z^f z_t \int_0^1 t_{it}^f di = z^f z_t l_{it}^f.
\]

### 2.2.2 Retailers

In the retail sector, there is a continuum of monopolistic competitive retailers indexed by \( j \) on the unit interval. Let \( y_j \) be the quantity of output sold by retailer \( j \). Retail firms use an aggregate of intermediate goods to produce a final differentiated good. The aggregate intermediate good, is a composite of formal and informal goods, according to the Constant Elasticity of Substitution (CES) aggregator:

\[
y_{jt} = \left[ \left( \frac{y_{jt}^f}{\gamma} \right)^{\gamma - 1} + \left( \frac{y_{jt}^i}{\gamma - 1} \right)^{\gamma - 1} \right]^{\gamma - 1}, \tag{39}
\]

where \( \gamma \) is the elasticity of substitution between formal and informal produced goods.

To determine the demand for \( y_{jt}^f \) and \( y_{jt}^i \), retailers solve the following minimization cost problem:

\[
\min p_i^f y_{jt}^f + p_i^i y_{jt}^i,
\]
subject to (39).

The First Order Conditions (F.O.C) imply:

\[ y_{it}^f = \left( \frac{mc_t}{p_f^t} \right)^\gamma y_{jt}, \quad y_{it}^i = \left( \frac{mc_t}{p_i^t} \right)^\gamma y_{jt}, \]

where

\[ mc_t = \left[ (p_f^t)^{1-\gamma} + (p_i^t)^{1-\gamma} \right]^{\frac{1}{1-\gamma}} \]

represents the real marginal cost of producing and additional unit of \( y_t \).

Total production of final goods is equal to the following composite of individual retail goods:

\[ y_t = \left[ \int_o^1 \left( y_{jt}^{\frac{\alpha}{\alpha-1}} \right) \right]^{\frac{\alpha}{\alpha-1}}, \]

where \( \Theta \) is the elasticity of substitution between differentiated goods. In line with Calvo (1983), retail firms can change their prices optimally every period with a probability \( (1 - \omega) \) and set a price \( P_t^* \). With probability \( \omega \) the firm will set the price of the previous period \( P_{t-1} \).

In the case the firm has the chance to set prices optimally, it will choose the price that maximize the present discounted value of the firm’s benefits, as follows:

\[
\max_{P_t^*} \quad E_t \sum_{\ell=0}^\infty \Gamma_{t+\ell} (\omega^p)\ell \left[ (1 + \tau^m) P_t^* y_{t+\ell/t} - MC_{t+\ell/t} y_{t+\ell/t} \right]
\]

subject to the sequence of demand constraints:

\[ y_{t+\ell/t} = \left( \frac{P_t^*}{P_{t+\ell}} \right)^{-\Theta} y_{t+\ell}. \] (40)

where \( y_{t+\ell/t} \) and \( MC_{t+\ell/t} \) denote, respectively, the output and nominal marginal cost in period \( t + \ell \) for a firm whose last reset of prices was in period \( t \).

To offset the distortion caused by monopolistic competition in the retail sector and to ensure the steady-state equilibrium is efficient, I assume that the firm’s output is subsidized at the fixed rate \( \tau^m = \frac{1}{\Theta} \). The optimal firm’s price-setting decision is given by (see Appendix B2 for the derivation):
\[
\frac{P'_t}{P_t} = \frac{E_t \sum_{\ell=0}^{\infty} \beta^\ell (\omega^p)^\ell \left( \frac{c_{i+\ell}}{c_t} \right)^{1-\sigma} \left( \frac{P_i}{P_t} \right)^{-\Theta} m c_{i+\ell}}{E_t \sum_{\ell=0}^{\infty} \beta^\ell (\omega^p)^\ell \left( \frac{c_{i+\ell}}{c_t} \right)^{1-\sigma} \left( \frac{P_i}{P_t} \right)^{1-\Theta} m c_{i+\ell}}.
\]

(41)

### 2.2.3 Households

The representative household consist of an extended family that contains a continuum of members. In this household, a fraction \( l'_{fi} = \int_0^1 l'_{di} di \) of its members are employed in the formal sector, where \( l'_{di} \) represents the number of workers in a firm \( i \). A fraction \( l'_i \) is working in the informal sector (self-employed), and the remaining fraction \( l''_i = 1 - l'_{fi} - l'_i \) is unemployed and searching for a job in the formal sector. Following most of the literature, I assume the existence of a representative infinitely-lived household, where all members pool their income and consumption is equalized across members. The representative household maximizes the following utility function:

\[
U_t = \sum_{i=0}^{\infty} \left\{ u(c_t) - \psi(l'_{fi} + l'_i) \right\},
\]

where

\[
c_t = \left[ \int_0^1 (c_{jt})^{\frac{\alpha-1}{\alpha}} \, dj \right]^{\frac{\alpha}{\alpha-1}}
\]

is an aggregate of differentiated final goods purchased from the continuum of retail firms, indexed by \( j \in [0, 1] \). The function \( u(c_t) \) is strictly increasing and strictly concave. Following Ravena and Walsh (2011) and Tomas (2008), I introduce a fixed component of the dis-utility of working \( \psi^{10} \).

The demand for each differentiated consumption good is determined by the intratemporal optimal choice across goods. It implies:

\[
c_{jt} = \left( \frac{P_{jt}}{P_t} \right)^{-\Theta} c_t,
\]

(42)

where

\[
P_t = \left[ \int_0^1 (P_{jt})^{\frac{\alpha-1}{\alpha}} \, dj \right]^{\frac{\alpha}{\alpha-1}}
\]

(43)

\[^{10} Under this framework it is possible to assume a linearity in work dis-utility due to the assumption of risk sharing and labor supply changing at the extensive margin (see Rogerson, 1988)
is the Dixit–Stiglitz aggregate price index. The law of motion for the price level is given by:

\[ P_t^{1-\Theta} = \omega^p (P_{t-1})^{1-\Theta} + (1 - \omega^p) (P_t^*)^{1-\Theta}. \]

Each period, the household faces the following budget constraint:

\[
\int_0^1 w_i^f l_i^f di + w_i^l l_i^l + div_i^h + \frac{(1 + r_{t-1})}{(1 + \pi_t)} b_{t-1} = c_t + b_t, \tag{44}
\]

where \( div_i^h \) are the dividends from the intermediary formal firms, \( r_t \) is the nominal interest rate, \( b_t \) are bonds and \( \pi_t \) is the inflation rate. The household chooses \( c_t \) and \( b_t \) that maximize their expected discounted utility, subject to their budget constraint. The first-order condition for this optimization problem results in the standard Euler equation:

\[ u'(c_t) = \beta E_t u'(c_{t+1}) \frac{(1 + r_t)}{(1 + \pi_{t+1})}, \tag{45} \]

where \( u'(c_t) \) is the marginal utility of consumption.

In equilibrium, total supply of the final good \( y_t \) must equal total demand by households \( \int_0^1 c_j d_j \). This condition can be written as follows:

\[ y_t = \Delta_t c_t, \tag{46} \]

where \( \Delta_t = \int_0^1 \left( \frac{p_{it}}{p_t} \right)^{-\Theta} d_j \) is a measure of the price dispersion.

### 2.2.4 Worker’s Value functions

The present discounted value for a worker in the formal sector is:

\[ Q_{it}^f = w_{it} - \frac{\varphi}{u'(c_t)} + E_t \gamma_{t+1} \left( (1 - \rho)Q_{it+1}^f + \rho \max \left[ Q_{it+1}^{lu} + Q_{it+1}^{iu} \right] \right). \tag{47} \]

Equation (47) implies that a worker hired in the formal sector receives a real wage \( w_{it}^f = \frac{w_{it}}{p_t} \) and has a disutility of working equal to \( \frac{-\varphi}{u'(c_t)} \). In the next period, she will continue working in this sector with probability \( (1 - \rho) \), in which case she will obtain an expected value of \( Q_{it+1}^f \). The probability that a formal worker loses her job is \( \rho \), in which case she will decide whether to become unemployed or work in the informal sector. This decision will depend on the maximal value between \( Q_{it+1}^{lu} \) and \( Q_{it+1}^i \).
Additionally, the present discounted value for a worker in the informal sector is:

\[ Q_{it}^u = w_{it} - \frac{\varphi}{u'(c_t)} + E_t \Gamma_{t, t+1} \max \left[ Q_{t+1}^u, Q_{t+1}^i \right]. \]  

(48)

In this case, a worker in the informal sector receives a real wage \( w_{it} = \frac{W_{it}}{P_t} \) and has a disutility of working equal to \( \frac{\varphi}{u'(c_t)} \). To apply for formal jobs, informal workers have to become unemployed. Therefore, next period workers in this sector will become unemployed or will continue working in the informal sector depending on the \( \max \left[ Q_{t+1}^u, Q_{t+1}^i \right] \).

Finally, the present discounted value for an unemployed worker is equal to:

\[ Q_{t}^u = E_t \Gamma_{t, t+1} \left( p(\theta_t) \bar{Q}_{F, t+1}^f + (1 - p(\theta_t)) \max \left[ Q_{t+1}^u, Q_{t+1}^i \right] \right), \]  

(49)

where \( \bar{Q}_{F, t}^f = \int_0^1 Q_{it}^f d\theta \) is the average value of employment in the formal sector. The probability of finding a job in the formal sector in period \( t \) is \( p(\theta_t) \). In this case, they will start working in the next period and obtain an expected value of \( \bar{Q}_{F, t+1}^f \).

With probability \( (1 - p(\theta_t)) \) they do not find a job in the formal sector, in which case they either will continue to be unemployed or will go to work in the informal sector, depending on the \( \max \left[ Q_{t+1}^u, Q_{t+1}^i \right] \). In equilibrium, the present discounted value for an unemployed equals the present discounted value of a worker in the informal sector, \( Q_{t}^u = Q_{t}^i \). Note that in the presence of staggered wage bargaining, the present value of finding a formal job in the next period for a worker who is currently unemployed is \( \bar{Q}_{F, t+1}^f \). This is because the unemployed worker does not know in advance which firm would be paying higher wages next period. The unemployed agent can only choose randomly among the formal firms posting vacancies.

The surplus derived by the worker at the firm paying a real wage \( w_{it}^f \), is denoted as \( \mathbb{H}_{it}^f \), and \( \bar{H}_{F, t}^f \) denotes the average formal worker’s surplus conditional on being a new hire, which are defined as follows:

\[ \mathbb{H}_{it}^f = Q_{it}^f - Q_{t}^u \]

\[ \bar{H}_{F, t}^f = \bar{Q}_{F, t}^f - Q_{t}^u. \]

Worker’s surplus in the formal sector can be expressed as:

\[ \mathbb{H}_{it}^f = w_{it}^f - \frac{\varphi}{u'(c_t)} + E_t \Gamma_{t, t+1} \left[ (1 - \rho) \mathbb{H}_{it+1}^f - p(\theta_t) \bar{H}_{F, t+1}^f \right]. \]  

(50)
Additionally, in equilibrium, the value of being unemployed, equation (49), equals the value of being informal, equation (48). This condition implies:

\[ p(\theta_t)E_t\Gamma_{t,t+1}\mathbb{E}^f_{t+1} + \frac{\varphi}{u'(c_t)} = w_i. \]  

(51)

The opportunity cost of being in the informal sector, which is equal to the sum of the expected value of searching for a job in the formal sector plus the labor disutility, equals the labor income in this sector.

2.3 Efficient and Flexible Wage Equilibrium

For comparative purposes, and before determining formal wages in the decentralized economy, I consider both the equilibrium of the model under flexible wages, hereafter referred to as the flexible wage equilibrium, and the social planner solution that is referred to as the efficient equilibrium.

2.3.1 Efficient equilibrium

In this section, I consider the social planner solution. The efficient allocation will be the benchmark relative to which monetary policy results will be evaluated.

In a scenario of perfect competition in goods and labor markets, the social planner chooses the state-contingent path of \( c_t, l^f_t, l^i_t \) and \( v_t \) to maximize the following joint welfare of households and managers:

\[ U_t = E_t \sum_{t=1}^{\infty} \beta^t \left( u(c_t) - \varphi \left( l^f_t + l^i_t \right) - \frac{\kappa}{2} F^2 l^f_t \right), \]

subject to the law of motion of employment, \( l^f_{t+1} = (1 - \rho) l^f_t + m(v_t, l^u_t) \), and the aggregate resource constraints: \( 1 = l^c_t + l^f_t + l^i_t \), and \( y_t = c_t \).

with \( y_t = \left[ (u'_t)^{\frac{\gamma - 1}{\gamma}} + (y'_t)^{\frac{\gamma - 1}{\gamma}} \right]^{\frac{\gamma}{\gamma - 1}} \) and \( m(v_t, l^u_t) = N(v_t)^{1-\mu} (l^u_t)^{\mu} \).

The first-order conditions with respect to \( v_t, l^f_{t+1} \) and \( l^i_t \) are given, respectively, by:
\[
\kappa \left( \frac{v_t}{l_t^f} \right) = \gamma_l^f m_v(v_t, l_t^v),
\]

\[
\gamma_l^f = \beta E_t \left[ u'(c_{t+1}) \frac{\partial y_{t+1}}{\partial y_{t+1}} - \varphi + \frac{\kappa}{2} \left( \frac{v_{t+1}}{l_{t+1}^f} \right)^2 + \gamma_{t+1}^f \left( (1 - \rho) - m_{l^v} \left( v_{t+1}, l_{t+1}^v \right) \right) \right],
\]

\[
u'(c_t) \frac{\partial y}{\partial y^i_t} - \varphi = \gamma_l^f m_{l^v} \left( v_t, l_t^v \right),
\]

where \( m_v(v_t, l_t^v) = (1 - \mu) q(\theta_t) \), \( m_{l^v}(v_t, l_t^v) = \mu p(\theta_t) \), and \( p(\theta_t) = \theta_d q(\theta_t) \).

\( \gamma_l^f \) represents the social value of an additional worker in the formal sector. From equation (54), the social value of an additional worker in the informal sector \( \gamma_i^f \) is equal to \( u'(c_t) \frac{\partial y}{\partial y^i_t} - \varphi \).

Reorganizing and combining equations (52), (53) and (54), I obtain the following efficient job creation condition (the algebra is given in Appendix B4):

\[
\frac{\kappa F_t}{q(\theta_t)} = \beta E_t \left[ (1 - \mu) u'(c_{t+1}) \left( \frac{\partial y_{t+1}}{\partial y_{t+1}} m_{l^v}^t + \frac{\partial y_{t+1}}{\partial y_{t+1}} m_{l^v}^t + \frac{\kappa F_{t+1}^i}{2u'(c_{t+1}} \right) + (1 - \rho) \frac{\kappa F_{t+1}^i}{q(\theta_{t+1})} \right].
\]

In what follows, I denote \( \hat{x} \) as the log deviation of variable \( x_t \) from its steady-state value \( x \).

To gain some intuition, I derive the log-linear version of equations (52) and (54):

\[
\frac{2}{\rho(1 - \mu)} s_v \left( \mu \hat{\theta}_t + \hat{F}_t \right) = \gamma_l^f \gamma_l^f, \tag{56}
\]

\[
\frac{\mu^2 s_v}{(1 - \mu)} \left( \hat{F}_t + \hat{\theta}_t \right) = \gamma_i^f \gamma_i^f, \tag{57}
\]

where \( s_v = \frac{2F_{2l}^f}{u'(c)c} \), and \( \gamma_i^f = u'(c) \frac{\partial y}{\partial y^i} - \varphi \).

Equation (56) implies that the social value of an additional job in the formal sector \( \gamma_l^f \gamma_l^f \) equals the marginal cost for a formal firm of adding a new worker. Similarly, equation (57) implies that the social value of an additional worker in the informal sector \( \gamma_i^f \gamma_i^f \) equals the social value of an additional unemployed worker. These two
equations, together with the efficient job creation condition, are the benchmark relative to which the flexible wage equilibrium and the staggered wage bargained equilibrium will be compared.

2.3.2 Equilibrium under flexible wages

In this section, I derive the three main equations of the model that govern the labor market dynamics outside the steady-state, assuming that wages are flexible.

In an environment with search and matching frictions, wages are determined through a negotiation process between firms and workers. Once wages are set, firms choose the level of employment that maximizes their benefit. I assume that firms renegotiate their nominal wages every period according to the Nash Bargaining Solution. The conventional sharing rule implies:

\[(1 - \phi) H^f_t = \phi J^f_t, \quad (58)\]

where \(\phi\) measures the worker’s relative bargaining power. \(H^f_t\) and \(J^f_t\) are the worker’s and firm’s surplus, defined in equation (50) and (37) respectively.

Then, replacing the expressions for \(H^f_t\) and \(J^f_t\), and equation (51) into equation (58), I find that under a flexible wage setting, all firms in the formal sector set the following real wage every period (see Appendix B3 for the derivation):

\[w^f_t = w^o_t = \phi \left( p^f_t m^f_t \frac{x^2_t}{u'(c_t)} + \frac{\nu^f_t}{u'(c_t)} \theta_t \right) + (1 - \phi) \left( \phi u'(c_t) \right). \quad (59)\]

The negotiated wage is a combination of what a worker contributes to the match and what the worker loses by accepting a job, weighted by relative bargaining power. When all formal wages are renegotiated every period, all formal firms set the same wage. This is why the subscript \(i\) disappears.

From the equilibrium condition \(Q^u_t = Q^f_t\) in equation (51), combined with equation (36), gives:

\[\frac{\kappa F^i_t \theta_t}{u'(c_t)} \frac{\phi}{1 - \phi} + \frac{v^i_t}{u'(c_t)} = w^f_t. \quad (60)\]

Replacing (60) into (59), I obtain an expression for the average formal wage as a linear combination between the firm’s income from having a job filled and the outside option
for workers:

\[ w_f^i = w_i^o = \phi \left( p_f^i m p_f^{i+1} + \frac{\kappa F^2}{2 u'(c)} \right) + (1 - \phi) (w_i^1). \]  

(61)

Different from the case without informality, the outside option for workers depends on wages in the informal sector \( w_i^1 \). Therefore, after an adverse aggregate productivity shock, wages in the informal sector decrease, decreasing the outside option for formal workers and therefore increasing the negative effect of the shock on formal wages. Additional to this, the effect of a productivity shock on the labor market tightness is also going to be exacerbated by the presence of informality. Indeed, the informal sector works as a buffer that absorbs workers in bad times, and vice-versa. Therefore, after a negative productivity shock, a proportion of unemployed workers will find it more profitable to go to the informal sector, decreasing the probability of a formal vacancy being filled. This, in turn, pins down the firm’s surplus and, therefore, the incentive to hire. The decrease in the firm’s surplus will have an even larger impact on the hiring rate in the formal sector.

Finally, replacing equation (61) into equation (38), I obtain the following job creation condition:

\[
\frac{\kappa F_t}{q(\theta_t)} = \beta E_t \left[ (1 - \varphi) u'(c_{t+1}) \left( \frac{\partial y_{t+1}}{\partial y_f^{t+1}} m p_f^{t+1} - \frac{\partial y_{t+1}}{\partial y_i^{t+1}} m p_i^{t+1} + \frac{\kappa F^2_{t+1}}{2 u'(c_{t+1})} \right) + (1 - \rho) \frac{\kappa F_{t+1}}{q(\theta_{t+1})} \right],
\]

(62)

where \( m p_i^{t+1} \) is the marginal productivity of labor in the informal. The total wholesale output is equal to \( y_t = \left[ (y_f^t)^{\frac{\gamma - 1}{\gamma}} + (y_i^t)^{\frac{\gamma - 1}{\gamma}} \right]^{\frac{\gamma}{\gamma - 1}} \). Under flexible price and wage setting, prices in the formal and in the informal sector are equal to the marginal increase in production due to one unit increase in \( y_f^t \) and in \( y_i^t \) respectively, this is: \( p_f^t = \frac{\partial y_f^t}{\partial y_f^t} \) and \( p_i^t = \frac{\partial y_i^t}{\partial y_i^t} \).

Now it is possible to compare the flexible wage equilibrium with the efficient equilibrium found in the previous section. Note that equation (55) is equivalent to (62) when \( \mu = \phi \), which means the elasticity of the matching function with respect to vacancies \( (1 - \mu) \) is equal to the firm’s bargaining power \( (1 - \phi) \). This is known as the Hosios condition necessary to achieve the constrained Pareto efficiency in an economy with search and matching frictions (Hosios, 1990).

In addition to assuming an optimal subsidy that eliminates the distortion caused by monopolistic competition, efficiency also requires that equation (55) holds, together
with the elimination of inefficient dispersion on prices $\Delta_t = 1$. The absence of price dispersion requires keeping the price level constant, which can be attained by a policy that stabilizes the marginal cost in the retail sector $mc_t$ at the level consistent with the firm’s desire mark-up.

It implies that when wages in the formal sector are negotiated every period and the steady-state is efficient, a zero price inflation policy is optimal.

### 2.4 Equilibrium under wage rigidities in the formal sector

In this section, I determine the equilibrium conditions assuming wage rigidities in the formal sector. In line with Gertler and Trigari (2009) and Tomas (2008), I suppose staggered wage contracting, where every period, each firm in the formal sector has a fixed probability $(1 - \omega^w)$ of renegotiating salaries. When the firm has the chance to renegotiate its nominal wage, it negotiates with both the existing workers and the new hires, so that all workers in the firm receive the same wage. For firms that can not renegotiate wages, they will maintain the nominal wage from the previous period, and new hires will receive the same wage.

I denote $W^{f*}_{it}$ as the nominal wage of a formal firm $i$ that renegotiates their salary in period $t$. I assume that, in renegotiating firms, managers and workers split the match surplus as follows:

\[
(1 - \phi) \mathbb{H}_f^f \left( W^{f*}_{it} \right) = \phi J_f^f \left( W^{f*}_{it} \right). \tag{63}
\]

For simplicity, I assume that in renegotiating firms, the match surplus is split in the same way as in period-by-period Nash bargaining\(^\text{11}\).

\(^{11}\)The other option is to maximize the weighted average of the firm and worker surplus (see Gertler and Trigari, 2009). The Nash bargaining solution is given by:

\[
\left[ 1 - \chi_t(W^{f*}_{it}) \right] \mathbb{H}_f^f(W^{f*}_{it}) = \chi_t(W^{f*}_{it}) J_{it},
\]

where $\chi_t(W^{f*}_{it}) = \phi f \left( \phi + (1 - \phi) \frac{\mu_t(W^{f*}_{it})}{\epsilon_t} \right)$. $\epsilon_t$ is the cumulative discount factor that workers use to value the contract wage, while $\mu_t(W^{f*}_{it})$ is the cumulative discount for the firm. $\chi_t(W^{f*}_{it})$ is the relative share that depends not only on the bargaining power but also on the horizon over which the worker and the firm value the impact of the contract wage. According to Gertler and Trigari (2009), this is called the "horizon effect" that influences the bargained wage. Under this setting, firms account for the implications of the contract wage for future hires, but workers care about wages only during his or her working time at the firm. They find that, while the horizon effect is interesting from a theoretical perspective, it turns out not to be quantitatively important in their baseline calibration.
I next characterize the relation between the contract wage $W^{f*}_{it}$, and the evolution of the average nominal wage $W^f_t$ across workers in the formal sector, which is given by:

$$W^f_t = \int_0^1 W^{f^*}_{it} \, di.$$  \hspace{1cm} (64)

Given the constant returns to scale, all firms renegotiating wages face the same optimization problem and set the same contract wage, then $W^{f^*}_{it} = W^{f^*}_{it}$. Additionally, since firms that can renegotiate wages are randomly chosen, equation (64) can be expressed recursively as

$$W^f_t = (1 - \omega^w)W^{f^*}_t + \omega^w W^f_{t-1}. \hspace{1cm} (65)$$

Finally, the aggregate job creation condition can be expressed as follows:

$$\frac{\kappa F_t}{q(\theta_t)} = \beta E_t \Gamma_{t,t+1} \left[ u'(c_{t+1}) \left( p^f_{t+1} m^f_{t+1} - \frac{W^f_{t+1}}{P_{t+1}} + \frac{\kappa F^2_{t+1}}{2u'(c_{t+1})} \right) + (1 - \rho) \frac{\kappa F_{t+1}}{q(\theta_{t+1})} \right]. \hspace{1cm} (66)$$

The job creation condition is the same as in the case of the flexible wage scenario. As such, under wage rigidities, efficiency requires that the Hosios condition holds, together with the elimination of inefficient dispersion on prices and wages.

