DYNAMIC MODELING IN SUSTAINABLE OPERATIONS AND SUPPLY CHAIN MANAGEMENT

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DECLARATION

I hereby declare that this thesis has been composed by myself and has not been presented or accepted in any previous application for a degree. The work, of which this is a record, has been carried out by myself unless otherwise stated and where the work is mine, it reflects personal views and values. All quotations have been distinguished by quotation marks and all sources of information have been acknowledged by means of references.

Baolong Liu

June 4, 2018

To my parents and mentors.

不以物喜，

不以己悲；

先天下之忧而忧，

后天下之乐而乐。

—— 范仲淹《岳阳楼记》

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GENERAL INTRODUCTION

"The International Union for Conservation of Nature (IUCN) Program 2013-16 is driven by two features of life today: Global production and consumption patterns are destroying our life support system - nature - at persistent and dangerously high rates." (The IUCN World Conservation Congress, 2012)

As indicated by IUCN, the world's attention on sustainability has already been aroused and keeps increasing without ceasing. In particular, business activities are implicitly alluded: global production and consumption. Companies that are bringing convenient service, innovative products and even creative ideas are at the same time directly and indirectly eroding our natural environment gradually. Therefore, balancing profitability and sustainability is a question puzzling every company that truly respects social responsibility or simply attempts to fulfill environmental regulations. Although achieving the balance is a challenge, companies benefit "win-win" opportunities both economically and environmentally (Porter & Van der Linde, 1995). For instance, an end-of-service aircraft actually holds great value. Not only can the engines be recycled, the high-quality metals also possess great values and potentials for future use in various industries. The Aircraft Fleet Recycling Association (AFRA) announces that by the year 2034, approximately 12,000 aircrafts will be out of service (Llopis, 2015). In lieu of landfill which potentially results in serious damage to our environment and enormous amount of land consumption, many companies have started the business of dismantling the retired aircrafts, recycling high-quality metals, engines, etc., and investing green innovations to save the lands and create future values. Typically, Boeing, as the founding member of AFRA, has been continuously and successfully raising its sustainability performance by e.g., recycling, waste reduction (Boeing, 2017).

Not only are the sustainable operations carried out within a firm, but also create opportunities for cooperation in supply chain settings. For instance, Jaguar Land Rover, together with one of its aluminum suppliers, Novelis, University of Cambridge Institute for Sustain-

ability Leadership and the UK government, launched a project, namely, REALCAR, to recycle aluminum from end-of-use vehicles (Cassell et al., 2016). By utilizing the technology of Novelis, the collaboration of the supply members and other parties manages to generate post-consumer aluminum for manufacturing Jaguar Land Rover again, which saved more than 30,000 tons' aluminum, 95% energy consumption compared with using new aluminum and helped Novelis reduce 13% greenhouse gas emissions in the years 2014 to 2015. More surprisingly, the collaboration of Ford and Toyota exemplifies that two companies that are actually competitors can cooperate in order to develop sustainable products: hybrid systems for light trucks and SUVs (Bunkley, 2011).

In addition to practical evidences, academic research has seen the growth and expansion of sustainable operations and supply chain management. Kunreuther and Kleindorfer (1980) was among the first to shed light on this field by understanding institutional arrangements and decision processes and emphasizing the necessity of making alternative plans to cope with climatic change and weather modification. Naturally, the growth of population and consumers' needs as well as the depletion of natural resources attract the attention of the society, which leads to the growing interest of sustainable operations management research (Drake & Spinler, 2013). To address the sustainability issue, research was basically directed to two major standing points from two agents: the social planner/government and the firm. With different viewpoints, their objectives are not consistent: the former aims at maximizing social benefits/welfare while the latter's interests lie at maximizing profits under environmental regulation/legislation. Examples of the former research include Atasu et al. (2009) showing that the efficient policy under take-back regulation makes producers responsible for their own waste to avoid fairness concerns and favor eco-design producers to create stronger environmental benefits, Atasu and Van Wassenhove (2012) which points out the shortcoming of high-level policy, gives suggestions on e-waste operations management and analyzes take-back regulation's influence on social welfare, etc.

The second stream of literature, highlighting the influence and roles of sustainable op-

erations and supply chain management to fulfill the goal of firm/supply chain level, is the direction which this thesis is in line with. The reason that we explore sustainability topic in this micro-angle in lieu of the macro-angle of social benefits is that understanding the decision making of firm/supply chain level operations is the key element to realize the macro-objective set by the government. Moreover, the findings and insights can serve as important tools for firms/supply chains to implement green practice more efficiently. From environmental collaboration (Cellini & Lambertini, 2005; Cellini & Lambertini, 2009; Zhang et al., 2016) to products cannibalization effect (Debo et al., 2006; De Giovanni & Ramani, 2017; Ramani & De Giovanni, 2017); from profitability to sustainability (Akçalı & Çetinkaya, 2011), from recycling to remanufacturing (Zhou & Yu, 2011; Vercraene et al., 2014), and built on the existing literature, e.g., Fleischmann and Kuik (2003), Benjaafar and El Hafsi (2006), Zhao et al. (2008), this thesis includes three essays covering three important sustainable issues as follows.

1. Essay 1, entitled *Environmental Collaboration and Process Innovation in Supply Chain Management with Coordination*, investigates in a supply chain coordination problem when green process innovation is a way to stimulate sales and affect production costs. To coordinate the supply chain, we design several types of contracts in a differential game theoretical setting.

2. Essay 2, entitled *Remanufacturing of Multi-Component Systems with Product Substitution*, models a hybrid manufacturing/remanufacturing system in a product and component substitution setting so as to realize cost efficiency and sustainability benefits in inventory management setting. This essay is modeled by a continuous Markov Decision Process to incorporate uncertainty and explore multi-layer flexibility.

3. Essay 3, entitled *Joint Dynamic Pricing and Return Quality Strategies Under Demand Cannibalization*, solves the trade-off of cannibalization effect by selecting certain portion of upper-level quality returns to remanufacture. The problem is modeled by dynamic programming as an inventory management problem.

The framework of this thesis is *de facto* consistent with the approaches to practice sustainable operations and supply chain management classified and summarized in Kleindorfer et al. (2005):

(a) Green product and process development;

(b) Remanufacturing and closed-loop supply chains;

(c) Lean and green operations management.

Essay 1 is corresponding with (a) since we investigate a green innovation supply chain coordination problem to reach the goal of profitability and environmental performance; Essay 2 explores opportunities to benefit from a remanufacturing setting, which echoes point (b); and Essay 3 takes quality into account in order to extract values from returned products, which is the implementation of green operations management in point (c). Moreover, we choose dynamic models for modeling the respective problems because of two reasons. First, both static models and two-period models are widely explored. However, the major limitation of the two types of models is that their insights can be short-sighted but not enough strategic. Dynamic models further contribute to the long-term decision making insights. Second, dynamic models possess the feature of strategic viewpoints. Sometimes sacrificing current profit possibly yields higher total gains, which is not usually reflected by static or two-period models. In the following, we briefly introduce the problem, modeling, findings and contributions regarding the three essays.

Essay 1 (Chapter 2) investigates a dynamic supply chain model in which a supplier decides both the wholesale price and the green process innovation investments while a manufacturer sets the retail price. The green innovation investments not only contribute positively to the environmental performance, but also lead to marginal production cost reductions. The environmental performance positively influences the demand, which in turn damages the environmental performance via negative externalities (e.g., emissions). We resolve this operational trade-off and compare an uncoordinated setting to a revenue sharing contract complemented by a collaborative program. We show that the overall benefit of environmen-

tal cooperation in green process innovation entails the existence of a profit-Pareto-improving region. Nevertheless, the maximum environmental performance fails to occur in the profit-Pareto-improving region, which shows the mismatch between economic and environmental performance. Moreover, supply chains might prefer a wholesale price contract to maximize the environmental performance and a revenue sharing contract or vertical integrated chain to maximize profits.

Essay 2 (Chapter 3) investigates the inventory and production management of a hybrid manufacturing/remanufacturing system. The system serves the demand for new and for remanufactured products and allows for substitution between these types of products. The objective of the system is to minimize "strategic" cost, i.e., the weighted sum of economic cost and environmental impact. First, we analyze single-component products and show that the optimal policy is of the threshold-type under certain conditions. Then, we analyze multi-component products to achieve flexibility by manipulating the portion of remanufactured components in a product. To address problems of high dimensions, we develop a close-to-optimal heuristic. Our results indicate that our model of partial substitution outperforms existing models with no or complete substitution. We also find that management at the component level reduces strategic cost compared to management at the product level and that a stronger weight on economic cost increases the need for product substitution. Our analysis leads to managerial insights on how remanufacturing can support a company's business strategy.

In Essay 3 (Chapter 4), we investigate a hybrid manufacturing/remanufacturing system receiving returns of used products and producing and selling new and remanufactured products to customers. Given different remanufacturing costs based on different quality levels, the decision maker observes the quality of the returns and decides the amounts to remanufacture and to recycle for raw materials in order to maximize the profit under cannibalization. We model the problem as a stochastic programming problem with the decision maker determining the production quantity and the quality threshold for returns to enter remanufacturing

process, as well as the pricing of the two types of products. Then we look at the multi-period setting with either exogenous prices or endogenous prices. Our findings show that with an additive form of demand functions, the optimal strategies are of threshold-type. We also find that the quality strategy (1) dominates the "*remanufacture-all*" and the "*recycle-all*" strategies; (2) counteracts (compensates) the negative (positive) effect of cannibalization effect on profit.

This thesis articulates several important issues in sustainable operations and supply chain management not only to provide insights for enhancing the performance of firms but also to appeal to the enterprises to adopt appropriate means for a better environment of our society. The link from firm level to society level is that, to improve the green performance through better operations management efficiency in firms and supply chains, is an indispensable element to ameliorate the environment in our society. Taking China as an example. Since a few years ago (The Straitstimes, 2017; Stanway & Perry, 2018), the government started to spare no effort in resolving the air pollution problems. An important and useful means is to put strict regulations and monitoring the efforts of firms which will face serious fine if certain standards are not met by random inspection. Therefore, firms have to cooperate for the betterment of its profitability and, more importantly, the environmental impacts. Throughout the endeavor, despite the uncertain future situation, the air quality has gradually improved in China (Zheng, 2018). This thesis, in a more general setting, aims to provide important insights to firms so that they are not only able to meet the regulations but genuinely to make contributions to building a better environment for our future generations.

Basically, our goal is to obtain deep understanding of the trade-offs with which companies are faced, and to model the problems for seeking possible solutions and helping firms/supply chains to enhance their performance from a theoretical point of view. Then, indirectly, the work will help firms to realize the importance of developing sustainable operations and supply chain management means on our society.

The structure of the thesis is organized as follows. Chapter 2 introduces the thesis in

French. Chapter 3 is the first essay, *Environmental Collaboration and Process Innovation in Supply Chain Management with Coordination.* Chapter 4 includes the contents of the second essay, *Remanufacturing of Multi-Component Systems with Product Substitution*, and the third essay, *Joint Dynamic Pricing and Return Quality Strategies Under Demand Cannibalization*, is introduced in Chapter 5. Chapter 6 gives the general concluding remarks of the three essays which is followed by the reference list and the appendices.

CHAPTER 2

INTRODUCTION GÉNÉRALE

« The International Union for Conservation of Nature (IUCN) Program 2013-16 is driven by two features of life today: Global production and consumption patterns are destroying our life support system - nature - at persistent and dangerously high rates. » (The IUCN World Conservation Congress, 2012)

(Le Programme 2013-2016 de l'Union internationale pour la Conservation de la Nature est axé sur deux caractéristiques de la vie actuelle : les schémas de production et de consommation mondiale sont en train de détruire notre système de survie - la nature - dans une proportion à la fois persistante et dangereusement élevée.)

Comme l'indique l'UICN, la conscience du monde en matière de durabilité s'est déjà éveillée et continue d'augmenter sans cesse. En particulier, les activités commerciales sont implicitement impliquées : la production et la consommation mondiales. Dans le même temps, les entreprises qui apportent des services pratiques, des produits innovants et encore des idées créatives, érodent progressivement, directement et indirectement notre environnement naturel. Par conséquent, équilibrer la rentabilité et la durabilité des produits est une question qui intéresse toute entreprise qui prend vraiment sa part de responsabilité sociale ou tente simplement de respecter les réglementations environnementales. Bien que la réalisation de cet équilibre soit un défi, elle profite aux entreprises avec des opportunités « gagnant-gagnant », à la fois économiquement et écologiquement bénéfiques. Par exemple, un avion en fin de service a une grande valeur : non seulement ses moteurs peuvent être recyclés, mais les métaux de haute qualité possèdent aussi une grande valeur et des potentialités pour une utilisation future dans diverses industries. L'AFRA annonce que d'ici 2034, environ 12 000 appareils seront hors service. Au lieu de les enfouir, ce qui pourrait potentiellement causer de graves dommages à notre environnement et une énorme consommation de terres, de nombreuses entreprises ont commencé à les démanteler, à recycler leurs métaux de haute qualité, leurs moteurs, entre autres, et à promouvoir des innovations vertes, pour sauver des

terres et créer de la valeur future. Boeing, membre fondateur de l'AFRA, a continuellement augmenté sa performance de durabilité, par exemple en matière de recyclage et de réduction des déchets.

Non seulement des opérations visant la durabilité sont réalisées au sein des entreprises, mais celles-ci créent également des opportunités de coopération dans le cadre de la chaîne d'approvisionnement. Par exemple, Jaguar Land Rover, avec la coopération de l'un de ses fournisseurs d'aluminium, Novelis, ainsi que de l'University of Cambridge institute for Sustainability Leadership et le gouvernement britannique, a lancé un projet : REALCAR (Cassell et coll., 2016). Celui-ci vise à recycler l'aluminium des véhicules en fin de vie. En utilisant la technologie de Novelis, la collaboration des fournisseurs et d'autres parties parvient à générer de l'aluminium de post-consommation pour la fabrication de Jaguar Land Rover, ce qui a économisé plus de 30 000 tonnes d'aluminium, 95 % de l'énergie consommée pour produire la même quantité d'aluminium neuf, et a aidé l'entreprise à réduire ses émissions de gaz à effet de serre de 13 % dans les années 2014 à 2015. Plus étonnamment, la collaboration de Ford et Toyota illustre le fait que deux entreprises concurrentes peuvent coopérer pour développer des produits durables : des systèmes hybrides pour camionnettes et SUV.

En plus des preuves pratiques, les recherches académiques ont pris pour cible la croissance et l'expansion des opérations durables et la gestion de la chaîne d'approvisionnement. Kunreuther et Kleindorfer (1980) étaient parmi les premiers à faire la lumière sur ce domaine en soulignant les arrangements institutionnels et les processus décisionnels tout en insistant sur la nécessité d'élaborer des plans alternatifs pour faire face au changement climatique. Naturellement, la croissance de la population et des besoins des consommateurs ainsi que l'épuisement des ressources naturelles attirent l'attention de la société, ce qui conduit à l'intérêt croissant de la recherche en gestion des opérations durables. Pour adresser la question de la durabilité, la recherche a essentiellement été orientée vers deux points majeurs de deux agents : le planificateur social/le gouvernement et l'entreprise. Avec les points de vue différents, leurs objectifs ne sont pas cohérents : le premier vise à maximiser

les avantages/bien-être sociaux alors que les intérêts de ces derniers visent à maximiser les profits en vertu de la réglementation/législation environnementale. Exemples de recherches antérieures : Atasu et coll. (2009) montre que la politique efficace de la réglementation en matière de reprise rend les producteurs responsables de leurs déchets afin d'éviter les problèmes d'équité et favorise les producteurs d'écodesign proposant des avantages environnementaux plus importants ; Atasu et VanWassenhove (2012) attirent l'attention sur les lacunes de la politique de haut niveau, donnent des suggestions sur la gestion des opérations des déchets électroniques et analyse l'influence de la réglementation en matière de reprise sur le bien-être social, etc.

Le deuxième volet de la littérature, soulignant l'influence et les rôles des opérations durables et de la gestion de la chaîne d'approvisionnement pour atteindre l'objectif de niveau entreprise/chaîne d'approvisionnement, est la direction avec laquelle cette thèse est constituée. La raison pour laquelle nous explorons le sujet de la durabilité dans ce microangle de vue, au lieu de celui macro des avantages sociaux, est que la compréhension de la prise de décision concernant les opérations au niveau des entreprises/chaînes d'approvisionnement est l'élément clé pour atteindre le macro-objectif fixé par le gouvernement. De plus, les conclusions et les remarques perspicaces peuvent être des outils importants pour les entreprises/chaînes d'approvisionnement afin de mettre en œuvre une pratique plus efficace. De la collaboration environnementale à l'effet de cannibalisation des produits, de la rentabilité à la durabilité, du recyclage au reconditionnement, et bâtie sur la littérature existante, la thèse de Fleischmann et Kuik (2003), Benjaafar et El Hafsi (2006), Zhao et coll. (2008) inclut trois essais couvrant trois importantes questions durables comme suit.

1. L'essai 1, intitulé *Environmental Collaboration and Process innovation in Supply Chain Management with Coordination*, étudie un problème de coordination de la chaîne d'approvisionnement lorsque l'innovation environnementale est un moyen de stimuler les ventes et d'affecter les coûts de production. Pour coordonner la chaîne d'approvisionnement, nous concevons plusieurs types de contrats dans un cadre de jeu différentiel.

2. L'essai 2, intitulé *Remanufacturing of Multi-Component Systems with Product Substitution*, modélise un système hybride de fabrication/reconditionnement dans un environnement de substitution de produits et de composants, afin d'obtenir un rapport coût-efficacité dans la gestion des stocks. Cet essai est modélisé par un processus décisionnel de Markov continu, afin d'incorporer l'incertitude et de tirer parti de la théorie de la file le d'attente.

3. L'essai 3, intitulé *Joint Dynamic Pricing and Return Quality Strategies Under Demand Cannibalization*, permet de résoudre le problème de la cannibalisation en sélectionnant une partie des produits qualité supérieure retournés au reconditionnement. Le problème est également stochastique, modélisé par la programmation dynamique en tant que problème de gestion des stocks.

Le cadre de cette thèse est *de facto* cohérent avec les approches de la pratique des opérations durables et de la gestion de la chaîne d'approvisionnement classées et résumées par Kleindorfer et coll. (2005) :

(a) Développement de produits et de processus écologiques ;

(b) Reconditionnement et chaîne d'approvisionnement en circuit fermé ;

(c) Gestion des opérations Lean et Green.

Introduction de l'Essai 1.

Les consommateurs d'aujourd'hui apprécient davantage les produits respectueux de l'environnement que par le passé et la performance environnementale des entreprises a une influence considérable sur les décisions d'achat des consommateurs. Par conséquent, les entreprises sont invitées à mettre en œuvre des pratiques de production écologiques, le marché ayant des attentes élevées en ce qui concerne les questions environnementales, et la législation imposant d'importantes restrictions sur l'impact général d'une entreprise. Les entreprises sont ainsi appelées à investir continuellement dans les innovations vertes pour réduire les émissions, économiser l'énergie en production, réduire les déchets, tirer parti des produits recyclés et, plus généralement, améliorer la performance environnementale. Bien que les entreprises s'engagent à adopter des stratégies respectueuses de l'environnement, leur développement

peut avoir l'effet indésirable d'augmenter les dommages environnementaux globaux. La littérature de l'économie environnementale soutient pleinement l'hypothèse selon laquelle une demande plus grande génère un plus grand volume d'émissions et de pollution, détériorant ainsi la performance environnementale globale (par exemple, Jeux de pollution de Jørgensen et coll., 2010).

Pour résoudre ce compromis, nous caractérisons un jeu d'innovation entre un fournisseur de composants et un fabricant, dans lequel le fournisseur investit dans un type d'innovation de processus pour rendre le processus de production plus écologique, c'est ce que l'on nomme innovation de processus vert. Les efforts d'innovation verte ont un triple rôle :

1. Les entreprises peuvent réduire les émissions générées par des biens individuels, répondant ainsi à la fois aux attentes des consommateurs quant àla performance environnementale des produits, et aux législations.

2. Les entreprises peuvent fabriquer des produits plus écologiques, car les consommateurs préfèrent acheter des produits verts plutôt que bruns. Cependant, plus de ventes impliquent davantage de pollution, car il existe généralement une quantité moyenne de pollution par produit.

3. Les efforts d'innovation de processus écologiques ont un impact imprécis sur les coûts de production. La littérature du domaine se caractérise par deux écoles de pensée différentes, selon lesquelles l'innovation de processus peut avoir un impact positif ou négatif sur le coût marginal de production.

Dans notre modèle, nous cherchons à étudier l'impact des efforts d'innovation des processus verts sur les bénéfices des membres de la chaîne d'approvisionnement, en analysant son influence sur les coûts de production et en vérifiant comment les entreprises ajustent leurs stratégies en conséquence. Nous modélisons d'abord un scénario de référence dans lequel les deux entreprises ne coopèrent pas dans le cadre d'un programme d'innovation écologique, mais plutôt dans la concurrence pour leurs stratégies de prix. Le fournisseur décide du prix de gros avec les efforts d'innovation verte, tandis que le fabricant fixe le prix de détail. Ainsi,

la chaîne d'approvisionnement adopte un contrat de prix de gros pour coordonner les flux financiers. Néanmoins, parce qu'elle laisse la question de la double marginalisation, la littérature a proposé la mise en œuvre d'un contrat de partage des recettes comme mécanisme de coordination alternatif pour surmonter les limitations du contrat de prix de gros.

Dans chacun des scénarios proposés, nous caractérisons un programme de coopération dans lequel le fabricant soutient une fraction des effort d'innovation verte du fournisseur en mettant en œuvre un programme collaboratif. Dans notre modèle, nous permettons au fabricant de payer une partie des investissements dans l'innovation verte, ce qui lui permet également de mieux contrôler le niveau d'innovation et de favoriser les avantages opérationnels du fournisseur en matière de réduction des coûts. Nous pouvons ensuite compléter notre analyse en examinant comment la coopération affecte la sélection d'un mécanisme de coordination ainsi que les stratégies des entreprises, la performance environnementale et les profits.

Notre analyse donne les résultats suivants. Tout d'abord, la présence d'un programme collaboratif offre une région de profit-Pareto-improving pour les deux acteurs, dans les deux situations de contrat de prix de gros et contrat de partage des recettes. Ainsi, les deux joueurs peuvent s'améliorer si le programme collaboratif est conçu correctement. Deuxièmement, malgré l'excellence d'une chaîne verticale intégrée en termes de bénéfice de la chaîne d'approvisionnement, la performance environnementale dans les scénarios décentralisés peut surpasser celle de la chaîne intégrée verticale avec certains programmes collaboratifs en raison de l'augmentation des ventes, et compromet la performance environnementale. Parce que cette région bénéfique du programme de collaboration ne se situe pas dans la région de profit-Pareto-improving, les décideurs doivent se référer à leur stratégie d'entreprise pour comprendre quel contrat correspond le mieux à la stratégie de l'entreprise. Nous découvrons qu'un décalage entre la performance économique et la performance environnementale existe toujours. Troisièmement, un contrat de partage des recettes ne fonctionne pas toujours mieux qu'un contrat de prix de gros pour la rentabilité ou la performance environnementale.

Cela s'explique par les coûts administratifs et les effets complexes des paramètres du partage des revenus et du programme collaboratif. Enfin, nous étendons l'analyse dans deux directions. D'une part, nous analysons le coût de production dans le cas où celui-ci augmente avec l'innovation de processus vert. Nous obtenons finalement des modèles de résultats cohérents. D'un autre côté, nous incluons une contrainte sur les paramètres du partage des revenus et du programme collaboratif. Cela ne garantit pas des profits plus élevés ni une performance environnementale plus élevée par rapport aux autres mécanismes de coordination que nous explorons, mais dispose du potentiel d'offrir de meilleurs résultats. Néanmoins, l'effet de discordance ne disparaît pas.

Introduction de l'Essai 2.

Le reconditionnement fait référence à la fabrication de produits à partir d'une combinaison de composants neufs et réutilisés. il est déjà largement utilisé dans certaines industries telles que l'automobile, l'aéronautique, la machinerie lourde et l'électronique. Par exemple, Ford Motor Corporation a introduit son étiquette *Ford Authorized Remanufactured*, qui a couvert plus de 120 millions de livres de composants reconditionnés dans les dix premières années après sa création. Le reconditionnement améliore la rentabilité, car les coûts des composants pour les produits réusinés sont généralement inférieurs de 50 à 65 % par rapport à ceux des nouveaux produits. En outre, il réduit également les effets environnementaux négatifs de la fabrication, en évitant le gaspillage de matériaux tels que l'acier ou les produits chimiques et en réduisant la consommation d'énergie et d'eau. Un impact environnemental typique comprend 80 % de consommation d'énergie et d'eau en moins et 70 % de déchets en moins.

Akaçalı et Çetinkaya (2011) et Govindan et coll. (2015) recommandent d'utiliser le reconditionnement comme un moyen de soutenir une stratégie d'entreprise complète, y compris des considérations économiques et environnementales. En effet, les entreprises considèrent de plus en plus les aspects environnementaux dans leurs stratégies d'affaires. Par exemple, Jeffrey R. immelt, directeur général de General Electric, écrit « Sustainability is not an ini-

tiative for us; it is integrated into our core business strategy... » (La durabilité n'est pas une initiative, pour nous. Elle est intégrée à notre stratégie commerciale fondamentale). Dans cet article, nous considérons donc la valeur stratégique créée à partir du reconditionnement. La valeur stratégique comprend l'impact économique et l'impact environnemental. L'impact économique fait référence au coût de production, à la détention de stocks, aux pénalités pour les clients et à l'acquisition des retours. L'impact environnemental fait référence à la consommation d'énergie et d'eau ainsi qu'aux déchets des matériaux. La pondération entre les deux types d'impact représente la stratégie commerciale de l'entreprise en ce qui concerne l'équilibre entre rentabilité économique et durabilité.

Les produits remanufacturés doivent être étiquetés pour permettre au client de faire la différence entre les produits reconditionnés et nouveaux. Alors que les premiers doivent satisfaire des normes de qualité strictes et que leur qualité et garantie dépassent souvent celles des seconds, de nombreux clients perçoivent encore leur valeur différemment de celle des nouveaux produits. Par exemple, certains clients croient obstinément que la qualité des produits remanufacturés est inférieure à celle des nouveaux produits, et certains clients soucieux de l'environnement peuvent, à leur tour, préférer les produits remanufacturés aux nouveaux produits pour leur moindre impact environnemental. Par conséquent, la demande en nouveaux produits n'est pas identique à celle en produits remanufacturés.

Même si les produits nouveaux et remanufacturés ne sont pas identiques en ce qui concerne la valeur perçue pour les clients, les fabricants ont une certaine flexibilité pour servir ces derniers. Si une incitation financière est offerte, certains clients acceptent un produit (nouveau) remanufacturé, même s'ils ont demandé un nouveau produit (remanufacturé). Nous nous référons à cette flexibilité comme product substitution (remplacement du produit). Et même lorsque la demande du produit est desservie comme requis, les produits remanufacturés contiennent rarement 100 % de composants remanufacturés. Dans la plupart des cas, ils contiennent un mélange de composants remanufacturés et d'autres nouveaux, et le fabricant ne garantit qu'une part minimale des composants remanufacturés. Il en va de même

pour les nouveaux produits, qui peuvent contenir certains composants remanufacturés, à condition qu'un minimum de composants soit neuf. Cette définition permet aux fabricants de jouer de manière flexible avec la part des composants remanufacturés dans les produits avec lesquels ils servent les clients.

Nous nous référons à cette flexibilité comme component substitution (remplacement de composant). Nous cherchons à répondre aux questions de recherche suivantes : (1) Quelle est la valeur de la substitution (produit et composant) dans le reconditionnement ? (2) Comment ajuster les décisions opérationnelles pour faire un usage optimal de la substitution ? (3) Comment la stratégie commerciale d'une entreprise affecte-t-elle ses décisions opérationnelles ?

Pour développer l'analyse, nous commençons par le cas d'un seul composant avant de l'étendre au cas des composants multiples. Dans le cas d'un composant unique, nous formulons le modèle dynamique sous la forme d'un processus décisionnel de Markov continu dans le but de minimiser le coût stratégique moyen. Nous montrons que la politique optimale a une structure de base-stock dépendant de l'état lorsque le coût de substitution pour servir une demande reconditionnée avec un nouveau produit est de zéro. Pour le cas des composants multiples, nous dérivons plusieurs propriétés structurelles de la politique optimale et développons une procédure heuristique. Enfin, nous étendons le cas du seul composant à la corrélation entre la demande et le retour, ce qui se produit notamment lorsque les clients retournent un produit ancien en même temps qu'ils achètent un produit.

Nous constatons que la substitution a un effet significatif sur le coût stratégique. La substitution de produit entraîne une réduction de 18,26 % du coût stratégique dans le cas d'un composant unique par rapport à la politique de référence qui ne tient pas compte de la substitution de produit. L'effet diminue avec un poids plus fort sur l'environnement dans la stratégie commerciale. La substitution de composants réduit le coût stratégique de 1,40 % (environ 1 million d'euros) dans le cas de trois composants, par rapport au cas de niveau du système. En ce qui concerne la stratégie commerciale, nous constatons qu'un poids fort de

l'impact sur l'environnement nécessite une substitution moins fréquente des produits et des composants qu'un poids faible. Enfin, non seulement la gestion du reconditionnement au niveau des composants réduit le coût stratégique, mais elle nécessite aussi une substitution de produits moins fréquente que la gestion du reconditionnement au niveau du système.

La contribution de cet article est triple. Premièrement, nous sommes les premiers auteurs à considérer la substitution flexible des produits et des composants en reconditionnement, ce qui est une situation couramment observée dans la pratique. La littérature existante considère soit un marché intégré unique (substitution complète) soit deux marchés séparés (pas de substitution). Nous dérivons des propriétés structurelles et une politique heuristique efficace pour les systèmes hybrides de fabrication/reconditionnement avec substitution. Deuxièmement, nous considérons le reconditionnement comme un moyen de soutenir une stratégie commerciale globale qui inclut la rentabilité et la durabilité. La littérature précédente sur le reconditionnement s'est principalement concentrée sur le seul profit économique. Nous analysons explicitement l'impact du poids stratégique sur les résultats et les décisions optimales. Troisièmement, nous comparons la gestion du reconditionnement au niveau du système avec la gestion du reconditionnement au niveau des composants.

Introduction de l'Essai 3

Des fabricants comme Apple, Asus, Dell, Sony et Xerox ont lancé leurs programmes de recyclage il y a des années. Les consommateurs sont encouragés à renvoyer les produits en fin d'utilisation ou en fin de vie aux fabricants ou à des équipementiers tiers, afin qu'ils soient réutilisés. En plus des appareils électriques, par exemple, les téléphones portables, tablettes et ordinateurs portables, les produits d'occasion y compris les distributeurs automatiques, les pièces automobiles et même les moteurs d'avion sont également collectés. Alors que certains des produits retournés, qui satisfont encore à certaines normes, passeraient par le processus de reconditionnement pour servir le marché à une autre occasion, d'autres ne peuvent pas être reconditionnés, soit en raison de leur qualité médiocre ou du coût élevé de reconditionnement. Habituellement, les retours non reconditionnables sont soit vendus à des

usines capables de recycler les matériaux de valeur, soit simplement envoyés à la décharge à l'incinération, ce qui peut être préjudiciable pour l'environnement.

Alors que la remise à neuf et le recyclage jouent un rôle dans la protection de l'environnement et la réduction des coûts de production et conduisent à

L'économie circulaire, la mise en œuvre massive des programmes concernés est également préjudiciable à la rentabilité des entreprises. D'une part, la disposition reconditionnée cannibalise les ventes des nouveaux produits à marge bénéficiaire plus élevée, ce qui oblige les entreprises à équilibrer le compromis. Pendant ce temps, plus la qualité des retours est faible, plus il en coûte pour le reconditionnement. D'un autre côté, de nombreuses études montrent que l'activité de recyclage peut être assez inefficace pour plusieurs types de produits, mais que les coûts d'enfouissement/d'incinération sont beaucoup moins élevés.

Par conséquent, les entreprises doivent faire face à un compromis : l'incertitude quant à la qualité et la quantité des retours, aux coûts de reconditionnement éventuellement plus élevés pour des retours inférieurs, aux exigences du marché et du gouvernement en matière de reconditionnement et de recyclage, et à la rentabilité en poursuivant à des coûts plus bas et des revenus plus élevés. Par conséquent, la réponse pour choisir la bonne combinaison de stratégies de qualité et de prix pour maximiser les revenus des entreprises n'est pas très claire.

Dans cet article, nous nous intéressons à la stratégie de tarification dynamique pour les produits neufs et reconditionnés ainsi qu'à la stratégie de sélection de qualité qui maximisent les bénéfices tirés de la vente des produits et du recyclage des retours non reconditionnables. Nous analysons aussi le modèle pour comprendre les questions de recherche suivantes : (1) La stratégie de sélection du classement de la qualité est-elle supérieure à la simple stratégie de « tout reconditionner »ou de « tout recycler » ? (2) La stratégie de qualité atténue-t-elle plus efficacement l'effet de la cannibalisation par rapport aux stratégies faciles telles que « tout reconditionner » ou « tout recycler » ? (3) Comment expliquons-nous les différences entre les stratégies optimales pour différents types de produits ? Beaucoup de produits de faible

valeur, par exemple, le cuivre dans les fils électriques décorant les arbres de Noël, gagnent à être recyclés plutôt que reconditionnés dans la pratique. Pour répondre aux questions ci-dessus, nous modélisons un système d'inventaire à révision périodique et à la commande pour la production de produits nouveaux et reconditionnés. Lorsque des incertitudes subsistent, quant à la quantité et à la qualité de demande et de retour, un décideur fixe dynamiquement les prix de vente, choisit le seuil de qualité optimale de reconditionnement et décide de la quantité de production dans chaque période.

Les contributions théoriques et pratiques de cet article résident dans les aspects suivants. Tout d'abord, la plupart des littératures existantes sur les domaines de tarification dynamique et de classement de qualité des retours n'ont pas mis en lumière la cannibalisation de la demande et la stratégie qualité, ce qui nous permet de combler le déficit des recherches. Deuxièmement, de nombreux chercheurs ont envisagé de compenser l'effet négatif de la cannibalisation de la demande par des stratégies de type, entre autres, renouvellement de permis, publicité et service, mais n'ont pas étudié l'influence du classement de qualité de retour. Troisièmement, en réponse à la suggestion de Kumar and Ramachandran (2016), notre modèle concerne la question de considérer le reconditionnement comme une stratégie d'entreprise dans la gestion des revenus grâce à une stratégie de qualité. Finalement, le modèle peut aider les entreprises à contrôler la quantité de reconditionnement, à bien maintenir l'inventaire des nouveaux produits et à obtenir des revenus élevés de manière plus flexible que de simples stratégies telles que « tout reconditionner », « tout recycler » et/ou « garder des prix constants d'une période à l'autre ».

En caractérisant la politique optimale concernant le modèle monophasé, le modèle dynamique de tarification exogène et le modèle dynamique de tarification endogène, nous constatons que la sélection flexible de classement de qualité non seulement profite aux entreprises en adoptant une quantité optimale de retours, mais sert aussi de stratégie pour contrer les influences négatives de l'effet de cannibalisation. Cependant, selon notre analyse, il est possible que la cannibalisation soit un outil positif pour augmenter les profits malgré son effet

de réduction des ventes de nouveaux produits. Dans une telle circonstance, la sélection du grade de qualité compense la cannibalisation au lieu de la contre-action. Nous montrons également que sous un régime de tarification exogène, le problème peut être décomposé en deux sous-problèmes distincts. Optimiser chacun d'entre eux individuellement conduit à l'optimisation du problème principal. En comparant avec la littérature examinée, nous trouvons que, dans une forme additive de la fonction de demande, la stratégie de tarification des nouveaux produits augmente avec le niveau de stock des produits reconditionnés, ce qui est intuitif, mais incompatible avec la recherche par Yan et coll. (2017) utilisant une forme de fonction d'utilité perçue sans classement de qualité.

Cette thèse articule plusieurs questions importantes dans les opérations durables et la gestion de la chaîne d'approvisionnement, non seulement afin de fournir des idées pour améliorer la performance des entreprises, mais aussi pour inciter ces dernières à adopter les moyens appropriés pour un meilleur environnement de notre société. Le lien entre le niveau de l'entreprise et le niveau de la société est que l'amélioration de la performance écologique par une meilleure gestion des opérations dans les entreprises et les chaînes d'approvisionnement est un élément indispensable pour améliorer l'environnement dans notre société. Prenons la Chine comme exemple. Depuis quelques années, le gouvernement a commencé à favoriser toutes les initiatives pour résoudre les problèmes de pollution de l'air. Un moyen important et utile est de mettre en place une réglementation stricte et de surveiller les efforts des entreprises qui seront passibles d'amendes sérieuses si certaines normes ne sont pas respectées par des inspections aléatoires. Par conséquent, les entreprises doivent coopérer pour améliorer leur rentabilité et, plus important encore, leurs impacts environnementaux. Grâce à cet effort prolongé, malgré le fait que la situation future est incertaine, la qualité de l'air s'est progressivement améliorée en Chine. Cette thèse, dans un cadre plus général, vise à fournir aux entreprises des informations importantes afin qu'elles soient non seulement en mesure de respecter la réglementation, mais aussi en mesure d'apporter véritablement leur contribution à la construction d'un environnement meilleur pour les générations futures.

Notre objectif fondamental est d'obtenir une compréhension approfondie des compromis auxquels les entreprises sont confrontées, de modéliser les problèmes de recherche de solutions possibles et d'aider les entreprises/chaînes d'approvisionnement à améliorer leur performance d'un point de vue théorique. Ensuite, la thèse aidera indirectement les entreprises à réaliser l'importance du développement de moyens de gestion durable des opérations et de la chaîne d'approvisionnement sur notre société.

La thèse est organisée comme la structure suivante. Le chapitre 3 est le premier essai, *Environmental Collaboration and Process Innovation in Supply Chain Management with Coordination*. Le chapitre 4 comprend le contenu du deuxième essai, *Remanufacturing of Multi-Component Systems with Product Substitution*, et le troisième essai, *Joint Dynamic Pricing and Return Quality Strategies Under Demand Cannibalization*, est présenté au chapitre 5. Le chapitre 6 donne les remarques finales générales des trois essais, suivies de la liste de référence, et les annexes.

CHAPTER 3

ENVIRONMENTAL COLLABORATION AND PROCESS INNOVATION IN SUPPLY CHAIN MANAGEMENT WITH COORDINATION

3.1 Abstract

This chapter investigates a dynamic supply chain model in which a supplier decides both the wholesale price and the green process innovation investments while a manufacturer sets the retail price. The green innovation investments not only contribute positively to the environmental performance, but also lead to marginal production cost reductions. The environmental performance positively influences the demand, which in turn damages the environmental performance via negative externalities (e.g., emissions). We resolve this operational trade-off and compare an uncoordinated setting to a revenue sharing contract complemented by a collaborative program. We show that the overall benefit of environmental cooperation in green process innovation entails the existence of a profit-Pareto-improving region. Nevertheless, the maximum environmental performance fails to occur in the profit-Pareto-improving region, which shows the mismatch between economic and environmental performance. Moreover, supply chains might prefer a wholesale price contract to maximize the environmental performance and a revenue sharing contract or vertical integrated chain to maximize profits.

Key words: Supply chain management, Environmental performance, Environmental process innovation, Collaborative program, Supply chain coordination

3.2 Résumé

Cet article étudie un modèle de chaîne d'approvisionnement dynamique dans lequel un fournisseur décide à la fois du prix de gros et des investissements dans l'innovation en matière de processus vert, tandis qu'un fabricant fixe le prix de détail. Les investissements dans l'innovation verte contribuent non seulement positivement à la performance environnementale, mais conduisent aussi à des réductions marginales des coûts de production. La performance environnementale influe positivement sur la demande, ce qui nuit à la performance environnementale via des externalités négatives (par exemple, les émissions). Nous résolvons ce compromis opérationnel et comparons un paramètre non coordonné à un contrat de partage des recettes complété par un programme collaboratif. Nous montrons que l'avantage global de la coopération environnementale dans l'innovation de processus vert implique l'existence d'une région de profit-Pareto-improving. Néanmoins, la performance environnementale maximale ne parvient pas à se produire dans cette région, ce qui montre l'inadéquation entre la performance économique et la performance environnementale. De plus, les chaînes d'approvisionnement pourraient préférer un contrat de prix de gros pour améliorer la performance environnementale et un contrat de partage des recettes ou une chaîne intégrée verticale pour maximiser les profits.

Mots clés: Gestion de la supply chain, Performance environnementale, Innovation de processus environnemental, Programme de collaboration, Coordination de la supply chain

3.3 Introduction

Today's consumers value environment-friendly products more than in the past and firms' environmental performance has a considerable influence on consumers' purchasing decisions (Grimmer & Bingham, 2013). In the car manufacturing industry, this is exemplified by the United States (US) Office of Energy Efficiency & Renewable Energy (2014) announcing an exponential sales increase for both hybrid and plug-in electric vehicles about 45 months after their market introduction. Meanwhile, consumers have shown the same type of behavior in other industries. According to a survey in the UK Department of Energy & Climate Change (2014), consumers position appliances' energy efficiency as a top priority when considering the purchase of new devices, despite their life span or price. For instance, 65% of the surveyed consumers purchases a refrigerator based on its energy efficiency and 64% ranks "good energy rating" as the top priority for new laundry appliances. For both refrigerators and laundry appliances, brand is the second feature that consumers evaluate. The survey also illustrates that the sales of advanced energy-saving products has gradually taken the major market and replaced the regular energy-saving ones, which corroborates that consumers put higher value on green products.

According to these findings, firms are asked to establish green production practices because the market has high expectations with respect to environmental issues and legislation imposes important restrictions on the general impact of a business. Firms are thus called to continuously invest in green innovations to reduce emissions, save energy in production, cut down wastes, take advantage of recycled products (Panda et al., 2017), and, more generally, enhance environmental performance (Schiederig et al., 2012). Although firms commit to environment-friendly strategies, business development can bring the undesired effect of increasing the overall environmental damage. The literature of environmental economics fully supports the assumption that larger demand generates a larger volume of emissions and pollution, thus worsening the overall environmental performance (e.g., pollution games of Jørgensen et al., 2010; El Ouardighi et al., 2016).

This evidence is also confirmed in the car industry, as displayed in Fig. 3.1. In the UK market, car sales and the emissions level (total CO_2 emitted, measured by grams/kilometer) show interesting patterns from 2005 to 2014, which covers the years of the Euro-4 and Euro-5 emission standards (SMMT, n.d.-b) implemented in Europe. In particular, the evidence shows a strong correlation between the number of new cars (British Car Auctions, n.d.) and the emissions level (SMMT, n.d.-a) between 2005 and 2009 in the years of the Euro-4 standards. Although firms invested in green innovation and successfully reduced the single-vehicle emissions in that period, a larger number of cars sold generated more pollution. Starting in 2009, in accordance with Euro-5 standards, firms invested considerably in green process innovations to manufacture greener cars. Therefore, cars emitted even less than before, especially after 2011. However, the emissions level still ramps up with increasing sales. This evidence highlights an interesting operational trade-off we seek to investigate in this research: Investments in green innovation improve the single-product environmental performance but do not improve the overall emissions in the planet due to a larger consumer willingness to purchase cars and then a higher number of cars in a given territory.

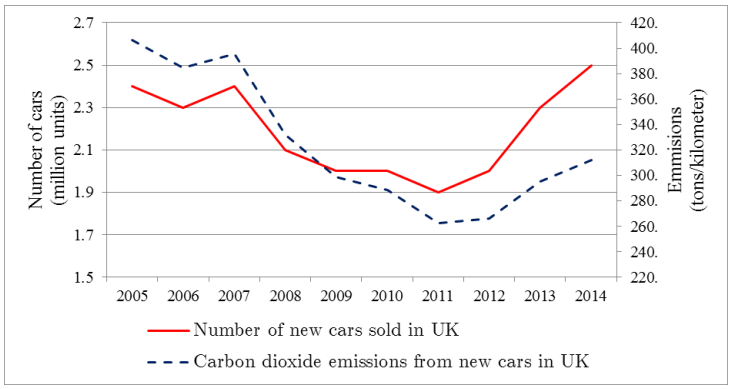


Figure 3.1: UK Cars sales and emissions 2005 to 2014

To address this trade-off, we characterize an innovation game between one component supplier and one manufacturer, in which the supplier invests in a type of process innovation efforts to make the production process greener, namely, *green process innovation*. The green innovation efforts have a triple role:

1. Companies can abate the emissions generated by single goods, thus fulfilling both the consumers' expectations regarding the products' environmental performance as well as legislations.

2. Companies can produce greener products, as consumers prefer to purchase green over brown products. However, the larger sales translates into more pollution as, in general, there exists an average per-product amount of pollution. For example, each unit of Apple's products generates an average of 98 kg CO_2 emissions, including manufacturing and transportation (Cole, 2015). Another example from the US Energy Information and Administration (2016) shows that 1 million British thermal units of energy generated by anthracite lead to 228.6 pounds (103.69 kg) CO_2 emitted.

3. Green process innovation efforts have an unclear impact on the production cost. The literature is characterized by two different schools of thought, according to which process innovation can have a positive impact or negative impact on the marginal production cost. On the one hand, the literature has highlighted the positive role that process innovation has on operations, specifically, on marginal production cost reduction (D'Aspremont & Jacquemin, 1988). For example, Cellini and Lambertini (2009) characterize a marginal production cost as a state variable that decreases according to the research and development (R&D) investments. On the other hand, another part of the literature discusses the negative implications of process innovation efforts on operational efforts. Twenty years ago, Bhoovaraghavan et al. (1996) compared the case of producing conventional cars vs. electric cars and claimed that process innovation strategies generate operational benefits for the production of conventional vehicles but not necessarily for electric cars. More recently, Carrillo-Hermosilla et al. (2010) show that an "end of pipe" green process innovation has a detrimental effect on the production cost as compared to a conventional innovation process (as an example of car painting, see Geffen & Rothenberg, 2000). According to Bhoovaraghavan et al. (1996), any time the process innovation efforts imply a very strong upgrade to a product such that the boundaries between product and process innovation are less clear, firms might expect a

deterioration of the production cost. To emphasize this issue, Genc and De Giovanni (2017) show that investments in green R&D can substantially increase the marginal production cost and thus postpone the decisions to invest in process innovation.

In our model, we seek to investigate the impact of green process innovation efforts on the supply chain (SC) members' profits by analyzing its influence on production cost and checking how firms adjust their strategies accordingly. We first model a benchmark scenario in which the two firms do not cooperate on a green innovation program but rather compete on their pricing strategies. The supplier decides the wholesale price along with the green innovation efforts, while the manufacturer sets the retail price; thus, the SC adopts a wholesale price contract (WPC) to coordinate the financial flows. This setting is very much used in the SC context where the manufacturers are mainly represented by large producers or assemblers (e.g., computer manufacturers, car manufacturers, electronics) that simply purchase components or modules from some suppliers and resell at a larger transfer price after adding a markup (De Giovanni, 2017). The WPC is also popular in SC research as it presents several interesting properties: It is intuitive and easily understood and implemented (Cachon, 2003), it does not require SC members to be formally integrated, and it does not require any additional administrative burdens such as negotiation, transaction, and monitoring costs (De Giovanni, 2014). Nevertheless, because it leaves the double marginalization issue, the literature has proposed the implementation of a revenue sharing contract (RSC) as an alternative coordination mechanism to overcome the WPC's limitations (Moorthy, 1988). An RSC allows a downstream SC member to set the retail price and transfer a share of its revenues to an upstream SC member, which fixes a wholesale price very close to the marginal production cost (e.g., Cachon & Lariviere, 2005). This allows the manufacturer to set a lower retail price and, consequently, boost the demand (Mortimer, 2008). Although both the concept and the mechanism behind an RSC are intuitive and the benefits are highly promising, several doubts remain regarding its real effectiveness. Even in the SC coordination literature, two different schools of thought exist. On the one hand, several contributions

support the adoption of an RSC by virtue of its coordination efficiency (He et al., 2009; Chen et al., 2012). On the other hand, the literature also demonstrates that the adoption of an RSC coordinates an SC only in a few cases. For example, Sluis and De Giovanni (2016) empirically show that lacking a trustful relationship and consumers' integration through demand management lead to failure to adopt an RSC. Moreover, according to Wang and Shin (2015), an RSC is not always beneficial to all SC members although it coordinates the SC. The downstream manufacturer can extract all the SC profit through a quality-dependent W-PC when the product innovation cost is low because this contract can effectively incentivize the supplier to innovate, and sharing revenue is not a preferred option for the manufacturer.

Within each of the proposed scenarios, we characterize a cooperation program in which the manufacturer supports a fraction of the supplier's green innovation efforts by implementing a collaborative program. The marketing literature indicates that adoption of collaborative programs is extremely successful; one firm pays a part of another firm's advertising efforts to boost sales and be economically better off (e.g., He et al., 2009; Jørgensen & Zaccour, 2014). This program has also been successful in operational and closed-loop SC contexts (e.g., De Giovanni & Zaccour, 2013). Nevertheless, relying solely on collaborative programs does not necessarily help in reaching the vertical integrated solution (Zhou & Lin, 2014), but results are usually better than without any cooperative program (De Giovanni, 2011). In our model, we allow the manufacturer to pay a fraction of the green innovation investments, thus also having more control over the innovation level and fostering the supplier's operational benefits of cost reduction. We can then complement our analysis by investigating how cooperation affects the selection of a coordination mechanism as well as the firms' strategies, environmental performance and profits.

Indeed, we seek to identify a coordination mechanism that better mimics the SC integration solution. As documented in the literature, the vertical integrated solution always performs better than a decentralized solution. Thus, we compare all combinations of coordination mechanisms and collaborative programs to the vertical integrated solution to suggest

the best package to adopt.

Our analysis yields the following findings. First, the presence of a collaborative program provides a profit-Pareto-improving region for both players, in both WPC and RSC situations. Thus, both players are likely to be better off if the collaborative program is designed properly. Second, despite the excellence of a vertical integrated chain in terms of SC profit, the environmental performance in decentralized scenarios can surpass that in the vertical integrated chain with certain collaborative programs due to the increasing sales trade-off in which higher sales increases profits but compromises the green performance. Since this greenness beneficial region of the collaborative program parameter does not locate in the profit-Pareto-improving region, decision makers must refer to their corporate business strategy to understand which contract best conforms to the business strategy. We discover that a mismatch between economic and environmental performance always exists. Third, an RSC does not consistently perform better than a WPC for either profit or environmental performance. The reasons locate in the administrative costs as well as the complex effects of both revenue sharing and the collaborative program parameters. Finally, we further extend the analysis in two directions. On the one hand, we analyze production cost in the case in which it increases with green process innovation. We eventually obtain consistent patterns of results. On the other hand, we include a constraint on the revenue sharing and collaborative program parameters. This does not guarantee higher profits and environmental performance when compared with other coordination mechanisms that we explore, but it has the potential to provide better outcomes. Nevertheless, the mismatch effect does not vanish.

The remainder of this essay is organized as follows. Section 3.4 proposes the model and the underlying assumptions, and Section 3.5 presents the equilibria in WPC, RSC, as well as the vertical integrated chain. In Section 3.6, we compare the scenarios and derive managerial implications. Section 3.7 and Section 3.8 provide special variations of the model and concluding remarks, respectively.

3.4 The Model

We consider an SC consisting of a car manufacturer that is in charge of assembly and its supplier that produces engines in automotive sector SCs in which selling more vehicles generates not only higher profits but also excessive emissions. Meanwhile, the supplier is capable of conducting green process innovation to provide greener engines to the manufacturer so as to realize satisfactory environmental performance and profits. The model that we explore fits with the setting of automotive sector SCs and can be generalized to other industries. We therefore consider a generalized SC with two players, a supplier, S, and a manufacturer, M. In the rest of the manuscript, we refer to player S as *he* and player M as *she*. S produces under an infinite production capacity and adopts a just-in-time production philosophy, thus following a make-to-order approach. He produces and sells components (e.g., engines) to M after having realized the demand, $D(t)$. S optimally sets the wholesale price, $w(t)$, at which goods are sold to M and faces a marginal production cost, c_p. He invests in some environmental process innovation efforts, $I(t)$, to achieve higher environmental performance, $E(t)$ for two reasons. On the one hand, S must respect the legislation imposed by the government; on the other hand, today's consumers are environmentally and socially conscious and evaluate firms' environmental performance before making their purchasing decisions. M purchases the components with $w(t)$ and sets the retail price $p(t)$ to downward SC members, where $p(t) > w(t)$. Thus, S's and M's marginal revenues are denoted by $\pi_S = w(t) - c_p$ and $\pi_M = p(t) - w(t)$, respectively. The demand function takes a linear formulation that depends on the retail price, $p(t)$, and the environmental performance of the entire SC, $E(t)$, according to the following formulation:

$$D(p(t), E(t), t) = \alpha - \beta p(t) + \eta E(t) \tag{3.1}$$

where $\alpha > 0$ represents the market potential and $\beta, \eta > 0$ denote the market price elasticity and environmental performance sensitivity, respectively. Eq. (3.1) identifies the existence of

a market independent of the environmental performance. Nevertheless, good environmental performance pushes up sales, thus becoming an interesting marketing lever. This linear demand formulation follows the stylized models presented by Zhang et al. (2016), De Giovanni (2014) and De Giovanni (2016a). It allows one to capture the pricing and environmental performance effects within the SC. The environmental performance, $E(t)$, is described by a differential equation whose motion takes the following form:

$$\dot{E}(t) = mI(t) - g[\alpha - \beta p(t) + \eta E(t)] - nE(t),\ E(0) = E_0 \geq 0 \tag{3.2}$$

The interpretation of Eq. (3.2) is as follows. The environmental performance, $E(t)$, increases with the green process innovation efforts, $I(t)$, according to the environmental efficiency, m. The green process innovation allows S to reduce the impact of his production process on the environment by polluting less. This linear relationship is widely used in the literature of environmental games and innovation games (e.g., Jørgensen & Zaccour, 2001; Breton et al., 2005). Meanwhile, the pollution generated by consumers' product usage (e.g., car emissions, raw materials depletion) drags down the environmental performance. We denote by g the environmental performance deceleration rate due to sales. A similar structure is presented in many papers, such as Jørgensen and Zaccour (2001) and El Ouardighi et al. (2016), when dealing with pollution accumulation. However, the emissions or pollutants are not included in their studies' constructions to affect the demand, which differentiates their models from ours. Finally, the SC environmental performance decreases naturally according to the decay parameter n, representing technology obsolescence.

The green process innovation efforts have influence on the production cost. The literature offers two different approaches in this regard: $I(t)$ can either reduce (Cellini & Lambertini, 2005) or increase (De Giovanni, 2011; Esfahbodi et al., 2016) the marginal production cost depending on the practices employed. In this essay, we introduce the cost factor c, which takes positive values when it reduces the production cost and negative values otherwise.

Conventionally, process innovation aims at cost reduction; thus, $c > 0$. For example, adding a recycling and remanufacturing process for a car body shell reduces waste in aluminum, thus lowering the overall production cost as well as enhancing the environmental performance (Guide Jr et al., 2003; Cassell et al., 2016). However, environmental process innovation efforts do not always reduce the production cost. When a hybrid car engine manufacturer enhances the product's hybrid electric vehicle (HEV) level for better fuel economy, it usually costs more due to the additional purchase/production efforts of greener components such batteries. In these cases, $c < 0$. Hence, we model the dynamic production cost influence as $c_p - cI(t)$. In this essay, we mainly discuss the case with $c > 0$ and treat $c < 0$ in Section 3.7.1 as a special case. Hereafter, the *green process innovation* is by default the type such that $c > 0$. Note that the production cost is reduced but cannot be negative; hence, we impose the following constraint:

$$c_p - cI(t) \geq 0 \tag{3.3}$$

The constraint expresses that the marginal production cost can be reduced no further than 0. In our game, S invests in some environmental process innovation efforts, $I(t)$, while the players can collaborate by negotiating a collaborative program, $B \in (0, 1)$, which informs on the fraction of S's efforts $I(t)$ paid by M. The collaborative program parameter B highlights a promising means that motivates S to invest more in green process innovation to achieve the goals of environmental performance and profits. The green process innovation efforts take a classical quadratic convex form, specifically:

$$C_S(I(t)) = \frac{\mu(1-B)}{2} I(t)^2, C_M(I(t)) = \frac{\mu B}{2} I(t)^2 \tag{3.4}$$

where $\mu > 0$ is an innovation efficiency parameter. Note that we allow firms to negotiate the support parameter before the game starts; thus, both firms have full knowledge on the support parameter B when setting their strategies. When $B = 0$, M does not support the green process innovation efforts and the firms do not collaborate on any green program.

Assume that both players are profit maximizers. The objective functions of S and M in the WPC scenario (W-Scenario) are as follows.

$$J_S^W = \int_0^{+\infty} e^{-rt} \left\{ [\alpha - \beta p(t) + \eta E(t)] \left[w(t) - \left(c_p - cI(t) \right) \right] - \frac{\mu(1-B)}{2} I(t)^2 \right\} dt \quad (3.5)$$

$$J_M^W = \int_0^{+\infty} e^{-rt} \left\{ [\alpha - \beta p(t) + \eta E(t)] [p(t) - w(t) - c_0] - \frac{\mu B}{2} I(t)^2 \right\} dt \quad (3.6)$$

where $r > 0$ is the common discount factor and c_0 represents M's production cost. As c_0 is marginal compared with other costs and it does not affect the insights of our analysis, we neglect c_0 by setting $c_0 = 0$ hereafter without loss of generality. With Eq. (3.2), we maximize both profit functions over infinite time horizon *à la* Stackelberg. S in our game is assumed to possess higher channel power according to his upstream position in the SC, which makes him the Stackelberg leader. In the first stage, S announces the wholesale price and environmental process innovation efforts simultaneously. This is followed by M's pricing strategies in the second stage. The game is solved by feedback strategies.

Then, we compare the W-Scenario with an RSC scenario (R-Scenario) as the firms use a sharing mechanism to coordinate the chain. Here, M shares a part of her marginal profits with S sharing according to the sharing parameter $\phi \in (0, 1)$. We assume that the SC coordinates through an RSC where the sharing parameter is pre-committed; thus, the firms negotiate it before the game starts. This negotiation is highly important for S whose wholesale price becomes null and whose marginal profits will largely depend on M's pricing strategy and the sharing parameter.

Accordingly, the firms' objective functions are modified as follows.

$$J_S^R = \int_0^{+\infty} e^{-rt} \left\{ [\alpha - \beta p(t) + \eta E(t)] \left[p(t)\phi - c_S - \left(c_p - cI(t) \right) \right] - \frac{\mu(1-B)}{2} I(t)^2 \right\} dt \quad (3.7)$$

$$J_M^R = \int_0^{+\infty} e^{-rt} \left\{ [\alpha - \beta p(t) + \eta E(t)] [p(t)(1-\phi) - c_M] - \frac{\mu B}{2} I(t)^2 \right\} dt \quad (3.8)$$

The R-Scenario framework also applies when, for example, the automotive assembler is

cooperating with a research technology center devoted to green process innovation for engines. Thus, our model is extendable to other settings, such as, collaboration with technology institutions. Note that firms also create more complexity by negotiating on the collaborative parameter in the R-Scenario. According to the Transaction Cost Theory and the recent developments in sharing contracts (e.g., De Giovanni, 2014), two transaction cost components affect the efficiency of an RSC: The first is a bargaining cost, which includes all costs required to define an acceptable agreement with the other party to a transaction, or drawing up an appropriate contract; the second consists of the policing and enforcement costs to ensure that the other party sticks to the terms of the contract. In sum, implementing an RSC involves the costs of controlling and monitoring for the SC participants (De Giovanni, 2011). We capture these efforts by means of the transaction costs coefficients c_S and c_M associated with S and M, respectively. From now on, the time argument is omitted.

3.5 Equilibria

3.5.1 The wholesale price contract scenario

In the W-Scenario, the game is played *à la* Stackelberg and S, which is the leader, announces the adoption of a WPC with wholesale price w as well as innovation efforts I; then M reacts to this announcement by optimally setting her pricing strategy, p. S takes in to account the M's strategy and sets both the wholesale price and the green process innovation efforts. Corresponding with the automotive industry as well as the framework we are referring to, the component supplier invests in green process innovation to supply a greener component to the assembler and fixes an optimal wholesale price accordingly; the vehicle assembler decides purchases the greener component and fixes an optimal price that guarantees a certain markup. This setting is consistent with, for example, De Giovanni (2011) and Zhang et al. (2016), which have studied similar settings. Accordingly, firms' optimal strategies can be summarized in the following proposition.

Proposition 1. *The firms' strategies in the W-Scenario are given as follows.*

$$w^W = \frac{c\beta\left(c\alpha + c\eta E - cg\beta V_M^{W\prime} + 2mV_S^{W\prime}\right) - 2\mu\left(1-B\right)\left(\alpha + \beta c_p + \eta E + g\beta V_S^{W\prime} - g\beta V_M^{W\prime}\right)}{c^2\beta^2 - 4\beta\mu\left(1-B\right)} \tag{3.9}$$

$$p^W = \frac{c\beta\left(c\alpha + c\eta E + mV_S^{W\prime}\right) - \mu\left(1-B\right)\left(3\alpha + 3\eta E + \beta c_p + g\beta\left(V_S^{W\prime} + V_M^{W\prime}\right)\right)}{c^2\beta^2 - 4\beta\mu\left(1-B\right)} \tag{3.10}$$

$$I^W = \frac{c\left(\alpha - \beta c_p\right) + c\eta E + \left(4m - cg\beta\right)V_S^{W\prime} - cg\beta V_M^{W\prime}}{4\mu\left(1-B\right) - c^2\beta} \tag{3.11}$$

where S_1, S_2, M_1 and M_2 are the coefficients of the quadratic value functions: $V_S^W = \frac{S_1}{2}E^2 + S_2E + S_3$ and $V_M^W = \frac{M_1}{2}E^2 + M_2E + M_3$.

Proof. See Appendix A. □

All firms' strategies are state-dependent, so both players observe the environmental performance, E, before setting their strategies. However, the analysis of firms' strategies with respect to the state variable is not trivial. As reported in Appendix A, the Riccati system of equations shows that the polynomial terms (S_i, M_i) with $i = 1, 2, 3$ from Eqs. (3.9) to (3.11) are heavily coupled, thus compromising any type of analytical developments. Before proceeding with our analysis, we need to assess the signs of (S_i, M_i). We fix the following baseline parameters as: $\alpha = 1.25$, $\beta = 0.475$, $\eta = 0.125$, $c_p = 0.25$, $c = 0.2$, $m = 1.4$, $g = 0.3$, $n = 0.3$, $\mu = 0.8$, $r = 0.1$. We initially look at the solutions when $B = 0$ and extend the discussion of varying B later on. As three of four solutions to the Riccati system violate either positivity assumptions or constraint Eq. (3.3), or have unsatisfactory robustness when parameter B varies, we select the fourth solution. All the solution candidates are shown in Appendix B. Here, we summarize the results in the following proposition.

Proposition 2. *At the steady-state, the environmental performance E_{SS}^W is positive and $E(t) \in (E_0, E_{SS}^W)$ if $E_0 < E_{SS}^W$ or $E(t) \in (E_{SS}^W, E_0)$ if $E_0 > E_{SS}^W$.*

Proof. Substituting Eqs. (3.9) - (3.11) in Eq. (3.2) leads to $\dot{E}(t) = mI^W(t) - g(\alpha - \beta p^W(t) + \eta E(t)) - nE(t)$. The environmental performance at the steady-state is given by:

$$E_{SS}^{W} = \frac{cgm\beta(2S_2 + M_2) - 4m^2S_2 - cm(\alpha - \beta c_p) + \mu g(1-B)(\alpha - \beta(c_p + gS_2 + gM_2))}{4m^2S_1 + cm\eta + c^2n\beta + \mu g^2\beta(1-B)(S_1 + M_1) - \mu(1-B)(4n + g\eta) - cgm\beta(2S_1 + M_1)} \tag{3.12}$$

where the $NUM(E_{SS}^W) = \Omega_1 < 0$ and $DEN(E_{SS}^W) = \Omega_2 < 0$; thus, $E_{SS}^W > 0$. To check the stability of E_{SS}^W, $\forall t \in (0, \infty)$, solve Eq. (3.2) to get $E(t) = \left(1 - e^{-\frac{\Omega_2}{\Omega_3}t}\right) E_{SS}^W + e^{-\frac{\Omega_2}{\Omega_3}t} E_0$, where $\Omega_3 = c^2\beta - 4\mu(1-B) < 0$. The stability condition ensures that $E(t)$ either increases from E_0 to E_{SS}^W when $E_0 < E_{SS}^W$, or decreases from E_{SS}^W to E_0 when $E_{SS}^W < E_0$. □

We also numerically check the signs of expressions: $c\beta\,(c\eta + mS_1) - \mu(1-B)(3\eta + g\beta(S_1 + M_1)) < 0$, $c\eta + (4m - cg\beta)S_1 - cg\beta M_1 > 0$ and $c\beta(c\eta - cg\beta M_1 + 2mS_1) - 2\mu(1-B)(\eta + g\beta S_1 - g\beta M_1) < 0$ to assess the behavior of strategies, sales and profits with respect to E at the steady-state.

Proposition 3. *Under a WPC, it results that $w(E_{SS}^W)$, $p(E_{SS}^W)$ and $I(E_{SS}^W)$ increase with E_{SS}^W; $V_M^W(E_{SS}^W)$ and $V_S^W(E_{SS}^W)$ increase with E_{SS}^W; the sales $D(E_{SS}^W)$ increases with E_{SS}^W and the double marginalization markup $p(E_{SS}^W) - w(E_{SS}^W)$ increases with E_{SS}^W.*

Proof. Check $\frac{\partial I(E_{SS}^W)}{\partial E_{SS}^W} = \frac{c\eta + (4m - cg\beta)S_1 - cg\beta M_1}{4\mu(1-B) - c^2\beta} > 0$,
$\frac{\partial p(E_{SS}^W)}{\partial E_{SS}^W} = \frac{c\beta(c\eta + mS_1) - \mu(1-B)(3\eta + g\beta(S_1 + M_1))}{c^2\beta^2 - 4\beta\mu(1-B)} > 0$,
$\frac{\partial w(E_{SS}^W)}{\partial E_{SS}^W} = \frac{c\beta(c\eta - cg\beta M_1 + 2mS_1) - 2\mu(1-B)(\eta + g\beta S_1 - g\beta M_1)}{c^2\beta^2 - 4\beta\mu(1-B)} > 0$,
$\frac{\partial D(E_{SS}^W)}{\partial E_{SS}^W} = \frac{\mu\eta(1-B) + c\beta m S_1 - \mu g\beta(1-B)(S_1 + M_1)}{4\mu(1-B) - c^2\beta} > 0$,
$\frac{\partial (p(E_{SS}^W) - w(E_{SS}^W))}{\partial E_{SS}^W} = \frac{c^2\beta^2 g M_1 - c\beta m S_1 - \mu\eta(1-B) + \mu(1-B)g\beta(S_1 - 3M_1)}{c^2\beta^2 - 4\beta\mu(1-B)} > 0$ and finally,
$\frac{\partial V_S^W(E_{SS}^W)}{\partial E_{SS}^W} = S_1 E_{SS}^W + S_2 > 0$ and $\frac{\partial V_M^W(E_{SS}^W)}{\partial E_{SS}^W} = M_1 E_{SS}^W + M_2 > 0$. □

The results from Proposition 3 highlight the key role played by environmental performance, E, to optimally set firms' strategies and assess their profits. High environmental performance is largely beneficial for both firms, if they seek higher stock of E. Thus, in general, it is in the best interest of both firms to push up the stock of E. This result directly

links to the positive effect of E on sales, as displayed in Eq. (3.1). Accordingly, consumers are aware of the companies' environmental performance and prefer to purchase green products (e.g., Grimmer & Bingham, 2013). In the case of high performing products, firms can price higher because of greenness, which entails the consumers' purchasing decisions. In particular, despite the nature of environmental process innovation efforts which reduce the firms' production cost, S sets the wholesale price higher when E is higher, thus resulting in a higher margin as indicated by $\pi_S = w(t) - c_p$. In such a case, as the Stackelberg leader, S is not willingly to lower w^W since this action may provide less incentive for him to invest in the green innovation and E would be lower accordingly. Firms will face a loss in both profit and environmental performance. In conclusion, green process innovation stimulates the increase of environmental performance.

