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INTERACTION BETWEEN MONETARY AND FISCAL POLICY IN A NON-RICARDIAN ECONOMY

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Resumé

L'objectif de cette thèse est double. Premièrement, nous analysons l'interaction entre politique monétaire et fiscale dans un cadre non-Ricardien où la politique monétaire est contrainte par la positivité des taux d'intérêt nominaux. Deuxièmement, nous étudions les implications de la dette publique sur les agrégats macroéconomiques.

Tout d'abord, nous étudions l'interaction entre politique monétaire et fiscale du point de vue de l'analyse globale dans une économie Ricardienne avec capital et tenant en compte la contrainte de positivité du taux d'intérêt nominal. Nous démontrons que, dans ce cadre, quatre équilibres stationnaires peuvent coexister. Ces équilibres ont les mêmes caractéristiques dynamiques que les quatre configurations d'équilibre, décrites par Leeper (1991). Mais alors que dans Leeper (1991), chaque équilibre correspond à une configuration particulière des paramètres qui décrivent les politiques fiscale et monétaire comme active ou passive, nous obtenons ces quatre équilibres pour un ensemble unique de paramètres. Nous montrons en particulier que l'équilibre de trappe à liquidité (dette-déflation), qui est également caractérisé par un fort ratio dette publique/PIB, un faible stock de capital et un faible niveau de consommation, possède les propriétés habituellement requises pour la détermination locale de l'équilibre, ainsi que l'équilibre plus traditionnel ciblé par les autorités monétaire et fiscale. Le modèle est calibré en se basant sur des données annuelles européennes et simulé pour évaluer qualitativement les conséquences d'un choc d'anticipation.

Ensuite, ce cadre a l'avantage de regrouper deux équilibres localement déterminés, qui représentent deux environnements très différents: l'environnement "ciblés" et l'environnement de "dette-déflation". Par conséquent, nous pouvons analyser et comparer les effets des réductions d'impôt financées par la dette dans les deux environnements. Nous montrons que, dans l'environnement ciblé, les réductions d'impôt financées par la dette entraîne un effet de richesse positif, induisant une consommation plus élevée, une augmentation du taux d'intérêt réel, une baisse de l'investissement et l'output chute. En revanche, dans l'environnement de dette-déflation, la réduction des impôts implique un effet de

richesse négatif, entraînant la baisse de la consommation et des taux d'intérêt réels, une augmentation de l'investissement et une hausse de l'output.

Finalement, nous étudions les effets de la dette publique sur les agrégats macroéconomiques dans un cadre non-Ricardien. Nous développons un cadre micro-fondé qui combine des taux de marge variables dans le temps, une offre de travail endogène et des générations imbriquées basées sur des familles à durée de vie infinie. La principale contribution de cette étude est de fournir un nouveau mécanisme de transmission de la dette publique à travers les mouvements countra-cycliques des taux de marge induits par les formations d'habitude extérieures profondes. Nous analysons les effets des réductions d'impôts financées par la dette publique. Nous montrons que l'augmentation du taux d'intérêt réel, entraînant des taux de marge plus élevés, ce qui implique une baisse de l'emploi et la consommation. Il est particulièrement intéressant de noter que, même sans capital, un effet d'éviction lié à la dette publique est obtenue dans le long terme. Toutefois, on ne retrouve pas l'effet expansionniste de court terme lié à des réductions d'impôts financées par l'endettement public, ce qui serait éventuellement prévu dans un cadre non-Ricardien. Cela est dû à l'hypothèse de flexibilité des prix. D'un autre côté, nous montrons que l'introduction de l'hypothèse de rigidité des prix dans notre modèle, implique que les réductions d'impôt financées par la dette publique ont un effet expansionniste à court terme tout en préservant l'effet de contraction de long terme.

Summary

The focus of this doctoral thesis is two fold. First, we analyze the interaction between monetary and fiscal policy in a non-Ricardian framework where monetary policy is constrained by the zero lower bound on nominal interest rates. Second, we investigate the implications of government debt on macroeconomic aggregates.

First, we study the interaction between monetary and fiscal policies from the perspective of global analysis in a non-Ricardian economy with capital and a zero bound on the nominal interest rate. We demonstrate that, in such a framework, four steady state equilibria may coexist. These equilibria have the same dynamic characteristics as the four equilibrium configurations, described by Leeper (1991). But whereas in Leeper (1991), each equilibrium corresponds to a particular configuration of the parameters which describe fiscal and monetary policies as active or passive, we get these four equilibria for a single set of parameters. We show in particular that a liquidity trap—debt-deflation—equilibrium, which is also characterized by a high public debt-to-GDP ratio, a low capital stock and a low consumption level, owns the usually required properties for local determinacy, as well as the more traditional equilibrium targeted by the monetary and fiscal authorities. The model is calibrated based on European annual data and simulated in order to qualitatively asses the implications of a self-fulfilling expectation shock.

Next, this framework has the advantage of gathering two locally determinate equilibria, which represent to two very different environments: the "targeted" environment and the "debt-deflation" environment. Therefore, we can analyze and compare the effects of debt-financed tax cuts in both environments. We show that, in the targeted environment, debt-financed tax cuts entail a positive wealth effect, inducing higher consumption, an increase in the real interest rate, a decline in the investment and the output falls. On the other hand, in the debt-deflation environment, the tax reduction implies a negative wealth effect, entailing a decline in consumption, a lower real interest rate, an increase in the investment and the output goes up.

Finally, we study the effects of government debt on macroeconomic aggregates in

a non-Ricardian framework. We develop a micro-founded framework which combines time-varying markups, an endogenous labor supply and overlapping generations based on infinitely-lived families. The main contribution of this study is to provide a new transmission mechanism of public debt through the countercyclical markup movements induced by external deep habits. We analyze the effects of debt-financed tax cuts. We show that the interest rate rises, entailing higher markups, which imply a fall in employment and consumption. It is particularly noteworthy that, even without capital, a crowding out effect of government debt is obtained in the long run. However, the short-run expansionary effect of debt-financed tax cuts, which would eventually be expected in a non-Ricardian framework fails to occur. This is due to the flexible price assumption. On the other hand, we show that incorporating sticky prices in our model, causes debt-financed tax cuts to have a short-run expansionary effect while preserving the long-run contractionary effect.

Chapter 1

Introduction

The issue of the effectiveness of fiscal policy in stimulating economic activity has received renewed attention during the last global recession of 2008-2010. During the last two years, the short-term nominal interest rates have reached (or near) a lower level below which monetary authorities are unable to stimulate the economy by lowering the interest rate further. For instance, in the US, since December 2008, the Fed's target for the fed funds rate has been essentially zero. In this situation, the lack of monetary policy effectiveness has pushed many governments to intervene actively by using fiscal instruments in order to stimulate their economies and fight the recession. Government actions will consequently lead to substantial increase in the level of public debt. In fact, according to the OECD, total industrialized country public debt is now expected to exceed 100% of GDP in 2011. This is the highest level ever reached in peacetime. Since all these events, the interaction between fiscal and monetary policies in terms of macroeconomic stability and the implications of the government debt are part of the current macroeconomic debate.

This dissertation takes part in the debate. It addresses two main questions: *i)*- How do monetary and fiscal policies interact in a non-Ricardian framework when monetary policy is constrained by the zero lower bound (henceforth ZLB) on the nominal interest rate? and *ii)*- What are the effects of government debt on macroeconomic aggregates?

Thereby, aiming to answer these questions, we develop non-Ricardian models, where government debt is non-neutral, offering rich interaction between monetary and fiscal policy and taking into account the risks of the liquidity trap. We wish to present an innovative analysis with respect to the work already accomplished in this area.

In this introductory chapter, we first give an overview of the issues addressed in this doctoral dissertation. In other words, we develop the points made above, that is *i)* and *ii).* Second, we introduce the main topics covered in this thesis, which include liquidity trap, relative importance of monetary and fiscal policy and deep habit formation. We briefly restate what some of the economic literature says on each topic. We provide a connection between these topics, which seem to be unrelated at first glance. We point out that this connection is the key to answer our questions. Then, we give a brief account of the key results that will emerge from our analysis. Finally, we introduce the structure of the thesis.

1.1 Issues

In this section, we highlight the issues addressed in this doctoral thesis. As the title of the thesis suggests, we aim to explore the interaction between monetary and fiscal policy in an economy which departs from Ricardian equivalence.

First, we are interested in an environment where monetary policy is constrained by the ZLB on nominal interest rates. We believe that a non-Ricardian environment, combining wealth effects with ZLB on nominal interest rates, shed lights on theoretical mechanisms that have not been explored before.

Actually, our work is motivated by the deflation experienced during the global economic crisis of 2008-2010. It is observed that most of the advanced countries are stuck in the liquidity trap. In the next section, the liquidity trap will be explained in details. However, we provide now a brief definition[1] of it, for a better understanding. In the liquidity trap, the economy is satiated with liquidity and the nominal interest rate is

[1] See also Krugman (1998), Svensson (1999), among many others.

zero. Therefore, the conventional open-market operations are no longer able to stabilize the economy as money and short-term government bonds become perfect substitutes.

In the liquidity trap, fiscal instruments are required to stimulate aggregate demand. Obviously, this explains the high public debt that accompanies deflation and recession. In short, we may say that a higher public debt is a consequence of the liquidity trap. However, we want to show that there exists another link between deflation and public debt, which is deeper and more sustainable. In other words, is there a liquidity trap equilibrium characterized by a recession, a deflation and a high public debt? Could the expectations of high public debt cause the liquidity trap?

Second, the fast growth of public debt in most of the advanced economies raises concern about the negative effects of debt burden. As summarized by Bernheim (1989) and Elmendorf and Mankiw (1999), the economic effects of lump-sum fiscal policy are expansionary in the short run (the traditional Keynesian view) and contractionary in the long run (the Neoclassical view). Indeed, an increase in public debt to finance tax cuts (or increase in transfers) should stimulate aggregate demand, entailing output increase, when prices (and/or wages) are sticky. This is the short-run effect. However, the real interest rate must rise to bring securities market into balance. Consequently, the investment is crowded out. Accordingly, the capital and the output eventually decrease. This is the long run effect. If we abstract capital from a standard neoclassical model then the long-run negative effects of the government debt disappear.

From the theoretical point of view, can we have a model, without capital and default risk, capable of reproducing the short-run expansionary effect of public debt, while preserving the long-run negative effects?

In Section 1.2 and 1.3, we focus on the topics of the liquidity trap and relative importance of monetary and fiscal policy, respectively. We also provide more details about the issue of the link between deflation and public debt. And, we give insights about the connection with the topic of deep habits.

1.2 The liquidity trap

This "global" liquidity trap experience, in addition to the Japanese deflation experience, shows that the liquidity trap is not just a theory but also a risk that must be taken seriously. Indeed, a number of policymakers and academics are increasingly concerned about the possibility that the deflation observed in the US (as in many other countries) persists, like in the case of Japan. It is noteworthy, at this stage, to shed light on the causes and consequences of a liquidity trap emphasized in the literature.

The main goal of many central banks is to stabilize inflation around a low level. They also aim to keep output close to its potential level. Because of the uncertainty about the state of the economy, central banks are forward looking and construct forecasts of inflation and output. And they use available information about the economy and anticipated shocks to make its forecasts. In particular, a central bank needs to lower its instrument, i.e. the short-term nominal interest rate, when its forecasts display low inflation and output. The intuition behind lowering its instrument is the following. A lower short-term nominal interest rate, combined with sluggish private-agents inflation expectations, will induce a decline in the short-term real interest rate. Expectations of lower future short-term real interest rate imply a decrease in long-term real interest rates. As consumption and investment decisions are influenced by long-term interest rates, the fall in long-term interest rates implies an increase in consumption and investment. Thereby stimulates aggregate demand. The rise in aggregate demand and the increase in expectations of future inflation push up the current inflation to go up.

If the current and expected future inflation rates are low then lowering the nominal interest rate may induce the nominal interest rate to hit its ZLB. Consequently, the central bank does not have much room to further lower the nominal interest rate. More in particular, conventional open-market operations to expand the monetary base, by buying treasury bonds, seem to be inefficient. As a consequence, the economy is mired into recession and deflation.

The recent literature on the liquidity trap situation, associated to the ZLB on the nominal interest rate, has been prompted in part by the Japanese experience[2]. The main explanations of the liquidity trap are given by Krugman (1998) and Benhabib, Schmitt-Grohé and Uribe (2001a, 2001b). Krugman (1998) explains the liquidity trap by a fall in the natural rate of interest, which is defined as the equilibrium real interest rate. Benhabib et al. (2001a, 2001b) present another approach in which the liquidity trap results from self-fulfilling deflationary expectations. Indeed, they show that, when the Taylor rule is constrained by the ZLB on the nominal interest rate, a second low-inflation equilibrium appears besides the desirable target equilibrium. The second equilibrium is indeterminate so there is a continuum of "liquidity trap" paths. Consequently, a wave of pessimistic expectations can bring the economy into the liquidity trap.

The explanation of Benhabib et al. (2001a, 2001b) has largely influenced our thinking about the relationship between deflation and public debt. What we have in mind is also a link based on expectations. This will be clearer in what follows. The analysis of Benhabib et al. (2001a, 2001b) is conducted in a representative agent framework. One question may arises about how do their results change if we adopt a non-Ricardian model? Annicchiarico *et al.* (2009) answer to this question. Their work was motivated by the original contributions by Pigou (1943, 1947) and Patinkin (1965), according to which wealth effects are a channel to avoid the liquidity trap. So they wanted to investigate whether in a micro-founded model wealth effects can rule out the indeterminate (liquidity trap) equilibrium find by Benhabib et al. (2001a). They find instead that, in a continuous-time Blanchard (1985)-Yaari (1965) framework, taking into account the ZLB on the nominal interest rate leads to the existence of four equilibria. They analyze the local dynamic characteristics of these four equilibria. They find that two equilibira are locally determined, one of which corresponds to a liquidity trap situation. By contrast to the liquidity trap situation described by Benhabib et al. (2001a), which is locally indeterminate, this locally determinate liquidity trap equilibrium is characterized by a

[2] Yates (2004) provides a thorough survey of this literature, including the implications for the conduct of monetary policy.

unique and stable trajectory. But, it is noteworthy to mention that Annicchiarico *et al.* (2009) assume an exchange economy. It follows that, in their model, there are no transmission mechanism.

Our intuition is that introducing capital and government debt into a non-Ricardian model with ZLB on nominal interest rates leads to a new mechanism that brings about a liquidity trap. What are the long-run characteristics of this new liquidity trap equilibrium? What if this locally determinate liquidity trap is characterized by a high long-run level of public debt? Do expectations of high public debt bring the economy to a liquidity trap situation?

In Chapter 2 and 3, we propose to answer to these questions by developing a model of overlapping families of infinitely-lived agents, including an interest rate rule *à la* Leeper (1991)-Taylor (1993), the ZLB on the nominal interest rate, and physical capital. We examine the long-run characteristics of each equilibrium. In addition, we focus on the local dynamic characteristic around each equilibrium. Moreover, we investigate whether we can draw an analogy between these four equilibria and the four configurations described by Leeper (1991). This point will be developed in more details in the following section.

1.3 Relative importance of monetary and fiscal policy

Monetary policy rules, specified as a simple relationship between nominal interest rates and endogenous variables like inflation or output[3], have been widely analyzed. In contrast, fiscal policy rules based on lump sum taxation have received less attention. The reason of this asymmetric treatment is connected to the popular Ricardian equivalence proposition [Barro (1974)]. According to this proposition, it does not matter whether

[3] See Taylor (1993).

public spending are financed by collecting (lump sum) taxes or issuing public debt. It results that lump sum fiscal policy is neutral. Hence, representative agent models, where Ricardian equivalence proposition holds, are commonly used to analyze monetary policy, justifying thus the abstraction from fiscal policy.

Dynamic stochastic general equilibrium models (DSGE, henceforth) have become an essential tool in modern macroeconomic analysis. These models are widely used by both academic researchers and policy makers, replacing the traditional "ad-hoc" macroeconomic models. The most popular DSGE models used to analyze monetary and fiscal issues can be classified into two categories: infinitely-lived representative agent framework and overlapping generations framework.

In the first category, households are represented by homogeneous family of infinitely-lived individuals, with intergenerational altruism. This assumption leads to the validity of the Ricardian equivalence and thus the neutrality of lump sum fiscal policy. Nevertheless, in this framework, monetary and fiscal policy may interact if *i)* fiscal policy is distortionary[4], *ii)* fiscal policy is non-Ricardian, in the sense of Woodford (1994), or *iii)* part of consumers are "rule of thumb" consumers (liquidity-constrained consumers), as described by Gali et al (2004, 2007).

In the second category, Ricardian equivalence breaks down because the assumption of intergenerational altruism is relaxed, leading to the interaction between monetary and fiscal policy. In such framework the distinction between debt-financing and tax-financing budget deficits is relevant, and public debt represent real wealth for the agents.

Following Benassy (2007), we will call the first category "Ricardian" Economy and the second category "non-Ricardian" Economy[5].

[4] See Schmitt-Grohe an Uribe (2007) and the reference therein.

[5] This terminology is different to the one used by Woodford(1996). The distinction between the two terminology will be clear later on.

1.3.1 Ricardian Economy

In this subsection, we only focus on non-Ricardian Framework, i.e. the condition *ii)*. It has been long recognized that it is sufficient for a fully independent monetary authority to adjust the interest rate instrument more than one-for-one with inflation—this proposition is often referred to as "Taylor principle"—in order to guarantee price stability (McCallum (2003); Woodford (2003)). However, sustainable public finances are important for monetary policy to achieve price stability. Such a connection is notably reflected in the Maastricht Treaty and the Stability and Growth Pact of the European Union. The general government deficit should not exceed the 3% to GDP reference value and the debt to GDP ratio should be below 60% or, if above, tend to that reference value at a satisfactory pace.

Sargent and Wallace (1981) were the first ones to point out the fact that monetary and fiscal policy coordination is important for price stability. They introduce the notion of "monetary dominant" and "fiscal dominant" regimes. In the "monetary dominant" regime, monetary authority is not constrained by fiscal policy conduct, so that money supply and demand determine the price level. As counterpart, the fiscal authority adjusts the primary surpluses in order to assure fiscal solvency, for any path of the price level. By contrast, under a "fiscal dominant" regime, the fiscal authority independently sets its budget from its liabilities, announcing current and future surpluses. Therefore, the path of price level and money supply must satisfy the needs of fiscal solvency.

Following the contribution of Sargent and Wallace (1981), the fiscal explanation of inflation was explicitly presented by Leeper (1991) and then Sims (1994), Woodford (1994, 1995, 2003) and Cochrane (2005) formulated this idea into the Fiscal Theory of the Price Level (FTPL, henceforth).

The FTPL has examined how prices might move to ensure that the government's intertemporal budget constraint holds. Price level determinacy emerges from particular combinations of monetary and fiscal policy behavior. As a matter of fact, two combinations are recognized. The first combination stems from the quantity theory of money.

According to which the monetary authority sets the price level paths independently of the fiscal authority, which guarantee a balanced intertemporal budget for any price path. In this framework, the Taylor principle is locally satisfied. In the sense of Leeper (1991), this configuration corresponds to an "active" monetary policy and "passive" fiscal policy. Second, the combination based on the FTPL corresponds to a "passive" monetary policy and "active" fiscal policy. Notably, there is an equilibrium situation, with respect to the requirement of a rational expectations equilibrium, where price level adjusts to balance the intertemporal government budget.

Woodford (1995) argues that fiscal policy affects inflation rates if and only if the government follows "non-Ricadian" (or "active", in the terms of Leeper (1991)) fiscal policies under which the intertemporal budget constraint is satisfied, for some but not all, prices path. It turns out to consider that the government can behave in a fundamentally different way from households. Recall that households must satisfy intertemporal budget constraints, no matter what price paths they face[6]. In order to understand the FTPL, Cochrane (1999) propose to interpret the intertemporal budget constraint as a valuation equation instead of a constraint. If surpluses are not sufficient, the government must default on debt or inflation it away. Therefore, we can determine the price level via the valuation equation for government debt.

1.3.2 Non-Ricardian Economy

In an overlapping generations model without intergenerational altruism the Ricardian equivalence does not hold. Therefore, fiscal policy is non-neutral, implying the interaction between monetary and fiscal policy. The most common micro-founded general equilibrium models used in this literature are the finite-lifetime approach, first highlighted by Yaari (1965) and Blanchard (1985), and the overlapping families of infinitely-lived agents, developed by Weil (1987, 1989).

[6] See Kocherlakota and Phelan (1999), Woodford (2001) and Gordon and Leeper (2006) for surveys of the FTPL

The Blanchard (1985)-Yaari (1965) model is a model of uncertain lifetime where agents face a constant probability of death. However agents are non-altruistic, which raises the problem of leaving unintended bequest when they die. To avoid it, the authors assume a perfectly competitive life insurance market where insurance companies collect financial wealth from the deceased agents and pay insurance premiums to alive agents.

Weil (1987, 1989)'s model has the appeal of overlapping generations models while keeping the infinite horizon assumption. In other words, in this model we also have the traditional results of overlapping generations models that is debt non-neutality, the possibility of the existence of asset bubbles and the possibility of having dynamically inefficient competitive equilibria. This is due to the existence of operative intergenerational linkages between some but not all agents.

It is noteworthy to point out that the planning horizon of households is not the cause of the validity (or not) of the Ricardian equivalence. Indeed, deferring tax payment until some date in the future means that it will be paid by new generations not yet alive when the tax payment was deferred. This holds true independently of the life-time of each agent; however only the positive birth rate matters. Indeed, Buiter (1988) and Weil (1989) is a necessary condition to guarantee the non-neutrality of debt .

The non-Ricardian Framework has been very useful to solve some puzzles such as nominal indeterminacy, which was pointed out by Sargent and Wallace (1975). Before going through the details, it is important to distinguish between two cases of indeterminacy: nominal indeterminacy and solution multiplicity. This point was analyzed in detail in McCallum (1986) and summarized in Benassy (2000). In brief, Benassy (2000) defines the nominal indeterminacy as multiple equilibria with different (and usually proportional) nominal prices, but all corresponding to the same real allocation. Multiplicity was in turn defined as multiple equilibria with both different nominal prices and different real allocations. Sargent and Wallace (1975) show that pure interest rate pegging provides no monetary anchor and no mechanism to determine the price level. Benassy (2000) argues that this is quite bothering since, from a normative point of view, many optimal policy packages include the “Friedman rule”: the nominal interest rate should be equal to zero. This means that such policies could lead to price indeterminacies. Cushing (1999), Benassy (2000) and Annicchiarico and Marini (2006) show that non-Ricadian framework eliminates nominal indeterminacy but multiplicity of equilibrium paths towards steady state still exist. However, Benassy (2000) obtains full determinacy[7] by allowing fiscal policy to be non-Ricadian (in the sense of Woodford(1994)).

Among the solutions recommended to exit the liquidity trap, the use of fiscal policy instruments has received intention. Recent examples are Christiano, Eichenbaum, and Rebelo (2009), Devereux (2010), Mertens and Ravn (2010), and Corsetti, Kuester, Meier and Muller (2010), among others. The literature on the ZLB is very extensive.

1.4 Deep habit formation

There has been a growing recognition of the role of consumption externalities in macroeconomics. Studies on this issue showed that consumption externalities can solve the equity premium puzzle, in particular, Abel (1990), Constantinides (1990), Gali (1994), and Campbell and Cochrane (1999). In addition, consumption externalities have also been used to analyze other issues such as, equilibrium efficiency; Alonso-Carrera et al (2003), Liu and Turnovsky (2003), and Turnovsky and Monteiro (2007), indeterminacy; Weder(2000), and Mino (2008), the effects of demographic shocks; Fisher and Heijdra (2009), etc.

More Recently, Ravn, Schmitt-Grohe, and Uribe (2006) offer an alternative form of consumption externalities, which is labeled "external deep habit". Accordingly, households form habits at the level of aggregate consumption of a specific good, rather than the level of aggregate consumption basket. In the terminology of Ravn et al. (2006), external superficial habit formation corresponds to the catching-up with the Joneses specification

[7] The full determinacy corresponds to the case where, at the same time, nominal indeterminacy and multiplicity are eliminated.

as in Abel (1990), and external deep habit formation corresponds to the catching-up with the Joneses on a good-by-good basis.

The contribution by Ravn et al. (2006) presents a micro-founded model where the price markup policy is countercyclical. As a result, deep habits models reconcile DSGE models with empirical evidence of countercyclical markups. In such a framework, the demand for a specific good is composed of a price elastic component (as in standard monopolistic competition models) and an inelastic component that does not react to price changes. An increase in aggregate demand raises the weight of the elastic component, inducing producers to lower their prices. This is what they label *the price-elasticity effect.* Also, an increase in the present value of per unit profits generates an incentive for firms to invest in consumer base, entailing a decrease in the markup. This is what they label *the intertemporal effect.*

Deep habits models are useful to understand some puzzles related to monetary and fiscal policy shocks. The literature on the dynamic impact of monetary policy shocks has identified two puzzles. The first is the so-called "inflation persistence puzzle". According to which, an expansionary monetary policy shock entails a slow and delayed rise in inflation. The second corresponds to a temporary drop in the price level in response to an expansionary monetary policy shock: the "price puzzle". Ravn, Schmitt-Grohe, Uribe and Uusküla (2010), show that deep habits model solve these two puzzles.

Empirical studies as in Blanchard and Perotti (2002) or Gali et al. (2007) show that autonomous increase in government spending implies a short-run rise in private consumption in the short-run. Standard neoclassical models cannot account for this effect and, in contrast, predict a negative relationship between government spending and private consumption. However, Ravn et al. (2006) show that deep habits model solves this puzzle. They show that an increase in public consumption increases the aggregate demand, inducing the decline of markups through the *price-elasticity effect.* Consequently, the labor demand increases. So, wages go up to balance the labor market, entailing higher consumption because individuals substitute away from leisure towards consumption.

The countercyclicality of markup behavior in turn implies that output, consumption and wages can respond positively to positive demand shocks. In other words, the fiscal policy transmission mechanism can be quite different from that when habits are either absent or superficial.

In a standard non-Ricardian model the absence of physical capital make the long-run crowding out effect of government debt disappear. Is it possible to have a DSGE model without physical capital able to generate a short-run expansion and a long-run crowding out effect of public debt? Our intuition is that external deep habits combined with wealth effects may give an answer. To this purpose, in Chapter 4, we develop an overlapping generations model à la Weil (1987, 1991) with endogenous labor supply and external deep habits.

1.5 Main Results

In Chapter 2, we introduce non-linear Taylor rule—incorporating the ZLB on the nominal interest rate—into non-Ricardian DSGE model à la Weil(1987, 1991) with capital accumulation. We find four steady state equilibria as in Annicchiarico et al. (2009). In particular, the four steady state equilibria have the same dynamic characteristics as the four types of equilibria described by Leeper (1991), but for one set of policy parameter space. Our result emerges from the double non-linearity associated to the presence of wealth effects and the zero lower bound on the nominal interest rate. In short, when the nominal interest rate rule is bounded below by zero, the presence of wealth effects emphasizes the global indeterminacy problem identified by Benhabib, Schmitt-Grohé and Uribe (2001a, 2001b).

We notably show that a liquidity trap equilibrium, also characterized by a high public debt-to-GDP ratio, a low capital stock and a low consumption level, possesses the usually required properties of determinacy, like a more traditional equilibrium targeted by the monetary and fiscal authorities.

We show that a change in agents' expectations may lead the economy to the debt-deflation equilibrium[8]. We find that the mechanism that allows to reach the debt-deflation equilibrium corresponds to the one described by the FTPL. The liquidity trap is reached thanks to and initial deflation of about 50%. This result is of course unrealistic. Chapter 3 aims to solve this limit.

In Chapter 3, we introduce deep habits into the model developed in Chapter 2. We show that this deep habit specification can reduce the gap between the two steady state values of government debt while retaining a reasonable gap between the two steady state values of output. In other words, the initial deflation is reduced considerably (11%) while the recessionary effect is still significant.

In addition, we analyze and compare the effects of debt-financed tax cut in both environments. We show that in the "targeted" environment, debt-financed tax cuts entail a positive wealth effect. As a consequence, consumption increases but investment decreases because the real interest rate goes up, inducing a fall in output. However in the "debt-deflation" environment, tax reductions imply a negative wealth effect. As a result, consumption decreases but investment increases, entailing an increase in output.

In our model, in the "debt-deflation" environment, tax reduction triggers the decrease of real government debt, which is achieved thanks to a price increase. In this case there is a negative wealth effect inducing a fall in consumption. On the other hand, lower real government debt entails a decrease in the real interest rate and consequently the investment increases, stimulating aggregate demand . Accordingly, output increases.

In Chapter 4, we analyze the implications of public debt shocks in a non-Ricardian model with external deep habits. We show that an increase in government debt to finance tax reduction, has a long-run contractionary effect despite the lack of capital. On the other hand, the short-run effect of debt-financed tax cut is contractionary in a flexible-price framework, while it is expansionary in a sticky-price framework.

[8] The liquidity trap equilibrium which is locally determinate.

The novelty in this chapter is the introduction of external deep habits into a non-Ricardian framework. In other words, we bring together wealth effects, which imply a non-neutral fiscal policy, and time-varying markups which are countercyclical to output. This offers a new transmission mechanism of government debt through the countercyclical markup movements. The transmission mechanism can be summarized as follows. Debt-financed tax cuts raise the interest rate, entailing higher markups, which in turn induce a fall in employment and consumption.

1.6 Structure of the thesis

The rest of this thesis is organized as follows. In Chapter 1, we introduce the analytical framework from which we shall depart in the remaining chapters of the thesis. As explained above, this is a standard overlapping families of infinitely-lived agents à la Weil (1987, 1989). In Chapter 2, we extend the analysis to include ZLB on the nominal interest rate in overlapping families of infinitely-lived agents with capital. In Chapter 3, we introduce monopolistic competition and external deep habits into the model developed in Chapter 3. In Chapter 4, we develop a non-Ricadian model à la Weil (1987, 1989) without capital and with external deep habit formation. Finally, Chapter 5 concludes and outlines a possible research agenda along the lines of the analysis presented in the thesis.

Chapter 2

A Simple Public Debt-Deflation Theory : Leeper Revisited[1]

2.1 Introduction

Macroeconomic policy discussions recognize the intimate connection between monetary and fiscal policy. A representative example of such a connection is the Maastricht Treaty and the Stability and Growth Pact[2]. On the other hand, the study of the interaction between monetary and fiscal policies has been the object of vigorous interest since the seminal works of Sargent and Wallace (1981), Aiyagari and Gertler (1985), and more recently, Leeper (1991), Sims (1994), Woodford (1994, 2003) and Cochrane (2005) around the FTPL.

The main contribution of this literature is an explicit specification of the conditions under which the monetary and fiscal policies interact, contrasting, thus, the traditional configuration—the quantity theory of money—where no interaction of fiscal policy *with* monetary policy is allowed. In fact, the fiscal policy is neutral if the following con-

[1] **This chapter is an adaptation of Aloui and Guillard (2009).**

[2] The statement of the Masstricht Treaty and the Stability and Growth Pact is that the general government deficit should not exceed the 3% to GDP reference value and the debt to GDP ratio should be below 60%.

ditions are fulfilled : there is *i)* no fiscal distortions, *ii)* no wealth effects or financial constraint, and *iii)* the fiscal policy is Ricardian in the sense of Woodford (1995), *i.e.* the fiscal authority ensures the government solvency by respecting its intertemporal budget constraint for any sequence of the price level and other endogenous variables.

Accordingly, if the last condition is unfulfilled, then the fiscal policy is non-Ricardian which means that the intertemporal budget constraint of the government needs an adjustment of the price level to be balanced. In the sense of Leeper (1991), fiscal policy is said to be "active" and then monetary policy must be "passive"[3]. This interaction between monetary and fiscal policy corresponds to the FTPL.

When the condition *ii)* is not satisfied then the economy is non-Ricardian. In this case, wealth effects can emerge and make the fiscal policy non neutral. As spelled out by Leith and Wren-Lewis (2000), even when fiscal policy is passive—does not constrain the active monetary policy—it can still influence prices.

With a different objective from that of Leith and Wren-Lewis, Cushing (1999) and Bénassy (2000) study the consequences of pegging the nominal interest rate in a non-Ricardian economy. But, while Cushing (1999) tries to show that the price level is always indeterminate in the presence of these effects, Bénassy (2000) shows, more clearly according to us, that the nominal indeterminacy[4] described by Sargent and Wallace (1975) disappears around the steady state which is locally determinate. There is nevertheless, as a general rule, another stationary equilibrium locally indeterminate towards which converge the multiple trajectories emphasized by Cushing (1999). The link between these results and those of the FTPL is not immediate. The presence of wealth effects does not any more allow to consider a simple rule as satisfying or not the criteria of a fiscal Ricardian policy. The difference between the conclusions of Cushing (1999)

[3] An active monetary policy arises when the Taylor principle is fulfilled and passive monetary policy arises at the opposite case, when the response of the nominal interest rate is less than one-for-one to inflation. Similarly, passive fiscal policy occurs when the local convergence of the government debt is guaranteed and active fiscal policy happens when taxes do not respond sufficiently to debt to cover real interest payments and public spending.

[4] Benassy explains clearly the difference between multiplicity of equilibrium and nominal indeterminacy.

and Bénassy (2000) actually results from the little operational character of this concept in a non-Ricardian Economy[5].

The model developed in this chapter proposes a generalization of Cushing (1999) and Bénassy (2000) to a more complex economy, including: an interest rate rule *à la* Leeper (1991)-Taylor (1993), a ZLB on the nominal interest rate, and the presence of capital in the production process. On the other hand, we can see our contribution as an extension of Leeper (1991) to take into account the presence of wealth effects in a non-Ricardian economy with capital and with a ZLB on *the* interest rate.

Before summarizing our results let us recall the main findings of Leeper (1991). The interaction between simple monetary and fiscal policies yields four configurations depending on the policy parameters set by monetary and fiscal authorities. A determinate equilibrium then requires one active and one passive policy.

The results we obtain are the following: we can see coexisting the four types of equilibria described by Leeper (1991), but for one set of policy parameter space. This means that in our framework the determinacy region is no longer specified by the policy parameter space. We notably show that a liquidity trap equilibrium, also characterized by a high public debt-to-GDP ratio, a low capital stock and a low consumption level, possesses the usually required properties of determinacy, like a more traditional equilibrium targeted by the monetary and fiscal authorities.

Our result emerges from the double non-linearity associated to the presence of wealth effects and the ZLB on the nominal interest rate. In short, when the nominal interest rate rule is bounded below by zero, the presence of wealth effects emphasizes the global indeterminacy problem identified by Benhabib, Schmitt-Grohé and Uribe (2001a, 2001b).