**2.4.1 The linearized model**

In this section, I derive the log-linear approximation of the rational expectations equilibrium around the efficient steady-state. I start by deriving the log-linear version of the three central equations that govern labor market dynamics outside the steady-state: the relation for formal wages, the job creation condition in the formal sector, and the equilibrium condition in the informal sector (the log-linear and the steady-state equations are presented in Appendix B1).

In Appendix B5, I show that log-linearizing and combining equations (63), (50), (51) and (37) results in the following law of motion for the average real wage in the formal sector:

$$\hat{w}^f_t = \psi_0 \hat{w}^o_t + \psi_1 E_t \left( \hat{w}^f_{t+1} + \hat{\pi}_{t+1} \right) + \psi_2 \left( \hat{w}^f_{t-1} - \hat{\pi}_t \right), \hspace{1cm} (67)$$

Therefore, in the same line with Thomas (2008), I assume that managers and workers split the match surplus the same as in the flexible-wage equilibrium.
where

\[ \hat{w}_t^q = \phi \left[ \gamma_{a} \hat{\omega}_t + \gamma_{\text{w}} \left( 2 \hat{F}_t - \hat{w}'(c_t) \right) \right] + (1 - \phi) \left( \gamma_{\text{w}} \hat{w}_t^q \right) \]  

(68)

is the real formal wage that would arise under period-by-period Nash bargaining, \( a = \frac{p^f m}{} \), \( \gamma_{a} = \frac{a}{w} \), \( \gamma_{\text{w}} = \frac{w^f}{w} \), \( \psi_0 + \psi_1 + \psi_2 = 1 \), and \( \hat{\omega}_t = \hat{w}_t^f + mpl_t^f \). Due to staggered wage negotiation, the average formal wage \( \hat{w}_t^f \) depends on its lagged value \( \hat{w}_{t-1}^f \) as well as on the expected future wage \( E_t \hat{w}_{t+1}^f \). Under a flexible wage-setting where \( \omega^w = 0 \), both \( \psi_1 \) and \( \psi_2 \) become equal to zero, and \( \psi_0 \) equal to 1, thus \( \hat{w}_t^f = \hat{w}_t^? \).

Additionally, log-linearizing the aggregate job creation condition in equation (66), results in:

\[ \hat{\omega}_t - \hat{\theta}_t = \frac{1}{f} \left( \hat{a}_t - w^f \hat{w}_{t+1}^f \right) + \gamma_{\text{f}} \hat{\omega}_{t+1} + \gamma_{\text{f}} (1 - \rho) \hat{\theta}_t + E_t (a - w) \frac{1}{f} \hat{w}'(c_{t+1}). \]  

(69)

Finally, log-linearizing the equilibrium condition in the informal sector, equation (51), yields (see Appendix B5 for details):

\[ E_t \hat{\gamma}_t = \frac{1}{f} \left( \hat{\omega}_t - \hat{w}'(c_t) \right) - \frac{\nabla \omega^w}{1 - \omega^w} E_t \left[ \hat{w}_{t+1}^f - \hat{w}_t^f + \hat{\pi}_{t+1} \right] = w^f \hat{w}_t^f + \frac{\varphi}{u'(c_t)} \hat{w}'(c_t), \]  

(70)

where \( \nabla = \frac{\omega^w}{1 - \omega^w} \left( \frac{\omega^w}{1 - \omega^w} + 1 \right) \).

To gain some intuition, I next express the job creation condition (69) and the equilibrium condition in the informal sector (70) in terms of the inefficient fluctuations on the marginal cost and on the formal wage gap, as follows (the algebra is presented in Appendix B6):

\[ \frac{2 s_v}{\rho (1 - \mu)} \left( \hat{\theta}_t + \hat{F}_t \right) - \gamma_{\text{f}} \hat{y}_{t} = \beta E_t \left[ \left( \frac{y_{/o}}{y} \right)^{1 - \gamma} m^c_t + \frac{s_v}{(1 - \mu)} \left( \hat{\omega}_{t+1}^f - \hat{w}_{t+1}^f \right) \right], \]  

(71)

and

\[ \frac{\mu 2 s_v}{(1 - \mu)} \left( \hat{F}_t + \hat{\theta}_t \right) - \gamma_{\text{f}} \hat{y}_{t} = \left( \frac{y_{/o}}{y} \right)^{1 - \gamma} \frac{l^u}{l^v} m^c_t + \frac{\mu 2 s_v}{(1 - \mu)} \frac{\nabla \omega^w}{1 - \omega^w} E_t [\hat{\pi}_{t+1} - \hat{\pi}_{t+1}], \]  

(72)

where \( s_v = \frac{w^f l_f}{y} \) is the steady-state formal labor income share, \( s_v = \frac{w^f y}{w (c_t) c} \) is the vacancy posting cost in consumption units as a fraction of total consumption, and \( \hat{\pi}_{uw} \) is the wage inflation. \( \gamma_{\text{f}} \) is the social value of an additional job in the formal sector, defined in equation (56), and \( \gamma_{\text{f}} \) is the social value of an additional worker in the informal
sector, defined in equation (57). Note that the left-hand side (LHS) of equation (71) represents the difference between the marginal cost for a formal firm of adding a worker and the social value of an additional job in the formal sector. This difference depends on the expected fluctuations in the marginal cost and on the formal wage gap. In the same way, the LHS of equation (72) represents the difference between the social value of an unemployed worker and the social value of an additional worker in the informal sector. This difference also depends on the fluctuations in the marginal cost and on the formal wage gap.

The Phillips curve, wage inflation equation and IS curve

By log-linearizing and combining equation (41) and (43) it is possible to obtain the standard expression of the price inflation that is known as the Phillips curve (see Appendix B7):

$$\hat{\pi}_t = \kappa_{px} (\hat{m}c_t) + \beta E_t \hat{\pi}_{t+1},$$

(73)

where $\kappa_{px} = (1-\omega_p)(1-\omega_p \beta)$. $\hat{m}c_t$ denotes the log-deviation of the real marginal cost from its steady-state value.

I next derive an expression for the wage inflation. From equation (67) and the average real wage dynamics $\hat{w}_f^t - \hat{w}_f^{t-1} = \hat{\pi}_{wt} - \hat{\pi}_t$, I obtain the following expression for the wage inflation (see Appendix B8 for the derivation):

$$\hat{\pi}_{wt} = \frac{\hat{w}_0}{\hat{w}_2} \left( \hat{w}_0^t - \hat{w}_2^t \right) + \frac{\psi_1}{\psi_2} E_t (\hat{\pi}_{wt+1}),$$

(74)

where $\frac{\hat{w}_0}{\hat{w}_2} = \frac{1-\omega}{\omega (1+(1-\rho+\mu) \frac{\phi}{\theta}) \omega \beta \phi}$, and $\frac{\psi_1}{\psi_2} = \frac{(1-\rho) \phi}{1+(1-\rho+\mu) \omega \beta \phi}$.

According to equation (74), wage inflation depends on the gap between the target and the actual average real wage $\left( \hat{w}_0^t - \hat{w}_2^t \right)$. The intuition behind this result is as follows: when the average formal wage in the economy $\hat{w}_f^t$ is below (above) its target level $\hat{w}_t^o$, renegotiating firms will tend to increase (decrease) their nominal wages, thus generating positive (negative) wage inflation. Consequently, an aggregate productivity shock in the economy will affect $\hat{w}_t^o$, and formal real wages will converge slowly towards their target levels. In this case, the gap $\left( \hat{w}_0^t - \hat{w}_2^t \right) \neq 0$ generates an inefficient wage dispersion that translates into hiring rate dispersion in the formal sector.
Finally, by log-linearizing the Euler equation, equation (45), I obtain an standard expression for the IS curve:

\[ \hat{y}_t = E_t (\hat{y}_{t+1}) - \sigma (i_t - E_t \hat{\pi}_{t+1}) . \]  

(75)

### 2.5 Optimal Monetary policy

In this section, I analyze optimal monetary policy in an economy with informality. I first derive the second-order approximation of the welfare criterion, which will be the objective function in the Central Bank’s optimal monetary policy problem. To keep the analysis simple, I assume that the steady state of this economy is efficient. It implies that the Hosios condition holds \((\mu = \phi)\), and there is a subsidy to monopoly firms (finance by a lump-sum tax to the same firms) that eliminates the monopoly distortion.

In Appendix B9, I show that the second-order approximation of the household’s welfare can be written as follows:

\[ \sum_{t=0}^{\infty} \beta^t U_t = - \sum_{t=0}^{\infty} \frac{\beta^t u(c)}{2} L_t + t.i.p, \]

with

\[ L_t = \Psi_{\pi} \tilde{\pi}_t^2 + \Psi_{\pi w} \tilde{\pi}_{w t}^2 + \mathcal{L}^{l,h}_t, \]

(76)

where

\[ \mathcal{L}^{l,h}_t = (\sigma^{-1} - 1) \tilde{y}_t^2 + 2s_v \left[ \mu \tilde{\pi}_t^2 + \tilde{\pi}_t^2 \right] + \psi_{y_{f}} \left( \tilde{l}_t^2 \right)^2 + \psi_{y_{l}} \left( \tilde{l}_t^2 \right)^2, \]

and

\[ \Psi_{\pi} = \frac{\phi}{\gamma}, \quad \Psi_{\pi w} = s_v \frac{h^2}{w}, \quad h = \frac{\beta \omega^w n w}{(1-\beta \omega^w) s_v}, \quad \gamma_{w} = \frac{(1-\omega^w)(1-\beta \omega^w)}{\beta \omega^w}, \quad \psi_{y_{f}} = \left( \frac{y}{\gamma} \right)^{1-\gamma}, \quad \psi_{y_{l}} = \left( \frac{y}{\gamma} \right)^{1-\gamma}. \]

\( \mathcal{L}^{l,h}_t \) measures the success of monetary policy in stabilizing output and labor market variables around their efficient steady-state value. \( t.i.p \) are the terms independent of policy. Note that in the case of a logarithmic utility function, \( \sigma = 1 \), and taking into account that the steady-state is efficient, the value of \( \mathcal{L}^{l,h}_t \) does not depend on
output, and the efficient allocation of employment remains constant after an aggregate productivity shock. In this specific case, the variables in $L_t$ are measured in terms of deviations from their efficient values. Hereafter I assume $\sigma = 1$.

Equation (76) shows that welfare decreases with price and wage inflation volatility. Under this framework, price inflation causes inefficient dispersion on prices across retail firms, and wage inflation creates inefficient dispersion on wages across formal firms.12 Welfare also decreases with labor market tightness volatility. Indeed, the composition of total production between formal and informal goods can be inefficient if the labor market tightness in the formal sector differs from its efficient value. Additionally, since the utility cost of hiring is convex in hiring rates, dispersion in $F$ increases the welfare cost involved in job creation in the formal sector. Finally, inefficient fluctuations in employment, and hence on unemployment, are an additional source of welfare losses.

The contribution of inflation volatility to welfare losses $\Psi_\pi$ is increasing in the degree of price stickiness. Similarly, the contribution of wage inflation volatility to the welfare losses is increasing in the degree of wage stickiness and the steady-state formal labor income share $s_w$. In the presence of an informal sector, the proportion of firms in the economy facing wage rigidities is lower, together with the formal labor income share.

As a result, the contribution of wage inflation volatility to the welfare loss, relative to the contribution of price inflation, is lower for the case with informality. Consequently, in the presence of an informal sector, the optimal policy will result in a lower price inflation volatility for a given level of wage inflation volatility.

---

12I show in Appendix B5 the hiring rate can be expressed as follows:

$$E_t \left[ \hat{F}_{it} - \hat{F}_i \right] = -\omega_w \frac{\mu}{Q'} \left[ \hat{W}_{it} - \hat{W}_i^* \right].$$

This implies that wage dispersion creates dispersion in hiring rates: $\text{var}_i \left( \hat{F}_{it} \right) = \left( \omega_w \frac{\mu}{Q'} \right)^2 \text{var}_i \left( \hat{W}_{it}^* \right)$

$$\text{var}_i \left( \hat{F}_{it} \right) = \left( \frac{\beta \omega w s_w}{(1 - \beta w) \rho v} \right)^2 \text{var}_i \left( \hat{W}_{it}^* \right)$$

$$\text{var}_i \left( \hat{F}_{it} \right) = \sigma^2 \text{var}_i \left( \hat{W}_{it}^* \right)$$

Since the hiring rates cost is convex and is measure in terms of utility, dispersion in the formal hiring rate increases the welfare cost involved in aggregate job creation.
2.5.1  Policy trade-offs

From the Phillips curve in equation (73), the wage inflation in equation (74), the hiring rate condition in the formal sector, equation (71), and the equilibrium condition in the informal sector, equation (72), is possible to analyze the trade-offs faced by the monetary authority in an economy with a dual labor market and price and wage rigidities.

I first consider equations (73) and (74):

\[
\hat{\pi}_t = \kappa px (\hat{mc}_t) + \beta E_t \hat{\pi}_{t+1},
\]

(77)

\[
\hat{\pi}_{wt} = \psi_o \psi_2 (\hat{w}_o t - \hat{w}_f t) + \psi_1 \psi_2 E_t (\hat{\pi}_{wt+1}).
\]

(78)

According to equation (78), wage inflation depends on the gap between the target and the average real wage. In response to an aggregate productivity shock, the average real wage in the formal sector is affected, but not as much as its natural (target) wage. Because of the presence of wage rigidities, formal real wages will converge slowly towards their target levels. As a result, the gap \((\hat{w}_o t - \hat{w}_f t) \neq 0\) translates into a formal wage inflation that results in inefficient wage dispersion.

When price inflation is equal to zero, \(\hat{\pi}_t = 0\), equation (67) can be expressed as

\[
\hat{w}_f^f = \psi_o \hat{w}_o t + \psi_1 E_t \hat{w}_{f+1} + \psi_2 \hat{w}_{f-1}.
\]

(79)

Equation (79) implies that in response to a real shock in the economy, \(\hat{w}_f^f\) differs from \(\hat{w}_o^o\) when price inflation is equal to zero. Therefore, under a zero price inflation policy, the central bank is not able to close the gap between actual and desired wages in the formal sector. It follows that the central bank faces a trade-off between price inflation and wage inflation.

I next consider the job creation condition in the formal and the informal sector sector, defined in equations (71) and (72) respectively as follows:

\[
\frac{2 \rho \pi}{\rho (1 - \mu)} (\mu \bar{\theta}_t + \hat{F}_t) - \Upsilon_f \hat{Y}_f = \beta E_t \left[ \left( \frac{\psi}{y} \right)^{-\gamma} \bar{mc}_{t+1} + \frac{\psi}{(1 - \mu)} (\hat{w}_o^o - \hat{w}_f^f) \right],
\]

75
\[
\frac{\mu^2 s_v}{(1 - \mu)} (\bar{F}_t + \bar{\theta}_t) - \Upsilon \hat{\Upsilon}_t = \left( \frac{y^*}{y} \right)^{\frac{\nu^*}{\nu}} \frac{l^u}{l^i} m^c_t + \frac{\mu^2 s_v}{(1 - \mu)} \frac{\nabla \omega^w}{1 - \omega^w} E_t [\pi_{w} t - \hat{\pi}_{t+1}].
\]

Efficiency requires that equations (56) and (57) hold. However, when either \((\hat{\omega}_w - \hat{\omega}_f) \neq 0\) or \(\hat{m}^c_t \neq 0\), the job creation condition in the formal and the informal sector is inefficient. Therefore, under this framework, both price and wage inflation generates a distortion in the formal hiring rate and the job creation condition in the informal sector, which translates to inefficient fluctuations in unemployment. As a consequence, the Central Bank also faces a trade-off between price inflation and unemployment.

Note that the higher the gap \((\hat{\omega}_w - \hat{\omega}_f)\), the higher the inefficiencies fluctuations in the labor market variables. In response to an aggregate productivity shock, only a fraction of firms in the formal sector can adjust their nominal wages. This wage rigidity generates a gap between the average formal wage \(\hat{\omega}_f\) and the target wage \(\hat{\omega}_w\). Equation (68) shows that in the presence of an informal sector, the target wage \(\hat{\omega}_w\) depends, apart from productivity, on the informal wage (the outside option) and on the hiring rate (which depends on the labor market tightness). As I noted in section 2.3.2, after an aggregate productivity shock, the effect in both variables is higher than in the case without informality.

Consequently, for a given level of inflation, the wage gap is higher in the presence of an informal sector. On that account, the inefficient fluctuations in the labor market variables such as labor market tightness, hiring rate, and unemployment are also higher. It follows that the trade-off between price inflation and unemployment faced by the Central Bank increases in the presence of an informal sector.

In summary, the existence of a large informal sector has two opposite implications for optimal monetary policy. On the one side, the optimal policy results in a lower price inflation volatility for a given level of wage inflation volatility. This is because wages in the informal sector are flexible, and hence the proportion of firms in the economy facing wage rigidities is lower. On the other side, in the presence of informality, wage inflation and unemployment volatility are higher. As a result, the Central Bank should use price inflation to avoid excessive unemployment volatility and excessive dispersion in the formal hiring rate. The aggregate effect on price inflation volatility would depend on which effect dominates.
2.5.2 Responses Under Optimal Monetary Policy and Quantitative Analysis

In this section, I use numerical methods to characterize optimal monetary policy with informality. For simplicity, I focus only on the volatility generated by exogenous aggregate productivity shocks\footnote{When the utility function is logarithmic \((\sigma = 1)\), the value of \(L^l_t,h\) does not depend on output, and the efficient allocation of employment remains constant after an aggregate productivity shock. In this specific case, the variables in \(L_t\) are measured in terms of deviations from their efficient values.}.

Calibration

A summary of the calibration of the model is reported in Table 2.1 The model is calibrated at a quarterly frequency to reproduce some key metrics for the Colombian economy\footnote{I choose Colombia as a benchmark country given the weight and persistence of the informal sector, and the availability of information on labor market flows and wage differentials across sectors}. The first set of parameters correspond to standard values in the business cycle literature. I set the quarterly discount factor \(\beta = 0.99\), which implies a quarterly real rate of interest of approximately 1%. I assume an intertemporal elasticity of substitution equal to 1, \(\sigma = 1\). In the same line with Restrepo-Echavarría (2014), Fernández and Meza (2015), and Alberola and Urrutia (2021) I set the elasticity of substitution between formal and informal inputs, \(\gamma\), equal to 8. The markup of prices on marginal costs is assumed to be on average 20 percent. This amount is obtained by setting \(\Theta\) equal to 6.

In addition, based on most of the existing literature, where the bargaining power has typically been set either to satisfy the Hosios (1990) condition or to achieve symmetric Nash bargaining, in which the surplus is equally shared, I set the worker’s bargaining power parameter, \(\phi\), and the elasticity of matches with respect to vacancies, \(\mu\), equal to 0.5. This assumption ensures the efficiency of the equilibrium in the flexible version of the model (Hosios, 1990). Finally, I assume an average duration of wage contracts of one year, and price contracts of one semester, i.e. \(\omega^w = 0.75\) and \(\omega^p = 0.5\) respectively.

The second set of parameters, \((\rho, \kappa, N, \varphi, z^a, \text{and } z^f)\) are jointly calibrated so that the steady-state of the model matches the long-term properties of the data: an unemployment rate of 11\%, a share of informally employed workers equal to 41\%, a probability to fill a vacancy of 0.894, the relative productivity of the informal sector equal to 0.635,
and a job-finding rate in the formal sector of 0.1371.\textsuperscript{15} I obtain the job destruction rate in the formal sector $\rho = 0.0314$, the adjustment cost parameter $\kappa = 155.17$, the efficiency parameter of the matching function $N = 0.35$, the fixed component of labor dis-utility $\varphi = 0.048$, and the labor productivity in the informal sector $z^i = 0.635$, with $z^f$ normalized to 1.

\begin{table}[h]
\centering
\caption{Parameters for the baseline economy}
\begin{tabular}{llll}
\hline
Description & Symbol & Value & Justification \\
\hline
\textbf{Fixed parameters} & & & \\
Discount rate & $\beta$ & 0.99 & Standard values in the RBC literature \\
Intertemporal elasticity of substitution & $\sigma$ & 1 & Standard values in the RBC literature \\
Elasticity of substitution formal-informal inputs & $\gamma$ & 8 & Restrepo-Echavarria (2014) \\
Elasticity of substitution between varieties & $\Theta$ & 6 & Standard value in the NK literature \\
Bargaining worker’s power & $\phi$ & 0.5 & Standard value in the literature \\
Elasticity of matches with respect to vacancies & $\mu$ & 0.5 & Standard value in the literature \\
Fraction of formal firms not changing wages & $\omega^w$ & 0.75 & Average wage contracts of one year \\
Fraction of retail firms not changing prices & $\omega^p$ & 0.5 & Average price contracts of one semester \\
Labor productivity in the formal sector & $z^f$ & 1 & Normalized to one \\
\textbf{Calibrated to steady-state moments} & & & \\
Job destruction rate in the formal sector & $\rho$ & 0.0314 & Job finding rate in formal sector: 0.1371 \\
Adjustment cost parameter & $\kappa$ & 155.17 & Informal employment rate: 0.41 \\
Efficiency parameter of the matching function & $N$ & 0.35 & Unemployment rate: 0.11 \\
Fixed component of labor dis-utility & $\varphi$ & 0.0166 & Probability to fill a vacancy : 0.894 \\
Labor productivity in the informal sector & $z^i$ & 0.635 & Productivity gap: $z^i/z^f = 0.635$ \\
\textbf{Calibrated to business cycle moments} & & & \\
Persistence of the aggregate productivity & $\rho_z$ & 0.89 & Standard deviation of unemployment: 0.017 \\
Std deviation of aggregate productivity shocks & $\sigma^z$ & 0.0062 & Standard deviation of GDP: 0.00698 \\
\hline
\end{tabular}
\end{table}

\footnote{The size of the informal sector measures the share of the urban labor force working in the informal sector in Colombia. The rate of unemployment and informal employment are calculated using data from the Household Survey (GEIH for its acronym in Spanish) and taking into account the definition of informality used by the Colombian System of National Accounts (DANE for its acronym in Spanish). DANE’s definition of informal employment includes the group of employees and employers working in firms with less than 10 workers, unpaid family workers, domestic household workers, and self-employed individuals who are not professionals or technicians. Data on the probability to fill a vacancy in the formal sector comes from Cardozo (2019), who estimates that the time to fill a vacancy in Colombia is approximately 1.35 months. It will imply a monthly probability to fill a vacancy of 0.528 (i.e. a quarterly probability of 0.894). Data on the differential in productivity between the formal and the informal sector is taken from Fernandez (2018), who finds that informal firms have between 54\% and 73\% of the formal firm’s productivity. Finally, the probability that an unemployed worker finds a job in the formal sector, is estimated using data from the GEIH that allows estimating the allocation of workers between occupations (formal, informal, and unemployed) as well as calculating the transition rate between them.}
The rest of the parameters are associated with the aggregate productivity shock and the Taylor Rule and are jointly calibrated in order to match the volatility of the Gross Domestic Product (GDP) and unemployment. I set the persistence of the aggregate productivity $\rho_z = 0.89$, and the standard deviation of aggregate productivity shock $\sigma^z = 0.0062$ to match the volatility of GDP and unemployment to 0.0068 and 0.017, respectively, under a Taylor rule of the form $\hat{i}_t = 3.5\hat{\pi}_t + 0.09\hat{y}_t$.

**Optimal Monetary Policy under Commitment**

I begin the quantitative analysis by simulating the behavior of the decentralized economy when the central bank implements the optimal monetary policy in an economy with and without an informal sector.

At time 0, the central bank chooses the state-contingent plan that minimizes:

$$\sum_{t=0}^{\infty} \beta^t W L_t = -\sum_{t=0}^{\infty} \beta^t \frac{u'(c)}{c} L_t + t.p.i + \sigma^3,$$

with

$$L_t = \Psi_\pi \hat{\pi}_t^2 + \Psi_\pi \hat{\pi}_w^2 + L_t^{I,h},$$

$$L_t^{I,h} = (\sigma^{-1} - 1) \hat{g}_t^2 + 2\sigma \left[ \mu \hat{\theta}_t^2 + \hat{F}_t^2 \right] + \Psi_{\theta_f} \left( \hat{\theta}_t \right)^2 + \Psi_{\theta_f} \left( \hat{\theta}_t \right)^2,$$

subject to the Phillips curve, equation (73), the law of motion of labor, equation (34), and the equilibrium condition in the informal sector, equation (72).

