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Abstract

According to the global humanitarian overview published by the UNOCHA [1], 274 million people needed some form of humanitarian assistance across the world in 2022. In the same year, 387 natural disasters were recorded world wide, affecting 187 million people [2]. According to an estimate, in 2022, 27 million people were affected by labor-related human trafficking globally [3]. Now, more than ever, human race faces a tough challenge, and this calls for renewed focus on research into humanitarian operations. This thesis deals with two humanitarian applications in operations research and operations management. The first application pertains to optimizing the logistics of post-disaster relief efforts and the second application pertains to analysing buyer-agent contracts to reduce the incidence of modern slavery in supply chains. The body of the thesis is divided into 5 chapters. The first chapter is introductory and explores the need for research into humanitarian operations in a detailed manner and briefly explores the broad state-of-the-art.

Chapter 2 explores the first humanitarian application - crowdsourced humanitarian relief vehicle routing problem (CHR-VRP). This section defines the problem and positions the application in the

[1] Global Humanitarian Overview 2022, UNOCHA https://2022.gho.unocha.org/

[2] 2022 Disasters in numbers https://reliefweb.int/report/world/2022-disasters-numbers#:~:text=In%202022%2C%20the%20Emergency%20Event,totaled%20around%20US%24%20223.8%20billion.

[3] International Labor Organization https://www.ilo.org/global/topics/forced-labour/lang--en/index.htm#:~:text=Facts%20and%20figures,22%20million%20in%20forced%20marriage.

context of existing literature. A mixed integer linear programming model is suggested to optimize the delivery of relief efforts using crowdsourcing to a geography affected by natural disaster. Due to the complex nature of the problem, an iterated local search heuristic is suggested to solve large instances. For conducting numerical experiments, multiple instances are generated (both random locations and locations that mimic real life cities). A discussion on the choice of the objective explores alternative objective functions and benchmark them against each other. Some managerial insights from the numerical experiments are also included in the chapter. The paper based on this work was published in Computers and Operations Research journal as Parappathodi and Archetti (2022). This work was also presented in EURO 2021 conference held in Athens, Greece.

Chapter 3 explores solving the CHR-VRP using exact solution approaches. The chapter deals with different possible formulations of the CHR-VRP while using both the min-max rescue time objective function as well as the average rescue time objective function. Three methods are explored and compared with each other. The first method is to solve the compact formulation model using the default `Cplex` solver, while the second and third methods are decomposition based methods. The second method method uses generalized sub-tour elimination constraints to project out flow variables while the third method uses tailored Benders cuts. Multiple lower bounds are suggested to improve the formulation and their effectiveness is studied and compared. The paper based on the work presented in this chapter is currently in the final stages of completion for submission. This work was presented in EURO 2022 conference held in Helsinki, Finland and IFORS 2023 conference held in Santiago, Chile.

Chapter 4 takes a slightly different direction. This chapter studies labor exploitation. Exploitation related to labor recruitment is especially prevalent in 'labor supply chains' where buying companies rely on independent 'labor agents' to recruit and manage workers for them. The market and economic

factors that influence the occurrence of human trafficking in supply chains are studied. Optimal contracts are developed with which buying companies can prevent forced labor. Game theoretic models are used to study the interactions in labor supply chain. The case of information asymmetry is also studied where the buying company has limited information about the labor agents. A 'sequential' menu of contracts is developed when buyer has to choose from several candidates to ensure no coercion when maximizing buyer profit. Numerical analysis of the model is conducted using H2A labor contractor data from US Department of Labor. The paper based on the work presented in this chapter has been submitted to Manufacturing and Service Operations Management in August 2023. This work was also presented in POMS 2023 conference held in Paris, France.

Chapter 5 is a concluding chapter that recaps the major findings in the previous chapters and discusses briefly the potential research extensions and future directions.

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1

Introduction

1.1 A world in crisis

Human race has achieved marvellous feats during the last few years. From inventing cars in the early 1900s to artificial intelligence in 2020s, the breakneck speed of progress, sadly, came at a cost. Somewhere along the way, human suffering and oppression started being considered as collateral damage and an acceptable cost of progress. With each passing year, industrial growth encroached more and more into nature's territory. Socially and ecologically, life on earth is as precarious as it has ever been for a huge number of people. A deep dive into some of the statistics behind this threat that the world faces, paints a bleak future for us.

COVID-19 shook the world beyond what we thought was possible. It was a reminder that for all the advances in medicine and technology, we are still quite vulnerable to a lot of dangers around us. In 2004, an earthquake that recorded 9.1 on the Richter scale unleashed destructive power of the sea in Indian Ocean. 225,000 people across multiple countries lost their lives in the most deadly tsunami in recorded history. The scale of death and destruction was so high that a large part of rescue efforts had to be carried out by civilians while emergency services had their hands full with people needing critical care. The 2010 Haiti earthquake was another large scale natural disaster that cost 227,000 lives in a much more concentrated area. As recently as this year, the Turkey-Syria earthquake cost 60,000 lives. According to World Disaster Report, in 2021, a total of 378 major natural disasters were recorded-roughly a 12% increase over the average during the preceding 20 years. This increase is not an accidental spike. A look at the numbers from 1970 as shown in Fig 1.1 IFRC (2023) shows a clear increasing trend in the number of natural disasters reported from 1970-2020 bar a spike in the turn of the millennium. With the increasing threat to ecological balance due to human activity, it stands to reason that this trend is going to continue in the near future.

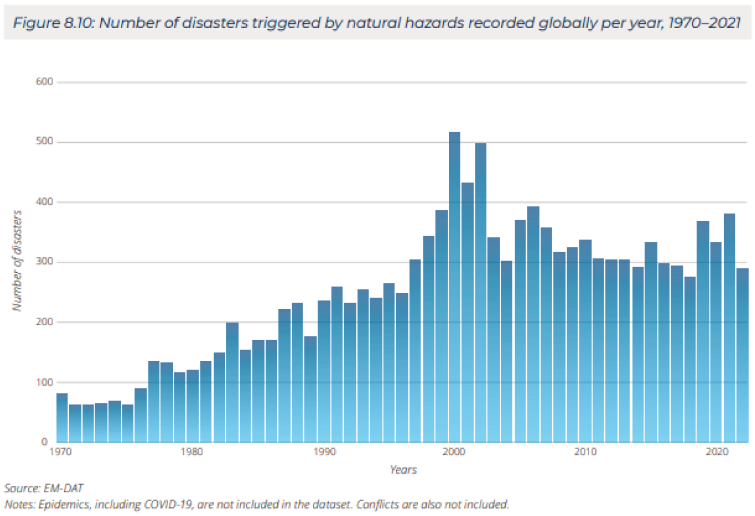


Figure 1.1: Number of natural disasters reported IFRC

The 'war to end all wars' was fought between 1914-1918. In a little more than 100 years since that, countless wars have been fought between countries as well as within countries claiming millions of human lives and livelihoods. According to the data from Peace Research Institute of Oslo (PRIO), the number of countries facing civil conflict and internationalized civil conflict has been on a rise since the 1940s as given in Fig 1.2 Obermeier and Rustad (2022). The world had almost forgotten what a full blown interstate war was, as evidenced by the decreasing interstate conflict numbers as per the graph. However, the Russian aggression on Ukraine in 2022 was a rude reminder that the complex geopolitical balance is never far away from an all out war. In 2021 alone, 85,000 battle related deaths were recorded according to PRIO. The suffering is not just limited to the death and destruction that wars and other armed conflicts cause. The displacement and mass destitution that results from such conflicts is usually in a scale that cannot be tackled by any government or established systems working in isolation.