In addition, in our model, a detailed analytical analysis is made and allows to give the dynamic characteristics of each equilibrium. We find that two equilibria are locally determinate, one equilibrium is overdeterminate and one equilibrium is locally indeter-

[5] The Cushing's result is linked to a supplementary condition that he imposes. Despite the use of a simple fiscal rule, the author does not allow the fiscal policy to be non-Ricardian in the sense of Woodford (1995).

minate. To resume, the four equilibria have, locally, the same dynamic characteristics as the equilibria described by Leeper (1991). Consequently, two convergent paths coexist. This finding becomes more interesting because of the capital accumulation. Indeed, the presence of capital stock allows the wealth effects of public debt to cause significant supply and demand effects. Accordingly, the debt-liquidity trap, or "public debt-deflation" equilibrium, corresponding to a lower level of capital stock, is associated to a recessionary trajectory.

Furthermore, from the perspective of global analysis, the existence of two locally convergent paths arises the question of self-fulfilling prophecies. An expectation shock can lead the economy from a virtuous trajectory to the debt-liquidity-trap trajectory. Therefore, our model could provide an alternative or complementary explanation for some episodes of deflation—like the Japanese recession of the 90s[6]—based on agents' expectations change. In order to evaluate the possibility and the implications of a self-fulfilling expectation shock, the model is calibrated on annual data and permits us to simulate the effects of such a shock. We find that the liquidity trap and the increase in the public debt level are both the consequences of an initial deflation caused by a change in expectations.

Related literature

A recent literature has focused on the interaction between monetary an fiscal policy issue in a New Keynesian framework in the presence of wealth effects and with capital accumulation. Annicchiarico (2007), Annicchiarico *et al.* (2006), among others, study the effect of shocks when fiscal policy is non neutral because of wealth effects. They do not focus, however, on the matter of multiple equilibria. This issue was already analyzed within the framework of an exchange economy by Annicchiarico *et al.* (2009) in a continuous-time model. They point out the existence of four equilibria when wealth

[6] It is worth noting that, during the Japanese liquidity-trap episode, the government debt has increased from 60% of GDP, in 1993, to 160%, in 2003.

effects and ZLB on the nominal interest rate are taken into account[7].

Closest to our work in ideas and motivation is the paper of Leith and von Thadden (2008). The main finding of their work is that the local determinacy region is not solely specified by the policy parameter space but also by the steady state government debt level.

Their framework differs from ours in two points. First, it ignores the lower bound on nominal interest rate and considers therefore only one source of non-linearity associated to the presence of wealth effects. Although, despite the presence of this non-linearity, they don't have multiple equilibria. The reason is that—and this is the second difference with our chapter—they use a fiscal rule in which the government debt target corresponds to its endogenous steady state level. In a model à la Blanchard-Yaari[8], this particular fiscal rule leads a second equilibrium—that is likely to appear due to the non-linearity—to correspond to the golden rule, and to be associated to a negative government debt level. This second equilibrium does not present any interest to their analysis. On the other hand, the extended version of the Weil's (1987, 1991) model we use can exhibit two positive values of stationary government debt, as in more traditional OLG models.

The chapter proceeds as follows. In section 2.2, we build the model of a non Ricardian economy with money and capital and we introduce simple monetary and fiscal rules. In Section 2.3, we turn our attention to steady state equilibria. Section 2.4 is devoted to the study of the local properties of these equilibria and proposes a discussion about the global dynamics. In section 2.5, the model is calibrated based on Euro area annual data and is simulated in order to qualitatively asses the implications of a self-fulfilling expectation shock. Section 2.6 concludes.

[7] Guillard (2004) found the same results in a discrete-time model: "Politique monétaire et fiscale dans un monde non-Ricardien: une théorie fiscale de l'inflation", mimeo *Université d'Evry val d'Essonne.*

[8] The basic version of the overlapping generation model *à la* Blanchard-Yaari is always characterized by under-accumulation.

2.2 The Model

We use an expanded version of Weil's (1987, 1991) overlapping-generations structure. The economy consists of many infinitely-lived families (or dynasties). Each period new and identical infinitely-lived families appear in the economy without initial wealth. The economy also consists of identical infinitely-lived firms using capital and labor to produce a unique good, of the fiscal authority (the government) and of the monetary authority (the central bank). We use a stochastic framework and we assume that markets are complete.

2.2.1 Households

In period t, the economy is populated by a large number N_t of agents. Each period a new dynasty appears consisting of $(N_t - N_{t-1}) = nN_{t-1}$ agents where $n \geq 0$ represents at the same time the population growth rate and the birth rate.

Each household belonging to the dynasty $j \leq t$ has preferences defined over consumption and real money balances described by the utility function:

$$E_t \sum_{s=t}^{\infty} \beta^{s-t} U \left(c_{j,s}, \frac{M_{j,s}}{P_s} \right), \tag{2.1}$$

where E_t denotes the mathematical expectation operator conditional on information available at time t, $\beta \in [0,1]$ represents a subjective discount factor, and $U(\cdot,\cdot)$ is a period utility index assumed to be strictly increasing in its two arguments and strictly concave. The variables, $c_{j,t}$, P_t and $M_{j,t}$, represent the consumption of the household j in period $t \geq j$, the price of consumption good, and the nominal money balances held by household j in period $t \geq j$, respectively.

At the beginning of the period t, the household $j < t$ holds the initial nominal wealth, $V_{j,t}$, defined by:

$$V_{j,t} = M_{j,t-1} + (1 - \delta + \kappa_t) P_t k_{j,t} + A_{j,t}, \tag{2.2}$$

where $(1-\delta+\kappa_t) P_t k_{j,t}$ is the nominal value of the capital stock, including capital incomes net of depreciation, and $A_{j,t}$ is the beginning-of-period state-contingent value of all other financial assets, whether privately issued or claims on the government.

In each period, agents supply an inelastic and constant amount of labour and receive a real wage, $w_{j,t}$. Each agent uses his total financial wealth augmented by the wage incomes net of taxes, $P_t\tau_{j,t}$, to consume and to reconstitute his financial holdings. We can write the household's flow budget constraint as follows:

$$P_t c_{j,t} + M_{j,t} + E_t \Lambda_{t,t+1} A_{j,t+1} + P_t k_{j,t+1} \leq V_{j,t} + P_t w_{j,t} - P_t \tau_{j,t}, \tag{2.3}$$

where $\Lambda_{t,t+1}$, is the stochastic discount factor[9].

Markets are assumed to be complete. This assumption implies the existence of the one-period risk-free nominal interest rate defined by:

$$1 + R_t = \left[E_t \Lambda_{t,t+1}\right]^{-1}. \tag{2.4}$$

Finally, the household is subject to an appropriate set of borrowing limits that rule out "Ponzi Games". Let us define:

$$h_{j,t} = \frac{1}{P_t} E_t \sum_{s=t}^{\infty} \Lambda_{t,s} P_s \left[w_{j,s} - \tau_{j,s}\right], \tag{2.5}$$

the household j's human wealth which corresponds to the discounted value of future labor incomes net of taxes. In the absence of financial market frictions, the borrowing constraint takes the form:

$$V_{j,t+1} \geq -P_{t+1} h_{j,t+1} \qquad \forall j, \forall t. \tag{2.6}$$

[9] To be more precise, $\Lambda_{t,t+1}$ is the asset price in period t, that gives one unit of money in a given state of the world in period $t+1$, weighted by the probability (or density function) of such state. $E_t \Lambda_{t,t+1} A_{j,t+1}$ can be rewritten as $\sum p_{t,t+1} A_{j,t+1}$ (where $p_{t,t+1}$ is an asset price) and represents the state-contingent assets portofolio. We have more generally: $\Lambda_{t,T} = \Lambda_{t,t+1} \times \Lambda_{t+1,t+2} \times .... \times \Lambda_{T-1,T}$ and $\Lambda_{t,t} = 1$.

This constraint implies that the household has to be able to reimburse his debt contracted in period t in each state of the world that may be realized at date $t+1$.

The representative household of generation j maximizes his intertemporal utility (2.1) subject to the budget constraint (2.3) and the borrowing constraint (2.6), where $V_{j,t}$ is defined by equation (2.2).

Denoting $U_x(t) = \partial U(\cdot)/\partial x_t$, the first-order conditions for this maximizing problem can be written as follows:

$$\beta \frac{U_{c_j}(t+1)}{U_{c_j}(t)} = \Lambda_{t,t+1} \frac{P_{t+1}}{P_t}, \tag{2.7}$$

$$\frac{U_{m_j}(t)}{U_{c_j}(t)} = \frac{R_t}{1+R_t}, \tag{2.8}$$

$$\left(E_t \Lambda_{t,t+1} \frac{P_{t+1}}{P_t} \right)^{-1} - 1 = \kappa_{t+1} - \delta \equiv r_t, \tag{2.9}$$

$$E_t \Lambda_{t,t+1} V_{j,t+1} + P_t c_{j,t} + \frac{R_t}{1+R_t} M_{j,t} = V_{j,t} + P_t \left(w_{j,t} - \tau_{j,t} \right), \tag{2.10}$$

$$\lim_{T \to +\infty} E_t \Lambda_{t,T} V_{j,T} = 0. \tag{2.11}$$

Equation (2.7) is a stochastic Euler equation summarizing the intertemporal arbitrage between present and future consumptions in each possible state of the world. Equation (2.8) represents an arbitrage condition between real money balances and present consumption. Equation (2.9) is a no-arbitrage condition relative to the saving choice in terms of capital accumulation or in terms of nominal state-contingent assets. Note that the net return on capital, $\kappa_{t+1} - \delta$, is not associated to an expectation operator because we assume a risk-free production. Thus, r_t represents the real risk-free interest rate, and it is known in period t. Equation (2.10) is the household j's balanced budget constraint obtained by combining equations (2.2), (2.3) and (2.9). Finally, Equation (2.11) corresponds to the transversality condition and states that the discounted value of the financial wealth (or debt) tends to zero when time goes to infinity.

Iterating Equation (2.10) forward, with the use of (2.11), leads to the following household j's intertemporal budget constraint:

$$V_{j,t} = E_t \sum_{s=t}^{+\infty} \Lambda_{t,s} \left[P_s c_{j,s} + \frac{R_s}{1+R_s} M_{j,s} - P_s \left(w_{j,s} - \tau_{j,s} \right) \right]. \tag{2.12}$$

In order to obtain an explicit outcome for individual consumption, one specifies the utility function as follows:

$$U\left(c_{j,t}, \frac{M_{j,t}}{P_t} \right) = \xi \ln c_{j,t} + (1-\xi) \ln \frac{M_{j,t}}{P_t}.$$

Defining

$$q_{t,t+1} = \left(\Lambda_{t,t+1} \frac{P_{t+1}}{P_t} \right)^{-1}, \tag{2.13}$$

as the stochastic gross real interest rate corresponding to real return of the state-contingent nominal asset[10], equations (2.7) and (2.8) can then be rewritten as:

$$c_{j,t} = \beta^{-1} \frac{c_{j,t+1}}{q_{t,t+1}} \tag{2.14}$$

and

$$c_{j,t} = \xi \left[c_{j,t} + \frac{R_t}{1+R_t} \frac{M_{j,t}}{P_t} \right].$$

Introducing these results into equation (2.12) and using (2.5), one can easily show that the optimal consumption of agent j is a constant fraction of his consolidated wealth (financial wealth + human wealth).

$$c_{j,t} = \xi (1-\beta) \left(v_{j,t} + h_{j,t} \right), \tag{2.15}$$

where $v_{j,t} = V_{j,t}/P_t$.

[10]Note that according to (2.9), we have: $r_t = \left[E_t \left(1/q_{t,t+1} \right) \right]^{-1} - 1$.

2.2.2 Aggregation

Noting that the generation j is composed of $N_j - N_{j-1}$ agents, the following aggregation rule is applied to get *per capita* aggregate variables:

$$z_t = \sum_{j \leq t} \frac{(N_j - N_{j-1})}{N_t} z_{j,t}, \tag{2.16}$$

for $z_{j,t} = c_{j,t}$, $v_{j,t}$, and $h_{j,t}$.

We assume that the agent's inelastic supply of labor corresponds to one unit of labor, whatever the agent's age and we assume that taxes are independent of the age. Therefore, $h_{j,t} = h_t \quad \forall j$.

Finally, notice that applying the aggregate rule (2.16) in period t to the variable $v_{j,t+1}$, we get:

$$\begin{aligned}
\sum_{j \leq t} \frac{(N_j - N_{j-1})}{N_t} v_{j,t+1} &= \frac{N_{t+1}}{N_t} \sum_{j \leq t} \frac{(N_j - N_{j-1})}{N_{t+1}} v_{j,t+1} \\
&= (1+n) \left[\sum_{j \leq t+1} \frac{(N_j - N_{j-1})}{N_{t+1}} v_{j,t+1} - \frac{n}{1+n} v_{t+1,t+1} \right] \\
&= (1+n)\, v_{t+1},
\end{aligned}$$

since $v_{t+1,t+1} = 0$, the dynasty $j = t+1$ having no financial wealth in period $t+1$.

Using this result and applying the aggregate rule (2.16) to equation (2.14) where we replace $c_{j,t+1}$ by its expression given by equation (2.15) expressed in $t+1$, we obtain:

$$c_t = \xi (1-\beta) \beta^{-1} \frac{(1+n)\, v_{t+1} + h_{t+1}}{q_{t,t+1}}.$$

Finally, by incorporating (2.15) expressed in $t+1$ in the previous equation, it can be rewritten:

$$c_t = \beta^{-1} \frac{c_{t+1}}{q_{t,t+1}} + \zeta \frac{v_{t+1}}{q_{t,t+1}}, \tag{2.17}$$

where $\zeta = n\xi \left(\beta^{-1} - 1\right) \geq 0$ if $n \geq 0$.

This equation is the aggregate stochastic Euler equation which differs from the individual Euler condition (2.14) as long as the population growth rate is different from zero[11]. It includes a real wealth effect which is characteristic of a non-Ricardian economy. In each state of nature, the growth rate of individual consumption is greater than the aggregate growth rate, reflecting the heterogeneity of individual wealth. An increase in the expected beginning-of-period financial wealth in $t+1$ benefits only to currently alive consumers in period t and thus it can not be proportionally distributed amongst present and future aggregate consumptions.

2.2.3 Firms

It is assumed that there exists a large number of competitive firms with access to a standard neoclassical technology: $Y_t = F(K_t, L_t)$, where Y_t, K_t and L_t denote the aggregate levels of production, physical capital and labour demand, respectively. The production function is homogeneous of degree one, concave, twice continuously differentiable and satisfies the Inada conditions. Firms are price takers in input and output markets. Let $k_t = K_t/L_t$ denote the *per capita* capital stock, the *per capita* output level $y_t = Y_t/L_t$ is given by: $y_t = F(k_t, 1) \equiv f(k_t)$.

Competitive profit-maximizing firms leads to the standard conditions that factor prices equal their respective marginal products:

$$\kappa_t = f_k(k_t), \tag{2.18}$$

$$w_t = f(k_t) - k_t f_k(k_t). \tag{2.19}$$

Given the constant return to scale, factor payments exhaust firm revenues.

[11] Recall that in Weil's model the population growth rate couldn't be negative since the absence of death.

2.2.4 Monetary and Fiscal Authorities

The government collects lump-sum taxes in the amount of P_tT_t, spends P_tG_t, prints money M_t and issues one-period nominally risk-free bonds B_t at the nominal price of $(1+R_t)^{-1}$. Denoting:

$$\Omega_t = M_{t-1} + B_{t-1},$$

the total beginning-of-period t government debt, including money balances, the government flow budget constraint can be written:

$$\frac{\Omega_{t+1}}{(1+R_t)} + \frac{R_t}{1+R_t}M_t + P_tT_t = \Omega_t + P_tG_t \tag{2.20}$$

Fiscal Rule

We assume that in order to determine the amount of the lump-sum taxes, the fiscal authority applies the following simple rule:

$$T_t = \bar{\tau}_t Y_t + \gamma\frac{\Omega_t}{P_t} - \frac{R_t}{1+R_t}\frac{M_t}{P_t} \tag{2.21}$$

The first term on the right-hand side of equation (2.21), $\bar{\tau}_t Y_t$, represents the part of taxes proportional to the output. $\bar{\tau}_t$ is a choice variable of fiscal authority, but it can be perceived by private agents as stochastic. The second component reflects the fact that the government debt is partially backed by direct taxes. It generalizes the rule proposed by Leeper (1991) to the total government debt, $\Omega_t = M_{t-1} + B_{t-1}$, instead of B_{t-1} alone. The parameter γ verifies : $0 \leq \gamma \leq 1$. Finally, the government transfers all its seigniorage revenues, $\frac{R_t}{1+R_t}M_t/P_t$, to agents. The last two assumptions will considerably simplify the model by neutralizing the effects of seigniorage on the total government debt dynamic.

The government expenditures are assumed to be proportional to the output:

$$G_t = \bar{g}_t Y_t \tag{2.22}$$

where $\bar{g}_t$ is determined by fiscal authority but it can also be perceived as stochastic by private agents.

Inserting (2.21) and (2.22) into the budget constraint (2.20) and using the definition of the nominal interest rate (2.4) and the definition of the stochastic gross real interest rate (2.13), we obtain the following equation:

$$E_t \left(\frac{\omega_{t+1}}{q_{t,t+1}} \right) = \frac{1}{1+n} \left[(1-\gamma)\, \omega_t + (\bar{g}_t - \bar{\tau}_t)\, y_t \right] \tag{2.23}$$

which describes the dynamic of $\omega_t = \Omega_t / P_t N_t$, the total *per capita* government debt in real terms.

To simplify the analysis, we will assume that in long run the fiscal authority imposes the condition: $\bar{g} = \bar{\tau}$, in order to *guarantee* that the primary deficit can equal zero when the debt is entirely paid back.

Monetary Rule

Taking up the assumption introduced by Leeper (1991) and then generalized and popularized by Taylor (1993) we assume that monetary authority has, in the short-run, leverage over the nominal interest rate that responds to the deviation (or the ratio) of inflation from its long-run target, $\bar{\pi}$.

In order to take into account a lower bound constraint on the nominal interest rate, we specify the following class of non-linear monetary rules[12]:

$$1 + R_t = \Phi\left(\bar{r}_t, \pi_t; \bar{\pi} \right) \tag{2.24}$$

where $\bar{r}_t$ is a real interest rate target and the function $\Phi(\cdot)$ is assumed to be continuous

[12] This point was analysed particularly by Benhabib, Schmitt-Grohé and Uribe (2001a, 2001b).

and to have the following properties:

$$\begin{array}{rcll}\Phi\left(r,\bar{\pi};\bar{\pi}\right) & = & \left(1+r\right)\left(1+\bar{\pi}\right) \quad \forall r,\ \forall\bar{\pi} \text{ s.t. } \left(1+r\right)\left(1+\bar{\pi}\right)\geq 1+\underline{R}\longrightarrow 1^{+}, & \\ \Phi_{\pi}\left(r,\bar{\pi};\bar{\pi}\right) & > & \left(1+r\right) \qquad \forall r,\ \forall\bar{\pi} \text{ s.t. } \left(1+r\right)\left(1+\bar{\pi}\right)\geq 1+\underline{R}\longrightarrow 1^{+}, & \text{(H1)} \\ \Phi\left(\cdot\right) & \geq & 1+\underline{R}\longrightarrow 1^{+}, & \\ \Phi_{\pi}\left(\cdot\right) & \geq & 0, \quad \Phi_{r}\left(\cdot\right)\geq 0, \quad \Phi_{\pi^{2}}\left(\cdot\right)\geq 0. & \end{array}$$

The first condition helps to guarantee that the inflation target $\bar{\pi}$ can be reached at stationary state when the real interest rate target, $\bar{r}$, equal the long run value of the real interest rate, r, as long as the resulting nominal interest rate is strictly positive[13]. The second condition is the Leeper's condition (Leeper, 1991) for an active monetary policy when the inflation target is reached. Note that, by combining this condition with the first one, we obtain: $\phi_{\bar{\pi}} > 1$, where $\phi_{\bar{\pi}}$ is the elasticity of $\Phi\left(\cdot\right)$ with respect to π_t when $\pi_t = \bar{\pi}$, that is, the more popular "Taylor Principle". The third condition generalizes the ZLB on the nominal interest rate constraint to all possible values of the real interest rate target and the inflation rate. The three last conditions help to preclude atypical rules.

The case where $\bar{r}_t$ represents a constant target, *i.e.* $\bar{r}_t = \bar{r}\ \forall t$, is often used in the literature, notably by Taylor (1993). Nevertheless we will analyze the case where $\bar{r}_t$ is equal to the current real interest rate, *i.e.* $\bar{r}_t = r_t$, that could be wise to stabilize inflation around its target when the stationary level of capital is not yet reached.

[13] Taking into account the logarithmic form of the utility function, the zero bound on nominal interest rate can never be reached. A positive lower bound, $\underline{R} > 0$, has to be defined (see Alstadheim and Henderson [2006]). The limit case $\underline{R} = 0$ can be considered in a cashless economy, when $\xi = 1$.

2.2.5 Market Clearing

In equilibrium, the surplus of state-contingent assets supplied by agents equals zero, thus their financial holdings are composed of government bonds, money and capital:

$$v_t = \frac{M_{t-1} + B_{t-1}}{P_t N_t} + (1 + r_{t-1}) k_t = \omega_t + (1 + r_{t-1}) k_t.$$

It follows that the stochastic aggregate Euler equation (2.17) takes the form:

$$c_t = \beta^{-1} \frac{c_{t+1}}{q_{t,t+1}} + \zeta \frac{\omega_{t+1} + (1 + r_t) k_{t+1}}{q_{t,t+1}}.$$

Using (2.9) and (2.18), we define the function $\tilde{q}(k_{t+1})$ that determines the value of the gross real interest rate according to the capital accumulated in t :

$$1 + r_t = 1 - \delta + f_k(k_{t+1}) \equiv \tilde{q}(k_{t+1}). \tag{2.25}$$

We can then describe an equilibrium by the following set of equations:

$$c_t = \beta^{-1} \frac{c_{t+1}}{q_{t,t+1}} + \zeta \frac{\omega_{t+1} + \tilde{q}(k_{t+1}) k_{t+1}}{q_{t,t+1}}, \tag{2.26}$$

$$k_{t+1} = \frac{1}{1+n} \left[(1 - \delta) k_t + (1 - \bar{g}_t) \cdot f(k_t) - c_t \right], \tag{2.27}$$

$$E_t \left(\frac{\omega_{t+1}}{q_{t,t+1}} \right) = \frac{1}{1+n} \left[(1 - \gamma) \omega_t + (\bar{g}_t - \bar{\tau}_t) f(k_t) \right], \tag{2.28}$$

$$E_t \left(\frac{1}{q_{t,t+1}} \right) = \frac{1}{\tilde{q}(k_{t+1})}, \tag{2.29}$$

$$E_t \left(\frac{1}{q_{t,t+1} (1 + \pi_{t+1})} \right) = \frac{1}{1 + R_t}, \tag{2.30}$$

$$1 + R_t = \Phi(\bar{r}_t, \pi_t; \bar{\pi}). \tag{2.31}$$

Equation (2.27) is the good market clearing condition. (2.28) is the real *per capita* government budget constraint (2.23). Equation (2.29) comes from (2.9), (2.13) and (2.25).

Finally, equation (2.30) is the one-period risk-free nominal interest rate described in equation (2.4) where we have used (2.13).

If the period $t+1$ is characterized by S_{t+1} possible states of the world then the later system of equations is composed by $5+S_{t+1}$ equations allowing to find the values of c_t, k_{t+1}, ω_t, π_t, R_t and the S_{t+1} values of $q_{t,t+1}$, subject to equilibrium existence and uniqueness. Notice that it is possible, in theory at least, to eliminate the variables R_t and $q_{t,t+1}$—both non-predetermined and non-dynamic—in order to reduce the size of the system. So we can consider a representation[14] composed of four dynamic equations where two variables, c_t and π_t, are non-predetermined and two variables, k_t and $\varkappa_t$, are predetermined, with $\varkappa_t = (1+\pi_t)\,\omega_t = (M_{t-1}+B_{t-1})\,/N_t P_{t-1}$. This choice would theoretically permit to solve the problem posed by the dynamic status of $\omega_t = (M_{t-1}+B_{t-1})\,/N_t P_t$ and $1+\pi_t = P_t/P_{t-1}$, whose values can jump but not independently of each other. This representation is more satisfactorily from a conceptual point of view but is not sufficiently malleable on a technical level. Later on we will take a roundabout way to analyze the previous model.

2.3 Steady State Equilibria

A deterministic steady state equilibrium is a vector (c,k,ω,π) verifying a four-equations system which is obtained from equations (2.26) to (2.31) where we delete the indications of time and uncertainty and we merge the deterministic version of (2.30) with (2.31). In addition, assuming $\bar{g}=\bar{\tau}$, we obtain:

$$\left[\tilde{q}(k)-\beta^{-1}\right]c = \zeta\left[\tilde{q}(k)\,k+\omega\right], \tag{2.32}$$

$$\left[\frac{1+n}{\tilde{q}(k)}-(1-\gamma)\right]\omega = 0, \tag{2.33}$$

$$c = (1-\bar{g})\,f(k)-(n+\delta)\,k, \tag{2.34}$$

[14] Appendix (A.4) gives details of such a representation.

and

$$\tilde{q}(k)(1+\pi) = \Phi(\bar{r}, \pi). \tag{2.35}$$

The first three equations are independent of π. The system is then dichotomous and allows to find (c, k, ω) independently of the monetary policy. For a given value of k, equation (2.35) allows to find the equilibrium value(s) of π according to the target, $\bar{r}$ which can (or cannot) be chosen to be equal to the actual steady state value of $1 - \tilde{q}(k)$.

Notice that this long run dichotomy is not a fundamental characteristic of such a model. It is the consequence of *i)* the simple monetary and fiscal rules that we use, and *ii)* the adoption of the variable ω, the beginning-of-period real debt, rather than $\varkappa = (1+\pi)\omega$, the end-of-period real debt.

2.3.1 Equilibrium Inflation

We begin this subsection by analyzing the monetary part of the steady state. According to assumption (H1), and when the real interest rate target coincides with the long run real interest rate : $\bar{r} = \tilde{q}(k) - 1$, equation (2.35) has at least one solution corresponding to the inflation target, $\bar{\pi}$.

Benhabib, Schmitt-Grohé and Uribe (2001a, 2001b) show that the possibility of the existence of a second steady state equilibrium is one of the unexpected consequences of the ZLB on the nominal interest rate. It is notably the case when the rule is active in the sense of Leeper (1991) around the inflation target $\bar{\pi}$, as we have supposed in (H1). In this case, a second equilibrium appears, corresponding to a lower inflation rate, potentially negative and remembering the Keynesian liquidity trap. Figure 2.1 illustrates this case, where we assume that $\bar{r} = \tilde{q}(k) - 1$.

Figure 2.1 corresponds to the case where the function $\Phi(\cdot)$ crosses the horizontal axis defined by $1+\underline{R}$ for a value of π greater than $[(1+\underline{R})/(1+\bar{r})]-1$, which determines the lower equilibrium value in $\pi^{**} = [(1+\underline{R})/(1+\bar{r})] - 1$. The associated nominal interest rate, R, is at its minimum value, $\underline{R}$, and then the liquidity trap is reached.

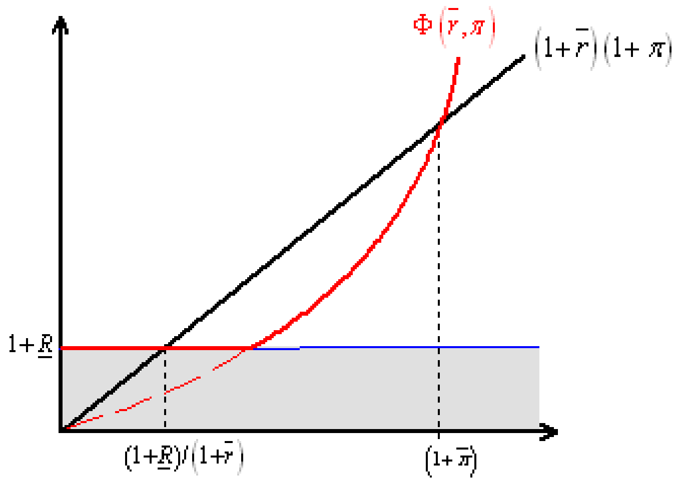

Figure 2.1.

2.3.2 Debt, Capital, and Interest

We now analyze the real part of the deterministic steady state. Equation (2.33) admits two evident solutions, $\omega^* = 0$ and $\tilde{q}(k^{**}) = \frac{1+n}{(1-\gamma)}$, corresponding to two stationary equilibrium vectors of the variables c, k, and ω.

"Autarkic Equilibria"

First, we study the solution corresponding to a zero public debt in the steady state. Equation (2.32) together with equation (2.34), allow to obtain the value of the per capita capital stock and the per capita consumption in an implicit form. We get:

$$\omega^* = 0, \tag{2.36}$$

$$1 + r^* = \tilde{q}(k^*) = \frac{\beta^{-1}}{1 - \zeta k^*/c^*}, \tag{2.37}$$

$$c^* = (1 - \bar{g}) f(k^*) - (n + \delta) k^*. \tag{2.38}$$

The second equation allows to make sure that the equilibrium interest rate, r^*, verifies:

$$r^* > \beta^{-1} - 1,$$

where β^{-1} is the gross interest rate in the Ricardian economy, obtained by assuming that $\zeta = n = 0$.

Because the parameter ζ, given by $n\xi\left(\beta^{-1} - 1\right)$, is small, the gap between r^* and $\beta^{-1} - 1$ is likely to be low. Besides, the value of the equilibrium-debt level equals zero, as in the Ricardian case[15]. With reference to the standard OLG model, where this kind of equilibria corresponds to the absence of exchange among generations, we call these first equilibria: "*autarkic equilibria*".

"Debt Equilibria"

The second solution of equation (2.33) allows to compute the equilibrium value of the real public debt according to equation (2.32). One obtains:

$$\omega^{**} = \zeta^{-1}\left(\frac{1+n}{1-\gamma} - \beta^{-1}\right)c^{**} - \frac{1+n}{1-\gamma}k^{**}, \tag{2.39}$$

where the values of k^{**} and c^{**} are respectively given by (2.40), implicitly, and by (2.41):

$$\begin{aligned} r^{**} &= \tilde{q}\left(k^{**}\right) - 1 = \frac{\gamma + n}{1-\gamma} &(2.40)\\ c^{**} &= \left(1-\bar{g}\right) f\left(k^{**}\right) - \left(n+\delta\right) k^{**} &(2.41) \end{aligned}$$

Comparing autarkic and debt equilibria we obtain the following proposition whose proof is provided in appendix A.1:

Proposition 1 *The real value of the* per capita *public debt is positive in a debt equilibrium if and only if the associated real interest rate is greater than the autarkic real*

[15]Since the equation (2.33) has to be verified when $\zeta = 0$, the stationary debt level is: $\omega^R = 0$ in the ricardian case.

interest rate, i.e.:

$$r^{**} = \frac{\gamma + n}{1 - \gamma} \geq r^* \Longleftrightarrow \omega^{**} \geq 0.$$

The intuition of this proposition is straightforward. In presence of wealth effects, an increase in the real public debt level increases the net wealth of the agents and leads them to increase their level of consumption. Accordingly, their savings does not grow sufficiently to absorb the new issued debt, which led to an increase in the real interest rate. The reverse is also true and a higher interest rate is necessarily associated to a higher level of the real public debt.

A graphical representation

The two kinds of equilibria can easily be represented in a $(k; c)$ plan. The hump-shaped curve in Figure 2.2 is the steady state resources constraint of the economy, given by equation (2.38) or (2.41). The top of this curve corresponds to a modified golden rule which is reached for a *per capita* capital stock k^g implicitly defined by: $(1 - \bar{g}) f_k (k^g) - (n + \delta) = 0$, or equivalently, by using (2.25) :

$$r^g \equiv r(k^g) = \frac{n + \delta \bar{g}}{1 - \bar{g}}. \qquad (2.42)$$

The Ricardian *per capita* capital stock k^r is given by equation (2.37) when $\zeta = n = 0$, *i.e.* $\tilde{q}(k^r) = \beta^{-1}$. The upward sloping curve corresponds to equation (2.37) and intersects with (2.38) to give the locus (k^*, c^*). The vertical k^{**} is the steady state *per capita* capital stock in a debt equilibrium. It is implicitly defined by (2.40). Figure 2.2 represents the case where: $k^{**} < k^* < k^g$ (and $k^g < k^r$) or, equivalently, $r^g < r^* < r^{**}$ (and $\beta^{-1} - 1 < r^g$). In this case, a debt equilibrium is necessarily characterized by a positive level of public debt (Proposition 1) and a lower steady state consumption level c^{**} than the one obtained in the autarkic equilibrium, c^*. Afterward, we will assume a weaker assumption:

$$r^{**} = \frac{\gamma + n}{1 - \gamma} \geq \max(r^*; r^g), \qquad \text{(H2)}$$

which nevertheless guarantees the positivity of ω^{**}.

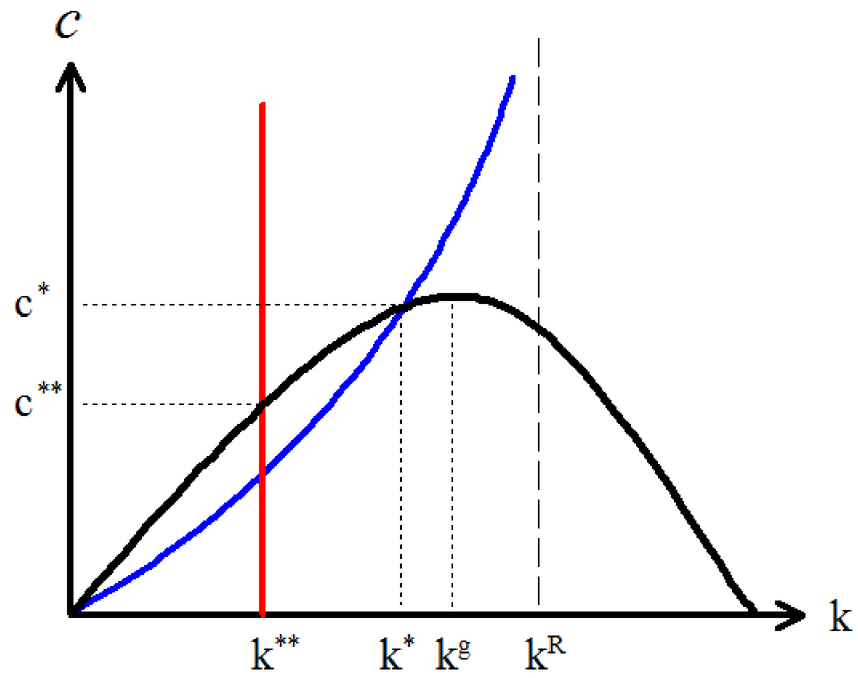

Figure 2.2.