Nevertheless, high demand worsens the environmental performance, thus entailing a challenging trade-off that firms within the SC must solve. In fact, a large stock of E pushes S to invest even more in process innovation to resolve that trade-off. This finding is attributed to two reasons. First, higher innovation efforts are invested to increase the environmental performance, thus counteract the negative effect on sales resulting from the enhanced retail price. This conforms to the results in the SC and distribution channel literature, according to which investments for a strategy (e.g., advertising or quality) that contributes to a state variable (e.g., goodwill or design quality) increases according to the evolution of the state due to increasing prices (e.g., De Giovanni, 2011). Second, as sales is enlarged with the stock of E, higher innovation efforts aim at counteracting the impact of sales in declining E. Thus, we complement the literature such that the increasing innovation offsets the extra absorbing effects by the sales, as in Eq. (3.2).

Our approach allows firms to look at the stock of environmental performance before setting their decisions. Indeed, when the stock takes large values, firms sponsor their green orientation by making additional innovation investments until $I^W = c_p/c$. This allows them to fix enhanced prices although higher prices hurt sales.

Corollary 3.5.1. *When $E_0 < E_{SS}^W$, both the wholesale price and the retail price increase over time, along with the innovation efforts and the firms' profits.*

As the stock $E_0 < E_{SS}^W$. Firms attempt to increase it by investing in process innovation at the beginning and increasing this expenditure over time. Meanwhile, both the wholesale and retail prices increase over time according to the innovation efforts and along with firms' sales and profits increase over time according to E as all constant terms inside the value functions are positive. In contrast, if the environmental performance is already sufficiently large in the beginning of the planning horizon, S is induced to invest less. This leads to lower prices over time and, consequently, to decreasing profits. In sum, when firms already achieve a high level of environmental performance through process innovation, they should look at other strategies to further improve their profits.

Proposition 4. *Under a WPC with a collaborative program, $B^W \in (0,1)$, it results that $w\left(E_{SS}^W\right)$, $p\left(E_{SS}^W\right)$, $I\left(E_{SS}^W\right)$, and $V_S^W(E_{SS}^W)$ increase in B^W while $V_M^W(E_{SS}^W)$ increases in $B^W \iff B^W \in \left(0, B^{W*}\right]$. Complementing a WPC with a collaborative program is profit-Pareto-improving for $B^W \in \left(0, \overline{B}^W\right)$.*

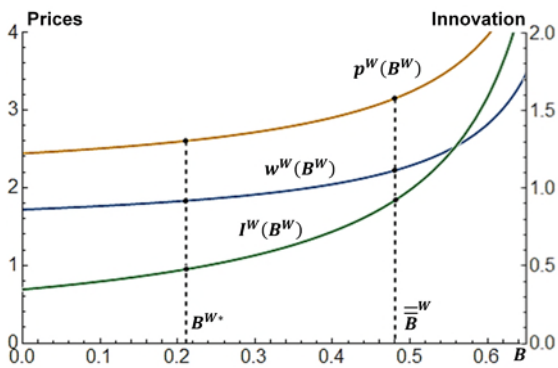
We here refer to B in the W-Scenario as B^W. The results in Proposition 4 have been derived numerically and are displayed in Fig. 3.2 due to the heavy network of relationships existing among all parameter values. Analysis of the results reveals three important ranges of values for B^W, which are evaluated according to M's profits since she is in charge of offering the program fraction B^W of process innovation efforts. For S, in fact, the implementation of a collaborative program is always profit improving. Thus, S would drive the negotiation process by proposing the largest possible B^W, which would be $B^W = \overline{\overline{B}}^W$. $\overline{\overline{B}}^W$ represents the upper bound of B^W above which the game is not feasible due to the boundary constraint Eq. (3.3). When $B^W \in \left(0, B^{W*}\right]$, M's profit increases in the collaborative program until it reaches the maximum profit at B^{W*}, which represents the optimal collaborative program level for M to maximize her profit. If we had modeled an endogenous collaborative program

where M decides B^W, she would fix $B^W = B^{W*}$. Consequently, during the negotiation process, M's best deal would be fixing $B^W = B^{W*}$. Furthermore, when $B^W \in \left(B^{W*}, \overline{B}^W\right]$, M's profit decreases in B^W. Nevertheless, she is economically better off with respect to the non-collaboration case (i.e., $B^W = 0$). Thus, M still prefers the implementation of a collaborative program within this range. Finally, when $B^W \in \left(\overline{B}^W, \overline{\overline{B}}^W\right]$, M's profit continues to decrease in B^W hence resulting not at all convenient. If the negotiation on the collaborative program ends up within this range, M will not be willing to collaborate. In conclusion, a collaborative program based on environmental process innovation is profit-Pareto-improving when $B^W \in \left(0, \overline{B}^W\right]$, while M would not opt for collaboration if $B^W \in \left(\overline{B}^W, \overline{\overline{B}}^W\right]$.

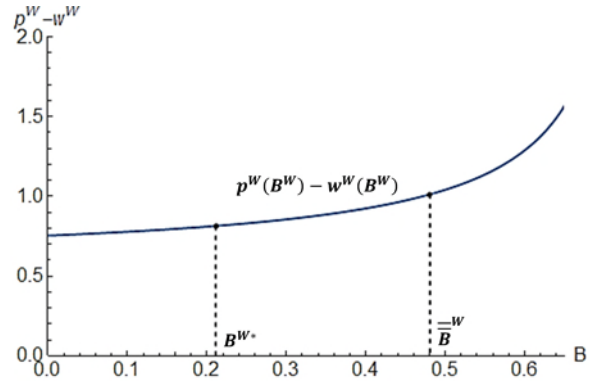
Interestingly, a collaborative program is considerably beneficial for environmental performance, E^W, resulting in $\frac{\partial E^W_{SS}}{\partial B^W} > 0$. Thus, firms within the SC should fix a maximum level of B^W when the optimization of the environmental performance is in the menu. Thus, $B^W = \overline{\overline{B}}^W$. This finding highlights the mismatch existing between economic and environmental performance. Here, M is the challenged firm. When the support B^W is too large, collaboration is not economically convenient, while it turns out to be considerably beneficial for the environment. The maximization of the environmental performance passes through M's decisions and willingness to sacrifice profits to achieve larger environmental benefits.

Intuitively, under a collaborative program, S invests more in environmental process innovation as a part of these investments will be paid by M. Thus, $I^W = c_p/c$ is the maximum investment in environmental process innovation. Furthermore, the existence of a collaborative program allows both firms to price higher. When evaluating a collaborative program, I^W increases in B^W and both w^W and p^W increase in B^W as well. Therefore, high levels of collaboration as well as a larger stock of E suggest that S should invest more in environmental process innovation. In contrast, the firms' pricing strategies are more challenging because collaboration and environmental performance result in a trade-off. Finally, as displayed in Fig. 3.2, the double marginalization effect increases with B^W, highlighting the

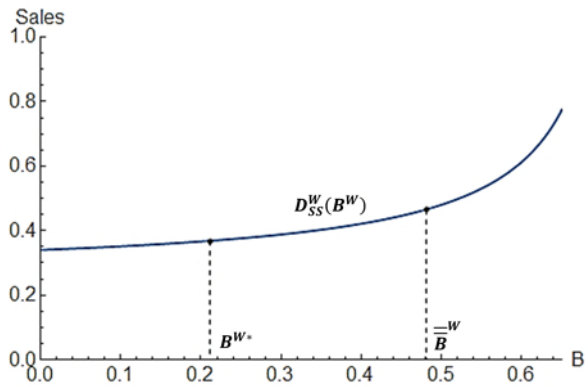
need to implement other coordination mechanisms when targeting to remove the double marginalization issue.



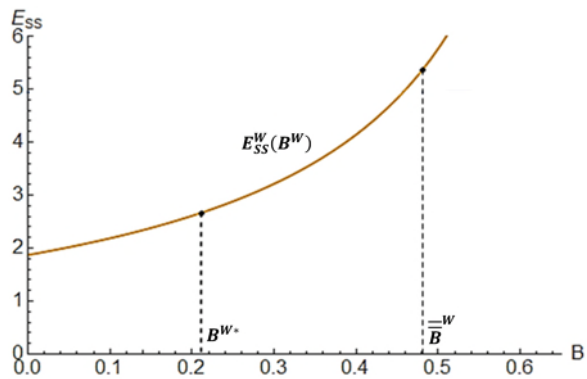
(a) Strategies



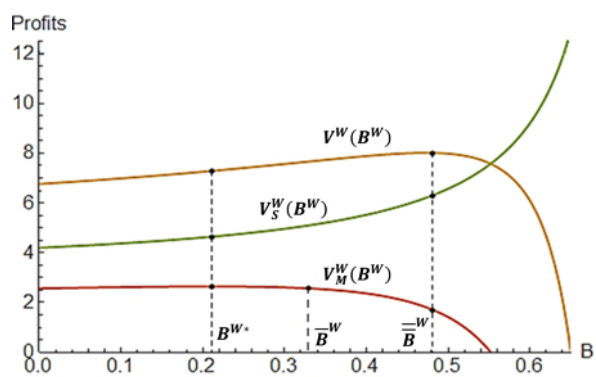
(b) M's profit margin



(c) Sales



(d) Environmental performance



(e) Profits

Figure 3.2: Strategies, M's profit margin, sales, environmental performance and profits in W-Scenario

3.5.2 The revenue sharing contract scenario

In the R-Scenario, the game is played *à la* Stackelberg. S decides the innovation efforts I in the first stage and M sets the retail price p in the second stage. As a powerful mechanism to remedy the double marginalization problem, an RSC has been widely investigated and is commonly used between research institutions and industrial partners in innovation investment (De Giovanni, 2016b). Within our framework, the car assembler retains a certain share of revenue from the sales, and transfers a share to the upstream engine supplier. This mechanism allows the supply chain to seize a larger market and avoid the double marginalization. Again, the car assembler can collaborate to innovate. Moreover, according to Cachon and Lariviere (2005), an RSC induces higher administrative costs than a WPC. S needs to track the sales status of M and she has to cooperate to obtain trust from S. Hence, the administrative cost of S (c_S) is usually higher than that of M (c_M). Denoting the sharing parameter as ϕ and following the modeling technique of De Giovanni (2014) on administrative costs, we take the objective functions Eqs. (3.7) and (3.8), with the conditions that $c_S > c_M$ and ϕ is an exogenous variable.

After solving the game, we summarize the results in the following proposition.

Proposition 5. *The firms' strategies in the R-Scenario are given as follows.*

$$p^R = \frac{\alpha(1-\phi) + \beta c_M + \eta(1-\phi)E + g\beta\left(\widetilde{M}_1 E + \widetilde{M}_2\right)}{2\beta(1-\phi)} \tag{3.13}$$

$$I^R = \frac{c\alpha(1-\phi) - c\beta c_M + c\eta(1-\phi)E + 2m(1-\phi)\left(\widetilde{S}_1 E + \widetilde{S}_2\right) - cg\beta\left(\widetilde{M}_1 E + \widetilde{M}_2\right)}{2\mu(1-B)(1-\phi)} \tag{3.14}$$

where $\widetilde{S}_1$, $\widetilde{S}_2$, $\widetilde{M}_1$ *and* $\widetilde{M}_2$ *are the coefficients of the quadratic value functions:* $V_S^R = \frac{\widetilde{S}_1}{2}E^2 + \widetilde{S}_2 E + \widetilde{S}_3$ *and* $V_R^R = \frac{\widetilde{M}_1}{2}E^2 + \widetilde{M}_2 E + \widetilde{M}_3$.

Proof. See Appendix A. □

In the R-Scenario, the retail price and innovation efforts are state-dependent as well. In Appendix A, the proof of Proposition 5 demonstrates the high complexity of solving the Riccati system of equations explicitly as the coefficients, $\left(\widetilde{S}_i, \widetilde{M}_i\right)$ with $i = 1, 2, 3$, are heavily coupled. We thus reach analytical conclusions by applying the same baseline parameters as in the W-Scenario. In addition, we set $c_S = 0.2$ and $c_M = 0.1$. Similarly, we initially look at the solutions when $B = 0$ and extend the discussion of varying B later on. Moreover, we take an initial $\phi = \phi_0 = 0.65$ such that both players' profits are non-inferior to those in the W-Scenario without any collaborative program ($B = 0$) introduced, that is, to ensure $V_S^R \geq V_S^W$, $V_M^R \geq V_M^W$ at $B = 0$. By solving the Riccati system, we choose the second solution (see Appendix B) which is feasible and has better robustness. We then explore the stability of the steady-state and summarize in the following proposition.

Proposition 6. *At the steady-state, the environmental performance E_{SS}^R is positive and $E(t) \in \left(E_0, E_{SS}^R\right)$ if $E_0 < E_{SS}^R$ or $E(t) \in \left(E_{SS}^R, E_0\right)$ if $E_0 > E_{SS}^R$.*

Proof. Substituting Eqs. (3.13) and (3.14) in Eq. (3.2) leads to $\dot{E}(t)$ $= mI^R(t) - g\left(\alpha - \beta p^R(t) + \eta E(t)\right) - nE(t)$. The environmental performance at the steady-state is given by:

$$E_{SS}^R = \frac{m\left(2m\widetilde{S}_2 + c\alpha\right)(1-\phi) + \beta\left(\mu g(1-B) - cm\right)\left(c_M + g\widetilde{M}_2\right) - g\alpha\mu(1-\phi)(1-B)}{cgm\beta\widetilde{M}_1 + \mu(2n + g\eta)(1-\phi)(1-B) - m\left(2m\widetilde{S}_1 + c\eta\right)(1-\phi) - \mu g^2\beta\widetilde{M}_1(1-B)} \tag{3.15}$$

where $NUM\left(E_{SS}^R\right) = \Pi_1 > 0$ and $DEN\left(E_{SS}^R\right) = \Pi_2 > 0$, thus, $E_{SS}^R > 0$. To check the stability of E_{SS}^R, $\forall t \in (0, \infty)$, solve Eq. (3.2) to get $E(t) = \left(1 - e^{-\frac{\Pi_2}{\Pi_3}t}\right)E_{SS}^R + e^{-\frac{\Pi_2}{\Pi_3}t}E_0$, where $\Pi_3 = 2\mu(1-B)(1-\phi) > 0$. The stability condition ensures that $E(t)$ either increases from E_0 to E_{SS}^R when $E_0 < E_{SS}^R$, or decreases from E_{SS}^R to E_0 when $E_{SS}^R < E_0$. □

We also numerically check the signs of the expressions: $\eta(1-\phi) + g\beta\widetilde{M}_1 > 0$ and $c\eta(1-\phi) + 2m(1-\phi)\widetilde{S}_1 - cg\beta\widetilde{M}_1 > 0$ to assess the behavior of strategies and sales with

respect to E at the steady-state. We summarize the results in the following proposition.

Proposition 7. *Under RSC, it results that* $p\left(E_{SS}^{R}\right)$ *and* $I\left(E_{SS}^{R}\right)$ *increase with* E_{SS}^{R}*;* $V_{M}^{R}(E_{SS}^{R})$ *and* $V_{S}^{R}(E_{SS}^{R})$ *increase with* E_{SS}^{R} *and the sales* $D\left(E_{SS}^{R}\right)$ *increases with* E_{SS}^{R}*.*

Proof. Check $\frac{\partial p(E_{SS}^{R})}{\partial E_{SS}^{R}} = \frac{\eta(1-\phi)+g\beta\widetilde{M}_1}{2\beta(1-\phi)} > 0$, $\frac{\partial I(E_{SS}^{R})}{\partial E_{SS}^{R}} = \frac{\left(c\eta+2m\widetilde{S}_1\right)(1-\phi)-cg\beta\widetilde{M}_1}{2\mu(1-\phi)(1-B)} > 0$, $\frac{\partial D(E_{SS}^{R})}{\partial E_{SS}^{R}} = \frac{\eta(1-\phi)-g\beta\widetilde{M}_1}{2(1-\phi)} > 0$ and finally, $\frac{\partial V_{S}^{R}(E_{SS}^{R})}{\partial E_{SS}^{R}} = \widetilde{S}_1 E_{SS}^{R} + \widetilde{S}_2 > 0$ and $\frac{\partial V_{M}^{W}(E_{SS}^{W})}{\partial E_{SS}^{W}} = \widetilde{M}_1 E_{SS}^{R} + \widetilde{M}_2 > 0$. □

Consistent with the arguments in the W-Scenario, both strategies $p\left(E_{SS}^{R}\right)$ and $I\left(E_{SS}^{R}\right)$ increase with the steady-state E_{SS}^{R}, which indicates that once high environmental performance is observed, firms price more and invest more in green process innovation. Under an RSC, firms' sales and profits increase with E_{SS}^{R}, which highlights the firms' goal of reaching a certain level of environmental performance and customers' preference for greener products. Before reaching an agreement on an RSC, firms observe their initial stock of E to set strategies over time and, more importantly, the evolving behavior of profits. We therefore summarize the results in the following corollary.

Corollary 3.5.2. *When* $E_0 < E_{SS}^{R}$*, the retail price, the innovation efforts and the firms' profits increase over time.*

Corollary 3.5.2 expresses the same idea as Corollary 3.5.1 in the W-Scenario. Under RSC, after agreeing on a sharing parameter ϕ, firms with lower initial environmental performance stock ($E_0 < E_{SS}^{R}$) are expected to increase green innovation efforts over time until reaching the steady-state. At the same time, M gradually prices the product higher. Consequently, both firms' profits increase with E from E_0 to E_{SS}^{R} even with higher innovation expenditures. This is also consistent with the nature of an RSC such that large downstream player's profit makes the upstream better off, given a fixed share ϕ. On the other hand, if the initial environmental performance stock is high ($E_0 > E_{SS}^{W}$), we reverse the above analysis as in the W-Scenario. These firms are motivated to find other strategies for better outcomes.

Basically, two options are available: to vary the collaborative program parameter B or to alter the RSC parameter ϕ in the contract. To this end, we state the following two propositions discussing the effect of the two parameters. Here, we refer to parameter B as B^R for the R-Scenario.

Proposition 8. *Under an RSC with a collaborative program, $B^R \in (0,1)$, it results that $p\left(E_{SS}^R\right)$, $I\left(E_{SS}^R\right)$, and $V_S^R(E_{SS}^R)$ increase in B^R while $V_M^R(E_{SS}^R)$ increases in $B^R \iff B^R \in \left(0, B^{R*}\right]$. Complementing an RSC with a collaborative program is profit-Pareto-improving for $B^R \in \left(0, \overline{B}^R\right)$.*

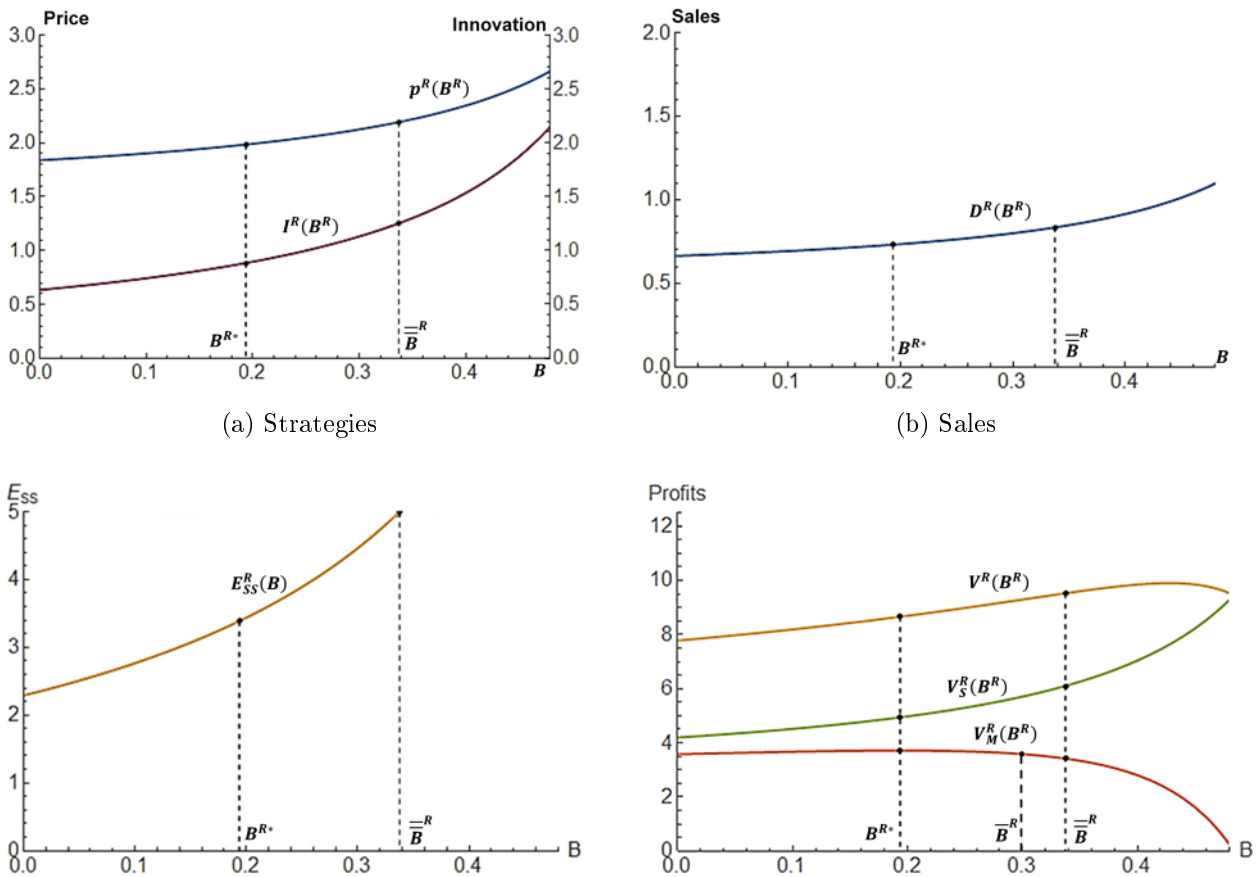
The results in Proposition 8 have been derived numerically and are displayed in Fig. 3.3, due to the heavy network of relationships existing among all parameter values. The information conveyed here is in fact consistent with the W-Scenario.

In the R-Scenario, three important values of B^R, denoted by B^{R*}, $\overline{B}^R$, and $\overline{\overline{B}}^R$, indicate different decision-making prescriptions. In $\left(0, \overline{\overline{B}}^R\right]$, S always prefers large B^R as $V_S^R(E_{SS}^R)$ increases with it and $\overline{\overline{B}}^R$ maximizes his profit. If we look at the innovation efforts strategy, the collaboration program motivates S to invest more as M supports a part of the expense: the more profit M gains, the more she shares. Her profit ceases to increase further because any value of B^R above $\overline{\overline{B}}^R$ violates the constraint Eq. (3.3). Assuming B^R continues to increase, this is equivalent to extracting profit from M and transferring it to S, while S does not utilize the extra money for innovation. Therefore, M never agrees and the collaborative program fails.

On the side of M, $\left(0, \overline{B}^R\right)$ is profit-Pareto-improving, compared to the case of $B^R = 0$. She is therefore inclined to take $B^R = B^{R*} \in \left(0, \overline{B}^R\right)$, which maximizes her profit. If we assume B^R is an endogenous variable decided by M, she would take B^{R*} as the reaction to the proposal for a collaborative program in green process innovation.

From the perspective of environmental performance, a collaborative program can ameliorate the stock of E_{SS}^R, which results in $\frac{\partial E_{SS}^R}{\partial B^R} > 0$. The maximum stock occurs at the point of $B^R = \overline{\overline{B}}^R$, which does not belong to the profit-Pareto-improving region. M is again

challenged by the trade-off between profit and environmental performance like she is in the W-Scenario. Thus, the mismatch of economic and environmental performance exists in RSC as well. To further explore whether altering ϕ unravels this trade-off, we summarize the results in the following proposition.



(a) Strategies

(b) Sales

(c) Environmental performance

(d) Profits

Figure 3.3: Strategies, sales, environmental performance and profits in R-Scenario

Proposition 9. *Under an RSC with a collaborative program where $B^R = 0$, it results that E_{SS}^R increases in $\phi \iff \phi \in \left(\underline{\underline{\phi}}, \underline{\phi}\right]$, $p\left(E_{SS}^R\right)$ increases in ϕ, $V_M^R(E_{SS}^R)$ decreases in ϕ, $I\left(E_{SS}^R\right)$ increases in $\phi \iff \phi \in \left(\underline{\underline{\phi}}, \widehat{\phi}\right]$ while $V_S^R(E_{SS}^R)$ increases in $\phi \iff \phi \in \left(\underline{\underline{\phi}}, \phi^*\right]$, $\underline{\phi} > \widehat{\phi} > \phi^*$.*

The results in Proposition 9 have been derived numerically and are displayed in Fig. 3.4. The feasible region of ϕ is basically divided into four parts, as follows.

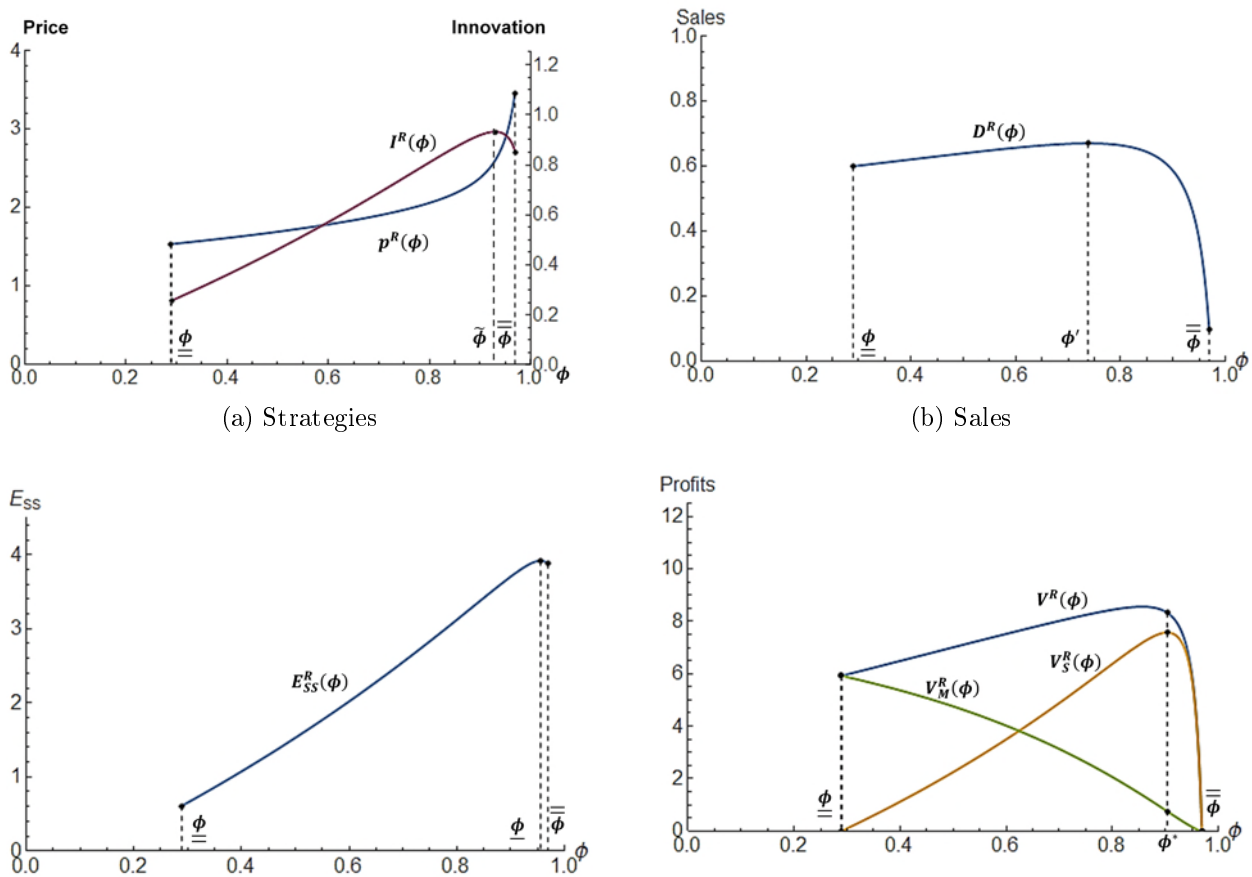
- When $\phi \in \left(\underline{\underline{\phi}}, \phi^*\right]$, where $\underline{\underline{\phi}}$ represents the lower bound of ϕ for keeping both players profitable and ϕ^* is the value maximizing $V_S^R(E_{SS}^R)$. S's profit initially grows with ϕ because M transfers an increasing portion of revenue to him. However, a sufficiently large ϕ brings down his profit drastically because the total revenue generated by M is too low. The threshold is thus characterized by ϕ^*. This indeed creates a dilemma for S. On the one hand, he prefers high ϕ to share the revenue as much as possible; on the other hand, he does not want it too high so that the downstream player stops making enough profit from the business.

- When $\phi \in \left(\underline{\underline{\phi}}, \widehat{\phi}\right]$, where $\widehat{\phi}$ denotes the value to maximize $I\left(E_{SS}^R\right)$. This threshold point $\widehat{\phi}$ provides an interesting trade-off for both players. As ϕ is exogenous, after observing the steady-state under $\widehat{\phi}$, that is $E_{SS}^R\left(\widehat{\phi}\right)$, firms realize that they can increase their profits but end up by worsening the environmental performance if a lower $\phi = \phi^*$ is permitted; or increase the environmental performance with sacrificing the profits if higher $\phi = \overline{\overline{\phi}}$ is permitted. Surprisingly, in either case, S is going to invest in greenness less than when $\phi = \widehat{\phi}$. The former case implies that less innovation brings lower environmental performance, but the latter case leads to the opposite result. In fact, by jointly looking at the pricing strategy, we observe that the price goes up steeply by increasing ϕ from $\widehat{\phi}$ and thus decreases sales. As indicated in Eq. (3.2), less sales results in less production but better environmental performance. Hence, firms must evaluate their business strategy toward greenness to decide on their strategies.

- When $\phi \in \left(\underline{\underline{\phi}}, \underline{\phi}\right]$, where $\underline{\phi}$ is the value maximizing E_{SS}^R. At $\phi = \underline{\phi}$, the goal of greenness is maximally pursued but the retail price is extremely high, which makes this point dangerous because of the declining sales and the negligible profits that both players obtain. Although green products are still well accepted if price is increased within an acceptable range, they stop serving as a good option if the price is too high to afford (Grimmer & Bingham, 2013). Therefore, the mismatch of economic and environmental performance is not resolved by only evaluating ϕ.

- When $\phi \in \left(\underline{\underline{\phi}}, \overline{\overline{\phi}}\right)$, where $\overline{\overline{\phi}}$ highlights the upper bound of ϕ to keep both players making profits because any point onward causes at least one player to have a negative profit.

In sum, we observe that M's profit keeps dropping with higher ϕ in $\left(\underline{\underline{\phi}}, \overline{\overline{\phi}}\right)$, which is consistent with the rationale of RSC. Meanwhile, S's preference on ϕ is not monotone as he first prefers to share the revenue with a larger percentage. But too large a value is detrimental to M's profit so that M does not well cooperate on the investment and eventually decreases S's profit as well. Moreover, the mismatch between Pareto-improving profits and maximum environmental performance does not vanish in an RSC.



(a) Strategies

(b) Sales

(c) Environmental performance

(d) Profits

Figure 3.4: Strategies, sales, environmental performance and profits in R-Scenario (ϕ)

3.5.3 The vertical integrated scenario

We here analyze the V-Scenario where we no longer deal with either wholesale price or innovation collaborative program. Accordingly, the V-Scenario introduces a benchmark that removes all economic inefficiencies. We aim to compare later the W-Scenario and the R-Scenario to the V-scenario to assess whether the simultaneous presence of a certain contract complemented with a cooperative program allows a decentralized supply chain to reach the centralized solution while investigating both the economic and the environmental performance. As a single decision maker is determining both the innovation and retail price strategies, we solve an optimal control problem. The objective function is as follows.

$$J^V = J_S^W + J_M^W = \int_0^{+\infty} e^{-rt} \left\{ [\alpha - \beta p(t) + \eta E(t)] \left[p(t) - \left(c_p - cI(t) \right) \right] - \frac{\mu I(t)^2}{2} \right\} dt \tag{3.16}$$

Together with the state equation Eq. (3.2), we solve the problem by feedback strategies and the results are displayed in the following proposition.

Proposition 10. *The strategies in the V-Scenario are as follows.*

$$p^V = \frac{\mu\left(\alpha + \beta c_p\right) - c^2\alpha\beta + \eta\left(\mu - c^2\beta\right)E + \beta\left(g\mu - cm\right)\left(k_1 E + k_2\right)}{\beta\left(2\mu - c^2\beta\right)} \tag{3.17}$$

$$I^V = \frac{c\left(\alpha - \beta c_p\right) + c\eta E + \left(2m - cg\beta\right)\left(k_1 E + k_2\right)}{2\mu - c^2\beta} \tag{3.18}$$

where k_1, k_2 are the coefficients of the quadratic value function: $V^V = \frac{k_1}{2}E^2 + k_2 E + k_3$.

Proof. See Appendix A. □

Both retail price strategy and innovation strategy are state-dependent in the vertical integrated chain. As displayed in Appendix A, both strategies in Eqs. (3.17) and (3.18) include the coefficients of the value function, k_1 and k_2. Moreover, k_1, k_2 and k_3 are highly coupled with the regarding parameters. Again, solving the Riccati system explicitly is extremely complex and can hardly lead to clear analytical insights. We thus carry out the

analysis by using the same baseline parameters as in the W-Scenario. We choose the second solution (see Appendix B) with higher robustness from the solutions to the Riccati system and summarize in the following proposition the evolving behavior toward the steady-state.

Proposition 11. *At the steady-state, the environmental performance E_{SS}^V is positive and $E(t) \in \left(E_0, E_{SS}^V\right)$ if $E_0 < E_{SS}^V$ or $E(t) \in \left(E_{SS}^V, E_0\right)$ if $E_0 > E_{SS}^V$.*

Proof. Substituting Eqs. (3.17) and (3.18) in Eq. (3.2) leads to $\dot{E}(t) = mI^V(t) - g(\alpha - \beta p^V(t) + \eta E(t)) - nE(t)$. The environmental performance at the steady-state is given by:

$$E_{SS}^V = \frac{\left(2m^2 - 2cgm\beta + g^2\beta\mu\right)k_2 + (cm - g\mu)\left(\alpha - \beta c_p\right)}{\left(2cgm\beta - 2m^2 - g^2\beta\mu\right)k_1 - (cm - g\mu)\eta + \left(2\mu - c^2\beta\right)n} \tag{3.19}$$

where $NUM\left(E_{SS}^V\right) = \Phi_1 > 0$ and $DEN\left(E_{SS}^V\right) = \Phi_2 > 0$, thus, $E_{SS}^V > 0$. To check the stability of E_{SS}^V, $\forall t \in (0, \infty)$, solve Eq. (3.2) to get $E(t) = \left(1 - e^{-\frac{\Phi_2}{\Phi_3}t}\right)E_{SS}^V + e^{-\frac{\Phi_2}{\Phi_3}t}E_0$, where $\Phi_3 = 2\mu - c^2\beta > 0$. The stability condition ensures that $E(t)$ either increases from E_0 to E_{SS}^V when $E_0 < E_{SS}^V$, or decreases from E_{SS}^V to E_0 when $E_{SS}^V < E_0$. □

Proposition 11 verifies the convergence of the state variable to stability such that wherever the environmental performance starts, it converges to the steady-state. We numerically check the signs of expressions: $\eta\left(\mu - c^2\beta\right) + \beta(g\mu - cm)k_1 > 0$ and $c\eta + (2m - cg\beta)k_1 > 0$ to assess the behavior of strategies and sales with E at the steady-state. We can thus state the following proposition.

Proposition 12. *Under V-Scenario, it results that $p(E_{SS}^V)$ and $I(E_{SS}^V)$ increase with E_{SS}^V; $V^V(E_{SS}^V)$ increases with E_{SS}^V while the sales $D(E_{SS}^V)$ increases with E_{SS}^V.*

Proof. Check $\frac{\partial p(E_{SS}^V)}{\partial E_{SS}^V} = \frac{\eta(\mu - c^2\beta) + \beta(g\mu - cm)k_1}{\beta(2\mu - c^2\beta)} > 0$, $\frac{\partial I(E_{SS}^V)}{\partial E_{SS}^V} = \frac{c\eta + (2m - cg\beta)k_1}{2\mu - c^2\beta} > 0$, $\frac{\partial D(E_{SS}^R)}{\partial E_{SS}^R} = \frac{\mu\eta - \beta(g\mu - cm)k_1}{2\mu - c^2\beta} > 0$ and finally, $\frac{\partial V^V(E_{SS}^V)}{\partial E_{SS}^V} = k_1 E_{SS}^V + k_2 > 0$. □

Proposition 12 shows that the retail price increases with the steady-state environmental performance (E_{SS}^V). Investing in E_{SS}^V provides the possibility of raising the retail price

without losing sales, while customers have higher willingness to pay for greener products (Grimmer & Bingham, 2013). Moreover, higher environmental performance motivates the firm to increase innovation efforts $I\left(E_{SS}^{V}\right)$ and the channel profit goes up with E_{SS}^{V}. In the following corollary, we assess how the strategies and profits evolve over time starting from E_0 and converging to E_{SS}^{V}.

Corollary 3.5.3. *When $E_0 < E_{SS}^{V}$, the retail price increases over time, while the innovation efforts and the SC profit increase over time.*

Corollary 3.5.3 expresses the same idea with both Corollary 3.5.1 and 3.5.2 but by taking the SC's viewpoint.

3.6 Comparison among scenarios

In this section, we first perform the comparisons among the W-Scenario, R-Scenario, and V-Scenario regarding the strategies, state variable, sales, and profits. The comparisons provide information on the contracts' advantages and disadvantages, thus giving insights to managers regarding the mechanism to be adopted. Following that, we analyze the RSC parameter ϕ to understand the players' outcomes and evaluate the mismatch between economic and environmental performance.

3.6.1 Comparison of strategies

In Fig. 3.5, we display the strategies that firms set according to the contract they adopt. For the retail pricing strategy, if the collaborative program is not adopted, we obtain that $p_{B^W=0}^{W} > p^V > p_{B^R=0}^{R}$. Here, retail price of the W-Scenario ranks the highest due to the existence of the wholesale price so that M must set a high retail price to keep the business profitable. This indeed implies the drawback of double marginalization in WPC. In contrast, the retail price of the R-Scenario is lower than p^V. Despite that, the innovation efforts are

not invested to the maximum value so that M needs to set lower price to guarantee sales and profits.

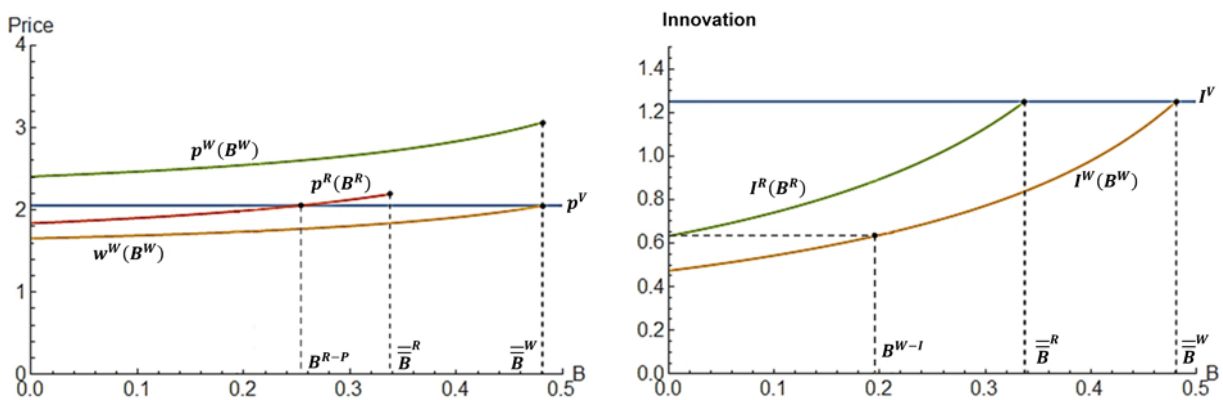
Regarding the innovation efforts strategy, we have $I^V > I^R_{B^R=0} > I^W_{B^W=0}$. Recall that we choose a value of ϕ such that both players' profits in the R-Scenario are not inferior to those in the W-Scenario when $B^W = B^R = 0$. Therefore, by switching from WPC to RSC, both players are better off by investing more, which also contributes to weaken the double marginalization. Indeed, the maximum efforts occur in the V-Scenario where the single decision maker prefers to invest more, to the optimal amount.

When a collaborative program complements the contract, it provides some changes for firms' strategies. First, with M sharing the innovation expense, she sets the retail price equal to p^V when $B^R = B^{R-P}$ (B^{R-P} is the value taken by B^R when the retail price of RSC equals to the vertical integrated price) and continues to increase it until $B^R = \overline{\overline{B}}^R$. Nevertheless, the maximum profit of M occurs when $B^R = B^{R*}$, which is lower than B^{R-P};thus, $p^R_{B^R=B^{R*}} < p^V$. That is, when M's profit is maximized, the retail price she sets in the R-Scenario is lower than that in the V-Scenario. This finding highlights that with $B^R = B^{R*}$, since the decentralized SC under RSC invests less in green process innovation than a centralized SC, M has to set a lower retail price than p^V to ensure sufficient sales to maintain profitability. When B^R further increases, M can set a higher retail price but her profit starts to decline.

Second, the W-Scenario retail price is always larger than p^V. This result is consistent with the widely documented finding that the WPC decentralized optimal retail price under a collaborative program is higher than the centralized retail price (e.g., He et al., 2009).

Eventually, $I^V = I^R_{B^R=B^{R*}} = I^W_{B^W=B^{W*}}$ where the innovation efforts are invested maximally. For the V-Scenario, the boundary solution when $I^V = c_p/c$ coincides with the interior solution. However, for the other two scenarios, constraint Eq. (3.3) is activated and hinders the investment from increasing. With collaborative program, $I^R_{B^R=B} > I^W_{B^W=B}$, for any value of B in the feasible region, indicates that an RSC provides higher motivation for S to

invest. Moreover, when complemented by a collaborative program of $B^W \in \left[B^{W-I}, \overline{\overline{B}}^W\right]$ in WPC (B^{W-I} refers to the value taken by B^W when the innovation efforts in WPC equals to the innovation efforts in RSC at $B^R = 0$), we can always identify a B^R in RSC that makes the investment efforts in the two contracts equivalent (e.g., $I^R_{B^R=0} = I^W_{B^W=B^{W-I}}$).



(a) Pricing strategies (b) Innovation efforts

Figure 3.5: Strategies comparison among W-Scenario, R-Scenario and V-Scenario

3.6.2 Comparison of environmental performance

Fig. 3.6 shows the environmental performance that firms can achieve depending on the contract adopted as well as the presence of a collaborative program. Consistent with the literature, the vertical integrated solution represents a benchmark that contracts alone can never mimic. In fact, when a collaborative program is not in place, the stock at the steady-state gives $E^V_{SS} > E^R_{SS|B^R=0} > E^W_{SS|B^W=0}$. Accordingly, firms should aim to become fully integrated to achieve high environmental performance while departing from traditional contract mechanisms that turn out to be less effective. Furthermore, if SC integration is not a feasible option, firms should coordinate by adopting an RSC. Without any type of collaboration on environmental process innovation, firms can achieve better environmental performance when aiming at SC coordination through an RSC.

The presence of a collaborative program allows firms to completely change the previous

assessment. As mentioned in Propositions 4 and 8 and displayed in Figs. 2 and 3, the stock of E increases with B^W and B^R, respectively. According to Fig. 3.6, firms' preferences regarding the selection of contracts and collaborative programs change according to the support rate, B:

- When $B \in \left(\underline{B}^R, \overline{\overline{B}}^R\right]$, firms maximize the environmental performance by adopting an RSC.

- When $B \in \left(\underline{B}^W, \overline{\overline{B}}^W\right]$, firms maximize the environmental performance by adopting a WPC.

- Firms can maximize the environmental performance by integrating the SC in all other cases.

These ranges supply interesting findings, especially in accordance with the literature. First, the vertical integrated solution represents a benchmark suggesting that firms can hit high performance when complementing a coordination mechanism with a collaborative program. This finding deviates from the widely accepted extant literature in collaborative programs arguing that the vertical integrated steady-state (e.g., goodwill) always outperforms the decentralized cases (Liu et al., 2015; Zhang et al., 2013) even with a collaborative program. This is attributable to the trade-off of stimulating profit and harming environmental performance based on higher sales. Among the three scenarios, the V-Scenario offers the highest sales and, thus, in fact, considerably damages the environmental performance. When the innovation efforts of decentralized scenarios reach a certain level, the greenness increases to a larger extent.

Second, a generally accepted result is that the RSC works better than the WPC (Cachon & Lariviere, 2005). However, we show that the WPC can provide more opportunities to complement the issue of coordination with a collaborative program due to a larger space available for B^W before violating constraint Eq. (3.3). Overall, firms can identify a region of B^W in which WPC works better than RSC in achieving higher environmental performance. Therefore, if firms aim to maximize environmental performance, they should adopt a WPC

complemented by a collaborative program with $B^W \in \left(\underline{B}^W, \overline{\overline{B}}^W\right]$.

Finally, although the RSC works better than the WPC when $B \in \left(\underline{B}^R, \overline{\overline{B}}^R\right]$, it is always possible to use a larger value of B for a WPC to make the two contracts indifferent from an environmental viewpoint. For example, if a WPC is complemented with $B = \widehat{B}^W$, we have $E^R_{SS|B^R=0} = E^W_{SS|B^W=\widehat{B}^W}$. Thus, the presence of a collaborative program can allow firms to enhance their environmental performance through coordination with a WPC.

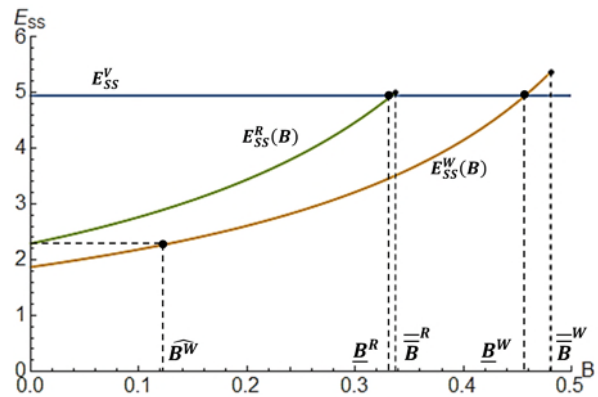


Figure 3.6: Environmental performance comparison among W-Scenario, R-Scenario and V-Scenario

3.6.3 Comparison of sales

In Fig. 3.7, we observe that the sales of the W-Scenario are the lowest while that of the V-Scenario are at the highest position when no collaborative program is introduced. By referring to the results of retail pricing strategy and the environmental status, together with the demand function Eq. (3.1), M sets the retail price much higher in a WPC than in the other two and the corresponding environmental performance ranks the lowest. Thus, the sales statuses are easy to understand.

With a collaborative program, the findings are summarized as follows.

- The players expect higher sales under either WPC or RSC with a collaborative program.
- Within any feasible B for both B^W and B^R, $D^R_{B^R} > D^W_{B^W}$ if $B^R = B^W$.
- Neither contract manages to reach the sales in the vertical integrated chain.

The rationale behind the last finding is as follows. According to Section 3.6.2, although the best environmental performance in either WPC or RSC is superior to the V-Scenario at the upper bound of B, (i.e., $E^R_{SS|B^R=\overline{\overline{B}}^R} > E^V_{SS}$ and $E^W_{SS|B^W=\overline{\overline{B}}^W} > E^V_{SS}$), the associated retail prices of the two scenarios are also much higher. Considering Eq. (3.1), we conclude that when B grows, as consumers are highly price sensitive, the influence of the price increase becomes stronger than the environmental performance increase so that the total sales is lower than in the V-Scenario.

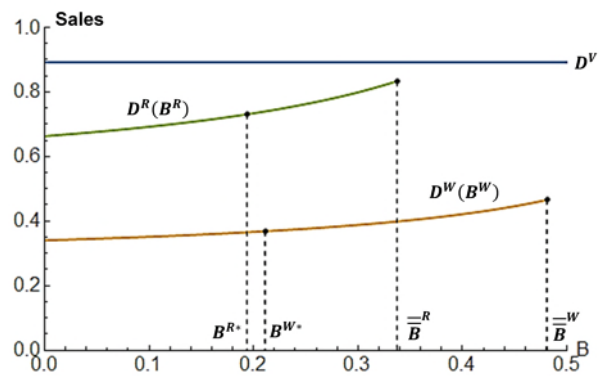


Figure 3.7: Sales comparison among W-Scenario, R-Scenario and V-Scenario

3.6.4 Comparison of firms' profits

In Fig. 3.8, we compare the profits among the three scenarios. Recall that we deliberately set $\phi = \phi_0$ such that $V^R_S \geq V^W_S$, $V^R_M \geq V^W_M$ when $B^R = B^W = 0$. Therefore, without a collaborative program, an RSC is preferred to a WPC as both players are economically better off. Nevertheless, the vertical integrated profit is still unreachable. Thus, the double marginalization is remedied but not fully eliminated by RSC without a collaborative program.

If we let B increase, the double marginalization still exists. For WPC, this result is consistent with previous literature discussing WPC (e.g., He et al., 2009; El Ouardighi, 2016). For RSC, the key to its inefficiency is that we take administrative costs into account, which drags down the efficiency of an RSC. If we remove the administrative costs, the SC

profit V^R approaches and finally becomes very close to V^V by raising B^R, but is still not capable of reaching it. This unreachability results from the maximum innovation efforts stemming from the constraint Eq. (3.3), which limits the chances of further increasing SC profit. Therefore, in accordance with the literature (De Giovanni, 2011), the vertical integrated profit is unreachable in either the WPC or RSC considering administrative costs even with the collaborative programs.

However, increasing B creates decision-making thresholds in choosing between WPC and RSC in a decentralized setting. We observe that:

- When $B \in \left(0, \widehat{B}^{W-P}\right)$ ($\widehat{B}^{W-P}$ is the value taken by B^W making S's profit in WPC equal to S's profit in RSC when $B^R = \overline{\overline{B}}^R$), the firms maximize each of their profits by adopting an RSC.

- When $B \in \left(0, \overline{\overline{B}}^W\right]$, S maximizes his profit by adopting an WPC as $V^W_{S|B^W} > V^R_{S|B^R}$ where $B^W \in \left(\widehat{B}^{W-P}, \overline{\overline{B}}^W\right]$ and $B^R \in \left(0, \overline{\overline{B}}^R\right]$.

The results are interesting for several reasons. First, WPC provides more opportunities to complement the issue of coordination with a collaborative program and S can identify a region such that $B \in \left(\widehat{B}^{W-P}, \overline{\overline{B}}^W\right]$ where a WPC economically works better than an RSC. Thus, if S's economic benefits are under strong protection, the exogenous B is likely to be designed in $\left(\widehat{B}^{W-P}, \overline{\overline{B}}^W\right]$ so as to encourage the employment of a WPC with such a B^W.

Moreover, M always benefits from participating in an RSC as $V^W_{M|B=B^{W*}} < V^R_{M|B=\overline{\overline{B}}^R}$ where the latter item is the minimum profit when taking an RSC, and RSC provides both players with a profit-Pareto-improving region $B \in \left(0, \overline{\overline{B}}^R\right]$ by applying the same B for RSC and WPC.

3.6.5 Comparisons of environmental performance and profits with varying ϕ

In this section we first analyze the changes in the findings in the R-Scenario and the different positioning with respect to both the W-Scenario and the V-Scenario, when ϕ takes lower values. Moreover, we investigate the existence of a mismatch between economic and envi-

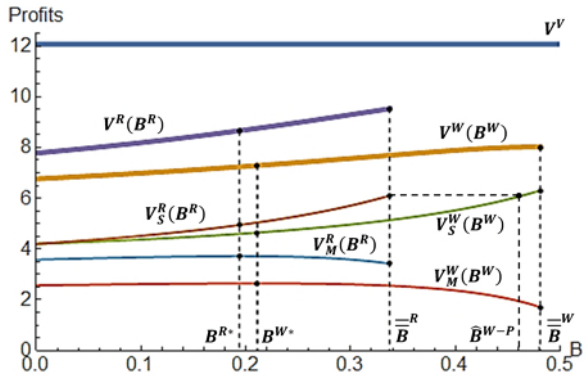


Figure 3.8: Profits comparison among W-Scenario, R-Scenario and V-Scenario

ronmental performance in this setting. We refer to this scenario as the R_L-Scenario, which indicates that S agrees to receive a lower share of M's revenue, although he pays higher administrative costs. Intuitively, S gets lower profits and has a lower willingness to invest in environmental process innovation. The latter implies lower environmental performance as compared to the W-Scenario, independent of the collaborative program B, as displayed in Fig. 3.9. For the sake of clarity, we keep the notation of the support rate as B^R when specifying this parameter in the R_L-Scenario.

With sharing the lower portion of revenue (in the analysis $\phi = 0.4$), S is unwilling to invest in green innovation as much as in the W-Scenario without a collaborative program. By increasing B, we characterize B's feasible regions into three parts and obtain findings as follows.

- When $B \in \left(\underline{B}^W, \overline{\overline{B}}^W\right]$, firms maximize the environmental performance by adopting a WPC.

- When $B \in \left(\underline{B}^R, \overline{\overline{B}}^R\right]$, firms maximize the environmental performance by adopting an RSC.

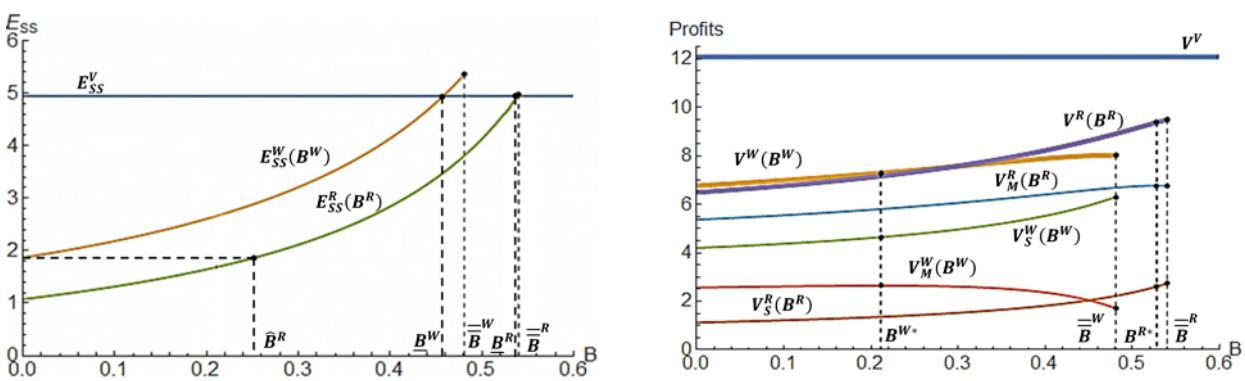
- Firms maximize the environmental performance by integrating the SC in all other cases.

The findings here with low ϕ differ from the case of $\phi = \phi_0$. First, with a collaborative program, a WPC is generally preferred to such an RSC given that if $B \in \left(0, \overline{\overline{B}}^W\right)$, $E^W_{SS|B} > E^R_{SS|B}$. Moreover, for $B \in \left[\widehat{B}^R, \overline{\overline{B}}^R\right]$, firms can always find a pair of values to B^R and B^W

that make the two contracts indifferent. For instance, $E^W_{SS|B^W=0} = E^R_{SS|B^R=\widehat{B}^R}$.

Regarding the profits, the low ϕ creates a dilemma for the players such that S becomes worse off and M is better off in the R_L-Scenario regardless of the value of B^R. In business, if the downstream manufacturer's industry such as M's commonly has stronger bargaining power over her supplier S, S is likely to agree with the low ϕ RSC despite lower profitability. However, if M does not possess such strong power, or she cares very much about the environmental performance, the agreement can be altered to a WPC, or an RSC with a higher ϕ, such as $\phi = \phi_0$ or even higher. Moreover, it is worth mentioning that the mismatch phenomenon still occurs in the R_L-Scenario.

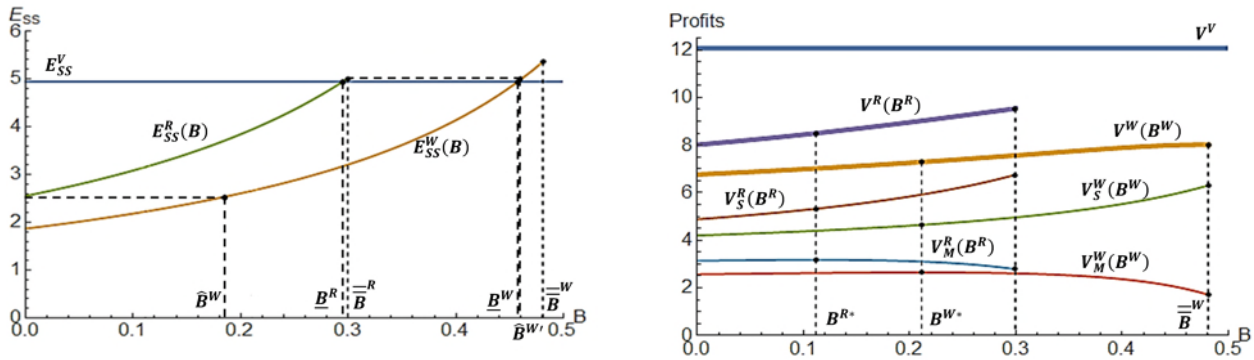
To guarantee S's satisfaction toward profitability and environmental performance, we analyze the situation where ϕ is enhanced, for example, to $\phi = 0.7$ in our analysis. We refer to this scenario as the R_H-Scenario. To reach an agreement, S who possesses high bargaining power obtains a large share of the profit under the RSC. With this design, we derive much the same findings on the steady-state as in the R-Scenario. The difference is that both players always prefer an RSC under high ϕ rather than a WPC; see Fig. 3.10. With the previous analysis, we omit the redundant description.



(a) Environmental performance

(b) Profits

Figure 3.9: Environmental performance and profits comparisons among W-Scenario, R_L-Scenario and V-Scenario



(a) Environmental performance

(b) Profits

Figure 3.10: Environmental performance and profits comparisons among W-Scenario, R_H-Scenario and V-Scenario

3.7 Special cases

In this section, we consider two special cases of the green process innovation model. First, it is also interesting to see the type of production cost increase where $c < 0$. Second, as both B and ϕ vary, it can be very inefficient to achieve an agreement due to this high degree of freedom. Therefore, motivated by the rationale that the player that obtains the higher share of profit should pay more in the innovation efforts, we propose a constraint on ϕ and B. These two variations are considered as extensions of the above main analysis.

3.7.1 Special case 1 - Green process innovation of production cost increase

To extend the insights obtained from the environmental process innovation where $c > 0$, we explore the case for the type of cost increase where $c < 0$. As the fundamental difference between the two types is the sign of c, the corresponding analytical solutions are identical, which are shown in Appendix A and summarized in Proposition 1, 5 and 10. Moreover, here constraint Eq. (3.3) is no longer valid as $c_p - cI(t)$ never results in negative values. By maintaining other parameters constant, we set here $c = -0.1$ to solve the Riccati system of the W-Scenario and select the fourth solution (see Appendix B) to demonstrate the corresponding results.

The results are consistent with Propositions 2 through 4 as well as Corollary 3.5.1. Apart from that, we solve for both the R-Scenario and the V-Scenario under the same set of parameters, which have consistent results with the cost reduction type in terms of steady-state, strategies and profits. We therefore omit the computations and interpretations and show in Fig. 3.11 the corresponding environmental performance and profits of the W-Scenario and the V-Scenario.

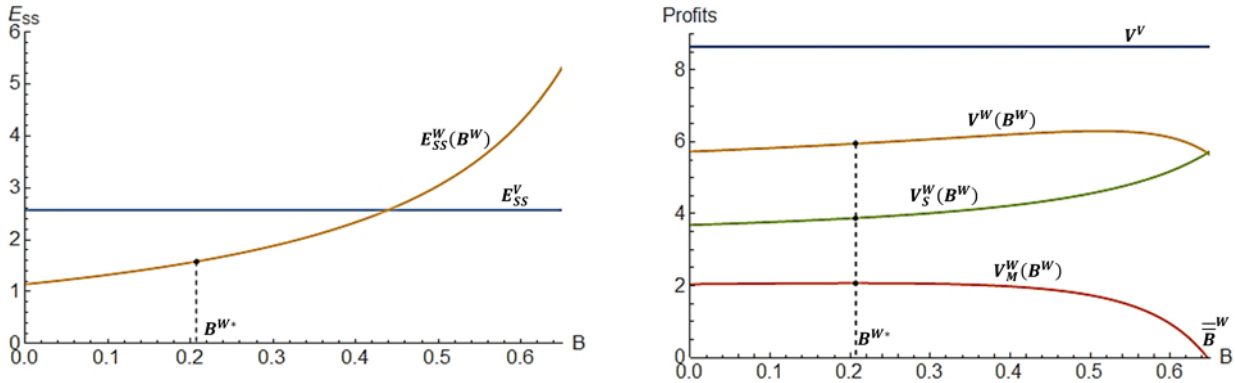
In Fig. 3.11, as stated, similar patterns of steady-state and profits are discovered. First, if S is only capable of employing cost-increasing green process innovation practices, the B and also ϕ need to be designed in the same manner as in the cost reduction case. Selecting one contract over another also means choosing between profit maximization and environmental performance maximization in a certain region of B. Therefore, the mismatch issue still exists.

Second, the environmental performance and the profits are relatively low in this type. This results from the raising production cost brought by green innovation efforts. The higher production cost encourages S and M to set relatively higher prices and discourages S (or the SC in the V-Scenario) from sparing much efforts in innovation.

Eventually, the cost-increasing type does not restrict B by constraint Eq. (3.3). However, B does not lack an upper bound. Beyond a certain B^W or B^R, M's profit starts to drop quickly to negative values. Meanwhile, the solution becomes unstable when sufficiently large B is present. The upper bound of B hence takes the minimum B of the occurrence of either of the two mentioned situations.

3.7.2 Special case 2 - $\phi = f(1 - B)$

Letting the revenue sharing parameter ϕ and collaborative program parameter B vary freely at the same time may result in high coordination efforts for both players. By recalling the objective functions in Eqs. (3.7) and (3.8), S prefers both values to be high while M prefers them low. Thus, the collaborative program is difficult to design and probably does not



(a) Environmental performance

(b) Profits

Figure 3.11: Environmental performance and profits comparisons between W-Scenario and V-Scenario of $c < 0$

work because no agreement is reached. Moreover, it is reasonable for a player who gains a higher share of the revenue to pay a higher innovation expense. To this end, we introduce a constraint $\phi = f(B)$ to limit their freedom. For the sake of simplicity, we introduce a linear constraint that reads as follows.

$$\phi = f(B) = 1 - B \tag{3.20}$$

This formulation is intended to express the viewpoint that if S takes a large share (ϕ) of the revenue generated from the retailing process, he should take charge in the green process innovation by a higher share of the cost $(1 - B)$, and vice versa. With the parameters set previously, we conduct analysis accordingly and demonstrate the results in Fig. 3.12.

In the presence of constraints Eqs. (3.3) and (3.20), the solution is divided into two regions characterized by three values of ϕ.

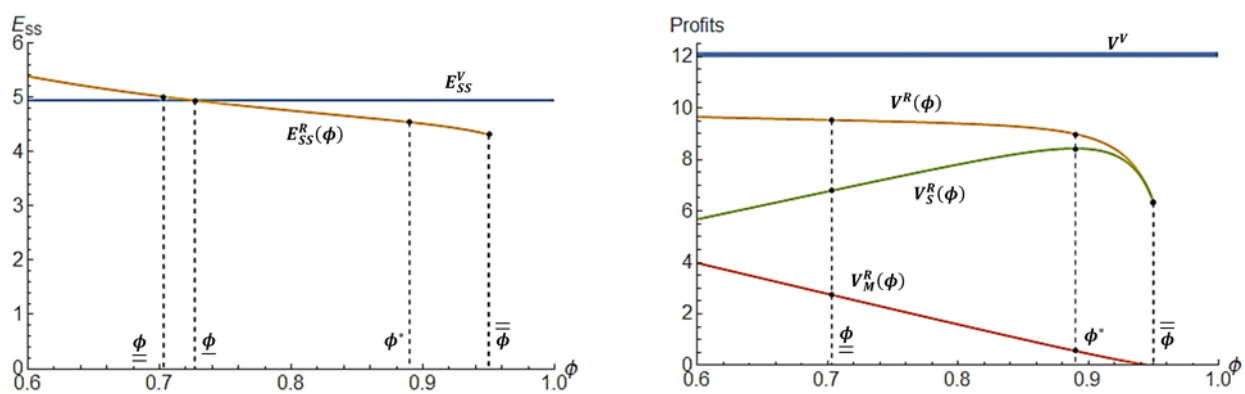
- When $\phi \in \left[\underline{\underline{\phi}}, \phi^{*}\right]$, where $\underline{\underline{\phi}}$ denotes the lower bound of ϕ in this case as any smaller value activates constraint Eq. (3.3) and causes infeasibility. Moreover, the environmental performance and the profit of M are maximized at this point. ϕ^{*} takes the value that maximizes S's profit.

- When $\phi \in \left(\phi^{*}, \overline{\overline{\phi}}\right]$, where $\overline{\overline{\phi}}$ represents the upper bound of ϕ as any value onward leads

to a negative profit for M and the equilibrium is off the table.

In this setting, that $\phi > \phi^*$ is not interesting because both players' profits and their environmental performance decline. $\phi = \underline{\underline{\phi}}$ allows M to achieve a higher profit, support more innovation (low ϕ means high B), and get $E_{SS} > E^V_{SS}$. Meanwhile, S prefers $\phi = \phi^*$ with the highest profit. If we compare Figs. 4, 5, and 12, we find that none of them consistently performs better than the others. Ultimately, the mismatch between economic and environmental performance always exists.

With this constraint, the profit in the V-Scenario still cannot be reached. With this constraint strongly imposed by, for example, regulation, the SC is not coordinated efficiently. However, we can derive a point of maximum profit for each player and for the best environmental performance.



(a) Environmental performance

(b) Profits

Figure 3.12: Environment performance and profits comparisons between V-Scenario and R-Scenario with $\phi = 1 - B$

3.8 Conclusions

Motivated by the existing trade-off between economic and environmental performance in the automotive industry, our main findings are summarized as follows. First, the model confirms the diverging role of the stock of environmental performance. Higher stock is preferred to increase sales which in return hurts environmental performance. Meanwhile, enhancing the

stock certainly incurs higher innovation expenses. This results in the mismatch of economic and environmental performance as the most beneficial region of the collaborative program parameter for greenness, in fact, does not locate in the profit-Pareto-improving region. Thus, decision makers may refer to their corporate-level business strategy toward greenness so that firms can take advantage of the diverging role of E for balancing the performance.

Second, to achieve greenness as with the development of new energy vehicles, it is always preferable to invest more in green process innovation and apply a collaborative program, regardless of the adopted contract type (WPC or RSC). When the current amount of E stock is low, sparing efforts in green process innovation contributes to the accumulation of E, so that firms can attract higher sales and keep sufficient profits even with pricing the product higher. Consistent with Klassen and Vachon (2003), investing jointly with another SC member leads to higher profit and better environmental performance than investing individually. When the current amount of E stock is already high, firms can further increase it together with profits through collaborative programs. When a car assembler shares a part of the innovation expenses, the engine supplier is motivated to invest more, which gives the SC better environmental performance and higher profits. In this case, the car assembler can keep enhancing the retail price to a larger extent.