John Maynard Keynes predicted almost hundred years ago that humanity will solve its economic

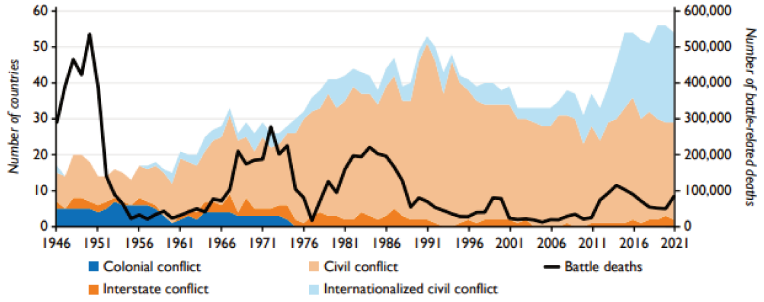


Figure 1.2: Number of countries facing conflict

problem and would gradually move towards a society where 15 hour work week is a norm and love of money will be considered to be a disgusting morbidity. Not only are we quite far from the utopia he had envisioned, since then, humanity as a whole has committed some of the most sinister crimes possible, towards its fellow humans, more often than not in the pursuit of money and profits. As per the estimates of international Labor Organization, in 2021, 50 million people are victims of modern slavery on any given day. That is one in every 150 people living a life of misery. Although slavery in its conventional form has been deemed illegal across the world, many forms of exploitative business practices exist to this day. The Palermo Protocol(2000) by UN was instrumental in demarcating what constitutes modern slavery, and brought world-wide attention to the term and its associated practices. The three key aspects of modern slavery are *coercion, dehumanization and commodification of labour* according to Dowlah (2021). Any exploitative relationship on any of the above aspects between a business owner and a worker can be deemed as a form of modern slavery. These people are forced to work against their wishes in unimaginable conditions for unfairly low wages in labor bondage lasting years, or forced into marital relationships which are, more often than not, a lifetime of misery. Out of the roughly 50 million people in modern slavery, 27.6 million people are victims of forced labor. Out of this, 3.3 million are children. This is as much as the population of Uruguay. Dwelling on this statistic really drives the point home - as many children as an entire country's population find themselves a victim of unethical labor practices, and the numbers have been on the rise over the last

few years. This points to the urgency with which this challenge has to be confronted head-on.

1.1.1 'Hell hath no fury like nature scorned'

Human beings have inhabited this planet for a remarkably short period of time in the ecological scale. Cambrian explosion is termed as the evolutionary event when the first form of complex lifeforms similar to what we see around us established themselves as the main inhabitants of this planet. This evolutionary event happened around 500 million years ago. In comparison, *homo sapiens* started appearing on the planet no earlier than 500,000 years ago. According to the Intergovernmental Panel on Climate Change (IPCC), the 50 year period since 1970 has witnessed the highest global temperature increase when compared to any other 50 year period in the last 2000 years. There has been an increase of 20cm in global average sea levels since the 1900s. At the current rate we stand the risk of having major cities to completely submerge in not too distant a future. From 2001 to 2022, Brazil has lost 13% of its tree cover, equivalent to 36.1 *G-tons* of carbon dioxide equivalent emissions [1]. There is a strong correlation between the increased impact of humans on the environment and the increase in intensity and scale of natural disasters. There has also been an uptick in the number of countries that are facing a combination of natural disasters at the same time.

The 2020 wildfires in Australia was estimated to cost the Australian healthcare system a total of $1 billion AUD. The destruction of wildlife along with the increase health impact due to smoke and particulate matter inhalation can have long term impact on a significant proportion of the population. The 2022 floods in Pakistan[2] affected 33 million people costing $14.9 billion in damages and $15.2 billion in economic cost. California was brought to its knees by the atmospheric river (AR)

[1] Global Forest Watch https://www.globalforestwatch.org/dashboards

[2] Disaster Philanthropy https://disasterphilanthropy.org/disasters/2022-pakistan-floods/

phenomenon that left major parts of the state flooded [3]. More prosperous countries are usually better equipped to deal with crises of such magnitude. However, poorer countries often find themselves deep in trouble when trying to handle the situation. For instance, around 2 million people in Pakistan are living in close proximity to contaminated floodwater in Pakistan, six months after the government declared an emergency. Sometimes climate-based emergency affects some of the most vulnerable sections of the population. Controlling nature is something that is outside the realms of possibility, the best that we can hope for is to reduce our impact on the climate and prepare for extreme events in the best ways possible.

1.1.2 Modern slavery and labor coercion

One of the most natural reaction to any discussion on modern slavery is to classify it as a problem faced by the 'developing' countries that purely arises out of extreme poverty. This statement could not be further from the truth. A study by ILO shows that as a proportion of population, modern slavery is most prevalent in the Arab states. This geographical area is home to some of the richest countries on the planet in terms of per capita income. The 'kafala' system of employment, which was rigged to be misused, was the bedrock on which Qatar World Cup 2022 was built. This event single handedly witnessed mass exploitation of hundreds of thousands of low-skilled laborers who were recruited *en masse* from countries such as India, Pakistan, Nepal and Bangladesh. Prevalence of forced labor is independent of geographical area and wealth of a country. Typically, it has also been observed that migrants face a higher risk of being exploited. H2A visa system in the US sees a high number of workers who are exploited and made to work unreasonable hours in inhuman conditions.

[3] Center for western weather and water extremes https://cw3e.ucsd.edu/cw3e-event-summary-29-december-2022-1-january-2023/

Between 2016 and 2021, in absolute terms and as a proportion of population, labor coercion has increased globally - an increase of 2.7 million people which translates to an increase in prevalence from 3.4 to 3.5 per thousand people. Roughly 20% of people involved in forced labor is in some form of debt bondage. Often the exploitation starts with an offer of job, sometimes in a rich country offering a significantly higher standard of living, and is an offer that is too good to refuse. Once the workers get to their destination, their documents are often taken away and the actual wages and conditions revealed to them. At this point of time, either they have already paid such a huge amount of money to their 'agent' that there is no sense in going back or they do not have the means to travel back home. Sometimes the debts can even last generations.

Yet another fertile avenue for such unethical practices to take root is in areas where some form of armed conflict takes place. War Child estimates that in 2020, 8521 children were recruited by armed groups [4]. Beber and Blattman (2013) study the recruitment of child soldiers in conflict zones. They note that child soldiers are an attractive recruitment option for armed groups as it is easier to *"intimidate, indoctrinate and misinform children"* compared to adults. The authors note, with the need for concerted efforts to fight child exploitation from governmental and non governmental organizations, there is also a need to provide ample rehabilitation opportunities for children who are rescued from the clutches of miscreants.

1.2 The need for change

It is, however, not all doom and gloom. A significant proportion of the world recognizes the monumental challenge that lies in front of us. There has been a concerted effort by governmental, quasi governmental and volunteer organizations to develop systems and processes that ensure well being of

[4] War Child https://www.warchild.org.uk/

the people. With the increase in awareness, there has also been an increase in a lot of companies being held accountable for their actions that put the world around them in jeopardy. According to a study by Simon-Kutcher [5], 34% of consumers expressed their willingness to pay higher price for sustainable products.

Lesk et al. (2016) discusses the impact of natural disasters on crop production. The authors note a 7% higher effect of natural disasters on crop production in recent years. Unless we actively identify and mitigate the effect of natural disasters and extreme weather events, their effects will be felt across multiple industries. While businesses have increasingly made attempts to be a part of sustainable economy, more needs to be done to preserve lives and livelihoods.

Almost all large organizations in manufacturing sector have extremely fragmented and globalized supply chains in order to remain competitive. A consequence of this inter-connectivity is possible links with unethical suppliers either directly or indirectly. This in turn increases litigation risk and public outrage due to ever increasing scrutiny of organizations especially in developed countries. There have been quite a few examples of NGOs or other organizations intervening in the event of a firm being associated with suppliers indulging in modern slavery. A recent such instance was in 2020, when more than 172 rights groups and NGOs jointly called out international brands to stop being complicit in conducting business with cotton suppliers from Xinjiang Uyghur Autonomous Region (XUAR). As a response, H&M, the fashion retailer, suspended the licensing of Better Cotton Initiative in the region and stopped their procurement from the region altogether [6]. Yet another example of media intervention is that of the '*boohoo*' scandal. In 2020, a workers rights organization called 'Labor be-

[5] Global Sustainability Study 2021 https://www.simon-kucher.com/sites/default/files/studies/Simon-Kucher_Global_Sustainability_Study_2021.pdf

[6] H&M statement https://hmgroup.com/sustainability/fair-and-equal/human-rights/h-m-group-statement-on-due-diligence/

hind the label' published a report citing the abysmal conditions of workers during peak pandemic time in garment factories around Leicester [7]. The lack of safety measures and the paltry wages paid were highlights of the report. This was followed by an undercover story on the same topic by The Sunday Times, a British newspaper. The high amount of backlash and the risk of litigation due to the UK Modern Slavery Act (2015) forced an immediate reaction from the company. '*boohoo*' published a statement promising to reevaluate its entire UK supply chain and allocating an incremental fund of £10 million to eradicate supply chain malpractices.

