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## Résumé

Cest dernières décennies, la plupart des pays européens ont connu un ralentissement de leur croissance économique ainsi qu'un taux de chômage élevé et persistant. Cette évolution, dite de long terme, a été accompagnée d'une série de fluctuations économiques de court terme. Dans ce contexte, cette thèse analyse le fonctionnement du marché du travail et son incidence sur la performance des économies développées. Plus précisément, nous analysons les effets de court et de long terme de certaines distorsions jugées représentatives du marché du travail des pays européens, tels que la fiscalité, les systèmes d'indemnisation du chômage et les mécanismes de fixation du salaire.

Le premier chapitre présente le modèle canonique de cycle réel dans un contexte international. Il s'agit de déterminer un ensemble d'hypothèses visant à pallier aux défaillances du modèle original dans l'explication des fluctuations du marché du travail. L'incorporation de ces hypothèses dans ce cadre théorique fait l'objet de la première partie du chapitre 2. Même si ces amendements du cadre canonique conduisent à une meilleure compréhension des déterminants des fluctuations économiques et de leur synchronisation entre pays, les faits concernant la dynamique des heures et du salaire ne sont pas expliqués. Ceci justifie le développement d'une modélisation alternative du marché du travail, présenté dans la deuxième partie de ce chapitre. Au centre de ce modèle prennent place le chômage et les liens économiques entre pays.

Ce cadre est étendu au chapitre 3 pour intégrer la fiscalité, ce qui nous permet de rendre compte de la plupart des faits de court terme. Finalement, les chapitres 4 et 5 s'intéressent à la problématique liée à la croissance économique ainsi qu'à l'évolution tendancielle du temps du travail d'équilibre. En tenant compte des rigidités présentes sur le marché du travail, nous fournissons une explication des phénomènes de long terme.

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### Introduction

Last decades, continental European countries have experienced high and persistent unemployment, and a slowdown of economic growth. In parallel, aggregate hours of market work exhibit dramatic differences across industrialized countries, whereas the aggregate hours worked have decrease relative to the United States. Moreover, this evolution has been accompanied by recurrent fluctuations in the economies' incomes, products, and factor inputs, especially labor, that are due to nonmonetary sources. Against this background, this dissertation tries to gain insight on the identification of the key factors that shape the short-run and the long-run evolution of the labor market of the industrialized economies.

To this goal, two parts are distinguished. Part I focuses on the short run issues and is divided in three chapters. Chapter 1 sets up the basis of our study on international fluctuations and the labor market. We expose there the properties of an international general equilibrium model in which all markets are assumed to be walrasian and fluctuations are solely driven by stochastic technological impulsions. The detailed analysis of this framework, that we regard as the canonical international real business cycle (thereafter, IRBC) model, let us identify its limits and is essential to appreciate the empirical relevance of each new hypothesis incorporated along the subsequent chapters.

This chapter is as well a methodological one. We present the standard solution method, and we conduct sensitivity analysis to get a better understanding of the basic mechanisms at work. This let us assess the role played by two key parameters: the first one related to the adjustment costs of capital, and the second one to the elasticity of labor. As well, at each time we compare the implications from two specifications of the agents' preferences. The first one assumes a standard separability between consumption and leisure, whereas the second one assumes a non-separability between them. The canonical IRBC model developed in chapter 1 appears to be insufficient to account for most of the international features of business cycles. Moreover, it has

the same limitations as its closed-economy counterpart regarding the dynamics of the real wage, the labor productivity and the total hours.

According to these results, chapter 2 exposes a survey of several standard amendments intended to improve the predictions of the model. This survey is restricted to issues that are directly related to the real economy. The first extension aims to deep the link between the home and the foreign countries. To this end we introduce an additional consumption/investment good by considering national specialization (Backus, Kehoe, and Kydland 1994). This richer structure adds a new mechanism by which the expansion of output experienced in the country receiving the shock, may induce an expansion of output in the other country. This potentially allows for positive cross-correlations for labor inputs and investments. Even if this ameliorates the theoretical predictions relative to the international facts, the model is far to be sufficient. Following Galí (1994), we also distinguish the composite good for consumption from the composite good for investment. However, this does not change the predictions of the model since we allow for perfect competitive markets.

The second extension aims to reduce the international correlation of consumptions. This is done by restricting international trade to non-contingent bonds (Baxter 1995). This limitation in the agent's ability to risk pooling country-specific shocks produces more realistic international correlations of outputs and consumptions. However, the correlation of outputs still larger than the one of consumptions.

The last extension in the pure walrasian framework that we consider is the introduction of a realistic potential for intra- and international capital flows by the disaggregation of the economy into internationally traded and non-traded sectors (Stockman and Tesar 1995). This is justified by the empirical evidence that roughly a half of the typical G-10 country's output consists of non-traded goods and services. It must be enhanced that, conversely to traditional IRBC models with only technological shocks, as the Stockman and Tesar (1995)'s model, our model predicts positive international comovements of production inputs, which is more in accordance with the empirical correlations, but they are overstated particularly when shocks are highly persistent. Nevertheless, because at this point we have not yet modified the walrasian nature of the labor market, all models still fail in reproducing the fluctuations of the employment, the hours worked and the real wage. Hence, the next step is to modify the walrasian labor market by introducing search and matching in the labor market. This is the core of the second part of this survey, in which we take as starting point the Hairault (2002)'s two-country, two-good search economy

to going ahead in the study of some stylized facts of the US labor market. Next, we make a reduction to the single-good case to assess the role of the key hypotheses in the Hairault's economy. Namely, (i) the non-separability between consumption and leisure in the agents' preferences, (ii) the existence of two goods in the world and so of one relative price, and (iii) search and bargaining in the labor market. In this single-good search framework we also evaluate the predictions from the model with restricted international trade to non-contingent bonds. However, we do not extend the search model to include two sectors because the results from the walrasian economy are discouraging.

There we show that in the single-good economy, the combination of search and matching in the labor market with the non-separability is enough to predict positive comovements of labor inputs and investments as well as a large dissociation of consumptions. Moreover, the procyclicality of real wage rate is reduced, and the correlation of total hours with both output and labor productivity is lower. Then, the three puzzles are partially solved. However, consumptions correlation still larger than outputs correlation, even if the incompleteness of financial markets produces more realistic international correlations of outputs and consumptions. Then, we show that the gain from including two goods in that framework is that the model is able now to replicate a correlation of outputs bigger than the one of consumptions (Hairault 2002). However, the price dynamics provoked by a positive productivity shock decrease the agent's purchasing power, leading to a stronger vindication of salary and so to a slightly more procyclical real wage. To sum up the first two chapters, we can say that, relative to the data, in the walrasian extensions the variability of consumption, hours of work, and output is too low, and the variability of investment is too high. But maybe the main failure is the predicted correlation of real wages with both hours worked and output. In such a models, variations in technology shifts the labor demand curve but not the labor supply curve, thus inducing a strong positive correlation between wages and hours. The introduction of search and matching in the labor market (Andolfatto 1996) outperforms the model predictions. But the volatility of total hours still underestimated, and the real wage still procyclical.

This line of reasoning naturally suggests that to improve the predictions from the real business cycle models one must include something that shifts labor supply. If both labor demand and labor supply shift, then the strong positive correlation between hours and wages can probably be reduced. So, in chapter 3 we study the short run effects of fiscal policy in a search framework. In the Keynesian tradition, fiscal policy, and therefore taxation, is one of the main instruments

to stabilize the economy. However, in the 1990s, several pioneering works considered taxation as a source of business cycle fluctuations (Christiano and Eichenbaum (1992), Braun (1994), McGrattan (1994), ?)). This feeds the criticisms about the possibility to use taxes as stabilization tool. These pioneering articles have shown that stochastic fiscal policy improves the performance of real business cycle models. Intuitively, shocks to income and payroll taxes can be interpreted as shocks to labor supply, as opposed to technology shocks which may be interpreted as shocks to labor demand. Thus, tax rates provide another mechanism for explaining the observed correlation between hours and wages.

In quantitative terms, these former works yield to predictions for the correlation between hours and real wages, as measured by average productivity, closer to the empirical correlation. Likewise, the predicted variability of hours worked and consumption are much closer to their empirical values when fiscal policy is included (even if in general the relative volatility of aggregate hours is overstated). Nevertheless, these former papers show two drawbacks. The first one is that all of them consider a closed economy, so that the possible variability in the macro aggregates passing through the international trade is not accounted for. The second one is that the theoretical real wage is measured by the average productivity. This obviously prevents from analyzing other features of the US labor market, such as the lower volatility of the real wage with respect to the volatility of the labor productivity.

By contrast, in chapter 3 we show that fluctuations in distortive taxes can account for some of the puzzling features of the U.S. business cycle. Namely, the observed real wage rigidity, the international comovement of investment and labor inputs, and the so-called consumption correlation puzzle (according to which cross-country correlations of output are higher than the one of consumption). This is done in a two-country search and matching model with fairly standard preferences, extended to include a tax/benefit system. In this simple framework, the tax side is represented by taxation on labor income, employment (payroll tax) and consumption, whereas the benefit side is resumed by the unemployment benefits and the worker's bargaining power.

Then, the main departures from the former literature on taxation as a source of business cycle fluctuations are twofold. First, we consider a two-country general equilibrium model, so that we are able to discuss the effects on the observed international fluctuations. Second, we assume search and matching in the labor market. Our model is close to the Hairault (2002)'s one, who develops a two-country, two-good search model, able to explain the puzzling facts of international

fluctuations once a non-separability in the agents' preferences is considered. Our model is also close to the Chéron and Langot (2004)'s model, who explain the real wage rigidity in a closed-economy search model by means of a particular set of non-separable preferences.

Either in the Hairault (2002)'s paper or in the Chéron and Langot (2004)'s paper, the non-separability of preferences plays a main role. However, this hypothesis is unable to simultaneously account for the real wage rigidity and for the observed international fluctuations. Conversely, we show that all those facts can be accounted in a single framework with fairly standard separable preferences. These new results concerning business cycle theory provide support to the matching models.

Part II is concerned with the long term issues and is composed of two chapters. In chapter 4 we investigate the issue of the long run link between growth and unemployment at two levels. First, we conduct an empirical analysis to explore the heterogeneity of growth and unemployment experiences across 183 European regions from 1980 to 2003 and we evaluate how much of this heterogeneity is accounted by the national labor market institutions. One originality of this approach is to take into account the large heterogeneity between regions among a country. Second, we construct a theoretical economy to assess the explicative role of labor-market variables on the bad performance of European countries. The main hypotheses of our model are the following: (i) Innovations are the engine of growth. This implies a "creative destruction" process generating jobs reallocation. (ii) Agents have the choice of being employed or being trying their hand at R&D; and (iii) Unemployment is caused both by the wage-setting behavior of unions, and by the labor costs associated to the tax/benefit system.<sup>1</sup>

The advises from the empirical exercise are that: (i) The tax wedge and the unemployment benefits are positively correlated with the regional unemployment rates. Conversely, the employment protection and the level of coordination in the wage bargaining process are negatively correlated with the regional unemployment rates. (ii) The tax wedge and the unemployment benefits are negatively correlated with the regional growth rates of the Gross Domestic Product (GDP) per capita. Conversely, more coordination in the wage bargaining process diminishes the regional growth rates of GDP per capita. This last result points to the existence of an arbitration between unemployment and growth, if we focus on the impact of coordination in the wage bargaining process. These results are in accordance with the country-level results of Daveri and Tabellini (2000).

<sup>&</sup>lt;sup>1</sup>The two first hypotheses are the same as those of Aghion and Howitt (1994).

On the other side, the implications of the theoretical model are the following: (i) The bargaining power of unions, the unemployment compensation, the taxes on labor and the employment protection have a positive effect on unemployment and a negative effect on the economic growth. (ii) A more coordinated bargaining process increases employment, at the price of a lower economic growth. The first result clearly contrast with the results of Lingens (2003) and Mortensen (2005). Lingens (2003) treats the impact of unions in a model with two kind of skills, and shows that the bargain over the low-skilled labor wage causes unemployment but the growth effect is ambiguous. Similarly, in a matching model of schumpeterian growth, Mortensen (2005) finds a negative effect of labor market policy on unemployment, but an ambiguous effect on growth. Finally, chapter 5 studies the dynamics of aggregate hours of market work, which exhibit dramatic differences across industrialized countries, either at points of time across countries, or within a country over time. In the current literature, there are two candidate approaches allowing to explain these differences. A first set of contributions focuses on the decline of the average hours worked per employee (the intensive margin) in European countries since 1960. Prescott (2004) studies the role of taxes in accounting for differences in labor supply across time and across countries. He finds that the effective marginal tax rate on labor income explains most of the differences at points of time and the large change in relative (to US) labor supply over time. On this line of research, Rogerson (2006) shows that the aggregate hours worked in Continental European countries such as Belgium, France, Germany and Italy are roughly one third less than in the US. This fact results from a diverging process in the hours worked per employee in each zone: between 1960 and 1980, whereas in Europe we observe a large decrease, in the US this decline is very small; and after 1980, we observe in the two zones a stable number of hours worked per employee. This evolution of the hours worked per employee is strongly correlated to the dynamics of the taxes. Hence, as it is suggested by Prescott (2004), Rogerson (2006) or Ohanian, Raffo, and Rogerson (2006), a theory providing a link between the hours worked per employee and taxes seems to be sufficient to explain why Europeans work less than Americans. However, since 1980 a notable feature of the data is that differences across countries in aggregate hours are due to quantitatively important differences along the extensive margin. Hence, a second set of contributions (see e.g. Jackman, Layard, and Nickell (1991), Mortensen and Pissarides (1999a), Blanchard and Wolfers (2000) or Ljungqvist and Sargent (2007b)) considers that the large decrease of the employment rate observed after 1980 in the European countries, is an important factor of the dynamics of total hours. These works show that different labor market

institutions lead to different labor market outcomes after a common shock. In these previous papers, there is fairly robust evidence that (i) the level and duration of unemployment benefits and (ii) the union's bargaining power have a significant positive impact on unemployment.

To sum up, the main factors explaining the decline in the hours worked per employee differ from those explaining the decline in the employment rate: the taxes for the former, and the labor market institutions, such as the unions' power or the unemployment benefits, for the second. Clearly, all together contribute to the dynamics of the two margins of the total hours.

From a theoretical point of view, the aim of this chapter is to provide a theory allowing to account for the impact, of both taxes and labor market institutions, on the two margins of the aggregate hours worked. To this end, we follow the empirical methodology presented in Ohanian, Raffo, and Rogerson (2006): the quantitative evaluation of the several models and the impact of distortions is based on the computation of series for the gap between the marginal cost and the marginal return of labor that is produced using actual data and model restrictions<sup>2</sup>. Furthermore, we extend the theoretical investigation: beyond the usual neo-classical growth model which allows to predict the hours worked per employee, we explore the ability of the Hansen (1985)-Rogerson (1988) model to reproduce the dynamics of the employment rate. Finally, we develop a general equilibrium matching model, close to the one proposed by Andolfatto (1996), Fève and Langot (1996) and Chéron and Langot (2004), allowing to explain the dynamics of both the hours worked per employee and the employment rate. This last model is rich enough to allow the evaluation of the relative contribution of the tax/benefit systems and unions in the explanation of the observed allocation of time.

The main findings of last chapter are the following. First, the long-run decline in the hours worked per employee is mainly due to the increase of the taxes, as it is suggested by Prescott (2004), Rogerson (2006) and Ohanian, Raffo, and Rogerson (2006). Second, the employment rate is affected by institutional aspects of the labor market, such as the bargaining power and the unemployment benefits, rather than by taxes, conversely to the individual work effort. Finally, this behavior of the two margins of the aggregate hours is well accounted by our search model, when it includes the observed heterogeneity of the tax/benefit systems and the labor market indicators of the wage-setting process across countries. These findings give some support to the two explanations of the European decline in total hours: the important role of taxes through the intensive margin and the large contribution of the labor market institutions through the

<sup>&</sup>lt;sup>2</sup>The closer these gaps are to zero, the better the model accounts for the observed labor behavior.

extensive margin. Because these findings come from an unified framework, they also give a strong support to the matching models.

# Chapter 1

The canonical international real business cycle model

#### Introduction

This chapter is attempted to set up the basis of our study on international fluctuations and the labor market. To this end, we expose the properties of an international general equilibrium model in which all markets are assumed to be walrasian and fluctuations are solely driven by stochastic technological impulsions. The detailed analysis of this framework, that we regard as the canonical international real business cycle (thereafter, IRBC) model, let us identify its limits and is essential to appreciate the empirical relevance of each new hypothesis incorporated along the subsequent chapters.

This chapter is as well a methodological one. We present the standard solution method, and we conduct several sensitivity analysis to get a better understanding of the basic mechanisms at work. This let us assess the role played by two key parameters: the first one related to the adjustment costs of capital, and the second one to the elasticity of labor. Moreover, at each time we compare the results obtained from two specifications of the agents' preferences. The first one assumes a standard separability between consumption and leisure, whereas the second one assumes a non-separability between them.

Since there is a single good, international trade takes place only to smooth consumption and to ensure that capital is allocated in the most productive country. We show that, regarding the international context, the canonical IRBC model is able to reproduce two characteristics of developed economies:

- The net exports and the trade balance (measured as the ratio of net exports to output) are counter-cyclical.
- Saving and investment rates are highly correlated.

However, the model is unable to replicate two major facts of developed economies:

- Interdependency (Baxter 1995): the cross correlations for production, consumption, investment and labor input are positive across countries.
- (Backus, Kehoe, and Kydland 1995) The cross-country correlation of outputs is larger than the one of consumptions.

Moreover, regarding within country business-cycle facts, the striking limits of the model concern, as its close-economy counterpart, the labor market fluctuations:

- The dynamics of the hours worked is not reproduced by the model.
- The real wage is highly pro-cyclical in the model, conversely to the data.

In addition, due to the single-good nature of the canonical model, the facts involving international prices, such as the terms of trade or the real exchange rate, are obviously left unexplained.

#### 1.1 The model

The world economy consists of two countries (country 1 or home country and country 2 or foreign country), each represented by a large number of identical consumers and a production technology. The countries produce the same final good, which is used for consumption and investment purposes, and their preferences and technologies have the same structure and parameter values. Although, the technologies differ in two important aspects: in each country, the labor input consists only of domestic labor, and production is subjected to idiosyncratic shocks to productivity. Markets are complete: agents may trade any contingent claims they wish. Since there is a single good, international trade takes place only to smooth consumption and to ensure that capital is allocated in the most productive country.

#### 1.1.1 The representative Firm

The representative firm in country i = 1, 2 produce the single good with a constant returns to scale technology using capital  $K_{i,t}$  and labor  $H_{i,t}$  as inputs<sup>2</sup>,

$$Y_{i,t} = a_{i,t} K_{i,t}^{\alpha} H_{i,t}^{1-\alpha} \tag{1.1}$$

The variables  $a_{i,t}$  represent the stochastic component of the productivity variable and are assumed to follow the stationary vector-autoregressive process given by

$$\begin{bmatrix}
\log a_{1,t} \\
\log a_{2,t}
\end{bmatrix} = \begin{bmatrix}
\rho_{a,1} & \rho_{12}^{a} \\
\rho_{12}^{a} & \rho_{a2}
\end{bmatrix} \begin{bmatrix}
\log a_{1,t-1} \\
\log a_{2,t-1}
\end{bmatrix} + \begin{bmatrix}
1 - \rho_{a1} & -\rho_{12}^{a} \\
-\rho_{12}^{a} & 1 - \rho_{a2}
\end{bmatrix} \begin{bmatrix}
\log a_{1} \\
\log a_{2}
\end{bmatrix} + \begin{bmatrix}
1 & \psi \\
\psi & 1
\end{bmatrix} \begin{bmatrix}
\varepsilon_{1,t} \\
\varepsilon_{2,t}
\end{bmatrix} (1.2)$$

were the innovations  $\epsilon = [\epsilon_1, \epsilon_2]'$  are serially independent:  $E(\varepsilon_1) = E(\varepsilon_2) = 0$ ,  $E(\varepsilon_1^2) = \sigma_{\varepsilon_1}^2$ ,  $E(\varepsilon_2^2) = \sigma_{\varepsilon_2}^2$ , and  $E(\varepsilon_1 \varepsilon_2) = 0$  for all t. Under this specification, innovations to productivity that originate in one country ( $\varepsilon_1$  or  $\varepsilon_2$ ) are transmitted to the other country if the "spill-over"

<sup>&</sup>lt;sup>1</sup>This model is very close to the Baxter and Crucini (1993)'s model.

<sup>&</sup>lt;sup>2</sup>To simplify, we abstract from the deterministic growth rate.

parameters,  $\rho_{12}^a$ ,  $\rho_{21}^a$  and  $\psi$  are different from zero. Because of the symmetry assumption we impose  $\rho_{a1} = \rho_{a2}$  and  $\rho_{12}^a = \rho_{21}^a$ .

New capital goods are internationally mobile and all investment is subject to adjustment costs. Capital adjustment costs have been incorporated to slowdown the response of investment to location-specific shocks, due to the strong incentive of capital owners to locate new investment in the most productive place. Since we are interested on the dynamics of the model near to the steady state, we do not need to specify a particular functional form for adjustment costs. However, to simplify the computations we suppose the following quadratic expression for the adjustment costs:

$$C_{i,t} = \frac{\phi}{2} (K_{i,t+1} - K_{i,t})^2 \tag{1.3}$$

Capital accumulates over time according to

$$K_{i,t+1} = (1 - \delta)K_{i,t} + I_{i,t} \tag{1.4}$$

Then, the Firms' program is dynamic and consists of maximizing the expected discounted sum of profit flows, contingent to the state  $A_{t+1}$ ,

$$\max_{H_{i,t},I_{i,t}} E_0 \sum_{t=0}^{\infty} \int v_t (Y_{i,t} - \mathcal{C}_{i,t} - I_{i,t} - w_{i,t} H_{i,t}) dA_{t+1}$$
(1.5)

subject to the production constraint (1.1) and to the capital constraint (1.4).  $v_t = v(A_{t+1})$  is the factor actualization of the Firm and  $w_i$  the wage rate in country i. This program can be written in recursive form and the solution satisfies the Bellman's equation,

$$\mathcal{W}(K_{i,t}) = \max_{H_{i,t}, K_{i,t+1}} \left\{ Y_{i,t} - \mathcal{C}_{i,t} - K_{i,t+1} + (1-\delta)K_{i,t} - w_{i,t}H_{i,t} + \int v_t \mathcal{W}(K_{i,t+1}) dA_{t+1} \right\}$$
(1.6)

The optimal demands for labor and capital are

$$w_{i,t} = (1 - \alpha) \frac{Y_{i,t}}{H_{i,t}} \tag{1.7}$$

$$q_{i,t} = \int v_t \frac{\partial \mathcal{W}(K_{i,t+1})}{\partial K_{i,t+1}} dA_{t+1}$$
(1.8)

with  $q_{i,t}$  defined by

$$q_{i,t} \equiv 1 + \phi(I_{i,t} - \delta K_{i,t}) \tag{1.9}$$

Using the envelop condition for the state variable,  $\frac{\partial W(K_{i,t})}{\partial K_{i,t}} = \alpha \frac{Y_{i,t}}{K_{i,t}} + q_{i,t} - \delta$ , equation (1.8) becomes

$$q_{i,t} = \int v_t \left( \alpha \frac{Y_{i,t+1}}{K_{i,t+1}} + q_{i,t+1} - \delta \right) dA_{t+1}$$
(1.10)

Finally, we impose the following transversality condition.

$$\lim_{j \to \infty} E_t[q_{i,t+j+1} K_{t+j+1}] = 0 \tag{1.11}$$

#### 1.1.2 The representative household

As in the canonical model for a closed economy, the dynamics of the model rely on savings and on the labor supply behavior following a technological shock. The labor supply behavior depends in turn on the household's preferences:

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_{i,t}, 1 - H_{i,t})$$
(1.12)

where  $U(C_{i,t}, L_{i,t}) \equiv U_{i,t}$  denotes the instantaneous utility function.  $C_{i,t}$  stands for the household's consumption, whereas  $L_{i,t} = 1 - H_{i,t}$  stands for the amount of leisure enjoyed at period t.<sup>3</sup>

Financial markets are complete. At each date t households have access to contingent claims, at price  $v_t = v(A_{t+1})$ , providing one unit of the single good if the state A occurs at t+1. We denote by  $f(A) \equiv f(A_{t+1}, A_t)$  the density function describing the evolution from the state  $A_t$  to the state  $A_{t+1}$ .

So, given the wage rate proposed by the Firm,  $w_{i,t}$ , the representative household's aims at choosing a contingency plan  $\{C_{i,t}, H_{i,t}\}$  that maximizes (1.12) subject to the budget constraint

$$C_{i,t} + \int v_t B_i(A_{t+1}) dA_{t+1} \le B_i A_t + w_{i,t} H_{i,t} \qquad (\lambda_{i,t})$$
(1.13)

were  $B_{i,t} \equiv B_i(A_t)$  denotes the household's portfolio of contingent bonds, and  $\lambda_{i,t}$  the shadow price associated to the budget constraint.

The households' program can be written in a recursive way and its optimal solution verifies the Bellman equation

$$\mathcal{V}(B_{i,t}) = \max_{C_{i,t}, H_{i,t}, B_{i,t+1}} \left\{ U_{i,t} + \beta \int \mathcal{V}(B_{i,t+1}) f(A) dA_{t+1} \right\}$$
(1.14)

subject to the budget constraint (1.13). The optimality conditions are,

$$\frac{\partial U_{i,t}}{\partial C_{i,t}} = \lambda_{i,t} \tag{1.15}$$

$$\frac{\partial U_{i,t}}{\partial H_{i,t}} = \lambda_{i,t} w_{i,t} \tag{1.16}$$

 $<sup>^{3}</sup>$ The function U is assumed to be strictly increasing, concave, twice continuously differentiable and to satisfy the Inada conditions. Moreover, C and L are assumed to be normal goods in order to guarantee the existence of a saddle point at the general equilibrium.

Then, the household's labor supply is such that the marginal utility of leisure is equal to the wage, expressed in utility terms (since  $\lambda_{i,t}$  is equal to the marginal utility of consumption), i.e., equal to the marginal value of one hour worked. The optimal choice of contingent bonds determines the interest rate on the international financial market:

$$\beta \frac{\partial \mathcal{V}(B_{i,t+1})}{\partial B_{i,t+1}} = v_t \lambda_{i,t} \tag{1.17}$$

Using the envelop condition for  $B_{i,t}$ :  $\frac{\partial \mathcal{V}(B_{i,t})}{\partial B_{i,t}} = \lambda_{i,t}$ , equation (1.17) can be written as

$$v_t = \beta \frac{\lambda_{i,t+1}}{\lambda_{i,t}} f(A_{t+1}) \tag{1.18}$$

Lastly, the transversality condition ensures that the marginal value of bonds holdings, in utility units, is null at the end of the household's life:

$$\lim_{j \to \infty} E_t[\beta^{t+j} \lambda_{i,t+j} B_{i,t+j+1}] = 0 \tag{1.19}$$

#### 1.1.3 General Equilibrium

The equilibrium of this economy consists of a set of households' optimal decision rules  $\{C_i(\cdot), H_i^s(\cdot), B_i(\cdot)\}$ , the firms' optimal demands of capital and labor  $\{K_i^d(\cdot), H_i^d(\cdot)\}$  and a vector of prices equilibrating the goods market, the labor market and the financial market.

Goods Market: The world constraint for the single good of this economy satisfies:

$$Y_{1,t} + Y_{2,t} = C_{1,t} + C_{2,t} + I_{1,t} + I_{2,t} + C_{1,t} + C_{2,t}$$
(1.20)

**Labor Market:** Together, equations (1.15) and (3.24) determine the instantaneous rate of substitution between leisure and consumption as a function of the real wage,

$$\frac{\frac{\partial U_{i,t}}{\partial H_{i,t}}}{\frac{\partial U_{i,t}}{\partial C_{i,t}}} = w_{i,t} \tag{1.21}$$

Thus, the wage rate corresponds to the marginal gain of leisure expressed in consumption units. Notice that, as consumption and leisure are normal goods, both vary in the same way for a given wage, which is in stark contradiction with data.

Moreover, the equilibrium in the labor market implies that labor is remunerated at its marginal productivity,

$$(1 - \alpha) \frac{Y_{i,t}}{H_{i,t}} = w_{i,t} \tag{1.22}$$

Equation (1.17) implies that  $\frac{\lambda_{1,t+1}}{\lambda_{1,t}} = \frac{\lambda_{2,t+1}}{\lambda_{2,t}} = \Lambda \Leftrightarrow \frac{\lambda_{2,t}}{\lambda_{1,t}} = \frac{\lambda_{2,t+1}}{\lambda_{1,t+1}} = \Lambda' \Leftrightarrow$ Financial Market:  $\lambda_{2,t} = \Lambda' \lambda_{1,t}$ . If we suppose that the initial wealth is the same for each individual (i.e.  $\Lambda' = 1$ ) then  $\lambda_{2,t} = \lambda_{1,t} \equiv \lambda_t$ . Then, we can rewrite the evolution of the firm's implicit price (equation (1.9)) as

$$q_{i,t} = \beta \int \frac{\lambda_{t+1}}{\lambda_t} \left( \alpha \frac{Y_{i,t+1}}{K_{i,t+1}} + q_{i,t+1} - \delta \right) f(A_{t+1}) dA_{t+1} \equiv \beta E_t \left[ \frac{\lambda_{t+1}}{\lambda_t} \left( \alpha \frac{Y_{i,t+1}}{K_{i,t+1}} + q_{i,t+1} - \delta \right) \right]$$
(1.23)

#### **Empirical results** 1.2

First of all, we have to specify the utility function. Throughout this thesis, we will consider two quite standard functions: a separable utility between consumption and leisure:

$$U_{i,t} = \log(C_{i,t}) + \sigma \frac{(1 - H_{i,t})^{1-\eta}}{1 - \eta}, \quad \eta > 0,$$
(1.24)

and a non-separable utility:

$$U_{i,t} = \log\left(C_{i,t} + \tilde{\sigma} \frac{(1 - H_{i,t})^{1-\eta}}{1 - \eta}\right), \quad \eta > 0$$
(1.25)

In the first case, the household's optimal choices take the form

$$\frac{1}{C_{i,t}} = \lambda_t \tag{1.26}$$

$$\frac{1}{C_{i,t}} = \lambda_t$$

$$C_{i,t}\sigma(1 - H_{i,t})^{-\eta} = w_{i,t}$$
(1.26)

whereas in the second case,

$$\frac{1}{C_{i,t} + \tilde{\sigma} \frac{(1 - H_{i,t})^{1 - \eta}}{1 - \eta}} = \lambda_t \tag{1.28}$$

$$\tilde{\sigma}(1 - H_{i,t})^{-\eta} = w_{i,t} \tag{1.29}$$

The theoretical implications of this utility specifications will be analyzed below. However, at this stage we remark that, at equilibrium, the separability of preferences implies  $C_{1,t} = C_{2,t} \ \forall t$ (see equations (1.26)). Conversely, the equalization of consumption across countries does not longer hold with non-separable preferences (see equations (1.28)).

#### 1.2.1Solution and simulation of the model

The resolution strategy is to approximate the optimality and equilibrium conditions linearly around the steady state and to solve the resulting dynamic system. The approximate solution can then be written in the state-space form as

$$\begin{pmatrix}
\hat{K}_{1,t+1} \\
\hat{K}_{2,t+1} \\
\hat{a}_{1,t+1} \\
\hat{a}_{2,t+1}
\end{pmatrix} = \underbrace{\begin{pmatrix}
\mu_{1} & \mu_{2} & \pi_{Ka}^{1} & \pi_{Ka}^{2} \\
\mu_{2} & \mu_{1} & \pi_{Ka}^{2} & \pi_{Ka}^{1} \\
0 & 0 & \rho_{a} & \rho_{12} \\
0 & 0 & \rho_{12} & \rho_{a}
\end{pmatrix}}_{M_{SS}} \begin{pmatrix}
\hat{K}_{1,t} \\
\hat{K}_{2,t} \\
\hat{a}_{1,t} \\
\hat{a}_{2,t}
\end{pmatrix} + \underbrace{\begin{pmatrix}
0 & 0 \\
0 & 0 \\
1 & \psi \\
\psi & 1
\end{pmatrix}}_{M_{SE}} (1.30)$$

and

$$\begin{pmatrix}
\hat{C}_{1,t} \\
\hat{C}_{2,t} \\
\hat{I}_{1,t} \\
\hat{I}_{2,t} \\
\hat{H}_{1,t} \\
\hat{H}_{2,t} \\
\hat{Y}_{1,t} \\
\hat{Y}_{2,t} \\
\hat{\lambda}_{t} \\
\hat{w}_{1,t} \\
\hat{w}_{2,t}
\end{pmatrix} = \begin{pmatrix}
\Pi_{C_{1}K_{1}} & \Pi_{C_{1}K_{2}} & \Pi_{C_{1}a_{1}} & \Pi_{C_{1}a_{2}} \\
\Pi_{C_{2}K_{1}} & \Pi_{C_{2}K_{2}} & \Pi_{C_{2}a_{1}} & \Pi_{C_{2}a_{2}} \\
\Pi_{I_{1}K_{1}} & \Pi_{I_{1}K_{2}} & \Pi_{I_{1}a_{1}} & \Pi_{I_{1}a_{2}} \\
\Pi_{H_{1}K_{1}} & \Pi_{H_{1}K_{2}} & \Pi_{H_{2}a_{1}} & \Pi_{H_{2}a_{2}} \\
\Pi_{H_{2}K_{1}} & \Pi_{H_{2}K_{2}} & \Pi_{H_{2}a_{1}} & \Pi_{H_{2}a_{2}} \\
\Pi_{Y_{1}K_{1}} & \Pi_{Y_{1}K_{2}} & \Pi_{Y_{1}a_{1}} & \Pi_{Y_{1}a_{2}} \\
\Pi_{Y_{2}K_{1}} & \Pi_{Y_{2}K_{2}} & \Pi_{Y_{2}a_{1}} & \Pi_{Y_{2}a_{2}} \\
\Pi_{X_{1}K_{1}} & \Pi_{X_{1}K_{2}} & \Pi_{X_{1}a_{1}} & \Pi_{X_{1}a_{2}} \\
\Pi_{W_{1}K_{1}} & \Pi_{W_{1}K_{2}} & \Pi_{W_{1}a_{1}} & \Pi_{W_{1}a_{2}} \\
\Pi_{W_{2}K_{1}} & \Pi_{W_{2}K_{2}} & \Pi_{W_{2}a_{1}} & \Pi_{W_{2}a_{2}}
\end{pmatrix}$$

$$(1.31)$$

The matrices  $M_{SS}$  and  $\Pi$  are composed of instantaneous elasticities, which are non-linear combinations of the structural parameters of the model. Then, for a given set of parameter values, we will be able to analyze the responses of the variables to an idiosyncratic technological shock, as well as to compute the cyclical properties of the model (*i.e.*, the second order moments). The Impulse Response Functions (IRF) are computed from the two last expressions.

#### 1.2.2 Qualitative Analysis

#### Steady State and calibration of the structural parameters

H is fixed to 1/3 and we calculate  $\eta$  such that the average individual labor supply elasticity is equal to  $\frac{1}{\eta}\frac{1-H}{H}=\frac{2}{3}\Rightarrow \eta=3$ , a value consistent with the bulk of empirical estimates. The Tobin's q is set equal to unity (Baxter and Crucini 1993).  $\phi$ , the capital adjustment cost parameter, is calibrated in order to replicate the volatility of investment in the economy with non-separable preferences and international transmission of the shock (IRBC1b-NSP). We keep it constant across the other simulations in order to isolate the intrinsic properties of the different models. We normalize a=1, then we can compute the steady-state values for the remaining variables

as

$$K = \left(\frac{1/\beta - 1 + \delta}{\alpha H^{1-\alpha}}\right)^{\frac{1}{\alpha - 1}}$$

$$Y = K^{\alpha}H^{1-\alpha}$$

$$I = \delta K$$

$$C = Y - I$$

$$w = (1 - \alpha)\frac{Y}{K}$$

If preferences are separable between consumption and leisure, then

$$\lambda = 1/C$$

$$\sigma = \frac{w}{C(1-H)^{-\eta}}$$

Else,

$$\tilde{\sigma} = \frac{w}{(1-H)^{-\eta}}$$

$$\lambda = \frac{1}{C + \tilde{\sigma} \frac{(1-H)^{1-\eta}}{1-\eta}}$$

Table 1.1: Benchmark calibration.

$\alpha$	β	Н	η	$\phi$	δ	$\rho_a$	$ ho_{12}^a$	$\psi$	$\sigma_{arepsilon^a}$	$\sigma$	$\tilde{\sigma}$
0.36	0.99	1/3	3	0.056	0.025	0.906	0	0	0.00852	0.7508	0.6804

#### Responses to a technological shock

Matter of clarity, we take as benchmark the simplest case in which a positive 1% productivity shock arrives to country 1, but it is no transmitted to country 2:  $\psi = \rho_{12}^a = 0$ . The values for  $\rho_a$  and  $\sigma_{\varepsilon^a}$  are taken from Backus, Kehoe, and Kydland (1994), while the remaining parameters come from Hairault (2002) (See Table 1). This corresponds to the framework analyzed by Devereux, Gregory, and Smith (1992). The IRF are shown in figure 1.2, for separable preferences (IRBC1a-SP), and in figure 1.4, for non-separable preferences (IRBC1a-NSP). The responses of the country 1 variables at impact are as follows:

The instantaneous response of output ( $\approx 1.28\%$ ) is due to the direct effect of the productivity shock and to the increase in labor, as can be seen from the log-linear expression of output:

$$\hat{Y}_{1,t} = \hat{a}_{1,t} + (1-\alpha)\hat{H}_{1,t} + \alpha\hat{K}_{1,t}$$
  
 $1.28\% \approx 1\% + 0.64(0.45\%) + 0.36(0\%)$ 

The positive response of labor is in turn the total outcome of several effects affecting simultaneously the labor demand and the labor supply, as is argued below. By log-linearizing the equilibrium condition (1.22), the labor demand in country 1 is expressed as

$$\hat{w}_{1,t} = \hat{a}_{1,t} + \alpha \hat{K}_{1,t} - \alpha \hat{H}_{1,t}$$

The arrival in country 1 of a positive innovation directly increases the marginal productivity of labor in that country. This encourages firms to increase their demand for labor. The labor-supply response is more complicate because the household's trade-offs also change at impact (equation (1.21)), so that her labor supply results from the combination of one instantaneous effect and two intertemporal effects. The *instantaneous substitution* effect corresponds to the substitution between current consumption and current leisure: the higher wages incentive the household to work more.

On the other hand, the intertemporal effects reflect the household's dynamic behavior, who faces a trade-off between current leisure and future leisure. This lead to two opposite phenomena: a wealth effect that induces the household to work less today<sup>4</sup>, and a substitution effect, related to the temporary nature of the technological shock, that motivates the household to work more today. It is rational to substitute current leisure, expensive in consumption terms for future leisure, with smaller opportunity cost (the current wage is high relative to expected future wages).

**Separable preferences.** With separable preferences, the log-linearization of the labor supply equilibrium condition gives:

$$\hat{C}_{i,t} + \eta \frac{H}{1 - H} \hat{H}_{i,t} = \hat{w}_{i,t}$$

Then, the instantaneous substitution effect is determined by the value of  $\eta$ : for a given consumption (i.e., for a given intertemporal effect  $\lambda_t$ ), un increase in the wage rate proposed

<sup>&</sup>lt;sup>4</sup>The productivity shock increases the household's expected gains. This reduces the weight of the budget constraint on the household's objective (*i.e.*  $\lambda_t$ ). Then, from equation (3.24) one can see that this increases the marginal utility of current leisure.

by the firm incentives the household to augment her labor supply. The more the labor supply elasticity  $\epsilon_H \equiv \frac{1}{\eta} \left( \frac{1-H}{H} \right)$  is elevate (i.e., the more  $\eta$  is small), the more the instantaneous effect is important.<sup>5</sup>

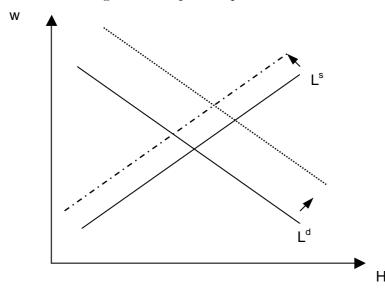


Figure 1.1: Separable preferences.

Instantaneous perturbation of the labor market equilibrium following a positive productivity shock.

The intertemporal wealth and substitution effects are captured by the term  $C_{i,t}$ . That is, the labor supply depends on the household's consumption behavior (see figure 1.1). With the benchmark calibration, the substitution effects predominate, which explains the positive response of labor at impact (figure 1.2).

**Non-separable preferences.** With non-separable preferences, the log-linearization of the labor supply equilibrium condition gives:

$$\eta \frac{H}{1-H}\hat{H}_{i,t} = \hat{w}_{i,t}$$

In this case the labor supply does not depend on the household's consumption behavior (see figure 1.3). In other words, the wealth and intertemporal substitution effects of wage changes on labor supply are exactly offsetting. Thus, the labor supply is static and is only determined by the instantaneous substitution effect.

<sup>&</sup>lt;sup>5</sup>This is because  $\epsilon_H \hat{C} + \hat{H} = \epsilon_H \hat{w}$ .

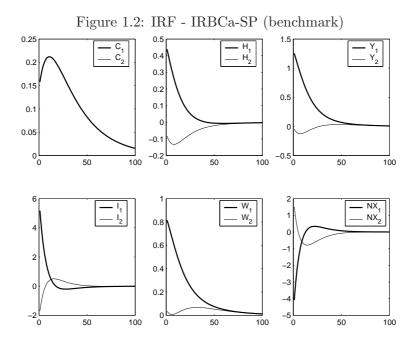
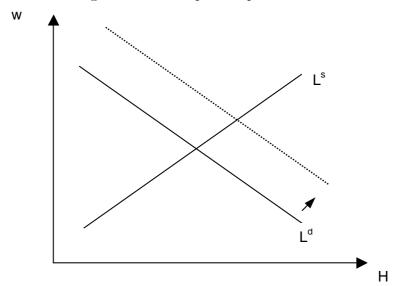


Figure 1.3: Non-separable preferences.



 $In stantaneous\ perturbation\ of\ the\ labor\ market\ equilibrium\ following\ a\ positive\ productivity\ shock.$ 

This is reflected by a stronger response of labor at impact (figure (1.4)). This explains in turn the bigger gain in output and then the larger wealth effect, which increases consumption by more than with separable preferences. This also accounts for the weaker increase in investment than with separable preferences.

At the same time, the household must choose what the economy will do with the additional output. One possibility is to consume all at impact, but this would be inefficient due to the concavity of the utility function. The decreasing nature of the marginal utility of consumption induces a preference for smooth paths of consumption: it is optimal to increase consumption both now and in the future, thus only a small fraction of the output will be consumed instantaneously and the remaining will be invested. The completeness of international markets and the high degree of physical capital mobility amplify the positive response of investment. The higher total factor productivity in country 1  $(a_{1,t})$  increases the capital returns in that country. This shifts investment from country 2 to country 1. However, the sum of the increase in consumption and that in investment is greater than the gain in output. This causes a deficit in country 1: the net exports, computed as NX = Y - C - I, fall at impact and during all the period of high output. A striking effect of the non-separability between consumption and leisure is that the link between home and foreign consumption is largely broken. One the one hand, this results from the equalization of the marginal consumption across countries at equilibrium:

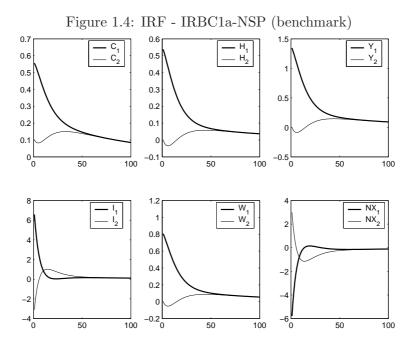
$$C_{1,t} + \frac{\tilde{\sigma}(1 - H_{1,t})^{1-\eta}}{1 - \eta} = C_{2,t} + \frac{\tilde{\sigma}(1 - H_{2,t})^{1-\eta}}{1 - \eta}$$
(1.32)

One the other hand, this results from the equilibrium condition (1.29), which can be expressed as:

$$H_{i,t} = f(\alpha, \tilde{\sigma}, \eta, a_{i,t}, K_{i,t}) \tag{1.33}$$

Then, conditional on a given capital stock, the total hours worked in each country respond positively to current domestic productivity shocks, but are orthogonal to the productivity shocks to the other country. So, while  $H_1$  responds positively to the shock arising in country 1,  $H_2$  does not changes at impact. Thus, to equation (1.32) still being verified, consumption in country 1 must increase by more than consumption in country 2.

The dynamics of most of the country 2 variables are pretty the converse. In plain words, this is due to the shift of capital to the more productive location (i.e., country 1). This induces a fall in the marginal productivity of labor in country 2 and then in the wage rate. As consequence, the



labor supply falls. On the other hand, the completeness of the financial market guarantees full risk sharing. This means that the increased wealth directly implied by the productivity shock (more output was produced at impact with the same input quantities) is equally shared among all the households in the world.

Nonetheless, the instantaneous responses of hours and output are different for both specifications of the utility function. When preferences are separable, the wealth effect is higher because the risk sharing condition implies that the increase of consumption is the same in the two countries. This incentives the country 2's household to work less at impact. Then, output also falls at impact. However, when preferences are non-separable, and without international transmission of the shock, the hours worked in country 2 are not affected. By consequence, output does not reacts at impact. Indeed, the labor supply and the production of country 2 react over time as investment responds to the productivity disturbance.