Third, the trade-off between profit and environmental performance due to sales can be resolved by applying a collaborative program in a decentralized chain. This finding deviates from previous research in games with collaborative programs (e.g., Zhang et al., 2013), according to which the vertical integrated chain's results cannot be mimicked only by adopting a collaborative program. In fact, we show that if the car assembler and its engine supplier are environmental performance maximizers, they prefer a decentralized chain to a vertical integrated chain.

Fourth, an RSC does not consistently perform better than a WPC regarding profits or environmental performance for two reasons. First, the supplier and the manufacturer have to put in extra effort in administrating an RSC (Cachon & Lariviere, 2005). Second, the feasible

range of B can be too narrow in an RSC to allow profits and environmental performance to develop. Thus, either contract has the potential to outperform the other.

Finally, we explore two special cases of our model. On the one hand, we analyze the production cost-increasing type of green process innovation, which yields result patterns similar to the production cost-reducing type. On the other hand, we introduce a constraint on B and ϕ to express the rationale that the player who obtains higher revenue also pays more for innovation efforts. If the latter constraint applies in the car industry, the SC is unlikely to be coordinated in a more efficient manner but it helps the SC achieve satisfactory environmental performance and profits. In fact, this does not guarantee better profits and environmental performance than other contracts. Moreover, since both the manufacturer's profit and the environmental performance decrease with ϕ but the supplier has the chance to become better off, the players do not eliminate the mismatch effects between economic and environmental performance.

Our model can be generalized and applied to other industrial settings. Firms in the cosmetic industry that seek green process innovation opportunities with their raw material suppliers and toy makers (e.g., Lego) that gain competitiveness through collaborating with suppliers on green process innovation. More applications can be found in electronic device manufacturers, the chemical industry, the pharmaceutical industry, and others.

Future extensions of this essay can be carried out from four aspects. First, to address the strategic issue on profitability and sustainability more effectively, we can introduce environmental performance in the objective function. For instance, we can introduce an expression $-\gamma\left(E^{*}-E\right)^{+}$ where E^{*} represents the greenness standard by legislation and γ denotes the unit penalty for unsatisfactory performance. Second, based on our model, we can investigate incentive strategies so that the vertical integrated solution is reached efficiently (Jørgensen & Zaccour, 2001; De Giovanni et al., 2016). Third, we can integrate a green product innovation strategy and check whether our findings still hold. Finally, future research can examine multiple players (e.g., multi-channel manufacturing), where horizontal competitions are in-

cluded in green activities (e.g., Mitra, 2016, and include governmental subsidy (e.g., Han et al., 2017).

CHAPTER 4

REMANUFACTURING OF MULTI-COMPONENT SYSTEMS WITH PRODUCT SUBSTITUTION

4.1 Abstract

This essay investigates the inventory and production management of a hybrid manufacturing/remanufacturing system. The system serves the demand for new and for remanufactured products and allows for substitution between these types of products. The objective of the system is to minimize "strategic" cost, i.e., the weighted sum of economic cost and environmental impact. First, we analyze single-component products and show that the optimal policy is of the threshold-type under certain conditions. Then, we analyze multi-component products to achieve flexibility by manipulating the portion of remanufactured components in a product. To address problems of high dimensions, we develop a close-to-optimal heuristic. Our results indicate that our model of partial substitution outperforms existing models with no or complete substitution. We also find that management at the component level reduces strategic cost compared to management at the product level and that a stronger weight on economic cost increases the need for product substitution. Our analysis leads to managerial insights on how remanufacturing can support a company's business strategy.

Key words: Remanufacturing, Product substitution, Multi-component systems, Operational flexibility, Markov decision process

4.2 Résumé

Cet article étudie la gestion des stocks et de la production d'un système hybride de fabrication/reconditionnement. Le système répond à la demande de produits nouveaux et de produits reconditionnés et permet la substitution entre les deux types de produits. L'objectif du système et de minimiser le coût « stratégique », c'est-à-dire la somme pondérée des coûts économiques et de l'impact environnemental. Tout d'abord, nous analysons les produits composants seuls et montrons que la politique optimale est de type seuil sous certaines conditions. Ensuite, nous analysons les produits multicomposants pour obtenir une flexibilité en manipulant la partie des composants reconditionnés dans un produit. Pour résoudre les problèmes de grandes dimensions, nous développons une heuristique proche de l'optimum. Nos résultats indiquent que notre modèle de substitution partielle surpasse ceux existants avec une substitution nulle ou complète. Nous constatons également que la gestion au niveau des composants réduit le coût stratégique par rapport à la gestion au niveau des produits et qu'un poids plus important sur le coût économique augmente le besoin de substitution des produits. Notre analyse nous amène à comprendre comment le reconditionnement peut soutenir la stratégie commerciale d'une entreprise.

Mots clés: Remanufacturing, Substitution de produits, Systèmes multi-composants, flexibilité Opérationnel, Processus de décision de Markov

4.3 Introduction

In this essay, we analyze a hybrid manufacturing/remanufacturing system with multiple components and substitution between new and remanufactured parts.

Remanufacturing refers to the manufacturing of products from a combination of new and re-used components. Remanufacturing is widely used in some industries, such as automotive, aerospace, heavy-machinery, and electronics (Goodall et al., 2014; D'Adamo & Rosa, 2016). For example, Ford Motor Corporation has introduced its *Ford Authorized Remanufactured* label, which has been applied to more than 120 million pounds of remanufactured components in the first ten years after its creation (Recycling Today, 2013). Remanufacturing improves profitability, because component costs for remanufactured products are typically 50-65% less than those for new products (Munot, 2015). In addition, remanufacturing also reduces the negative environmental effects of manufacturing by avoiding the waste of materials such as steel or chemicals and by reducing energy and water consumption. A typical environmental impact is 80% less energy and water consumption and 70% less waste (Perkins, 2017).

Akçalı and Çetinkaya (2011) and Govindan et al. (2015) recommend using remanufacturing as a means of supporting a comprehensive business strategy that includes both economic as well as environmental considerations. In fact, companies are increasingly considering environmental aspects in their business strategies. For example, Jeffrey R. Immelt, Chief Executive Officer of General Electric, writes *"Sustainability is not an initiative for us; it is integrated into our core business strategy..."*(Immelt, 2017). In this essay, we therefore consider the *strategic* value that is created from remanufacturing. The strategic value comprises the economic impact and the environmental impact. Economic impact refers to the cost of production, inventory holding, customer penalties, and returns acquisition. Environmental impact refers to energy and water consumption as well as material waste. The relative weighting of these two types of impact represents the company's business strategy with respect to the balance of economic profitability and sustainability (Graafland et al., 2004).

Remanufactured products must be labeled to allow the customer to differentiate between remanufactured and new products. Although remanufactured products must meet strict quality standards and their quality and warranty often exceed those of new products (Parkinson & Thompson, 2003; Savaskan et al., 2004), many customers still perceive that their value differs from that of new products (Fleischmann & Kuik, 2003; Akçalı & Çetinkaya, 2011). For example, some customers stubbornly believe that the quality of remanufactured products is lower than that of new products (Debo et al., 2005), whereas some environmentally conscious customers may prefer remanufactured products over new products due to their reduced environmental impact. Therefore, the demand for new products is not identical to the demand for remanufactured products.

Even though new and remanufactured products are not identical in the perceived value to customers, manufacturers have some flexibility in serving customers. If a financial incentive is offered, some customers may accept a remanufactured (new) product despite initially requesting a new (remanufactured) product. We refer to this flexibility as *product substitution*. Examples of product substitution include Ford (Ward, 2013; SAP, 2016) and General Motors (General Motors, 2017). In addition, even when product demand is served as requested, remanufactured products rarely contain 100% remanufactured components. In most cases, they contain a mix of remanufactured and new components and the manufacturer only guarantees a minimum share of remanufactured components (Stahel, 1995). Caterpillar, for example, only uses an average of 60% of remanufactured components in its remanufactured products (Yates & Castro-Lacouture, 2015). This definition allows manufacturers to flexibly play with the share of remanufactured components in the products that they sell to customers (Munot, 2015). We refer to this flexibility as *component substitution*.

We seek to answer the following research questions: (1) What is the value of (product and component) substitution in remanufacturing? (2) How can operational decisions be adjusted to make optimal use of substitution? (3) How does the business strategy of a company affect its operational decisions?

Remanufacturing systems have received considerable attention in the research literature. Fleischmann and Kuik (2003) were among the first to investigate a single-component product inventory system with returns and showed that a stationary (s, S) order policy is optimal. Zhou and Yu (2011) analyzed product acquisition, pricing and the inventory management decisions in remanufacturing. Zhou et al. (2011) considered multiple types of cores returned from customers. Kim et al. (2013) examined a single-component product system considering production, remanufacturing and disposal decisions and showed that the optimal policy has a state-dependent base-stock structure. Vercraene et al. (2014) analyzed a model in which remanufactured products were considered by the market as identical to new products and showed that the optimal policy was characterized by state-dependent base-stock thresholds. Cai et al. (2014) investigated the quality of cores and found that a state-dependent base-stock policy is optimal. Gayon et al. (2017) investigated a setting in which disposals could be decided either upon arrival of the returns or after they become serviceable products. Few contributions consider multiple components. Examples are DeCroix and Zipkin (2005), DeCroix (2005), DeCroix et al. (2009) who considered multi-product and multi-component assemble-to-order systems with returns in which inventory is managed at the component level. We extend this work on multi-component inventory management by analyzing the value of product and component substitution between remanufactured and new components.

Our research also builds on two other streams in inventory management: product substitution and transshipment. With respect to product substitution, Bayındır et al. (2005) showed the benefits of revenue management by downward substitution in remanufacturing. Nagarajan and Rajagopalan (2008) examined a retailing system with two partially substitutable products in which a consumer accepts the other product with probability γ if the demanded product is out of stock. Xu et al. (2011) investigated an inventory system that produces two mutually substitutable products over a finite selling season. More detailed information regarding substitution is available in the review paper of Shin et al. (2015). The

literature on transshipment considers the possibility of serving a local demand from a site other than the one that regularly serves this location. Conceptually, transshipment follows a logic similar to that of two-way substitution in situations in which the transshipped products are partial substitutes. Transshipment is a means of gaining flexibility. Zhao et al. (2008) characterized the optimal policy for managing the production planning and inventory for a two-location make-to-order system. Hu et al. (2008) considered capacity uncertainty in a setting with multiple production facilities and characterize the optimal policy. Abouee-Mehrizi et al. (2015) considered several types of lost sales functions in the transshipment model. A review of the literature on transshipment can be found in Paterson et al. (2011).

Our model differs from the literature on product substitution and the literature on transshipment in two fundamental aspects: (1) Contrary to the existing literature, we consider two-way substitution between new and remanufactured products and (2) our hybrid manufacturing/remanufacturing system contains an uncontrolled returns process in addition to the traditional, controlled production process used in the literature.

To develop the analysis, we begin with the case of a single-component durable product before extending it to the case of multi-component durable product. For the case of a single-component product, we formulate the dynamic model as a continuous-time Markov decision process (MDP) with the objective of minimizing the average strategic cost. We show that the optimal policy has a state-dependent base-stock structure when the substitution cost of serving a remanufactured demand with a new product is zero. For the case of multiple components, we derive several structural properties of the optimal policy and develop a heuristic procedure. Finally, we extend the single-component case to the correlation between demand and return, which arises in particular if customers return an old product at the same time that they buy another unit for continued usage.

We find that substitution has a significant effect on the strategic cost. In our data set, product substitution leads to a reduction of 18.3% of the strategic cost in the single-component case compared with the benchmark policy that does not consider product sub-

stitution. The reduction decreases with a stronger weight on environmental concerns in the business strategy. Component substitution reduces the strategic cost by 1.4% (approximately 1 million euros) in our case, with three components compared with the system-level case. With regard to the business strategy, we find that a strong weight of the environmental impact requires less frequent product and component substitution than a low weight. Finally, managing remanufacturing at the component level not only reduces the strategic cost but also requires less frequent product substitution than managing remanufacturing at the product level.

The contributions of this essay are threefold. First, our essay is the first to consider flexible product and component substitution in remanufacturing, which is a situation that is commonly observed in practice. The existing literature considers either a single integrated market (full substitution) or two separated markets (no substitution). We derive structural properties and an effective heuristic policy for hybrid manufacturing/remanufacturing systems with substitution. Second, we consider remanufacturing as a means for supporting a comprehensive business strategy that includes profitability as well as sustainability aspects. The previous remanufacturing literature has mainly focused on economic profit only. We explicitly analyze the impact of the strategic weight on the results and the optimal decisions. Third, we compare the management of remanufacturing at the system level with the management of remanufacturing at the component level.

The rest of the essay is organized as follows. Section 4.4 develops the model of a single-component product and characterizes its optimal policy. Section 4.4.1 develops a model for a multi-component product and derives a heuristic procedure to solve it. In Section 4.5, we numerically analyze the model and derive managerial implications. We conclude the essay in Section 4.7.

4.4 The Single-Component Case

In this section, we consider the case of products with a single component. This is the case if products are not disassembled but treated as one, non-separable entity. We first describe the model as a Markov decision process and then analyze its optimal policy.

4.4.1 Model Description

We consider a make-to-order (MTO) system that manufactures and remanufacturers a single-component product.

The demand for new products and for remanufactured products is uncertain and follows Poisson processes with rates λ_N and λ_R, respectively. A demand for a new product can be served either with a new product incurring zero cost or with a remanufactured product incurring a substitution penalty $\eta_{RN} \geq 0$. If a new product is not in stock, the demand can either be satisfied with a remanufactured product, or be rejected. The rejection incurs a lost sales penalty $\eta_L > \eta_{RN}$. The penalty of per unit of lost sales induces a potential loss of goodwill and that may exceed the pure financial loss (Flapper et al., 2012; Shi et al., 2014). Similarly, a demand for a remanufactured product can be served with a remanufactured product, incurring zero cost, or with a new product, incurring a substitution penalty $\eta_{NR} \geq 0$. If a remanufactured product is not available, the decision maker can decide whether to serve the demand with a new product or to reject the the demand, which leads to a lost sales penalty $\eta_L > \eta_{NR}$.

The system comprises three different inventories: the inventory of returned products that are currently in remanufacturing x_A; the inventory of ready-to-use remanufactured products x_R; and the inventory of ready-to-use new products x_N. The respective inventory holding costs are denoted by h_A, h_R, h_N.

To re-supply inventory, the decision maker can either produce a new product or acquire and remanufacture a returned, used product. New products can be manufactured at a cost

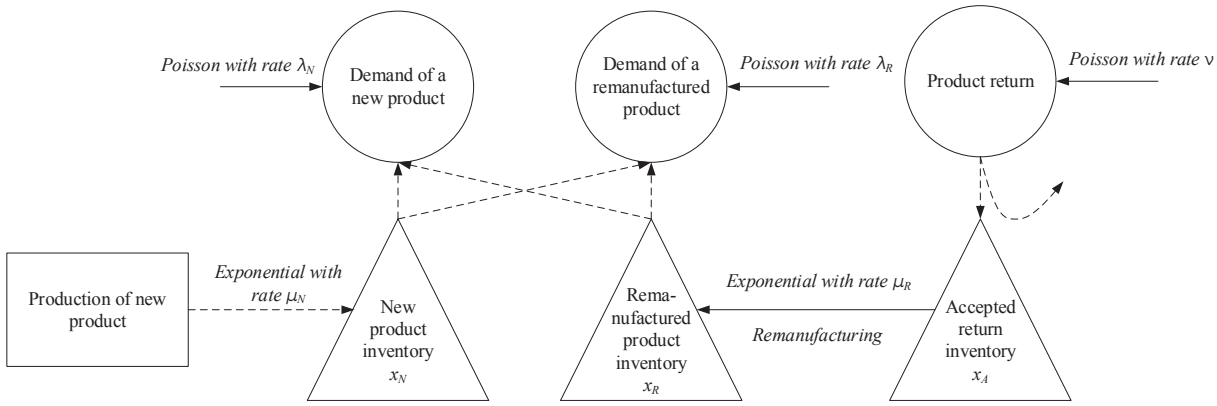


Figure 4.1: Events, decisions and product flow in the single-component product model.

Circles are uncontrolled events; the rectangle represents a controlled event; triangles are the inventory of products; solid lines are uncontrolled flows; and the dash lines are controlled flows that require decision making.

of c_N and an environmental impact of ε per unit. The manufacturing lead time follows an exponential distribution with rate μ_N. To remanufacture products, the decision maker must acquire a used product at an acquisition cost whenever a customer returns a product. The return rate of used products follows a Poisson process with rate ν. We refer to this situation as a *regular return* and to these customers as *returners*. After acquisition, the remanufacturing process starts immediately and incurs a cost and environmental impact $\theta\varepsilon$, where $\theta \in [0, 1]$ indicates the relation of the environmental impact between manufacturing new products and remanufacturing used products. We assume that the sum of the cost for return acquisition and for remanufacturing is incurred immediately upon return acceptance, and we denote this cost by c_{RM}. We assume $c_{RM} < c_N$ to model the cost benefit of remanufacturing. The remanufacturing processing lead time follows an exponential distribution with rate μ_R. The events, decisions, and product flows of the model are summarized in Figure 4.1.

Next, we formulate the model as an MDP with continuous-time and infinite horizon. The objective of the MDP is to minimize the average, expected *strategic cost* per time unit, i.e.,

to minimize the weighted sum of the economic cost C and the environmental impact E,

$$\min \ \omega C + (1 - \omega)E, \tag{4.1}$$

where ω is the weighting factor that describes the company's strategy for balancing economic and environmental aspects.

Taking the weighted sum of multiple indicators as an aggregated index for expressing comprehensive strategic goals is a standard approach in the literature on *Triple Bottom Line (3BL)* and *Corporate Social Responsibility (CSR)*; see, for example, Graafland et al. (2004). The weighted sum allows a company to specify that, for example, it considers the reduction of a certain amount of waste or resource consumption to be as important as earning 1 euro of profit.

The state of the MDP is given by vector $\mathbf{x} = (x_A, x_R, x_N) \in \mathbb{N}^3$. The actions of the decision maker can be summarized as follows:

1. Accept a return from a returner or not $\longrightarrow (x_A + 1, x_R, x_N)$ or (x_A, x_R, x_N);

2. Manufacture a new product or not $\longrightarrow (x_A, x_R, x_N + 1)$ or (x_A, x_R, x_N);

3. Satisfy a demand with a new product or a remanufactured product $\longrightarrow (x_A, x_R, x_N - 1)$ or $(x_A, x_R - 1, x_N)$ when $x_N, x_R > 0$;

4. When the demanded type of product is unavailable, satisfy the demand with a substituted product or reject the demand $\longrightarrow (x_A, x_R, x_N - 1)$ or (x_A, x_R, x_N) when a demand for a remanufactured product arrives but $x_R = 0$, $(x_A, x_R - 1, x_N)$ or (x_A, x_R, x_N) when a demand for a new product arrives but $x_N = 0$.

We denote the action space in state $\mathbf{x}$ by $A(\mathbf{x})$ and the action that the decision maker has to take at each decision epoch by $a \in A(\mathbf{x})$. $a = 1$ refers to accepting a return; $a = 2$ to initiating the manufacturing of a new product; $a = 3$ ($a = 4$) to substituting demand when both types of products are available, i.e., to serving the demand for a new (remanufactured) product with a remanufactured (new) product; $a = 5$ to rejecting a demand when the original

type is out of stock; and $a = 0$ to not taking any of the above actions.

The transition rate of the system is given by $\tau = \nu + \mu_R + \mu_N + \lambda_R + \lambda_N$. The probability to transit from state $\mathbf{x}$ to $\mathbf{x}'$ under action a is given by $p_{\mathbf{x},\mathbf{x}'}(a)$. Furthermore, we define I_a as an indicator function such that $I_a = 1$ if action a is taken and 0 otherwise.

We denote the optimal policy that minimizes the strategic cost by π^*. This policy is given by the following Bellman equation

$$\begin{aligned} g^* + v^*(\mathbf{x}) = \min_{a \in A(\mathbf{x})} \Big\{ & h(\mathbf{x}) + \nu I_{\{a=1\}} \Big(\omega c_{RM} + (1-\omega)\theta\varepsilon \Big) + \mu_N I_{\{a=2\}} \Big(\omega c_N + (1-\omega)\varepsilon \Big) \\ & + \lambda_N \Big(I_{\{a=3\}} \omega \eta_{RN} + I_{\{x_N = x_R = 0 \vee a=5\}} \omega \eta_L \Big) + \lambda_R \Big(I_{\{a=4\}} \omega \eta_{NR} + I_{\{x_N = x_R = 0 \vee a=5\}} \omega \eta_L \Big) \\ & + \sum_{\mathbf{x}' \in N^3} p_{\mathbf{x},\mathbf{x}'}(a) v^*(\mathbf{x}') \Big\}, \end{aligned} \tag{4.2}$$

in which v^* denotes the optimal value function and $h(\mathbf{x}) = \omega(h_A x_A + h_R x_R + h_N x_N)$ for notational convenience. Rewriting the Bellman equation in iterative form leads to

$$Tv(\mathbf{x}) = h(\mathbf{x}) + \nu T_A v(\mathbf{x}) + \mu_R T_R v(\mathbf{x}) + \mu_N T_N v(\mathbf{x}) + \lambda_N T_{dN} v(\mathbf{x}) + \lambda_R T_{dR} v(\mathbf{x}), \tag{4.3}$$

with the operators T_i, $i = A, R, N, dN, dR$ being defined as

$$T_A v(\mathbf{x}) = \min\left\{v(x_A+1, x_R, x_N) + \omega c_{RM} + (1-\omega)\theta\varepsilon, v(x_A, x_R, x_N)\right\}, \tag{4.4}$$

and

$$T_R v(\mathbf{x}) = \begin{cases} v(x_A-1, x_R+1, x_N), & \text{if } x_A \geq 1, \\ v(x_A, x_R, x_N), & \text{otherwise,} \end{cases} \tag{4.5}$$

and

$$T_N v(\mathbf{x}) = \min\left\{v(x_A, x_R, x_N+1) + \omega c_N + (1-\omega)\varepsilon, v(x_A, x_R, x_N)\right\}, \tag{4.6}$$

and

$$T_{dN} v(\mathbf{x}) = \begin{cases} \min\left\{v(x_A, x_R, x_N-1), v(x_A, x_R-1, x_N) + \omega\eta_{RN}\right\}, & \text{if } x_N, x_R \geq 1, \\ v(x_A, x_R, x_N-1), & \text{if } x_R = 0 \text{ and } x_N \geq 1, \\ \min\left\{v(x_A, x_R-1, x_N) + \omega\eta_{RN}, v(x_A, x_R, x_N) + \omega\eta_L\right\}, & \text{if } x_N = 0 \text{ and } x_R \geq 1, \\ v(x_A, x_R, x_N) + \omega\eta_L, & \text{otherwise.} \end{cases} \tag{4.7}$$

Operator T_{dR} is similarly defined as operator T_{dN} for serving a remanufactured demand.

4.4.2 Analysis of the Optimal Policy

This section first identifies the conditions that the value function must satisfy to characterize the optimal policy, i.e., the policy for which $g^* + v^* = Tv^*$ holds. Then, we derive the structure of the optimal policy under some conditions and discuss several propositions of the optimal policy.

First, we define

$$\Delta_i v(\mathbf{x}) = v(\mathbf{x} + \mathbf{e}_i) - v(\mathbf{x}), i \in \{A, R, N\}, \tag{4.8}$$

where $\mathbf{e}_i$ is a vector with a value of 1 in dimension i and zero in the other dimensions.

Let $\mathcal{V}$ denote the set of real-valued functions v defined on N^3 that satisfy the following conditions:

C1: $\Delta_i v(\mathbf{x}+\mathbf{e}_i) \geq \Delta_i v(\mathbf{x}), i \in \{A, R, N\}$,

C2: $\Delta_i v(\mathbf{x}+\mathbf{e}_j) \geq \Delta_i v(\mathbf{x}), i, j \in \{A, R, N\}, i \neq j$,

C3: $\Delta_i v(\mathbf{x}+\mathbf{e}_i) \geq \Delta_i v(\mathbf{x}+\mathbf{e}_j), i, j \in \{A, R, N\}, i \neq j$,

C4: $v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i) - v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j) \geq v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_A) - v(\mathbf{x}+\mathbf{e}_j+\mathbf{e}_A), i, j \in \{R, N\}, i \neq j$,

C5: $\Delta_A v(\mathbf{x}+\mathbf{e}_R) \geq \Delta_A v(\mathbf{x}+\mathbf{e}_N)$,

C6: $\Delta_N v(\mathbf{x}+\mathbf{e}_R) \geq \Delta_N v(\mathbf{x}+\mathbf{e}_A)$,

C7: $v(\mathbf{x}+\mathbf{e}_A) \geq v(\mathbf{x}+\mathbf{e}_R)$,

C8: $\Delta_N v(\mathbf{x}) \geq -\omega\eta_L, \Delta_R v(\mathbf{x}) \geq -\omega\eta_L$.

Condition 1 means that $v(\mathbf{x})$ is convex in x_i. Condition 2 implies that $v(\mathbf{x})$ is supermodular in x_i and x_j. Condition 3 specifies that by holding an extra unit of j, the value of having one more unit of i does not increase as much as the increment of having one more unit of i if that extra unit of j is replaced by i. Condition 4 specifies that by holding an extra unit of A and j, the increase in value by replacing the unit of j with i is not as large as the increment of the case of holding an extra unit of i and j with the extra unit of j replaced with i, for $i, j = R, N, i \neq j$. Condition 5(6) implies that by holding an extra unit of $N(A)$, the value of having one more unit of $A(N)$ does not increase as much as the increment of having one more unit of $A(N)$ if the extra unit of $N(A)$ is replaced by R. Condition 7 indicates that it is always preferable to remanufacture a return into a servable remanufactured product. Condition 8 indicates that regardless of the type of demand arrival, it is always better to serve the demand by the original type of product than to reject it.

Given C1 to C8, we state the main technical result of our analysis in Lemma 4.4.1.

Lemma 4.4.1. *For $\eta_{NR} = 0$, if $v \in \mathcal{V}$, $Tv \in \mathcal{V}$. The optimal cost v^* is therefore an element of $\mathcal{V}$, keeping the properties of C1 to C8.*

The proof of Lemma 4.4.1 and all other proofs are contained in the Appendix. Lemma 4.4.1 applies to the situation where $\eta_{NR} = 0$, i.e., the situation that serving a demand for a

remanufactured product with a new product is costless. This is the case if a customer accepts a new product as being equal to remanufactured products. According to our discussions in Section 4.3, we expect this case to hold in many settings. For $\eta_{NR} > 0$, the monotone threshold optimal policy structure does not hold because conditions C4 to C6 do not carry over from one period to the next. Next, we can describe the optimal policy for the case of $\eta_{NR} = 0$. We denote $\mathbf{x}_{-i} = \{x_j, x_k\}, i, j, k = A, R, N, i \neq j \neq k$.

Theorem 4.4.1. *Optimal Policy*

For $\eta_{NR} = 0$, the optimal policy includes the following switching-curve conditions.

(1) Accept the return from a returner if

$x_A < s_A(\mathbf{x}_{-A}) = \min\{x_A \geq 0 | v^*(\mathbf{x} + \mathbf{e}_A) - v^*(\mathbf{x}) \geq -\omega c_{RM} - (1-\omega)\theta\varepsilon\}$;

(2) Produce a new product if

$x_N < s_N(\mathbf{x}_{-N}) = \min\{x_N \geq 0 | v^*(\mathbf{x} + \mathbf{e}_N) - v^*(\mathbf{x}) \geq -\omega c_N - (1-\omega)\varepsilon\}$;

(3) When a demand for new product arrives,

(3a) Satisfy the new demand by a remanufactured product if

$x_R \geq r_R(\mathbf{x}_{-R}) = \min\{x_R \geq 1 | v^*(\mathbf{x} - \mathbf{e}_N) - v^*(\mathbf{x} - \mathbf{e}_R) \geq \omega\eta_{RN}, x_N \geq 1\}$ *and* $x_R \geq r'_R(\mathbf{x}_{-R}) = \min\{x_R \geq 1 | v^*(\mathbf{x}) - v^*(\mathbf{x} - \mathbf{e}_R) \geq -\omega\eta_L + \omega\eta_{RN}, x_N = 0\}$;

(3b) Reject the new demand if both $x_N = 0$ and

$x_R < r'_R(\mathbf{x}_{-R}) = \min\{x_R \geq 1 | v^*(\mathbf{x}) - v^*(\mathbf{x} - \mathbf{e}_R) \geq -\omega\eta_L + \omega\eta_{RN}\}$;

(3c) Otherwise, satisfy the new demand by a new product.

(4) When a demand for remanufactured product arrives,

(4a) Satisfy a remanufactured demand by a new product if

$x_N \geq r_N(\mathbf{x}_{-N}) = \min\{x_N \geq 1 | v^*(\mathbf{x} - \mathbf{e}_R) - v^*(\mathbf{x} - \mathbf{e}_N) \geq \omega\eta_{NR}, x_R \geq 1\}$ *and* $x_N \geq r'_N(\mathbf{x}_{-N}) = \min\{x_N \geq 1 | v^*(\mathbf{x}) - v^*(\mathbf{x} - \mathbf{e}_N) \geq -\omega\eta_L + \omega\eta_{NR}, x_R = 0\}$;

(4b) Reject a remanufactured demand if both $x_R = 0$ *and*

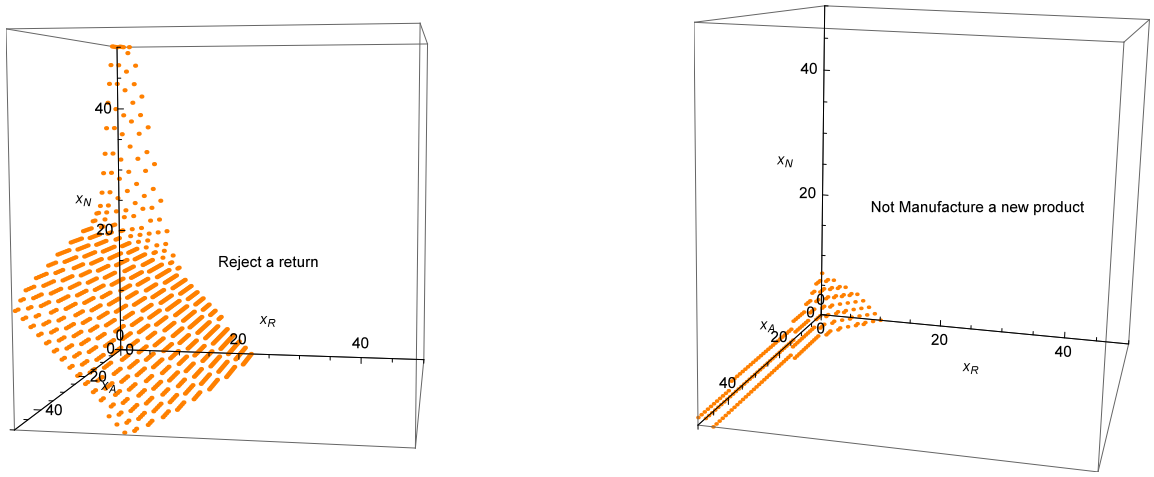
$x_N < r'_N(\mathbf{x}_{-N}) = \min\{x_N \geq 1 | v^*(\mathbf{x}) - v^*(\mathbf{x} - \mathbf{e}_N) \geq -\omega\eta_L + \omega\eta_{NR}\}$;

(4c) Otherwise, satisfy the remanufactured demand by a remanufactured product.

Theorem 4.4.1 implies that the optimal policy takes the shape of a state-dependent base-stock policy for its decisions of accepting returns, manufacturing products, and serving demand. State-dependent base-stock policies have been used for a variety of operational settings, such as Benjaafar and El Hafsi (2006) in the MTO assembly setting, Gayon et al. (2009) in the Make-to-Stock setting, Alp et al. (2013) and Benjaafar et al. (2017) in continuous state variable setting, and Kim et al. (2013), Gayon et al. (2017) in remanufacturing settings. Theorem 4.4.1 serves as a general optimal policy which also covers the case of full substitution if $\eta_{NR} = \eta_{RN} = 0$ (Zhou & Yu, 2011; Gayon et al., 2017).

Figures 4.2a to 4.3b illustrate the behavior of the switching-curve-policy as described in Theorem 4.4.1 with a numerical example. In Figure 4.2a, if we fix x_A, all points on that plane below the switching curves (including the points themselves) indicate that it is optimal to accept a return. In Figure 4.2b, the points form three-dimensional, monotone stairs. For any point under these stairs (including the points themselves) it is optimal to trigger the manufacturing of a new product. In Figure 4.3a, for any states above the switching curve formed by the points, it is optimal to use a new product to satisfy the demand for a new product (no substitution). For any other point, it is optimal to substitute. Figure 4.3b shows that it is not optimal to serve the demand for a remanufactured product with a new product unless the remanufactured product is out of stock at the dotted states when $x_R = 0$ (substitution is preferred) or the three inventory levels are in the region located in the upper-right corner formed by the points (sufficient inventory levels of returns and products are available).

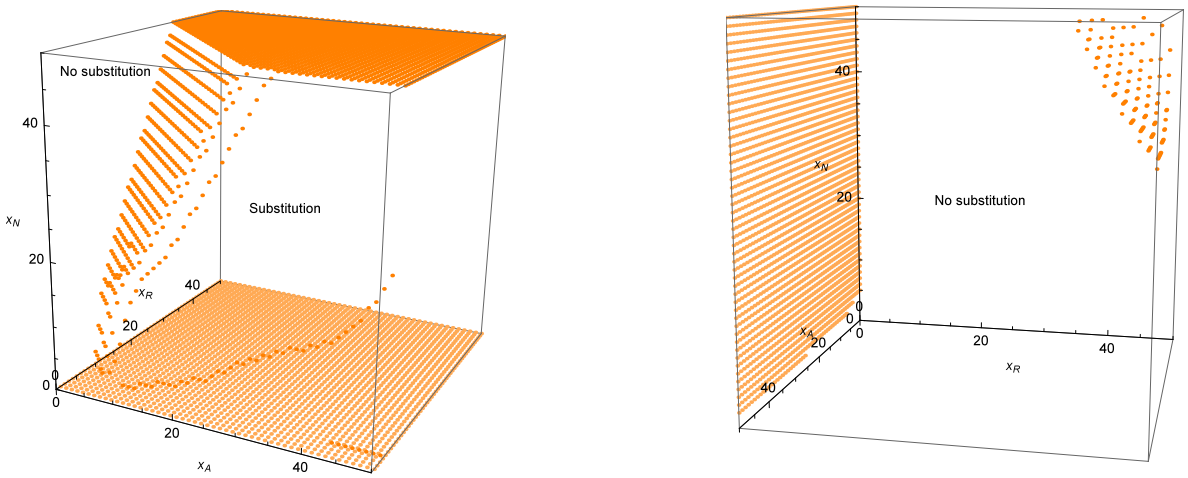
In the next proposition, we describe how the base-stock levels of the optimal policy depend on the system parameters.



(a) Return acquisition.

(b) Production decision of new products.

Figure 4.2: Acceptance and production decisions for a numerical example.



(a) Satisfy a new demand.

(b) Satify a remanufactured demand.

Figure 4.3: Satisfy a new or a remanufactured demand for a numerical example.

Proposition 4.4.1. *The optimal policy has the following propositions.*

(a) The threshold $s_i(\mathbf{x}_{-i})$ is non-increasing in x_j, $i, j = A, R, N$, $i \neq j$.

(b) The threshold $r_i(\mathbf{x}_{-i})$ is non-increasing in $x_j - \mathbf{e}_A$, $i, j = R, N$, $i \neq j$.

(c) The threshold $r_i^{'}(\mathbf{x}_{-i})$ is non-increasing in x_j, $i = R, N$, $j = A, R, N$, $i \neq j$.

(d) $s_A(\mathbf{x}_{-A} + \mathbf{e}_R - \mathbf{e}_N) \leq s_A(\mathbf{x}_{-A})$ and $s_N(\mathbf{x}_{-N} + \mathbf{e}_R - \mathbf{e}_A) \leq s_N(\mathbf{x}_{-N})$.

Part (a) of Proposition 4.4.1 shows that under the optimal policy, the switching curves for return acquisition and manufacturing are monotone-decreasing in any of the inventory levels. In part (b) of Proposition 4.4.1, the switching curve $r_i(\mathbf{x}_{-i})$ is monotone in the direction $x_j - \mathbf{e}_A$, where $i, j = R, N, i \neq j$, which implies that the value of $r_i(\mathbf{x}_{-i})$ will decrease if we replace an A with a j. In part (c), the switching curves for accepting demand are monotone-decreasing in any of the inventory levels. Part (d) of Proposition 4.4.1 indicates that the switching curve for accepting returns does not increase if remanufactured products are converted into new products. Furthermore, finishing the remanufacturing of a product (i.e., a transition of an inventory item from x_A to x_R) does not increase the switching curve for manufacturing.

Next, we evaluate the optimal policy and optimal value function with respect to the strategic priority parameter ω.

Proposition 4.4.2. *The optimal value function g^* is concave in strategic weight ω.*

Proposition 4.4.2 indicates that the strategic value increases sub-proportionally or even decreases in the strategic weight ω.

Proposition 4.4.3. *If $c_N \geq \varepsilon$ and $c_{RM} \geq \theta\varepsilon$, the objective value function is non-decreasing with ω, i.e. $g^*(\omega_1) \geq g^*(\omega_2)$ given $\omega_1 \geq \omega_2$.*

Proposition 4.4.3 indicates that the optimal strategic value increases in strategic weight ω under certain conditions. This result holds if $c_N \geq \varepsilon$ and $c_{RM} \geq \theta\varepsilon$, which is typically the case if the production cost exceeds the environmental impact.

We can write the strategic cost by the two components of economic and environmental costs, i.e., $g^* = g^{C*} + g^{E*}$. The economic and environmental costs are thus denoted by g^{C*}/ω and $g^{E*}/(1-\omega)$, respectively. Next, we separately analyze the economic and environmental cost.

Proposition 4.4.4. *Given $c_N \geq \varepsilon$ and $c_{RM} \geq \theta\varepsilon$, the economic cost is non-increasing with ω, i.e., $\frac{g^{C*}(\omega_1)}{\omega_1} \leq \frac{g^{C*}(\omega_2)}{\omega_2}$ if $\omega_1 \geq \omega_2$. If $\omega = 0$, the infinite horizon average economic cost is $(\lambda_N + \lambda_R)\eta_L$; if $\omega = 1$, the economic cost is minimized and $\arg\min_{\pi(a)} g^* = \arg\min_{\pi(a^C)} \frac{g^{C*}}{\omega}$.*

Proposition 4.4.4 indicates that the true economic cost is non-increasing in strategic weight ω, which indicates that the economic cost does not increase as more importance is attached to it. If $\omega = 0$, it is never preferred to conduct any production or remanufacturing activities since they incur environmental cost. Thus, as we focus on the infinite horizon average strategic cost, the entire system is only faced with lost sales. As the environmental cost is weighted 0 if $\omega = 1$, minimizing the strategic cost is equivalent to minimizing the economic cost. Therefore, for $\omega = 1$ the policy that minimizes $v^*(\mathbf{x})$ also minimizes g^{C*}/ω.

Proposition 4.4.5. *Given $c_N \geq \varepsilon$ and $c_{RM} \geq \theta\varepsilon$, the environmental cost is non-decreasing with ω, i.e., $\frac{g^{E*}(\omega_1)}{1-\omega_1} \geq \frac{g^{E*}(\omega_2)}{1-\omega_2}$ if $\omega_1 \geq \omega_2$. If $\omega = 0$, the infinite horizon average economic cost is 0, and $\arg\min_{\pi(a)} g^* = \arg\min_{\pi(a^E)} \frac{g^{E*}}{1-\omega}$.*

Proposition 4.4.5 illustrates that the environmental cost, in contrast to the economic cost, is non-decreasing in strategic weight ω. From a managerial viewpoint, this is intuitive since the more sustainability is valued, the more it is optimal to avoid environmental cost. If profit is prioritized, firms tend to expend more efforts in decreasing economic cost, which may lead to higher environmental cost. Therefore, we observe a non-decreasing behavior of the environmental cost. For $\omega = 0$ the economic cost is entirely ignored. Hence, minimizing the strategic cost is equivalent to minimizing the environmental cost, and the policy that minimizes g^* also minimizes $g^{E*}/(1-\omega)$.

In Appendix D, we present an extension to our model, in which the return of used products and the demand for remanufactured products are correlated. This represents the case in which a customer returns an old product to replace it with a remanufactured one.

4.5 The Multi-Component Case

In the previous section, we analyzed the case in which the remanufactured product is handled as a single, non-divisible system. However, in many situations, a product, such as an engine, is composed of different components, and the manufacturer may prefer to handle the components separately. Management at the component level permits greater flexibility in serving the markets for remanufactured and new products. In this section, we extend our hybrid manufacturing/remanufacturing system to allow for multiple components that constitute the final product.

4.5.1 Model Description

We assume that the final product is assembled from n different components, such that one unit of the final product contains $\xi_i \geq 1$ units of component $i = 1, ..., n$. We write $\boldsymbol{\xi} = (\xi_1, ...\xi_n)$.

The manufacturing (remanufacturing) times for component i are independent and identically distributed with an exponential distribution of rate μ_{Ni}^M (μ_{Ri}^M), and we write $\boldsymbol{\mu}_N^M = \left(\mu_{N1}^M, ..., \mu_{Nn}^M\right)$ and $\boldsymbol{\mu}_R^M = \left(\mu_{R1}^M, ..., \mu_{Rn}^M\right)$. We assume that all components of a returned product are remanufacturable because our focus of analysis is on the fulfillment of market demands rather than on the return process itself.

As discussed in Section 4.3, remanufactured products may contain some new components, as long as the majority are remanufactured. We denote the minimum number of remanufactured components in a remanufactured product by $\underline{Q}$. For simplicity, we assume that the components are weighted equally. The model can be extended to components with different

weights. Even that new products technically contain 100% of new components ($\overline{Q} = 0$), the model has the flexibility to allow for $\overline{Q} > 0$ for cases where some remanufactured components are permitted in new products. To satisfy a demand, the decision maker has to decide a column vector $\mathbf{q}, (q_i \leq \xi_i, i = 1, \ldots, n)$, in which each element represents the number of remanufactured component i used in serving the customer and the vector $\boldsymbol{\xi} - \mathbf{q}$ denotes the complementary new components. In order to serve a demand for new (remanufactured) product, the $\mathbf{q}$ should satisfy $\sum_{i=1}^{n} q_i \leq \overline{Q}$ to be a new product for original (substitute) serving, or satisfy $\sum_{i=1}^{n} q_i \geq \underline{Q}$ to be a remanufactured product for substitute (original) serving. We define $\overline{Q} \leq \underline{Q}$ and any $\sum_{i=1}^{n} q_i$ locating in between results in an infeasible combination which is neither a new nor a remanufactured product.

For example, for a product consisting of 10 components, if new products can contain no remanufactured component and remanufactured products have to contain at least 70% remanufactured components, we have $\overline{Q} = 0$ and $\underline{Q} = 7$. The decision maker can choose different combinations for $\mathbf{q}$ in which $\sum_{i=1}^{10} q_i = 0$ for a new product and $\sum_{i=1}^{10} q_i \geq 7$ for a remanufactured product.

The state has to be amended to allow for the three inventories for each component. We use $n \times 3$ matrix $\mathbf{X}^M = \left(\mathbf{x}_A^M, \mathbf{x}_R^M, \mathbf{x}_N^M\right)$ to denote the system state, where each vector in $\mathbf{X}^M$ contains the respective inventory levels, x_{Ai}, x_{Ri}, and x_{Ni} of components i. Note that all vectors in this essay are column vectors.

We also amend the action space by the composition of the product that serves the market demand, i.e., $\mathbf{a}^M \in A^M(\mathbf{X}^M)$, where $\mathbf{A}^M(\mathbf{X}^M)$ also takes into account the constraints defined by $\underline{Q}$ and $\overline{Q}$.

We define vectors $\mathbf{c}_N$ as the economic costs of manufacturing the components, $\boldsymbol{\varepsilon}$ and $\boldsymbol{\theta\varepsilon}$ as the environmental costs of manufacturing and of remanufacturing, respectively, of the different components. Same to the single-component model, we define the sum of the acquisition and remanufacturing economic cost as c_{RM}. Vectors $\mathbf{h}_A$, $\mathbf{h}_R$, and $\mathbf{h}_N$ represent the respective inventory holding costs, and thus, the total holding costs per time period can be written

as $h^M(\mathbf{X}^M) = \mathbf{h}_A^T\mathbf{x}_A^M + \mathbf{h}_R^T\mathbf{x}_R^M + \mathbf{h}_N^T\mathbf{x}_N^M$, where e.g., $\mathbf{h}_A^T = (\omega h_{A1}, \omega h_{A2}, \cdots, \omega h_{An})^T$. The transition rate becomes $\tau^M = \nu + \mathbf{e}^T\boldsymbol{\mu}_{Ri}^M + \mathbf{e}^T\boldsymbol{\mu}_{Ni}^M + \lambda_R + \lambda_N \equiv 1$. $\mathbf{e}$ represents a vector of ones of corresponding size.

Let v^M be the value function of the multi-component model. We write the optimality equation:

$$\begin{aligned} T^M v^M(\mathbf{X}^M) =& h^M(\mathbf{X}^M) + \nu T_A^M v^M(\mathbf{X}^M) + \sum_{i=1}^n \xi_i \mu_{Ri}^M T_{Ri}^M v^M(\mathbf{X}^M) + \sum_{i=1}^n \xi_i \mu_{Ni}^M T_{Ni}^M v^M(\mathbf{X}^M) \\ &+ \lambda_N T_{dN}^M v^M(\mathbf{X}^M) + \lambda_R T_{dR}^M v^M(\mathbf{X}^M), \end{aligned} \tag{4.9}$$

where,

$$\begin{aligned} T_A^M v^M(\mathbf{X}^M) =& \min\Bigg\{ v^M\left(\mathbf{x}_A^M + \sum_{i=1}^n \xi_i \mathbf{e}_i, \mathbf{x}_R^M, \mathbf{x}_N^M\right) + \omega c_{RM} + \sum_{i=1}^n (1-\omega)\theta_i \varepsilon_i, \\ & v^M(\mathbf{x}_A^M, \mathbf{x}_R^M, \mathbf{x}_N^M) \Bigg\}, \end{aligned} \tag{4.10}$$

and

$$T_{Ri}^M v^M(\mathbf{X}^M) = \begin{cases} v^M(\mathbf{x}_A^M - \mathbf{e}_i, \mathbf{x}_R^M + \mathbf{e}_i, \mathbf{x}_N^M), & \text{if } x_{Ai} \geq 1, \\ v^M(\mathbf{x}_A^M, \mathbf{x}_R^M, \mathbf{x}_N^M), & \text{otherwise}, \end{cases} \tag{4.11}$$

and

$$T_{Ni}^M v^M(\mathbf{X}^M) = \min\Bigg\{ v^M(\mathbf{x}_A^M, \mathbf{x}_R^M, \mathbf{x}_N^M + \mathbf{e}_i) + \omega c_{Ni} + (1-\omega)\varepsilon_i, v^M(\mathbf{x}_A^M, \mathbf{x}_R^M, \mathbf{x}_N^M) \Bigg\}, \tag{4.12}$$

and

$$
T_{dN}^{M} v^{M}(\mathbf{X}^{M}) = \begin{cases}
\min\limits_{0 \le q_i \le \min\{\xi_i, x_{Ri}\}} \Big\{ v^{M}\Big(\mathbf{x}_A^M, \mathbf{x}_R^M - \sum\limits_{i=1}^{n} q_i \mathbf{e}_i, \mathbf{x}_N^M - \sum\limits_{i=1}^{n} (\xi_i - q_i)\mathbf{e}_i\Big) \\
\qquad\qquad + \mathbf{1}\Big(\sum\limits_{i=1}^{n} q_i \ge \underline{Q}\Big)\omega\eta_{RN}\Big\}, \text{ if } \forall i,\ x_{Ni} \ge \xi_i, \\
\min\limits_{0 \le q_i \le \min\{\xi_i, x_{Ri}\}} \Big\{ v^{M}\Big(\mathbf{x}_A^M, \mathbf{x}_R^M - \sum\limits_{i=1}^{n} q_i \mathbf{e}_i, \mathbf{x}_N^M - \sum\limits_{i=1}^{n} (\xi_i - q_i)\mathbf{e}_i\Big)\Big\}, \\
\qquad\qquad \text{if } \exists\, \mathbf{q} \mid \sum\limits_{i=1}^{n} q_i \le \overline{Q} \text{ but } \nexists\, \mathbf{q} \mid \sum\limits_{i=1}^{n} q_i \ge \underline{Q}, \\
v^{M}(\mathbf{x}_A^M, \mathbf{x}_R^M, \mathbf{x}_N^M) + \omega\eta_L, \text{ if } \nexists\, \mathbf{q} \mid (\sum\limits_{i=1}^{n} q_i \le \overline{Q} \ \wedge \sum\limits_{i=1}^{n} q_i \ge \underline{Q}), \\
\min\Big\{ \min\limits_{0 \le q_i \le \min\{\xi_i, x_{Ri}\}} \Big\{ v^{M}\Big(\mathbf{x}_A^M, \mathbf{x}_R^M - \sum\limits_{i=1}^{n} q_i \mathbf{e}_i, \mathbf{x}_N^M - \sum\limits_{i=1}^{n} (\xi_i - q_i)\mathbf{e}_i\Big) \\
\qquad + \mathbf{1}\Big(\sum\limits_{i=1}^{n} q_i \ge \underline{Q}\Big)\omega\eta_{RN}\Big\}, v^{M}(\mathbf{x}_A^M, \mathbf{x}_R^M, \mathbf{x}_N^M) + \omega\eta_L\Big\}, \text{ otherwise.}
\end{cases} \tag{4.13}
$$

Operator T_{dR} is defined analogously to operator T_{dN}.

Proposition 4.5.1. *In the multi-component setting,*

(a) g^{M*} is concave in strategic weight ω.

(b) Under the optimal policy, if $c_{Ni} > \varepsilon_i$ for any component $i \in \{1, \ldots, n\}$ and $c_{RM} > \sum_{i=1}^{n} \theta_i \varepsilon_i$, the strategic cost g^{M*} and the environmental cost $\frac{g^{ME*}}{1-\omega}$ are non-decreasing in ω, and the economic cost $\frac{g^{MC*}}{\omega}$ is non-increasing in ω.

Proposition 4.5.1 expresses the same ideas as Propositions 4.4.2-4.4.5 of the single-component product model with regard to monotonicity and concavity of the strategic cost and monotonicity of the economic cost and environmental cost in ω.

4.5.2 Development of a Heuristic Policy

In this subsection, we develop a heuristic policy for the multi-component case, because the size of the state space grows exponentially in the number of components, which renders the

application of numerical procedures to obtain the optimal policy feasible only for a small number of components.

The optimal policy of the single-component model (see Section 4.4) has the structure of a base-stock policy (see Proposition 4.4.1). We base our heuristic on the assumption that this structure is preserved in the multi-component case and propose an *Aggregated Level Base-stock Policy* (ALBP). This policy uses a set of constant base-stock levels to trigger the (re-)manufacturing of components and the allocation of products to demands. The intuition behind "aggregated" is to integrate all the resources together, i.e., the returns, remanufactured and new products, to determine whether to acquire a return, produce a new product, etc. Its decisions are defined as follows:

1. Accept a return if and only if the total inventory of a component is less than what is required for the assembly of s_A products, i.e., if

$$\min_{i\in\{1,\ldots,n\}}\left\{\frac{x_{Ai}+x_{Ri}+x_{Ni}}{\xi_i}\right\} < s_A.$$

2. Produce one new unit of component i if and only if

$$x_{Ai}+x_{Ni}+x_{Ri} < s_{Ni}.$$

3. Within the feasible actions $\mathbf{A}^M(\mathbf{X}^M)$, satisfy a demand for a new (remanufactured) product using the components:

(a) if at least one pure[1] product of a new (remanufactured) product is available[2] and $x_{Ri} \geq r_{Ri}$ ($x_{Ni} \geq r_{Ni}$), use remanufactured (new) component(s) with the amount of $\min\{\xi_i, x_{Ri}\}$ ($\min\{\xi_i, x_{Ni}\}$), $i = 1, \ldots, n$ and complement by new (remanufactured) components with the amount of $\max\{\xi_i - x_{Ri}, 0\}$ ($\max\{\xi_i - x_{Ni}, 0\}$) if necessary;

(b) if at least one pure product of a new (remanufactured) product is available and

1. A pure product refers to a product that contains 100% new/remanufactured components.

2. It means that, $\forall i$, $x_{Ni} \geq \xi_i$ for a new product and $x_{Ri} \geq \xi_i$ for a remanufactured product.

$x_{Ri} < r_{Ri}$ ($x_{Ni} < r_{Ni}$), use new (remanufactured) components with the amount of ξ_i, $i = 1, \ldots, n$;

(c) if the pure product of the demanded new (remanufactured) type is unavailable,

(c-1) for the new (remanufactured) components such that $x_{Ni} < \xi_i$ ($x_{Ri} < \xi_i$), use $\min\{\xi_i, x_{Ri}\}$ ($\min\{\xi_i, x_{Ni}\}$) units of the remanufactured (new) component(s) if $x_{Ri} \geq r'_{Ri}$ ($x_{Ni} \geq r'_{Ni}$) and complement by new (remanufactured) components with the amount of $\max\{\xi_i - x_{Ri}, 0\}$ ($\max\{\xi_i - x_{Ni}, 0\}$) if necessary;

(c-2) if $\exists i$ such that its new (remanufactured) components satisfy $x_{Ni} < \xi_i$ ($x_{Ri} < \xi_i$) and the remanufactured (new) components have $x_{Ri} < r'_{Ri}$ ($x_{Ni} < r'_{Ni}$), reject the demand;

(c-3) for the new (remanufactured) components such that $x_{Ni} \geq \xi_i$ ($x_{Ri} \geq \xi_i$), use $\min\{\xi_i, x_{Ri}\}$ ($\min\{\xi_i, x_{Ni}\}$) units of the remanufactured (new) component(s) if $x_{Ri} \geq r_{Ri}$ ($x_{Ni} \geq r_{Ni}$) and complement by new (remanufactured) components with $\max\{\xi_i - x_{Ri}, 0\}$ ($\max\{\xi_i - x_{Ni}, 0\}$) if necessary; otherwise, use $\min\{\xi_i, x_{Ni}\}$ ($\min\{\xi_i, x_{Ri}\}$) units of the new (remanufactured) component(s) if $x_{Ri} < r_{Ri}$ ($x_{Ni} < r_{Ni}$) and complement by remanufactured (new) components with $\max\{\xi_i - x_{Ni}, 0\}$ ($\max\{\xi_i - x_{Ri}, 0\}$) if necessary.

In the return acquisition and the production actions, we aggregate all the available resources together to decide whether we accept a return or produce a new product. This is more strategic than only looking at one single inventory level. For instance, in the return acquisition process, we accept a return when the aggregate resource is insufficient by taking in to consideration of the other two states.

Let us illustrate the ALBP heuristic with an example with $n = 4$ components, flexibility constraints $\overline{Q} = 2$ and $\underline{Q} = 3$, and base-stock levels $\mathbf{r}_R = \{1, 1, 2, 2\}$. In state $\mathbf{x}_N = \{1, 1, 1, 1\}$ and $\mathbf{x}_R = \{3, 3, 4, 4\}$, we serve a demand for a new product with a system of only remanufactured components, by paying the penalty cost for substitution. In state $\mathbf{x}_N = \{1, 1, 1, 1\}$ and $\mathbf{x}_R = \{0, 0, 1, 1\}$, we satisfy a demand for a new product with a system that contains only new components.

Value s_A and vectors $\mathbf{s}_N, \mathbf{r}_R, \mathbf{r}_N, \mathbf{r}'_R, \mathbf{r}'_N \in \mathbb{N}^n$ are the parameters of the heuristic. There-

fore, for the n-component case, the ALBP requires the selection of values for the $5n + 1$ parameters such that the average strategic costs are minimized. We propose a numerical search procedure with simulation to obtain the optimal policy parameters based on consecutive greedy searches of each base-stock level (see Appendix E.4 for details).

4.6 Numerical Experiments

In this section, we first describe the data used for the numerical analysis. Then, we evaluate the performance of the ALBP heuristic policy by comparing the results of the ALBP with those of the optimal policy for the single-component and the 2-component cases. Finally, we perform sensitivity analyses with regard to four system parameters: (1) component-level structure, (2) strategic weight ω, (3) market substitution (η_{NR}, η_{RN}), and (4) component flexibility $\underline{Q}$.

4.6.1 Data Description

For the numerical experiments, we consider a six-cylinder diesel internal combustion engine as in Sutherland et al. (2008).

For the single-component case, we use the following data values. The manufacturing cost is €3,500 per unit (Kochhan et al., 2014). The energy consumption for manufacturing is 18,100 MJ for a new product and 1,850 MJ for a remanufactured product (Sutherland et al., 2008). The electricity price is assumed to be €0.18/kWh = €0.05/MJ Kochhan et al. (2014). The cost to remanufacture is approx. 50% of the cost of manufacturing new engines (Xiong et al., 2014). For return acquisition, we assume an acquisition price of €200, which corresponds to the selling prices of used engines on Autogator (Autogator, 2017), taking into consideration economies of scale of the remanufacturer. Annual holding costs for inventory are estimated to be 25% of the product cost Azzi et al. (2014). The remaining parameters for lost sales penalties, substitution, strategic weight ω, and demand and lead times are

Table 4.1: Parameters of the single-component product used in the numerical experiments.

c_{RM}	c_N	h_A	h_R	h_N	ε	η_N	η_R	η_{NR}	η_{RN}
€1,950	€3,500	€50	€437.5	€875	€905	€10,000	€12,000	€2,000	€2,500
	θ	ω	ν	μ_R	μ_N	λ_N	λ_R		
	0.1	0.7	4000	10000	8000	12000	8000		

estimated. All time-dependent values are annual values. All parameters are summarized in Table 4.1.

Following Sutherland et al. (2008), a six-cylinder diesel engine consists of five main components in total: 1 engine block, 1 cylinder head, 1 crankshaft, 6 connecting rods, and 6 pistons.

For the 2-component case, we use the cylinder head as Component 1 and the remaining parts as Component 2; for the 3-component case, we consider the engine block as Component 1, the cylinder head and crankshaft as Component 2, and the remaining parts as Component 3. The parameters for the different multi-component cases are shown in Table 4.2. We rescale the time to $\tau^M \equiv 1$ year. We use eBay (2017a), eBay (2017b) and eBay (2017c) to find proper prices of the original parts and estimate the component-level parameters.

We use various sets for the values $\overline{Q}$ and $\underline{Q}$ in our the analysis. In the 2-component case, we use $\overline{Q} = 0$ and $\underline{Q} = 1$; in the 3-component case, $\overline{Q} = 0$ and $\underline{Q} = 1$ given that the remanufactured product consists at least of Component 1 or 2 of the remanufactured options.

We use simulation and the numerical procedure of Appendix E.4 to determine the optimal parameter values. The run-length of each simulation is chosen such that the half-width of the 99% confidence interval is less than 1% of the average value.

Table 4.2: Parameters of the multi-item product numerical experiments.

Product	No.	Component	Unit	Remanufactured cost(€)	Manufactured cost(€)	Remanufactured energy cost(€)	Manufactured energy cost(€)	θ
2-Component	1	Cylinder head	1	760	1,520	55.5	222.25	0.25
	2	Others	1	990	1,980	35	682.75	0.05
3-Component	1	Engine block	1	725	1,450	30	498.5	0.06
	2	Cylinder head & Crankshaft	1	860	1,720	59	362.25	0.16
	3	Others	6	27.5	55	0.25	7.375	0.03

4.6.2 Performance Analysis of the Heuristic

We analyze the effectiveness of the ALBP heuristic by comparing its strategic cost to the optimal strategic cost. Due to the exponential increase of the size of the state space, we can derive the optimal cost in a reasonable run time only for the single-component case and for the 2-component case. The results are shown in Table 4.3.

In the single-component case, the ALBP heuristic achieves close-to-optimal results. The percentage difference, which is defined as $\frac{g^{ALBP}-g^*}{g^*} \times 100\%$, is less than 0.02%. In the 2-component case, Table 4.3 indicates that the average gap is 0.89% and the maximum gap is 1.44%. We conclude that the ALBP performs well, for cases with few components.

Table 4.3: Optimality Gap of the Heuristic.

Comp. No.	ν	$\boldsymbol{\mu}_R$	$\boldsymbol{\mu}_N$	λ_N	λ_R	Opt.(k€)	ALBP (k€)	Gap%	$(s_A, \mathbf{s}_N, \mathbf{r}_R, \mathbf{r}_N, \mathbf{r}'_R, \mathbf{r}'_N)$
		(1×10^3)							
1	2	**10**	**8**	12	8	102,957	102,957	0.00%	(22,44,23,25,21,1)
1	4					88,942	88,942	0.00%	(51,51,37,46,51,1)
1	6					74,927	74,927	0.00%	(65,78,51,47,51,1)
1	8					61,339	61,351	0.02%	(146,148,51,49,50,1)
1	4	**6**	**8**	12	8	88,942	88,942	0.00%	(51,51,36,43,40,1)
1		**8**				88,942	88,942	0.00%	(51,51,35,44,39,1)
1		**12**				88,942	88,942	0.00%	(57,62,35,43,51,1)
1	4	**10**	**6**	12	8	97,498	97,498	0.00%	(51,51,30,34,39,1)
1			**10**			80,385	80,385	0.00%	(64,74,42,39,51,1)
1			**12**			71,829	71,830	0.00%	(142,94,51,5,49,41)
1	4	**10**	**8**	6	8	46,944	46,944	0.00%	(150,68,51,6,51,1)
1				8		60,942	60,942	0.00%	(147,89,50,5,48,1)
1				10		74,942	74,942	0.00%	(61,69,45,39,51,1)
1	4	**10**	**8**	12	4	55,581	55,590	0.02%	(82,99,51,51,50,1)
1					6	72,142	72,142	0.00%	(60,70,44,46,51,1)
1					10	105,741	105,741	0.00%	(51,51,31,39,33,1)
2	2	**10**	**8**	12	8	101,769	102,508	0.73%	(18,(68,68),(17,27),(82,82),(17,14),(1,1))
2	4					87,301	88,426	1.23%	(15,(46,47),(13,16),(73,74),(32,47),(1,1))
2		**8**				87,327	87,725	0.46%	(48,(63,52),(57,44),(14,1),(9,43),(4,1))
2		**10**	**10**			79,060	79,428	0.47%	(21,(82,89),(38,43),(72,68),(25,29),(1,1))
2			**8**	10		73,254	74,010	1.03%	(22,(92,94),(15,18),(78,79),(16,11),(1,1))
2				12	10	103,140	104,625	1.44%	(14,(93,118),(13,21),(123,97),(20,10),(1,1))

4.6.3 Sensitivity Analysis

Next, we perform sensitivity analysis with respect to some of the model parameters.

The Effect of the Number of Component Levels

Table 4.4 displays the results for a single-component product and 2- and 3-component prod-

Table 4.4: Results for single-component and 2- and 3-component products ($\omega = 0.7$).

No.	$\nu, \boldsymbol{\mu}_R, \boldsymbol{\mu}_N, \lambda_N, \lambda_R$ (1×10^3)	Single-Component	2-Component			3-Component		
			Cost (k€)	Imprvmt.[a] (k€)	Imprvmt.[b] (%)	Cost (k€)	Imprvmt. (k€)	Imprvmt. (%)
1	4,**10**,**8**,12,8	88,942	88,426	516	0.58	87,696	730	1.40
2	2,**10**,**8**,12,8	102,957	102,508	449	0.44	102,263	245	0.67
3	4,**10**,**8**,12,10	105,741	104,625	1,089	1.03	103,845	780	1.77

The parameters of the Poisson processes' and exponential distributions' rates are as follows: (1) 2-component, $\mathbf{10} \rightarrow \mu_{R1} = \mu_{R2} = 10$ and $\mathbf{8} \rightarrow \mu_{N1} = \mu_{N2} = 8$; (2) 3-component, $\mathbf{10} \rightarrow \mu_{R1} = \mu_{R2} = 10, \mu_{R3} = 60$ and $\mathbf{8} \rightarrow \mu_{N1} = \mu_{N2} = 8, \mu_{N3} = 48$.

[a] The improvement is compared between i-component product and $(i-1)$-component product.
[b] The improvement is compared between i-component product with the single-component product $(\frac{g^{\text{single-component}} - g^{i\text{-component}}}{g^{\text{single-component}}})$.

ucts. The results illustrate the flexibility provided by breaking products into components. We observe that the strategic cost decreases with the number of components n, which implies that a company can take advantage of greater flexibility by breaking products into smaller components. The greater the number of components that a product is broken down to, the lower the strategic cost.

In Table 4.4, by separating the entire product into two components (cylinder head and others, including the engine block), the strategic cost decreases by 449-1,089k€. In the 3-component case, in which we take one additional step to separate the engine block from the rest, the strategic cost only decreases by 245-780k€. We observe that separating the less valuable components (in terms of both economic and environmental costs), such as the connecting rods and pistons, from the valuable components does not lead to significant improvement. By contrast, treating the valuable components independently and flexibly serving the demands with these components represents an effective way of taking advantage of component flexibility. Hence, separating the valuable components from the others yields the largest benefits when implementing multi-component models.

The Effect of the Strategic Weight ω

By taking the business's strategic view of sustainability, we analyze the influence of the strategic weight ω on the decisions of the ALBP policies. The results are shown in Figure

4.4.

Recall that ω represents the strategic weight of the economic cost while $1-\omega$ represents the weight of the environmental cost. Small values of ω indicate that the firm is more sustainability-oriented and less economy-oriented. The solid line in Figure 4.4 indicates that the strategic cost increases with ω and is concave in ω. As analyzed in Propositions 4.4.4 and 4.4.5, the economic cost (environmental cost) is non-increasing (non-decreasing) with respect to ω (dashed lines in Figure 4.4). In our instance, the economic cost decreases sharply with the increase in ω when ω is small but remains unchanged for $\omega \geq 0.2$. This pattern implies that the optimal policy for the strategic cost seems particularly sensitive to changes of small values of ω.

Figure 4.5 indicates that the substitution policy changes with respect to ω. We define the substitution rates γ_{NR} as the ratio of the times a new product is used to satisfy a demand for remanufactured product and the total number of remanufactured demand arrivals. The rate of the other direction of substitution is γ_{RN}.

For γ_{NR}, the single-component case substitutes for $\omega \geq 0.2$ as shown in Figure 4.5, which implies that for $\omega < 0.2$, no substitution of new products for remanufactured demand is more strategically profitable. The reason is that when the value of ω is sufficiently small, only sustainability matters and it is preferable to suspend the production of new products to reduce negative environmental impacts. Moreover, when the number of components increases, the value of γ_{NR} decreases, which demonstrates the advantage of breaking a single-component product into components such that the high component level provides sufficient flexibility in managing inventory, which leads to lower substitution rates and higher frequency of satisfying customers with the type of product they demand.

For γ_{RN}, the policy indicates that, in our numerical instances, using remanufactured products to substitute new demand is not strategically beneficial. Therefore, for all cases, $\gamma_{RN} = 0$. The system should either satisfy the remanufactured demand type with the corresponding original products or reject it if the original product is unavailable.