1.3 Envisioning a better world

United Nations has published the Sustainable Development Goals (SDGs), a set of 17 objectives discussed and agreed to, by the world leaders with the goal to make the earth a better place to live by 2030. The goals cover multiple facets of our lives from eradicating hunger to fostering world peace, the SDGs attempt to ensure decent standard for living for every human being, irrespective of their background. While these goals only provide a canvas, the onus is on us to work towards making this a reality. This means increased focus on research that work towards attaining these goals and increased participation by businesses with specific sustainable goals in mind.

Research on humanitarian relief and rescue is all the more relevant in the context of increased frequency and scale of natural disasters. The resources available for relief and rescue have to be optimized to ensure that the benefits reach as many people as possible. Social media has revolutionized how content is consumed in the post-internet era. It is high time that we use the opportunity to complement our rescue efforts using social media in providing relief and rescue. Haiti earthquake and Kerala floods of 2018 provide concrete examples of how general public can effectively complement the efforts of

[7] Labor behind the label https://labourbehindthelabel.org/report-boohoo-covid-19-the-people-behind-the-profit/

emergency responders.

With increasing attention being paid to the ethics of business by both governmental bodies as well as general public, it is imperative that organizations understand the risks that modern slavery poses to their supply chains and take steps to mitigate them. There have been multiple efforts by lawmakers to implement legislation that fights modern slavery and forced labor. The *'Lieferkettengesetz'* (translation: 'Supply Chain Act') introduced by Germany in 2023 is indicative of the increasingly strict scope and accountability of businesses. Looking at how the acts have evolved from US California Transparency in Supply Chains Act in 2010, it is obvious that businesses have to understand the dynamics of unethical and unsustainable businesses and take action to work toward a more equal world.

1.4 Purpose driven research- origins of this thesis

The existence of humanitarian crises is an irrefutable truth. It could be argued that the need for more research into humanitarian logistics is not rooted in conventional motivations of business and economics. It does not deal with profits and revenues. This is exactly why academic research has to champion 'purpose driven' research rather than 'profit driven' research. Unless a conscious effort is undertaken to focus on specific humanitarian problems, the topic stands at risk of being talked about only in a superficial manner.

The first application focuses on a basic operational aspect of using crowdsourcing in humanitarian rescue operation. The research presented in the subsequent chapters study the problem from both practical and theoretical perspectives. The second application studied in this thesis deals with a systemic issue of labor exploitation in labor supply chains. While these are just small ventures into the vast possibilities of humanitarian logistics, the objective behind picking specific applications is to

ensure the right balance between practical relevance and academic rigor.

2

Crowdsourced humanitarian relief vehicle routing problem

joint work with **Claudia Archetti**

2.1 Introduction

Over the last two decades, there has been an increase in both the number and the scale of natural disasters. In 2019 alone, 361 different natural disasters were reported across the globe, which included 170 floods, 85 extreme weather events and 32 earthquakes. Natural disasters and climate change affect 350 million people annually on an average, according to the UNOCHA (2018). An interesting recent development in the field of humanitarian logistics is the increase in acceptance of crowdsourcing as a high potential tool. The potential of crowdsourcing in disaster relief came into the fore during the Haiti earthquake according to Schimak et al. (2015). During the 2018 floods in India as well, crowdsourcing played an important role during relief operations, as reported by Aravind (2018). While the former is an example of how social media can be used as an effective information gathering tool, the latter is an example of how public participation can be crucial in ensuring timely disaster response. In line with the second example is where we can identify a direct application of crowdsourcing, that of post-disaster emergency relief vehicle routing. In the context of this chapter, we look at crowdsourcing as defined by Estellés-Arolas and González-Ladrón-De-Guevara (2012):

> Crowdsourcing is a type of participative online activity in which an individual, an institution, a non-profit organization, or company proposes to a group of individuals of varying knowledge, heterogeneity, and number, via a flexible open call, the voluntary undertaking of a task. The undertaking of the task, of variable complexity and modularity, and in which the crowd should participate bringing their work, money, knowledge and/or experience, always entails mutual benefit. The user will receive the satisfaction of a given type of need, be it economic, social recognition, self-esteem, or the development of individual skills, while the crowdsourcer will obtain and utilize to their advantage what the user has brought to the venture, whose form will depend on the type of activity undertaken.

During the 2018 floods in India, large swathes of coastal area in southern India were inundated with water. During this time, even emergency services themselves could not reach out to many people in need because of shortage of resources and because of the fact that most of the roads were flooded. Consequently, the fishermen living in the coastal areas volunteered to use their boats to navigate the flooded roads and reach out to the victims. The relief materials that the fishermen were supposed to distribute were collected in different locations by administrative officials and volunteers. There was central command unit comprising of both government officials and public volunteers in each of the districts that would coordinate the rescue efforts and would plan the rescue routes. Because of the evolutionary nature of the situation, the rescue runs were done multiple times in a day as per Sajin (2018). It is from this relief and rescue model created with successful cooperation between authorities and public citizens that we find inspiration for this chapter. Though the scenario mentioned is rather specific, with some minor modifications, this model can be applied to a variety of scenarios in humanitarian logistics.

Time and efficiency is of paramount importance in disaster relief and any small improvement in operations can potentially save lives. Post-disaster humanitarian logistics has to ensure that the relief material (food, water, batteries or other essentials) reach as many people in as little time as possible. In a crowdsourced disaster relief system, vehicles and relief materials are provided by multiple people spread across the geography. Consequently, relief vehicles have to start from their location, then travel to a supply point and pick up the required supplies, and finally visit the demand points and drop off relief materials. This makes the routing decisions quite complex.

Crowdsourcing has a few possible advantages over a centralized disaster response. In this context, a centralized disaster response system would be one that has vehicles that start from a central depot. In such a centralized scenario, time is spent in mobilizing the vehicles to a few central locations, which

then pick up supplies to be distributed among the people. This can be an inefficient system as the mobilization time can be used to reach out to more people. Second, the amount of non value added time (time spent by vehicles in reaching the first delivery node etc.) can be cut down if the supply points and vehicle locations are more geographically dispersed. Third, not all locations might be equally accessible from a central depot. In scenarios where the natural disasters cause obstruction to road network, crowdsourcing can be effective in reaching areas that have been cut-off from the main road network, provided that supply points are properly geographically dispersed.

In this chapter we study the crowdsourced humanitarian relief vehicle routing problem (CHR-VRP) which can be defined as follows. The situation considered is that of a post-natural disaster relief operation. There are people in need of supplies spread out over the geography. A set of drivers with vehicles (crowdsourced vehicles) have volunteered to pick up and deliver the emergency goods and are waiting for instructions. The success of the operation is defined as minimizing the time by which the last person in need receives emergency kits that contain items like medicines, batteries, food and water. It is assumed that though there are multiple commodities required, the rescue kits can be considered to be one unit which in turn contains a fixed number of the aforementioned commodities. The vehicles are assumed to be homogeneous and have a finite capacity in terms of volume. Because we consider the rescue kit to be one item and we assume that each demand point has a demand of one unit, the question of split deliveries does not arise in this context. We also assume that each supply points have sufficient kits to service the entire demand by itself. This assumption is not too far from the reality as resource mobilization would have started during the disaster preparedness phase. This also ensures tractability of the problem. Each vehicle starts from a vehicle node, visits a supply point to collect rescue kits, and deliver those kits to the demand points. It then returns to its respective vehicle node at the end of the route. It can be noted that some simplifying assumptions about the nature of the problem have been made. This is to ensure tractability of the problem and also due to the fact that this

chapter is seen as a starting point to future research that includes relaxation of a few key assumptions. The objective of the CHR-VRP is to build vehicle routes in such a way that:

- all demand points are served,
- vehicle capacity is not exceeded,
- the length of the longest route, computed up to the last demand point visited, is minimized.

The main contributions of this chapter are the following. First, we formally define the CHR-VRP problem and present a MILP formulation.The presence of supply points that can be visited by multiple vehicles and the existence of multiple depots make the problem significantly different from the situations addressed in the literature already. Second, we develop an Iterated Local Search (ILS) heuristic to solve reasonable sized instances of the CHR-VRP. Third, we generate realistic instances mimicking the shape of big cities. Analysis of the performance of the heuristic are done on two sets of instances and managerial levers are analyzed as tools at the disposal of the crisis manager. Also, a comparison of different KPIs used as objective functions is done to assess the performance of the chosen objective function (minimizing the length of the longest route).