2.3.3 Multiple Equilibria

Recall that in Subsection 2.3.1, we noted that for a given level of real interest rate, there were two stationary state values for the rate of inflation, one of the two being associated to an active monetary policy and the other to a passive policy. Accordingly, each of the two real interest rates r^* and r^{**} can be associated with two possible inflation rates, and our economy potentially admits four equilibria. These equilibria are represented on the figures 2.3a and 2.3b, each representing a particular version of the monetary rule.

In the first case, represented on figure 2.3a, the monetary rule depends on a constant real interest rate target[16], corresponding to the autarkic equilibrium: $\bar{r}_t = r^*$. If the actual real interest rate is r^{**} then the target $\bar{\pi}$ is not reached and an inflationary bias appears.

[16]The rule used in this case is similar to the Leeper-Taylor's rule.

In the second case, represented on figure 2.3b, the monetary rule depends on the current real interest rate: $\bar{r}_t = r_t$. As we can note, this rule presents the advantage of not making the long term inflation rate depending on the equilibrium value of the real interest rate, except in the liquidity trap. So, the inflation target $\bar{\pi}$ can be reached for r^* and for r^{**}. On the other hand, the assumption adopted about the representation of a liquidity trap does not allow us to obtain the uniqueness of the lower inflation rate.

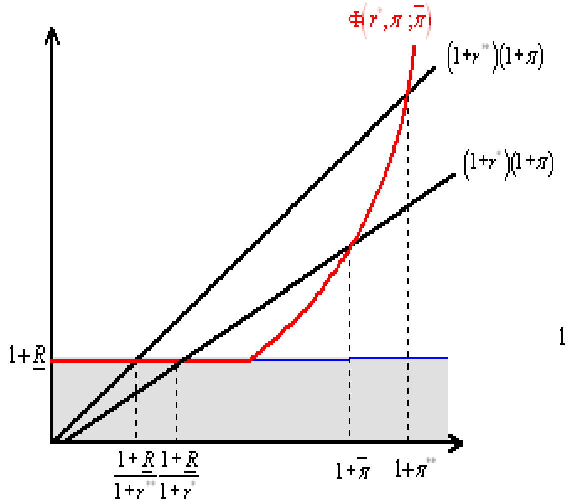

Figure 2.3a

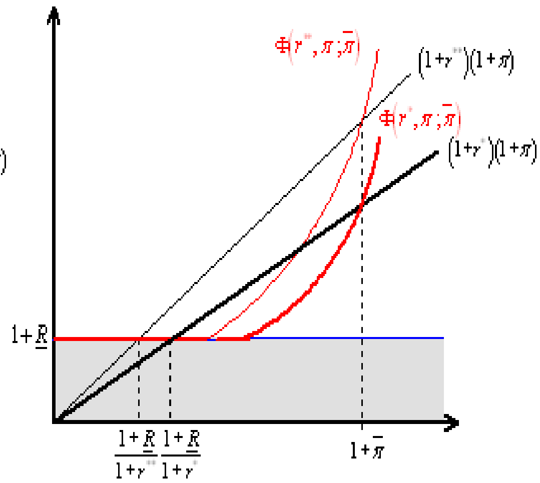

Figure 2.3b

2.4 From Local to Global Dynamics

As we will verify in this section, the four steady state equilibria we obtained correspond to—and have, locally, the same dynamics properties as—those analyzed by Leeper (1991), but unlike the case of the Ricardian economy without liquidity trap considered by Leeper, these four equilibria can exist for a unique set of the fundamental parameters.

2.4.1 The Linearized Model

In order to study the dynamics of the economy, we start by analyzing the local stability around each stationary equilibrium. As we have already noted, the most relevant linearized model would be the one constituted of the predetermined variables k_t and

$\varkappa_t = (M_{t-1} + B_{t-1}) / N_t P_{t-1}$ and the non predetermined variables c_t and π_t. Then the Blanchard and Kahn (1981) conditions would theoretically allow to characterize the local dynamics of the four stationary equilibria. We shall adopt this procedure in the last section in order to simulate numerically the model, but the dimension of the system does not allow us to characterize analytically the equilibria. On the other hand, the system composed of the variables c_t, k_t, ω_t and π_t offers some interesting possibilities that we are going to investigate.

Because one of the variables, ω_t, could equal zero in the long run, we linearize the equations (2.26) to (2.31) around any stationary equilibrium, by defining each variable in difference : $\hat{u}_t = u_t - u$, where u represents the variable u_t evaluated in one of the stationary equilibria. We obtain :

$$\begin{aligned}
\hat{c}_t &= \frac{\beta^{-1}}{1+r} E_t \hat{c}_{t+1} + \frac{\zeta}{1+r} E_t \hat{\omega}_{t+1} + \left(\zeta - \frac{\beta^{-1} c + \zeta \omega}{(1+r)^2} f_{kk} \right) \hat{k}_{t+1}, && (2.43) \\
\hat{k}_{t+1} &= \frac{1}{1+n} \Big([1 + r - \bar{g} f_k] \hat{k}_t - \hat{c}_t - f \cdot \hat{g}_t \Big), && (2.44) \\
E_t \hat{\omega}_{t+1} &= \frac{1+r}{1+r^{**}} \hat{\omega}_t + \frac{\omega f_{kk}}{1+r^{**}} \hat{k}_{t+1} + \frac{1+r}{1+n} f \cdot (\hat{g}_t - \hat{\tau}_t), && (2.45) \\
E_t \hat{\pi}_{t+1} &= \phi_\pi \hat{\pi}_t + (\phi_r - 1) \frac{1+\pi}{1+r} f_{kk} \hat{k}_{t+1}, && (2.46)
\end{aligned}$$

where $\phi_\pi = (1+\pi) \Phi_\pi / \Phi$ and $\phi_r = (1+r) \Phi_r / \Phi$ are the elasticity of the function $\Phi(\cdot)$ and where we have used $\frac{\gamma+n}{1-\gamma} = r^{**}$.

Denoting $\hat{Y}_t = \begin{bmatrix} \hat{k}_t & \hat{\omega}_t & \hat{c}_t & \hat{\pi}_t \end{bmatrix}'$, the vector of the endogenous variables, using the long run equations (2.32) to (2.35) and neglecting the shocks $\hat{g}_t$ and $\hat{\tau}_t$, the equations (2.43) to (2.46) could be combined in order to get the following state-space form :

$$E_t \hat{Y}_{t+1} = J_4 (k, \omega, c, \pi) \cdot \hat{Y}_t, \qquad (2.47)$$

where the Jacobian matrix $J_4(k, \omega, c, \pi)$ is given by:

$$J_4(\cdot) = \begin{pmatrix} \frac{1+r-\bar{g}f_k}{1+n} & 0 & -\frac{1}{1+n} & 0 \\ \frac{1+r-\bar{g}f_k}{1+n} f_{kk} \frac{\omega}{1+r^{**}} & \frac{1+r}{1+r^{**}} & -\frac{1}{1+n} f_{kk} \frac{\omega}{1+r^{**}} & 0 \\ \frac{1+r-\bar{g}f_k}{1+n} \left(c\frac{f_{kk}}{1+r} - \zeta\beta(1+r) \right) & -\zeta\beta\frac{1+r}{1+r^{**}} & \beta(1+r) - \frac{c\frac{f_{kk}}{1+r} - \zeta\beta(1+r)}{1+n} & 0 \\ \frac{1+r-\bar{g}f_k}{1+n} (\phi_r - 1) \frac{1+\pi}{1+r} f_{kk} & 0 & -(\phi_r - 1)\frac{1+\pi}{1+r}\frac{f_{kk}}{1+n} & \phi_\pi \end{pmatrix}. \tag{2.48}$$

The vector $\hat{Y}_t$ is composed of a predetermined variable, $\hat{k}_t$, a non predetermined variable, $\hat{c}_t$, and the two variables $\hat{\omega}_t$ and $\hat{\pi}_t$, both potentially non predetermined but linked to one another by the relation:

$$\hat{\omega}_t = \frac{1}{1+\pi}\hat{\varkappa}_t - \frac{\omega}{1+\pi}\hat{\pi}_t,$$

where $\hat{\varkappa}_t$ is predetermined. It is therefore necessary, in order to apply the Blanchard and Kahn conditions, to consider one of the two variables ($\hat{\pi}_t$ or $\hat{\omega}_t$) as predetermined and the other one ($\hat{\omega}_t$ or $\hat{\pi}_t$) as non predetermined. The matrix $J_4(k, \omega, c, \pi)$, evaluated in one of the stationary states, has to possess two eigenvalues inside the unit circle and two eigenvalues outside in order to let the associated equilibrium locally determinate.

The interest of the matrix $J_4(k, \omega, c, \pi)$, with regard to the Jacobian matrix $\mathcal{J}_4(k, \varkappa, c, \pi)$ which would be associated to the vector variables $\hat{Z}_t = \begin{bmatrix} \hat{k}_t & \hat{\varkappa}_t & \hat{c}_t & \hat{\pi}_t \end{bmatrix}'$, lies in its decomposition property. The three first lines of the last column are composed of zero, what means that we can study the properties (the eigenvalues) of the sub-systems $J_3(k, \omega, c)$ and $J_1(\pi)$ independently of each other, with :

$$J_3(k, \omega, c) = \begin{pmatrix} \frac{1+r-\bar{g}f_k}{1+n} & 0 & -\frac{1}{1+n} \\ \frac{1+r-\bar{g}f_k}{1+n} f_{kk} \frac{\omega}{1+r} & \frac{1+r}{1+r^{**}} & -\frac{1}{1+n} f_{kk} \frac{\omega}{1+r} \\ \frac{1+r-\bar{g}f_k}{1+n} \left(c\frac{f_{kk}}{1+r} - \zeta\beta(1+r) \right) & -\zeta\beta\frac{1+r}{1+r^{**}} & \beta(1+r) - \frac{c\frac{f_{kk}}{1+r} - \zeta\beta(1+r)}{1+n} \end{pmatrix} \tag{2.49}$$

and :

$$J_1\left(\pi\right) = \phi_\pi.$$

The eigenvalue associated to $J_1\left(\pi\right)$ is its unique component, ϕ_π. If the function $\Phi\left(\cdot\right)$ is of the form used in the figures 2.3a or 2.3b, we have $\phi_\pi > 1$ in $\bar{\pi}$, as well as in π^{**}, and $\phi_\pi = 0$, around the liquidity trap equilibria in $\left(1+\underline{R}\right)/\left(1+r^{*}\right)-1$ and in $\left(1+\underline{R}\right)/\left(1+r^{**}\right)-1$.

The sub-system $J_3\left(k,\omega,c\right)$ is easier to study when the type of the considered steady state is specified.

2.4.2 Autarkic Equilibria

In an autarkic steady state equilibrium, the real debt equals zero, which allows to simplify the matrix $J\left(k,\omega,c\right)$:

$$J_3\left(k^*,\omega^*,c^*\right) = \begin{pmatrix} \frac{1+r^*-\bar{g}f_k^*}{1+n} & 0 & -\frac{1}{1+n} \\ 0 & \frac{1+r^*}{1+r^{**}} & 0 \\ \frac{1+r^*-\bar{g}f_k^*}{1+n}\left(c^*\frac{f_{kk}^*}{1+r^*}-\zeta\beta\left(1+r^*\right)\right) & -\zeta\beta\frac{1+r^*}{1+r^{**}} & \beta\left(1+r^*\right)-\frac{c\frac{f_{kk}^*}{1+r}-\zeta\beta(1+r^*)}{1+n} \end{pmatrix}.$$

Rearranging the variables, it is once again possible to decompose this matrix into two sub-systems $J_2\left(k^*,c^*\right)$ and $J_{1'}\left(\omega^*\right)$, with :

$$J_2\left(k^*,c^*\right) = \begin{pmatrix} \frac{1+r^*-\bar{g}f_k^*}{1+n} & -\frac{1}{1+n} \\ \frac{1+r^*-\bar{g}f_k^*}{1+n}\left(c^*\frac{f_{kk}^*}{1+r^*}-\zeta\beta\left(1+r^*\right)\right) & \beta\left(1+r^*\right)-\frac{c^*\frac{f_{kk}^*}{1+r^*}-\zeta\beta(1+r^*)}{1+n} \end{pmatrix}$$

and:

$$J_{1'}\left(\omega^*\right) = 1+r^*/1+r^{**}.$$

Under assumption (H2), the eigenvalue $1+r^*/1+r^{**}$ is strictly less than 1 and we show, in appendix A.2, that the condition :

$$(1-\bar{g})(r^*-r^g)(\beta(1+r^*)-1)-\zeta\beta(1+r^*)<-c^*\frac{f_{kk}^*}{1+r^*} \tag{H3}$$

is necessary and sufficient for the matrix $J_2(k^*,c^*)$ to admit one and only one eigenvalue less than unity in absolute value[17]. These results are summarized by the following proposition whose proof is provided in Appendix A.2:

Proposition 2 *Under assumptions (H1), (H2) and (H3), the autarkic equilibrium associated to the inflation target, $\bar{\pi}$, is locally determinate and the autarkic liquidity trap equilibrium is locally indeterminate.*

Equivalent results are obtained for Ricardian economies by putting $n=0$ and by replacing r^* with $\beta^{-1}-1$.

2.4.3 Debt Equilibria

The matrix $J_3^{**}=J_3(k^{**},\omega^{**},c^{**})$ corresponding to the debt equilibria is obtained by putting $r=r^{**}$ in (2.49). One obtains :

$$J_3^{**}=\begin{pmatrix} \frac{1+r^{**}-\bar{g}f_k^{**}}{1+n} & 0 & -\frac{1}{1+n} \\ \frac{1+r^{**}-\bar{g}f_k^{**}}{1+n}f_{kk}^{**}\frac{\omega^{**}}{1+r^{**}} & 1 & -\frac{1}{1+n}f_{kk}^{**}\frac{\omega^{**}}{1+r^{**}} \\ \frac{1+r^{**}-\bar{g}f_k^{**}}{1+n}\left(c^{**}\frac{f_{kk}^{**}}{1+r^{**}}-\zeta\beta(1+r^{**})\right) & -\zeta\beta & \beta(1+r^{**})-\frac{c^{**}\frac{f_{kk}^{**}}{1+r^{**}}-\zeta\beta(1+r^{**})}{1+n} \end{pmatrix}$$

In appendix A.3, we analyze the characteristic polynomial $\mathcal{P}^{**}(\lambda)$ associated to the matrix J_3^{**} which allows us to show that it admit one eigenvalue in absolute value less than unity and two eigenvalues, greater than unity. One can deduce the following proposition whose proof is provided in appendix A.3:

[17] Notice that assumption (H3) is always verified in a Ricardian economy, when $\beta(1+r)=1$ (and $\zeta=0$). An other case verifying (H3) is when $r^*<r^g$ which is a stronger version of assumption (H2) but which *excludes* the representation adopted in Figure 2.2.

Proposition 3 *Under the assumption (H1) and (H2), the debt equilibrium associated to the higher inflation rate, $\bar{\pi}$ or π^{**}, is locally overdeterminate and the debt-liquidity-trap equilibrium is locally determinate*

2.4.4 From Local Determinacy to Global Indeterminacy

Based on propositions 2 and 3, we can conclude that the four potential stationary equilibria of our economy have, locally, the properties of the four equilibria associated to the four configurations of fiscal and monetary policies identified by Leeper (1991).

Within the framework considered by Leeper, monetary and fiscal policies simultaneously passive lead to indeterminacy and active policies[18] to overdeterminacy (instability). Only the configurations where one of the two policies is active and the other passive provide the determinacy—*i.e.* the local uniqueness—of the equilibrium.

If we are to interpret the local stability properties of our equilibria with the same concepts, we must use a local definition of active vs passive monetary and fiscal policies, and we must explain why a policy cannot be globally active or passive.

The difficulty with the definition of an interest rate policy corresponding to a globally active monetary rule was already noted by Benhabib, Schmitt-Grohé and Uribe (2001b). The ZLB on the nominal interest rate (the liquidity trap) fails to ensure the application of an interest rate rule sufficiently reactive to inflation (active) when the rates are low. We have seen that the required non-linearity of the monetary rule doubled the number of stationary equilibria and no longer ensured the determinacy of the autarkic equilibrium when the fiscal policy was locally passive (reactive to the level of debt), which is the main result of Benhabib, Schmitt-Grohé and Uribe (2001b).

The second source of difficulty arises from the accumulation of debt. The exchange economy considered by Leeper permits to characterize a simple fiscal rule whose properties do not depend on the level of the initial real public debt[19]. The mere presence of

[18] Recall that for Leeper, a fiscal policy is called active when the fiscal authority pays no attention to the debt stabilization objective. The rule is then not very reactive to the level of debt.

[19] The term "initial" can be misleading, insofar as the general level of prices can jump so that the real

production and capital accumulation is not sufficient to modify this result. In a Ricardian economy, the Barro-equivalence (Barro, 1974) insulates the real interest rate from the real public debt level. However, in a non-Ricardian economy, the presence of wealth effects results in the dependence of the real interest rate level on the public debt. In this case, a too simple fiscal policy (linear) is not sufficient to offset the increased debt burden associated to a high real public debt, even if it is sufficiently responsive (passive in terms of Leeper) for a lower level of debt. This finding has already been reported by Cushing (1999) and Benassy (2000) when the monetary authorities set the nominal interest rate to a constant value.

Ironically, the characteristics of both rules which explain the multiplicity of equilibria are diametrically opposed. The monetary rule would not be a problem if it was linear[20]. On the other hand, a non-linear fiscal rule, becoming more responsive to the level of debt as the real interest rate rises, would easily allow to ensure the convergence of debt to its targeted value. From another point of view, the double multiplicity of equilibria results from a double non-linearity: *i)* an interest rate rule which respects a lower bound, and *ii)* the existence of wealth effects with a too simple fiscal rule.

The most original result of our model lies probably in the coexistence of two steady state equilibria locally determinate, *i.e.* associated with saddle *i.e.*, locally unique, trajectories[21]. The first one is the targeted autarkic equilibrium. Since the Taylor principle is verified around this equilibrium, the associated monetary policy is said to be active. On the other hand, the fiscal policy is locally passive. This last point is easy to verify by rewriting the linearized government constraint (2.45) when $r = r^*$, $\omega = 0$, and $\bar{g}_t = \bar{\tau}_t$. One obtains: $E_t\hat{\omega}_{t+1} = \left(\left(1 + r^*\right) / \left(1 + r^{**}\right)\right) \hat{\omega}_t$. Then, according to assumption (H2), the real public debt converges to 0, its long run value. The second considered equilibrium is the debt-liquidity trap (or public debt-deflation) equilibrium. Because the nominal in-

government debt is just covered by expected income, as in the highly controversial "Fiscal Theory of the Price Level".

[20]But this would require the possibility of negative nominal interest rates...

[21]It is in fact about a stable variety of dimension 2, *i.e.* "saddle plans".

terest rate is stuck at its ZLB, the monetary policy is forced to be passive. On the other hand, the fiscal policy is locally active because the low value of γ does not compensate for the public debt burden associated with a high real interest rate. This configuration corresponds to the FTPL, but the properties of this equilibrium also recall those of a "Samuelson equilibrium" in a traditional OLG model. We return to this point in the next section.

According to the existence of two saddle trajectories the issue of self-fulfilling expectations becomes particularly interesting. The economy by being situated on one of these two saddle trajectories could jump, thanks to a likely important shock affecting agents' expectations, on the other saddle trajectory.

2.5 Self-fulfilling Expectations: the Peril of Public Debt-Deflation

In this section, we will verify our last conjecture by simulating an expectation shock. For this purpose, we calibrate the structural parameters of the model based on Euro Area data. Then, assuming that the predetermined variables are in halfway between the two considered steady states, we investigate the effects of an expectation shock bringing the economy—that we suppose to be initially on a virtuous trajectory towards the autarkic equilibrium—on a public debt-deflation trajectory.

2.5.1 Functional Forms and Calibration

We assume that the production function is of the kind: $f(k_t) = Ak_t^{\alpha}$, where α is the capital share and A, a scaling parameter. We define the monetary rule as

$$\Phi(r_t, \pi_t; \bar{\pi}) = \max\left\{(1+r_t)(1+\bar{\pi})\left(\frac{(1+\pi_t)}{(1+\bar{\pi})}\right)^{\phi}; 1+\underline{R}\right\} \tag{2.50}$$

where $\underline{R} > 0 \quad$ and $\phi \geq 1$. This rule respects (H1).

We assume that each period corresponds to a year. The parameter values we use in the numerical analysis are shown in Table 2.1. Most of them are taken from Smets and Wouters (2002) and Fagan *et al* (2001). We set the discount factor β to 0.96 implying an annual discount rate approximately 4%. The capital share α is chosen to be equal to 0.3 and the depreciation of capital, δ, to 0.1. Public expenditure share, $\bar{g}$, is set equal to 0.2. In order to have zero primary public deficit at the zero-debt steady state, we calibrate taxes-to-GDP ratio $\bar{\tau}$ to 0.2. The consumption weight in utility function, ξ, is set equal to 0.95. The scaling parameter, A, is calibrated such that we get, at the autarkic steady state, a value of the output equals to 100[22].

While Melitz (2000) estimates the weight of public debt in the fiscal rule at 0.03, Gali and Perotti (2003) estimate this parameter at 0.05. In order to obtain reasonable values both for the public debt-to-GDP ratio and the real interest rate at the debt equilibrium, we follow Melitz (2000) and we set $\gamma = 0.03$. The population growth rate, n, is set equal to 0.014 in order to get a steady state value of government debt-to-GDP ratio at the debt equilibrium, ω^{**}, approximately equal to 160%. This parameter value is slightly larger than the value observed in the data but it is assumed to capture all the wealth effects which would affect the real economy. The lower bound for the nominal interest rate is set at 0.00009%. The reason is that, as we have a logarithmic utility function, choosing zero as the lower bound for nominal interest rate will imply an infinite demand of money.

According to the parameter values displayed in Table 2.1. We easily obtain the values of the real interest rate in the Ricardian, autarkic and debt equilibria, and the value of the modified golden rule interest rate that are summarized in Table 2.2. Notice that the ranking of steady state real interest rate values is consistent with the representation in Figure 2.2.

[22] Our calibration leads to $A = 20.1467$.

Table 2.1: Parameters Values.

Definition	Parameter	Value
Discount factor:	β	0.96
Weight of consumption in the utility function:	ξ	0.95
Capital share of output:	α	0.3
Depreciation rate of capital:	δ	0.1
Population growth rate:	n	0.014
Nominal interest rate lower bound	$\underline{R}$	0.00009%
Public expenditure-to-GDP ratio:	$\bar{g}$	0.2
GDP parameter in the fiscal rule:	$\bar{\tau}$	0.2
Debt parameter in the fiscal rule:	γ	0.03

Table 2.2: Real Interest Steady State Values.

Real Interest Rate	Parameter	Value
Ricardian Equilibria Interest Rate:	$\beta^{-1} - 1$	4.17%
Modified-Golden Rule Interest Rate	r^g	4.25%
Autarkic Equilibria Interest Rate:	r^*	4.38%
Debt Equilibria Interest Rate:	r^{**}	4.54%

2.5.2 Simulation and Discussion

Now, we assume that both capital stock k_t and real public debt $\varkappa_t$ (the predetermined variables) are at a half distance between the targeted autarkic equilibrium and the public debt-deflation equilibrium. We then study the convergence of the economy towards each steady state. We have made this exercise both with the linearized version of the model and with a non-linearized one, using the DYNARE package for Matlab (see Juillard [2004]). We found identical results with the two versions, which signifies that the linearization does not affect the convergence towards the steady state. We report on figure 2.4 the results of the non linearized model.

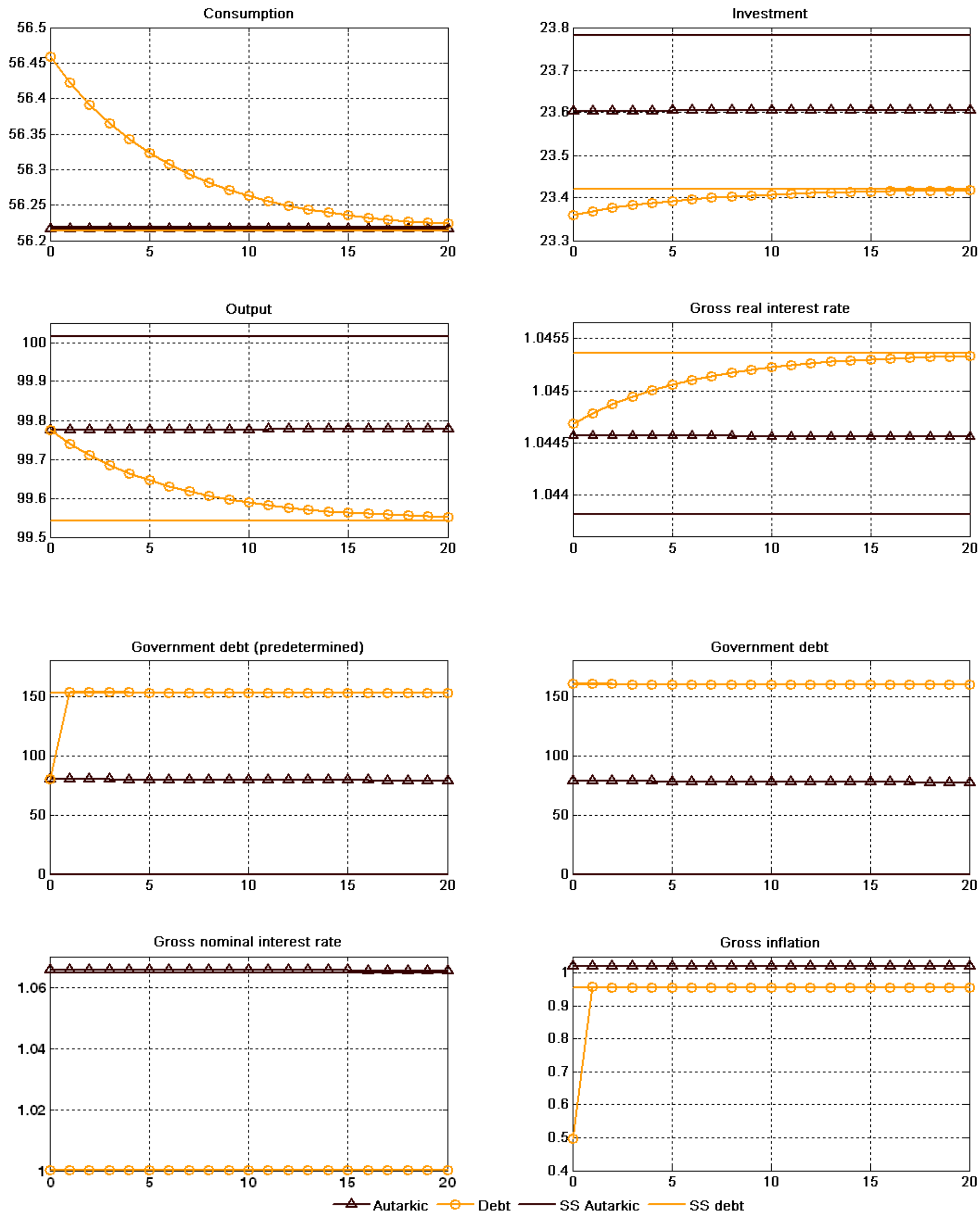

Figure 2.4

The convergence toward the autarkic equilibrium takes a very long time to reach the steady state. This is due to the relatively low value of γ. The adjustment of investment, output, and the real interest rate is expected since the initial value of the capital stock is below its steady-state level. Nevertheless, consumption converges faster towards the targeted autarkic equilibrium.

Now assume that the expectations lead the economy towards the public debt-deflation equilibrium. We note first an increased consumption, a reduced investment and a sharp decline in the rate of inflation which leads the nominal interest rate to the zero bound. Then, output and consumption declines and the real interest rate rises toward its higher steady state value.

Since the economy is non Ricardian, an expected increase in public debt entails a positive wealth effect. Consequently, agents reduce their savings so as to increase their consumption. As a result, the real interest rate increases and investment decreases.

The sharp deflation can be explained by the FTPL that Leeper (1991), Sims (1994) and Woodford (1994) presented and analyzed, but with a slight extension. Referring to the "stock analogy" used by Cochrane (2005), the *per capita* government budget constraint (2.28) can be rewritten as a valuation equation:

$$\omega_t = \frac{\bar{\tau}_t - \bar{g}_t}{1-\gamma} y_t + \frac{1+n}{1-\gamma} E_t \left(\frac{\omega_{t+1}}{q_{t,t+1}} \right)$$

where $\omega_t = \Omega_t / P_t N_t$. Using again the assumption $\bar{\tau}_t = \bar{g}_t \; \forall t$, made in the simulation, and supposing, for sake of simplicity, that there is no more uncertainty after the expectation shock, this equation becomes:

$$\omega_t = \frac{(1+r^{**})\, \omega_{t+1}}{1+r_t} \tag{2.51}$$

where we have used $r^{**} = \frac{\gamma+n}{1-\gamma}$. Because the exogenous component of the primary public surplus is zero, the right-hand term of the valuation equation is only constituted by the bubble, *i.e.* the unbaked part of the public debt. The OLG structure of our model permits the existence of an equilibrium where this right-hand term is positive at the steady state. This is the case at the public debt-deflation equilibrium.

In order to understand this last point, it can be useful to consider a simplified version of our model by supposing an endowment economy. Let y be the *per capita* endowment and $c = (1-\bar{g})\,y$, the equilibrium consumption. In absence of capital, the equilibrium aggregate Euler equation (2.26) can be rewritten as:

$$1 + r_t = \beta^{-1} + \frac{\zeta}{c}\omega_{t+1} \tag{2.52}$$

Combining (2.51) and (2.52), we easily obtain the dynamic equation:

$$\omega_{t+1} = \frac{\beta^{-1}\omega_t}{1 + r^{**} - \frac{\zeta}{c}\omega_t}$$

which accepts $\omega^* = 0$ and $\omega^{**} = \left(1 + r^{**} - \beta^{-1}\right) c/\zeta$ as steady state values and which can be represented on the following figure:

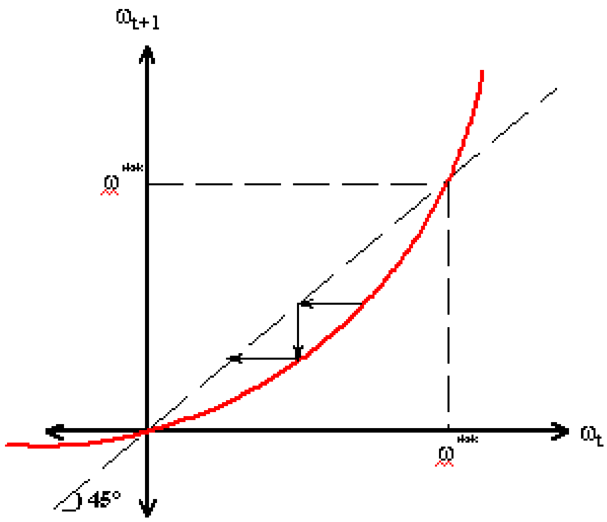

Figure 2.5.

Starting from a value $\omega_0 = \Omega_0/P_0N_0$ between ω^* and ω^{**}, the initial value of P_0 being determined by an active monetary policy, the economy converges towards the autarkic equilibrium ω^*. Thanks to an expectation shock, the economy can then jump to the public debt-deflation equilibrium ω^{**}, without any transition in this simple exchange economy. Since the nominal debt Ω_t is predetermined, the real value of the public debt adjusts by a price fall which throw the economy into the liquidity trap and the deflation. Interestingly, this equilibrium resembles to the Samuelson equilibrium of a standard OLG economy. In the case[23]: $\gamma = 0$, this equilibrium would be a pure bubble. More generally, the steady state real public debt ω^{**} is a growing function of r^{**} and thus of γ and this is a characteristic of a FTPL equilibrium when part of the primary surplus is function of the level of the real public debt. The sole difference is that the standard FTPL needs the existence of a positive exogenous primary surplus that is not necessary in our economy, this role being played by the bubble component of the debt.

In the non Ricardian economy with capital, the public debt-deflation steady state is not immediately reached. In particular, consumption and investment take time—around 20 years—to reach theirs long run lower values. Unfortunately, we have to recognize that the adjustment of the real public debt is much more—actually too—sharp. In our simulation, the initial deflation is around 50% of the initial price level which, of course, is unrealistic. Without this last disadvantage and the initial increase of the consumption level, our model could offer an alternative—or, at least, a complementary—explanation to the more traditional reading brought by Krugman (1998), Svensson (2001), and Eggertson and Woodford (2004) of some deflation episodes like the Japanese recession of the 90s. These authors argue that the Japanese liquidity trap was the consequence of a very negative shock on the natural interest rate in a context of inflation stabilization around a maybe too low target. The hypothesis of the liquidity trap of Benhabib, Schmitt-Grohé and Uribe (2001a, 2001b) explains the weakness of the nominal interest rates and the incapacity of the monetary authority to stabilize the economy in such a context, but does not allow to explain the entrance in recession and the persistence of this one[24]. Our public debt-deflation equilibrium does not have this flaw. The higher level of the real interest rate provokes a crowding out effect of the private investment and reduces

[23] Which would require $1 + n > \beta^{-1}$.

[24] We could even expect a more important level of activity in a model where the weakness of the nominal interest rates reduces the level of the monetary distortions and increases the labor supply.

the level of the production, which seems to be a strong characteristic of a deflationary episode. For this reason, the main objective of further researches would be to make this scenario more plausible, by both reducing the initial price jump and the initial increased consumption, but without affecting our long run results. Among other assumptions, the introduction of a learning process could be an interesting perspective.

2.6 Conclusion

The focus of this chapter is the study of the interaction between monetary and fiscal policy in the presence of non Ricardian consumers and with capital accumulation. In the economic environment considered, the Ricardian equivalence breaks down and government debt spawn wealth effects.

To this end, we develop an extended version of Weil's(1987, 1991) overlapping generations model in which government debt affects the behavior of consumers that is the fiscal policy is no longer neutral. This model has the Ricardian equivalence as a special case, when the population is constant. Assuming a simple fiscal policy, in the spirit of Leeper (1991) and a non linear monetary rule namely a Taylor rule taking into account the ZLB on the nominal interest rate, in the spirit of Benhabib, Schmitt-Grohé and Uribe (2001a), our analysis of the steady state exhibits the presence of four equilibria. We analyze local steady states dynamics. Comparing the four equilibria with the four configurations described by Leeper (1991) yield the same dynamics characteristics. As a consequence, the determinacy region is no longer specified by the policy parameter space. In short, in the presence of wealth effects, a nominal interest rate rule bounded below by zero emphasizes the global indeterminacy problem identified by Benhabib, Schmitt-Grohé and Uribe (2001a, 2001b). Accurately, four equilibria are founded regardless of the policy parameter space.