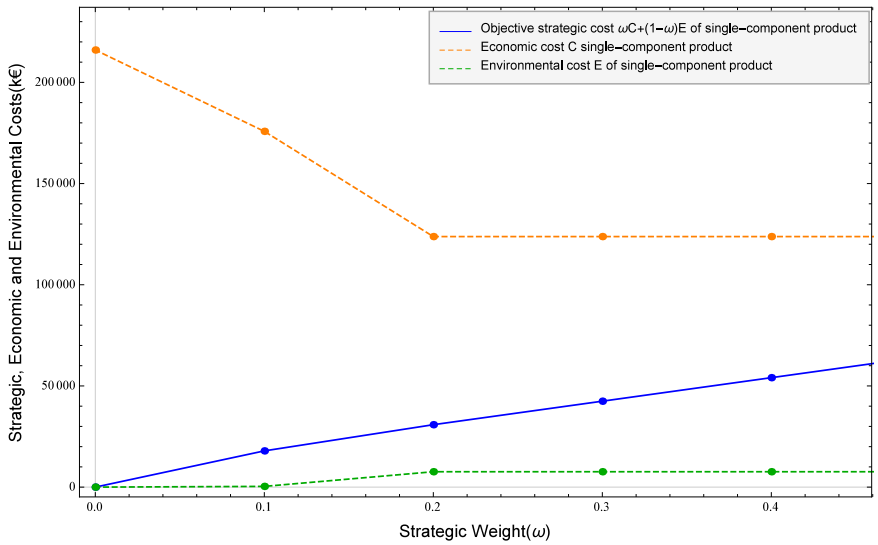


Figure 4.4: Cost sensitivity with respect to ω of the strategic, economic and environmental costs for the single-component case.

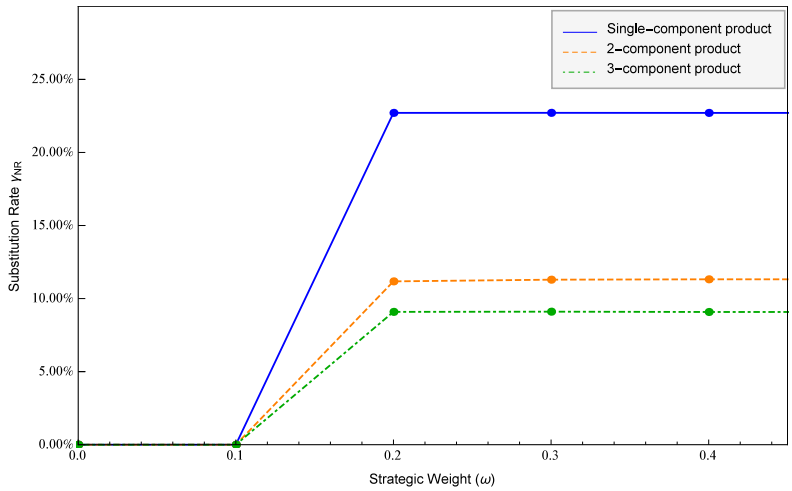


Figure 4.5: The sensitivity of substitution rate with respect to ω for serving new products to remanufactured demands, γ_{NR}.

Table 4.5: Optimal policies of different substitution costs.

Cases (k€)		Cost (k€)	Optimal Policy r_R, r_N [c]	Cases (k€)		Cost (k€)	Optimal Policy r_R, r_N
$\eta_{NR} = 0$	$\eta_{RN} = 2.5$	84,463	50, 1	$\eta_{NR} = 2$	$\eta_{RN} = 0$	88,942	50, 15
$\eta_{NR} = 2$	$\eta_{RN} = 2.5$	88,942	50, 15	$\eta_{NR} = 2$	$\eta_{RN} = 2.5$	88,942	50, 15
$\eta_{NR} = 6$	$\eta_{RN} = 2.5$	88,942	X, 50	$\eta_{NR} = 2$	$\eta_{RN} = 5$	88,942	X, 15
$\eta_{NR} = 12$	$\eta_{RN} = 2.5$	88,942	X, X	$\eta_{NR} = 2$	$\eta_{RN} = 10$	88,942	X, 15

[c] The optimal base-stocks are state dependent, we thus choose $r_i(x_j, x_k)$ where $i \in \{R, N\}, j, k \in \{A, R, N\}$, $i \neq j \neq k$ and x_j, x_k are fixed for each base-stock level. For r_N, we choose $x_j = x_k = 10$ and for r_R we choose $x_A = 50$ and $x_N = 45$. The truncated inventory capacity we set is 50.

The Effect of Market Substitution

In this part, we analyze the effect of market substitution by varying the substitution costs η_{RN} and η_{NR}. We focus on the single-component case, which allows us to better understand the effect of substitution on the decisions of the optimal policy by analyzing the different values $\eta_{RN} = \{0; 2,500; 5,000; 10,000\}$ and $\eta_{NR} = \{0; 2,000; 6,000; 12,000\}$. We also compare the performance of our ALBP policy to the two benchmark policies of always or never substituting.

The results are shown in Table 4.5. The optimal strategic cost increases with the substitution cost regardless of the substitution direction. We observe that thresholds $r_i(\mathbf{x}_{-i}), i \in \{R, N\}$ are, however, non-decreasing with the substitution costs. This result implies that the higher the substitution cost, the more reluctantly substitution is considered as an option. In particular, when $\eta_{RN} = 12,000$€, the "X" in Table 4.5 implies that in our examples, no substitution is preferred.

Table 4.6 illustrates the benefits of partial substitution. Moreover, compared to the benchmark policies that either always substitute or never substitute, Figure 4.6 indicate that using our ALBP heuristic and substitution reduce the strategic cost and that this effect leads to on average 7.6% (perfect substitution) and 14.1% (no substitution) cost reduction.

The Effect of the Component Flexibility

Next, we explore the effect of flexibility in the product configuration on the strategic cost and the base-stock levels. We do so by varying the authorized minimum numbers of reman-

Table 4.6: Comparisons between the case with substitution flexibility and two benchmarks.

No.	ALBP (k€)	$(s_A, s_N, r_R, r_N, r'_R, r'_N)$	Perfect Substitution (p_Sub, k€)	$(s^1_A, s^1_N, r^1_R, r^1_N, r^{1\prime}_R, r^{1\prime}_N)$	$\frac{p\_Sub}{ALBP}$ ×100%	No Substitution (n_Sub, k€)	(s^2_A, s^2_N)	$\frac{n\_Sub}{ALBP}$ ×100%
1	102,957	(22,44,23,25,21,1)	102,957	(49,51,51,1,51,1)	100%	102,958	(57,59)	100%
2	88,942	(51,51,37,46,51,1)	88,950	(57,65,43,1,51,1)	100%	88,942	(60,66)	100%
3	74,927	(65,78,51,47,51,1)	78,738	(65,81,51,1,27,1)	105%	74,927	(50,76)	100%
4	61,351	(146,148,51,49,50,1)	70,998	(78,97,51,1,20,1)	116%	62,273	(48,82)	102%
5	88,942	(51,51,36,43,40,1)	88,944	(60,72,51,1,51,1)	100%	88,942	(55,73)	100%
6	88,942	(51,51,35,44,39,1)	88,945	(57,65,50,1,50,1)	100%	88,942	(60,66)	100%
7	88,942	(57,62,35,43,51,1)	88,945	(57,64,50,1,51,1)	100%	88,942	(60,68)	100%
8	97,498	(51,51,30,34,39,1)	97,499	(51,51,46,1,51,1)	100%	97,499	(59,61)	100%
9	80,385	(64,74,42,39,51,1)	80,994	(61,72,47,1,51,1)	101%	80,388	(67,77)	100%
10	71,830	(142,94,51,5,49,41)	76,081	(61,72,30,1,51,1)	106%	72,849	(67,77)	101%
11	46,944	(150,68,51,6,51,1)	51,181	(141,150,51,1,51,1)	109%	55,516	(100,31)	118%
12	60,942	(147,89,50,5,48,1)	61,649	(147,89,50,1,51,1)	101%	61,628	(78,74)	101%
13	74,942	(61,69,45,39,51,1)	74,987	(147,78,50,1,51,1)	100%	74,943	(66,74)	100%
14	55,590	(82,99,51,51,50,1)	61,647	(66,65,51,1,14,1)	111%	56,009	(46,82)	101%
15	72,142	(60,70,44,46,51,1)	74,258	(63,73,51,1,40,1)	103%	72,142	(52,71)	100%
16	105,741	(51,51,31,39,33,1)	105,742	(56,57,51,1,51,1)	100%	105,742	(59,65)	100%

[d] The cases correspond to the first 16 cases of Table 4.3 with $\omega = 0.7$.

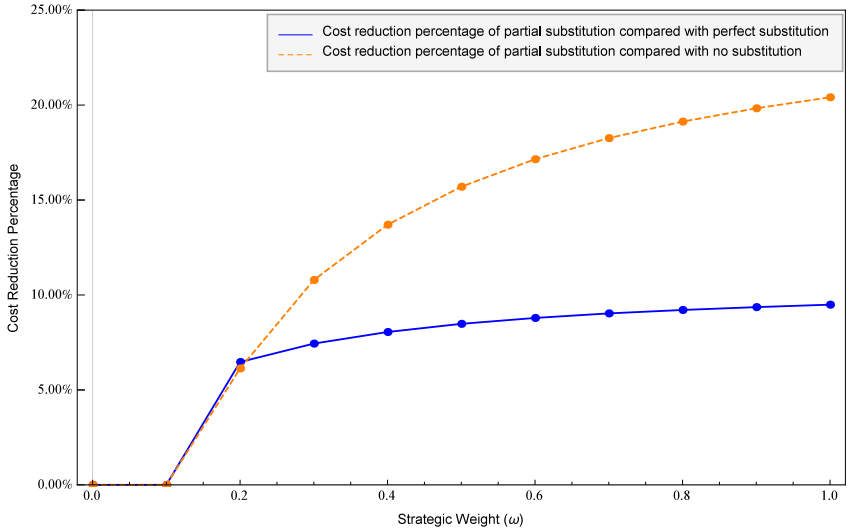


Figure 4.6: Reduction of strategic cost by using ALBP compared to perfect substitution and no substitution, as a function of the strategic weight.

Table 4.7: Comparison of thresholds and strategic costs for different values of $\underline{Q}$.

$\underline{Q}$	ALBP cost (k€)	$(s_A, (s_{N1}, s_{N2}, s_{N3}), (r_{R1}, r_{R2}, r_{R3}), (r_{N1}, r_{N2}, r_{N3}), (r'_{R1}, r'_{R2}, r'_{R3}), (r'_{N1}, r'_{N2}, r'_{N3}))$
1	87,696	(11,(112,157,46),(12,80,13),(118,94,2),(14,11,10),(1,1,1))
2	89,149	(23,(216,201,161),(27,17,3),(403,228,105),(23,47,6),(1,1,1))
8	89,138	(31,(203,202,205),(200,212,200),(205,214,200),(12,25,81),(1,1,1))

ufactured components in the products. We focus our analysis on values $\underline{Q} = \{1, 2, 8\}$ [3] with $\omega = 0.7$. Recall that, for example, $\underline{Q} = 2$ implies that a remanufactured product contains at least 2 of the 3 components, excluding the connecting rods and pistons. The results are shown in Table 4.7.

If $\underline{Q}$ increases, the condition for a product to be considered as a remanufactured product becomes stricter, and the component flexibility decreases. Table 4.7 indicates that the strategic cost increases with $\underline{Q}$ between $\underline{Q} = 1$ and $\underline{Q} = 2$ and does not increase for $\underline{Q} = 8$. Hence, in the least flexible case, firms seem to no longer need component separation, as the single-component product case is much easier to manage and leads to the same results.

For companies, the $\underline{Q}$ is usually not a decision variable to be optimized but is exogenously given by industry or firm standards. Nevertheless, our analysis highlights that it is important for the decision maker to adjust the base-stock levels to the degree of component flexibility to serve customers and to avoid penalty costs.

4.7 Conclusions

we developed a model of a hybrid manufacturing/remanufacturing system with two-way substitution, component-level inventory management, and sustainability objectives in a market differentiating new and remanufactured products. We derived the structure of the optimal policy for the single-component case when the substitution from new to remanufactured

3. Since the 6 connecting rods and pistons are of low economic and environmental values, we do not separate them to analyze $\underline{Q} = \{3, 4, 5, 6, 7\}$.

products is costless. To permit additional operational flexibility, we also considered a case with multiple components in which the decision maker can dynamically manipulate the proportion of remanufactured components in its products. We derived analytic properties of the optimal policy for the multi-component case and develop a heuristic procedure, the ALBP heuristic. We found that the heuristic performs close to optimality. We also considered an extension of the single-component case to correlated demand and return.

We compared different schemes of substitution (no, partial, and perfect substitution) and found that our model of partial substitution achieves the lowest strategic cost. We also found that economy-oriented firms tend to benefit more than sustainability-oriented firms from partial substitution. Our results indicate that while economy-oriented firms should make use of substitution of new products to satisfy remanufactured demands, sustainability-oriented firms should avoid substitution because taking advantage of remanufacturing is less environmentally costly than producing new products and substitution is less cost-efficient in this case. Our analysis also indicates that a hybrid manufacturing/remanufacturing system with a better fine-grained component level helps to reduce strategic cost. Firms should substitute less frequently if products are managed at the component level than as if they are managed at the product level.

Our analysis provides important insights for production managers. For example, Ford and General Motors, which we cite in Section 4.3, could focus on managing their remanufacturing on detailed component level and actively manage partial substitution to reduce their strategic costs. For General Electric, on the other hand, partial substitution may be less effective, given the orientation of its business strategy to sustainability targets (see Section 4.3). Although we have used a case from machinery manufacturing as numerical example, our insights are generalizable to a variety of industry settings.

There are also a few limitations of this research. First, our model assumes that all the components in a return is remanufacturable, while some components may not be remanufacturable (Jin et al., 2013). Second, accepted returns and servable remanufactured products

can also be disposed of if needed (Gayon et al., 2017). In our model, to emphasize the two-way substitution, we simplify the remanufacturing process and neglect the two decisions. Third, to highlight the substitution issue, we assume customers always accept the alternative type of products given the substitution cost. However, some customers may turn away instead of accepting it.

We hope that our research sparks interest in further analyzing the impact of remanufacturing on multi-objective business strategies. One direction for future research is to consider consumer preferences and choices. Another direction is to integrate the pricing problem for returns acquisition. As noted in the literature Weatherford and Bodily (1992), Hahler and Fleischmann (2017), the acquisition price of returns may depend on the quality of the returned product and current inventory level, and the interaction between pricing and substitution may reveal interesting new insights.

CHAPTER 5

JOINT DYNAMIC PRICING AND RETURN QUALITY STRATEGIES UNDER DEMAND CANNIBALIZATION

5.1 Abstract

In this essay, we investigate a hybrid manufacturing/remanufacturing system receiving returns of used products and producing and selling new and remanufactured products to customers. Given different remanufacturing costs based on different quality levels, the decision maker observes the quality of the returns and decides the amounts to remanufacture and to recycle for raw materials in order to maximize the profit under cannibalization. We model the problem as a stochastic programming problem with the decision maker determining the production quantity and the quality threshold for returns to enter remanufacturing process, as well as the pricing of the two types of products. Then we look at the multi-period setting with either exogenous prices or endogenous prices. Our findings show that with an additive form of demand functions, the optimal strategies are of threshold-type. We also find that the quality strategy (1) dominates the "*remanufacture-all*" and the "*recycle-all*" strategies; (2) counteracts (compensates) the negative (positive) effect of cannibalization effect on profit.

Key words: Quality strategy, Dynamic pricing, Remanufacturing, Demand cannibalization, Dynamic programming

5.2 Résumé

Dans cet article, nous étudions un système de fabrication/reconditionnement hybride recevant des retours, produisant et vendant des produits neufs et reconditionn és aux clients. Compte tenu des différents coûts de réusinage basés sur différents niveaux de qualité, le décideur observe la qualité des retours et décide des quantités à reconditionner et à recycler pour les matières premières afin de maximiser les profits dans le cadre de la cannibalisation. Nous modélisons d'abord un problème de programmation stochastique sur une période unique, où le décideur détermine la quantité de production et le seuil de qualité pour l'acceptation des retours, ainsi que les prix des deux produits. Ensuite, nous examinons le problème à travers plusieurs périodes avec des prix exogènes et endogènes. Nos résultats montrent qu'avec une forme additive des fonctions de demande, la stratégie optimale est de type seuil. Nous constatons également que la stratégie de sélection des grades de qualité (1) domine les stratégies « tout reconditionner » et « tout recycler », qui sont largement adoptées ; (2) peut neutraliser (compenser) l'influence négative (positive) de l'effet de cannibalisation sur le profit.

Mots Clés: Stratégie de qualité, Tarification dynamique, Remanufacturing, Cannibalisation de la demande, Programmation dynamique

5.3 Introduction

Manufacturers such as Apple, Asus, Dell, Sony and Xerox (Electronics TackBack Coalition, 2016) have initiated their recycling programs years ago. End-of-use or end-of-life products are encouraged to be taken back by the manufacturers or third party OEMs for the purpose of reusing. In addition to electric devices, for instances, cellphones, tablets and laptops, used products including vending machines (Royal Remanufacturing, LLC, n.d.), automotive parts (Oakwille Voklswagen, n.d.) and even airplane engines (Rolls-Royce, n.d.) are also collected for future usages. Some of the returned products that can still satisfy certain standard would go through the remanufacturing process for serving the market again, whereas some others are not able to be remanufactured, either because of low quality or high remanufacturing cost (Liu et al., 2017). Usually, the non-remanufacturable returns are either sold to factories that are able to recycle valuable materials from the returns, or simply sent to landfill/incineration which can be detrimental to the environment (Guide & Van Wassenhove, 2001).

Although remanufacturing and recycling play a role in protecting the environment, saving production cost and leading to circular economy, massive implementation of the regarding programs is also harmful to firms' profitability. On the one hand, the remanufactured provision cannibalizes the sales of the new products, which requires firms to balance the trade-off. Meanwhile, higher cost of remanufacturing a return signifies the return's lower quality. On the other hand, many investigations highlight that the recycling activity can be quite cost-inefficient for some types of products but landfill/incineration costs much less (Tierney, 2015; Gradus et al., 2017).

Therefore, firms are facing a trade-off to solve, regarding (1) the uncertainty of returns' quality and amount, (2) higher remanufacturing costs for lower quality returns, (3) the market's and government's requirements for remanufacturing and recycling, and (4) their own profitability by lowering costs and/or higher sales. Here, the answer of choosing the optimal combination of quality and pricing strategies to maximize firms' profit is not quite clear.

In this essay, we investigate a manufacturing/remanufacturing firm which seeks to maximize its profit. In particular the firm is interested in using dynamic pricing strategy for new and remanufactured products, as well as choosing the returns' quality threshold, to maximize the profit which is collected from selling both new and remanufactured products and recycling the non-remanufacturable returns. We would also like to analyze the model to understand the following research questions: (1) How much improvement does the quality strategy achieve compared to the simple "remanufacture all" or "salvage all" strategy? (2) Whether the flexible quality strategy mitigates cannibalization effects more effectively as the easy strategies such as "remanufacture all" or "salvage all"? (3) How do we explain the differences of the optimal strategies for various types of products? Many low value products, for example, the copper in the electric wires decorating Christmas trees (Minter, 2011), are preferred in practice to be recycled rather than remanufactured. To answer the above questions, we model a period-review make-to-order inventory system for producing new and remanufactured products, where facing demand and return quantity and quality uncertainties, a decision maker dynamically sets the selling prices, chooses the optimal remanufacturing quality threshold and decides the production quantity in each period.

The theoretical and practical contributions of this essay lie in the following aspects. First, most of the existing literature in the dynamic pricing and return quality grading streams has not shed lights on demand cannibalization and quality strategy, which renders us the opportunity to fill the research gap. Second, many researchers have considered to contract the negative effect of demand cannibalization by strategies of e.g., relicensing (Oraiopoulos et al., 2012), advertising (De Giovanni, 2017) and service (Ramani & De Giovanni, 2017), yet have not investigated the influence of return quality strategy. Third, in response to the suggestion of Kumar and Ramachandran (2016), our model addresses the issue of viewing remanufacturing as a business strategy in revenue/profit management through quality strategy. Finally, the model can assist firms to control the remanufacturing quantity, well maintain the inventory of new products and gain high profit by a more flexibly manner than

simple strategies such as “remanufacture all” and “salvage all”.

By characterizing the optimal policy regarding the single-period model, exogenous pricing dynamic model and endogenous pricing dynamic model, we find that a flexible quality strategy does not only benefit firms by adopting optimal amount of returns but also serves as a strategy to counteract the negative influence of cannibalization. However, according to our analysis, it is possible that cannibalization serves as a positive tool to increase profits despite its effect of cutting down the new products sales. In such a circumstance, the quality strategy compensates the cannibalization instead of counteracting. We also show that under exogenous pricing scheme, the problem can be decomposed into two separate subproblems. Optimizing each individually leads to the optimization of the primary problem. By comparing to the reviewed literature, we find that, in an additive form of demand function, the pricing strategy of new products increases with the inventory level of remanufactured products, which is intuitive but inconsistent with the finding by Yan et al. (2017) from a customers' perceived utility angle without quality grading.

The rest of the essay is organized as follows. Section 5.4 and Section 5.5 respectively reviews related literature and presents the model. Following that, we analyze the structure of the model in Section 5.6 and Section 5.7 for single-period and multi-period scenarios, respectively. In Section 5.8, we use numerical experiments to derive important managerial insights and analyze the model with different types of products. We in the end conclude the essay in Section 5.9.

5.4 Literature Review

For discussing the related trade-offs, three main streams of literature are relevant to the problem: returns of heterogeneous quality, dynamic pricing and demand cannibalization under the remanufacturing setting. Particularly, we look at the literature using dynamic modeling to emphasize the strategic elements.

Most of the existing literature treats the quality of returns either homogeneously or

heterogeneously but exogenously. Aras et al. (2004) use a dynamic model which characterizes returns into quality levels of "high" and "low" and point out that it is always more beneficial to remanufacture high quality returns first and that to differentiate the quality helps firms lower the cost of remanufacturing. Taking further steps, Cai et al. (2014) take pricing and return acquisition quantity strategies into account based on the "high" and "low" quality levels. They highlight that the difference between the two quality levels affects the strategies greatly. The larger it is, the higher (lower) acquisition price of the high (low) quality returns is. However, the two-category quality grading system is disputable. In Ferguson et al. (2009), the authors point out the benefits of quality grading for returns in bringing down costs and discuss the disadvantages of two categorizes and the unnecessity of more than five. In addition to dynamic models, there are plenty of research addressing quality issue using static deterministic/stochastic methods such as Galbreth and Blackburn (2010) and Örsdemir et al. (2014). Particularly, Liu et al. (2017) connect the pricing strategy with the cannibalization effect under the consideration of returns' quality in order to understand the influence of Chinese Fund Policy onto the remanufacturing/recycling strategies. However, to the best of our knowledge, the previous research has not considered quality as a flexible strategy to investigate in its influence on demand cannibalization in a dynamic setting, and this essay aims to fill this research gap.

Dynamic pricing is recognized as an approach for revenue/profit management (Akçay et al., 2010). Federgruen and Heching (1999) investigate in the dynamic pricing strategy interfaced with production inventory management and characterize the optimal policy when facing demand uncertainty. Furthermore, Chen and Simchi-Levi (2004) include fixed ordering cost in the model and explore serval demand function types, and Allon and Zeevi (2011) take capacity investment decisions into account to optimally manage the inventory system. Instead of a single product, (Maglaras & Meissner, 2006) consider a multiple-product setting and discuss the optimal capacity allocation and dynamic pricing strategies. In the remanufacturing setting, Zhou and Yu (2011) consider a comprehensive case with deciding the

acquisition effort, product selling pricing strategy and inventory management policy for a remanufacturing system. Their findings show that when price is endogenous, the optimal selling price decreases but the optimal acquisition effort increases in the serviceable product inventory level, and both decisions decrease with the aggregate inventory level. As new and remanufactured products can be viewed as partially substitutable products, we review the literature that prices substitutable products dynamically. Dong et al. (2009) examine a dynamic pricing and inventory management problem in which a retailer is facing long supply lead time but a short selling season for substitute goods. The findings show the enormous benefits of dynamic pricing strategy when inventory scarcity and/or products' quality differences occur. Instead of taking substitute goods as customers' choices, Yu et al. (2017) evaluate a system providing a high-end product and a low-end product, where the latter ones can be substituted by the former ones. The results show that the dynamic pricing strategy and substitution strategy are complementary when (1) the utilization of high-end production facility increases; (2) the price sensitivity of high-end product decreases; and (3) the production cost of high-end product increases. This stream of research sufficiently studies the interface between dynamic pricing and inventory management yet still lacks the discussion of demand cannibalization and the quality of returns.

In contrast to stochastic dynamic pricing, demand cannibalization effect which means each sold unit of secondary products causes a lost sales of one unit new product (Debo et al., 2006), has been explored in various types of research. Abbey et al. (2015) conduct an empirical and experimental research for advising firms to make strategic decisions on whether to enter remanufacturing business by evaluating the cannibalization effect. When firms implement remanufacturing processes, the art of designing mechanisms to overcome cannibalization and/or taking cannibalization as a strategy to protect firms' profits becomes indispensable. Ferguson and Toktay (2006) find that when remanufacturing is carried out by the manufacturer itself, the revenue increases from remanufacturing may exceed the impact of cannibalization effect. If a third party remanufacturing OEM tries to enter the

competition, the manufacturer may choose remanufacturing or preemptively collect its used products to deter the entry. Atasu et al. (2008) evaluate the interaction among competition with OEM competitors, product life cycle, and cannibalization in determining the profitability of remanufacturing and find that despite the cannibalization, remanufacturing can serve as a good marketing strategy against low-cost OEM competitors entering the market. Oraiopoulos et al. (2012) takes relicensing as an effective strategy leading to gain revenue to counteract the cannibalization effect. Motivated by DellReconnect project, De Giovanni (2017) and Ramani and De Giovanni (2017) find a service strategy and an advertising strategy, respectively, can mitigate the cannibalization effect in a supply chain consisting of a manufacturer and a collector who refurbishes used products. However, provided the existence of a group of consumers who do not buy the remanufactured product regardless of the price, two competing OEMs may benefit from the entry of third part remanufacturers (Wu & Zhou, 2016), which leads to a contrasting conclusion that secondary market should not be always prohibited to avoid cannibalization effect. Other than the single manufacturer case, Mitra (2016) shows that in a duopoly environment, one of the manufacturers who performs remanufacturing gain higher profit even with the presence of cannibalization. Moving from static or two-period modeling, Yan et al. (2017) formulate a multi-period cannibalization setting and show the optimal policy to dynamically price new and remanufactured products to maximize revenue. However, the literature except Ovchinnikov (2011), Liu et al. (2017) and Yan et al. (2017) hardly connects cannibalization with dynamic pricing strategies, not to mention quality strategy. Yet none of the three exceptions addresses the flexible quality strategy and only the last one includes dynamic pricing as well as demand and return uncertainty.

5.5 Model Formulation

We consider a firm which collects used products for remanufacturing while producing new products for a selling season of a finite horizon. We denote the time period as t, $t = 1, \ldots, T$

and the final period $T+1$. The demand of each type of product depends on the price and is uncertain. Let two random variables D_t^N and D_t^R be the demands of the products in period t, which follow the demand structure of Zhou and Yu (2011), we have:

$$D_t^N(p_t^N, p_t^R) = \alpha_N - \beta_N p_t^N - \gamma(p_t^N - p_t^R) + \epsilon_t^N, \text{ and} \tag{5.1}$$

$$D_t^R(p_t^N, p_t^R) = \alpha_R - \beta_R p_t^R + \gamma(p_t^N - p_t^R) + \epsilon_t^R, \tag{5.2}$$

where p_t^N and p_t^R are the prices decided by the firm for new and remanufactured products bounded by $[\underline{p}^N, \overline{p}^N]$ and $[\underline{p}^R, \overline{p}^R]$, respectively. α_N and α_R are the market fundamentals for the products and β_N and β_R highlight the price sensitivity. γ represents the cannibalization effect due to the price competition of the two products. Both ϵ_t^N and ϵ_t^R are *i.i.d.* random variables. They are nonnegative variables following cumulative distribution functions (CDF) of $F^N(\cdot)$ and $F^R(\cdot)$, respectively. The realizations of the demands are noted by d_t^N and d_t^R.

The amount of returned products R_t of each period t is an *i.i.d.* random variable, whose distribution is denoted by $F^r(R)$ (CDF) and its realization is r_t. In each period, the quality of each return is graded based on a continuous scale of $[0, 1]$ where 1 is equivalent to useless and 0 indicates a untouched returned new product, i.e., the smaller the grade is, the higher the quality is. To become a candidate for remanufacturing, a return by regulation satisfies a quality grade of at most i^S. In the rest of the essay, we let $i^S = 1$ without loss of generality. To take advantage of flexible grading to maximize profit, firms in our model can choose a threshold i_t to dynamically control the remanufacturing quantity as long as $i_t \leq i^S$ is satisfied. The quality threshold is announced at the beginning of t but the true quality of returns is unknown before their arrival. However, the decision maker has the knowledge of the distribution of the return quality, which is denoted as a uniform distribution $F^q(i)$, $i \in [0, 1]$ and the random variable is *i.i.d.* throughout periods (Ferguson et al., 2009; Teunter & Flapper, 2011). Meanwhile, the unit remanufacturing cost $c_R(i_t)$ is increasing on i_t to express the non-decreasing behavior when quality declines. To produce a new product, the

unit production cost is represented by c_N.

At the beginning of period t, the decision maker sets the prices p_t^N and p_t^R, chooses a quality threshold $i_t \leq i^S$ and decides the manufacturing quantity Q_t^N according to the inventory levels of the two products before the realization of demands and return quantity. Each recycled return generates a salvage value of s which can be either positive or negative. ρ is the discount factor. We let $\mathbf{x} = (x_t^N, x_t^R)$ be the state variable containing the elements of new and remanufactured servable products inventory. We assume the manufacturing and remanufacturing processes are completed within the time period and before the realization of randomness. For clarification, we use y_t^N and y_t^R as the inventory levels right after the demands have been satisfied and unsatisfied demand is backlogged. To maximize the total profit, we have the dynamic programming formulation as follows. Note that we have $[x]^+ = \max\{x, 0\}$ and $[x]^- = \min\{x, 0\}$.

$$\begin{aligned} v_t(\mathbf{x}) = &\max_{p_t^N, p_t^R, Q_t^N, i_t} V_t(x_t^N, x_t^R) = \max_{p_t^N, p_t^R, Q_t^N, i_t} \mathbb{E}\Big\{ p_t^N D_t^N - c_N Q_t^N - G_t^N(y_t^N) \\ &+ p_t^R D_t^R - c_R(i_t) Q_t^R - G_t^R(y_t^R) + s(R_t - Q_t^R) + \rho v_{t+1}(y_t^N, y_t^R) \Big\}, \text{ and} \\ v_{T+1}(\mathbf{x}) = &g(\mathbf{x}), \end{aligned} \tag{5.3}$$

where $x_t^N, x_t^R \geq 0$ and $c_R(i_t)$ is a nonnegative, continuous, differentiable non-decreasing and convex function in i_t where $i_t \leq i^S$, which indicates $c_R^{'}(i) \geq 0$ and $c_R^{''}(i) \geq 0$. We also have $Q_t^R = F^q(i_t) r_t$, $y_t^N = x_t^N + Q_t^N - D_t^N$ and $y_t^R = x_t^R + Q_t^R - D_t^R$. Following Federgruen and Heching (1999), we assume $G_t^i(y_t^i)$ is a convex function in y_t^i, $i = N, R$. For $i = N, R$, if $y_t^i \geq 0$, holding costs $G_t^i(y_t^i) = h_t^i [y_t^i]^+$ are incurred whereas backlogging results in costs $G_t^i(y_t^i) = -b_t^i [y_t^i]^-$. Both h_t^i and b_t^i are non-negative. For time period $T+1$, we assume that $g(\mathbf{x}) = g_N(x_{T+1}^N) + g_R(x_{T+1}^R)$ which is the final collected value of the remaining new and remanufactured products at the last period. $g_i(x_{T+1}^i)$, $i = N, R$ is a twice continuously differentiable function with respect to x_{T+1}^i. This signifies the decreasing marginal utility if

new and remanufactured products are left in stock in the last period. We focus on finding an optimal policy Π^* in a finite time horizon that maximizes the discounted profit at $t = 0$, i.e., $v_0(x_0^N, x_0^R)$. Note that if the cannibalization effect vanishes, i.e., $\gamma = 0$, the problem is degraded into solving two separate inventory management problems.

5.6 Analysis of Single-Period Scenario

By analyzing the single-period version of the problem, we actually look at the model with $T = 1$ and $v_{T+1}(\mathbf{x}) = 0$. We are interested in the optimal strategies to compare with the multi-period analysis in order to compare the policy differences. In this case, the inventory of the two products are filled only once. By disregarding the time index t for readability, the model is modified as follows.

$$\begin{aligned} v^{\text{single}}(\mathbf{x}) = \max_{p^N, p^R, Q^N, i} V^{\text{single}}(x^N, x^R) = \max_{p^N, p^R, Q^N, i} \mathbb{E}\Big\{ & p^N D^N - c_N Q^N \\ & - G_t^N(y^N) + p^R D^R - c_R(i) Q^R - G_t^R(y^R) + s(R - Q^R) \Big\}. \end{aligned} \tag{5.4}$$

Let $u^N = x^N + Q^N$ and $u^R = x^R + Q^R$, the problem is equivalent to:

$$\begin{aligned} v^{\text{single}}(\mathbf{x}) &= \max_{p^N, p^R, u^N, i} V^{\text{single}}(x^N, x^R) \\ &= \max_{p^N, p^R, u^N, i} \mathbb{E}\Big\{ p^N D^N - G_t^N(u^N - D^N) - c_N(u^N - x^N) \\ &\quad + p^R D^R - c_R(i) Q^R - G_t^R(u^R - D^R) + s(R - Q^R) \Big\} \\ &= \max_{p^N, p^R, u^N, i} \Big\{ -c_N(u^N - x^N) - [(c_R(i) + s) F^q(i) - s] \mathbb{E}[R] + L(x^N, x^R) \Big\}, \end{aligned} \tag{5.5}$$

where $L(x^N, x^R) = p^N \mathbb{E}[D^N] - \mathbb{E} G_t^N(u^N - D^N) + p^R \mathbb{E}[D^R] - \mathbb{E} G_t^R(x^R + (1 - F^q(i))r - D^R)$. Denote $K(x^N, x^R) = -c_N(u^N - x^N) - [(c_R(i) + s) F^q(i) - s] \mathbb{E}[R]$.

Assumption 5.6.1. *The salvage value is always less than or equal to both remanufacturing and manufacturing profit margins within a certain range of quality grades, i.e.,* $\exists \hat{i} \in [0,1]$, *such that* $s \leq \min\{p_R - c_R(\hat{i}), p_N - c_N\}$ *if* $i \leq \hat{i}$.

Assumption 5.6.1 is to avoid the situation that salvaging is always the best policy. And it is sufficient to reach the goal by requiring the existence of $\hat{i}$ for a certain range of values of i, instead of asking $\hat{i} = 1$ which is too strong to be necessary. Moreover, previous research also points out that salvaging can be costly (Tierney, 2015; Gradus et al., 2017), which implies s can be either positive or negative.

Lemma 5.6.1. *The objective function has the following features:*

(a) $V(x^N, x^R)$ *is jointly concave in* (p^N, p^R, u^N, i);

(b) $V(x^N, x^R)$ *is jointly concave in* x^N *and* x^R.

With Lemma 5.6.1, we can derive the main result in the following theorem.

Theorem 5.6.1. *The structure of the optimal strategies in the single-period model are:*

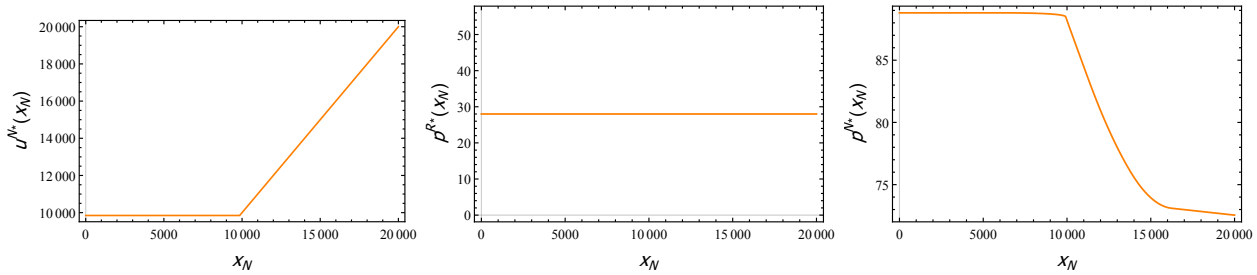
$$p^{N*} = \begin{cases} \overline{p}^N & \text{if } \pi^{N*} \geq \overline{p}^N \\ \pi^{N*} & \text{if } \underline{p}^N < \pi^{N*} < \overline{p}^N \\ \underline{p}^N & \text{if } \underline{p}^N \leq \pi^{N*} \end{cases}, \; p^{R*} = \begin{cases} \overline{p}^R & \text{if } \pi^{R*} \geq \overline{p}^R \\ \pi^{R*} & \text{if } \underline{p}^R < \pi^{R*} < \overline{p}^R \\ \underline{p}^R & \text{if } \underline{p}^R \leq \pi^{R*} \end{cases}, \; i^* = \begin{cases} 1 & \text{if } \iota^* \geq 1 \\ \iota^* & \text{if } 0 < \iota^* < 1 \\ 0 & \text{if } \iota^* \leq 0 \end{cases}$$

and produce up to ν^{N*} *if* $x^N < \nu^{N*}$, *otherwise do not produce, where*

$$\pi^{N*} = \arg\max_{p^N, p^{R*}, u^{N*}, i^*} V^{single}(x^N, x^R), \qquad \pi^R = \arg\max_{p^{N*}, p^R, u^{N*}, i^*} V^{single}(x^N, x^R),$$

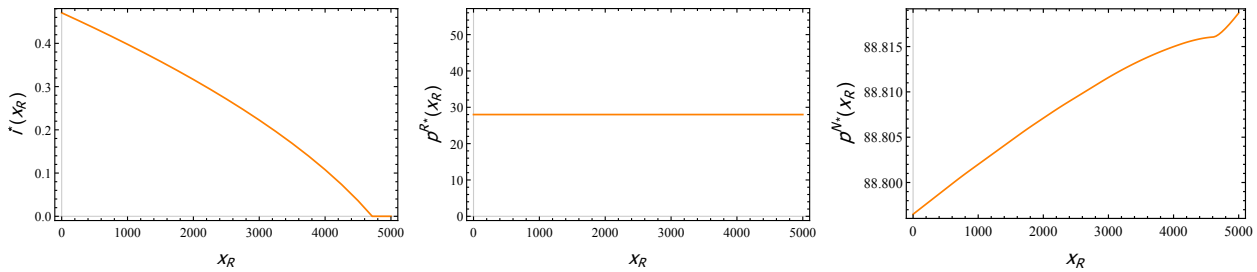
$$\nu^{N*} = \arg\max_{p^{N*}, p^{R*}, u^N, i^*} V^{single}(x^N, x^R), \qquad \iota^* = \arg\max_{p^{N*}, p^{R*}, u^{N*}, i} V^{single}(x^N, x^R).$$

Theorem 5.6.1 states the structure of the optimal strategies to the single-period model. By solving the stochastic optimization model, we obtain a "*produce-up-to*" strategy for new product manufacturing. To balance the remanufacturing cost, cannibalization effect and profit, the remanufacturing quantity can also be viewed an remanufactured-up-to strategy



(a) (b) (c)

Figure 5.1: Optimal quality threshold and pricing strategies with respect to x_N.



(a) (b) (c)

Figure 5.2: Optimal quality threshold and pricing strategies with respect to x_R.

based on the current on-hold inventory x^R and the return quantity. Both pricing strategies are obtained accordingly and bounded by their respective upper bound and lower bound.

In Figure 5.1 and Figure 5.2, we show that the strategies vary with the initial inventory levels of the new and remanufactured products. As x_N increases, u^{N*} does not change when $x_N < \nu^{N*}$ which implies the system always needs to bring the inventory level of new products to ν^{N*}. However, when $x_N \geq \nu^{N*}$, we do not manufacture at all, i.e., $u^{N*} = x_N$. The price of new products is decreasing with x_N. p^{R*} remains the same because it is its upper bound that maximizes the profit in this case, i,e,, $p^{R*} = \overline{p}^R$. Therefore, given the non-increasing p^{N*}, the higher x_N is, the smaller the difference of the two prices is, and the weaker the cannibalization effect impacts on the sales, which stimulates the sales of the new products to deplete the corresponding inventory. For the remanufactured products, the quality threshold decreases with x_R indicating higher inventory requires few to be remanufactured. The price of new product increases with x_R in order to take advantage

of cannibalization effect for selling more unit of remanufactured products. Later, we look at the multi-period case in which the optimal policy has the same structure with the solutions of different initial states demonstrated in Figure 5.1 and Figure 5.2. Therefore, the figures provide a sketch of graphical illustrations of the optimal policy in multi-period setting shown in Proposition 5.7.2 and Proposition 5.7.4 in the following discussions.

5.7 Analysis of Multi-Period Scenario

In this section, we first discuss the problem in an exogenous prices situation, noting that (1) some products, e.g., iPhone, in most of the time, has fixed price when new generations of products are released, and (2) many products' prices are controlled by the market in lieu of the firm itself, as low end personal computers, routers, etc. Following that, we let the prices be endogenous.

5.7.1 Case with Exogenous Price

When prices are exogenous, the model removes the decision variables p_t^N and p_t^R. Here the prices are still dynamic along time but decided by the market rather than the firm.

$$\begin{aligned} v_t(\mathbf{x}) &= \max_{Q_t^N, i_t} V_t(x_t^N, x_t^R) \\ &= \max_{Q_t^N, i_t} \mathbb{E}\Big\{ p_t^N D_t^N - c_N Q_t^N - G_t^N(y_t^N) + p_t^R D_t^R - c_R(i_t) Q_t^R - G_t^R(y_t^R) + s(R_t - Q_t^R) \\ &\quad + \rho V_{t+1}(y_t^N, y_t^R) \Big\}, \text{ and} \\ v_{T+1}(\mathbf{x}) &= g(\mathbf{x}). \end{aligned} \tag{5.6}$$

Denoting $u_t^N = x_t^N + Q_t^N$ as the "*produce-up-to*" inventory level, the problem can be

reformulated as:

$$
\begin{aligned}
v_t(\mathbf{x}) = \max_{u_t^N, i_t} V_t(x_t^N, x_t^R) = \max_{u_t^N, i_t} \mathbb{E}\Big\{ & p_t^N D_t^N - c_N(u_t^N - x_t^N) - G_t^N(u_t^N - D_t^N) \\
& + p_t^R D_t^R - c_R(i_t) F^q(i_t) R_t - G_t^R(x_t^R + F^q(i_t) R_t - D_t^R) + s[R_t - F^q(i_t) R_t] \\
& + \rho v_{t+1}(u_t^N - D_t^N, x_t^R + F^q(i_t) R_t - D_t^R) \Big\}. \qquad (5.7)
\end{aligned}
$$

We first provide the structural properties of the objective function in order to characterize the optimal policy.

Lemma 5.7.1. *When the selling prices of new and remanufactured products are exogenous,*

(a) $V_t(x_t^N, x_t^R)$ *is jointly concave in* x_t^N *and* x_t^R*;*

(b) $V_t(x_t^N, x_t^R)$ *is concave in* (u_t^N, i_t)*.*

Lemma 5.7.1 states the key properties that help to understand the structure of the optimal policy.

Proposition 5.7.1. *When the prices are exogenous,* $v_t(x_t^N, x_t^R)$ *can be decomposed as a sum of two independent concave functions* $M_t(x_t^N)$ *and* $N_t(x_t^R)$*, where*

$$
\begin{aligned}
M_t(x_t^N) &= \max_{u_t^N} \mathbb{E}\Big\{ p_t^N D_t^N - c_N(u_t^N - x_t^N) - G_t^N(u_t^N - D_t^N) + \rho v_{t+1}(u_t^N - D_t^N) \Big\}; \\
N_t(x_t^R) &= \max_{i_t} \mathbb{E}\Big\{ p_t^R D_t^R - c_R(i_t) F^q(i_t) R_t - G_t^R(x_t^R + F^q(i_t) R_t - D_t^R) \\
&\qquad + s[R_t - F^q(i_t) R_t] + \rho v_{t+1}(x_t^R + F^q(i_t) R_t - D_t^R) \Big\}. \qquad (5.8)
\end{aligned}
$$

In Proposition 5.7.1, the objective function is decomposed into two individual parts which can be solved separately. This structural property, together with Lemma 5.7.1, helps in characterizing the optimal policy shown in Theorem 5.7.1.

Theorem 5.7.1. *When the prices are exogenous, for period* t*, given the state of the system* $\mathbf{x}$*, the optimal manufacturing and remanufacturing of the two products follow the structure*

below.

$$u_t^{N*}(x_t^N) = \arg\max_{u_t^N} M_t(x_t^N) \qquad = \begin{cases} x_t^N & \text{if } x_t^N \geq \nu_t^{N*}(x_t^N), \\ \nu_t^{N*}(x_t^N, x_t^R) & \text{if } x_t^N < \nu_t^{N*}(x_t^N); \end{cases}$$

$$\iota^*(x_t^R) = \arg\max_{i_t} N_t(x_t^R) \qquad = \begin{cases} 1 & \text{if } \iota^*(x_t^R) \geq 1, \\ \iota^*(x_t^N, x_t^R) & \text{if } 0 < \iota^*(x_t^R) < 1, \\ 0 & \text{if } \iota^*(x_t^R) \leq 0. \end{cases}$$

The optimal level of manufacturing is a '*produce-up-to*' policy that leads the inventory level of new products close to $\nu_t^*(x_t^N)$ as much as possible. For the remanufacturing part, it directly gives the quantity and quality level to process for remanufacturing. Moreover, the optimal policy has the following proposition.

Proposition 5.7.2. *When prices are exogenous, the optimal quality level policy $i_t^*(x_t^R)$ is non-increasing in x_t^R and the optimal "produce-up-to" policy $u_t^{N*}(x_t^N)$ is non-decreasing in x_t^N.*

Proposition 5.7.2 firstly implies that the higher the remanufactured servable inventory level is, the lower the quality threshold value is. It is intuitive that if we have enough remanufactured products in stock, we are not in need of remanufacturing many cores as the processing and the inventory costs would be high. For the "*produce-up-to*" level, when the inventory level of new products is lower than the ν^{N*}, the system always produce up to this level. When the inventory level becomes higher than this threshold, the system does not produce. Therefore, u_t^{N*} is non-decreasing in x_t^N. The proof of this proposition follows the sufficient conditions in Topkis (1998).

Corollary 5.7.1. *When prices are exogenous, the "produce-up-to" level u_t^N is independent of the remanufactured products inventory level x_t^R and the quality level i_t is independent of the the new products inventory level x_t^N.*

Corollary 5.7.1 originates from the decomposability property of the objective function, shown in Proposition 5.7.1. As the prices are exogenous, production and quality threshold decisions can be made separately for the manufacturing and remanufacturing processes, respectively.

Proposition 5.7.3. *The single-period myopic "produce-up-to" policy $\nu_t^{N*}(x_N)$ with exogenous price is optimal to the exogenous pricing in the multi-period model.*

Proposition 5.7.3 illustrates that the optimal production policy of new products in the exogenous pricing model can be easily computed by maximizing the profit by taking any period of the model.

5.7.2 Case with Endogenous Price

In this section, we take pricing decisions into account to investigate in the situation where firm decides new and remanufactured products' prices and to understand the optimal policy structure. Here we refer to the model of Eq.(5.3) shown in Section 5.5.

To derive the optimal policy, certain properties are shown in Lemma 5.7.2.

Lemma 5.7.2. *When the selling prices of new and remanufactured products are endogenous,*

(a) $V_t(x_t^N, x_t^R)$ is jointly concave in x_t^N and x_t^R;

(b) $V_t(x_t^N, x_t^R)$ is jointly concave in pairs in $(p_t^N, p_t^R, u_t^N, i_t)$.

Note that, in part (b) of Lemma 5.7.2, the value function is jointly concave in any two of the decision variables. Next, with Lemma 5.7.2, we summarize the optimal policy in the following remarks.

When the prices are endogenous, for period t, given the state of the system $\mathbf{x}$, the optimal manufacturing and remanufacturing of the two products and the pricing strategies follow the results below.

$\pi_t^{N*} = \arg\max_{p_t^N, p_t^{R*}, u_t^{N*}, i_t^*} V_t(x_t^N, x_t^R)$, $\pi_t^{R*} = \arg\max_{p_t^{N*}, p_t^R, u_t^{N*}, i_t^*} V_t(x_t^N, x_t^R)$,

$\nu_t^{N*} = \arg\max_{p_t^{N*}, p_t^{R*}, u_t^N, i_t^*} V_t(x_t^N, x_t^R)$ and $\iota_t^* = \arg\max_{p_t^{N*}, p_t^{R*}, u_t^{N*}, i_t} V_t(x_t^N, x_t^R)$.

where $p_t^N, p_t^{N*} \in [\underline{p}_N, \overline{p}_N]$, $p_t^R, p_t^{R*} \in [\underline{p}_R, \overline{p}_R]$. The optimal policy structure resembles the optimal solution in the single-period case with an additional time argument.

Proposition 5.7.4. *When prices are endogenous,*

(a) The optimal quality level policy $i_t^*(x_t^R)$ is non-increasing in x_t^R;

(b) The optimal "produce-up-to" policy $u_t^{N*}(x_t^N)$ is non-decreasing in x_t^N;

(c) The optimal pricing strategies p_t^{N*} is non-decreasing in x_t^R;

(d) The optimal pricing strategies p_t^{R*} is non-decreasing in x_t^N.

Part (a) and (b) of Proposition 5.7.4 coincide with the statements in Proposition 5.7.2. Part (c) points out the behavior of pricing strategy regarding the inventory levels. When the inventory level of remanufactured products is higher, the price of new products increases, so that the decision maker takes advantage of cannibalization effect to sell more remanufactured products in stock. Similarly, in part (d), when the inventory level of new products increases, the price of remanufactured products goes up. In this case, the cannibalization effect is shrunk instead of amplified. Therefore, it becomes beneficial as it increases the sales of new products and depletes the corresponding inventory. The proof of this proposition follows the sufficient conditions in Topkis (1998).

Proposition 5.7.5. *If $s > 0$, the holding/backlogging costs are stationary across periods, i.e., for $i = N, R$, $h_t^i = h_i$ and $b_t^i = b_i$, the optimal profits under the policies of dynamic quality threshold, "remanufacture-all" and "recycle-all" satisfy the following inequalities:*

$$v_t^*(\mathbf{x}) - v_t^{1*}(\mathbf{x}) \leq \frac{1-\rho^{T+1-t}}{1-\rho}(h_t^R + c_R(1))\mathbb{E}[R]$$

and

$$v_t^*(\mathbf{x}) - v_t^{0*}(\mathbf{x}) \leq \frac{1-\rho^{T+1-t}}{1-\rho}(b_t^R + s)\mathbb{E}[R]$$

where we denote v^{1*} as the optimal solution when "Remanufacture-all" is deployed and correspondingly, v^{0*} as the optimal solution when "Recycle-all" is deployed. $\mathbb{E}[R]$ is the expectation of return quantity in any period (on account of the stationarity).

Proposition 5.7.5 addresses the upper bounds between the flexible optimal quality threshold and the "*remanufacture-all*" policy, "*recycle-all*" policy, respectively. Particularly, in the single-period model, the fractional item in the right hand sides vanishes.

5.8 Numerical Analysis

In this section, we first use the single-period model to numerically derive managerial insights that are worth exploring since the multi-period model basically coincides with the conclusions. To confirm the consistency, we take another example of two-period. In terms of analysis, we first carry out sensitivity analysis on several parameters including the remanufacturing cost, salvage value and the cannibalization factor. Following that, we compare the optimal policy with the widely used "*Remanufacture-all*" as well as the "*Recycle-all*" strategies in both single-period and multi-period analysis.

Table 5.1: Parameters in the numerical analysis for the single-period model.

α_N	β_N	$\overline{\epsilon}_N$	$\underline{\epsilon}_N$	α_R	β_R	$\overline{\epsilon}_R$	$\underline{\epsilon}_R$	γ	$\overline{R}$	$\underline{R}$
10,000	100	10,000	0	5,000	150	5,000	0	0.1	7,000	4,000
h_N	h_R	b_N	b_R	c_N	c	s	$\overline{p}_N$	$\underline{p}_N$	$\overline{p}_R$	$\underline{p}_R$
1.5€	1€	$\overline{p}_N$	$\overline{p}_R$	20€	18€	10€	90€	70€	28€	22€

The baseline parameters are shown in Table 4.1 and we use uniform distributions for all the analysis. Specifically, $\epsilon_N \sim U[\underline{\epsilon}_N, \overline{\epsilon}_N]$, $\epsilon_R \sim U[\underline{\epsilon}_R, \overline{\epsilon}_R]$ and $R \sim U[\underline{R}, \overline{R}]$. The supports of the prices are denotes as $p_t^N \in [\underline{p}^N, \overline{p}^N]$ and $p_t^R \in [\underline{p}^R, \overline{p}^R]$. Moreover, the inventory costs are corresponding to one period and the remanufacturing cost function is assumed to be a quadratic convex function written as $c_R(i) = ci^2$. Without of loss of generality, we assume $i^s = 1$ implying that all returns are legitimately remanufacturable.

5.8.1 Sensitivity Analysis

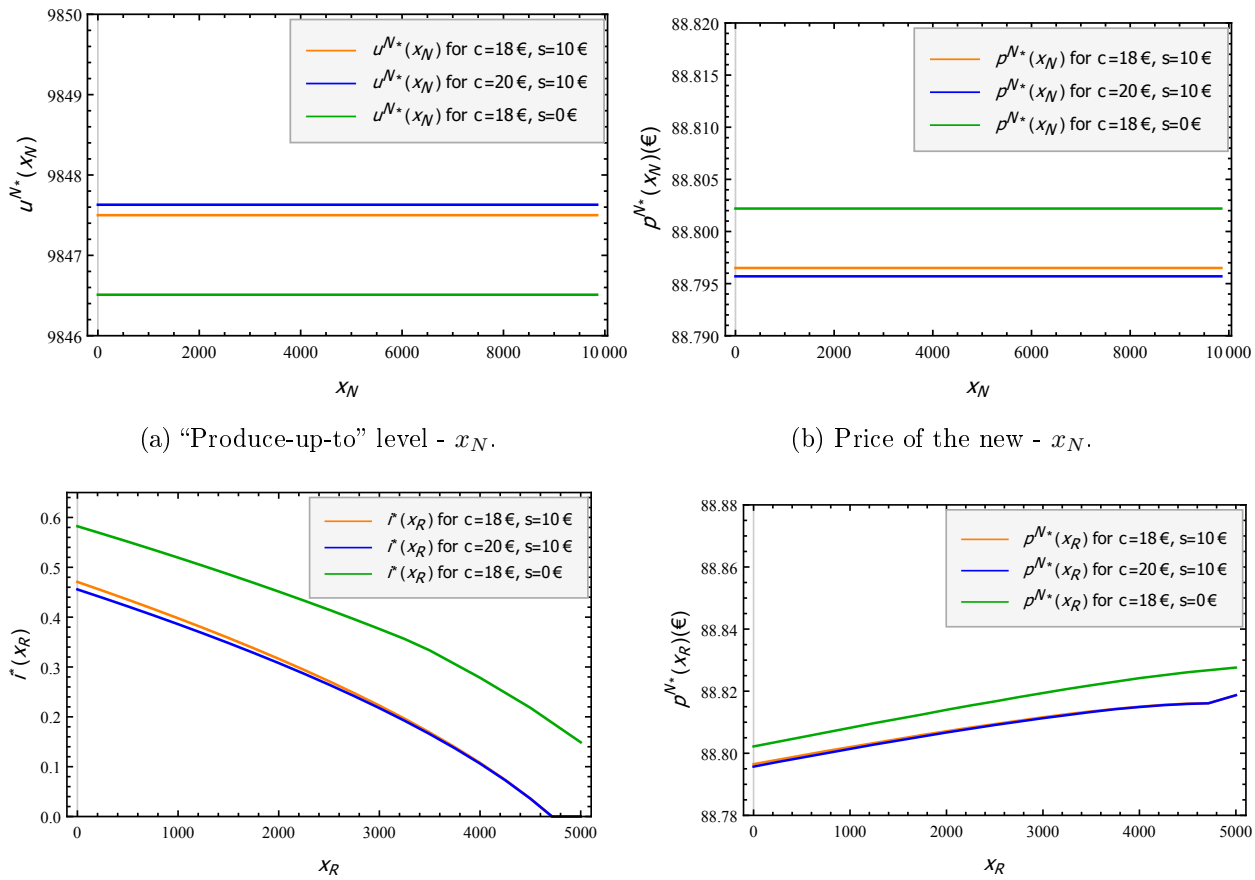
First, we look at the sensitivity analysis by the remanufacturing cost.

The Effect of Unit Remanufacturing Cost

The results are demonstrated in Figure 5.3, the orange lines are the results of the baseline parameters, and the blue ones correspond to the higher remanufacturing cost scenario.

Intuitively, higher remanufacturing cost discourages decision makers from adopting large amount of returns and convert them to servable products. In Figure 5.3(c), i^* decreases when remanufacturing cost increases, which means the decision maker has to choose stricter quality threshold in order to reduce remanufacturing quantity. The value gradually coincides with the baseline case as higher x^R lets the necessity of remanufacturing disappear. The price of new products decreases slightly whereas the price of remanufactured products remains the same since our example always has $\overline{p}^R$ as the optimal solution to p^R. To compensate the profit loss from larger remanufacturing cost, the lower p^{N*} reduces the cannibalization effect but enhances the sales of new products. Therefore, a higher u^{N*} is expected.

For the products of low values, such as the copper recycled from used Christmas trees (Section 5.3), they can be seen as products of high remanufacturing cost. To remanufacture them would cost lots of efforts and seriously hurt the profitability. Therefore, according to our analysis, the optimal threshold i^* would decrease and eventually discourage all remanufacturing practice. Firms thus opt recycling over remanufacturing for these types of products.



(a) "Produce-up-to" level - x_N.

(b) Price of the new - x_N.

(c) Quality standard - x_R.

(d) Price of the new - x_R.

Figure 5.3: Optimal policy comparisons among (1) $c = 18$€, $s = 10$€, (2) $c = 20$€, $s = 10$€ of higher remanufacturing cost and (3) $c = 18$€, $s = 0$€ of lower salvage value.

Next, we look at the case where the salvage value is ignorable.

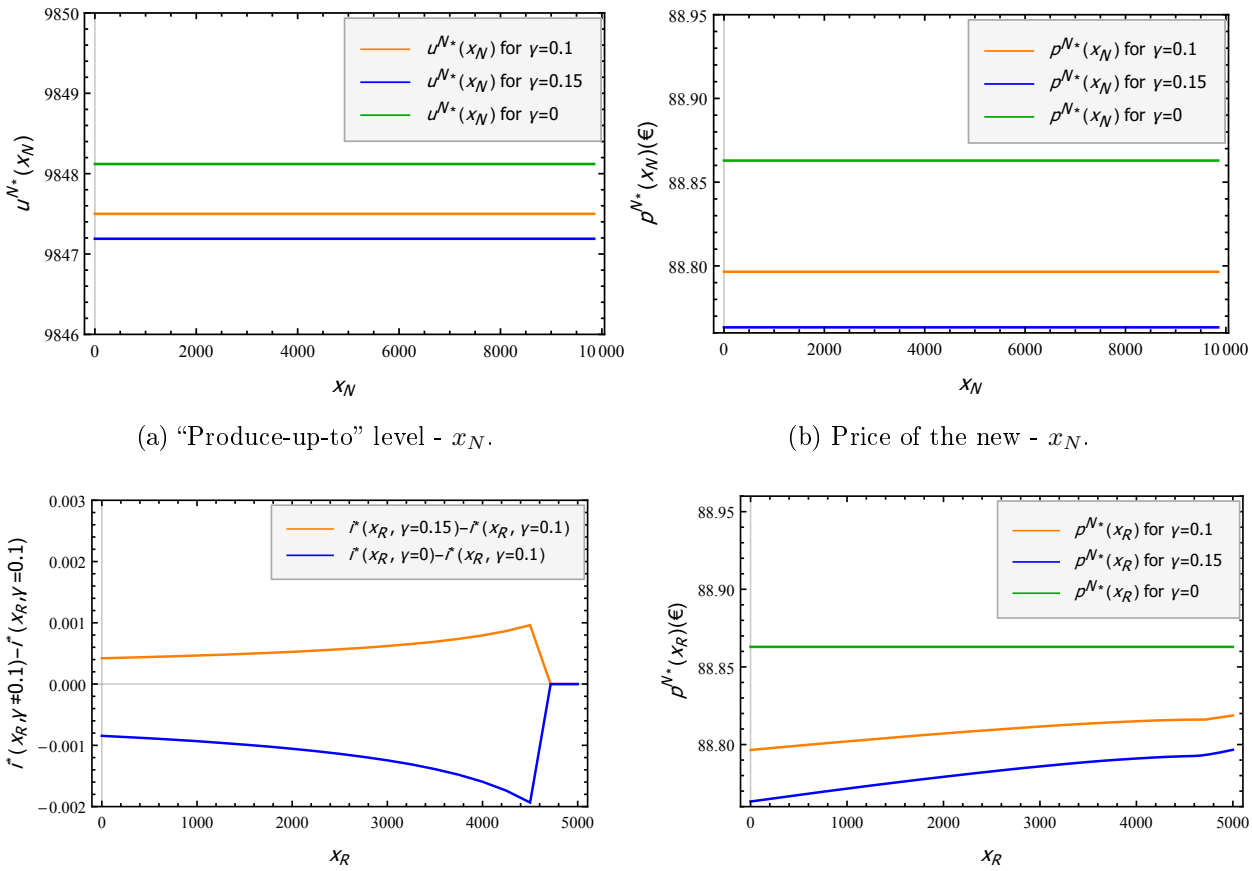
The Effect of Salvage Value

In Figure 5.3, the effect of low salvage value is the opposite of high remanufacturing cost (green lines). When salvage does not provide profit for the firm, the decision maker tends to adopt as many returned products as possible, as long as the market for remanufactured products is large enough. Therefore, we observe a much higher i^*. p^{N*} goes up in this case to enlarge the cannibalization effect so that the remanufactured market is stimulated whereas the production of new products (u^{N*}) has to be lowered to avoid overstock.

The Effect of Cannibalization

In this part, we investigate the impact of cannibalization effect. Particulary, we compare

the baseline scenario with a lower cannibalization factor ($\gamma = 0$) where cannibalization vanishes, and with a higher one ($\gamma = 0.15$).



(a) "Produce-up-to" level - x_N.

(b) Price of the new - x_N.

(c) Quality threshold difference - x_R.

(d) Price of the new - x_R.

Figure 5.4: Optimal policy comparisons between (1) $\gamma = 0.1$ and (2) $\gamma = 0.15$ which indicates stronger cannibalization effect.

Figure 5.4 displays the strategies regarding different values of γ. When cannibalization effect disappears, Figure 5.4(c) indicates that the i^* strategy is slightly lower than when $\gamma = 0.1$. It implies that without cannibalization, firms should not adopt that many returns to remanufacture. Therefore, those customers remain in the market of new products, which leads to higher u^{N*} shown in Figure 5.4(a). Moreover, without the worry of amplifying cannibalization effect by enlarging the gap of prices, the decision maker can price the new products higher (Figure 5.4(b)) and this price is not related to the stock of remanufactured products any more (Figure 5.4(d)).

If the cannibalization factor is higher in the market, we observe the opposite results against the arguments above. From an overall view, we conclude that, regardless of the value of γ, the true cannibalization is also affected by the price difference, i.e., the cannibalization effect is measured by $\gamma(p^N - p^R)$. Therefore, firms price their products to avoid strong cannibalization but not counteract it so that it can be taken advantage of to reach the maximum total profit.

In Figure 5.5, we illustrate a special case which shows that the profit does not necessarily decrease with the cannibalization effect γ. Generally, the profit drops with the increase of γ. However, in this case, if the profit margins of the two types of products are on similar level, and the market is very sensitive to the price of new product, we observe that stronger cannibalization effect actually increases the profit. Therefore, demand cannibalization effect does not always hurt the profit. And by deploying flexible quality strategy, the demand cannibalization effect can be either compensated or mitigated to reach satisfactory profit level. Note that, the single-period model always shows a linear result since we do not need a concave function $g(\mathbf{x})$ as in multi-period model.

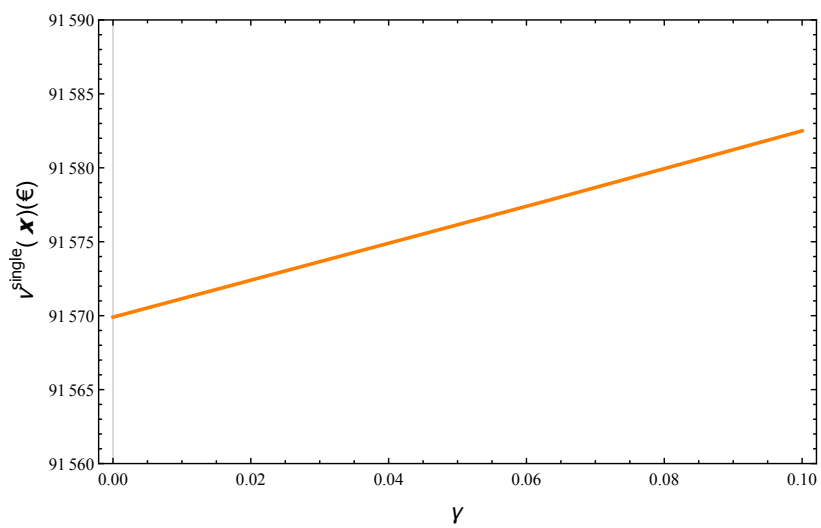


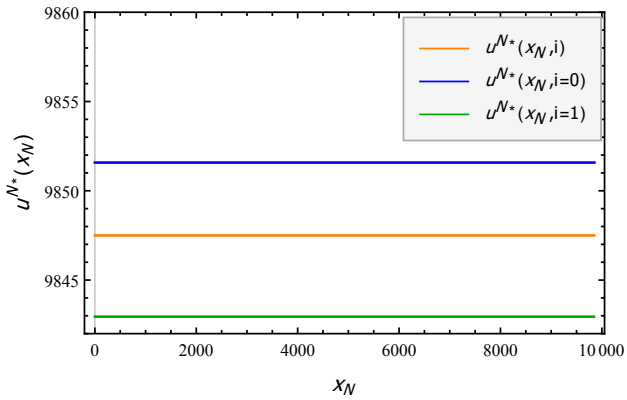
Figure 5.5: Optimal profit with γ if the profit margins of the two products are on the same level and the sensitivity to the price of new product is large.

5.8.2 Optimal, Remanufacture-All and Recycle-All Policies

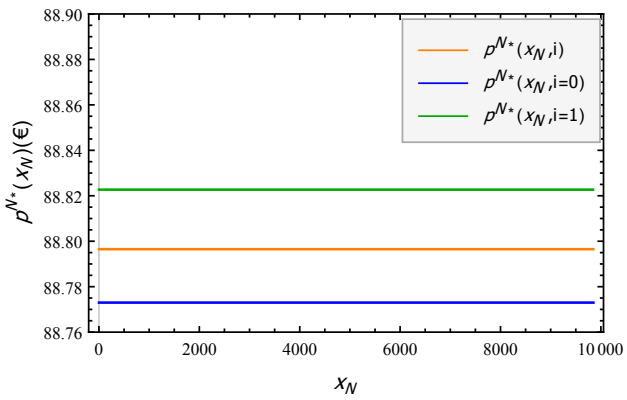
In this section, we compare three strategies regarding our problem, including the optimal strategy, the "*remanufacture-all*"(RM) as well as the "*recycle-all*"(RC) strategy. Since the later two strategies serve as two feasible-but-non-optimal policies of our model, they certainly lead to lower profit compared to that of using the optimal policy. Nonetheless, we are interested in the profit differences and the changes of each strategy under these three regimes.