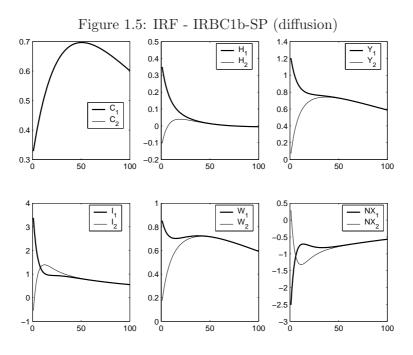
Now, let us analyze the model dynamics when the shock is diffused from country 1 to country 2. In this case we set the spill-over parameter  $\rho_{12}^a$  equal to 0.088 (i.e., the productivity shock is transmitted at a 8.8% rate per period), and the instantaneous diffusion parameter  $\psi$  equal to 0.133. This value corresponds to a instantaneous cross-country correlation of innovations to productivity equal to 0.258 (Backus, Kehoe, and Kydland 1994). The IRF functions from this

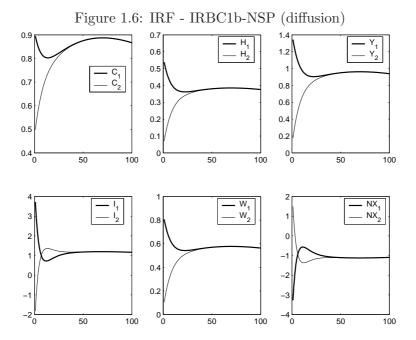
 $<sup>^{6}\</sup>rho(\varepsilon_{1,t},\varepsilon_{2,t})=\frac{2\psi}{1+\psi^{2}}.$ 

calibration are shown in figure 1.5 and figure 1.6. With this parameterization, the innovation that impacts country 1 has an immediate effect,  $(\psi)$ , as well as a delayed effect,  $(\rho_{12}^a)$ , on the country 2 productivity. This is easily seen from the log-linear expression of the productivity process:

$$\hat{a}_{2,t+1} = \rho_{12}^a \hat{a}_{1,t} + \rho_a \hat{a}_{2,t} + \psi \varepsilon_{1,t+1}$$

Then, the current productivity in country 2 also increases when a positive innovation impacts country 1. The higher productivity leads immediately to a positive response of total hours in both countries (equation (1.33)). The resulting output gain in country 2 induces a higher wealth effect than with the benchmark calibration: there is more production to be shared among all the households in the world. This explains the enhanced response of consumption in both countries. Thus, the spill-over of the shock produces more symmetric consumption paths across countries. This is true even with non-separable preferences: to equation (1.32) still holds, given that now both  $H_1$  and  $H_2$  increase at impact,  $C_2$  must increase by more than with independent shocks. The transmission of the productivity shock also curtails the capital flows to country 1 because the differential gap across countries, between the marginal productivity of labor and the marginal productivity of capital, is smaller than without spill-over. This is shown by the lower response of net exports.





#### 1.2.3 Quantitative Properties

The intuitions given by the IRF functions are reinforced by the results reported in table 1.2. The non-separability of the utility function improves the model's predictions regarding the international comovements. For the benchmark calibration (columns 2 and 3 of table 1.2), the cross-country correlation of outputs and total hours are less negative than with separable preferences. More strikingly, the cross-correlation of consumptions falls from 1 to 0.26. In both cases, due to the high capital mobility and to the completeness of financial markets, investments are negatively correlated.

Regarding the within-country statistics, we observe that the non-separability of the household's preferences augments the relative volatility of consumption and that of the total hours. Conversely, it diminishes the relative volatility of both investment and the labor productivity, but the persistence of the variables still virtually unchanged.

Turning to the procyclicality of the variables, we remark that only the correlation of consumption with output seems sensible to the specification of preferences. As is expected from the previous analysis, this correlation is lower with separable preferences. Finally, as soon as the shock is transmitted (last two columns of table 1.2), the cross-country correlations of output and labor input increase. However, this largely increases the cross-correlation of consumptions (from 0.26 to 0.84).

Table 1.2: Business-cycles statistics - Standard IRBC Model

	Data	IRBC1a	IRBC1a	IRBC1b	IRBC1b
		SP	NSP	SP	NSP
$\overline{\text{International}^{a,e}}$					
$\rho(Y_1, Y_2)$	0.51	-0.15	-0.10	0.09	0.24
$\rho(C_1^C, C_2^C)$	0.40	1.00	0.26	1.00	0.84
$ ho(H_1,H_2)$	0.36	-0.42	-0.10	-0.42	0.24
$\rho(I_1^C, I_2^C)$	0.38	-0.67	-0.76	-0.50	-0.78
$\sigma_{ au}/\sigma_{Y}$	1.90	_	_	_	_
$\sigma_{\Gamma}/\sigma_{Y}$	3.28	_	_	_	_
$\sigma_{NX}/\sigma_{Y}$	0.69	0.80	0.89	0.42	0.43
$\mathrm{USA}^{c,e}$					
$\sigma_Y$ (in %)	1.91	1.49	1.50	1.29	1.46
$\sigma_C/\sigma_Y$	0.40	0.19	0.41	0.41	0.75
$\sigma_I/\sigma_Y$	3.07	5.29	5.14	3.67	3.05
$\sigma_H/\sigma_Y$	0.86	0.36	0.40	0.30	0.40
$\sigma_{LP}/\sigma_{Y}$	0.57	0.64	0.60	0.72	0.60
$\sigma_W/\sigma_Y$	0.45	0.64	0.60	0.72	0.60
$\rho(Y_t, Y_{t-1})$	0.85	0.71	0.69	0.67	0.69
$\rho(C_t, C_{t-1})$	0.86	0.78	0.69	0.71	0.70
$\rho(H_t, H_{t-1})$	0.84	0.72	0.69	0.66	0.69
$ ho(I_t,I_{t-1})$	0.81	0.62	0.61	0.59	0.58
$\rho(LP_t, LP_{t-1})$	0.52	0.71	0.70	0.68	0.69
$\rho(Y,C)$	0.83	0.57	0.97	0.72	0.92
$\rho(Y, H)$	0.82	0.98	1.00	0.92	1.00
$\rho(Y, I)$	0.97	0.83	0.81	0.87	0.77
$\rho(Y, LP)$	0.51	0.99	1.00	0.98	1.00
$\rho(Y, W)$	0.28	0.99	1.00	0.98	1.00
$\rho(Y, LS)$	-0.30	-	-	_	-
$\rho(U,V)$	-0.89	_	_	_	_
$\rho(H, LP)$	-0.07	0.96	1.00	0.85	1.00
$\rho(H,W)$	0.03	0.96	1.00	0.85	1.00
$\rho(S,I)^d$	0.86	0.85	0.83	0.91	0.89
$\frac{\rho(Y, NX)^b}{}$	-0.29	-0.22	-0.42	-0.02	-0.28

IRBC1a-SP: The standard model with separable preferences and independent productivity processes across countries. IRBC1a-NSP: The standard model with non-separable preferences and independent productivity processes across countries. IRBC1b-SP: The standard model with separable preferences and diffusion of the shock. IRBC1b-NSP: The standard model with non-separable preferences and diffusion of the shock. The moments reported are computed from Hodrick-Prescott filtered artificial time series. <sup>a</sup> Backus, Kehoe, and Kydland (1995). <sup>b</sup> Hairault (2002). <sup>c</sup> Chéron and Langot (2004). <sup>d</sup> Baxter and Crusini (1993). <sup>e</sup> Hairault (1995).

#### 1.2.4 Sensibility analysis

To complete the analysis, in this subsection we conduct a sensibility analysis of variations in two key parameters of the model:  $\eta$ , that determines the labor supply elasticity, and  $\phi$ , the parameter governing the capital adjustment costs. In particular, the first case can be though off as a test over the agents preferences. Whereas the second test is conducted just to assess the role of this new parameter with respect to the canonical closed-economy framework.

#### Separable preferences

Sensibility to changes in  $\eta$  ( $\epsilon_H$ ). From Table 1.3 we can see that as long as the elasticity of labor  $\epsilon_H$  increases, the cross-country correlation of outputs, investments and total hours falls, whereas the cross correlation of consumptions still equal to one. The standard deviation of output, total hours and investment increases. By contrast, the standard deviation of consumption and that of the labor productivity fall. The persistence and the procyclicality still roughly unchanged but the correlation of labor productivity with both the total hours and output falls. For a better understanding of these results we also analyze the changes on the instantaneous elasticities. Particularly, we concentrate on the third column of the  $\Pi$  matrix in equation 1.31, which captures the direct effect of the productivity shock to country 1 (Table A.1). As we can see, the response of all the country 1 variables increases as  $\epsilon_H$  increases, apart from the real wage (see the log-linear expressions of the labor supply). The converse is true for the country 2 variables, consumption excepted.

Sensibility to changes in  $\phi$ . As we can see from Table A.3, when  $\phi$  increases, the response of investment is lower in country 1 but higher in country 2, so that the gap between them becomes smaller. This leads to larger cross-country correlation of investments. Finally, the response of total hours is decreasing in  $\phi$  for both countries, whereas the response of the labor productivity increases.

#### Non-separable preferences

Sensibility to changes in  $\eta$  ( $\epsilon_H$ ). The most striking differences with what happens with separable preferences, are the lower cross-country correlation of consumption and the equalization between the cross-country correlation of total hours and the one of outputs. This comes from the insensibility of  $H_2$  and  $Y_2$  to the productivity shock to country 1 (Table A.2). In

addition, the correlation of output with both total hours and labor productivity, as well as the correlation of total hours with labor productivity, are all equal to 1.

Sensibility to changes in  $\phi$ . The cross-country correlations increase as the adjustment costs become larger. This is remarkable for investment. The relative volatility of both total hours and labor productivity does not change. This is explained by the insensibility of their instantaneous elasticities to changes in  $\phi$  (Table A.3).

### 1.3 Conclusions

The canonical IRBC model developed in this chapter appears to be insufficient to account for most of the international features of business cycles. Moreover, it has the same limitations as its closed-economy counterpart regarding the dynamics of the real wage, the labor productivity and the total hours. In addition, due to its single-good nature, the model is obviously silent concerning the international facts involving relative prices. Nonetheless, we can point out the following:

Given a world economy composed of two symmetric countries which trade a single-homogeneous good then,

- When productivity is identically and independently distributed both across time and across countries, the non-separability between consumption and leisure induces a low cross-country correlation of consumption and a negative cross-country correlation of hours worked, investment and output.<sup>7</sup>
- As soon as we allow for positive international correlation of contemporaneous innovations
  to productivity, the cross-country correlation of consumptions, hours worked and outputs
  increase.

According to these results, chapter 2 exposes a survey of several standard amendments intended to improve the predictions of the model.

<sup>&</sup>lt;sup>7</sup>This is the particular case analyzed by Devereux, Gregory, and Smith (1992).

Table 1.3: Business-cycles statistics - Sensitivity analysis to changes in  $\eta$ 

	Data	IRBC1a-SP				IRBC1a-NSP	
$\eta  \left( \epsilon_H  ight)$		5 (0.4)	3 (2/3)	0.5(4)	5 (0.4)	3 (2/3)	0.5 (4)
$\rho(Y_1, Y_2)$	0.51	-0.13	-0.16	-0.34	-0.07	-0.05	-0.14
$ \rho(C_1, C_2) $	0.40	1.00	1.00	1.00	0.47	0.31	-0.58
$ ho(H_1,H_2)$	0.36	-0.40	-0.43	-0.55	-0.07	-0.05	-0.14
$ ho(I_1,I_2)$	0.38	-0.64	-0.67	-0.82	-0.70	-0.74	-0.72
$\sigma_{Y_1}$ (in %)	1.91	1.32	1.43	2.08	1.39	1.52	2.43
$\sigma_{C_1}/\sigma_{Y_1}$	0.40	0.20	0.19	0.16	0.34	0.41	0.57
$\sigma_{H_1}/\sigma_{Y_1}$	0.86	0.25	0.36	0.74	0.28	0.40	0.80
$\sigma_{I_1}/\sigma_{Y_1}$	3.07	5.22	5.38	6.58	4.98	5.02	6.10
$\sigma_{LP_1}/\sigma_{Y_1}$	0.57	0.74	0.64	0.28	0.71	0.60	0.20
$\sigma_{NX_1}/\sigma_{Y_1}$	0.69	0.78	0.84	1.21	0.82	0.89	1.35
$\rho(Y_{1,t},Y_{1,t-1})$	0.85	0.69	0.70	0.72	0.71	0.71	0.74
$\rho(C_{1,t},C_{1,t-1})$	0.86	0.76	0.77	0.76	0.72	0.71	0.80
$\rho(H_{1,t},H_{1,t-1})$	0.84	0.70	0.71	0.73	0.71	0.71	0.74
$\rho(I_{1,t},I_{1,t-1})$	0.81	0.61	0.62	0.61	0.62	0.61	0.64
$\rho(LP_{1,t}, LP_{1,t-1})$	0.52	0.69	0.70	0.72	0.71	0.71	0.74
$\rho(Y_{1,t}, C_{1,t})$	0.83	0.58	0.56	0.50	0.94	0.97	0.87
$\rho(Y_{1,t}, H_{1,t})$	0.82	0.98	0.98	0.98	1.00	1.00	1.00
$\rho(Y_{1,t},I_{1,t})$	0.97	0.85	0.83	0.78	0.83	0.81	0.77
$\rho(Y_{1,t}, LP_{1,t})$	0.51	0.99	0.99	0.92	1.00	1.00	1.00
$\rho(H_{1,t}, LP_{1,t})$	-0.07	0.96	0.95	0.85	1.00	1.00	1.00
$\rho(S_{1,t},I_{1,t})$	0.86	0.86	0.85	0.80	0.85	0.83	0.69
$\rho(Y_{1,t}, NX_{1,t})$	-0.29	-0.26	-0.24	-0.30	-0.36	-0.40	-0.45

IRBC1a-SP: The standard model with separable preferences and independent productivity processes across countries.

IRBC1a-NSP: The standard model with non-separable preferences and independent productivity processes across countries.

The moments reported are computed from Hodrick-Prescott filtered artificial time series.

Table 1.4: Business-cycles statistics - Sensitivity analysis to changes in  $\phi$ 

	Data	IRBC1a-SP		IRBC1a-NSP			
$\phi$		0.001	0.056	1.00	0.001	0.056	1.00
$\rho(Y_1, Y_2)$	0.51	-0.49	-0.13	-0.23	-0.40	-0.08	-0.02
$\rho(C_1,C_2)$	0.40	1.00	1.00	1.00	-0.07	0.27	0.47
$ ho(H_1,H_2)$	0.36	-0.65	-0.40	-0.88	-0.40	-0.08	-0.02
$ ho(I_1,I_2)$	0.38	-0.98	-0.66	0.56	-0.99	-0.75	0.76
$\sigma_{Y_1}$ (in %)	1.91	1.93	1.44	1.30	1.85	1.50	1.45
$\sigma_{C_1}/\sigma_{Y_1}$	0.40	0.13	0.20	0.43	0.38	0.41	0.47
$\sigma_{H_1}/\sigma_{Y_1}$	0.86	0.38	0.35	0.32	0.40	0.40	0.40
$\sigma_{I_1}/\sigma_{Y_1}$	3.07	20.25	5.37	1.38	21.00	5.03	1.72
$\sigma_{LP_1}/\sigma_{Y_1}$	0.57	0.62	0.65	0.71	0.60	0.60	0.60
$\sigma_{NX_1}/\sigma_{Y_1}$	0.69	5.69	0.82	0.76	5.95	0.89	0.46
$\rho(Y_{1,t}, Y_{1,t-1})$	0.85	0.80	0.70	0.66	0.78	0.70	0.68
$\rho(C_{1,t},C_{1,t-1})$	0.86	0.81	0.79	0.66	0.76	0.70	0.68
$\rho(H_{1,t}, H_{1,t-1})$	0.84	0.81	0.70	0.67	0.78	0.70	0.68
$\rho(I_{1,t},I_{1,t-1})$	0.81	0.17	0.62	0.65	0.18	0.61	0.67
$\rho(LP_{1,t}, LP_{1,t-1})$	0.52	0.80	0.71	0.66	0.78	0.70	0.68
$\rho(Y_{1,t},C_{1,t})$	0.83	0.42	0.57	0.60	0.98	0.98	0.96
$\rho(Y_{1,t}, H_{1,t})$	0.82	0.99	0.98	0.90	1.00	1.00	1.00
$\rho(Y_{1,t},I_{1,t})$	0.97	0.24	0.84	0.90	0.22	0.81	0.89
$\rho(Y_{1,t}, LP_{1,t})$	0.51	0.99	0.99	0.98	1.00	1.00	1.00
$\rho(H_{1,t}, LP_{1,t})$	-0.07	0.97	0.95	0.98	1.00	1.00	1.00
$\rho(S_{1,t},I_{1,t})$	0.86	0.24	0.86	0.73	0.22	0.83	0.83
$\rho(Y_{1,t}, NX_{1,t})$	-0.29	-0.04	-0.26	0.77	-0.08	-0.40	0.71

IRBC1a-SP: The standard model with separable preferences and independent productivity processes across countries.

IRBC1a-NSP: The standard model with non-separable preferences and independent productivity processes across countries.

The moments reported are computed from Hodrick-Prescott filtered artificial time series.

# Chapter 2

A survey on international real business cycles and the labor market

# Introduction

In this chapter we review several standard extensions that were conceived to improve the predictions of the canonical model discussed in chapter 1. We still evaluate the performance of the different economies with respect to the three puzzling facts early described, reproduced here for easier reference. Two of them refer to observed international co-movements: (i) the cross correlations for production, consumption, investment and labor input are positive across countries, and (ii) the cross-correlation of consumptions tends to be lower than that of productions. The last one concerns the observed rigidity of the real wage: (iii) the contemporaneous correlation of the aggregate real wage with both output and labor input is very weak.

In stark contradiction, former international RBC models, as the one presented in chapter 1, tends to predict negative cross-country correlations of labor input, investment and eventually of output (Baxter 1995). Moreover, the theoretical correlation of consumptions is very close or equal to unity, roughly two times than in the data.

Nonetheless, as will be discussed along this chapter, posterior amendments have improved the predictions of these canonical models. Basically by studying the mechanisms able either to reduce the cross-correlation of consumptions, or to enhance the cyclical synchronization of productions.<sup>3</sup> On the other hand, one important weakness of RBC models for a closed economy is the predicted high contemporaneous correlation of aggregate real wage with both output and labor input, which contradicts the observed rigidity of the aggregate real wage.

In the first part of this chapter we survey three standard extensions of the basic walrasian framework exposed in chapter 1. The first one aims to deep the link between the home and the foreign countries. To this end we introduce an additional consumption/investment good by considering national specialization (Backus, Kehoe, and Kydland 1994). This richer structure adds a new mechanism by which the expansion of output experimented in the country receiving

<sup>&</sup>lt;sup>1</sup>To cite some examples: Arvanitis and Mikkola (1996), Baxter (1995), Baxter and Crucini (1993), Backus, Kehoe, and Kydland (1994).

<sup>&</sup>lt;sup>2</sup>See, for instance, the seminal works of Backus, Kehoe, and Kydland (1992) and Baxter and Crucini (1993).

<sup>&</sup>lt;sup>3</sup>For instance, Devereux, Gregory, and Smith (1992) show that the non separability of consumption and leisure in the agents' preferences can generate a realistic international correlation between consumptions, while Stockman and Tesar (1995) obtain similar results by incorporating non-traded goods and taste shocks. Baxter and Crucini (1995) build a model with incomplete asset markets which, by reducing the incentive for risk sharing, improves a little the correlation of consumptions. Kehoe and Perri (2000) almost solve the consumption-correlation puzzle with a special incompleteness of financial markets in which each country may opt for the non-payment of his debt. In this case, the country is excluded from financial markets and rests in autarky.

the shock, may induce an expansion of output in the other country. This potentially allows for positive cross-correlations for labor inputs and investments.

This new mechanism pass through two channels. On the one side, agents demand a basket of the two goods produced in the world for consumption and investment purposes. Then, the additional wealth that results from a positive technological shock (that allows to produce more with the same inputs), together with the perfect risk sharing, make agents of both countries to increase their level of consumption at impact. This increases the demand for the two goods and leads to an expansion of output in both countries. The other channel concerns the change in the relative price of goods that follows the idiosyncratic innovation. Even if all this ameliorates the theoretical predictions relative to the international facts, the model is far to be sufficient. Following Galí (1994), we also distinguish the composite good for consumption from the composite good for investment. However, this does not change the predictions of the model since we allow for perfect competitive markets.

The second extension aims to reduce the international correlation of consumption. This is done by restricting international trade to non-contingent bonds (Baxter 1995). This limitation in the agent's ability to risk pooling country-specific shocks produces more realistic international correlations of outputs and consumptions. However, the correlation of outputs still larger than the one of consumptions.

The last extension in the pure walrasian framework that we consider is the introduction of a realistic potential for intra- and international capital flows by the disaggregation of the economy into internationally traded and non-traded sectors (Stockman and Tesar 1995). This is justified by the empirical evidence that roughly a half of the typical G-10 country's output consists of non-traded goods and services. It must be enhanced that, conversely to traditional IRBC models with only technological shocks, as the Stockman and Tesar (1995)'s model, our model predicts positive international input co-movements, which is more in accordance with empirical correlations, but they are overstated particularly when shocks are highly persistent.

Nevertheless, because at this point we have not yet modified the walrasian nature of the labor market, all models still fail in reproducing the fluctuations of the employment, the hours worked and the real wage. Hence, the next step is to modify the walrasian labor market by introducing search and matching in the labor market. This is the core of the second part of this survey, in which we take as starting point the Hairault (2002)'s two-country, two-good search economy to going ahead in the study of some stylized facts of the US labor market. Next, we make a

reduction to the single-good case to assess the role of each key hypotheses in the Hairault's economy. Namely, (i) the non-separability between consumption and leisure in the agents' preferences, (ii) the existence of two goods in the world and so one relative price, and (iii) search and bargaining in the labor market. In this single-good search framework we also evaluate the predictions from the model with restricted international trade to non-contingent bonds. However, we do not extend the search model to include two sectors because the results from the walrasian economy are discouraging.

Regarding the search economies, we show that in the single-good economy, the combination of search and matching in the labor market with the non-separability is enough to predict positive comovements of labor inputs and investments as well as a large dissociation of consumptions. Moreover, the procyclicality of real wage rate is reduced, and the correlation of total hours with both output and labor productivity is low. Then, the three puzzles are partially solved. However, consumptions correlation still larger than outputs correlation, even if the incompleteness of financial markets produces more realistic international correlations of outputs and consumptions. Then, we show that the gain from including two goods in that framework is that the model is able now to replicate a correlation of outputs bigger than that of consumptions (Hairault 2002). However, the price dynamics provoked by a positive productivity shock decrease the agent's purchasing power, leading to a stronger vindication of salary and so to a slightly more procyclical real wages.

# 2.1 National Specialization

The world economy consists of two countries (country 1 or home country and country 2 or foreign country), each represented by a large number of identical consumers and a production technology. Population size is normalized to unity. Each country specializes in the production of a single good affected by persistent shocks A to productivity that are diffused internationally. Agents demand constant elasticity of substitution (CES) baskets of the two goods for consumption C and investment I purposes. Finally, the good produced in country 1 is taken as accounting unit.

# 2.1.1 Firms

Each country specializes in the production of a single good. The goods are produced with a constant returns to scale technology using capital  $K_i^C$ , which is a composite of good 1 and good

2, and labor  $H_i$  as inputs,

$$Y_{i,t} = a_{i,t} (K_{i,t}^C)^{\alpha} H_{i,t}^{1-\alpha}$$
(2.1)

As before, the variables  $a_{i,t}$  stand for the stochastic component of the productivity variable and follow the vector-autoregressive process described in the previous chapter. New capital goods are internationally mobile and all investment is subject to quadratic adjustment costs:  $C_{i,t} = \frac{\phi}{2}(K_{i,t+1}^C - K_{i,t}^C)^2$ . Investment to country i = 1, 2 is a CES index of the two goods,

$$I_{i,t}^{C} = \left[ \gamma_I^{\frac{1}{\theta_I}} I_{i,t}^{\frac{\theta_I - 1}{\theta_I}} + (1 - \gamma_I)^{\frac{1}{\theta_I}} I_{j \neq i,t}^{\frac{\theta_I - 1}{\theta_I}} \right]^{\frac{\theta_I}{\theta_I - 1}}$$
(2.2)

with the price index defined at each date as:

$$P_i^I = \left[ \gamma_I P_i^{1-\theta_I} + (1 - \gamma_I) P_{j \neq i}^{1-\theta_I} \right]^{\frac{1}{1-\theta_I}}$$
 (2.3)

where  $\theta_I$  denotes the elasticity of substitution between the national and the foreign goods when they are used to production purposes,  $\gamma_I$  defines the share of the national good in the investment basket, and  $P_{i,t}$  is the production price of good i. Capital accumulates over time according to

$$K_{i,t+1}^C = (1 - \delta)K_{i,t}^C + I_{i,t}^C$$
(2.4)

The wage rate payed by firms is expressed in units of the national good (*i.e.*, firms pay a "product wage"). Then, by normalizing the price of good 1 to  $P_{1,t} = 1 \,\forall t$ , the dynamic problem of each firm in country i = 1, 2 can be written, in units of the good 1 as follows:

$$\mathcal{W}(K_{1,t}^C) = \max_{H_{1,t}, I_{1,t}^C} \left\{ Y_{1,t} - \mathcal{P}_{1,t}^I(\mathcal{C}_{1,t} + I_{1,t}^C) - w_{1,t}H_{1,t} + \int v_t \mathcal{W}(K_{1,t+1}^C) dA_{t+1} \right\}$$
(2.5)

$$\mathcal{W}(K_{2,t}^C) = \max_{H_{2,t}, I_{2,t}^C} \left\{ p_t Y_{2,t} - \mathcal{P}_{2,t}^I(\mathcal{C}_{2,t} + I_{2,t}^C) - p_t w_{2,t} H_{2,t} + \int v_t \mathcal{W}(K_{2,t+1}^C) dA_{t+1} \right\}$$
(2.6)

subject to constraints (2.1) and (2.4). In last expressions  $p_t$  is the relative price of good 2 and  $\mathcal{P}_{i,t}^I \equiv \frac{P_{i,t}^I}{P_{1,t}}$ , and  $v_t = v(A_{t+1})$  stands for the firm's actualization rate. The optimal demands for labor and capital are then:

$$w_{i,t} = (1 - \alpha) \frac{Y_{i,t}}{H_{i,t}}, \text{ for i=1,2}$$
 (2.7)

$$q_{i,t} \equiv \mathcal{P}_{i,t}^{I} + \mathcal{P}_{i,t}^{I} \frac{\partial \mathcal{C}_{i,t}^{C}}{\partial I_{i,t}^{C}}, \text{ for i=1,2}$$
 (2.8)

$$q_{1,t} = \int v_t \left( \alpha \frac{Y_{1,t+1}}{K_{1,t+1}^C} + q_{1,t+1} - \delta \mathcal{P}_{1,t}^I \right) dA_{t+1}, \text{ for } i=1$$
 (2.9)

$$q_{2,t} = \int v_t \left( \alpha p_{t+1} \frac{Y_{2,t+1}}{K_{2,t+1}^C} + q_{2,t+1} - \delta \mathcal{P}_{2,t}^I \right) dA_{t+1}, \text{ for } i=2$$
 (2.10)

Finally, we impose the following transversality condition for capital, where  $q_{i,t}$  is the shadow price of capital:

$$\lim_{j \to \infty} E_t[q_{i,t+j+1} K_{t+j+1}^C] = 0 \tag{2.11}$$

# 2.1.2 Households

Given the wage rate proposed by firms, the representative household's objective is to choose a contingency plan  $\{C_{i,t}^C, H_{i,t}\}$  that maximizes her expected lifetime utility

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_{i,t}^C, 1 - H_{i,t})$$
 (2.12)

 $C_{i,t}^C$  stands for the household's consumption of the composite goods  $C^C$ . Similar than investment, consumption is assumed to have the following CES structure:

$$C_i^C = \left[ \gamma_C^{\frac{1}{\theta_C}} C_i^{\frac{\theta_C - 1}{\theta_C}} + (1 - \gamma_C)^{\frac{1}{\theta_C}} C_{j \neq i}^{\frac{\theta_C - 1}{\theta_C}} \right]^{\frac{\theta_C}{\theta_C - 1}}$$
(2.13)

with the price index defined as

$$P_i^C = \left[ \gamma_C P_i^{1-\theta_C} + (1 - \gamma_C) P_{j \neq i}^{1-\theta_C} \right]^{\frac{1}{1-\theta_C}}$$
 (2.14)

where  $\theta_C$  is the elasticity of substitution between the goods and  $\gamma_C$  the share of good 1 in the consumption basket.

Financial markets are complete and we assume perfect international risk sharing: households in the two countries have access to contingent claims  $B_{i,t} = B_i(A_t)$  at prices  $v_t = v(A_{t+1})$  providing one unit of good 1 (i.e., of the accounting unit) if the state A occurs at t+1. The households' budget constraints are then

$$P_{i,t}^{C}C_{i,t}^{C} + \int v_{t}P_{1,t+1}B_{i,t+1}dA_{t+1} \le P_{1,t}B_{i,t} + P_{i,t}w_{i,t}H_{i,t} \qquad (\lambda_{i,t})$$
(2.15)

which are written in terms of good 1 as:

$$\mathcal{P}_{1,t}^C C_{1,t}^C + \int v_t B_{1,t+1} dA_{t+1} \le B_{1,t} + w_{1,t} H_{1,t} \qquad (\lambda_{1,t})$$
(2.16)

$$\mathcal{P}_{2,t}^{C}C_{2,t}^{C} + \int v_{t}B_{2,t+1}dA_{t+1} \le B_{2,t} + p_{t}w_{2,t}H_{2,t} \qquad (\lambda_{2,t})$$
(2.17)

with  $\mathcal{P}_{i,t}^C \equiv \frac{P_{i,t}^C}{P_{1,t}}$ . Let  $f(A) \equiv f(A_{t+1}, A_t)$  denote the density function describing the transition from the state  $A_t$  to the state  $A_{t+1}$ . Then, the representative household in country i = 1, 2 solves the dynamic problem:

$$\mathcal{V}(B_{i,t}) = \max_{C_{i,t}^C, H_{i,t}, B_{i,t+1}} \left\{ U(C_{i,t}^C, 1 - H_{i,t}) + \beta \int \mathcal{V}(B_{i,t+1}) f(A) dA \right\}$$
(2.18)

subject to the respective budget constraint ((2.16) for i = 1, or (2.17) for i = 2).

If the agents' preferences exhibit a standard separability between consumption and leisure,<sup>4</sup> the instantaneous utility is defined as:

$$U(C_{i,t}^C, 1 - H_{i,t}) = \log(C_{i,t}^C) + \sigma \frac{(1 - H_{i,t})^{1-\eta}}{1 - \eta}, \quad \eta > 0$$

and the optimality conditions are:

$$\frac{1}{C_{i,t}^C} = \mathcal{P}_{i,t}^C \lambda_{i,t}, \quad \text{for i=1,2}$$
(2.19)

$$\sigma(1 - H_{1,t})^{-\eta} \mathcal{P}_{1,t}^C C_{1,t}^C = w_{1,t}, \text{ for i=1}$$
 (2.20)

$$\sigma(1 - H_{2,t})^{-\eta} \mathcal{P}_{2,t}^C C_{2,t}^C = p_t w_{2,t}, \text{ for i=2}$$
 (2.21)

Then, with two imperfect-substitutable goods in the world, the separability hypothesis not longer guarantees the equalization of consumptions among countries.

Else, if preferences exhibit a non-separability,<sup>5</sup> the instantaneous utility is:

$$U(C_{i,t}^C, 1 - H_{i,t}) = \log\left(C_{i,t} + \tilde{\sigma} \frac{(1 - H_{i,t})^{1-\eta}}{1 - \eta}\right), \quad \eta > 0$$

which implies:

$$\frac{1}{C_{i,t}^C + \tilde{\sigma} \frac{(1 - H_{i,t})^{1 - \eta}}{1 - \eta}} = \mathcal{P}_{i,t}^C \lambda_{i,t}, \quad \text{for i=1,2}$$
(2.22)

$$\tilde{\sigma}(1 - H_{1,t})^{-\eta} \mathcal{P}_{1,t}^C = w_{1,t}, \text{ for i=1}$$
 (2.23)

$$\tilde{\sigma}(1 - H_{2,t})^{-\eta} \mathcal{P}_{2,t}^C = p_t w_{2,t}, \text{ for i=2}$$
 (2.24)

Either at odds with the predictions of the single-good economy, the non-separability combined with the two-good hypothesis, is not longer helpful in disentangle national- from foreign consumption.

Finally, the optimal choice of contingent bonds and the transversality condition are:

$$v_t = \beta \frac{\lambda_{i,t+1}}{\lambda_{i,t}} f(A) \tag{2.25}$$

$$\lim_{j \to \infty} E_t[\beta^{t+j} \lambda_{i,t+j} B_{i,t+j+1}] = 0$$
(2.26)

With respect to the canonical model developed in chapter 1, the marginal utility of either consumption and labor, is now affected by the evolution of prices, which react in turn to changes in productivity.

 $<sup>^4</sup>$ See Chapter 1 for a detailed analysis of this kind of preferences.

 $<sup>^5</sup> Ibidem.$ 

# 2.1.3 General Equilibrium

The equilibrium of this economy consists of a set of households' optimal decision rules  $\{C_i(\cdot), H_i^s(\cdot), B_i(\cdot)\}$ , the firms' optimal demands for capital and labor  $\{K_i^d(\cdot), H_i^d(\cdot)\}$  and a set of prices  $\{p_t, \mathcal{P}_{i,t}^C, \mathcal{P}_{i,t}^I, w_{i,t}, v_t(A_{t+1})\}$  equilibrating the goods market, the labor market and the financial market. Then, for each country:

**Equilibrium on the Goods Market:** The equilibrium on the goods market is given by the accounting equations for output,<sup>6</sup>

$$Y_{1,t} = C_{1,t}^{1} + C_{1,t}^{2} + I_{1,t}^{1} + I_{1,t}^{2} + C_{1,t}^{1} + C_{1,t}^{2}$$

$$= \gamma_{C} \left( \mathcal{P}_{1,t}^{C} \right)^{\theta_{C}} C_{1,t}^{C} + \gamma_{I} \left( \mathcal{P}_{1,t}^{I} \right)^{\theta_{I}} I_{1,t}^{C} + (1 - \gamma_{C}) \left( \mathcal{P}_{2,t}^{C} \right)^{\theta_{C}} C_{2,t}^{C} + (1 - \gamma_{I}) \left( \mathcal{P}_{2,t}^{I} \right)^{\theta_{I}} \left( \mathcal{C}_{2,t} + I_{2,t}^{C} \right)$$

$$Y_{2,t} = C_{2,t}^{1} + C_{2,t}^{2} + I_{2,t}^{1} + I_{2,t}^{2} + C_{2,t}^{1} + C_{2,t}^{2}$$

$$= (1 - \gamma_{C}) \left( \frac{\mathcal{P}_{1,t}^{C}}{p_{t}} \right)^{\theta_{C}} C_{1,t}^{C} + (1 - \gamma_{I}) \left( \frac{\mathcal{P}_{1,t}^{I}}{p_{t}} \right)^{\theta_{I}} I_{1,t}^{C} + \gamma_{C} \left( \frac{\mathcal{P}_{2,t}^{C}}{p_{t}} \right)^{\theta_{C}} C_{2,t}^{C} + \gamma_{I} \left( \frac{\mathcal{P}_{2,t}^{I}}{p_{t}} \right)^{\theta_{I}} \left( I_{2,t}^{C} + \mathcal{C}_{2,t} \right)$$

$$= (1 - \gamma_{C}) \left( \frac{\mathcal{P}_{1,t}^{C}}{p_{t}} \right)^{\theta_{C}} C_{1,t}^{C} + (1 - \gamma_{I}) \left( \frac{\mathcal{P}_{1,t}^{I}}{p_{t}} \right)^{\theta_{I}} I_{1,t}^{C} + \gamma_{C} \left( \frac{\mathcal{P}_{2,t}^{C}}{p_{t}} \right)^{\theta_{C}} C_{2,t}^{C} + \gamma_{I} \left( \frac{\mathcal{P}_{2,t}^{I}}{p_{t}} \right)^{\theta_{I}} \left( I_{2,t}^{C} + \mathcal{C}_{2,t} \right)$$

 $\kappa_{j,t}^i$ , for  $\kappa = \{C, I, C\}$  stands for the demand for good j = 1, 2 from country i = 1, 2 for consumption and investment purposes.

**Equilibrium on the Labor Market:** When preferences are separable, the desired hours of work in each country are determined by the following conditions:

$$\mathcal{P}_{1,t}^C C_{1,t}^C \sigma (1 - H_{1,t})^{-\eta} = (1 - \alpha) \frac{Y_{1,t}}{H_{1,t}}$$
(2.29)

$$\left(\frac{\mathcal{P}_{2,t}^C}{p_t}\right)C_{2,t}^C\sigma(1-H_{2,t})^{-\eta} = (1-\alpha)\frac{Y_{2,t}}{H_{2,t}}$$
(2.30)

and, either the following condition when preferences are non-separable:

$$\mathcal{P}_{1,t}^{C}\tilde{\sigma}(1-H_{1,t})^{-\eta} = (1-\alpha)\frac{Y_{1,t}}{H_{1,t}}$$
(2.31)

$$\left(\frac{\mathcal{P}_{2,t}^C}{p_t}\right)\tilde{\sigma}(1-H_{2,t})^{-\eta} = (1-\alpha)\frac{Y_{2,t}}{H_{2,t}}$$
(2.32)

The properties of these conditions are discussed in next section.

Equilibrium on the Financial and Capital Market: Equation (2.25) implies that the initial wealth distribution in one country is proportional to the initial wealth distribution in the

<sup>&</sup>lt;sup>6</sup>See Annex B for details.

other country  $(\frac{\lambda_{1,t+1}}{\lambda_{1,t}} = \frac{\lambda_{2,t+1}}{\lambda_{2,t}})$ . Making the simplifying assumption that all households have the same initial wealth distribution, we deduce that  $\lambda_{1,t} = \lambda_{2,t} = \lambda_t$ . Then, the evolution of the firm's implicit price  $q_{i,t}$  can be written as:

$$q_{2,t} = \beta E_t \left[ \frac{\lambda_{t+1}}{\lambda_t} \left( \alpha \frac{Y_{1,t+1}}{K_{1,t+1}^C} + q_{1,t+1} - \delta \mathcal{P}_{1,t+1}^I \right) \right]$$
 (2.33)

$$q_{2,t} = \beta E_t \left[ \frac{\lambda_{t+1}}{\lambda_t} \left( \alpha p_{t+1} \frac{Y_{2,t+1}}{K_{2,t+1}^C} + q_{2,t+1} - \delta \mathcal{P}_{2,t+1}^I \right) \right]$$
 (2.34)

with  $\lambda_t$  given by equation (2.19), if preferences are separable; and by equation (2.22), if preferences are non-separable. In both cases  $\lambda$  is also affected by the dynamics of the prices of goods. As is showed below, this largely dampen the negative response of investment in country 2 (it is "less negative").

### 2.1.4 Qualitative Analysis

In what follows, to highlight the new effects associated with the two-good assumption, we does not discuss again the dynamics that are common to this model and the model in the previous section:

Table 2.1: Benchmark calibration.

$\alpha$	β	Н	η	$\phi$	δ	$\rho_a$	$ ho_{12}^a$	$\psi$	$\sigma_{arepsilon^a}$	$\sigma$	$\tilde{\sigma}$
0.36	0.99	1/3	3	0.056	0.025	0.906	0	0	0.00852	0.7508	0.6804

# Steady State and calibration of the structural parameters

The additional parameters with respect to the canonical model (chapter 1), are the elasticities of substitution between foreign and domestic goods:  $\theta_C$  and  $\theta_I$ ; and the shares of national good in the CES baskets:  $\gamma_C$  and  $\gamma_I$ . Following previous literature, we first study the model's behavior under three different configurations of elasticities (Galí 1994) but equal shares  $\gamma$  (Backus, Kehoe, and Kydland 1994): (i)  $\theta_C = \theta_I$ , labelled BKK; (ii)  $\theta_C > \theta_I$ , labelled GALI1; and (iii)  $\theta_C < \theta_I$ , labelled GALI2. In these cases,  $\gamma_C = \gamma_I$  is determined from the observed ratios of imports and exports to Gross Domestic Product (GDP), using the first order conditions from the optimal composition of the CES baskets. Next, we propose a similar exercise regarding the home bias parameters to evaluate the model responses to a different composition of the consumption basket

from the investment basket. The experiences are: (iv)  $\gamma_C < \gamma_I$ , labeled EXP1, and the opposite case (v)  $\gamma_C > \gamma_I$ , labeled EXP2.

From the optimal determination of the composite goods (see appendix B), we have that <sup>7</sup>

$$\frac{P_2}{P_1} = \left(\frac{1 - \gamma_C}{\gamma_C}\right)^{1/\theta_C} (C_1^1/C_2^1)^{1/\theta_C} \\
= \left(\frac{1 - \gamma_I}{\gamma_I}\right)^{1/\theta_I} (I_1^1/I_2^1)^{1/\theta_I}$$

At the steady state Y = C + I. Since we are interested in a symmetric equilibrium, then  $Y_1 = Y_2$ ,  $C_2^1 = C_1^2$ ,  $I_2^1 = I_1^2$ . This implies that the terms of trade for country 1,  $\frac{P_2}{P_1}$ , are equal to one. Then,

$$\frac{C_1^1}{C_2^1} = \frac{\gamma_C}{1 - \gamma_C} \Rightarrow \gamma_C = \frac{(C_1^1/C_2^1)}{1 + (C_1^1/C_2^1)}$$

$$\frac{I_1^1}{I_2^1} = \frac{\gamma_I}{1 - \gamma_I} \Rightarrow \gamma_I = \frac{(I_1^1/I_2^1)}{1 + (I_1^1/I_2^1)}$$

On the other hand, from the accounting equation for  $Y_1$  we get<sup>8</sup>

$$\frac{Y_1^1}{Y_2^1} = \frac{1 - (Y_2^1/Y_1)}{(Y_2^1/Y_1)}$$

where  $(Y_2^1/Y_1)$  is the ratio of imports to output in country 1. Whereas there are some estimations for this share, this is not the case for the share of the imported goods destined to consumption to total consumption (i.e. ,  $C_2^1/C_1$ ). Nor for the share of the imported goods destined to investment to total investment (i.e. ,  $I_2^1/I_1$ ). So, we take  $Y_2^1/Y_1 = C_2^1/C_1 = I_2^1/I_1$ . This implies  $\gamma_C = \gamma_I \equiv \gamma$ . For an imports to output share of 0.2 (a very standard value), one gets  $\gamma = 0.8$ . The values of these additional parameters are resumed in Table 8. For the remaining parameters, we retain the calibration from chapter 1.

#### Model dynamics

We analyze the responses of the model variables to a positive 1% technological shock to country 1. The basic dynamics described in chapter 1 still at work. However, the two-good assumption induces two additional effects: an *Volume Effect (VE)*, due to the linkages imposed by the CES structure of the consumption and investment baskets; and a *Price Effect (PE)*, due to the variations in the relative price of goods following a positive productivity shock in country 1.

 $<sup>{}^{7}</sup>C_{i}^{i}$  and  $I_{i}^{i}$  denote the demand of goods j from country i.

 $<sup>^8\</sup>text{At the steady state, } Y_1 = \left(C_1^1 + I_1^1\right) + \left(C_1^2 + I_1^2\right) \equiv Y_1^1 + Y_1^2, \text{ which implies } \frac{Y_1^1}{Y_1^2} = \frac{1 - \left(Y_1^2/Y_1\right)}{\left(Y_1^2/Y_1\right)}, \text{ but } Y_1^2 = Y_2^1.$ 

Table 2.2: Additional parameters

	$\gamma_C$	$\gamma_I$	$\theta_C$	$\theta_I$	$\phi$
BKK	0.8	0.8	1.5	1.5	0.1
GALI 1	0.8	0.8	1.75	0.79	0.1
GALI 2	0.8	0.8	0.6	5.4	0.1
EXP 1	0.6	0.8	0.6	5.4	0.1
EXP 2	0.8	0.6	0.6	5.4	0.1

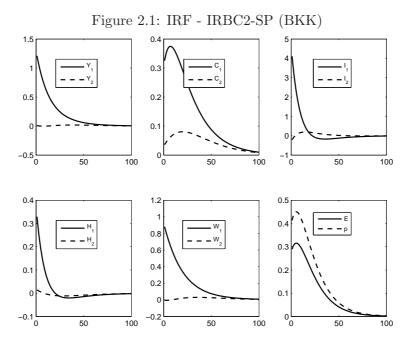
The VE implies that, since  $\gamma_C \neq 1$ , the positive wealth effect induced by the shock itself makes households in both countries to increase their demands for the two goods. In turn, the PE modifies the real wage in country 2 via the deterioration of the terms of trade of country 1 (i.e.  $p_t$ ).

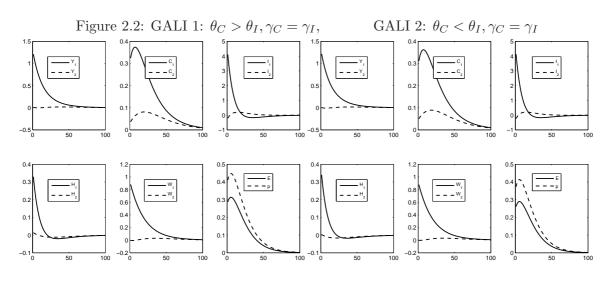
As in the canonical model, the household's labor supply results from the combination of an instantaneous substitution effect between current consumption and current leisure, and two opposite intertemporal effects that reflect the household's dynamic trade-off between current leisure and future leisure. The total outcome is contingent on the parameterization of the model and on the specification of the utility function. However, with respect to the single-good canonical model, these new effects lead to quiet different dynamics for consumption and hours, as it is discussed below.