The remainder of this chapter is organized as follows. Section 2.2 presents a detailed literature review relevant to the problem. Section 2.3 formally defines the problem and proposes the MILP formulation. Section 2.4 describes the ILS heuristic. Section 2.5 presents the details of the numerical analysis of the heuristic including results on real-life instances and finally, Section 2.6 presents our conclusions.

2.2 Literature Review

We look at literature pertaining to two aspects of the problem. The first aspect is the choice of the objective function for the problem and the second is the existing literature on the nature of the problem.

2.2.1 Choice of the objective function

One of the most critical elements of any analytical model of a post disaster humanitarian logistics problem is the choice of the objective function. Holguín-Veras et al. (2013) argue that social deprivation costs have to be the most important consideration when studying such problems. The authors examine two philosophical implications and compare their fairness in different situations. They argue that when the resources are available to rescue all members in a group, the "Rule of Rescue", which is based on the theory of deontological ethics, suits the best. This school of thought advocates that there is a moral obligation to help everyone who is in an immediate need even if it means high costs and economic inefficiencies. This perspective allows for tolerating some inefficiencies in the system to ensure that the group as a whole is better off. This is in contrast to the views of followers of the consequentialist school of thought, who hold that the criteria in such a situation should necessarily be individualistic and utilitarian. In the current problem, we assume that we have enough crowdsourced vehicles to tend to all the demand points. This aspect of the problem works in favor of adopting the deontological school of thought.

A clear parallel can be drawn between the discussion above and the problem at hand. From a deontological perspective, it can be argued that the objective function should reflect how we minimize the deprivation cost of the entire group as a whole. As Gutjahr and Nolz (2016) notes, a good proxy for social deprivation is the response time in reaching out to people. This would translate to an objective function that minimizes the time required to reach out to the last person in need. However, from a

consequentialist perspective, the objective function should reflect how we minimize the deprivation cost of each individual. A suitable proxy objective function in this case can be the mean cumulative response time. However, Taurek (1977) argues that in social policies involving survival of a group of people, equal importance should be given to the chance of survival of every single member in the group. The author also puts forward another argument that in such circumstances, collective pain or social deprivation does not make sense. This is a very strong argument against using mean cumulative rescue time as an objective function.

Gutjahr and Fischer (2018) prove how usage of utilitarian objective function randomly results in extreme inequality based on the instance. Gutjahr and Fischer (2018) and Eisenhandler and Tzur (2019a) present strong arguments for using an objective function based on the well known *Gini coefficient* as a measure of social deprivation. However, it may be noted that using a min-max response time as the objective function preserves the need for equity while being extremely parsimonious. Hence, the min-max function has been utilized as the objective function for the problem at hand. However, to ensure completeness, we also explore two other objective functions during our numerical study and compare the performance of each of the three key performance indicators (KPIs) in each scenario. The details of this study is given in Section 2.5.2

2.2.2 Literature on nature of the problem

As far as the structure of the problem is concerned, the CHR-VRP belongs to the family of Vehicle Routing Problems (VRPs). The area is very prolific in terms of research and there have been multiple studies and surveys that summarize the taxonomy and notable research in vehicle routing. Toth and Vigo (2014) and Golden et al. (2008) provide a summary of multiple classifications of VRPs. The set of applicable literature to the current problem can be classified into three main categories.

The first category of literature deals with multi-depot VRP problems (MDVRP). An MDVRP has multiple depots from which multiple vehicles start as opposed to the single vehicle scenario in a conventional VRP. A survey of MDVRP literature is provided by Montoya-Torres et al. (2015). Taking the variation one step further, if the vehicle routes in the MDVRP do not end in the start depot itself, we have a multi depot open VRP (MDOVRP) as discussed in Brandão (2020). The current problem can be considered as a variant of the MDOVRP. It primarily differs from a conventional MDOVRP in two ways. First, there is a requirement for all vehicles to first pick up supplies from any of the supply points. Second, the objective function in this problem is not to minimize the total sum of time (distance), but to minimize the time of the maximum route. The minimization of the maximum route duration implies minimizing the longest service time over all the affected people. As discussed in previous section, this is a common objective function frequently used in humanitarian logistics (see Bish (2011) and De la Torre et al. (2012)). Also, the rescue time is calculated only until the last delivery node is visited for the delivery of relief items.

The second category of literature deals with humanitarian logistics. There have been extensive studies on optimization of humanitarian logistics problems. Özdamar and Ertem (2015) provides a detailed survey on the research on humanitarian logistics from an operations research perspective. A broader discussion on humanitarian supply chain management can be found in Behl and Dutta (2019), where the authors considered the entire gamut of humanitarian relief and rescue from disaster preparedness to post-disaster recovery. Campbell et al. (2008) develop an ILP formulation for a multi trip setting during humanitarian relief. While this chapter considers both objective functions discussed above, it does not consider existence of multiple depots as well as existence of supply points. Two recent papers of particular note in the context of humanitarian aid are Eisenhandler and Tzur (2019a) and Eisenhandler and Tzur (2019b). In both these papers, the authors have considered a re-

lief delivery setting with multiple supply points. The main point of difference with our research lies in the fact that, in the latter papers, demand and pickup points can be visited in any order, contrary to the case analysed in the current paper where the supply point is the first point visited in any route. In addition, Eisenhandler and Tzur (2019a,b) considered demand points with different demand sizes. Finally, they used the *Gini coefficient* as objective function.

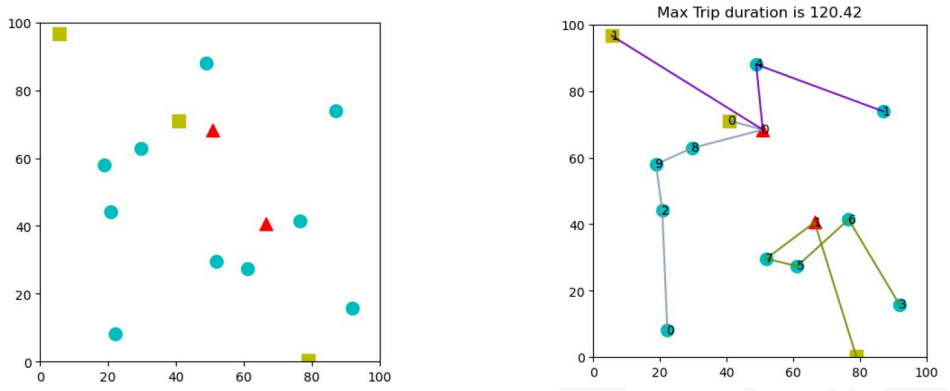
The third category of literature primarily deals with crowdsourcing and its applications in logistics. In the context of VRP, Archetti et al. (2016b) discusses a crowdshipping problem with the presence of occasional drivers. The VRP with Occasional Drivers (VRP-OP) has gained growing attention after the introductory paper of Archetti et al. (2016b). We refer to Archetti et al. (2021) for a recent contribution reviewing the literature on the topic. It can be noted that there has been a slight amount of hesitation in adopting crowdshipping in business applications, possibly because of a lower on-time performance observed in such a setting when compared to a dedicated fleet. An interesting observation as per Castillo et al. (2018) is that the total deliveries are higher for a crowdsourced logistics network when compared to dedicated fleet. The fact that the current research problem has to ensure that the relief supplies reach the maximum number of people as quickly as possible, further strengthens the argument in favor of crowdsourcing. In general, crowdsourcing has been gaining acceptance after the turn of the century, with the idea being a popular notion among entrepreneurs for crowdfunding. For instance, Du et al. (2020) discusses the problem of optimum choice of crowdfunding mechanism. Crowdsourcing for leveraging collective creativity and innovation has also been explored in the area of new product development, as discussed by Schemmann et al. (2016). Even in the area of urban freight logistics, the concept of crowdsourcing is increasingly being considered as a more sustainable means of transportation (see Buldeo Rai et al. (2017)). Li et al. (2014) explores two multi commodity sharing models in an urban setting. The concept of dynamic ride sharing has been explored by Lee and Savelsbergh (2015), a situation similar to taxi sharing platforms. Furuhata et al. (2013) pro-

vides a detailed state-of-the-art in ride-sharing and discusses future research directions. Trentini and Mahléné (2010) explores different focus areas using the concept of crowdsourcing in an urban setting.