In our numerical example, the profits of the three policies are: 4.150×10^5€ (Flexible OPT), 3.884×10^5€ (RC) and 3.236×10^5€ (RM). The improvements of taking advantage of quality strategy are $6.41\%(\frac{v^*-v^{0*}}{v^*} \times 100\%)$ and $22.02\%(\frac{v^*-v^{1*}}{v^*} \times 100\%)$.

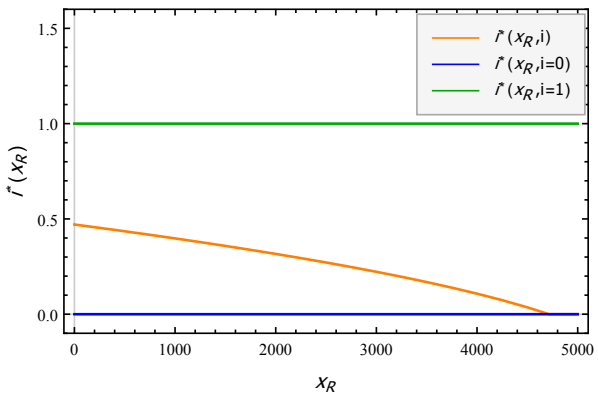
The results of the each case are shown in Figure 5.6. When nothing is adopted, the customers purchasing remanufactured products can be only fulfilled by the on-hand remanufactured inventory and the firm collects all the profit from salvaging the returns. In such a case, the decision maker prices its products in a way that the purchase of new products is encouraged since the expected profit from remanufactured sector is too limited due to shortage of products. Therefore, the price of new products is lower and the "*product-up-to*" level increases to cope with the higher demand. When the initial inventory level x^R increases, so does the price of new products. Then the cannibalization effect is stronger to divert a part of the customers from buying new products to remanufactured ones.



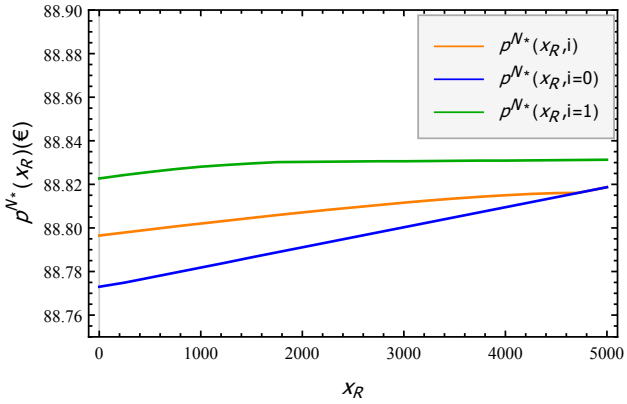
(a) "Produce-up-to" level - x_N.



(b) Price of the new - x_N.



(c) Quality threshold - x_R.



(d) Price of the new - x_R.

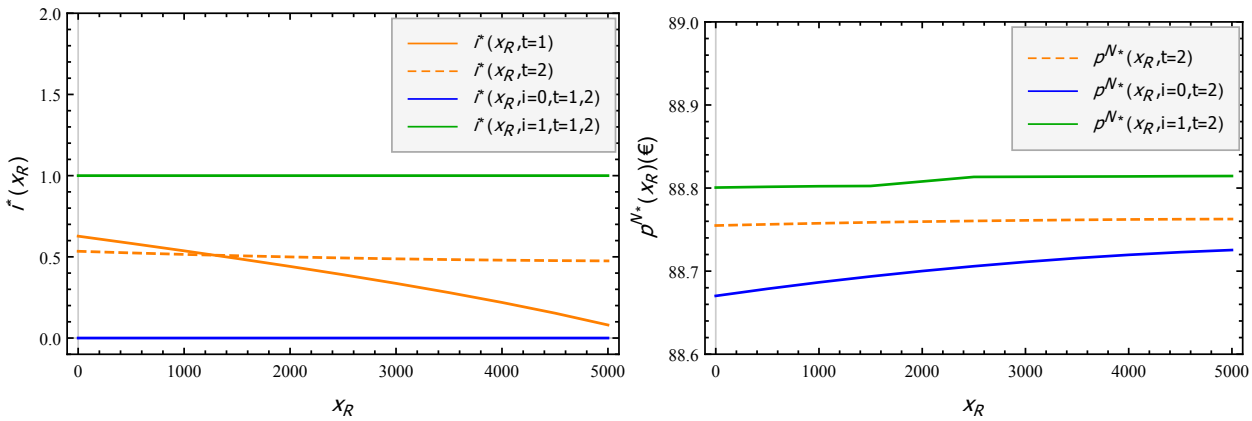
Figure 5.6: Comparisons among the optimal policy, "*remanufacturing-all*" policy where $i^* = 1$ and "*recycle-all*" policy where $i^* = 0$.

On the contrary, if the policy of RM is deployed, the unit remanufacturing cost would be always the highest and quite a large amount of remanufactured products would be in stock for selling. Since the highest unit remanufactured cost (18€) is much higher than the unit salvage payoff (10€), the profit in the RM case is much lower than that in the RC case.

In conclusion, to take advantage of quality as a strategy is beneficial for firms in terms of profit maximization. Applying simple strategies can cause great loss of profit. In a dynamic setting, the quality threshold can always change across periods so that the advantage can be more significant than using RM or RC policy.

5.8.3 An Example of Multi-Period Model

In this section, we use the same baseline parameters in Table 5.1, take discount rate $\rho = 0.9$ and demonstrate the propositions in a multi-period model of $T = 2$ which is sufficient enough to confirm the consistency with the analysis in single-period model by bearable computing efforts for personal computers.



(a) Quality standard - x_R.

(b) Price of the new - x_R.

Figure 5.7: Comparisons among the optimal policy, "*remanufacturing-all*" policy where $i^* = 1$ and "*recycle-all*" policy where $i^* = 0$ in a two-period model.

In both periods, Figure 5.7 shows similar pattern for the pricing strategy of the new product and the quality strategy in x_t^R, with the single-period analysis in Figure 5.6. The missing first period pricing strategy in Figure 5.7(b) is a constant $p^{N*} = \overline{p}^N = 90$€. And p^{R*} always take $\overline{p}^R$. Regarding the changes in the strategies across periods, we summarize the results in Table 5.2 in details.

Table 5.2: The results of the two-period numerical analysis.

	p_1^N(€)	u_1^N	p_2^N(€)	u_2^N
$i*$	90	9529.37	88.7549	9848.65
$i = 0$	90	9529.37	88.6702	9863.38
$i = 1$	90	9529.37	88.8006	9840.71

Note that in Table 5.2, we keep $x_1^R = 0$ all the time. The insights are similar to the analysis in the single-period model of Section 5.8.2.

5.9 Conclusions

In this essay, we mainly address the issue of taking advantage of quality as a strategy to flexibly separate the returns into the remanufacturable group and the salvage group and to counteract or compensate the cannibalization effect for gaining profit. To this end, we initially establish a single-period stochastic programming model considering pricing strategies and analyze the basic properties. Following that, we look at the multi-period dynamic setting and investigate in (1) exogenous pricing setting, and (2) endogenous pricing setting. We characterize the optimal policy structure and analyze the properties of the optimal policy. To better understand the sensitivity effect on strategies as well as the gain and policies of quality strategy, we take the single-period model and a two-period model to use numerical means for managerial insights.

We find that under the condition of additive demand functions, uniform distribution of the returns' quality and convex non-decreasing remanufacturing cost form, the optimal policy of the dynamic models follows a state-dependent threshold structure. Particularly, if the prices are exogenous, the objective function can be decomposed into two additive subproblems and optimized by each subproblem individually. Managerially, we find that the quality serves as a direct strategy to control the inventory of remanufactured products and also as an indirect tool to counteract/compensate the negative/positive impact of cannibalization effect on profit.

For future study, the model can be extended to a multiplicative demand function form and consider the quality aspect as well. In this direction, the conclusion may disagree with some of the propositions in this essay as Yan et al. (2017) which points out that higher inventory of remanufactured products does not imply lower price of new product. Moreover, the distribution of returns' quality can be extended to a more general form. Furthermore,

the model can take other elements into account such as back-order costs, effort-dependent return quantity, environmental innovation and acquisition pricing of returns. Eventually, a multi-player problem is also an interesting direction to explore.

CHAPTER 6

GENERAL CONCLUSIONS

The three essays in this thesis, in general, explore different dimensions of flexibility to better manage the operations in firms/supply chains for higher economic and environmental performance.

Essay one looks at the diverging role of the stock of environmental performance, which increases sales and also results in higher production volume which hurts the environmental performance. Due to this flexible attribute shown by the environmental performance, we compare several types of contracts together with a cooperative program together to improve the supply chain profit and highlight the parameters' region where high environmental performance is located. We always observe a mismatch between the supply chain economic performance and environmental performance. And in terms of the environmental performance, using a cooperative program in a wholesale price contract can be more beneficial than a revenue sharing contract or an integrated chain.

Essay two investigates multiple layers of flexibility in a multi-component system. The first layer is shown by using component substitution in producing a final product for managing an inventory system. Multi-component systems achieve lower strategic cost and substitution rates than single-component system, which implies that this layer of flexibility contributes to the cost efficiency. The second layer reflects the advantage of partial substitution. Compared to full and no substitution, partial substitution offers significant reduction of strategic cost. The third layer is corresponding to the objective function which is the weighted average of economic cost and environmental cost. By adjusting the weight, firms are able to adopt suitable policies to reach their economic and environmental goals.

Essay three emphasizes the importance of taking flexible strategies toward the quality of returns. By selecting the higher quality ones into remanufacturing and the rest to recycling, firms are able to dynamically choose operational policies to reach higher profit and also tackle the demand cannibalization effect. Compared to the simple policies adopted by many

firms/industries, e.g., “Remanufacture all", “Recycle all", to consider quality strategy is certainly more flexible and it is beneficial to the overall profit. In the meantime, demand cannibalization effect is not always a negative factor since it does not necessarily decrease the profit.

The economic and environmental performance translated from taking advantage of the above dimensions of flexibility is not only beneficial to the firms and supply chains but also the entire society. The flexibility assists firms to e.g., control pollution, save raw materials, manage energy consumption and well cooperate with supply chain members. All the possible contributions lead to a sustainable environment for our future generations.

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APPENDIX A

PROOFS OF CHAPTER 3.

A.1 Proof of Proposition 1

Proof. For the W-Scenario, we search for a pair of bounded and continuously differentiable value functions V_S^W and V_M^W for which a unique solution for $E^W(t)$ exists, and the Hamilton–Jacobi–Bellman (HJB) equations:

$$rV_S^W = \max_{w,I} \left\{ (\alpha - \beta p + \eta E)\left(w - c_p + cI\right) - \frac{\mu B I^2}{2} + \frac{\partial V_S^W}{\partial E}\left[mI - g\left(\alpha - \beta p + \eta E\right) - nE\right] \right\} \tag{A.1}$$

$$rV_M^W = \max_{p} \left\{ (\alpha - \beta p + \eta E)(p - w) - \frac{\mu (1 - B) I^2}{2} + \frac{\partial V_M^W}{\partial E}\left[mI - g\left(\alpha - \beta p + \eta E\right) - nE\right] \right\} \tag{A.2}$$

are satisfied for any value of E. As S is the Stackelberg leader, we adopt the backward induction and maximize the M's HJB for the pricing strategy, which gives:

$$p^W = \frac{\alpha + w\beta + \eta E + g\beta \frac{\partial V_M^W}{\partial E}}{2\beta}$$

We plug the pricing strategy back to S's value function and derive the wholesale price and innovation strategy simultaneously, which gives:

$$w^W = W_1 + W_2 E + W_3 \frac{\partial V_S^W}{\partial E} + W_4 \frac{\partial V_M^W}{\partial E}$$

$$I^W = W_5 + W_6 E + W_7 \frac{\partial V_S^W}{\partial E} + W_8 \frac{\partial V_M^W}{\partial E}$$

while the pricing strategy turns out to be:

$$p^W = W_9 + W_{10}E + W_{11}\frac{\partial V_S^W}{\partial E} + W_{12}\frac{\partial V_M^W}{\partial E}$$

where,

$$W_1 = \frac{c^2\alpha\beta - 2\mu\left(\alpha + \beta c_p\right)(1-B)}{c^2\beta^2 - 4\beta\mu(1-B)}, W_2 = \frac{c^2\beta\eta - 2\mu\eta(1-B)}{c^2\beta^2 - 4\beta\mu(1-B)}, W_3 = \frac{2\left(cm - g\mu(1-B)\right)}{c^2\beta - 4\mu(1-B)}$$

$$W_4 = \frac{g\left(2\mu(1-B) - c^2\beta\right)}{c^2\beta - 4\mu(1-B)}, W_5 = \frac{c\left(\alpha - \beta c_p\right)}{4\mu(1-B) - c^2\beta}, W_6 = \frac{c\eta}{4\mu(1-B) - c^2\beta}$$

$$W_7 = \frac{4m - cg\beta}{4\mu(1-B) - c^2\beta}, W_8 = \frac{-cg\beta}{4\mu(1-B) - c^2\beta}, W_9 = \frac{c^2\alpha\beta - \mu\left(3\alpha + \beta c_p\right)(1-B)}{c^2\beta^2 - 4\beta\mu(1-B)}$$

$$W_{10} = \frac{c^2\beta\eta - 3\mu\eta(1-B)}{c^2\beta^2 - 4\beta\mu(1-B)}, W_{11} = \frac{cm - g\mu(1-B)}{c^2\beta - 4\mu(1-B)}, W_{12} = \frac{-g\mu(1-B)}{c^2\beta - 4\mu(1-B)}$$

are constant terms. By making conjecturing quadratic value functions,

$$V_S^W = \frac{S_1}{2}E^2 + S_2E + S_3, \frac{\partial V_S^W}{\partial E} = S_1E + S_2$$

$$V_M^W = \frac{M_1}{2}E^2 + M_2E + M_3, \frac{\partial V_M^W}{\partial E} = M_1E + M_2$$

We rewrite the value functions of S and M as follows:

$$r\left(\frac{S_1}{2}E^2 + S_2E + S_3\right) = (W_{13} + W_{14}E)(W_{15} + W_{16}E) + W_{17}(W_{18} + W_{19}E)^2$$
$$+ (S_1E + S_2)(W_{20} + W_{21}E)$$
$$r\left(\frac{M_1}{2}E^2 + M_2E + M_3\right) = (W_{13} + W_{14}E)(W_{22} + W_{23}E) + W_{24}(W_{18} + W_{19}E)^2$$
$$+ (M_1E + M_2)(W_{20} + W_{21}E)$$

where,

$$W_{13} = \alpha - \beta(W_9 + W_{11}S_2 + W_{12}M_2), W_{14} = \eta - \beta(W_{10} + W_{11}S_1 + W_{12}M_1)$$

$$W_{15} = W_1 + W_3S_2 + W_4M_2 - c_p + c(W_5 + W_7S_2 + W_8M_2)$$

$$W_{16} = W_2 + W_3S_1 + W_4M_1 + c(W_6 + W_7S_1 + W_8M_1)$$

$$W_{17} = -\frac{\mu(1-B)}{2}, W_{18} = W_5 + W_7S_2 + W_8M_2, W_{19} = W_6 + W_7S_1 + W_8M_1$$

$$W_{20} = m(W_5 + W_7S_2 + W_8M_2) - g(\alpha - \beta(W_9 + W_{11}S_2 + W_{12}M_2))$$

$$W_{21} = m(W_6 + W_7S_1 + W_8M_1) - g(\eta - \beta(W_{10} + W_{11}S_1 + W_{12}M_1)) - n$$

$$W_{22} = W_9 + W_{11}S_2 + W_{12}M_2 - (W_1 + W_3S_2 + W_4M_2)$$

$$W_{23} = W_{10} + W_{11}S_1 + W_{12}M_1 - (W_2 + W_3S_1 + W_4M_1), W_{24} = -\frac{\mu B}{2}$$

are constant terms. By identification, we obtain the following equations:

$$r\frac{S_1}{2} = W_{14}W_{16} + W_{17}W_{19}^2 + W_{21}S_1$$

$$rS_2 = W_{13}W_{16} + W_{14}W_{15} + 2W_{17}W_{18}W_{19} + W_{20}M_1 + W_{21}S_2$$

$$rS_3 = W_{13}W_{15} + +W_{17}W_{18}^2 + W_{20}S_2$$

$$r\frac{M_1}{2} = W_{14}W_{23} + W_{24}W_{19}^2 + W_{21}M_1$$

$$rM_2 = W_{13}W_{23} + W_{14}W_{22} + 2W_{24}W_{18}W_{19} + W_{20}M_1 + W_{21}M_2$$

$$rM_3 = W_{13}W_{22} + W_{24}W_{18}^2 + W_{20}M_2$$

We finally solve the Riccati equation system to get the related coefficients. □

A.2 Proof of Proposition 5

Proof. For the R-Scenario, we search for a pair of bounded and continuously differentiable value functions V_S^R and V_M^R for which a unique solution for $E^R(t)$ exists, and the Hamilton–Jacobi–Bellman (HJB) equations:

$$\begin{aligned} rV_S^R &= \max_I \left\{ D(\cdot)\left(p\phi - c_S - c_p + cI\right) - \frac{\mu B I^2}{2} + \frac{\partial V_S^R}{\partial E}\left[mI - gD(\cdot) - nE\right] \right\} \\ rV_M^R &= \max_p \left\{ D(\cdot)\left(p\left(1-\phi\right) - c_M\right) - \frac{\mu\left(1-B\right)I^2}{2} + \frac{\partial V_M^R}{\partial E}\left[mI - gD(\cdot) - nE\right] \right\} \end{aligned}$$

are satisfied for any value of E, where $D(\cdot) = (\alpha - \beta p + \eta E)$.

We solve the game backward and derive the pricing strategy from the M's HJB to get:

$$p^R = \frac{\alpha(1-\phi) + \beta c_M + \eta(1-\phi)E + g\beta\frac{\partial V_M^R}{\partial E}}{2\beta(1-\phi)}$$

By plugging the pricing strategy back to the value function of S, we derive the innovation strategy as follows.

$$I^R = \frac{c\alpha(1-\phi) - c\beta c_M + c\eta(1-\phi)E + 2m(1-\phi)\frac{\partial V_S^R}{\partial E} - cg\beta\frac{\partial V_M^R}{\partial E}}{2\mu(1-B)(1-\phi)}$$

We rewrite the strategies for attaining clearer equations

$$p^R = A_1 + A_2 E + A_3\frac{\partial V_M^R}{\partial E}$$

$$I^R = A_4 + A_5 E + A_6\frac{\partial V_S^R}{\partial E} + A_7\frac{\partial V_M^R}{\partial E}$$

where,

$$A_1 = \frac{\alpha}{2\beta} + \frac{c_M}{2(1-\phi)}, A_2 = \frac{\eta}{2\beta}, A_3 = \frac{g}{2(1-\phi)}$$

$$A_4 = \frac{c\alpha(1-\phi) - c\beta c_M}{2\mu(1-B)(1-\phi)}, A_5 = \frac{c\eta}{2\mu(1-B)}, A_6 = \frac{m}{\mu(1-B)}, A_7 = \frac{-cg\beta}{2\mu(1-B)(1-\phi)}$$

are constant terms. By conjecturing quadratic value functions we obtain,

$$V_S^R = \frac{\widetilde{S}_1}{2}E^2 + \widetilde{S}_2 + \widetilde{S}_3, \frac{\partial V_M^R}{\partial E} = \widetilde{S}_1 E + \widetilde{S}_2$$

$$V_M^R = \frac{\widetilde{M}_1}{2}E^2 + \widetilde{M}_2 E + \widetilde{M}_3, \frac{\partial V_R^R}{\partial E} = \widetilde{M}_1 E + \widetilde{M}_2$$

By plugging the above equations and their derivatives back to the value functions of S and M and rewrite them as follows:

$$r\left(\frac{\widetilde{S}_1}{2}E^2+\widetilde{S}_2+\widetilde{S}_3\right)=(A_8+A_9E)(A_{12}+A_{13}E)-A_{19}(A_{16}+A_{17}E)^2$$
$$+\left(\widetilde{S}_1E+\widetilde{S}_2\right)(A_{14}+A_{15}E)$$
$$r\left(\frac{\widetilde{M}_1}{2}E^2+\widetilde{M}_2E+\widetilde{M}_3\right)=(A_8+A_9E)(A_{10}+A_{11}E)-A_{18}(A_{16}+A_{17}E)^2$$
$$+\left(\widetilde{M}_1E+\widetilde{M}_2\right)(A_{14}+A_{15}E)$$

where,

$$A_8=\alpha-\beta\left(A_1+A_3\widetilde{M}_2\right),A_9=\eta-\beta\left(A_2+A_3\widetilde{M}_1\right)$$

$$A_{10}=\left(A_1+A_3\widetilde{M}_2\right)(1-\phi)-c_M,A_{11}=\left(A_2+A_3\widetilde{M}_1\right)(1-\phi)$$

$$A_{12}=\phi\left(A_1+A_3\widetilde{M}_2\right)-c_S-c_p+c\left(A_4+A_6\widetilde{S}_2+A_7\widetilde{M}_2\right)$$

$$A_{13}=\phi\left(A_2+A_3\widetilde{M}_1\right)+c\left(A_5+A_6\widetilde{S}_1+A_7\widetilde{M}_1\right)$$

$$A_{14}=m\left(A_4+A_6\widetilde{S}_2+A_7\widetilde{M}_2\right)-g\left(\alpha-\beta\left(A_1+A_3\widetilde{M}_2\right)\right)$$

$$A_{15}=m\left(A_5+A_6\widetilde{S}_1+A_7\widetilde{M}_1\right)-g\left(\eta-\beta\left(A_2+A_3\widetilde{M}_1\right)\right)-n$$

$$A_{16}=A_4+A_6\widetilde{S}_2+A_7\widetilde{M}_2,A_{17}=A_5+A_6\widetilde{S}_1+A_7\widetilde{M}_1$$

$$A_{18}=\frac{\mu B}{2},A_{19}=\frac{\mu(1-B)}{2}$$

are constant terms. By identification, we obtain the following equations.

$$r\frac{\widetilde{S}_1}{2} = A_9A_{13} - A_{19}A_{17}^2 + A_{15}\widetilde{S}_1$$

$$r\widetilde{S}_2 = A_8A_{13} + A_9A_{12} - 2A_{19}A_{16}A_{17} + A_{14}\tilde{M}_1 + A_{15}\widetilde{S}_2$$

$$r\widetilde{S}_3 = A_8A_{12} - A_{19}A_{16}^2 + A_{14}\widetilde{S}_2$$

$$r\frac{\widetilde{M}_1}{2} = A_9A_{11} - A_{18}A_{17}^2 + A_{15}\widetilde{M}_1$$

$$r\widetilde{M}_2 = A_8A_{11} + A_9A_{10} - 2A_{18}A_{16}A_{17} + A_{14}\widetilde{M}_1 + A_{15}\widetilde{M}_2$$

$$r\widetilde{M}_3 = A_8A_{10} - A_{18}A_{16}^2 + A_{14}\widetilde{M}_2$$

Solving the Riccati equation system we get the related coefficients. □

A.3 Proof of Proposition 10

Proof. When characterizing the V-scenario, we search for a bounded and continuously differentiable value function V^V for which a unique solution for $E^V(t)$ exists, and the Hamilton–Jacobi–Bellman (HJB) equation:

$$rV^V = \max_{I,p}\left\{(\alpha - \beta p + \eta E)\left(p - c_p + cI\right) - \frac{\mu I^2}{2} + \frac{\partial V^V}{\partial E}\left[mI - g\left(\alpha - \beta p + \eta E\right) - nE\right]\right\}$$

is satisfied for any value of E. The corresponding strategies are solved as follows.

$$p^V = \frac{\alpha\mu + \beta\mu c_p - c^2\alpha\beta + \eta\left(\mu - c^2\beta\right)E + \beta\left(g\mu - cm\right)\frac{\partial V^V}{\partial E}}{\beta\left(2\mu - c^2\beta\right)}$$

$$I^V = \frac{c\left(\alpha - \beta c_p\right) + c\eta E + \left(2m - cg\beta\right)\frac{\partial V^V}{\partial E}}{2\mu - c^2\beta}$$

For the sake of clarity, we rewrite the strategies as follows.

$$p^V = C_1 + C_2 E + C_3 \frac{\partial V^V}{\partial E}$$

$$I^V = C_4 + C_5 E + C_6 \frac{\partial V^V}{\partial E}$$

where,

$$C_1 = \frac{\alpha\mu + \beta\mu c_p - c^2\alpha\beta}{\beta\left(2\mu - c^2\beta\right)}, C_2 = \frac{\eta\left(\mu - c^2\beta\right)}{\beta\left(2\mu - c^2\beta\right)}, C_3 = \frac{g\mu - cm}{2\mu - c^2\beta}$$

$$C_4 = \frac{c\left(\alpha - \beta c_p\right)}{2\mu - c^2\beta}, C_5 = \frac{c\eta}{2\mu - c^2\beta}, C_6 = \frac{2m - cg\beta}{2\mu - c^2\beta}$$

Then, we conjecture a quadratic value function:

$$V^V = \frac{k_1}{2}E^2 + k_2 E + k_3, \frac{\partial V^V}{\partial E} = k_1 E + k_2$$

Plugging the above expression and its derivatives to the HJB equation, we obtain the following equation:

$$r\left(\frac{k_1}{2}E^2 + k_2 E + k_3\right) = \left(C_7 + C_8 E\right)\left(C_9 + C_{10} E\right) - C_{11}\left(C_{12} + C_{13} E\right)^2 + \left(k_1 E + k_2\right)\left(C_{14} + C_{15} E\right) \tag{A.3}$$

where,

$$C_7 = \alpha - \beta\left(C_1 + C_3 k_2\right), C_8 = \eta - \beta\left(C_2 + C_3 k_1\right)$$

$$C_9 = C_1 + C_3 k_2 - c_p + c\left(C_4 + C_6 k_2\right), C_{10} = C_2 + C_3 k_1 + c\left(C_5 + C_6 k_1\right)$$

$$C_{11} = \frac{\mu}{2}, C_{12} = C_4 + C_6 k_2, C_{13} = C_5 + C_6 k_1$$

$$C_{14} = m\left(C_4 + C_6 k_2\right) - g\left(\alpha - \beta\left(C_1 + C_3 k_2\right)\right)$$

$$C_{15} = m\left(C_5 + C_6 k_1\right) - g\left(\eta - \beta\left(C_2 + C_3 k_1\right)\right) - n$$

are constant terms. By identification, we obtain the following equations.

$$r\frac{k_1}{2} = C_8 C_{10} - C_{11} C_{13}^2 + C_{15} k_1$$

$$r k_2 = C_7 C_{10} + C_8 C_9 - 2 C_{11} C_{12} C_{13} + C_{14} k_1 + C_{15} k_2$$

$$r k_3 = C_7 C_9 - C_{11} C_{12}^2 + C_{14} k_2$$

Solving the Riccati equation system gives the related coefficients. □

APPENDIX B

SOLUTION CANDIDATES OF SCENARIOS OF CHAPTER 3.

B.1 Coefficient solutions to W-Scenario

- Solution 1: $S_1 = 0.1522$, $S_2 = 0.0582$, $M_1 = 3.0545$, $M_2 = 8.0165$.

- Solution 2: $S_1 = 0.1359$, $S_2 = -0.1067$, $M_1 = -1.2541$, $M_2 = 8.2296$.

- Solution 3: $S_1 = 0.2718$, $S_2 = -0.3027$, $M_1 = -0.0071$, $M_2 = -0.1623$.

- Solution 4: $S_1 = 0.0122$, $S_2 = 0.1992$, $M_1 = 0.0067$, $M_2 = 0.1141$.

B.2 Coefficient solutions to R-Scenario

- Solution 1: $\widetilde{S}_1 = 0.2664$, $\widetilde{S}_2 = -0.3453$, $\widetilde{M}_1 = -0.0094$, $\widetilde{M}_2 = -0.2288$.

- Solution 2: $\widetilde{S}_1 = 0.0167$, $\widetilde{S}_2 = 0.2287$, $\widetilde{M}_1 = 0.0093$, $\widetilde{M}_2 = 0.1584$.

B.3 Coefficient solutions to V-Scenario

- Solution 1: $k_1 = 0.2566$, $k_2 = -0.6810$.

- Solution 2: $k_1 = 0.0265$, $k_2 = 0.4561$.

B.4 Coefficient solutions to Special Case - 1

- Solution 1: $S_1 = 0.1356$, $S_2 = 0.0624$, $M_1 = 2.8211$, $M_2 = 8.0232$.

- Solution 2: $S_1 = 0.1517$, $S_2 = -0.0927$, $M_1 = -1.5202$, $M_2 = 8.1990$.

- Solution 3: $S_1 = 0.2828$, $S_2 = -0.1292$, $M_1 = -0.0015$, $M_2 = -0.0672$.

- Solution 4: $S_1 = 0.0116$, $S_2 = 0.1854$, $M_1 = 0.0060$, $M_2 = 0.0980$.

APPENDIX C

TABLES OF THRESHOLD VALUES OF CHAPTER 3.

The following tables show the threshold values shown in the figures of Chapter 3.

Table C.1: W-Scenario - B

$B^{W*} = 0.211$	$\overline{B}^{W} = 0.330498$	$\overline{\overline{B}}^{W} = 0.48081$

Table C.2: R-Scenario - B

$\phi_0 = 0.651842$	$B^{R*} = 0.1936$	$\overline{B}^{R} = 0.300916$	$\overline{\overline{B}}^{R} = 0.337314$

Table C.3: R-Scenario - ϕ

$\underline{\underline{\phi}} = 0.2892$	$\phi^{'} = 0.7376$	$\phi^{*} = 0.9037$	$\widetilde{\phi} = 0.929$	$\underline{\phi} = 0.9558$	$\overline{\overline{\phi}} = 0.96904626$

Table C.4: Comparisons of W-Scenario, R-Scenario and V-Scenario

$\widehat{B}^{W} = 0.127932$	$B^{W-I} = 0.195613$	$B^{R-P} = 0.25262$
$\underline{B}^{R} = 0.333604$	$\underline{B}^{W} = 0.4570265$	$\widehat{B}^{W-P} = 0.46407$

Table C.5: Comparisons of W-Scenario, R_L-Scenario and V-Scenario

$\widehat{B}^{R} = 0.253316$	$B^{R*} = 0.5279$	$\underline{B}^{R} = 0.538679$	$\overline{\overline{B}}^{R} = 0.540035$

Table C.6: Comparisons of W-Scenario, R_H-Scenario and V-Scenario

$B^{R*} = 0.1114$	$\widehat{B}^W = 0.188075$	$\underline{B}^R = 0.2947375$	$\overline{\overline{B}}^R = 0.299337$	$\widehat{B}^{W'} = 0.4610705$

Table C.7: Comparisons of W-Scenario and V-Scenario of special case - 1

$\phi_0 = 0.7144021$	$B^{W*} = 0.207$

Table C.8: Comparisons of V-Scenario and R-Scenario of special case - 2

$\underline{\underline{\phi}} = 0.70306$	$\underline{\phi} = 0.726726$	$\phi^* = 0.8899$	$\overline{\overline{\phi}} = 0.95$

APPENDIX D

EXTENSION TO GREEN CUSTOMERS OF CHAPTER 4.

In this section, we extend the single-component model of Section 2 to allow for "*green*" customers. By green customers, we refer to customers who return their old product when buying a remanufactured product, i.e., customers who replace their products for continued usage. We refer to this case as *correlated demand and return.*

Green customers arrive according to a Poisson process with parameter ν_G. An arrival triggers two events simultaneously, a return arrival and a demand for a remanufactured product. The case is visualized in Figure D.1.

We assume that if the decision maker accepts a return from a green customer, the customer is always served a product. Therefore, we discard the cases of accepting a return without satisfying the demand and of rejecting a return but satisfying the demand, since these cases change a green customer's initial purpose of "exchanging" the product for continued service.

The state $\mathbf{x}$ transits to $(\mathbf{x} + \mathbf{e}_A - \mathbf{e}_R)$ or $(\mathbf{x} + \mathbf{e}_A - \mathbf{e}_N)$ or remains $\mathbf{x}$ if the product is unavailable. We scale the transition rate to $\tau^G = \nu + \mu_R + \mu_N + \lambda_R + \lambda_N + \nu_G \equiv 1$. The value function v^G can be written as

$$\begin{aligned} Tv^G(\mathbf{x}) = h(\mathbf{x}) &+ \nu T_A v^G(\mathbf{x}) + \mu_R T_R v^G(\mathbf{x}) + \mu_N T_N v^G(\mathbf{x}) \\ &+ \lambda_N T_{dN} v^G(\mathbf{x}) + \lambda_R T_{dR} v^G(\mathbf{x}) + \nu_G T_G v^G(\mathbf{x}), \end{aligned} \tag{D.1}$$

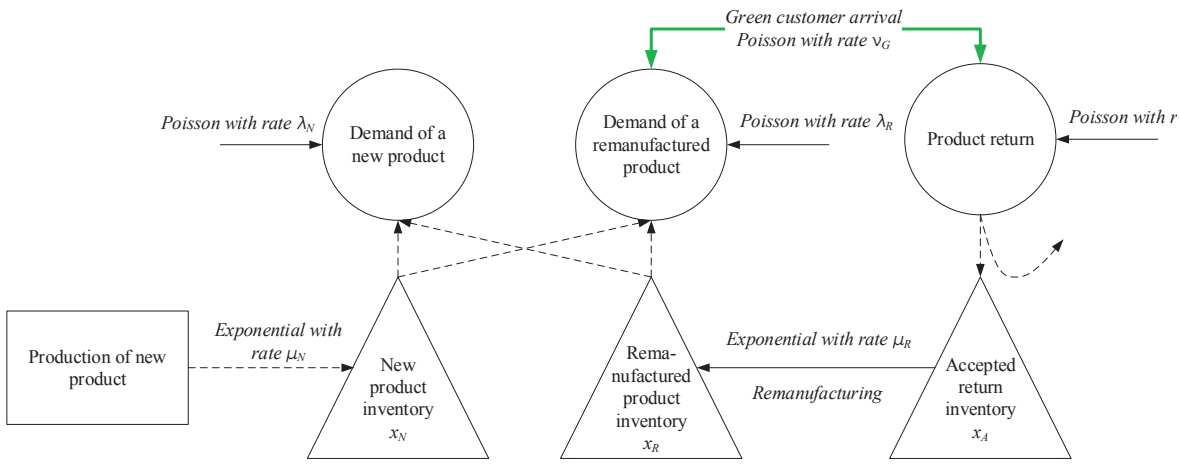

Figure D.1: Events, decisions and product flows in the single-component product model with correlated demand and return.

where operators are the same as in the single-component product model with regular returns. The additional operator T_G is defined as follows.

$$T_G v^G(\mathbf{x}) = \begin{cases} \min\{v^G(x_A+1, x_R-1, x_N) + \omega c_{RM} + (1-\omega)\theta\varepsilon, & \\ \qquad v^G(x_A+1, x_R, x_N-1) + \omega(c_{RM} + \eta_{NR}) + (1-\omega)\theta\varepsilon\}, & \text{if } x_R, x_N \geq 1, \\ v^G(x_A+1, x_R-1, x_N) + \omega c_{RM} + (1-\omega)\theta\varepsilon, & \text{if } x_R \geq 1 \text{ and } x_N = 0, \\ \min\{v^G(x_A+1, x_R, x_N-1) + \omega(c_{RM} + \eta_{NR}) + (1-\omega)\theta\varepsilon, & \\ \qquad v^G(x_A, x_R, x_N) + \omega\eta_L\}, & \text{if } x_N \geq 1 \text{ and } x_R = 0, \\ v^G(x_A, x_R, x_N) + \omega\eta_L, & \text{otherwise.} \end{cases} \tag{D.2}$$

In the numerical analysis we use the same parameter values as in the numerical analysis of the single-component case with regular returns (see Table 1) by subtracting part of the regular returns to formulate the green returns. We make this assumption to model that part of the demand for remanufactured products comes from green customers, i.e., we keep $\nu + \nu_G$

Table D.1: Sensitivity analysis for the model of correlated demand and return under different values of ν_G.

No.	ν	μ_R	μ_N	λ_N	λ_R	ν_G	Cost (k€)
1	4	10	8	12	8	0	88,942
2	3	10	8	12	7	1	89,564
3	2	10	8	12	6	2	90,411
4	1	10	8	12	5	3	91,608
5	0	10	8	12	4	4	93,409

constant. The results are shown in Table D.1.

By keeping the total return rate $\nu + \nu_G$ constant, the results of Table D.1 illustrate that the strategic cost increases with the share of green customers. This is counter-intuitive since we would have expected lower cost for higher ν_G due to the certainty of demanding a remanufactured product by a green customer. However, lower regular return rates reduce the flexibility to satisfy regular customers in the meantime, which eventually leads to higher cost.

In Table D.2, we compare the results with the corresponding cases of returns only from regular customers shown in Table 4.3. We find that if the rates of both returning Poisson processes are small compared to the arrival rate of remanufactured demand, the cost per demand is higher than the costs in the situation of only having regular customers. The reason is that the company incurs costs for remanufacturing the returned products but may not be able to satisfy the customer in time due to limited resources and timeliness, which eventually incurs penalty or substitution cost. However, when the returning rates are sufficiently high, the strategic cost per demand is lower than that in the case of only regular customers since a portion of the customers who return the products certainly demand a new one. This correlation offsets part of the uncertainty of the demand arrivals and reduces the

Table D.2: Comparisons between the results of Table 4.3 and the results for the single-component product model with correlated demand and return.

No.	Case of Green customers								Case of Regular customers							ΔCost per demand (€)
	ν	μ_R	μ_N	λ_N	λ_R	ν_G	Cost (k€)	Cost per demand (€)	ν	μ_R	μ_N	λ_N	λ_R	Cost (k€)	Cost per demand (€)	
1	1	10	8	12	7	1	104,905	5,245	2	10	8	12	8	102,957	5,148	97
2	2				6	2	90,411	4,521	4					88,942	4,447	74
3	3				5	3	74,935	3,747	6					74,927	3,746	1
4	4				4	4	61,180	3,059	8					61,339	3,067	-8
5	2	6	8	12	6	2	90,416	4,521	4	6	8	12	8	88,942	4,447	74
6		8					90,413	4,521		8				88,942	4,447	74
7		12					90,409	4,520		12				88,942	4,447	73
8	2	10	6	12	6	2	100,644	5,032	4	10	6	12	8	97,498	4,875	157
9			10				80,491	4,025			10			80,385	4,019	6
10			12				71,839	3,592			12			71,829	3,591	1
11	2	10	8	6	6	2	46,955	3,354	4	10	8	6	8	46,944	3,353	1
12				8			61,043	3,815				8		60,942	3,809	6
13				10			75,704	4,206				10		74,942	4,163	43
14	2	10	8	12	2	2	55,444	3,465	4	10	8	12	4	55,581	3,474	-9
15					4		72,152	4,008					6	72,142	4,008	0
16					8		108,823	4,947					10	105,741	4,806	141

overall cost.

Furthermore, we observe that the strategic cost per demand decreases in ν, μ_R, μ_N and increases in λ_N and λ_R. This is intuitive because with more demand and/or returns, the company can pool its inventory. This observation implies that by enlarging the manufacturing/remanufacturing efficiency and/or by more frequent return flow, we seize more resources to serve the demand, which results in lower strategic cost.

APPENDIX E
PROOFS OF CHAPTER 4.

For notation convenience, we refer to T_A as T_1, T_R as T_2, T_N as T_3, T_{dN} as T_4, T_{dR} as T_5 and T_G as T_6 throughout the online appendix.

E.1 Proof of Lemma 4.4.1

For Lemma 1, we prove the preservation of C1 to C8 for operators T_A, T_R, T_N, T_{dN} and t_{dR} given the parameter constraint specified in Lemma 1.

Proof. Proof of C1 In this section, we prove $T_s v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i) - T_s v(\mathbf{x}+\mathbf{e}_i) \geq T_s v(\mathbf{x}+\mathbf{e}_i) - T_s v(\mathbf{x})$, $s = A, R, N, dN, dR$ and $i = A, R, N$.

Operator T_A

(1) If $T_A v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i) = v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i+\mathbf{e}_A) + \omega c_{RM} + (1-\omega)\theta\varepsilon$ and $T_A v(\mathbf{x}) = v(\mathbf{x}+\mathbf{e}_A) + \omega c_{RM} + (1-\omega)\theta\varepsilon$,

$$\begin{aligned} & T_A v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i) - T_A v(\mathbf{x}+\mathbf{e}_i) \\ \geq\ & v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i+\mathbf{e}_A) + \omega c_{RM} + (1-\omega)\theta\varepsilon - v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_A) - \omega c_{RM} - (1-\omega)\theta\varepsilon \\ \geq\ & v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_A) + \omega c_{RM} + (1-\omega)\theta\varepsilon - v(\mathbf{x}+\mathbf{e}_A) - \omega c_{RM} - (1-\omega)\theta\varepsilon \geq T_A v(\mathbf{x}+\mathbf{e}_i) - T_A v(\mathbf{x}) \end{aligned}$$

(2) If $T_A v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i) = v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i+\mathbf{e}_A) + \omega c_{RM} + (1-\omega)\theta\varepsilon$ and $T_A v(\mathbf{x}) = v(\mathbf{x})$,

$$\begin{aligned} & T_A v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i) - T_A v(\mathbf{x}+\mathbf{e}_i) \\ \geq\ & v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i+\mathbf{e}_A) + \omega c_{RM} + (1-\omega)\theta\varepsilon - v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_A) - \omega c_{RM} - (1-\omega)\theta\varepsilon \\ \geq\ & v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_A) - v(\mathbf{x}+\mathbf{e}_A) \geq v(\mathbf{x}+\mathbf{e}_i) - v(\mathbf{x}) \geq T_A v(\mathbf{x}+\mathbf{e}_i) - T_A v(\mathbf{x}) \end{aligned}$$

(3) If $T_A v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i) = v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i)$ and $T_A v(\mathbf{x}) = v(\mathbf{x}+\mathbf{e}_A) + \omega c_{RM} + (1-\omega)\theta\varepsilon$,

$$\begin{aligned} & T_A v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i) - T_A v(\mathbf{x}+\mathbf{e}_i) \geq v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i) - v(\mathbf{x}+\mathbf{e}_i) \\ \geq\ & v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_A) + \omega c_{RM} + (1-\omega)\theta\varepsilon - v(\mathbf{x}+\mathbf{e}_A) - \omega c_{RM} - (1-\omega)\theta\varepsilon \geq T_A v(\mathbf{x}+\mathbf{e}_i) - T_A v(\mathbf{x}) \end{aligned}$$

(4) If $T_A v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i) = v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i)$ and $T_A v(\mathbf{x}) = v(\mathbf{x})$,

$$T_A v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i) - T_A v(\mathbf{x}+\mathbf{e}_i) \geq v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i) - v(\mathbf{x}+\mathbf{e}_i) \geq v(\mathbf{x}+\mathbf{e}_i) - v(\mathbf{x}) \geq T_A v(\mathbf{x}+\mathbf{e}_i) - T_A v(\mathbf{x})$$

Operator T_R

(1) If $x_A > 0$, $T_R v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i) = v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i-\mathbf{e}_A+\mathbf{e}_R)$ and $T_R v(\mathbf{x}) = v(\mathbf{x}-\mathbf{e}_A+\mathbf{e}_R)$,

$$\begin{aligned} & T_R v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i) - T_R v(\mathbf{x}+\mathbf{e}_i) = v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i-\mathbf{e}_A+\mathbf{e}_R) - v(\mathbf{x}+\mathbf{e}_i-\mathbf{e}_A+\mathbf{e}_R) \\ \geq\ & v(\mathbf{x}+\mathbf{e}_i-\mathbf{e}_A+\mathbf{e}_R) - v(\mathbf{x}-\mathbf{e}_A+\mathbf{e}_R) = T_R v(\mathbf{x}+\mathbf{e}_i) - T_R v(\mathbf{x}) \end{aligned}$$

(2) If $x_A = 0$, $i = A$, $T_R v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i) = v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i-\mathbf{e}_A+\mathbf{e}_R)$ and $T_R v(\mathbf{x}) = v(\mathbf{x})$,

$$\begin{aligned} & T_R v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i) - T_R v(\mathbf{x}+\mathbf{e}_i) = v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i-\mathbf{e}_A+\mathbf{e}_R) - v(\mathbf{x}+\mathbf{e}_i-\mathbf{e}_A+\mathbf{e}_R) \\ =\ & v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_R) - v(\mathbf{x}+\mathbf{e}_R) \geq v(\mathbf{x}+\mathbf{e}_i) - v(\mathbf{x}), \text{ use C7.} \\ \geq\ & v(\mathbf{x}+\mathbf{e}_R) - v(\mathbf{x}) = v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_R-\mathbf{e}_A) - v(\mathbf{x}) \geq T_R v(\mathbf{x}+\mathbf{e}_i) - T_R v(\mathbf{x}) \end{aligned}$$

(3) If $x_A = 0$, $i \neq A$, $T_R v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i) = v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i)$ and $T_R v(\mathbf{x}) = v(\mathbf{x})$,

$$T_R v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i) - T_R v(\mathbf{x}+\mathbf{e}_i) \geq v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i) - v(\mathbf{x}+\mathbf{e}_i) \geq v(\mathbf{x}+\mathbf{e}_i) - v(\mathbf{x}) \geq T_R v(\mathbf{x}+\mathbf{e}_i) - T_R v(\mathbf{x})$$

Operator T_N

(1) If $T_N v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i) = v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i+\mathbf{e}_N) + \omega c_N + (1-\omega)\varepsilon$ and $T_N v(\mathbf{x}) = v(\mathbf{x}+\mathbf{e}_N) + \omega c_N + (1-\omega)\varepsilon$,

$$\begin{aligned} & T_N v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i) - T_N v(\mathbf{x}+\mathbf{e}_i) \\ \geq\ & v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i+\mathbf{e}_N) + \omega c_N + (1-\omega)\varepsilon - v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_N) - \omega c_N - (1-\omega)\varepsilon \\ \geq\ & v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_N) + \omega c_N + (1-\omega)\varepsilon - v(\mathbf{x}+\mathbf{e}_N) - \omega c_N - (1-\omega)\varepsilon \geq T_N v(\mathbf{x}+\mathbf{e}_i) - T_N v(\mathbf{x}) \end{aligned}$$

(2) If $T_N v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i) = v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i+\mathbf{e}_N) + \omega c_N + (1-\omega)\varepsilon$ and $T_N v(\mathbf{x}) = v(\mathbf{x})$,

$$\begin{aligned} & T_N v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i) - T_N v(\mathbf{x}+\mathbf{e}_i) \\ \geq\ & v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i+\mathbf{e}_N) + \omega c_N + (1-\omega)\varepsilon - v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_N) - \omega c_N - (1-\omega)\varepsilon \\ \geq\ & v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i) - v(\mathbf{x}+\mathbf{e}_i) \geq v(\mathbf{x}+\mathbf{e}_i) - v(\mathbf{x}) \geq T_N v(\mathbf{x}+\mathbf{e}_i) - T_N v(\mathbf{x}) \end{aligned}$$

(3) If $T_N v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i) = v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i)$ and $T_N v(\mathbf{x}) = v(\mathbf{x}+\mathbf{e}_N) + \omega c_N + (1-\omega)\varepsilon$,

$$\begin{aligned} & T_N v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i) - T_N v(\mathbf{x}+\mathbf{e}_i) \geq v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i) - v(\mathbf{x}+\mathbf{e}_i) \geq v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_N) - v(\mathbf{x}+\mathbf{e}_N) \\ =\ & v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_N) + \omega c_N + (1-\omega)\varepsilon - v(\mathbf{x}+\mathbf{e}_N) - \omega c_N - (1-\omega)\varepsilon \geq T_N v(\mathbf{x}+\mathbf{e}_i) - T_N v(\mathbf{x}) \end{aligned}$$

(4) If $T_N v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i) = v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i)$ and $T_N v(\mathbf{x}) = v(\mathbf{x})$,

$$T_N v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i) - T_N v(\mathbf{x}+\mathbf{e}_i) \geq v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i) - v(\mathbf{x}+\mathbf{e}_i) \geq v(\mathbf{x}+\mathbf{e}_i) - v(\mathbf{x}) \geq T_N v(\mathbf{x}+\mathbf{e}_i) - T_N v(\mathbf{x})$$

Operator T_{dN} (T_{dR} follows the same structure, thus omitted)

(1) If $x_N > 0$, $x_R \geq 0$, $T_{dN} v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i) = v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i-\mathbf{e}_N)$ and $T_{dN} v(\mathbf{x}) = v(\mathbf{x}-\mathbf{e}_N)$,

$$\begin{aligned} & T_{dN} v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i) - T_{dN} v(\mathbf{x}+\mathbf{e}_i) \geq v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i-\mathbf{e}_N) - v(\mathbf{x}+\mathbf{e}_i-\mathbf{e}_N) \geq v(\mathbf{x}+\mathbf{e}_i-\mathbf{e}_N) - v(\mathbf{x}-\mathbf{e}_N) \\ \geq\ & T_{dN} v(\mathbf{x}+\mathbf{e}_i) - T_{dN} v(\mathbf{x}) \end{aligned}$$

(2) If $x_N > 0$, $x_R > 0$, or $x_N = 0$, $x_R > 0$, $i = N$, $T_{dN} v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i) = v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i-\mathbf{e}_N)$ and $T_{dN} v(\mathbf{x}) = v(\mathbf{x}-\mathbf{e}_R)+\omega\eta_{RN}$,

$$\begin{aligned} & T_{dN} v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i) - T_{dN} v(\mathbf{x}+\mathbf{e}_i) \\ & \text{if } i = N, \\ \geq\ & v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i-\mathbf{e}_N) - v(\mathbf{x}+\mathbf{e}_i-\mathbf{e}_N) = v(\mathbf{x}+\mathbf{e}_i) - v(\mathbf{x}) \geq v(\mathbf{x}+\mathbf{e}_i-\mathbf{e}_R) - v(\mathbf{x}-\mathbf{e}_R) \\ =\ & v(\mathbf{x}+\mathbf{e}_i-\mathbf{e}_R) + \omega\eta_{RN} - v(\mathbf{x}-\mathbf{e}_R) - \omega\eta_{RN} \geq T_{dN} v(\mathbf{x}+\mathbf{e}_i) - T_{dN} v(\mathbf{x}) \\ & \text{if } i \neq N, \\ \geq\ & v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i-\mathbf{e}_N) - v(\mathbf{x}+\mathbf{e}_i-\mathbf{e}_N) \geq v(\mathbf{x}+\mathbf{e}_i) - v(\mathbf{x}) \geq v(\mathbf{x}+\mathbf{e}_i-\mathbf{e}_R) - v(\mathbf{x}-\mathbf{e}_R) \\ =\ & v(\mathbf{x}+\mathbf{e}_i-\mathbf{e}_R) + \omega\eta_{RN} - v(\mathbf{x}-\mathbf{e}_R) - \omega\eta_{RN} \geq T_{dN} v(\mathbf{x}+\mathbf{e}_i) - T_{dN} v(\mathbf{x}) \end{aligned}$$

(3) If $x_N = 0$, $x_R > 0$, $i = N$, $T_{dN}v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i) = v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i-\mathbf{e}_N)$ and $T_{dN}v(\mathbf{x}) = v(\mathbf{x})+\omega\eta_L$,

$$\begin{aligned}
& T_{dN}v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i) - T_{dN}v(\mathbf{x}+\mathbf{e}_i) \\
\geq\ & v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i-\mathbf{e}_N) - v(\mathbf{x}+\mathbf{e}_i-\mathbf{e}_N) = v(\mathbf{x}+\mathbf{e}_i) - v(\mathbf{x}), \text{ use C8, } \geq v(\mathbf{x}) - \omega\eta_L - v(\mathbf{x}) \\
=\ & v(\mathbf{x}+\mathbf{e}_i-\mathbf{e}_N) - v(\mathbf{x}) - \omega\eta_L \geq T_{dN}v(\mathbf{x}+\mathbf{e}_i) - T_{dN}v(\mathbf{x})
\end{aligned}$$

(4) If $x_N > 0$, $x_R > 0$, or $x_N > 0$, $x_R = 0$, $i = R$, $T_{dN}v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i) = v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i-\mathbf{e}_R)+\omega\eta_{RN}$ and $T_{dN}v(\mathbf{x}) = v(\mathbf{x}-\mathbf{e}_N)$,

$$\begin{aligned}
& T_{dN}v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i) - T_{dN}v(\mathbf{x}+\mathbf{e}_i) \\
& \text{if } i = R, \\
\geq\ & v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i-\mathbf{e}_R) + \omega\eta_{RN} - v(\mathbf{x}+\mathbf{e}_i-\mathbf{e}_R) - \omega\eta_{RN} \\
=\ & v(\mathbf{x}+\mathbf{e}_i) - v(\mathbf{x}) \geq v(\mathbf{x}+\mathbf{e}_i-\mathbf{e}_N) - v(\mathbf{x}-\mathbf{e}_N) \geq T_{dN}v(\mathbf{x}+\mathbf{e}_i) - T_{dN}v(\mathbf{x}) \\
& \text{if } i \neq R, \\
\geq\ & v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i-\mathbf{e}_R) + \omega\eta_{RN} - v(\mathbf{x}+\mathbf{e}_i-\mathbf{e}_R) - \omega\eta_{RN} \\
\geq\ & v(\mathbf{x}+\mathbf{e}_i) - v(\mathbf{x}) \geq v(\mathbf{x}+\mathbf{e}_i-\mathbf{e}_N) - v(\mathbf{x}-\mathbf{e}_N) \geq T_{dN}v(\mathbf{x}+\mathbf{e}_i) - T_{dN}v(\mathbf{x})
\end{aligned}$$

(5) If $x_N \geq 0$, $x_R > 0$, $T_{dN}v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i) = v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i-\mathbf{e}_R)+\omega\eta_{RN}$ and $T_{dN}v(\mathbf{x}) = v(\mathbf{x}-\mathbf{e}_R)+\omega\eta_{RN}$,

$$\begin{aligned}
& T_{dN}v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i) - T_{dN}v(\mathbf{x}+\mathbf{e}_i) \geq v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i-\mathbf{e}_R) + \omega\eta_{RN} - v(\mathbf{x}+\mathbf{e}_i-\mathbf{e}_R) - \omega\eta_{RN} \\
\geq\ & v(\mathbf{x}+\mathbf{e}_i-\mathbf{e}_R) + \omega\eta_{RN} - v(\mathbf{x}-\mathbf{e}_R) - \omega\eta_{RN} \geq T_{dN}v(\mathbf{x}+\mathbf{e}_i) - T_{dN}v(\mathbf{x})
\end{aligned}$$

(6) If $x_N = 0$, $x_R > 0$ or $x_N = 0$, $x_R = 0$, $i = R$, $T_{dN}v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i) = v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i-\mathbf{e}_R)+\omega\eta_{RN}$ and $T_{dN}v(\mathbf{x}) = v(\mathbf{x})+\omega\eta_L$,

$$\begin{aligned}
& T_{dN}v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i) - T_{dN}v(\mathbf{x}+\mathbf{e}_i) \geq v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i-\mathbf{e}_R) + \omega\eta_{RN} - v(\mathbf{x}+\mathbf{e}_i-\mathbf{e}_R) - \omega\eta_{RN} \\
\geq\ & v(\mathbf{x}+\mathbf{e}_i) + \omega\eta_L - v(\mathbf{x}) - \omega\eta_L \geq T_{dN}v(\mathbf{x}+\mathbf{e}_i) - T_{dN}v(\mathbf{x})
\end{aligned}$$

(7) If $x_N = 0$, $x_R > 0$, $i \neq N$, $T_{dN}v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i) = v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i)+\omega\eta_L$ and $T_{dN}v(\mathbf{x}) = v(\mathbf{x}-\mathbf{e}_R)+\omega\eta_{RN}$,

$$\begin{aligned}
& T_{dN}v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i) - T_{dN}v(\mathbf{x}+\mathbf{e}_i) \geq v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i) + \omega\eta_L - v(\mathbf{x}+\mathbf{e}_i) - \omega\eta_L \\
\geq\ & v(\mathbf{x}+\mathbf{e}_i-\mathbf{e}_R) + \omega\eta_{RN} - v(\mathbf{x}-\mathbf{e}_R) - \omega\eta_{RN} \geq T_{dN}v(\mathbf{x}+\mathbf{e}_i) - T_{dN}v(\mathbf{x})
\end{aligned}$$

(8) If $x_N = 0$, $x_R > 0$, $i \neq N$, $T_{dN}v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i) = v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i) + \omega\eta_L$ and $T_{dN}v(\mathbf{x}) = v(\mathbf{x}) + \omega\eta_L$,

$$\begin{aligned} & T_{dN}v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i) - T_{dN}v(\mathbf{x}+\mathbf{e}_i) \geq v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i) + \omega\eta_L - v(\mathbf{x}+\mathbf{e}_i) - \omega\eta_L \\ \geq\ & v(\mathbf{x}+\mathbf{e}_i) + \omega\eta_L - v(\mathbf{x}) - \omega\eta_L \geq T_{dN}v(\mathbf{x}+\mathbf{e}_i) - T_{dN}v(\mathbf{x}) \end{aligned}$$

(9) If $x_N = 0$, $x_R = 0$, $i = N$, $T_{dN}v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i) = v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i-\mathbf{e}_N)$ and $T_{dN}v(\mathbf{x}) = v(\mathbf{x}) + \omega\eta_L$

$$\begin{aligned} & T_{dN}v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i) - T_{dN}v(\mathbf{x}+\mathbf{e}_i) \\ =\ & v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i-\mathbf{e}_N) - v(\mathbf{x}+\mathbf{e}_i-\mathbf{e}_N) = v(\mathbf{x}+\mathbf{e}_i) - v(\mathbf{x}), \text{ use C8.} \\ \geq\ & v(\mathbf{x}+\mathbf{e}_i-\mathbf{e}_N) - v(\mathbf{x}) - \omega\eta_L = T_{dN}v(\mathbf{x}+\mathbf{e}_i) - T_{dN}v(\mathbf{x}) \end{aligned}$$

(10) If $x_N = 0$, $x_R = 0$, $i = R$, $T_{dN}v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i) = v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i-\mathbf{e}_R) + \omega\eta_{RN}$ and $T_{dN}v(\mathbf{x}) = v(\mathbf{x}) + \omega\eta_L$

$$\begin{aligned} & T_{dN}v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i) - T_{dN}v(\mathbf{x}+\mathbf{e}_i) \\ \geq\ & v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i-\mathbf{e}_R) + \omega\eta_{RN} - v(\mathbf{x}+\mathbf{e}_i-\mathbf{e}_R) - \omega\eta_{RN} \\ =\ & v(\mathbf{x}+\mathbf{e}_i) - v(\mathbf{x}) = v(\mathbf{x}+\mathbf{e}_i) + \omega\eta_L - v(\mathbf{x}) - \omega\eta_L \geq T_{dN}v(\mathbf{x}+\mathbf{e}_i) - T_{dN}v(\mathbf{x}) \end{aligned}$$

(11) If $x_N = 0$, $x_R = 0$, $i = R$, $T_{dN}v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i) = v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i) + \omega\eta_L$ and $T_{dN}v(\mathbf{x}) = v(\mathbf{x}) + \omega\eta_L$

$$\begin{aligned} & T_{dN}v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i) - T_{dN}v(\mathbf{x}+\mathbf{e}_i) \geq v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i) + \omega\eta_L - v(\mathbf{x}+\mathbf{e}_i) - \omega\eta_L \\ \geq\ & v(\mathbf{x}+\mathbf{e}_i) + \omega\eta_L - v(\mathbf{x}) - \omega\eta_L \geq T_{dN}v(\mathbf{x}+\mathbf{e}_i) - T_{dN}v(\mathbf{x}) \end{aligned}$$

(12) If $x_N = 0$, $x_R = 0$, $i = A$, $T_{dN}v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i) = v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i)$ $+\omega\eta_L$ and $T_{dN}v(\mathbf{x}) = v(\mathbf{x}) + \omega\eta_L$,

$$\begin{aligned} & T_{dN}v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i) - T_{dN}v(\mathbf{x}+\mathbf{e}_i) \geq v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i) + \omega\eta_L - v(\mathbf{x}+\mathbf{e}_i) - \omega\eta_L \\ \geq\ & v(\mathbf{x}+\mathbf{e}_i) + \omega\eta_L - v(\mathbf{x}) - \omega\eta_L = T_{dN}v(\mathbf{x}+\mathbf{e}_i) - T_{dN}v(\mathbf{x}) \end{aligned}$$

Proof. Proof of C2 In this section, we prove $T_s v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j) - T_s v(\mathbf{x}+\mathbf{e}_i) \geq T_s v(\mathbf{x}+\mathbf{e}_j) - T_s v(\mathbf{x})$, $s = A, R, N, dN, dR$ and $i, j = A, R, N$, $i \neq j$.