Separable Preferences. The impulse response functions (IRF) following a technological shock to country 1 are showed in figure 2.1 for the BKK calibration, in figure 2.2 for the GALI calibrations, and in figure 2.3 for the own EXP calibrations (since we analyse the orthogonal responses to the shock, we set the diffusion parameters to  $\psi = \rho_{12}^a = 0$  to compute the IRF from the model).

With separable preferences, the log-linearization of the labor supply equilibrium condition gives

That is, the higher productivity at impact in country 1 provokes a fall in the production price of good 1, and thus an increase in  $p_t$ .





 $\ensuremath{\mathsf{IRF}}$  -  $\ensuremath{\mathsf{IRBC2}\text{-}\mathsf{SP}}.$ 

(a hat stands for the deviation of the log-linearized variable from their steady-state value):

$$\hat{C}_{1,t}^C + \hat{\mathcal{P}}_{1,t}^C + \eta \frac{H}{1 - H} \hat{H}_{1,t} = \hat{w}_{1,t}$$

$$\hat{C}_{2,t}^C + \hat{\mathcal{P}}_{2,t}^C + \eta \frac{H}{1 - H} \hat{H}_{2,t} = \hat{w}_{2,t} + \hat{p}_t$$

Given the normalization of the production price of good 1, the (log-linear) price index become:  $\hat{P}_{1,t}^C = (1 - \gamma_C)\hat{p}_t$  and  $\hat{P}_{2,t}^C = \gamma_C\hat{p}_t$ . Then, the (log-linear) labor supply condition can be written as:

$$\hat{C}_{1,t}^C + (1 - \gamma_C)\hat{p}_t + \eta \frac{H}{1 - H}\hat{H}_{1,t} = \hat{w}_{1,t}$$
(2.35)

$$\hat{C}_{2,t}^C - (1 - \gamma_C)\hat{p}_t + \eta \frac{H}{1 - H}\hat{H}_{2,t} = \hat{w}_{2,t}$$
(2.36)

Responses in the BKK and GALI scenarios. First of all, note that the IRF from these three configurations are virtually indistinguishable to each other, apart from the slightly larger increase in the terms of trade, and in the labor supply in country 1, with the GALI2 calibration (right-hand side panel in figure 2.1). In all cases, as in the canonical model, the instantaneous substitution effect is determined by the value of  $\eta$ : for a given consumption, an increase in the wage rate proposed by the country 1's firm incentives the household in that country to augment her labor supply (equation (2.35)). The intertemporal wealth and substitution effects are captured by the term  $\hat{C}_{1,t}$ . With the benchmark calibration, the substitution effects predominate, which explains the positive response of labor at impact. However, the response of labor is lower than in the canonical setting because the increase in the real wage is damped by the increase in the relative price p.

The shift of capital from country 2 to country 1 reduces the marginal productivity in country 2, and so the real wage (equation (2.36)). Then, for a given consumption, the labor supply must increase in country 2 to offset the negative effect of the increase in p. Under our calibration, the response of both  $H_2$  and  $w_2$  are close to zero but vary in the right way.

However, the striking differences with respect to the canonical framework concern the responses of consumption. At equilibrium, separability implies

$$C_{1,t}^C = E_t C_{2,t}^C$$

with  $E \equiv \frac{\mathcal{P}_{2,t}^C}{\mathcal{P}_{1,t}^C} = 1$ . Then, the equalization of consumption across countries holds only for E = 1  $\Leftrightarrow \mathcal{P}_{2,t}^C = \mathcal{P}_{1,t}^C = 1 \Leftrightarrow P_{1,t} = P_{2,t}$ . But the arrival of the technological shock rules out this

possibility.<sup>10</sup> Then, on the one hand national and foreign consumption are disentangled because of the price effect. And on the other hand, because of the home-biased demands: the more  $\gamma_C$  is bigger than 1/2, the larger is the dissociation of consumptions.

**Responses in the** EXP1 and EXP2 scenarios. The dynamics described above still at work. Nevertheless,

- **EXP1.** If the share of national good on the consumption basket is lower than the one on the investment share (i.e.  $\gamma_C < \gamma_I$ ), the response of consumption is lower in the two countries (see the left hand panel in figure 2.3). But the fall in  $C_1^C$  is larger than the fall in  $C_2^C$ . This reduce the consumption gap between countries. Concerning country 2, the apparent reason is the lower deterioration of the country 1's terms of trade (p), which implies a lower wealth effect in country 2. Concerning country 1, the reduction in consumption seems to be due to the lower weight of the wealth effect relative to the substitution effects in the household's trade-off, which results from the lower p.
- **EXP2.** Conversely, if  $\gamma_C > \gamma_I$ , the instantaneous response of consumption in country 2 is negative while the response of labor supply is largely positive. This is explained by the higher increase of the relative price p, which implies a "negative wealth effect" in country 2.

It is worth to note the explanative power of our EXP2 configuration. It deals better with the major puzzle of predicting higher cross-country correlations for outputs than for consumptions (see table 2.3). And this is done in the fairly standard context of separable preferences.

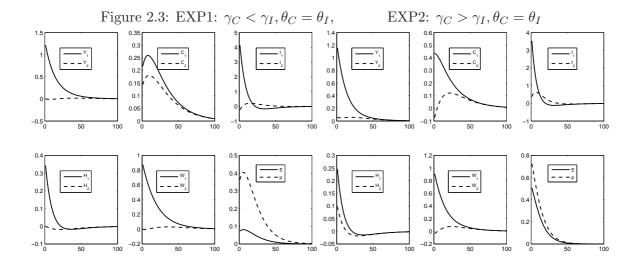
Non-separable Preferences. Finally, in stark contrast with the predictions from the canonical economy, the non-separability between consumption and leisure in the agents' preferences is not longer helpful to disentangle national and foreign consumptions. And this is true regardless of the scenario (see figure 2.4 for the BKK calibration, figure figure 2.5 for the GALI calibration, and figure 2.6 for our EXP calibrations).

In this case, the log-linearization of the labor supply equilibrium condition can be written as:

$$(1 - \gamma_C)\hat{p}_t + \eta \frac{H}{1 - H}\hat{H}_{1,t} = \hat{w}_{1,t}$$
(2.37)

$$-(1 - \gamma_C)\hat{p}_t + \eta \frac{H}{1 - H}\hat{H}_{2,t} = \hat{w}_{2,t}$$
 (2.38)

<sup>&</sup>lt;sup>10</sup>That is,  $p = P_2/P_1 = 1$ , which implies a null response of the linearized terms of trade, rather than the positive increase observed after the shock.



IRF - IRBC2-SP.

As in the canonical model, the instantaneous substitution effect explains the large and positive response of labor in country 1 to the increase in the real wage. Whereas in country 2 the labor supply increase to compensate, on the one hand, the fall in the real wage and, on the other hand the negative impact of the increase in p.

In other terms, the equilibrium condition for total hours in country 2 depends now also on p:

$$H_{2,t} = f(\alpha, \tilde{\sigma}, \eta, a_{2,t}, K_{2,t}, p_t)$$
(2.39)

Since a positive productivity shock impacting country 1 always induces an increase in p,  $H_2$  responds positively even if shocks are independent across countries. As consequence,  $C_2^C$  must increase much more than in the single-good model, to the equality of marginal consumptions across countries still holds:

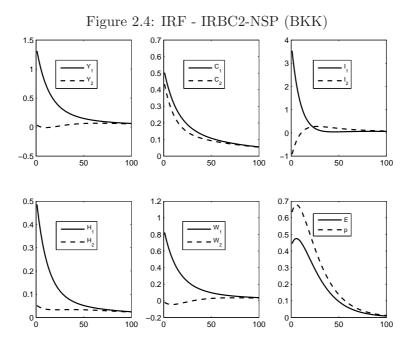
$$C_{1,t}^{C} + \frac{\hat{\sigma}(1 - H_{1,t})^{1-\eta}}{1 - \eta} = \frac{\mathcal{P}_{2,t}^{C}}{\mathcal{P}_{1,t}^{C}} \left( C_{2,t}^{C} + \frac{\tilde{\sigma}(1 - H_{2,t})^{1-\eta}}{1 - \eta} \right)$$
(2.40)

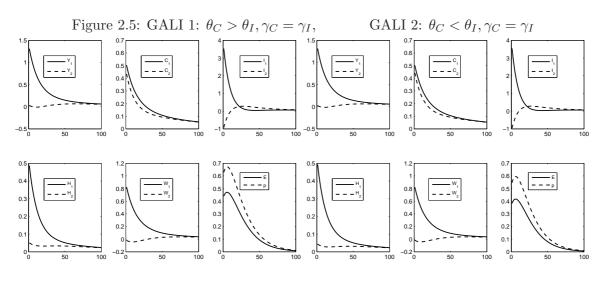
Indeed, the log-linearization of last equality yields to:

$$\hat{C}_{2,t}^C + \eta \frac{H}{1 - H} (\hat{H}_{1,t} - \hat{H}_{2,t}) - (2\gamma_C - 1)\hat{p}_t = \hat{C}_{1,t}^C$$
(2.41)

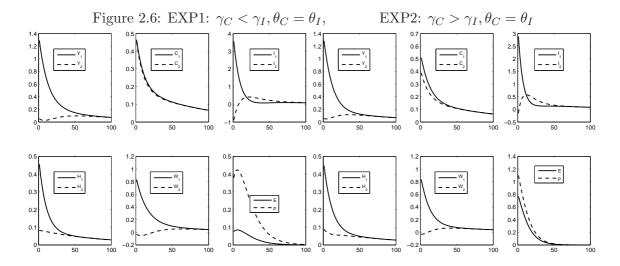
Then, the increase of consumption is similar in the two countries because the gap between national and foreign labor supply is largely damped by the increase in the relative price p.<sup>11</sup> Under non-separability, the only important difference that we remark among scenarios concerns the negative response at impact of productivity in country 2 (*i.e.*, of  $w_2$  in the right hand panel of figure 2.6).

 $<sup>^{11}2\</sup>gamma - 1 \ge 0 \text{ for } \gamma_C \ge 1/2.$ 





IRF - IRBC2-NSP.



IRF - IRBC2-NSP.

## 2.1.5 Quantitative Properties

To complete the analysis, we report the business cycle statistics of the several configurations. Results are summarized in table 2.3.

#### 2.1.6 Restricted Asset Markets

In this variant of the model agents are restricted to trade only goods and non-contingent real debt. As in Baxter and Crucini (1995), we assume that the single debt instrument is a one-period discount bond. Let  $P_t^B \equiv \frac{1}{1+r_t}$  denote the price per unit of one-period discount bonds purchased in period t,  $B_{i,t+1}$  (maturing in t+1).  $r_t$  denotes the world rate of return. Then, the flow budget constraints are:

$$\mathcal{P}_{1,t}^C C_{1,t}^C + P_t^B B_{1,t+1} \leq B_{1,t} + w_{1,t} H_{1,t} \qquad (\lambda_{1,t})$$
(2.42)

$$\mathcal{P}_{2,t}^{C}C_{2,t}^{C} + P_{t}^{B}B_{2,t+1} \leq B_{2,t} + p_{t}w_{2,t}H_{2,t} \qquad (\lambda_{2,t})$$
(2.43)

The relevant aspect of this asset market structure is that it plays a role in mitigating with the co-movement puzzle (Baxter and Crucini 1995) In particular, this structure limits agents' ability to engage in international risk-pooling, thereby reducing the incentive to increase labor input in response to wealth-increasing productivity shocks. Consequently, this eliminates a contributor of the negative investment and labor cross-country correlations predicted by most IRBC models.

General Equilibrium with Restricted Asset Markets The optimal choices for consumption, total hours and capital still unchanged, whereas the optimal choices for the non-contingent

Table 2.3: Cyclical properties - Two-good model

				SP					NSP		
	Data	BBK	GALI1	GALI2	EXP1	EXP2	BBK	GALI1	GALI2	EXP1	EXP2
${\rm International}^{a,e}$											
$ ho(Y_1,Y_2)$	0.51	0.214	0.161	0.204	0.225	0.322	0.355	0.350	0.337	0.336	0.345
$\rho(C_1^C, C_2^C)$	0.40	0.795	0.779	0.818	0.985	0.484	0.998	0.997	0.998	0.999	0.990
$ ho(H_1,H_2)$	0.36	-0.204	-0.264	-0.240	-0.233	0.548	0.465	0.460	0.437	0.540	0.579
$\rho(I_1^C, I_2^C)$	0.38	-0.246	-0.300	-0.239	-0.220	0.093	-0.620	-0.626	-0.630	-0.613	-0.448
$\sigma_{ au}/\sigma_{Y}$	1.90	0.294	0.303	0.272	0.075	0.549	0.357	0.352	0.316	0.071	0.704
$\sigma_{\Gamma}/\sigma_{Y}$	3.28	0.420	0.434	0.388	0.376	0.784	0.511	0.503	0.452	0.356	1.005
$\mathrm{USA}^{c,e}$											
$\sigma_Y$ (in %)	1.91	1.250	1.220	1.270	1.250	1.190	1.440	1.440	0.145	1.380	1.390
$\sigma_C/\sigma_Y$	0.40	0.463	0.458	0.456	0.445	0.535	0.780	0.772	0.766	0.767	0.775
$\sigma_H/\sigma_Y$	0.86	0.223	0.227	0.230	0.230	0.167	0.385	0.384	0.386	0.373	0.370
$\sigma_I/\sigma_Y$	3.07	3.055	3.090	3.043	3.024	2.653	2.070	2.071	2.051	2.051	1.730
$\sigma_{LP}/\sigma_{Y}$	0.57	0.788	0.785	0.782	0.783	0.838	0.616	0.617	0.615	0.632	0.637
$\sigma_W/\sigma_Y$	0.45	0.788	0.785	0.782	0.783	0.838	0.616	0.617	0.615	0.632	0.637
$\rho(Y_t, Y_{t-1})$	0.85	0.682	0.660	0.693	0.665	0.675	0.693	0.678	0.684	0.679	0.670
$\rho(C_t, C_{t-1})$	0.86	0.729	0.706	0.728	0.721	0.704	0.713	0.700	0.700	0.694	0.685
$\rho(H_t, H_{t-1})$	0.84	0.652	0.634	0.667	0.632	0.669	0.695	0.680	0.685	0.681	0.675
$\rho(I_t, I_{t-1})$	0.81	0.650	0.631	0.665	0.631	0.640	0.646	0.632	0.637	0.640	0.622
$\rho(LP_t, LP_{t-1})$	0.52	0.693	0.670	0.702	0.677	0.679	0.692	0.677	0.683	0.679	0.667
$\rho(Y,C)$	0.83	0.917	0.910	0.910	0.815	0.987	0.925	0.926	0.926	0.905	0.928
$\rho(Y, H)$	0.82	0.961	0.960	0.960	0.959	0.974	0.998	0.998	0.999	0.992	0.991
$\rho(Y, I)$	0.97	0.960	0.958	0.961	0.960	0.966	0.890	0.893	0.898	0.892	0.909
$\rho(Y, LP)$	0.51	0.997	0.997	0.997	0.996	0.999	0.999	0.999	0.999	0.998	0.997
$\rho(Y, W)$	0.28	0.997	0.997	0.997	0.996	0.999	0.999	0.999	0.999	0.998	0.997
$\rho(Y, LS)$	-0.30	-	-	-	-	-	_	-	-	-	-
$\rho(U, V)$	-0.89	-	-	-	-	-	_	-	-	-	-
$\rho(H, LP)$	-0.07	0.937	0.934	0.935	0.931	0.963	0.994	0.994	0.995	0.979	0.977
$\rho(H,W)$	0.03	0.937	0.934	0.935	0.931	0.963	0.994	0.994	0.995	0.979	0.977
$\rho(S,I)^d$	0.86	0.994	0.993	0.993	0.988	0.977	0.982	0.982	0.980	0.975	0.929
$\rho(Y, NX)^b$	-0.29	-0.347	-0.337	-0.254	0.180	-0.150	0.078	0.106	0.161	0.253	0.385

The moments reported are computed from Hodrick-Prescott filtered artificial time series. <sup>a</sup> Backus, Kehoe, and Kydland (1995). <sup>b</sup> Hairault (2002). <sup>c</sup> Chéron and Langot (2004). <sup>d</sup> Baxter and Crucini (1993). <sup>e</sup> Hairault (1995).

bonds, together with the envelop condition, lead in the two countries to

$$P_t^B = \beta E_t \left[ \frac{\lambda_{i,t+1}}{\lambda_{i,t}} \right] \tag{2.44}$$

Then,  $P_t^B$  defines the interest rate prevailing in the international financial market. We have in addition the following accumulation constraints:

$$P_t^B B_{1,t+1} + \mathcal{P}_{1,t}^C C_{1,t}^C + \mathcal{P}_{1,t}^I (I_{1,t}^C + \mathcal{C}_{1,t}) = Y_{1,t} + B_{1,t}$$
 (2.45)

$$P_t^B B_{2,t+1} + \mathcal{P}_{2,t}^C C_{2,t}^C + \mathcal{P}_{2,t}^I (I_{2,t}^C + \mathcal{C}_{2,t}) = p_t Y_{2,t} + B_{2,t}$$
 (2.46)

The key issue now is how to compute the world general equilibrium. To close the equilibrium we follow the Baxter and Crucini (1995)'s procedure: First, since the financial-market equilibrium conditions imply that only one of the financial stocks is independent, we drop one of the asset accumulation equations. Then, we treat the country 1's shadow price  $\lambda_{1,t}$  as an additional control variable. This determines the world control vector as a function of the world controlled state vector  $[K_{1,t}^C, K_{2,t}^C, B_{2,t}^C]$ , the world costate vector  $[q_{1,t}, q_{2,t}, \lambda_{2,t}]$ , and the exogenous variables  $[a_{1,t}, a_{2,t}]$ . Finally, we impose the equilibrium condition (2.44) for i = 1 into the remaining asset accumulation equation. So,

$$\beta E_t \left[ \frac{\lambda_{1,t+1}}{\lambda_{1,t}} \right] B_{2,t+1} + \mathcal{P}_{2,t}^C C_{2,t}^C + P_{2,t}^I (I_{2,t}^C C_{2,t}) - p_t Y_{2,t} - B_{2,t} = 0$$
 (2.47)

Empirical results When financial markets are complete, the wealth effects are equally shared between the households of the two countries. This implies an asymmetric evolution of the labor supply. This, together with the separability of preferences lead to highly correlated consumptions. That is why, when the risk-sharing (and so the wealth sharing) is restricted, we obtain lower consumptions correlation and less negative cross-country correlation of labor input and investment, with separable preferences. Conversely, the non-separability yields to larger negative correlations of investments and labor. This seems in part due to the price effect at work.

#### 2.1.7 Section conclusion

The distinction of the elasticity of substitution of goods in the consumption basket from that in the investment basket does not change the model dynamics (labels BKK and GALI). This is because, converse to the Galí (1994)'s, framework with monopolistic competition, we assume perfect competition in all markets.

Table 2.4: Cyclical properties - Two-good model with incomplete markets

	Data	BKK(SP)	BKK(NSP)
International <sup>a</sup>			
$ ho(Y_1,Y_2)$	0.51	0.27	0.19
$\rho(C_1^C, C_2^C)$	0.40	0.80	0.87
$\rho(H_1, H_2)$	0.36	-0.04	-0.88
$\rho(I_1^C,I_2^C)^b$	0.38	-0.18	-0.63
$\sigma_ au/\sigma_Y$	1.78	0.44	0.47
$\sigma_{\Gamma}/\sigma_{Y}$	3.28	0.26	0.28
$\mathrm{USA}^{c,e}$			
$\sigma_Y$ (in %)	1.91	1.25	1.18
$\sigma_C/\sigma_Y$	0.40	0.47	0.70
$\sigma_I/\sigma_Y$	3.07	2.88	2.46
$\sigma_H/\sigma_Y$	0.86	0.20	0.15
$\sigma_{LP}/\sigma_{Y}$	0.57	0.79	0.88
$\sigma_W/\sigma_Y$	0.45	0.79	0.88
$\rho(Y_t, Y_{t-1})$	0.85	0.69	0.67
$\rho(C_t, C_{t-1})$	0.86	0.73	0.69
$\rho(H_t, H_{t-1})$	0.84	0.66	0.62
$\rho(I_t, I_{t-1})$	0.81	0.65	0.62
$\rho(LP_t, LP_{t-1})$	0.52	0.70	0.68
$\rho(Y,C)$	0.83	0.93	0.90
$\rho(Y, H)$	0.82	0.97	0.79
$\rho(Y, I)$	0.97	0.95	0.88
$\rho(Y, LP)$	0.51	0.99	0.99
$\rho(Y, W)$	0.28	0.99	0.99
$\rho(Y, LS)$	-0.30	_	_
$\rho(U,V)$	-0.89	_	_
$\rho(H, LP)$	-0.07	0.95	0.73
$\rho(H, W)$	0.03	0.95	0.73
$\rho(S,I)^d$	0.86	0.99	0.98
$\rho(Y, NX)^a$	-0.22	-0.32	-0.27

The moments reported are computed from Hodrick-Prescott filtered artificial time series.  $^a$  Backus, Kehoe, and Kydland (1995).  $^b$  Hairault (2002).  $^c$  Chéron and Langot (2004).  $^d$  Baxter and Crucini (1993).  $^e$  Hairault (1995).

On the other hand, the distinction of the consumption basket from the production basket, at the level of the share of the national good, improves the prediction of the model regarding the major puzzle of predicting higher cross-country correlations for outputs than for consumptions when  $\gamma$ . An this in the fairly standard context of separable preferences. However, we have not empirical data on the issue.

Finally, even if the restricted asset markets structure plays a role in mitigating with the comovement puzzle, it is not enough to replicate the observed international fluctuations.

# 2.2 The two-sector economy

The economy is disaggregated into internationally traded and non-traded sectors. Each country is now specialized in the production of two goods: a tradable good T and a non-tradable good NT. Fluctuations arise from persistent shocks to productivity in both sectors that are diffused either across sectors and internationally. The productivity shock vector  $A = [a_1^T, a_2^T, a_1^{NT}, a_2^{NT}]'$  follows a VAR(1) process in natural logarithms:

$$\ln A_{t+1} = \Omega \ln A_t + \Psi \epsilon_{t+1}, \epsilon_{t+1} \quad \sim \quad iid \quad \mathcal{N}(0, \mathbf{I}). \tag{2.48}$$

The vector  $\epsilon_t^a = [\epsilon_1^T, \epsilon_2^T, \epsilon_1^{NT}, \epsilon_2^{NT}]'$  represents the innovations to productivity variables.  $\Omega$  is a (4x4) matrix describing the autoregressive component of the disturbance, whereas  $\Psi$  is a (4x4) matrix governing the instantaneous and the delayed transmission of the shock.

#### 2.2.1 Households

Households consume a nested CES basket of the three goods available to them: the national tradable good, the foreign tradable good, and the country-specific non-tradable good. The composite-consumption good  $C_{i,t}^C$  has now the following two-stages CES structure:

$$C_i^C = \left[ \gamma^{\frac{1}{\theta}} (C_i^T)^{\frac{\theta - 1}{\theta}} + (1 - \gamma)^{\frac{1}{\theta}} (C_i^{NT})^{\frac{\theta - 1}{\theta}} \right]^{\frac{\theta}{\theta - 1}}$$

$$(2.49)$$

whit

$$C_{1}^{T} = \left[ \gamma_{T}^{\frac{1}{\theta_{T}}} (CT_{1}^{1})^{\frac{\theta_{T}-1}{\theta_{T}}} + (1 - \gamma_{T})^{\frac{1}{\theta_{T}}} (CT_{2}^{1})^{\frac{\theta_{T}-1}{\theta_{T}}} \right]^{\frac{\theta_{T}}{\theta_{T}-1}}$$
(2.50)

and

$$C_2^T = \left[ \gamma_T^{\frac{1}{\theta_T}} (CT_2^2)^{\frac{\theta_T - 1}{\theta_T}} + (1 - \gamma_T)^{\frac{1}{\theta_T}} (CT_1^2)^{\frac{\theta_T - 1}{\theta_T}} \right]^{\frac{\theta_T}{\theta_T - 1}}$$
(2.51)

 $CT_{m,t}^n$ , for m, n = 1, 2, denotes the demand for the tradable-good produced in country m from agents in country n. This time  $\theta$  corresponds to the elasticity of substitution between tradable and non-tradable goods, whereas  $\theta_T$  stands for the elasticity of substitution between the national and the foreign tradable goods. The price index of the final-consumption good is given by:

$$P_i^C = \left[ \gamma(P_i^T)^{1-\theta} + (1-\gamma)(P_i^{NT})^{1-\theta} \right]^{\frac{1}{1-\theta}}$$
 (2.52)

where  $P_i^{NT}$  is the production price of the non-tradable goods and  $P_i^T$  is the price of the composite tradable goods. Similar to the one-sector, two-good economy, this price index is given by:

$$P_i^T = \left[ \gamma_T(P_i)^{1-\theta_T} + (1 - \gamma_T)(P_{j \neq i})^{1-\theta_T} \right]^{\frac{1}{1-\theta_T}}$$
 (2.53)

where  $P_{i,t}$  denotes as before the production price of the tradable good produced in country i. In this economy, we still take the tradable good 1 as accounting unit, as well as we still assume that wages are paid in units of the national tradable good. With two sectors in the economy, there are two representative firms per country: the one in the tradable-goods sector (thereafter,  $Firm_i^T$ ), and the one on the non-tradable-goods sector (thereafter,  $Firm_i^{NT}$ ).

At each time, the representative household in country i works  $H_{i,t}^T$  hours in the  $Firm_i^T$  and the remaining  $H_{i,t}^{NT} = H_{i,t} - H_{i,t}^T$  in the  $Firm_i^{NT}$ , at the wage rate  $w_{i,t}$ , which is given in units of the national tradable good. Then, once we normalize the production price of the tradable good  $1 (P_{1,t} \equiv 1 \ \forall t)$ , the budget constraints can be written as:

$$\mathcal{P}_{1,t}^C C_{1,t}^C + \int v_t B_{1,t+1} dA_{t+1} \le B_{1,t} + w_{1,t} H_{1,t} \qquad (\lambda_{1,t})$$
(2.54)

$$\mathcal{P}_{2,t}^{C}C_{2,t}^{C} + \int v_{t}B_{2,t+1}dA_{t+1} \le B_{2,t} + p_{t}w_{2,t}H_{2,t} \qquad (\lambda_{2,t})$$
(2.55)

Then, the dynamic problem solved for the representative household inhabiting country i = 1, 2 is:

$$W_{i,t}^{H} \equiv \max_{C_{i,t}^{C}, B_{i,t+1}} \left\{ U(C_{i,t}^{C}, 1 - H_{i,t}) + \beta \int W_{i,t+1}^{H} f(A) dA \right\}$$
 (2.56)

subject to her budget constraint ((2.54) or (2.55)). Given the two specifications of the instantaneous utility, the optimal conditions for consumption and labor supply are,

Separable preferences: Non-separable preferences: for 
$$i = 1, 2$$
:  $\left(\mathcal{P}_{i,t}^{C} C_{i,t}^{C}\right)^{-1} = \lambda_{i,t}, \qquad \left(\mathcal{P}_{i,t}^{C} \left(C_{i,t}^{C} + \tilde{\Gamma}_{i,t}\right)\right)^{-1} = \lambda_{i,t}$  for  $i = 1$ :  $w_{1,t} = \mathcal{P}_{1,t}^{C} C_{1,t}^{C} \sigma (1 - H_{1,t})^{-\eta}, \qquad w_{1,t} = \mathcal{P}_{1,t}^{C} \tilde{\sigma} (1 - H_{1,t})^{-\eta}$  for  $i = 2$ :  $p_{t}w_{2,t} = \mathcal{P}_{2,t}^{C} C_{2,t}^{C} \sigma (1 - H_{2,t})^{-\eta}, \quad p_{t}w_{2,t} = \mathcal{P}_{2,t}^{C} \tilde{\sigma} (1 - H_{2,t})^{-\eta}$ 

Finally, the equilibrium condition of the asset holdings implies  $v_t = \beta \frac{\lambda_{i,t+1}}{\lambda_{i,t}} f(A)$ . Even if these conditions look very similar to those obtained in the national-specialization economy, the main difference is in the structure of the price index  $\mathcal{P}_{i,t}^C$ , which now is a function of the production price of the three goods available to the household. Since all relative prices vary following a productivity shock, the "aggregate price effect" may lead to different dynamics than in the previous model.

#### 2.2.2 Firms

Matter of simplicity, and given the previous results for the national specialization economy, we assume that investment has the same CES structure as consumption. That is, investment to each sector is a fraction of the total composite capital good.

The representative firm in sector j = T, NT of country i = 1, 2 produces output with a constant returns to scale technology using composite capital  $K_i^j$  and labor  $H_i^j$  as inputs,

$$Y_{i,t}^{j} = a_{i,t}^{j} (K_{i,t}^{j})^{\alpha_{j}} (H_{i,t}^{j})^{1-\alpha_{j}}$$
(2.58)

New capital goods are internationally mobile and all investment is subject to quadratic costs of adjustment  $C_{i,t}^j = \frac{\phi}{2}(K_{i,t+1}^j - K_{i,t}^j)^2$  Matter of simplicity, we assume that investment has the same structure as consumption. Capital evolves according to

$$K_{i,t+1}^{j} = (1 - \delta)K_{i,t}^{j} + I_{i,t}^{j}$$
(2.59)

Thus, investment to industry j = T, NT is a fraction of the total composite capital.

#### Tradable-goods sector

The representative  $Firm_1^T$  aims at maximizing the expected discounted sum of profit flows, given in units of tradable good 1:

$$\mathcal{W}^{F,T}(K_{1,t}^T) = \max \left\{ Y_{1,t}^T - \mathcal{P}_{1,t}^C \mathcal{C}_{1,t}^T - \mathcal{P}_{1,t}^C I_{1,t}^T - w_{1,t} H_{1,t}^T + \int v_t \mathcal{W}(K_{1,t+1}^T) dA_{t+1} \right\}$$
(2.60)

subject to constrains (2.58) and (2.59), for i = 1, j = T. Then, the firm's optimal choices imply,

$$w_{1,t} = (1 - \alpha_T) \frac{Y_{1,t}^T}{H_{1,t}^T} \tag{2.61}$$

$$q_{1,t}^T \equiv \mathcal{P}_{1,t}^C \left[ 1 + \phi \left( I_{1,t}^T - \delta K_{1,t}^T \right) \right]$$
 (2.62)

$$q_{1,t}^T = \int v_t \left( \alpha_T \frac{Y_{1,t+1}^T}{K_{1,t+1}^T} + q_{1,t+1}^T - \delta \mathcal{P}_{1,t+1}^C \right) dA_{t+1}$$
 (2.63)

The program of the representative  $Firm_2^T$ , expressed in units of tradable good 1, is analogous:

$$\mathcal{W}^{F,T}(K_{2,t}^T) = \max \left\{ p_t Y_{2,t}^T - \mathcal{P}_{2,t}^C \mathcal{C}_{2,t}^T - \mathcal{P}_{2,t}^C I_{2,t}^T - p_t w_{2,t} H_{2,t}^T + \int v_t \mathcal{W}(K_{2,t+1}^T) dA_{t+1} \right\}$$
(2.64)

subject to constrains (2.58) and (2.59), for i = 2, j = 2. Then,

$$w_{2,t} = (1 - \alpha_T) \frac{Y_{2,t}^T}{H_{2,t}^T} \tag{2.65}$$

$$q_{2,t}^T \equiv \mathcal{P}_{2,t}^C \left[ 1 + \phi \left( I_{2,t}^T - \delta K_{2,t}^T \right) \right]$$
 (2.66)

$$q_{2,t}^T = \int v_t \left( \alpha_T \frac{p_{t+1} Y_{2,t+1}^T}{K_{2,t+1}^T} + q_{2,t+1}^T - \delta \mathcal{P}_{2,t+1}^C \right) dA_{t+1}$$
 (2.67)

## Non-tradable-goods sector

Similarly, the profit flows of the representative  $Firm_1^{NT}$  are given in units of tradable good 1, so the dynamic problem is written as:

$$\mathcal{W}^{F,NT}(K_{1,t}^{NT}) = \max \left\{ P_{1,t}^{NT} Y_{1,t}^{NT} - \mathcal{P}_{1,t}^{C} \mathcal{C}_{1,t}^{NT} - \mathcal{P}_{1,t}^{C} I_{1,t}^{NT} - w_{1,t} H_{1,t}^{NT} + \int v_{t} \mathcal{W}(K_{1,t+1}^{NT}) dA_{t+1} \right\}$$
(2.68)

subject to constrains (2.58) and (2.59), for i = 1, j = NT. Then,

$$w_{1,t} = P_{1,t}^{NT} (1 - \alpha_{NT}) \frac{Y_{1,t}^{NT}}{H_{1,t}^{NT}}$$
(2.69)

$$q_{1,t}^{NT} \equiv \mathcal{P}_{1,t}^{C} \left[ 1 + \phi \left( I_{1,t}^{NT} - \delta K_{1,t}^{NT} \right) \right]$$
 (2.70)

$$q_{1,t}^{NT} = \int v_t \left( \alpha_{NT} \frac{P_{1,t+1}^{NT} Y_{1,t+1}^{NT}}{K_{1,t+1}^{NT}} + q_{1,t+1}^{NT} - \delta \mathcal{P}_{1,t+1}^C \right) dA_{t+1}$$
 (2.71)

Likewise, the program of the representative  $Firm_2^{NT}$ , expressed in units of tradable good 1, is:

$$W^{F,NT}(K_{2,t}^{NT}) = \max \left\{ P_{2,t}^{NT} Y_{2,t}^{NT} - \mathcal{P}_{2,t}^{C} \mathcal{C}_{2,t}^{NT} - \mathcal{P}_{2,t}^{C} I_{2,t}^{NT} - p_{t} w_{2,t} H_{2,t}^{NT} + \int v_{t} \mathcal{W}(K_{2,t+1}^{NT}) dA_{t+1} \right\}$$
(2.72)

subject to constrains (2.58) and (2.59), for i = 2, j = NT. Then,

$$p_t w_{2,t} = P_{2,t}^{NT} (1 - \alpha_{NT}) \frac{Y_{2,t}^{NT}}{H_{2,t}^{NT}}$$
(2.73)

$$q_{2,t}^{NT} \equiv \mathcal{P}_{2,t}^{C} \left[ 1 + \phi \left( I_{2,t}^{NT} - \delta K_{2,t}^{NT} \right) \right]$$
 (2.74)

$$q_{2,t}^{NT} = \int v_t \left( \alpha_{NT} \frac{P_{2,t+1}^{NT} Y_{2,t+1}^{NT}}{K_{2,t+1}^{NT}} + q_{2,t+1}^{NT} - \delta \mathcal{P}_{2,t+1}^C \right) dA_{t+1}$$
 (2.75)

# 2.2.3 Equilibrium

**Equilibrium on the Goods Market:** The equilibrium on the goods market is given by the accounting equations for the output of each of the four goods,

$$Y_{1,t}^{T} = CT_{1,t}^{1} + CT_{1,t}^{2} + IT_{1,t}^{1} + IT_{1,t}^{2} + \mathcal{C}_{1,t}^{1,T} + \mathcal{C}_{1,t}^{2,T}$$

$$= \gamma_{T}\gamma \left(P_{1,t}^{T}\right)^{\theta_{T}-\theta} \left(\mathcal{P}_{1,t}^{C}\right)^{\theta} \left(C_{1,t}^{C} + I_{1,t}^{C} + \mathcal{C}_{1,t}^{T}\right) + \gamma(1-\gamma_{T}) \left(P_{2,t}^{T}\right)^{\theta_{T}-\theta} \left(\mathcal{P}_{2,t}^{C}\right)^{\theta} \left(C_{2,t}^{C} + I_{2,t}^{C}\right)$$

$$Y_{2,t}^{T} = CT_{2,t}^{1} + CT_{2,t}^{2} + IT_{2,t}^{1} + IT_{2,t}^{2} + \mathcal{C}_{2,t}^{1,T} + \mathcal{C}_{2,t}^{2,T}$$

$$= \gamma(1-\gamma_{T}) \left(P_{1,t}^{T}\right)^{\theta_{T}-\theta} \left(\mathcal{P}_{1,t}^{C}\right)^{\theta} p_{t}^{-\theta_{T}} \left(C_{1,t}^{C} + I_{1,t}^{C}\right) + \gamma_{T}\gamma \left(P_{2,t}^{T}\right)^{\theta_{T}-\theta} \left(\mathcal{P}_{2,t}^{C}\right)^{\theta} p_{t}^{-\theta_{T}} \left(C_{2,t}^{C} + I_{2,t}^{C} + \mathcal{C}_{2,t}^{T}\right)$$

$$Y_{i,t}^{NT} = CNT_{i,t} + INT_{i,t} + \mathcal{C}_{i,t}^{NT} = (1-\gamma) \left(\frac{\mathcal{P}_{i,t}^{C}}{P_{i,t}^{NT}}\right)^{\theta} \left(C_{i,t}^{C} + I_{i,t}^{C} + \mathcal{C}_{i,t}^{NT}\right)$$

$$(2.78)$$

where the subindex denotes the origin of goods, whereas the exponent denotes their destination.

**Equilibrium on the Labor Market:** The equilibrium on the labor market implies in particular the equalization of the marginal productivity of labor across sectors:

Separable preferences:

for 
$$i = 1$$
: 
$$\mathcal{P}_{1,t}^{C} C_{1,t}^{C} \sigma (1 - H_{1,t})^{-\eta} = (1 - \alpha_{T}) \frac{Y_{1,t}^{T}}{H_{1,t}^{T}}$$

$$(1 - \alpha_{T}) \frac{Y_{1,t}^{T}}{H_{1,t}^{T}} = (1 - \alpha_{NT}) \frac{Y_{1,t}^{NT}}{H_{1,t}^{NT}}$$

$$(2.79)$$
for  $i = 2$ : 
$$\mathcal{P}_{2,t}^{C} C_{2,t}^{C} \sigma (1 - H_{2,t})^{-\eta} = p_{t} (1 - \alpha_{T}) \frac{Y_{1,t}^{T}}{H_{1,t}^{T}}$$

$$p_{t} (1 - \alpha_{T}) \frac{Y_{1,t}^{T}}{H_{1,t}^{T}} = P_{2,t}^{NT} (1 - \alpha_{NT}) \frac{Y_{2,t}^{NT}}{H_{2,t}^{NT}}$$

Separable preferences:

for 
$$i = 1$$
: 
$$\mathcal{P}_{1,t}^{C}\tilde{\sigma}(1 - H_{1,t})^{-\eta} = (1 - \alpha_{T}) \frac{Y_{1,t}^{T}}{H_{1,t}^{T}}$$
$$(1 - \alpha_{T}) \frac{Y_{1,t}^{T}}{H_{1,t}^{T}} = (1 - \alpha_{NT}) \frac{Y_{1,t}^{NT}}{H_{1,t}^{NT}}$$
$$(2.80)$$
for  $i = 2$ : 
$$\mathcal{P}_{2,t}^{C}\tilde{\sigma}(1 - H_{2,t})^{-\eta} = p_{t}(1 - \alpha_{T}) \frac{Y_{1,t}^{T}}{H_{1,t}^{T}}$$
$$p_{t}(1 - \alpha_{T}) \frac{Y_{1,t}^{T}}{H_{1,t}^{T}} = P_{2,t}^{NT}(1 - \alpha_{NT}) \frac{Y_{2,t}^{NT}}{H_{2,t}^{NT}}$$

Equilibrium on the Financial and Capital Market: As before, from the equation guarantying the equilibrium on the financial market, and under the assumptions that: (i) financial assets are in units of tradable good 1, and (ii) all households have the same initial wealth distribution, we deduce that  $\lambda_{1,t} = \lambda_{2,t} = \lambda_t$ . Then, the evolution of the implicit prices of capital

 $q_{i,t}^{j}$ , for j=T,NT and i=1,2, can be written as:

$$q_{1,t}^{T} = \beta E_{t} \left[ \frac{\lambda_{t+1}}{\lambda_{t}} \left( \alpha_{T} \frac{Y_{1,t+1}^{T}}{K_{1,t+1}^{T}} + q_{1,t+1}^{T} - \delta \mathcal{P}_{1,t+1}^{C} \right) \right]$$
(2.81)

$$q_{2,t}^{T} = \beta E_{t} \left[ \frac{\lambda_{t+1}}{\lambda_{t}} \left( \alpha_{T} p_{t+1} \frac{Y_{2,t+1}^{T}}{K_{2,t+1}^{T}} + q_{2,t+1}^{T} - \delta \mathcal{P}_{2,t+1}^{C} \right) \right]$$
(2.82)

$$q_{1,t}^{NT} = \beta E_t \left[ \frac{\lambda_{t+1}}{\lambda_t} \left( \alpha_{NT} P_{1,t+1}^{NT} \frac{Y_{1,t+1}^{NT}}{K_{1,t+1}^{NT}} + q_{1,t+1}^{NT} - \delta \mathcal{P}_{1,t+1}^C \right) \right]$$
 (2.83)

$$q_{2,t}^{NT} = \beta E_t \left[ \frac{\lambda_{t+1}}{\lambda_t} \left( \alpha_{NT} P_{2,t+1}^{NT} \frac{Y_{2,t+1}^{NT}}{K_{2,t+1}^{NT}} + q_{2,t+1}^{NT} - \delta \mathcal{P}_{2,t+1}^C \right) \right]$$
 (2.84)

with  $\lambda_t$  given by equations in the first line of expressions (2.57), according to the specification of the utility function. Once again, the dynamics of  $\lambda$ , and then the response of consumptions, are affected by the variations in the prices of goods, but this time there are more than one relative price of goods, and all of them change following a productivity shock.

# 2.2.4 Empirical results

#### **Parameterization**

We are interested in the properties of a zero net exports symmetrical equilibrium in which both countries are equally sized and have identical preferences and technology. In this case, the terms of trade p are equal to unity. This, together with the normalization of  $P_1$ , imply that  $P_2 = 1$ . Then  $P^T = 1$ . The steady state value of Tobin's  $q^j$  is set to unity so that the model with adjustment costs has the same deterministic steady state as the model without adjustment costs (Baxter and Crucini 1995). Then we deduce  $\mathcal{P}^C = 1$  and  $P^{NT} = 1$ . So, at the steady state all the prices are equal to unity. The discount factor  $\beta$ , the depreciation rate  $\delta$  and the capital adjustment cost parameter  $\phi$  are still with the same values as in the two-good economy.

The two production function parameters match the labor's shares of income in goods and services over the 1964-1987 period (Stockman and Tesar 1995):  $1 - \alpha^T = 0.74$  and  $1 - \alpha^{NT} = 0.64$ . So,  $K^T = \alpha^T Y^T / (1/\beta - 1 + \delta)$  and  $K^{NT} = \alpha^{NT} Y^{NT} / (1/\beta - 1 + \delta)$ . Then,  $I^T = \delta K^T$ ,  $I^{NT} = \delta K^{NT}$  and  $I^C = \delta(K^T + K^{NT})$ . Total output is normalized to unity and  $Y^T = Y^{NT} = Y/2$  (Corsetti, Dedola, and Leduc 2004), (Stockman and Tesar 1995).

At the steady state,  $C^C = Y - I^C$ . Then,  $\gamma = 1 - \frac{Y^{NT}}{C^C + I^C}$ . We set the elasticity of substitution between foreign and domestic tradable goods  $\theta_T$  equal to 1.5 (Backus, Kehoe, and Kydland 1994), whereas the elasticity of substitution between tradable and non-tradable goods  $\theta$  is set

equal to 0.74 (Corsetti, Dedola, and Leduc 2004). 12

For the productivity shocks, we adopt either the Stockman and Tesar' and Corsetti, Dedola, and Leduc (2004)'s calibrations. In both cases technology shocks are identified with Solow residuals in each sector, using annual data on manufacturing and services. The Stockman and Tesar's estimates of technology shocks are weakly persistent, particularly in the tradable-good sector. Conversely, the Corsetti *et al.*'s estimates are fairly persistent in both sectors, and the impact of the non-tradable good sector on the tradable good-sector is insignificantly different from zero.<sup>13</sup> In both cases, spill-overs across countries and sectors are not negligible.

Stockman and Tesar (1995) Corsetti et al.(2004) 0.154-0.1990.2620.110.190.310.2620.0400.190.1990.1540.110.780.31-0.0150-0.1100.6320.125-0.040.01 0.990.05-0.0150.12550.632 0.01 -0.040.05 0.99 -0.1100.01230.015 0.03620.00510.0123 0.026 0.0540.015 0.003 0 0.0123 0.0027 0.0199 0.003 0.008

Table 2.5: Technological shocks - Two-sector economy

The share of national tradable good in the composite consumption/investment tradable basket is set so that allows to replicate the 15% stationary ratio of imports to GDP (Backus, Kehoe, and Kydland 1994):  $Y_2^1/Y = 0.15 \Rightarrow Y_2^1/Y^T = 0.15Y/Y^T$ . From the household's optimal choices of the composite baskets we have that  $p^{-\theta_T} = \frac{\gamma_T}{1-\gamma_T} \frac{Y_2^1}{Y_1^1} 14$ . But at the steady state p=1 so  $\gamma_T = \frac{Y_1^1/Y_2^1}{1+Y_1^1/Y_2^1}$ . To determine the ratio  $Y_1^1/Y_2^1$  we use:  $Y^T = Y_1^1 + Y_1^2 \Leftrightarrow Y_1^1/Y^T = 1 - Y_1^2/Y^T$ . Symmetry implies that  $Y_1^2 = Y_2^1$  so  $Y_1^1/Y_2^1 = \frac{1-\omega^T V^T - Y_2^1/Y^T}{Y_2^1/Y^T}$ . All this imply a roughly value of 0.7 for  $\gamma_T$ . The households' optimal choices between tradable and non-tradable goods imply that  $C^{NT} = (1-\gamma)(\mathcal{P}^C/P^{NT})^\theta C^C$  and  $C^T = \gamma(P^C/P^T)^\theta C^C$ . Then,  $C^T/C^C = \gamma$  and  $\frac{C^{NT}}{C^C} = 1 - \gamma$ . Finally, total hours are fixed to 1/3.