To better understand the effect of informality on optimal monetary policy design, I compare the predictions of the model against the case where there is no informal sector in the economy, in which case I assume $z^I = 0$. Figure 2.1 shows the Impulse Response Functions (IRFs) of all the variables in $L_t$, unemployment, and the interest rate, in response to one standard deviation ($\sigma^z = 0.0062$) negative productivity shock ($z_t \downarrow$) under the optimal monetary policy. Relative to the situation without informality, the optimal policy features significant deviations from price stability. After a negative productivity shock, only a fraction of firms in the formal sector can adjust their nominal wages. As a result, the gap between the target wage and the average wage in the formal sector decreases, which in turn generates negative wage inflation. By increasing the
inflation rate, the Central Bank can affect the real value of nominal wages and then bring real formal wages closer to their flexible-wage levels.

As noted in section 2.5.1, for a given level of price inflation, the wage gap is higher in the presence of an informal sector. On that account, the inefficient fluctuations in the labor market variables such as labor market tightness, hiring rate, and unemployment are also higher. It follows that the presence of an informal sector requires a higher adjustment on inflation to reduce this gap.

**Figure 2.1 Impulse response functions to a 0.62% negative productivity shock under the optimal policy**

From the welfare loss function, equation (76), I find that the contribution of wage inflation volatility to the welfare loss, relative to the contribution of inflation \( \frac{\psi_{\pi w}}{\psi_{\pi}} \), is lower in the case with informality. It implies that in the presence of an informal sector, the optimal policy results in a lower price inflation volatility for a given level of wage inflation volatility. However, I also find that for a given level of price inflation, the presence of an informal sector amplifies the effect of an aggregate productivity shock on wage inflation, and on the inefficient fluctuations in employment. It implies that the
Central Bank has to move further away from a full-price stabilization policy in order to reduce the formal wage gap. Figure 2.1 shows that under the baseline calibration the second effect dominates, and the optimal policy features significant deviations from price stability in the presence of an informal sector.  \(^{16}\)

Table 2.2 shows the relative standard deviation (relative to the standard deviation of output) of price and wage inflation, output, employment, and unemployment under the optimal monetary policy, relative to the case without informality. The optimal volatility of price inflation is about four times higher for the case with informality. This result suggests that for emerging countries, characterized by the presence of a large informal labor market, it is optimal to allow more inflation volatility.

**Table 2.2. Relative standard deviations under Optimal monetary policy: with and without informality**

<table>
<thead>
<tr>
<th>Standard Deviations(^{\circ})</th>
<th>(p = 0.41)</th>
<th>(p = 0)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price Inflation</td>
<td>0.0116</td>
<td>0.0035</td>
</tr>
<tr>
<td>Wage Inflation</td>
<td>0.1769</td>
<td>0.1360</td>
</tr>
<tr>
<td>Output</td>
<td>0.0078</td>
<td>0.0110</td>
</tr>
<tr>
<td>Formal employment</td>
<td>0.0278</td>
<td>0.0032</td>
</tr>
<tr>
<td>Informal Employment</td>
<td>0.1003</td>
<td>–</td>
</tr>
<tr>
<td>Unemployment</td>
<td>0.3442</td>
<td>0.0138</td>
</tr>
</tbody>
</table>

\(^{\circ}\) The standard deviation of output is expressed in absolute terms. The standard deviation of all other variables is divided by the standard deviation of output.

**Zero price inflation and Taylor Rule policy**

To illustrate the implications of the trade-offs faced by the central bank, I analyze the behavior of the decentralized economy when the monetary authority implements a zero price inflation policy and a Taylor Rule.

\(^{16}\) This result is robust to changes in the parameter values and model specifications. I repeat the same exercise with different values of the elasticity of substitution between formal and informal inputs, \(\gamma\), different combinations of price and wage rigidities, and also different values of the job destruction rates in the formal sector, \(\rho\). I also develop different versions of the model: (i) with effort and changes in employment at the intensive and extensive margin and (ii) with search and matching friction also in the informal sector. For all cases, I found that optimal policy features significant deviations from price stability in the presence of informality. Only for the case when both sectors are independent of each other and wages in the formal sector do not depend directly on informal labor market variables, the first effect dominates, and optimal policy results in a lower price inflation volatility in the presence of an informal sector.
Figure 2.2 plots the response of the economy variables to a 0.62% negative productivity shock under a zero price inflation policy. The decrease in the aggregate productivity reduces the target wage in the formal sector, $\hat{w}_t^o$, via a fall in the marginal product of labor and a reduction in informal wages and the labor market tightness. Note that under this policy, relative to the situation under the optimal policy, the decrease in wage inflation as well as the fluctuations in the rest of the labor market variables, is much higher for the case with informality. Notice that unemployment decreases due to the larger increase in informal employment.

Table 2.3 shows the relative standard deviation of wage inflation, output, employment, and unemployment under a zero inflation policy for the cases with and without informality. By comparing Table 2.3 and Table 2.2, it is also possible to notice that under a zero inflation policy, wage inflation, formal and informal employment, and unemployment are much more volatile than under the optimal monetary policy, especially for the case with informality. By allowing some price inflation, the Central Bank can
significantly reduce the inefficient fluctuations in the labor market variables.

Table 2.3 Relative standard deviations under a zero price inflation policy

<table>
<thead>
<tr>
<th>Standard Deviations®</th>
<th>$P = 0.41$</th>
<th>$P = 0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price inflation</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Wage inflation</td>
<td>0.1941</td>
<td>0.1380</td>
</tr>
<tr>
<td>Output</td>
<td>0.0075</td>
<td>0.0110</td>
</tr>
<tr>
<td>Formal employment</td>
<td>0.0618</td>
<td>0.0033</td>
</tr>
<tr>
<td>Informal Employment</td>
<td>0.2034</td>
<td>–</td>
</tr>
<tr>
<td>Unemployment</td>
<td>0.7217</td>
<td>0.0141</td>
</tr>
</tbody>
</table>

® The standard deviation of output is expressed in absolute terms. The standard deviation of all other variables is divided by the standard deviation of output.

Additionally, I analyze the behavior of the decentralized economy when Central Bank follows a Taylor Rule. Figure 2.3 shows the response of all the variables in the welfare loss function ($L_t$), unemployment, and the interest rate, to a 0.62% negative productivity shock under the Taylor Rule used in the benchmark calibration (i.e. $\hat{\pi}_t = 3.5\hat{\pi}_t + 0.09\hat{y}_t$). Relative to the situation without informality, the increase in price inflation is lower, while the decrease in wage inflation is higher. In both cases (with and without informality) output decreases, formal employment and the interest rate increase. Unemployment increases without informality and decreases in the presence of an informal sector. The decrease in unemployment is explained by the large increase in informal employment. Notice that all the labor market variables (formal employment, unemployment, wage inflation, hiring rate, etc.) are much more volatile in the presence of an informal sector.
Figure 2.3. Impulse response functions to a 0.62% negative productivity shock under the benchmark Taylor Rule*

\[ \hat{i}_t = 3.5\hat{\pi}_t + 0.09\hat{y}_t \]

Table 2.4 shows the relative standard deviation of price and wage inflation, output, employment, and unemployment for different Taylor Rules. The first rule considers a response of the interest rate to inflation of 3.5 and the response to output gap of 0.09 \( \hat{i}_t = 3.5\hat{\pi}_t + 0.09\hat{y}_t \). The second rule considers a response of the interest rate only to inflation of 5 \( \hat{i}_t = 5\hat{\pi}_t \). Finally, the third Taylor considers a response to inflation of 5 and to unemployment gap of 2 \( \hat{i}_t = 5\hat{\pi}_t + 2\hat{l}_u \). In all cases, the volatility of unemployment and the rest of the labor market variables are higher in the presence of an informal sector. Particularly under the Taylor Rule that responds to inflation and output, the volatility of price and wage inflation, and the labor market variables are much higher for the case with informality. Therefore, in this model, a policy rule that targets output at the extent of wage inflation generates too much volatility in unemployment,
especially for the case with informality. By targeting only price inflation, or price inflation along with unemployment, the Central Bank can considerably reduce the labor market volatility.

Table 2.4 Relative standard deviations under different Taylor Rules and Optimal Policy

<table>
<thead>
<tr>
<th>Standard Deviations</th>
<th>$i_t = 3.5\hat{\pi}_t + 0.09\hat{y}_t$</th>
<th>$i_t = 5\hat{\pi}_t$</th>
<th>$i_t = 5\hat{\pi}_t + 2\hat{\pi}_t^w$</th>
<th>Optimal Policy</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\rho = 0.41$</td>
<td>$\rho = 0$</td>
<td>$\rho = 0.41$</td>
<td>$\rho = 0.41$</td>
</tr>
<tr>
<td>Price inflation</td>
<td>0.0576</td>
<td>0.0199</td>
<td>0.1516</td>
<td>0.0116</td>
</tr>
<tr>
<td>Wage inflation</td>
<td>0.1998</td>
<td>0.1928</td>
<td>0.1020</td>
<td>0.1769</td>
</tr>
<tr>
<td>Output</td>
<td>0.0068</td>
<td>0.0072</td>
<td>0.0078</td>
<td>0.0078</td>
</tr>
<tr>
<td>Formal employment</td>
<td>0.1510</td>
<td>0.0894</td>
<td>0.0197</td>
<td>0.0278</td>
</tr>
<tr>
<td>Informal employment</td>
<td>0.6698</td>
<td>0.3458</td>
<td>0.0817</td>
<td>-</td>
</tr>
<tr>
<td>Unemployment</td>
<td>2.4997</td>
<td>1.2699</td>
<td>0.3019</td>
<td>0.3442</td>
</tr>
</tbody>
</table>

*The standard deviation of output is expressed in absolute terms. The standard deviation of all other variables is divided by the standard deviation of output.*

Welfare loss analysis

In line with Tomas (2008), I also consider a simple targeting rule that stabilizes a weighted average of price and wage inflation, with the same relative weights as in the welfare loss function. It writes:

$$\frac{\Psi_\pi}{\Psi_\pi + \Psi_{\pi w}} \hat{\pi}_t + \frac{\Psi_{\pi w}}{\Psi_\pi + \Psi_{\pi w}} \hat{\pi}_w t = 0.$$

Table 2.5 shows that any deviation from the optimal monetary policy under commitment generates higher welfare losses in the presence of an informal sector. A zero price inflation policy induces a substantial welfare cost under a staggered wage setting and in the presence of an informal sector, due to the excessive variation in wage inflation and unemployment. The welfare loss under the zero inflation policy is about 1.26 times as large as under the optimal policy, while for the case without informality the welfare loss under zero inflation is only about 0.015 times as large as under the optimal policy.

Different from Thomas (2008), who finds that the targeting rule that stabilizes a weighted average of price and wage inflation performs almost as well as the optimal...
policy, I find that in the presence of an informal sector, the same targeting rule generates significant welfare losses. By comparing equations (71) and (72), one can observe that the weighted average of price and wage inflation in the RHS of equation (71) is not equal to the weighted average of price and wage inflation in the RHS of equation (72). As a result, a simple targeting rule that stabilizes a weighted average of price and wage inflation is not enough to stabilize the formal hiring rate and informal employment at the same time.

The last two columns of Table 2.5 show that for the case where there are not wage rigidities, $\omega_w = 0$, and the economy’s steady-state is efficient, the Central Bank can replicate the efficient equilibrium with a full price inflation stabilization policy, even in the presence of informality.

| Table 2.5. Welfare losses due to deviations from the optimal policy |
|----------------|----------------|
| $\omega_w = 0.75$ | $\omega_w = 0$ |
| **Monetary policy** | **Monetary policy** |
| $p_t = 0.41$ | $p_t = 0.41$ |
| $\hat{\pi}_t = 0$ | $\hat{\pi}_t = 0$ |
| $1.2579$ | $1.2003$ |
| $0.0146$ | $0.0063$ |
| $0$ | $-$ |
| $0$ | $-$ |

### 2.6 Conclusions

This paper analyzes optimal monetary policy in the presence of informality. I develop a closed economy model with nominal price and wage rigidities, search and matching frictions, and a dual labor market. I found that in the absence of wage rigidities and under an efficient steady-state a zero price inflation policy is optimal. In a more realistic scenario, where both price and formal wage rigidities are present, a trade-off between inflation and unemployment emerges. I compare the predictions of the model against the case in which there is no informal sector in the economy. I found that the trade-off between price inflation and unemployment increases with the presence of an informal sector.

Under this framework, optimal monetary policy with informality features significant deviations from price stability in response to productivity shocks. In the presence of informality, wage inflation is more responsive to productivity shocks. Higher wage
inflation generates a higher dispersion on wages in the formal sector. This wage dispersion translates into inefficient fluctuations in formal and informal employment and thus on unemployment. Therefore, by controlling the price inflation rate, the central bank is able to affect the real value of nominal wages and then bring real formal wages closer to their flexible-wage levels. The presence of an informal sector requires a higher adjustment of inflation in order to reduce this gap.

To illustrate the implications of the trade-off faced by the Central Bank, I analyze the behavior of a decentralized economy when the monetary authority implements a policy of full inflation stabilization. I found that the welfare loss under the zero inflation policy is about 1.26 times as large as under the optimal policy, while for the case without informality the welfare loss under the zero inflation policy is about 0.015 times as large as under the optimal policy. These results show that a policy designed to minimize inflation volatility can generate significant welfare losses in the presence of formal wage rigidities and informality, as is the case for most emerging countries.

2.7 Appendix

Appendix B1: Steady State and log-linearized equations

Steady State

\[ q(\theta) F = \rho \]
\[ m(v, l^u) = N(l^u)^\mu (v)^{1-\mu} \]
\[ q(\theta) = N(\theta)^{-\mu} \]
\[ \frac{\kappa F}{w(c)} = \beta E_t \left[ u'(c) \left( p^f mpl^f - w^f \right) + \frac{\kappa}{2} F^2 + (1 - \rho) \frac{\kappa F}{\eta(\theta)} \right] \]
\[ \frac{\kappa F}{w(c)} = \left( w^a - \frac{\phi}{u(c)} \right) \]
\[ w^f = \phi \left( p^f mpl^f + \frac{\kappa}{2} F^2 \right) + (1 - \phi) (w^a) \]
\[ w^a = p^s z^s z \]
\[ y^f = z z^f y^f \]
\[ y^a = z z^a y^a \]
\[ y = \left[ (y^f)^{\frac{1}{\gamma}} + (y^a)^{\frac{1}{\gamma}} \right]^{\gamma} \]
\[ y^f = \left( \frac{1}{p^f} \right)^\gamma y \]
\[ y' = \left( \frac{1}{p'} \right)^\gamma y \]
\[ y = c \]

**log-linearized equations**

\[ \hat{y}_t = \hat{z}_t + \hat{l}_t \]
\[ \hat{w}_t = \hat{p}_t + \hat{z}_t \]
\[ \hat{y}_t = \hat{z}_t + \hat{l}_t \]
\[ \hat{a}_t = \hat{p}_t + \hat{y}_t - \hat{l}_t \]
\[ \hat{y}_t = \hat{c}_t \]
\[ \hat{y}_t = \gamma (\hat{mc}_t - \hat{p}_t) + \hat{y}_t \]
\[ \hat{y}_t = \gamma (\hat{mc}_t - \hat{p}_t^f) + \hat{y}_t \]

\[ \hat{y}_t = \Psi_{yf} \hat{y}_t^f + \Psi_{yi} \hat{y}_i, \text{ where } \Psi_{yf} = \left( \frac{y_f'}{y_f} \right)^{\frac{\gamma - 1}{\gamma}}, \Psi_{yi} = \left( \frac{y_i'}{y_i} \right)^{\frac{\gamma - 1}{\gamma}}, \]
\[ \hat{m}_t = \mu (\hat{t}_t^u) + (1 - \mu) (\hat{t}_t) \]
\[ \hat{l}_{t+1}^f = \hat{l}_t^f + q(\theta) \mathcal{F} (\hat{g}(\theta_t) + \hat{F}_t) \]
\[ \hat{q}(\theta_t) = \hat{m}_t - \hat{v}_t \]
\[ \hat{p}(\theta_t) = (1 - \mu) \hat{\theta} \]
\[ \hat{\theta} = \hat{v}_t - \hat{l}_t^u \]
\[ 0 = \Lambda^u \hat{l}_t + \Lambda^v \hat{l}_t + J^f \hat{l}_t \]
\[ \mathcal{F}_t = \hat{v}_t - \hat{l}_t^f \]
\[ 0 = \mathcal{E}_t \hat{\Gamma}_{t,t+1} + \hat{t}_t - \hat{\pi}_{t+1} \]
\[ \mathcal{E}_t \hat{\Gamma}_{t,t+1} = \hat{\lambda}_{t+1} - \hat{\lambda}_t \]
\[ \hat{u}'(c) = -\frac{1}{\sigma} \hat{c}_t = \hat{\lambda}_t \]
\[ \left( \mathcal{F}_t - \hat{q}(\theta_t) \right) = \frac{1}{\mathcal{F}_t} \left( a\hat{t}_{t+1} - w^f \hat{w}_{t+1}^f \right) + \Gamma \mathcal{F}_{t+1} + \Gamma (1 - \rho) \hat{q}(\theta_{t+1}) + E_t (u - w) \frac{1}{\mathcal{F}_t} \hat{u}'(c_{t+1}) \]
\[ \hat{w}_t^\phi = \phi \left[ \Upsilon_{\phi} \hat{a}_t + \Upsilon_{\psi} \left( 2 \mathcal{F}_t - \hat{u}'(c_t) \right) \right] + (1 - \phi) (\Upsilon_{\psi} \hat{w}_t^i) \]
\[ E_t \Gamma p(\theta) \left[ \hat{\theta}_t + \mathcal{F}_t - \hat{u}'(c_t) - \frac{w^u}{1 - \omega^u} E_t \left[ \hat{w}_{t+1}^f - w^f + \hat{\pi}_{t+1} \right] \right] = w^u \hat{w}_t^i + \frac{\phi}{u^\phi} \hat{u}'(c) \]
\[ \hat{w}_t^\psi = \psi_0 \hat{w}_t^\phi + \psi_1 E_t \left( \hat{w}_{t+1}^f + \hat{\pi}_{t+1} \right) + \psi_2 \left( \hat{w}_{t-1}^f - \hat{\pi}_t \right) \]
\[ \hat{\pi}_t = \kappa_{px} (\hat{mc}_t) + \beta E_t \hat{\pi}_{t+1} \]
\[ \hat{w}_t^f = \hat{w}_{t-1}^f + \hat{\pi}_{ut} - \hat{\pi}_t \]
\[ \hat{z}_t^f = \rho \hat{z}_{t-1}^f + \varepsilon_t^f \]
Appendix B2: Price settings

Total production of final goods in the informal sector, denoted with $y_t^f$ is the following composite of individual retail goods:

$$y_t = \left[ \int_0^1 \left( \frac{y_{t+1}}{y_{t+1}^f} \right)^{\frac{\Theta}{1-\sigma}} \right]^{ \frac{1}{\sigma-1}}$$

In the case that the firm has the chance to set prices optimally, it will choose the price that maximize the present discounted value of the firm’s benefits, as follows:

$$\max_{p_t^*} E_t \sum_{\ell=0}^{\infty} \Gamma_{t,t+\ell} \omega^{\ell} \left[ P_t^* y_{t+\ell}^f - MC_{t+\ell} y_{t+\ell}^f \right]$$

subject to the sequence of demand constraints:

$$y_{t+\ell} = \left( \frac{P_t^*}{P_{t+\ell}} \right)^{-\Theta} y_{t+\ell}.$$  \hspace{1cm} (80)

The maximization problem can be written as follows

$$\max_{p_t^*} E_t \sum_{\ell=0}^{\infty} \Gamma_{t,t+\ell} \omega^{\ell} y_{t+\ell} \left[ (1-\Theta) \left( \frac{P_t^*}{P_{t+\ell}} \right)^{-\Theta} + \Theta(1-\tau^m)MC_{t+\ell} \left( \frac{P_t^*}{P_{t+\ell}} \right)^{-\Theta} \right]$$

The First order condition (FOC) with respect to $p_t^*$ writes

$$E_t \sum_{\ell=0}^{\infty} \Gamma_{t,t+\ell} \omega^{\ell} y_{t+\ell} \left[ (1-\Theta) \left( \frac{P_t^*}{P_{t+\ell}} \right)^{-\Theta} + \Theta(1-\tau^m)MC_{t+\ell} \left( \frac{P_t^*}{P_{t+\ell}} \right)^{-\Theta} \right] = 0$$

$$\left( P_t^* \right)^{1-\Theta} E_t \sum_{\ell=0}^{\infty} \beta^\ell \omega^{\ell} \left( \frac{c_{t+\ell}}{c_t} \right)^{1-\sigma} \left( \frac{P_t}{P_{t+\ell}} \right)^{-\Theta} P_t^{\Theta} = \Theta(1-\tau^m) E_t \sum_{\ell=0}^{\infty} \beta^\ell \omega^{\ell} \left( \frac{c_{t+\ell}}{c_t} \right)^{1-\sigma} \left( \frac{P_t}{P_{t+\ell}} \right)^{1-\Theta} MC_{t+\ell}$$

$$P_t^* = \Theta(1-\tau^m) E_t \sum_{\ell=0}^{\infty} \beta^\ell \omega^{\ell} \left( \frac{c_{t+\ell}}{c_t} \right)^{1-\sigma} \left( \frac{P_t}{P_{t+\ell}} \right)^{-\Theta} MC_{t+\ell} P_t$$

Divining by $P_t$, and with $p_t^* = P_t^* P_t$,

$$p_t^* = \Theta(1-\tau^m) E_t \sum_{\ell=0}^{\infty} \beta^\ell \omega^{\ell} \left( \frac{c_{t+\ell}}{c_t} \right)^{1-\sigma} \left( \frac{P_t}{P_{t+\ell}} \right)^{-\Theta} MC_{t+\ell}$$
Finally, the general price index in the formal sector is equal to:

\[ P_t = \left( \omega^p (P_{t-1})^{1-\Theta} + (1 - \omega^p) (P^*_t)^{1-\Theta} \right) \frac{1}{1-\Theta}. \]

dividing both sides by \( P_t \)

\[ 1 = \left( \omega^p (\pi_t)^{1-\Theta} + (1 - \omega^p) (p^*_t)^{1-\Theta} \right) \frac{1}{1-\Theta}. \]

### Appendix B3: Wage bargaining under a flexible wage setting

\[ \max_{W^*_t} \Phi_t = \left[ J^f_{it} \right]^{1-\phi} \left[ H^f_{it} \right]^{\phi} \tag{81} \]

subject to:

\[ W^f_{it} = \begin{cases} W^f_{it-1} \quad \text{with probability } \omega^w \\ W^f_{it}^* \quad \text{with probability } (1 - \omega^w) \end{cases} \tag{82} \]

\( J^f_{it} \) and \( H^f_{it} \) can be expressed as follows:

\[ H^f_{it} = \frac{W^f_{it}}{p_t} - \frac{\varphi_t}{u'(c)} + E_t \Gamma_{t,t+1} \left[ (1 - \rho) H^f_{it+1} - p(\theta_t) H^f_{F,t+1} \right] \tag{83} \]

\[ J^f_{it} = a_t - w^f_{it} + \frac{\kappa F^2_{it}}{2u'(c_t)} + (1 - \rho) E_t \Gamma_{t,t+1} J^f_{it+1} \tag{84} \]

where

\[ E_t \Gamma_{t,t+1} \left[ p(\theta_t) H^f_{F,t+1} \right] = w^i_t - \frac{\varphi_t}{u'(c)} \]

The first order necessary condition for the Nash bargaining solution is given by:

\[ (1 - \phi) H^f_{it}(W^f_{it}^*) = \phi J^f_{it}(W^f_{it}^*) \tag{85} \]