Our results show that a liquidity trap, also characterized by a higher real interest rate and a higher level of real debt, possesses the usually required properties of determinacy, like the more traditional equilibrium targeted by the monetary and fiscal authorities.

Furthermore, from the perspective of global analysis, the existence of two paths locally convergent arises the question of a self-fulfilling expectations shock. Indeed a self-fulfilling expectations shock can lead the economy from one trajectory to another.

To this end, the model is calibrated on annual data and allows to evaluate the implications of a self-fulfilling expectations shock. Our results show that for a given initial level of predeterminate variables, the convergence towards the debt-liquidity-trap equilibrium is fulfilled by an initial deflation and an increase in the real interest rate, caused by a change in agents' expectations. We thus give an other endogenous explanation of the liquidity trap where the public debt play a crucial role.

If these results can be empirically supported, then some deflationary episodes—like the Japanese recession of the 90s—could be reinterpreted as the consequences of a change in agents' expectations. But this proposition needs a deeper empirical arguments which are not treated in the present chapter. Thereby, in our simulation, the initial deflation accounts for 50% that is not realistic. For this reason, the main objective of further researches would be to make this scenario more plausible, by both reducing the initial price jump and the initial increased consumption, but without affecting our long run results. Among other assumptions, the introduction of a learning process could be an interesting perspective.

Another question arises: how to avoid deflation? Extrapolating the results of Bénassy and Guillard (2005) who study the case of a non-Ricardian exchange economy, the control of the growth rate of nominal debt should simultaneously ensure the uniqueness and determinacy of the equilibrium. Again, this issue deserves further research.

Chapter 3

Monetary and Fiscal Policy interactions in a non-Ricardian Model with Deep Habits

3.1 Introduction

The recent deflationary episodes, observed during the 2008-2010 global economic crisis, have revived the question of how policy should be conducted when short-term nominal interest rates reach their lower bound. In order to fight the recession and stimulate the economy, many countries have used intensively fiscal instruments. Accordingly, the debate on the effectiveness of fiscal instruments in influencing demand is back on the table.

In this chapter we study the interaction between monetary and fiscal policy, taking into account the possibility that short-term nominal interest rate reaches its ZLB and we analyze the effects of debt-financed tax cuts. In order to do so the departure from the Ricardian equivalence is essential because otherwise lump-sum fiscal policy is neutral. The most common ways are distortionary taxation, and overlapping generation structure. In this chapter we develop a model based on overlapping generation structure because we are interested on fiscal policies that have direct effects on the aggregate demand. Actually, in order to fight the recession, it is needed to stimulate the aggregate demand. In contrast, distortionary taxes operate mainly through the supply side.

The Ricardian equivalence failure based on overlapping generation structure leads to rich interactions between monetary and fiscal policy. Many authors, such as Leith and Wren-Lewis (2000), Annicchiarico (2009), Leith and von Thadden (2008) and Devereux (2010), among others, analyze the interaction between monetary and fiscal policy in such a framework. Particularly, this chapter is in line with the work of Annicchiarico et al (2006), and a continuation of our work started in chapter 2. These two works show that in a non Ricardian framework, based on overlapping generation structure, taking into account the ZLB on the nominal interest rate entails multiple steady state equilibria. More precisely, they found four steady state equilibria. Among, these equilibria, two are locally stable. In addition, pushing the argument further, we show in Chapter 2 that the first locally determined equilibrium is the equilibrium targeted by monetary and fiscal authorities[1], while the second locally determined equilibrium is an equilibrium characterized by higher government debt, deflation, lower output, and where the short-term nominal interest rate is at its lower bound.

Undoubtedly, this framework has the advantage of including two very different environments. The first corresponds to the situation targeted by monetary and fiscal authorities. The second is an environment characterized by debt-deflation and recessionary situation. Clearly, it allows us to study the behavior of the economy on the deflationary trajectory. In addition, we can analyze and compare the effect of fiscal shock with regard to both environments.

First, the existence of two locally convergent trajectories raises the following question. What is the response of the economy if agents believe that they will be on the debt-deflation trajectory? Answering this question could provide alternative or complementary

[1] The steady state levels of inflation and government debt are those targeted by monetary and fiscal authorities.

explanation of deflationary episodes based on change in agents' expectations. In Chapter 2, we showed that liquidity trap and high government debt are both consequences of an initial deflation caused by a change in agents' believes. Although, we found that the necessary initial deflation to reach the liquidity trap is approximately 50%, which is unrealistic. In this chapter, we try to render the initial deflation realistic. To do so, we introduce external deep habit specification, as in Ravn Schmitt-Grohe and Uribe (2006), into our non-Ricardian framework[2]. We show that this deep habit specification can reduce the gap between the two steady state values of government debt while retaining a reasonable gap between the two steady state values of output. In other words, the initial deflation is reduced considerably (11%) while the recessionary effect is still significant.

Second, our framework has the appeal that government debt is non-neutral. Thus the distinction between tax financed and debt-financed fiscal policy is relevant. Actually, many authors, such as Krugman (1998) and Eggertsson and Woodford (2003, 2005) among others, explored how to usefully employ monetary and fiscal policy, even when monetary authorities can not reduce short-term nominal interest rates. But, they used infinitely lived representative household framework, so there is no difference between tax financed and debt financed fiscal policy. This is because Ricardian equivalence holds. In this chapter, we focus on the effects of debt-financed fiscal policy rather than tax financed fiscal policy which has been widely studied. So our chapter aims to analyze and compare the effects of debt-financed tax cuts in both environments. We show that in the "targeted" environment debt-financed tax cuts entail a positive wealth effect. Consumption increases but investment decreases inducing a fall in output. On the other hand, in the "debt-deflation" environment, tax reductions imply a negative wealth effect. Consumption decreases but investment increases, entailing an increase in output.

Recently, Devereux (2010) analyzed the effect of debt financed and tax financed fiscal stimulus in New Keynesian model based on the Blanchard (1985) and Yaari (1969) struc-

[2] In this paper deep habits refer to external deep habits. It is the catching-up-with-the-Joneses of Abel (1990) but on a good-by-good basis.

ture and where monetary authority is constrained by the lower bound on the nominal interest rate. He argued that his model predicts that government debt issue has substantial wealth effects in a liquidity trap. These wealth effects stimulate aggregate demand and private consumption, and play an expansionary macroeconomic role. Even though, we conclude that debt-financed tax cuts have an expansionary effect in the liquidity trap, the mechanism is very different from the one outlined by Devereux (2010). Indeed, in our model, in the "debt-deflation" environment, tax reduction triggers the decrease of real government debt which is achieved thanks to a price increase. In this case there is a negative wealth effect inducing a fall in consumption. On the other hand, lower real government debt entails a decrease in the real interest rate and consequently the investment increases, stimulating aggregate demand . Accordingly, output increases.

The remainder of the chapter is organized in the following way. Section 3.2 develops our non-Ricardian model with external deep habits and capital accumulation. Section 3.3 investigates the effects of expectation shock and also analyzes the effect of debt-financed tax cut in both environments. Section 3.4 concludes.

3.2 The Model

This chapter develops a micro-founded general equilibrium model based on operlapping generations model of infinitely-lived families *à la* Weil (1987, 1991), featuring monopolistic competition, external habit formation and capital accumulation, under standard conditions of uncertainty and taking explicitly into account the existence of lower bound on the nominal interest rate.

In this economy there are many infinitely-lived families, firms, the government and the central bank. There are two types of firm: many monopolistically competitive firms, producing differentiated goods and a representative firm producing investment good, which is rented out to monopolistically competitive firms.

The population grows at a positive rate n. Each period new families appear in the economy without financial wealth and with a family firm (a monopolistically competitive firm). Hence, the profit of the family firm is totally transferred to the agents.

The economy is assumed to be a cashless economy as in Woodford (2003) where money serves uniquely as a unit of account[3]. There is uncertainty in the economy. However, we adopt a complete market assumption.

3.2.1 Consumers

A generation j consists of many identical infinitely-lived families (or agents) of type j, where j belongs to the interval $[1, N_t]$. Accordingly, we can consider a representative agent framework into a generation. Agents derive utility from consumption of many differentiated goods. Specifically, for each specific good, they care about their own consumption of a specific good compared to the benchmark level of the consumption of that specific good, as in Ravn, Schmitt-Grohe and Uribe (2006). In our environment characterized by heterogeneous agents, we need to specify an aggregation rule to obtain aggregate variables and define the consumption reference. The aggregation rule is the following:

$$z_t = \sum_{j \leq t-1} \frac{(N_j - N_{j-1})}{N_{t-1}} z_{j,t-1}, \tag{3.1}$$

where z is a generic variable. Notice that $N_j - N_{j-1}$ is the number of agents compound of the representative generation j, where N_j is the number of agents born in period $j \leq t$. Following Ravn et al (2006), we define $x^c_{j,t}$ as the CES habit-adjusted consumption index with elasticity of substitution, $\varepsilon > 1$:

$$x^c_{j,t} = M_t^{\frac{1}{1-\varepsilon}} \left(\sum_{m=1}^{M_t} \left(c_{j,t}\left(m\right) - \theta_c \tilde{c}_{t-1}\left(m\right) \right)^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{\varepsilon}{\varepsilon-1}}, \tag{3.2}$$

[3]Recall that a version of the model with money demand has been analyzed in Chapter 2. We use cashless economy assumption only to simplify the exposition of the model.

where $c_{j,t}(m)$ is the individual consumption of good $m \in [1, M_t]$ by agent j born in period $j \leq t$. The parameter $\theta_c \in [0,1]$ measures the degree of external habit formation in consumption of each variety. $\tilde{c}_{t-1}(m)$ denote the consumption reference which is defined by

$$\tilde{c}_{t-1}(m) = \begin{cases} c_{t-1}(m) & m \leq M_{t-1} \\ c_{t-1} & \forall m \in]M_{t-1}, M_t] \end{cases} .$$

$c_{t-1}(m)$ is the *per capita* aggregate consumption of good m[4]. $c_{j,t-1}$ and c_{t-1} denote the individual consumption of the basket of goods in period $t-1$ and the *per capita* aggregate consumption of the basket of goods in time $t-1$, respectively[5].

Notice that the consumption reference used in (3.2) differs from the one used in Ravn et al (2006). The reason is the following. Remember that each agent is owner of a monopolistically-competitive firm so the number of specific goods is growing at the same rate as the population. The appearance of new specific goods in each period raises a new difficulty to develop a deep-habits non-Ricardian model. Indeed, new goods appearing in period t have not been consumed in period $t-1$. Consequently, the benchmark level cannot be the average level of past consumption of those goods. Therefore, we assume that agents observe the *per capita* aggregate consumption of the basket of goods in period $t-1$, which will be considered as the benchmark level of the consumption of goods appearing between periods $t-1$ and t. This assumption is important to develop a deep-habits non-Ricardian model precluding the life cycle of goods, and eliminating any discontinuity between the first period and the next periods. This is also helpful to restore symmetry in the firm's decisions.

Letting β denotes the constant subjective discount factor and E_t the mathematical expectations operator conditional on information available in period t, agent $j's$ preferences

[4] The aggregation rule (3.1) is used to obtained all aggregate variables.

[5] We might use Dixit-Stiglitz aggregator to obtain the consumption basket of goods. In other words, the consumption basket of goods results from (3.2) when θ_c equals zero.

are described by the following utility function:

$$E_t \sum_{s=t}^{\infty} \beta^{s-t} \ln \left(x_{j,s}^c \right). \tag{3.3}$$

The representative agent j maximizes (3.3) subject to the dynamic budget constraint and the borrowing constraint, described by

$$\begin{aligned} & \sum_{m=1}^{M_t} P_t(m) c_{j,t}(m) + \frac{B_{j,t+1}}{1+R_t} + E_t \Lambda_{t,t+1} A_{j,t+1} + S_t \varpi_{j,t+1} \\ \leq \ & B_{j,t} + (S_t + D_t) \varpi_{j,t} + A_{j,t} + \Psi_{j,t} - P_t \tau_{j,t} \quad , \end{aligned} \tag{3.4}$$

and

$$V_{j,t+1} \geq -P_{t+1} h_{j,t+1} \qquad \forall j, \forall t, \tag{3.5}$$

respectively. $P_t(m)$ denotes the nominal price of variety m, and P_t denotes the overall price index defined by:

$$P_t \equiv \left(\frac{1}{M_t} \sum_{m=1}^{M_t} P_t(m)^{1-\varepsilon} \right)^{\frac{1}{1-\varepsilon}}.$$

The agent j holds three types of asset: shares in capital rental firms, government bonds, and state-contingent securities. Letting $\varpi_{j,t}$, $B_{j,t}$,and $A_{j,t}$ be the share in capital rental firms held by the agent j entering period t, the beginning-of-period government bonds, and state-contingent securities, respectively, the beginning-of-period financial wealth of the agent j is given by:

$$V_{j,t} = B_{j,t} + (S_t + D_t) \varpi_{j,t} + A_{j,t}, \tag{3.6}$$

where D_t denotes nominal dividends, i.e. $D_t \equiv P_t d_t$ and S_t denotes the value of the share in capital rental firms held by the agent j, i.e.$S_t \equiv P_t s_t$. New government bonds are purchased at a nominal price $\frac{1}{1+R_t}$, where R_t denotes the risk-free nominal interest rate between t and $t+1$.

$$h_{j,t} = \frac{1}{P_t} E_t \sum_{s=t}^{\infty} \Lambda_{t,s} \left(\Psi_{j,s} - P_s \tau_{j,s} \right), \tag{3.7}$$

denotes the household j's non-financial wealth, which corresponds to the discounted value of future profit incomes net of taxes, where $\Psi_{j,t}$ and $P_t \tau_{j,t}$ denote the nominal average profit of the representative agent j and the lump-sum taxes paid to the government, respectively. $\Lambda_{t,t+1}$ is the stochastic discount factor defined by

$$\Lambda_{t,T} = \Lambda_{t,t+1} \times \Lambda_{t+1,t+2} \times .... \times \Lambda_{T-1,T} \text{ with } \Lambda_{t,t} = 1. \tag{3.8}$$

The problem above can be solved in two stages. In the first stage, for a given level of $x^c_{j,t}$, the agent j chooses the purchases of each variety m in period t, in order to minimize the total expenditure, $\sum_{m=1}^{M_t} P_t(m) c_{j,t}(m)$ subject to the aggregate constraint (3.2). Hence, the demand of good $m \in [1, M_t]$ for consumption purpose is

$$c_{j,t}(m) = \frac{1}{M_t} \left(\frac{P_t(m)}{P_t} \right)^{-\varepsilon} x^c_{j,t} + \theta_c \tilde{c}_{t-1}(m), \quad \text{for all } m \in [1, M_t]. \tag{3.9}$$

Note that the demand of good m, $c_{j,t}(m)$, features a dynamic component, as it depends not only on current period habit-adjusted consumption, $x^c_{j,t}$, but also on the lagged value of consumption of good m. This, in turn, makes the pricing decision of the firm $m \in [1, M_t]$ intertemporal. This is the consequence of using external deep habits. Moreover, from (3.9) we can represent the expenditure on habit-adjusted consumption goods of agent j by

$$P_t x^c_{j,t} = \sum_{m=1}^{M_t} P_t(m) c_{j,t}(m) - \theta_c \sum_{m=1}^{M_t} P_t(m) \tilde{c}_{t-1}(m). \tag{3.10}$$

The second stage of the household $j's$ problem consists of choosing the set of processes $\left\{ x^c_{j,s} \right\}_{s=t}^{\infty}$ and assets $\{B_{j,s+1}, A_{j,s+1}, \varpi_{j,s+1}\}_{s=t}^{\infty}$, taking as given the set of processes $\{\Lambda_{s,s+1}, P_s\}_{s=t}^{\infty}$

and the initial quantity of shares $\varpi_{j,0}$ so as to maximize its utility (3.3) subject to (3.4), (3.5), and (3.6).

The first-order conditions for this maximizing problem yield the following optimality conditions:

$$\beta \frac{x_{j,t}^c}{x_{j,t+1}^c} = \Lambda_{t,t+1} \frac{P_{t+1}}{P_t}, \qquad \forall j, \forall s^t, \tag{3.11}$$

$$\frac{1}{1+R_t} = E_t \Lambda_{t,t+1}, \tag{3.12}$$

$$S_t = E_t \Lambda_{t,t+1} \left(S_{t+1} + D_{t+1} \right), \tag{3.13}$$

$$P_t x_{j,t}^c + \theta_c \sum_{m=1}^{N_t} P_t (m) \, \tilde{c}_{t-1} (m) + E_t \Lambda_{t,t+1} V_{j,t+1} = V_{j,t} + \Psi_{j,t} - P_t \tau_{j,t}, \tag{3.14}$$

$$\lim_{T \to +\infty} E_t \Lambda_{t,T} V_{j,T} = 0. \tag{3.15}$$

Equation (3.11) is a stochastic Euler equation summarizing the intertemporal arbitrage between present and future consumptions in each possible state of the world. This equation gives the price of the Arrow-Debreu security. Equation (3.12) represents the no-arbitrage condition between bonds and state-contingent securities. Equation (3.13) is a no-arbitrage condition relative to the saving choice in terms of share holdings of capital rental firms or in terms of nominal state-contingent assets. Equation (3.14) is the household j's balanced budget constraint obtained by combining equations (3.4), (3.6), (3.10) (3.13) and (3.12). Equation (3.15) states that the discounted value of the financial wealth tends to zero when time goes to infinity. This prevents agents from rolling their debt forever.

Iterating Equation (3.14) forward, with the use of (3.15), leads to the following household j's intertemporal budget constraint:

$$V_{j,t} = E_t \sum_{s=t}^{+\infty} \Lambda_{t,s} P_s \left(x_{j,s}^c + \omega_t - h_{j,t} \right), \tag{3.16}$$

where

$$\omega_t = \theta_c \frac{1}{P_t} E_t \sum_{s=t}^{+\infty} \Lambda_{t,s} \sum_{m=1}^{N_t} P_s(m) \tilde{c}_{s-1}(m),$$

denotes the future time path of the per capita reference consumption.

Let us define a stochastic gross real interest corresponding to real return of the state-contingent nominal asset[6], as the following:

$$q_{t,t+1} = \left(\Lambda_{t,t+1} \frac{P_{t+1}}{P_t} \right)^{-1}. \tag{3.17}$$

Accordingly, equation (3.11) can then be rewritten as:

$$x_{j,t}^c = \beta^{-1} \frac{x_{j,t+1}^c}{q_{t,t+1}}, \qquad \forall j, \forall t. \tag{3.18}$$

Introducing (3.18) into equation (3.16), one can easily show that the optimal habit-adjusted consumption of agent j is a constant fraction of his consolidated wealth (financial wealth + human wealth) and the future time path of the *per capita* reference consumption:

$$x_{j,t}^c = (1-\beta) \left[v_{j,t} + h_{j,t} - \omega_t \right], \tag{3.19}$$

where $v_{j,t} = V_{j,t}/P_t$.

So far we have focused on individual variables. Let us now focus on aggregate variables. Actually, aggregate variables are obtained by applying the aggregation rule used in (3.1). Moreover, we assume that agents receive profits from the ownership of monopolistic firms, whatever the age of the agent and we assume that taxes are independent of

[6]Note that according to (3.12), we have: $1+r_t = \left[E_t\left(1/q_{t,t+1}\right)\right]^{-1}$, where r_t is the riskless real interest rate

age. Therefore, $h_{j,t} = h_t \quad \forall j$.

Notice that applying the aggregation rule used in (3.1) in period t to the variable $v_{j,t+1}$, yields:

$$\sum_{j \leq t} \frac{(N_j - N_{j-1})}{N_t} v_{j,t+1} = (1+n)\, v_{t+1},$$

as $v_{t+1,t+1} = 0$, the dynasty $j = t+1$ has no financial wealth in period $t+1$.

Using this result and applying the aggregation rule to equation (3.18) where we replace $P_{t+1} x^c_{j,t+1}$ by its expression given by equation (3.19) expressed in $t+1$, one obtains:

$$x^c_t = \beta^{-1} \frac{x^c_{t+1}}{q_{t,t+1}} + n \left(\beta^{-1} - 1\right) \frac{v_{t+1}}{q_{t,t+1}}. \tag{3.20}$$

This equation (3.20) is the aggregate stochastic Euler equation which differs from the individual Euler condition (3.18) as long as the population growth rate is different from zero[7]. It includes a real wealth effect which is characteristic of a non-Ricardian economy. Moreover, the *per capita* aggregate demand of good m is given by

$$c_t (m) = \frac{1}{M_t} \left(\frac{P_t (m)}{P_t} \right)^{-\varepsilon} x^c_t + \theta_c \tilde{c}_{t-1} (m) . \tag{3.21}$$

Notice that the aggregate demand of good m given in (3.21) has two components. The first term in the right hand side is a price-elastic component that depends on aggregate demand. The second term in the right hand side is perfectly price-inelastic. Accordingly, an increase in aggregate demand increases the share of the price-elastic component. Consequently, the elasticity of demand increases, entailing a decrease in the markups. This is what was referred by Ravn et al (2006) as the price-elasticity effect of deep habits.

[7]Recall that in Weil's model the population growth rate couldn't be negative since the absence of death.

3.2.2 Monetary and Fiscal Authorities

The public sector consists of the government and the central bank. The government is assumed to specify a fiscal rule while the central bank is assumed to specify an interest rate rule.

Fiscal Authority

In period t, the government purchases (and spends) on goods, collects a nominal lump-sum taxes from the households, and issues one-period nominally risk-free bonds. The government flow budget identity is thus

$$\frac{B_{t+1}}{1+R_t} = B_t + \sum_{m=1}^{M_t} P_t(m) G_t(m) - T_t,$$

or combined with (3.17) and (3.12) to get the following real terms form:

$$(1+n) E_t \frac{b_{t+1}}{q_{t,t+1}} = b_t - \tau_t + \sum_{m=1}^{M_t} \frac{P_t(m)}{P_t} g_t(m), \tag{3.22}$$

where $b_t = \frac{B_t}{P_t N_t}$ denotes *per capital* real debt, $\tau_t = \frac{T_t}{P_t N_t}$ is the *per capita* real taxes, and $\sum_{m=1}^{M_t} \frac{P_t(m)}{P_t} g_t(m)$ represents the *per capita* real government spending. In the line of Ravn et al (2006), the government is assumed, like households, to attempt to catch up with the Joneses in public consumption. Ravn et al (2006), motivate the external deep habit formulation in public spending by the fact that if the households consume public goods then it may exhibits a catching up with a Joneses behavior as it is the case with private consumption[8]. Alternatively, this assumption may be justified by the fact that the government may prefer transactions with vendors who provided public goods in the past.

The government allocates spending over the differentiated good m so as to maximize

[8] They give the example of the situation in which the provision of public services in one community creates the desire in other communities to have access to the same type of service.

the quantity of composite good produced using good m as input according to:

$$X_t^g = M_t^{\frac{1}{1-\varepsilon}} \left(\sum_{m=1}^{M_t} \left(G_t(m) - \theta_g G_{t-1}(m) \right)^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{\varepsilon}{\varepsilon-1}},$$

where $G_{t-1}(m)$ denotes the level of government expenditures for good m in period $t-1$. Accordingly, the government's demand of the variety $m \in [1, M_t]$ is given by

$$G_t(m) = \frac{1}{M_t} \left(\frac{P_t(m)}{P_t} \right)^{-\varepsilon} X_t^g + \theta_g G_{t-1}(m), \quad \text{for all } m \in [1, M_t]. \tag{3.23}$$

For the government to remain solvent, the following no Ponzi game condition must hold

$$\lim_{s \to \infty} E_t \frac{b_s}{q_{t,s}} = 0.$$

First, we assume that the fiscal policy follows a fiscal debt targeting rule which aims at stabilizing government debt at some target $\bar{b}$. To achieve this target, lump-sum taxes get adjusted according to the following simple rule:

$$\tau_t = (1-\rho) \left(\gamma \left(b_t - \bar{b} \right) + \bar{\tau} y_t + \frac{\bar{r} - n}{1 + \bar{r}} \bar{b} \right) + \rho \tau_{t-1} + \xi_t^\tau, \tag{3.24}$$

where the parameter $\gamma \in [0, 1]$. In equation (3.24), $\bar{\tau} y_t$ represents the part of taxes proportional to the *per capita* output. The third term on the right hand side of (3.24), states that lump-sum taxes also finance the interest on the long-run government debt $\bar{b}$ when the real interest rate is $\bar{r}$. The parameter $\rho \in [0, 1]$ reflects the autocorrelation of budget decisions. In addition, the innovation ξ_t^τ distributes i.i.d. with mean zero and standard deviation σ^τ.

Second, we assume that *per capita* government expenditures denoted by g_t are assumed to be exogenous, stochastic and evolves according to an autoregressive process (AR(1)):

$$g_t = (1-\rho) \bar{g} y_t + \rho g_{t-1} + \xi_t^g, \tag{3.25}$$

where ξ_t^g is a white noise disturbance with standard deviation σ^g, and $\bar{g}$ the part of government spending proportional to the *per capita* output.

Monetary Authority

We assume that monetary authority has, in the short-run, leverage over the nominal interest rate that responds to the deviation (or the ratio) of inflation from its long-run target, $\bar{\pi}$. In order to take into account a lower bound constraint on the nominal interest rate[9], we specify the following class of non linear monetary rules:

$$1 + R_t = \Phi\left(\pi_t; \bar{r}; \bar{\pi}\right), \tag{3.26}$$

where $\bar{r}$ is the long-run level of real interest rate when the real government debt reaches its long-run target $\bar{b}$.

The function $\Phi\left(\cdot\right)$ is assumed to have the following form:

$$\Phi\left(\pi_t; \bar{r}; \bar{\pi}\right) = \max\left\{\left(1+\bar{\pi}\right)\left(1+\bar{r}\right)\left(\frac{1+\pi_t}{1+\bar{\pi}}\right)^{\phi}; 1\right\}. \tag{3.27}$$

3.2.3 Firms

The production sector consists of two types of firms. The first type concerns representative firm producing investment good which is rented out to monopolistic firm. The second type concerns monopolistically-competitive firms, each of which producing a unique variety of good.

Capital Rental Firm

In this sector the capital is accumulated by the representative firm according to the following law of motion:

$$K_{t+1} = \left(1-\delta\right) K_t + X_t^i, \tag{3.28}$$

[9] This point was analysed particularly by Benhabib, Schmitt-Grohé and Uribe (2001).

where K_{t+1} is the stock of capital accumulated in period t. The capital stock depreciates at the rate $\delta > 0$. The homogeneous investment good X_t^i is produced using a differentiated good inputs m. We assume that the production technology is given by:

$$X_t^i = M_t^{\frac{1}{1-\varepsilon}} \left(\sum_{m=1}^{M_t} \left(I_t(m) - \theta_i I_{t-1}(m) \right)^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{\varepsilon}{\varepsilon-1}}, \tag{3.29}$$

where $I_t(m)$ is the input of the variety m used to produce X_t^i unites of homogenous investment good. $I_{t-1}(m)$ is the total inputs m used in this sector in period $t-1$. In order to control the investment volatility, we need to have inertial response in investment. We assume external deep habit in investment rather than adjustment cost in investment to have the symmetry of exposition. Indeed, according to Giannoni and Woodford (2003), using habit persistence here should be understood as a proxy for adjustment costs in investment[10]. Note that as Ravn et al (2006) do, we use the same constant elasticity of substitution, $\varepsilon > 1$, for investment goods as for consumption goods[11].

Each period, given the level of effective investment X_t^i, the representative firm minimizes the level of its investment expenditures $\sum_{m=1}^{M_t} P_t(m) I_t(m)$, subject to the production technology (3.29). This yields the demand of good $m \in [1, M_t]$ by the representative firm for investment purpose:

$$I_t(m) = \frac{1}{M_t} \left(\frac{P_t(m)}{P_t} \right)^{-\varepsilon} X_t^i + \theta_i I_{t-1}(m), \quad \text{for all } m \in [1, M_t]. \tag{3.30}$$

During period t, the capital rental firm accumulates capital that is then rented out to the monopolistically-competitive firms at a real price κ_t. Iterating (3.13) from t to

[10] Giannoni and Woodford (2003) apply superficial habit formation to total aggregate (private) demand and not solely to total aggregate consumption.

[11] In general the elasticity of substitution in consumption may be different from that in production since both activities are of different nature. But, for simplicity, we follow Kiyotaki (1988) and assume the same elasticity of substitution in production and consumption.

infinity, and using (3.17) and the following definition of dividends in real term:

$$\tilde{D}_t \equiv \kappa_t K_t - X_t^i,$$

the real value of the capital rental firm at time t (in units of consumption) is given by

$$\tilde{S}_t + \tilde{D}_t = E_t \sum_{s=t}^{\infty} q_{t,t+1}^{-1} \left(\kappa_s K_s - X_s^i \right). \tag{3.31}$$

At time t the representative firm chooses the paths of capital stock $\{K_{s+1}\}_{s=t}^{\infty}$ and investment $\{X_s^i\}_{s=t}^{\infty}$ to maximize (3.31) subject to (3.28) described in *per capita* terms, taking as given the rental real price, κ_t, and the initial level of capital stock, $K_0 > 0$. The first-order condition with respect to X_t^i and K_t yield the following equilibrium equation:

$$1 = E_t q_{t,t+1}^{-1} \left(\kappa_{t+1} + 1 - \delta \right). \tag{3.32}$$

Introducing (3.32) into (3.31) and using the transversality condition yield the following equation

$$\tilde{S}_t = K_{t+1}. \tag{3.33}$$

Monopolistic Firms

The differentiated goods are produced by monopolistically competitive firms indexed by $m \in [1, M_t]$. At each point in time, firm $m \in [1, M_t]$ uses capital stock $K_t(m)$ and specific human capital—normalized to one—to produce a quantity $Y_t(m)$ of differentiated good according to a standard neoclassical technology:

$$Y_t(m) = F(K_t(m)). \tag{3.34}$$

The production function is concave, twice continuously differentiable and satisfies the Inada conditions.

A good m may be used by private sector for consumption and investment or by public sector for government expenditure. Given (3.21), (3.23), and (3.30), the total aggregate demand faced by the monopolistically-competitive firm m is given by:

$$Y_t(m) = C_t(m) + I_t(m) + G_t(m). \tag{3.35}$$

Assuming $\theta_c(1+n) = \theta_g = \theta_i = \theta(1+n)$ and letting $X_t = X_t^c + X_t^i + X_t^g$, it turns out from (3.35) that

$$Y_t(m) = \frac{1}{M_t}\left(\frac{P_t(m)}{P_t}\right)^{-\varepsilon} X_t + \theta(1+n)\, Y_{t-1}(m), \tag{3.36}$$

where $Y_{t-1}(m) = C_{t-1}(m) + I_{t-1}(m) + G_{t-1}(m)$.

Letting

$$\Psi_t(m) = P_t(m)\, Y_t(m) - P_t \kappa_t K_t(m),$$

defines the firm's profit in period t and using (3.8), the owner-manager $m's$ problem is to maximize the discounted value of the sum of its present and future cash-flows,

$$E_t \sum_{s=t}^{T} \Lambda_{t,s} \Psi_s(m), \tag{3.37}$$

subject to (3.34) and (3.36). Letting

$$\eta_t(m) \equiv \frac{P_t(m)}{P_t} - \frac{\kappa_t}{f'[K_t(m)]} \tag{3.38}$$

defines the relative markup that is the ratio between profit margin (prices charged by firm m minus nominal marginal cost) and price index and defining the elasticity of demand

$$\epsilon_t(m) \equiv \varepsilon \frac{P_t}{P_t(m)}\left[1 - \theta(1+n)\frac{Y_{t-1}(m)}{Y_t(m)}\right], \tag{3.39}$$

the first-order conditions corresponding to the owner-manager m's optimization problem are (3.36),

$$\lambda_t(m) = \epsilon_t^{-1}(m) \tag{3.40}$$

and

$$\eta_t(m) = \epsilon_t^{-1}(m) - \theta(1+n) E_t q_{t,t+1}^{-1} \epsilon_{t+1}^{-1}(m). \tag{3.41}$$

$\lambda_t(m)$ is the Lagrange multiplier associated to (3.36). Equation (3.41) states that relative markup has two components. The first component is the inverse of the elasticity of demand. The second component is the discounted value of the inverse of the elasticity of demand in time $t+1$, which reflects future expected profits associated with selling an extra unit of good m in the current period[12].

At this stage, we distinguish two effects of deep habits on markup dynamics. As it was referred by Ravn et al (2006), the first effect is the price-elasticity effect summarized as follows. An increase in the aggregate demand implies an increase in the elasticity of demand, inducing a decline in markups. The second effect is the intertemporal effect described as the following. Firms take into account the fact that today's price decisions will affect future demand. So when the present value of future per unit profit are expected to be high, firms have an incentive to invest in the customer base today. This is achieved by a decline in markups today. In addition, when the real interest rate is expected to be high, firms have less incentive to invest in the customer base today because future profits are discounted more. As a result, markups increase in the current period.

3.2.4 Symmetric Equilibrium and Market Clearing

Firms are different because of the date of appearance. Recall that, in our model, even firms appearing in t face a dynamic (backward) demand of goods. Thus, assuming that all firms make the same decisions in period $t-1$ implies that firms display the same behavior and make the same decisions also in period t. We recall that $M_t = N_t$. Accordingly, we set $G_t(m) = g_t$, $I_t(m) = i_t$, $C_t(m) = c_t$, $Y_t(m) = y_t$, $K_t(m) = k_t$.

[12] See Appendix B.2 for further detail on firms' maximization program.

Furthermore, by assuming assets to be perfectly substitutes in the household portfolio, the state-contingent securities' supply to equal zero, i.e. $A_t = 0$, $\varpi = 1$, and introducing (3.13) and (3.33), into the *per capita* aggregate financial holdings (3.6), yield

$$E_t \frac{V_{t+1}}{q_{t,t+1}} = E_t \frac{b_{t+1}}{q_{t,t+1}} + k_{t+1}$$

For further reference, the symmetric equilibrium can be summarized as follows.