In terms of the solution methodologies presented in existing literature, Brandão (2020) uses a memory based iterated local search. However, the presence of the supply points that can be visited by multiple vehicles pose a challenge in adapting this heuristic for the problem at hand. Li et al. (2016) uses a hybrid genetic algorithm with local search for studying a variant of the MDVRP and Uit Het Broek et al. (2021) uses a branch and cut algorithm as the exact solution approach. Both these papers too, do not have the intermediate supply points to visit, while testing on considerably smaller instances. In summary, the CHR-VRP addresses the knowledge gap in a multi-depot crowdsourced scenario for relief distribution. Ultimately, the role of crowdsourcing in relief operations is only going to increase with the increase in magnitude and frequency of humanitarian disasters. Crowdsourcing can be used to quickly deploy disaster relief while eliminating the time wasted in mobilizing the operation in a centralized location first. The decision point in CHR-VRP focuses on ensuring relief aid reaches everyone in as quick a time as possible.

2.3 Problem Definition

The crowdsourced humanitarian relief vehicle routing problem (CHR-VRP) can be defined in the following manner. $G = (N, A)$ is a complete graph with a node set N and arc set A. The node set N is defined as $N' \cup V$. Node set N' is in turn comprised of two types of nodes; the supply point nodes N_s where there is a supply of commodities; and demand nodes N_d where deliveries are to be made. We assume that each supply point has an infinite availability of commodities. There are also V vehicle nodes where vehicles are ready to be deployed. Each vehicle depot $d(v)$ is associated with a vehicle



(a) Sample Instance of CHR-VRP

(b) Feasible Solution for the instance

Figure 2.1: Sample instance of CHR-VRP

v. Each delivery node i has a requirement equal to 1. The travel time between each pair of nodes i, j is t_{ij}, which is known apriori. The vehicles available at the vehicle nodes are assumed to have a finite capacity limit of Q. We assume that there are enough crowdsourced vehicles available to serve all the demand points. Moreover, each rescue kit takes up a capacity of 1 unit in the vehicle. The demand points (people in need of rescue kits) have to be served by delivering rescue kits which are available at supply points. The distribution is performed by crowdsourced drivers (i.e., private citizens) who start the delivery activity from their place (i.e., citizen's home), visit a supply point to collect a number of rescue kits, and deliver these rescue kits to the demand points. The objective is to define the delivery plan that minimizes the maximum time to serve all demand points. Figure 2.1a shows a sample instance as per the above definition. The square nodes are vehicle depots, the triangle nodes are supply points and the circular nodes are demand points. In this example, there are 3 vehicles, 2 supply points and 10 demand points. The maximum capacity of the vehicle is 4. Figure 2.1b shows a feasible solution (and the optimal one in this case) as per the constraints imposed by the problem.

The mathematical formulation of the CHR-VRP makes use of the following decision variables:

$$x_{ij}^{v} = \begin{cases} 1 & \text{if arc } (i,j) \text{ is served by vehicle } v \\ 0 & \text{otherwise} \end{cases} \qquad \forall i,j \in N$$

$$w_{i}^{v} = \begin{cases} 1 & \text{if node } i \text{ is visited by vehicle } v \\ 0 & \text{otherwise} \end{cases} \qquad \forall i \in N$$

$$f_{ij}^{mv} = \begin{cases} 1 & \text{if vehicle } v \text{ traverses arc } (i,j) \text{ to serve node } m \\ 0 & \text{otherwise} \end{cases} \qquad \forall i,j \in N, \forall m \in N', \forall v \in V$$

The objective function minimizes the time required for serving the last demand node.

$$\text{minimize} \qquad z \tag{2.1}$$

subject to:

$$\sum_{i \in N} x_{ij}^{v} - \sum_{k \in N} x_{jk}^{v} = 0 \qquad \forall j \in N, \forall v \in V \tag{2.2}$$

$$\sum_{v \in V} \sum_{i \in N} x_{ij}^{v} = 1 \qquad \forall j \in N_d \tag{2.3}$$

$$\sum_{j \in N_s} x_{d(v)j}^{v} \leq 1 \qquad \forall v \in V \tag{2.4}$$

$$x_{ij}^{v} \leq w_{i}^{v} \qquad \forall i,j \in N, \forall v \in V \tag{2.5}$$

$$x_{ij}^{v} \leq w_{j}^{v} \qquad \forall i,j \in N, \forall v \in V \tag{2.6}$$

$$\sum_{j\in N} f_{ij}^{mv} - \sum_{j\in N} f_{ji}^{mv} = \begin{cases} w_m^v & \text{if } i = d(v) \\ -w_m^v & \text{if } i = m \\ 0 & \text{otherwise} \end{cases} \qquad \forall v \in V, \forall m \in N', i \in N \qquad (2.7)$$

$$\sum_{i\in N'} w_i^v \leq Q + 2 \qquad \forall v \in V \qquad (2.8)$$

$$f_{ij}^{mv} \leq x_{ij}^v \qquad \forall i,j \in N, \forall m \in N', \forall v \in V \qquad (2.9)$$

$$\sum_{i\in N}\sum_{j\in N'} x_{ij}^v t_{ij} \leq z \qquad \forall v \in V \qquad (2.10)$$

$$w_i^v \leq w_{d(v)}^v \qquad \forall i \in N, v \in V \qquad (2.11)$$

$$x_{ij}^v \in \{0,1\} \qquad \forall i,j \in N, v \in V \qquad (2.12)$$

$$w_i^v \in \{0,1\} \qquad \forall i \in N, v \in V \qquad (2.13)$$

$$f_{ij}^{mv} \geq 0 \qquad \forall i,j \in N, m \in N', v \in V \qquad (2.14)$$

Constraints (2.2)–(2.4) are flow constraints for vehicles. Constraints (2.2) ensure that if a vehicle comes into a node, then it has to leave. This is valid for all nodes including the vehicle nodes as all vehicles are expected to end their trips in the starting node. Constraints (2.3) ensure that each demand node is visited by a single vehicle. Constraints (2.4) ensure that each vehicle performs one route at most. Constraints (2.5) and (2.6) connect the visit variables with the arc variables. Constraints (2.7) are flow constraints that act as sub-tour elimination constraints. These constraints ensure for each node (other than a depot) the outflow minus the inflow at that node will be equal to the negative of the visit variable. If the node is not visited, the RHS take the value of 0, else it takes the value of 1. Constraints (2.8) ensure that capacity of each vehicle is not exceeded. Constraints (2.9) link flow variables with arc variables. Constraints (2.10) specify the min-max nature of the objective function. Constraints (2.11) ensure that unless a vehicle visits its depot node (in other words, unless the vehicle

is active), it cannot visit any node. (2.12) to (2.14) define variable domain. Formulation (2.1)–(2.14) can be strengthened by introducing the following constraints which discard arcs that cannot belong to any route. The arc between two vehicle nodes is set to 0 as a vehicle cannot visit a depot other than its own, as denoted by (2.15a), while (2.15b) ensures that there are no arcs going into supply nodes except from depot nodes, and finally the arcs from depot nodes directly to demand points as given by (2.15c) are set to 0. These constraints become redundant if the corresponding arc variables are explicitly declared to be equal to 0 during pre-processing (as per the implementation of the model, the results of which are presented in Section 2.5).

$$\sum_{i \in N} x^v_{id(v')} = 0 \qquad \forall v, v' \in V, v \neq v' \tag{2.15a}$$

$$\sum_{i \in N \setminus d(v)} x^v_{ij} = 0 \qquad \forall v \in V, \forall j \in N_s \tag{2.15b}$$

$$\sum_{j \in N_d} x^v_{d(v)j} = 0 \qquad \forall v \in V \tag{2.15c}$$

It can be noted that though the model is defined on a closed circuit, the definition of z, while not taking into account the last arc back to the vehicle depot ($j \in N'$) ensures that the objective function reflects the open path nature of the problem.