Replacing (83) and (84) into (85), I obtain:
\[(1 - \phi) \left( \frac{W_i^F}{p_t} - \frac{\varphi_t}{u'(c)} \right) + E_i \Gamma_{t+1} \left[ (1 - \rho - p(\theta_t)) \Pi_{t+1}^t \right) = \phi \left( a_t - w^i_t + \frac{\kappa F^2_i}{2u'(c_t)} + (1 - \rho) \frac{\kappa F^t}{u'(c_t)q(\theta_t)} \right) \]

\[
\frac{W_i^F}{p_t} = \phi \left( a_t + \frac{\kappa F^2_i}{2u'(c_t)} + \frac{\kappa F_i}{u'(c_t)} \right) + (1 - \phi) \left( \frac{\varphi_t}{u'(c_t)} \right)
\]

with \[\left[ \frac{\kappa F_i}{u'(c_t)} \right] = u_t^i - \frac{\varphi_t}{u'(c)} \]

\[
\frac{W_i^F}{p_t} = \phi \left( a_t + \frac{\kappa F^2_i}{2u'(c_t)} \right) + (1 - \phi) \left( u_t^i \right)
\]

**Appendix B4. Efficient Equilibrium**

The social planner chooses the state-contingent path of \( c, l^f, l^i \) and \( v_t \) to maximize the joint welfare of households and managers, subject to the law of motion of employment and the aggregate resource constraint:\[ l^f_t = (1 - \rho) l^f_t + m(v_t, l^v_t), \]

with \( y_t = \left[ \left( y^f_t \right)^{\frac{1}{\gamma-1}} + \left( y^i_t \right)^{\frac{1}{\gamma-1}} \right]^{\frac{\gamma-1}{\gamma}} \) and \( m(v_t, l^v_t) = N(v_t)^{1-\mu} (l^v_t)^\mu \). The first-order conditions with respect to \( v_t, l^f_{t+1} \) and \( l^i_t \) are given by

\[ [v_t] \quad \kappa \left( \frac{v_t}{l^i_t} \right) = \mathcal{T}_t^f m_v (v_t, l^v_t) \quad (86) \]

\[ \left[ l^f_{t+1} \right] \quad \mathcal{T}_t^f = \beta \left[ u'(c_{t+1}) \frac{\partial y_{t+1}}{\partial y_{t+1}} \frac{\partial y_{t+1}}{\partial y_{t+1}} - \varphi + \frac{\kappa}{2} \left( \frac{y_{t+1}}{l^f_{t+1}} \right) \right] + \mathcal{T}_t^f \left[ (1 - \rho) - m_{l^v} (v_{t+1}, l^v_{t+1}) \right] \quad (87) \]

\[ [l^i_t] \quad u'(c_t) \frac{\partial y_t}{\partial y_t} \frac{\partial y_t}{\partial l^i_t} - \varphi = \mathcal{T}_t^f m_{l^v} (v_t, l^v_t) \quad (88) \]

where \( m_v (v_t, l^v_t) = (1 - \mu) q(\theta_t) \) and \( m_{l^v} (v_t, l^v_t) = \mu p(\theta_t), \quad p(\theta_t) = \theta_t q(\theta_t) \) and \( 1 - \mu = \frac{\partial m_v}{\partial v_t} \). \( \mathcal{T}_t^f \) is known as the social value of an additional worker in the formal sector.

reorganizing and replacing \( \kappa \left( \frac{v_t}{l^i_t} \right) \frac{1}{(1-\mu)q_t} = \mathcal{T}_t^f \) and \( u'(c_t) \frac{\partial y_t}{\partial y_t} \frac{\partial y_t}{\partial l^i_t} - \varphi = \mathcal{T}_t^f (m_2(v_t, l^v_t)) \) into (87), I obtain the following expression for the efficient job creation condition:
log-linearizing this equation and iterating forward, we have:

\[ \kappa \left( \frac{v_t}{l^t_1} \right) \frac{1}{(1 - \mu) q^t} = \beta \left[ u'\left(c_{t+1}\right) \frac{\partial y_{t+1}}{\partial y_{t+1}} m_l t_{t+1} - \varphi + \frac{\kappa}{2} \left( \frac{v_{t+1}}{l_{t+1}^t} \right)^2 + \gamma_{t+1}^f \left( (1 - \rho) + m_2 v_{t+1} l_{t+1}^t \right)(-1) \right] \]

\[ \kappa \left( \frac{v_t}{l^t_1} \right) \frac{1}{(1 - \mu) q^t} = \beta \left[ u'\left(c_{t+1}\right) \frac{\partial y}{\partial y^*} m_l t_{t+1} - \varphi + \frac{\kappa}{2} \left( \frac{v_{t+1}}{l_{t+1}^t} \right)^2 + \kappa \left( \frac{v_{t+1}}{l_{t+1}^t} \right) \frac{1}{(1 - \mu) q \left( \theta_{t+1} \right)} (1 - \rho) - \left( u'\left(c_{t+1}\right) \frac{\partial y}{\partial y^*} m_l t_{t+1} - \varphi \right) \right] \]

\[ \frac{\kappa F_t}{q(\theta_t)} = \beta \left[ (1 - \mu) u'\left(c_{t+1}\right) \left( \frac{\partial y}{\partial y^*} m_l t_{t+1} - \kappa \frac{\kappa}{2} f_{t+1} \right) + (1 - \rho) \frac{\kappa F_t}{q(\theta_t)} \right] \tag{89} \]

Appendix B5: Wage dynamics in the formal sector under staggered wage bargaining

The worker’s surplus can be written as:

\[ \mathbb{H}^f_t \left( W^*_t \right) = \frac{W^*_t}{p_t} - \varphi_t + E_t \gamma \left( \begin{array}{c} (1 - \rho) \mathbb{H}^f_{t+1} (w_{t+1}^f) - p_t \mathbb{H}^f_{t+1} \\ (1 - \rho) \omega^w \left[ \mathbb{H}^f_{t+1} (W^*_t) - \mathbb{H}^f_{t+1} (W^*_t) \right] \end{array} \right) \tag{90} \]

the term \( E_t \left[ \mathbb{H}^f_{t+1} (W^*_t) - \mathbb{H}^f_{t+1} (W^*_t) \right] \) writes as follows:

\[ E_t \left[ \mathbb{H}^f_{t+1} (W^*_t) - \mathbb{H}^f_{t+1} (W^*_t) \right] = E_t \left[ \frac{W^*_t}{\gamma_{t+1}} - \frac{W^*_t}{\gamma_{t+1}} \right] + (1 - \rho) \omega^w E_t \gamma \left[ \mathbb{H}^f_{t+2} (W^*_t) - \mathbb{H}^f_{t+2} (W^*_t) \right] \]

log-linearizing this equation and iterating forward, we have:

\[ E_t \left[ \mathbb{H}^f_{t+1} (W^*_t) - \mathbb{H}^f_{t+1} (W^*_t) \right] = E_t w^f \left[ \hat{W}^f_t - \hat{W}^f_t \right] \]

\[ + (1 - \rho) \omega^w E_t \gamma \left[ \hat{W}^f_{t+1} (W^*_t) - \hat{W}^f_{t+1} (W^*_t) \right] \]

\[ E_t \left[ \mathbb{H}^f_{t+1} (W^*_t) - \mathbb{H}^f_{t+1} (W^*_t) \right] = \frac{w^f}{\mu} \left[ \hat{W}^f_t - \hat{W}^f_t \right] \]

with \( w^f = \frac{w^f}{\mu} \) and \( \epsilon = \frac{1}{(1 - \beta (1 - \rho) w^w)} \)

In this way, the complete log linearization of (90) takes the form:
Replacing (93) into (92), and iterating forward I obtain:

The producer surplus can be written as:

\[
\hat{J}_t^f (W_t^{f*}) = \frac{w_t^{f*}}{\rho_t} - \frac{\kappa}{2} \frac{F_t^2}{\omega(c)} + \left(1 - \rho + q f \mathcal{F}_t (W_t^{f*})\right) E_t \Gamma_{t,t+1} J_{t+1}^f
\]

\[
J_t^f (W_t^{f*}) = \frac{w_t^{f*}}{\rho_t} - \frac{\kappa}{2} \frac{F_t^2}{\omega(c)} + \left(1 - \rho + q f \mathcal{F}_t (W_t^{f*})\right) E_t \Gamma_{t,t+1} J_{t+1}^f
\]

\[
J_t^f (W_t^{f*}) = \frac{w_t^{f*}}{\rho_t} + \frac{\kappa}{2} \frac{F_t^2}{\omega(c)} + \left(1 - \rho + q f \mathcal{F}_t (W_t^{f*})\right) E_t \Gamma_{t,t+1} J_{t+1}^f
\]

\[
J_t^f (W_t^{f*}) = \frac{w_t^{f*}}{\rho_t} + \frac{\kappa}{2} \frac{F_t^2}{\omega(c)} + E_t \Gamma_{t,t+1} J_{t+1}^f
\]

The term \( E_t \left[ J_{t+1}^f (W_t^{f*}) - J_{t+1}^f (W_t^{f*}) \right] \) can be written as follows:

\[
E_t \left[ J_{t+1}^f (W_t^{f*}) - J_{t+1}^f (W_t^{f*}) \right] = \frac{\kappa \mathcal{F}_t}{\omega(c)q_t}\]

From \( E_t \Gamma_{t,t+1} J_t^f = \frac{\kappa \mathcal{F}_t}{\omega(c)q_t} \), I can obtain an expression for \( \mathcal{F}_t (W_t^{f*}) - \mathcal{F}_t (W_t^{f*}) \)

\[
E_t \Gamma_{t,t+1} \omega w \left[ J_{t+1}^f (W_t^{f*}) - J_{t+1}^f (W_t^{f*}) \right] = \frac{\kappa}{\omega(c)q_t} \mathcal{F}_t (W_t^{f*}) - \mathcal{F}_t (W_t^{f*})
\]

Replacing (93) into (92), and iterating forward I obtain:

\[
E_t \left[ J_{t+1}^f (W_t^{f*}) - J_{t+1}^f (W_t^{f*}) \right] = -\frac{w_t^{f*}}{\rho_t} \left[ W_t^{f*} - W_t^{f*} \right]
\]

93
\[
E_t \left[ \tilde{F}_{t+1} \left( W_t^f \right) - \tilde{F}_{t+1} \left( W_{t+1}^f \right) \right] = -\omega w^f \mu / \tilde{F} \left[ \tilde{W}_t^f - \tilde{W}_{t+1}^f \right]
\]

where \( \mu = \frac{1}{1 - \omega w^T} \).

In this way the log-linearized version of the formal firm's and worker's surplus can be written, respectively, as follows:

\[
\bar{J}_t^f (w_t^f) = \frac{a}{\tilde{F}} \tilde{a}_t - \frac{w_t^f}{\tilde{F}} \left[ \tilde{W}_t^f - \tilde{F}_t + (1 - \rho) \Gamma \omega w \mu E_t \left[ \tilde{W}_t^f - \tilde{W}_{t+1}^f \right] + \frac{\bar{a} \tilde{F} - \frac{1}{2} \tilde{F}_{t+1} \left( w_t^f \right) - \tilde{a}'(c) \right] + (1 - \rho) \Gamma E_t \left[ \bar{J}_{t+1}^f \left( w_{t+1}^f \right) + \tilde{F}_{t+1} \right]
\]

\[
\bar{H}_t^f (w_t^f) = \frac{w}{\tilde{F}} \left[ \tilde{W}_t^f + (1 - \rho) \omega w \Gamma \left[ \tilde{W}_t^f - \tilde{W}_{t+1}^f - \tilde{\pi}_{t+1} \right] \right] + E_t \Gamma \left\{ (1 - \rho) \left( \bar{H}_{t+1}^f (w_{t+1}^f) + \tilde{F}_{t+1} \right) - \rho \left( \tilde{\theta}_t \right) \left( \bar{H}_{t+1}^f \left( w_{t+1}^f \right) + \tilde{F}_{t+1} \right) \right\}
\]

where \( a_t \) is the marginal productivity of labor \( \tilde{a}_t = \tilde{a}_t + \tilde{a}_t - \tilde{a}_t \).

The wage contract would be in the following way:

Managers and workers split the match surplus in the same way as in the case of period-by-period Nash negotiation:

\[
(1 - \phi) \bar{J}_t^f = \phi \bar{J}_t^f
\]

log-linearizing this equation gives

\[
\bar{J}_t^f (W_t^f) = \bar{J}_t^f (W_t^f)
\]

replacing the expression for \( \bar{J}_t^f (W_t^f) \) and \( \bar{J}_t^f (W_t^f) \)

\[
\frac{w}{\tilde{F}} \left[ \tilde{W}_t^f + (1 - \rho) \omega w \Gamma \left[ \tilde{W}_t^f - \tilde{W}_{t+1}^f - \tilde{\pi}_{t+1} \right] \right] + \frac{w}{\tilde{F}} \left[ \tilde{W}_t^f + (1 - \rho) \Gamma \omega w \mu E_t \left[ \tilde{W}_t^f - \tilde{W}_{t+1}^f \right] + \frac{\bar{a} \tilde{F} - \frac{1}{2} \tilde{F}_{t+1} \left( w_t^f \right) - \tilde{a}'(c) \right] + (1 - \rho) \Gamma E_t \left[ \bar{J}_{t+1}^f \left( w_{t+1}^f \right) + \tilde{F}_{t+1} \right]
\]

replacing \( \bar{H}_{t+1}^f (W_{t+1}^f) = \bar{H}_{t+1}^f (W_{t+1}^f) \) and \( \frac{\bar{\phi}}{1 - \bar{\phi}} = \frac{\tilde{F}}{\tilde{F}} \)
\[
\begin{align*}
\frac{w}{\mu} \left( \dot{w}_t^f + (1 - \rho) w \epsilon \Gamma \left[ \hat{w}_t^f - \hat{w}_{t+1}^f - \pi_{t+1} \right] \right) \\
- \frac{1}{\mu} \left( w^a \hat{w}_t^a \right) + E_t \Gamma \left\{ \left( 1 - \rho \right) \left( \hat{H}_{w}^f (w_{t+1}^f) + \hat{\Gamma}_{t+1} \right) \right\} \\
= \frac{\sigma}{\mu} \hat{a}_t - \frac{w^f}{\mu} \left( \dot{w}_t^f + (1 - \rho) \Gamma c \hat{w}_t^f \right) \\
+ \frac{\beta \gamma^f x}{2} \left( \frac{2 \hat{F}}{w_t^f} - \hat{u}'(c) \right) + (1 - \phi)^{-1} \hat{\phi}_t (w_t^f) \\
+ (1 - \rho) \Gamma E_t \left( \hat{\Gamma}_{t+1} \hat{H}_{t+1}^f (w_{t+1}^f) - (1 - \phi)^{-1} \hat{\phi}_{t+1} (W_t^f) \right) \\
\end{align*}
\]

then

\[
\begin{align*}
(1 - \phi) w \left( \dot{w}_t^f + (1 - \rho) w^{\epsilon} \Gamma \left[ \hat{w}_t^f - \hat{w}_{t+1}^f - \pi_{t+1} \right] \right) &- \frac{(1 - \phi)}{w^f} \left( w^a \hat{w}_t^a \right) \\
= (1 - \phi) \left( w^a \hat{w}_t^a \right) + \phi \hat{a}_t \\
+ \phi \frac{\beta \gamma^f x}{2} \left( \frac{2 \hat{F}}{w_t^f} - \hat{u}'(c) \right)
\end{align*}
\]

rearranging

\[
\begin{align*}
w \dot{w}_t^f + ((1 - \phi) \epsilon + \phi \mu) (1 - \rho) w^{\epsilon} \Gamma w^f \left[ \hat{w}_t^f - \hat{w}_{t+1}^f - \pi_{t+1} \right] \\
= (1 - \phi) \left( w^a \hat{w}_t^a \right) + \phi \hat{a}_t \\
+ \phi \frac{\beta \gamma^f x}{2} \left( \frac{2 \hat{F}}{w_t^f} - \hat{u}'(c) \right)
\end{align*}
\]

where \( \Theta = (1 - \phi) \epsilon + \phi \mu \)

\[
\begin{align*}
(1 + \Psi) \dot{w}_t^f - \Psi E_t \left[ \hat{w}_t^f + \pi_{t+1} \right] \\
= \frac{\phi}{w^a} \hat{a}_t + (1 - \phi) \left( w^a \hat{w}_t^a \right) + \phi \frac{\beta \gamma^f x}{2} \left( \frac{2 \hat{F}}{w_t^f} - \hat{u}'(c) \right)
\end{align*}
\]

where \( \Psi = (1 - \rho) w^{\epsilon} \Gamma \Theta \) and \( \dot{w}_t^\circ \) is the target wage given by:

\[
\dot{w}_t^\circ = \frac{1}{(1 + \Psi)} \hat{w}_t^\circ + \frac{\Psi}{(1 + \Psi)} E_t \left[ \hat{w}_t^f + \hat{\pi}_{t+1} \right]
\]

(95)

with, \( a = p^f m^{pl} \), \( \gamma_a = \frac{a}{w^f} \), \( \gamma_f = \frac{\gamma^f x}{w^f \hat{u}'(c)} \), \( w^a = \frac{w^i}{w^f} \), \( \psi_0 + \psi_1 = 1 \), and \( \hat{a}_t = \hat{\mu}_t^f + m^{pl} \).

**Formal wage and Hiring dynamics**

the target wage is
\[
\hat{w}_t^* (w_t^f) = \left[ \Theta_0 \hat{a}_t + \Theta_{w,t} w_t^f + \Theta_{x,t} \left( 2\hat{F} \left( w_t^f \right) - \hat{a}'(c) \right) \right]
\]

Let’s find expressions for \( \hat{F} \left( w_t^f \right) \) and \( \hat{H}_{x,t+1} \) in terms of gaps between contract and average wages. Previously, I found \( E_t \left[ \hat{F}_{t+1} \left( W_{t+1}^f \right) - \hat{F}_{t+1} \left( W_{t+1}^f \right) \right] = -w^w w^f \frac{\partial}{\partial F} \left[ \hat{W}_{t+1} - \hat{W}_{t+1}^f \right] \) then

\[
E_t \left[ \hat{F}_t \left( W_t^f \right) - \hat{F}_t \left( W_t^f \right) \right] = -w^w w^f \frac{\partial}{\partial F} \left[ \hat{W}_t^f - \hat{W}_t^f \right]
\]

where \( \hat{F}_t \left( w_t^f \right) \) is the average hiring rate

Using the results in previous section

\[
E_t \left[ \hat{f}_{t+1}(W_{t+1}^f) - \hat{f}_{t+1}(W_{t+1}^f) \right] = -w^f \frac{\partial}{\partial F} \left[ \hat{W}_{t+1}^f - \hat{W}_{t+1}^f \right]
\]

\[
E_t \left[ \hat{f}_{t+1}(W_{t+1}^f) - \hat{f}_{t+1}(W_{t+1}^f) \right] = w^f \frac{\partial}{\partial F} \left[ \hat{W}_{t+1}^f - \hat{W}_{t+1}^f \right]
\]

with \( \phi_w = \frac{w^f}{\hat{F}} \)

\[
E_t \left[ \hat{f}_{t+1}(W_{t+1}^f) - \hat{f}_{t+1}(W_{t+1}^f) \right] = -\phi_w \left[ \hat{W}_{t+1}^f - \hat{W}_{t+1}^f \right]
\]

\[
E_t \left[ \hat{f}_{t+1}(W_{t+1}^f) - \hat{f}_{t+1}(W_{t+1}^f) \right] = \phi_w \frac{1 - \phi}{\phi} \left[ \hat{W}_{t+1}^f - \hat{W}_{t+1}^f \right]
\]

Starting from the Nash first order condition in \( t + 1 \):

\[
E_t \hat{f}_{t+1}(W_{t+1}^f) = E_t \hat{f}_{t+1}(W_{t+1}^f)
\]

\[
\hat{f}_{t+1}(W_{t+1}^f) + \frac{w^f}{\hat{F}} \left[ \hat{W}_{t+1}^f - \hat{W}_{t+1}^f \right] = \hat{f}_{t+1}(W_{t+1}^f) - \frac{w^f}{\hat{F}} \left[ \hat{W}_{t+1}^f - \hat{W}_{t+1}^f \right]
\]

\[
\hat{f}_{t+1}(W_{t+1}^f) + \left( \frac{w^f}{\hat{F}} + \frac{w^f}{\hat{F}} - (q^f F \omega^w \Gamma) (\omega^w \mu \phi_w) \mu \right) \left[ \hat{W}_{t+1}^f - \hat{W}_{t+1}^f \right] = \hat{f}_{t+1}(W_{t+1}^f)
\]

\[
\hat{f}_{t+1}(W_{t+1}^f) + \nabla \left[ \hat{W}_{t+1}^f - \hat{W}_{t+1}^f \right] = \hat{f}_{t+1}(W_{t+1}^f)
\]
where \( \nabla = \phi_w \mu \left( \frac{f'}{H\mu} + 1 \right) \). Using \( E_t \Gamma_{t+1} F_{t+1} = \frac{\kappa_{r_t}}{w(c)\eta_t} \)

\[
\left( \tilde{F}_{t+1} + f_{t+1}(W_{t+1}^f) \right) = \tilde{F}_t \left( W_t^f \right) - \dot{u}'(c_t) - \dot{q}(\theta_t)
\]

we have then:

\[
\tilde{h}_{t+1}^f(w_{t+1}^n) + \nabla \left[ \tilde{W}_{t+1}^f - \tilde{W}_t^f \right] = \tilde{F}_t \left( W_t^f \right) - \dot{u}'(c_t) - \dot{q}(\theta_t) - \tilde{F}_{t+1} +
\]

\[
E_t \left[ \tilde{h}_{t+1}^f(w_{t+1}^n) + \tilde{F}_{t+1} \right] = \tilde{F}_t \left( W_t^f \right) - \dot{u}'(c_t) - \dot{q}(\theta_t) - \nabla E_t \left[ \tilde{W}_{t+1}^f - \tilde{W}_t^f \right]
\]

where \( \tilde{h}_{t+1}^f(w_{t+1}^n) = \tilde{h}_{r+1}^f \). Then

\[
E_t \left[ \tilde{h}_{r+1}^f + \tilde{F}_{r+1} \right] = \tilde{F}_t \left( W_t^f \right) - \dot{u}'(c_t) - \dot{q}(\theta_t) - \nabla E_t \left[ \tilde{W}_{r+1}^f - \tilde{W}_t^f \right]
\]

substituting in the target wage and rearranging

\[
\dot{w}_t^o = \phi \left[ Y_a \dot{a}_t + Y_f \left( 2 \tilde{F}_t - \dot{u}'(c) \right) \right] + \left( 1 - \phi \right) (Y_w \dot{w}_t^o)
\]

\[
\left[ \tilde{F}_t \left( W_t^f \right) - \tilde{F}_t \left( W_t^f \right) \right] = -\omega^w w_t^f \frac{\mu}{\tilde{F}_t} \left[ \tilde{W}_{t+1}^f - \tilde{W}_t^f \right]
\]

\[
E_t \left[ \tilde{h}_{r+1}^f + \tilde{F}_{r+1} \right] = \tilde{F}_t \left( W_t^f \right) - \dot{u}'(c_t) - \dot{q}(\theta_t) - \nabla E_t \left[ \tilde{W}_{r+1}^f - \tilde{W}_t^f \right]
\]

\[
\dot{w}_t^o = \phi \left[ Y_a \dot{a}_t + Y_f \left( 2 \tilde{F}_t - \dot{u}'(c) \right) \right] + \left( 1 - \phi \right) (Y_w \dot{w}_t^o)
\]

\[
\dot{w}_t^o \left( W_t^f \right) = \left[ \phi Y_a \dot{a}_t + (1 - \phi) Y_w \dot{w}_t^o + \phi Y_f \left( 2 \tilde{F}_t \left( \dot{w}_t^o - \dot{w}_t^f \right) - \omega^w w_t^f \right) \frac{\mu}{\tilde{F}_t} \left[ \dot{w}_t^o - \dot{w}_t^f \right] - \dot{u}'(c_t) \right]
\]

in real terms

\[
\dot{w}_t^o \left( w_t^f \right) = \dot{w}_t^o + \omega_2 \left[ \dot{w}_t^f - \dot{w}_t^o \right]
\]

where \( \omega_2 = \omega^w \mu \phi_w \left( \phi Y_f \right) \). \( \dot{w}_t^o = \phi Y_a \dot{a}_t + (1 - \phi) Y_w \dot{w}_t^o + \phi Y_f \left( 2 \tilde{F}_t - \dot{u}'(c) \right) \). Additionally, the average wage in the formal sector is defined as:

\[
\tilde{W}_t^f = (1 - \omega^w) \tilde{W}_{t+1}^f + \omega^w \tilde{W}_t^f
\]

Combining this expression with the equations that define the evolution of the contract wage, then yields the following second order difference equation for the aggregate wage:
\[
\hat{w}_t f = \frac{1}{(1 + \Psi)} \hat{w}_t^o + \Psi E_t \hat{W}_t f^* + 1\]

\[
\hat{w}_t^o (\hat{W}_t f^*) = \hat{w}_t^o + \varpi_2 [\hat{W}_t f - \hat{W}_t f^*]
\]

\[
\hat{W}_t f = (1 - \omega^w) \hat{W}_t f^* + \omega^w \hat{W}_{t-1}
\]

\[
\hat{w}_t^o \hat{W}_t f = (1 - \omega^w) \hat{W}_t f^* + 1 + (1 + \Psi) \hat{W}_{t-1}
\]

\[
(1 + \Psi) \hat{W}_t f = (1 - \omega^w) \hat{w}_t^o + (1 - \omega^w) \Psi E_t \hat{W}_t f^* + 1 + (1 + \Psi) \omega^w \hat{W}_{t-1}
\]

\[
\hat{W}_t f = \psi_o \hat{w}_t^o + \psi_1 E_t \hat{W}_t f^* + \psi_2 \hat{W}_{t-1}
\]

where \(\varsigma = 1 + \Psi + (\varpi_2 + \Psi) \omega^w\), \(\psi_o = \frac{(1 - \omega^w)}{\varsigma}\), \(\psi_1 = \frac{\Psi}{\varsigma}\), \(\psi_2 = \frac{(\varpi_2 + \Psi) \omega^w}{\varsigma}\). \(\psi_o + \psi_1 + \psi_2 = 1\)

**Appendix B6: Job creation condition and equilibrium condition in the informal sector**

In this section, I express the job creation condition in the formal sector and the equilibrium condition in the informal sector as a function of the marginal cost and the formal wage gap.