This value corresponds to an estimation referred to a sample of industrialized countries. Stockman and Tesar (1995) estimate a lower elasticity (0.44) but their sample includes both developed and developing countries.

<sup>&</sup>lt;sup>13</sup>Then, for simulations the cross-correlation pairs  $\rho(NT_{i,t},T_{i,t})$  for 1=1,2, where set equal to zero.

 $<sup>^{14}</sup>Y_i^i$  denotes uses (consumption and investment) of tradable goods j in country i.

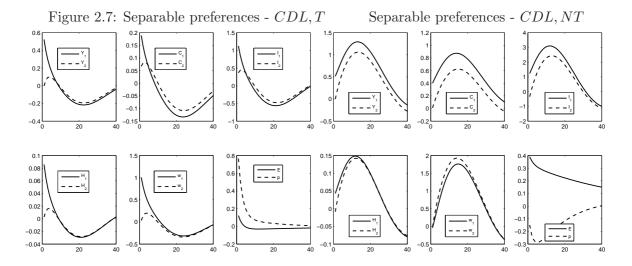
<sup>&</sup>lt;sup>15</sup>This value is very close to the Corsetti *et al.*(2004) estimation (0.72) and no far from the value given in Backus *et al.*(1994) (0.76).

#### Models evaluation

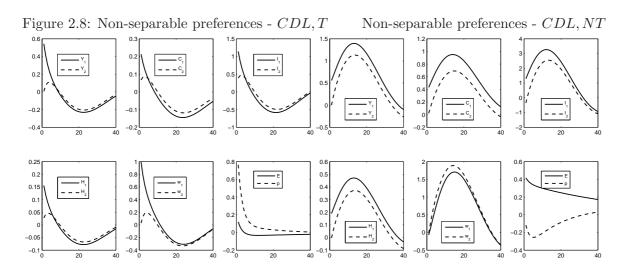
First of all, we note the general poor performance of this economy in almost all dimensions. In particular regarding the US standard deviation of real per-capita output. Whereas with the Stockman and Tesar's (thereafter, ST) calibration it is understated, with the Corsetti et al.'s (thereafter, CDL) calibration it is largely overstated.

Regarding the international fluctuations, the striking implication from the ST calibration, where shocks are weakly persistent, is the large dissociation of consumptions, for whatever specification of preferences. However, even with weak persistent shocks (i.e. the TS calibration) the model is able to reproduce positive cross-country correlations of investments and labor inputs. This is at odds with the implications from the Stockman and Tesar (1995)'s pioneer model. The key difference between our model and theirs is the structure of investment to each sector. In the Stockman and Tesar's model, each good is used for consumption and investment in its own sector, so that capital is industry-specific. Conversely, we assume important linkages of goods on both the demand side and the supply side. This explains why the quasi-perfect synchronization of national and foreign economies under the CDL calibration, with larger persistence of shocks. Finally, the intuitions we can drawn from the IRF functions (in particular in the CDL case) are roughly the same as those from the two-goods economy. That is, the basic mechanisms of the walrasian economy still at work following a positive shock to productivity in some sector. Look, for instance, at figures 2.7 and 2.8, for the CDL calibration. The main differences with respect to the IRF from national specialization economy are the hump-shaped response of country 2 variables and the instantaneous response of consumptions. The first one is due to either the international diffusion of the shock and the international linkages of productions. On the other hand, the dynamics of consumption are much harder to understand, mainly because their profile is pretty the same for the two specifications of preferences.

The analogous IRF functions from the ST calibration are shown in figures B.1 and B.2 (see appendix B.2). With respect to the IRF from the CDL calibration we remark that, because shocks are less persistent, all variables come back to their steady state values in a few periods. This is particularly true for the responses to a positive shock to the non-tradable-goods sector.



International two-country, two-good real business cycle model with separable preferences. Left-hand side panel: IRF to a positive 1% productivity shock to the country 1 tradable-goods sector. Right-hand side panel: IRF to a positive 1% productivity shock to the country 1 non-tradable-goods sector.



International two-country, two-good real business cycle model with non-separable preferences. Left-hand side panel: IRF to a positive 1% productivity shock to the country 1 tradable-goods sector. Right-hand side panel: IRF to a positive 1% productivity shock to the country 1 non-tradable-goods sector.

Table 2.6: Cyclical properties

		Separable	preferences	Non-separable	preferences	
	Data	CDL	ST	CDL	ST	
International $^{a,e}$						
$ ho(Y_1,Y_2)$	0.51	0.94	0.35	0.94	0.35	
$\rho(C_1^C, C_2^C)$	0.40	0.88	0.66	0.90	0.63	
$ \rho(H_1, H_2) $	0.36	0.91	0.42	0.96	0.54	
$\rho(I_1^C, I_2^C)$	0.38	0.94	0.67	0.94	0.66	
$\mathrm{USA}^{c,e}$						
$\sigma_Y$ (in %)	1.91	0.98	3.61	1.02	3.81	
$\sigma_C/\sigma_Y$	0.40	0.55	0.46	0.56	0.48	
$\sigma_H/\sigma_Y$	0.86	0.15	0.16	0.33	0.31	
$\sigma_I/\sigma_Y$	3.07	2.69	2.53	2.63	2.46	
$\sigma_{LP}/\sigma_{Y}$	0.57	0.84	0.84	0.67	0.69	
$\sigma_W/\sigma_Y$	0.45	1.75	1.54	1.63	1.43	
$\rho(Y_t, Y_{t-1})$	0.85	0.76	0.04	0.76	0.03	
$\rho(C_t, C_{t-1})$	0.86	0.78	0.06	0.77	0.05	
$\rho(H_t, H_{t-1})$	0.84	0.75	0.04	0.76	0.04	
$\rho(I_t, I_{t-1})$	0.81	0.77	0.06	0.77	0.05	
$\rho(LP_t, LP_{t-1})$	0.52	0.76	0.03	0.76	0.03	
$\rho(Y,C)$	0.83	0.96	0.98	0.97	0.98	
$\rho(Y, H)$	0.82	0.99	1.00	0.99	0.99	
$\rho(Y, I)$	0.97	0.98	0.98	0.98	0.98	
$\rho(Y, LP)$	0.51	1.00	1.00	1.00	1.00	
$\rho(Y, W)$	0.28	0.96	0.98	0.92	0.98	
$\rho(H, LP)$	-0.07	0.99	1.00	0.99	0.98	
$\rho(H,W)$	0.03	0.97	0.98	0.92	0.97	

International two-country, two-sector real business cycle model. CDL: the model calibrated with the Corsetti, Dedola, and Leduc (2004)'s estimates for the productivity shock. ST: the model calibrated with the Stockman and Tesar (1995)'s estimates for the productivity shock. The moments reported are computed from Hodrick-Prescott filtered artificial time series. <sup>a</sup> Backus, Kehoe, and Kydland (1995). <sup>b</sup> Hairault (2002). <sup>c</sup> Chéron and Langot (2004). <sup>d</sup> Baxter and Crucini (1993). <sup>e</sup> Hairault (1995).

# 2.3 Labor market search economies

In this section, we introduce search and matching on the labor market, so we focus mainly on the new aspects added to the corresponding walrasian economies.

#### 2.3.1 National specialization

In this context, agents still demand a CES basket of the two goods for consumption and investment purposes. They participate in the trade on the labor market. Employees  $(N_{i,t})$  work  $h_{i,t}$  units of time at a wage rate of  $w_{i,t}$  and unemployed workers  $(1 - N_{i,t})$  devote  $e_i$  units of time to seeking employment. The rate  $M_{i,t}$  at which new job matches are formed is governed by a standard aggregate matching function while separations of job-worker pairs occurs at the exogenous rate  $s_i$ . Then, employment in each country evolves according to:

$$N_{i,t+1} = (1 - s_i)N_{i,t} + M_{i,t}, \ M_{i,t} = \Upsilon V_{i,t}^{\psi} [e(1 - N_{i,t})]^{1-\psi}$$
(2.85)

with  $\Upsilon > 0$  and  $0 < \psi < 1$ .  $V_{i,t}$  is the aggregate number of vacancies posted by the representative firm in country i. The dynamic problem solved by the representative household in each country is,

$$W^{H}(B_{i,t}) = \max_{C_{i,t}^{C,S}, B_{i,t+1}} \left\{ N_{i,t} U_{i,t}^{n} + (1 - N_{i,t}) U_{i,t}^{u} + \beta \int W^{H}(B_{i,t+1}) f(A) dA \right\}$$
(2.86)

subject to the budget constraint (with dynamic multiplicator  $\lambda_{i,t}$ , and in units of good 1):

$$N_{1,t}\mathcal{P}_{1,t}^{C}C_{1,t}^{C,n} + (1 - N_{1,t})\mathcal{P}_{1,t}^{C}C_{1,t}^{C,u} + \int v_{t}B_{1,t+1}dA \le B_{1,t} + N_{1,t}w_{1,t}h_{1,t}$$
(2.87)

$$N_{2,t}\mathcal{P}_{2,t}^{C}C_{2,t}^{C,n} + (1 - N_{2,t})\mathcal{P}_{2,t}^{C}C_{2,t}^{C,u} + \int v_{t}B_{2,t+1}dA \le B_{2,t} + N_{2,t}p_{t}w_{2,t}h_{2,t}$$

$$(2.88)$$

and to the labor constraint (2.85), with dynamic multiplicator  $\zeta_{i,t}$ .  $U^s$ , for s=n,u, denotes the Household's utility function according to her labor market situation: employed (n) or unemployed (u). For separable preferences (SP), the instantaneous utility is given by  $U^s_{i,t} = \log C^s_{i,t} + \Gamma^s_{i,t}$ , where  $\Gamma^n_{i,t} \equiv \sigma_n \frac{(1-h_{i,t})^{1-\eta}}{1-\eta}$  and  $\Gamma^u_{i,t} = \Gamma^u \ \forall t$ . For non-separable preferences (NSP),  $\tilde{U}^s_{i,t} = \log(C^s_{i,t} + \tilde{\Gamma}^s_{i,t})$ , where  $\tilde{\Gamma}^n_{i,t} = \tilde{\sigma}_n \frac{(1-h_{i,t})^{1-\eta}}{1-\eta}$  and  $\tilde{\Gamma}^u_{i,t} = \tilde{\Gamma}^u \ \forall t$ .

Production is given by a constant return to scale technology with composite capital of two goods  $K_{i,t}^C$  and labor  $N_{i,t}h_{i,t}$  as inputs,

$$Y_{i,t} = a_{i,t} (K_{i,t}^C)^{\alpha} (h_{i,t} N_{i,t})^{1-\alpha}$$
(2.89)

Investment has the same CES structure as consumption and is subject to quadratic adjustment costs (as in the walrasian economy). Then, the Bellman equation of the representative firm in each country can be written, in units of good 1 as:

$$\mathcal{W}^{F}(K_{1,t}^{C}, N_{1,t}) = \max_{N_{1,t+1}, I_{1,t}^{C}} \left\{ Y_{1,t} - \omega V_{1,t} - w_{1,t} h_{1,t} N_{1,t} - \mathcal{P}_{1,t}^{C}(\mathcal{C}_{1,t} + I_{1,t}^{C}) + \int v_{t} \mathcal{W}^{F}(K_{1,t+1}^{C}, N_{1,t+1}) dA \right\}$$
(2.90)

$$W^{F}(K_{2,t}^{C}, N_{2,t}) = \max_{N_{2,t+1}, I_{2,t}^{C}} \left\{ p_{t}(Y_{2,t} - \omega V_{2,t} - w_{2,t} h_{2,t} N_{2,t}) - \mathcal{P}_{2,t}^{C}(\mathcal{C}_{2,t} + I_{2,t}^{C}) + \int v_{t} W^{F}(K_{2,t+1}^{C}, N_{2,t+1}) dA \right\}$$
(2.91)

subject to the capital accumulation equation, with dynamic multiplicator  $q_{i,t}$ :

$$K_{i,t+1}^C = (1 - \delta)K_{i,t}^C + I_{i,t}^C$$

and to the labor constraint (2.85), with dynamic multiplicator  $\xi_{i,t}$ .

Finally, the bargained-hours and wage contracts are given by the solution to the generalized Nash criterion

$$\max_{w_{i,t},h_{i,t}} (\lambda_{i,t} \Omega_{i,t}^F)^{\epsilon} (\Omega_{i,t}^H)^{1-\epsilon}$$
(2.92)

with  $\Omega_{i,t}^F = \frac{\partial \mathcal{W}^F(K_{i,t}^C, N_{i,t})}{\partial N_{i,t}}$  the marginal value of a match for a firm and  $\Omega_{i,t}^H = \frac{\mathcal{W}^H(B_{i,t})}{\partial N_{i,t}}$  the marginal value for a match for a worker.  $\epsilon$  denotes the firm's share of a jobs value.

# Equilibrium

Household's optimal choice of portfolio:

$$v_t = \beta \frac{\lambda_{i,t+1}}{\lambda_{i,t}} f(A) \tag{2.93}$$

This implies that  $\frac{\lambda_{1,t+1}}{\lambda_{1,t}} = \frac{\lambda_{2,t+1}}{\lambda_{2,t}} = \Lambda \Leftrightarrow \frac{\lambda_{2,t}}{\lambda_{1,t}} = \frac{\lambda_{2,t+1}}{\lambda_{1,t+1}} = \Lambda' \Leftrightarrow \lambda_{2,t} = \Lambda'\lambda_{1,t}$ . If we suppose that the initial wealth is the same for each individual (i.e.  $\Lambda' = 1$ ) then  $\lambda_{2,t} = \lambda_{1,t} \equiv \lambda_t$ . Hence, the search equilibrium is characterized by the following system of equations. With separable preferences, the optimal choice of consumption is:

$$(C_{i,t}^C)^{-1} = \lambda_t \mathcal{P}_{i,t}^C \tag{2.94}$$

Whereas with non-separable preferences the optimal choice of consumption is contingent to the household's situation on the labor market. In this case the optimality conditions imply:

$$(C_{i,t}^{C,u} + \widetilde{\Gamma}_{i,t}^{u})^{-1} = \lambda_t \mathcal{P}_{i,t}^{C}$$
(2.95)

$$C_{i,t}^{C,n} - C_{i,t}^{C,u} = \widetilde{\Gamma}_i^u - \widetilde{\Gamma}_i^n \tag{2.96}$$

Moreover, the aggregate consumption in each country must satisfy:

$$C_{i,t}^{C} = N_{i,t}C_{i,t}^{C,n} + (1 - N_{i,t})C_{i,t}^{C,u}$$
(2.97)

On the other hand, with separable preferences the hours contracts are:

$$(1-\alpha)\frac{Y_{1,t}}{N_{1,t}h_{1,t}} = \frac{\sigma_n}{\lambda_t}(1-h_{1,t})^{-\eta}$$
(2.98)

$$(1-\alpha)\frac{Y_{2,t}}{N_{2,t}h_{2,t}} = \frac{\sigma_n}{p_t \lambda_t} (1-h_{2,t})^{-\eta}$$
(2.99)

and the wage contracts are:

$$(1 - \epsilon) \left\{ (1 - \alpha) \frac{Y_{1,t}}{N_{1,t}} + \frac{\omega V_{1,t}}{1 - N_{1,t}} \right\} + \epsilon \frac{\left[\Gamma_1^u - \Gamma_{1,t}^n\right]}{\lambda_t} = w_{1,t} h_{1,t}$$
 (2.100)

$$(1 - \epsilon) \left\{ (1 - \alpha) \frac{Y_{2,t}}{N_{2,t}} + \frac{\omega V_{2,t}}{1 - N_{2,t}} \right\} + \epsilon \frac{\left[\Gamma_2^u - \Gamma_{2,t}^n\right]}{p_t \lambda_t} = w_{2,t} h_{2,t}$$
 (2.101)

Whereas with non-separable preferences we have:

$$(1 - \alpha) \frac{Y_{1,t}}{N_{1,t} h_{1,t}} = \mathcal{P}_{1,t}^C \tilde{\sigma}_n (1 - h_{1,t})^{-\eta}$$
(2.102)

$$(1-\alpha)\frac{Y_{2,t}}{N_{2,t}h_{2,t}} = \frac{\mathcal{P}_{2,t}^C \tilde{\sigma}_n}{p_t} (1-h_{2,t})^{-\eta}$$
(2.103)

and,

$$(1 - \epsilon) \left\{ (1 - \alpha) \frac{Y_{1,t}}{N_{1,t}} + \omega \frac{V_{1,t}}{1 - N_{1,t}} \right\} + \epsilon \mathcal{P}_{1,t}^{C} [\tilde{\Gamma}^{u} - \tilde{\Gamma}_{1,t}^{n}] = w_{1,t} h_{1,t}$$
 (2.104)

$$(1 - \epsilon) \left\{ (1 - \alpha) \frac{Y_{2,t}}{N_{2,t}} + \omega \frac{V_{2,t}}{1 - N_{2,t}} \right\} + \epsilon \frac{\mathcal{P}_{2,t}^C [\tilde{\Gamma}^u - \tilde{\Gamma}_{2,t}^n]}{p_t} = w_{2,t} h_{2,t}$$
 (2.105)

Then, the worker's wage bill (equations (3.17), (3.19) and (2.104), (2.105)) is a weighted average of (i) her contribution to output plus the hiring costs per unemployed worker, and (ii) her outside opportunities. Remark that with separable preferences the outside opportunities are affected by income and intertemporal substitution effects, through variations in  $\lambda_t$ .

The intertemporal allocation of consumption and leisure is such that the marginal contribution of one hour of labor for the firm is equal to the marginal cost of one worked hour for the household. As the wage bill, the allocation of time is affected by income and intertemporal substitution effects in the case of separable preferences.

Let us think to the impact of an economic boom relying on a positive temporary, orthogonal technological shock to country i. With separable preferences, the productivity shock leads to a higher labor productivity and hiring costs, whereas the decrease in  $\lambda_t$  which arises at economic

boom increases the outside options. That is, the dynamics of the real wage in this context is a combination of two procyclical components. Conversely, as the non-separable preferences have the property that, from an expost perspective, employed agents are actually better off than unemployed agents, and can take more advantage of an economic boom. This depresses the outside options, in putting a downward pressure on the real wage (see Chéron and Langot (2004) for a deeper discussion in a closed-economy context).

The firm's optimal choices of employment and capital are respectively:

$$\xi_{1,t} \equiv \frac{\omega V_{1,t}}{M_{1,t}} \tag{2.106}$$

$$\xi_{2,t} \equiv \frac{p_t \omega V_{2,t}}{M_{2,t}} \tag{2.107}$$

$$\xi_{1,t} = \beta E_t \left[ \frac{\lambda_{t+1}}{\lambda_t} \left( (1-\alpha) \frac{Y_{1,t+1}}{N_{1,t+1}} - w_{1,t+1} h_{1,t+1} + (1-s) \xi_{1,t+1} \right) \right]$$
(2.108)

$$\xi_{2,t} = \beta E_t \left[ \frac{\lambda_{t+1}}{\lambda_t} \left( p_{t+1} (1 - \alpha) \frac{Y_{2,t+1}}{N_{2,t+1}} - p_{t+1} w_{2,t+1} h_{2,t+1} + (1 - s) \xi_{2,t+1} \right) \right]$$
(2.109)

$$q_{i,t} = \mathcal{P}_{i,t}^{C} (1 + \phi(I_{i,t}^{C} - \delta K_{i,t}^{C}))$$
(2.110)

$$q_{1,t} = \beta E_t \left[ \frac{\lambda_{t+1}}{\lambda_t} \left( \alpha \frac{Y_{1,t+1}}{K_{1,t+1}^C} + q_{1,t+1} - \delta \mathcal{P}_{1,t+1}^C \right) \right]$$
 (2.111)

$$q_{2,t} = \beta E_t \left[ \frac{\lambda_{t+1}}{\lambda_t} \left( \alpha \frac{p_t Y_{2,t+1}}{K_{2,t+1}^C} + q_{2,t+1} - \delta \mathcal{P}_{2,t+1}^C \right) \right]$$
 (2.112)

The labor decision (equations (2.106)-(2.109)) are very similar to the capital ones (equations (2.110)-(2.112)). That is, firms view their existing workforce as a capital asset because finding new workers takes time and effort. Together mean that the number of vacancies posted by each firm is such that the average cost of creating a job at date t,  $\xi_{i,t}$ , is equal to the expected discounted marginal value of one job at date t+1: the marginal productivity net of the marginal cost of labor plus the average cost of creating a job at t+1.

In this setting, the equilibrium in the goods market is given by:

$$Y_{1,t} = C_{1,t}^1 + C_{1,t}^2 + I_{1,t}^1 + I_{1,t}^2 + C_{1,t} + \omega_1 V_{1,t}$$
(2.113)

$$Y_{2,t} = C_{2,t}^1 + C_{2,t}^2 + I_{2,t}^1 + I_{2,t}^2 + C_{2,t} + \omega_2 V_{2,t}$$
(2.114)

#### 2.3.2 The single good economy

Both countries produce the same final good. Since there is a single good, international trade takes place only to smooth consumption and to ensure that capital is allocated in the most productive country. In the variant of the model with restricted asset markets agents only can trade goods and non-contingent real debt, as was done in the walrasian economy. Finally, since the consumption/investment good is internationally mobile, there is a single resource constraint for this good:

$$Y_{1,t} + Y_{2,t} = C_{1,t} + C_{2,t} + I_{1,t} + I_{2,t} + C_{1,t} + C_{2,t}$$
(2.115)

#### 2.3.3 Empirical results

Calibration. We briefly describe the calibration of the parameters and values specific to the search economies. The detailed procedure and main sources are given in next chapter. Aggregate expenditures on search activity are set in each sector equal to 1 % of output:  $\omega V/Y = 0.01$ . The quarterly rate of transition from employment to nonemployment is set equal to N = 0.6.  $\eta$  is fixed at 5, a fairly standard value. To guaranty that the equilibrium unemployment is socially-efficient we set both, the elasticity of the matching function with respect to vacancies and the firms' bargaining power, at  $\epsilon = \psi = 0.6$ . We fix h = 1/3 and the average fraction of time that unemployed households devote to search equal to e = h/2. The rate at which vacancies become productive is m = 0.9. The remaining values of endogenous variables are computed from the equilibrium equations evaluated at the steady state.

Table 2.7 reports statistics concerning the international fluctuations of industrialized countries and the US real wage dynamics. These results correspond to average values from actual and theoretical series filtered with the Hodrick-Prescott filter. Models include the one-good labor market search economy (LMS1) and the Hairault's type two-good labor market search economy (LMS2). Simulations results are completed by Impulse Response Functions (IRF).

We first concentrate on the effects of two hypotheses: the non-separability of consumption and leisure in the utility function, and the search in the labor market (LMS1). From table 2.7 we point out that (see also the IRF showed in figure 2.9 and figure 2.10): the positive cross-country correlations of outputs and labor inputs, as well as the dissociation of national and foreign consumptions seem mostly explained by the non-separability hypothesis. On the other hand, the search hypothesis accounts for the positive correlation of investments and the additional

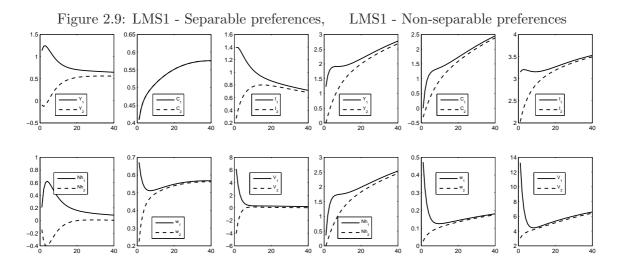
Table 2.7: Cyclical properties - Search economies

	Data	LMS1 LMS1 LI		LMS1-IM	LMS2	LMS2
		SP	NSP	NSP	SP	NSP
$\overline{\text{International}^{a,e}}$						
$\rho(Y_1, Y_2)$	0.51	-0.03	0.48	0.50	0.29	0.57
$\rho(C_1^C, C_2^C)$	0.40	1.00	0.58	0.54	0.69	0.49
$ ho(H_1,H_2)$	0.36	-0.88	0.64	0.66	0.05	0.77
$\rho(I_1^C, I_2^C)$	0.38	0.64	0.95	0.95	0.35	0.86
$\sigma_ au/\sigma_Y$	1.90	_	_	_	0.76	0.43
$\sigma_{\Gamma}/\sigma_{Y}$	3.28	_	-	_	0.53	0.30
$\mathrm{USA}^{c,e}$						
$\sigma_Y$ (in %)	1.91	1.42	1.98	2.00	1.22	2.03
$\sigma_C/\sigma_Y$	0.40	0.54	0.66	0.69	0.68	0.70
$\sigma_H/\sigma_Y$	0.86	0.52	0.85	0.85	0.21	0.85
$\sigma_I/\sigma_Y$	3.07	1.26	2.34	2.37	1.61	2.46
$\sigma_{LP}/\sigma_{Y}$	0.57	0.68	0.40	0.39	0.83	0.40
$\sigma_W/\sigma_Y$	0.45	0.59	0.22	0.21	0.78	0.23
$\rho(Y_t, Y_{t-1})$	0.85	0.77	0.85	0.85	0.72	0.86
$\rho(C_t, C_{t-1})$	0.86	0.74	0.85	0.87	0.73	0.85
$\rho(H_t, H_{t-1})$	0.84	0.89	0.91	0.91	0.89	0.91
$\rho(I_t, I_{t-1})$	0.81	0.74	0.71	0.72	0.73	0.74
$\rho(LP_t, LP_{t-1})$	0.52	0.62	0.43	0.46	0.64	0.48
$\rho(Y,C)$	0.83	0.70	0.75	0.80	0.97	0.77
$\rho(Y, H)$	0.82	0.78	0.92	0.92	0.85	0.92
$\rho(Y, I)$	0.97	0.92	0.84	0.84	1.00	0.88
$\rho(Y, LP)$	0.51	0.88	0.54	0.54	0.99	0.55
$\rho(Y, W)$	0.28	0.83	0.60	0.60	0.99	0.62
$\rho(Y, LS)$	-0.30	-0.60	-0.47	-0.46	-0.59	-0.43
$\rho(H, LP)$	-0.07	0.38	0.17	0.16	0.77	0.18
$\rho(H,W)$	0.03	0.34	0.24	0.23	0.81	0.27

The moments reported are computed from Hodrick-Prescott filtered artificial time series.  $^a$  Backus, Kehoe, and Kydland (1995).  $^b$  Hairault (2002).  $^c$  Chéron and Langot (2004).  $^d$  Baxter and Crucini (1993).  $^e$  Hairault (1995).

increase in the correlation of labor inputs. Finally, the combination of the two allows for a low procyclical real wage and for a weak correlation of total hours with both output and labor productivity.

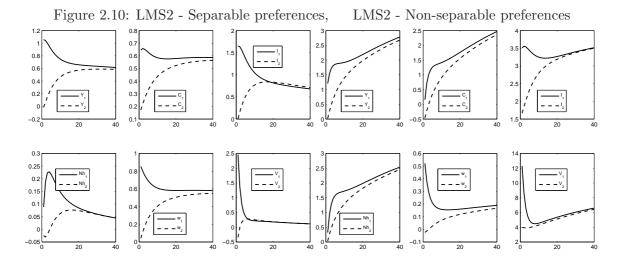
With a single good, the non-separability of preferences largely reduces the wealth effects in household's labor decisions. This means that the labor supply is static and responds positively to increases on productivity. Moreover, the international diffusion of the productivity shock gives incentives for firms of two countries to post more vacancies at same time. Total hours rise slowly in both countries, displaying a typical hump-shaped profile. This hours dynamics interact with the capital investment, which exhibits a hump-shaped response too, leading to a positive cross-country correlation of investment. All this explains why these two hypotheses tends to reproduce the interdependency across countries and to reduce the real wage procyclicality. However, converse to data, outputs correlation still slightly below than consumptions correlation.



Two-country, one-good labor market search economy.  $\psi = 0$ .

As in the walrasian economy, the limitation of the agents' ability to engage in international risk-pooling slightly improves the estimation of the cross correlation of consumptions, yet it still bigger than the cross correlation of outputs. The correlation of consumption with output also improves, while the remaining measure do not change.

Finally, we turn to the effects from the two-good hypothesis. Concerning international fluctuations, we observe that the LMS2 economy produces a cross-country correlation of consumption smaller than the cross-country correlation of output, as in the data. On the supply side, the



Two-country, two-good labor market search economy.  $\psi = 0$ .

non-separability in the preferences reduces the wealth effects in the household's labor decisions, leading to a dissociation of national consumption from foreign consumption in the LMS2 economy. This seems in contradiction with the analysis presented in the corresponding walrasian economy, where we enhanced the fact that the increase in the relative price p tends to offset the reduction in the income effects due to the non-separability of preferences. This point repose on the analysis of the (log-linear) equilibrium condition for consumption, namely:

$$\hat{C}_{2,t}^C + \eta \frac{h}{1-h}(\hat{h}_{1,t} - \hat{h}_{2,t}) - (2\gamma_C - 1)\hat{p}_t = \hat{C}_{1,t}^C$$

Then, the more the gap in individuals hours is dampened by the increase in the relative price p, the more the increase of consumption is similar in the two countries. However, with the calibration retained, this gap is large enough to allow the wealth effect to predominate.

Nonetheless, the non-separability tends to increase the procyclicality of the real wage rate with respect to the single-good economy. This is explained by the price effect discussed in the two-good walrasian economy. After a positive technological shock in country 1, the purchasing power in country 1 falls. This leads to a stronger vindication of salary and so to a more procyclical real wages.

# 2.4 Conclusion

The main points to highlight from this analysis are:

- The international comovement of the macro aggregates is well reproduced by the standard walrasian model once international linkages between the outputs are added *via* the introduction of more than one good.
- However, in the two-good economy the variations in the relative price of goods offset the reduction in the wealth effect allowed by the non-separability of preferences, which in turn may lead to worst predictions for the cross-country correlation of consumption than with standard additively separable preferences.
- Even if the restricted asset markets structure plays a role in mitigating with the comovement puzzle, it is not enough to replicate the observed international fluctuations.
- Regarding the search economies, we show that in the single-god economy, the combination of search and matching in the labor market with the non-separability is enough to predict positive comovements of labor inputs and investments as well as a large dissociation of consumptions. Moreover, the procyclicality of real wage rate is reduced, and the correlation of total hours with both output and labor productivity is low. Then, the three puzzles are partially solved.
- Finally we show that the gain from including two goods in that framework is that the model is able now to replicate a correlation of outputs bigger than that of consumptions (Hairault 2002). However, the price dynamics provoked by a positive productivity shock decrease the agent's purchasing power, leading to a stronger vindication of salary and so to a slightly more procyclical real wages.

# Chapter 3

Tax/Benefit system and labor market search: reconciling the standard separable preferences with the real wage dynamics and the international business cycles

This chapter is built on the basis of a joint working paper with François Langot

# Introduction

Traditional real business cycle models assume that technological change is the driving force behind growth and fluctuations observed in developed economies, in particular the U.S.. While these models have been successful in accounting for a large fraction of the variability and comovements of the aggregate time series, they do not do well along some dimensions<sup>1</sup> As is well known, relative to the data, the variability of consumption, hours of work, and output is too low, and the variability of investment is too high. But maybe the main failure is the predicted correlation of real wages with both hours worked and output. In such a models, variations in technology shifts the labor demand curve but not the labor supply curve, thus inducing a strong positive correlation between wages and hours. The introduction of search and matching in the labor market (Andolfatto 1996) outperforms the model predictions along these lines<sup>2</sup>.

This line of reasoning naturally suggests that to improve the predictions from the real business cycle models one must include something that shifts labor supply. If both labor demand and labor supply shift, then the strong positive correlation between hours and wages can probably be reduced.

Several candidate labor supply shifters have already been considered, such as home production (Benhabib, Rogerson, and Wright 1991) or government consumption (Christiano and Eichenbaum 1992). As it is shown by Burnside, Eichenbaum, and Fisher (2004), we also observe large changes in the tax rates. In the Keynesian tradition, fiscal policy, and therefore taxation, is one of the main instruments to stabilize the economy. However, in the 1990s, several pioneering works considered taxation as a source of business cycle fluctuations. This feeds the criticisms about the possibility to use taxes as stabilization tool.

As Christiano and Eichenbaum (1992) show, the inclusion of a public sector has the potential to improve some of the predictions of the standard real business cycle model. In particular, they study a real business cycle model in which government purchases affect the agents' utility. The expenditures are financed through lump-sum taxes. In this case, shocks to expenditures shift the labor supply curve. However, they predict that while the hours and wage correlation comes closer to that observed, it is significantly positive. But Christiano and Eichenbaum (1992) do not allow for distortionary taxation. Intuitively, like government expenditures, shocks to income and payroll taxes can be interpreted as shocks to labor supply, as opposed to technology shocks which

<sup>&</sup>lt;sup>1</sup>See chapter 1 and chapter 2 of this thesis.

<sup>&</sup>lt;sup>2</sup>See chapter 2.

may be interpreted as shocks to labor demand. Thus, tax rates provide another mechanism for explaining the observed correlation between hours and wages.

In this line of research, some pioneering articles have shown that stochastic fiscal policy improves the performance of real business cycle models. McGrattan (1994) finds that a significant portion of the variance of the aggregate consumption, output, hours worked, capital stock, and investment can be attributed to the factor tax (*i.e.* on capital and labor income) and government spending processes. Similarly, Braun (1994) shows that modelling fluctuations in personal and corporate income tax rates increases the model's predicted relative variability of hours and decreases its predicted correlation between hours and average productivity. Finally, using Swedish data, ?) find that the empirical fit of a simple stochastic growth model is significantly improved when it is amended to include imperfectly predictable fluctuations in payroll taxes, consumption taxes and government consumption.<sup>3</sup>

In all cases, the basic mechanisms at work are as follows. Taxes to labor alter the leisure/labor supply decision, highlighting the volatility of hours worked. In plain words, if income and payroll taxes fluctuate over time, it is optimal to work hard when taxes are relatively low and to take time off when they are relatively high. Then, as labor taxes fluctuate, so do hours worked. Similarly, a temporarily high tax rate on consumption provides an incentive to postpone consumption to a later date, when the tax rate is likely to be lower. Hence, as the consumption tax fluctuates, so does consumption. Consequently, the inclusion of such a taxes should increase the predicted volatility of hours and consumption, bringing the implications of theory closer to the facts. Finally, the variability in investment and capital increases either because of increases in the capital tax, or indirectly by the complementarity of capital and labor, and even though the agents' trade-off between consumption and saving following a consumption tax shock.

In quantitative terms, these models yield to predictions for the correlation between hours and real wages, as measured by average productivity, close to the empirical correlation. Likewise, the predicted variability of hours worked and consumption are much closer to their empirical values when fiscal policy is included (even if in general the relative volatility of aggregate hours is overstated). Nevertheless, these former papers show two drawbacks. The first one is that all of them consider a closed economy, so that the possible variability in the macro aggregates passing through the international trade is not accounted for. The second one is that the theoretical

<sup>&</sup>lt;sup>3</sup>Moreover, they find that for large sets of conventional moments, models with stochastic fiscal policy cannot be statistically rejected, whereas a model without it is always rejected.

real wage is measured by average productivity. This obviously prevents from analyzing other features of the US labor market, such as the lower volatility of the real wage than the one of the labor productivity.

Then, in this chapter we show that fluctuations in distortive taxes can account for some of the puzzling features of the U.S. business cycle. Namely, the observed real wage rigidity, the international comovement of investment and labor inputs, and the so-called consumption correlation puzzle (according to which cross-country correlations of output are higher than the one of consumption). This is done in a two-country search and matching model with fairly standard preferences, extended to include a tax/benefit system. In this simple framework, the tax side is represented by taxation on labor income, employment (payroll tax) and consumption, whereas the benefit side is resumed by the unemployment benefits and the worker's bargaining power.

The main departures from the former literature on taxation as a source of business cycle fluctuations are twofold. First, we consider a two-country general equilibrium model, so that we are able to discuss the effects on the observed international fluctuations. Second, we assume search and matching in the labor market. Our model is close to the Hairault (2002)'s one, who develops a two-country, two-good search model, able to explain the puzzling facts of international fluctuations once a non-separability in the agents' preferences is considered<sup>4</sup>.

Our model is also close to the Chéron and Langot (2004)'s model, who explain the real wage rigidity in a closed-economy search model by means of a particular set of non-separable preferences.

Either in the Hairault (2002)'s paper or in the Chéron and Langot (2004)'s paper, the non-separability of preferences plays a main role. However, this hypothesis is unable to simultaneously account for the real wage rigidity and for the observed international fluctuations. Conversely, in this work we show that all those facts can be accounted in a single framework with fairly standard preferences. On the one side, an economic boom accompanied of a positive shock to the labor taxes leads to countercyclical outside options, which dampens the procyclicality of the real wage. On the other side, under the national specialization hypothesis, the equalization of consumptions across countries following a productivity shock does not hold anymore, (even in presence of separable preferences), and the gap between domestic and foreign consumption increases as long as the consumption tax is shocked too.

<sup>&</sup>lt;sup>4</sup>See chapter 2.

# 3.1 The Model

The world economy consists of two countries (country 1 or home country and country 2 or foreign country), each represented by a large number of identical consumers and a production technology. Population size is normalized to unity. Each country specializes in the production of a single good. The main source of fluctuations are persistent shocks to productivity that are internationally diffused. Additionally, both countries are affected by shocks to taxes on consumption and labor (*i.e.*, taxes on labor income and payroll taxes). The countries are linked either on the consumption and the production side since agents demand a CES basket of the two goods for consumption and investment purposes. Finally, agents participate in the trade on the labor market.

#### 3.1.1 Labor market flows

Employment in country i = 1, 2 is predetermined at each time and changes only gradually as workers separate from jobs, at the exogenous rate  $s_i$ , or unemployed agents find jobs, at the hiring rate  $M_{i,t}$ . Let  $N_{i,t}$  and  $V_{i,t}$ , respectively be the number of workers and the total number of new jobs made available by firms, then employment evolves according to

$$N_{i,t+1} = (1 - s_i)N_{i,t} + M_{i,t}, \quad M_{i,t} = V_{i,t}^{\phi}[e_i(1 - N_{i,t})]^{1-\phi}, \quad 0 < \phi < 1$$

where  $e_i > 0$  and  $0 < s_i < 1$  are the constant search effort and the exogenous separation rate of job-worker pairs.

#### 3.1.2 Households

At any period,  $N_i$  agents are employed while the remaining  $1-N_i$  are unemployed. Unemployed agents are randomly matched with job vacancies. Employees work  $h_i$  units of time at a wage rate of  $w_i$ . Unemployed workers devote  $e_i$  units of their time seeking employment and receive the unemployment benefits  $b_i$ . In both cases they pay a labor income tax levied at rate  $\tau_w^i$ . Markets are complete, so we can derive the intertemporal decision rules by solving the program of a representative household. This agent consumes a CES basket of the two goods (Hairault 2002):

$$C_i^{C,z} = \left[ \gamma^{\frac{1}{\theta}} C_i^{\frac{\theta-1}{\theta}} + (1-\gamma)^{\frac{1}{\theta}} C_{j\neq i}^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}}, \quad for \quad z=n,u, \quad i=1,2$$

where  $C^{C,n}$  and  $C^{C,u}$  respectively stand for the consumption of employed and unemployed agents.  $\theta$  is the elasticity of substitution between the two goods and  $\gamma$  is the share of the national good in the consumption basket. The price index of the composite goods is defined as:

$$P_i^C = \left[ \gamma P_i^{1-\theta} + (1-\gamma) P_{j\neq i}^{1-\theta} \right]^{\frac{1}{1-\theta}}, \text{ for } i = 1, 2.$$

with  $P_i$  the production price of good i.

We still assume perfect international risk sharing: households in the two countries have access to contingent claims  $B_{i,t} = B_i(A_t)$  at prices  $v_t = v(A_{t+1})$  providing one unit of good 1 if the state A occurs at t+1. That is, we take the good produced in country 1 as accounting unit, and we normalize its value to 1. The representative household in country i is assumed to maximize the expected discounted sum of its utility flows,

$$W^{H}(B_{i,t}) = \max_{C_{i,t}^{C,n}, C_{i,t}^{C,u}, B_{i,t}} \left\{ N_{i,t} U(C_{i,t}^{C,n}, 1 - h_{i,t}) + (1 - N_{i,t}) U(C_{i,t}^{C,u}, 1 - e_i) + \beta \int W^{H}(B_{i,t+1}) f(A) dA_{t+1} \right\}$$
(3.1)

subject to the labor constraint:

$$N_{i,t+1} = (1-s)N_{i,t} + \Psi_{i,t}(1-N_{i,t}), \text{ for i=1,2}$$
 (3.2)

where  $\Psi_{i,t} \equiv M_{i,t}/(1-N_{i,t})$  is the rate at which unemployed agents find jobs, and subject to the budget constraint:

$$(1 + \tau_{c,t}^{1}) \left[ N_{1,t} \mathcal{P}_{1,t}^{C} C_{1,t}^{C,n} + (1 - N_{1,t}) \mathcal{P}_{1,t}^{C} C_{1,t}^{C,u} \right] + \int v_{t} B_{1,t+1} dA_{t+1} \leq B_{1,t} +$$

$$(1 - \tau_{w,t}^{1}) \left[ N_{1,t} w_{1,t} h_{1,t} + (1 - N_{1,t}) b_{1,t} \right] + L_{1,t}, \text{ for } i=1$$

$$(1 + \tau_{c,t}^{2}) \left[ N_{2,t} \mathcal{P}_{2,t}^{C} C_{2,t}^{C,n} + (1 - N_{2,t}) \mathcal{P}_{2,t}^{C} C_{2,t}^{C,u} \right] + \int v_{t} B_{2,t+1} dA_{t+1} \leq B_{2,t} +$$

$$(1 - \tau_{w,t}^{1}) p_{t} \left[ N_{2,t} w_{2,t} h_{2,t} + (1 - N_{2,t}) b_{2,t} \right] + L_{2,t}, \text{ for } i=2$$

$$(3.3)$$

and given some initial conditions  $(N_{i,0}, B_{i,0})$ .  $p_t \equiv \frac{P_{2,t}}{P_{1,t}}$ ,  $\mathcal{P}_{i,t}^C \equiv \frac{P_{i,t}^C}{P_{1,t}}$  and  $L_{i,t}$  are lump-sum transfers from the government to be defined below.  $f(A) \equiv f(A_{t+1}, A_t)$  denotes the density function describing the transition from the state  $A_t$  to the state  $A_{t+1}$ .  $\tau_c^i$ ,  $\tau_w^i$  stand respectively for the consumption tax rate and the labor income tax rate prevailing in country i.

The contemporaneous utility function is assumed to be increasing and concave in both arguments and exhibits conventional additive separability between consumption  $(C_{i,t}^{C,z})$  and leisure  $(L_{i,t}^z)$ :

$$U_{i,t}(C_{i,t}^{C,z}, L_{i,t}^z) = \log C_{i,t}^{C,z} + \Gamma_{i,t}^z \equiv U_{i,t}^z, \quad z = n, u$$
(3.5)

where  $\Gamma_{i,t}^n \equiv \sigma \frac{(1-h_{i,t})^{1-\eta}}{1-\eta}$  with  $\sigma, \eta > 0$  and  $\Gamma_{i,t}^u = \Gamma_i^u \ \forall t$ . The parameter  $\sigma$  can be interpreted as reflecting differences in the efficiency of household's home production technology across different states of employment opportunities.

#### 3.1.3 Firms

Each country specializes in the production of a single good. The representative firm in country i = 1, 2 has a constant returns-to-scale technology that uses composite capital and labor hours to produce output,

$$Y_{i,t} = a_{i,t} K_{i,t}^{\alpha} (N_{i,t} h_{i,t})^{1-\alpha}, \quad 0 < \alpha < 1$$
(3.6)

The primary source of fluctuations are persistent shocks to aggregate productivity, represented by  $a_{i,t}$ . The stochastic productivity vector  $a_t = [a_{1,t}, a_{2,t}]'$  is assumed to follow a VAR(1) process in natural logarithms:

$$\ln a_{t+1} = \Omega \ln a_t + \vartheta \epsilon_{t+1}$$
, where  $\epsilon_{t+1} \sim iid \mathcal{N}(0, \mathbf{I})$ 

The vector  $\epsilon_t = [\epsilon_1, \epsilon_2]'$  represents the innovations to productivity variables. The matrix  $\Omega$ , defined by

$$\Omega = \left(\begin{array}{cc} \rho_{1,1} & \rho_{1,2} \\ \rho_{2,1} & \rho_{2,2} \end{array}\right)$$

describes the autoregressive component of the disturbance. Finally, the covariances between the elements are given by the matrix  $\vartheta$ , defined by

$$\vartheta = \left(\begin{array}{cc} 1 & \upsilon_{1,2} \\ \upsilon_{2,1} & 1 \end{array}\right)$$

This matrix reflects the extent to which the shocks are idiosyncratic or global in nature.