Operator T_A

(1) If $T_A v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j) = v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j+\mathbf{e}_A) + \omega c_{RM} + (1-\omega)\theta\varepsilon$ and $T_A v(\mathbf{x}) = v(\mathbf{x}+\mathbf{e}_A) + \omega c_{RM} + (1-\omega)\theta\varepsilon$,

$$\begin{aligned}
& T_A v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j) - T_A v(\mathbf{x}+\mathbf{e}_i) \\
\geq\ & v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j+\mathbf{e}_A) + \omega c_{RM} + (1-\omega)\theta\varepsilon - v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_A) - \omega c_{RM} - (1-\omega)\theta\varepsilon \\
\geq\ & v(\mathbf{x}+\mathbf{e}_j+\mathbf{e}_A) + \omega c_{RM} + (1-\omega)\theta\varepsilon - v(\mathbf{x}+\mathbf{e}_A) - \omega c_{RM} - (1-\omega)\theta\varepsilon \geq T_A v(\mathbf{x}+\mathbf{e}_j) - T_A v(\mathbf{x})
\end{aligned}$$

(2) If $T_A v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j) = v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j+\mathbf{e}_A) + \omega c_{RM} + (1-\omega)\theta\varepsilon$ and $T_A v(\mathbf{x}) = v(\mathbf{x})$,

$$\begin{aligned}
& T_A v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j) - T_A v(\mathbf{x}+\mathbf{e}_i) \\
\geq\ & v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j+\mathbf{e}_A) + \omega c_{RM} + (1-\omega)\theta\varepsilon - v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_A) - \omega c_{RM} - (1-\omega)\theta\varepsilon \\
\geq\ & v(\mathbf{x}+\mathbf{e}_j+\mathbf{e}_A) - v(\mathbf{x}+\mathbf{e}_A) \geq v(\mathbf{x}+\mathbf{e}_j) - v(\mathbf{x}) \geq T_A v(\mathbf{x}+\mathbf{e}_j) - T_A v(\mathbf{x})
\end{aligned}$$

(3) If $T_A v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j) = v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j)$ and $T_A v(\mathbf{x}) = v(\mathbf{x}+\mathbf{e}_A) + \omega c_{RM} + (1-\omega)\theta\varepsilon$,

$$\begin{aligned}
& T_A v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j) - T_A v(\mathbf{x}+\mathbf{e}_i) \\
& \text{if } j = A, \\
\geq\ & v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j) - v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_A) - \omega c_{RM} - (1-\omega)\theta\varepsilon \\
=\ & v(\mathbf{x}+\mathbf{e}_j) - v(\mathbf{x}+\mathbf{e}_A) - \omega c_{RM} - (1-\omega)\theta\varepsilon \geq T_A v(\mathbf{x}+\mathbf{e}_j) - T_A v(\mathbf{x}) \\
& \text{if } i = A, \\
\geq\ & v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j) - v(\mathbf{x}+\mathbf{e}_i) = v(\mathbf{x}+\mathbf{e}_A+\mathbf{e}_j) - v(\mathbf{x}+\mathbf{e}_A) \\
=\ & v(\mathbf{x}+\mathbf{e}_j+\mathbf{e}_A) + \omega c_{RM} + (1-\omega)\theta\varepsilon - v(\mathbf{x}+\mathbf{e}_A) - \omega c_{RM} - (1-\omega)\theta\varepsilon \geq T_A v(\mathbf{x}+\mathbf{e}_j) - T_A v(\mathbf{x}) \\
& \text{if } i = R,\ j = N, \\
\geq\ & v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j) - v(\mathbf{x}+\mathbf{e}_i), \text{ use C6.} \\
\geq\ & v(\mathbf{x}+\mathbf{e}_j+\mathbf{e}_A) + \omega c_{RM} + (1-\omega)\theta\varepsilon - v(\mathbf{x}+\mathbf{e}_A) - \omega c_{RM} - (1-\omega)\theta\varepsilon \geq T_A v(\mathbf{x}+\mathbf{e}_j) - T_A v(\mathbf{x}) \\
& \text{if } i = N,\ j = R, \\
\geq\ & v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j) - v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_A) - \omega c_{RM} - (1-\omega)\theta\varepsilon, \text{ use C6.} \\
\geq\ & v(\mathbf{x}+\mathbf{e}_j) - v(\mathbf{x}+\mathbf{e}_A) - \omega c_{RM} - (1-\omega)\theta\varepsilon \geq T_A v(\mathbf{x}+\mathbf{e}_j) - T_A v(\mathbf{x})
\end{aligned}$$

(4) If $T_A v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j) = v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j)$ and $T_A v(\mathbf{x}) = v(\mathbf{x})$,

$$T_A v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j) - T_A v(\mathbf{x}+\mathbf{e}_i) \geq v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j) - v(\mathbf{x}+\mathbf{e}_i) \geq v(\mathbf{x}+\mathbf{e}_j) - v(\mathbf{x}) \geq T_A v(\mathbf{x}+\mathbf{e}_j) - T_A v(\mathbf{x})$$

Operator T_R

(1) If $x_A > 0$, $T_R v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j) = v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j-\mathbf{e}_A+\mathbf{e}_R)$ and $T_R v(\mathbf{x}) = v(\mathbf{x}-\mathbf{e}_A+\mathbf{e}_R)$,

$$\begin{aligned} & T_R v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j) - T_R v(\mathbf{x}+\mathbf{e}_i) = v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j-\mathbf{e}_A+\mathbf{e}_R) - v(\mathbf{x}+\mathbf{e}_i-\mathbf{e}_A+\mathbf{e}_R) \\ \geq\ & v(\mathbf{x}+\mathbf{e}_j-\mathbf{e}_A+\mathbf{e}_R) - v(\mathbf{x}-\mathbf{e}_A+\mathbf{e}_R) = T_R v(\mathbf{x}+\mathbf{e}_j) - T_R v(\mathbf{x}) \end{aligned}$$

(2) If $x_A = 0$, $i \vee j = A$, $T_R v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j) = v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_k-\mathbf{e}_A+\mathbf{e}_R)$ and $T_R v(\mathbf{x}) = v(\mathbf{x})$,

$$\begin{aligned} & T_R v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j) - T_R v(\mathbf{x}+\mathbf{e}_i) \\ & \text{if } i = A, \\ =\ & v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j-\mathbf{e}_A+\mathbf{e}_R) - v(\mathbf{x}+\mathbf{e}_i-\mathbf{e}_A+\mathbf{e}_R) \\ =\ & v(\mathbf{x}+\mathbf{e}_j+\mathbf{e}_R) - v(\mathbf{x}+\mathbf{e}_R) \geq v(\mathbf{x}+\mathbf{e}_j) - v(\mathbf{x}) = T_R v(\mathbf{x}+\mathbf{e}_j) - T_R v(\mathbf{x}) \\ & \text{if } j = A, \\ =\ & v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j-\mathbf{e}_A+\mathbf{e}_R) - v(\mathbf{x}+\mathbf{e}_i) = v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j-\mathbf{e}_A+\mathbf{e}_R) - v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j-\mathbf{e}_A) \\ \geq\ & v(\mathbf{x}+\mathbf{e}_j-\mathbf{e}_A+\mathbf{e}_R) - v(\mathbf{x}+\mathbf{e}_j-\mathbf{e}_A) = T_R v(\mathbf{x}+\mathbf{e}_j) - T_R v(\mathbf{x}) \end{aligned}$$

(3) If $x_A = 0$, $i, j \neq A$, $T_R v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j) = v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j)$ and $T_R v(\mathbf{x}) = v(\mathbf{x})$,

$$T_R v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j) - T_R v(\mathbf{x}+\mathbf{e}_i) = v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j) - v(\mathbf{x}+\mathbf{e}_i) \geq v(\mathbf{x}+\mathbf{e}_j) - v(\mathbf{x}) = T_R v(\mathbf{x}+\mathbf{e}_j) - T_R v(\mathbf{x})$$

Operator T_N

(1) If $T_N v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j) = v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j+\mathbf{e}_N) + \omega c_N + (1-\omega)\varepsilon$ and $T_N v(\mathbf{x}) = v(\mathbf{x}+\mathbf{e}_N) + \omega c_N + (1-\omega)\varepsilon$,

$$\begin{aligned}
& T_N v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j) - T_N v(\mathbf{x}+\mathbf{e}_i) \\
\geq\ & v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j+\mathbf{e}_N) + \omega c_N + (1-\omega)\varepsilon - v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_N) - \omega c_N - (1-\omega)\varepsilon \\
\geq\ & v(\mathbf{x}+\mathbf{e}_j+\mathbf{e}_N) + \omega c_N + (1-\omega)\varepsilon - v(\mathbf{x}+\mathbf{e}_N) - \omega c_N - (1-\omega)\varepsilon \\
\geq\ & T_N v(\mathbf{x}+\mathbf{e}_j) - T_N v(\mathbf{x})
\end{aligned}$$

(2) If $T_N v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j) = v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j+\mathbf{e}_N) + \omega c_N + (1-\omega)\varepsilon$ and $T_N v(\mathbf{x}) = v(\mathbf{x})$,

$$\begin{aligned}
& T_N v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j) - T_N v(\mathbf{x}+\mathbf{e}_i) \\
\geq\ & v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j+\mathbf{e}_N) + \omega c_N + (1-\omega)\varepsilon - v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_N) - \omega c_N - (1-\omega)\varepsilon \\
\geq\ & v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j) - v(\mathbf{x}+\mathbf{e}_i) \geq v(\mathbf{x}+\mathbf{e}_j) - v(\mathbf{x}) \geq T_N v(\mathbf{x}+\mathbf{e}_j) - T_N v(\mathbf{x})
\end{aligned}$$

(3) If $T_N v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j) = v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j)$ and $T_N v(\mathbf{x}) = v(\mathbf{x}+\mathbf{e}_N) + \omega c_N + (1-\omega)\varepsilon$,

$$
\begin{aligned}
& T_N v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j) - T_N v(\mathbf{x}+\mathbf{e}_i) \\
& \text{if } j = N, \\
\geq\ & v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j) - v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_N) - \omega c_N - (1-\omega)\varepsilon \\
=\ & v(\mathbf{x}+\mathbf{e}_j) - v(\mathbf{x}+\mathbf{e}_N) - \omega c_N - (1-\omega)\varepsilon \geq T_N v(\mathbf{x}+\mathbf{e}_j) - T_N v(\mathbf{x}) \\
& \text{if } i = N, \\
\geq\ & v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j) - v(\mathbf{x}+\mathbf{e}_i) = v(\mathbf{x}+\mathbf{e}_j+\mathbf{e}_N) - v(\mathbf{x}+\mathbf{e}_N) \\
=\ & v(\mathbf{x}+\mathbf{e}_j+\mathbf{e}_N) + \omega c_N + (1-\omega)\varepsilon - v(\mathbf{x}+\mathbf{e}_N) - \omega c_N - (1-\omega)\varepsilon \\
\geq\ & T_N v(\mathbf{x}+\mathbf{e}_j) - T_N v(\mathbf{x}) \\
& \text{if } i = A,\ j = R, \\
\geq\ & v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j) - v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_N) - \omega c_N - (1-\omega)\varepsilon, \text{ use C5.} \\
\geq\ & v(\mathbf{x}+\mathbf{e}_j) - v(\mathbf{x}+\mathbf{e}_N) - \omega c_N - (1-\omega)\varepsilon \geq T_N v(\mathbf{x}+\mathbf{e}_j) - T_N v(\mathbf{x}) \\
& \text{if } i = R,\ j = A, \\
\geq\ & v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j) - v(\mathbf{x}+\mathbf{e}_i), \text{ use C5, } \geq v(\mathbf{x}+\mathbf{e}_N+\mathbf{e}_A) - v(\mathbf{x}+\mathbf{e}_N) \\
=\ & v(\mathbf{x}+\mathbf{e}_j+\mathbf{e}_N) + \omega c_N + (1-\omega)\varepsilon - v(\mathbf{x}+\mathbf{e}_N) - \omega c_N - (1-\omega)\varepsilon \geq T_N v(\mathbf{x}+\mathbf{e}_j) - T_N v(\mathbf{x})
\end{aligned}
$$

(4) If $T_N v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j) = v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j)$ and $T_N v(\mathbf{x}) = v(\mathbf{x})$,

$$
T_N v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j) - T_N v(\mathbf{x}+\mathbf{e}_i) \geq v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j) - v(\mathbf{x}+\mathbf{e}_i) \geq v(\mathbf{x}+\mathbf{e}_j) - v(\mathbf{x}) \geq T_N v(\mathbf{x}+\mathbf{e}_j) - T_N v(\mathbf{x})
$$

Operator T_{dN} (T_{dR} follows the same structure, thus omitted)

(1) If $x_N > 0$, $x_R \geq 0$, $T_{dN} v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j) = v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j-\mathbf{e}_N)$ and $T_{dN} v(\mathbf{x}) = v(\mathbf{x}-\mathbf{e}_N)$,

$$
\begin{aligned}
& T_{dN} v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j) - T_{dN} v(\mathbf{x}+\mathbf{e}_i) \geq v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j-\mathbf{e}_N) - v(\mathbf{x}+\mathbf{e}_i-\mathbf{e}_N) \\
\geq\ & v(\mathbf{x}+\mathbf{e}_j-\mathbf{e}_N) - v(\mathbf{x}-\mathbf{e}_N) \geq T_{dN} v(\mathbf{x}+\mathbf{e}_j) - T_{dN} v(\mathbf{x})
\end{aligned}
$$

(2) If $x_N > 0$, $x_R > 0$, or $x_N \geq 0$, $x_R > 0$, $i \vee j = N$, $T_{dN}v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j) = v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j-\mathbf{e}_N)$ and $T_{dN}v(\mathbf{x}) = v(\mathbf{x}-\mathbf{e}_R)+\omega\eta_{RN}$,

$$
\begin{aligned}
& T_{dN}v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j) - T_{dN}v(\mathbf{x}+\mathbf{e}_i) \\
& \text{if } i = N, \\
\geq\ & v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j-\mathbf{e}_N) - v(\mathbf{x}+\mathbf{e}_i-\mathbf{e}_N) = v(\mathbf{x}+\mathbf{e}_j) - v(\mathbf{x}) \\
\geq\ & v(\mathbf{x}+\mathbf{e}_j-\mathbf{e}_R) - v(\mathbf{x}-\mathbf{e}_R) = v(\mathbf{x}+\mathbf{e}_j-\mathbf{e}_R) + \omega\eta_{RN} - v(\mathbf{x}-\mathbf{e}_R) - \omega\eta_{RN} \\
\geq\ & T_{dN}v(\mathbf{x}+\mathbf{e}_j) - T_{dN}v(\mathbf{x}) \\
& \text{if } j = N, \\
\geq\ & v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j-\mathbf{e}_N) - v(\mathbf{x}+\mathbf{e}_i-\mathbf{e}_R) - \omega\eta_{RN} \\
=\ & v(\mathbf{x}+\mathbf{e}_i) - v(\mathbf{x}+\mathbf{e}_i-\mathbf{e}_R) - \omega\eta_{RN} \geq v(\mathbf{x}) - v(\mathbf{x}-\mathbf{e}_R) - \omega\eta_{RN} \\
=\ & v(\mathbf{x}+\mathbf{e}_j-\mathbf{e}_N) - v(\mathbf{x}-\mathbf{e}_R) - \omega\eta_{RN} \geq T_{dN}v(\mathbf{x}+\mathbf{e}_j) - T_{dN}v(\mathbf{x}) \\
& \text{if } i = A,\ j = R, \\
\geq\ & v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j-\mathbf{e}_N) - v(\mathbf{x}+\mathbf{e}_i-\mathbf{e}_R) - \omega\eta_{RN}, \text{ use C5.} \\
\geq\ & v(\mathbf{x}+\mathbf{e}_j-\mathbf{e}_N) + v(\mathbf{x}+\mathbf{e}_i) - v(\mathbf{x}) - v(\mathbf{x}+\mathbf{e}_i-\mathbf{e}_R) - \omega\eta_{RN} \\
\geq\ & v(\mathbf{x}+\mathbf{e}_j-\mathbf{e}_N) - v(\mathbf{x}-\mathbf{e}_R) - \omega\eta_{RN} \geq T_{dN}v(\mathbf{x}+\mathbf{e}_j) - T_{dN}v(\mathbf{x}) \\
& \text{if } i = R,\ j = A, \\
\geq\ & v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j-\mathbf{e}_N) - v(\mathbf{x}+\mathbf{e}_i-\mathbf{e}_N), \text{ use C5.} \\
\geq\ & v(\mathbf{x}+\mathbf{e}_j) - v(\mathbf{x}) \geq v(\mathbf{x}+\mathbf{e}_j-\mathbf{e}_R) + \omega\eta_{RN} - v(\mathbf{x}-\mathbf{e}_R) - \omega\eta_{RN} \geq T_{dN}v(\mathbf{x}+\mathbf{e}_j) - T_{dN}v(\mathbf{x})
\end{aligned}
$$

(3) If $x_N = 0$, $x_R \geq 0$, $i \vee j = N$, $T_{dN}v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j) = v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j-\mathbf{e}_N)$ and $T_{dN}v(\mathbf{x}) = v(\mathbf{x}) + \omega\eta_L$,

$$
\begin{aligned}
& T_{dN}v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j) - T_{dN}v(\mathbf{x}+\mathbf{e}_i) \\
& \text{if } i = N, \\
\geq\ & v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j-\mathbf{e}_N) - v(\mathbf{x}+\mathbf{e}_i-\mathbf{e}_N) \\
=\ & v(\mathbf{x}+\mathbf{e}_j) - v(\mathbf{x}) = v(\mathbf{x}+\mathbf{e}_j) + \omega\eta_L - v(\mathbf{x}) - \omega\eta_L \geq T_{dN}v(\mathbf{x}+\mathbf{e}_j) - T_{dN}v(\mathbf{x}) \\
& \text{if } j = N, \\
\geq\ & v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j-\mathbf{e}_N) - v(\mathbf{x}+\mathbf{e}_i) - \omega\eta_L = v(\mathbf{x}+\mathbf{e}_i) - v(\mathbf{x}+\mathbf{e}_i) - \omega\eta_L \\
=\ & v(\mathbf{x}+\mathbf{e}_j-\mathbf{e}_N) - v(\mathbf{x}) - \omega\eta_L \geq T_{dN}v(\mathbf{x}+\mathbf{e}_j) - T_{dN}v(\mathbf{x})
\end{aligned}
$$

(4) If $x_N > 0$, $x_R > 0$, or $x_N > 0$, $x_R \geq 0$, $i \vee j = R$, $T_{dN}v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j) = v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j-\mathbf{e}_R)+\omega\eta_{RN}$ and $T_{dN}v(\mathbf{x}) = v(\mathbf{x}-\mathbf{e}_N)$,

$$
\begin{aligned}
& T_{dN}v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j) - T_{dN}v(\mathbf{x}+\mathbf{e}_i) \\
& \text{if } i = R, \\
\geq\ & v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j-\mathbf{e}_R) + \omega\eta_{RN} - v(\mathbf{x}+\mathbf{e}_i-\mathbf{e}_R) - \omega\eta_{RN} \\
=\ & v(\mathbf{x}+\mathbf{e}_j) - v(\mathbf{x}) \geq v(\mathbf{x}+\mathbf{e}_j-\mathbf{e}_N) - v(\mathbf{x}-\mathbf{e}_N) \geq T_{dN}v(\mathbf{x}+\mathbf{e}_j) - T_{dN}v(\mathbf{x}) \\
& \text{if } j = R,\ i = A, \\
\geq\ & v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j-\mathbf{e}_R) + \omega\eta_{RN} - v(\mathbf{x}+\mathbf{e}_i-\mathbf{e}_N) \\
=\ & v(\mathbf{x}+\mathbf{e}_i) + \omega\eta_{RN} - v(\mathbf{x}+\mathbf{e}_i-\mathbf{e}_N) \geq v(\mathbf{x}) + \omega\eta_{RN} - v(\mathbf{x}-\mathbf{e}_N) \\
\geq\ & v(\mathbf{x}+\mathbf{e}_j-\mathbf{e}_R) + \omega\eta_{RN} - v(\mathbf{x}-\mathbf{e}_N) \geq T_{dN}v(\mathbf{x}+\mathbf{e}_j) - T_{dN}v(\mathbf{x}) \\
& \text{if } j = R,\ i = N, \\
\geq\ & v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j-\mathbf{e}_R) + \omega\eta_{RN} - v(\mathbf{x}+\mathbf{e}_i-\mathbf{e}_N) \\
=\ & v(\mathbf{x}+\mathbf{e}_i) + \omega\eta_{RN} - v(\mathbf{x}) \\
\geq\ & v(\mathbf{x}+\mathbf{e}_i-\mathbf{e}_N) + \omega\eta_{RN} - v(\mathbf{x}-\mathbf{e}_N) = v(\mathbf{x}) + \omega\eta_{RN} - v(\mathbf{x}-\mathbf{e}_N) \\
=\ & v(\mathbf{x}+\mathbf{e}_j-\mathbf{e}_R) + \omega\eta_{RN} - v(\mathbf{x}-\mathbf{e}_N) \geq T_{dN}v(\mathbf{x}+\mathbf{e}_j) - T_{dN}v(\mathbf{x}) \\
& \text{if } j = N,\ i = A, \\
\geq\ & v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j-\mathbf{e}_R) + \omega\eta_{RN} - v(\mathbf{x}+\mathbf{e}_i-\mathbf{e}_R) - \omega\eta_{RN} \\
\geq\ & v(\mathbf{x}+\mathbf{e}_j-\mathbf{e}_R) - v(\mathbf{x}-\mathbf{e}_R) \geq v(\mathbf{x}) - v(\mathbf{x}-\mathbf{e}_N) = v(\mathbf{x}+\mathbf{e}_j-\mathbf{e}_N) - v(\mathbf{x}-\mathbf{e}_N) \\
\geq\ & T_{dN}v(\mathbf{x}+\mathbf{e}_j) - T_{dN}v(\mathbf{x}) \\
& \text{if } j = A,\ i = N, \\
\geq\ & v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j-\mathbf{e}_R) + \omega\eta_{RN} - v(\mathbf{x}+\mathbf{e}_i-\mathbf{e}_N) \\
\geq\ & v(\mathbf{x}+\mathbf{e}_j-\mathbf{e}_R) + \omega\eta_{RN} + v(\mathbf{x}+\mathbf{e}_i-\mathbf{e}_R) - v(\mathbf{x}-\mathbf{e}_R) - v(\mathbf{x}) \\
\geq\ & v(\mathbf{x}+\mathbf{e}_j-\mathbf{e}_R) + \omega\eta_{RN} + v(\mathbf{x}-\mathbf{e}_N) \geq T_{dN}v(\mathbf{x}+\mathbf{e}_j) - T_{dN}v(\mathbf{x})
\end{aligned}
$$

(5) If $x_N \geq 0$, $x_R > 0$, $T_{dN}v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j) = v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j-\mathbf{e}_R) + \omega\eta_{RN}$ and $T_{dN}v(\mathbf{x}) = v(\mathbf{x}-\mathbf{e}_R) + \omega\eta_{RN}$,

$$
\begin{aligned}
& T_{dN}v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j) - T_{dN}v(\mathbf{x}+\mathbf{e}_i) \geq v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j-\mathbf{e}_R) + \omega\eta_{RN} - v(\mathbf{x}+\mathbf{e}_i-\mathbf{e}_R) - \omega\eta_{RN} \\
\geq\ & v(\mathbf{x}+\mathbf{e}_j-\mathbf{e}_R) + \omega\eta_{RN} - v(\mathbf{x}-\mathbf{e}_R) - \omega\eta_{RN} \geq T_{dN}v(\mathbf{x}+\mathbf{e}_j) - T_{dN}v(\mathbf{x})
\end{aligned}
$$

(6) If $x_N = 0$, $x_R > 0$, or $x_N = 0$, $x_R \geq 0$, $i \vee j = R$, $T_{dN}v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j) = v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j-\mathbf{e}_R)+\omega\eta_{RN}$ and $T_{dN}v(\mathbf{x}) = v(\mathbf{x})+\omega\eta_L$,

$$
\begin{aligned}
& T_{dN}v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j) - T_{dN}v(\mathbf{x}+\mathbf{e}_i) \\
& \text{if } i = A,\ j = N, \\
\geq\ & v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j-\mathbf{e}_R) + \omega\eta_{RN} - v(\mathbf{x}+\mathbf{e}_i-\mathbf{e}_R) - \omega\eta_{RN}, \text{ use C8.} \\
\geq\ & v(\mathbf{x}+\mathbf{e}_i-\mathbf{e}_R) - v(\mathbf{x}+\mathbf{e}_i-\mathbf{e}_R) - \omega\eta_L \\
=\ & v(\mathbf{x}+\mathbf{e}_j-\mathbf{e}_N) - v(\mathbf{x}) - \omega\eta_L \geq T_{dN}v(\mathbf{x}+\mathbf{e}_j) - T_{dN}v(\mathbf{x}) \\
& \text{if } i = A,\ j = R, \\
\geq\ & v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j-\mathbf{e}_R) + \omega\eta_{RN} - v(\mathbf{x}+\mathbf{e}_i) - \omega\eta_L \\
=\ & v(\mathbf{x}+\mathbf{e}_j-\mathbf{e}_R) + \omega\eta_{RN} - v(\mathbf{x}) - \omega\eta_L \geq T_{dN}v(\mathbf{x}+\mathbf{e}_j) - T_{dN}v(\mathbf{x}) \\
& \text{if } i = N,\ j = A, \\
\geq\ & v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j-\mathbf{e}_R) + \omega\eta_{RN} - v(\mathbf{x}+\mathbf{e}_i-\mathbf{e}_N) \\
=\ & v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j-\mathbf{e}_R) + \omega\eta_{RN} - v(\mathbf{x}), \text{ use C8, } \geq v(\mathbf{x}+\mathbf{e}_j-\mathbf{e}_R) - \omega\eta_L + \omega\eta_{RN} - v(\mathbf{x}) \\
=\ & v(\mathbf{x}+\mathbf{e}_j-\mathbf{e}_R) + \omega\eta_{RN} - v(\mathbf{x}) - \omega\eta_L \geq T_{dN}v(\mathbf{x}+\mathbf{e}_i) - T_{dN}v(\mathbf{x}) \\
& \text{if } i = N,\ j = R, \\
\geq\ & v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j-\mathbf{e}_R) + \omega\eta_{RN} - v(\mathbf{x}+\mathbf{e}_i-\mathbf{e}_N) \\
=\ & v(\mathbf{x}+\mathbf{e}_i) + \omega\eta_{RN} - v(\mathbf{x}), \text{ use C8, } \geq v(\mathbf{x}) - \omega\eta_L + \omega\eta_{RN} - v(\mathbf{x}) \\
=\ & v(\mathbf{x}+\mathbf{e}_j-\mathbf{e}_R) + \omega\eta_{RN} - v(\mathbf{x}) - \omega\eta_L \geq T_{dN}v(\mathbf{x}+\mathbf{e}_i) - T_{dN}v(\mathbf{x}) \\
& \text{if } i = R,\ j = N, \\
\geq\ & v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j-\mathbf{e}_R) + \omega\eta_{RN} - v(\mathbf{x}+\mathbf{e}_i-\mathbf{e}_R) - \omega\eta_{RN} \\
=\ & v(\mathbf{x}+\mathbf{e}_j) - v(\mathbf{x}), \text{ use C8, } \geq v(\mathbf{x}) - v(\mathbf{x}) - \omega\eta_L \\
=\ & v(\mathbf{x}+\mathbf{e}_j-\mathbf{e}_N) - v(\mathbf{x}) - \omega\eta_L \geq T_{dN}v(\mathbf{x}+\mathbf{e}_j) - T_{dN}v(\mathbf{x}) \\
& \text{if } i = R,\ j = A, \\
\geq\ & v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j-\mathbf{e}_R) + \omega\eta_{RN} - v(\mathbf{x}+\mathbf{e}_i-\mathbf{e}_R) - \omega\eta_{RN} \\
=\ & v(\mathbf{x}+\mathbf{e}_j) + \omega\eta_L - v(\mathbf{x}) - \omega\eta_L \geq T_{dN}v(\mathbf{x}+\mathbf{e}_j) - T_{dN}v(\mathbf{x})
\end{aligned}
$$

(7) If $x_N = 0$, $x_R \geq 0$, $i, j \neq N$, $T_{dN}v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j) = v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j) + \omega\eta_L$ and $T_{dN}v(\mathbf{x}) = v(\mathbf{x}) + \omega\eta_L$,

$$\begin{aligned} & T_{dN}v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j) - T_{dN}v(\mathbf{x}+\mathbf{e}_i) \geq v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j) + \omega\eta_L - v(\mathbf{x}+\mathbf{e}_i) - \omega\eta_L \\ \geq\ & v(\mathbf{x}+\mathbf{e}_j) + \omega\eta_L - v(\mathbf{x}) - \omega\eta_L \geq T_{dN}v(\mathbf{x}+\mathbf{e}_j) - T_{dN}v(\mathbf{x}) \end{aligned}$$

(8) If $x_N = 0$, $x_R > 0$, $i, j \neq N$, $T_{dN}v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j) = v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j) + \omega\eta_L$ and $T_{dN}v(\mathbf{x}) = v(\mathbf{x}-\mathbf{e}_R) + \omega\eta_{RN}$,

$$\begin{aligned} & T_{dN}v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j) - T_{dN}v(\mathbf{x}+\mathbf{e}_i) \geq v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j) + \omega\eta_L - v(\mathbf{x}+\mathbf{e}_i) - \omega\eta_L \\ \geq\ & v(\mathbf{x}+\mathbf{e}_j) - v(\mathbf{x}) \geq v(\mathbf{x}+\mathbf{e}_j-\mathbf{e}_R) + \omega\eta_{RN} - v(\mathbf{x}-\mathbf{e}_R) - \omega\eta_{RN} \geq T_{dN}v(\mathbf{x}+\mathbf{e}_j) - T_{dN}v(\mathbf{x}) \end{aligned}$$

Proof. Proof of C3

In this section, we prove $T_s v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i) - T_s v(\mathbf{x}+\mathbf{e}_i) \geq T_s v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j) - T_s v(\mathbf{x}+\mathbf{e}_j)$, $s = A, R, N, dN, dR$ and $i, j = A, R, N$, $i \neq j$.

Operator T_A

(1) If $T_A v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i) = v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i+\mathbf{e}_A) + \omega c_{RM} + (1-\omega)\theta\varepsilon$ and $T_A v(\mathbf{x}+\mathbf{e}_j) = v(\mathbf{x}+\mathbf{e}_j+\mathbf{e}_A) + \omega c_{RM} + (1-\omega)\theta\varepsilon$,

$$\begin{aligned} & T_A v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i) - T_A v(\mathbf{x}+\mathbf{e}_i) \\ \geq\ & v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i+\mathbf{e}_A) + \omega c_{RM} + (1-\omega)\theta\varepsilon - v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_A) - \omega c_{RM} - (1-\omega)\theta\varepsilon \\ \geq\ & v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j+\mathbf{e}_A) + \omega c_{RM} + (1-\omega)\theta\varepsilon - v(\mathbf{x}+\mathbf{e}_j+\mathbf{e}_A) - \omega c_{RM} - (1-\omega)\theta\varepsilon \\ \geq\ & T_A v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j) - T_A v(\mathbf{x}+\mathbf{e}_j) \end{aligned}$$

(2) If $T_A v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i) = v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i+\mathbf{e}_A) + \omega c_{RM} + (1-\omega)\theta\varepsilon$ and $T_A v(\mathbf{x}+\mathbf{e}_j) = v(\mathbf{x}+\mathbf{e}_j)$,

$$\begin{aligned} & T_A v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i) - T_A v(\mathbf{x}+\mathbf{e}_i) \\ \geq\ & v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i+\mathbf{e}_A) + \omega c_{RM} + (1-\omega)\theta\varepsilon - v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_A) - \omega c_{RM} - (1-\omega)\theta\varepsilon \\ \geq\ & v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j+\mathbf{e}_A) - v(\mathbf{x}+\mathbf{e}_j+\mathbf{e}_A) \geq v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j) - v(\mathbf{x}+\mathbf{e}_j) \geq T_A v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j) - T_A v(\mathbf{x}+\mathbf{e}_j) \end{aligned}$$

(3) If $T_A v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i) = v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i)$ and $T_A v(\mathbf{x}+\mathbf{e}_j) = v(\mathbf{x}+\mathbf{e}_j+\mathbf{e}_A) + \omega c_{RM} + (1-\omega)\theta\varepsilon$,

$$\begin{aligned}
& T_A v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i) - T_A v(\mathbf{x}+\mathbf{e}_i) \\
& \text{if } i = A, \\
\geq\ & v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i) - v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_A) - \omega c_{RM} - (1-\omega)\theta\varepsilon \\
=\ & v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j) - v(\mathbf{x}+\mathbf{e}_j+\mathbf{e}_A) - \omega c_{RM} - (1-\omega)\theta\varepsilon \geq T_A v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j) - T_A v(\mathbf{x}+\mathbf{e}_j) \\
& \text{if } j = A, \\
\geq\ & v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i) - v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_A) - \omega c_{RM} - (1-\omega)\theta\varepsilon \\
\geq\ & v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_A) + v(\mathbf{x}+\mathbf{e}_i) - v(\mathbf{x}+\mathbf{e}_A) - v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_A) - \omega c_{RM} - (1-\omega)\theta\varepsilon \\
\geq\ & v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j) - v(\mathbf{x}+\mathbf{e}_j+\mathbf{e}_A) - \omega c_{RM} - (1-\omega)\theta\varepsilon \geq T_A v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j) - T_A v(\mathbf{x}+\mathbf{e}_j) \\
& \text{if } i, j \neq A, \\
\geq\ & v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i) - v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_A) - \omega c_{RM} - (1-\omega)\theta\varepsilon, \text{ use C4.} \\
\geq\ & v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j) - v(\mathbf{x}+\mathbf{e}_j+\mathbf{e}_A) - \omega c_{RM} - (1-\omega)\theta\varepsilon \geq T_A v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j) - T_A v(\mathbf{x}+\mathbf{e}_j)
\end{aligned}$$

(4) If $T_A v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i) = v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i)$ and $T_A v(\mathbf{x}+\mathbf{e}_j) = v(\mathbf{x}+\mathbf{e}_j)$,

$$\begin{aligned}
& T_A v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i) - T_A v(\mathbf{x}+\mathbf{e}_i) \geq v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i) - v(\mathbf{x}+\mathbf{e}_i) \\
\geq\ & v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j) - v(\mathbf{x}+\mathbf{e}_j) \geq T_A v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j) - T_A v(\mathbf{x}+\mathbf{e}_j)
\end{aligned}$$

Operator T_R

(1) If $x_A > 0$, $T_R v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i) = v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i-\mathbf{e}_A+\mathbf{e}_R)$ and $T_R v(\mathbf{x}+\mathbf{e}_j) = v(\mathbf{x}+\mathbf{e}_j-\mathbf{e}_A+\mathbf{e}_R)$,

$$\begin{aligned}
& T_R v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i) - T_R v(\mathbf{x}+\mathbf{e}_i) \geq v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i-\mathbf{e}_A+\mathbf{e}_R) - v(\mathbf{x}+\mathbf{e}_i-\mathbf{e}_A+\mathbf{e}_R) \\
\geq\ & v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j-\mathbf{e}_A+\mathbf{e}_R) - v(\mathbf{x}+\mathbf{e}_j-\mathbf{e}_A+\mathbf{e}_R) \geq T_R v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j) - T_R v(\mathbf{x}+\mathbf{e}_j)
\end{aligned}$$

(2) If $x_A = 0$, $i = A$, $j = R$, $T_R v(\mathbf{x} + \mathbf{e}_i + \mathbf{e}_i) = v(\mathbf{x} + \mathbf{e}_i + \mathbf{e}_i - \mathbf{e}_A + \mathbf{e}_R)$ and $T_R v(\mathbf{x} + \mathbf{e}_j) = v(\mathbf{x} + \mathbf{e}_j)$,

$$\begin{aligned}
& T_R v(\mathbf{x} + \mathbf{e}_i + \mathbf{e}_i) - T_R v(\mathbf{x} + \mathbf{e}_i) = v(\mathbf{x} + \mathbf{e}_i + \mathbf{e}_i - \mathbf{e}_A + \mathbf{e}_R) - v(\mathbf{x} + \mathbf{e}_i - \mathbf{e}_A + \mathbf{e}_R) \\
= \; & v(\mathbf{x} + \mathbf{e}_i + \mathbf{e}_R) - v(\mathbf{x} + \mathbf{e}_R), \text{ use C7. } \geq v(\mathbf{x} + \mathbf{e}_R + \mathbf{e}_j) - v(\mathbf{x} + \mathbf{e}_j) \\
= \; & v(\mathbf{x} + \mathbf{e}_i + \mathbf{e}_j - \mathbf{e}_A + \mathbf{e}_R) - v(\mathbf{x} + \mathbf{e}_j) = T_R v(\mathbf{x} + \mathbf{e}_i + \mathbf{e}_j) - T_R v(\mathbf{x} + \mathbf{e}_j)
\end{aligned}$$

(3) If $x_A = 0$, $i = A$, $j = N$, $T_R v(\mathbf{x} + \mathbf{e}_i + \mathbf{e}_i) = v(\mathbf{x} + \mathbf{e}_i + \mathbf{e}_i - \mathbf{e}_A + \mathbf{e}_R)$ and $T_R v(\mathbf{x} + \mathbf{e}_j) = v(\mathbf{x} + \mathbf{e}_j)$,

$$\begin{aligned}
& T_R v(\mathbf{x} + \mathbf{e}_i + \mathbf{e}_i) - T_R v(\mathbf{x} + \mathbf{e}_i) = v(\mathbf{x} + \mathbf{e}_i + \mathbf{e}_i - \mathbf{e}_A + \mathbf{e}_R) - v(\mathbf{x} + \mathbf{e}_i - \mathbf{e}_A + \mathbf{e}_R) \\
= \; & v(\mathbf{x} + \mathbf{e}_i + \mathbf{e}_R) - v(\mathbf{x} + \mathbf{e}_R), \text{ use C5.} \\
\geq \; & v(\mathbf{x} + \mathbf{e}_i + \mathbf{e}_N) - v(\mathbf{x} + \mathbf{e}_N) \geq v(\mathbf{x} + \mathbf{e}_R + \mathbf{e}_j) - v(\mathbf{x} + \mathbf{e}_j) \\
= \; & v(\mathbf{x} + \mathbf{e}_i + \mathbf{e}_j - \mathbf{e}_A + \mathbf{e}_R) - v(\mathbf{x} + \mathbf{e}_j) = T_R v(\mathbf{x} + \mathbf{e}_i + \mathbf{e}_j) - T_R v(\mathbf{x} + \mathbf{e}_j)
\end{aligned}$$

(4) If $x_A = 0$, $j = A$, $T_R v(\mathbf{x} + \mathbf{e}_i + \mathbf{e}_i) = v(\mathbf{x} + \mathbf{e}_i + \mathbf{e}_i)$ and $T_R v(\mathbf{x} + \mathbf{e}_j) = v(\mathbf{x} + \mathbf{e}_j - \mathbf{e}_A + \mathbf{e}_R)$,

$$\begin{aligned}
& T_R v(\mathbf{x} + \mathbf{e}_i + \mathbf{e}_i) - T_R v(\mathbf{x} + \mathbf{e}_i) \\
= \; & v(\mathbf{x} + \mathbf{e}_i + \mathbf{e}_i) - v(\mathbf{x} + \mathbf{e}_i) = v(\mathbf{x} + \mathbf{e}_i + \mathbf{e}_i + \mathbf{e}_j - \mathbf{e}_A) - v(\mathbf{x} + \mathbf{e}_i + \mathbf{e}_j - \mathbf{e}_A) \\
\geq \; & v(\mathbf{x} + \mathbf{e}_i + \mathbf{e}_j - \mathbf{e}_A + \mathbf{e}_R) - v(\mathbf{x} + \mathbf{e}_j - \mathbf{e}_A + \mathbf{e}_R) = T_R v(\mathbf{x} + \mathbf{e}_i + \mathbf{e}_j) - T_R v(\mathbf{x} + \mathbf{e}_j)
\end{aligned}$$

(5) If $x_A = 0$, $i, j \neq A$, $T_R v(\mathbf{x} + \mathbf{e}_i + \mathbf{e}_i) = v(\mathbf{x} + \mathbf{e}_i + \mathbf{e}_i)$ and $T_R v(\mathbf{x} + \mathbf{e}_j) = v(\mathbf{x} + \mathbf{e}_j)$,

$$\begin{aligned}
& T_R v(\mathbf{x} + \mathbf{e}_i + \mathbf{e}_i) - T_R v(\mathbf{x} + \mathbf{e}_i) \\
= \; & v(\mathbf{x} + \mathbf{e}_i + \mathbf{e}_i) - v(\mathbf{x} + \mathbf{e}_i) \geq v(\mathbf{x} + \mathbf{e}_i + \mathbf{e}_j) - v(\mathbf{x} + \mathbf{e}_j) = T_R v(\mathbf{x} + \mathbf{e}_i + \mathbf{e}_j) - T_R v(\mathbf{x} + \mathbf{e}_j)
\end{aligned}$$

Operator T_N

(1) If $T_N v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i) = v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i+\mathbf{e}_N) + \omega c_N + (1-\omega)\varepsilon$ and $T_N v(\mathbf{x}+\mathbf{e}_j) = v(\mathbf{x}+\mathbf{e}_j+\mathbf{e}_N) + \omega c_N + (1-\omega)\varepsilon$,

$$\begin{aligned}
& T_N v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i) - T_N v(\mathbf{x}+\mathbf{e}_i) \\
\geq\ & v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i+\mathbf{e}_N) + \omega c_N + (1-\omega)\varepsilon - v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_N) - \omega c_N - (1-\omega)\varepsilon \\
\geq\ & v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j+\mathbf{e}_N) + \omega c_N + (1-\omega)\varepsilon - v(\mathbf{x}+\mathbf{e}_j+\mathbf{e}_N) - \omega c_N - (1-\omega)\varepsilon \\
\geq\ & T_N v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j) - T_N v(\mathbf{x}+\mathbf{e}_j)
\end{aligned}$$

(2) If $T_N v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i) = v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i+\mathbf{e}_N) + \omega c_N + (1-\omega)\varepsilon$ and $T_N v(\mathbf{x}+\mathbf{e}_j) = v(\mathbf{x}+\mathbf{e}_j)$,

$$\begin{aligned}
& T_N v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i) - T_N v(\mathbf{x}+\mathbf{e}_i) \\
\geq\ & v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i+\mathbf{e}_N) + \omega c_N + (1-\omega)\varepsilon - v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_N) - \omega c_N - (1-\omega)\varepsilon \\
\geq\ & v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j+\mathbf{e}_N) - v(\mathbf{x}+\mathbf{e}_j+\mathbf{e}_N) \\
\geq\ & v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j) - v(\mathbf{x}+\mathbf{e}_j) \geq T_N v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j) - T_N v(\mathbf{x}+\mathbf{e}_j)
\end{aligned}$$

(3) If $T_N v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i) = v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i)$ and $T_N v(\mathbf{x}+\mathbf{e}_j) = v(\mathbf{x}+\mathbf{e}_j+\mathbf{e}_N) + \omega c_N + (1-\omega)\varepsilon$,

$$\begin{aligned}
& T_N v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i) - T_N v(\mathbf{x}+\mathbf{e}_i) \\
& \text{if } i = N, \\
\geq\ & v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i) - v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_N) - \omega c_N - (1-\omega)\varepsilon \\
=\ & v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j) - v(\mathbf{x}+\mathbf{e}_j+\mathbf{e}_N) - \omega c_N - (1-\omega)\varepsilon \geq T_N v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j) - T_N v(\mathbf{x}+\mathbf{e}_j) \\
& \text{if } i = R,\ j = A, \\
\geq\ & v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i) - v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_N) - \omega c_N - (1-\omega)\varepsilon, \text{ use C4.} \\
\geq\ & v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_A) - v(\mathbf{x}+\mathbf{e}_A+\mathbf{e}_N) - \omega c_N - (1-\omega)\varepsilon \\
=\ & v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j) - v(\mathbf{x}+\mathbf{e}_j+\mathbf{e}_N) - \omega c_N - (1-\omega)\varepsilon \geq T_N v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j) - T_N v(\mathbf{x}+\mathbf{e}_j) \\
& \text{if } i = R,\ j = N, \text{ or if } i = A, \\
\geq\ & v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i) - v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_N) - \omega c_N - (1-\omega)\varepsilon \\
\geq\ & v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j) + v(\mathbf{x}+\mathbf{e}_i) - v(\mathbf{x}+\mathbf{e}_j) - v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_N) - \omega c_N - (1-\omega)\varepsilon \text{ (if } i = A,\ j = R, \text{ use C6.)} \\
\geq\ & v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j) - v(\mathbf{x}+\mathbf{e}_j+\mathbf{e}_N) - \omega c_N - (1-\omega)\varepsilon \geq T_N v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j) - T_N v(\mathbf{x}+\mathbf{e}_j)
\end{aligned}$$

(4) If $T_N v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i) = v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i)$ and $T_N v(\mathbf{x}+\mathbf{e}_j) = v(\mathbf{x}+\mathbf{e}_j)$,

$$\begin{aligned} & T_N v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i) - T_N v(\mathbf{x}+\mathbf{e}_i) \geq v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i) - v(\mathbf{x}+\mathbf{e}_i) \\ \geq \; & v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j) - v(\mathbf{x}+\mathbf{e}_j) \geq T_N v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j) - T_N v(\mathbf{x}+\mathbf{e}_j) \end{aligned}$$

Operator T_{dN} (T_{dR} follows the same structure, thus omitted)

(1) If $x_N > 0$, $x_R \geq 0$, $T_{dN} v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i) = v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i-\mathbf{e}_N)$ and $T_{dN} v(\mathbf{x}+\mathbf{e}_j) = v(\mathbf{x}+\mathbf{e}_j-\mathbf{e}_N)$,

$$\begin{aligned} & T_{dN} v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i) - T_{dN} v(\mathbf{x}+\mathbf{e}_i) \geq v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i-\mathbf{e}_N) - v(\mathbf{x}+\mathbf{e}_i-\mathbf{e}_N) \\ \geq \; & v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j-\mathbf{e}_N) - v(\mathbf{x}+\mathbf{e}_j-\mathbf{e}_N) \geq T_{dN} v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j) - T_{dN} v(\mathbf{x}+\mathbf{e}_j) \end{aligned}$$

(2) If $x_N > 0$, $x_R > 0$ or $x_N \geq 0$, $x_R > 0$, $i = N$, or $x_N > 0$, $x_R \geq 0$, $j = R$, or $x_N \geq 0$, $x_R \geq 0$, $i = N$, $j = R$,

$T_{dN}v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i) = v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i-\mathbf{e}_N)$ and $T_{dN}v(\mathbf{x}+\mathbf{e}_j) = v(\mathbf{x}+\mathbf{e}_j-\mathbf{e}_R)+\omega\eta_{RN}$,

$$
\begin{aligned}
& T_{dN}v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i) - T_{dN}v(\mathbf{x}+\mathbf{e}_i) \\
& \text{if } i=R, \\
\geq\ & v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i-\mathbf{e}_N) - v(\mathbf{x}+\mathbf{e}_i-\mathbf{e}_N) \geq v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j-\mathbf{e}_N) - v(\mathbf{x}+\mathbf{e}_j-\mathbf{e}_N) \\
=\ & v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j-\mathbf{e}_R+\mathbf{e}_R-\mathbf{e}_N) - v(\mathbf{x}+\mathbf{e}_j-\mathbf{e}_R+\mathbf{e}_R-\mathbf{e}_N) \geq v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j-\mathbf{e}_R) - v(\mathbf{x}+\mathbf{e}_j-\mathbf{e}_R) \\
=\ & v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j-\mathbf{e}_R) + \omega\eta_{RN} - v(\mathbf{x}+\mathbf{e}_j-\mathbf{e}_R) - \omega\eta_{RN} \geq T_{dN}v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j) - T_{dN}v(\mathbf{x}+\mathbf{e}_j) \\
& \text{if } i=N,\ j=R, \\
\geq\ & v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i-\mathbf{e}_N) - v(\mathbf{x}+\mathbf{e}_i-\mathbf{e}_N) = v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j-\mathbf{e}_R) - v(\mathbf{x}+\mathbf{e}_j-\mathbf{e}_R) \\
=\ & v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j-\mathbf{e}_R) + \omega\eta_{RN} - v(\mathbf{x}+\mathbf{e}_j-\mathbf{e}_R) - \omega\eta_{RN} \geq T_{dN}v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j) - T_{dN}v(\mathbf{x}+\mathbf{e}_j) \\
& \text{if } i=A,\ j=N, \\
\geq\ & v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i-\mathbf{e}_N) - v(\mathbf{x}+\mathbf{e}_i-\mathbf{e}_N) \geq v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j-\mathbf{e}_N) - v(\mathbf{x}+\mathbf{e}_j-\mathbf{e}_N) \\
=\ & v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_R-\mathbf{e}_R) - v(\mathbf{x}+\mathbf{e}_R-\mathbf{e}_R), \text{ use C5.} \\
\geq\ & v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j-\mathbf{e}_R) + \omega\eta_{RN} - v(\mathbf{x}+\mathbf{e}_j-\mathbf{e}_R) - \omega\eta_{RN} \geq T_{dN}v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j) - T_{dN}v(\mathbf{x}+\mathbf{e}_j) \\
& \text{if } i=A,\ j=R, \\
\geq\ & v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i-\mathbf{e}_N) - v(\mathbf{x}+\mathbf{e}_i-\mathbf{e}_N) \geq v(\mathbf{x}+\mathbf{e}_i) - v(\mathbf{x}) \\
=\ & v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j-\mathbf{e}_R) + \omega\eta_{RN} - v(\mathbf{x}+\mathbf{e}_j-\mathbf{e}_R) - \omega\eta_{RN} \geq T_{dN}v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j) - T_{dN}v(\mathbf{x}+\mathbf{e}_j) \\
& \text{if } i=N,\ j=A, \\
\geq\ & v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i-\mathbf{e}_N) - v(\mathbf{x}+\mathbf{e}_i-\mathbf{e}_N) = v(\mathbf{x}+\mathbf{e}_i) - v(\mathbf{x}) \\
=\ & v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_R-\mathbf{e}_R) - v(\mathbf{x}+\mathbf{e}_R-\mathbf{e}_R), \text{ use C6, } \geq v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_A-\mathbf{e}_R) - v(\mathbf{x}+\mathbf{e}_A-\mathbf{e}_R) \\
=\ & v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j-\mathbf{e}_R) + \omega\eta_{RN} - v(\mathbf{x}+\mathbf{e}_j-\mathbf{e}_R) - \omega\eta_{RN} \geq T_{dN}v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j) - T_{dN}v(\mathbf{x}+\mathbf{e}_j)
\end{aligned}
$$

(3) If $x_N = 0$, $x_R \geq 0$, $i = N$, $T_{dN}v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i) = v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i-\mathbf{e}_N)$ and $T_{dN}v(\mathbf{x}+\mathbf{e}_j) = v(\mathbf{x}+\mathbf{e}_j)+\omega\eta_L$,

$$
\begin{aligned}
& T_{dN}v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i) - T_{dN}v(\mathbf{x}+\mathbf{e}_i) \geq v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i-\mathbf{e}_N) - v(\mathbf{x}+\mathbf{e}_i-\mathbf{e}_N) \\
=\ & v(\mathbf{x}+\mathbf{e}_i) - v(\mathbf{x}), \text{ use C8, } \geq v(\mathbf{x}) - \omega\eta_L - v(\mathbf{x}) \\
=\ & v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j-\mathbf{e}_N) - v(\mathbf{x}+\mathbf{e}_j) - \omega\eta_L \geq T_{dN}v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j) - T_{dN}v(\mathbf{x}+\mathbf{e}_j)
\end{aligned}
$$

(4) If $x_N > 0$, $x_R > 0$ or $x_N > 0$, $x_R \geq 0$, $i = R$, or $x_N \geq 0$, $x_R > 0$, $i = N$, or $x_N \geq 0$, $x_R \geq 0$, $i = R$, $j = N$,

$T_{dN}v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i) = v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i-\mathbf{e}_R)+\omega\eta_{RN}$ and $T_{dN}v(\mathbf{x}+\mathbf{e}_j) = v(\mathbf{x}+\mathbf{e}_j-\mathbf{e}_N)$,

$$
\begin{aligned}
& T_{dN}v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i) - T_{dN}v(\mathbf{x}+\mathbf{e}_i) \\
& \text{if } i=R,\ j=N, \\
\geq\ & v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i-\mathbf{e}_R)+\omega\eta_{RN} - v(\mathbf{x}+\mathbf{e}_i-\mathbf{e}_R)-\omega\eta_{RN} = v(\mathbf{x}+\mathbf{e}_i)-v(\mathbf{x}) \\
=\ & v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j-\mathbf{e}_N)-v(\mathbf{x}+\mathbf{e}_j-\mathbf{e}_N) \geq T_{dN}v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j)-T_{dN}v(\mathbf{x}+\mathbf{e}_j) \\
& \text{if } i=R,\ j=A, \\
\geq\ & v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i-\mathbf{e}_R)+\omega\eta_{RN} - v(\mathbf{x}+\mathbf{e}_i-\mathbf{e}_N) = v(\mathbf{x}+\mathbf{e}_i)+\omega\eta_{RN}-v(\mathbf{x}+\mathbf{e}_i-\mathbf{e}_N), \text{ use C6.} \\
\geq\ & v(\mathbf{x}+\mathbf{e}_A)+\omega\eta_{RN}-v(\mathbf{x}+\mathbf{e}_A-\mathbf{e}_N) = v(\mathbf{x}+\mathbf{e}_A+\mathbf{e}_i-\mathbf{e}_R)+\omega\eta_{RN}-v(\mathbf{x}+\mathbf{e}_A-\mathbf{e}_N) \\
=\ & v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j-\mathbf{e}_R)+\omega\eta_{RN}-v(\mathbf{x}+\mathbf{e}_j-\mathbf{e}_N) \geq T_{dN}v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j)-T_{dN}v(\mathbf{x}+\mathbf{e}_j) \\
& \text{if } i=N, \\
\geq\ & v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i-\mathbf{e}_R)+\omega\eta_{RN}-v(\mathbf{x}+\mathbf{e}_i-\mathbf{e}_R)-\omega\eta_{RN} \\
\geq\ & v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j-\mathbf{e}_R)-v(\mathbf{x}+\mathbf{e}_j-\mathbf{e}_R) \\
=\ & v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j-\mathbf{e}_R+\mathbf{e}_N-\mathbf{e}_N)-v(\mathbf{x}+\mathbf{e}_j-\mathbf{e}_R+\mathbf{e}_N-\mathbf{e}_N) \\
\geq\ & v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j-\mathbf{e}_N)-v(\mathbf{x}+\mathbf{e}_j-\mathbf{e}_N) \geq T_{dN}v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j)-T_{dN}v(\mathbf{x}+\mathbf{e}_j) \\
& \text{if } i=A,\ j=N, \\
\geq\ & v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i-\mathbf{e}_R)+\omega\eta_{RN}-v(\mathbf{x}+\mathbf{e}_i-\mathbf{e}_R)-\omega\eta_{RN} \geq v(\mathbf{x}+\mathbf{e}_i)-v(\mathbf{x}) \\
=\ & v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j-\mathbf{e}_N)-v(\mathbf{x}+\mathbf{e}_j-\mathbf{e}_N) \geq T_{dN}v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j)-T_{dN}v(\mathbf{x}+\mathbf{e}_j) \\
& \text{if } i=A,\ j=R, \\
\geq\ & v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i-\mathbf{e}_R)+\omega\eta_{RN}-v(\mathbf{x}+\mathbf{e}_i-\mathbf{e}_N) \\
\geq\ & v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j-\mathbf{e}_R)+v(\mathbf{x}+\mathbf{e}_i-\mathbf{e}_R)-v(\mathbf{x}+\mathbf{e}_j-\mathbf{e}_R)+\omega\eta_{RN}-v(\mathbf{x}+\mathbf{e}_i-\mathbf{e}_N), \text{ use C4.} \\
\geq\ & v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j-\mathbf{e}_R)+\omega\eta_{RN}-v(\mathbf{x}+\mathbf{e}_j-\mathbf{e}_N) \geq T_{dN}v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j)-T_{dN}v(\mathbf{x}+\mathbf{e}_j)
\end{aligned}
$$

(5) If $x_N \geq 0$, $x_R > 0$, $T_{dN}v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i) = v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i-\mathbf{e}_R)+\omega\eta_{RN}$ and $T_{dN}v(\mathbf{x}+\mathbf{e}_j) = v(\mathbf{x}+\mathbf{e}_j-\mathbf{e}_R)+\omega\eta_{RN}$,

$$
\begin{aligned}
& T_{dN}v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i) - T_{dN}v(\mathbf{x}+\mathbf{e}_i) \\
\geq\ & v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i-\mathbf{e}_R)+\omega\eta_{RN}-v(\mathbf{x}+\mathbf{e}_i-\mathbf{e}_R)-\omega\eta_{RN} \\
\geq\ & v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j-\mathbf{e}_R)+\omega\eta_{RN}-v(\mathbf{x}+\mathbf{e}_j-\mathbf{e}_R)-\omega\eta_{RN} \geq T_{dN}v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j)-T_{dN}v(\mathbf{x}+\mathbf{e}_j)
\end{aligned}
$$

(6) If $x_N = 0$, $x_R > 0$, $j \neq N$, or $x_N = 0$, $x_R \geq 0$, $i = R$, $j \neq N$ $T_{dN}v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i) = v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i-\mathbf{e}_R)+\omega\eta_{RN}$ and $T_{dN}v(\mathbf{x}+\mathbf{e}_j) = v(\mathbf{x}+\mathbf{e}_j)+\omega\eta_L$,

$$
\begin{aligned}
& T_{dN}v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i) - T_{dN}v(\mathbf{x}+\mathbf{e}_i) \\
& \text{if } i = N,\ j = A, \\
\geq\ & v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i-\mathbf{e}_R)+\omega\eta_{RN} - v(\mathbf{x}+\mathbf{e}_i-\mathbf{e}_R) - \omega\eta_{RN} \\
\geq\ & v(\mathbf{x}+\mathbf{e}_i) - v(\mathbf{x}), \text{ use C8, } \geq v(\mathbf{x}+\mathbf{e}_i) - v(\mathbf{x}+\mathbf{e}_i) - \omega\eta_L \\
\geq\ & v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j-\mathbf{e}_N) - v(\mathbf{x}+\mathbf{e}_j) - \omega\eta_L \geq T_{dN}v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j) - T_{dN}v(\mathbf{x}+\mathbf{e}_j) \\
& \text{if } i = R,\ j = A, \\
\geq\ & v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i-\mathbf{e}_R)+\omega\eta_{RN} - v(\mathbf{x}+\mathbf{e}_i) - \omega\eta_L \\
=\ & v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j-\mathbf{e}_R)+\omega\eta_{RN} - v(\mathbf{x}+\mathbf{e}_j) - \omega\eta_L \geq T_{dN}v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j) - T_{dN}v(\mathbf{x}+\mathbf{e}_j) \\
& \text{if } i = N,\ j = R, \\
\geq\ & v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i-\mathbf{e}_R)+\omega\eta_{RN} - v(\mathbf{x}+\mathbf{e}_i-\mathbf{e}_R) - \omega\eta_{RN} \\
\geq\ & v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j-\mathbf{e}_R) - v(\mathbf{x}+\mathbf{e}_j-\mathbf{e}_R) = v(\mathbf{x}+\mathbf{e}_i) - v(\mathbf{x}), \text{ use C8.} \\
\geq\ & v(\mathbf{x}) - v(\mathbf{x}) - \omega\eta_L = v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j-\mathbf{e}_N) - v(\mathbf{x}+\mathbf{e}_j) - \omega\eta_L \\
\geq\ & T_{dN}v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j) - T_{dN}v(\mathbf{x}+\mathbf{e}_j) \\
& \text{if } i = A,\ j = R, \\
\geq\ & v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i-\mathbf{e}_R)+\omega\eta_{RN} - v(\mathbf{x}+\mathbf{e}_i) - \omega\eta_L \\
\geq\ & v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j-\mathbf{e}_R)+\omega\eta_{RN} + v(\mathbf{x}+\mathbf{e}_i-\mathbf{e}_R) - v(\mathbf{x}+\mathbf{e}_j-\mathbf{e}_R) - v(\mathbf{x}+\mathbf{e}_i) - \omega\eta_L \\
\geq\ & v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j-\mathbf{e}_R)+\omega\eta_{RN} - v(\mathbf{x}+\mathbf{e}_j) - \omega\eta_L \geq T_{dN}v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j) - T_{dN}v(\mathbf{x}+\mathbf{e}_j)
\end{aligned}
$$

(7) If $x_N = 0$, $x_R \geq 0$, $i \neq N$, $j = N$, $T_{dN}v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i) = v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i)+\omega\eta_L$ and $T_{dN}v(\mathbf{x}+\mathbf{e}_j) = v(\mathbf{x}+\mathbf{e}_j-\mathbf{e}_N)$,

$$
\begin{aligned}
& T_{dN}v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i) - T_{dN}v(\mathbf{x}+\mathbf{e}_i) \geq v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i)+\omega\eta_L - v(\mathbf{x}+\mathbf{e}_i) - \omega\eta_L \\
\geq\ & v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j) - v(\mathbf{x}+\mathbf{e}_j) \geq v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j-\mathbf{e}_N) - v(\mathbf{x}+\mathbf{e}_j-\mathbf{e}_N) \geq T_{dN}v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j) - T_{dN}v(\mathbf{x}+\mathbf{e}_j)
\end{aligned}
$$

(8) If $x_N = 0$, $x_R > 0$, $i \neq N$, or $x_N = 0$, $x_R \geq 0$, $i \neq N$, $j = R$, $T_{dN}v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i) = v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i)+\omega\eta_L$ and

$T_{dN}v(\mathbf{x}+\mathbf{e}_j) = v(\mathbf{x}+\mathbf{e}_j-\mathbf{e}_R)+\omega\eta_{RN}$,

$$\begin{aligned}
& T_{dN}v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i) - T_{dN}v(\mathbf{x}+\mathbf{e}_i) \geq v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i)+\omega\eta_L - v(\mathbf{x}+\mathbf{e}_i)-\omega\eta_L \\
\geq\ & v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j) - v(\mathbf{x}+\mathbf{e}_j) \geq v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j-\mathbf{e}_R)+\omega\eta_{RN} - v(\mathbf{x}+\mathbf{e}_j-\mathbf{e}_R)-\omega\eta_{RN} \\
\geq\ & T_{dN}v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j) - T_{dN}v(\mathbf{x}+\mathbf{e}_j)
\end{aligned}$$

(9) If $x_N = 0$, $x_R \geq 0$, $i,j \neq N$, $T_{dN}v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i) = v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i)+\omega\eta_L$ and $T_{dN}v(\mathbf{x}+\mathbf{e}_j) = v(\mathbf{x}+\mathbf{e}_j)+\omega\eta_L$,

$$\begin{aligned}
& T_{dN}v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i) - T_{dN}v(\mathbf{x}+\mathbf{e}_i) \geq v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i)+\omega\eta_L - v(\mathbf{x}+\mathbf{e}_i)-\omega\eta_L \\
\geq\ & v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j)+\omega\eta_L - v(\mathbf{x}+\mathbf{e}_j)-\omega\eta_L \geq T_{dN}v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j) - T_{dN}v(\mathbf{x}+\mathbf{e}_j)
\end{aligned}$$

Proof. Proof of C4 In this section, we prove $T_s v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i) - T_s v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j) \geq T_s v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_A) - T_s v(\mathbf{x}+\mathbf{e}_j+\mathbf{e}_A)$, $s = A, R, N, dN, dR$ and $i,j = R, N,\ i \neq j$.

Operator T_A

(1) If $T_A v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i) = v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i+\mathbf{e}_A)+\omega c_{RM}+(1-\omega)\theta\varepsilon$ and $T_A v(\mathbf{x}+\mathbf{e}_j+\mathbf{e}_A) = v(\mathbf{x}+\mathbf{e}_j+\mathbf{e}_A+\mathbf{e}_A)+\omega c_{RM}+(1-\omega)\theta\varepsilon$,

$$\begin{aligned}
& T_A v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i) - T_A v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j) \\
\geq\ & v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i+\mathbf{e}_A)+\omega c_{RM}+(1-\omega)\theta\varepsilon - v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j+\mathbf{e}_A)-\omega c_{RM}-(1-\omega)\theta\varepsilon \\
\geq\ & v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_A+\mathbf{e}_A)+\omega c_{RM}+(1-\omega)\theta\varepsilon - v(\mathbf{x}+\mathbf{e}_j+\mathbf{e}_A+\mathbf{e}_A)-\omega c_{RM}-(1-\omega)\theta\varepsilon \\
\geq\ & T_A v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_A) - T_A v(\mathbf{x}+\mathbf{e}_j+\mathbf{e}_A)
\end{aligned}$$

(2) If $T_A v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i) = v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i+\mathbf{e}_A)+\omega c_{RM}+(1-\omega)\theta\varepsilon$ and $T_A v(\mathbf{x}+\mathbf{e}_j+\mathbf{e}_A) = v(\mathbf{x}+\mathbf{e}_j+\mathbf{e}_A)$,

$$\begin{aligned}
& T_A v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i) - T_A v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j) \\
\geq\ & v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i+\mathbf{e}_A)+\omega c_{RM}+(1-\omega)\theta\varepsilon - v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j+\mathbf{e}_A)-\omega c_{RM}-(1-\omega)\theta\varepsilon \\
\geq\ & v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_A) - v(\mathbf{x}+\mathbf{e}_j+\mathbf{e}_A) \geq T_A v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_A) - T_A v(\mathbf{x}+\mathbf{e}_j+\mathbf{e}_A)
\end{aligned}$$

(3) If $T_A v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i) = v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i)$ and $T_A v(\mathbf{x}+\mathbf{e}_j+\mathbf{e}_A) = v(\mathbf{x}+\mathbf{e}_j+\mathbf{e}_A+\mathbf{e}_A) + \omega c_{RM} + (1-\omega)\theta\varepsilon$,

$$\begin{aligned}
& T_A v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i) - T_A v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j) \geq v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i) - v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j+\mathbf{e}_A) - \omega c_{RM} - (1-\omega)\theta\varepsilon \\
\geq\ & v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i) - v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i+\mathbf{e}_A) + v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_A+\mathbf{e}_A) - v(\mathbf{x}+\mathbf{e}_j+\mathbf{e}_A+\mathbf{e}_A) - \omega c_{RM} - (1-\omega)\theta\varepsilon \\
\geq\ & v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_A) - v(\mathbf{x}+\mathbf{e}_j+\mathbf{e}_A+\mathbf{e}_A) - \omega c_{RM} - (1-\omega)\theta\varepsilon \geq T_A v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_A) - T_A v(\mathbf{x}+\mathbf{e}_j+\mathbf{e}_A)
\end{aligned}$$

(4) If $T_A v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i) = v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i)$ and $T_A v(\mathbf{x}+\mathbf{e}_j+\mathbf{e}_A) = v(\mathbf{x}+\mathbf{e}_j+\mathbf{e}_A)$,

$$\begin{aligned}
& T_A v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i) - T_A v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j) \geq v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i) - v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j) \\
\geq\ & v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_A) - v(\mathbf{x}+\mathbf{e}_j+\mathbf{e}_A) \geq T_A v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_A) - T_A v(\mathbf{x}+\mathbf{e}_j+\mathbf{e}_A)
\end{aligned}$$

Operator T_R

(1) If $x_A > 0$, or $x_A = 0$, $i = A$, $T_R v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i) = v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i-\mathbf{e}_A+\mathbf{e}_R)$ and $T_R v(\mathbf{x}+\mathbf{e}_j+\mathbf{e}_A) = v(\mathbf{x}+\mathbf{e}_j+\mathbf{e}_A-\mathbf{e}_A+\mathbf{e}_R)$,

$$\begin{aligned}
& T_R v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i) - T_R v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j) \geq v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i-\mathbf{e}_A+\mathbf{e}_R) - v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j-\mathbf{e}_A+\mathbf{e}_R) \\
\geq\ & v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_A-\mathbf{e}_A+\mathbf{e}_R) - v(\mathbf{x}+\mathbf{e}_j+\mathbf{e}_A-\mathbf{e}_A+\mathbf{e}_R) \geq T_R v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_A) - T_R v(\mathbf{x}+\mathbf{e}_j+\mathbf{e}_A)
\end{aligned}$$

(2) If $x_A = 0$, $i \neq A$, $T_R v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i) = v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i)$ and $T_R v(\mathbf{x}+\mathbf{e}_j+\mathbf{e}_A) = v(\mathbf{x}+\mathbf{e}_j+\mathbf{e}_A-\mathbf{e}_A+\mathbf{e}_R)$,

$$\begin{aligned}
& T_R v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i) - T_R v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j) \\
& \text{if } i = R, \\
=\ & v(\mathbf{x}+\mathbf{e}_R+\mathbf{e}_R) - v(\mathbf{x}+\mathbf{e}_R+\mathbf{e}_N) = v(\mathbf{x}+\mathbf{e}_R+\mathbf{e}_A-\mathbf{e}_A+\mathbf{e}_R) - v(\mathbf{x}+\mathbf{e}_N+\mathbf{e}_A-\mathbf{e}_A+\mathbf{e}_R) \\
=\ & T_R v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_A) - T_R v(\mathbf{x}+\mathbf{e}_j+\mathbf{e}_A) \\
& \text{if } i = N, \\
=\ & v(\mathbf{x}+\mathbf{e}_N+\mathbf{e}_N) - v(\mathbf{x}+\mathbf{e}_N+\mathbf{e}_R) \\
\geq\ & v(\mathbf{x}+\mathbf{e}_N+\mathbf{e}_R) + v(\mathbf{x}+\mathbf{e}_N) - v(\mathbf{x}+\mathbf{e}_R) - v(\mathbf{x}+\mathbf{e}_N+\mathbf{e}_R) \geq v(\mathbf{x}+\mathbf{e}_N+\mathbf{e}_R) - v(\mathbf{x}+\mathbf{e}_R+\mathbf{e}_R) \\
=\ & v(\mathbf{x}+\mathbf{e}_N+\mathbf{e}_A-\mathbf{e}_A+\mathbf{e}_R) - v(\mathbf{x}+\mathbf{e}_R+\mathbf{e}_A-\mathbf{e}_A+\mathbf{e}_R) = T_R v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_A) - T_R v(\mathbf{x}+\mathbf{e}_j+\mathbf{e}_A)
\end{aligned}$$

Operator T_N

(1) If $T_N v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i) = v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i+\mathbf{e}_N)+\omega c_N+(1-\omega)\varepsilon$ and $T_N v(\mathbf{x}+\mathbf{e}_j+\mathbf{e}_A) = v(\mathbf{x}+\mathbf{e}_j+\mathbf{e}_A+\mathbf{e}_N)+\omega c_N+(1-\omega)\varepsilon$,

$$\begin{aligned}
& T_N v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i) - T_N v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j) \\
\geq\ & v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i+\mathbf{e}_N)+\omega c_N+(1-\omega)\varepsilon - v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j+\mathbf{e}_N)-\omega c_N-(1-\omega)\varepsilon \\
\geq\ & v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_A+\mathbf{e}_N)+\omega c_N+(1-\omega)\varepsilon - v(\mathbf{x}+\mathbf{e}_j+\mathbf{e}_A+\mathbf{e}_N)-\omega c_N-(1-\omega)\varepsilon \\
\geq\ & T_N v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_A) - T_N v(\mathbf{x}+\mathbf{e}_j+\mathbf{e}_A)
\end{aligned}$$

(2) If $T_N v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i) = v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i+\mathbf{e}_N)+\omega c_N+(1-\omega)\varepsilon$ and $T_N v(\mathbf{x}+\mathbf{e}_j+\mathbf{e}_A) = v(\mathbf{x}+\mathbf{e}_j+\mathbf{e}_A)$,

$$\begin{aligned}
& T_N v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i) - T_N v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j) \\
& \text{if } i=N, \\
\geq\ & v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i+\mathbf{e}_N)+\omega c_N+(1-\omega)\varepsilon - v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j+\mathbf{e}_N)-\omega c_N-(1-\omega)\varepsilon \\
\geq\ & v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_N) - v(\mathbf{x}+\mathbf{e}_j+\mathbf{e}_N) \geq v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_A) - v(\mathbf{x}+\mathbf{e}_j+\mathbf{e}_A) \geq T_N v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_A) - T_N v(\mathbf{x}+\mathbf{e}_j+\mathbf{e}_A) \\
& \text{if } i=R, \\
\geq\ & v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i+\mathbf{e}_N)+\omega c_N+(1-\omega)\varepsilon - v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j) \\
\geq\ & v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_A+\mathbf{e}_N)+\omega c_N+(1-\omega)\varepsilon - v(\mathbf{x}+\mathbf{e}_A+\mathbf{e}_j) \geq T_N v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_A) - T_N v(\mathbf{x}+\mathbf{e}_j+\mathbf{e}_A)
\end{aligned}$$

(3) If $T_N v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i) = v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i)$ and $T_N v(\mathbf{x}+\mathbf{e}_j+\mathbf{e}_A) = v(\mathbf{x}+\mathbf{e}_j+\mathbf{e}_A+\mathbf{e}_N)+\omega c_N+(1-\omega)\varepsilon$,

$$
\begin{aligned}
& T_N v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i) - T_N v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j) \\
& \text{if } i=N, \\
\geq\ & v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i) - v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j), \text{ use C5, } \geq v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i+\mathbf{e}_A) - v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j+\mathbf{e}_A) \\
=\ & v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_A+\mathbf{e}_N)+\omega c_N+(1-\omega)\varepsilon - v(\mathbf{x}+\mathbf{e}_j+\mathbf{e}_A+\mathbf{e}_N)-\omega c_N-(1-\omega)\varepsilon \\
\geq\ & T_N v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_A) - T_N v(\mathbf{x}+\mathbf{e}_j+\mathbf{e}_A) \\
& \text{if } i=R, \\
\geq\ & v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i) - v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j) \geq v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i+\mathbf{e}_N) - v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j+\mathbf{e}_j) \\
=\ & v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i+\mathbf{e}_N) - v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j+\mathbf{e}_N) \geq v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_A+\mathbf{e}_N) - v(\mathbf{x}+\mathbf{e}_j+\mathbf{e}_A+\mathbf{e}_N) \\
=\ & v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_A+\mathbf{e}_N)+\omega c_N+(1-\omega)\varepsilon - v(\mathbf{x}+\mathbf{e}_j+\mathbf{e}_A+\mathbf{e}_N)-\omega c_N-(1-\omega)\varepsilon \\
\geq\ & T_N v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_A) - T_N v(\mathbf{x}+\mathbf{e}_j+\mathbf{e}_A)
\end{aligned}
$$