From the Nash wage bargaining under flexible wages, and the formal job creation condition I have, respectively:

\[
w_t^o = \phi \left[ a_t + \frac{\kappa}{2} \mathcal{F}_t \frac{1}{w(c_t)} + \theta_t \frac{\kappa}{w(c_t)} \right] + [1 - \phi] \left[ \frac{\hat{w}_t}{w(c_t)} \right] \tag{96}
\]

\[
\frac{\kappa \mathcal{F}_{it}}{q(\theta_t)} = \beta E_t \left[ u'(c_{t+1}) \left( \frac{l_{t+1} m p_{t+1}}{w(c_{t+1})} - w_{it+1} + \frac{\kappa \mathcal{F}_{it+1}^2}{2u'(c_{t+1})} \right) + (1 - \rho) \frac{\kappa \mathcal{F}_{it+1}}{q(\theta_{t+1})} \right], \tag{97}
\]

Equations (96) and (97) at the steady-state (SS) can be written as follows

\[
s_w = \phi \left[ \frac{l'}{y} a + s_v + \frac{\kappa}{m} s_v \right] + \left[ 1 - \phi \right] \left[ \frac{\hat{w}_t}{w(c_t)} \frac{l'}{y} \right] \tag{98}
\]

\[
s_v \left[ \beta^{-1} - 1 + \frac{\rho}{2} \right] = \frac{\rho}{2} E_t \left[ \left( \frac{l'}{y} a - s_w \right) \right]
\]
where \( s_w = w_t^{lI}/y \), \( \rho l^f = q(\theta) v \), \( s_v = \frac{hc}{w(c)c} = \frac{s^2 \rho^2 I^f}{w(c)c} \).

Log-linearizing equations (96) and (97) around the steady-state gives:

\[
\frac{2}{\rho} s_v (\mu \theta_t + \tilde{F}_{it}) = \beta \left\{ \left( \frac{lI}{y} a \tilde{w}_{it+1} - s_w \tilde{w}_{it+1} + \left( \frac{lI}{y} a - s_w \right) \tilde{u}'(c_{t+1}) \right) \right. + \frac{2}{\rho} s_v \tilde{F}_{it+1} + \frac{(1 - \rho)2}{\rho} s_v \mu \tilde{\theta}_{it+1} \right\}
\]

(99)

\[
s_w u_t^p = \frac{lI}{y} a \tilde{w}_{it} - (1 - \phi) \frac{lI}{y} u_t(\tilde{c}_t) + \phi \left( p(\theta) \frac{2}{\rho} s_v + 2s_v \right) \tilde{F}_{it} - \phi \left( p(\theta) \frac{2}{\rho} s_v + s_v \right) \tilde{u}'(c_{t+1}) + \phi p(\theta) \frac{2}{\rho} s_v \theta_t
\]

(100)

Combining both equations in the way that it is possible to express equation (99) in terms of \( \tilde{w}_{it+1}^2 - w^p_{it+1} \) I obtain:

\[
\frac{2}{\rho^3} s_v \left( \mu \tilde{\theta}_t + \tilde{F}_{it} \right) = \left\{ \left( (1 - \phi) \frac{lI}{y} a \tilde{w}_{it+1} - s_w \left( \tilde{w}_{it+1} - w^p_{it+1} \right) \right) \tilde{u}'(c_{t+1}) \right. + \frac{2}{\rho} s_v \tilde{F}_{it+1} + \frac{(1 - \rho)2}{\rho} s_v \mu \tilde{\theta}_{it+1} \right\}
\]

\[
- \left[ \left( s_w - \phi \left[ \frac{lI}{y} a + s_v + \frac{\rho(\theta)}{\rho} s_v \right] \right) \tilde{u}'(c_{t+1}) + \phi \left( p(\theta) \frac{2}{\rho} s_v + 2s_v \right) \tilde{F}_{it+1} - \phi \left( p(\theta) \frac{2}{\rho} s_v + s_v \right) \tilde{u}'(c_{t+1}) + \phi p(\theta) \frac{2}{\rho} s_v \tilde{\theta}_{it+1} \right]
\]

reorganizing

\[
\frac{2}{\rho^3} s_v \left( \mu \tilde{\theta}_t + \tilde{F}_{it} \right) = \left\{ \left( (1 - \phi) \frac{lI}{y} a \tilde{w}_{it+1} + \frac{lI}{y} a \tilde{u}'(c_{t+1}) \right) - s_w \left( \tilde{w}_{it+1} - w^p_{it+1} \right) \tilde{u}'(c_{t+1}) \right. + \frac{2}{\rho} s_v \tilde{F}_{it+1} + \frac{(1 - \rho)2}{\rho} s_v \mu \tilde{\theta}_{it+1} \right\}
\]

\[
- \left[ \phi \left( p(\theta) \frac{2}{\rho} s_v + 2s_v \right) \tilde{F}_{it+1} + \phi p(\theta) \frac{2}{\rho} s_v \tilde{\theta}_{it+1} \right]
\]

then with \( s_w - \phi \left[ \frac{lI}{y} a + s_v + \frac{\rho(\theta)}{\rho} s_v \right] = [1 - \phi] \left[ \frac{lI}{y} a \right] \)

\[
\frac{2}{\rho^3} s_v \left( \mu \tilde{\theta}_t + \tilde{F}_{it} \right) = \left\{ \left( (1 - \phi) \frac{lI}{y} a \tilde{w}_{it+1} + \frac{lI}{y} a \tilde{u}'(c_{t+1}) \right) - s_w \left( \tilde{w}_{it+1} - w^p_{it+1} \right) \tilde{u}'(c_{t+1}) \right. + \frac{2}{\rho} s_v \tilde{F}_{it+1} + \frac{(1 - \rho)2}{\rho} s_v \mu \tilde{\theta}_{it+1} \right\}
\]

\[
- \left[ \phi \left( p(\theta) \frac{2}{\rho} s_v + 2s_v \right) \tilde{F}_{it+1} + \phi p(\theta) \frac{2}{\rho} s_v \tilde{\theta}_{it+1} \right]
\]

99
I have that prices are equal to the marginal cost, that in perfect competition they should be equal to the marginal income \( \frac{\partial y}{\partial y} f \). \[
\frac{2}{\rho} s_v (\mu \hat{\theta} + \hat{F}_t) = \left\{ (1 - \phi) \left( \frac{y}{y} \right)^{-1} \left( \hat{p}_t f - \frac{1}{\gamma} (\hat{y}_t f - \hat{y}_t f) \right) \right\}
\]
From the Hosios condition I have \( \phi = \mu \), then previous equation becomes \[
\frac{2}{\rho} s_v (\mu \hat{\theta} + \hat{F}_t) = \left\{ (1 - \phi) \left( \frac{y}{y} \right)^{-1} \left( \hat{p}_t f - \frac{1}{\gamma} (\hat{y}_t f - \hat{y}_t f - p (\hat{c}_t f + 1)) \right) \right\}
\]
reorganizing \[
\frac{2}{\rho} s_v (\mu \hat{\theta} + \hat{F}_t) = \left\{ (1 - \phi) \left( \frac{y}{y} \right)^{-1} \left( \hat{p}_t f - \frac{1}{\gamma} (\hat{y}_t f - \hat{y}_t f - \hat{u} c + 1)) \right) \right\}
\]
where \( \hat{Y}_t f \) is the social value of an additional job in the formal sector found in the social planner solution.

**Appendix B7: Phillips curve**

from the optimal price setting we have
\[
p^*_t = \frac{E_t \sum_{\ell=0}^{\infty} \beta^\ell \omega^\ell \left( \frac{c_{t+\ell}}{c_t} \right)^{1-\sigma} \left( \frac{p_{t+\ell}}{p_{t+\ell}} \right)^{-\Theta} m_{c_{t+\ell}}}{E_t \sum_{\ell=0}^{\infty} \beta^\ell \omega^\ell \left( \frac{c_{t+\ell}}{c_t} \right)^{1-\sigma} \left( \frac{P_{t+\ell}}{P_{t+\ell}} \right)^{1-\Theta}}
\]
\[
p^*_t = \Theta (1 - \tau^m) N_t \quad \hat{p}_t = \frac{\Theta (1 - \tau^m) N_t}{(1 - \Theta) \hat{D}_t}
\]
\[
N_t = E_t \sum_{\ell=0}^{\infty} \beta^\ell \omega^\ell \left( \frac{c_{t+\ell}}{c_t} \right)^{1-\theta} \left( \frac{P_{t+\ell}}{P_{t+\ell}} \right)^{-\Theta} m_{c_{t+\ell}}
\]
\[ D_t = E_t \sum_{\ell=0}^{\infty} \beta^\ell \omega^\ell \left( \frac{c_{t+\ell}}{c_t} \right)^{1-\frac{1}{\sigma}} \left( \frac{P_t}{P_{t+\ell}} \right)^{1-\Theta} \]

log-linearizing \( N_t \) and \( D_t \)

\[ \dot{D}_t = \omega \beta \left( (1 - \Theta) (\hat{p}_t - \hat{p}_{t+1}) + (1 - \frac{1}{\sigma}) (\hat{c}_{t+1} - \hat{c}_t) + \dot{D}_{t+1} \right) \]

\[ \hat{N}_t = \frac{mc}{N} \hat{m}c_t + \omega \beta \left( -\Theta (\hat{p}_t - \hat{p}_{t+1}) + \left( 1 - \frac{1}{\sigma} \right) (\hat{c}_{t+1} - \hat{c}_t) + \hat{N}_{t+1} \right) \]

with \( \hat{p}_t^* = \hat{N}_t - \hat{D}_t \) I obtain

\[ \hat{p}_t^* = (1 - \omega \beta) \hat{m}c_t + \omega \beta \left( \hat{p}_{t+1}^* + \pi_{t+1} \right) \] (101)

Additionally, the general price index in the formal sector is equal to:

\[ P_t = \left( \omega (P_{t-1})^{1-\Theta} + (1 - \omega) (P_t^*)^{1-\Theta} \right)^{\frac{1}{1-\Theta}} \]

\[ P_t^{1-\Theta} = \left( \omega (P_{t-1})^{1-\Theta} + (1 - \omega) (P_t^*)^{1-\Theta} \right). \]

dividing both sides by \( \frac{1}{P_t^{1-\Theta}} \)

\[ \frac{P_t^{1-\Theta}}{(P_{t-1})^{1-\Theta}} = \omega + (1 - \omega) \left( \frac{P_t^*}{P_{t-1}} \right) \frac{1}{P_t}. \]

log-linearizing around the steady-state

\[ \hat{\pi}_t = (1 - \omega) (\hat{p}_t^* + \hat{\pi}_t). \] (102)

replacing (101) into (102) I obtain

\[ \hat{\pi} = \kappa \hat{m}c_t + \beta E_t \hat{\pi}_{t+1} \]

with \( \kappa = \frac{(1-\omega_p)(1-\omega_p \Gamma)}{\omega_p} \)

**Appendix B8: wage inflation**

By definition, real wage inflation is equal to nominal formal wage inflation minus price inflation,
\[ \dot{w}_t^f = \dot{w}_{t-1}^f + \pi_{wt} - \pi_t \]

From equation (67) I have

\[ \dot{w}_t^f = \psi_o \dot{w}_t^o + \psi_1 E_t \left( \dot{w}_{t+1}^f + \pi_{t+1} \right) + \psi_2 \left( \dot{w}_{t-1}^f - \pi_t \right) \]

\[ \dot{w}_t^f - \dot{w}_{t-1}^f = \psi_o \left( \dot{w}_t^o - \dot{w}_{t-1}^o \right) + \psi_1 E_t \left( \dot{w}_{t+1}^f - \dot{w}_{t-1}^f + \pi_{t+1} \right) + \psi_2 \left( \dot{w}_{t-1}^f - \pi_{t-1} \right) \]

\[ \pi_{wt} - \pi_t = \psi_o \left( \dot{w}_t^o - \dot{w}_{t-1}^o \right) + \psi_1 E_t \left( \dot{w}_{t+1}^f - \dot{w}_{t-1}^f + \pi_{t+1} \right) + \psi_2 (\pi_{t-1}) \]

\[ \psi_2 \pi_{wt} = \psi_o \left( \dot{w}_t^o - \dot{w}_{t-1}^o \right) + \psi_1 E_t \left( \dot{w}_{t+1}^f - \dot{w}_{t-1}^f + \left( -\dot{w}_{t+1}^f + \dot{w}_{t+1}^f + \pi_{t+1} \right) \right) + \psi_2 (\pi_{t-1}) \left( \dot{w}_t^f - \dot{w}_{t-1}^f \right) \]

Since \( \psi_o + \psi_1 + \psi_2 = 1 \)

\[ \psi_2 \pi_{wt} = \psi_o \left( \dot{w}_t^o - \dot{w}_{t-1}^o \right) + \psi_1 E_t (\pi_{wt+1}) \]

\[ \pi_{wt} = \frac{\psi_o}{\psi_2} \left( \dot{w}_t^o - \dot{w}_{t-1}^o \right) + \frac{\psi_1}{\psi_2} E_t (\pi_{wt+1}), \]

where \( \psi_o = \frac{(1-\omega^w)}{\varsigma}, \psi_1 = \frac{(\psi - \pi_t \omega^w)}{\varsigma}, \) and \( \psi_2 = \frac{(\pi_{t+1} + \Psi) \omega^w}{\varsigma}. \)

Then \( \frac{\psi_0}{\psi_2} = \frac{1-\omega^w}{\omega^w (1+(1-\rho+\mu \tau^F) \omega^w \beta \phi)}, \) and \( \frac{\psi_1}{\psi_2} = \frac{(1-\rho \phi)}{1+(1-\rho+\mu \tau^F) \omega^w \beta \phi}. \)

**Appendix B9: Welfare loss function**

The second-order approximation of the welfare criterion

\[ U_t = E_t \sum_{i=1}^{\infty} \beta^i (\varepsilon_t), \]

\[ U_t = E_t \sum_{i=1}^{\infty} \beta^i \left( \frac{c_i}{1-\frac{1}{1-\tau}} - \left( l_i^f + l_i^l \right) \phi - \frac{\kappa}{2} \int_{0}^{l_i^f} F_i^2 l_i^f d_i \right), \]
\[ U_t = E_t \sum_{t=1}^{\infty} \beta^t \left( u_t(c_t) - \left( l^f_t + l^l_t \right) \varphi - \int_0^1 h_{cit} d_i \right). \]

We can then expand every function in the logarithm of its arguments around their steady-state levels,

\[ u_t(c_t) = u(c) (1 - \sigma^{-1}) \left( \hat{c}_t + \frac{1 - \sigma^{-1}}{2} \hat{c}_t^2 \right) + t.i.p \]

where \( t.i.p \) represents the terms independent of policy. By using \( u(c) (1 - \sigma^{-1}) = u'(c) c \) and \( c = s y \) I obtain

\[ u_t(c_t) = u'(c) \left( s \hat{c}_t + \frac{1 - \sigma^{-1}}{2} s \hat{c}_t^2 \right) + t.i.p \]

Similarly, I do the following approximation

\[ \left( l^f_t + l^l_t \right) \varphi = u'(c) c \left[ \frac{l^f l^f \varphi}{u'(c) c} \left( l^f_t + \frac{1}{2} \left( l^f_t \right)^2 \right) + \frac{l^f l^l \varphi}{u'(c) c} \left( l^l_t + \frac{1}{2} \left( l^l_t \right)^2 \right) \right] + t.i.p \]

In order to eliminate the linear terms in the previous equation, we need to approximate the aggregate resource constraint.

Individual hiring costs can be written as

\[ hc_{it} = \kappa \mathcal{F}^2_{it} l^f_t = hc \left[ 2 \hat{\mathcal{F}}_{it} + l^f_t + \frac{1}{2} \left( l^f_t \right)^2 + 2(2) \hat{\mathcal{F}}_{it} l^f_t \right] + t.i.p \]

employment in the formal sector \( l^f_t = \int l^f_{it} d_i \) and the average hiring rate \( \hat{\mathcal{F}}_t = \int \mathcal{F}_{it} l^f_{it} d_i \) can be approximated respectively by

\[ \hat{l}^f_t = E_l \hat{l}^f_{it} + \frac{1}{2} Var_{l^f_{it}} + t.i.p \]

\[ \hat{\mathcal{F}}_t = E_l \hat{\mathcal{F}}_{it} + \frac{1}{2} Var(\hat{\mathcal{F}}_{it}) + E_l l^f_{it} \hat{\mathcal{F}}_{it} - l^f_{it} \hat{\mathcal{F}}_{it} + t.i.p \]

where for any variable \( \varepsilon_{it}, E_l \varepsilon_{it} = \int_0^1 \varepsilon_{it} d_i \) and \( Var(l_{it}) = E_l (\varepsilon_{it} - E_l \varepsilon_{it})^2 \) denote its cross-sectional average and variance, respectively. I have also used the identity \( E_l (\hat{l}^f_{it})^2 = \)}
$Var_i \hat{t}_i + \left( E_i \hat{t}_i \right)^2$ and the fact that $\left( \hat{t}_i \right)^2 = \left( E_i \hat{t}_i \right)^2$ (and similarly for $\hat{F}_t$). On the other hand, the average hiring rate can also be written as $F_t = \frac{\hat{V}_t}{\hat{t}_i}$ which allows me to write $\hat{F}_t = \hat{\nu}_t - \hat{t}_i$

then

combining the previous three equations, the total hiring costs can be written as follows

$$\int \frac{p_t \kappa}{2} \hat{F}_t^2 \hat{t}_i \, dt = hc \left[ 2 \int \hat{F}_t \, dt + \int \hat{t}_i \, dt + \frac{1}{2} \int 2 \hat{F}_t^2 + 2 \left( 2 \hat{t}_i \right)^2 + 2 \left( 2 \hat{F}_t \hat{t}_i \right) \right] + t.i.p$$

$$= hc \left[ 2 \left( \hat{\nu}_t + \frac{1}{2} \left( Var \hat{F}_t + \hat{t}_i \right) \right) + \frac{1}{2} \left( 2 \hat{F}_t + \hat{t}_i \right)^2 \right] + t.i.p$$

with $\hat{F}_t = \hat{\nu}_t - \hat{t}_i$

$$\int_0^1 \frac{\kappa}{2} \hat{F}_t^2 \hat{t}_i d_t = u'(c)cs \left\{ \left( 2 \hat{\nu}_t - \hat{t}_i \right) + \frac{1}{2} \left( 2 \hat{\nu}_t - \hat{t}_i \right)^2 + 2 Var \hat{F}_t \right\} + t.i.p \quad (103)$$

$$\int_0^1 \frac{\kappa}{2} \hat{F}_t^2 \hat{t}_i d_t = u'(c)cs \left\{ \left( 2 \hat{\nu}_t - \hat{t}_i \right) + \frac{1}{2} \left( 2 \hat{\nu}_t - \hat{t}_i \right)^2 + 2 Var \hat{F}_t \right\} + t.i.p$$

where $s_v = \frac{hc}{u(c)c} = \frac{\kappa F^2 \hat{t}_i}{u(c)c}$ is the vacancy posting cost in consumption units as a fraction of GDP

therefore

$$U_t = u'(c) c \left( \hat{\nu}_t + \frac{1}{2} \frac{\sigma - 1}{\sigma} \hat{\nu}_t^2 - \hat{t}_i + \frac{2}{u(c)c} \left( \hat{t}_i + \frac{1}{2} \left( \hat{t}_i \right)^2 \right) - \frac{1}{2} \frac{\sigma - 1}{\sigma} \hat{\nu}_t^2 - \hat{t}_i + \frac{1}{2} \left( \hat{t}_i \right)^2 \right) + t.i.p$$

$$-u'(c)cs \left\{ \left( 2 \hat{\nu}_t - \hat{t}_i \right) + \frac{1}{2} \left( 2 \hat{\nu}_t - \hat{t}_i \right)^2 + 2 Var \hat{F}_t \right\} + t.i.p$$

we have $y_t = \Delta_t c_t$, then $\hat{y}_t = \Delta_t + \hat{c}_t$ and from the equilibrium in the intermediate good market

$$\left( \hat{t}_i + t.p.i \right) = \hat{y}_i$$

$$\left( \hat{t}_i \right) + t.p.i = \hat{y}_i$$

$\hat{y}_t = \Psi y_f \hat{y}_t + \Psi y_{y} \hat{y}_t = \Delta_t + \hat{c}_t$

$\hat{y}_t = \Psi y_f \hat{y}_t + \Psi y_{y} \hat{y}_t + t.i.p = \Delta_t + \hat{c}_t$

$$U_t = u'(c) c \left( \left( \Psi y_f - \hat{t}_i + \frac{2}{u(c)c} \hat{t}_i \right) \hat{y}_i + \left( \Psi y_{y} - \hat{t}_i + \frac{2}{u(c)c} \hat{t}_i \right) \hat{y}_i - \Delta_t + \frac{1}{2} \frac{\sigma - 1}{\sigma} \hat{y}_t^2 - \hat{t}_i + \frac{1}{2} \left( \hat{t}_i \right)^2 - \hat{t}_i + \frac{1}{2} \left( \hat{t}_i \right)^2 \right)$$

$$-u'(c)cs \left\{ \left( 2 \hat{\nu}_t - \hat{t}_i \right) + \frac{1}{2} \left( 2 \hat{\nu}_t - \hat{t}_i \right)^2 + 2 Var \hat{F}_t \right\} + t.i.p$$

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The Beveridge Curve and law of motion of the employment in the informal sector

In order to eliminate the linear terms in the previous equation, I perform the following second order approximation of the law of motion of employment in the formal and in the informal sector

\[
I_{t+1}^f = (1 - \rho) I_t^f + \Omega (I_t^u) (\nu_t)^{1-\mu}
\]

then

\[
\dot{I}_{t+1} + \frac{1}{2} (\dot{I}_{t+1})^2 = (1 - \rho) \left( \dot{I}_t + \frac{1}{2} (\dot{I}_t)^2 \right) + \rho \left[ \mu \dot{I}_t + (1 - \mu) \dot{\nu}_t + \frac{1}{2} (\mu \dot{I}_t + (1 - \mu) \dot{\nu}_t)^2 \right] + \sigma^3
\]

or

\[
\ddot{I}_t = -\frac{\nu}{t_1} \left( \dot{I}_t + \frac{1}{2} (\dot{I}_t)^2 \right) - \frac{\nu}{t_1} \left( \ddot{I}_t + \frac{1}{2} (\ddot{I}_t)^2 \right) - \frac{1}{2} (\dot{\nu}_t)^2 + \sigma^3
\]

replacing in

\[
\dot{I}_{t+1} + \frac{1}{2} (\dot{I}_{t+1})^2 = (1 - \rho) \left( \dot{I}_t + \frac{1}{2} (\dot{I}_t)^2 \right) + \rho \left[ \mu \dot{I}_t + (1 - \mu) \dot{\nu}_t + \frac{1}{2} (\mu \dot{I}_t + (1 - \mu) \dot{\nu}_t)^2 \right] + \sigma^3
\]

multiplying by \( \beta^t \) and iterating across \( t \)

\[
(\beta^{-1} - (1 - \rho)) \sum_{t=0}^{\infty} \beta^t \left( \dot{I}_t + \frac{1}{2} (\dot{I}_t)^2 \right) = \sum_{t=0}^{\infty} \beta^t \rho \left[ \mu \dot{I}_t + (1 - \mu) \dot{\nu}_t + \frac{1}{2} (\mu \dot{I}_t + (1 - \mu) \dot{\nu}_t)^2 \right] + \sigma^3
\]