New capital goods are internationally mobile. Investment has the same CES structure as consumption (and then, the same price  $Pi^{C}$ ) and is subject to quadratic adjustment costs:

$$C_{i,t} = \frac{\hat{\phi}}{2} (K_{i,t+1} - K_{i,t})^2$$

Let  $\omega_i$  be the unitary cost of a vacancy job. Firms seek to maximize the discounted value of the dividend flows,

$$W^{F}(K_{i,t}, N_{i,t}) = \max_{V_{i,t}, I_{i,t}} \left\{ \pi_{i,t} + \int v_{t} W^{F}(K_{i,t+1}, N_{i,t+1}) dA_{t+1} \right\}$$
for  $\pi_{i,t} = P_{i,t} \left( Y_{i,t} - \omega_{i} V_{i,t} - (1 + \tau_{f,t}^{i}) w_{i,t} N_{i,t} h_{i,t} - \mathcal{C}_{i,t} \right) - P_{i,t}^{c} I_{i,t}^{c}$  (3.7)

subject to the constraints,

$$N_{i,t+1} = (1 - s_i)N_{i,t} + \Phi_{i,t}V_{i,t}$$
(3.8)

$$K_{i,t+1} = (1-\delta)K_{i,t} + I_{i,t}^c \tag{3.9}$$

and given some initial conditions  $(N_{i,0}, K_{i,0})$ , where  $0 < \delta < 1$  is the depreciation rate of capital.  $\Phi_{i,t} \equiv M_{i,t}/V_{i,t}$  is the rate at which vacancies and workers are matched and  $\tau_f^i$  stands for the country-specific payroll tax payed by firms.

# 3.1.4 Government

The government levies taxes to finance expenditures. We assume a balanced budget at each period, so that any revenue that is not used to finance current purchases is transferred to households in a lump-sum payment. Thus, real transfers to country i households are given by:

$$L_{1,t} = \tau_{c,t}^1 \mathcal{P}_{1,t}^C [N_{1,t} C_{1,t}^{C,n} + (1 - N_{1,t}) C_{1,t}^{C,u}] + (\tau_{f,t}^1 + \tau_{w,t}^1) w_{1,t} h_{1,t} N_{1,t} - b_{1,t} (1 - \tau_{w,t}^1) (1 - N_{1,t})$$
(3.10)

$$L_{2,t} = \tau_{c,t}^2 \mathcal{P}_{2,t}^C [N_{2,t} C_{2,t}^{C,n} + (1 - N_{2,t}) C_{2,t}^{C,u}] + p_t (\tau_{f,t}^2 + \tau_{w,t}^2) w_{2,t} h_{2,t} N_{2,t} - b_{2,t} p_t (1 - \tau_{w,t}^2) (1 - N_{2,t})$$

$$(3.11)$$

In order to be coherent with our estimations (section 3.2), we assume that the stochastic process governing tax rate  $\tau_j^i$ , for j = c, w, f, follows an AR(1),

$$\tau_{i,t+1}^i = (1 - \rho_i^i)\bar{\tau}_i^i + \rho_i^i \tau_{i,t}^i + \epsilon_{i,t+1}^i, \quad \text{with} \quad \epsilon_{i,t}^i \sim \mathcal{N}(0, \sigma_i^i)$$
(3.12)

with  $\bar{\tau}^i_j$  denoting the mean value of the tax variable  $\tau^i_{j,t}$ .

#### 3.1.5 Nash bargaining

Wages and hours worked are derived from the generalized Nash-bargaining model (Pissarides 2000), with firm's relative bargaining power  $\epsilon_i$ :

$$\max_{w_{i,t},h_{i,t}} (\lambda_t \mathcal{V}_t^F)^{\epsilon_i} (\mathcal{V}_{i,t}^H)^{1-\epsilon_i} \tag{3.13}$$

with  $\mathcal{V}_{i,t}^F = \frac{\partial \mathcal{W}(\Omega_{i,t}^F)}{\partial N_{i,t}}$  the marginal value of a match for a firm and  $\mathcal{V}_{i,t}^H = \frac{\mathcal{W}(\Omega_{i,t}^F)}{\partial N_{i,t}}$  the marginal value for a match for a worker. Matter of simplicity, we define the labor tax:  $\tau_{i,t}^n \equiv \frac{1+\tau_{f,t}^i}{1-\tau_{w,t}^i}$ .

It is worth stressing that, under the separability assumption, optimal households' decision rules imply

$$C_{i,t}^{C,n} = C_{i,t}^{C,u} \equiv C_{i,t}^{C}$$
 (3.14)

$$U_{i,t}^u = U_{i,t}^n + \left(\Gamma_i^u - \Gamma_{i,t}^n\right) \tag{3.15}$$

This entails the following hours worked contracts, for i = 1:

$$\left(\frac{1-\tau_{w,t}^{1}}{1+\tau_{f,t}^{1}}\right)(1-\alpha)\frac{Y_{1,t}}{N_{1,t}h_{1,t}} = \sigma\frac{(1-h_{1,t})^{-\eta}}{\lambda_{1,t}}$$

$$\Leftrightarrow \left(\frac{1-\tau_{w,t}^{1}}{(1+\tau_{f,t}^{1})(1+\tau_{c,t}^{1})}\right)(1-\alpha)\frac{Y_{1,t}}{N_{1,t}h_{1,t}} = \sigma(1-h_{1,t})^{-\eta}\mathcal{P}_{i,t}^{C}C_{i,t}^{C}$$
(3.16)

and the following wages contracts, for i = 1:

 $Labor\ cost$ 

$$(1 + \tau_{f,t}^{1})w_{1,t}h_{1,t} = (1 - \epsilon_{1})\underbrace{\left[(1 - \alpha)\frac{Y_{1,t}}{N_{1,t}} + SC_{1,t}\right]}_{\text{Bargained Surplus}}$$

$$+\epsilon_{1}\left(\frac{1 + \tau_{f,t}^{1}}{1 - \tau_{w,t}^{1}}\right)\underbrace{\left[\frac{\Gamma_{1}^{u} - \Gamma_{1,t}^{n}}{\lambda_{1,t}} + (1 - \tau_{w,t}^{1})b_{1,t}\right]}_{\text{Outside option}}$$

$$(3.17)$$

Net wage

$$(1 - \tau_{w,t}^1)w_{1,t}h_{1,t} = (1 - \epsilon_1)\left(\frac{1 - \tau_{w,t}^1}{1 + \tau_{f,t}^1}\right)\left[(1 - \alpha)\frac{Y_{1,t}}{N_{1,t}} + SC_{1,t}\right] + \epsilon_1\left[\frac{\Gamma_1^u - \Gamma_{1,t}^n}{\lambda_{1,t}} + (1 - \tau_{w,t}^1)b_{1,t}\right]$$

Gross wage

$$w_{1,t}h_{1,t} = \frac{1-\epsilon_1}{1+\tau_{f,t}^1} \left[ (1-\alpha)\frac{Y_{1,t}}{N_{1,t}} + SC_{1,t} \right] + \frac{\epsilon_1}{1-\tau_{w,t}^1} \left[ \frac{\Gamma_1^u - \Gamma_{1,t}^n}{\lambda_{1,t}} + (1-\tau_{w,t}^1)b_{1,t} \right]$$

where, for i = 1, 2 the search costs SC are defined by<sup>5</sup>:

$$SC_{i,t} = \omega_{i} \left\{ \underbrace{\frac{V_{i,t}}{1 - N_{i,t}} E_{t} \left[ \left( \frac{1 + \tau_{i,t}^{f}}{1 + \tau_{i,t+1}^{f}} \right) \left( \frac{1 - \tau_{i,t+1}^{w}}{1 - \tau_{i,t}^{w}} \right) \right]}_{\mathbf{a} = \text{Outsiders}} + \underbrace{\frac{1 - s_{i}}{\Phi_{i,t}} \left( 1 - E_{t} \left[ \left( \frac{1 + \tau_{i,t}^{f}}{1 + \tau_{i,t+1}^{f}} \right) \left( \frac{1 - \tau_{i,t+1}^{w}}{1 - \tau_{i,t}^{w}} \right) \right] \right)}_{\mathbf{b} = \text{Insiders}} \right\}$$

$$\lambda_{i,t} = \underbrace{\frac{1}{\pi G G G \left( t - \frac{1}{2} \right)}}_{\mathbf{D} = \mathbf{D} G G \left( t - \frac{1}{2} \right)}$$
(3.20)

$$(1-\alpha)\frac{Y_{2,t}}{N_{2,t}h_{2,t}} = \frac{\tau_{2,t}^n}{p_t \lambda_{2,t}} \sigma (1-h_{2,t})^{-\eta}$$
(3.18)

$$(1 + \tau_{f,t}^2)w_{2,t}h_{2,t} = (1 - \epsilon_2)\left[(1 - \alpha)\frac{Y_{2,t}}{N_{2,t}} + SC_{2,t}\right] + \epsilon_2\tau_{2,t}^n\left[\frac{\Gamma_2^u - \Gamma_{2,t}^n}{p_t\lambda_{2,t}} + (1 - \tau_{w,t}^2)b_{2,t}\right]$$
(3.19)

<sup>&</sup>lt;sup>5</sup>Similarly, for i = 2 we have:

where  $\lambda_{i,t}$  denotes the Lagrange multiplier associated with the representative households' budget constraint, and  $SC_i$  the average search and matching costs per hiring incurred by firms.

As in the standard search framework discussed in chapter 2, the wage bill turns to be some average, weighted by the relative bargaining power, of (i) the worker's contribution to output plus the average costs per hiring, and (ii) the worker's endogenous outside options. Nonetheless, this time the search costs are affected by the dynamics of the labor tax. Likewise, the income and intertemporal effects that shape the outside opportunities (through variations in  $\lambda_{i,t}$ ) depend not only on variations in the relative price of goods, but also on variations in the consumption tax. All this together could potentially lead to counter-cyclical wage dynamics, converse to the standard setting.

#### The impact of taxes on the labor supply

Taxes have an impact on the number of hours worked and on wages.

- Hours worked and taxes. The equation (3.16) shows that the marginal productivity net of the labor taxes (payroll tax and labor income tax) is equal to the marginal disutility of labor. The introduction of these two taxes reduces the labor supply: households prefer leisure because this good is not taxed. Moreover, the tax on consumption also decrease the number of hours worked because it increases the consumption cost and then reduce the incentives to work.
- Wage bargained and taxes. In the bargaining process, either the firm or the worker try to incorporate in the wage their personal taxes, respectively the payroll tax for the firm, and the labor income tax and the consumption tax for the worker. The equations (3.17) show that when a tax increases ( $\tau_w$  or  $\tau_f$ ), we observe a decrease of the fraction of the bargained surplus distributed to the workers. This clearly reduces the net wage and then decreases the labor supply. Another way to interpret the impact of the labor taxes on the wages is reported in the last wage equation: an increase of the payroll taxes decreases the bargained surplus distributed to the workers, by decreasing the workers bargained power, whereas an increase of the labor income tax acts as an increase of the bargaining power of the firms. The consumption tax has an impact on wages through the higher cost of consumption, leading to a higher value of the outside options. This also reduces the labor supply.

• Search costs and taxes. The search costs integrate the intertemporal substitution of labor. Indeed, if for example, the labor income tax increases tomorrow  $(\tau_{w,t+1}^i > \tau_{w,t}^i)$ , the firm anticipates that its bargaining power will be higher than today: **b** increases in the equation (3.20). This leads to a higher valuation of the rent from keep an insider (the probability to keep an insider is  $1 - s_i$  and the search costs saved are  $\omega_i/\Phi_{i,t}$ ). At the opposite, this reduce the value of hiring today an outsider: **a** decrease in the equation (3.20). Likewise, if  $\tau_{f,t+1}^i > \tau_{f,t}^i$  the term **a** in equation (3.20) will increase, whereas the term term **b** will decrease. In this case the intuition is that insiders have lower salary vindication today because they anticipate a fall in their purchasing power tomorrow. Conversely, the outsiders accept lower wages today because they anticipate that their outside options will be lower tomorrow (remember that the income of unemployed workers is also taxed).

Hence, since in both cases the insiders effect and the outsiders effect offset each other, the total outcome on the search costs is ambiguous.

#### 3.1.6 Equilibrium

The optimal households' choices of contingent bonds lead to:  $v_t = \beta \frac{\lambda_{i,t+1}}{\lambda_{i,t}} f(A)$ . Under the assumption that all households have the same initial wealth distribution, we deduce that  $\lambda_{1,t} = \lambda_{2,t} = \lambda_t$ . Then, the search equilibrium in country i = 1, 2 is characterized by the following system of equations:

$$N_{i,t+1} = (1 - s_i)N_{i,t} + V_{i,t}^{\phi} [e_i(1 - N_{i,t})]^{1 - \phi}$$
(3.22)

$$((1 + \tau_{c,t}^i) P_{i,t}^C C_{i,t}^C)^{-1} = \lambda_t$$
(3.23)

As discussed in chapter 2, the equalization of consumptions across countries following an idiosyncratic shock does not hold anymore even in presence of separable preferences. This is due to the change in the relative price of goods. In addition, in the present case the consumption gap increases as long as the consumption tax is shocked as well.

$$\frac{(1-\alpha)Y_{i,t}}{(1+\tau_{f,t}^i)N_{i,t}h_{i,t}} = \frac{\sigma(1-h_{i,t})^{-\eta}}{(1-\tau_{w,t}^i)P_{i,t}\lambda_t}$$
(3.24)

The intertemporal allocation of consumption and leisure is such that the marginal contribution of one hour of labor for the firm is equal to the marginal cost of one worked hour for the worker.

$$w_{i,t}h_{i,t} = \frac{1}{1 - \epsilon_i \rho_i} \left\{ \frac{1 - \epsilon_i}{1 + \tau_{f,t}^i} \left[ (1 - \alpha) \frac{Y_{i,t}}{N_{i,t}} + SC_{i,t} \right] + \frac{\epsilon_i}{1 - \tau_{w,t}^i} \left( \frac{\Gamma_i^u - \Gamma_{i,t}^n}{P_{i,t} \lambda_t} \right) \right\}$$
(3.25)

This expression for the wage contract is obtained by assuming that, at equilibrium, the unemployment benefits are a fraction of the wage bill, with the fraction given by the average replacement rate  $(\rho_{i,t})$ :  $b_{i,t} = \rho_{i,t} w_{i,t} h_{i,t}$ .

Let us now complete the analyze of the effects on the wage bill of the tax/benefit system. The impact transiting through the average replacement rate:  $\rho_i$  is as follows. Higher unemployment benefit yields to higher wages because they now compensate the higher outside options of the worker.

Given these contracts on the hours worked and wages, the labor demand is summarized by:

$$\xi_{i,t} \equiv \frac{P_{i,t}\omega_i}{\Phi_{i,t}} \tag{3.26}$$

$$\xi_{i,t} = \beta E_t \left[ \frac{P_{i,t+1}\lambda_{t+1}}{\lambda_t} \left( (1-\alpha) \frac{Y_{i,t+1}}{N_{i,t+1}} + (1-s_i)\xi_{i,t+1} - (1+\tau_{f,t+1}^i) w_{i,t+1} h_{i,t+1} \right) \right] (3.27)$$

$$q_{i,t} \equiv P_{i,t}^c + P_{i,t}\hat{\phi}(I_{i,t}^C - \delta K_{i,t})$$
 (3.28)

$$q_{i,t} = \beta E_t \left[ \frac{\lambda_{t+1}}{\lambda_t} \left( \alpha P_{i,t+1} \frac{Y_{i,t+1}}{K_{i,t+1}} - \delta P_{i,t}^C + q_{i,t+1} \right) \right]$$
(3.29)

The firm optimal choices of employment (equations (3.26) and (3.27)) are very similar to those of capital (equations (3.28) and (3.29)) because finding new workers takes time and effort, so that the existing labor force is viewed by the firm as an capital asset. Nonetheless, the firm's tends to reduce the number of vacancies as the payroll tax increases. Moreover, either the capital or the employment decisions are affected by the consumption tax trough  $\lambda_t$ .

The equilibrium on the goods market is given by the accounting equations for output,

$$Y_{1,t} = C_{1,t}^1 + C_{1,t}^2 + I_{1,t}^1 + I_{1,t}^2 + C_{1,t} + \omega_1 V_{1,t}$$
(3.30)

$$= \gamma \left(\frac{P_{1,t}^c}{P_{1,t}}\right)^{\theta} \left(C_{1,t}^c + I_{1,t}^c\right) + (1 - \gamma) \left(\frac{P_{2,t}^c}{P_{1,t}}\right)^{\theta} \left(C_{2,t}^c + I_{2,t}^c\right) + \mathcal{C}_{1,t} + \omega_1 V_{1,t}$$
(3.31)

$$Y_{2,t} = C_{2,t}^1 + C_{2,t}^2 + I_{2,t}^1 + I_{2,t}^2 + C_{2,t} + \omega_2 V_{2,t}$$
(3.32)

$$= (1 - \gamma) \left(\frac{P_{1,t}^c}{P_{2,t}}\right)^{\theta} \left(C_{1,t}^c + I_{1,t}^c\right) + \gamma \left(\frac{P_{2,t}^c}{P_{2,t}}\right)^{\theta} \left(C_{2,t}^c + I_{2,t}^c\right) + \mathcal{C}_{2,t} + \omega_2 V_{2,t}$$
(3.33)

 $C_{j,t}^i$ ,  $I_{j,t}^i$  respectively denote the demands for good j from country (i = 1, 2, j = 1, 2) for consumption and investment. The job filling probability is given by

$$\Phi_{i,t} = \left(\frac{V_{i,t}}{e_i(1 - N_{i,t})}\right)^{\psi - 1} \tag{3.34}$$

Finally,

$$Y_{i,t} = A_{i,t} K_{i,t}^{\alpha} (h_{i,t} N_{i,t})^{1-\alpha}$$
(3.35)

$$K_{i,t+1} = (1 - \delta)K_{i,t} + I_{i,t}^c \tag{3.36}$$

# 3.2 Empirical results

As we aim to shed new light on old debates, for numerous parameters, as well as for the stylized facts, we use standard values. The most taken from the Backus, Kehoe, and Kydland (1994)' and Andolfatto (1996)' works. We also adopt a symmetrical calibration between the two countries with a null net exports steady state. This facilitates the comparison of our results with a bulk of previous literature on international fluctuations. For the additional parameters, in particular those regarding taxes, the procedure for calibrating is quiet traditional. However, matter of consistency, estimations and average values are based on the 1970:1-1986:4 period, as in Backus et al.(1994).

#### 3.2.1 Parameterization

The technology parameters are calibrated as follows (Backus *et al.*(1994)). The autocorrelation parameter  $\rho_{1,1} = \rho_{2,2} = \rho$  is set at 0.906. The cross-country diffusion parameter  $\rho_{1,2} = \rho_{2,1} = \rho^*$  is fixed at 0.088 and  $v_{1,2} = v_{2,1} = v$  is calibrated in order to get a correlation between technology innovations of 0.258. The depreciation rate  $\delta$  is set at 0.025.  $\alpha$ , which no longer corresponds to the labor share of output, is calibrated in order to get a labor share of 64%.

Following Backus et al.(1994),  $\theta$ , the elasticity of substitution between domestic and foreign goods, is set at 1.5, while the discount factor  $\beta$  is set at 0.99. The value of the home bias  $\gamma$  is set at 0.85. The capital adjustment cost parameter,  $\hat{\phi}$  is calibrated in order to replicate the volatility of investment in each model configuration.

For the labor-market parameters, the calibration is symmetric across countries and rely mostly on (Andolfatto 1996). The elasticity of the matching function with respect to vacancies and the firm's bargaining power are set to  $\epsilon = \psi = 0.6$ . The value of  $\chi$  is chosen to be consistent with the stationary values (in the non-taxation economy) for the probability that a vacancy position becomes a productive job, the employment ratio and the fraction of time spent working:  $\Phi = 0.9$ , N = 0.57 and h = 1/3. The ratio of aggregate recruiting expenditures to output is fixed at  $\omega V^*/Y^* = 1\%$ , and the average fraction of time that nonemployed households devote to search to  $e = h^*/2$ . Following Hairault (2002), we choose  $\eta = 5$  and the quarterly rate of transition from employment to non-employment equal to s = 0.10. Lastly, parameters  $\sigma$  and  $\Gamma^u$  are computed to be consistent with steady-state restrictions.

The last set of values concerns the evolution of the tax rates (equation 3.12). We estimate the

Table 3.1: Tax rates stochastic processes:  $\tau^i_{us,t+1} = \rho^i_{us} \tau^i_{us,t} + \epsilon^i_{us,t+1}$ 

	$ au_{w,t+1}^{us}$			$ au_{f,t+1}^{us}$			$ au_{c,t+1}^{us}$		
	Coef.	t-stat.	t-prob.	Coef.	t-stat.	t-prob.	Coef.	t-stat.	t-prob.
$\overline{ au}_w^{us}$	0.2078								
$ ho_{us}^w$	0.7790	10.5179	0.0000						
$\overline{\tau}_f^{us}$				0.1974					
$ ho_{us}^f$				0.6479	6.5543	0.0000			
$\overline{\tau}_c^{us}$							0.0832		
$ ho_{us}^c$							0.7868	10.0143	0.0000
$R^2$		0.6446			0.4132			0.6156	

OLS estimations. Hodrick-Prescott filtered data. The log-linear interpolation process is simple: the average annual growth  $\gamma$  in the variable  $\tau$  is calculated as  $\gamma = \exp(\ln(\tau_{Q4}/\tau_{Q0})/3)$ , where  $\tau_{Q4}$  and  $\tau_{Q0}$  are the observed values 3 quarters apart. The resulting average quarterly growth rate between two consecutive years is applied to generate the interpolated series.

AR(1) processes for the US in the 1970-1986 period using the annual series of average tax rates constructed by McDaniel (2007). However, following McGrattan (1994), data are log-linearly interpolated to obtain quarterly observations. See table 3.1.

The equilibrium can now be computed numerically.

### 3.2.2 Models evaluation

The equilibrium decision rules are used to simulate the time paths for the variables of interest. The statistical properties of these simulated time series are then compared to the statistics summarizing the cyclical properties of the US and the model economies. Statistics are reported in Table 3.2. Models include the Hairault (2002)'s international search model with non-separable preferences<sup>6</sup> (LMS0), and our tax/benefit, international search model with standard preferences under six configurations: (i) fluctuations are only driven by productivity shocks (LMS1), (ii) fluctuations are driven as well by consumption tax shocks (LMS2), (iii) fluctuations are driven by both technological- and labor income tax shocks (LMS3), (iv) fluctuations are driven by both technological- and payroll tax shocks (LMS4), and (v) fluctuations are driven by simultaneous shocks to technology and to all tax rates (LMS5). In explaining the instantaneous responses of variables, we shall focus primarily on the home economy (country 1).

<sup>&</sup>lt;sup>6</sup>See chapter 3 for a detailed presentation of this model.

Table 3.2: Cyclical properties

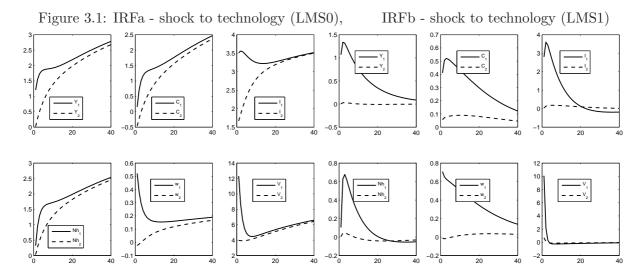
	Data	LMS1	LMS2	LMS3	LMS4	LMS5	ILMS0 (NSP)
		$(a_1)$	$(a_1, \tau_c^1)$	$(a_1, \tau_w^1)$	$(a_1,  au_f^1)$	$(a_1, \tau_c^1, \tau_w^1, \tau_f^1)$	$(a_1, taxes = 0)$
		$\phi = .6$	$(\phi = 1)$	$(\phi=0.9)$	$(\phi = 1.1)$	$(\phi = 1.4)$	$(\phi = 1.63)$
$\overline{\text{International}^{a,e}}$							
$\rho(Y_1, Y_2)$	0.51	0.42	0.48	0.47	0.50	0.52	0.55
$\rho(C_1^C, C_2^C)$	0.40	0.99	0.37	0.95	0.90	0.38	0.53
$ ho(H_1,H_2)$	0.36	-0.34	-0.04	0.23	0.51	0.62	0.74
$\rho(I_1^C, I_2^C)$	0.38	-0.65	-0.27	-0.44	-0.31	0.03	0.75
$\mathrm{USA}^{c,e}$							
$\sigma_Y$ (in %)	1.91	1.18	1.15	1.33	1.55	1.64	2.05
$\sigma_C/\sigma_Y$	0.40	0.38	0.82	0.39	0.36	0.59	0.69
$\sigma_H/\sigma_Y$	0.86	0.53	0.52	0.89	1.11	1.18	0.86
$\sigma_I/\sigma_Y$	3.07	3.00	3.07	3.02	3.08	3.07	3.05
$\sigma_{LP}/\sigma_{Y}$	0.57	0.67	0.70	0.67	0.64	0.64	0.39
$\sigma_W/\sigma_Y$	0.45	0.53	0.58	0.64	0.50	0.59	0.23
$\rho(Y_t, Y_{t-1})$	0.85	0.64	0.62	0.63	0.59	0.60	0.86
$\rho(C_t, C_{t-1})$	0.86	0.56	0.48	0.58	0.58	0.51	0.79
$\rho(H_t, H_{t-1})$	0.84	0.66	0.66	0.65	0.57	0.59	0.91
$\rho(I_t, I_{t-1})$	0.81	0.67	0.56	0.63	0.57	0.54	0.73
$\rho(LP_t, LP_{t-1})$	0.52	0.26	0.30	0.35	0.39	0.42	0.46
$\rho(Y,C)$	0.83	0.98	0.60	0.96	0.95	0.65	0.60
$\rho(Y, H)$	0.82	0.78	0.74	0.76	0.82	0.83	0.92
ho(Y,I)	0.97	0.99	0.86	0.99	0.99	0.94	0.88
$\rho(Y, LP)$	0.51	0.87	0.86	0.50	0.13	0.01	0.54
$\rho(Y,W)$	0.28	0.94	0.91	0.44	0.65	0.38	0.61
$\rho(H, LP)$	-0.07	0.38	0.30	-0.20	-0.46	-0.54	0.17
$\rho(H, W)$	0.03	0.54	0.43	-0.15	0.22	-0.002	0.26

The moments reported are computed from Hodrick-Prescott filtered artificial time series.  $^a$  Backus, Kehoe, and Kydland (1995).  $^b$  Hairault (2002).  $^c$  Chéron and Langot (2004).  $^d$  Baxter and Crucini (1993).  $^e$  Hairault (1995).

#### Only technological shocks (LMS0 and LMS1).

The combination of trading frictions and non-separable preferences (LMS0) allows the international search economy to match the stylized facts better than the tax/benefit economy (LMS1). In particular the US standard deviation of real per-capita output: 1.07% vs 58%, and the facts regarding the international fluctuations.<sup>7</sup> It also does better concerning the persistence. However, both economies fails in reproducing the low procyclicality of the real wage.

Responses to productivity shock to country 1 are similar in both economies (figure 3.1), excepting for consumption. As discussed in the previous chapter, this is a consequence of the specification of preferences. Let us turn now to the analysis of the effects from shocks only to taxes. This is



ILMS0: Labor market search economy with national specialization and non-separable preferences but without taxation, as in Hairault (2002). ILMS1: Our Tax/Benefit, labor market search economy. In both cases, country 1 receives a positive 1% shock to productivity ( $\rho_{12} = \psi = 0$ ).

useful to understand the aggregate effect of simultaneous (positive) shocks to both productivity and taxes.

#### Consumption tax shock (LMS2).

An increase in the consumption tax reduces the demand for consumption. A temporarily high tax rate on consumption provides an incentive to postpone consumption to a later date, when the tax rate is likely to be lower. Hence, as the consumption tax fluctuates, so does consumption. Furthermore, because such a tax lowers the purchasing power of an hour worked, it also reduces

<sup>&</sup>lt;sup>7</sup>See chapter 2.

the labor supply. The increase in saving raises the agent's wealth and then her outside option. This reduces the incentives to post vacancies. Remark that conversely to what we expected from the analysis of the wage equation, the consumption tax shock largely dampens the search costs. Nevertheless, the dynamics of the outside options through the decrease in  $\lambda$  (figure 3.2) dominate, so that the real wage rate responds positively in the two countries. This reduces too the incentives to post vacancies. The aggregate hours worked go down in both countries, bringing output down below trend. This explains the positive international correlation of labor input.

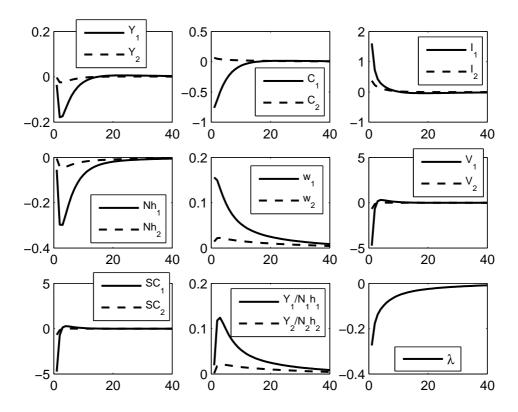
Then, an additional positive shock to consumption tax in country 1 diminishes the cross-country correlation of consumption, and it is even lower than the cross-correlation of outputs. This also enhances both, the volatility and the procyclicality of consumption. Because the consumption tax encourages saving, this leads to an accumulation of capital and so to a positive response of investment in the first periods, allowing for a slightly positive response of output few periods later, after which the economy slowly goes back to the steady state. This increases the predicted volatility of hours and consumption, bringing the implications of theory closer to the facts.

#### Labor income tax shocks (LMS3).

A positive 1% shock to labor income tax in country 1 yields either to a higher international correlation of hours and to a higher volatility of this variable. But the striking effect is the reduction of the correlation of real wage with both output and labor input. The IRF functions to a positive orthogonal 1% shock to labor income tax are found in figure 3.3. This shock produces a large response in aggregate hours, which falls about 1%. This is due to the deeper rise in the real wage (figure 3.3). Indeed, the direct impact that labor income tax have on wages is larger than the indirect adjustments of productivity and wealth. Whereas the labor income tax leads to an increase of the real wage through the bargaining process, it also leads to a decrease of labor input, implying an increase in productivity (figure 3.3) and a decrease of the reservation wage due to the lower agent's wealth ( $\lambda$  increases, figure 3.3). Then, we observe a larger fall in vacancies and in the search costs than with consumption tax shock. Basically, the leisure/labor supply decision is affected by an instantaneous substitution effect which induces households to reduce current consumption and work effort. The fall in aggregate hours, in turn, raises the average productivity in the early periods.

Finally, the larger instantaneous response of the hourly wage is also explained by the stronger

Figure 3.2: IRF - consumption tax shock to country 1



Country 1 receives a positive 1% shock to consumption tax.

effect on the outside options due to the fall in the relative prices of goods (p and E, figure 3.5), which offsets in part the fall in  $\lambda$ .

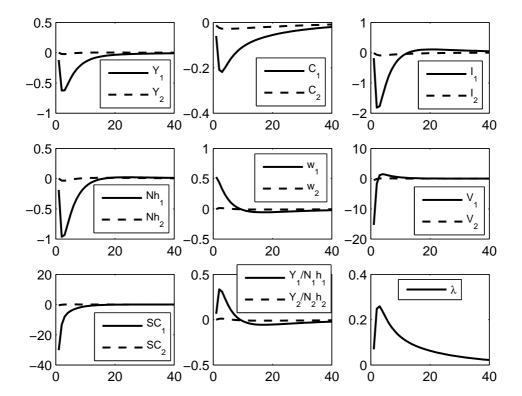


Figure 3.3: IRF - labor income tax shock to country 1

Country 1 receives a positive 1% shock to labor income tax.

#### Payroll tax shocks (LMS4).

The effects of a positive shock to payroll tax in country 1 are plotted in figure 3.4. Apart from the real wage, the instantaneous responses of variables are qualitatively similar to those produced by the shock to the labor income tax. However, the volatility of hours is larger, as well as the international correlation of labor input. But now the international correlation of investment is non-negative, whereas the correlation of real wage with both output and labor input decreases by more.

In contrast with the labor income tax, the real wage instantaneous response of a payroll tax shock is negative in country 1. This shows that a part of the taxes paid by firms is supported by workers. This decrease of the purchasing power of an hour worked leads to a fall in both consumption

and saving (investment). Nevertheless, even if the gross wage decreases, the labor cost increases. This explains why aggregate hours fall. From the employment equation (equations (3.26) we see the direct negative effect of the higher payroll tax on the firm's employment decision. This adds to the larger fall in vacancies and on the search costs. With the calibration retained, all this makes real wage in country 1 to reduce.

Furthermore, taxes to labor highlight the volatility of hours worked. In plain words, if income and payroll taxes fluctuate over time, it is optimal to work hard when taxes are relatively low and to take time off when they are relatively high. Then, as labor taxes fluctuate, so do hours worked.

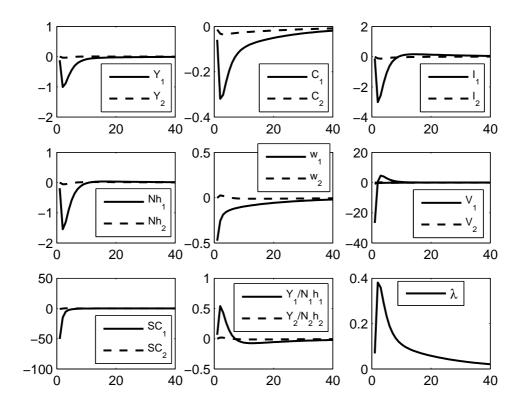
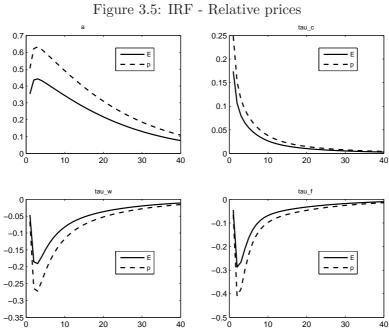


Figure 3.4: IRF - payroll tax shock to country 1

Country 1 receives a positive 1% shock to payroll tax.

#### All shocks at work (LMS5).

Finally, we add the effects of all the shocks to better understand the quantitative implications of the LMS5 economy. In this case the model reproduces quite well the facts regarding the



IRF of the relative prices of goods E and p to the several shocks. a: tax/benefit economy when country 1 receives a positive 1% shock to productivity ( $\rho_{12} = \psi = 0$ ). tau\_c: tax/benefit economy with a positive shock to consumption tax in country 1. tau\_w: tax/benefit economy with a positive shock to labor income tax in country 1. tau\_f: tax/benefit economy with a positive shock to payroll tax in country 1.

international comovements, because the consumption tax allows to reduce the international correlation of consumption, whereas the two production inputs (aggregate hours and investment) are now positively correlated, contrary two the model with only technological shock.

Concerning the labor market dynamics, the introduction of fiscal shocks allows to significantly reduce the correlation of wages with the output and the aggregate hours (the Dunlop-Tarshis puzzle). The model allows to match the relative volatility of real wages. However, the relative standard deviation of hours is slightly overstated. Nevertheless, results are encouraging.

#### 3.3 Conclusion

In this chapter the effects of distortionary taxation are studied in the context of a two-country general equilibrium model with search and matching on the labor market. We show that distortive taxes on labor and consumption have important effects on the quantitative properties of the model, allowing to outperform the predictions from the model without taxation in several lines. In particular, we show that the fluctuations in distortive taxes provide a plausive explanation from the three empirical puzzles concerning the real wage dynamics and the international fluctuations.

Moreover, our framework reconciliate the standard separable preferences with either the observed U.S. real wage rigidity and the international fluctuations, by taking into account the tax/benefyt system. This provides simultaneously an explanation to the real wage rigidity puzzle, alternative to that of Chéron and Langot (2004); and an explanation to the quantities puzzle (concerning the ranking of the outputs correlation relative to the consumptions correlation), alternative to the Hairault (2002)'s one. In the two cases, the authors base their explanation on the non-separability of agents' preferences between consumption and leisure.

Nevertheless, the problem of modelling income taxes has not been resolved in a fully satisfactory way. The volatility of labor input is exaggerated, whereas the persistence of output and the other macro aggregates is underestimated. Despite our model's shortcomings, it is striking how much we are able to explain by amending a basic two-country search model with fairly standard preferences to include fiscal policy variables.

# Chapter 4

# Growth, Unemployment and Tax/Benefit system in European Countries

This chapter is in part based on a joint working paper with Stéphane Adjemian and François Langot (European Commission, TAXBEN SCS8-CT-2004-502639, 2006.)

# Introduction

The observed high unemployment in continental Europe and the slowdown in economic growth in the last decades naturally raise the question of whether these two phenomena are related. On the empirical side, there is no consensus regarding the sign of the correlation between growth and unemployment, either across countries or over time within a country. The same is true on the theoretical side. Nevertheless, the endogenous growth theory predicts that distortions due to fiscal instruments lead to a lower growth whereas the equilibrium unemployment theory predicts that these distortions lead to a higher unemployment rate. This suggests that the link between growth and unemployment can be viewed through the simultaneous link of growth and unemployment with the labor market institutions.

In this chapter we investigate the issue of the long run link between growth and unemployment at two levels. First, we conduct an empirical analysis to we explore the heterogeneity of growth and unemployment experiences across 183 European regions and we evaluate how much of this heterogeneity is accounted by the national labor market institutions. The originality of this approach is to take into account the large heterogeneity between regions among a country. Second, we construct a theoretical economy to assess the explicative role of labor-market variables on the bad performance of European countries. The main hypotheses of our model are the following: (i) Innovations are the engine of growth. This implies a "creative destruction" process generating jobs reallocation. (ii) Agents have the choice of being employed or being trying their hand at R&D; and (iii) Unemployment is caused both by the wage-setting behavior of unions, and by the labor costs associated to the tax/benefit system.<sup>3</sup> In addition, in the appendix to this chapter, we conduct a social welfare exercise using a simplified version of this model.

The advises from the empirical exercise are that: (i) The tax wedge and the unemployment benefits are positively correlated with the regional unemployment rates. Conversely, the employment protection and the level of coordination in the wage bargaining process are negatively correlated with the regional unemployment rates. (ii) The tax wedge and the unemployment

<sup>&</sup>lt;sup>1</sup>See Mortensen (2005) for a wide review of the empirical literature, which shows the diversity of results about the correlation between growth and unemployment.

<sup>&</sup>lt;sup>2</sup>This is due to the offsetting nature of two main effects: a higher rate of growth in productivity will reduce unemployment trough a positive "capitalization" effect on investment in job creation; whereas the "creative destruction effect", inherent to the growth process, leads to a faster obsolescence of technologies and so to a faster rate of job destruction.

<sup>&</sup>lt;sup>3</sup>The two first hypotheses are the same as those of Aghion and Howitt (1994).

benefits are negatively correlated with the regional growth rates of the Gross Domestic Product (GDP) per capita. Conversely, more coordination in the wage bargaining process diminishes the regional growth rates of GDP per capita. This last result points to the existence of an arbitration between unemployment and growth, if we focus on the impact of coordination in the wage bargaining process. These results are in accordance with those of Daveri and Tabellini (2000). Using national level data, Daveri and Tabellini (2000) find that most continental European countries exhibit a strong positive correlation between the unemployment rate and both, the effective tax rate on labor income and the average replacement rate. Conversely, they find a strong negative correlation between the growth rate of per capita GDP and the tax on labor income, either over time and across countries.

On the other side, the implications of the theoretical model are the following: (i) The bargaining power of unions, the unemployment compensation, the taxes on labor and the employment protection have a positive effect on unemployment and a negative effect on the economic growth. (ii) A more coordinated bargaining process increases employment, at the price of a lower economic growth. The first result clearly contrast with the results of Lingens (2003) and Mortensen (2005). Lingens (2003) treats the impact of unions in a model with two kind of skills, and shows that the bargain over the low-skilled labor wage causes unemployment but the growth effect is ambiguous. Similarly, in a matching model of schumpeterian growth, Mortensen (2005) finds a negative effect of labor market policy on unemployment, but an ambiguous effect on growth. Finally, in the welfare exercise, we show that the optimal growth rate can be reached by compensating the distortions on the goods-sector due to the growth process with the distortions induced by the labor market rigidities.

# 4.1 Empirical Analysis

The observed high unemployment in continental Europe and the slowdown in economic growth in lasts decades naturally raised the question of whether these two phenomena are related. On the empirical side, no consensus was found regarding the sign of the correlation between growth and unemployment, either across countries or over time within a country.

Whereas the institutions causing elevate labor costs are accepted in the empirical literature as the primary cause for high unemployment (Blanchard and Wolfers 2000), or for low hours worked and/or low participation in European countries (Kaitila 2006), the statistical relation between

unemployment-causing variables and long run economic growth is a moot point. For instance, Layard and Nickell (1999) and Kaitila (2006) show that the link between unemployment-causing variables and TFP growth is weak or nonexistent. Conversely, Daveri et al. (2000) or Alonso et al. (2004) report a negative significant impact of these labor market institution variables on the growth rate of a large panel of OECD countries. These recent empirical findings constitute an interesting point to be investigated deeply. With this aim, in this section we explore if the heterogeneity of growth and unemployment experiences across European countries prevails at a regional level and, if that is the case, how much of this is accounted by the labor market institutions.

#### 4.1.1 The data

Disaggregated data come from the Eurostat European Regional Database (Summer 2006, NUTS 2 regions).<sup>4</sup>

The selection criterium of regions was the availability of data for the 1980-2003 period.<sup>5</sup> So, we end with 183 regions belonging to Austria (AT), Belgium (BE), Germany (DE), Denmark (DK), Spain (ES), Finland (FI), France (FR), Ireland (IE), Italy (IT), Netherlands (NL), Portugal (PT), Sweden (SE) and the United Kingdom (UK). The disaggregated data we use comes from the Eurostat European Regional Database (2005).

Concerning the labor market institution indicators, we use the data provided by Blanchard and Wolfers (2000): Tax wedge (TW), Unemployment benefit (BRR), Employment protection (PE), Coordination (CO), Active labor market policies (ActPol) and Collective bargaining coverage (CbC).

#### 4.1.2 Growth and Unemployment at a regional level: a descriptive analysis

To shed some light on the relation between the growth rate of the Gross Domestic Product (GDP) per capita and unemployment, we estimate the joint density of these two variables (figure 4.1). Looking at the regional level, we do not find a clear relation between the GDP per capita growth and unemployment.

<sup>&</sup>lt;sup>4</sup>The Statistical regions of Europe correspond to the second level of the Nomenclature of Territorial Units for Statistics (NUTS 2 regions). The average size of the regions in this category is between 800 000 and 3 million. Details on this classification can be found at European Union's web site: <a href="http://europa.eu.int/comm/eurostat/ramon/nuts">http://europa.eu.int/comm/eurostat/ramon/nuts</a>

<sup>&</sup>lt;sup>5</sup>In particular, this excluded Norway from the sample.

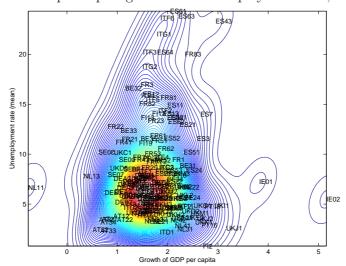


Figure 4.1: GDP per capita growth and unemployment rate, 1980-2003\*

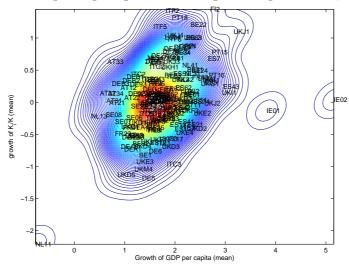
Joint distribution. The contour plots correspond to the kernel (non-parametric) estimator of the bivariate density.

\*: N.U.T.S. 2 regions (BE, DK, DE, FR, IE, IT, NL, ES, PT, SE, AT, FI, UK).

Nevertheless, the joint distribution of the growth rate of the regional capital share  $(k_j)$  with both, the growth of GDP per capita (figure 4.2), and unemployment rate (figure 4.3) suggest an interesting result. The correlation between the regional capital share and the GDP per capita is clearly positive, whereas the correlation between the regional capital share and the unemployment rate is slightly negative. Then, the regional development, measured by the growth rate of  $k_j$ , leads to more output per capita and less unemployment. In the latter case, the negative relationship is not strong enough to imply a clear link between growth of GDP and unemployment.

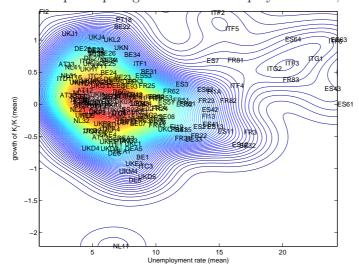
The same stronger result is suggested by the joint distribution of the growth of the Total Factor Productivity (TFP) and the growth of GDP per capita (figure 4.4), and by the joint distribution of the TFP growth and the relative unemployment rate (figure 4.5). The correlation between the growth of the TFP and the growth of the GDP per capita is clearly positive, whereas the correlation between the growth of the TFP and the unemployment rate is negative. Hence, the regional development, in this case measured by the growth of TFP, leads to more output per capita and less unemployment. As with the capital share, the negative relationship is not strong enough to imply a clear link between growth of GDP and unemployment.