2.4 Solution Approach

It can be observed that CHR-VRP is an *NP-hard* problem. Indeed, for the special case in which there is one vehicle only with capacity equal to the number of demand points and depot location corresponding to supply location, the CHR-VRP corresponds to the minimum cost Hamiltonian path, which is proved to be *NP-hard*. In the specific application discussed in this chapter, due to the fact

that the instance sizes are big and having a reasonably short solution time is crucial, we propose an Iterated Local Search (ILS) heuristic as the solution method, as it proved to be effective in solving complex routing problems (see, for example, Cattaruzza et al. (2014), Ibaraki et al. (2005) and Coelho et al. (2016)). It is composed of two main procedures: initialization and local search. Sub-sections 2.4.1 and 2.4.2 discuss these procedures respectively. The ILS iterates through these procedures for multiple runs and returns the best solution found. The general scheme of the ILS is given in Algorithm 1. The local search is conducted using 3 operators, Clustered Removal Reinsert (`CRR`), `Swap` and `2-OPT`. The outline of each operator is given in sub-section 2.4.2. In preliminary experiments, it was found that instead of cycling through each operator one after another, if we randomize the sequence of operators to be applied, the solution quality improved. Hence, we use a randomly generated parameter α to decide the sequence of application of the operators. The probability of selection of each operator is split evenly between the three operators. Additionally, every 10th cycle, the supply point reassignment operator `S-assign` is applied. The stopping criterion is set as 20 cycles of the ILS without improvement in the best solution. During preliminary experiments, it was noticed that extending the stopping criterion beyond 20 runs did not result in any significant improvement of the final solution. So, this stopping criterion was chosen.

Different initial solutions are generated for performing the local search by changing the vehicle capacity used in line 2 of Algorithm 1. Local search is run for 20 iterations. For each run, the capacity value is randomly picked between theoretical minimum capacity and actual vehicle capacity. Theoretical minimum capacity is calculated as $Q'_{run} = |N_d|/|V|$. In case of a fractional value, the number is rounded up. The solution with the best objective function is chosen among 20 runs as the final solution.

We now provide a detailed explanation of the ILS procedures.

Algorithm 1: ILS for the CHR-VRP

Input : $\mathcal{N}$ = Node Locations, $\mathcal{A}$ = Arcs
Output: Set of routes $\mathbf{R}$ = min($\mathbf{R}_{run}$)

```
repeat
    select Q'_run ∈ [mincap, Q]
    R_curr = Initialize (N, A, Q'_run)
    repeat
        α ← rand(0,1)
        if α ≤ 0.33 then
            Run CRR
        else if 0.33 < α ≤ 0.66 then
            Run Swap
        else
            Run 2OPT
        Run S-assign once every 10 cycles
    until no improvement in max(R_curr) for 20 cycles;
until 20 runs;
```

2.4.1 Generation of Initial Solutions

The generation of initial solutions is done by using a combination of two assignment problems. The first is a demand-supply assignment that links demand nodes with supply points. The other is a supply-vehicle assignment where the supply points are linked to the available vehicle fleet. For both assignment problems, we have used minimization of sum of travel times between demand and supply points and between customers and supply points respectively as the objective functions. This assignment serves as an input for generating good initial solutions. Based on which assignment is done first, we generate two different families of initial solutions. Out of the 20 runs mentioned in Algorithm 1, 10 runs use the initial solutions generated when the demand-supply assignment is done first and 10 runs use the initial solutions generated when the supply-vehicle assignment is done first. These two procedures are explained in detail below.

Demand first-Vehicle second

Demand-Supply Assignment. The aim of this procedure is to assign all demand nodes to supply point nodes in such a way that the sum of travel times is the least between linked nodes while meeting the demand. The MILP formulation of the problem is as given below. We define a binary variable u_{is} that takes value 1 if delivery node i is assigned to the supply point s. We also define an integer variable G_s which represents the number of vehicles that need to be assigned to the supply point s. The capacity of the vehicle is assumed to be Q'_{run}. The other parameters used in the formulation have the same definitions as given above. The problem is formulated as:

$$\text{minimize} \qquad \sum_{i \in N_d} \sum_{s \in N_s} u_{is} t_{is} \tag{2.16}$$

subject to:

$$\sum_{s \in N_s} u_{is} = 1 \qquad \forall i \in N_d \tag{2.17}$$

$$\sum_{s \in N_s} G_s \leq |V| \tag{2.18}$$

$$\sum_{i \in N_d} u_{is} \leq G_s \times Q'_{run} \qquad \forall s \in N_s \tag{2.19}$$

$$u_{is} \in \{0, 1\} \qquad \forall i \in N_d, \forall s \in N_s \tag{2.20}$$

$$G_s \in \mathbb{Z}^+ \qquad \forall s \in N_s \tag{2.21}$$

Objective function (2.16) minimizes the sum of travel times from all delivery nodes to the supply points the delivery node is assigned to. It can be noted that this objective function does not serve any other purpose here than to generate a good initial solution. In this context, (2.16) can be seen as a proxy objective function to the main objective of the problem, while not being computationally

expensive. Same applies to (2.22) as well. Constraints (2.17) ensure that a delivery node is assigned to a single supply point. Constraint (2.18) ensures that the number of vehicles assigned to each supply points do not exceed the total number of available vehicles. Constraints (2.19) ensure that the total number of delivery nodes assigned to a supply point does not exceed the total available capacity over all vehicles assigned to that supply point. Constraints (2.20) and (2.21) define the variable domains.

Vehicle-Supply Assignment. Following the demand-supply assignment, the next step is to assign vehicles to supply points. The assignment takes place in the same manner as the previous step, with the objective function trying to minimize the sum of travel times between vehicle depots and the supply points. The MILP formulation of the problem is given below. We define a binary variable z_{sv} that takes value 1 if vehicle v is assigned to supply point s. The other parameters used in the formulation have the same definitions as given above. The problem is formulated as:

$$\text{minimize} \qquad \sum_{s \in N_s} \sum_{v \in V} z_{sv} t_{sd(v)} \tag{2.22}$$

subject to:

$$\sum_{v \in V} z_{sv} = G_s \qquad \forall s \in N_s \tag{2.23}$$

$$\sum_{s \in N_s} z_{sv} = 1 \qquad \forall v \in V \tag{2.24}$$

$$z_{sv} \in \{0, 1\} \qquad \forall s \in N_s, \forall v \in V \tag{2.25}$$

The objective function seeks to minimize the sum of travel times between each vehicle-supply point pair. During demand-supply assignment, demand points are linked to supply points. When vehicle-

supply assignment is done, it has to be feasible in terms of capacity. If demand points are assigned to a supply point in the previous stage, enough vehicles have to be assigned to that supply point in this stage for the solution to be feasible. Constraint (2.23) ensures this is done. Each supply point can only be assigned as many vehicles as G_s, a value defined by the demand-supply assignment discussed above. Equation (2.24) ensures that each vehicle is assigned to one supply point. Constraints (2.25) define the variable domain.

Vehicle first-Demand second

In this case, formulation (2.22)–(2.25) is solved first without constraint (2.23). Then, formulation (2.16)–(2.20) is solved, without considering constraint (2.18) and where G_s represents the number of vehicles assigned to supply point s in the solution of the vehicle-supply assignment formulation (and, thus, it is not a decision variable).

Initial Vehicle Routes

Once demand nodes and vehicle are assigned to supply points, separate routes are created for all vehicles in the next step. So far we have demand points linked to supply points and also vehicles linked to supply points. The procedure to generate vehicle routes works as follows. We first create a giant route visiting all demand points assigned to a supply point. For this, we solve an Asymmetric Traveling Salesman Problem (ATSP) for each supply point, over the demand points assigned to the supply point considered. This entire route then has to be divided among all the vehicles that are assigned to the supply point. For this purpose, we divide the the route equally between all vehicles with each vehicle taking a part of the total route. This distribution is randomized at each supplier point between vehicles, i.e, a random vehicle already assigned to the supply point is chosen, then that vehicle is given the first portion of the route to serve and the procedure is iterated over all vehicles assigned to the sup-

ply point. The number of demand points assigned to a vehicle is such that all vehicles get roughly the same number of demand points.

We used the implementation of the Lin-Kernighan heuristic (LKH) as given in Helsgaun (2000) for the solution of the ATSP. In order to ensure that the initial solution is also in tune with the objective function used, we modify the travel times by setting to 0 the time from each delivery node to the supply point, as this is not accounted for in the objective function of the CHR-VRP.