Reorganizing

\[
(\beta^{-1} - (1 - \rho)) \sum_{t=0}^{\infty} \beta^t \left( \dot{I}_t + \frac{1}{2} (\dot{I}_t)^2 \right) = \sum_{t=0}^{\infty} \beta^t \rho \left[ \mu \dot{I}_t + (1 - \mu) \dot{\nu}_t + \frac{1}{2} (\mu \dot{I}_t + (1 - \mu) \dot{\nu}_t)^2 \right] + t.i.p
\]

\[
\sum_{t=0}^{\infty} \beta^t \left[ (\beta^{-1} - (1 - \rho)) \dot{I}_t - \rho \left( (1 - \mu) \dot{\nu}_t + \mu \dot{I}_t \right) \right] = \sum_{t=0}^{\infty} \beta^t \left[ \rho \left( \mu \dot{I}_t + (1 - \mu) \dot{\nu}_t \right)^2 - (\beta^{-1} - (1 - \rho)) (\dot{I}_t)^2 \right] + t.i.p
\]

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from the efficient job creation condition in the steady-state I have

\[(\beta^{-1} - (1 - \rho)) = (1 - \mu) \frac{\rho}{2} \left[ s_v^{-1} \left( \left( \frac{y}{y'} \right)^{\frac{1-\gamma}{\gamma}} - \left( \frac{y}{y'} \right)^{\frac{1-\gamma}{\gamma}} \frac{\mu}{\mu} \right) \right] + 1 \]

\[(1 - \alpha) \left( \frac{y}{y'} \right)^{\frac{1-\gamma}{\gamma}} \frac{\mu}{\mu} = 2s_v \frac{\mu}{(1 - \mu)} \]

combining the two following equations

\[(\beta^{-1} - (1 - \rho)) = (1 - \mu) \frac{\rho}{2} \left[ s_v^{-1} \left( \left( \frac{y}{y'} \right)^{\frac{1-\gamma}{\gamma}} - \left( \frac{y}{y'} \right)^{\frac{1-\gamma}{\gamma}} \frac{\mu}{\mu} \right) \right] + 1 \]

\[
\sum_{t=0}^{\infty} \beta^t \left[ (\beta^{-1} - (1 - \rho)) l_t^2 - \rho \left( (1 - \mu) \hat{v}_t + \mu \hat{v}_t^2 \right) \right] = \sum_{t=0}^{\infty} \beta^t \frac{1}{2} \left[ \rho \left( \mu \hat{v}_t^2 + (1 - \mu) \hat{v}_t \right) - (\beta^{-1} - (1 - \rho)) (l_t^2)^2 \right] + t.i.p
\]

I obtain

\[
\sum_{t=0}^{\infty} \beta^t \frac{1}{2} \left[ s_v^{-1} \left( \left( \frac{y}{y'} \right)^{\frac{1-\gamma}{\gamma}} - \left( \frac{y}{y'} \right)^{\frac{1-\gamma}{\gamma}} \frac{\mu}{\mu} \right) \right] l_t^2 - 2 \left( \hat{v}_t + \frac{\mu}{(1 - \mu)} \hat{v}_t^2 \right) \]

\[
= \sum_{t=0}^{\infty} \beta^t \frac{1}{2} \left( \mu \hat{v}_t^2 + (1 - \mu) \hat{v}_t \right)^2 - \frac{1}{2} \left[ s_v^{-1} \left( \left( \frac{y}{y'} \right)^{\frac{1-\gamma}{\gamma}} - \left( \frac{y}{y'} \right)^{\frac{1-\gamma}{\gamma}} \frac{\mu}{\mu} \right) \right] l_t^2 + t.i.p
\]

and combining with

\[
\sum_{t=1}^{\infty} \beta^t U_t = u'(c) cs \left\{ -\triangle \right\}
\]

\[- \sum_{t=1}^{\infty} \beta^t u'(c) cs \left\{ - (1 - \sigma^{-1}) \hat{g}_t^2 + (\Psi_{yt}) (l_t^2)^2 + (\Psi_{iy}) (l_t^2)^2 \right\}
\]

\[- \sum_{t=1}^{\infty} \beta^t u'(c) cs \left\{ \frac{\mu (\theta_t)^2 + (\mathcal{F}_t)^2 + Var \mathcal{F}_t} {Var \mathcal{F}_t} \right\} + t.i.p \]

then, reorganizing I have

\[
\sum_{t=1}^{\infty} \beta^t U_t = \sum_{t=1}^{\infty} \beta^t u'(c) cs \left\{ \frac{-2\hat{\triangle} - (\sigma^{-1} - 1) \hat{g}_t^2 - 2s_v \left[ 2\hat{\mu}^2 + \mathcal{F}_t^2 + Var \mathcal{F}_t \right]} {2} \right\}
\]

\[- (\Psi_{iy}) (l_t^2)^2 + t.i.p \]

**Price dispersion and inflation**

The second order Taylor expansion of \( \triangle = \int_{0}^{1} \left( \frac{p_{it}}{p_{i}} \right)^{-\Theta} dj \) writes:

\[
\hat{\triangle} + \frac{1}{2} \hat{\triangle}^2 = -\Theta \left( E_j \hat{p}_{jt} - \frac{\Theta}{2} E_j (\hat{p}_{jt})^2 \right) + \Theta^3
\]

where

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\[ \hat{p}_{jt} = \log \left( \frac{p_{jt}}{p_j} \right) \] and we have \( \Delta = 1. \Delta_t \) is proportional to the cross-sectional variance of relative prices. Therefore, \( \Delta_t \approx \frac{\Theta}{2} \var_i \{ p_t(i) \} \)

In Woodford (2003, chapter 6) is proved that

\[
\sum_{t=0}^{\infty} \beta^t \var_i \{ p_t(i) \} = \frac{\omega}{(1 - \beta \omega)(1 - \omega)} \sum_{t=0}^{\infty} \beta^t \pi_t^2
\]

then

\[
\sum_{t=0}^{\infty} \beta^t \Delta_t \approx \sum_{t=0}^{\infty} \beta^t \frac{\Theta}{2} \var_i \{ p_t(i) \}
\]

\[
\sum_{t=0}^{\infty} \beta^t \Delta_t \approx \sum_{t=0}^{\infty} \beta^t \frac{\Theta}{2} \var_i \{ p_t(i) \}
\]

with \( \Upsilon = \frac{(1 - \beta \omega)(1 - \omega)}{\omega} \) we can express \( U_t \) as follows

\[
\sum_{t=1}^{\infty} \beta^t U_t = \sum_{t=1}^{\infty} \beta^t \var_i \log \left( w_{it}^f \right) = \omega w \var_i \var \left( w_{it-1}^f \right) + \frac{\omega^w}{1 - \omega^w} \pi_{wt}^2
\] (105)

**Wage inflation and hiring rate dispersion**

Analogously, the cross-sectional variance of nominal wages can be approximated by

\[
\var_i \log \left( w_{it}^f \right) = \omega w \var_i \log \left( w_{it-1}^f \right) + \frac{\omega^w}{1 - \omega^w} \pi_{wt}^2
\] (106)

Multiplying (106) by \( \beta^t \) integrating forward and using the fact that \( \var_i \log \left( w_{it-1}^f \right) \) is independent of policy as of time 0, I obtain

\[
\sum_{t=0}^{\infty} \beta^t \var_i \log \left( w_{it}^f \right) = \frac{\omega w}{(1 - \omega) (1 - \beta \omega w)} \sum_{t=0}^{\infty} \beta^t \pi_{wt}^2 + t.i.p
\]

By using Lemma 1 in Thomas (2008) I found that

\[
\var_i \var \left( F_{it} \right) = h^2 \var_i \log \left( w_{it}^f \right)
\]

where \( h = \frac{\beta \omega^w s_w}{(1 - \beta \omega^w)^2 s_v} \), \( s_w = \frac{t f}{y} \) is the steady state formal labor income share, \( s_v \) is the steady state ratio of vacancy posting cost (in consumption units) to output

then it is possible to write

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\[
\sum_{t=0}^{\infty} \beta^t \text{Var}_t \hat{X}_{it} = \frac{h^2 \omega^w}{(1 - \omega^w)(1 - \beta \omega^w)} \sum_{t=0}^{\infty} \beta^t \pi_{wt}^2 + t.i.p \tag{107}
\]

\[
\sum_{t=0}^{\infty} \beta^t \text{Var}_t \hat{X}_{it} = \frac{h^2}{\Upsilon_w} \sum_{t=0}^{\infty} \beta^t \pi_{wt}^2 + t.i.p \tag{108}
\]

with \(\Upsilon_w = \frac{(1 - \omega^w)(1 - \beta \omega^w)}{\omega^w}\)

finally inserting (107) into (105)

\[
\sum_{t=1}^{\infty} \beta^t U_t = \sum_{t=1}^{\infty} \beta^t \frac{u'(c) c}{2} \left\{ -\frac{\pi_t^2}{\gamma} - \left(\sigma - 1\right) \tilde{y}_t^2 - 2s_v \left[ \mu \beta_t^2 + \mathcal{F}^2 + \frac{h^2}{\omega^w} \pi_{wt}^2 \right] - \left(\Psi_yf\right) \left(\dot{l}_t^2\right)^2 - \left(\Psi_yi\right) \left(\dot{t}_i\right)^2 + t.i.p \right\}
\]

\[
\sum_{t=1}^{\infty} \beta^t U_t = \sum_{t=1}^{\infty} \beta^t \frac{u'(c) c}{2} \left\{ -\Psi_x \pi_t^2 - \Psi_{\pi w} \pi_{wt}^2 - \left(\sigma - 1\right) \tilde{y}_t^2 - 2s_v \left[ \mu \beta_t^2 + \mathcal{F}^2 \right] - \left(\Psi_yf\right) \left(\dot{l}_t^2\right)^2 - \left(\Psi_yi\right) \left(\dot{t}_i\right)^2 + t.i.p \right\}
\]

\[
\sum_{t=1}^{\infty} \beta^t U_t = -\sum_{t=1}^{\infty} \beta^t \frac{u'(c) c}{2} \left\{ \Psi_x \pi_t^2 + \Psi_{\pi w} \pi_{wt}^2 + \left(\sigma - 1\right) \tilde{y}_t^2 + 2s_v \left[ \mu \beta_t^2 + \mathcal{F}^2 \right] - \left(\Psi_yf\right) \left(\dot{l}_t^2\right)^2 + t.i.p \right\}
\]

where \(\Psi_x = \frac{\pi_t}{\gamma}\) and \(\Psi_{\pi w} = s_v \frac{h^2}{\Upsilon_w}, \Psi_yf = \left(\frac{\hat{y}}{\gamma}\right)^{\frac{1}{1-\gamma}}, \Psi_yi = \left(\frac{\hat{y}}{\gamma}\right)^{\frac{2-\gamma}{1-\gamma}}\)

\[
\sum_{t=0}^{\infty} \beta^t U_t = -\sum_{t=0}^{\infty} \beta^t \frac{u'(c) c}{2} L_t + t.i.p
\]

with

\[
L_t = \Psi_x \pi_t^2 + \Psi_{\pi w} \pi_{wt}^2 + \mathcal{L}_t^{1,h}
\]

\[
\mathcal{L}_t^{1,h} = (\sigma - 1) \tilde{y}_t^2 + 2s_v \left[ \mu \beta_t^2 + \mathcal{F}^2 \right] + \Psi_yf \left(\dot{l}_t^2\right)^2 + \Psi_yi \left(\dot{t}_i\right)^2.
\]

2.8 References


3 Job Polarization and the Informal Labor Market

Abstract

This paper analyses the incidence of job polarization in developing and emerging countries, where a substantial fraction of the urban labor force works in the informal sector. I build a general equilibrium model with informality and endogenous occupational choice. Workers in the informal sector do not pay taxes, are less productive, and have the same ability to perform manual tasks. The analytical solution of the model shows that job polarization, driven by a Routine-Biased Technological Change (RBTC), increases the size of the informal sector. It also could lead to a decrease in the share of employment in the informal sector and a reduction in the wage inequality at the bottom of the skill distribution.

Keywords: Informality, Job polarization, Technological change, Wage distribution

JEL classification: E26, J24, J31, J46.

3.1 Introduction

The polarization of employment in industrialized countries has been a widely studied phenomenon over the last couple of decades. It documents a simultaneous growth in employment and wages of high-skill (problem-solving, creativity, situational adaptability, and in-person interactions) occupations and low-skill (personal services) occupations, compared to middle-skill (production, clerical, and sales) occupations. The main explanation about the drivers of job polarization is the Routine-Biased Technological Change (RBTC) hypothesis, first introduced by Autor et al. (2003), which suggests that technological progress tends to substitute for workers who operate routine tasks. Simultaneously, it increases the relative demand for workers who perform complementary non-routine tasks (abstract and manual tasks). According to Acemoglu and Autor (2011) routine tasks are characteristic of many middle-skilled cognitive and manual jobs (such as bookkeeping, clerical work, repetitive production, and monitoring jobs). Considering that the core job tasks of these occupations follow precise and well-understood procedures, they can be codified in computer software and performed by machines (or they can be sent electronically outsourced to foreign worksites). As a result, this process of automation and offshoring of routine tasks raises relative demand for workers
who can perform a complementary non-routine task, i.e. problem-solving, situational adaptability, creativity, and in-person interactions.

Additional to the routinization hypothesis, some authors have also pointed to offshoring as a driver of job polarization. Blinder (2009) states that since many tasks can be automated, they can also be suitable for offshoring to a low-cost producer in a different country, without a deterioration in quality. As a result, international trade in routine tasks may also have a polarizing effect on the labor market of countries that engage in offshoring (Reijnders and de Vries, 2018). Some studies (Goos et al. 2014, Michaels et al. 2014, and Reijnders and de Vries, 2018) use cross-country empirical data to quantify the importance of RBTC and offshoring in explaining job polarization, find that RBTC drives job polarization in almost all countries, with offshoring having a much less significant impact. However, recent detailed country studies, Author et. al, (2015) for the US and Keller and Utar (2016) for Denmark, find that local labor markets with greater exposure to trade competition experience large declines in manufacturing employment.

A sizable body of literature dealing with polarization of the labor market (see Autor et al. 2003, Autor et al. 2006, Autor and Dorn 2013, Goos et al. 2014, Michaels et al. 2014, Feng and Graetz 2015) focus on analyzing this phenomenon in developed countries, specifically in the United States and several European countries. However, the phenomenon of job polarization is not limited to developed countries. According to the World Development Report (2016), there are signs that employment is also polarizing in several low and middle-income countries. This study finds that the average decline in the share of routine employment has been 7.8 percentage points for the period 1995-2012. Reijnders and de Vries (2018) also find evidence of an increase in the share of non-routine jobs in total employment for a group of advanced and major emerging countries during the period 1999-2007. They find that for all these countries, technological change was the main force behind employment changes.

One of the main features of labor markets in emerging economies is the existence of a large informal sector\textsuperscript{17}. Informality refers to activities that are outside the regulatory frameworks, most workers in this sector are self-employed, and their income comes from operating small unincorporated enterprises. These include activities such as trading on

\textsuperscript{17}Informal employment accounts for more than half of non-agricultural employment in most developing countries: around 72 percent in Africa, 63 percent in Asia and the Pacific, 64 percent in the Arab States, 50 percent in Latin America, and 30 percent in Europe and Central Asia. In the case of developed countries, only 17 percent of the urban labor force is employed in informal activities (International Labor Office, 2018).
the streets or in markets; sales of cooked food from kiosks; the transport of people or goods by pedal-power or motorbikes; repairing clothes, shoes, or motor scooters; dwelling construction or adding extensions to them; scavenge for reusable waste; or providing a range of personal services like hairdressing, shoe cleaning, street theater, house cleaning, and the like (Blades et al. 2011). In sum, the informal sector can be described as a labor-intensive sector with poor working conditions and relatively lower productivity compared with the formal economy.

The informal sector contributes significantly to employment creation, production, and income generation in developing countries. Nevertheless, a large informal sector has negative consequences for competitiveness and growth and may also be the source of further economic retardation (Loayza 1996, Loayza, Oviedo, and Serven 2004). But on the other side, empirical evidence also shows that as countries become richer, the share of informal employment falls (see La Porta and Shleifer 2014, Docquier et. al 2017). Elgin and Birinci (2016) use a panel dataset of 161 advanced and emerging market economies over the period from 1950 to 2010 and find an inverted-U relationship between informal sector size and growth of GDP per capita. This result implies that small and large sizes of the informal economy are associated with little growth and medium levels of the size of the informal economy are associated with higher levels of economic growth. As a result, the relationship between informality and growth is not linear and it depends on the stage of development and characteristics of each country. Similarly, Wu and Schneider (2019) use a dataset of 158 countries over the period from 1996 to 2015. They find a robust U-shaped relationship between the shadow economy size and GDP per capita.

Since the job polarization process involves a significant reallocation and mobility of workers in the labor market, it is very likely to produce different performances in the presence of a large informal sector. According to the ILO (2018b) “technology is likely to have both positive and negative effects on informality. Productivity and economic structure influence the income distribution and labor outcomes, including informality. Technology can increase informality via the probable increase in the productivity gaps among economic units, especially when access to technologies is not equal, or via the spreading of new forms of work, especially those where informality is higher. But the question is, can it also reduce informality? ” There is not a consensus and enough evi-

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18 On average, economies with larger informal sectors tend to have lower productivity, slower physical and human capital accumulation, higher poverty and inequality, and smaller fiscal resources (World Bank, 2019)
dence about the implications of job polarization on informality. More studies, empirical and theoretical, would be required to analyze the effect of job polarization on informal employment and wages.

This paper contributes to this literature by analyzing the incidence of job polarization in developing countries on informality, using a structural model. I develop a general equilibrium model with informality and endogenous occupational choice, based on Autor and Dorn (2013). I consider a labor market in which some workers are low-skill while others are high-skill. I assume that there are three sectors in this economy: the goods sector uses capital and employs workers to perform abstract and routine tasks; the formal service sector employs workers to perform manual tasks, and; the informal service sector employs workers also to perform manual tasks. Workers in the informal service sector can avoid taxation, but are less productive.

A key feature of the model is that households can produce services in the informal sector, which are substitutes for services produced in the formal sector, and complement for goods. Additionally, each worker is characterized by a set of skills in performing abstract, routine, and manual tasks. High-skill workers only perform abstract tasks, and low-skill workers can perform both routine and manual tasks. I assume that low-skill workers have the same ability to accomplish manual tasks in the informal sector, while they are heterogeneous in their ability to perform a routine task or a manual task in the formal sector. This feature implies that workers moving from the goods sector to the formal service sector can keep some of their abilities, while the ones moving to the informal sector will have the same ability as all informal workers.

Additionally, the lack of taxation in the informal sector leads to an inefficient reallocation of employment between the formal and informal service sectors. At the same time, it allows the fiscal policy to have an asymmetric effect on the reallocation of labor between these two sectors.

The analytical solution (asymptotic solution) of the model shows that when the elasticity of substitution between capital and routine labor is higher than the elasticity of substitution between goods and services, the constant decrease in prices of automating routine tasks eventually causes low-skill labor flows from routine tasks to manual tasks. In this case, Routine-biased technological change (RBTC), affecting mainly the production of goods, can increase aggregate demand for services and eventually increase

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19 Since informal sector is labor-intensive, I assume that informal workers are employed in manual task occupations.
employment and wages in service occupations. Additionally, when goods and services are complements, wages also polarize. These conditions for job and wage polarization are the same found by Autor and Dorn (2013), in the case without informality.

I find that employment and wages in both the formal and the informal service sectors increase due to the increased demand for services. The allocation of labor in the service sector depends on the level of labor income taxes, the degree of substitution between the two types of services, and the level of efficiency in the formal service sector. I find that the relative units of efficient labor in the informal sector (as well as the share of informal employment in the service sector) decrease with technological progress. This result is driven by the assumption that workers, previously working in routine tasks, still can use some of their skills when they move to the formal service sector, while all workers in the informal sector have the same ability to accomplish manual tasks. Therefore, the increasing demand for labor in the informal service sector requires a higher increase in wages to compensate for the loss of skills from working in this sector. As a consequence, relative wages in the informal sector increase, as well as their relative prices. It follows that the increase in relative prices of informal services decreases their relative demand and therefore the relative demand for workers in the service sector.

The remainder of the chapter is organized as follows. Section 3.2 presents the model. Section 3.3 derives the analytical solution of the model for the labor allocation. Section 3.4 derives the analytical solution of the model for the relative wages. Section 3.5 presents the results of the model simulations, and Section 3.6 concludes.

### 3.2 Model

In this section, I develop a general equilibrium model with three sectors and endogenous occupational choice based on Autor and Dorn (2013). The goods sector employs high-skilled workers in abstract jobs $L_a$, low-skill workers in routine tasks $L_r$, and Capital $K$. The service sector is composed of two sectors: formal and informal, which use only unskilled labor in manual tasks, $L_{sf}$ and $L_{si}$ respectively. I assume that high-skilled workers have homogeneous skills at performing abstract tasks, while their skills are heterogeneous in performing routine tasks and manual tasks in the formal sector. As in Autor and Dorn (2013), Routine-biased technological change is modeled as an exogenous fall in the price of capital $P_k$.  

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3.2.1 The goods sector

The goods sector is perfectly competitive and uses abstract tasks $L_a$, routine tasks $L_r$, and capital $K$ to produce $Y_g$ units of goods. As in Autor and Dorn (2013), I assume the good sector uses the following technology:

$$Y_g = A_g (L_a)^{(1-\beta)} X^\beta = A_g (L_a)^{(1-\beta)} \left[ ((1 - \mu_k) L_r)^\nu + (\mu_k K)^\nu \right]^{\frac{\beta}{\nu}}, \quad (109)$$

with $\mu_k$ the capital input share, and $\beta \in (0, 1)$ and $\nu \in (0, 1)$. From equation (109), the elasticity of substitution between abstract labor and the total routine task is equal to 1, while the elasticity of substitution between routine labor and computer capital is $\frac{1}{1-\nu} > 1$. As a result, $K$ is a relative complement to abstract labor and a relative substitute for routine labor.

Firms in the goods sector solve the following maximization problem:

$$\max \Pi^G_t = P_g A_g L_a^{1-\beta} X^\beta - P_k K - w_r L_r - w_a L_a. \quad (110)$$

The first-order conditions for problem (110) with respect to abstract labor $L_a$, routine labor $L_r$, and capital $K$ respectively are given by

$$P_g (1 - \beta) \frac{Y_g}{L_a} = w_a, \quad (111)$$

$$\kappa_R \frac{Y_g}{L_r} \left( \frac{L_r}{X} \right)^\nu = w_r, \quad (112)$$

$$\kappa_k \frac{Y_g}{K} \left( \frac{K}{X} \right)^\nu = P_k, \quad (113)$$

where $\kappa_R = \beta \mu_r^\nu$ and $\kappa_k = \beta \mu_k^\nu$. I have normalized $P_g = 1$.

3.2.2 The service sector

The service sector uses manual tasks to produce services in a competitive environment. There are two types of firms in this sector: formal and informal. Workers and firms in the formal service sector have to pay labor income taxes, but due to the better
employment conditions workers are more productive. Workers in the informal sector can avoid taxes but are less productive.

**Formal service sector**

The formal service sector uses only manual labor as input. The production function writes:

\[ Y_{sf} = A_{sf} L_{sf}, \]

where \( L_{sf} \) is the total efficient units of manual labor employed in the formal service sector, and \( A_{sf} \) is the common labor productivity in this sector.