Market clearing:

$$g_t + i_t = f(k_t) - c_t. \tag{3.42}$$

Consumers:

$$x_t^c = c_t - \tilde{\theta} c_{t-1}, \tag{3.43}$$

$$x_t^c = \beta^{-1} E_t \frac{x_{t+1}^c}{q_{t,t+1}} + \zeta \left(E_t q_{t,t+1}^{-1} b_{t+1} + k_{t+1} \right). \tag{3.44}$$

Firms:

$$(1+n) k_{t+1} = (1-\delta) k_t + i_t - \tilde{\theta} i_{t-1}, \tag{3.45}$$

$$1 = E_t q_{t,t+1}^{-1} \left(f'(k_{t+1}) \left(1 - \eta_{t+1}\right) + 1 - \delta \right), \tag{3.46}$$

$$\eta_t = \epsilon^{-1}(k_t, k_{t-1}) - \tilde{\theta} E_t q_{t,t+1}^{-1} \epsilon^{-1}(k_{t+1}, k_t). \tag{3.47}$$

Government:

$$E_t \left(\frac{b_{t+1}}{q_{t,t+1}} \right) = \frac{1}{1+n} \left[b_t - \tau_t + g_t \right], \tag{3.48}$$

$$\tau_t = (1-\rho) \left(\gamma \left(b_t - \bar{b} \right) + \bar{\tau} \cdot f(k_t) + \frac{\bar{r} - n}{1 + \bar{r}} \bar{b} \right) + \rho \tau_{t-1} + \xi_t^{\tau}, \tag{3.49}$$

$$g_t = (1-\rho) \bar{g} \cdot f(k_t) + \rho g_{t-1} + \xi_t^g. \tag{3.50}$$

Central bank:

$$1 + R_t = \max \left\{ (1+\bar{\pi})(1+\bar{r}) \left(\frac{(1+\pi_t)}{(1+\bar{\pi})} \right)^{\phi} ; 1 \right\}, \tag{3.51}$$

$$1+R_t=\left(E_t\frac{1}{q_{t,t+1}\left(1+\pi_{t+1}\right)}\right)^{-1}. \tag{3.52}$$

Here $\tilde{\theta}=\theta\left(1+n\right)$, $\zeta=n\left(\beta^{-1}-1\right)$ and

$$\epsilon\left(k_t,k_{t-1}\right)\equiv\varepsilon\left[1-\tilde{\theta}\frac{f\left(k_{t-1}\right)}{f\left(k_t\right)}\right]. \tag{3.53}$$

(3.43) and (3.44) come from (3.21) and (3.20), respectively. Equation (3.21) is the aggregate Euler equation which is modified in two ways. First, it is expressed in terms of aggregate habit-adjusted consumption rather than aggregate consumption. Second it depends on the government debt as long as the population growth rate is positive. Equation (3.45) comes from (3.28)combined with (3.30). Equation (3.46) is obtained by introducing the definition of relative markups (3.38) into (3.32). Equation (3.52) is the Fisher equation. Equation (3.47) states that relative markup is time-varying. This equation is specific to deep habits models.

3.2.5 Steady State Equilibria

Consider dropping out the indication of time and assuming that $\bar{g}=\bar{\tau}$ and $q=1+r$, the above system, (3.45)-(3.52), becomes:

$$\left(\gamma-\frac{r-n}{1+r}\right)b=\left(\gamma-\frac{\bar{r}-n}{1+\bar{r}}\right)\bar{b} \tag{3.54}$$

$$f'\left(k\right)=\frac{\left(r+\delta\right)}{1-\eta\left(r\right)} \tag{3.55}$$

$$b=\zeta^{-1}\left(1-\tilde{\theta}\right)\left[\left(1+r\right)-\beta^{-1}\right]c\left(k\right)-\left(1+r\right)k, \tag{3.56}$$

$$\left(1+r\right)\left(1+\pi\right)=\max\left\{\left(1+\bar{\pi}\right)\left(1+\bar{r}\right)\left(\frac{\left(1+\pi_t\right)}{\left(1+\bar{\pi}\right)}\right)^{\phi};1\right\}, \tag{3.57}$$

where

$$\eta(r) = \frac{1 - \frac{\tilde{\theta}}{1+r}}{\varepsilon\left(1 - \tilde{\theta}\right)}$$

and

$$c(k) \equiv (1 - \bar{g}) f(k) - \frac{(\delta + n)}{\left(1 - \tilde{\theta}\right)} k.$$

Equation (3.54) comes from the introduction of (3.49) and (3.50) into (3.48). Equation (3.55) is obtained by combining (3.46) with (3.47) and using (3.53). Equation (3.56) comes from (3.44) combined with (3.42) and (3.45). Equation (3.57) obtained by combining (3.51) with (3.52).

In Chapter 2, we have shown that the double non linearity, due to the presence of wealth effects and the ZLB on the nominal interest rate, implies the multiplicity of equilibrium. Precisely, we found four steady state equilibrium. Our intuition is that the introduction of deep habits does not affect this result. We expect to find four steady state equilibria in this chapter. Actually, Fisher and Heijdra (2009) find a unique steady state equilibrium in a Blanchard (1985)-Yaari (1965) model incorporating consumption externalities (or external superficial habit). In addition, the deep habits model à la Ravn et al (2006) admits a unique steady state equilibrium. It seems, a priori, that deep habits do not affect the number of steady state equilibrium.

Let us focus on the steady state system described by (3.54)-(3.57). The system is dichotomous. Equations (3.54)-(3.56) determine the steady state values of r, b and k. Then, introducing the steady state value(s) of r into (3.57), we get the steady state value(s) of inflation. Indeed, as shown by Benhabib, Schmitt-Grohe and Uribe (2001), the combination of the Fisher equation (3.52) and the Taylor rule (3.51), i.e. equation (3.57) induces two steady state values of inflation, for a given value of the real interest rate. Now, solving the sub-system (3.54)-(3.56) gives two steady state values of r. The first, evident, solution is $\bar{r}$. The second solution can not be given explicitly. So rather than solving the model analytically which is infeasible, we propose simulating it. Thus, in the next section, we will calibrate the parameter values of the model. We will verify the local dynamic properties of the each equilibrium using Blanchard and Kahn (1980) conditions. Finally, we will analyze the behavior of the economy on the debt-deflationary trajectory, and will give the response of the economy to fiscal shock.

3.3 Aggregate Dynamics

3.3.1 Calibration

In order to simulate the model, we define the following explicit form of the production technology:

$$f(k_t) = A_t k_t^{\alpha} \tag{3.58}$$

where the technology shock A_t denotes the aggregate technology shock.

To analyze the behavior of the model in the presence of shocks, we simulate the model using DYNARE package for Matlab (see Juillard [2004]). The strategy for calibrating the model is based on the calibration of Ravn, Schmitt-Grohe, Uribe and Uusküla (2009). The parameterization of the model are displayed in Table 3.1.

The time unit is meant to be a year. The rate of capital depreciation, δ, and the discount factor, β, and the elasticity of substitution across varieties, ε are exactly the same as in Ravn, Schmitt-Grohe, Uribe and Uusküla (2009).

The capital share parameter, α, is 0.3. The long-run level of public debt-GDP ratio is 60%. The public spending's share $\bar{g}$ is calibrated at 0.2. The taxes-GDP ratio, $\bar{\tau}$, is set equal to $\bar{g}$ so as to obtain a 60% for public debt-GDP ratio at the steady state targeted by monetary and fiscal authorities. This calibration induces a long-run target level of real interest rate, $\bar{r}$, of about 5.2%. Monetary authority sets the inflation rate target at 2%. In addition, the deep habit degree, θ, is set at 0.6 so that a public debt-GDP ratio equals approximately the value of 120% at the second steady state. The value of γ equals 0.04, is consistent with the values estimated, by Melitz (2000) and Gali and Perotti (2003), at 0.03, and 0.05, respectively. The population growth rate is set at 1.5% in order to reflect a significant wealth effects. The parameter, A, is calibrated such that we obtain, at the targeted steady state, a value of output equals to 100[13]. Finally, we set the persistence parameter ρ to 0.9.

Table 3.1: Benchmark Calibration.

Symbol	Value	Description
β	0.96	Subjective discount factor
α	0.3	Capital share of output
δ	0.1	Depreciation rate of capital
n	0.015	Population growth rate
$\bar{g}$	0.2	Public spending-GDP ratio
$\bar{\tau}$	0.2	Taxes-GDP ratio
γ	0.04	Debt parameter in the fiscal rule
ε	5.3	Elasticity of substitution across varieties
θ	0.6	Degree of deep habit formation
ρ	0.9	Persistence of government spending and taxes
$\bar{\pi}$	0.02	Inflation rate target
$\bar{r}$	0.052	Long-run target level of real interest rate
$\bar{b}$	0.6	Long-run target level of government debt

Moreover, according to our calibration, we find as in Chapter 2 four steady state equilibria. There are two positive values of real government debt and real interest rate. The first steady state solution is consistent with the long-run level of real government debt targeted by fiscal authorities (set at 0.6) and with the long-run level of real interest rate (5.2%). The second steady state solution corresponding to a higher level of real government debt about 121% and a higher level of real interest rate approximately 5.5%. Combining those values of real interest rate with the Taylor rule, (3.57), one obtains four equilibria. Furthermore, the analysis of the dynamic properties of each equilibrium

[13]Our calibration leads to $A \approx 22$.

reveals that only, two steady state equilibria verify the Blanchard and Kahn conditions. These conditions state that a necessary condition for the uniqueness of a stable solution in the neighborhood of the steady state is that there are as many eigenvalues larger than one in modulus as there are non-predetermined variables in the model. We call the first determined equilibrium, "targeted" equilibrium, where the targeted levels of real government debt, real interest rate and inflation rate are reached. The second determined equilibrium, is called "debt-deflation" equilibrium because of the higher level of real government debt and real interest rate and the negative value of inflation.

In Chapter 2 we have simulated an expectation shock that brought the economy towards the debt-deflation equilibrium. This simulation exercise was to set the values of predetermined variables in halfway between the two considered steady states. Then assuming that agents' believes changed—for a reason that we did not specify—which pushed the economy to be on the debt-deflationary trajectory. We found that the initial deflation necessary to reach the debt-deflation situation was 50%. This figure is doubtless unrealistic. Thus, this chapter also aims to solve this limit. Indeed, we try to give a plausible scenario to deflationary episodes. A possible solution is to reduce the gap between both steady state levels of government debt, by reducing the value of γ. But, in doing so, the recessionary effect becomes insignificant. However, by introducing deep habits we solve this problem. Indeed, we can reduce the gap between both steady state levels of government debt while keeping a significant recessionary effect. Table 3.2 summarizes the effect of deep habit on steady state levels of government debt and output.

3.3.2 Expectation Shock

This section focuses on the behavior of the economy on the debt-deflationary trajectory. This simulation exercise is identical to the one made in Chapter 2. We assume that predetermined variables (capital and real government debt) are approximately in halfway between the locally-determined steady states. Accordingly, the level of real government

Table 3.2: Deep Habit Effects on the Steady State.

Debt-deflation steady state	With DH ($\theta = 0.04$)	Without DH ($\theta = 0.04$)	Without DH ($\theta = 0.02903$)
Government debt (%)	121	1280	121
Output	99.2	96	99.8
Targeted steady state	**With DH ($\theta = 0.04$)**	**Without DH ($\theta = 0.04$)**	**Without DH ($\theta = 0.02903$)**
Government debt (%)	60	60	60
Output	100	100	100

debt is assumed to be equal to 100%. For some reason unknown to us, agents believe that deflation is on the horizon. So this change in agents' expectations leads the economy to the debt-deflationary steady state equilibrium[14].

Figure 3.1 displays the behavior of macroeconomic aggregates when the economy converges toward the debt-deflation steady state. We notice, a consumption increase, a rise of the real interest rate, a decline in investment and output, and a fall in the inflation rate. As the economy is non Ricardian, an increase in government debt is perceived as an increase in wealth for currently alive agents. Consequently, consumption increases while desired saving fall. As a result, the real interest rate increases and investment decreases. But the change in the real interest rate affect the relative markup which increases. It turns out that the rise of the markup reduces the consumption, but this is not strong enough to offset the positive wealth effect on the consumption. On the other hand, the sharp deflation may be explained based on the Cochrane's explanation of the FTPL. Indeed, according to the stock valuation equation of the government bonds, if the public debt is expected to increase then the present discounted value of the government's future surpluses increases too entailing a fall of inflation (because the nominal government debt is predetermined)[15]. The initial deflation leads the economy to the liquidity trap.

[14] Here we do not focus on why expectations change. In order to analyze this issue, we need to modelize expectation shock which modifies agents 'expectations and brings the economy into the liquidity trap.

[15] It is worthnoting that even though taxes are not indexed to the real value of the public debt, $\theta = 0$,

Furthermore, comparing this result with that in Chapter 2 we see that the introduction of deep habits makes the initial deflation acceptable. The initial deflation is approximately 11% instead of 50%. A deflation of 11% remains too sharp compared to the latter deflation in US and Europe, and even in Japan. However, it may find support in deflationary episodes observed in the 1930s associated with the Great Depression. In addition, we notice that the recessionary effect is significant and persistence. It takes approximately 20 years to reach its debt-deflation steady state level. This improves considerably our later result in Chapter 2 but is still insufficient with regard to inflation dynamics.

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the result of deflation still exist. For further discussion see Aloui and Guillard (2009).

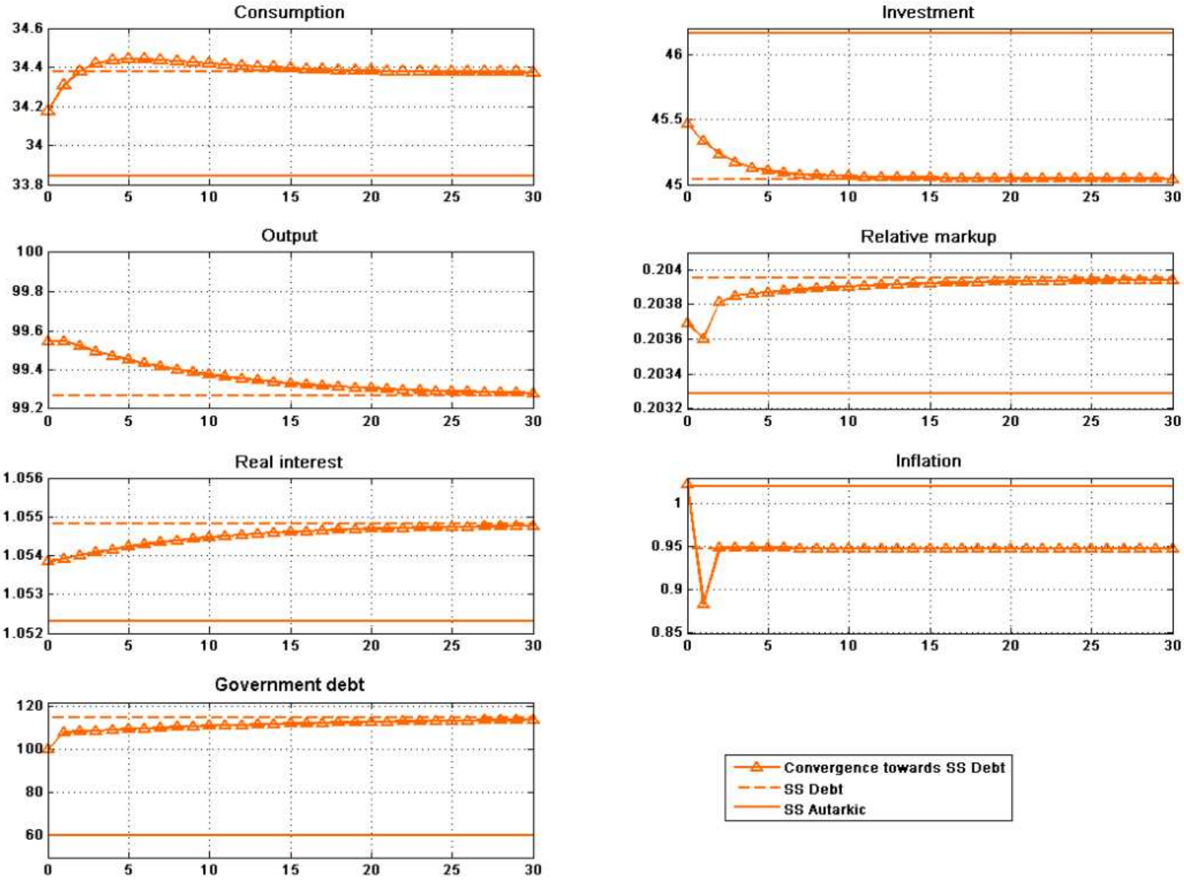


Figure 3.1

3.3.3 The Effects of Debt-Financed Tax Cut

In this section, we analyze and compare the effects of debt-financed tax cuts in two very different environments. The first environment is the situation targeted by monetary and fiscal authorities. The second environment corresponds to a crisis characterized by a deflation, recession and high government debt.

Figure 3.2 depicts the impulse responses of government debt, consumption, investment, output, relative markups, real interest, inflation, and nominal interest, to a 5 percent decrease in taxes. These responses are represented by a solid line with triangles when the shock hits the economy at targeted equilibrium path, while solid line with circles delimit the response of the economy around debt-deflation equilibrium. Notice that each variable is in deviation from its steady state value. As a consequence the steady state of the variables will be zero.

From Figure 3.2, we notice that the response of the economy to debt-financed tax cut is very different depending on the equilibrium. The economy takes much more time to reach its steady state value in the targeted equilibrium regarding the debt-deflation equilibrium. Moreover, in the targeted situation, tax reduction entails an increase in consumption, government debt and real interest rate, and a fall in investment on impact. The relative markup decreases on impact then increases. On the other hand, in the debt-deflation situation, the effect of fiscal shock is reversed. We observe a decline in consumption and government debt, a fall in real interest rate, an increase in investment, on impact. The relative markup increases on impact then decreases. Also, the inflation rate jumps on impact.

The intuition is the following. When the economy is in the targeted environment, an increase in government debt to finance tax reduction induces a positive wealth effect as the economy is non Ricardian. The increase in the desire of consuming more today reduces the desire of saving. Consequently, the real interest rate increases implying a fall in the investment. The output remains unchanged as it only depends on the capital stock accumulated in the last period. However, in the next periods the output decreases progressively following the decline of capital. The output decrease has an impact on the markup through its effect on the elasticity of demand. Indeed, an expected decrease in future output entails a lower expected future elasticity of demand. Firms have the incentive to invest in the customer base today, inducing a lower markup in the current period[16]. It should be noted that, the increase in the real interest rate have a positive effect on the markup, all things remain constant. As we have already mentioned in Section 3.2, future profits are discounted more when real interest rate increases. This

[16] This relationship between the markup and the future elasticity of demand is clear in equation (3.47).

implies higher markups because firms have less incentive to invest in the customer base in the current period. It is clear that the effect of expected lower elasticity of demand dominates the effect of the higher real interest rate on the markup in the first period[17]. But this result is reversed in the next period, because we have in addition the elasticity effect, which enhances the intertemporal effect related to the real interest rate. Indeed, in the next period, the lower elasticity of demand implies higher markups.

On the other hand, around the debt-deflation equilibrium, things are different. The real government debt decreases in the first period. In order to understand this result, we should note that this equilibrium has the characteristics of an FTPL equilibrium. Indeed, a decrease in government revenues must be offset by an equivalent decrease in the real public debt, so as to ensure that the intertemporal budget constraint of the government holds. This is allowed by a jump of the price level in the first period and consequently a temporary jump in the inflation rate. In this case there is a negative wealth effect inducing a fall in consumption on impact. Agents want to save more implying a decrease in the real interest rate and consequently the investment increases on impact. Accordingly, output increases in the next periods, entailing higher today's markup through the intertemporal effect based on the expected future elasticity of demand which increases. Next period markup decreases because the elasticity effect and the decrease of the real interest rate.

[17]Notice that in the first period there is no effect on the elasticity of demand as the output is not affected.

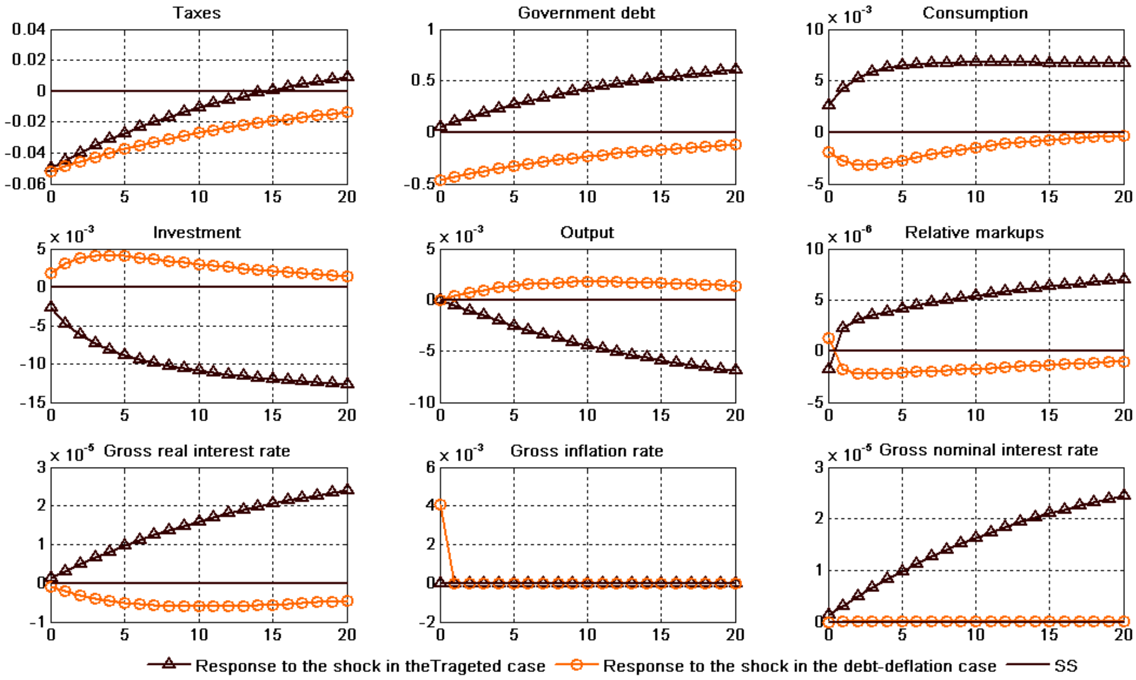

Taxes
Government debt
x 10^-3 Consumption
x 10^-3 Investment
x 10^-3 Output
x 10^-6 Relative markups
x 10^-5 Gross real interest rate
x 10^-3 Gross inflation rate
x 10^-5 Gross nominal interest rate
Response to the shock in theTrageted case
Response to the shock in the debt-deflation case
SS

Figure 3.2

3.4 Conclusion

We have developed a dynamic, general equilibrium model with monopolistically competitive firms and infinitely-lived families, where preferences feature external deep habits and monetary policy is constrained by the ZLB on the nominal interest rate. This model displays a double nonlinearity arising from the non-Ricardian structure coupled with the ZLB on the nominal interest rate constraint. This double nonlinearity implies the existence of multiple steady state equilibrium. We found four steady state equilibria, where two are locally determined, as in Chapter 2. Our framework has the advantage of including two very different environments. The first corresponds to the situation targeted by monetary and fiscal authorities. The second is an environment characterized by debt-deflation and recessionary situation. We showed that if agents' expectations change, the convergence of the economy towards the debt-deflation equilibrium is achieved by an initial deflation of approximately 11%.

Furthermore, we have analyzed and compared the effects of debt-financed tax cuts in both environments. We found that the responses of the economy are very different. In the targeted environment, debt-financed tax cut is contractionary. In fact, government debt increase entails a positive wealth effect inducing higher consumption and lower investment. As a result, output decreases. In the debt-deflation environment, debt-financed tax cut is expansionary. Indeed, tax reductions imply less government revenues and consequently real government debt decreases. This implies a negative wealth effect, entailing lower consumption and higher investment. As a result, output increases. The negative wealth effect found in debt-deflation environment is in contrast with Devereux (2010) who finds a positive wealth effect in the liquidity trap. Although, we reach the same conclusion as Devereux (2010) that is debt-financed tax cut is expansionary. We should point out that in Devereux (2010) there is abstraction from capital accumulation. At this stage, we ask the following question: do we find the expansionary effect of debt-financed tax cut if we ignore capital in our model? We might guess that the answer is no, if prices are flexible. On the other hand, if prices are sticky we expect to find the same result as Devereux (2010).

Chapter 4

Deep Habits and the Macroeconomic Effects of Government Debt

4.1 Introduction

The latest economic crises in Europe and the United States have pushed many governments to intervene to fight the recession. The active use of fiscal policy has raised concern about debt and revived the old debate on the impact of government debt on economic activity. This is equivalent to asking the following question: how does debt-financed lump-sum fiscal policy affect macroeconomic aggregates? This paper contributes to answering that question by studying the effects of debt-financed tax cuts in a micro-founded, general equilibrium, non-Ricardian model.

As summarized by Bernheim (1989) and Elmendorf and Mankiw (1999), the economic effects of lump-sum fiscal policy are expansionary in the short run (the traditional Keynesian view) and contractionary in the long run (the Neoclassical view). Indeed, a raise in public debt to finance tax cuts (or raise transfers) stimulates aggregate demand, causing output to increase when prices (and/or wages) are sticky. This is the short-run effect. However, the real interest rate must rise to bring securities market into balance. Consequently, investment is crowded out. Accordingly, capital and output eventually decrease. This is the long-run effect. However, this old (traditional) literature was not based on micro-founded behavior. For instance, in the Keynesian view, illustrated by the undergraduate IS-LM, there is no role for expected future income.

Furthermore, the vast majority of micro-founded literature on fiscal policy has focused on the economic effects of distortionary fiscal policy. By contrast, the economic effects of lump-sum fiscal policy have not received much attention because the assumption of infinitely-lived representative households is usually adopted. In other words, Ricardian equivalence holds in those models, implying the neutrality of lump-sum fiscal policy.

Nevertheless, given the recent macroeconomic events, i.e. the global recession of 2008-2010, it is of great interest to focus on lump-sum fiscal policy, which directly affects aggregate demand. For this, the departure from the Ricardian equivalence is necessary. There are two alternatives: Overlapping generation structure or rule of thumb (liquidity-constrained consumers) structure, developed by Gali et al (2007). We discard the last alternative from our analysis because the rule of thumb is an ad-hoc assumption. Hence, in our paper we adopt the overlapping generation structure. Specifically, we develop a micro-founded general equilibrium model with overlapping generation structure, monopolistic competition and external deep habit formation. In addition, our model abstracts from capital accumulation. We show that an increase in government debt to finance tax cuts has a long-run contractionary effect despite the lack of capital. On the other hand, the short-run effect of debt-financed tax cuts is contractionary in a flexible-price framework, while it is expansionary in a sticky-price framework.

At this stage, it is of interest to notice that, in the non-Ricardian framework, the short-run and the long-run effects on output depend on the assumptions made about price adjustment, labor supply, and capital.

First, if labor is supplied inelastically, there is no short-run effect on output even when prices are sticky. In the long run, output decreases because of capital adjustment. Indeed, Annicchiarico (2007) shows that the increase in aggregate demand caused by the rise in government debt entails higher consumption and higher real interest rates in the short run. The real interest rate rise crowds out investment and output falls in the long run.

Second, if labor supply is endogenous and physical capital is absent, when prices are flexible, the higher government debt will have no short-run or long-run effects on output. So, government debt is neutral despite the non-Ricardian framework. However, if prices are sticky, the short-run expansionary effect on output is evident but there is no long-run effect. This result is found in Devereux (2010). He analyzes the effect of government debt increase in a non-Ricardian framework without capital and with sticky prices and shows that higher government debt leads to the consumption and output rise in the short run.

In this paper, in the flexible-price model, we find that the government-debt neutrality expected to occur, because of the lack of capital, does not hold. Actually, the crowding out effect of government debt on output is based on the countercyclical markup movements induced by the assumption of external deep habits. Indeed, the novelty in this paper is the introduction of external deep habits into a non-Ricardian framework. In other words, we bring together wealth effects, which imply a non-neutral fiscal policy, and time-varying markups which are countercyclical to output. This offers a new transmission mechanism of government debt through the countercyclical markup movements. The transmission mechanism can be summarized as follows. Debt-financed tax cuts raise the interest rate, entailing higher markups, which in turn induce a fall in employment and consumption.

As shown by Ravn, Schmitt-Grohe and Uribe (2006), the assumption of external deep habits profoundly alters the supply side of the economy. Under external deep habits, households do not simply form habits from a benchmark consumption level, but rather feel the need to catch up with the Joneses on a good-by-good basis[1]. Households that consume a lot of a particular good today are more likely to buy this kind of good in the future by force of habit. Such behavior influences firms' pricing strategy. Indeed,

[1] In this paper deep habits refer to external deep habits. It is the catching up with the Joneses described by Abel (1990) but on a good-by-good basis.

under deep habits, the demand for goods faced by firms becomes dynamic, implying time-varying markups. So a higher real interest rate implies higher markups because firms discount future profits more. As a consequence, labor demand declines, and output and consumption decrease. In addition, the decline in aggregate demand entails lower elasticity of demand, inducing higher markups. This is a price elasticity of demand effect which strengthens the decline in output.

In the sticky-price model, an increase in government debt to finance tax reduction induces an increase in consumption and aggregate demand. As prices cannot fully adjust to balance the goods market, output increases. Thus the short-run expansionary effect is obtained.

Our paper offers an alternative channel for debt-financed lump-sum fiscal policy through countercyclical movements of markups that give rise to short-run expansionary and long-run contractionary effects.

The remainder of the chapter is organized as follows. The next section develops the flexible-price model. Sections 4.3 and 4.4 study the steady state equilibrium and the implications of the increase in government debt in the long run. Section 4.5 investigates the impact of temporary and permanent public debt shocks. Section 4.6 develops the sticky-price model and describes the effects of a temporary public debt shock. Section 4.7 concludes.

4.2 The Model

The economy consists of three types of agents: infinitely-lived dynasties (or families), monopolistically competitive firms, and the fiscal authority. Each period, new and identical infinitely-lived families (component of a generation) appear in the economy without financial wealth and owing a monopolistically competitive firm producing a specific good using labor. It is assumed that the firm's ownership is not transferable. Therefore, the profit of the family firms is transferred in full to the owner-manager (the infinitely-lived family). On the other hand, labor moves freely in this economy.

Moreover, there is uncertainty in the economy caused by fiscal shocks. However, we assume that agents have access to complete markets. In addition, as in most of the recent New Keynesian literature, we assume a cashless economy à la Woodford (2003). Here, money is only a unit of account.

4.2.1 Consumers

In this economy agents care about their own consumption of a specific good compared to the benchmark level of the consumption of that specific good. The deep habit specification in this chapter is identical to the one described in Chapter 3. Thus, we start by given the aggregation rule which will be used to aggregate individual variables:

$$z_t = \sum_{j \leq t-1} \frac{(N_j - N_{j-1})}{N_{t-1}} z_{j,t-1}, \tag{4.1}$$

where z is a generic variable. Notice that $N_j - N_{j-1}$ is the number of agents compound of the representative generation $j \leq t$, where N_j is the number of agents born in period $j \leq t$.

We adopt the same specification of the CES habit-adjusted consumption index, $x_{j,t}$, as in Chapter 3:

$$x_{j,t} = M_t^{\frac{1}{1-\varepsilon}} \left(\sum_{m=1}^{M_t} \left(c_{j,t}(m) - \theta \tilde{c}_{t-1}(m)\right)^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{\varepsilon}{\varepsilon-1}}, \tag{4.2}$$

where

$$\tilde{c}_{t-1}(m) = \begin{cases} c_{t-1}(m) & \forall m \leq M_{t-1} \\ c_{t-1} & \forall m \in \left]M_{t-1}, M_t\right] \end{cases}.$$

Here $\varepsilon > 1$, is the elasticity of substitution across varieties and the parameter θ measures the degree of external habit formation in consumption of each variety. $c_{j,t}(m)$ is the consumption of good $m \in [1, M_t]$ by agent j born in period $j \leq t$ and $c_{t-1}(m)$

denotes the *per capita* aggregate consumption of good m in period $t-1$. $c_{j,t-1}$ and c_{t-1} denote the individual consumption of a basket of good in period $t-1$ and *per capita* aggregate consumption of the basket of goods in period $t-1$, respectively.

In order the preclude the life cycle of goods, and eliminates any discontinuity between the first period and the next periods, we adopt the same assumption as in Chapter 3. we assume that agents observe the *per capita* aggregate consumption of a basket of goods in period $t-1$, which will be considered as the benchmark level of the consumption of goods appearing between periods $t-1$ and t[2]. In addition, this assumption is helpful to restore symmetry in the firm's decisions.

For any given level of $x_{j,t}$, agent $j's$ demand for individual goods varieties must solve the cost minimization problem:

$$\min \sum_{m=1}^{M_t} p_t(m)\, c_{j,t}(m)$$

subject to the aggregate constraint (4.2). Solving this problem yields the demand functions:

$$c_{j,t}(m) = \frac{1}{M_t}(p_t(m))^{-\varepsilon} x_{j,t} + \theta \tilde{c}_{t-1}(m), \quad \text{for all } m \in [1, M_t]. \tag{4.3}$$

The price index is defined by:

$$P_t \equiv \left(\frac{1}{M_t}\sum_{m=1}^{M_t} P_t(m)^{1-\varepsilon}\right)^{\frac{1}{1-\varepsilon}}, \tag{4.4}$$

where $P_t(m)$ denotes nominal price of good m. For simplicity, we assume that each period's nominal price index is normalized to unity, so all remaining prices are expressed in terms of a basket of goods (the numéraire).

Using equation (4.3) and the definition of price index (4.4), we define the total ex-

[2]For further details see Subsection 3.2.1.

penditure on habit-adjusted consumption as[3]:

$$x_{j,t} = \sum_{m=1}^{M_t} p_t(m) c_{j,t}(m) - \theta \sum_{m=1}^{M_t} p_t(m) \tilde{c}_{t-1}(m). \tag{4.5}$$

Notice that the demand for good m by agent j, equation (4.3), features a dynamic component, as it depends not only on current period habit-adjusted consumption, $x_{j,t}$, but also on the lagged value of consumption of good m. This, in turn, makes the pricing decision of the firm $m \in [1, M_t]$ intertemporal. Indeed, as pointed out by Ravn et al (2006), the deep habits assumption makes the price elasticity of demand procyclical. From equation (4.3), we can easily see that an increase in the level of $x_{j,t}$ raises the relative importance of the price-elastic term $\frac{1}{M_t}(p_t(m))^{-\varepsilon} x_{j,t}$, and reduces the relative importance of the price-inelastic, demand component, $\theta\tilde{c}_{t-1}(m)$. As a result, the price elasticity of demand for good m increases with aggregate demand.