2.4.2 Local Search

Once an initial solution is constructed, the local search is called to possibly improve it. It is based on three operators: Clustered Removal Reinsert (`CRR`), `Swap` and `2-OPT`. Each of the three operators has the same probability ($\frac{1}{3}$) of being applied. The sequence of application of operators is randomized using the parameter α as application of the operators in a random sequence provided better results than a deterministic sequence during initial tests. Note that each operator works on nodes associated with demand points only. We now describe them in detail.

Clustered Removal Reinsert (CRR)

The general outline of the Clustered Removal Reinsert (`CRR`) operator can be found in Algorithm 2. The idea behind the operator is to first identify a neighborhood surrounding a node defined by the parameter *threshold*, referred to as the *cluster* of a node. Any two nodes are considered to be in the same cluster if and only if the distance between the nodes is less than the threshold and they are both visited by the same vehicle. Then the operator selects the vehicle route with the longest duration, as this route determines the objective function value. From the selected vehicle route, clusters around each of the nodes visited by the vehicle is removed from the route and are reinserted into the route

Algorithm 2: CRR

Input : $\mathbf{R}_{\text{curr}}$

$\mathbf{P}_{\text{out}} \leftarrow$ longest route in $\mathbf{R}_{\text{curr}}$
$z_0 = z(\mathbf{R}_{\text{curr}})$
$\mathbf{R}_{\text{mod}} \leftarrow \mathbf{R}_{\text{curr}}$
foreach $i \in P_{out}$ **do**
    $\mathscr{N}_{\text{out}} \leftarrow$ `Cluster`(i,*threshold*)
    $\mathscr{C}_{\text{out}} \leftarrow \mathscr{N}_{\text{out}} \cap \mathbf{P}_{\text{out}}$
    $j \leftarrow$ node $\notin \mathbf{P}_{\text{out}}$ which is closest to i
    shortest $\leftarrow$ shortest route in $\mathbf{R}_{\text{curr}}$
    nearest $\leftarrow$ route visiting j
    $\mathbf{P}_{\text{s}}$ = *shortest* $+ \mathscr{C}_{\text{out}}$
    $\mathbf{P}_{\text{n}}$ = *nearest*$+$ insert $\mathscr{C}_{\text{out}}$ after j
    $\mathbf{P}_{\text{in}} \leftarrow \min(\mathbf{P}_{\text{n}}, \mathbf{P}_{\text{s}})$
    $\mathbf{P}_{\text{out}} \leftarrow \mathbf{P}_{\text{out}} - \mathscr{C}_{\text{out}}$
    $\mathbf{R}_{\text{mod}}$ = Update ($\mathbf{P}_{\text{in}}$, $\mathbf{P}_{\text{out}}$)
    $z = z(\mathbf{R}_{\text{mod}})$
    if $z_0 > z$ **then**
        $\mathbf{R}_{\text{curr}} \leftarrow \mathbf{R}_{\text{mod}}$
        $z_0 \leftarrow z(\mathbf{R}_{\text{mod}})$
return $\mathbf{R}_{\text{curr}}$

of another vehicle. The reinsertion could be either on the vehicle with the lowest trip length or on the vehicle that passes closest to the cluster that is removed. Among these two alternatives, the better option is chosen as the final destination, if there is an improvement in the objective function value. It should also be noted that the reinsertion preserves the visit sequence within the cluster from the original route.

Figure 1 presents an example of how the operator works. The square nodes represent vehicle nodes, the triangles represent supply points, and circles are demand points. The longest route is the route traveled by vehicle 1. In this route, nodes 5, 6 and 7 belong to the same cluster. Removing the entire cluster leads to an improvement in this route. These removed nodes are then reinserted in the same order as it was in the source route to the route that is closest to the removed cluster, which, in this case, is route 2. The reinserting position is the node in the destination route that is to closest to node 6, which, in this case, is node 10.

CRR is applied to each node of the longest route and the best removal is selected.

Swap

The general outline of the Swap operator is given in Algorithm 3. The route with the longest duration is selected. Each node in the selected route is swapped with its nearest node that belongs to a different route. Whenever the objective function improves, the modification is accepted. For the Swap operator, best improvement strategy and first improvement strategy were both tried. However, it was noticed that best improvement strategy did not greatly improve the solution quality, while the time taken was quite high. So, it was decided to continue with first improvement strategy for Swap in contrast to CRR which uses best improvement strategy.

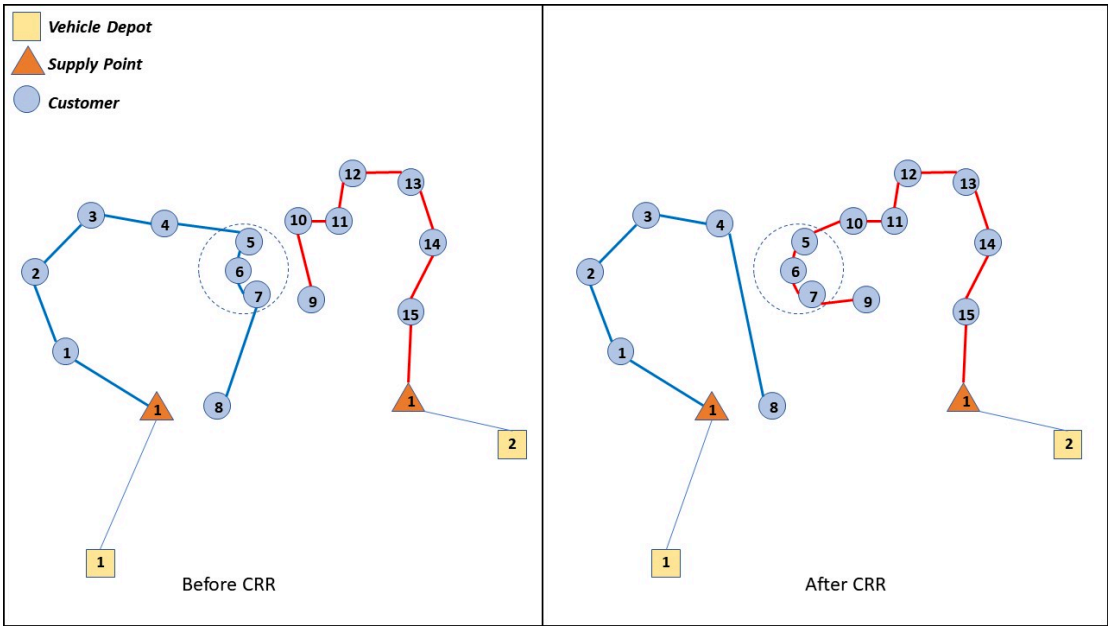


Figure 2.2: Example of Clustered Removal Reinsert Operator

2-OPT

The third operator is the classical `2-OPT` algorithm for the Traveling Salesman Problem, (Rosenkrantz et al. (1977)). It is applied to all routes. The operator iterates over all pairs of points and accepts all improvements.

Supply point reassignment

Every 10 cycles of the three operators described above, the supply point assigned to each vehicle is possibly changed with the aim of improving the objective function. The outline of the operator is given in Algorithm 4. For a given solution, for each route, the operator removes the first two nodes, i.e., the vehicle node and the supply point, and sorts the resulting routes (set $\mathscr{R}$) in descending order of length (line 1). The longest route is chosen first. The operator creates a list of all supply point-vehicle node combinations and calculates the distance between each supply point-vehicle node combination and

Algorithm 3: Swap

Input : $\mathbf{R}_{\text{curr}}$

$\mathbf{P}_{\text{out}} \leftarrow$ longest route in $\mathbf{R}_{\text{curr}}$
$z_0 = z(\mathbf{R}_{\text{curr}})$
for $i \in \boldsymbol{P}_{out}$ **do**
    $\mathbf{P}_{\text{in}} \leftarrow \mathbf{P}$ (neighbor(*i-node*) not in $\mathbf{P}_{\text{out}}$)
    for $j \in \boldsymbol{P}_{in}$ **do**
        $\mathbf{P}_{\text{out}} \leftarrow \mathbf{P}_{\text{out}} - i + j$
        $\mathbf{P}_{\text{in}} \leftarrow \mathbf{P}_{\text{in}} + i - j$
        $\mathbf{R}_{\text{mod}} = \text{Update}(\mathbf{P}_{\text{in}}, \mathbf{P}_{\text{out}})$
        $z = \max(\mathbf{R}_{\text{mod}})$
        if $z_0 > z$ ***or*** *mean(z_0) > mean(z)* **then**
            return $\mathbf{R}_{\text{mod}}$
            break
return $\mathbf{R}_{\text{curr}}$

the first demand point of the route under consideration. The vehicle node-supply point combination with the minimum distance to the first node is selected for that route. The operator is iterated over all routes. When the entire cycle is done, in case of an improvement, the modification is implemented.