Firms solve the following maximization problem:

\[ \max \Pi_{sf} = P_{sf} Y_{sf} - w_{sf} L_{sf}, \]

In equilibrium, wage per efficiency unit of labor in the formal service sector is equal to their marginal productivity:

\[ w_{sf} = P_{sf} A_{sf}. \]  

Since workers in the formal service sector differs in their skill to perform manual task activities, the wage of a worker with \( \eta_{i}^{\theta} \) efficiency units of manual labor is \( \eta_{i}^{\theta} w_{sf}. \)

**Informal service sector**

The informal sector is labor-intensive, and uses only manual labor as input. The total amount of informal services produced in the economy is:

\[ Y_{si} = A_{si} L_{si}, \]

where \( L_{si} \) is the total units of low-skill labor employed in the informal service sector, and \( A_{si} \) is the labor productivity in this sector. With \( A_{si} < A_{sf} \). I assume that each worker is equally talented in providing low-skilled informal services.
The profit maximization problem writes:

\[
\text{Max } \Pi_{si} = P_{si}Y_{si} - w_{si}L_{si}.
\]

The first-order condition with respect to \( L_{si} \) implies:

\[
w_{si} = P_{si}A_{si}.
\] (115)

Note that since everyone has the same skill in this sector, everyone working in the low-skilled informal service sector earns the same wage.

### 3.2.3 Capital

Capital is produced and supplied in a competitive framework. As in Autor and Dorn (2013), The production technology of capital is described by

\[
K = Y_k \frac{e^{\delta_k t}}{\Theta},
\] (116)

where \( Y_k \) is the amount of final goods used to produce capital, \( \delta_k > 0 \), and \( \Theta = e^{\delta_k} \) is an efficiency term. Capital fully depreciates at each period. \( \delta_k \) represents the growth rate of capital productivity, and \( t \) represents the period of time.

The price of capital is equal to its marginal cost.

\[
P_k = \frac{Y_k}{K} = e^{-\delta_k(t-1)}.
\] (117)

Note that as time passes, the price of capital falls to zero asymptotically.

### 3.2.4 Occupational choice

I assume that every member of the household works full-time in one of the three market sectors. Low-skill workers are heterogeneous in their endowment of efficiency units of labor \( \eta \), which is drawn from a time-invariant distribution \( f(\eta) \). The endowment \( \eta \) determines the productivity of each individual in each sector. I assume that \( \eta \) denotes
the worker’s efficiency units of labor in routine tasks, while $\eta^\theta$ denotes the worker’s efficiency units of labor in manual tasks in the formal service sector. Workers in the informal service sector have homogeneous skills in performing manual tasks, each worker in this sector supply a unit mass of manual labor. This assumption implies that a worker with endowment $\eta$ has individual productivity, measured in efficiency units, of $\eta$ performing routine tasks, $\eta^\theta$ performing manual tasks in the formal service sector, with $\theta \in (0, 1)$, and 1 performing manual tasks in the informal service sector. Note that workers with an endowment $\eta > 1$ are more efficient in the formal service sector than in the informal service sector. This assumption is motivated by the fact that most of the workers in the formal sector work in larger companies with better working conditions, and in most cases, they have access to training programs. While workers in the informal sector are self-employed. Therefore, it is realistic to assume that some workers are more skilled performing manual task activities in the formal sector than in the informal sector.

I assume that workers in the formal service sector (goods and formal services) have to pay labor income taxes, while workers in the informal sector can avoid taxation. Since any low-skill worker can work in any of the three sectors, it is optimal for each worker to choose the type of work and the sector that provides her with the highest wage. Therefore, it is optimal for an individual $i$ endowed with $\eta_i$ efficiency units of labor to work in the goods sector only if:

$$\eta_i(1 - \tau)w_r \geq \max \left[ \eta_i^\theta(1 - \tau)w_{sf}, w_{si} \right].$$

$$\eta^* = \left( \frac{w_{sf}}{w_r} \right)^{\frac{1}{1-\theta}}.$$  \hspace{1cm} (118)

Workers whose efficiency level is lower than $\eta^*$ have to decide whether to work in the formal service sector or the informal service sector. Then, it is optimal for an individual to work in the formal service sector only if:

$$\eta_i^\theta(1 - \tau)w_{sf} \geq w_{si}.$$
\[ \eta^{**} = \left( \frac{w_{si}}{(1 - \tau)w_{sf}} \right)^{\frac{1}{\pi}}. \tag{119} \]

Figure 3.1 shows this endogenous occupational choice. Low-skill workers whose efficiency units of labor are higher than \( \eta^* \) sort themselves into the goods to perform routine tasks, and those with efficiency units of labor lower than \( \eta^{**} \) sort themselves into the informal service sector.

Figure 3.1. Optimal Labor Choice

I assume that \( \eta \) is distributed Uniform on the interval \([0, \eta^{max}]\), with density and distribution functions \( F(\eta) \) and \( f(\eta) \) defined as follows:

\[
F(\eta) = \begin{cases} 
\frac{\eta}{\eta^{max}} & 0 \leq \eta \leq \eta^{max} \\
1 & \eta > \eta^{max}
\end{cases}
\tag{120}
\]

\[
f(\eta) = \begin{cases} 
\frac{1}{\eta^{max}} & 0 \leq \eta \leq \eta^{max} \\
0 & \eta < 0 \text{ or } \eta > \eta^{max}
\end{cases}
\]

The endogenous occupational choice of low-skill workers determines the effective labor supply in each sector. The aggregate efficiency units supplied to the routine tasks,
manual tasks in the formal and informal service sector can be written, respectively, as follows:

\[
L_r = \int_{\eta^*}^{\eta_{\text{max}}} \frac{\eta}{\eta_{\text{max}}} d\eta = \left[ \frac{(\eta_{\text{max}})^2}{2} - \frac{(\eta^*)^2}{2\eta_{\text{max}}} \right],
\]

\[
L_{sf} = \int_{\eta^{**}}^{\eta^*} \frac{\eta^\theta}{\eta_{\text{max}}} d\eta = \left[ \frac{(\eta^*)^{\theta+1}}{(\theta + 1) \eta_{\text{max}}} - \frac{(\eta^{**})^{\theta+1}}{(\theta + 1) \eta_{\text{max}}} \right],
\]

\[
L_{si} = \int_{0}^{\eta^{**}} \frac{1}{\eta_{\text{max}}} d\eta = \frac{\eta^{**}}{\eta_{\text{max}}},
\]

### 3.2.5 Households

I assume a representative household, whose members derive utility from the consumption of goods and formal and informal services. The household collects the wages of all its members and allocate total income to maximize the following utility function:

\[
U = \ln \left( \left[ \gamma_g C_g^\rho + \gamma_s C_s^\rho \right]^{\frac{1}{\rho}} \right),
\]

with

\[
C_s = \left( a_f C_{sf}^\psi + a_i C_{si}^\psi \right)^{\frac{1}{\psi}}
\]

subject to the budget constraint.

\[
C_g + P_{sf} C_{sf} + P_{si} C_{si} = (1 - \tau) \left( w_a L^a + w_r L^r + w_{sf} L_{sf} \right) + w_{si} L_{si} + T.
\]

The elasticity of substitution between \( C_g \) and \( C_s \) is equal to \( \sigma_c = \frac{1}{1-\rho} \). The elasticity of substitution between \( C_{sf} \) and \( C_{si} \) is equal to \( \sigma_s = \frac{1}{1-\psi} \). I assume that goods and services are complements, \( \sigma_c < 1 \), while formal and informal services are substitutes, \( \sigma_s > 1 \).

The First order conditions of this maximization problem are as follows:

\[
\frac{C_g}{C_s} = \left( C_{sf}^{1-\psi} \frac{\gamma_g}{\psi a_f \gamma_s} \frac{P_{sf}}{P_g} \right)^{\sigma_c}
\]
\[
\frac{C_{si}}{C_{sf}} = \left( \frac{a_i}{a_f} \frac{P_{sf}}{P_{si}} \right)^{\sigma_s}.
\] (127)

The left-hand side of the equation (126) represents the relative supply, while the right-hand side is the relative demand for goods compared to services. In the same way, equation (127) equals the relative supply of informal services, compared to formal services, with its relative demand.

### 3.2.6 Government

Government always runs a balanced budget. Therefore, in each period, government budget constraint is as follows:

\[
T = \tau (w_a L_a + w_r L_r + w_{sf} L_{sf}).
\] (128)

### 3.2.7 Clearing conditions

\[
C_g = Y^g - P_k K
\] (129)
\[
C_{sf} = Y_{sf}
\] (130)
\[
C_{si} = Y_{si}
\] (131)

### 3.3 Asymptotic Labor Allocation

In this section, I determine the log run allocation (asymptotic equilibrium) of low-skill workers in the goods, formal and informal service sectors. Given that the price of computer capital \(P_k(t)\) converges to zero asymptotically, computer capital converges to infinity:

\[
\lim_{t \to \infty} K(t) = \infty.
\] (132)
Since the maximum value of $L_r$ is $\left(\eta_{\text{max}}\right)^2$, the production of $X$ will be asymptotically determined by the capital level ($X \sim \alpha_kK$). It implies that:

$$\lim_{t\to\infty} \frac{X}{\mu_r} = 1,$$  \hspace{1cm} (133)

and

$$Y_g \sim (\mu_kK)^{\beta}.$$  \hspace{1cm} (134)

Replacing equation (134) into equation (129), and using equations (132) and (133), I show in Appendix C1, that the solution for the asymptotic supply of low-skilled labor in the goods and formal and informal service sector is as follows:

$$L_r = \begin{cases} 
0 & \text{if } \frac{1}{\sigma_c} > \frac{\beta - \nu}{\beta}, \\
\bar{L}_r & \text{if } \frac{1}{\sigma_c} = \frac{\beta - \nu}{\beta}, \\
\eta_{\text{max}}^2 & \text{if } \frac{1}{\sigma_c} < \frac{\beta - \nu}{\beta}
\end{cases}, \hspace{1cm} (135)$$

where $\bar{L}_r = gg(\bar{L}_{si})$.

$$L_{si} = \begin{cases} 
\hat{L}_{si} & \text{if } \frac{1}{\sigma_c} > \frac{\beta - \nu}{\beta}, \\
\bar{L}_{si} & \text{if } \frac{1}{\sigma_c} = \frac{\beta - \nu}{\beta}, \\
0 & \text{if } \frac{1}{\sigma_c} < \frac{\beta - \nu}{\beta}
\end{cases}, \hspace{1cm} (136)$$

$$L_{sf} = \begin{cases} 
\hat{L}_{sf} & \text{if } \frac{1}{\sigma_c} > \frac{\beta - \nu}{\beta}, \\
\bar{L}_{sf} & \text{if } \frac{1}{\sigma_c} = \frac{\beta - \nu}{\beta}, \\
0 & \text{if } \frac{1}{\sigma_c} < \frac{\beta - \nu}{\beta}
\end{cases} = \Psi, \hspace{1cm} (137)$$

where $\hat{L}_{sf} = \left(\frac{L_{si}}{C_{11}}\right)^{\frac{1+\theta-\psi}{1-\nu}}$ and $\bar{L}_{sf} = \left(\frac{L_{si}}{C_{11}}\right)^{\frac{1+\theta-\psi}{1-\nu}}$. $C_{11} = \left(\frac{A_{sf}}{\lambda_{sf}}\right)^{\psi} \frac{a_{g}}{a_{f}(\eta_{\text{max}})^{\psi}}(1-\tau)$.

Equations (136), (137), and (135) show that, as in Autor and Dorn (2013), the allocation of low-skill labor between manual tasks and routine tasks depends on the relative
magnitudes of the consumption and production elasticities, scaled by the share of the routine aggregate in goods production ($\beta$). When $\frac{1}{\sigma_c} > \frac{\beta - \nu}{\beta}$ (the production elasticity, scaled by $\beta$, exceeds the consumption elasticity) the demand for routine labor decreases with technological progress, and the relative demand for low-skill labor in both the formal and the informal service sector increases. Hence, the constant decrease in prices of automating routine tasks eventually causes all low-skill labor to flow from routine tasks to manual tasks. As a result, employment in the formal and informal service sectors increases.

The allocation of labor between the formal and the informal service sector is determined by the equation $L_{sf} = \left(\frac{A_{si}}{A_{sf}}\right)^{\frac{\psi \left(1+\psi\right)}{\left(\psi\right)^2}}$. It depends on the differences in productivity ($A_{si}$ and $A_{sf}$), the level of labor income taxes paid by formal workers $\tau$, consumer preferences ($\psi$, $a_f$, and $a_i$), and efficiency of labor in the formal sector ($\eta_{max}^\theta$). The higher the aggregate labor productivity and efficiency in the formal service sector, and the lower the labor income taxes, the higher the relative allocation of labor in the formal service sector.

Replacing equations (132) and (133) into equation (127), I have that the evolution of the ratio between the efficient units of labor in the informal and the formal sector depends exclusively on the evolution of the wage ratio between the two sectors.

$$\frac{L_{si}}{L_{sf}} = \left(\frac{A_{si}}{A_{sf}}\right)^{\frac{\psi \left(1+\psi\right)}{\left(\psi\right)^2}} \left(\frac{a_i}{a_f} \frac{w_{sf}}{w_{si}}\right)^{\sigma_s}. \quad (138)$$

Additionally, from equation (119) the wage ratio $\frac{w_{si}}{w_{sf}}$ can be expressed as follows:

$$\frac{w_{si}}{w_{sf}} = (\eta^*)^\theta (1 - \tau). \quad (139)$$

The wage ratio between the formal and the informal service sector depends on the level of labor income tax paid by formal workers and the efficiency level of the marginal worker, $(\eta^*)^\theta$. The assumption that workers in the formal service sector are heterogeneous in their skill to perform manual tasks implies that this ratio is not constant and varies with the efficiency units of labor in the formal service sector. The higher the value of $\theta$, the higher the worker’s skills in the formal service sector (for those workers whose skill level is $\eta > 1$) compared with their skills in the informal service sector, and therefore the higher the relative informal wage. Note that for the case when all workers have the same ability to perform manual tasks activities in both service sectors (when
\( \theta = 0 \), the wage ratio in the service sector \( \frac{w_{sf}}{w_{sf}} \) and hence the employment ratio \( \frac{L_{sf}}{L_{sf}} \) are constant and independent from technological progress.

Low-skill workers leaving the goods sector and entering the formal service sector are the ones that have relatively high efficiency, and those leaving the formal service sector are the ones that have relatively low efficiency. As a consequence, the average efficiency in the formal service sector increases.

### 3.4 Asymptotic wage inequality.

In this section, I study the evolution of wage inequality, measured by the evolution of manual to abstract, and manual to routine wage ratios, as well as the evolution of formal to informal wage ratios in the service sector. Using equations (112) and (126), and replacing the optimal conditions for wages in the service sector, (114) and (115), I show in Appendix C2 that the wage ratio \( \frac{w_{sf}}{w_{r}} \) can be written as follows:

\[
\frac{w_{sf}}{w_{r}} = \left( (\theta + 1) \eta^{\max} CC_1 L_{sf} + (\eta^{\max} L_{si})^{\theta+1} \right)^{-1}.
\]

Given the asymptotic labor allocation when \( K \to \infty \):

\[
\frac{w_{sf}}{w_{r}} = \begin{cases} 
(\eta^{\max})^{1-\theta} & \text{if } \frac{1}{\sigma_c} > \frac{\beta - \nu}{\beta} \\
0 & \text{if } \frac{1}{\sigma_c} = \frac{\beta - \nu}{\beta} \\
(\eta^{\max})^{1-\theta} & \text{if } \frac{1}{\sigma_c} < \frac{\beta - \nu}{\beta} \end{cases}.
\]

(140)

On the other side, the wage ratio \( \frac{w_{si}}{w_{r}} \) can be written as follows (see Appendix C2 for the complete derivation):

\[
\frac{w_{si}}{w_{r}} = \left( \frac{L_{si}}{L_{sf}} \right)^{\psi-1} \Psi_1 \left( (\theta + 1) \eta^{\max} L_{sf} + (\eta^{\max} L_{si})^{\theta+1} \right)^{-1}.
\]

Similarly, given the asymptotic labor allocation when \( K \to \infty \) I obtain:
Equations (140) and (141) imply that the relative wage paid to manual tasks versus routine tasks increases with technological progress, for the case when the production elasticity (scaled by $\beta$) excess the consumption elasticity. Since Task-biased technological progress affects mainly the production of goods, and goods and services are complements, the aggregate demand for services also increases, consequently increasing the unit wage in service occupations relative to the unit wage of routine labor in the goods sector.

Additionally, from equations (139) and (123), the unit wage of manual labor in the informal service sector relative to the unit wage of manual labor in the formal service sector can be expressed as follows:

$$
\frac{w_{si}}{w_{sf}} = \left\{ \begin{array}{ll}
\left(\frac{(\theta + 1) \eta_{\text{max}} L_{s f} + (\eta_{\text{max}} L_{s i})^{\theta+1}}{(\theta + 1) \eta_{\text{max}} L_{s f} + (\eta_{\text{max}} L_{s i})^{\theta+1}}\right)^{-1} & \psi^{-1} \Psi_1, \\
0 & \psi^{-1} \Psi_1
\end{array} \right. \quad \text{if} \quad \frac{1}{\sigma_c} > \frac{\beta - \nu}{\beta}.
$$

The evolution of the wage ratio $\frac{w_{si}}{w_{sf}}$ is thus determined by the evolution of the employment share in the informal service sector $L_{si}$. Scaled by the level of taxes ($\tau$) and the efficiency levels in the formal service sector ($\theta$). From equations (142) and (136), the asymptotic wage ratio $\frac{w_{si}}{w_{sf}}$ is determined as follows

$$
\frac{w_{si}}{w_{sf}} = \left\{ \begin{array}{ll}
\left(\eta_{\text{max}} L_{s i}\right)^{\theta} (1 - \tau) & \psi^{-1} \Psi_1, \\
\left(\eta_{\text{max}} L_{s f}\right)^{\theta} (1 - \tau) & \psi^{-1} \Psi_1
\end{array} \right. \quad \text{if} \quad \frac{1}{\sigma_c} > \frac{\beta - \nu}{\beta}.
$$

Given that $\hat{L}_{si} > L_{si}$, equation (143) implies that the wage ratio between informal and formal occupations in the service sector increases with technological progress, for the case when $\frac{1}{\sigma_c} > \frac{\beta - \nu}{\beta}$. When the production elasticity (scaled by $\beta$) is bigger than the consumption elasticity, technological progress raises the relative demand for low-skill labor in the formal and the informal service sectors, which requires a rise in wages in both sectors. Note that a worker endowed with $\eta_i$ efficiency units of labor has individual labor productivity of $\eta_i^\theta$ when he works in the formal service sector, while the individual labor productivity of the same worker in the informal service sector is equal to 1. Since
a worker with $\eta > 1$, is more efficient when he works in the formal service sector, the increase in wages in the informal service sector has to be higher than the one in the formal service sector in order to compensate for this loss of skills. The higher the worker’s skill in the formal service sector ($\theta$), the higher the increase in the wage ratio $\frac{w_a}{w_s}$. 

From equation (138), it is now possible to analyze the evolution of the employment composition in the service sector. This equation shows that when formal and informal services are substitutes (when $1 - \psi > 1$), the efficiency units of labor employed in the informal sector relative to those employed in the formal service sector $\frac{L_{si}}{L_{sf}}$ decrease with the relative wage in the informal sector. The increase in $\frac{w_s}{w_f}$ increases the relative price of informal services, thus lowering the relative demand for these services and therefore the relative demand for informal labor.

Finally, I determine the evolution of the wage ratio between wages in the service sector and wages in the abstract task. As Autor and Dorn (2013), wage polarization will occurs when $\frac{w_a}{w_{sf}}$ and $\frac{w_a}{w_{si}}$ rises, and $\frac{w_a}{w_{sf}}$ and $\frac{w_a}{w_{si}}$ are either stable or declining.

In Appendix C2, I show that the wage ratios $\frac{w_a}{w_{sf}}$ and $\frac{w_a}{w_{si}}$ can be written as follows:

$$\frac{w_a}{w_{sf}} = \kappa_{af} K^{\beta(\frac{\sigma_c - 1}{\sigma_c})} C_s^{1-\rho} L_{sf}^{1-\psi},$$

$$\frac{w_a}{w_{si}} = \kappa_{ai} K^{\beta(\frac{\sigma_c - 1}{\sigma_c})} C_s^{1-\rho} L_{si}^{1-\psi},$$

with $\kappa_{af} = \frac{1 - \beta(\mu)(\mu_k)^{\beta} a_f (1 - \beta(\mu_k)^{\beta})^{\nu - 1}}{(\sigma_c) a_f \psi A_{sf}^D}$, and $\kappa_{ai} = \kappa_{af} \left( A_{sf} / A_{si} \right)^{\psi}$.

As capital converge to infinity, the evolution of $\frac{w_a}{w_{sf}}$ is determined by the elasticity of substitution between goods and services, $\sigma_c$. Specifically,

$$\frac{w_a}{w_{sf}} = \begin{cases} \infty & \text{if } \sigma_c > 1, \\ \kappa_{af} C_s^{1-\rho} L_{sf}^{1-\psi} & \text{if } \sigma_c = 1 \text{ with } \frac{1}{\sigma_c} > \frac{\beta - \nu}{\rho} \\ 0 & \text{if } \sigma_c < 1 \end{cases}$$

(144)
Equations (144) and (145) show that when goods and services are gross complements $\sigma_c < 1$, the ratio between abstract tasks wages and manual tasks wages converge to zero. As a result, the model implies overall wage and job polarization due to a task-biased technological change.

### 3.5 Model simulations

In this section, I analyze the evolution of employment shares and sectoral wages as the price of capital converges to zero. I simulate the evolution of employment and wages under different values of the elasticity of substitution between formal and informal services $\sigma_s$. As equation (138) shows, the elasticity of the relative employment in the informal sector to the relative informal wages depends on $\sigma_s$. I also consider a different fiscal policy where labor income taxes are variable. The purpose of this exercise is to analyze how the job polarization process in developing and emerging countries, would affect the informal sector depending on preferences, and tax policies within each country.

All parameters are time-invariant, and the only exogenous change over time is the price of capital. I simulate the model for the case when the production elasticity (scaled by $\beta$) excess the consumption elasticity (it means when $\frac{1}{\sigma_c} > \frac{\beta - \nu}{\beta}$). Under this scenario, independent from the values $\sigma_c$, $\beta$, and $\nu$, technological progress always leads to job polarization. I choose standard values of the parameters commonly used in the literature. For the parameters describing preferences, I set the elasticity of substitution between goods and services at $\sigma_c = 0.4$, and the elasticity of substitution between formal and informal services at $\sigma_s = \frac{1}{0.7}$. These values imply goods and services are complements, and formal and informal services are substitutes. The share of routine aggregate in goods production is set at $\beta = 0.67$, as in Bock (2018). I normalize the aggregate labor productivity in the informal service to 1 and set aggregate labor productivity in the informal service sector equal to 0.71. It implies that the labor productivity in the formal sector is 39% higher than in the informal sector. This value is consistent with La Porta and Shleifer (2008) who find that the value-added per employee for registered firms is 39% higher than for their unregistered counterparts. I assumed similar weighs of routine labor and capital in the production of goods, and of goods and services in the utility function, this is $\mu_k = 0.5$, $\gamma_g = \gamma_s = 0.5$ and $a_f = a_i = 0.5$. The labor income tax is set at $\tau = 0.205$. This value is calibrated to
obtain a level of tax revenue of 15% of GDP. According to Besley and Persson (2014), low-income countries typically collect taxes of between 10 to 20 percent of GDP, while the average for high-income countries is more like 40 percent. Finally I set arbitrary values for $\delta_k$, $\eta^{max}$ and $\theta$. Changes in these parameters do not affect the main result of the model. Table 3.1 summarizes the parameter values of the model.