Figure 4.2: GDP per capita growth and regional capital share, 1980-2003\*



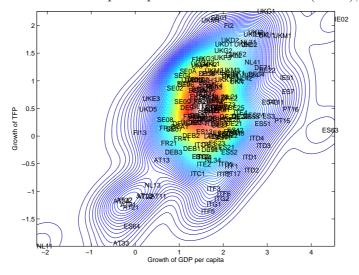
Joint distribution. \*: N.U.T.S. 2 regions (BE, DK, DE, FR, IE, IT, NL, ES, PT, SE, AT, FI, UK). The share of the capital stock in region j of country i is given by  $k_j \equiv \frac{K_{j,i}}{Ki}$ , where  $K_{j,i}$  and  $K_i$  respectively denote the regional capital stock, and the national capital stock.

Figure 4.3: GDP per capita growth and Unemployment rate, 1980-2003\*



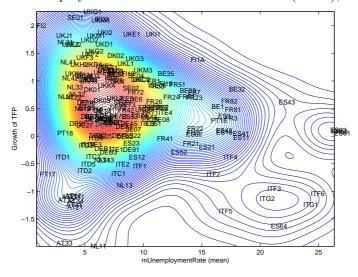
Joint distribution. \*: N.U.T.S. 2 regions (BE, DK, DE, FR, IE, IT, NL, ES, PT, SE, AT, FI, UK). The share of the capital stock in region j of country i is given by  $k_j \equiv \frac{K_{j,i}}{Ki}$ , where  $K_{j,i}$  and  $K_i$  respectively denote the regional capital stock, and the national capital stock.

Figure 4.4: Growth of GDP per capita and Growth of TFP (mean), 1980–1995\*.



Joint distribution. \*: N.U.T.S. 2 regions (BE, DK, DE, FR, IE, IT, NL, ES, PT, SE, AT, FI, UK).

Figure 4.5: Unemployment rates and Growth of TFP (mean), 1980–1995\*.



Joint distribution. \*: N.U.T.S. 2 regions (BE, DK, DE, FR, IE, IT, NL, ES, PT, SE, AT, FI, UK).

#### 4.1.3 Recovering the missing link: an econometric analysis

At a disaggregated level, the GDP per capita growth and the unemployment rate seem to be very weakly related. According to Daveri and Tabellini (2000), the relation between these two variables at the national level has mainly to be explained by common job-market-related national policies, and more precisely by taxes on wages. In this section we propose a formal statistical test allowing to evaluate the impact of national labor market institutions (taxes on wages, union density, unemployment benefits, employment protection, etc...) on the regional GDP per capita growth and the unemployment rate. The originality of the approach is to take into account the large heterogeneity between regions among a country.

The specificity of each European region is accounted by three variables: the growth rates of the regional capital share  $(\mathcal{K}_j)$ , the regional employment on the energy and manufacturing sector  $(\mathcal{E}_j^{e\&m})$ , and the mean of the growth rate of its Solow residual, which is computed assuming that the technology in each region is Cobb-Douglas. These indicators can be viewed as a close measure of the specific technology available in a specific region<sup>6</sup>. The first two are defined as follows:

$$\mathcal{K}_{j} = \frac{K_{j,i}}{Ki} 
\mathcal{E}_{j}^{e\&m} = \frac{E_{j,i}^{e\&m}}{Ei}$$

where  $K_{j,i}$  and  $K_i$  are respectively the regional capital stock, and the national capital stock. Similarly,  $E_{j,i}^{e\&m}$  and  $E_i$  are respectively the regional employment in the energy and manufacturing sector, and the national employment.

# **Empirical models**

Let  $\mathcal{X}_c$  be a  $1 \times k$  vector gathering the policy variables of country  $c = 1, \ldots, C$ . Each country c is divided in  $N_c$  regions  $i = 1, \ldots, N_c$  and we define  $N = \sum_{c=1}^{C} N_c$  the total number of European regions in our sample. Let  $\mathbf{c}$  be a mapping from the regional indices to the national indices:

$$\mathbf{c}: \{1, \dots, N\} \mapsto \{1, \dots, C\}$$

$$j \to \mathbf{c}(j)$$

<sup>&</sup>lt;sup>6</sup>The theoretical model can be viewed as a regional economy with specific innovation process.

Our empirical models are defined by the two following pairs of equations:

$$g_{j} = \alpha^{g} + \mathcal{X}_{\mathbf{c}(j)}\beta^{g} + \mathcal{S}R_{j}\gamma^{g} + \varepsilon_{j}^{g}$$

$$u_{j} = \alpha^{u} + \mathcal{X}_{\mathbf{c}(j)}\beta^{u} + \mathcal{S}R_{j}\gamma^{u} + \varepsilon_{j}^{u}$$

$$(4.1)$$

and

$$g_{j} = \alpha^{g} + \mathcal{X}_{\mathbf{c}(j)}\beta^{g} + \mathcal{K}_{j}\gamma^{g} + \mathcal{E}_{j}^{e\&m}\delta^{g} + \varepsilon_{j}^{g}$$

$$u_{j} = \alpha^{u} + \mathcal{X}_{\mathbf{c}(j)}\beta^{u} + \mathcal{K}_{j}\gamma^{u} + \mathcal{E}_{j}^{e\&m}\delta^{g} + \varepsilon_{j}^{u}$$

$$(4.2)$$

where  $g_j$  and  $u_j$  are respectively the growth rate of GDP per capita and the unemployment rate (average) of region j,  $\alpha^g$  and  $\alpha^u$  are two constants that will eventually be replaced by the following set of dummy variables:  $dum_1$ : DK, SE, NL, FI;  $dum_2$ : BE, DE, FR, ES, PT, AT, IT; and  $dum_3$ : IE, UK. These dummy variables regroup countries according to an specific socioeconomic organisation which is not included in our set of explanatory variables (Nordic, Anglo-saxon and Continental countries).  $\varepsilon_j^g$  and  $\varepsilon_j^u$  are two zero expectation random variables such that  $E\left[\varepsilon_j^s\varepsilon_j^s\right] = \sigma_s^2$ ,  $E\left[\varepsilon_j^s\mathcal{X}_{\mathbf{c}(j)}\right] = 0$  for s = u, g and  $E\left[\varepsilon_j^u\varepsilon_j^g\right] = 0^7$ . Finally, the growth rate of the Solow residual is denoted by  $\mathcal{S}R_j$ .

#### **Empirical strategy**

The estimation of models (4.1) and (4.2) may be done using OLS equation by equation, but this approach would eventually be sensible to the existence of outliers. Figures 4.1, 4.2 and 4.3 suggest that there is a number of such observations, so a more robust approach is needed. In order to obtain point estimates less sensible to outliers we use a median-regression (LAD) instead of mean-regression (OLS). For instance, in the case of the growth equation this estimator is defined as follows:

$$\begin{split} \widehat{b}_{LAD,N}^g & \equiv \left( \widehat{\alpha}_{LAD,N}^g, \widehat{\beta}_{LAD,N}^g, \widehat{\gamma}_{LAD,N}^g \right) \\ & = & \arg \min_{\{\alpha^s,\beta^s,\gamma^s\}} \sum_{j=1}^N \left| g_j - \alpha^g - \mathcal{X}_{\mathbf{c}(j)} \beta^g - \mathcal{S} R_j \gamma^g \right| \end{split}$$

we minimize the sum of the absolute values of the residuals instead of the sum of the squared residuals. The asymptotic distribution of this estimator is given by:

$$\sqrt{N} \left( \widehat{b}_{LAD,N}^g - \beta \right)_{N \to \infty} \Longrightarrow \mathcal{N} \left( 0, \frac{1}{2f_{\varepsilon^g}(0)} (X'X)^{-1} \right)$$

<sup>&</sup>lt;sup>7</sup>Under these assumptions we can estimate (4.1) and (4.2) equation by equation.

where X is a  $N \times (k+2)$  matrix gathering the constant, the set of policy variables and the growth rate of the Solow residual, and  $f_{\varepsilon^g}$  the density function associated to the error term. As a consequence, to test if a parameter significantly differs from zero we have first to evaluate the density of the error term at zero. To evaluate the variance of  $\hat{b}_{LAD,N}^g$  we can (i) impose a parametric shape to the error term, (ii) use a nonparametric (kernel) estimate of the density at zero or (iii) use a bootstrap approach as described in Greene (2002). In what follows we consider the latter solution, which has the advantage over (i) and (ii) to be exact at finite distance.

#### Results

Estimations from the specification in (4.1) are reported in table 4.1, whereas those from the specification in (4.2) are reported in table 4.2. In both cases, we estimate two regressions: a first one where the endogenous variable is the growth rate of GDP per capita for each European Region (labeled Growth) and a second one where the endogenous variable is the Regional unemployment rate (labeled Unemployment).

First specification. In the growth equation, excepting for the PE (Employment protection), the Actpol (active labor market policies) and the CbC (collective bargaining coverage), all the point estimates significantly differs from zero at a 5% level. Finally, the positive link between the growth rate of the regional TFP and the growth rate of GDP per capita, suggested by figure 4.4, is confirmed by this statistical analysis. Concerning the unemployment equation, all the variables have the expected signs, except ActPol (active labor market policies) and are significant, except CbC (collective bargaining coverage).

Second specification. In the growth equation, the point estimates significantly differs from zero at a 5%, and have the expected sign for the following variables: the regional capital share, the tax wedge (TW), the replacement rate (BRR), and the coordination on the wage bargaining (CO). Finally, the positive link between the growth rate of the regional capital stock and the growth rate of GDP per capita, suggested by figure 4.2, is confirmed by this statistical analysis. Concerning the unemployment equation, all the variables have the expected signs, except ActPol (active labor market policies) and are significant at 5% or 10% level.

#### **Summary:**

Table 4.1: First specification.

	Gro	wth	Unemployment		
	β	p-value	β	p-value	
gTFP	0.7983	0.0000	-0.9349	0.0070	
TW	-3.0425	0.0000	5.1462	0.0250	
BRR	-0.5436	0.0000	2.8232	0.0000	
PE	0.4098	0.1006	-7.7997	0.0000	
CO	-2.0250	0.0000	-20.453	0.0000	
ActPol	0.2215	0.0718	4.3593	0.0000	
CbC	-0.2311	0.6081	0.5911	0.8058	
dum1	5.1820	0.0153	156.33	0.0000	
dum2	8.4435	0.0152	279.67	0.0000	
dum3	-1.5131	0.0179	17.819	0.0000	
Fischer	232.04	0.0000	81.07	0.0000	
$R^2$	0.6789	_	0.3484	_	
# Observations	183	_	183	_	

LAD estimation. The dependent variables are annual mean GDP per capita growth rate for the **Growth** regression and mean unemployment rate for the **Unemployment** regression. Student and associated p-values are computed with a bootstrap procedure as advocated by Greene (2002).

Table 4.2: Second Specification.

Table 4.2. Second Specification.							
	Grov	$\operatorname{wth}$	Unemployment				
	β	p-value	β	p-value			
$\mathcal{K}_{j}$	0.4487	0.0000	-1.1516	0.0001			
$\mathcal{E}_{j}^{e\&m}$	-0.0015	0.9138	-0.1278	0.0685			
TW	-1.2368	0.0002	2.7331	0.0996			
BRR	-0.1379	0.0320	2.6579	0.0000			
PE	0.0037	0.9847	-3.9600	0.0001			
CO	-1.4539	0.0000	-16.5395	0.0000			
ActPol	0.1208	0.2149	3.8073	0.0000			
CbC	0.2634	0.4732	4.0794	0.0305			
dum1	12.2149	0.0000	116.2032	0.0000			
dum2	18.8026	0.0000	213.3097	0.0000			
dum3	1.9634	0.0001	16.7360	0.0000			
Fischer	218.2335	0.0000	71.3733	0.0000			
$R^2$	0.44314	_	0.28323	_			
# Observations	183	_	183	_			

LAD estimation. Student and associated p-values are computed with a bootstrap procedure as advocated by Greene (2002).

- The tax wedge (TW) and the unemployment benefits (BRR) lower the growth rates but increase the unemployment rates,
- The coordination of the wage bargaining (CO) lowers the growth rates and the unemployment rates.
- Either the growth rate of the regional capital share, or the growth rate of the TFP, increase (decrease) the GDP per capita growth (the unemployment).
- The bargaining power increases the unemployment in the second specification.

Finally, for the growth and unemployment equations, in the first specification the  $R^2$  are respectively 44% and 28%, meaning that our collection of labor market related policy variables and the growth rate of the two regional-specific variables explains about 1/2 of the heterogeneity of the growth rates and roughly 1/3 of the heterogeneity of the unemployment rates. Likewise, in the second specification these values are respectively 68% and 35%, meaning that our collection of labor market related policy variables and the growth rate of the TFP explains more than 2/3 of the heterogeneity in growth rates and roughly 1/3 of the heterogeneity in unemployment rates. As expected, the role of Solow residuals is much more important explaining growth than unemployment.

## Counterfactuals

In this section, we propose to evaluate the marginal impact of both national (each labor market institution) and regional (the growth rate of the TFP) components on the predicted growth and unemployment rate of an European region.

## The methodology

Let considers the following experience. We assume that a Region j' in France has the same environment than a region j in UK excepting for one of its national specific variables (labor market policies) or its specific regional one. Using the estimation of the growth and unemployment rate, this experience allows us to evaluate the marginal impact of the national/regional specific variables.

More precisely, we construct these counterfactual experiences as follows:

• Predicted GDP per capita growth of Region j in UK is defined by:

$$\widehat{g}_{j,UK} = \widehat{c}_g + \mathcal{X}_{UK} \widehat{\beta}_g + \mathcal{S} R_{j,UK} \widehat{\beta}_g$$
 with  $\mathcal{X}_{UK} \equiv (TW_{UK}, BRR_{UK}, PE_{UK}, CO_{UK}, ActPol_{UK}, CbC_{UK})$ 

• Suppose that Region j' in France is as Region j in UK with respect to all the conditioning variables except Tax Wedge. Hence Region j' in France counterfactual GDP per capita growth will be:

$$\widetilde{g}_{j',FR}^{TW} = \hat{c}_g + \widetilde{\mathcal{X}}_{FR}^{TW} \hat{\beta}_g + \mathcal{S}R_{j',UK} \hat{\beta}_g$$
 with  $\widetilde{\mathcal{X}}_{FR}^{TW} \equiv (TW_{FR}, BRR_{UK}, PE_{UK}, CO_{UK}, ActPol_{UK}, CbC_{UK})$ 

The gap between  $\hat{g}_{j',FR}$  and  $\tilde{g}_{j',FR}^{TW}$  gives a measure of the marginal effect of the French fiscal policy.

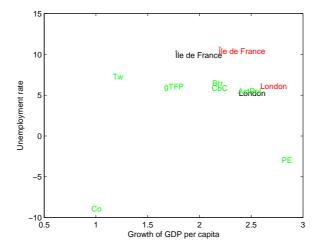
#### The results

Due to the high number of Regions (183), we focus only on typical cases. Then, we assume that the reference is London, and we choose to evaluate the marginal impact of typical European labor market experience. Then, we choose a north continental country (France), a south continental country (Spain) and a Nordic country (Sweden). In the two first countries, we propose to evaluate the marginal impacts of the explanatory variable in two Regions: a Region highly developed and a poor one. For France, we choose "Ile de France" because this Region encompasses Paris, and "Corse". For Sapin, we choose "Madrid" and "Andalucia".

Figures 4.6 and 4.7 present the results for the French economy. First in figure 4.6, we show that the predictions of the econometric model are close to the observed values. The point TW represents the prediction of the model if all the explanatory variables, except the taxes, are the same than in London. Hence, the gap between the prediction for London and this point gives a measure of the marginal impact of the French tax<sup>8</sup>. The higher unemployment and the lower growth in Paris than in London are mainly due to the higher tax (TW) and to a lower growth in TFP (gTFP). Moreover, the wage bargaining coordination (CO) in France leads to less unemployment but at the price of a lower growth rate of the GDP per capita. Second, in figure 4.7, we show that the predictions of the model are quit poor for Corse, the poorest French Region.

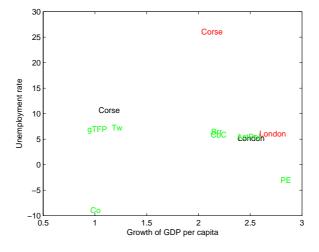
 $<sup>^8</sup>$ The same is tue for all the explanatory variables: employment protection (PE), unemployment benefits (Brr), etc...

Figure 4.6: The French case (I): London versus Paris (Ile de France).



Observed and predicted London are respectively denoted "London" and "London". We use the same color convention for Île de France. The marginal effects of our explanatory variables are in soft color (CbC, Tw, etc...).

Figure 4.7: The French case (II): London versus Corse



This clearly suggests that this region gets specific policies which lead to a higher unemployment than its model predictive value. Nevertheless, this experience for Corse underlines that, beyond the national component as the high tax (TW) already mentioned for Paris, it is the lack of R&D investments, measured by the growth rate of the TFP (gTFP) that largely explains the lower performance of this Region.

Figure 4.8 gives an illustration of our estimation for a Nordic Region, the Region of Stockholm. The results show that higher taxes in Sweden than in UK lead to more unemployment and less growth. Nevertheless, contrary than for the French Region, the level of the growth rate of the TFP leads this Nordic Region to converge toward the Region of London. Moreover, as the coordination of the wage bargaining is higher than in the French economy, this leads to largely decrease the unemployment rate, whereas the impact of this labor market institution is negligible in the growth equation.

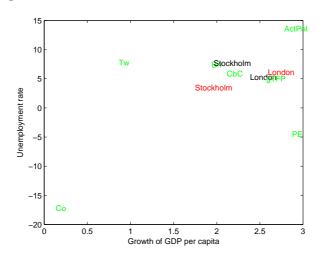


Figure 4.8: The Nordic case: London versus Stockholm

What do we learn from the Spanish cases? Figures 4.9 and 4.10 show that these higher unemployment rates are mainly due to the low level of TFP growth. If the growth rate of the GDP per capita is high, it is not explained by a high level of technology (gTFP). Then, these Regions have a high level of growth (equal or higher than the one observed in the Region of London), but this growth can be explained only by a catch-up phenomena. The poor performances measured by the growth rate of the TFP, even in Madrid, would lead the Spanish government to give some incentives in the R&D sector. The estimation also shows that the labor market institutions in

Figure 4.9: The Spanish case (I): London versus Madrid

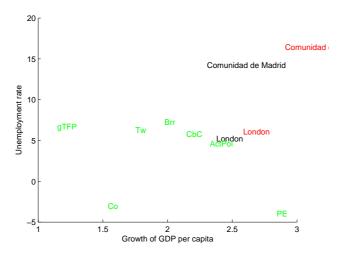
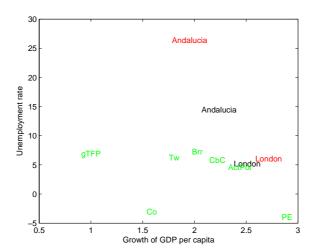


Figure 4.10: The Spanish case (II): London versus Andalucia



Spain lead to better economic performances than in France, for exemple.

# 4.2 The model

At the light of the empirical results, we develop the next theoretical model.

## 4.2.1 Preferences

The economy is populated by L identical agents, each endowed with one unit flow of labor. At each time, they may be employed (x), trying their hand at R&D (n) or unemployed (u): L = x + n + u. When employed, workers pay a tax  $\tau^w$  on their labor income. When unemployed, they receive the unemployment benefits B.

All individuals have the same linear preferences over lifetime consumption of a single final good:

$$U(C_t) = E_0 \int_0^\infty C_t e^{-\rho t} dt \tag{4.3}$$

where  $\rho > 0$  is the subjective rate of time preference and  $C_t$  is the per capita consumption of the final good at time t. Each household is free to borrow and lend at interest rate  $r_t$ . However, given linear preferences, the optimal household's behavior implies  $\rho = r_t \,\forall t$ . Hence, the level of consumption is undefined. A standard solution to this problem is to assume that households consume all their wage income. This assumption allows us to analyze the impact of the unemployment benefit system.

## 4.2.2 Goods sector

The final good is produced by perfectly competitive firms that use the latest vintage of a continuum of intermediate inputs  $x_j$ ,

$$C_t = \int_0^1 A_{j,t} x_{j,t}^{\alpha} dj, \quad 0 < \alpha < 1, \quad j \in [0,1]$$
(4.4)

 $A_j$  represents the productivity of the intermediate good j and is determined by the number of technical improvements realized up to date t, knowing that between two consecutive innovations the gain in productivity is equal to q > 1 ( $A_{t+1} = qA_t$ ).

In turn, intermediate goods are produced by monopolistic firms. Production of one unit of intermediate good requires one unit of labor as input. Since the final-good sector is perfectly competitive, the price of each intermediate good,  $p(x_j)$ , is equal to the value of its marginal

product:

$$p(x_{j,t}) = \frac{\partial C}{\partial x_{j,t}} = \alpha A_{j,t} x_{j,t}^{\alpha - 1} \quad \forall j$$
(4.5)

## 4.2.3 R&D sector

Technology improvements lead to good-specific public knowledge allowing to start improvement efforts upon the current vintage v. Innovations on good j arrive randomly at a Poisson rate  $hn_j$ , where  $n_j$  is the amount of labor used in R&D, and h > 0 a parameter indicating the productivity of the research technology. Finally, the size of the R&D sector is given by the arbitrage condition:<sup>9</sup>

$$\frac{(1-\tau^w)W_{j',v}}{h} \le \min_{j} V_{j,v+1} \quad \forall j, j' \in [0,1]$$
(4.6)

That is, the opportunity cost of R&D is the hourly net wage prevailing in the production sector, industry j,  $(1-\tau^w)W_{j',v}$ , times the expected duration of the innovation process, 1/h.<sup>10</sup> On the other hand, the expected payoff of next innovation,  $V_{j,v+1}$ , is equal to the net discounted value of an asset yielding  $\Pi_{j,v+1}$  per period until the arrival of next innovation, at the arrival rate  $hn_{j,v+1}$ .

We assume that the employment protection laws imply a cost E of shutting down a firm, which occurs as current producers are replaced by next ones. Then:

$$V_{j,v+1} = \frac{\prod_{j,v+1} - hn_{j,v+1} E_{v+1}}{r + hn_{j,v+1}}$$
(4.7)

Assuming that Firms pay a proportional payroll tax  $\tau$  over employment, the instantaneous monopolistic profits earned by the successful innovator are:

$$\Pi_{j,v+1} = p(x_{j,v+1})x_{j,v+1} - W_{j,v+1}(1+\tau)x_{j,v+1}$$
(4.8)

Normalizing the lasts expressions by the productivity level associated to the  $(v+1)^{th}$  innovation, and using equation (4.5) we obtain:

$$\pi_{i,v+1} = \alpha x_{i,v+1}^{\alpha} - w_i (1+\tau) x_{i,v+1} \tag{4.9}$$

<sup>&</sup>lt;sup>9</sup>Equivalently, the entry condition also reflects the fact that labor can be freely allocated between production and research:  $(1 - \tau^w)W_{j',v}$  is the net value of an hour in production while  $hV_{j,v+1}$  is the expected value of an hour in research.

<sup>&</sup>lt;sup>10</sup>Equivalently, we can assume that the opportunity cost amounts to the unemployment benefits, or even to a linear combination of both, the earnings of employed and those of unemployed workers.

hence the free entry (D.3) condition becomes:

$$(1 - \tau^{w})w_{j',v} \leq qh\nu_{j,v+1}$$

$$= qh\left(\frac{\pi_{j,v+1} - hn_{j,v+1}e}{r + hn_{j,v+1}}\right)$$
(4.10)

for 
$$\pi \equiv \frac{\Pi}{A}$$
,  $w \equiv \frac{W}{A}$ ,  $e \equiv \frac{E}{A}$  and  $\nu \equiv \frac{V}{A}$ .

## 4.2.4 Government

The government faces the following budget constraint:

$$Bu + T = (\tau + \tau^w) \int_0^1 w_j x_j dj + Eh \int_0^1 n_j dj$$
 (4.11)

Any change in the revenue caused by changes in taxes and subsidies is rebated to household through the lump-sum transfer T.

## 4.2.5 Wage bargaining and labor demand

The wage rate is the solution to the bargaining problem between the monopolistic producer of good j and the trade union representing the workers' interests. We model the bargaining process as a generalized Nash bargaining game, with union's relative bargaining power  $\beta$ . If they don't agree, workers get the unemployment benefits and the monopolist pays the firing costs E. Given the bargained wages, the firm chooses the level of employment that maximizes her profit flow. That is,

$$W_{j,v+1} = \arg\max\left\{ \left[ ((1-\tau^w)W_{j,v+1} - B_{j,v+1})x(W_{j,v+1}) \right]^{\beta} (\Pi_{j,v+1} - hn_{j,v+1}E - \bar{\pi}_{j,v+1})^{1-\beta} \right\}$$
(4.12)

 $\bar{\pi}_{j,v+1} \equiv -hn_{j,v+1}E$  denotes the firm's disagreement point.

## 4.2.6 Equilibrium

Given  $\rho > 0$ , for all intermediate good sector j and for all vintage v a **steady-state** (or balanced growth path) equilibrium is defined as follows:

$$w = \frac{\beta_1 b}{1 - t}, \quad \beta_1 \equiv 1 + \frac{\beta(1 - \alpha)}{\alpha} \tag{4.13}$$

for  $w \equiv \frac{W}{A}$ 

## (ii) Labor demand:

$$x = \left(\frac{\alpha^2 (1 - \tau^w)}{(1 + \tau)\beta_1 b}\right)^{\frac{1}{1 - \alpha}} \tag{4.14}$$

## (*iii*) **R&D**

The symmetry on wages and so on labor demand imply that the expected gains from an innovation are identical across industries:  $V_{j'} = V_j \ \forall j, j' \in [0, 1]$ . By consequence the amount of labor allocated to R&D is the same for any intermediate good j:  $n_j = n$ . Hence, from the free entry condition we deduce:

$$n = \left(\frac{1}{h}\right) \left(\frac{qh\pi - r\beta_1 b}{\beta_1 b + qhe}\right) \tag{4.15}$$

where

$$\pi = \frac{(1-\alpha)(1+\tau)\beta_1 b}{\alpha(1-\tau^w)} x \tag{4.16}$$

## (iv) Unemployment:

Unemployment u is deduced from the employment identity given the endowment of labor L, the labor demand for production x and the aggregate number of potential innovators n:

$$u = L - x - n \tag{4.17}$$

## (v) Government:

The balanced budget of government is:

$$bu + T = (\tau + \tau^w)wx + ehn \tag{4.18}$$

were  $b \equiv \frac{B}{A}$ , and  $\top \equiv \frac{T}{A}$ .

(vi) **Economic growth:** Between two consecutive innovations final output is augmented a fixed amount q:  $C_{v+1} = qC_v$ . Then, between date t and date t+1 expected output is:

$$E[C_{t+1}] = q^{\int_0^1 h n_t dt} C_t$$

By taking logarithms and arranging terms we get:

$$g_t \equiv E[\ln C_{t+1} - \ln C_t] = hn_t \ln(q)$$

Then, at the steady state  $(n_t = n)$ :

$$g = hn\ln(q) \tag{4.19}$$

# 4.3 The impact of labor market institutions on growth and unemployment

## 4.3.1 Labor market policies

In this section we analyze the consequences for growth and unemployment of, (ii) a more generous unemployment insurance, (ii) higher taxes on labor incomes, and (iii) the employment protection.

**Proposition.** 1 An increase in the unemployment compensation (b), or in the payroll taxes ( $\tau$ ), or in the taxes on labor income ( $\tau^w$ ) or in the employment protection (e), leads to (i) higher unemployment and (ii) lower rate of growth.

This result is very intuitive (see the proof in the appendix): higher labor costs imply higher wages (equation (4.13)) and so a decline in the labor demand (equation (4.14)). This contracts the monopolistic profits and reduces the expected value of an innovation. Moreover, the higher wages make production more attractive than R&D. As the size of R&D decline, the growth rate falls. Since neither the wage rates nor the labor demands change, the only effect is a contraction of profits. This reduces the workers' incentives to engage in R&D. Then the growth rate falls and the unemployment raises.

## 4.3.2 The wage bargaining processes

The impact of unions is analyzed in two steps. First, for an uncoordinated wage bargaining process we derive the implications of a higher bargaining power. Second, we can compare the outcome of an efficient bargaining process (that is, with simultaneous bargain of wages and labor demand) with the inefficient outcome computed above.

## The bargaining power

**Proposition. 2** An increase in the unions' bargaining power leads to an increase in the unemployment level and to a decrease in the economic growth.

The economic intuition is the following (see the proof in the appendix): a bigger bargaining power implies higher wages. Then the labor demand for production declines, this contracts the monopolistic profits and so the expected value of an innovation. This discourages workers from R&D. The total outcome is higher unemployment and lower economic growth.

## Inefficient v.s. efficient bargain

If in each industry the monopolistic firm and the trade union bargain jointly over the labor demand and the wage rate, the outcome is the efficient one (E). In formal terms, the wage and the firm size pairs are the solution to the following problem:

$$(w_{j,v+1}^E, x_{j,v+1}^E) = \arg\max \left\{ [((1-\tau^w)w_{j,v+1}^E - b)x_{j,v+1}^E]^\beta \right.$$
$$(\pi_{j,v+1}^E - hn_{v+1}^E e - \bar{\pi}_{v+1}^E)^{1-\beta} \left. \right\}$$

The firm's disagreement points and the instantaneous profit flow are respectively:

$$\bar{\pi}_{v+1} \equiv -hn_{v+1}e$$

$$\bar{\pi}_{j,v+1}^E = \alpha (x_{j,v+1}^E)^{\alpha} - w_{j,v+1}^E (1+\tau) x_{j,v+1}^E$$

Then at equilibrium, for all j and for all vintage v:

$$w_E = \frac{\beta_1 b}{1 - \tau^w} \tag{4.20}$$

$$x_E = \left(\frac{(1-\tau^w)\alpha^2}{(1+\tau)b}\right)^{\frac{1}{1-\alpha}} \tag{4.21}$$

$$n_E = \left(\frac{1}{h}\right) \left(\frac{qh\pi_E - r\beta_1 b}{\beta_1 b + qhe}\right)$$

$$\pi_E = \frac{(1 - \alpha\beta_1)(1 + \tau)b}{\alpha(1 - \tau^w)} x_E$$
(4.22)

**Proposition. 3** Under efficient bargaining, employment levels are larger but the rate of economic growth is also lower than under uncoordinated bargaining. However, the comparison is ambiguous for unemployment.

The gain in employment is due to the coordination in the setting of wages and the labor demand for production. The decreasing returns to research and the unchanged opportunity cost of R&D explain why economic growth is lower under efficient bargaining (see the proof in the appendix).

**Summary:** Most of the theoretical results are in accordance with our empirical approach. The few exceptions are:

• Converse to the empirical model, the theoretical model predicts an ambiguous link between unemployment and coordination.

• Even if the link between the bargaining power and the GDP growth is not significant, it has the unambiguous sign predicted by our theoretical model. These results can be explained by the poor approximation of our statistical measure (collective bargaining coverage (CbC)) to the workers' bargaining power.

## 4.4 Conclusion

We have constructed a general equilibrium model in which economic growth and unemployment are endogenously determined by the number of innovations made in the economy, which in turn is determined by the workers' incentive to engage in R&D activities. We have shown that high labor costs or powerful trade unions lead to bigger unemployment and to a slowdown of the economic growth whereas an efficient bargain allows to higher employment, at the price of a lower growth rate.

Using a cross-section of European regions and a large set of labor market variables, we find that national institutions on the labor market are highly correlated with unemployment. Hence, the tax wedge and the unemployment benefits increase the regional unemployment rates whereas the employment protection and a high level of coordination in the wage bargaining process decrease the regional unemployment rates. On the other hand, we find that increases in the tax wedge and in the unemployment benefits decrease the regional growth rate of GDP per capita. Nevertheless, a high level of coordination in the wage bargaining process decreases the regional growth rate of GDP per capita. This last result shows that there is an arbitration between unemployment and growth if we focus on the impact of the coordination in the wage bargaining process. Finally, the empirical results concerning the active labor market policies (ActPol) suggest to include them into the theoretical model because they have positive impact on the unemployment rate.

# Chapter 5

Explaining the evolution of hours worked and employment across OECD countries: an equilibrium search approach

This chapter is based on a joint paper with François Langot (IZA DP No. 3364, February 2008)

# Introduction

Aggregate hours of market work exhibit dramatic differences across industrialized countries. What accounts for these differences? In the current literature, there are two candidate approaches allowing to explain these differences.

A first set of contributions focus on the decline of the average hours worked per employee (the intensive margin) in European countries since 1960. Prescott (2004) studies the role of taxes in accounting for differences in labor supply across time and across countries. He finds that the effective marginal tax rate on labor income explains most of the differences at points of time and the large change in relative (to US) labor supply over time. On this line of research, Rogerson (2006) shows that the aggregate hours worked in Continental European countries such as Belgium, France, Germany and Italy are roughly one third less than in the US. This fact results from a diverging process in the hours worked per employee in each zone: between 1960 and 1980, whereas in Europe we observe a large decrease, in the US this decline is very small; and after 1980, we observe in the two zones a stable number of hours worked per employee. This evolution of the hours worked per employee is strongly correlated to the dynamics of the taxes. Hence, as it is suggested by Prescott (2004), Rogerson (2006) or Ohanian, Raffo, and Rogerson (2006), a theory providing a link between the hours worked per employee and taxes seems to be sufficient to explain why Europeans work less than Americans.

However, since 1980 a notable feature of the data is that differences across countries in aggregate hours are due to quantitatively important differences along the extensive margin. Hence, a second set of contributions (see e.g. Jackman, Layard, and Nickell (1991), Mortensen and Pissarides (1999a), Blanchard and Wolfers (2000) or Ljungqvist and Sargent (2007b)) considers that the large decrease of the employment rate observed after 1980 in the European countries, is an important factor of the dynamics of total hours. These works show that different labor market institutions lead to different labor market outcomes after a common shock. In these previous papers, there is fairly robust evidence that (i) the level and duration of unemployment benefits and (ii) the union's bargaining power have a significant positive impact on unemployment<sup>1</sup>

To sum up, the main factors explaining the decline in the hours worked per employee differ from

<sup>&</sup>lt;sup>1</sup>There is less consensus on the effects of the employment protection legislation. At the opposite, there is some labor market institutions associated with lower unemployment: highly centralized and/or coordinated wage bargaining systems, as well as some categories of public spending on active labor market programmes. See Daveri and Tabellini (2000) or Bassanini and Duval (2006) who provide a review of recent literature on this topic.

those explaining the decline in the employment rate: the taxes for the former, and the labor market institutions, such as the unions' power or the unemployment benefits, for the second. Clearly, all together contribute to the dynamics of the two margins of the total hours.

From a theoretical point of view, the aim of this chapter is to provide a theory allowing to account for the impact, of both taxes and labor market institutions, on the two margins of the aggregate hours worked. To this end, we follow the empirical methodology presented in Ohanian, Raffo, and Rogerson (2006): the quantitative evaluation of the model and the impact of distortions is based on the computation of series for the gap between the marginal cost and the marginal return of labor that is produced using actual data and model restrictions<sup>2</sup>. Furthermore, we extend the theoretical investigation: beyond the usual neo-classical growth model which allows to predict the hours worked per employee, we explore the ability of the Hansen (1985)-Rogerson (1988) model to reproduce the dynamics of the employment rate. Finally, we develop a general equilibrium matching model, close to the one proposed by Andolfatto (1996), Fève and Langot (1996) and Chéron and Langot (2004), allowing to explain the dynamics of both the hours worked per employee and the employment rate. This last model is rich enough to allow the evaluation of the relative contribution of the tax/benefit systems and unions in the explanation of the observed allocation of time.

Our sample consists of six countries: Belgium, Spain, France, Italy, United Kingdom and the United States. Depending on the availability of data, the analysis covers the 1980-2003 or the 1960-2003 period. The main findings are the following. First, the long-run decline in the hours worked per employee is mainly due to the increase of the taxes, as it is suggested by Prescott (2004), Rogerson (2006) and Ohanian, Raffo, and Rogerson (2006). Second, the employment rate is affected by institutional aspects of the labor market, such as the bargaining power and the unemployment benefits, rather than by taxes, conversely to the individual work effort. Finally, this behavior of the two margins of the aggregate hours is well accounted by our search model, when it includes the observed heterogeneity of the tax/benefit systems and the labor market indicators of the wage-setting process across countries. These findings give some support to the two explanations of the European decline in total hours: the important role of taxes through the intensive margin and the large contribution of the labor market institutions through the extensive margin. Because these findings come from an unified framework, they also give a strong support to the matching models.

<sup>&</sup>lt;sup>2</sup>The closer these gaps are to zero, the better the model accounts for the observed labor behavior.

In the first section of the chapter, we present some stylized facts concerning the total hours worked, the contrasted dynamics of the hours worked per employee (the intensive margin) and those relative to the employment rate (the extensive margin). The second section is devoted to explore the implications of two walrasian growth models: in the first one only the intensive margin is endogenous, whereas in the second one, only the extensive margin is endogenous. This extension of the Ohanian, Raffo, and Rogerson (2006)'s work clearly shows that the increase in the divergence between theory and data is explained by two factors: the taxes for the intensive margin, and the labor market institutions for the employment rate. In the third section, we propose a model where both margins are endogenous. Moreover, this framework, by introducing search and wage bargaining, allows to measure the relative contribution of the labor market institutions and taxes. Last section gives the concluding remarks.

# 5.1 Stylized Facts

In this part we establish some facts concerning the allocation of time in the countries of our sample: Belgium, Spain, France, Italy, United Kingdom, United States<sup>3</sup>. To this goal, we decompose the aggregate number of hours between the average hours worked per employee (intensive margin) and the employment rate (extensive margin):

$$\underbrace{\frac{Nh}{A}}_{\text{hours}} = \underbrace{h}_{\text{hours}} \times \underbrace{\frac{N}{A}}_{\text{employment}}$$
(5.1)

where A denotes the active population (i.e., employed plus unemployed), h the yearly hours worked per employee and N the total civilian employment. As a first overview of the labor behavior, we compute the sample mean of each variable in equation (5.1) over the 1960 to 2003 period (table 5.1). We observe notable differences in the total hours of work (Nh/A). Moreover, countries with similar performances, measured by the aggregate hours, show different work efforts (h) and employment rates (N/A). For instance, the average total hours worked in Spain and France are very close to the total hours worked in the US. However, while in France employees work as much as in the US, in Spain the individual work effort is high enough to compensate its lower employment rate with respect to France and the US.

Since the heterogeneity in the total number of hours worked is driven by the heterogeneity of its components, we estimate their weight in the variance of the mean total hours (last line of table

 $<sup>{}^3\</sup>mathrm{See}$  appendix E.1 for details on the data.

Table 5.1: Averages over 1960 - 2003

	$\frac{Nh}{A}$	h	$\frac{N}{A}$
Belgium	1682	1806	0.928
Spain	1756	1958	0.892
France	1745	1861	0.933
Italy	1598	1738	0.917
United Kingdom	1921	2033	0.943
United States	1760	1868	0.941
Variance decomposition	V[h]	V[N/A]	Cov(h, N/A)
	0.50556	0.42814	0.066302

To avoid distortions associated to the dependence of the variance to the dimension of the variables, we normalize the hours per employee h as follows:  $h^* = \frac{h}{h_{max}}$ , where  $h_{max} = 14 * 365$ ; is the maximum number of hours per year to be shared between work and leisure. See appendix E.1 for details on the data.

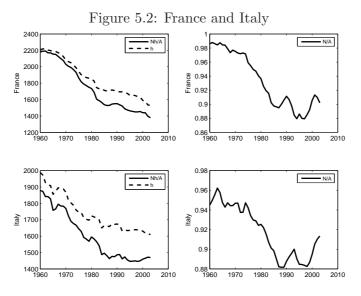
5.1). Results enhance the relevance of taking into account both margins: the hours worked per employee and the employment rate have similar weights.

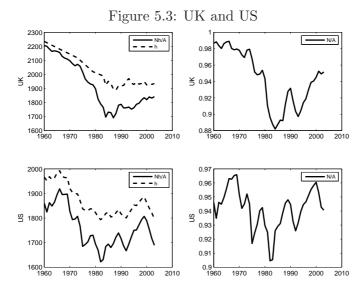
Next, we turn to the the evolution over time of the aggregate hours and its components, displayed in figures 5.1 to 5.3. To avoid scale problems, the total number of hours and the individual effort are displayed on the left hand panel of each figure, and the employment rate on the right hand panel.

Aggregate hours. Most countries experience a sustained decrease in the aggregate hours until the early 80s. The only exception is Spain, where the decline starts around 1970. It is worth to notice that before 1975, the aggregate hours worked in the US are lower than in the European countries.

After 1982, the aggregate hours worked remain roughly stable in Belgium and Italy, and still decreasing in France. Conversely, they increase in Spain, the UK, and the US. Finally, even if the UK and the US display a similar evolution, the aggregate hours in the UK still higher than in the US.

Figure 5.1: Belgium and Spain \_\_\_\_ N/A B 1800 8 0.94 0.92 0.88 1960 0.86 1960 ----- N/A 15 0.9 0.8 1960 0.75 <u>–</u> 1960 





Hours per employee. The hours worked per employee exhibit a sustained decline along the whole period in Belgium, France, Italy and the UK. In Spain, the decline starts around 1970. This decline is particularly sharp before 1980. By contrast, in the US the hours per employee decrease until the early 80s and then levels off. But it is still lower than in the UK.

**Employment rate.** Before 1985, all countries experience a steady decline in the employment rate of roughly 10%. Then there is virtually no trend in Belgium, Spain, France and Italy. Whereas, in the US and the UK, the employment rate (in tendency) increases ever since.

# 5.2 Walrasian growth model

In this section, we test the ability of two walrasian models to account for the long run dynamics of the labor market in OECD countries. The first model focus on the dynamic of the intensive margin (the number of hours worked per employee), whereas the second only explain the dynamic of the extensive margin (the employment rate).

## 5.2.1 When only the intensive margin is endogenous

In this first section we propose to analyze the link between the hours worked per employee and the labor market taxes. Similarly to Prescott (2004), Rogerson (2006) and Ohanian, Raffo, and Rogerson (2006), we use the traditional walrasian growth model where the hours worked per

employee are divisible: full-employment insures that the employment rate is constat and that all the labor market adjustments are driven by the intensive margin.

## **Behaviors**

The economy is populated by a large number of identical households whose measure is normalized to one. Each household consists of a continuum of infinitely-lived agents. At each period there is full employment:  $N_t = 1$ ,  $\forall t$ . The representative household's preferences are

$$\sum_{t=0}^{\infty} \beta^t U(C_t, 1 - h_t) \tag{5.2}$$

where  $0 < \beta < 1$  is the discount factor.  $C_t$  stands for per capita consumption and  $1 - h_t$  for the leisure time. The contemporaneous utility function is assumed to be increasing and concave in both arguments and it shows conventional separability between consumption and leisure:

$$U(C_t, 1 - h_t) = \ln C_t + \sigma \ln(1 - h_t) \ \sigma > 0$$

The capital stock  $K_t$  is rented to firms at net price  $(r_t + \delta)$ , where  $0 < \delta < 1$  is the depreciation rate of capital. Each household chooses  $\{C_t, h_t, K_{t+1} | t \geq 0\}$  to maximize (5.2) subject to the budget constraint

$$(1+R_t)K_t + (1-\tau_{w,t})w_th_t + L_t + \pi_t - K_{t+1} - (1+\tau_{c,t})C_t \ge 0 \quad (\lambda_t)$$

$$(5.3)$$

where  $R_t = (1 - \tau_k)r$  is the effective interest rate,  $\tau_k$  is the capital income tax rate,  $\tau_c$  is the consumption tax rate,  $\tau_w$  the labor income tax rate, and b are the unemployment benefits. L is a lump-sum transfer from the government. We assume a balanced budget at each period. w and  $\pi$  are the real wage and lump-sum dividends remitted by firms.

Each firm has access to the Cobb-Douglas production technology to produce output. Each producer maximizes the following profit flow:

$$\pi_t = A_t K_t^{\alpha} (h_t)^{1-\alpha} - (1 + \tau_{f,t}) w_t h_t - (r_t + \delta) K_t$$
(5.4)

where  $0 < \alpha < 1$  and  $\tau_f$  stands for the payroll taxes. Then, we have  $\max \pi = 0$ .

#### The equilibrium and parameterization

Let  $\lambda_t$  be the shadow price of the budget constraint, the optimality conditions of these problems lead to:

$$\lambda_t = ((1 + \tau_{c,t})C_t)^{-1} \tag{5.5}$$

$$w_t = \frac{\sigma(1 - h_t)^{-1}}{(1 - \tau_{w,t})\lambda_t} \tag{5.6}$$

$$1 = \beta \left[ \frac{\lambda_{t+1}}{\lambda_t} \left( 1 + (1 - \tau_{k,t+1}) \left( \alpha \frac{Y_{t+1}}{K_{t+1}} - \delta \right) \right) \right]$$
 (5.7)

$$w_t = \frac{(1-\alpha)Y_t}{(1+\tau_{f,t})h_t} \tag{5.8}$$

Therefore, the labor market equilibrium is then determined by:

$$(1 - \alpha) \frac{Y_t}{h_t} = \left\{ \frac{(1 + \tau_{f,t})(1 + \tau_{c,t})}{1 - \tau_{w,t}} \right\} \sigma (1 - h_t)^{-1} C_t$$

$$\Leftrightarrow MPH_t^w = (1 + TW_t) \times MRS(H/C)_t$$
(5.9)

where  $MPH_t^w$  and MRS(H/C) denote respectively the marginal product of an hour worked and the marginal rate of substitution between hours worked and consumption. The tax wedge is defined by:

$$1 + TW_t = \frac{(1 + \tau_{f,t})(1 + \tau_{c,t})}{1 - \tau_{w,t}}$$

Following Ohanian, Raffo, and Rogerson (2006), one can compute the gap between the return and the cost of the marginal hours worked as follows:

$$MRS(H/C)_t = (1 - \Delta_t^{h,w})MPH_t^w \text{ for } TW = 0 \Rightarrow \Delta_t^{h,w} = 1 - \frac{MRS(H/C)_t}{MPH_t^w}$$

In this case, the measure of  $\Delta_t^{h,w}$  includes the restriction of full employment  $(N_t = 1)$ . Hence,  $Y_t$  is measured by the aggregate production per capita,  $C_t$  by the aggregate consumption per capita and  $h_t$  by the average number of hours worked per employee. Finally, in order to compute the empirical counterpart of  $\Delta_t^{h,w}$ , we choose the same parameters of the structural model than in Ohanian, Raffo, and Rogerson (2006):  $\alpha = .4$  and  $\sigma = 2$ .