Letting β denotes the constant subjective discount factor and E_t the mathematical expectations operator conditional on information available in period t, the life-time utility of a representative agent j is:

$$E_t \sum_{s=t}^{\infty} \beta^{s-t} \ln(x_{j,s} - d(l_{j,s})) \tag{4.6}$$

where $d(l_{j,t})$ is an increasing and convex function giving disutility of labor supply of agent j, $l_{j,t}$. More specifically, the functional form that will be used later on is:

$$d(l_{j,t}) \equiv \alpha \frac{l_{j,t}^{1+\frac{1}{\sigma}}}{1+\frac{1}{\sigma}}, \tag{4.7}$$

with $\sigma > 0$ representing the Frisch elasticity of labor supply.

(4.6) features preferences à la Greenwood, Hercowitz and Huffman (1988) (henceforth "GHH"). The reason is twofold. First, it is helpful to make the aggregation feasible. We

[3]See Appendix C.1 for further details.

will show latter that the GHH specification makes labor age independent, which is necessary to aggregate individual human wealth. Second, the labor supply is endogenous that raises a potential problem of negative labor supply, since we have overlapping generations structure. Actually, if leisure is a normal good, so wealthier agents supply less labor. Indeed, if labor is not constrained by a lower positive bound then labor supply may be negative. As shown by Ascari and Rankin (2007), the GHH specification makes labor supply independent of wealth[4].

In each period agents supply labor, $l_{j,t}$, in a competitive market and receive real wages, w_t, which are independent of the agents' age.

The agent j maximizes its expected utility subject to its intertemporal budget constraint:

$$\sum_{m=1}^{M_t} p_t(m)\, c_{j,t}(m) + E_t q_{t,t+1} v_{j,t+1} \leq v_{j,t} + w_t l_{j,t} - \tau_{j,t} + \psi_{j,t}. \tag{4.8}$$

where $p_t(m)$ is the relative price of the differentiated good m. The agent j receives an average profit $\psi_{j,t}$ from the family's ownership of a monopolistic firm and pays lump-sum taxes $\tau_{j,t}$. $v_{j,t}$ is the agent $j's$ initial financial wealth. $q_{t,t+1}$ is the stochastic discount factor.

In addition, as markets are complete, there is a risk-free one-period interest rate defined by:

$$1 + r_t = \left[E_t q_{t,t+1}\right]^{-1}. \tag{4.9}$$

The household j's problem consists in choosing its demand for $x_{j,t}$ and its financial asset holdings $v_{j,t+1}$, resulting from the maximization of life-time utility (4.6) subject to the dynamic budget identity (4.8). The first-order conditions for this maximizing problem yield the following optimality conditions:

$$x_{j,t} - d(l_{j,t}) = \beta^{-1} q_{t,t+1}\left(x_{j,t+1} - d(l_{j,t+1})\right), \qquad \forall j \text{ and } \forall s^t \tag{4.10}$$

[4] This issue is discussed in more details in Ascari and Rankin (2007).

$$d_l\left(l_{j,t}\right)=w_t, \tag{4.11}$$

$$x_{j,t}+\theta\sum_{m=1}^{M_t}p_t\left(m\right)\tilde{c}_{t-1}\left(m\right)+E_t q_{t,t+1}v_{j,t+1}=v_{j,t}+w_t l_{j,t}-\tau_{j,t}+\psi_{j,t}, \tag{4.12}$$

$$\lim_{T\rightarrow+\infty}E_t q_{t,T}v_{j,T}=0. \tag{4.13}$$

We note from equation (4.11) that labor is independent of the agent's age and also independent of the agent's consumption. This is a consequence of the GHH preferences, which feature no wealth effect on hours. Equation (4.12) is the intertemporal budget constraint of agent j, which is obtained by introducing (4.5) into (4.8). Equation (4.13) represents the transversality condition.

Moreover, we notice, from (4.10), that the standard Euler equation is modified in two ways. First it is expressed in term of individual habit-adjusted consumption $x_{j,t}$ rather than individual consumption $c_{j,t}$. Second, the term $d\left(l_{j,t}\right)$ is subtracted from the individual habit-adjusted consumption $x_{j,t}$. As we have already mentioned, the term $d\left(l_{j,t}\right)$ is independent of agents' age i.e. it is identical for all agents. Consequently, we can drop the subscript j. In addition, Ascari and Rankin (2007) consider that $d\left(l_t\right)$ acts as a "subsistence" level of consumption. For this reason, they define "adjusted" consumption as the individual habit-adjusted consumption minus its subsistence level $\left(d\left(l_t\right)\right)$. We follow Ascari and Rankin (2007) and define adjusted consumption as[5]:

$$a_{j,t}\equiv x_{j,t}-d\left(l_t\right). \tag{4.14}$$

In addition, we define human wealth (discounted present value of labor income and

[5]From (4.6), we note that individuals' preferences are undefined for habit-adjusted consumption values below the subsistence level. $a_{j,t}$ needs to be positive for all j,t.

profits minus taxes) as:

$$h_{j,t} = E_t \sum_{s=t}^{\infty} q_{t,s} \left[w_s l_{j,s} + \psi_{j,s} - \tau_{j,s} \right]. \tag{4.15}$$

Iterating the budget constraint (4.12) forward (from t to infinity), taking into account the No Ponzi restriction, using (4.10) iterated forward (from t to infinity), applying the definition of adjusted consumption (4.14), and the definition of human wealth (4.15), yields[6]:

$$a_{j,t} = (1-\beta)\left(v_{j,t} + h_{j,t} - \chi_t\right) \tag{4.16}$$

where

$$\chi_t = \theta E_t \sum_{s=t}^{\infty} q_{t,s} \sum_{m=1}^{N_t} p_s(m)\, \tilde{c}_{s-1}(m),$$

denotes the future time path of the reference consumption.

We note from equation (4.16) that in the absence of a consumption externality ($\theta = 0$ and thus $\chi_t = 0$), individuals condition their consumption solely on their consolidated wealth $(v_{j,t} + h_{j,t})$, with a ratio of $(1-\beta)$ of total wealth. With a non-zero consumption externality, however, individual adjusted consumption is also directly affected by the future time path of economy-wide, *per capita* consumption of good m.

So far, we have focused on individual variables. Now we consider aggregate variables. Variables without the subscript "j" represent a *per capita* aggregate value. We apply the aggregation rule used in (4.1) to $x_{j,t}$ and $v_{j,t}$. Agents are assumed to pay the same amount of taxes independently of the age, so $\tau_{j,t} = \tau_t$. Moreover, we will show latter that as firms have the same behavior, the average profits received from firms are independent of the agents' age, i.e. $\psi_{j,t} = \psi_t$. Accordingly, since $l_{j,t}$ is the same for all age cohorts, human wealth is also the same for all, namely $h_{j,t} = h_t$.

Finally, notice that applying the aggregation rule used in (4.1) in period t to the

[6] See Appendix 1 for further details.

variable $v_{j,t+1}$ yields:

$$\sum_{j\leq t}\frac{(N_j - N_{j-1})}{N_t}v_{j,t+1} = (1+n)\,v_t,$$

resulting from the fact that the generation $j = t+1$ has no financial wealth in period $t+1$, i.e. $v_{t,t} = 0$. Here, n denotes the population growth rate, i.e. $N_t = (1+n)\;N_{t-1}$.

Using this result and aggregating equation (4.10), where we replace $a_{j,t+1}$ by its expression given by equation (4.16) expressed in $t+1$, one obtains[7]:

$$a_t = \beta^{-1}q_{t,t+1}a_{t+1} + n\left(\beta^{-1} - 1\right)q_{t,t+1}v_{t+1}. \tag{4.17}$$

This equation is the aggregate Euler equation, which differs from the individual Euler condition (4.10) as long as the population growth rate is different from zero. The last term on the right hand side reflects a real wealth effect, which is characteristic of a non Ricardian economy. Indeed, the growth rate of aggregate adjusted consumption depends negatively on the aggregate financial wealth. An increase in beginning-of-period financial wealth in period $t+1$ cannot be proportionally allocated to present and future aggregate adjusted consumption because those consumers alive during this period benefit.

4.2.2 Firms

This section focuses on the supply side. Here we describe the problem of a firm m appeared before $t-1$. Later on, we will show that new firms behave as old firms.

The differentiated good $m \in [1, M_{t-1}]$ is produced by a monopolist, m, who uses labor input $l_t\,(m)$ and specific human capital—normalized to one—to produce a quantity $y_t\,(m)$ using linear production technology:

$$y_t\,(m) = l_t\,(m)\,. \tag{4.18}$$

[7] See Appendix 2 for further details.

Firms are assumed to be price setters. Letting

$$\psi_t(m) = p_t(m)\, y_t(m) - w_t l_t(m) \tag{4.19}$$

defines the firm m's profits in period t, using (**??**), the owner-manager m's problem is to maximize the discounted value of the sum of its present and future cash flows[8],

$$E_t \sum_{s=t}^{T} q_{t,s} \psi_s(m),$$

subject to (4.18), and

$$y_t(m) = (p_t(m))^{-\varepsilon} \frac{N_t x_t}{M_t} + \theta(1+n)\, \tilde{y}_{t-1}(m), \quad \forall m \tag{4.20}$$

where equation (4.20) is given from the aggregation of (4.3) expressed in level terms. x_t is the *per capita* habit-adjusted consumption. Notice that $\tilde{y}_{t-1}(m)$ is defined as $\tilde{c}_{t-1}(m)$.

Note that the marginal costs of firm m are equal to real wages, w_t. The first-order conditions corresponding to firm m's optimization problem give the following equilibrium equations: (4.20),

$$\lambda_t(m) = p_t(m) - w_t + \theta(1+n)\, E_t q_{t,t+1} \lambda_{t+1}(m), \tag{4.21}$$

and

$$\lambda_t(m) = \frac{M_t}{N_t} \frac{y_t(m)}{\varepsilon x_t} (p_t(m))^{\varepsilon+1}. \tag{4.22}$$

$\lambda_t(m)$ is the Lagrangian multiplier associated with (4.20) and represents the shadow value of selling an extra unit of good m in period t. Equation (4.21) states that the value of selling an extra unit of good m in period t, $\lambda_t(m)$, has two components. The first term on the right hand side represents the short-run profit margin of firm m in period t. The second term on the right hand side corresponds to future expected profits associated

[8]Notice that because markets are complete, we use $q_{t,t+1}$ in the firm's program.

with selling an extra unit of good m in period t.

Letting $\eta_t(m)$ denote the relative markup, in other words the ratio between profit margin (prices minus marginal cost) and prices charged by firm m :

$$\eta_t(m) \equiv \frac{p_t(m) - w_t}{p_t(m)} \tag{4.23}$$

and defining $\epsilon_t(m)$ as the elasticity of demand using (4.20)

$$\epsilon_t(m) \equiv \varepsilon \left[1 - \theta (1+n) \frac{y_{t-1}(m)}{y_t(m)} \right]. \tag{4.24}$$

Rearranging equation (4.22) using (4.20) and the definition (4.24) yields:

$$\lambda_t(m) = \epsilon_t^{-1}(m). \tag{4.25}$$

Equation (4.25) states that the value of selling an extra unit of good m in period t equals the inverse of the price elasticity of demand. Now, combining (4.21) and (4.25) leads to:

$$\eta_t(m) = \epsilon_t(m)^{-1} - \theta(1+n) E_t q_{t,t+1} \frac{p_{t+1}(m)}{p_t(m)} \epsilon_{t+1}(m)^{-1} \tag{4.26}$$

Notice that in the absence of deep habits, i.e. $\theta = 0$, the price elasticity of demand and the relative markup lose their dynamic component. Equation (4.26) becomes $\eta_t(m) = \varepsilon^{-1}$

Equation (4.26) shows that the short-run profit margin of the firm m in period t is negatively related to the price elasticity of demand for good m, $\epsilon_t(m)$, and it is negatively related to future expected profits associated with selling an extra unit of good m in period t, $\lambda_{t+1}(m)$. Also, it is negatively related to the discount factor $q_{t,t+1}$. Moreover, the deep habit assumption gives rise to two sorts of effect, a price elasticity effect and an intertemporal effect. Ravn et al (2006) explain these effects clearly .

First, when the aggregate demand for good m, $y_t(m)$ increases, the price elasticity of demand, $\epsilon_t(m)$ increases too, inducing a decline in the short-run profit margin of firm m in period t, and thus a decline in markups: this is what Ravn et al (2006) call the price-elasticity effect of deep habits on markup. Second, today's price decisions will affect future demand, and so when the present value of future per unit profit is expected to be high—or future price elasticity of demand, $\epsilon_{t+1}(m)$, is expected to be low—, firms have an incentive to invest in the customer base today. Therefore, they induce higher current sales by lowering the current markups. Ravn et al (2006) call this effect: the intertemporal effect of deep habits on markup. The intertemporal effect is also driven by the change in the real interest rate. Indeed if the real interest rate goes up, then the firm discounts future profits more, and thus has less incentive to invest in market share today.

4.2.3 Government

In period t, the government collects lump-sum taxes from households, and issues one-period risk-free government bonds. Government expenditures are assumed to be null. Therefore, government revenues are obtained from net tax receipts and debt issue. The flow budget constraint of the government, expressed in *per capita* terms, reads as

$$(1+n)\frac{b_{t+1}}{R_t} = b_t - \tau_t, \tag{4.27}$$

where b_t, R_t, and τ_t are the number of *per capita* government bonds issued at the start of period $t-1$, the risk-free return and the *per capita* lump-sum taxes, respectively[9]. For the government to remain solvent, the No Ponzi condition must be satisfied.

In this chapter, we focus on the effects on public debt change. Actually, the fiscal shock used in the analysis is a public debt shock. For this reason, we specify a fiscal rule such that a law of motion of public debt follows a first-order autoregressive process:

$$b_{t+1} = \rho b_t + (1-\rho)\bar{b} + \xi_t, \tag{4.28}$$

[9] The public debt is a predetermined value.

where ξ_t reflects a public debt shock, $\bar{b}$ is the target level of long-run debt, and $0 < \rho < 1$ denotes the speed of debt adjustment.

Precisely, the debt-stabilizing fiscal rule is given by:

$$\tau_t = \left(1 - \rho \frac{(1+n)}{R_t}\right) b_t - \frac{(1+n)}{R_t} \left((1-\rho)\bar{b} + \xi_t\right). \tag{4.29}$$

4.3 Symmetric Equilibrium

Firms are different because of the date of appearance. Recall that, in our model, even firms appearing in t face a dynamic (backward) demand of goods. Thus, assuming that all firms make the same decisions in period $t-1$ implies that firms display the same behavior and make the same decisions also in period t. As we have already mentioned, agents are owner-managers of monopolistically competitive firms, i.e. $M_t = N_t$. Accordingly, $p_t(m) = 1$, $c_t(m) = c_t$, $y_t(m) = y_t$, $l_t(m) = l_t$, $\eta_t(m) = \eta_t$, and $\epsilon_t(m) = \epsilon_t$. In addition, the equilibrium in the financial market, in the goods market, and labor market are given by :

$$v_t = b_t,$$

$$y_t = c_t,$$

$$l_t = y_t.$$

It follows that we can describe the symmetric equilibrium using the following set of equations:

$$a_t = \beta^{-1} q_{t,t+1} a_{t+1} + \zeta q_{t,t+1} b_{t+1}, \tag{4.30}$$

$$\eta(y_t) = \epsilon_t(y_t, y_{t-1}) - \tilde{\theta} E_t q_{t,t+1} \epsilon_{t+1}(y_{t+1}, y_t), \tag{4.31}$$

$$a_t = y_t - \tilde{\theta} y_{t-1} - d(y_t), \tag{4.32}$$

$$b_{t+1} = \rho b_t + (1-\rho)\bar{b} + \xi_t, \tag{4.33}$$

where

$$\epsilon_t\left(y_t, y_{t-1}\right) \equiv \varepsilon^{-1}\left(\frac{y_t}{y_t-\tilde{\theta} y_{t-1}}\right), \tag{4.34}$$

$$\eta\left(y_t\right) \equiv 1-d_l\left(y_t\right), \tag{4.35}$$

$\tilde{\theta}=\theta(1+n)$ and $\zeta=n\left(\beta^{-1}-1\right)$. (4.32) gives the definition of adjusted consumption. (4.32) is obtained by replacing *per capita* habit-adjusted consumption by its expression given by (4.20) into the definition of aggregate adjusted consumption given by the aggregation of (4.14). (4.30) is the modified aggregate Euler equation. (4.33) states that the government debt is stabilized, in each period, with an adjustment speed ρ. (4.34) is obtained from (4.24), states that the elasticity of demand is positively related to aggregate demand, y_t. In the symmetric equilibrium (4.26) becomes (4.31), states that the relative markup is dynamic.

In the absence of deep habits, i.e.$\theta=0$, the relative markup is invariant and equals ε^{-1}. We note that using the definition (4.35) with equation (4.31) give the equilibrium level of labor which is time independent. As a consequence, the level of output is determined as is consumption. In this case, fiscal policy is neutral despite the non-Ricardian structure . Accordingly, wealth effects are irrelevant. In fact, a change in government debt affects only the real interest rate.

In the deep habit case, i.e. $\theta \neq 0$, equation (4.31) does not solely determine the equilibrium level of employment. We notice, from equation (4.31), that the markup depends on the present value of future marginal profits induced by a unit increase in current sales, $\tilde{\theta} E_t q_{t,t+1} \epsilon_{t+1}\left(y_{t+1}, y_t\right)$, and the short-run price elasticity of demand, ϵ_t. In this case, wealth effects are significant. For instance, an increase in debt to finance tax cuts in period t, implies a rise in the real interest rate, which has an impact on the markup. The description of this new mechanism is illustrated in section 5 which gives the result of fiscal shock simulations.

4.4 Steady State Equilibrium

In this section we analyze the long-run effects of fiscal policy on the steady state levels of consumption, output and real interest rates. Consider dropping out the indication of time and using (4.9), the system of equations (4.30)-(4.33) becomes:

$$R = \beta^{-1} + \zeta \frac{\bar{b}}{a}, \tag{4.36}$$

$$d_l(y) = 1 - \left(\frac{1 - \frac{\tilde{\theta}}{R}}{1 - \tilde{\theta}} \right) \varepsilon^{-1}, \tag{4.37}$$

$$a = \left(1 - \tilde{\theta}\right) y - d(y), \tag{4.38}$$

$$b = \bar{b}, \tag{4.39}$$

with

$$d(y) = \alpha \frac{y^{1+\frac{1}{\sigma}}}{1 + \frac{1}{\sigma}}, \tag{4.40}$$

and

$$d_l(y) = \alpha y^{\frac{1}{\sigma}}. \tag{4.41}$$

First of all, we notice that $\eta \geq \varepsilon^{-1}$. The steady state markup in the presence of deep habits, i.e. when $\theta \neq 0$, is greater than the steady state markup in the absence of deep habits, i.e. when $\theta = 0$. Firms have more market power in the presence of deep habits. Indeed, charging a low markup in the short run implies high market power in the long run because of the habit effect.

The above steady state system, (4.36)-(4.37), can be rewritten as:

$$R \equiv \Re(y) = \beta^{-1} + \zeta \frac{\bar{b}}{a}, \tag{4.42}$$

$$y \equiv \Upsilon(R) = \left[\left(1 - \frac{1 - \frac{\tilde{\theta}}{R}}{\varepsilon \left(1 - \tilde{\theta}\right)} \right) \alpha^{-1} \right]^{\sigma}, \tag{4.43}$$

with

$$a = \left(1 - \tilde{\theta}\right) y - \alpha \frac{y^{1+\frac{1}{\sigma}}}{1 + \frac{1}{\sigma}}$$

where (4.43) is given by substituting the derivative of $d(l)$ in (4.40) into (4.37).

We show in Appendix C.3 that the necessary and sufficient condition for the existence and the uniqueness of the steady state equilibrium is

$$0 < \underline{y} < \overline{y}, \tag{4.44}$$

where

$$\underline{y} \equiv \left[\frac{1 - \frac{1}{\varepsilon\left(1 - \tilde{\theta}\right)}}{\alpha} \right]^{\sigma},$$

and

$$\overline{y} \equiv \left[\frac{\left(1 - \tilde{\theta}\right)\left(1 + \frac{1}{\sigma}\right)}{\alpha} \right]^{\sigma},$$

Equivalently to (4.44), we have

$$\begin{cases} \tilde{\theta} < \tilde{\theta}_{\max} \equiv \left(1 - \varepsilon^{-1}\right), & \text{for } \sigma < 4\left(\varepsilon - 4\right)^{-1}, \\ \tilde{\theta} \in \left[0, \tilde{\theta}_1\right) \cup \left(\tilde{\theta}_2, \tilde{\theta}_{\max}\right), & \text{for } \sigma < 4\left(\varepsilon - 4\right)^{-1}, \end{cases}$$

where

$$\tilde{\theta}_1, \tilde{\theta}_2 = \frac{2d - 1}{2d} \mp \sqrt{\frac{\varepsilon - 4d}{4\varepsilon d^2}}, \qquad \text{with } d = 1 + \frac{1}{\sigma}.$$

Equation (4.42) and (4.43) are graphed in Figure 4.1. Function Υ and $\Re$ are represented by the dashed-line curve and the solid-line curve in yRplane, respectively. We easily see that in the interval $\left(\underline{y}, \overline{y}\right)$, the two curves intersect once. The steady state equilibrium is given by E.

Figure 4.1 also displays the qualitative effects of a change in the long-run level of public debt, $\bar{b}$. If $\bar{b}$ increases ($\Delta\bar{b} > 0$), the $\Re$ curve moves upward, entailing an increase in long-run gross interest rates, R, and a decrease in long-run output, y. The new steady state equilibrium is given by E'.

Remember that in a Non-Ricardian framework the crowding out effect on output obtained is due to the presence of physical capital. So, if we make abstraction from capital accumulation, the crowding out effect is expected to disappear. We need to point out that in our model, the crowding out effect of a debt-financed tax cut is obtained even without capital. At this stage, we emphasize a new transmission mechanism of fiscal policy. This new channel is based on a lack of Ricardian equivalence and the countercyclical movement of the markup. In the next section, we analyze the effects of temporary and permanent debt-financed tax cuts.

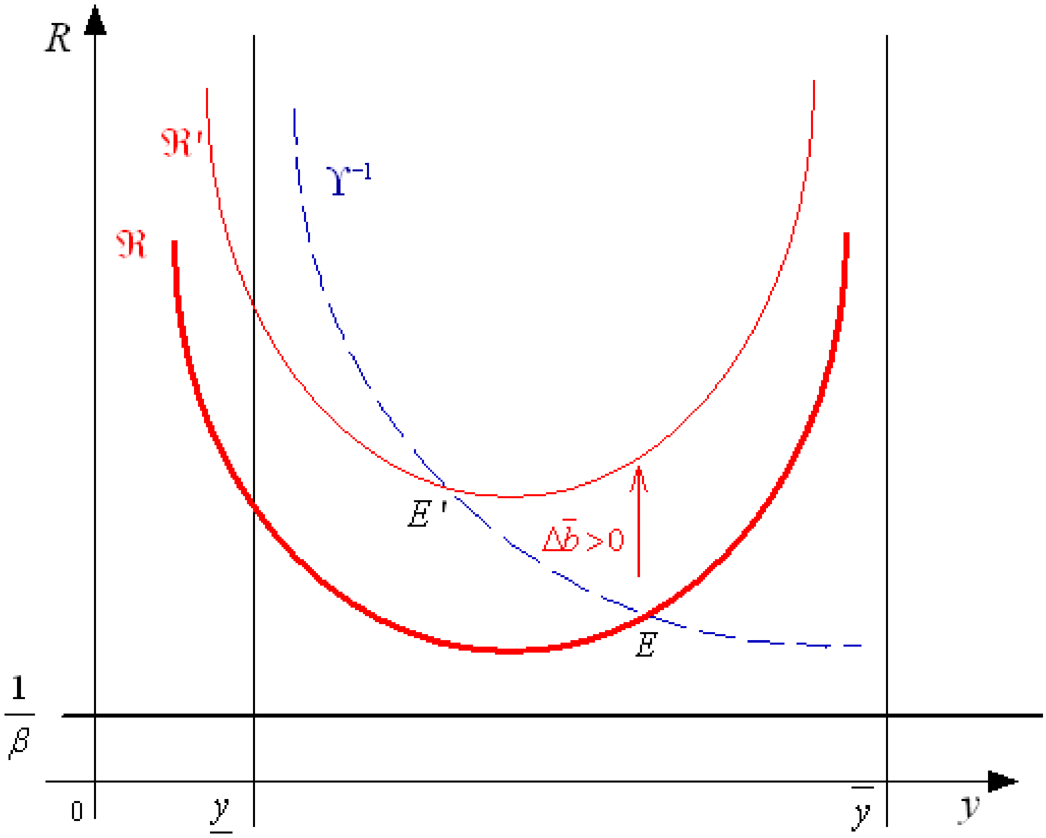


Figure 4.1

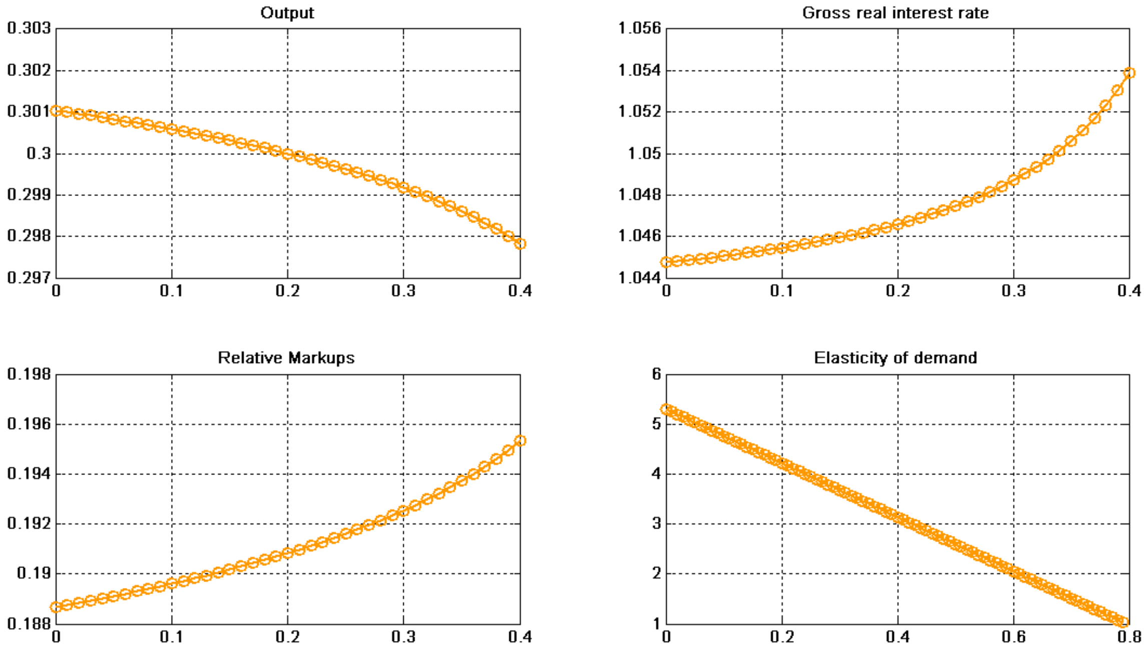
Numerical illustration:

We also give a numerical illustration of the long-run effects of deep habits. So we give values to the parameters. We adopt the calibration used in Ravn et al (2006)[10]. Notice that accordingly, $\sigma < 4(\varepsilon - 4)^{-1}$, so the necessary and sufficient condition for the steady state equilibrium is $\tilde{\theta} < \tilde{\theta}_{\max}$. Figure 4.2 displays the effects of the variation of θ from 0 to 0.4.

Figure 4.2 shows that a higher degree of habit formation implies lower long-run levels of consumption and output, and higher long-run levels of markup and real interest rates and lower elasticity of demand. We observe that the variation is non-linear. In fact, the variation is sharp for values of ϕ between approximately 0.2 and 0.4.

The intuition behind the effects of the change in the degree of deep habits is the following. The higher the degree of habit formation, the more agents care about the difference between their consumption of a specific brand and the average consumption of that brand in the last period. This is a catching up with the Joneses mechanism on a specific brand basis. Agents who have low consumption (the young) are willing to sacrifice future consumption to increase their consumption today. They do so by lowering their savings today in order to catch up with the benchmark level of consumption. The decrease in savings entails higher real interest rates, implying a higher markup. As a result employment decreases, entailing lower consumption and output. This result is in line with Fisher and Heijdra (2009) who show that in a Blanchard-Yaari framework with exogenous labor supply, consumption externalities cause the long-run level of consumption and capital to drop. In our framework this result is preserved even without capital because of the effects of the endogenous markup.

[10] We assume that, $\bar{b} = 0.6$, $\varepsilon = 5.3$, $\sigma = 1.3$, $n = 0.02$ and $\beta = 0.96$. We will give more details about the calibration exercise in the next section.



Output
0.303
0.302
0.301
0.3
0.299
0.298
0.297
0
0.1
0.2
0.3
0.4
Gross real interest rate
1.056
1.054
1.052
1.05
1.048
1.046
1.044
0
0.1
0.2
0.3
0.4
Relative Markups
0.198
0.196
0.194
0.192
0.19
0.188
0
0.1
0.2
0.3
0.4
Elasticity of demand
6
5
4
3
2
1
0
0.2
0.4
0.6
0.8

Figure 4.2

4.5 Public Debt Shocks

In this section, we calibrate our model and investigate the implications of temporary and permanent debt-financed tax cuts. This exercice is just an illustration of the effects of fiscal shocks. We specify that this paper does not aim to show that the model fits the data. We want to shed lights on the response of the economy to public debt shock in a deep-habits non-Ricardian model.

In Table 4.1 we summarize the information on our calibrated parameters. We assume that each period corresponds to a year. We set the discount factor β to 0.96, implying an annual discount rate of approximately 4%. We follow Ravn et al (2006) and set the elasticity of substitution, ε, equal to 5.3, and the Frisch labor supply elasticity, σ, equal to 1.3. In addition, the parameter α is calibrated such that the long-run level of labor equals 0.3. The population growth rate, n, is set equal to 0.02, which larger than the values observed in the data. The reason is that, this value is supposed to take into account all the wealth effects which would affect the real economy. The degree of habit formation θ is set to 0.2. This value is lower than the value used in Ravn, Schmitt-Grohe and Uribe (2006) which is 0.86. The reason is that $\theta = 0.86$ induces a value of the gross real interest of 3. This value is unrealistic. Moreover, the eigenvalues depend on the parameter θ. As a consequence the determinacy of the equilibrium also depends on θ. As shown in Blanchard and Kahn (1980), a necessary condition for the uniqueness of a stable solution in the neighborhood of the steady state is that there are as many eigenvalues larger than one in modulus as there are non-predetermined variables in the model. Therefore, we choose the value of 0.2 which allows to verify the Blanchard and Kahn's conditions. In addition, $\theta = 0.2$ gives a plausible value for r, namely 4.6%.

We solve the model and simulate the model using DYNARE[11].

[11] See Juillard (2004).

Table 4.1: Parameter Values.

Definition	Parameter	Value
Discount factor	β	0.96
Elasticity of substitution across varieties	ε	5.3
Population growth rate	n	0.02
Frisch elasticity of labor supply	σ	1.3
Degree of habit formation	ϕ	0.2
Public debt adjustment speed	ρ	0.9
Long-run level of labor	$\bar{l}$	0.3
Public debt long-run target level	$\bar{b}$	0.6

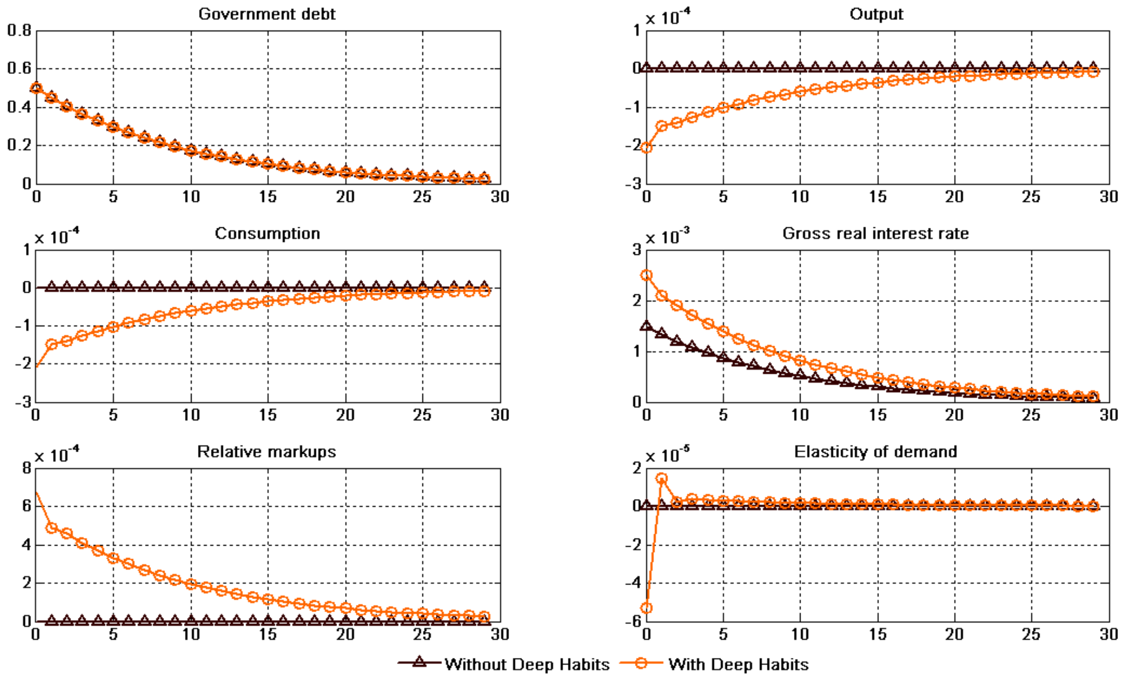
4.5.1 Temporary Public Debt Increase

Here we simulate a temporary tax cut financed by an increase in public debt. We assume that the public debt rises from 60% to 90%. In other words, ξ is set equal to 0.3. Notice that all variables are expressed in deviation (percentage) from the steady state. Figure 4.3 represents the time paths in response to a one-period debt-financed tax cut. We also contrast the effect of shock with and without deep habits.

First, in the absence of deep habits, the public debt increase only affects the real interest rate, which rises. In this case, fiscal policy is neutral despite the non-Ricardian framework. This is a consequence of the using the GHH preference. In fact, the usual wealth effect on labour supply has been eliminated. Thus an increase in government debt does not affect labor supply or output.

Second, in the presence of deep habits, higher public debt entails lower consumption and consequently output. Consumption, employment and output fall on impact. Relative markups and the real interest rate jump on impact. In addition, the elasticity of demand decreases, then increases and then falls to reach its steady state value.