<table>
<thead>
<tr>
<th>Name</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Share of routine aggregate in goods production</td>
<td>$\beta$</td>
<td>0.67</td>
</tr>
<tr>
<td>Elasticity of substitution between consumption and services</td>
<td>$\sigma_c$</td>
<td>0.4</td>
</tr>
<tr>
<td>Elasticity of substitution between formal and informal services</td>
<td>$\sigma_s$</td>
<td>1/0.7</td>
</tr>
<tr>
<td>Inverse of the elasticity of substitution between routine labor and capital</td>
<td>$\nu$</td>
<td>0.5</td>
</tr>
<tr>
<td>Parameter associated to the skill level of formal workers in the service sector</td>
<td>$\theta$</td>
<td>0.7</td>
</tr>
<tr>
<td>Maximum skill level</td>
<td>$\eta^{max}$</td>
<td>2</td>
</tr>
<tr>
<td>Labor income tax</td>
<td>$\tau$</td>
<td>0.205</td>
</tr>
<tr>
<td>Parameter reflecting technological progress</td>
<td>$\delta_k$</td>
<td>0.01</td>
</tr>
<tr>
<td>Relative weigh of goods and services in the utility function</td>
<td>$\gamma_g = \gamma_s$</td>
<td>0.5</td>
</tr>
<tr>
<td>Relative weigh of formal and informal services in the utility function</td>
<td>$a_f = a_i$</td>
<td>0.5</td>
</tr>
<tr>
<td>Relative weigh of capital in the production function of goods</td>
<td>$\mu_k$</td>
<td>0.5</td>
</tr>
<tr>
<td>Aggregate labor productivity in the formal service sector</td>
<td>$A_{sf}$</td>
<td>1</td>
</tr>
<tr>
<td>Aggregate labor productivity in the informal service sector</td>
<td>$A_{si}$</td>
<td>0.71</td>
</tr>
</tbody>
</table>

Note that $L_j$ represents the total amount of efficiency unit of labor employed in sector $j$, and $w_j$ represents the wage per efficiency unit of labor in that sector, $j \in (r, si, sf)$. For the simulation, I also analyze the evolution of the employment shares and the average wages in each sector.

The low-skill employment share $N_j$ is the mass of individuals who supply their labor in sector $j$, which are defined as follows:

$$N_r = \int_{\eta^*}^{\eta^{max}} f(\eta)d\eta = \left[1 - \frac{\eta^*}{\eta^{max}}\right],$$ (146)

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\[ N_{sf} = \int_{\eta^*}^{\eta^{**}} f(\eta) d\eta = \frac{1}{\eta^{max}} [\eta^* - \eta^{**}] , \]  
\[ (147) \]

\[ N_{si} = \int_{0}^{\eta^{**}} f(\eta) d\eta = \frac{\eta^{**}}{\eta^{max}}. \]  
\[ (148) \]

Additionally, the average wage in sector \( j \) is defined as the total labor income in sector \( j \), divided by the mass of people working in this sector:

\[ \bar{w}_j = \frac{w_j L_j}{N_j}, \quad \text{for} \quad j \in (r,sf,si), \]

and in terms of the wage ratio between different sectors:

\[ \frac{\bar{w}_j}{\bar{w}_i} = \frac{w_j L_j}{w_i N_j} \frac{L_i}{N_i}, \quad \text{for} \quad j \neq i. \]

Figure 3.2 shows the evolution of low-skill employment in the goods and the service sectors, under the benchmark calibration, when the price of capital converges to zero. In the goods sector, routine labor is substituted with capital, which decreases employment and the efficiency units of routine labor \( (N_r \text{ and } L_r) \) in this sector. At the same time, the increase in the aggregate demand for services increases both raw employment \( (N_{si} \text{ and } N_{sf}) \) and the efficiency units of labor in the formal and informal sectors \( (L_{si} \text{ and } L_{sf}) \). Consequently, the constant decrease in prices of automating routine tasks eventually causes all low-skill labor to flow from routine tasks to manual tasks. Moreover, it is worth noticing from Figure 3.2 that under this scenario, job polarization leads to a decrease in the ratio between the efficient units of labor in the informal and the formal sector \( L_{si}/L_{sf} \). This decrease is due to the increase of relative informal unit wages in the service sector (see equation \((138)\) and Figure 3.3). Additionally, by comparing the evolution of \( L_{si}/L_{sf} \) versus the relative employment share of the informal sector \( N_{si}/N_{sf} \), it is possible to notice that the decrease on \( L_{si}/L_{sf} \) is higher than the decrease on \( N_{si}/N_{sf} \), which implies that the formal service sector becomes more efficient.
Figure 3.2. Evolution of employment

Figure 3.3 shows the evolution of relative prices and wages as the price of capital decreases. It shows that the ratio between informal and formal wages in the service sector \( \frac{w_{si}}{w_{sf}} \) increases. As was stated previously in section 3.4, technological progress raises the relative demand for low-skill labor in both service sectors, which requires a rise in wages in each sector. Workers with ability \( \eta > 1 \) are more skilled when they work in the formal sector. Therefore, the increase in wages in the informal service sector has to be higher than the increase in the formal service sector to compensate for this loss of skills. As a result, the increase in the relative informal wage increases the relative price of informal services and, as a consequence, it reduces their relative demand.

In terms of average wages, it is worth noticing that the relative average wage in the informal service sector versus the average wage in the formal sector, \( \bar{w}_{si}/\bar{w}_{sf} \), is almost constant when the price of capital decreases. It implies that, on average, the wage differences between formal and informal workers remain constant with technological progress. Notice also that the wage ratio between manual tasks in the service sector and routine tasks increases, while the wage ratio between abstract tasks and manual tasks increases initially and then converges to zero. This result shows that when \( \sigma_c < 1 \),
wages polarize in the long run (see equation (144)).

Figure 3.3 Evolution of relative wages and prices in the service sector

In order to better understand the effect of technological progress and tax policy on the composition of employment between sectors, I analyze the evolution of the share of low skill employment in the goods and the service sector when the price of capital decreases ($\downarrow P_k$) and labor income taxes in the formal sector increases ($\uparrow \tau$). Figure 3.4 shows that a decrease in the price of capital decreases the share of routine labor ($N_r$) in the goods sector, almost independent from the level of taxes. The decrease in taxes however affects the distribution of workers in the service sector. For a given level of taxes, the decrease in the price of capital increases employment in both the formal and the informal service sectors, $N_{sf}$ and $N_{si}$ respectively. An increase in the level of taxes increases the share of employment in the informal service sector and decreases the share of employment in the formal service sector. These results suggest that technological progress combined with an important decrease in taxes could lead to a decrease in the size of the informal sector.
Figure 3.4 Effect of a decrease on the price of capital and an increase on labor income taxes on employment

Figure 3.5 shows the evolution of relative employment, prices, and wages in the service sector when the price of capital decreases and the degree of substitution between formal and informal services is higher. When $\sigma_s$ is higher (increases from 1.43 to 2), the elasticity of the relative informal employment with respect to relative informal wages increases. Therefore, the decrease in the relative demand of informal labor in the service sector is higher, since consumer’s demand for formal services is higher.
3.6 Conclusions

This paper analyzes how the incidence of job polarization affects the distribution of employment and wages in the presence of a large informal sector. I develop a general equilibrium model with informality and endogenous occupational choice, based on Autor and Dorn (2013). I assume that there are three sectors in this economy: the goods sector, the formal service sector, and the informal service sector. Workers in the informal service sector are at the bottom of the skill distribution, are less productive, and can avoid taxation.

The analytical solution of the model implies that when the elasticity of substitution between capital and routine labor is higher than the elasticity of substitution between goods and services, the constant decrease in the price of capital eventually causes low-skill labor flows from routine tasks to manual tasks. This condition for job polarization
is the same found by Autor and Dorn (2013). In this case, Task-biased technological progress can increase aggregate demand for services and eventually increase employment and wages in service occupations. Additionally, when goods and services are complements, wages also polarize.

Additionally, I find that the efficient units of labor, as well as the number of workers, hired in the informal sector increase as a result of the increasing demand for informal services. However, the optimal composition of employment in the service sector depends on the level of taxes in the formal service sector $\tau$, the degree of substitution between the two types of services $\sigma_s$, and the level of efficiency in the formal service sector $\eta^\theta$. I find that share of informal employment in the service sector decreases with technological progress. This result is explained by the fact that some workers, whose skill level is $\eta > 1$, are more skilled when they work in the formal sector. Therefore, the increase in the demand for labor in the informal service sector requires a higher increase in wages to compensate for this loss of skills. As a consequence, the increase in relative wages in the informal sector increases their relative prices, which in turn decreases their relative demand and hence the relative demand for workers in the informal service sector.

I simulate the model for different values of the elasticity of substitution between formal and informal services and also for the case when labor income taxes are variable. I find that, when the elasticity of substitution between formal and informal goods increases, the effect of technological progress on the reduction of the share of informal employment in the service sector is higher. This is because consumers are more likely to substitute informal services when their relative price increases. I also analyze the evolution of employment in the goods and the service sector for different values of the price of capital and labor income taxes. I find that technological progress (i.e. a decrease in the price of capital) combined with a significant decrease in taxes could lead to a reduction in the size of the informal sector.

The previous results are mainly driven by the assumption of flexible labor markets in all sectors and by the fact that some workers are more skilled when they work in the formal sector. One interesting extension of the model will be the introduction of wage and employment rigidities in the formal sector, while the informal sector is frictionless. Under this scenario, the positive effect of job polarization outlined in this paper can be diminished or even reversed. When task-biased technological progress leads to an increase in labor demand in the service sector, the presence of real wage rigidities and search and matching frictions in the formal sector, could lead to a higher increase in
employment in the informal service sector. Since wages in the formal sector could not increase that much, and also there are some restrictions to enter into the formal sector. Another interesting extension would be to consider the fact that workers can receive education and acquire specific skills. Under this scenario, technological progress could lead to a decrease in the share of informal employment in the whole economy. Because more educated workers would either work in the goods sector performing abstract tasks or stay in the formal service sector where workers are remunerated according to their skills, the size of the informal sector will decrease.

3.7 Appendix

Appendix C1: Asymptotic labor allocation

From the optimization conditions for the households I have

$$\max_{\{C_g, C_{sf}, C_{si}\}} U = \ln(C)$$

$$U = \ln(C), \quad (149)$$

with

$$C = [\gamma_g C_g^o + \gamma_s C_s^o]^{1/\rho}, \quad (150)$$

$$C_s = (a_f C_{sf}^\psi + a_i C_{si}^\psi)^{1/\psi}$$

$$P_g C_g + P_{sf} C_{sf} + P_{si} C_{si} = (1 - \tau) (w_a L^a + w_r L^r + w_{sf} L_{sf}) + w_{si} L_{si} + T$$

The Lagrangian of this problem writes:

$$\mathcal{L} = \ln \left[ \gamma_g C_g^o + \gamma_s \left( a C_{sf}^\psi + (a_i) C_{si}^\psi \right)^{1/\psi} \right]^{1/\rho}$$

$$+ \lambda \left( (1 - \tau) (w_a L^a + w_r L^r + w_{sf} L_{sf}) + w_{si} L_{si} + T - P_g C_g - P_{sf} C_{sf} - P_{si} C_{si} \right)$$

F.O.C
\{C_g\} \quad [... \frac{1}{\gamma_g} \gamma_g C_g^{\rho - 1} = P_g \lambda \quad (151)

\{C_{sf}\} \quad [... \frac{1}{\gamma_s} \gamma_s C_{sf}^{\rho - 1} a_f \psi C_{sf}^{\psi - 1} = \lambda P_{sf} \quad (152)

\{C_{si}\} \quad [... \frac{1}{\gamma_s} \gamma_s C_{si}^{\rho - 1} (a_i) \psi C_{si}^{\psi - 1} = \lambda P_{si} \quad (153)

By dividing equations (151) and (152), and equations (153) and (152) I obtain:

\begin{align*}
\frac{C_g}{C_s} &= \left( \frac{C_{sf}^{1-\psi}}{\psi a_f \gamma_s P_g} \right)^{\sigma_e} \\
\frac{C_{si}}{C_{sf}} &= \left( \frac{a_i P_{sf}}{a_f P_{si}} \right)^{\sigma_s}
\end{align*}

(154)

(155)

Replacing \(C_{si} = A_{si} L_{si}\) and \(C_{sf} = A_{sf} L_{sf}\) into equation (151) I obtain

\[ L_{si} = L_{sf} \left( \frac{A_{si}}{A_{sf}} \right)^{\frac{\psi}{1-\psi}} \left( \frac{a_i}{a_f} \frac{w_{sf}}{w_{si}} \right)^{\frac{1}{1-\psi}} \]

from equation (119) I have \(\frac{w_{sf}}{w_{si}} = \frac{1}{(\eta^{**})^\theta (1-\tau)}\). Replacing this expression into the previous equation:

\[ L_{si} = L_{sf} \left( \frac{A_{si}}{A_{sf}} \right)^{\frac{\psi}{1-\psi}} \left( \frac{a_i}{a_f} \frac{1}{(\eta^{**})^\theta (1-\tau)} \right)^{\frac{1}{1-\psi}} \]

Using equation (123) I can express \(\eta^{**} = \eta_{max} L_{si}\), then

\[ L_{si} = L_{sf} \left( \frac{A_{si}}{A_{sf}} \right)^{\frac{\psi}{1-\psi}} \left( \frac{a_i}{a_f} \frac{1}{(\eta_{max} L_{si})^\theta (1-\tau)} \right)^{\frac{1}{1-\psi}} \]

\[ L_{si} = L_{sf}^{\frac{\phi}{1-\psi}} C_{11} \quad (156) \]

where \(C_{11} = \left( \frac{A_{si}}{A_{sf}} \right)^{\frac{\psi}{1-\psi}} \left( \frac{a_i}{a_f (\eta_{max})^\theta (1-\tau)} \right)^{\frac{1}{1-\psi}} \)

On the other side, replacing equations (112), (129), and (134) into equation (154) I have:

138
\[
\frac{C_g}{C_s} = \left( C_{sf} \frac{\gamma_g}{\psi_a f \gamma_s} \frac{P_{sf}}{P_g} \right)^{\sigma_c}
\]

\[
\gamma_g (Y^g - P_k K) \frac{w_{sf}}{w_r} = \gamma_s C_s \sigma_c \psi_a f C_{sf}^{\psi - 1}
\]

\[
\gamma_g (Y^g - P_k K)^{\rho - 1} \frac{\kappa R A_g L_a^{(1 - \beta)}}{A_{sf}} X^{\beta - \nu} L_{r}^{\nu - 1} w_{sf} w_r = \gamma_s (C_s(L_{si}))^{(\rho - 1)} \psi_a f (A_{sf} L_{sf})^{\psi - 1}
\]

From (118) I have that \(\frac{w_{sf}}{w_r} = (\eta^*)^{1 - \theta}\). Replacing this equation into the previous equation I obtain:

\[
\gamma_g (Y^g - P_k K)^{\rho - 1} \frac{\kappa R A_g L_a^{(1 - \beta)}}{A_{sf}} X^{\beta - \nu} L_{r}^{\nu - 1} (\eta^*)^{1 - \theta} = \gamma_s (C_s(L_{si}))^{(\rho - 1)} \psi_a f (A_{sf} L_{sf})^{\psi - 1}
\]

with \(Y^g - P_k K = (1 - \beta) (\mu_k K)^{\beta}\) and \( \frac{X_{k}}{\mu_k K} = 1 \), \(L_{si} = L_{sf}^{1 - \nu} C_{11}\)

\[
\gamma_g \left( (1 - \beta) (\mu_k K)^{\beta} \right)^{\rho - 1} \frac{\kappa R A_g L_a^{(1 - \beta)}}{A_{sf}} (\mu_k K)^{\beta - \nu} L_{r}^{\nu - 1} (\eta^*)^{1 - \theta} = \gamma_s (C_s(L_{si}))^{(\rho - 1)} \psi_a f (A_{sf} L_{sf})^{\psi - 1}
\]

\[
\frac{L_r^{1 - \nu}}{(\eta^*)^{1 - \beta}} (C_s(L_{si}))^{(\rho - 1)} L_{sf}^{\psi - 1} = K^{-(\beta)(1 - \rho) + (\beta - \nu)} \Psi_1
\]

with \(\Psi_1 = \frac{\gamma_g \kappa R A_g \mu_k^{\beta(\rho - 1) + (\beta - \nu)} (1 - \beta)^{-1 + \theta} \psi_a f \gamma_s A_{sf}}{(1 - \beta) \kappa R A_g \mu_k^{\beta(\rho - 1) + (\beta - \nu)} (1 - \beta)^{-1 + \theta} \psi_a f \gamma_s A_{sf}}\).

Now I want to express equation (157) in terms of \(L_{si}\) only.

Combining equations (147), (148) and (156) I obtain:

\[
\eta^* = \left( (\theta + 1) \eta_{\text{max}} \left( \frac{L_{si}}{C_{11}} \right)^{1 + \theta - \psi} + (\eta_{\text{max}} L_{si})^{(\theta + 1)} \right)^{\frac{1}{\theta + 1}}
\]

Replacing (158) into (121) I am able to express \(L_r\) as a function of \(L_{si}\):

\[
L_r = \left[ \left( \frac{\eta_{\text{max}}^{\eta_{\text{max}}}}{2} - \left( \left( \theta + 1 \right) \eta_{\text{max}} \left( \frac{L_{si}}{C_{11}} \right)^{1 + \theta - \psi} + (\eta_{\text{max}} L_{si})^{(\theta + 1)} \right)^{\frac{1}{\theta + 1}} \right)^{2} \right] = gg(L_{si})
\]

Therefore, equation (157) can be written as follows:
\[
\frac{gg(L_{si})^{1-\nu} \left(C_s(L_{si})\right)^{\theta-1} \left(\frac{L_{si}}{C_{si}}\right)^{\frac{1+\theta-\psi}{1-\nu}}}{\left(\theta + 1\right) \eta^{\max} \left(\frac{L_{si}}{C_{si}}\right)^{\frac{1+\theta-\psi}{1-\nu}} + \left(\eta^{\max} L_{si}\right)^{\theta+1}} = K^{-\left(\beta(1-\rho) + (\beta-\nu)\right)} \Psi_1
\]

with \(\sigma_c = \frac{1}{1-\rho}\)

\[
\frac{gg(L_{si})^{1-\nu} \left(C_s(L_{si})\right)^{\theta-1} \left(\frac{L_{si}}{C_{si}}\right)^{\frac{1+\theta-\psi}{1-\nu}}}{\left(\theta + 1\right) \eta^{\max} \left(\frac{L_{si}}{C_{si}}\right)^{\frac{1+\theta-\psi}{1-\nu}} + \left(\eta^{\max} L_{si}\right)^{\theta+1}} = K^{-\left(\beta(1-\rho) + (\beta-\nu)\right)} \Psi_1
\]

Appendix C2: Asymptotic wage inequality

From equations (151) and (152) I find

\[
\left[...\right]^{1 \over 2-1} \left(\gamma_s\right) C_s^{\rho-1} a\psi C_s^{\psi-1} = \left[...\right]^{1 \over 2-1} \gamma_g C_g^{\rho-1} P_{sf}
\]

\[
\left(\gamma_s\right) C_s^{\rho-1} a\psi C_s^{\psi-1} = \gamma_g \left(\left(1-\beta\right) \left(\mu K\right)\right)^{\rho-1} \frac{w_{sf}}{A_{sf}}
\]

\[
w_{sf} = \frac{\gamma_s C_s^{\rho-1} a\psi C_s^{\psi-1} A_{sf}}{\gamma_g \left(\left(1-\beta\right) \left(\mu K\right)\right)^{\rho-1}}
\]

From the optimization problem for the firm in the goods and service sector, I have

\[w_r = \kappa_R \left(\mu_k K\right)^{\beta-\nu} g(L_{sf})^{\nu-1}
\]

\[w_a = P_g \left(1-\beta\right) \frac{Y_g}{I_a} = \left(1-\beta\right) \left(\mu K\right)^{\beta}
\]

\[w_{sf} = P_{sf} A_{sf}
\]

\[w_{si} = P_{si} A_{si}
\]

from equations (161) and (162) I obtain:
\[
\frac{w_{sf}}{w_r} = \frac{C^{\rho-1}_s (L_{sf})^{\psi-1} (g(L_{sf}))^{1-\nu}}{\kappa_{sf} (K)^{-\beta(1-\rho)+\beta-\nu}}
\]

with
\[
\kappa_{sf} = \frac{\alpha^{\rho-1} \kappa R (\mu_k)}{(A_{sf})^{1-\psi} (\mu_{A_{sf}})}
\]

Replacing the expression \( L_{si} = L_{sf}^{1-\psi} C_{11} \)

\[
\frac{w_{sf}}{w_r} = \frac{C^{\rho-1}_s \left( \left( \frac{L_{si}}{C_{11}} \right)^{\frac{1+\theta-\psi}{1-\psi}} \right) \psi^{-1} (g(L_{si}))^{1-\nu}}{\kappa_{sf} (K)^{-\beta(1-\rho)+\beta-\nu}}
\]

Replacing equation (160) I have

\[
\frac{w_r}{w_{sf}} = \frac{(\theta + 1) \eta_{A_{sf}} C_1 (L_{si})^{\frac{1+\theta-\psi}{1-\psi}} + (\eta_{A_{sf}} L_{si})^{\theta+1}}{\kappa_{sf} K^{-\beta(1-\rho)+\beta-\nu} (K_1)^{\beta(\rho-1)+\beta-\nu} \Psi_1}
\]

Similarly, the wage ratio between informal manual tasks and routine tasks \( \frac{w_{si}}{w_r} \) can be determined as follows:

By dividing equation (151) into (153) I obtain:

\[
\frac{\gamma C^{\rho-1}_g}{\gamma_s C^{\rho-1}_s \psi (\beta) C^{\psi-1}_{si}} = \frac{P_g}{P_{si}}
\]

Replacing \( w_{si} = P_{si} A_{si} \), and \( C_g = Y - P_k K = \alpha (\mu_k K)^\beta \) I have

\[
\frac{(1-\beta) (\mu_k K)^\beta}{C_s^{\rho-1} \psi (\beta) (A_{si} L_{si})^{\psi-1}} = \frac{\gamma_s A_{si}}{\gamma_w w_{si}}
\]

\[
\frac{w_{si}}{w_r} = C^{\rho-1}_s \psi \gamma_s \beta A_{si} L_{si}^{\psi-1} \frac{(1-\beta) (\mu_k K)^\beta}{\gamma_g w_{si}}
\]

Dividing equation (165) into (162), I obtain
with $\kappa_s = \frac{\psi \beta A_s \gamma_c}{\gamma_c \gamma_k} (1 - \beta) (\mu_k k)^{\beta - \nu} \nu$.

Replacing equation (160) I have

$$\frac{w_{s1}}{w_r} = \frac{C_s^{\rho - 1} \rho \gamma_k A_s \gamma_k^\nu L_{s1}^{\psi - 1}}{(1 - \beta) (\mu_k k)^{\beta - \nu} g(L_{sf})^{\nu - 1}}$$

$$\frac{w_{s1}}{w_r} = \frac{\kappa_s C_s^{\rho - 1} L_{s1}^{\psi - 1} g(L_{sf})^{\nu - 1}}{K^{-\beta(1 - \rho) + (\beta - \nu)}}$$

Finally, by dividing equation (163) into (161), and equation (163) into (165) I obtain the wage ratios $\frac{w_a}{w_{sf}}$ and $\frac{w_a}{w_{si}}$ respectively:

$$\frac{w_a}{w_{sf}} = \frac{(1 - \beta) (\mu_k k)^{\beta - \nu} \gamma_k (1 - \beta) (\mu_k k)^{\beta - \nu} K^{\beta(1 - \mu_k k)}}{(\gamma_s) C_s^{\rho - 1} \rho \gamma_k^\nu (A_s L_{sf})^{\psi - 1} A_{sf}}$$
where \( \kappa_{af} = \frac{(1-\beta)(\mu_k)^{1-\beta} \gamma_s (1-\beta)(\mu_k)^{1-\beta})^{1-\rho}}{(\gamma_s)_{af} \psi A_{sf}^{1-\rho}} \)

\[
\frac{w_a}{w_{si}} = \kappa_{af} K^{\beta} (\frac{\sigma C^1 - 1}{\sigma}) C_1^{1-\rho} L_{sf}^{1-\psi}
\]

\[
\frac{w_a}{w_{si}} = \frac{(1-\beta)(\mu_k)^{1-\beta}}{C_1^{1-\rho} \psi A_{si}^{1-\rho} \psi A_{si}^{1-\rho} \psi A_{si}^{1-\rho} \psi A_{si}^{1-\rho}} \left( (1-\beta)(\mu_k)^{1-\beta} \right)^{1-\rho}
\]

where \( \kappa_{si} = \frac{(1-\beta)(\mu_k)^{1-\beta} \gamma_s (1-\beta)(\mu_k)^{1-\beta})^{1-\rho}}{(\gamma_s)_{af} \psi A_{si}^{1-\rho}} \).

### 3.8 References