(4) If $T_N v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i) = v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i)$ and $T_N v(\mathbf{x}+\mathbf{e}_j+\mathbf{e}_A) = v(\mathbf{x}+\mathbf{e}_j+\mathbf{e}_A)$,

$$
\begin{aligned}
& T_N v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i) - T_N v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j) \geq v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i) - v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j) \\
\geq\ & v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_A) - v(\mathbf{x}+\mathbf{e}_j+\mathbf{e}_A) \geq T_N v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_A) - T_N v(\mathbf{x}+\mathbf{e}_j+\mathbf{e}_A)
\end{aligned}
$$

Operator T_{dN} (T_{dR} follows the same structure, thus omitted)

(1) If $x_N > 0$, $x_R \geq 0$, $T_{dN} v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i) = v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i-\mathbf{e}_N)$ and $T_{dN} v(\mathbf{x}+\mathbf{e}_j+\mathbf{e}_A) = v(\mathbf{x}+\mathbf{e}_j+\mathbf{e}_A-\mathbf{e}_N)$,

$$
\begin{aligned}
& T_{dN} v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i) - T_{dN} v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j) \geq v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i-\mathbf{e}_N) - v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j-\mathbf{e}_N) \\
\geq\ & v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_A-\mathbf{e}_N) - v(\mathbf{x}+\mathbf{e}_j+\mathbf{e}_A-\mathbf{e}_N) \geq T_{dN} v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_A) - T_{dN} v(\mathbf{x}+\mathbf{e}_j+\mathbf{e}_A)
\end{aligned}
$$

(2) If $x_N > 0$, $x_R > 0$, or $x_N \geq 0$, $x_R \geq 0$, $i=N$, $j=R$, $T_{dN} v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i) = v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i-\mathbf{e}_N)$and $T_{dN} v(\mathbf{x}+\mathbf{e}_j+\mathbf{e}_A) =$

$v(\mathbf{x}+\mathbf{e}_j+\mathbf{e}_A-\mathbf{e}_R)+\omega\eta_{RN}$,

$$
\begin{aligned}
& T_{dN}v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i)-T_{dN}v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j) \\
& \text{if } i=N, \\
\geq\ & v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i-\mathbf{e}_N)-v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j-\mathbf{e}_R)-\omega\eta_{RN}=v(\mathbf{x}+\mathbf{e}_i)-v(\mathbf{x}+\mathbf{e}_i)-\omega\eta_{RN} \\
=\ & v(\mathbf{x}+\mathbf{e}_A)-v(\mathbf{x}+\mathbf{e}_A)-\omega\eta_{RN}=v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_A-\mathbf{e}_N)-v(\mathbf{x}+\mathbf{e}_j+\mathbf{e}_A-\mathbf{e}_R)-\omega\eta_{RN} \\
\geq\ & T_{dN}v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_A)-T_{dN}v(\mathbf{x}+\mathbf{e}_j+\mathbf{e}_A) \\
& \text{if } i=R, \\
\geq\ & v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i-\mathbf{e}_N)-v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j-\mathbf{e}_N)\geq v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_A-\mathbf{e}_N)-v(\mathbf{x}+\mathbf{e}_j+\mathbf{e}_A-\mathbf{e}_N) \\
=\ & v(\mathbf{x}+\mathbf{e}_A+\mathbf{e}_i+\mathbf{e}_R-\mathbf{e}_N-\mathbf{e}_R)-v(\mathbf{x}+\mathbf{e}_j+\mathbf{e}_A-\mathbf{e}_N) \\
\geq\ & v(\mathbf{x}+\mathbf{e}_A+\mathbf{e}_i-\mathbf{e}_N-\mathbf{e}_R)+v(\mathbf{x}+\mathbf{e}_A+\mathbf{e}_i-\mathbf{e}_R)-v(\mathbf{x}+\mathbf{e}_A-\mathbf{e}_R)-v(\mathbf{x}+\mathbf{e}_j+\mathbf{e}_A-\mathbf{e}_N) \\
\geq\ & v(\mathbf{x}+\mathbf{e}_A+\mathbf{e}_i-\mathbf{e}_R)-v(\mathbf{x}+\mathbf{e}_N+\mathbf{e}_A-\mathbf{e}_R) \\
=\ & v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_A-\mathbf{e}_R)+\omega\eta_{RN}-v(\mathbf{x}+\mathbf{e}_j+\mathbf{e}_A-\mathbf{e}_R)-\omega\eta_{RN}\geq T_{dN}v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_A)-T_{dN}v(\mathbf{x}+\mathbf{e}_j+\mathbf{e}_A)
\end{aligned}
$$

(3) If $x_N=0$, $x_R\geq 0$, $j=R$, $T_{dN}v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i)=v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i-\mathbf{e}_N)$ and $T_{dN}v(\mathbf{x}+\mathbf{e}_j+\mathbf{e}_A)=v(\mathbf{x}+\mathbf{e}_j+\mathbf{e}_A)+\omega\eta_L$,

$$
\begin{aligned}
& T_{dN}v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i)-T_{dN}v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j) \\
\geq\ & v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i-\mathbf{e}_N)-v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j-\mathbf{e}_N)=v(\mathbf{x}+\mathbf{e}_i)-v(\mathbf{x}+\mathbf{e}_j),\ \text{use C5.} \\
\geq\ & v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_A)-v(\mathbf{x}+\mathbf{e}_j+\mathbf{e}_A),\ \text{use C8},\ \geq v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_A-\mathbf{e}_N)-v(\mathbf{x}+\mathbf{e}_j+\mathbf{e}_A)-\omega\eta_L \\
\geq\ & T_{dN}v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_A)-T_{dN}v(\mathbf{x}+\mathbf{e}_j+\mathbf{e}_A)
\end{aligned}
$$

(4) If $x_N>0$, $x_R>0$, or $x_N\geq 0$, $x_R\geq 0$, $i=R$, $j=N$, $T_{dN}v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i)=v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i-\mathbf{e}_R)+\omega\eta_{RN}$ and

$T_{dN}v(\mathbf{x}+\mathbf{e}_j+\mathbf{e}_A) = v(\mathbf{x}+\mathbf{e}_j+\mathbf{e}_A-\mathbf{e}_N)$,

$$\begin{aligned}
& T_{dN}v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i) - T_{dN}v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j) \\
& \text{if } i=R, \\
\geq\ & v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i-\mathbf{e}_R)+\omega\eta_{RN}-v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j-\mathbf{e}_N) = v(\mathbf{x}+\mathbf{e}_i)+\omega\eta_{RN}-v(\mathbf{x}+\mathbf{e}_i) \\
=\ & v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_A-\mathbf{e}_R)+\omega\eta_{RN}-v(\mathbf{x}+\mathbf{e}_j+\mathbf{e}_A-\mathbf{e}_N) \geq T_{dN}v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_A)-T_{dN}v(\mathbf{x}+\mathbf{e}_j+\mathbf{e}_A) \\
& \text{if } i=N, \\
\geq\ & v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i-\mathbf{e}_R)+\omega\eta_{RN}-v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j-\mathbf{e}_R)-\omega\eta_{RN} \\
\geq\ & v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_A-\mathbf{e}_R)-v(\mathbf{x}+\mathbf{e}_A+\mathbf{e}_j-\mathbf{e}_R) \\
=\ & v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i+\mathbf{e}_A-\mathbf{e}_R-\mathbf{e}_N)-v(\mathbf{x}+\mathbf{e}_A+\mathbf{e}_j-\mathbf{e}_R) \\
\geq\ & v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_A-\mathbf{e}_R-\mathbf{e}_N)+v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_A-\mathbf{e}_N)-v(\mathbf{x}+\mathbf{e}_A-\mathbf{e}_N)-v(\mathbf{x}+\mathbf{e}_A+\mathbf{e}_j-\mathbf{e}_R) \\
=\ & v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_A-\mathbf{e}_N)+v(\mathbf{x}+\mathbf{e}_A-\mathbf{e}_R)-v(\mathbf{x}+\mathbf{e}_A-\mathbf{e}_N)-v(\mathbf{x}+\mathbf{e}_A) \\
\geq\ & v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_A-\mathbf{e}_N)-v(\mathbf{x}+\mathbf{e}_A+\mathbf{e}_R-\mathbf{e}_N) \\
=\ & v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_A-\mathbf{e}_N)-v(\mathbf{x}+\mathbf{e}_j+\mathbf{e}_A-\mathbf{e}_N) \geq T_{dN}v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_A)-T_{dN}v(\mathbf{x}+\mathbf{e}_j+\mathbf{e}_A)
\end{aligned}$$

(5) If $x_N \geq 0$, $x_R > 0$, $T_{dN}v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i) = v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i-\mathbf{e}_R)+\omega\eta_{RN}$ and $T_{dN}v(\mathbf{x}+\mathbf{e}_j+\mathbf{e}_A) = v(\mathbf{x}+\mathbf{e}_j+\mathbf{e}_A-\mathbf{e}_R)+\omega\eta_{RN}$,

$$\begin{aligned}
& T_{dN}v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i)-T_{dN}v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j) \geq v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i-\mathbf{e}_R)+\omega\eta_{RN}-v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j-\mathbf{e}_R)-\omega\eta_{RN} \\
\geq\ & v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_A-\mathbf{e}_R)+\omega\eta_{RN}-v(\mathbf{x}+\mathbf{e}_j+\mathbf{e}_A-\mathbf{e}_R)-\omega\eta_{RN} \geq T_{dN}v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_A)-T_{dN}v(\mathbf{x}+\mathbf{e}_j+\mathbf{e}_A)
\end{aligned}$$

(6) If $x_N = 0$, $x_R > 0$, $j = R$, $T_{dN}v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i) = v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i-\mathbf{e}_R)+\omega\eta_{RN}$ and $T_{dN}v(\mathbf{x}+\mathbf{e}_j+\mathbf{e}_A) = v(\mathbf{x}+\mathbf{e}_j+\mathbf{e}_A)+\omega\eta_L$,

$$\begin{aligned}
& T_{dN}v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i)-T_{dN}v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j) \geq v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i-\mathbf{e}_R)+\omega\eta_{RN}-v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j-\mathbf{e}_N) \\
=\ & v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i-\mathbf{e}_R)-v(\mathbf{x}+\mathbf{e}_j)+\omega\eta_{RN}, \text{ use C5.} \\
\geq\ & v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i-\mathbf{e}_R)-v(\mathbf{x}+\mathbf{e}_j+\mathbf{e}_A)+v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_A)-v(\mathbf{x}+\mathbf{e}_N)+\omega\eta_{RN}, \text{ use C5.} \\
\geq\ & -v(\mathbf{x}+\mathbf{e}_j+\mathbf{e}_A)+v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_A+\mathbf{e}_i-\mathbf{e}_R)+\omega\eta_{RN}, \text{ use C8.} \\
\geq\ & v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_A-\mathbf{e}_R)+\omega\eta_{RN}-v(\mathbf{x}+\mathbf{e}_j+\mathbf{e}_A)-\omega\eta_L \geq T_{dN}v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_A)-T_{dN}v(\mathbf{x}+\mathbf{e}_j+\mathbf{e}_A)
\end{aligned}$$

(7) If $x_N = 0$, $x_R \geq 0$, $i = R$, $T_{dN}v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i) = v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i) + \omega\eta_L$ and $T_{dN}v(\mathbf{x}+\mathbf{e}_j+\mathbf{e}_A) = v(\mathbf{x}+\mathbf{e}_j+\mathbf{e}_A-\mathbf{e}_N)$,

$$\begin{aligned}
& T_{dN}v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i) - T_{dN}v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j) \geq v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i) + \omega\eta_L - v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j-\mathbf{e}_N), \text{ use C6.} \\
\geq\ & v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i) - v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j) + v(\mathbf{x}+\mathbf{e}_j+\mathbf{e}_A) - v(\mathbf{x}+\mathbf{e}_A) + \omega\eta_L, \text{ use C4.} \\
\geq\ & v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_A) + \omega\eta_L - v(\mathbf{x}+\mathbf{e}_j+\mathbf{e}_A-\mathbf{e}_N) \geq T_{dN}v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_A) - T_{dN}v(\mathbf{x}+\mathbf{e}_j+\mathbf{e}_A)
\end{aligned}$$

(8) If $x_N = 0$, $x_R > 0$, $i = R$, $T_{dN}v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i) = v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i)+\omega\eta_L$ and $T_{dN}v(\mathbf{x}+\mathbf{e}_j+\mathbf{e}_A) = v(\mathbf{x}+\mathbf{e}_j+\mathbf{e}_A-\mathbf{e}_R)+\omega\eta_{RN}$,

$$\begin{aligned}
& T_{dN}v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i) - T_{dN}v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j) \geq v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i) + \omega\eta_L - v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j-\mathbf{e}_R) - \omega\eta_{RN}, \text{ use C4.} \\
\geq\ & v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i) - v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i-\mathbf{e}_R) + v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_A-\mathbf{e}_R) - v(\mathbf{x}+\mathbf{e}_j+\mathbf{e}_A-\mathbf{e}_R) + \omega\eta_L - \omega\eta_{RN} \\
\geq\ & v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_A) + \omega\eta_L - v(\mathbf{x}+\mathbf{e}_j+\mathbf{e}_A-\mathbf{e}_R) - \omega\eta_{RN} \geq T_{dN}v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_A) - T_{dN}v(\mathbf{x}+\mathbf{e}_j+\mathbf{e}_A)
\end{aligned}$$

and we need to use the parameter constraint in Lemma 1 to complete this part of proof for T_{dR}, especially the cases corresponding to (3), (6), (7) and (8) in the proof of T_{dN}.

Proof. Proof of C5 In this section, we prove $T_s v(\mathbf{x}+\mathbf{e}_R+\mathbf{e}_A) - T_s v(\mathbf{x}+\mathbf{e}_R) \geq T_s v(\mathbf{x}+\mathbf{e}_N+\mathbf{e}_A) - T_s v(\mathbf{x}+\mathbf{e}_N)$, $s = A, R, N, dN, dR$.

Operator T_A

(1) If $T_A v(\mathbf{x}+\mathbf{e}_A+\mathbf{e}_R) = v(\mathbf{x}+\mathbf{e}_A+\mathbf{e}_R+\mathbf{e}_A) + \omega c_{RM} + (1-\omega)\theta\varepsilon$ and $T_A v(\mathbf{x}+\mathbf{e}_N) = v(\mathbf{x}+\mathbf{e}_N+\mathbf{e}_A) + \omega c_{RM} + (1-\omega)\theta\varepsilon$,

$$\begin{aligned}
& T_A v(\mathbf{x}+\mathbf{e}_A+\mathbf{e}_R) - T_A v(\mathbf{x}+\mathbf{e}_R) \\
\geq\ & v(\mathbf{x}+\mathbf{e}_A+\mathbf{e}_R+\mathbf{e}_A) + \omega c_{RM} + (1-\omega)\theta\varepsilon - v(\mathbf{x}+\mathbf{e}_R+\mathbf{e}_A) - \omega c_{RM} - (1-\omega)\theta\varepsilon \\
\geq\ & v(\mathbf{x}+\mathbf{e}_A+\mathbf{e}_N+\mathbf{e}_A) + \omega c_{RM} + (1-\omega)\theta\varepsilon - v(\mathbf{x}+\mathbf{e}_N+\mathbf{e}_A) - \omega c_{RM} - (1-\omega)\theta\varepsilon \\
\geq\ & T_A v(\mathbf{x}+\mathbf{e}_A+\mathbf{e}_N) - T_A v(\mathbf{x}+\mathbf{e}_N)
\end{aligned}$$

(2) If $T_A v(\mathbf{x}+\mathbf{e}_A+\mathbf{e}_R) = v(\mathbf{x}+\mathbf{e}_A+\mathbf{e}_R+\mathbf{e}_A) + \omega c_{RM} + (1-\omega)\theta\varepsilon$ and $T_A v(\mathbf{x}+\mathbf{e}_N) = v(\mathbf{x}+\mathbf{e}_N)$,

$$\begin{aligned}
& T_A v(\mathbf{x}+\mathbf{e}_A+\mathbf{e}_R) - T_A v(\mathbf{x}+\mathbf{e}_R) \\
\geq\ & v(\mathbf{x}+\mathbf{e}_A+\mathbf{e}_R+\mathbf{e}_A) + \omega c_{RM} + (1-\omega)\theta\varepsilon - v(\mathbf{x}+\mathbf{e}_R+\mathbf{e}_A) - \omega c_{RM} - (1-\omega)\theta\varepsilon \\
\geq\ & v(\mathbf{x}+\mathbf{e}_A+\mathbf{e}_R) - v(\mathbf{x}+\mathbf{e}_R) \geq v(\mathbf{x}+\mathbf{e}_A+\mathbf{e}_N) - v(\mathbf{x}+\mathbf{e}_N) \geq T_A v(\mathbf{x}+\mathbf{e}_A+\mathbf{e}_N) - T_A v(\mathbf{x}+\mathbf{e}_N)
\end{aligned}$$

(3) If $T_A v(\mathbf{x}+\mathbf{e}_A+\mathbf{e}_R) = v(\mathbf{x}+\mathbf{e}_A+\mathbf{e}_R)$ and $T_A v(\mathbf{x}+\mathbf{e}_N) = v(\mathbf{x}+\mathbf{e}_N+\mathbf{e}_A) + \omega c_{RM} + (1-\omega)\theta\varepsilon$,

$$\begin{aligned} & T_A v(\mathbf{x}+\mathbf{e}_A+\mathbf{e}_R) - T_A v(\mathbf{x}+\mathbf{e}_R) \geq v(\mathbf{x}+\mathbf{e}_A+\mathbf{e}_R) - v(\mathbf{x}+\mathbf{e}_A+\mathbf{e}_R) - \omega c_{RM} - (1-\omega)\theta\varepsilon \\ = \; & v(\mathbf{x}+\mathbf{e}_A+\mathbf{e}_N) - v(\mathbf{x}+\mathbf{e}_A+\mathbf{e}_N) - \omega c_{RM} - (1-\omega)\theta\varepsilon \geq T_A v(\mathbf{x}+\mathbf{e}_A+\mathbf{e}_N) - T_A v(\mathbf{x}+\mathbf{e}_N) \end{aligned}$$

(4) If $T_A v(\mathbf{x}+\mathbf{e}_A+\mathbf{e}_R) = v(\mathbf{x}+\mathbf{e}_A+\mathbf{e}_R)$ and $T_A v(\mathbf{x}+\mathbf{e}_N) = v(\mathbf{x}+\mathbf{e}_N)$,

$$\begin{aligned} & T_A v(\mathbf{x}+\mathbf{e}_A+\mathbf{e}_R) - T_A v(\mathbf{x}+\mathbf{e}_R) \geq v(\mathbf{x}+\mathbf{e}_A+\mathbf{e}_R) - v(\mathbf{x}+\mathbf{e}_R) \\ \geq \; & v(\mathbf{x}+\mathbf{e}_A+\mathbf{e}_N) - v(\mathbf{x}+\mathbf{e}_N) \geq T_A v(\mathbf{x}+\mathbf{e}_A+\mathbf{e}_N) - T_A v(\mathbf{x}+\mathbf{e}_N) \end{aligned}$$

Operator T_R

(1) If $x_A > 0$, $T_R v(\mathbf{x}+\mathbf{e}_A+\mathbf{e}_R) = v(\mathbf{x}+\mathbf{e}_A+\mathbf{e}_R-\mathbf{e}_A+\mathbf{e}_R)$ and $T_R v(\mathbf{x}+\mathbf{e}_N) = v(\mathbf{x}+\mathbf{e}_N-\mathbf{e}_A+\mathbf{e}_R)$,

$$\begin{aligned} & T_R v(\mathbf{x}+\mathbf{e}_A+\mathbf{e}_R) - T_R v(\mathbf{x}+\mathbf{e}_R) \geq v(\mathbf{x}+\mathbf{e}_A+\mathbf{e}_R-\mathbf{e}_A+\mathbf{e}_R) - v(\mathbf{x}+\mathbf{e}_R-\mathbf{e}_A+\mathbf{e}_R) \\ \geq \; & v(\mathbf{x}+\mathbf{e}_A+\mathbf{e}_N-\mathbf{e}_A+\mathbf{e}_R) - v(\mathbf{x}+\mathbf{e}_N-\mathbf{e}_A+\mathbf{e}_R) \geq T_R v(\mathbf{x}+\mathbf{e}_A+\mathbf{e}_N) - T_R v(\mathbf{x}+\mathbf{e}_N) \end{aligned}$$

(2) If $x_A = 0$, $T_R v(\mathbf{x}+\mathbf{e}_A+\mathbf{e}_R) = v(\mathbf{x}+\mathbf{e}_A+\mathbf{e}_R-\mathbf{e}_A+\mathbf{e}_R)$ and $T_R v(\mathbf{x}+\mathbf{e}_N) = v(\mathbf{x}+\mathbf{e}_N)$,

$$\begin{aligned} & T_R v(\mathbf{x}+\mathbf{e}_A+\mathbf{e}_R) - T_R v(\mathbf{x}+\mathbf{e}_R) = v(\mathbf{x}+\mathbf{e}_A+\mathbf{e}_R-\mathbf{e}_A+\mathbf{e}_R) - v(\mathbf{x}+\mathbf{e}_R) \\ = \; & v(\mathbf{x}+\mathbf{e}_R+\mathbf{e}_R) - v(\mathbf{x}+\mathbf{e}_R) \geq v(\mathbf{x}+\mathbf{e}_R+\mathbf{e}_N) - v(\mathbf{x}+\mathbf{e}_N) \\ = \; & v(\mathbf{x}+\mathbf{e}_A+\mathbf{e}_N-\mathbf{e}_A+\mathbf{e}_R) - v(\mathbf{x}+\mathbf{e}_N) = T_R v(\mathbf{x}+\mathbf{e}_A+\mathbf{e}_N) - T_R v(\mathbf{x}+\mathbf{e}_N) \end{aligned}$$

Operator T_N

(1) If $T_N v(\mathbf{x}+\mathbf{e}_A+\mathbf{e}_R) = v(\mathbf{x}+\mathbf{e}_A+\mathbf{e}_R+\mathbf{e}_N) + \omega c_N + (1-\omega)\varepsilon$ and $T_N v(\mathbf{x}+\mathbf{e}_N) = v(\mathbf{x}+\mathbf{e}_N+\mathbf{e}_N) + \omega c_N + (1-\omega)\varepsilon$,

$$\begin{aligned} & T_N v(\mathbf{x}+\mathbf{e}_A+\mathbf{e}_R) - T_N v(\mathbf{x}+\mathbf{e}_R) \\ \geq \; & v(\mathbf{x}+\mathbf{e}_A+\mathbf{e}_R+\mathbf{e}_N) + \omega c_N + (1-\omega)\varepsilon - v(\mathbf{x}+\mathbf{e}_R+\mathbf{e}_N) - \omega c_N - (1-\omega)\varepsilon \\ \geq \; & v(\mathbf{x}+\mathbf{e}_A+\mathbf{e}_N+\mathbf{e}_N) + \omega c_N + (1-\omega)\varepsilon - v(\mathbf{x}+\mathbf{e}_N+\mathbf{e}_N) - \omega c_N - (1-\omega)\varepsilon \\ \geq \; & T_N v(\mathbf{x}+\mathbf{e}_A+\mathbf{e}_N) - T_N v(\mathbf{x}+\mathbf{e}_N) \end{aligned}$$

(2) If $T_N v(\mathbf{x}+\mathbf{e}_A+\mathbf{e}_R) = v(\mathbf{x}+\mathbf{e}_A+\mathbf{e}_R+\mathbf{e}_N) + \omega c_N + (1-\omega)\varepsilon$ and $T_N v(\mathbf{x}+\mathbf{e}_N) = v(\mathbf{x}+\mathbf{e}_N)$,

$$\begin{aligned}
& T_N v(\mathbf{x}+\mathbf{e}_A+\mathbf{e}_R) - T_N v(\mathbf{x}+\mathbf{e}_R) \\
\geq\ & v(\mathbf{x}+\mathbf{e}_A+\mathbf{e}_R+\mathbf{e}_N) + \omega c_N + (1-\omega)\varepsilon - v(\mathbf{x}+\mathbf{e}_R+\mathbf{e}_N) - \omega c_N - (1-\omega)\varepsilon \\
=\ & v(\mathbf{x}+\mathbf{e}_A+\mathbf{e}_R+\mathbf{e}_N) - v(\mathbf{x}+\mathbf{e}_R+\mathbf{e}_N) \geq v(\mathbf{x}+\mathbf{e}_A+\mathbf{e}_N) - v(\mathbf{x}+\mathbf{e}_N) \geq T_N v(\mathbf{x}+\mathbf{e}_A+\mathbf{e}_N) - T_N v(\mathbf{x}+\mathbf{e}_N)
\end{aligned}$$

(3) If $T_N v(\mathbf{x}+\mathbf{e}_A+\mathbf{e}_R) = v(\mathbf{x}+\mathbf{e}_A+\mathbf{e}_R)$ and $T_N v(\mathbf{x}+\mathbf{e}_N) = v(\mathbf{x}+\mathbf{e}_N+\mathbf{e}_N) + \omega c_N + (1-\omega)\varepsilon$,

$$\begin{aligned}
& T_N v(\mathbf{x}+\mathbf{e}_A+\mathbf{e}_R) - T_N v(\mathbf{x}+\mathbf{e}_R) \geq v(\mathbf{x}+\mathbf{e}_A+\mathbf{e}_R) - v(\mathbf{x}+\mathbf{e}_R+\mathbf{e}_N) - \omega c_N - (1-\omega)\varepsilon, \text{ use C4.} \\
\geq\ & v(\mathbf{x}+\mathbf{e}_A+\mathbf{e}_N) - v(\mathbf{x}+\mathbf{e}_N+\mathbf{e}_N) - \omega c_N - (1-\omega)\varepsilon \geq T_N v(\mathbf{x}+\mathbf{e}_A+\mathbf{e}_N) - T_N v(\mathbf{x}+\mathbf{e}_N)
\end{aligned}$$

(4) If $T_N v(\mathbf{x}+\mathbf{e}_A+\mathbf{e}_R) = v(\mathbf{x}+\mathbf{e}_A+\mathbf{e}_R)$ and $T_N v(\mathbf{x}+\mathbf{e}_N) = v(\mathbf{x}+\mathbf{e}_N)$,

$$\begin{aligned}
& T_N v(\mathbf{x}+\mathbf{e}_A+\mathbf{e}_R) - T_N v(\mathbf{x}+\mathbf{e}_R) \geq v(\mathbf{x}+\mathbf{e}_A+\mathbf{e}_R) - v(\mathbf{x}+\mathbf{e}_R) \\
\geq\ & v(\mathbf{x}+\mathbf{e}_A+\mathbf{e}_N) - v(\mathbf{x}+\mathbf{e}_N) \geq T_N v(\mathbf{x}+\mathbf{e}_A+\mathbf{e}_N) - T_N v(\mathbf{x}+\mathbf{e}_N)
\end{aligned}$$

Operator T_{dN} (Part of T_{dR} follows the same structure, thus omitted)

(1) If $x_N > 0$, $x_R \geq 0$, $T_{dN} v(\mathbf{x}+\mathbf{e}_A+\mathbf{e}_R) = v(\mathbf{x}+\mathbf{e}_A+\mathbf{e}_R-\mathbf{e}_N)$ and $T_{dN} v(\mathbf{x}+\mathbf{e}_N) = v(\mathbf{x}+\mathbf{e}_N-\mathbf{e}_N)$,

$$\begin{aligned}
& T_{dN} v(\mathbf{x}+\mathbf{e}_A+\mathbf{e}_R) - T_{dN} v(\mathbf{x}+\mathbf{e}_R) \geq v(\mathbf{x}+\mathbf{e}_A+\mathbf{e}_R-\mathbf{e}_N) - v(\mathbf{x}+\mathbf{e}_R-\mathbf{e}_N) \\
\geq\ & v(\mathbf{x}+\mathbf{e}_A+\mathbf{e}_N-\mathbf{e}_N) - v(\mathbf{x}+\mathbf{e}_N-\mathbf{e}_N) \geq T_{dN} v(\mathbf{x}+\mathbf{e}_A+\mathbf{e}_N) - T_{dN} v(\mathbf{x}+\mathbf{e}_N)
\end{aligned}$$

(2) If $x_N > 0$, $x_R > 0$, $T_{dN} v(\mathbf{x}+\mathbf{e}_A+\mathbf{e}_R) = v(\mathbf{x}+\mathbf{e}_A+\mathbf{e}_R-\mathbf{e}_N)$ and $T_{dN} v(\mathbf{x}+\mathbf{e}_N) = v(\mathbf{x}+\mathbf{e}_N-\mathbf{e}_R) + \omega\eta_{RN}$,

$$\begin{aligned}
& T_{dN} v(\mathbf{x}+\mathbf{e}_A+\mathbf{e}_R) - T_{dN} v(\mathbf{x}+\mathbf{e}_R) \\
\geq\ & v(\mathbf{x}+\mathbf{e}_A+\mathbf{e}_R-\mathbf{e}_N) - v(\mathbf{x}+\mathbf{e}_R-\mathbf{e}_N) \geq v(\mathbf{x}+\mathbf{e}_A+\mathbf{e}_N-\mathbf{e}_N) - v(\mathbf{x}+\mathbf{e}_N-\mathbf{e}_N) \\
=\ & v(\mathbf{x}+\mathbf{e}_A+\mathbf{e}_R-\mathbf{e}_R) - v(\mathbf{x}+\mathbf{e}_R-\mathbf{e}_R) \geq v(\mathbf{x}+\mathbf{e}_A+\mathbf{e}_N-\mathbf{e}_R) - v(\mathbf{x}+\mathbf{e}_N-\mathbf{e}_R) \\
=\ & v(\mathbf{x}+\mathbf{e}_A+\mathbf{e}_N-\mathbf{e}_R) + \omega\eta_{RN} - v(\mathbf{x}+\mathbf{e}_N-\mathbf{e}_R) - \omega\eta_{RN} \geq T_{dN} v(\mathbf{x}+\mathbf{e}_A+\mathbf{e}_N) - T_{dN} v(\mathbf{x}+\mathbf{e}_N)
\end{aligned}$$

(3) If $x_N \geq 0$, $x_R \geq 0$, $T_{dN}v(\mathbf{x}+\mathbf{e}_A+\mathbf{e}_R) = v(\mathbf{x}+\mathbf{e}_A+\mathbf{e}_R-\mathbf{e}_R)+\omega\eta_{RN}$ and $T_{dN}v(\mathbf{x}+\mathbf{e}_N) = v(\mathbf{x}+\mathbf{e}_N-\mathbf{e}_N)$,

$$\begin{aligned} & T_{dN}v(\mathbf{x}+\mathbf{e}_A+\mathbf{e}_R) - T_{dN}v(\mathbf{x}+\mathbf{e}_R) \geq v(\mathbf{x}+\mathbf{e}_A+\mathbf{e}_R-\mathbf{e}_R)+\omega\eta_{RN} - v(\mathbf{x}+\mathbf{e}_R-\mathbf{e}_R)-\omega\eta_{RN} \\ = \; & v(\mathbf{x}+\mathbf{e}_A+\mathbf{e}_N-\mathbf{e}_N) - v(\mathbf{x}+\mathbf{e}_N-\mathbf{e}_N) \geq T_{dN}v(\mathbf{x}+\mathbf{e}_A+\mathbf{e}_N) - T_{dN}v(\mathbf{x}+\mathbf{e}_N) \end{aligned}$$

(4) If $x_R > 0$, $x_N \geq 0$, $T_{dN}v(\mathbf{x}+\mathbf{e}_A+\mathbf{e}_R) = v(\mathbf{x}+\mathbf{e}_A+\mathbf{e}_R-\mathbf{e}_R)+\omega\eta_{RN}$ and $T_{dN}v(\mathbf{x}+\mathbf{e}_N) = v(\mathbf{x}+\mathbf{e}_N-\mathbf{e}_R)+\omega\eta_{RN}$,

$$\begin{aligned} & T_{dN}v(\mathbf{x}+\mathbf{e}_A+\mathbf{e}_R) - T_{dN}v(\mathbf{x}+\mathbf{e}_R) \geq v(\mathbf{x}+\mathbf{e}_A+\mathbf{e}_R-\mathbf{e}_R)+\omega\eta_{RN} - v(\mathbf{x}+\mathbf{e}_R-\mathbf{e}_R)-\omega\eta_{RN} \\ \geq \; & v(\mathbf{x}+\mathbf{e}_A+\mathbf{e}_N-\mathbf{e}_R)+\omega\eta_{RN} - v(\mathbf{x}+\mathbf{e}_N-\mathbf{e}_R)+\omega\eta_{RN} \geq T_{dN}v(\mathbf{x}+\mathbf{e}_A+\mathbf{e}_N) - T_{dN}v(\mathbf{x}+\mathbf{e}_N) \end{aligned}$$

(5) If $x_N = 0$, $x_R \geq 0$, $T_{dN}v(\mathbf{x}+\mathbf{e}_A+\mathbf{e}_R) = v(\mathbf{x}+\mathbf{e}_A+\mathbf{e}_R)+\omega\eta_L$ and $T_{dN}v(\mathbf{x}+\mathbf{e}_N) = v(\mathbf{x}+\mathbf{e}_N-\mathbf{e}_N)$,

$$\begin{aligned} & T_{dN}v(\mathbf{x}+\mathbf{e}_A+\mathbf{e}_R) - T_{dN}v(\mathbf{x}+\mathbf{e}_R) \geq v(\mathbf{x}+\mathbf{e}_A+\mathbf{e}_R)+\omega\eta_L - v(\mathbf{x}+\mathbf{e}_R)-\omega\eta_L \\ \geq \; & v(\mathbf{x}+\mathbf{e}_A+\mathbf{e}_N-\mathbf{e}_N) - v(\mathbf{x}+\mathbf{e}_N-\mathbf{e}_N) \geq T_{dN}v(\mathbf{x}+\mathbf{e}_A+\mathbf{e}_N) - T_{dN}v(\mathbf{x}+\mathbf{e}_N) \end{aligned}$$

(6) If $x_N = 0$, $x_R > 0$, $T_{dN}v(\mathbf{x}+\mathbf{e}_A+\mathbf{e}_R) = v(\mathbf{x}+\mathbf{e}_A+\mathbf{e}_R)+\omega\eta_L$ and $T_{dN}v(\mathbf{x}+\mathbf{e}_N) = v(\mathbf{x}+\mathbf{e}_N-\mathbf{e}_R)+\omega\eta_{RN}$,

$$\begin{aligned} & T_{dN}v(\mathbf{x}+\mathbf{e}_A+\mathbf{e}_R) - T_{dN}v(\mathbf{x}+\mathbf{e}_R) \geq v(\mathbf{x}+\mathbf{e}_A+\mathbf{e}_R)+\omega\eta_L - v(\mathbf{x}+\mathbf{e}_R-\mathbf{e}_R)-\omega\eta_{RN}, \text{ use C5.} \\ \geq \; & v(\mathbf{x}+\mathbf{e}_N+\mathbf{e}_A) - v(\mathbf{x}+\mathbf{e}_N) + v(\mathbf{x}+\mathbf{e}_R)+\omega\eta_L - v(\mathbf{x})-\omega\eta_{RN} \\ \geq \; & v(\mathbf{x}+\mathbf{e}_N+\mathbf{e}_A)+\omega\eta_L - v(\mathbf{x}+\mathbf{e}_N-\mathbf{e}_R)-\omega\eta_{RN}, \text{ use C8, } \geq v(\mathbf{x}+\mathbf{e}_N+\mathbf{e}_A-\mathbf{e}_N) - v(\mathbf{x}+\mathbf{e}_N-\mathbf{e}_R)-\omega\eta_{RN} \\ \geq \; & T_{dN}v(\mathbf{x}+\mathbf{e}_A+\mathbf{e}_N) - T_{dN}v(\mathbf{x}+\mathbf{e}_N) \end{aligned}$$

Note: For Operator T_{dR}, the structural difference from proving T_{dN} happens when $x_R = 0$. Therefore (5) and (6) do not happen in T_{dR}, instead, we will have:

(5') If $x_R = 0$, $x_N > 0$, $T_{dR}v(\mathbf{x}+\mathbf{e}_A+\mathbf{e}_R) = v(\mathbf{x}+\mathbf{e}_A+\mathbf{e}_R-\mathbf{e}_N)+\omega\eta_{NR}$ and $T_{dR}v(\mathbf{x}+\mathbf{e}_N) = v(\mathbf{x}+\mathbf{e}_N)+\omega\eta_L$,

$$\begin{aligned} & T_{dR}v(\mathbf{x}+\mathbf{e}_A+\mathbf{e}_R) - T_{dR}v(\mathbf{x}+\mathbf{e}_R) \geq v(\mathbf{x}+\mathbf{e}_A+\mathbf{e}_R-\mathbf{e}_N)+\omega\eta_{NR} - v(\mathbf{x}+\mathbf{e}_R-\mathbf{e}_R), \text{ use C8,} \\ \geq \; & v(\mathbf{x}+\mathbf{e}_A-\mathbf{e}_N)+\omega\eta_{NR} - v(\mathbf{x}+\mathbf{e}_N-\mathbf{e}_N)-\omega\eta_L \geq v(\mathbf{x}+\mathbf{e}_A)+\omega\eta_{NR} - v(\mathbf{x}+\mathbf{e}_N)-\omega\eta_L \\ = \; & v(\mathbf{x}+\mathbf{e}_A+\mathbf{e}_N-\mathbf{e}_N)+\omega\eta_{NR} - v(\mathbf{x}+\mathbf{e}_N)-\omega\eta_L \geq T_{dN}v(\mathbf{x}+\mathbf{e}_A+\mathbf{e}_N) - T_{dN}v(\mathbf{x}+\mathbf{e}_N) \end{aligned}$$

(6') If $x_R = 0$, $x_N \geq 0$, $T_{dR}v(\mathbf{x}+\mathbf{e}_A+\mathbf{e}_R) = v(\mathbf{x}+\mathbf{e}_A+\mathbf{e}_R-\mathbf{e}_R)$ and $T_{dR}v(\mathbf{x}+\mathbf{e}_N) = v(\mathbf{x}+\mathbf{e}_N)+\omega\eta_L$,

$$\begin{aligned}
& T_{dR}v(\mathbf{x}+\mathbf{e}_A+\mathbf{e}_R) - T_{dR}v(\mathbf{x}+\mathbf{e}_R) \geq v(\mathbf{x}+\mathbf{e}_A+\mathbf{e}_R-\mathbf{e}_R) - v(\mathbf{x}+\mathbf{e}_R-\mathbf{e}_R) \\
= \; & v(\mathbf{x}+\mathbf{e}_A+\mathbf{e}_N-\mathbf{e}_N) - v(\mathbf{x}+\mathbf{e}_N-\mathbf{e}_N), \text{ use C8.} \\
\geq \; & v(\mathbf{x}+\mathbf{e}_A+\mathbf{e}_N-\mathbf{e}_N) - v(\mathbf{x}+\mathbf{e}_N) - \omega\eta_L = v(\mathbf{x}+\mathbf{e}_A+\mathbf{e}_N-\mathbf{e}_N) + \omega\eta_{NR} - v(\mathbf{x}+\mathbf{e}_N) - \omega\eta_L \\
\geq \; & T_{dN}v(\mathbf{x}+\mathbf{e}_A+\mathbf{e}_N) - T_{dN}v(\mathbf{x}+\mathbf{e}_N)
\end{aligned}$$

and we need to use the parameter constraint in Lemma 1 to complete this part of proof for T_{dR}.

Proof. Proof of C6

In this section, we prove $T_s v(\mathbf{x}+\mathbf{e}_R+\mathbf{e}_N) - T_s v(\mathbf{x}+\mathbf{e}_R) \geq T_s v(\mathbf{x}+\mathbf{e}_N+\mathbf{e}_A) - T_s v(\mathbf{x}+\mathbf{e}_A)$, $s = A, R, N, dN, dR$.

Operator T_A

(1) If $T_A v(\mathbf{x}+\mathbf{e}_R+\mathbf{e}_N) = v(\mathbf{x}+\mathbf{e}_R+\mathbf{e}_N+\mathbf{e}_A) + \omega c_{RM} + (1-\omega)\theta\varepsilon$ and $T_A v(\mathbf{x}+\mathbf{e}_A) = v(\mathbf{x}+\mathbf{e}_A+\mathbf{e}_A) + \omega c_{RM} + (1-\omega)\theta\varepsilon$,

$$\begin{aligned}
& T_A v(\mathbf{x}+\mathbf{e}_R+\mathbf{e}_N) - T_A v(\mathbf{x}+\mathbf{e}_R) \\
\geq \; & v(\mathbf{x}+\mathbf{e}_R+\mathbf{e}_N+\mathbf{e}_A) + \omega c_{RM} + (1-\omega)\theta\varepsilon - v(\mathbf{x}+\mathbf{e}_R+\mathbf{e}_A) - \omega c_{RM} - (1-\omega)\theta\varepsilon \\
\geq \; & v(\mathbf{x}+\mathbf{e}_A+\mathbf{e}_N+\mathbf{e}_A) + \omega c_{RM} + (1-\omega)\theta\varepsilon - v(\mathbf{x}+\mathbf{e}_A+\mathbf{e}_A) - \omega c_{RM} - (1-\omega)\theta\varepsilon \\
\geq \; & T_A v(\mathbf{x}+\mathbf{e}_A+\mathbf{e}_N) - T_A v(\mathbf{x}+\mathbf{e}_A)
\end{aligned}$$

(2) If $T_A v(\mathbf{x}+\mathbf{e}_R+\mathbf{e}_N) = v(\mathbf{x}+\mathbf{e}_R+\mathbf{e}_N+\mathbf{e}_A) + \omega c_{RM} + (1-\omega)\theta\varepsilon$ and $T_A v(\mathbf{x}+\mathbf{e}_A) = v(\mathbf{x}+\mathbf{e}_A)$,

$$\begin{aligned}
& T_A v(\mathbf{x}+\mathbf{e}_R+\mathbf{e}_N) - T_A v(\mathbf{x}+\mathbf{e}_R) \\
\geq \; & v(\mathbf{x}+\mathbf{e}_R+\mathbf{e}_N+\mathbf{e}_A) + \omega c_{RM} + (1-\omega)\theta\varepsilon - v(\mathbf{x}+\mathbf{e}_R+\mathbf{e}_A) - \omega c_{RM} - (1-\omega)\theta\varepsilon \\
\geq \; & v(\mathbf{x}+\mathbf{e}_N+\mathbf{e}_A) - v(\mathbf{x}+\mathbf{e}_A) \geq T_A v(\mathbf{x}+\mathbf{e}_A+\mathbf{e}_N) - T_A v(\mathbf{x}+\mathbf{e}_A)
\end{aligned}$$

(3) If $T_A v(\mathbf{x}+\mathbf{e}_R+\mathbf{e}_N) = v(\mathbf{x}+\mathbf{e}_R+\mathbf{e}_N)$ and $T_A v(\mathbf{x}+\mathbf{e}_A) = v(\mathbf{x}+\mathbf{e}_A+\mathbf{e}_A) + \omega c_{RM} + (1-\omega)\theta\varepsilon$,

$$\begin{aligned}
& T_A v(\mathbf{x}+\mathbf{e}_R+\mathbf{e}_N) - T_A v(\mathbf{x}+\mathbf{e}_R) \\
\geq\ & v(\mathbf{x}+\mathbf{e}_R+\mathbf{e}_N) - v(\mathbf{x}+\mathbf{e}_R+\mathbf{e}_A) - \omega c_{RM} - (1-\omega)\theta\varepsilon \\
\geq\ & v(\mathbf{x}+\mathbf{e}_R+\mathbf{e}_N) - v(\mathbf{x}+\mathbf{e}_A+\mathbf{e}_A) + v(\mathbf{x}+\mathbf{e}_A) - v(\mathbf{x}+\mathbf{e}_R) - \omega c_{RM} - (1-\omega)\theta\varepsilon \\
\geq\ & v(\mathbf{x}+\mathbf{e}_A+\mathbf{e}_N) - v(\mathbf{x}+\mathbf{e}_A+\mathbf{e}_A) - \omega c_{RM} - (1-\omega)\theta\varepsilon \geq T_A v(\mathbf{x}+\mathbf{e}_A+\mathbf{e}_N) - T_A v(\mathbf{x}+\mathbf{e}_A)
\end{aligned}$$

(4) If $T_A v(\mathbf{x}+\mathbf{e}_R+\mathbf{e}_N) = v(\mathbf{x}+\mathbf{e}_R+\mathbf{e}_N)$ and $T_A v(\mathbf{x}+\mathbf{e}_A) = v(\mathbf{x}+\mathbf{e}_A)$,

$$\begin{aligned}
& T_A v(\mathbf{x}+\mathbf{e}_R+\mathbf{e}_N) - T_A v(\mathbf{x}+\mathbf{e}_R) \geq v(\mathbf{x}+\mathbf{e}_R+\mathbf{e}_N) - v(\mathbf{x}+\mathbf{e}_R) \\
\geq\ & v(\mathbf{x}+\mathbf{e}_A+\mathbf{e}_N) - v(\mathbf{x}+\mathbf{e}_A) \geq T_A v(\mathbf{x}+\mathbf{e}_A+\mathbf{e}_N) - T_A v(\mathbf{x}+\mathbf{e}_A)
\end{aligned}$$

Operator T_R

(1) If $x_A > 0$, $T_R v(\mathbf{x}+\mathbf{e}_R+\mathbf{e}_N) = v(\mathbf{x}+\mathbf{e}_R+\mathbf{e}_N-\mathbf{e}_A+\mathbf{e}_R)$ and $T_R v(\mathbf{x}+\mathbf{e}_A) = v(\mathbf{x}+\mathbf{e}_A-\mathbf{e}_A+\mathbf{e}_R)$,

$$\begin{aligned}
& T_R v(\mathbf{x}+\mathbf{e}_R+\mathbf{e}_N) - T_R v(\mathbf{x}+\mathbf{e}_R) = v(\mathbf{x}+\mathbf{e}_R+\mathbf{e}_N-\mathbf{e}_A+\mathbf{e}_R) - v(\mathbf{x}+\mathbf{e}_R-\mathbf{e}_A+\mathbf{e}_R) \\
\geq\ & v(\mathbf{x}+\mathbf{e}_N+\mathbf{e}_A-\mathbf{e}_A+\mathbf{e}_R) - v(\mathbf{x}+\mathbf{e}_A-\mathbf{e}_A+\mathbf{e}_R) \geq T_R v(\mathbf{x}+\mathbf{e}_N+\mathbf{e}_A) - T_R v(\mathbf{x}+\mathbf{e}_A)
\end{aligned}$$

(2) If $x_A = 0$, $T_R v(\mathbf{x}+\mathbf{e}_R+\mathbf{e}_N) = v(\mathbf{x}+\mathbf{e}_R+\mathbf{e}_N)$ and $T_R v(\mathbf{x}+\mathbf{e}_A) = v(\mathbf{x}+\mathbf{e}_A-\mathbf{e}_A+\mathbf{e}_R)$,

$$\begin{aligned}
& T_R v(\mathbf{x}+\mathbf{e}_R+\mathbf{e}_N) - T_R v(\mathbf{x}+\mathbf{e}_R) \\
=\ & v(\mathbf{x}+\mathbf{e}_R+\mathbf{e}_N) - v(\mathbf{x}+\mathbf{e}_R) = v(\mathbf{x}+\mathbf{e}_N+\mathbf{e}_A-\mathbf{e}_A+\mathbf{e}_R) - v(\mathbf{x}+\mathbf{e}_A-\mathbf{e}_A+\mathbf{e}_R) \\
=\ & v(\mathbf{x}+\mathbf{e}_N+\mathbf{e}_A-\mathbf{e}_A+\mathbf{e}_R) - v(\mathbf{x}+\mathbf{e}_A-\mathbf{e}_A+\mathbf{e}_R) = T_R v(\mathbf{x}+\mathbf{e}_N+\mathbf{e}_A) - T_R v(\mathbf{x}+\mathbf{e}_A)
\end{aligned}$$

Operator T_N

(1) If $T_N v(\mathbf{x}+\mathbf{e}_R+\mathbf{e}_N) = v(\mathbf{x}+\mathbf{e}_R+\mathbf{e}_N+\mathbf{e}_N) + \omega c_N + (1-\omega)\varepsilon$ and $T_N v(\mathbf{x}+\mathbf{e}_A) = v(\mathbf{x}+\mathbf{e}_A+\mathbf{e}_N) + \omega c_N + (1-\omega)\varepsilon$,

$$\begin{aligned} & T_N v(\mathbf{x}+\mathbf{e}_R+\mathbf{e}_N) - T_N v(\mathbf{x}+\mathbf{e}_R) \\ \geq\ & v(\mathbf{x}+\mathbf{e}_R+\mathbf{e}_N+\mathbf{e}_N) + \omega c_N + (1-\omega)\varepsilon - v(\mathbf{x}+\mathbf{e}_R+\mathbf{e}_N) - \omega c_N - (1-\omega)\varepsilon \\ \geq\ & v(\mathbf{x}+\mathbf{e}_N+\mathbf{e}_A+\mathbf{e}_N) + \omega c_N + (1-\omega)\varepsilon - v(\mathbf{x}+\mathbf{e}_A+\mathbf{e}_N) - \omega c_N - (1-\omega)\varepsilon \\ \geq\ & T_N v(\mathbf{x}+\mathbf{e}_N+\mathbf{e}_A) - T_N v(\mathbf{x}+\mathbf{e}_A) \end{aligned}$$

(2) If $T_N v(\mathbf{x}+\mathbf{e}_R+\mathbf{e}_N) = v(\mathbf{x}+\mathbf{e}_R+\mathbf{e}_N+\mathbf{e}_N) + \omega c_N + (1-\omega)\varepsilon$ and $T_N v(\mathbf{x}+\mathbf{e}_A) = v(\mathbf{x}+\mathbf{e}_A)$,

$$\begin{aligned} & T_N v(\mathbf{x}+\mathbf{e}_R+\mathbf{e}_N) - T_N v(\mathbf{x}+\mathbf{e}_R) \\ \geq\ & v(\mathbf{x}+\mathbf{e}_R+\mathbf{e}_N+\mathbf{e}_N) + \omega c_N + (1-\omega)\varepsilon - v(\mathbf{x}+\mathbf{e}_R+\mathbf{e}_N) - \omega c_N - (1-\omega)\varepsilon \\ \geq\ & v(\mathbf{x}+\mathbf{e}_R+\mathbf{e}_N) - v(\mathbf{x}+\mathbf{e}_R) \geq v(\mathbf{x}+\mathbf{e}_N+\mathbf{e}_A) - v(\mathbf{x}+\mathbf{e}_A) \geq T_N v(\mathbf{x}+\mathbf{e}_N+\mathbf{e}_A) - T_N v(\mathbf{x}+\mathbf{e}_A) \end{aligned}$$

(3) If $T_N v(\mathbf{x}+\mathbf{e}_R+\mathbf{e}_N) = v(\mathbf{x}+\mathbf{e}_R+\mathbf{e}_N)$ and $T_N v(\mathbf{x}+\mathbf{e}_A) = v(\mathbf{x}+\mathbf{e}_A+\mathbf{e}_N) + \omega c_N + (1-\omega)\varepsilon$,

$$\begin{aligned} & T_N v(\mathbf{x}+\mathbf{e}_R+\mathbf{e}_N) - T_N v(\mathbf{x}+\mathbf{e}_R) \geq v(\mathbf{x}+\mathbf{e}_R+\mathbf{e}_N) - v(\mathbf{x}+\mathbf{e}_R+\mathbf{e}_N) - \omega c_N - (1-\omega)\varepsilon \\ =\ & v(\mathbf{x}+\mathbf{e}_N+\mathbf{e}_A) - v(\mathbf{x}+\mathbf{e}_N+\mathbf{e}_A) - \omega c_N - (1-\omega)\varepsilon \geq T_N v(\mathbf{x}+\mathbf{e}_N+\mathbf{e}_A) - T_N v(\mathbf{x}+\mathbf{e}_A) \end{aligned}$$

(4) If $T_N v(\mathbf{x}+\mathbf{e}_R+\mathbf{e}_N) = v(\mathbf{x}+\mathbf{e}_R+\mathbf{e}_N)$ and $T_N v(\mathbf{x}+\mathbf{e}_A) = v(\mathbf{x}+\mathbf{e}_A)$,

$$\begin{aligned} & T_N v(\mathbf{x}+\mathbf{e}_R+\mathbf{e}_N) - T_N v(\mathbf{x}+\mathbf{e}_R) \geq v(\mathbf{x}+\mathbf{e}_R+\mathbf{e}_N) - v(\mathbf{x}+\mathbf{e}_R) \\ \geq\ & v(\mathbf{x}+\mathbf{e}_N+\mathbf{e}_A) - v(\mathbf{x}+\mathbf{e}_A) \geq T_N v(\mathbf{x}+\mathbf{e}_N+\mathbf{e}_A) - T_N v(\mathbf{x}+\mathbf{e}_A) \end{aligned}$$

Operator T_{dN} (T_{dR} follows the same structure, thus omitted)

(1) If $x_N > 0$, $x_R \geq 0$, $T_{dN} v(\mathbf{x}+\mathbf{e}_R+\mathbf{e}_N) = v(\mathbf{x}+\mathbf{e}_R+\mathbf{e}_N-\mathbf{e}_N)$ and $T_{dN} v(\mathbf{x}+\mathbf{e}_A) = v(\mathbf{x}+\mathbf{e}_A-\mathbf{e}_N)$,

$$\begin{aligned} & T_{dN} v(\mathbf{x}+\mathbf{e}_R+\mathbf{e}_N) - T_{dN} v(\mathbf{x}+\mathbf{e}_R) \geq v(\mathbf{x}+\mathbf{e}_R+\mathbf{e}_N-\mathbf{e}_N) - v(\mathbf{x}+\mathbf{e}_R-\mathbf{e}_N) \\ \geq\ & v(\mathbf{x}+\mathbf{e}_A+\mathbf{e}_N-\mathbf{e}_N) - v(\mathbf{x}+\mathbf{e}_A-\mathbf{e}_N) \geq T_{dN} v(\mathbf{x}+\mathbf{e}_A+\mathbf{e}_N) - T_{dN} v(\mathbf{x}+\mathbf{e}_A) \end{aligned}$$

(2) If $x_N \geq 0$, $x_R > 0$, $T_{dN}v(\mathbf{x}+\mathbf{e}_R+\mathbf{e}_N) = v(\mathbf{x}+\mathbf{e}_R+\mathbf{e}_N-\mathbf{e}_N)$ and $T_{dN}v(\mathbf{x}+\mathbf{e}_A) = v(\mathbf{x}+\mathbf{e}_A-\mathbf{e}_R)+\omega\eta_{RN}$,

$$\begin{aligned}
& T_{dN}v(\mathbf{x}+\mathbf{e}_R+\mathbf{e}_N) - T_{dN}v(\mathbf{x}+\mathbf{e}_R) \\
\geq\ & v(\mathbf{x}+\mathbf{e}_R+\mathbf{e}_N-\mathbf{e}_N) - v(\mathbf{x}+\mathbf{e}_R-\mathbf{e}_R) - \omega\eta_{RN} = v(\mathbf{x}+\mathbf{e}_R) - v(\mathbf{x}) - \omega\eta_{RN} \\
\geq\ & v(\mathbf{x}+\mathbf{e}_R) - v(\mathbf{x}+\mathbf{e}_N) + v(\mathbf{x}+\mathbf{e}_N+\mathbf{e}_A-\mathbf{e}_R) - v(\mathbf{x}+\mathbf{e}_A-\mathbf{e}_R) - \omega\eta_{RN}, \text{ use C4.} \\
\geq\ & v(\mathbf{x}+\mathbf{e}_A) - v(\mathbf{x}+\mathbf{e}_A-\mathbf{e}_R) - \omega\eta_{RN} = v(\mathbf{x}+\mathbf{e}_A+\mathbf{e}_N-\mathbf{e}_N) - v(\mathbf{x}+\mathbf{e}_A-\mathbf{e}_R) - \omega\eta_{RN} \\
\geq\ & T_{dN}v(\mathbf{x}+\mathbf{e}_A+\mathbf{e}_N) - T_{dN}v(\mathbf{x}+\mathbf{e}_A)
\end{aligned}$$

(3) If $x_N = 0$, $x_R \geq 0$, $T_{dN}v(\mathbf{x}+\mathbf{e}_R+\mathbf{e}_N) = v(\mathbf{x}+\mathbf{e}_R+\mathbf{e}_N-\mathbf{e}_N)$ and $T_{dN}v(\mathbf{x}+\mathbf{e}_A) = v(\mathbf{x}+\mathbf{e}_A)+\omega\eta_L$,

$$\begin{aligned}
& T_{dN}v(\mathbf{x}+\mathbf{e}_R+\mathbf{e}_N) - T_{dN}v(\mathbf{x}+\mathbf{e}_R) \\
\geq\ & v(\mathbf{x}+\mathbf{e}_R+\mathbf{e}_N-\mathbf{e}_N) - v(\mathbf{x}+\mathbf{e}_R) - \omega\eta_L = v(\mathbf{x}+\mathbf{e}_R) - v(\mathbf{x}+\mathbf{e}_R) - \omega\eta_L \\
=\ & v(\mathbf{x}+\mathbf{e}_N+\mathbf{e}_A-\mathbf{e}_N) - v(\mathbf{x}+\mathbf{e}_A) - \omega\eta_L \geq T_{dN}v(\mathbf{x}+\mathbf{e}_A+\mathbf{e}_N) - T_{dN}v(\mathbf{x}+\mathbf{e}_A)
\end{aligned}$$

(4) If $x_N > 0$, $x_R \geq 0$, $T_{dN}v(\mathbf{x}+\mathbf{e}_R+\mathbf{e}_N) = v(\mathbf{x}+\mathbf{e}_R+\mathbf{e}_N-\mathbf{e}_R)+\omega\eta_{RN}$ and $T_{dN}v(\mathbf{x}+\mathbf{e}_A) = v(\mathbf{x}+\mathbf{e}_A-\mathbf{e}_N)$,

$$\begin{aligned}
& T_{dN}v(\mathbf{x}+\mathbf{e}_R+\mathbf{e}_N) - T_{dN}v(\mathbf{x}+\mathbf{e}_R) \\
\geq\ & v(\mathbf{x}+\mathbf{e}_R+\mathbf{e}_N-\mathbf{e}_R) + \omega\eta_{RN} - v(\mathbf{x}+\mathbf{e}_R-\mathbf{e}_R) - \omega\eta_{RN} = v(\mathbf{x}+\mathbf{e}_N+\mathbf{e}_N-\mathbf{e}_N) - v(\mathbf{x}+\mathbf{e}_N-\mathbf{e}_N) \\
\geq\ & v(\mathbf{x}+\mathbf{e}_A+\mathbf{e}_N-\mathbf{e}_N) - v(\mathbf{x}+\mathbf{e}_A-\mathbf{e}_N) \geq T_{dN}v(\mathbf{x}+\mathbf{e}_A+\mathbf{e}_N) - T_{dN}v(\mathbf{x}+\mathbf{e}_A)
\end{aligned}$$

(5) If $x_N \geq 0$, $x_R > 0$, $T_{dN}v(\mathbf{x}+\mathbf{e}_R+\mathbf{e}_N) = v(\mathbf{x}+\mathbf{e}_R+\mathbf{e}_N-\mathbf{e}_R)+\omega\eta_{RN}$ and $T_{dN}v(\mathbf{x}+\mathbf{e}_A) = v(\mathbf{x}+\mathbf{e}_A-\mathbf{e}_R)+\omega\eta_{RN}$,

$$\begin{aligned}
& T_{dN}v(\mathbf{x}+\mathbf{e}_R+\mathbf{e}_N) - T_{dN}v(\mathbf{x}+\mathbf{e}_R) \geq v(\mathbf{x}+\mathbf{e}_R+\mathbf{e}_N-\mathbf{e}_R) + \omega\eta_{RN} - v(\mathbf{x}+\mathbf{e}_R-\mathbf{e}_R) - \omega\eta_{RN} \\
\geq\ & v(\mathbf{x}+\mathbf{e}_A+\mathbf{e}_N-\mathbf{e}_R) + \omega\eta_{RN} - v(\mathbf{x}+\mathbf{e}_A-\mathbf{e}_R) - \omega\eta_{RN} \geq T_{dN}v(\mathbf{x}+\mathbf{e}_A+\mathbf{e}_N) - T_{dN}v(\mathbf{x}+\mathbf{e}_A)
\end{aligned}$$

(6) If $x_N = 0$, $x_R \geq 0$, $T_{dN}v(\mathbf{x}+\mathbf{e}_R+\mathbf{e}_N) = v(\mathbf{x}+\mathbf{e}_R+\mathbf{e}_N-\mathbf{e}_R)+\omega\eta_{RN}$ and $T_{dN}v(\mathbf{x}+\mathbf{e}_A) = v(\mathbf{x}+\mathbf{e}_A)+\omega\eta_L$,

$$\begin{aligned}
& T_{dN}v(\mathbf{x}+\mathbf{e}_R+\mathbf{e}_N) - T_{dN}v(\mathbf{x}+\mathbf{e}_R) \\
\geq\ & v(\mathbf{x}+\mathbf{e}_R+\mathbf{e}_N-\mathbf{e}_R) + \omega\eta_{RN} - v(\mathbf{x}+\mathbf{e}_R-\mathbf{e}_R) - \omega\eta_{RN} = v(\mathbf{x}+\mathbf{e}_N) - v(\mathbf{x}), \text{ use C8.} \\
\geq\ & v(\mathbf{x}+\mathbf{e}_N) - v(\mathbf{x}+\mathbf{e}_N) - \omega\eta_L = v(\mathbf{x}+\mathbf{e}_A+\mathbf{e}_N-\mathbf{e}_N) - v(\mathbf{x}+\mathbf{e}_A) - \omega\eta_L \geq T_{dN}v(\mathbf{x}+\mathbf{e}_A+\mathbf{e}_N) - T_{dN}v(\mathbf{x}+\mathbf{e}_A)
\end{aligned}$$

Note: For Operator T_{dR}, the structural difference from proving T_{dN} happens when $x_R = 0$. Therefore (3) and (6) do not

happen in T_{dR}, instead, we will have:

(3') If $x_N \geq 0$, $x_R = 0$, $T_{dR}v(\mathbf{x}+\mathbf{e}_R+\mathbf{e}_N) = v(\mathbf{x}+\mathbf{e}_R+\mathbf{e}_N-\mathbf{e}_N)+\omega\eta_{NR}$ and $T_{dR}v(\mathbf{x}+\mathbf{e}_A) = v(\mathbf{x}+\mathbf{e}_A)+\omega\eta_L$,

$$\begin{aligned}
& T_{dR}v(\mathbf{x}+\mathbf{e}_R+\mathbf{e}_N) - T_{dR}v(\mathbf{x}+\mathbf{e}_R) \\
\geq\ & v(\mathbf{x}+\mathbf{e}_R+\mathbf{e}_N-\mathbf{e}_N)+\omega\eta_{NR} - v(\mathbf{x}+\mathbf{e}_R-\mathbf{e}_R), \text{ use C8, } \geq v(\mathbf{x}+\mathbf{e}_R)+\omega\eta_{NR} - v(\mathbf{x}+\mathbf{e}_R) - \omega\eta_L \\
=\ & v(\mathbf{x}+\mathbf{e}_N+\mathbf{e}_A-\mathbf{e}_N)+\omega\eta_{NR} - v(\mathbf{x}+\mathbf{e}_A) - \omega\eta_L \geq T_{dR}v(\mathbf{x}+\mathbf{e}_A+\mathbf{e}_N) - T_{dR}v(\mathbf{x}+\mathbf{e}_A)
\end{aligned}$$

(6') If $x_N \geq 0$, $x_R = 0$, $T_{dR}v(\mathbf{x}+\mathbf{e}_R+\mathbf{e}_N) = v(\mathbf{x}+\mathbf{e}_R+\mathbf{e}_N-\mathbf{e}_R)$ and $T_{dR}v(\mathbf{x}+\mathbf{e}_A) = v(\mathbf{x}+\mathbf{e}_A)+\omega\eta_L$,

$$\begin{aligned}
& T_{dR}v(\mathbf{x}+\mathbf{e}_R+\mathbf{e}_N) - T_{dR}v(\mathbf{x}+\mathbf{e}_R) \\
\geq\ & v(\mathbf{x}+\mathbf{e}_R+\mathbf{e}_N-\mathbf{e}_R) - v(\mathbf{x}+\mathbf{e}_R-\mathbf{e}_R) = v(\mathbf{x}+\mathbf{e}_N) - v(\mathbf{x}), \text{ use C8, } \geq v(\mathbf{x}+\mathbf{e}_N) - v(\mathbf{x}+\mathbf{e}_N) - \omega\eta_L \\
=\ & v(\mathbf{x}+\mathbf{e}_A+\mathbf{e}_N-\mathbf{e}_N)+\omega\eta_{NR} - v(\mathbf{x}+\mathbf{e}_A) - \omega\eta_L \geq T_{dR}v(\mathbf{x}+\mathbf{e}_A+\mathbf{e}_N) - T_{dR}v(\mathbf{x}+\mathbf{e}_A)
\end{aligned}$$

and we need to use the parameter constraint in Lemma 1 to complete this part of proof for T_{dR}.

Proof. Proof of C7

In this section, we prove $T_s v(\mathbf{x}+\mathbf{e}_A) - T_s v(\mathbf{x}+\mathbf{e}_R) \geq \omega c_R + (1-\omega)\,\theta\varepsilon$, $s = A, R, N, dN, dR$.