These results can be explained as follows. First, the increasing government debt makes current agents feel wealthier and want to consume more today, all other things being equal. Second, in the securities market, the supply of public bonds outstrips demand for government bonds. As the economy is non-Ricardian, agents do not fear future taxes increases. Consequently, they do not lift the demand for government bonds by the same amount as the government bond supply rises. So, an interest rate increase is necessary to balance the securities market. Third, a higher real interest rate reduces the present value of future per unit profits. As a result, firms have less incentive to invest in the customer base today and hence they are willing to increase markups today. In addition, higher markups entail lower employment and consequently lower consumption. Besides, lower consumption today implies lower price elasticity of demand and thus higher markups today, all other things being equal. At the same time, lower consumption today implies higher elasticity of demand in $t+1$. As a result firms have less incentive to invest in the customer base and will increase their markups today. As we can easily notice, there is no ambiguity, an increase in government debt, $\xi_t > 0$, to finance tax reductions in t, implies an increase in the markup, a decrease in employment, and a drop in consumption. In the next period, firms facing lower demand for their products will set a lower markup in order to increase their demand for goods. Consequently, employment increases and consumption goes up. Finally, the economy converges towards the steady state equilibrium. However, the convergence takes time, because of the persistence of the government debt process (4.28).



Government debt
Output
Consumption
Gross real interest rate
Relative markups
Elasticity of demand
Without Deep Habits
With Deep Habits

Figure 4.3

4.5.2 Permanent Public Debt Increase

Here we simulate a permanent debt-financed tax cut. This exercise is slightly different than the one above. In fact, we assume that the long-run target level of public debt rises from 60% to 120%. Figure 4.4 shows that a permanent public debt shock entails a decline of consumption and output. On the other hand, the real interest rate and markups increase, while the elasticity of demand decreases.

As we have already mentioned in section 4.4, an increase in the supply of government bonds entails a higher real interest rate as the economy is non-Ricardian. The higher the real interest rate, the higher the markup. As the real interest rate increase is permanent, the increase in the markup is also permanent. In this economy markup is dynamic and depends on future sales. When the real interest rate goes up, future profits are discounted less so firms have less incentive to invest in customer base today. Consequently, they raise their markup, implying a decline of employment and consumption. It emerges from our result that the long-run crowding out effect of public debt is not only related to the presence of capital. We show that in a framework without capital, the lack of Ricardian equivalence coupled with a dynamic markup restore the long-run crowding out effect of debt-financed tax cut.

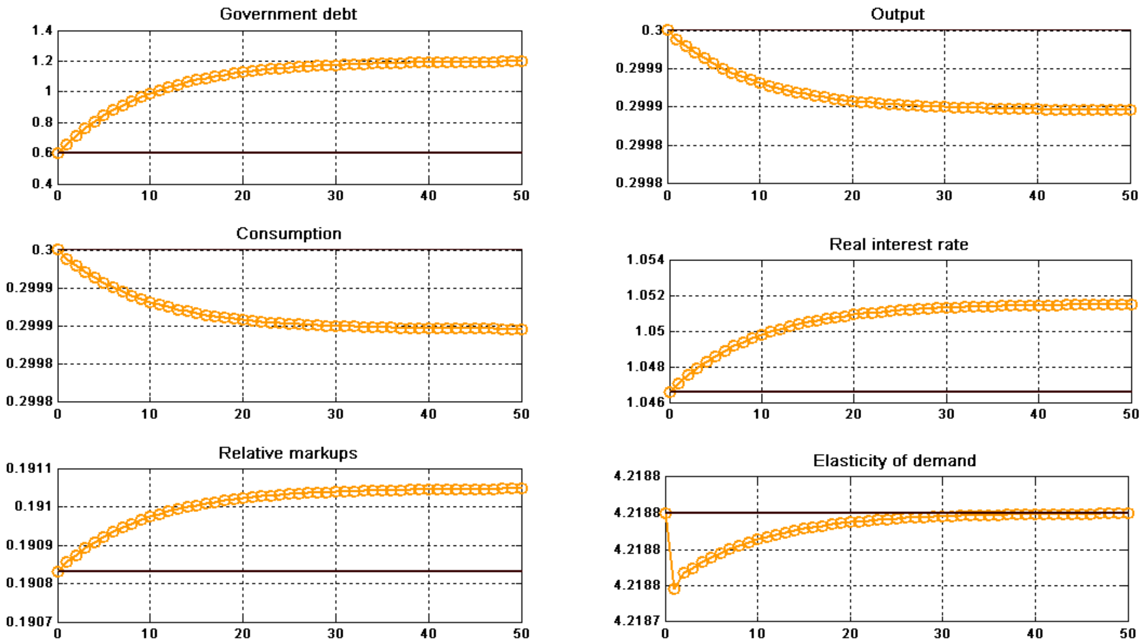


Figure 4.4

4.6 The Sticky Prices Model

In this section, we extend the simple version developed in Section 4.2 by incorporating sticky prices. The goal is to render the model capable of reproducing a short-run expansionary effect of lump-sum fiscal policy. In this extended version, the economy consists of four types of agents: infinitely-lived dynasties (or families), monopolistically competitive firms, the monetary authority and the fiscal authority. Consumption and labor supply decisions are the same as in Section 4.2. For this reason we will skip the description of households problem. On the other hand, firms program is altered by the assumption of price rigidities. Thus we will start this section by studying the behavior of firms. Then we will describe the program of the government and the central bank. Next, we will briefly describe the symmetric equilibrium. As price stickiness assumption does not alter the steady state, the long-run analysis is identical to the one in Section 4.4. Finally, we will study the effects of public debt shock.

4.6.1 Production and Price-Setting Decisions

As in Section 4.2, we assume that firm m transforms homogenous labor into differentiated good according to a linear production technology:

$$y_t(m) = l_t(m). \tag{4.45}$$

Notice that $l_t(m)$ is the quantity of labor used by the firm m to produce good m, in period t. Also, $y_t(m)$ is the production of the firm m of good $m \in [1, M_t]$, in period t.

We assume that monopolistic firms are subject to a Rotemberg's (1982) convex adjustment costs associated with changing nominal prices:

$$\frac{\phi_\pi}{2}\left(\frac{P_t(m)}{P_{t-1}(m)} - \bar{\Pi}\right)^2 \tag{4.46}$$

where $\bar{\Pi}$ denotes the steady state gross inflation rate, and $\phi_\pi \geq 0$ measures the degree of nominal rigidities. when $\phi_\pi = 0$ prices are flexible while positive values of ϕ_π implies that firms find it costless to adjust their prices in line with the central bank inflation target.

Letting

$$\psi_t(m) = \frac{P_t(m)}{P_t} y_t(m) - w_t y_t(m) - \frac{\phi_\pi}{2}\left(\frac{P_t(m)}{P_{t-1}(m)} - \bar{\Pi}\right)^2, \tag{4.47}$$

defines the firm m's profits in period t. The owner-manager m's problem is to maximize the discounted value of the sum of its present and future cash flows,

$$E_t \sum_{s=t}^{T} q_{t,s} \psi_s(m),$$

subject to (4.45),

$$y_t(m) = \left(\frac{P_t(m)}{P_t}\right)^{-\varepsilon} \frac{N_t x_t}{M_t} + \theta_c (1+n) y_{t-1}(m), \quad \forall m \leq M_{t-1} \tag{4.48}$$

where θ_c denotes the degree of external habit formation parameter. x_t is the *per capita* habit-adjusted consumption. Equation (4.48) is given from the aggregation of (4.3) expressed in level terms.

Note that the marginal costs of firm m are equal to real wages, w_t. The first-order conditions corresponding to firm m's optimization problem give the following equilibrium equations: (4.48),

$$\lambda_t(m) = \frac{P_t(m)}{P_t} - w_t + \theta_c (1+n) E_t q_{t,t+1} \lambda_{t+1}(m), \tag{4.49}$$

and

$$\begin{aligned} & y_t(m) - \theta_\pi \frac{P_t}{P_{t-1}(m)} \left(\frac{P_t(m)}{P_{t-1}(m)} - \bar{\Pi}\right) + \phi_\pi E_t q_{t,t+1} \frac{P_{t+1}(m)}{P_t(m)} \left(\frac{P_{t+1}(m)}{P_t(m)} - \bar{\Pi}\right) \\ & = \varepsilon \lambda_t(m) \frac{N_t}{M_t} x_t \left(\frac{P_t(m)}{P_t}\right)^{-\varepsilon-1}. \end{aligned} \tag{4.50}$$

Recall that $\lambda_t(m)$ is the Lagrangian multiplier associated with (4.48) and represents the shadow value of selling an extra unit of good m in period t. Equation (4.49) is identical to equation (4.21) in Section 4.2. On the other hand, (4.22) is altered to include the cost of price adjustment.

4.6.2 Government

The flow budget constraint of the government, reads as

$$\frac{B_{t+1}}{(1+R_t)} = B_t - T_t, \tag{4.51}$$

where B_t, and T_t are nominal government bonds issued at the start of period $t-1$ and total lump-sum taxes, respectively. R_t is a one-period risk-free nominal interest rate. For the government to remain solvent, the No Ponzi condition must be satisfied.

Letting $b_t = \frac{B_t}{N_t P_{t-1}}$, $\tau_t = \frac{T_t}{N_t P_t}$, government budget constraint is re-written in real terms, as follows:

$$b_{t+1} = \frac{1+R_t}{(1+n)} \left[\frac{b_t}{\Pi_t} - \tau_t \right], \tag{4.52}$$

where $\Pi_t = \frac{P_t}{P_{t-1}}$, denotes the gross inflation rate.

As in Section 4.2, we specify a fiscal rule such that a law of motion of public debt follows a first-order autoregressive process:

$$b_{t+1} = \rho_b b_t + (1-\rho_b)\,\bar{b} + \xi_t, \tag{4.53}$$

where ξ_t reflects a public debt shock, $\bar{b}$ is the target level of long-run debt, and $0 < \rho_b < 1$ denotes the speed of debt adjustment.

Using (4.52) and (4.53), our debt-stabilizing fiscal rule is as following:

$$\tau_t = \left(\Pi_t^{-1} - \rho_b \frac{1+n}{1+R_t} \right) b_t - \frac{1+n}{1+R_t} \left[(1-\rho_b)\,\bar{b} + \xi_t \right]. \tag{4.54}$$

4.6.3 Monetary Authority

The monetary authority controls the nominal interest rate. Specifically, monetary policy is assumed to be described by a simple Taylor rule, given by:

$$1 + R_t = \rho_i \left(1 + R_{t-1}\right) + (1-\rho_i) \left((1+\bar{r})\,\bar{\Pi} \left(\frac{\Pi_t}{\bar{\Pi}} \right)^{\phi} \right) \tag{4.55}$$

$\bar{\Pi}$ represents the inflation rate long-run target level. $\bar{R}$ is the steady state equilibrium gross real interest rate. Note that Taylor formulation (4.55) is modified to allow for interest rate smoothing, as proposed by Clarida et al. (1998). In particular, the parameter $\rho_i \in [0,1]$ captures the degree of interest rate smoothing. $\varphi > 1$ is the Taylor rule coefficient, describing the degree of responsiveness of interest rates to inflation.

Notice, that in this model, in contrast to the model in Aloui and Guillard (2009), we neglect the lower bound on the nominal interest constraint. Actually, here our focus is to analyze the new transmission mechanism of government debt through the time-varying markup. So we do local analysis around the targeted equilibrium. The reason why we omit the second (indeterminate) equilibrium of Benhabib Schmitt-Grohe and Uribe (2001).

4.6.4 Symmetric Equilibrium and Steady State

The equilibrium in the financial market is rewritten:

$$v_t = \frac{b_t}{\Pi_t},$$

The symmetric equilibrium described by (4.30)-(4.35) becomes:

$$a_t = \beta^{-1} q_{t,t+1} a_{t+1} + \zeta q_{t,t+1} \frac{b_{t+1}}{\Pi_{t+1}}, \tag{4.56}$$

$$\eta(y_t) = \lambda_t - \tilde{\theta}_c E_t q_{t,t+1} \lambda_{t+1}, \tag{4.57}$$

$$y_t - \phi_\pi \Pi_t \left(\Pi_t - \bar{\Pi}\right) + \phi_\pi E_t q_{t,t+1} \Pi_{t+1} \left(\Pi_{t+1} - \bar{\Pi}\right) = \varepsilon \lambda_t \left(y_t - \tilde{\theta}_c y_{t-1}\right), \tag{4.58}$$

$$a_t = y_t - \tilde{\theta}_c y_{t-1} - d(y_t), \tag{4.59}$$

$$b_{t+1} = \rho_b b_t + (1 - \rho_b) \bar{b} + \xi_t, \tag{4.60}$$

$$1 + R_t = \rho_i (1 + R_{t-1}) + (1 - \rho_i) \left((1 + \bar{r}) \bar{\Pi} \left(\frac{\Pi_t}{\bar{\Pi}} \right)^\phi \right) \tag{4.61}$$

$$1 + R_t = \left[E_t \frac{q_{t,t+1}}{\Pi_{t+1}} \right]^{-1} \tag{4.62}$$

Recall that

$$\eta(y_t) \equiv 1 - d_l(y_t),$$

and

$$d(y_t) = \alpha \frac{y_t^{1+\frac{1}{\sigma}}}{1+\frac{1}{\sigma}}.$$

$\tilde{\theta}_c = \theta_c(1+n)$, and $\zeta = n\left(\beta^{-1} - 1\right)$. Equation (4.62) is the Fisher equation. The steady state system is identical to the system of equation (4.36)-(4.37), which is analyzed in detail in Section 4.4.

4.6.5 Public Debt Shocks

As in Section 4.5, The numerical simulation is conducted using DYNARE (see Juillard (2004)). Recall that the equilibrium is locally determined so we can investigate the effects of temporary public debt shock around the steady state equilibrium. In Table 4.2 we summarize the information on our calibrated parameters.

We assume that the monetary authority reacts to the fluctuations in inflation. Thus we set the Taylor rule coefficient at 1.5. We follow Clarida et al (1998) and set the degree of interest rate inertia at 0.9. The degree of price stickiness is set equal to the value estimated by Ravn et al (2010), i.e. 14.5/4.

Table 4.2: Parameter Values.

Definition	Parameter	Value
Discount factor	β	0.96
Elasticity of substitution across varieties	ε	5.3
Population growth rate	n	0.02
Frisch elasticity of labor supply	σ	1.3
Degree of habit formation	ϕ_c	0.2
Public debt adjustment speed	ρ_b	0.9
Degree of interest rate inertia	ρ_i	0 or 0.9
Degree of price stickiness	ϕ_π	0 or 14.5/4
Taylor rule coefficient	φ	1.5
Labor long-run level	$\bar{l}$	0.3
Inflation long-run target level	$\bar{\Pi}$	1.02
Public debt long-run target level	$\bar{b}$	0.6

Figures 4.5 and 4.6 report the effects of the temporary public debt shock. We assume that the public debt rises from 60% to 90%. In other words, ξ is set equal to 0.3. Figure 4.5 contrasts the effect of temporary public debt shock when prices are full flexible with its effect when prices are sticky[12]. Figure 4.6 contrasts the effect of temporary public debt shock with and without nominal interest rate smoothing.

First, when prices are full flexible, tax cuts entail lower consumption and employment and output fall. Inflation increases the same as for the nominal interest rate. Relative markups and real interest rate rise[13]. The intuition is the following. Higher public debt makes current agents feel wealthier, increasing the desire to consume more today. Accordingly, the real interest rate increases, implying higher inflation. This is a consequence of our Taylor rule specification. Indeed, during the adjustment, the real interest rate targeted by monetary authority is below the natural real interest rate, implying inflationary bias. Furthermore, a higher real interest rate reduces the present value of future per-unit profit margin. As a result, firms have less incentive to invest in the customer base today and hence they are willing to increase markups today. In addition, higher markups entail lower employment and consequently lower output and consumption. Furthermore, the decline in consumption today implies lower demand for goods in the next periods. Thus the elasticity of demand increases, implying markups decline. Consequently, employment increases and output goes up.

Second, when prices are sticky, tax reduction leads to an increase in output on impact, while relative markups negatively deviate from their steady state level. After the initial jump the output decreases. Notice that output is then below its steady state level during the adjustment process, reflecting the crowding out effect of government debt. Real interest rate increases gradually and then starts to decrease again. Inflation increases on impact, but then decreases below its steady state value. Thus during the adjustment inflation is below its steady state level. Nominal interest rates increase on impact. These

[12]Notice that here we set $\rho_i = 0$.
[13]This effect is analysed in more detail in Section 4.5.

results can be explained as follows. As the economy is non-Ricardian, tax reduction stimulates the aggregate demand. For this reason total consumption jumps upwards on impact and then starts to decrease. As prices are sticky output jumps also on impact and then starts to decrease. At the same time, real interest rate increases gradually in order to balance the securities market. After the shock the nominal interest rate increases, then decreases below its steady state level along the adjustment path. This is consistent with the behavior of inflation rate. Consider now the effects on the markups. Here the elasticity effect dominates the intertemporal effect. Indeed, markups decrease on impact, despite the increase in the real interest rate. In fact, higher aggregate demand entails higher elasticity of demand, implying lower markups. But, as long as real interest rates rise and output decreases, intertemporal effect on markups starts to dominate elasticity effect, implying an increase in markup below its steady state level. It is clear that the introduction of sticky prices assumption restores the short-run expansionary effect of fiscal policy.

Moreover, we notice from Figure 5.6 that nominal interest rate smoothing strengthens the short-run expansionary effect. In fact, the real interest rate declines, strengthening the intertemporal effect on the markup and so the output increases more. The reason is the following. The increase in the nominal interest rate in response to the first period increase in inflation is smoothed over the time. As the first period increase in nominal interest rate is not sufficient to balance the Fisher equation, the real interest rate decreases. Consequently, markup declines more, entailing higher employment, output and consumption.

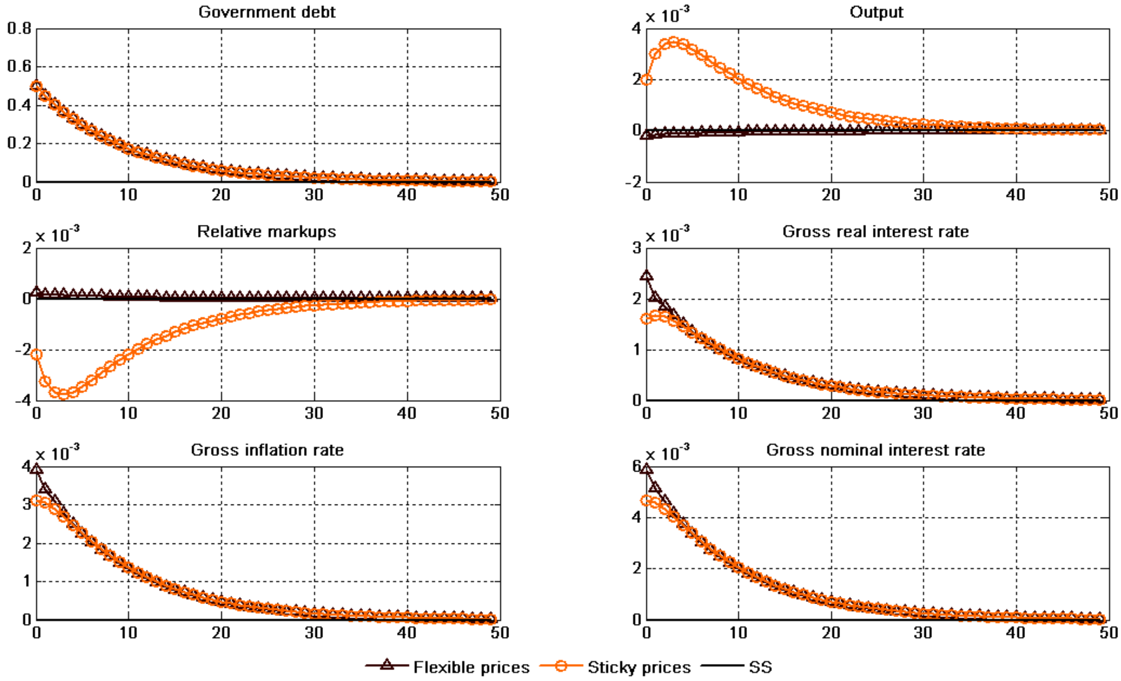
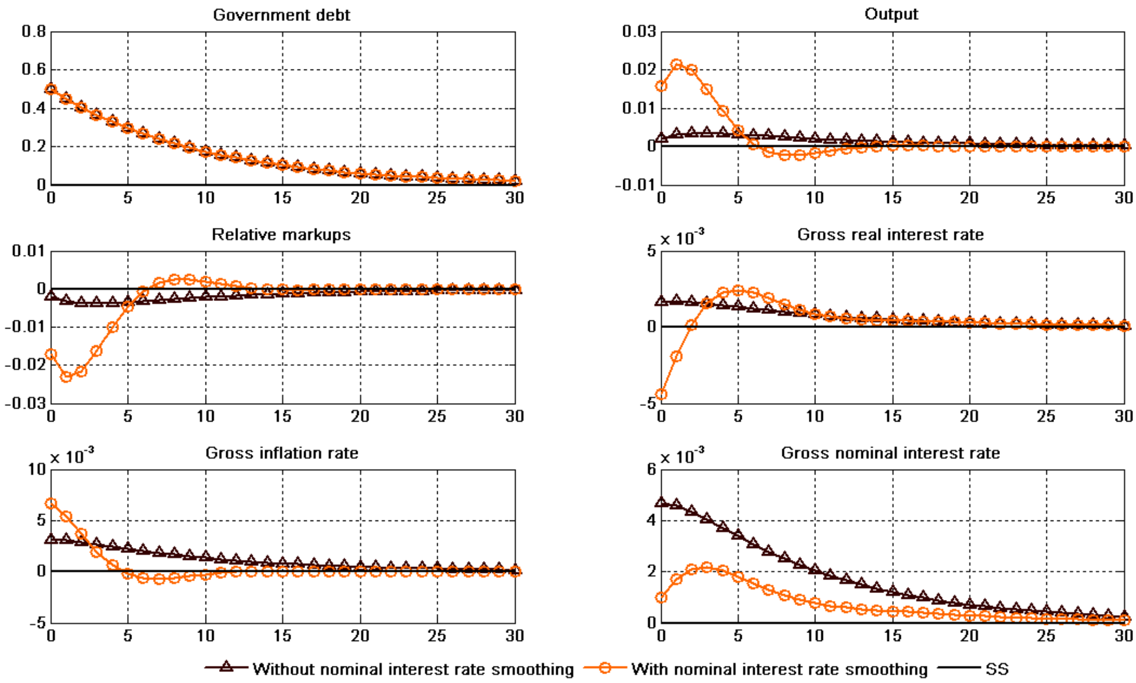

Government debt
Output
Relative markups
Gross real interest rate
Gross inflation rate
Gross nominal interest rate
x 10^-3
Flexible prices
Sticky prices
SS

Figure 4.5



Government debt
Output
Relative markups
Gross real interest rate
Gross inflation rate
Gross nominal interest rate
Without nominal interest rate smoothing
With nominal interest rate smoothing
SS

Figure 4.6

4.7 Conclusion

The goal of this chapter is to merge two recent strands in the macroeconomic literature: the OLG framework and time-varying markups. Our principal motivation in adopting the OLG approach is to break down Ricardian equivalence in order to study the impact of government debt on macroeconomic aggregates. We develop an extended stochastic version of overlapping generations à la Weil (1987) with monopolistically competitive structure, endogenous labor supply, and where agents' preferences feature external habit formation.

The main contribution of this chapter is to provide a new transmission mechanism of public debt through the countercyclical markup movements induced by external deep habits. We show that, when prices are sticky, debt-financed lump-sum fiscal policy is expansionary in the short run and contractionary in the long run.

Rather than reiterating the rest of our findings, let us briefly indicate some possible extensions of this model. One is to introduce monetary policy. It is worthwhile analyzing the interaction between monetary and fiscal policy in such a framework. In addition, given the recent economic crisis, such a model may be a useful tool to explore the role of government debt and deficits in an economy constrained by the ZLB on nominal interest rates.

Chapter 5

Summary and Further Research

This doctoral dissertation introduces original frameworks to analyze, the interaction between monetary and fiscal policy when the monetary policy is constrained by the ZLB on nominal interest rates, and the effects of government debt on macroeconomic aggregates. The aim has been to write a thesis that is innovative, and which provides a useful framework for thinking about a debt-deflation recession. In the introduction to this thesis, we set out an overview of some of the key issues surrounding the global recession of 2008-2010. We provide an overview of the topics covered in this work namely the liquidity trap, the relative importance of monetary and fiscal policy, and deep habits. These topics may seem unrelated at a first glance. Therefore, we point out the connection between them and show that this is helpful to answer some of our questions.

We have shown that assuming a simple fiscal policy, in the spirit of Leeper (1991) and a non linear monetary rule, in the spirit of Benhabib, Schmitt-Grohé and Uribe (2001a), in a non-Ricardian framework with capital accumulation exhibits four steady state equilibria. In addition, we found that these four equilibria have the same dynamics characteristics as the four configurations described by Leeper (1991). It turns out that the determinacy region is no longer specified by the policy parameter space. Our analysis shed light on the role of expectations in getting into a liquidity trap situation. Indeed, we have shown that a liquidity trap, also characterized by a higher real interest rate and a higher level of real debt, possesses the usually required properties of determinacy, like the more traditional equilibrium targeted by the monetary and fiscal authorities. Therefore, from the perspective of global analysis, the existence of two paths locally convergent arises the question of self-fulfilling expectation shocks. Indeed a change in agents' believes can lead the economy from one trajectory to another. We thus give another endogenous explanation of the liquidity trap based on a change in agents' expectations where the public debt play a crucial role.

However, in this doctoral thesis, we do not focus on how to avoid or exist liquidity traps. These two points are interesting avenues for further research. On the one hand, Bénassy and Guillard (2005) have studied the global determinacy conditions in the case of a non-Ricardian exchange economy with ZLB on nominal interest rates. They found that the control of the growth rate of nominal debt should simultaneously ensure uniqueness and determinacy of the equilibrium. An interesting avenue for further research would be to investigate the design of implementable fiscal policy rule that guarantees global determinacy.

On the other hand, Mertens and Ravn (2010) point out that in the basic New Keynesian model pessimistic expectations can set the economy on a deflationary path. They propose an exit solution based on supply side policies. They show, in contrary to a line of recent papers, that demand stimulating policies become less effective in a liquidity trap than in normal circumstances. The key reason is that demand stimulus leads agents to believe that things are even worse than they thought. However, in their model, the demand stimulating fiscal measures are an increase in government spending or sales tax cuts. It is noteworthy to verify these findings in the context of non-Ricardian economy. We believe than these results will be considerably altered, as government debt is non neutral in a non-Ricardian economy. Wealth effects become another channel through which debt-financed deficit policies will affect consumption.

Furthermore, we have analyzed and compared the effects of debt-financed tax cuts in both environments namely the targeted environment and the debt-deflation environment. We found that the responses of the economy are very different. In the targeted environment, debt-financed tax cuts are contractionary. Government debt increase entails a positive wealth effect inducing higher consumption and lower investment. As a result, output decreases. In the debt-deflation environment, debt-financed tax cut is expansionary. Indeed, tax reductions imply less government revenues and consequently real government debt decreases. This implies a negative wealth effect, entailing lower consumption and higher investment. As a result, output increases. The negative wealth effect found in debt-deflation environment is in contrast with Devereux (2010) who finds a positive wealth effect in the liquidity trap. Although, we reach the same conclusion as Devereux (2010) that is debt-financed tax cut is expansionary. It is noteworthy to point out that, in contrast to Devereux (2010), we do not have endogenous labor supply in the model where these shocks are analyzed.

However, in this thesis, we have also developed a non-Ricardian model where labor supply is endogenous in order to investigate the effects of government debt through time-varying markups. Our aim was to develop a model able to reproduce the crowding out effect of government debt regardless of capital. Thus we have abstracted from capital accumulation. We have shown that public debt increase is contractionary in the short and long run when prices are flexible. However, when prices are sticky, debt-financed lump-sum fiscal policy is expansionary in the short run and contractionary in the long run. It would be interesting to develop a non-Ricardian model with capital and time-varying markups, where labor supply is endogenous, and monetary policy is constrained by the ZLB on nominal interest rates. This would definitely provide a very rich framework where we may analyze and compare demand stimulating policies and supply side policies in both targeted environment and debt-deflation environment.

Appendix A

Appendix For Chapter 2

A.1 Proof Proposition 1

Proposition 1: *The real value of the per capita public debt is positive in the "debt equilibrium" if and only if the associated real interest rate is greater than the "autarkic" real interest rate, i.e.:*

$$r^{**} \geq r^{*} \Longleftrightarrow \omega^{**} \geq 0.$$

Proof: Using the concavity of the production function $f(k)$, and (consequently) the decrease of the function $\tilde{q}(k)$, equation (2.34): $c = (1-\bar{g}) f(k) - (n+\delta) k$, permits us to verify that :

$$r^{**} \geq r^{*} \Longrightarrow k^{**}/c^{**} \leq k^{*}/c^{*}.$$

By using (2.37) that we remind:

$$r^{*} = \tilde{q}(k^{*}) - 1 = \frac{\beta^{-1}}{1-\zeta k^{*}/c^{*}} - 1,$$

we easily find :

$$r^{**} \geq r^{*} \Longleftrightarrow \frac{\beta^{-1}}{1-\zeta k^{**}/c^{**}} \leq \frac{\beta^{-1}}{1-\zeta k^{*}/c^{*}}. \tag{A1.1}$$

Now, by using the equation (2.32) rewritten under the following form:

$$\tilde{q}(k) = \frac{\beta^{-1} + \zeta\omega/c}{1 - \zeta k/c}$$

and evaluated in the autarkic equilibrium and in the debt equilibrium, one observes that :

$$r^{**} \geq r^{*} \Longleftrightarrow \frac{\beta^{-1} + \zeta\omega^{**}/c^{**}}{1 - \zeta k^{**}/c^{**}} \geq \frac{\beta^{-1}}{1 - \zeta k^{*}/c^{*}}. \tag{A1.2}$$

By collecting the inequalities (A1.1) and (A1.2), we finally obtain:

$$r^{**} \geq r^{*} \Longleftrightarrow \frac{\beta^{-1}}{1 - \zeta k^{**}/c^{**}} \leq \frac{\beta^{-1}}{1 - \zeta k^{*}/c^{*}} \leq \frac{\beta^{-1} + \zeta\omega^{**}/c^{**}}{1 - \zeta k^{**}/c^{**}}.$$

Or, more simply:

$$r^{**} \geq r^{*} \Longleftrightarrow \omega^{**} \geq 0.$$

$\|$

A.2 Proof Proposition 2

Proposition 2: *Under assumptions (H1), (H2) and (H3), the autarkic equilibrium associated to the inflation target,* $\bar{\pi}$, *is locally determinate and the autarkic liquidity trap equilibrium is locally indeterminate.*

Proof: *a)* We show, at first, that the condition (H3) is sufficient so that the matrix $J_2(k^*, c^*)$ admits one and a single eigenvalue lower than the unity in absolute value. Let us remind, by convenience, $J_2(k^*, c^*)$:

$$J_2(k^*, c^*) = \begin{pmatrix} \frac{1+r^*-\bar{g}f_k^*}{1+n} & -\frac{1}{1+n} \\ \frac{1+r^*-\bar{g}f_k^*}{1+n}\left(c^*\frac{f_{kk}^*}{1+r^*} - \zeta\beta(1+r^*)\right) & \beta(1+r^*) - \frac{c^*\frac{f_{kk}^*}{1+r^*} - \zeta\beta(1+r^*)}{1+n} \end{pmatrix}.$$

Its characteristic polynomial is given by :

$$\begin{aligned} P^*(\lambda) &= \left(\frac{1+r^*-\bar{g}f_k^*}{1+n} - \lambda\right)\left(\beta(1+r^*) - \frac{c^*\frac{f_{kk}^*}{1+r^*} - \zeta\beta(1+r^*)}{1+n} - \lambda\right) \\ &\quad + \frac{1+r^*-gf_k^*}{1+n}\left(\frac{c^*\frac{f_{kk}^*}{1+r^*} - \zeta\beta(1+r^*)}{1+n}\right). \end{aligned}$$

Let us calculate the critical values of $P^*(\lambda)$. We find:

$$\begin{aligned} P^*(-1) &= (1+\beta(1+r^*))\left(1 + \frac{1+r^*-\bar{g}f_k^*}{1+n}\right) - \frac{\left(c^*\frac{f_{kk}^*}{1+r^*} - \zeta\beta(1+r^*)\right)}{(1+n)} > 0, \\ P^*(0) &= \frac{1+r^*-\bar{g}f_k^*}{1+n}\beta(1+r^*) > 0, \\ P^*(1) &= \left(1 - \frac{1+r^*-\bar{g}f_k^*}{1+n}\right)(1-\beta(1+r^*)) + \frac{\left(c^*\frac{f_{kk}^*}{1+r^*} - \Psi\beta(1+r^*)\right)}{(1+n)}. \end{aligned}$$

The signs of $P^*(-1)$ and of $P^*(0)$ are evident. Using the fact that : $r^* = f_k^* - \delta$, and remembering that $\frac{n+\delta\bar{g}}{1-\bar{g}} = r^g$, a necessary and sufficient condition to guarantee that

$P^*(1)$ is negative is given by :

$$(1-\bar{g})(r^*-r^g)(\beta(1+r^*)-1)-\zeta\beta(1+r^*)<-c^*\frac{f^*_{kk}}{1+r^*}. \tag{H3}$$

The polynomial $P^*(\lambda)$ is of degree 2, the condition $P^*(1)<0$ implied by (H3), jointly with $P^*(-1)>0$ and $P^*(0)>0$ is sufficient to guarantee the uniqueness of the eigenvalue inside the unit circle.

b) Notice that, by (H2), the eigenvalue of $J_{1'}(\omega^*)$ verifies : $(1+r^*)/(1+r^{**})<1$, one conclude that the initial matrix $J_4^*=J_4(k^*,\omega^*,c^*,\pi^*)$ has at least two eigenvalues less than the unit and one eigenvalue greater than the unit (in absolute value). According to the sign of $\phi_\pi-1$, the equilibrium is either locally determinate $(\phi_\pi>1)$, or locally indeterminate $(\phi_\pi<1)$. By (H1), the autarkic equilibrium associated to the inflation target, $\bar{\pi}$, is locally determinate and the autarkic liquidity-trap equilibrium is locally indeterminate. $\|$