If the labor supply is evaluated without the tax wedge, i.e. if TW = 0, we have, under the assumption that the model is able to generate the observed data  $MPH_t^w > MRS(H/C)_t$ . From the point of view of the econometrician, for a given known set of structural parameters,  $\Delta_t^{h,w}$  is the residual of the the first order equation estimated with an omitted variable, the tax wedge. If this omitted variable has a trend component, the estimation of  $\Delta_t^{h,w}$  has also this trend. Hence,

the measure of  $\Delta_t^{h,w}$  gives the impact of the tax wedge on the observed data, under the null assumption that the theory is not rejected. Then, in what follows we interpret  $\Delta_t^{h,w}$  as the wedge between the neo-classical growth model without taxes and data. In economic terms, this gap provides a measure of the under-utilization of the working time implied by the disincentive effect of taxes. Indeed, when taxes are large, the number of hours supplied decreases for a given wage. Hence, the larger the taxes, the larger is the gap between the labor demand (driven by the marginal productivity) and the labor supply as measured without taxes.

## The empirical implications

The time series of  $\Delta_t^{h,w}$  are computed for the six countries of our sample. The cross-country means of the wedges, relative to 1980, are showed in figure 5.4 (solid line). The first property documented by Ohanian, Raffo, and Rogerson (2006) seems to be verified: the average wedge produced by the model without taxes increases at a fairly steady rate from 1960 to the mid 80s, when it levels off.<sup>4</sup>

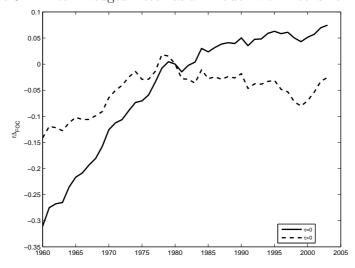
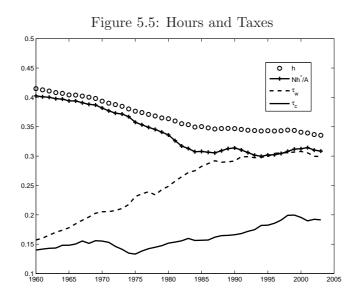


Figure 5.4: Mean wedges - Walrasian model with intensive margin

We take 1980 as normalization year. For details on the data see the appendix  $\mathrm{E.1.}$ 

Various factors can explain the labor wedges, including distorting taxation, product market regulation, non-competitive wage setting and labor market regulation. The role of taxes is remarkable from the beginning of the period until the mid 80s. (see figure 5.5 and table 5.2).

<sup>&</sup>lt;sup>4</sup>Given the normalization of the wedge to be zero in all countries in 1980, it is only the change in the wedge that has any significance. However, we kept this normalization to get series comparable with those of the authors.



 $h^* = \frac{h}{h_{max}}$ , where  $h_{max} = 14 * 365$  is the maximum number of hours per year to be shared between work and leisure.  $\tau_w$  and  $\tau_c$  are respectively the average tax rates on labor income and consumption. For details on the taxes see McDaniel (2007).

Table 5.2: Correlations with the labor income taxes

Absolute	1960-2003	1960-1980	1980-2003
Aggregate hours $(\frac{Nh}{L})$	-0.981	-0.970	-0.558
Hours per employee $(h)$	-0.991	-0.982	-0.885
Employment rate $(\frac{N}{A})$	-0.905	-0.868	0.185

Then, one may expect that the incorporation of distorting taxation reduces the size of the wedge over time and across the countries. With taxes, the wedges are computed as follows:

$$(1+TW_t)MRS(H/C)_t = (1-\Delta_t^{h,w})MPH_t^w \text{ for } TW > 0 \ \Rightarrow \ \Delta_{t,TW>0}^{h,w} = 1-(1+TW_t)\frac{MRS(H/C)_t}{MPH_t^w}$$

The dotted line in figure 5.4 confirms our intuition: the size of the gap is reduced over time and across countries when we consider distorting taxation.

On the other side, the negative impact of the labor market institutions on the performance of European labor markets after 1980 is well documented. In coherence with the search model developed in section 5.3, we conduct a simple statistical analysis to assess the impact of two institutional factors other than taxes that are typically thought to influence the labor market equilibrium. The strategy is to perform a panel regression to investigate the importance of taxes, the worker's bargaining power, and the average replacement rate in explaining the evolution in wedges. The specification is:

$$\ln(1 - \Delta_{i,t,TW=0}^{h,w}) = a_i + b\ln(TW_{i,t} - 1) + \gamma Barg_{i,t} + \beta ARR_{i,t} + \epsilon_{i,t}$$

where  $a_i$  is a country-specific fix effect,  $Barg_{i,t}$  is the workers' bargaining power, measured as the average of the union coverage and the union density (see section 5.3.2), and  $ARR_{i,t}$  is the average replacement rate. The results from the regression (table 5.3) show that taxes have a significant negative impact on the variation in the hours wedge. This result seems to be robust: for all the specifications (regressions (1) to (4)), the tax wedge has a negative significant impact at 5% level. At the opposite, the labor market institutions (LMI) have not significant impact on this wedge between theory and data. Regression (5) shows that the bargaining power, as the sole explanatory variable, has not significant effect. Finally, regressions (6) and (7) show that the average replacement ratio is correlated with taxes: in countries where taxes are high and follow an increasing path, the unemployment benefit follows this trend. Hence, if we omit taxes as an explanatory variable, the average replacement ratio has a significant negative impact on the dynamics of wedge between theory and data. These results give some support to the view that the increase in taxation is the main explanative factor of the large decrease in the hours worked in the OECD countries.

To sum up, the average hours per employee decrease since 1960. Without taxation, the wedge between the first order condition (equation (5.10)) and the data increases over time. Before the 80's, this negative correlation between the increasing taxes and the hours per employee is large and then explains the large increase in the wedge  $\Delta^{h,w}$ . After the 80's, our empirical results

Table 5.3: Regression results for the hours, 1980-2003

	ı						
	Reg (1)	Reg (2)	Reg (3)	Reg (4)	Reg (5)	Reg (6)	Reg (7)
$a_{be}$	.2055	.0370	.2690	.0047	.0348	.0119	.3177
	[0860;.4970]	[.0013;.0727]	[0016; .5397]	[0503 ; .0597]	[3031; .3728]	[0548; .0787]	[0332 ; .6686]
$a_{sp}$	.0773	.0054	.1364	0427	.1367	.1518	.3300
	[1061;.2606]	[0162;.0271 ]	[0166; .2893]	[1088 ; .0234]	[0576; .3310]	[ .0958 ; .2078]	[ .1216 ; .5384 ]
$a_{fr}$	.1383	.0236	.1829	.0002	0427	0359	.1755
	[0648;.3415]	[0160;.0632]	[0054; .3713]	[0496 ; .0500]	[2741; .1887]	[0954; .0236]	[0699 ; .4208]
$a_{it}$	.1510	0094	.1834	0100	0115	0904	.1568
	[0807;.3828]	[0409;.0220 ]	[0418; .4086]	[0413 ; .0213]	[2927;.2697]	[1201 ;0607]	[1233 ; .4370]
$a_{uk}$	.2237	.1353	.2712	.1032	.2872	.2576	.4376
	[.0453;.4021]	[ .1115;.1592 ]	[ .1123 ; .4300]	[.0552;.1512]	[ .0854 ; .4890]	[ .2213 ; .2939]	[ .2316 ; .6435 ]
$a_{us}$	.1045	.0843	.1387	.0519	.2462	.2484	.3239
	[.0123;.1967]	[.0509;.1178]	[ .0676 ; .2099]	[0018 ; .1056]	[ .1607 ; .3317]	[ .2187 ; .2780]	[ .2339 ; .4140 ]
TW	3571	3209	3254	3627			
	[4455;2687]	[3909;2509]	[3951;2557]	[4511 ;2744]			
Barg	2666		3196		1921		4062
	[6468;.1135]		[6891; .0500]		[6604; .2762]		[8639 ; .0515]
ARR	.0949			.1222		2785	3107
	[0680;.2577]			[0365 ; .2808]		[4305 ;1266]	[4658 ;1556]
N	144	144	144	144	144	144	144
$\mathbb{R}^2$	.9248	.9222	.9224	.9237	.8764	.8867	.8892

Ordinary least squares regression. "TW" is the tax wedge, "Barg" the bargaining power of the workers and "ARR" the average replacement rate. The confidence intervals (in brackets) are at the 95% level. See details on the data in the appendix E.1.

suggest that the dynamics of taxes is still correlated with  $\Delta^{h,w}$ , whereas it is not correlated with the LMI. Then the long-run decline in the hours worked per employee is mainly due to the increase in the taxes as it is suggested by Prescott (2004), Rogerson (2006) and Ohanian, Raffo, and Rogerson (2006).

## 5.2.2 When only the extensive margin is endogenous

Given that the employment rate and the average hours worked per employee follow different paths in some countries, it is interesting to explore the explanative role of taxation along the extensive margin. To this end, we compute an employment wedge to the equilibrium condition from the Hansen-Rogerson model. In this type of economy, the number of hours per employee is indivisible, and then, only the extensive margin can fluctuate.

## The Hansen-Rogerson economy

This configuration corresponds to the indivisible labor proposed by Hansen (1985) and Rogerson (1988). At the beginning of each period, the agent plays an employment lottery. If she wins, with probability  $N_t$ , she works h hours. In the opposite case, with probability  $1 - N_t$ , she does not work at all.

The utility function of the representative household is linear with respect to labor and is given by: $^5$ 

$$U_t = \ln C_t + N_t \sigma \ln(1-h) \Leftrightarrow U_t = \hat{\sigma} \ln C_t + (1-\hat{\sigma})(1-N_t)$$

Each household chooses  $\{C_t, N_t, K_{t+1} | t \ge 0\}$  to maximize

$$\sum_{t=0}^{\infty} \beta^t U(C_t, 1 - N_t)$$
 (5.10)

subject to the budget constraint

$$(1+R_t)K_t + (1-\tau_{w,t})w_tN_t + L_t + \pi_t - K_{t+1} - (1+\tau_{c,t})C_t > 0 \quad (\lambda_t)$$
 (5.11)

On the other side, the representative firm chooses  $\{K_t, N_t | t \geq 0\}$  to maximize the discounted

$$\frac{1 - \hat{\sigma}}{\hat{\sigma}} = \sigma \ln \left( \frac{1}{1 - h} \right)$$

<sup>&</sup>lt;sup>5</sup>The utility function is expressed in a simpler way using the ordinal property of utility functions. The transformation implies:

value of the dividend flow:

$$\max \pi = \max_{K_t, N_t} \{ Y_t - (r_t + \delta) K_t - (1 + \tau_{f,t}) w_t N_t \}$$
 (5.12)

subject to the technology constraint:

$$Y_t = A_t K_t^{\alpha} (N_t)^{1-\alpha}, \quad 0 < \alpha < 1$$
 (5.13)

implying that  $\max \pi = 0$ .

## The equilibrium and parameterization

The optimality conditions of these problems lead to:

$$\lambda_t = (\hat{\sigma}(1 + \tau_{c,t})C_t)^{-1} \tag{5.14}$$

$$w_t = \frac{1 - \hat{\sigma}}{(1 - \tau_{w,t})\lambda_t} \tag{5.15}$$

$$1 = \beta \left[ \frac{\lambda_{t+1}}{\lambda_t} \left( 1 + (1 - \tau_{k,t+1}) \left( \alpha \frac{Y_{t+1}}{K_{t+1}} - \delta \right) \right) \right]$$
 (5.16)

$$w_t = \frac{(1-\alpha)Y_t}{(1+\tau_{f,t})N_t} \tag{5.17}$$

Therefore, the labor market equilibrium is then determined by:

$$\underbrace{(1-\alpha)\frac{Y_t}{N_t}}_{MPN_t} = (1+TW_t)\underbrace{\left(\frac{1-\hat{\sigma}}{\hat{\sigma}}\right)C_t}_{MRS^{Hans}(N/C)_t}$$
(5.18)

where  $MPN_t$  and  $MRS^{Hans}(N/C)$  denote respectively the marginal product of an employee who works h hours and the marginal rate of substitution between employment and consumption. Note that, contrary to the previous model, equation (5.18) shows that, for a given wage, the variations in consumption are orthogonal to those in the employment. In this economy, the gap between the marginal return and the marginal cost of employment is computed as follows:

$$(1 + TW_t)MRS^{Hans}(N/C)_t = (1 - \Delta_t^{n,Hans})MPN_t \text{ for } TW \ge 0$$

$$\Rightarrow \Delta_t^{n,Hans} = 1 - (1 + TW_t)\frac{MRS^{Hans}(N/C)_t}{MPN_t}$$

In this case, there is not full employment but the measure of  $\Delta_t^{n,Hans}$  includes the restriction that employees work a fixed amount of time. Hence,  $Y_t$  is measured by the aggregate production per capita,  $C_t$  by the aggregate consumption per capita, and  $N_t$  by the total civilian employment. As in the divisible labor model, we choose the same parameters than in Ohanian, Raffo, and Rogerson (2006), i.e.  $\alpha = .4$  and  $\sigma = 2$ . Given that the (normalized) average number of hours worked by employee is equal to h = 0.3563, we deduce that  $\hat{\sigma} = 0.5316$ .

#### The empirical implications

Proceeding as before, we compute time series for the six countries of our sample. The cross-country means of the employment wedges, relative to 1980, are shown in figure 5.6. Contrarily than for the average hours worked per employee (previous section), the mean employment wedge, calculated without taxes, display virtually no trend (solid line). This, together with the results from the employment regressions (Table 5.4), suggests that taxes have a little or a not significant impact on employment. In other words, the correlation between the trend of the taxes and the cross-country means of employment wedges seems less robust. Indeed, when we incorporate taxes, the size of the mean wedge largely increases after the 70s, displaying a period of sharp decrease that is persistent in Europe. This suggest that the explanation of the labor market trend based only on the joint dynamics of hours worked per employee and taxes, as it is proposed in Prescott (2004), Rogerson (2006) or Ohanian, Raffo, and Rogerson (2006), must be completed.



Figure 5.6: Walrasian model with extensive margin

We take 1980 as normalization year. For details on the data see the appendix E.1.

The large literature on the European unemployment experience underlines that, conversely to the individual work effort, the employment rate is more likely to be affected by the institutional aspects of the labor market rather than by taxes. In order to test the relevance of this hypothesis, we conduct the same simple statistical analysis than for the hours worked per employee. We measure the influence of two institutional factors other than taxes, namely the bargaining power and the unemployment benefits, that are typically thought to influence the employment rate.

As before, the strategy is to perform panel regressions to investigate the importance of taxes, the worker's bargaining power, and the average replacement rate in explaining the evolution of employment wedges. Indeed, the average replacement rate is the product between the initial unemployment benefit replacement rate (RR1) and the duration of the unemployment benefits (UBD1). Then, we estimate the model:

$$\ln(1 - \Delta_{i,t,TW=0}^{n,Hans}) = a_i + b\ln(TW_{i,t} - 1) + \gamma Barg_{i,t} + \beta RR1_{i,t} + \delta UBD1_{i,t} + \epsilon_{i,t}$$

Results are presented in table 5.4. We observe that the effect of taxes is not significant when the tax wedge is the only regressor (regression (2)); or when only the bargaining power is included (regression (3)). However, taxes become significant in combination with the RR1 and the UDB1(regressions (1) and (4)). In contrast with the results obtained for individual hours, taxes have a negative effect on the employment wedge (equivalently, a positive effect on  $\log(1-\Delta_{i,t,TW=0}^{n,Hans})$ ): the observed increase in taxes induces a downward trend in the estimated gap between the theory and the data. Then, as is suggested by figure 5.6, the persistent decrease in the employment rate not explained by the walrasian model à la Hansen-Rogerson is not due to a shift in taxes. What about the labor market indicators? The results from this set of regressions show that the negative effect of the workers' bargaining power is robust: it remains negative and significant in all regressions. Finally, concerning the unemployment benefits, we notice that the initial replacement rate (RR1) has a significant negative effect on the employment wedges. This effect seems quite robust, even if, when no other regressor is included (regression (6)), it is significant only at the 20% level. At the opposite, the impact of the duration of the unemployment benefits (UBD1) is not robust: in the complete regression (regression (1)), its impact is negative and significant, whereas in the other regressions, it is not significant.

These statistical results support the view that the labor market institution shifts explain the wedge between theory and data when data.

To sum up, the employment rate is decreasing until the early 1980s. Then it remains stable in France and Italy, whereas in the other countries it shows a slight (Belgium and Spain) to moderate increasing trend (US and UK). Without taxation, the wedge between the first order condition and data displays virtually no significant trend. Finally, this wedge is negatively correlated with both the workers' bargaining power and the unemployment benefits, in particular with the initial replacement rate (RR1). Moreover, the tax wedge is not correlated with the gap between the theoretical model and the data  $(\Delta^{n,Hans})$ .

Table 5.4: Regression results for the employment, 1980-2003

	Reg (1)	Reg (2)	Reg (3)	Reg (4)	Reg (5)	Reg (6)	Reg (7)
$a_{be}$	-0.4583	-1.2336	7994	-1.184	7722	-1.173	5567
	[8418;0748]	[-1.282;-1.184]	[-1.151;4479]	[-1.250;-1.118]	[-1.120;4234]	[-1.239;-1.107]	[9457;1676]
$a_{sp}$	-0.7522	-1.3145	-1.065	-1.215	-1.071	-1.274	9005
	[-1.008;4962]	[-1.339;-1.289]	[-1.266;8638]	[-1.305;-1.125]	[-1.272;8700]	[-1.345;-1.202]	[-1.143;6578]
$a_{fr}$	-0.6809	-1.2328	9380	-1.183	9084	-1.162	7313
	[9509;4108]	[-1.286;-1.178]	[-1.180;6956]	[-1.252;-1.115]	[-1.145;6712]	[-1.228;-1.096]	[-1.007;4554]
$a_{it}$	-0.6528	-1.2554	8970	-1.237	8738	-1.217	7179
	[9607;3450]	[-1.298;-1.212]	[-1.187;6065]	[-1.284;-1.189]	[-1.161;5857]	[-1.261;-1.173]	[-1.031;4041]
$a_{uk}$	-0.4298	-0.9498	6955	8817	7063	9444	5812
	[6770;1825]	[9799;9196]	[9016;4895]	[9607;8028]	[9118;5008]	[9973;8915]	[8130;3495]
$a_{us}$	-0.5013	-0.7875	6838	7154	7049	7894	6307
	[6368;3659]	[8312;7439]	[7774;5903]	[7962;6345]	[7912;6186]	[8305;7484]	[7372;5243]
TW	0.1670	0.0559	.0546	.1234			
	[.0540; .2801]	[0403; .1522]	[0397; .1490]	[.0067; .2400]			
Barg	-0.9605		6021		6051		8116
	[-1.461;4598]		[-1.085;1192]		[-1.088;1217]		[-1.316;3064]
RR1	-0.1704			1033		0662	1118
	[2682;0726]			[1995;0072]		[1570; .0245]	[2039;0197]
UBD1	-0.0619			0362		0036	0155
	[1212;0026]			[0970; .0246]		[0567; .0495]	[0673; .0363]
N	144	144	144	144	144	144	144
$R^2$	.9329	.9268	.9310	.9275	.9310	.9269	.9320

Ordinary least squares regression. "TW" is the tax wedge, "Barg" the bargaining power of the workers, "RR1" the initial unemployment benefit replacement rate, and "UBD1" the unemployment benefit duration (years). The confidence intervals (in brackets) are at the 95% level. See details on the data in the appendix E.1.

Since 1980, a notable feature of the data is that differences across countries in aggregate hours are due to quantitatively important differences along the extensive margin and the intensive margin. Then, it seems relevant to analyze the labor market dynamics taking into account these two margins. Moreover, in this alternative quantitative model, the introduction of the labor market institutions is expected to reduce the gap between the marginal cost and the marginal return of labor. This is done in the search economy developed in next section.

# 5.3 Search model with intensive and extensive margins

In this section we propose a theoretical framework allowing to explain simultaneously the dynamics of the employment and the hours worked per employee. We also need for a theory explaining the impact of the labor market institutions such as the bargaining power and the unemployment benefits. The natural candidate is then the matching model, where both employment and hours are endogenous. With this type of model, we are able to quantify the relative importance of taxes and of labor market institutions in the total hours dynamics. More precisely, we expect that our model is able to explain that the dynamics of the hours worked per employee is mainly accounted by the taxes, whereas the employment dynamics is driven by the labor market institutions.

## 5.3.1 The equilibrium matching model

We present in this section a neo-classical growth model where the labor market equilibrium is determined by a search process and a wage bargaining process. This model is close to the one analyzed by Andolfatto (1996), Fève and Langot (1996) and Chéron and Langot (2004).

#### Labor market flows

Employment is predetermined at each time and changes only gradually as workers separate from jobs, at the exogenous rate s, or unemployed agents find jobs, at the hiring rate  $M_t$ . Let  $N_t$  and  $V_t$ , respectively be the number of workers and the total number of new jobs made available by firms, then employment evolves according to

$$N_{t+1} = (1 - s)N_t + M_t$$

with 
$$M_t = V_t^{\psi} (1 - N_t)^{1 - \psi}, \quad 0 < \psi < 1.$$

## Households

The economy is populated by a large number of identical households whose measure is normalized to one. Each household consists of a continuum of infinitely-lived agents. At any period, an agent can engage in only one of two activities: working or searching for a job. Employed agents (N) work h hours, while unemployed (1-N) expend their time searching a job. Unemployed agents are randomly matched with job vacancies. Individual idiosyncratic risks faced by each agent in his job match are smoothed by using employment lotteries. Hence, the representative household's preferences are:

$$\mathcal{W}(\Omega_0^H) = \sum_{t=0}^{\infty} \beta^t [N_t U(C_t^n, 1 - h_t) + (1 - N_t) U(C_t^u)]$$
(5.19)

where  $0 < \beta < 1$  is the discount factor and  $\Omega_t^H = \{K_t, N_t, \Psi_t, w_t, h_t, b_t, \pi_t, R_t, L_t\}$ ,  $\forall t. C_t^n$  and  $C_t^u$  stand for the consumption of employed and unemployed agents. The contemporaneous utility function is assumed to be increasing and concave in both arguments and it shows conventional separability between consumption and leisure, to know:

$$U(C_t^z, L_t^z) = \ln C_t^z + \Gamma_t^z, \quad z = n, u.$$

where  $\Gamma_t^n = \sigma \log(1 - h_t)$ , with  $\sigma > 0$ , and  $\Gamma_t^u = \Gamma^u = 0$ ,  $\forall t$ .

A household's employment opportunities evolve as follows:

$$N_{t+1} = (1-s)N_t + \Psi_t(1-N_t) \tag{5.20}$$

 $\Psi \equiv M_t/(1-N_t)$  is the rate at which unemployed agents find jobs.

The capital stock  $K_t$  is rented to firms at net price  $(r_t + \delta)$ , where  $0 < \delta < 1$  is the depreciation rate of capital. Each household chooses  $\{C_t^n, C_t^u, K_{t+1} | t \geq 0\}$  to maximize (5.19) subject to the labor supply constraint (5.20) and to the budget constraint

$$K_{t+1} = -(1+\tau_{c,t}) \left[ N_t C_t^n + (1-N_t) C_t^u \right]$$
  
+ 
$$\left[ 1 + (1-\tau_{k,t}) r_t \right] K_t + (1-\tau_{w,t}) \left[ N_t w_t h_t + (1-N_t) b_t \right] + L_t + \pi_t$$
 (5.21)

where b are the unemployment benefits. Then, the first order conditions with respect to consumption and capital are respectively,

$$(C_t^n)^{-1} = (C_t^u)^{-1} \equiv (C_t)^{-1} = (1 + \tau_{c,t})\lambda_t$$
 (5.22)

$$\beta[(1+R_{t+1})\lambda_{t+1}] = \lambda_t \tag{5.23}$$

## **Firms**

There are many identical firms in the economy. Each firm chooses a number  $V_t$  of job vacancies, produces goods and pays wages and capital services. The unit cost of maintaining an open vacancy is  $\omega$ . Each firm has access to a Cobb-Douglas production technology to produce output:

$$Y_t = A_t K_t^{\alpha} (N_t h_t)^{1-\alpha}, \quad 0 < \alpha < 1 \tag{5.24}$$

Job vacancies are matched at the constant rate  $\Phi_t = M_t/V_t$ . Hence, a firm's labor employment evolves as

$$N_{t+1} = (1-s)N_t + \Phi_t V_t \tag{5.25}$$

Each firm chooses  $\{N_{t+1}, K_t, V_t | t \ge 0\}$  to maximize the discounted value of the dividend flow, subject to the constraint (5.24), and to the labor constraint (5.25):

$$\mathcal{W}(\Omega_{t}^{F}) = \max_{V_{t}, N_{t+1}, K_{t}} \left\{ \pi_{t} + \frac{1}{1 + R_{t+1}} \mathcal{W}(\Omega_{t+1}^{F}) \right\} 
\pi_{t} = Y_{t} - (r_{t} + \delta) K_{t} - \omega_{t} V_{t} - (1 + \tau_{f,t}) w_{t} N_{t} h$$
(5.26)

with  $\Omega_t^F = \{N_t, \Phi_t, w_t, h_t, r_t\}$  and initial condition  $N_0$ .  $\tau_f$  stands for the payroll tax payed by firms. The first order conditions with respect to capital and employment are,

$$\alpha \frac{Y_t}{K_t} = r_t + \delta \tag{5.27}$$

$$\frac{\omega_t}{\Phi_t} = \left[ \frac{1}{1 + R_{t+1}} \left( (1 - \alpha) \frac{Y_{t+1}}{N_{t+1}} + \frac{\omega}{\Phi_{t+1}} (1 - s) - (1 + \tau_{f,t+1}) w_{t+1} h_{t+1} \right) \right]$$
(5.28)

## Nash bargaining

Wages and hours are determined via generalized Nash bargaining between individual workers and their firms:

$$\max_{w_t, h_t} (\lambda_t \mathcal{V}_t^F)^{\epsilon_t} (\mathcal{V}_t^H)^{1 - \epsilon_t} \tag{5.29}$$

with  $\mathcal{V}_t^F = \frac{\partial \mathcal{W}(\Omega_t^F)}{\partial N_t}$  the marginal value of a match for a firm and  $\mathcal{V}_t^H = \frac{\mathcal{W}(\Omega_t^H)}{\partial N_t}$  the marginal value for a match for a worker.  $\epsilon_t$  denotes the firm's bargaining power at date t. In coherence with our empirical measure of the worker's bargaining power (left panel of figure 5.7), this parameter varies over time and across countries.

The solution to this problem are the hours and wage contracts<sup>6</sup>. With an efficient bargaining over hours, the optimal choice of hours worked by employee is closed to the walrasian case. However,

<sup>&</sup>lt;sup>6</sup>See Chéron and Langot (2004) for more details on the wage bargaining process in the neo-classical growth model with matching

the wage contract takes into account the dynamic behavior of taxes and the unemployment benefits.

## The Equilibrium

Given the vector of taxes, unemployment benefits and bargaining powers  $\{\tau_{c,t}, \tau_{f,t}, \tau_{w,t}, b_t, \epsilon_t\}$ , the general equilibrium is defined by the set of functions  $\{C_t, V_t, K_{t+1}, N_{t+1}, w_t, h_t, L_t, M_t, Y_t\}_{t=0}^{\infty}$ , solution of the system formed by the optimality conditions, the equation of the employment dynamics and the condition for the equilibrium on the goods markets. Let define the market tightness as  $\theta_t = V_t/(1 - N_t)$ . Finally, to simplify notation, we define the employment tax:  $\tau_t^n \equiv \frac{1+\tau_{f,t}}{1-\tau_{w,t}}$ , and the relative bargaining power:  $\chi_t \equiv \frac{1-\epsilon_t}{\epsilon_t}$ . Then the system defining the equilibrium is:

$$N_{t+1} = (1-s)N_t + M_t (5.30)$$

$$M_t = \theta_t^{\psi} (1 - N_t) \tag{5.31}$$

$$Y_t = K_{t+1} - (1 - \delta)K_t + C_t + \omega V_t \tag{5.32}$$

$$L_t = \tau_{c,t}C_t + (\tau_{f,t} + \tau_{w,t})w_t h_t N_t + \tau_{k,t} r_t K_t - b(1 - \tau_{w,t})(1 - N_t)$$
(5.33)

$$Y_t = A_t K_t^{\alpha} (h_t N_t)^{1-\alpha} \tag{5.34}$$

$$1 = \beta \left[ \frac{\lambda_{t+1}}{\lambda_t} \left( 1 + (1 - \tau_{k,t+1}) \left( \alpha \frac{Y_{t+1}}{K_{t+1}} - \delta \right) \right) \right]$$
 (5.35)

$$\frac{\omega}{\Phi_t} = \beta \left[ \frac{\lambda_{t+1}}{\lambda_t} \left( (1 - \alpha) \frac{Y_{t+1}}{N_{t+1}} + (1 - s) \frac{\omega}{\Phi_{t+1}} - (1 + \tau_{f,t+1}) w_{t+1} h_{t+1} \right) \right]$$
(5.36)

$$((1+\tau_{c,t})C_t)^{-1} = \lambda_t \tag{5.37}$$

$$(1 - \alpha)\frac{Y_t}{N_t h_t} = \frac{1 + \tau_{f,t}}{1 - \tau_{w,t}} \frac{\sigma (1 - h_t)^{-1}}{\lambda_t}$$
(5.38)

$$w_{t}h_{t} = \frac{1 - \epsilon_{t}}{1 + \tau_{f,t}} \left\{ (1 - \alpha) \frac{Y_{t}}{N_{t}} + \left[ \frac{1 - s}{\Phi_{t}} \left( 1 - \frac{\chi_{t+1}}{\chi_{t}} \frac{\tau_{t}^{n}}{\tau_{t+1}^{n}} \right) + \frac{\chi_{t+1}}{\chi_{t}} \frac{\tau_{t}^{n}}{\tau_{t+1}^{n}} \theta_{t} \right] \omega \right\}$$

$$+ \frac{\epsilon_{t}}{1 - \tau_{w,t}} \left\{ \frac{\Gamma^{u} - \Gamma_{t}^{n}}{\lambda_{t}} + (1 - \tau_{w,t}) b_{t} \right\}$$
(5.39)

Finally, the unemployment benefits are computed as the product of the average replacement rate  $(\rho_t)$  and the hourly wage:  $b_t = \rho_t w_t h_t$ . The last equation can be rewritten as follows:

$$(1 + \tau_{f,t})w_{t}h_{t} = \frac{1 - \epsilon_{t}}{1 - \epsilon_{t}\rho_{t}} \left\{ (1 - \alpha)\frac{Y_{t}}{N_{t}} + \left[ \frac{1 - s}{\Phi_{t}} \left( 1 - \frac{\chi_{t+1}}{\chi_{t}} \frac{\tau_{t}^{n}}{\tau_{t+1}^{n}} \right) + \frac{\chi_{t+1}}{\chi_{t}} \frac{\tau_{t}^{n}}{\tau_{t+1}^{n}} \theta_{t} \right] \omega \right\} + \frac{\epsilon_{t}}{1 - \epsilon_{t}\rho_{t}} (1 + TW_{t})(\Gamma^{u} - \Gamma_{t}^{n})C_{t}$$

In the matching model, both the number of employees and the number of hours worked are endogenous. After the wage bargaining process, the right to manage assumption leads the firms to hire a number of workers given a bargained labor cost per employee  $((1 + \tau_{f,t})w_th_t)$ . Because we assume an efficient bargaining process, the equilibrium number of hours is determined by the same equation than in a walrasian economy (endogenous intensive margin). Then the labor market equilibrium conditions are:

$$\underbrace{(1+\tau_{f,t})w_{t}h_{t}}_{\text{MCN}} = \underbrace{(1-\epsilon_{t})\left\{(1-\alpha)\frac{Y_{t}}{N_{t}} + \overbrace{\left[\frac{1-s}{\Phi_{t}}\left(1-\frac{\chi_{t+1}}{\chi_{t}}\frac{\tau_{t}^{n}}{\tau_{t+1}^{n}}\right) + \frac{\chi_{t+1}}{\chi_{t}}\frac{\tau_{t}^{n}}{\tau_{t+1}^{n}}\theta_{t}\right]\omega\right\}}_{\text{Bargained surplus (BS)}}$$

$$+ \underbrace{\epsilon_{t}\left\{(1+TW_{t})\overbrace{(\Gamma^{u}-\Gamma_{t}^{n})C_{t}}^{\text{MRS(N/C)}}\underbrace{\text{UB}}_{\text{Reservation wage (RW)}}\right\}}_{\text{Reservation wage (RW)}}$$

$$\underbrace{(1+\tau_{f,t})w_{t}h_{t}}_{\text{MRN}} = \underbrace{(1-\alpha)\frac{Y_{t}}{N_{t}}}_{\text{Instantaneous returns}} + \underbrace{(1-s)\frac{\omega}{\Phi_{t}} - \left\{\frac{(1+\tau_{c,t})C_{t}}{\beta(1+\tau_{c,t-1})C_{t-1}}\right\}\frac{\omega}{\Phi_{t-1}}}_{\text{Intertemporal returns}}$$

$$\underbrace{(1-\alpha)\frac{Y_{t}}{N_{t}h_{t}}}_{\text{MPH}} = \underbrace{\left\{\frac{(1+\tau_{f,t})(1+\tau_{c,t})}{1-\tau_{w,t}}\right\}}_{\text{1+TW}}\underbrace{\sigma(1-h_{t})^{-1}C_{t}}_{\text{MRS(H/C)}}$$

where MRN denotes the marginal return of an employee and MCN her marginal cost. Similarly, MRS(N/C) denotes the marginal rate of substitution between employment and consumption whereas MPH stands for the marginal product of an hour worked. The previous system can be rewritten as:

$$MRN_t = MCN_t$$
  
 $MPH_t = (1 + TW_t) \times MRS(H/C)_t$ 

One can remark that the MCN has now two components: the bargained surplus and the reservation wage. Whereas the reservation wage is only affected by the tax/benefit system (tax wedge and unemployment benefits), the bargained surplus is also affected by the labor market frictions through the search cost.

Without taxes and without labor market institutions, i.e.  $TW = 1 - \epsilon = UB = 0$ , the marginal cost of employment MCN is given by the marginal rate of substitution between employment

and consumption MRS(N/C), as in the Hansen-Rogerson economy. Relatively to this walrasian model, the labor market institutions lead to an increase in the MCN, through the introduction of both an additional value of leisure (the unemployment benefits UB > 0) and a bargained surplus  $(1 - \epsilon > 0)$ . Hence, the shift across time of the labor market institutions can explain the dynamics of the employment by decreasing the wedge between the data and the theory. Finally, the increase in the tax wedge TW raises also the marginal cost of employment through the reservation wage. Via this last channel, one can also expect to reduce the gap between data and theory.

Following Ohanian, Raffo and Rogerson (2006), we compute the gap between the return and the cost of the marginal employee as follows:

$$MCN_t = (1 - \Delta_t^n)MRN_t \text{ for } TW \ge 0, \ UB \ge 0 \ \Rightarrow \ \Delta_t^n = 1 - \frac{MCN_t}{MRN_t}$$

Relatively to the Hansen-Rogerson's economy, the matching model leads to new evaluations of both the marginal return of employment (MRN) and the marginal cost of employment (MCN). The smaller the labor market frictions, the smaller is the gap between the Hansen-Rogerson economy and our matching model.

The condition allowing to generate the number of hours worked in the non-walrasian economy is the same than in the walrasian one, except that in the previous case the employment rate is not restricted to be equal to one. This expression does not introduce any labor market friction because we assume an efficient bargaining process over the hours worked. Then we have:

$$(1+TW_t)MRS(H/C)_t = (1-\Delta_t^h)MPH_t \text{ for } TW \ge 0, \ UB \ge 0 \ \Rightarrow \ \Delta_t^h = 1-(1+TW_t)\frac{MRS(H/C)_t}{MPH_t}$$

Contrary to the walrasian case, we do not impose  $N_t = 1$  (full-employment). The measure of  $\Delta_t^h$  is computed using the aggregate production per capita  $Y_t$ , the aggregate consumption per capita  $C_t$ , the number of hours worked by employee  $h_t$  and the employment rate  $N_t$ .

#### 5.3.2 Calibration and data

The aim of the empirical part is to measure the contribution of both the labor market indicators (LMI) and the taxes specific of each country. Taxes and LMI were not constant over the last two decades (see figure 5.7 and figure 5.9). Then, the dynamics of the gap between the marginal cost of employment and its marginal return is affected by changes in either the LMI and the taxes. In order to measure the impact of the tax/benefit system we construct the different hours and employment wedges under several configurations, using the data described in appendix E.1.

These measures are evaluated using the following parameterization of the structural parameters. We choose  $\alpha=0.4$  and  $\sigma=2$  as in (Ohanian, Raffo, and Rogerson 2006). The discount parameter is such that  $\beta=0.985$ . The elasticity of the matching function with respect to vacancies is equal to  $\psi=0.6$  (Blanchard and Diamond 1992). We assume a ratio of aggregate recruiting expenditures to output  $(\omega_t V_t/Y_t)$  equal 1% (Andolfatto 1996). We set  $\omega$  equal to the mean over time and countries. Finally, in order to make an easy comparison of the model with and without unemployment, we assume that an unemployed worker enjoys all her leisure time, which means that the search activity has no disutility cost.

The labor market indicators give the dynamics of the bargaining power of workers and the unemployment benefits (the replacement ratio) in each country. We have two statistical indicators which allow us to measure the bargaining power of the employee during the wage bargaining process: the union coverage and the union density. These two indicators are not directly the bargaining power, but are closely linked to it: a large union coverage or a high union density imply that the probability for the employee to be alone during the bargaining process is very low. Hence, the bargaining power is higher in an economy where the firm does not have a monopsony power. We choose to evaluate  $1 - \epsilon$  by the average of the union coverage and the union density. The dynamics of the LMI are represented in figure 5.7.

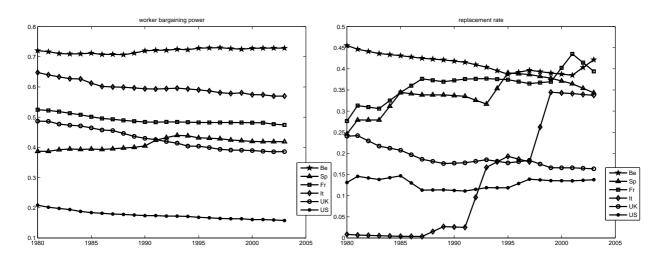


Figure 5.7: Bargaining power and replacement rate

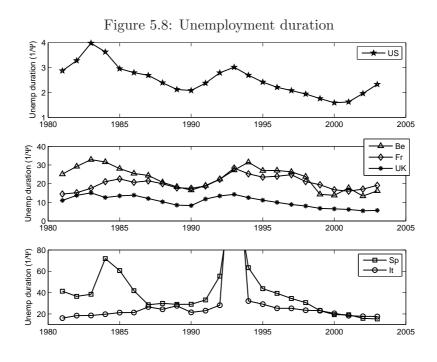
We also take into account the heterogeneity in the separation rate, but this parameter is constant over the time in order to be consistent with our theoretical framework. We calibrate the job destruction rates in order to reproduce the average unemployment duration for the 1985-1994

period estimated by Blanchard and Portugal (2001) (see appendix E.1). The average unemployment durations and the corresponding destruction rates are summarized in table 5.5, whereas the evolution of the expected unemployment duration across countries is displayed in figure 5.8. Notice that in Spain and Italy the unemployment duration shows a large peak (144 months for Spain and 208 for Italy) that corresponds to the economic crisis experienced in the early 90s.

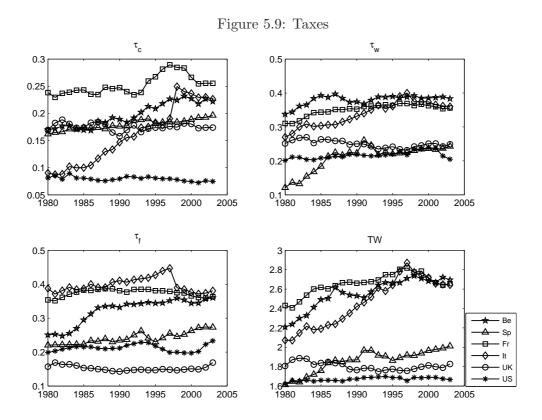
Table 5.5: Unemployment duration and the job destruction rate.

Country	Belgium	Spain	France	Italy	United Kingdom	United States
$\frac{1}{\Psi}^*$ (months)	23	41	20	30	10	2.5
s (%)	5.95	6.15	6.10	5.75	10.40	30.48

\*: Source: Blanchard and Portugal (2001). The authors construct monthly flows into unemployment as the average number of workers unemployed for less than one month, for the period 1985-1994, divided by the average labor force during the same period. The source of these data is the OECD duration database. Unemployment duration is constructed as the ratio of the average unemployment rate for the period 1985-1994 to the flow into unemployment.



Finally, figure 5.9 shows the dynamics in each country of the several taxes that define our tax wedge (TW): the payroll tax  $(\tau_f)$ , the tax on labor income  $(\tau_w)$  and the tax on consumption  $(\tau_c)$ . These figures clearly show that there are two groups of countries: the first, with Belgium,



France and Italy, where the tax wedge is high and the second, with Spain, the UK and the US, where the tax wedge is low. Figures show that these two groups of countries are the same if we focus on the labor tax or on the payroll tax. At the opposite, the consumption tax does not support this separation into two groups of countries.

## 5.3.3 Empirical results

In order to evaluate the impact of each deviation from the two neo-classical growth models presented in sections 5.2.1 and 5.2.2, we propose the following decomposition. First, we introduce only search costs in the measure of the marginal return of employment. Hence, in this first step (and as long as taxation is null), our theory on hours worked by employee is the same than in the neo-classical growth model with divisible hours, whereas the deviation between our theory on employment allocation and the Hansen-Rogerson' one comes from the country-specific dynamics of the labor market tightness. In a second step, our objective is to measure the impact of country-specific labor market institutions, to know: the separation rate, the bargaining power and the unemployment benefits. Once again, this does not change our theory on the hours worked allocation, but modifies both the marginal return of employment and the marginal

cost of employment. Next, we introduce country-specific dynamics of taxes, which change the allocation of both hours worked per employee and employment. This allows us to measure the relative impact of taxes. Finally, in order to asses the global impact of the tax/benefit systems, we consider simultaneously both sources of heterogeneity: the labor market institutions and taxation.

#### A world with search costs

Beyond the introduction of the labor market institutions such as bargaining and unemployment benefits, the matching model introduces search costs. The higher the unemployment rate, the higher is the probability to find a worker for a firm. Hence, in economies with high unemployment, search costs paid by firms are low. In order to evaluate the magnitude of the search costs, we set TW = UB = 0,  $1 - \epsilon = 0$  and  $\omega > 0$ . Because our simple model does not introduce endogenous job separation, we only introduce a constant heterogeneity in the separation rate:  $s = E_i[s_i]$ , where i denotes the country. Then we have:

$$\begin{split} MCN_{i,t} &= RW_{i,t} = MRS(N/C)_{i,t} \\ MRN_t &= (1-\alpha)\frac{Y_{i,t}}{N_{i,t}} + (1-s)\frac{\omega}{\Phi_{i,t}} - \frac{C_{i,t}}{\beta C_{i,t-1}}\frac{\omega}{\Phi_{i,t-1}} \\ \Delta^n_{i,t} &= 1 - \frac{MCN_{i,t}}{MRN_{i,t}} \end{split}$$

These expressions explain why we get closer results than in the Hansen-Rogerson economy (see the appendix E.2 where we report the results by country for the Hansen-Rogeson economy). The intuition is that agents face a similar employment lottery in both cases. Then, the MCN is the same in the two types of economy. However, in the search economy of figure 5.10 the gap is lower because we take into account that search is a costly process. This is measured though an evaluation of the marginal rate of return of an employee (MRN). In the US or in the UK, where the unemployment rate is significantly smaller, the search cost is higher for firms and then, the value of an employee is relatively higher than in the others countries. We observe the same phenomena for Spain at the end the sample period, where this country experienced a large decline of unemployment.

On the other side, the condition generating the number of hours worked in the non-walrasian economy (figure 5.11) is the same than in the walrasian one, except that in the former the employment rate is not restricted to be equal to one. This expression does not introduce any labor market friction because we assume an efficient bargaining process over the hours worked.

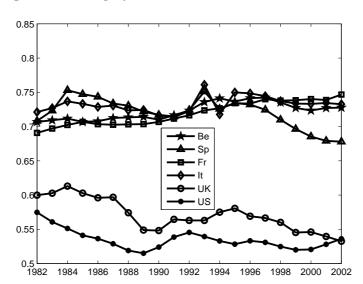


Figure 5.10: Employment in the model with search costs.