Operator T_A

(1) If $T_A v\,(\mathbf{x}+\mathbf{e}_A) = v\,(\mathbf{x}+\mathbf{e}_A+\mathbf{e}_A) + \omega c_{RM} + (1-\omega)\theta\varepsilon$,

$$\begin{aligned}
& T_A v\,(\mathbf{x}+\mathbf{e}_A) - T_A v\,(\mathbf{x}+\mathbf{e}_R) \\
\geq\ & v\,(\mathbf{x}+\mathbf{e}_A+\mathbf{e}_A) + \omega c_{RM} + (1-\omega)\theta\varepsilon - v\,(\mathbf{x}+\mathbf{e}_R+\mathbf{e}_A) - \omega c_{RM} - (1-\omega)\theta\varepsilon \\
\geq\ & v\,(\mathbf{x}+\mathbf{e}_A) - v\,(\mathbf{x}+\mathbf{e}_R) \geq 0
\end{aligned}$$

(2) If $T_A v\,(\mathbf{x}+\mathbf{e}_A) = v\,(\mathbf{x}+\mathbf{e}_A)$,

$$T_A v\,(\mathbf{x}+\mathbf{e}_A) - T_A v\,(\mathbf{x}+\mathbf{e}_R) \geq v\,(\mathbf{x}+\mathbf{e}_A) - v\,(\mathbf{x}+\mathbf{e}_R) \geq 0$$

Operator T_R

(1) If $x_A > 0$, $T_R v(\mathbf{x}+\mathbf{e}_A) = v(\mathbf{x}+\mathbf{e}_A-\mathbf{e}_A+\mathbf{e}_R)$,

$$\begin{aligned} & T_R v(\mathbf{x}+\mathbf{e}_A) - T_R v(\mathbf{x}+\mathbf{e}_R) \\ = \; & v(\mathbf{x}+\mathbf{e}_A-\mathbf{e}_A+\mathbf{e}_R) - v(\mathbf{x}+\mathbf{e}_R-\mathbf{e}_A+\mathbf{e}_R) \geq v(\mathbf{x}+\mathbf{e}_R-\mathbf{e}_A+\mathbf{e}_R) - v(\mathbf{x}+\mathbf{e}_R-\mathbf{e}_A+\mathbf{e}_R) = 0 \end{aligned}$$

(2) If $x_A = 0$, $T_R v(\mathbf{x}+\mathbf{e}_A) = v(\mathbf{x}+\mathbf{e}_A-\mathbf{e}_A+\mathbf{e}_R)$,

$$T_R v(\mathbf{x}+\mathbf{e}_A) - T_R v(\mathbf{x}+\mathbf{e}_R) = v(\mathbf{x}+\mathbf{e}_A-\mathbf{e}_A+\mathbf{e}_R) - v(\mathbf{x}+\mathbf{e}_R) = 0$$

Operator T_N

(1) If $T_N v(\mathbf{x}+\mathbf{e}_A) = v(\mathbf{x}+\mathbf{e}_A+\mathbf{e}_N) + \omega c_N + (1-\omega)\varepsilon$,

$$\begin{aligned} & T_N v(\mathbf{x}+\mathbf{e}_A) - T_N v(\mathbf{x}+\mathbf{e}_R) \geq v(\mathbf{x}+\mathbf{e}_A+\mathbf{e}_N) + \omega c_N + (1-\omega)\varepsilon - v(\mathbf{x}+\mathbf{e}_R+\mathbf{e}_N) - \omega c_N - (1-\omega)\varepsilon \\ \geq \; & v(\mathbf{x}+\mathbf{e}_R+\mathbf{e}_N) - v(\mathbf{x}+\mathbf{e}_R+\mathbf{e}_N) = 0 \end{aligned}$$

(2) If $T_N v(\mathbf{x}+\mathbf{e}_A) = v(\mathbf{x}+\mathbf{e}_A)$

$$T_N v(\mathbf{x}+\mathbf{e}_A) - T_N v(\mathbf{x}+\mathbf{e}_R) \geq v(\mathbf{x}+\mathbf{e}_A) - v(\mathbf{x}+\mathbf{e}_R) \geq 0$$

Operator T_{dN} (T_{dR} follows the same structure, thus omitted)

(1) If $x_N > 0$, $x_R \geq 0$, $T_{dN} v(\mathbf{x}+\mathbf{e}_A) = v(\mathbf{x}+\mathbf{e}_A-\mathbf{e}_N)$,

$$\begin{aligned} & T_{dN} v(\mathbf{x}+\mathbf{e}_A) - T_{dN} v(\mathbf{x}+\mathbf{e}_R) \geq v(\mathbf{x}+\mathbf{e}_A-\mathbf{e}_N) - v(\mathbf{x}+\mathbf{e}_R-\mathbf{e}_N), \text{ use C6.} \\ \geq \; & v(\mathbf{x}+\mathbf{e}_A) - v(\mathbf{x}+\mathbf{e}_R) \geq 0 \end{aligned}$$

(2) If $x_R > 0$, $x_N \geq 0$, $T_{dN} v(\mathbf{x}+\mathbf{e}_A) = v(\mathbf{x}+\mathbf{e}_A-\mathbf{e}_R) + \omega\eta_{RN}$,

$$\begin{aligned} & T_{dN} v(\mathbf{x}+\mathbf{e}_A) - T_{dN} v(\mathbf{x}+\mathbf{e}_R) \geq v(\mathbf{x}+\mathbf{e}_A-\mathbf{e}_R) + \omega\eta_{RN} - v(\mathbf{x}+\mathbf{e}_R-\mathbf{e}_R) - \omega\eta_{RN} \\ \geq \; & v(\mathbf{x}+\mathbf{e}_A) - v(\mathbf{x}+\mathbf{e}_R) \geq 0 \end{aligned}$$

(3) If $x_N = 0$, $x_R \geq 0$, $T_{dN} v(\mathbf{x} + \mathbf{e}_A) = v(\mathbf{x} + \mathbf{e}_A) + \omega\eta_L$,

$$T_{dN} v(\mathbf{x} + \mathbf{e}_A) - T_{dN} v(\mathbf{x} + \mathbf{e}_R) \geq v(\mathbf{x} + \mathbf{e}_A) + \omega\eta_L - v(\mathbf{x} + \mathbf{e}_R) - \omega\eta_L \geq 0$$

Proof. Proof of C8In this section, we prove $T_s v(\mathbf{x} + \mathbf{e}_N) - T_s v(\mathbf{x}) \geq -\omega\eta_L$ and $T_s v(\mathbf{x} + \mathbf{e}_R) - T_s v(\mathbf{x}) \geq -\omega\eta_L$.

Operator T_A

(1) If $T_A v(\mathbf{x} + \mathbf{e}_i) = v(\mathbf{x} + \mathbf{e}_i + \mathbf{e}_A) + \omega c_{RM} + (1-\omega)\theta\varepsilon$, $i = N, R$, $T_A v(\mathbf{x}) = v(\mathbf{x} + \mathbf{e}_A) + \omega c_{RM} + (1-\omega)\theta\varepsilon$,

$$\begin{aligned}
& T_A v(\mathbf{x} + \mathbf{e}_N) - T_A v(\mathbf{x}) \geq v(\mathbf{x} + \mathbf{e}_N + \mathbf{e}_A) + \omega c_{RM} + (1-\omega)\theta\varepsilon - v(\mathbf{x} + \mathbf{e}_A) - \omega c_{RM} - (1-\omega)\theta\varepsilon \\
\geq\ & v(\mathbf{x} + \mathbf{e}_N) - v(\mathbf{x}) \geq -\omega\eta_L \\
& T_A v(\mathbf{x} + \mathbf{e}_R) - T_A v(\mathbf{x}) \geq v(\mathbf{x} + \mathbf{e}_R + \mathbf{e}_A) + \omega c_{RM} + (1-\omega)\theta\varepsilon - v(\mathbf{x} + \mathbf{e}_A) - \omega c_{RM} - (1-\omega)\theta\varepsilon \\
\geq\ & v(\mathbf{x} + \mathbf{e}_R) - v(\mathbf{x}) \geq -\omega\eta_L
\end{aligned}$$

(2) If $T_A v(\mathbf{x} + \mathbf{e}_i) = v(\mathbf{x} + \mathbf{e}_i)$, $i = N, R$, $T_A v(\mathbf{x}) = v(\mathbf{x})$,

$$\begin{aligned}
T_A v(\mathbf{x} + \mathbf{e}_N) - T_A v(\mathbf{x}) &\geq v(\mathbf{x} + \mathbf{e}_N) - v(\mathbf{x}) \geq -\omega\eta_L \\
T_A v(\mathbf{x} + \mathbf{e}_R) - T_A v(\mathbf{x}) &\geq v(\mathbf{x} + \mathbf{e}_R) - v(\mathbf{x}) \geq -\omega\eta_L
\end{aligned}$$

Operator T_R

(1) If $x_A > 0$, $T_R v(\mathbf{x} + \mathbf{e}_i) = v(\mathbf{x} + \mathbf{e}_i - \mathbf{e}_A + \mathbf{e}_R)$, $i = N, R$,

$$\begin{aligned}
& T_R v(\mathbf{x} + \mathbf{e}_N) - T_R v(\mathbf{x}) \geq v(\mathbf{x} + \mathbf{e}_N - \mathbf{e}_A + \mathbf{e}_R) - v(\mathbf{x} - \mathbf{e}_A + \mathbf{e}_R) \\
\geq\ & v(\mathbf{x} + \mathbf{e}_N - \mathbf{e}_A) - v(\mathbf{x} - \mathbf{e}_A) \geq -\omega\eta_L \\
& T_R v(\mathbf{x} + \mathbf{e}_R) - T_R v(\mathbf{x}) \geq v(\mathbf{x} + \mathbf{e}_R - \mathbf{e}_A + \mathbf{e}_R) - v(\mathbf{x} - \mathbf{e}_A + \mathbf{e}_R) \\
\geq\ & v(\mathbf{x} + \mathbf{e}_R - \mathbf{e}_A) - v(\mathbf{x} - \mathbf{e}_A) \geq -\omega\eta_L
\end{aligned}$$

(2) If $x_A = 0$, $T_R v(\mathbf{x}+\mathbf{e}_i) = v(\mathbf{x}+\mathbf{e}_i)$, $i = N, R$,

$$\begin{aligned} T_R v(\mathbf{x}+\mathbf{e}_N) - T_R v(\mathbf{x}) &\geq v(\mathbf{x}+\mathbf{e}_N) - v(\mathbf{x}) \geq -\omega\eta_L \\ T_R v(\mathbf{x}+\mathbf{e}_R) - T_R v(\mathbf{x}) &\geq v(\mathbf{x}+\mathbf{e}_R) - v(\mathbf{x}) \geq -\omega\eta_L \end{aligned}$$

Operator T_N

(1) If $T_N v(\mathbf{x}+\mathbf{e}_i) = v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i) + \omega c_N + (1-\omega)\varepsilon$, $i = N, R$, $T_N v(\mathbf{x}) = v(\mathbf{x}+\mathbf{e}_N) + \omega c_N + (1-\omega)\varepsilon$,

$$\begin{aligned} & T_N v(\mathbf{x}+\mathbf{e}_N) - T_N v(\mathbf{x}) \geq v(\mathbf{x}+\mathbf{e}_N+\mathbf{e}_N) + \omega c_N + (1-\omega)\varepsilon - v(\mathbf{x}+\mathbf{e}_N) - \omega c_N - (1-\omega)\varepsilon \\ \geq\ & v(\mathbf{x}+\mathbf{e}_N) - v(\mathbf{x}) \geq -\omega\eta_L \\ & T_N v(\mathbf{x}+\mathbf{e}_R) - T_N v(\mathbf{x}) \geq v(\mathbf{x}+\mathbf{e}_R+\mathbf{e}_N) + \omega c_N + (1-\omega)\varepsilon - v(\mathbf{x}+\mathbf{e}_N) - \omega c_N - (1-\omega)\varepsilon \\ \geq\ & v(\mathbf{x}+\mathbf{e}_R) - v(\mathbf{x}) \geq -\omega\eta_L \end{aligned}$$

(2) If $T_N v(\mathbf{x}+\mathbf{e}_i) = v(\mathbf{x}+\mathbf{e}_i)$, $i = N, R$, $T_N v(\mathbf{x}) = v(\mathbf{x})$,

$$\begin{aligned} T_N v(\mathbf{x}+\mathbf{e}_N) - T_N v(\mathbf{x}) &\geq v(\mathbf{x}+\mathbf{e}_N) - v(\mathbf{x}) \geq -\omega\eta_L \\ T_N v(\mathbf{x}+\mathbf{e}_R) - T_N v(\mathbf{x}) &\geq v(\mathbf{x}+\mathbf{e}_R) - v(\mathbf{x}) \geq -\omega\eta_L \end{aligned}$$

Operator T_{dN} (T_{dR} follows the same structure, thus omitted)

$$
\begin{aligned}
& T_{dN} v(\mathbf{x}+\mathbf{e}_N) - T_{dN} v(\mathbf{x}) \\
1) \quad = \quad & v(\mathbf{x}+\mathbf{e}_N-\mathbf{e}_N) - v(\mathbf{x}) - \omega\eta_L = -\omega\eta_L, \text{ if } x_N = x_R = 0, \\
2) \quad \geq \quad & v(\mathbf{x}+\mathbf{e}_N-\mathbf{e}_N) - v(\mathbf{x}-\mathbf{e}_N) \geq -\omega\eta_L, \text{ if } x_N > 0, x_R \geq 0, \\
3) \quad \geq \quad & v(\mathbf{x}+\mathbf{e}_N-\mathbf{e}_N) - v(\mathbf{x}) - \omega\eta_L = -\omega\eta_L, \text{ if } x_N = 0, x_R > 0, \\
4) \quad \geq \quad & v(\mathbf{x}+\mathbf{e}_N-\mathbf{e}_R) + \omega\eta_{RN} - v(\mathbf{x}-\mathbf{e}_R) - \omega\eta_{RN} \geq -\omega\eta_L, \text{ if } x_N \geq 0, x_R > 0. \\
& T_{dN} v(\mathbf{x}+\mathbf{e}_R) - T_{dN} v(\mathbf{x}) \\
1) \quad = \quad & v(\mathbf{x}+\mathbf{e}_R-\mathbf{e}_R) + \omega\eta_{RN} - v(\mathbf{x}) - \omega\eta_L \geq -\omega(\eta_L - \eta_{RN}) \geq -\omega\eta_L, \text{ if } x_N = x_R = 0, \\
2) \quad = \quad & v(\mathbf{x}+\mathbf{e}_R) + \omega\eta_L - v(\mathbf{x}) - \omega\eta_L \geq -\omega\eta_L, \text{ if } x_N = 0,\ x_R \geq 0, \\
3) \quad = \quad & v(\mathbf{x}+\mathbf{e}_R-\mathbf{e}_R) - v(\mathbf{x}-\mathbf{e}_R) \geq -\omega\eta_L, \text{ if } x_N \geq 0,\ x_R > 0, \\
4) \quad \geq \quad & v(\mathbf{x}+\mathbf{e}_R-\mathbf{e}_R) + \omega\eta_{RN} - v(\mathbf{x}-\mathbf{e}_N) \geq -\omega(\eta_L - \omega\eta_{RN}) \geq -\omega\eta_L, \text{ if } x_N > 0, x_R \geq 0, \\
5) \quad \geq \quad & v(\mathbf{x}+\mathbf{e}_R-\mathbf{e}_N) - v(\mathbf{x}-\mathbf{e}_N) \geq -\omega\eta_L, \text{ if } x_N > 0, x_R \geq 0.
\end{aligned}
$$

and we used the parameter constraint in Lemma 1 to complete this part of proof for T_{dR}. □

E.2 Proof of Theorem 4.4.1

Proof. Proof of Theorem 1. We prove Theorem 1 in four parts as follow.

(1) Due to the convexity (C1) of the value function, $v^*(\mathbf{x}+\mathbf{e}_A) - v^*(\mathbf{x})$ increases with $\mathbf{e}_A$ and finally becomes greater than $-\omega c_{RM} - (1-\omega)\theta\varepsilon$ where the cost of accepting one more unit of return is cost-increasing. Therefore, we accept return with the minimum x_A of $v^*(\mathbf{x}+\mathbf{e}_A) - v^*(\mathbf{x}) \geq -\omega c_{RM} - (1-\omega)\theta\varepsilon$ given the inventory levels of new and remanufactured products.

(2) Similarly, with convexity (C1), $v^*(\mathbf{x}+\mathbf{e}_N) - v^*(\mathbf{x})$ increases with $\mathbf{e}_N$ and finally becomes greater than $-\omega c_N - (1-\omega)\varepsilon$. Therefore, we manufacture a new product with the minimum x_N of $v^*(\mathbf{x}+\mathbf{e}_N) - v^*(\mathbf{x}) \geq -\omega c_N - (1-\omega)\varepsilon$ given the inventory levels of returns and remanufactured products.

(3) From C3, we have $v^*(\mathbf{x}+\mathbf{e}_R+\mathbf{e}_R) - v^*(\mathbf{x}+\mathbf{e}_R+\mathbf{e}_N) \geq v^*(\mathbf{x}+\mathbf{e}_R) - v^*(\mathbf{x}+\mathbf{e}_N)$ which leads to $v^*(\mathbf{x}+\mathbf{e}_R-\mathbf{e}_N) - v^*(\mathbf{x}+\mathbf{e}_R-\mathbf{e}_R) \geq v^*(\mathbf{x}-\mathbf{e}_N) - v^*(\mathbf{x}-\mathbf{e}_R)$. It indicates that $v^*(\mathbf{x}-\mathbf{e}_N) - v^*(\mathbf{x}-\mathbf{e}_R)$ increases with $\mathbf{e}_R$. Thus, $v^*(\mathbf{x}-\mathbf{e}_N) - v^*(\mathbf{x}-\mathbf{e}_R)$ finally becomes greater than $\omega\eta_{RN}$ where the cost of using a remanufactured product to serve a demand of new product becomes cheaper than the cost of using a new product to serve. Therefore, we use a remanufactured product to satisfy a demand of new

product when the inventory level of remanufactured products is at least the minimum of $r_R(\mathbf{x}_{-R})$ which is the minimum value of x_R that satisfies $v^*(\mathbf{x}-\mathbf{e}_N)-v^*(\mathbf{x}-\mathbf{e}_R)\geq\omega\eta_{RN}$ if $x_R, x_N>0$ or when $x_N=0$ but $x_R>0$.

(4) Similarly to (1), due to the convexity (C1) of the value function, $v^*(\mathbf{x})-v^*(\mathbf{x}-\mathbf{e}_R)$ increases with $\mathbf{e}_R$ and finally becomes greater than $-\omega(\eta_L-\eta_{RN})$ where the cost of denying the demand is an inferior solution. Therefore, we stop denying demand when $v^*(\mathbf{x})-v^*(\mathbf{x}-\mathbf{e}_R)$ hits $-\omega(\eta_L-\eta_{RN})$.

(5) The same with (3).

(6) The same with (4). □

E.3 Proofs of propositions

Proof. Proof of Proposition 4.4.1(a). For acceptance decision, by looking at C2, we have $v(\mathbf{x}+\mathbf{e}_j+\mathbf{e}_A)-v(\mathbf{x}+\mathbf{e}_j)\geq v(\mathbf{x}+\mathbf{e}_A)-v(\mathbf{x})$. If $v(\mathbf{x}+\mathbf{e}_A)-v(\mathbf{x})\geq-\omega c_{RM}-(1-\omega)\theta\varepsilon$, then $v(\mathbf{x}+\mathbf{e}_j+\mathbf{e}_A)-v(\mathbf{x}+\mathbf{e}_j)$ must be greater than or equal to $-\omega c_{RM}-(1-\omega)\theta\varepsilon$. Then we have:

$$\begin{aligned}&\min\{x_A\geq 0|v^*(\mathbf{x}+\mathbf{e}_A)-v^*(\mathbf{x})\geq-\omega c_{RM}-(1-\omega)\theta\varepsilon\}\\ \geq\ &\min\{x_A^{'}\geq 0|v(\mathbf{x}+\mathbf{e}_j+\mathbf{e}_A)-v(\mathbf{x}+\mathbf{e}_j)\geq-\omega c_{RM}-(1-\omega)\theta\varepsilon\}\end{aligned}$$

which indicates that $s_A(\mathbf{x}_{-A})$ is non-increasing in x_j. This also applies for production decision.

Therefore, the optimal policy $s_i(\mathbf{x}_{-i})$ is non-increasing in x_i, $i=A,R,N$. □

Proof. Proof of Proposition 4.4.1(b). Use C4. $v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_i)-v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_j)\geq v(\mathbf{x}+\mathbf{e}_i+\mathbf{e}_A)-v(\mathbf{x}+\mathbf{e}_j+\mathbf{e}_A)$, which leads to $v(\mathbf{x}+\mathbf{e}_i-\mathbf{e}_j-\mathbf{e}_A)-v(\mathbf{x}-\mathbf{e}_A)\geq v(\mathbf{x}-\mathbf{e}_j)-v(\mathbf{x}-\mathbf{e}_i)$. We take satisfying a new demand by a remanufactured product as an example. If $v(\mathbf{x}-\mathbf{e}_N)-v(\mathbf{x}-\mathbf{e}_R)\geq\omega\eta_{RN}$, $v(\mathbf{x}+\mathbf{e}_R-\mathbf{e}_N-\mathbf{e}_A)-v(\mathbf{x}-\mathbf{e}_A)$ must be greater than or equal to $\omega\eta_{RN}$. Then we have:

$$\begin{aligned}&\min\{x_R\geq 1|v^*(\mathbf{x}-\mathbf{e}_N)-v^*(\mathbf{x}-\mathbf{e}_R)\geq\omega\eta_{RN}, x_N\geq 1\}\\ \geq\ &\min\{x_R^{'}\geq 1|v^*(\mathbf{x}+\mathbf{e}_R-\mathbf{e}_N-\mathbf{e}_A)-v^*(\mathbf{x}-\mathbf{e}_A)\geq\omega\eta_{RN}, x_N\geq 1\}\end{aligned}$$

which indicates that $r_i(\mathbf{x}_{-i})$ is non-increasing in $x_j-\mathbf{e}_A$, $(i,j=N,R,i\neq j)$. This also applies for other demand satisfaction decisions including the case with green customers. □

Proof. Proof of Proposition 4.4.1(c). The proof follows the proof of Proposition 1(a). □

Proof. Proof of Proposition 4.4.1(d). The proof follows C5, C6 and the proof of Proposition 1(a) and 1(b). □

Proof. Proof of Proposition 4.4.2 We here revise the model with the total cost and prove for each operator ($v^*_{total}(\mathbf{x}) = ng^* + h^*(\mathbf{x})$)), certain conditions hold for the successive periods by induction. We see that $h^*(\mathbf{x})/n$ diminishes when n goes to infinity. Therefore, we prove for the total cost case and the results apply for average cost (same for Proposition 3 to Proposition 5).

We here choose T_N to prove the concavity of the value function with respect to ω. All of the operators actually follow the same structure. Here we pick T_N as ω is involved with both economic and environmental costs. In the proof of concavity, we assume $\omega_1 > \omega_2$.

Starting from $v_j = v_{1,j} = v_{2,j} = 0$ when $j = 0$, it is clearly concave. We then prove it by induction. For period $j = s-1$, suppose the optimal cost is concave in the parameter ω, i.e. $v_{s-1}(\mathbf{x}, (1-\alpha)\omega_1 + \alpha\omega_2) \geq (1-\alpha)v_{1,s-1}(\mathbf{x}, \omega_1) + \alpha v_{2,s-1}(\mathbf{x}, \omega_2)$. Then we have:

(1) If $T_N v(\mathbf{x}, (1-\alpha)\omega_1 + \alpha\omega_2) = v(x_A, x_R, x_N + 1) + [(1-\alpha)\omega_1 + \alpha\omega_2]c_N + [(1-\alpha)(1-\omega_1) + \alpha(1-\omega_2)]\varepsilon$,

$$
\begin{aligned}
& v_s(\mathbf{x}, (1-\alpha)\,\omega_1 + \alpha\omega_2) \\
=& T_N v_{s-1}\,(\mathbf{x}, (1-\alpha)\,\omega_1 + \alpha\omega_2) \\
=& v_{s-1}\,(\mathbf{x} + \mathbf{e}_N, (1-\alpha)\omega_1 + \alpha\omega_2) + [(1-\alpha)\,\omega_1 + \alpha\omega_2]\,c_N + [(1-\alpha)\,(1-\omega_1) + \alpha\,(1-\omega_2)]\,\varepsilon \\
\geq& (1-\alpha)v_{1,s-1}(\mathbf{x} + \mathbf{e}_N, \omega_1) + (1-\alpha)(\omega_1 c_N + (1-\omega_1)\varepsilon) + \alpha v_{2,s-1}(\mathbf{x} + \mathbf{e}_N, \omega_2) + \alpha(\omega_1 c_N + (1-\omega_1)\varepsilon) \\
\geq& (1-\alpha)T_N v_{1,s-1}(\mathbf{x}, \omega_1) + \alpha T_N v_{2,s-1}(\mathbf{x}, \omega_2) \\
=& (1-\alpha)v_{1,s}(\mathbf{x}, \omega_1) + \alpha v_{2,s}(\mathbf{x}, \omega_2)
\end{aligned}
$$

(2) If $T_N v\,(\mathbf{x}, (1-\alpha)\,\omega_1 + \alpha\omega_2) = v\,(x_A, x_R, x_N)$,

$$
\begin{aligned}
& v_s\,(\mathbf{x}, (1-\alpha)\,\omega_1 + \alpha\omega_2) \\
=& T_N v_{s-1}\,(\mathbf{x}, (1-\alpha)\,\omega_1 + \alpha\omega_2) = v_{s-1}\,(\mathbf{x}, (1-\alpha)\,\omega_1 + \alpha\omega_2) \\
\geq& (1-\alpha)v_{1,s-1}(\mathbf{x}, \omega_1) + \alpha v_{2,s-1}(\mathbf{x}, \omega_2) \geq (1-\alpha)T_N v_{1,s-1}(\mathbf{x}, \omega_1) + \alpha T_N v_{2,s-1}(\mathbf{x}, \omega_2) \\
=& (1-\alpha)v_{1,s}(\mathbf{x}, \omega_1) + \alpha v_{2,s}(\mathbf{x}, \omega_2)
\end{aligned}
$$

□

Proof. Proof of Proposition 4.4.3. We here also choose T_N to prove the monotonicity as other operators follow exactly the same structure. Starting from $v_{1,j} = v_{2,j}$ when $j = 0$, it is clearly non-decreasing. We then prove it by induction. For period

$j = s-1$, suppose the optimal cost is non-decreasing in the parameter ω, i.e. $v_{1,s-1}(\mathbf{x},\omega_1) \geq v_{2,s-1}(\mathbf{x},\omega_2)$. Then we have:

$$
\begin{aligned}
& v_{1,s}(\mathbf{x},\omega_1) - v_{2,s}(\mathbf{x},\omega_2) \\
=& T_N v_{1,s-1}(\mathbf{x},\omega_1) - T_N v_{2,s-1}(\mathbf{x},\omega_2) \\
& \text{(1) if } T_N v_{1,s-1}(\mathbf{x},\omega_1) = v_{1,s-1}(\mathbf{x}+\mathbf{e}_N,\omega_1) + \omega_1 c_N + (1-\omega_1)\varepsilon \\
\geq& v_{1,s-1}(\mathbf{x}+\mathbf{e}_N,\omega_1) + \omega_1 c_N + (1-\omega_1)\varepsilon - v_{2,s-1}(\mathbf{x}+\mathbf{e}_N,\omega_2) - \omega_2 c_N - (1-\omega_2)\varepsilon \\
\geq& \omega_1 c_N + (1-\omega_1)\varepsilon - \omega_2 c_N - (1-\omega_2)\varepsilon \geq 0 \\
& \text{(2) if } T_N v_{1,s-1}(\mathbf{x},\omega_1) = v_{1,s-1}(\mathbf{x},\omega_1) \\
\geq& v_{1,s-1}(\mathbf{x},\omega_1) - v_{2,s-1}(\mathbf{x},\omega_2) \geq 0
\end{aligned}
$$

Since other operators follow the same structure, the optimal objective value $v_1^* \geq v_2^*$ by induction. □

Proof. Proof of Proposition 4.4.4. We here also choose T_N to prove the monotonicity as other operators follow exactly the same structure. First, it is obvious to see that the value function of $\frac{v^C(\mathbf{x},\omega)}{\omega}$ also satisfy Lemma 1 and Theorem 1. Now, let $\omega_1 > \omega_2$ Starting from $\frac{v^C_{1,j}}{\omega_1} = \frac{v^C_{2,j}}{\omega_2}$ when $j = 0$, it is clearly non-increasing. We then prove it by induction. For period $j = s-1$, suppose the optimal cost is non-decreasing in the parameter ω, i.e. $\frac{v^C_{1,s-1}(\mathbf{x},\omega_1)}{\omega_1} \leq \frac{v^C_{2,s-1}(\mathbf{x},\omega_2)}{\omega_2}$. Then we have:

$$
\begin{aligned}
& \frac{v^C_{1,s}(\mathbf{x},\omega_1)}{\omega_1} - \frac{v^C_{2,s}(\mathbf{x},\omega_2)}{\omega_2} \\
=& \frac{T_N v^C_{1,s-1}(\mathbf{x},\omega_1)}{\omega_1} - \frac{T_N v^C_{2,s-1}(\mathbf{x},\omega_2)}{\omega_2} \\
& \text{(1) if } T_N v^C_{2,s-1}(\mathbf{x},\omega_2) = v^C_{2,s-1}(\mathbf{x}+\mathbf{e}_N,\omega_2) + \omega_2 c_N + (1-\omega_2)\varepsilon \\
\leq& \frac{v^C_{1,s-1}(\mathbf{x}+\mathbf{e}_N,\omega_1)}{\omega_1} + c_N + \frac{1-\omega_1}{\omega_1}\varepsilon - \frac{v^C_{2,s-1}(\mathbf{x}+\mathbf{e}_N,\omega_2)}{\omega_2} - c_N - \frac{1-\omega_2}{\omega_2}\varepsilon \leq 0 \\
& \text{(2) if } T_N v^C_{2,s-1}(\mathbf{x},\omega_2) = v^C_{2,s-1}(\mathbf{x},\omega_2) \\
\leq& \frac{v^C_{1,s-1}(\mathbf{x},\omega_1)}{\omega_1} - \frac{v^C_{2,s-1}(\mathbf{x},\omega_2)}{\omega_2} \leq 0
\end{aligned}
$$

Since other operators follow the same structure, the optimal objective value $\frac{v_1^{C*}}{\omega_1} \leq \frac{v_2^{C*}}{\omega_2}$ by induction.

□

Proof. Proof of Proposition 4.4.5. We here also choose T_N to prove the monotonicity as other operators follow exactly the same structure. First, it is obvious to see that the value function of $\frac{v^E(\mathbf{x},\omega)}{1-\omega}$ also satisfy Lemma 1 and Theorem 1. Now, let $\omega_1 > \omega_2$ Starting from $\frac{v_{1,j}^E}{1-\omega_1} = \frac{v_{2,j}^E}{1-\omega_2}$ when $j = 0$, it is clearly non-decreasing. We then prove it by induction. For period $j = s-1$, suppose the optimal cost is non-decreasing in the parameter ω, i.e. $\frac{v_{1,s-1}^E(\mathbf{x},\omega_1)}{1-\omega_1} \geq \frac{v_{2,s-1}^E(\mathbf{x},\omega_2)}{1-\omega_2}$. Then we have:

$$
\begin{aligned}
&\frac{v_{1,s}^E(\mathbf{x},\omega_1)}{1-\omega_1} - \frac{v_{2,s}^E(\mathbf{x},\omega_2)}{1-\omega_2} \\
=&\frac{T_N v_{1,s-1}^E(\mathbf{x},\omega_1)}{1-\omega_1} - \frac{T_N v_{2,s-1}^E(\mathbf{x},\omega_2)}{1-\omega_2} \\
&(1) \text{ if } T_N v_{1,s-1}^E(\mathbf{x},\omega_1) = v_{2,s-1}^E(\mathbf{x}+\mathbf{e}_N,\omega_1) + \omega_1 c_N + (1-\omega_1)\varepsilon \\
\geq&\frac{v_{1,s-1}^E(\mathbf{x}+\mathbf{e}_N,\omega_1)}{1-\omega_1} + \frac{\omega_1}{1-\omega_1}c_N + \varepsilon - \frac{v_{2,s-1}^E(\mathbf{x}+\mathbf{e}_N,\omega_2)}{1-\omega_2} - \frac{\omega_2}{1-\omega_2}c_N - \varepsilon \geq 0 \\
&(2) \text{ if } T_N v_{1,s-1}^E(\mathbf{x},\omega_2) = v_{1,s-1}^E(\mathbf{x},\omega_1) \\
\geq&\frac{v_{1,s-1}^E(\mathbf{x},\omega_1)}{1-\omega_1} - \frac{v_{2,s-1}^E(\mathbf{x},\omega_2)}{1-\omega_2} \geq 0
\end{aligned}
$$

Since other operators follow the same structure, the optimal objective value $\frac{v_1^{E*}}{1-\omega_1} \geq \frac{v_2^{E*}}{1-\omega_2}$ by induction. □

Proof. Proof of Proposition 4.5.1. The proof of Proposition 6 strictly follows the structure in proving Proposition 2 to Proposition 5 by changing the single-component case to multi-component case. It is therefore omitted here. □

E.4 Heuristic algorithm

The ALBP heuristic procedure is summarized in Algorithm 1.

Algorithm 1 Procedure to derive optimal base-stock levels for the ALBP heuristic

procedure BASESTOCK($s_A, \mathbf{s}_N, \mathbf{r}_R, \mathbf{r}_N, \mathbf{r}'_R, \mathbf{r}'_N$)

Initialization. Set $\kappa = L$ ($L \in \mathbb{Z}^+$), $g = \Gamma + 1$, $g_{temp} = \Gamma$, where Γ is a vary large positive number. Let π^A and π^B be two vectors representing ($s_A, \mathbf{s}_N, \mathbf{r}_R, \mathbf{r}_N, \mathbf{r}'_R, \mathbf{r}'_N$) that minimize the cost in the following two loops, respectively.

repeat

while ($g - g_{temp} > \epsilon$, $g_{temp} > 0$ and $\kappa > 0$), OR ($\kappa = L$) **do** $\triangleright$ $\epsilon > 0$, represents the objective cost variation tolerance due to the results' volatility in simulation. $\kappa = L$ means the first iteration takes the initial values, although the initial $g - g_{temp} = 1$ may be smaller than ϵ.

$g \leftarrow g_{temp}$, simulate the Markov chain under ALBP using base-stock level κ for M times g^j_{temp}, $j = 1, 2, \ldots, M$;

$g_{temp} \leftarrow \frac{\sum_{j=1}^{M} g^j_{temp}}{M}$, $\kappa \leftarrow \kappa - 1$;

end while

until κ sequentially iterates over $s_A, s_{Ni}, r_{Ri}, r_{Ni}, r'_{Ri}, r'_{Ni}$ ($i = 1, \ldots, n$), $g_A(\pi^A) \leftarrow g$, $g_{temp} \leftarrow g$.

Set $g = \Gamma$, $\kappa = L$;

repeat

while $g - g_{temp} > \epsilon$ and $g_{temp} > 0$ **do**

$g \leftarrow g_{temp}$, simulate the Markov chain under ALBP using base-stock level κ for M times g^j_{temp}, $j = 1, 2, \ldots, M$;

$g_{temp} \leftarrow \frac{\sum_{j=1}^{M} g^j_{temp}}{M}$, $\kappa \leftarrow \kappa + 1$;

end while

until κ sequentially iterates over $s_A, s_{Ni}, r_{Ri}, r_{Ni}, r'_{Ri}, r'_{Ni}$ ($i = 1, \ldots, n$), $g_B(\pi^B) \leftarrow g$.

$g^* \leftarrow \min\{g_A(\pi^A), g_A(\pi^B)\}$, ($s_A, \mathbf{s}_N, \mathbf{r}_R, \mathbf{r}_N, \mathbf{r}'_R, \mathbf{r}'_N$) $\leftarrow \arg\min\{g_A(\pi^A), g_A(\pi^B)\}$.

return ($g^*, s_A, \mathbf{s}_N, \mathbf{r}_R, \mathbf{r}_N, \mathbf{r}'_R, \mathbf{r}'_N$).

end procedure

APPENDIX F

PROOFS OF CHAPTER 5.

F.1 Proof of Lemma 5.6.1.

Proof. Let us first show the joint concavity in (p^N, p^R, u^N, i) for $K(x^N, x^R)$ by looking at the second order conditions. We have:

$$\frac{\partial^2 K(x^N, x^R)}{\partial {p^N}^2} = \frac{\partial^2 K(x^N, x^R)}{\partial {p^R}^2} = \frac{\partial^2 K(x^N, x^R)}{\partial {u^N}^2} = 0$$

and

$$\frac{\partial^2 K(x^N, x^R)}{\partial i^2} = [-c_R^{''}(i)F^q(i) - 2c_R^{'}(i)f^q(i) - (c_R(i) + s)f^{q'}(i)]\mathbb{E}[R]$$

which is non-positive since $c_R(i)$ is convex non-decreasing function and $f^q(i)$ is the pdf of a uniform distribution.

Moreover, $\frac{\partial^2 K(x^N, x^R)}{\partial(\cdot)(\cdot)} = 0$ for all the decision variables. So $K(x^N, x^R)$ is jointly concave in (p^N, p^R, u^N, i).

For $L(x^N, x^R) = p^N\mathbb{E}[D^N] - \mathbb{E}G_t^N(u^N - D^N) + p^R\mathbb{E}[D^R] - \mathbb{E}G_t^R(x^R + F^q(i)r - D^R)$, we check the for each of the four individual item in the right hand side.

(1) In $p^N(\alpha_N - \beta_N p^N - \gamma(p^N - p^R) + \varepsilon^N) + p^R(\alpha_R - \beta_R p^R + \gamma(p^N - p^R) + \varepsilon^R)$, the second order conditions are $\frac{\partial^2 L(x^N, x^R)}{\partial {p^N}^2} = -2(\beta_N + \gamma)$, $\frac{\partial^2 L(x^N, x^R)}{\partial {p^R}^2} = -2(\beta_R + \gamma)$ and $\frac{\partial^2 L(x^N, x^R)}{\partial {u^N}^2} = \frac{\partial^2 L(x^N, x^R)}{\partial i^2} = 0$, which indicate separate concavity. $\frac{\partial^2 L(x^N, x^R)}{\partial(\cdot)(\cdot)} = 0$ for p^N and i, p^N and u^N, p^R and i, p^R and u^N, i and u^N, and $\frac{\partial^2 L(x^N, x^R)}{\partial p^N p^R} = 2\gamma$. The Hessian

matrix is

$$\{\{-2(\beta_N+\gamma), 2\gamma, 0, 0\}, \{2\gamma, -2(\beta_R+\gamma), 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}\}$$

in which we have the four eigenvalues

$$0, 0, -(\beta_N+\beta_R+2\gamma) \pm \sqrt{(\beta_N+\beta_R+2\gamma)^2 - 4[(\beta_N+\gamma)(\beta_R+\gamma)-\gamma^2]}$$

in which $(\beta_N+\gamma)(\beta_R+\gamma)-\gamma^2 \geq 0$ and $-(\beta_N+\beta_R+2\gamma) \pm \sqrt{\cdot} \leq 0$. As expectation preserves concavity (Federgruen & Heching, 1999), this item is jointly concave in (p^N, p^R, u^N, i).

(2) Since $u^N - (\alpha_N - \beta_N p^N - \gamma(p^N - p^R) + \varepsilon^N)$ is linear in (p^N, p^R, u^N, i) and maximization as well as expectation preserve convexity, $-h_t^N[u^N - (\alpha_N - \beta_N p^N - \gamma(p^N - p^R) + \varepsilon^N)]^+$ is concave in (p^N, p^R, u^N, i). Meanwhile, $(\alpha_N - \beta_N p^N - \gamma(p^N - p^R) + \varepsilon^N) - u^N$ is linear in (p^N, p^R, u^N, i) and minimization as well as expectation preserve concavity, $+b_t^N[u^N - (\alpha_N - \beta_N p^N - \gamma(p^N - p^R) + \varepsilon^N)]^-$ is jointly concave in (p^N, p^R, u^N, i).

(3) Since $x^R + F^q(i)r - (\alpha_R - \beta_R p^R + \gamma(p^N - p^R) + \varepsilon^R)$ has second order conditions as $\frac{\partial^2 L(x^N, x^R)}{\partial {p^N}^2} = \frac{\partial^2 L(x^N, x^R)}{\partial {p^R}^2} = \frac{\partial^2 L(x^N, x^R)}{\partial {u^N}^2} = 0$ and $\frac{\partial^2 L(x^N, x^R)}{\partial i^2} = f^{q'}(i)r = 0$ which clearly indicates the separate convexity on (p^N, p^R, u^N, i). Then by taking the same arguments with (2), this item is jointly concave in (p^N, p^R, u^N, i).

For the joint concavity in x^N and x^R, the structure follows the proof in Lemma 5.7.1 for proving its joint concavity in x_t^N and x_t^R. □

F.2 Proof of Theorem 5.6.1.

Proof. Given Lemma 5.6.1, we have both $K(x^N, x^R)$ and $L(x^N, x^R)$ concave in (p^N, p^R, u^N, i). Since the sum of the two concave functions defined on the same domain preserves concavity, we conclude that $V^{single}(x^N, x^R)$ is concave in (p^N, p^R, u^N, i). Therefore, each of the four strategies at the point $(p^{N*}, p^{R*}, u^{N*}, i^*)$ takes the value of either an interior solution within its corresponding domain or at the boundaries. (We first assume interior solutions and take the first order conditions with respect to the decision variables. Then, we evaluate whether the optimal point locate within the variables' domains and whether it is a saddle point.) Given the initial stock levels x_N and x_R, we can obtain the optimal strategies shown in Theorem 5.6.1. For Theorem 5.7.1 and the optimal policy in endogenous prices of multi-period model, the proofs follow the same structure given the proofs of Lemma 5.7.1 and Lemma 5.7.2, respectively, in the following appendix. □

F.3 Proof of Lemma 5.7.1.

Proof. We first prove part (a) of Lemma 5.7.1 by induction. Here $v_{T+1}(x^N_{T+1}, x^R_{T+1}) = g_N(x^N_{T+1}) + g_N(x^R_{T+1})$ is obviously concave in x^N_{T+1} and x^R_{T+1}. Assume $v_{t+1}(x^N_{t+1}, x^R_{t+1})$ is jointly concave in x^N_{t+1} and x^R_{t+1}, we prove that $V_t(x^N_t, x^R_t)$ is jointly concave in x^N_t and x^R_t.

$$\begin{aligned} v_t(x^N_t, x^R_t) =& \mathbb{E}\Big\{ p^N_t D^N_t - c_N(u^N_t - x^N_t) - G^N_t(u^N_t - D^N_t) + p^R_t D^R_t - c_R(i_t)F^q(i_t)R_t \\ & - G^R_t(x^R_t + F^q(i_t)R_t - D^R_t) + s(R_t - F^q(i_t)R_t) + \rho v_{t+1}(\cdot,\cdot) \Big\} \end{aligned}$$

It is easy to see that the coefficient c_N of x^N is positive, which indicates concavity. For

x_t^R, $-G_t^R[x_t^R + F^q(i_t)R_t - D_t^R]^+$ indicates concavity as $G_t^R[x_t^R + F^q(i_t)R_t - D_t^R]^+$ is convex in x_t^R. Because the value function is (discretely but concavely) linear in each state variable, the second order partial derivatives on the regarding sub-domain is zero, which implies joint concavity. For the last item, the joint concavity is indicated by induction. We have to show $v_t(\cdot,\cdot)''_{x_t^N, x_t^R} = 0$ and $v_t(\cdot,\cdot)''_{x_t^N} \leq 0$, $v_t(\cdot,\cdot)''_{x_t^R} \leq 0$ for four cases: (1) $u_t^N - D_t^N \geq 0$ and $x_t^R + F^q(i)R_t - D_t^R \geq 0$, (2) $u_t^N - D_t^N \geq 0$ and $x_t^R + F^q(i)R_t - D_t^R < 0$, (3) $u_t^N - D_t^N < 0$ and $x_t^R + F^q(i)R_t - D_t^R \geq 0$, and (4) $u_t^N - D_t^N < 0$ and $x_t^R + F^q(i)R_t - D_t^R < 0$. Since this is the same with the proof of Lemma 5.6.1, we omit the discussions of the cases and use the same structure to demonstrate that we take the second order condition with respect to each case. Given the convexity of $G_t^i(y_t^i)$, $i = N, R$, if we prove the concavity in each case, the value function is concave (S. X. Zhou & Yu, 2011). (The rest of the proof follows this argument.) We first show $v_t(\cdot,\cdot)''_{x_t^N, x_t^R} = 0$ and the cost of period $T+1$ is certainly jointly concave in x_{T+1}^N and x_{T+1}^R. Assume this holds for period $t+1$, we here prove it still holds

for period t.

$$
\begin{aligned}
&\frac{\partial^2 v_t(x_t^N, x_t^R)}{\partial x_t^N \partial x_t^R} \\
=&\frac{\partial^2 v_{t+1}(u_t^N - D_t^N, x_t^R + F^q(i)R_t - D_t^R)}{\partial x_t^N \partial x_t^R} \\
=&\left(\frac{\partial^2 v_{t+1}(x_{t+1}^N, x_{t+1}^R)}{\partial (x_{t+1}^N)^2}\frac{d(u_t^N - D_t^N)}{dx_t^R} + \frac{\partial^2 v_{t+1}(x_{t+1}^N, x_{t+1}^R)}{\partial x_{t+1}^N \partial x_{t+1}^R}\frac{d(x_t^R + F^q(i)R_t - D_t^R)}{dx_t^R}\right) \\
&\times \frac{d(u_t^N - D_t^N)}{dx_t^N} + \frac{\partial v_{t+1}(x_{t+1}^N, x_{t+1}^R)}{\partial x_{t+1}^N}\frac{\partial^2 (u_t^N - D_t^N)}{\partial x_t^N \partial x_t^R} + \\
&\left(\frac{\partial^2 v_{t+1}(x_{t+1}^N, x_{t+1}^R)}{\partial x_{t+1}^N \partial x_{t+1}^R}\frac{d(u_t^N - D_t^N)}{dx_t^R} + \frac{\partial^2 v_{t+1}(x_{t+1}^N, x_{t+1}^R)}{\partial (x_{t+1}^R)^2}\frac{d(x_t^R + F^q(i)R_t - D_t^R)}{dx_t^R}\right) \\
&\times \frac{d(x_t^R + F^q(i)R_t - D_t^R)}{dx_t^N} + \frac{\partial v_{t+1}(x_{t+1}^N, x_{t+1}^R)}{\partial x_{t+1}^R}\frac{\partial^2 (x_t^R + F^q(i)R_t - D_t^R)}{\partial x_t^N \partial x_t^R}
\end{aligned}
$$

Since both partial derivatives of $u_t^N - D_t^N, x_t^R + F^q(i)R_t - D_t^R$ are zero, and both first order condition of $u_t^N - D_t^N, x_t^R + F^q(i)R_t - D_t^R$ with respect to x_t^N are zero, the above equation equals to zero. Therefore, $v_t(\cdot,\cdot)''_{x_t^N, x_t^R} = 0$.

For $v_t(\cdot,\cdot)''_{x_t^N} \leq 0$, $v_t(\cdot,\cdot)''_{x_t^R} \leq 0$, we use chain rule to compute the second order condition and have:

$$v_{t+1}(\cdot,\cdot)''_{x_t^N} = 0$$

and

$$v_{t+1}(\cdot,\cdot)''_{x_t^R} = v_{t+1}(\cdot,\cdot)''_{x_{t+1}^R}\left(\frac{d(x_t^R + F^q(i)R_t - D_t^R)}{dx_t^R}\right) \leq 0.$$

Together with that the expectation preserve concavity, $V_t(x_t^N, x_t^R)$ is jointly concave in x_t^N and x_t^R.

For (b), we first decompose the objective function into two parts as done in the single-

period setting, $K_t^{exg}(x_t^N, x_t^R)$ and $L_t^{exg}(x_t^N, x_t^R)$, for the exogenous price case as in the single-period situation. Here we have:

$$K_t^{exg}(x_t^N, x_t^R) = - c_N(u_t^N - x_t^N) - [(c_R(i_t) + s)F^q(i_t) - s]\mathbb{E}[R_t]$$

$$L_t^{exg}(x^N, x^R) = p_t^N \mathbb{E}[D_t^N] - \mathbb{E}G_t^N(u_t^N - D_t^N) + p_t^R \mathbb{E}[D_t^R] - \mathbb{E}G_t^R(x_t^R + F^q(i_t)R_t - D_t^R)$$

For $K_t^{exg}(x_t^N, x_t^R)$, the Hessian matrix is $\{\{0,0\}, \{0, [-c_R''(i_t)F^q(i_t) - 2c_R'(i_t)f^q(i_t) - (c_R(i_t) + s)f^{q'}(i_t)]\mathbb{E}[R_t]\}\}$ and both eigenvalues are non-positive. Therefore, $K_t^{exg}(x_t^N, x_t^R)$ is jointly concave in u_t^N and i_t. For $L_t^{exg}(x_t^N, x_t^R)$, we evaluate each expectation individually. The first three items is clearly jointly concave in both decision variables. In the last one, the Hessian matrix of is $\{\{0,0\}, \{0, -h_R f^{q'}(i_t)\mathbb{E}[R_t]\}\}$ indicating that two eigenvalues are non-positive. As minimization and expectation preserve the concavity, this item is jointly concave in u_t^N and i_t. Therefore, the non-positive second order condition of $L_t^{exg}(x_t^N, x_t^R)$ in u_t^N and i_t indicates the separate concavity. To prove the concavities in the last item, we use induction. First in period $T+1$, the objective function is clearly concave in the two variable. Assume it holds for period $t+1$, we look at the last item.

In $v_{t+1}(u_t^N - D_t^N, x_t^R + F^q(i_t)R_t - D_t^R)$, we have

$$v_{t+1}(u_t^N - D_t^N, x_t^R + F^q(i_t)R_t - D_t^R)''_{u_t^N} = \frac{\partial^2 v_{t+1}}{\partial (x_{t+1}^N)^2}\left(\frac{d(u_t^N - D_t^N)}{du_t^N}\right)^2 \leq 0$$

and

$$v_{t+1}(u_t^N - D_t^N, x_t^R + F^q(i_t)R_t - D_t^R)''_{i_t} = \frac{\partial^2 v_{t+1}}{\partial (x_{t+1}^R)^2}\left(\frac{d(x_t^R + F^q(i_t)R_t - D_t^R)}{di}\right)^2 \leq 0$$

Therefore, $V_t(x_t^N, x_t^R)$ is separately concave in u_t^N and i_t. $\square$

F.4 Proof of Proposition 5.7.1.

Proof. We prove this proposition by induction. For $v_{T+1}(x_{T+1}^N, x_{T+1}^R)$, it is certainly can be decomposed into $M_{T+1}(x_{T+1}^N) = 0$ and $N_{T+1}(x_{T+1}^R) = 0$. Suppose $v_{t+1}(x_{t+1}^N, x_{t+1}^R) = M_{t+1}(x_{t+1}^N) + N_{t+1}(x_{t+1}^R)$, then

$$
\begin{aligned}
&V_t(x_t^N, x_t^R) \\
=&\mathbb{E}\Big\{p_t^N D_t^N - c_N Q_t^N - G_t^N(x_t^N + Q_t^N - D_t^N) + p_t^R D_t^R - c_R(i_t)F^q(i_t)R_t \\
&- G_t^R(x_t^R + F^q(i_t)R_t - D_t^R) + s(R_t - F^q(i_t)R_t) + \rho[M_{t+1}(x_{t+1}^N) + N_{t+1}(x_{t+1}^R)]\Big\} \\
=&\mathbb{E}\Big\{p_t^N D_t^N - c_N Q_t^N - G_t^N(x_t^N + Q_t^N - D_t^N) + \rho M_{t+1}(x_{t+1}^N)\Big\} \\
&+ \mathbb{E}\Big\{p_t^R D_t^R - c_R(i_t)F^q(i_t)R_t - G_t^R(x_t^R + F^q(i_t)R_t - D_t^R) + s(R_t - F^q(i_t)R_t) + \rho N_{t+1}(x_{t+1}^R)\Big\} \\
=&M_t(x_t^N) + N_t(x_t^R)
\end{aligned}
$$

where $M_t(x_t^N)$ and $N_t(x_t^R)$ are obviously concave functions in u_t^N and i_t, respectively. $\square$

F.5 Proof of Proposition 5.7.2.

Proof. To prove the first part of the proposition, it is sufficient to show that the objective function is submodular in x_R and i_t. That is, we need to show $\partial^2 V_t(x_t^N, x_t^R)/\partial i_t \partial x_t^R \leq 0$. We prove it by induction. For period $T+1$, the submodularity holds. Assume it holds for the $t+1$ period and we prove it for the t period.

$$
\begin{aligned}
V_t(x_t^N, x_t^R) =& \mathbb{E}\Big\{ p_t^N D_t^N - c_N(u_t^N - x_t^N) - G_t^N(u_t^N - D_t^N) \\
& + p_t^R D_t^R - c_R(i_t) F^q(i_t) R_t - G_t^R(x_t^R + F^q(i_t) R_t - D_t^R) \\
& + s(R_t - F^q(i_t) R_t) + \rho v_{t+1}(u_t^N - D_t^N, x_t^R + F^q(i_t) R_t - D_t^R) \Big\}
\end{aligned}
$$

Since $V_t(x_t^N, x_t^R)''_{i_t, x_t^R} = \rho \mathbb{E}[v_{t+1}(\cdot,\cdot)''_{x_t^R} f^q(i_t) R_t] \leq 0$, the submodularity is proven.

For the second part, apply the same structure. For the $T+1$ period, apparently the partial derivative is 0. Assume period $t+1$ has 0 partial derivative, we have:

$$
\begin{aligned}
V_t(x_t^N, x_t^R) =& \mathbb{E}\Big\{ p_t^N D_t^N - c_N(u_t^N - x_t^N) - G_t^N(u_t^N - D_t^N) + p_t^R D_t^R \\
& - c_R(i_t) F^q(i_t) R_t - G_t^R(x_t^R + F^q(i_t) R_t - D_t^R) + s(R_t - F^q(i_t) R_t) \\
& + \rho v_{t+1}(u_t^N - D_t^N, x_t^R + F^q(i_t) R_t - D_t^R) \Big\}
\end{aligned}
$$

Since $V_t(x_t^N, x_t^R)''_{u_t^N, x_t^N} = 0$, u_t^N does not change with x_t^N when $x_t^N \leq u_t^{N*}$. Afterwards, when $x_t^N > u_t^{N*}$, we have $u_t^{N*} = x_t^N$. Therefore, u_t^N is non-decreasing in x_t^N. □

F.6 Proof of Proposition 5.7.3.

Proof. We denote the optimal "*produce-up-to*" solution of the single-period exogenous price model as $\nu_N^*(x_N)$ and have:

$$
\begin{aligned}
M_t(x_t^N) =& \mathbb{E}\Big\{p_t^N D_t^N - c_N(u_t^N - x_t^N) - G_t^N(u_t^N - D_t^N) + \rho M_{t+1}(x_{t+1}^N)\Big\} \\
=& c_N x_t^N + \mathbb{E}\Big\{p_t^N D_t^N - c_N u_t^N - G_t^N(u_t^N - D_t^N) + \rho M_{t+1}([u_t^N - D_t^N]^+)\Big\}
\end{aligned}
$$

Let $m_t(x_t^N) = \mathbb{E}\Big\{p_t^N D_t^N - c_N u_t^N - G_t^N(u_t^N - D_t^N) + \rho m_{t+1}(u_t^N - D_t^N)\Big\}$. This equation is obviously unrelated to x_t^N, which implies that the optimal value is a constant as long as $x_t^N \leq u_t^N$ under the optimal u_t^N. Therefore, $m_t^{N*}(x_t^N) = m_{t+1}^{N*}(\nu_t^N - D_t^N)$ since $D_t^N \geq 0$ and $\nu_t^N - D_t^N \leq \nu_t^N$. Then we have:

$$
m_t^{N*} = \frac{p_t^N D_t^N - c_N \nu_t^{N*} - G_t^N(\nu_t^{N*} - D_t^N)}{1-\rho}
$$

which is equivalent to minimize the single-period model of exogenous price. □

F.7 Proof of Lemma 5.7.2.

Proof. We prove the concavity by induction. Since $V_{T+1}(x_{T+1}^N, x_{T+1}^R) = g_N(x_{T+1}^N) + g_R(x_{T+1}^R)$ which is clearly jointly concave in x_{T+1}^N and x_{T+1}^R (note that $\frac{\partial^2 v_{T+1}}{\partial\left(x_{T+1}^N\right)^2} \leq 0, \frac{\partial^2 v_{T+1}}{\partial\left(x_{T+1}^R\right)^2} \leq 0$) and concave in $(p_{T+1}^N, p_{T+1}^R, u_{T+1}^N, i_{T+1})$, we assume it holds for the $(t+1)$th period and look at the tth period.

In $V_t(x_t^N, x_t^R)$, the profit-to-go of the current period is jointly concave in x_t^N and x_t^R and concave in $(p_t^N, p_t^R, u_t^N, i_t)$, shown in the proof of Lemma 5.6.1. We only need to look at the last part, $\rho\mathbb{E}\{v_{t+1}(u_t^N - (\alpha_N - \beta_N p_t^N - \gamma(p_t^N - p_t^R)) + \varepsilon_t^N), x_t^R + F^q(i)R_t - (\alpha_R - \beta_R p_t^R + \gamma(p_t^N - p_t^R)) + \varepsilon_t^R)\}$.

The joint concavity in x_t^N and x_t^R is obvious since the Hessian matrix is

$$\{\{0,0\},\{0,\frac{\partial^2 v_{t+1}}{\partial {x_{t+1}^R}^2}\left(\frac{d(x_t^R+F^q(i)R_t-D_t^R)}{dx_t^R}\right)^2\}\}$$

which also keeps joint concavity.

For the joint concavity in $(p_t^N, p_t^R, u_t^N, i_t)$, the one period profit-to-go function is jointly concave in $(p_t^N, p_t^R, u_t^N, i_t)$ as shown in Lemma 5.6.1. For the expected profit of the period $t+1$, the second order conditions with respective to each decision variable are:

$$p_t^N : (\beta_N+\gamma)^2 v_{t+1}(\cdot,\cdot)''_{x_{t+1}^N} + \gamma^2 v_{t+1}(\cdot,\cdot)''_{x_{t+1}^R}, p_t^R : \gamma^2 v_{t+1}(\cdot,\cdot)''_{x_{t+1}^N} + (\beta_R+\gamma)^2 v_{t+1}(\cdot,\cdot)''_{x_{t+1}^R},$$

$$u_t^N : v_{t+1}(\cdot,\cdot)''_{x_{t+1}^N}, \text{ and } i_t : R_t^2 f^{q2}(i_t) v_{t+1}(\cdot,\cdot)''_{x_{t+1}^R}$$

which are non-positive and imply separate concavity. Now we evaluate the second order partial derivatives.

$$v_{t+1}(x_{t+1}^N, x_{t+1}^R)''_{p_t^N,p_t^R} = v_{t+1}(x_{t+1}^N, x_{t+1}^R)''_{x_{t+1}^N}(-\gamma)(\beta_N+\gamma) + v_{t+1}(x_{t+1}^N, x_{t+1}^R)''_{x_{t+1}^R}(\beta_R+\gamma)(-\gamma)$$

$$v_{t+1}(x_{t+1}^N, x_{t+1}^R)''_{p_t^N,u_t^N} = v_{t+1}(x_{t+1}^N, x_{t+1}^R)''_{x_{t+1}^N}(\beta_N+\gamma)$$

$$v_{t+1}(x_{t+1}^N, x_{t+1}^R)''_{p_t^N,i_t} = v_{t+1}(x_{t+1}^N, x_{t+1}^R)''_{x_{t+1}^R} f^q(i_t)(-\gamma)$$

$$v_{t+1}(x_{t+1}^N, x_{t+1}^R)''_{p_t^R,u_t^N} = v_{t+1}(x_{t+1}^N, x_{t+1}^R)''_{x_{t+1}^N}(-\gamma)$$

$$v_{t+1}(x_{t+1}^N, x_{t+1}^R)''_{p_t^R,i_t} = v_{t+1}(x_{t+1}^N, x_{t+1}^R)''_{x_{t+1}^R} f^q(i_t)(\beta_R+\gamma)$$

$$v_{t+1}(x^N_{t+1}, x^R_{t+1})''_{u^N_t, i_t} = 0$$

Denote $v_{t+1}(x^N_{t+1}, x^R_{t+1})''_{x^N_{t+1}} = \widetilde{N}$ and $v_{t+1}(x^N_{t+1}, x^R_{t+1})''_{x^R_{t+1}} = \widetilde{R}$, we have the Hessian matrix:

$$\begin{bmatrix} (\beta_N+\gamma)^2\widetilde{N}+\gamma^2\widetilde{R} & -\gamma[(\beta_N+\gamma)\widetilde{N}+(\beta_R+\gamma)\widetilde{R}] & (\beta_N+\gamma)\widetilde{N} & -\gamma f^q(i_t)\widetilde{R} \\ -\gamma[(\beta_N+\gamma)\widetilde{N}+(\beta_R+\gamma)\widetilde{R}] & \gamma^2\widetilde{N}+(\beta_R+\gamma)^2\widetilde{R} & -\gamma\widetilde{N} & (\beta_R+\gamma)f^q(i_t)\widetilde{R} \\ (\beta_N+\gamma)\widetilde{N} & -\gamma\widetilde{N} & \widetilde{N} & 0 \\ -\gamma f^q(i_t)\widetilde{R} & (\beta_R+\gamma)f^q(i_t)\widetilde{R} & 0 & R_t^2 f^{q2}(i_t)\widetilde{R} \end{bmatrix}$$

Since it is too obscure to have the joint concavity results, we step back and look in pairs. Note that both $\widetilde{N}$ and $\widetilde{R}$ are non-positive.

For p^N_t and p^R_t, the eigenvalues are

$$\frac{(\beta_N+\gamma)^2\widetilde{N}+(\beta_R+\gamma)^2\widetilde{R} \pm \sqrt{((\beta_N+\gamma)^2\widetilde{N}+(\beta_R+\gamma)^2\widetilde{R})^2-4(\beta_N\beta_R+\beta_N\gamma+\beta_R\gamma)^2\widetilde{N}\widetilde{R}}}{2} \leq 0$$

since $\sqrt{\cdot} \geq 0$, they are jointly concave.

For p^N_t and u^N_t, the eigenvalues are

$$\frac{((\beta_N+\gamma)^2+1)\widetilde{N}+\gamma^2\widetilde{R} \pm \sqrt{[((\beta_N+\gamma)^2+1)\widetilde{N}+\gamma^2\widetilde{R}]^2-4\gamma^2\widetilde{N}\widetilde{R}}}{2} \leq 0$$

since $\sqrt{\cdot} \geq 0$, they are jointly concave.

For p^N_t and i_t, the eigenvalues are

$$\frac{(\beta_N+\gamma)^2\widetilde{N}+\gamma^2+f^{q^2}(i_t)R_t^2\widetilde{R} \pm \sqrt{[(\beta_N+\gamma)^2\widetilde{N}+\gamma^2+f^{q^2}(i_t)R_t^2\widetilde{R}]^2-\Phi_1}}{2} \leq 0$$

where $\Phi_1 = 4f^{q^2}(i_t)\widetilde{R}\left(R_t^2(\beta_N+\gamma)^2\widetilde{N}+\gamma^2(R_t^2-1)\widetilde{R}\right) \geq 0$ and $\sqrt{\cdot}$, which means they are jointly concave.

For p_t^R and u_t^N, the eigenvalues are

$$\frac{(\beta_R+\gamma)^2\widetilde{R}+(1+\gamma^2)\widetilde{N} \pm \sqrt{((\beta_R+\gamma)^2\widetilde{R}+(1+\gamma^2)\widetilde{N})^2-4(\beta_R+\gamma)^2\widetilde{N}\widetilde{R}}}{2} \leq 0$$

since $\sqrt{\cdot} \geq 0$, which means they are jointly concave.

For p_t^R and i_t, the eigenvalues are

$$\frac{[(\beta_R+\gamma)^2+R_t^2 f^{q^2}(i_t)]\widetilde{R}+\gamma^2\widetilde{N} \pm \sqrt{\{[(\beta_R+\gamma)^2+R_t^2 f^{q^2}(i_t)]\widetilde{R}+\gamma^2\widetilde{N}\}^2-\Phi_2}}{2} \leq 0$$

where $\Phi_2 = 4f^{q^2}(i_t)\widetilde{R}[(\beta+\gamma)^2(R_t^2-1)\widetilde{R}+\gamma^2 R_t^2\widetilde{N}] \geq 0$ and $\sqrt{\cdot} \geq 0$, which means they are jointly concave.

For u_t^N and i_t, the eigenvalues are $N \leq 0$ and $f^{q^2}(i_t)R_t^2\widetilde{R} \leq 0$ which means they are jointly concave.

Therefore, for any two decision variable, the value function is jointly concave in them. □

F.8 Proof of Proposition 5.7.4.

Proof. For Part (a) and (b), the proofs follows the proof of Proposition 5.7.2. For Part (c) and (d), we use induction and check the sub- or supermodularity for each pair of variables. For $v_{T+1}(x_{T+1}^N, x_{T+1}^R) = g_N(x_{T+1}^N) + g_R(x_{T+1}^R)$, it is certainly sub- or supermodular in p_{T+1}^N and x_{T+1}^N; p_{T+1}^N and x_{T+1}^R; and p_{T+1}^R and x_{T+1}^N. Assume the submodularity and supermodularity hold for $v_{t+1}(x_{t+1}^N, x_{t+1}^R)$ and we check for the tth period. Note that in the

proof of Lemma 5.7.1 we show that $v_t(\cdot,\cdot)''_{x_t^N,x_t^R} = 0$.

$$v_t(x_t^N, x_t^R)''_{p_t^N,x_t^R} = -\gamma\rho\mathbb{E}v_{t+1}(x_{t+1}^N, x_{t+1}^R)''_{x_{t+1}^R} \geq 0;$$

$$v_t(x_t^N, x_t^R)''_{p_t^R,x_t^N} = 0 \text{ when } x_t^N \leq u_t^{N*}, \text{ and}$$

$$v_t(x_t^N, x_t^R)''_{p_t^R,x_t^N} = -\gamma\rho\mathbb{E}v_{t+1}(x_{t+1}^N, x_{t+1}^R)''_{x_{t+1}^R} \geq 0 \text{ when } x_t^N > u_t^{N*};$$

Therefore, the sub- and supermodularity are checked and both π_t^{N*} and π_t^{R*} are non-decreasing in x_t^R (x_t^N) and π_t^{N*} is non-increasing in x_t^N. □

F.9 Proof of Proposition 5.7.5.

Proof. We prove the proposition by induction. Given the holding/backlogging costs are stationary across periods, for period $T+1$, $v_t^*(\iota^*) - v_t^*(i=1) = v_t^*(\iota^*) - v_t^*(i=0) = 0$, hence the inequalities are satisfied. Suppose the inequalities hold for period $t+1$, we prove for period t. Denote $C_t(\cdot)$ as the reward-to-go function in the tth period. When $s > 0$,

$$\begin{aligned}
&v_t^*(\iota_t^*) - v_t^{1*}(i=1)\\
\leq& C_t^*(\iota_t^*) - C_t(i=1) + \rho(v_{t+1}^*(\iota_{t+1}^*) - v_{t+1}^{1*}(i=1))\\
\leq& \mathbb{E}\Big[-h_t^R[x_t^R + F^q(\iota_t^*)R_t - D_t^R]^+ + h_t^R[x_t^R + R_t - D_t^R]^+ + b_t^R[x_t^R + F^q(\iota_t^*)R_t - D_t^R]^-\\
&- b_t^R[x_t^R + R_t - D_t^R]^- + (c_R(1) - (c_R(\iota_t^*) + s)F^q(\iota_t^*))R_t\Big] + \rho\frac{1-\rho^{T+1-(t+1)}}{1-\rho}c_R(1)\mathbb{E}[R]\\
\leq& (h_t^R + c_R(1) - (c_R(\iota_t^*) + s)F^q(\iota_t^*))\mathbb{E}[R_t] + \rho\frac{1-\rho^{T+1-(t+1)}}{1-\rho}(h_t^R + c_R(1))\mathbb{E}[R]\\
\leq& c_R(1)\mathbb{E}[R_t] + \rho\frac{1-\rho^{T+1-(t+1)}}{1-\rho}c_R(1)\mathbb{E}[R] = \frac{1-\rho^{T+1-t}}{1-\rho}(h_t^R + c_R(1))\mathbb{E}[R].
\end{aligned}$$

Similarly, we have:

$$
\begin{aligned}
& v_t^*(\iota^*) - v_t^{0*}(i=0) \\
\leq & C_t^*(\iota_t^*) - C_t(i=0) + \rho(v_{t+1}^*(\iota_{t+1}^*) - v_{t+1}^{0*}(i=0)) \\
\leq & \mathbb{E}[-h_t^R[x_t^R + F^q(\iota_t^*)R_t - D_t^R]^+ + h_t^R[x_t^R - D_t^R]^+ + b_t^R[x_t^R + F^q(\iota_t^*)R_t - D_t^R]^- \\
& - b_t^R[x_t^R - D_t^R]^- + (s - (c_R(\iota_t^*) + s)F^q(\iota_t^*))\, R_t] + \rho\frac{1-\rho^{T+1-(t+1)}}{1-\rho}(b_t^R + s)\mathbb{E}[R] \\
\leq & (b_t^R + s)\mathbb{E}\left[R_t\right] + \rho\frac{1-\rho^{T+1-(t+1)}}{1-\rho}(b_t^R + s)\mathbb{E}[R] \\
= & \frac{1-\rho^{T+1-t}}{1-\rho}(b_t^R + s)\mathbb{E}[R].
\end{aligned}
$$

We therefore prove the upper bounds of the differences between the policies. □