A.3 Proof Proposition 3

Proposition 3: *Under the assumption (H1) and (H2), the debt equilibrium associated to the higher inflation rate, $\bar{\pi}$ or π^{**}, is locally overdeterminate and the* debt-*liquidity-trap equilibrium is locally determinate.*

Proof: The proof is twofold. First, we give a sufficient condition for the matrix $J_3^{**} = J_3(k^{**}, \omega^{**}, c^{**})$ to admit one eigenvalue in the absolute value less than the unit and two eigenvalues greater than the unit. Second, we deduce from (J_1^{**}) the dynamic characteristics of this equilibrium.

a) We show that the characteristic polynomial $\mathcal{P}^{**}(\lambda)$ of the matrix $J_3^{**} = J_3(k^{**}, \omega^{**}, c^{**})$ has one root in the interval $[-1, 1]$ and two outside the interval $[-1, 1]$. Let us remind, by convenience, J_3^{**} :

$$J_3^{**} = \begin{pmatrix} \frac{1+r^{**}-\bar{g}f_k^{**}}{1+n} & 0 & -\frac{1}{1+n} \\ \frac{1+r^{**}-\bar{g}f_k^{**}}{1+n} f_{kk}^{**} \frac{\omega^{**}}{1+r^{**}} & 1 & -\frac{1}{1+n} f_{kk}^{**} \frac{\omega^{**}}{1+r^{**}} \\ \frac{1+r^{**}-\bar{g}f_k^{**}}{1+n} \left(c^{**} \frac{f_{kk}^{**}}{1+r^{**}} - \zeta\beta(1+r^{**}) \right) & -\zeta\beta & \beta(1+r^{**}) - \frac{c^{**}\frac{f_{kk}^{**}}{1+r^{**}} - \zeta\beta(1+r^{**})}{1+n} \end{pmatrix}.$$

Its characteristic polynomial is given by :

$$\mathcal{P}^{**}(\lambda) = -\lambda^3 + T^{**}\lambda^2 - S^{**}\lambda + D^{**},$$

where T^{**}, S^{**} and D^{**} represent the trace, the sum of the principal minors of order two and the determinant of the matrix J^{**}, respectively, which are given by:

$$T^{**} = \frac{1 + r - \bar{g}f_k^{**}}{1+n} + 1 + \beta(1+r^{**}) - \frac{c^{**}\frac{f_{kk}^{**}}{1+r^{**}} - \zeta\beta(1+r^{**})}{1+n} > 0,$$

$$
\begin{aligned}
S^{**} &= \beta\left(1+r^{**}\right)-\frac{c^{**}\frac{f_{kk}^{**}}{1+r^{**}}-\zeta\beta\left(1+r^{**}\right)}{1+n}+ \\
&\quad \frac{1+r^{**}-\bar{g}f_k^{**}}{1+n}\left(1+\beta\left(1+r^{**}\right)\right)-\zeta\beta\frac{f_{kk}^{**}}{(1+n)}\frac{\omega^{**}}{1+r^{**}} \\
&> 0,
\end{aligned}
$$

$$
D^{**}=\frac{1+r^{**}-\bar{g}f_k^{**}}{1+n}\beta\left(1+r^{**}\right)>0.
$$

Let us calculate the critical values and the derivative of $\mathcal{P}^{**}(\lambda)$. We find:

$$
\begin{aligned}
\mathcal{P}^{**}(-1) &= 1+T^{**}+S^{**}+D^{**}>0, \\
\mathcal{P}^{**}(0) &= D^{**}>0, \\
\mathcal{P}^{**}(1) &= \zeta\beta\frac{f_{kk}^{**}}{(1+n)}\frac{\omega^{**}}{1+r^{**}}<0,
\end{aligned}
$$

and,

$$
\mathcal{P}_\lambda^{**}(\lambda)=-3\lambda^2+2T^{**}\lambda-S^{**}.
$$

We show, at first, that $\mathcal{P}^{**}(\lambda)$ does not admit a root in $[-\infty,0]$. Then we prove that it admits only odd roots in $[0,1]$; either 1 or 3. Finally, we give a sufficient condition to preclude the three-roots' case.

i) $P_\lambda^{**}(\lambda)$ is strictly negative in $[-\infty,0]$, accordingly the polynomial $\mathcal{P}^{**}(\lambda)$ is strictly decreasing in $[-\infty,0]$. Given the sign of $\lim_{\lambda\to-\infty}\mathcal{P}^{**}(\lambda)$, and $\mathcal{P}^{**}(0)$ we deduce that $\mathcal{P}^{**}(\lambda)\neq 0$ in $[-\infty,0]$. Therefore, the polynomial $\mathcal{P}^{**}(\lambda)$ does not admit a root in $[-\infty,0]$.

ii) According to $\mathcal{P}^{**}(0)>0$, and $\mathcal{P}^{**}(1)<0$ the polynomial $\mathcal{P}^{**}(\lambda)$ changes of sign between 0 and 1, thus it can have, either one, or three roots in $[0,1]$.

iii) If $\mathcal{P}^{**}(\lambda)$ admits three roots in $[0,1]$, then its derivative should cancel twice in $[0,1]$. A sufficient condition to preclude the later case, is to show that the polynomial of degree two $\mathcal{P}_\lambda^{**}(\lambda)$ admits a positive maximum outside the interval $[0,1]$ involving that $\mathcal{P}^{**}_{\lambda}(\lambda)$ has at most one root inside the interval $[0,1]$. Now, we have:

$$\mathcal{P}^{**}_{\lambda}(0) = -S^{**} < 0$$

and

$$\mathcal{P}^{**}_{\lambda\lambda}(\lambda) = -6\lambda + 2T^{**},$$

that equals zero when $\lambda = \frac{T^{**}}{3}$. Using $r^{**} = f^{**}_k - \delta$ and $r^g = \frac{n+\delta\bar{g}}{1-\bar{g}}$, the condition for $T^{**} > 3$ can be written:

$$(1-\bar{g})(r^{**} - r^g) + (1+n)(\beta(1+r^{**}) - 1) > c^* \frac{f^*_{kk}}{1+r^*} - \zeta\beta(1+r^*),$$

that is easily verified using $H2$. In fact, according to $H2$, we have $r^{**} > r^* > \beta^{-1} - 1$ and $r^{**} > r^g$ both involving that the left-hand term of the previous inequality is positive. This sufficient condition guarantees that $\mathcal{P}^{**}_{\lambda}(\lambda)$ cancels only once in $[0,1]$ and therefore the polynomial $\mathcal{P}^{**}(\lambda)$ admits only one root in $[0,1]$. We deduce that the matrix J^{**}_3 has one eigenvalue in the absolute value less than the unit and two, greater than the unit.

b) Finally, According to the sign of $\phi_\pi - 1$, the equilibrium is either locally determinate $(\phi_\pi < 1)$, or locally overdeterminate $(\phi_\pi > 1)$. By (H1), the debt equilibrium associated to the higher inflation rate, $\bar{\pi}$ or π^{**}, is locally overdeterminate and the debt-liquidity-trap equilibrium is locally determinate.‖

A.4 Linearization

In this appendix, we derive the state-space form of the model composed of the variables $\hat{c}_t$, $\hat{\pi}_t$, $\hat{k}_t$ and $\hat{x}_t$ (rather than $\hat{\omega}_t$). We remind, by convenience, the equations (2.26) to (2.31):

$$\begin{aligned}
c_t &= \beta^{-1}\frac{c_{t+1}}{q_{t,t+1}} + \zeta\left[\frac{\omega_{t+1}}{q_{t,t+1}} + \frac{\tilde{q}(k_{t+1})}{q_{t,t+1}}k_{t+1}\right], && \text{(A4.1)}\\
k_{t+1} &= \frac{1}{1+n}\left[(1-\delta)k_t + (1-\bar{g}_t)\cdot f(k_t) - c_t\right], && \text{(A4.2)}\\
E_t\left(\frac{\omega_{t+1}}{q_{t,t+1}}\right) &= \frac{1}{1+n}\left[(1-\gamma)\omega_t + (\bar{g}_t - \bar{\tau}_t) f(k_t)\right], && \text{(A4.3)}\\
E_t\left(\frac{1}{q_{t,t+1}}\right) &= \frac{1}{\tilde{q}(k_{t+1})}, && \text{(A4.4)}\\
E_t\left(\frac{1}{q_{t,t+1}(1+r_{t+1})}\right) &= \frac{1}{1+R_t}, && \text{(A4.5)}\\
1+R_t &= \Phi(\bar{r}_t, \pi_t). && \text{(A4.6)}
\end{aligned}$$

From (A4.1), we express the value of $q_{t,t+1}$:

$$q_{t,t+1} = \beta^{-1}\frac{c_{t+1}}{c_t} + \zeta\left[\frac{\omega_{t+1}}{c_t} + \frac{\tilde{q}(k_{t+1})}{c_t}k_{t+1}\right],$$

that we inject in (A4.3), (A4.4) and (A4.5). By using the value of $1+R_t$ given by (A4.6), defining the predetermined variable $\varkappa_t = (1+\pi_t)\omega_t = \frac{M_{t-1}+B_{t-1}}{N_t P_{t-1}}$, and rearranging the equations, we get:

$$\begin{aligned}
c_t &= \left[E_t\left[\beta^{-1}\frac{c_{t+1}}{\tilde{q}(k_{t+1})} + \zeta\left(\frac{\varkappa_{t+1}}{\tilde{q}(k_{t+1})(1+\pi_{t+1})} + k_{t+1}\right)\right]^{-1}\right]^{-1},\\
k_{t+1} &= \frac{1}{1+n}\left[(1-\delta)k_t + (1-\bar{g}_t)\cdot f(k_t) - c_t\right],\\
\varkappa_{t+1} &= \frac{\Phi(r_t,\pi_t)}{(1+n)}\left[(1-\gamma)\frac{\varkappa_t}{(1+\pi_t)} + (\bar{g}_t - \bar{\tau}_t) f(k_t)\right],\\
\Phi(r_t,\pi_t) &= \frac{\tilde{q}(k_{t+1})\,E_t\left[\beta^{-1}c_{t+1} + \zeta\left(\frac{\varkappa_{t+1}}{(1+\pi_{t+1})} + \tilde{q}(k_{t+1})\,k_{t+1}\right)\right]^{-1}}{E_t\left[(1+\pi_{t+1})\right]^{-1}\left[\beta^{-1}c_{t+1} + \zeta\left(\frac{\varkappa_{t+1}}{(1+\pi_{t+1})} + \tilde{q}(k_{t+1})\,k_{t+1}\right)\right]^{-1}},
\end{aligned}$$

which constitute a system of four dynamics, stochastic and non linear equations, with 2 predetermined variables, k_t and $\varkappa_t$, and non predetermined variables, c_t and π_t. It is necessary to clarify the processes followed by $\bar{g}_t$ and $\bar{\tau}_t$—2 additional predetermined variables—as well as the form of the function $\Phi(\cdot)$ and the value held for the real interest target r_t to obtain a completely specified system.

By linearizing the previous system around a some steady state, one obtains :

$$\begin{aligned}
\hat{c}_t &= \beta^{-1}\frac{E_t\hat{c}_{t+1}}{1+r} + \zeta\frac{E_t\hat{\varkappa}_{t+1}}{(1+\pi)(1+r)} - \zeta\omega\frac{E_t\hat{\pi}_{t+1}}{(1+\pi)(1+r)} \\
&+ \left(\zeta(1+r) - \frac{\beta^{-1}c+\zeta\omega}{(1+r)}f_{kk}\right)\frac{E_t\hat{k}_{t+1}}{1+r},
\end{aligned}$$

$$\hat{k}_{t+1} = \frac{1}{1+n}\left([1+r-\bar{g}f_k]\,\hat{k}_t - \hat{c}_t - f\cdot\hat{g}_t\right),$$

$$\begin{aligned}
E_t\hat{\varkappa}_{t+1} &= \omega E_t\hat{\pi}_{t+1} + \frac{(1+\pi)\,\omega f_{kk}}{1+r^{**}}\hat{k}_{t+1} + \frac{1+r}{1+r^{**}}\hat{\varkappa}_t \\
&\quad -\frac{1+r}{1+r^{**}}\omega\hat{\pi}_t + \frac{(1+\pi)(1+r)}{1+n}f\cdot(\hat{g}_t - \hat{\tau}_t),
\end{aligned}$$

$$E_t\hat{\pi}_{t+1} = \phi_\pi\hat{\pi}_t + (\phi_r - 1)\frac{(1+\pi)}{1+r}f_{kk}\hat{k}_{t+1},$$

where $\phi_\pi = (1+\pi)\,\Phi_\pi/\Phi$ and $\phi_r = (1+r)\,\Phi_r/\Phi$ are the elasticity of the function $\Phi(\cdot)$ and where we used $r^{**} = \frac{n+\gamma}{1-\gamma}$.

By denoting $\hat{Y}_{\varkappa,t} = \begin{bmatrix} \hat{k}_t & \hat{\varkappa}_t & \hat{c}_t & \hat{\pi}_t \end{bmatrix}'$, the vector of the endogenous variables and $\epsilon_t = \begin{bmatrix} \hat{g}_t & \hat{\tau}_t \end{bmatrix}'$, the vector of shocks, the previous equations can be combined to obtain the state-space form as follows :

$$E_t\hat{Y}_{\varkappa,t+1} = J_\varkappa\cdot\hat{Y}_{\varkappa,t} + J_\varepsilon\cdot\epsilon_t,$$

where the Jacobian matrix $J_\varkappa$ is given by:

$$J_\varkappa = \begin{pmatrix} \frac{1+r-gf_k}{1+n} & 0 & -\frac{1}{1+n} & 0 \\ \frac{\omega(1+\pi)}{1+r^{**}} f_{kk} \frac{1+r-gf_k}{1+n} \phi_r & \frac{1+r}{1+r^{**}} & -\phi_r \frac{(1+\pi)}{1+r} \omega \frac{f_{kk}}{(1+n)} & \left(\phi_\pi - \frac{1+r}{1+r^{**}}\right) \omega \\ \frac{1+r-gf_k}{(1+n)} \left(c\frac{f_{kk}}{r} - \zeta \beta r\right) & -\zeta\beta \frac{1+r}{1+r^{**}} & \beta(1+r) - \frac{c\frac{f_{kk}}{1+r} - \zeta\beta(1+r)}{1+n} & \beta\zeta \frac{1+r}{1+r^{**}} \frac{\omega}{(1+\pi)} \\ \frac{1+r-gf_k}{(1+n)} (\phi_r - 1) \frac{(1+\pi)}{1+r} f_{kk} & 0 & -(\phi_r - 1) \frac{(1+\pi)}{1+r} \frac{f_{kk}}{(1+n)} & \phi_\pi \end{pmatrix}$$

and J_ε by :

$$J_\varepsilon = \begin{pmatrix} -\frac{f(k)}{1+n} & 0 \\ (1+\pi)\left(1+r-\omega\frac{\phi_r}{1+r} f_{kk}\right) \frac{f(k)}{(1+n)} & -(1+\pi) \frac{1+r}{1+n} f(k) \\ -\frac{f_{kk} f(k)}{(1+r)(1+n)} c & (1+r) \beta \frac{\zeta}{1+n} f(k) \\ -(\phi_r - 1) \frac{(1+\pi)}{(1+r)} \frac{f_{kk}}{(1+n)} f(k) & 0 \end{pmatrix}.$$

The evolution of the variable $\hat{\omega}_t$ is obtained in a residual way:

$$\hat{\omega}_t = \frac{1}{(1+\pi)} \hat{\varkappa}_t - \frac{\omega}{(1+\pi)} \hat{\pi}_t$$

Appendix B

Appendix For Chapter 3

B.1 Optimality Conditions For The Consumer

Here we present the optimality conditions for the agents j.

The household j minimizes total expenditure $\sum_{m=1}^{M_t} p_t(m)\, c_{j,t}(m)$ subject to the aggregate constraint

$$x_{j,t} = M_t^{\frac{1}{1-\varepsilon}} \left(\sum_{m=1}^{M} \left(c_{j,t}(m) - \theta \tilde{c}_{t-1}(m) \right)^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{\varepsilon}{\varepsilon-1}}, \tag{B1.1}$$

where $p_t(m)$ denotes the relative price of good m at time t.

The Lagrangian for this problem is:

$$\min \sum_{m=1}^{M_t} p_t(m)\, c_{j,t}(m) + \zeta_t \left(x_{j,t} - M_t^{\frac{1}{1-\varepsilon}} \left(\sum_{m=1}^{M_t} \left(c_{j,t}(m) - \theta \tilde{c}_{t-1}(m) \right)^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{\varepsilon}{\varepsilon-1}} \right)$$

where ζ_t is the Lagrange multiplier.

The first order conditions of this problem for $c_{j,t}(m)$ and ζ_t are:

$$\frac{p_t(m)}{\zeta_t} = M_t^{\frac{1}{1-\varepsilon}} \left(c_{j,t}(m) - \theta \tilde{c}_{t-1}(m)\right)^{-\frac{1}{\varepsilon}} \left(\sum_{m=1}^{M_t} \left(c_{j,t}(m) - \theta \tilde{c}_{t-1}(m)\right)^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{1}{\varepsilon-1}}, \tag{B1.2}$$

$$x_{j,t} = M_t^{\frac{1}{1-\varepsilon}} \left(\sum_{m=1}^{M_t} \left(c_{j,t}(m) - \theta \tilde{c}_{t-1}(m)\right)^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{\varepsilon}{\varepsilon-1}}, \tag{B1.3}$$

Rearranging (B1.2) using (B1.3) yields:

$$c_{j,t}(m) = \frac{1}{M_t} \left(\frac{p_t(m)}{\zeta_t} \right)^{-\varepsilon} x_{j,t} + \theta \tilde{c}_{t-1}(m). \tag{B1.4}$$

From the definition of the composite level of consumption (B1.1), this implies

$$\zeta_t = \left(\frac{1}{M_t} \sum_{m=1}^{M} \left(p_t(m)\right)^{1-\varepsilon} \right)^{\frac{1}{1-\varepsilon}}.$$

We define P_t as a price index which verifies:

$$P_t c_t = \sum_{m=1}^{M} P_t(m) \sum_{j \leq t} \left(N_j - N_{j-1}\right) c_{j,t}(m),$$

where $P_t(m)$ is the nominal price of good m. The accounting definition of c_t is given by

$$c_t = M_t^{\frac{1}{1-\varepsilon}} \left(\sum_{m=1}^{M} c_t(m)^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{\varepsilon}{\varepsilon-1}},$$

which combined with (B1.2) and (B1.4) allows us to write:

$$P_t = \left(\frac{1}{M_t} \sum_{m=1}^{M} \left(P_t(m)\right)^{1-\varepsilon} \right)^{\frac{1}{1-\varepsilon}}.$$

Assuming that the price index equals one, the optimal level of $c_{j,t}(m)$ for $m \in [1, M_t]$

is given by

$$c_{j,t}(m) = \frac{1}{M_t} (p_t(m))^{-\varepsilon} x_{j,t} + \theta \tilde{c}_{t-1}(m). \tag{B1.5}$$

Moreover we multiply (B1.5) by $p_t(m)$

$$p_t(m) c_{j,t}(m) = \frac{1}{M_t} p_t(m)^{1-\varepsilon} x_{j,t} + \theta p_t(m) \tilde{c}_{t-1}(m),$$

then we sum the resulting equation over the variety goods m, which yields

$$\sum_{m=1}^{M} p_t(m) c_{j,t}(m) = \frac{1}{M_t} \sum_{m=1}^{M} p_t(m)^{1-\varepsilon} x_{j,t} + \theta \sum_{m=1}^{M} p_t(m) \tilde{c}_{t-1}(m).$$

Finally, using the definition of the price index, we obtain

$$x_{j,t} = \sum_{m=1}^{M} p_t(m) c_{j,t}(m) - \theta \sum_{m=1}^{M} p_t(m) \tilde{c}_{t-1}(m). \tag{B1.6}$$

B.2 Optimality Conditions For The Monopolistically-Competitive Firm

The firm's profit in period t is defined by:

$$\Psi_t(m) = P_t(m) Y_t(m) - P_t \kappa_t K_t(m). \tag{B2.1}$$

The firm m faces the following total demand for good m :

$$Y_t(m) = C_t(m) + G_t(m) + I_t(m). \tag{B2.2}$$

Remember that

$$\begin{aligned}
C_t(m) &= \frac{1}{M_t}\left(\frac{P_t(m)}{P_t}\right)^{-\varepsilon} \frac{X_t^c}{M_t} + \theta_c (1+n) C_{t-1}(m), \quad \forall m \\
I_t(m) &= \frac{1}{M_t}\left(\frac{P_t(m)}{P_t}\right)^{-\varepsilon} X_t^i + \theta_i I_{t-1}(m), \quad \forall m \\
G_t(m) &= \frac{1}{M_t}\left(\frac{P_t(m)}{P_t}\right)^{-\varepsilon} X_t^g + \theta_g G_{t-1}(m), \quad \forall m
\end{aligned}$$

Accordingly, assuming $\theta_c(1+n) = \theta_g = \theta_i = \theta(1+n)$ and letting $X_t = X_t^c + X_t^i + X_t^g$, equation (B2.2) becomes:

$$Y_t(m) = \frac{1}{M_t}\left(\frac{P_t(m)}{P_t}\right)^{-\varepsilon} X_t + \theta(1+n) Y_{t-1}(m), \tag{B2.3}$$

where $Y_{t-1}(m) = C_{t-1}(m) + I_{t-1}(m) + G_{t-1}(m)$. Firm m maximizes the discounted value of the sum of its present and future cash-flows, subject to (B2.3) and the production function:

$$Y_t(m) = F(K_t(m)).$$

The Lagrangian function corresponding to the firm's problem is:

$$E_t \sum_{s=t}^{T} q_{t,s} \left(\frac{P_s(m)}{P_s} Y_s - \kappa_s K_s(m) \right)$$
$$- \lambda_s(m) \left(Y_s(m) - \frac{1}{M_t} \left(\frac{P_s(m)}{P_s} \right)^{-\varepsilon} X_s - \theta(1+n) Y_{s-1}(m) \right)$$
$$- \delta_s(m) \left(Y_t(m) - f\left[K_t(m)\right] \right)$$

The First order conditions for this problem with regard to $Y_t(m)$, $K_t(m)$, $P_t(m)$, $\lambda_t(m)$ and $\delta_t(m)$ are:

$$\frac{P_t(m)}{P_t} - \lambda_t(m) - \delta_t(m) + \theta(1+n) E_t q_{t,t+1} \lambda_{t+1}(m) = 0, \tag{B2.4}$$

$$\delta_t(m) = \frac{\kappa_t}{f'\left[K_t(m)\right]}, \tag{B2.5}$$

$$Y_t(m) - \varepsilon \frac{1}{M_t} \lambda_t(m) \left(\frac{P_t(m)}{P_t} \right)^{-\varepsilon-1} X_t = 0, \tag{B2.6}$$

$$Y_t(m) = F\left(K_t(m)\right), \tag{B2.7}$$

and (B2.3).

Let us define the relative markup and the elasticity of demand by:

$$\eta_t(m) \equiv \left[\frac{P_t(m)}{P_t} - \frac{\kappa_t}{f'\left[K_t(m)\right]} \right], \tag{B2.8}$$

and

$$\epsilon_t(m) \equiv \varepsilon \frac{P_t}{P_t(m)} \left[1 - \theta(1+n) \frac{Y_{t-1}(m)}{Y_t(m)} \right], \tag{B2.9}$$

respectively. Using (B2.3) and (B2.9), (B2.6) becomes

$$\lambda_t(m) = \frac{1}{\epsilon_t(m)} \tag{B2.10}$$

Combining (B2.8), (B2.5) and (B2.10) with (B2.4), one obtains:

$$\eta_t(m) = \lambda_t(m) - \theta(1+n) E_t q_{t,t+1} \lambda_{t+1}(m) \tag{B2.11}$$

Appendix C

Appendix For Chapter 4

C.1 Optimality Conditions For The Consumer

We build the following Lagrangian function corresponding to the consumer's program:

$$E_t \sum_{s=t}^{\infty} \beta^{s-t} \ln \left(x_{j,s} - d\left(l_{j,s}\right)\right) -$$
$$\rho_s \left(x_{j,s} + \theta \sum_{m=1}^{M} p_s\left(m\right) \tilde{c}_{s-1}\left(m\right) - v_{j,s} - w_s l_{j,s} + \tau_{j,s} - \psi_{j,s} + q_{t,s+1} v_{j,s+1} \right)$$

where λ_t is a Lagrange multiplier.

The first order conditions of this problem for $x_{j,t}$, $l_{j,t}$, $v_{j,t+1}$ and ρ_t are:

$$\frac{1}{x_{j,t} - d\left(l_{j,t}\right)} = \rho_t, \tag{C1.7}$$

$$-\frac{d_l\left(l_{j,t}\right)}{x_{j,t} - d\left(l_{j,t}\right)} = -\rho_t w_t, \tag{C1.8}$$

$$E_t q_{t,t+1} \rho_t = \beta \rho_{t+1}, \tag{C1.9}$$

$$x_{j,t}+\theta\sum_{m=1}^{M}p_t\left(m\right)\tilde{c}_{t-1}\left(m\right)+E_tq_{t,t+1}v_{j,t+1}=v_{j,t}+l_{j,t}-\tau_{j,t}+\psi_{j,t}. \tag{C1.10}$$

Eliminating ρ_t by combining (C1.7) and (C1.9), we obtain the individual Euler equation:

$$\beta\frac{\left(x_{j,t}-d\left(l_{j,t}\right)\right)}{x_{j,t+1}-d\left(l_{j,t+1}\right)}=q_{t,t+1}. \tag{C1.11}$$

Then we combine (C1.7) and (C1.8) to get the labor supply function:

$$d_l\left(l_{j,t}\right)=w_t \tag{C1.12}$$

Let us call $a_{j,t}(\equiv x_{j,t}-d\left(l_{j,t}\right))$ the "adjusted consumption" of agent j, (C1.11) is rewritten:

$$a_{j,t}=\beta^{-1}q_{t,t+1}a_{j,t+1}. \tag{C1.13}$$

We remember that the individual "adjusted" consumption is defined by:

$$a_{j,t}=\left(1-\beta\right)\left(v_{j,t}+h_t-\chi_t\right). \tag{C1.14}$$

Iterating the equation (C1.14) once:

$$a_{j,t+1}=\left(1-\beta\right)\left(v_{j,t+1}+h_{j,t+1}-\chi_{t+1}\right) \tag{C1.15}$$

then introducing

$$a_{j,t}=\beta^{-1}q_{t,t+1}a_{j,t+1},$$

into (C1.15) leads to:

$$a_{j,t}=\left(1-\beta\right)\beta^{-1}q_{t,t+1}\left(v_{j,t+1}+h_{j,t+1}-\chi_{t+1}\right).$$

Now, aggregating this last equation, and using the fact that $h_{j,t+1}$ is age independent, yields:

$$a_t = (1-\beta)\,\beta^{-1} q_{t,t+1} \left((1+n)\, v_{t+1} + h_{t+1} - \chi_{t+1}\right). \tag{C1.16}$$

In addition, aggregating (C1.15) yields:

$$a_{t+1} = (1-\beta)\left(v_{t+1} + h_{t+1} - \chi_{t+1}\right). \tag{C1.17}$$

Finally, we obtain the aggregate Euler equation

$$a_t = \beta^{-1} q_{t,t+1} a_{t+1} + n\left(\beta^{-1} - 1\right) q_{t,t+1} v_{t+1},$$

by combining (C1.16) and (C1.17).

C.2 Optimality Conditions For The Firm

The Lagrangian function corresponding to the firm's problem is:

$$
\begin{aligned}
& E_t \sum_{s=t}^{T} q_{t,s} \left(p_s \left(m \right) y_s \left(m \right) - w_s y_s \left(m \right) \right) \\
& - \lambda_s \left(m \right) \left(y_s \left(m \right) - \left(p_s \left(m \right) \right)^{-\varepsilon} \frac{N_s}{M_s} x_s - \theta \left(1+n \right) y_{s-1} \left(m \right) \right).
\end{aligned}
$$

The first order conditions of this problem for y_t, p_t, and λ_t are:

$$
p_t \left(m \right) - w_t - \lambda_t \left(m \right) + \theta \left(1+n \right) E_t q_{t,t+1} \lambda_{t+1} \left(m \right) = 0, \tag{C2.1}
$$

$$
y_t \left(m \right) = \varepsilon \lambda_t \left(m \right) \frac{N_t}{M_t} x_t \left(p_t \left(m \right) \right)^{-\varepsilon-1}, \tag{C2.2}
$$

$$
y_t \left(m \right) = \left(p_t \left(m \right) \right)^{-\varepsilon} \frac{N_t}{M_t} x_t + \theta \left(1+n \right) y_{t-1} \left(m \right), \tag{C2.3}
$$

Let

$$
\eta_t \left(m \right) \equiv \frac{p_t \left(m \right) - w_t \left(m \right)}{p_t \left(m \right)} \tag{C2.4}
$$

denote the relative markup charged by firm m. Let us define ϵ_t as the elasticity of demand:

$$
\epsilon_t \equiv \varepsilon \left(1 - \theta \left(1+n \right) \frac{y_{t-1} \left(m \right)}{y_t \left(m \right)} \right).
$$

Equation (C2.2) becomes:

$$
\lambda_t \left(m \right) = \frac{p_t \left(m \right)}{\epsilon_t \left(m \right)}
$$

and equation (C2.1) becomes:

$$
\epsilon_t \left(m \right)^{-1} = \eta_t \left(m \right) + \theta \left(1+n \right) E_t q_{t,t+1} \epsilon_{t+1} \left(m \right)^{-1}. \tag{C2.5}
$$

C.3 Steady State Equilibrium

The aim of this appendix is to prove the existence and the uniqueness of the steady state equilibrium. The steady state system consists of the following main equations:

$$\Re\left(y\right) = \beta^{-1} + \zeta\frac{\bar{b}}{a}, \tag{C3.1}$$

$$\Upsilon\left(R\right) = \left[\left(1 - \frac{1 - \frac{\tilde{\theta}}{R}}{\varepsilon\left(1 - \tilde{\theta}\right)}\right)\alpha^{-1}\right]^{\sigma}, \tag{C3.2}$$

with

$$a = \left(1 - \tilde{\theta}\right)y - \alpha\frac{y^{1+\frac{1}{\sigma}}}{1 + \frac{1}{\sigma}},$$

where $\tilde{\theta} = \phi\left(1 + n\right)$ and $\zeta = n\left(\beta^{-1} - 1\right)$. First, a must be positive, because otherwise preferences are undefined. This implies the following necessary condition

$$0 < y < \overline{y} \equiv \left[\frac{\left(1 - \tilde{\theta}\right)\left(1 + \frac{1}{\sigma}\right)}{\alpha}\right]^{\sigma}. \tag{C3.3}$$

Second, we notice from (C3.2) that y can not be less than $\underline{y}$, defined by:

$$\underline{y} \equiv \left[\frac{1 - \frac{\varepsilon^{-1}}{\left(1-\tilde{\theta}\right)}}{\alpha}\right]^{\sigma} = \lim_{R\to+\infty}\Upsilon\left(R\right). \tag{C3.4}$$

According to (C3.4), (C3.3) becomes:

$$\underline{y} < \overline{y}, \tag{C3.5}$$

that is

$$\Phi\left(\tilde{\theta}\right) \equiv d\tilde{\theta}^{2} + \left(1 - 2d\right)\tilde{\theta} + d + \varepsilon^{-1} - 1 > 0, \tag{C3.6}$$

with $d = 1 + \sigma^{-1}$.

First, $\Phi\left(\tilde{\theta}\right)$ is always positive for $0 < \sigma < 4\left(\varepsilon - 4\right)^{-1}$. In fact, the discriminant of $\Phi\left(\tilde{\theta}\right)$ is negative, implying the positivity of $\Phi\left(\tilde{\theta}\right)$, since $\Phi\left(0\right) = \sigma^{-1} + \varepsilon^{-1} > 0$.

Second, for $\sigma > 4\left(\varepsilon - 4\right)^{-1}$, the discriminant of $\Phi\left(\tilde{\theta}\right)$ is positive. $\Phi\left(\tilde{\theta}\right)$ is positive only for $\tilde{\theta} \in \left[0, \tilde{\theta}_1\right) \cup \left(\tilde{\theta}_2, 1\right]$, where $\tilde{\theta}_1$, and $\tilde{\theta}_2$ denote the roots of $\Phi\left(\tilde{\theta}\right)$. i.e.

$$\tilde{\theta}_1, \tilde{\theta}_2 = \frac{2d-1}{2d} \mp \sqrt{\frac{\varepsilon - 4d}{4\varepsilon d^2}}. \tag{C3.7}$$

Now we have to check under which conditions the curves corresponding to (C3.1) and (C3.2), respectively, intersect in yRplane. So let us analyze $\Upsilon\left(\cdot\right)$ and $\Re\left(\cdot\right)$. We observe that the inverse of the function, $\Upsilon\left(\cdot\right)$, is strictly decreasing as its derivative is strictly negative in $\left(\underline{y}, +\infty\right)$. On the other hand, $\Re\left(y\right)$ is decreasing in $\left(\underline{y}, y_{\min}\right]$ and increasing in $\left[y_{\min}, \overline{y}\right)$. In fact, its derivative, i.e.

$$\Re_y\left(y\right) = \zeta\bar{b}\frac{y^{\frac{1}{\sigma}} - \left(1 - \tilde{\theta}\right)}{\left(\left(1 - \tilde{\theta}\right)y - \frac{y^{1+\frac{1}{\sigma}}}{1+\frac{1}{\sigma}}\right)^2}, \tag{C3.8}$$

vanishes for

$$y_{\min} = \left(1 - \tilde{\theta}\right)^{\sigma}$$

and is negative when $y < y_{\min}$, and positive when $y > y_{\min}$. Moreover, when y goes to zero $\Re\left(y\right)$ goes to infinity. In other words, $\Re\left(y\right)$ admits a vertical asymptote for $y = 0$. We deduce that, if condition (C3.5) is satisfied, it is sufficient that $\underline{y}$ is positive so that the two curves intersect once. In other words, the necessary and sufficient condition for the existence and the uniqueness of the steady state equilibrium is

$$0 < \underline{y} < \bar{y},$$

which can be rewritten as

$$\tilde{\theta} < \tilde{\theta}_{\max} \equiv \left(1 - \varepsilon^{-1}\right), \tag{C3.9}$$

for $\sigma < 4\left(\varepsilon - 4\right)^{-1}$,and

$$\tilde{\theta} \in \left[0, \tilde{\theta}_1\right) \cup \left(\tilde{\theta}_2, \tilde{\theta}_{\max}\right), \tag{C3.10}$$

for $\sigma > 4\left(\varepsilon - 4\right)^{-1}$.

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