As discussed before, the evaluation of this model without taxes enlarges the gap between the marginal productivity of labor and its marginal cost.

## A world with country-specific institutions

Is our theory able to explain the role of labor market institutions on the employment dynamics? In order to give a quantitative answer to this question, we introduce the country specific labor market indicators. Hence, in this configuration we set  $TW_{i,t} = 0$ ,  $1 - \epsilon_{i,t} > 0$ ,  $UB_{i,t} > 0$ ,  $s_i > 0$  and  $\omega > 0$ . This model allows us to evaluate the impact of the heterogeneity across countries of labor market institutions. As in the empirical literature (see e.g. Bassanini and Duval (2006)), we also take into account the country-specific dynamics of these labor market indicators (bargaining power and unemployment benefits). Then we have:

$$RW_{i,t} = MRS(N/C)_{i,t} + UB_{i,t}$$

$$BS_{i,t} = (1 - \alpha)\frac{Y_{i,t}}{N_{i,t}} + SC_{i,t}$$

$$MCN_{i,t} = (1 - \epsilon_{i,t})BS_{i,t} + \epsilon_{i,t}RW_{i,t}$$

$$MRN_{i,t} = (1 - \alpha)\frac{Y_{i,t}}{N_{i,t}} + (1 - s_i)\frac{\omega}{\Phi_{i,t}} - \frac{C_{i,t}}{\beta C_{i,t-1}}\frac{\omega}{\Phi_{i,t-1}}$$

$$\Delta^n_{i,t} = 1 - \frac{MCN_{i,t}}{MRN_{i,t}}$$

Since we do not introduce taxation, the heterogeneity comes only from the aggregates and

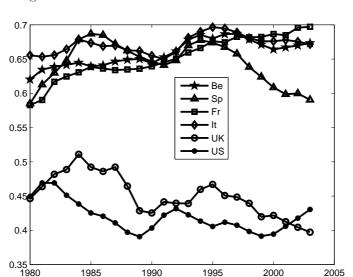


Figure 5.11: Hours in the model with search costs

from the labor market institutions. Results are reported in figure 5.12. The effect of the country-specific labor market indicators (worker's bargaining power, unemployment benefits and separation rates) is striking. Nevertheless, the results for the US economy are different from those for the European countries. Indeed, in the US, the observed labor market indicators are the lowest. Then, they have the smallest impact on the employment dynamics. At the opposite, when the labor indicators point to the existence of high real rigidities, the introduction of such variables in the theoretical model largely improves its fit. This is true for all European countries, included the UK. Indeed the labor market reforms in this last country have only a gradual impact on the employment dynamics after the 80s.

This experience clearly shows that in all countries, the gaps are largely damped when we take into account the country-specific heterogeneity of the labor market indicators. This gives some empirical support to our theoretical approach: a matching model with wage bargaining accounts quite well for the employment dynamics if we introduce country-specific labor market indicators. On the other hand, this experience gives also a theoretical foundation for our regression results presented in table 5.4.

## A world with country-specific taxation

Which is the relative weight of taxes on the observed employment dynamics? In order to give a quantitative answer to this question, we set the labor market indicators to zero and we allow for

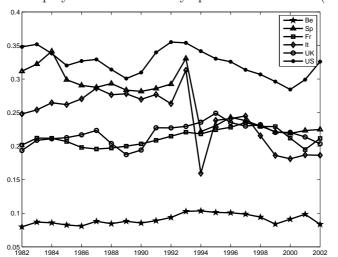


Figure 5.12: Employment with country-specific institutions (Taxes = 0)

positive taxation  $(TW_{i,t} > 0)$ . In this case, workers have not bargaining power  $(1 - \epsilon_{i,t} = 0)$  and, given that the unemployment benefits are equal to zero  $(\rho_{i,t} = 0)$ , the reservation wage is just the marginal rate of substitution between employment and consumption, net of taxes. Finally, if the search is a costless process  $(\omega = 0)$ , there are not intertemporal returns from labor. This configuration allows us to asses both the impact of the different tax systems across countries, and the relative weight of taxes with respect to the labor market institutions in shaping the employment behavior. Then we have:

$$MCN_{i,t} = (1 + TW_{i,t})MRS(N/C)_{i,t}$$
  

$$MRN_{i,t} = (1 - \alpha)\frac{Y_{i,t}}{N_{i,t}}$$

Results are shown in figure 5.13. The difference between this economy and the Hansen-Rogerson's one, is that the number of hours worked per employees varies over time and across countries. Then, the measure of the reservation wage in this economy is coherent with the other evaluations of the matching model. By comparing this simulation and the results reported in appendix E.2, we measure the impact of the observed time-varying heterogeneity in hours worked on the extensive margin. For countries where the number of hours worked per employee is higher than the average of the sample (the calibration retained for the Hansen-Rogerson economy), the reservation wage with flexible hours is bigger than its measure in the Hansen-Rogerson economy. Then, by taking into account the time-varying hours worked, we reduce the gap between theory and data for these countries (e.g. US and UK). The opposite is true for countries where the number of hours worked per employee is lower than the average.

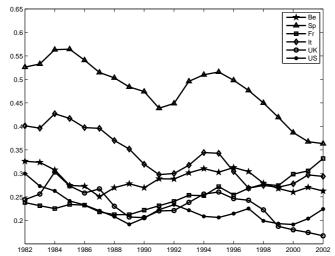


Figure 5.13: Employment with country-specific taxation (LMI = 0)

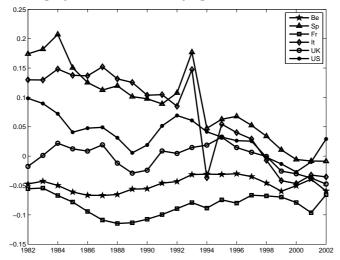
By comparing the figures 5.13 and 5.12, we deduce that even if the effect of taxes is not negligible, the wedges still larger than in economy where only the heterogeneity of labor market institutions is taking into account.

#### A world with country-specific taxation and institutions

Finally, in this configuration we allow for both taxation and labor market institutions. That is, we set  $TW_{i,t} > 0$ ,  $1 - \epsilon_{i,t} > 0$ ,  $UB_{i,t} > 0$ ,  $s_i > 0$  and  $\omega > 0$ . The corresponding gap gives a measure of the under-utilization of employment implied by the tax/benefit system. Results are shown in figure 5.14. The improvement with respect to the two previous economies is in part due to the fact that, without taxes, the real wage is underestimated.

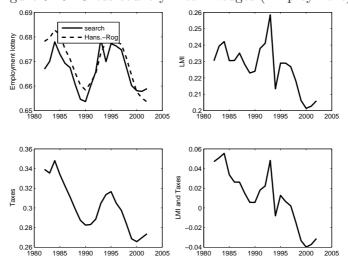
These results, concerning the relative contribution of taxes and labor market institution in the explanation of the employment dynamics, are well captured in figure 5.15. This figure shows the cross-country averages of the employment wedge for the various scenarios. In the top left panel we can asses the weight of unemployment. The gaps produced by the walrasian economy are very close to those produced by the search economy because the search costs are relatively small. The top right panel shows the marginal contribution of the labor market institutions whereas the bottom left panel shows the marginal contribution of taxes. Whereas the labor market institutions allow to divide the mean gap by 3, taxes allow to divide the mean gap by 2. The comparison of these two extreme cases enhances the preponderant role of the labor market institutions as determinant of the observed employment behavior. Finally, the bottom

Figure 5.14: Employment with country-specific taxation and institutions



right panel shows the average wedges produced by our model when we integrate both sources of cross-country heterogeneity: taxation and labor market institutions. In this case the (average) gap between the model and data is largely damped and very close to zero.

Figure 5.15: Cross-country mean wedges (Employment)



Finally, as expected, the gap between the theory and the data concerning the individual work effort diminishes because this setting captures the disincentive effect of taxes (figure 5.16).

$$\Delta_t^h = 1 - (1 + TW_t) \frac{MRS(H/C)_t}{MPH_t}$$

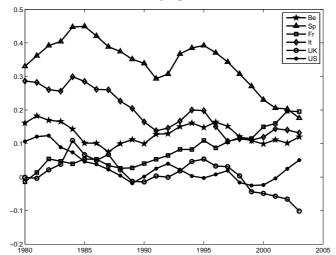


Figure 5.16: Hours with country-specific taxation and institutions

## 5.4 Conclusion

Aggregate hours of market work exhibit dramatic differences across industrialized countries. On the one hand, differences are large between Europe and the US. On the other hand, we observe large differences even among European countries. Moreover, since 1980 a notable feature of the data is that differences across countries in aggregate hours are due to quantitatively important differences along the extensive margin and the intensive margin.

The existing literature suggests that the main factors explaining the decline in the hours worked per employee differ from those explaining the decline in the employment rate: in the former case taxes play a prominent role (Prescott (2004), Rogerson (2006) and Ohanian, Raffo, and Rogerson (2006)), whereas labor market institutions, such as unions or unemployment benefits, explain the downturn in employment rates (Jackman, Layard, and Nickell (1991), Mortensen and Pissarides (1999a), Blanchard and Wolfers (2000) or Ljungqvist and Sargent (2007b)). In this paper, we show that all together contribute to the dynamics of the two margins of the total hours. We develop a model that includes the intensive and the extensive margins. The behavior of the two margins composing the aggregate hours is well accounted by our search model when it includes the observed heterogeneity across countries of both taxes and the labor market indicators (unemployment benefits and the bargaining power).

Relative to the walrasian economies, the general equilibrium matching model leads to new evaluations of both the marginal return of employment (MRN) and the marginal cost of employment (MCN). The labor market institutions lead to an increase in the MCN, through the introduc-

tion of both an additional value of leisure (the unemployment benefits) and a bargained surplus. Hence, we show that the shift across time of the labor market institutions explains approximately 2/3 of the dynamics of the employment rate. The increase of the tax wedge raises also the marginal cost of employment through the reservation wage. Through this channel, taxes explain about 1/3 of the employment rate dynamics. Finally, we show that we need only taxes for accounting for the observed shift in the average hours worked per employee.

In addition, our quantitative experiences put in evidence that the US economy is closer to the walrasian model than the European economies, because frictions on the labor market are smaller. Finally, our results suggest than in all cases, the matching model performs better in the labor market accounting exercise.

# Conclusion

This dissertation tries to gain insight on the identification of the key factors that shape the short-run and the long-run evolution of industrialized economies. Can we explain the observed international fluctuations and the U.S. labor market facts? Can we account for the economic slowdown of economic growth, and the high unemployment experienced by continental European countries last decades? How to rationalize the dramatic differences across industrialized countries of the aggregate hours of market work?

This dissertation propose plausible explanations to these issues. In particular, chapter 3 shows that that fluctuations in distortive taxes can account for some of the puzzling features of the U.S. business cycle. Namely, the observed real wage rigidity, the international comovement of investments and labor inputs, and the so-called consumption correlation puzzle (according to which cross-country correlations of output are higher than the one of consumption). This is done in a two-country search and matching model with fairly standard preferences, extended to include a tax/benefit system. In this simple framework, the tax side is represented by taxation on labor income, employment (payroll tax) and consumption, whereas the benefit side is resumed by the unemployment benefits and the worker's bargaining power.

In turn, chapter 4 argues that the link between economic growth and long term unemployment can be viewed through the simultaneous link of growth and unemployment with the labor market institutions. The empirical advises from this chapter are that: (i) The tax wedge and the unemployment benefits are positively correlated with the regional unemployment rates. Conversely, the employment protection and the level of coordination in the wage bargaining process are negatively correlated with the regional unemployment rates. (ii) The tax wedge and the unemployment benefits are negatively correlated with the regional growth rates of the Gross Domestic Product (GDP) per capita. Conversely, more coordination in the wage bargaining process diminishes the regional growth rates of GDP per capita. This last result points to the

existence of an arbitration between unemployment and growth, if we focus on the impact of coordination in the wage bargaining process. And the implications of the theoretical model are the following: (i) The bargaining power of unions, the unemployment compensation, the taxes on labor and the employment protection have a positive effect on unemployment and a negative effect on the economic growth. (ii) A more coordinated bargaining process increases employment, at the price of a lower economic growth.

Finally, chapter 5 points to the relevance of taking into account the intensive and the extensive margins on the labor market to better understand the dynamics of the aggregate hours of market work. From a theoretical point of view, this chapter also provides a theory allowing to account for the impact, of both taxes and labor market institutions, on the two margins of the aggregate hours worked. The main findings are the following. First, the long-run decline in the hours worked per employee is mainly due to the increase of the taxes, as it is suggested by Prescott (2004), Rogerson (2006) and Ohanian, Raffo, and Rogerson (2006). Second, the employment rate is affected by institutional aspects of the labor market, such as the bargaining power and the unemployment benefits, rather than by taxes, conversely to the individual work effort. Finally, this behavior of the two margins of the aggregate hours is well accounted by our search model, when it includes the observed heterogeneity of the tax/benefit systems and the labor market indicators of the wage-setting process across countries. These findings give some support to the two explanations of the European decline in total hours: the important role of taxes through the intensive margin and the large contribution of the labor market institutions through the extensive margin. Because these findings come from an unified framework, they also give a strong support to the matching models.

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# Appendix A

# Appendix to chapter 1

# A.1 Sensitivity analysis of the instantaneous elasticities

Table A.1: Sensitivity of  $\Pi_{\cdot a_i}$  elasticities to  $\eta$   $(\epsilon_H)$  - IRBC1a-SP

$\eta$ $(\epsilon_H)$	50 (0.04)	20 (0.1)	5 (0.4)	3 (2/3)	0.5 (4)
$\Pi_{C_1 a_1}$	0.139	0.142	0.153	0.161	0.335
$\Pi_{C_2 a_1}$	0.139	0.142	0.153	0.161	0.335
$\Pi_{I_1 a_1}$	5.098	5.293	6.184	6.878	7.742
$\Pi_{I_2 a_1}$	-1.767	-1.872	-2.379	-2.802	-4.292
$\Pi_{H_1 a_1}$	0.033	0.082	0.295	0.450	1.088
$\Pi_{H_2 a_1}$	-0.005	-0.013	-0.053	-0.087	-0.550
$\Pi_{Y_1 a_1}$	1.021	1.053	1.189	1.288	1.696
$\Pi_{Y_2 a_1}$	-0.003	-0.008	-0.034	-0.055	-0.352
$\Pi_{\lambda a_1}$	-0.139	-0.142	-0.153	-0.161	-0.335
$\Pi_{w_1 a_1}$	0.987	0.970	0.893	0.837	0.608
$\Pi_{w_2a_1}$	0.002	0.004	0.019	0.031	0.198

IRBC1a-SP: The standard model with separable preferences and independent productivity processes across countries.

Table A.2: Sensitivity of  $\Pi_{\cdot a_i}$  elasticities to  $\eta$   $(\epsilon_H)$  - IRBC1a-NSP

$\eta$ $(\epsilon_H)$	50 (0.04)	20 (0.1)	5 (0.4)	3 (2/3)	0.5(4)
$\Pi_{C_1 a_1}$	0.167	0.210	0.408	0.556	0.919
$\Pi_{C_2 a_1}$	0.133	0.129	0.113	0.102	-0.465
$\Pi_{I_1 a_1}$	5.078	5.243	5.990	6.584	12.723
$\Pi_{I_2 a_1}$	-1.788	-1.923	-2.573	-3.095	-5.686
$\Pi_{H_1 a_1}$	0.039	0.096	0.349	0.537	1.639
$\Pi_{H_2 a_1}$	0.000	0.000	0.000	0.000	0.000
$\Pi_{Y_1 a_1}$	1.025	1.061	1.223	1.344	2.049
$\Pi_{Y_2 a_1}$	0.000	0.000	0.000	0.000	0.000
$\Pi_{\lambda a_1}$	-0.138	-0.141	-0.196	-0.657	0.106
$\Pi_{w_1 a_1}$	0.985	0.965	0.874	0.806	0.409
$\Pi_{w_2a_1}$	0.000	0.000	0.000	0.000	0.000

IRBC1a-NSP: The standard model with non-separable preferences and independent productivity processes across countries.

Table A.3: Sensitivity of  $\Pi_{\cdot a_i}$  elasticities to  $\phi$ 

	IRBC1a-SP			IRBC1a-NSP		
$\phi$	0.001	0.056	1.00	0.001	0.056	1.00
$\Pi_{C_1 a_1}$	0.135	0.161	0.375	0.552	0.556	0.867
$\Pi_{C_2 a_1}$	0.135	0.161	0.375	0.098	0.102	-0.413
$\Pi_{I_1 a_1}$	33.892	6.878	1.615	33.491	6.584	1.077
$\Pi_{I_2 a_1}$	-29.577	-2.802	0.517	-29.977	-3.095	0.464
$\Pi_{H_1 a_1}$	0.464	0.450	0.335	0.537	0.537	0.537
$\Pi_{H_2a_1}$	-0.072	-0.087	-0.201	0.000	0.000	0.000
$\Pi_{Y_1a_1}$	1.297	1.288	1.214	1.344	1.344	1.344
$\Pi_{Y_2 a_1}$	-0.046	-0.055	-0.129	0.000	0.000	0.000
$\Pi_{\lambda a_1}$	-0.135	-0.161	-0.375	-0.631	-0.657	-2.661
$\Pi_{w_1 a_1}$	0.832	0.837	0.879	0.806	0.806	0.806
$\Pi_{w_2a_1}$	0.026	0.031	0.072	0.000	0.000	0.000

IRBC1a-SP: The standard model with separable preferences and independent productivity processes across countries.

IRBC1a-NSP: The standard model with non-separable preferences and independent productivity processes across countries.

# Appendix B

# Appendix to chapter 2

# B.1 National specialization economy

The optimal composition of the composite goods for investment and consumption can be viewed as a static choice. For given levels of  $C_{i,t}^C$  and  $I_{i,t}^{C,1}$ , Household in country i optimizes  $C_{i,t}^C$  by choosing  $C_{j,t}^i$ , for i = 1, 2 and j=1, 2, taking as given the prices  $P_{i,t}$ . Similarly, the Firm in each country optimizes  $I_{i,t}^C$  by choosing  $I_{j,t}^{i,2}$ . Then, the country 1's representative household determines her demand for consumption as follows:

$$\min_{C_{1,t}^1, C_{2,t}^1} P_{1,t} C_{1,t}^1 + P_{2,t} C_{2,t}^1 = P_{1,t}^C C_{1,t}^C$$

subject to

$$C_{1,t}^{C} = \left[ \gamma_C^{\frac{1}{\theta_C}} (C_{1,t}^1)^{\frac{\theta_C - 1}{\theta_C}} + (1 - \gamma_C)^{\frac{1}{\theta_C}} (C_{2,t}^1)^{\frac{\theta_C - 1}{\theta_C}} \right]^{\frac{\theta_C}{\theta_C - 1}} (\lambda_C)$$

for

$$P_{1,t}^{C} \equiv \left[ \gamma_C P_{1,t}^{1-\theta_C} + (1 - \gamma_C) P_{2,t}^{1-\theta_C} \right]^{\frac{1}{1-\theta_C}}$$
(B.1)

with  $P_i$  the production price of good i = 1, 2. The first order conditions are

$$P_{1,t} = \lambda_C (\gamma_C C_{1,t}^C)^{1/\theta_C} (C_{1,t}^1)^{\frac{\theta_C - 1}{\theta_C} - 1}$$

$$P_{2,t} = \lambda_C ((1 - \gamma_C) C_{1,t}^C)^{1/\theta_C} (C_{2,t}^1)^{\frac{\theta_C - 1}{\theta_C} - 1}$$

<sup>&</sup>lt;sup>1</sup>These levels will be chosen later by solving the intertemporal problems faced by agents.

 $<sup>{}^{2}</sup>C_{j,t}^{i}$ ,  $I_{j,t}^{i}$  denote the demands for good j from country  $(i=1,2,\ j=1,2)$  for consumption and investment respectively.

then,

$$\frac{P_{2,t}}{P_{1,t}} = \left(\frac{1 - \gamma_C}{\gamma_C}\right)^{1/\theta_C} \left(\frac{C_{1,t}^1}{C_{2,t}^1}\right)^{1/\theta_C} \tag{B.2}$$

so,

$$C_{1,t}^{1} = \left(\frac{\gamma_C}{1 - \gamma_C}\right) \left(\frac{P_{2,t}}{P_{1,t}}\right)^{\theta_C} C_{2,t}^{1}$$
(B.3)

 $\iff$ 

$$C_{2,t}^{1} = \left(\frac{1 - \gamma_{C}}{\gamma_{C}}\right) \left(\frac{P_{1,t}}{P_{2,t}}\right)^{\theta_{C}} C_{1,t}^{1}$$
(B.4)

By substituting (33) into the objective one gets

$$C_{2,t}^{1} = (1 - \gamma_C) \left(\frac{P_{1,t}^{C}}{P_{2,t}}\right)^{\theta_C} C_{1,t}^{C}$$
(B.5)

Similarly, by substituting (34),

$$C_{1,t}^{1} = \gamma_C \left(\frac{P_{1,t}^C}{P_{1,t}}\right)^{\theta_C} C_{1,t}^C$$
(B.6)

In a similar way, the representative household in country 2 determines his demand for consumption by solving

$$\min_{C_{2,t}^2, C_{1,t}^2} P_{1,t} C_{1,t}^2 + P_{2,t} C_{2,t}^2$$

subject to

$$C_{2,t}^{C} = \left[ \gamma_{C}^{\frac{1}{\theta_{C}}} (C_{2,t}^{2})^{\frac{\theta_{C}-1}{\theta_{C}}} + (1 - \gamma_{C})^{\frac{1}{\theta_{C}}} (C_{1,t}^{2})^{\frac{\theta_{C}-1}{\theta_{C}}} \right]^{\frac{\theta_{C}}{\theta_{C}-1}}$$

and given that

$$P_{2t}^{C} \equiv \left[ (1 - \gamma_C) P_1^{1 - \theta_C} + \gamma_C P_2^{1 - \theta_C} \right]^{\frac{1}{1 - \theta_C}}$$
(B.7)

Then,

$$\frac{P_{2,t}}{P_{1,t}} = \left(\frac{\gamma_C}{1 - \gamma_C}\right)^{1/\theta_C} \left(\frac{C_{1,t}^2}{C_{2,t}^2}\right)^{1/\theta_C} \tag{B.8}$$

so,

$$C_{2,t}^2 = \gamma_C \left(\frac{P_{2,t}^C}{P_{2,t}}\right)^{\theta_C} C_{2,t}^C \tag{B.9}$$

$$C_{1,t}^2 = (1 - \gamma_C) \left(\frac{P_{2,t}^C}{P_{1,t}}\right)^{\theta_C} C_{2,t}^C$$
(B.10)

The problems faced by firms are analogous, but the index prices are in this case

$$P_{i,t}^{I} = \left[ \gamma_{I} P_{i,t}^{1-\theta_{I}} + (1 - \gamma_{I}) P_{j \neq i,t}^{1-\theta_{I}} \right]^{\frac{1}{1-\theta_{I}}}$$

Thus, the optimal demands for investment addressed by firm 1 are deduced from

$$\frac{P_{2,t}}{P_{1,t}} = \left(\frac{1 - \gamma_I}{\gamma_I}\right)^{1/\theta_I} \left(\frac{I_{1,t}^1}{I_{2,t}^1}\right)^{1/\theta_I} \tag{B.11}$$

$$I_{2,t}^{1} = (1 - \gamma_I) \left(\frac{P_{1,t}^I}{P_{2,t}}\right)^{\theta_I} I_{1,t}^C$$
(B.12)

$$I_{1,t}^{1} = \gamma_{I} \left(\frac{P_{1,t}^{I}}{P_{1,t}}\right)^{\theta_{I}} I_{1,t}^{C}$$
(B.13)

while those from Firm 2 are

$$\frac{P_{2,t}}{P_{1,t}} = \left(\frac{\gamma_I}{1 - \gamma_I}\right)^{1/\theta_I} \left(\frac{I_{1,t}^2}{I_{2,t}^2}\right)^{1/\theta_I} \tag{B.14}$$

$$I_{2,t}^2 = \gamma_I \left(\frac{P_{2,t}^I}{P_{2,t}}\right)^{\theta_I} I_{2,t}^C$$
(B.15)

$$I_{1,t}^2 = (1 - \gamma_I) \left(\frac{P_{2,t}^I}{P_{1,t}}\right)^{\theta_I} I_{2,t}^C$$
(B.16)

Finally, as in Hairault and Portier (1995), to preserve a simple closing of the model, we assume that the capital accumulation costs  $C_{i,t}$  are paid by firms through the purchase of a CES basket with the same parameters as that for investment. The optimal composition of these baskets is identical to (B.12), (B.13), (B.15) and (B.16), and the demands for goods j from firm i are given by

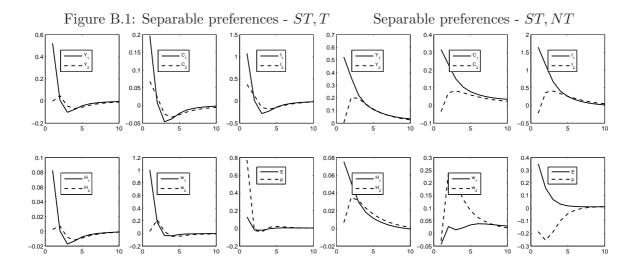
$$C_{2,t}^{1} = (1 - \gamma_I) \left(\frac{P_{1,t}^{I}}{P_{2,t}}\right)^{\theta_I} C_{1,t}$$
(B.17)

$$C_{1,t}^{1} = \gamma_{I} \left(\frac{P_{1,t}^{I}}{P_{1,t}}\right)^{\theta_{I}} C_{1,t}$$
(B.18)

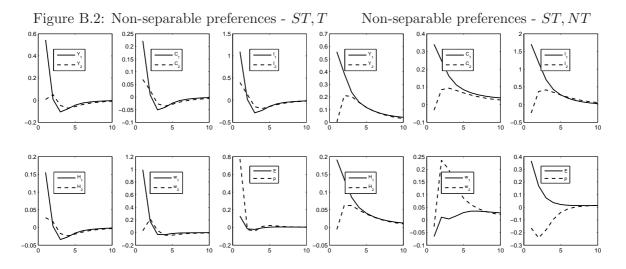
$$C_{2,t}^2 = \gamma_I \left(\frac{P_{2,t}^I}{P_{2,t}}\right)^{\theta_I} C_{2,t}$$
(B.19)

$$C_{1,t}^2 = (1 - \gamma_I) \left(\frac{P_{2,t}^I}{P_{1,t}}\right)^{\theta_I} C_{2,t}$$
(B.20)

# B.2 Two-sectors economy



International two-country, two-good real business cycle model with separable preferences. Left-hand side panel: IRF to a positive 1% productivity shock to the country 1 tradable-goods sector. Right-hand side panel: IRF to a positive 1% productivity shock to the country 1 non-tradable-goods sector.



International two-country, two-good real business cycle model with non-separable preferences. Left-hand side panel: IRF to a positive 1% productivity shock to the country 1 tradable-goods sector. Right-hand side panel: IRF to a positive 1% productivity shock to the country 1 non-tradable-goods sector.

# Appendix C

# Appendix to chapter 4

# C.1 Proofs

**Proposition 1. a.**  $\frac{\partial g}{\partial i}|_{i=b,\tau,\tau^w} = h \ln(q) \frac{\partial n}{\partial i}|_{i=b,\tau,\tau^w}$ . It is easy to show that:  $\frac{\partial x}{\partial i}|_{i=b,\tau,\tau^w} < 0$  So,

$$\frac{\partial n}{\partial b} = -\frac{\beta_1}{h(\beta_1 b + qhe)} \left( qh \left( \frac{1+\tau}{1-\tau^w} \right) x + r + n \right) < 0 \implies 
\frac{\partial g}{\partial b} < 0 \text{ and } \frac{\partial u}{\partial b} = -\left( \frac{\partial x}{\partial b} + \frac{\partial n}{\partial b} \right) > 0 
\frac{\partial n}{\partial \tau} = -\frac{q}{\beta_1 b + qhe} \left( \frac{\beta_1 b}{1-\tau^w} \right) x < 0 \implies \frac{\partial g}{\partial \tau} < 0 \text{ and } \frac{\partial u}{\partial \tau} > 0 
\frac{\partial n}{\partial \tau^w} = -\frac{q}{\beta_1 b + qhe} \left( \frac{1+\tau}{(1-\tau^w)^2} \right) x < 0 \implies \frac{\partial g}{\partial \tau} < 0 \text{ and } \frac{\partial u}{\partial \tau} > 0$$

In a similar way, we deduce: **b.**  $\frac{\partial x}{\partial e} = 0 \Rightarrow$ 

$$\frac{\partial g}{\partial e} = -\frac{qh \ln(q)n}{\beta_1 b + qhe} < 0$$

$$\frac{\partial u}{\partial e} = -\frac{\partial n}{\partial e} > 0$$

The first inequality comes from the fact that q > 1.

**Proposition 2.** Analogous to the proof of proposition 1:  $\frac{\partial x}{\partial \beta} < 0$  and  $\frac{\partial n}{\partial \beta} < 0$ . So,  $\frac{\partial g}{\partial \beta} < 0$  and  $\frac{\partial u}{\partial \beta} > 0$ .

**Proposition 3.** It is easy to verify that  $x_E = x\beta_1^{\frac{1}{1-\alpha}}$ . Since  $\beta_1 \geq 1$ , then  $x \leq x_E$ . On the other hand,  $\pi_E \leq \pi$ . This is due to the decreasing returns of the technology. Then,  $n_E \leq n \Rightarrow g_E \leq g$ . Because there are less researchers but more employed in production the total effect on u is ambiguous.

# Appendix D

# Reaching the Optimal Growth: Which is the role of the Labor Market Institutions?

In this part, we make a social welfare exercise using a simplified version of the general equilibrium model of endogenous growth developed in chapter 4. We show that the optimal growth rate can be reached by compensating the distortions on the goods-sector due to the growth process with the distortions induced by the labor market rigidities.

## Introduction

Creative destruction in the economic growth process could lead either to insufficient or excessive economic growth (Aghion and Howitt 1998). This is mainly explained by the distortions on the goods-sector induced by the monopolistic rents generated by R&D. However, we show that when the institutions and rigidities present in the labor market of many developed economies are acknowledged by the model, the optimal growth rate could be reached. Specifically, when the economic growth is excessive, the labor market rigidities are desirable because its negative impact on growth reduce the gap to the optimal rate. Conversely, when the economic growth is suboptimal, the fiscal policy gives the solution: the optimal rate can be reached by subsidizing labor.

# D.1 The model

The basics of the model are: (i) Innovations are the engine of growth. (ii) Agents have the choice of being employed or doing research and development activities (R&D). (iii) Unemployment is caused both by the wage-setting behavior of the unions representing the workers' interests, and by the labor costs associated to taxes and unemployment compensation.

#### D.1.1 Preferences

The economy is populated by L identical agents, each endowed with one unit flow of labor. At each time, they may be employed (x), trying their hand at R&D (n) or unemployed (u): L = x + n + u. When employed, workers pay a tax  $\tau^w$  on their labor income.

All individuals have the same linear preferences over lifetime consumption C of a single final good:

$$U(C) = E_0 \int_0^\infty C(t)e^{-\rho t}dt$$
 (D.1)

 $\rho > 0$  is the subjective rate of time preference and  $C_t$  is the individual's consumption of the final good at time t.

## D.1.2 Goods sector

The final good is produced by perfectly competitive firms that use the latest vintage v of intermediate input x, <sup>1</sup>

$$C(t) = A_{v,t} x_{v,t}^{\alpha}, \quad 0 < \alpha < 1 \tag{D.2}$$

 $A_v$  represents the productivity of the intermediate good and is determined by the number of technical improvements realized up to date t, knowing that between two consecutive innovations the gain in productivity is equal to q > 1 ( $A_{v+1} = qA_v$ ). Production of one unit of intermediate good requires one unit of labor as input. Since the final-good sector is perfectly competitive, the price of the intermediate good, p(x), is equal to the value of its marginal product.

<sup>&</sup>lt;sup>1</sup>Matter of simplicity, we assume just one homogeneous intermediate input. However, results are qualitatively the same if we assume instead a continuum of perfectly substitute intermediate inputs.

## D.1.3 R&D sector

Technology improvements lead to good-specific public knowledge allowing to start improvement efforts upon the current vintage. Innovations arrive randomly at a Poisson rate hn, where n is the amount of labor used in R&D, and h > 0 a parameter indicating the productivity of the research technology. Finally, the size of the R&D sector is given by the arbitrage condition:

$$\frac{(1-\tau^w)W_v}{h} = V_{v+1} \tag{D.3}$$

That is, the opportunity cost of R&D is the hourly net wage prevailing in the production sector,  $(1-\tau)W_v$ , times the expected duration of the innovation process, 1/h.<sup>2</sup> On the other hand, the expected payoff of next innovation,  $V_{v+1}$ , is equal to the net discounted value of an asset yielding  $\Pi_{v+1}$  per period until the arrival of next innovation, at the arrival rate  $hn_{v+1}$ . Assuming that Firms pay a proportional payroll tax  $\tau$  over employment, the instantaneous monopolistic profits earned by the successful innovator are:  $\Pi_{v+1} = p_{v+1}x_{v+1} - W_{v+1}(1+\tau)x_{v+1}$ .

## D.1.4 Government

The government faces the following budget constraint:

$$B_v u + T_v = (\tau + \tau^w) W_v x_v \tag{D.4}$$

B are the unemployment benefits, and any change in the revenue caused by changes in taxes and subsidies is rebated to household through the lump-sum transfer T.

## D.1.5 Wage bargaining and labor demand

The wage rate is the solution to the bargaining problem between the monopolistic producer and the trade union representing the workers' interests. We model the bargaining process as a generalized Nash bargaining game, with union's relative bargaining power  $\beta$ . If they don't agree, workers get the unemployment benefits and the monopolist makes zero profits. Given the bargained wages, the firm chooses the level of employment that maximizes her profit flow. That is,

<sup>&</sup>lt;sup>2</sup>Equivalently, we can assume that the opportunity cost amounts to the unemployment benefits, or even to a linear combination of both, the earnings of employed and those of unemployed workers.

$$W_{v+1} = \arg\max\left\{ \left[ ((1 - \tau^w)W_{v+1} - B_{v+1})x(W_{v+1}) \right]^{\beta} \Pi_{v+1}^{1-\beta} \right\}$$
 (D.5)

# D.1.6 Equilibrium

Given r > 0, for all "state of the art" v the equilibrium is defined as follows. The the wage rule, the labor demand and the research level satisfy the system of equations:

$$w = \frac{\beta_1 b}{1 - t}, \quad \beta_1 \equiv 1 + \frac{\beta(1 - \alpha)}{\alpha}$$
 (D.6)

$$x = \left(\frac{\alpha^2 (1 - \tau^w)}{(1 + \tau)\beta_1 b}\right)^{\frac{1}{1 - \alpha}} \tag{D.7}$$

$$n = \frac{q(1-\alpha)(1+\tau)x}{\alpha(1-\tau^w)} - \frac{r}{h}$$
 (D.8)

$$u = L - x - n \tag{D.9}$$

Finally, the average rate of growth in aggregate consumption is given by:  $g = hn \ln(q)$ . Remark that we have normalized lasts expressions by the productivity level associated to the  $(v+1)^{th}$  innovation (i.e.  $\pi \equiv \frac{\Pi}{A}$ ,  $w \equiv \frac{W}{A}$  and  $b = \frac{B}{A}$ ).

## D.1.7 The optimal economic growth

The optimal growth rate  $g^*$  is determined by the optimal level of research  $n^*$  that would be chosen by a social planner whose objective was to maximize the expected welfare E(U). Since consumption is a random variable that takes the values

 $\left\{A_0x^{\alpha},A_0qx^{\alpha},A_0q^2x^{\alpha},\ldots,A_0q^kx^{\alpha},\ldots\right\}_{k\in N}$ , the expected welfare E(U) is:

$$E(U) = \int_0^\infty e^{-rt} E(C_t) dt = \frac{A_0 x^{\alpha}}{r - hn(q - 1)}$$
 (D.10)

Hence the social planner will choose (x, n) to maximize the expected present value of lifetime consumption, subject to the labor constraint L = x + n.<sup>3</sup> Then,

$$n^* = \arg\max\left\{\frac{A_0(L-n)^{\alpha}}{r - hn(q-1)}\right\} = \frac{1}{1-\alpha}\left(L - \frac{\alpha r}{h(q-1)}\right)$$
 (D.11)

Given this level of research the optimal growth rate is  $g^* = hn^* \ln(q)$ .

<sup>&</sup>lt;sup>3</sup>Obviously, in an optimal setting there is no unemployment.

## D.1.8 Equilibrium growth v.s. optimal growth

Given that the average growth rate is proportional to the number of researchers, it is sufficient to compare the optimal level of research with the equilibrium level of our economy. In order to simplify the comparison between  $n^*$  and n we rewrite (D.11) and (D.8) respectively as:

$$1 = \frac{(q-1)h(\frac{1}{a})(L-n^*)}{r-hn^*(q-1)}$$
 (D.12)

$$1 = \frac{qh\left(\frac{1-\alpha}{\alpha}\right)(1+\top)(L-n-u)}{r+hn}$$
 (D.13)

where  $1 + T \equiv \frac{1+\tau}{1-\tau^w}$  can be thought as a proxy of the *Tax Wedge*. As in the ?)'s model, we find the following basic differences between  $n^*$  and n:

- **D1** The social discount rate  $r hn^*(q 1)$  is less than the private discount rate r + hn ("intertemporal-spillover effect").
- **D2** The private monopolist in unable to appropriate the whole output flow, but just a fraction  $(1-\alpha)$ .
- **D3** The factor (q-1) corresponds to the so-called "business-stealing" effect, whereby the successful monopolist destroys the surplus attributable to the previous generation of intermediate good by making it obsolete.

Whereas distortions D1 and D2 tend to make the average growth rate less than optimal, D3 tends to make it greater. Due to the offsetting nature of these effects, the market average growth rate may be more or less than optimal. These three distortions summarize the main welfare implications of introducing creative destruction in the process of economic growth: laissez-faire growth may be either insufficient or excessive. Additionally, we have two other differences due to the rigidities on the labor market, say:

- **D4** The optimal employment  $L n^*$  is bigger that the equilibrium employment L n u. This is directly due to the bargaining power of unions.
- D7 The equilibrium level of research is affected by the taxes on labor.

Clearly, D4 tends to make the average growth rate less than optimal. In contrast, D5 is growth enhancing only when 1 + T > 1, i.e., when T > 0. Nevertheless, the stark difference between

distortions due to D1 - D3 and those due to D4 - D5, is that the two lasts depend on labormarket policy variables that, at least theoretically, can be controlled by the policy deciders. This naturally suggest the question of whether variations in the policy variables, already present in the labor market, can reduce the gap between the optimal and the equilibrium growth rates caused by distortions D1 to D3. In other words, we are interested on issues as the following:

- $n > n^*$ : If the negative externality that new innovators exert upon incumbent firms (D3) dominates, which kind of policy adjustments could be done to converge to the optimum?
- $n < n^*$ : Conversely, if the intertemporal-spillover and the appropriability effects dominate (D1 and D2), which policy could foster growth?

To answer these questions, we look to the impact of the policy variables on the research level. Since  $\frac{\partial x}{\partial \Omega}|_{\Omega} = \{b, \beta, \tau^w, \tau\} < 0$ , then  $\frac{\partial n}{\partial \Omega}|_{\Omega} = \{b, \beta, \tau^w, \tau\} < 0$ . This suggest that when growth is excessive the labor market rigidities are desirable because they can help to reduce the gap between the equilibrium rate of growth and the optimal one. Moreover, when the economic growth is suboptimal the optimal rate can be reached by subsidizing labor.

# Appendix E

# Appendix to chapter 5

# E.1 Data

The sample is composed by Belgium, Spain, France, Italy, United Kingdom, United States. Depending on the availability of data, the analysis covers the 1980-2003 or the 1960-2003 period. Data on consumption, gross domestic product (GDP), employment, unemployment, population, wages and salaries, compensation of employees, the deflator of consumption and the defator of GDP (base year 2000) are from the OECD.<sup>1</sup>

Series for hours worked are from the Groningen Growth and Development Centre and the Conference Board,<sup>2</sup> whereas the mesures of institutional variables are taken from the Bassanini and Duval (2006) database. The Bassanini *et.al.*'s collection of labor market variables covers a large period (1970-2003 or 1982-2003) and mostly rely on indicators provided by the OECD.<sup>3</sup> Finally, we take the series of the average tax rates on labor, capital and consumption from the McDaniel (2007)'s dataset, which covers 15 OECD countries for the period 1950-2003.<sup>4</sup> The payroll tax is deduced from the ratio of the compensation of employees to the wage and salaries. Both mesures are taken from the OECD.

<sup>&</sup>lt;sup>1</sup>OECD Statistics, beta 1.0: http://stats.oecd.org/wbos/default.aspx

<sup>&</sup>lt;sup>2</sup> Total Economy Database, January 2007: http://www.ggdc.net

<sup>&</sup>lt;sup>3</sup>The OECD Secretariat has constructed several indicators of policies and institutions that are comparable both across countries and over time. These indicators have been used in a wide range of macro-econometric studies to explore the labor market effects of policies and institutions.

<sup>&</sup>lt;sup>4</sup>The McDaniel's tax estimates uses national account statistics as primary source and are in line with existing average tax rates calculated by Mendoza, Razzin, and Tessar (1994). In addition, these are the data used by Ohanian *et.al.* (2006).

**Job destruction.** For each country i, the average rate of job destruction  $s_i$  is computed such that the expected duration of unemployment  $(E_t[1/\Psi_t])$  is equal to the mean unemployment duration reported in table 5.5.

Job destruction in period t ( $d_{i,t}$ ) is defined as the sum of all net employment losses at establishments experiencing negative net employment gains between t-1 and t. Given the job destruction rate  $s_i$  and actual data for employment, we compute the job destruction series as:

$$d_{i,t} = s_i N_{i,t-1}$$

The Job creation series are obtained from equation (5.30):

$$M_{i,t} = N_{i,t} - N_{i,t-1} + d_{i,t}$$

that is, the job creation in period t is the sum of all net employment gains between t-1 and t. According with our model we compute series for the rate at which workers are matched with a vacant job as:

$$\Psi_t = \frac{M_t}{U_{t-1}}$$

where U is the observed unemployment level. Then, using the definition of the matching function we derive the market tightness  $(\theta)$  and the rate at which vacancies are matched with searching workers  $(\Phi)$ :

$$heta_t = \Psi_t^{rac{1}{\psi}}$$

$$\Phi_t = \theta_t^{\psi - 1}$$

# E.2 The Hansen-Rorgerson economy by country

Figure E.1: Hansen-Rogerson model (TW=0), 1980-2003

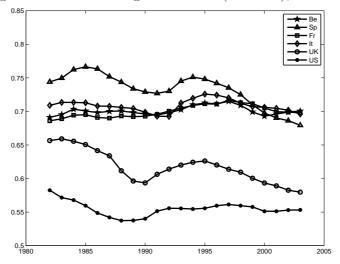
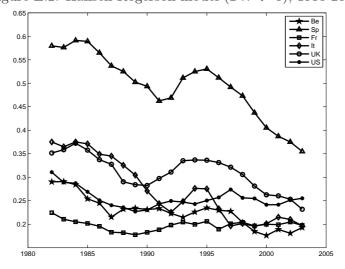


Figure E.2: Hansen-Rogerson model (TW > 0), 1980-2003



Essais sur les fluctuations économiques, la croissance et la performance du marché

du travail: l'impact de l'Etat providence

Cette thèse s'intéresse aux fluctuations économiques, au chômage et à la croissance économique. Ces

dernières décennies, la plupart des pays européens ont connu un ralentissement de leur croissance économique

ainsi qu'un taux de chômage élevé et persistant. Cette évolution, dite de long terme, a été accom-

pagnée d'une série de fluctuations économiques de court terme. Dans ce contexte, cette thèse analyse

le fonctionnement du marché du travail et son incidence sur la performance des économies développées.

Plus précisément, nous analysons les effets de court et de long terme de certaines distorsions jugées

représentatives du marché du travail des pays européens, tels que la fiscalité, les systèmes d'indemnisation

du chômage et les mécanismes de fixation du salaire.

Le premier chapitre présente le modèle canonique de cycle réel dans un contexte international. Il s'agit de

déterminer un ensemble d'hypothèses visant à pallier aux défaillances du modèle original dans l'explication

des fluctuations du marché du travail. L'incorporation de ces hypothèses dans ce cadre théorique fait

l'objet de la première partie du chapitre 2. Même si ces amendements du cadre canonique conduisent à

une meilleure compréhension des déterminants des fluctuations économiques et de leur synchronisation

entre pays, les faits concernant la dynamique des heures et du salaire ne sont pas expliqués. Ceci justifie

le développement d'une modélisation alternative du marché du travail, présenté dans la deuxième partie

de ce chapitre. Au centre de ce modèle prennent place le chômage et les liens économiques entre pays.

Ce cadre est étendu au chapitre 3 pour intégrer la fiscalité, ce qui nous permet de rendre compte de la

plupart des faits de court terme. Finalement, les chapitres 4 et 5 s'intéressent à la problématique liée à

la croissance économique ainsi qu'à l'évolution tendancielle du temps du travail d'équilibre. En tenant

compte des rigidités présentes sur le marché du travail, nous fournissons une explication des phénomènes

de long terme.

**Discipline:** Sciences Economiques.

Mots clés: Fluctuations internationales, marché du travail, croissance, chômage, taxes et in-

stitutions du marché du travail.

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