2.5 Computational results

For conducting numerical analysis, random locations for vehicle nodes, supply point and delivery nodes were generated. The instance generation procedure is explained in the next subsection. A summary of instances can be found in Tables 2.1-2.4. Experiments were conducted by changing the value of the parameters one at a time as given in the table. As the algorithm is based on pseudo-random parameter α, in order to ensure repeatability, the value of α has been set so that any repetition of the entire ILS will always return the same solution, as given in the tables. The algorithm was coded in Python 3.7 and run in a computer with Ryzen 5 2600 @ 3.4 GHz and 16 GB of RAM. Cplex 12.10

Algorithm 4: `S-assign`

Input : $\mathbf{R}_{\text{curr}}, V, S$

$\mathscr{R} \leftarrow$ sorted($\mathbf{R}_{\text{curr}}$ starting from first delivery node)
$z_0 = z(\mathbf{R}_{\text{curr}})$
$\mathscr{V}^* = V$
for $r \in \mathscr{R}$ **do**
    $c_1 \leftarrow$ first demand node in r
    $cost = t_{d(v)s} + t_{sc_1} \; \forall v \in \mathscr{V}^*, \forall s \in S$
    v* $\leftarrow$ v with min($cost$)
    s* $\leftarrow$ s with min($cost$)
    $\mathscr{V}^* \leftarrow \mathscr{V}^* -$ v*
    $r \leftarrow \{d(v^*, s^*, r)\}\}$
$z = z(\mathscr{R})$
if $z_0 > z$ **then**
    return $\mathscr{R}$
else
    return $\mathbf{R}_{\text{curr}}$

was used for solving the formulations in sections 2.4.1 and 2.4.1.

The computational study's objective is to:

- Evaluate the performance of ILS by comparing the value of the solutions provided with the optimal solutions obtained by solving formulation (2.1)–(2.14) (Section 2.5.3).
- Test the effectiveness of the operators and procedures of the ILS (Section 2.5.4).
- Check the impact of problem parameters on the objective value (Section 2.5.5).
- Evaluate the effect of using different KPIs as objective function. (Section 2.5.2)
- Evaluate the benefit of crowdsourcing with respect to a centralized system. (Section 2.5.6)

2.5.1 Instance Generation

A total of four types of instances were used in the numerical analysis. Three categories of instances were created and one set of instance was based on real life data as used by Eisenhandler and Tzur (2019b). The latter are based on the operations of an Israeli organization 'Latet' which runs the "Food For Life" program that collects food donations from suppliers and distributes it across multiple NGOs. In these instances, there are 62 customers and 8 suppliers with a central depot. The instances are asymmetric and the distance matrix is given. The authors had used multiple instances in their paper by varying the supply and demand numbers, while keeping the geography of distribution the same. In this chapter's context, as we have assumed the demand across all customers to be the same and equal to one, the different instances reduce to just one instance- hereafter referred to as `Latet` instance.

In the remaining three classes of instances that were created, distances are calculated as Euclidean distances, thus they are symmetrical. Details are reported below.

The first set of instances were small size instances for which the solution obtained using MILP formulation and the solution obtained using the ILS were compared. The locations of all nodes were randomly generated using a uniform distribution in a 100×100 grid. Details of these instances are found in Table 2.1, where *ID* is the instance number.

The second set of instances are medium size instances. For these instances as well, delivery, vehicle and supply point locations were randomly generated in a 100×100 grid. These are listed in Table 2.2.

In the third set of instances, delivery locations were generated to simulate the scenario of a real-

Table 2.1: Small Instances

ID	$\lvert N_d \rvert$	$\lvert N_s \rvert$	$\lvert V \rvert$	Q
S1	10	2	3	4
S2	10	2	3	4
S3	10	2	3	4
S4	10	2	3	4
S5	10	2	3	4
S6	10	2	3	4
S7	10	2	3	4
S8	10	2	3	4
S9	10	2	3	4
S10	10	2	3	4
S11	20	2	4	6
S12	20	2	4	6
S13	20	2	4	6
S14	20	2	4	6
S15	20	2	4	6
S16	20	2	4	6
S17	20	2	4	6
S18	20	2	4	6
S19	20	2	4	6
S20	20	2	4	6

Table 2.2: Medium Instances

ID	$\lvert N_d \rvert$	$\lvert N_s \rvert$	$\lvert V \rvert$	Q
M1	100	2	6	17
M2	100	2	6	18
M3	100	2	6	19
M4	100	2	6	20
M5	100	2	6	21
M6	100	2	6	22
M7	100	1	6	20
M8	100	3	6	20
M9	100	4	6	20
M10	100	5	6	20
M11	100	2	5	20
M12	100	2	7	20
M13	100	2	8	20
M14	100	2	9	20
M15	140	3	9	20
M16	160	4	10	20
M17	180	4	11	20
M18	200	5	12	20

life city. A very simple way of classifying different real-life cities is as circular and semi-circular cities. Circular cities are usually found inland with a high density of population in the center and decreasing density of population as we move outwards. Examples of such cities include Paris, London and Delhi. Semi-circular cities are usually found next to coastline or other geographical boundaries that restrict population growth in one direction. Examples of such cities include Toronto, Sydney and Mumbai. The locations of demand nodes and vehicle nodes are generated in a 100×100 grid with the help of a two-dimensional Gaussian distribution. Figure 2.3 provides a comparison of heat map of cities taken from CARTO (2021) and how it is mimicked by the instances generated. Supplier locations were generated using two different methods. In the first method, locations are randomly located, subject to the condition that they are no less than 30 minutes away from each other. Without any loss of generality, in the current context, we assume that 1 minute is equivalent to 1 unit of distance. In the second method, we use a simple planning heuristic to symmetrically place supply locations, which is explained below.

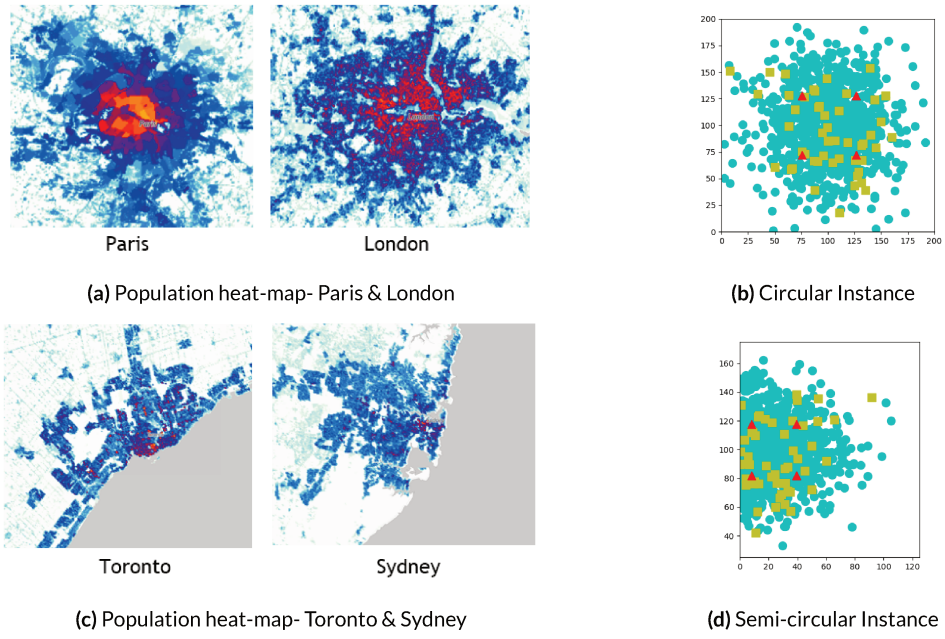
Table 2.3: Large Instances - Planned Supplier

ID	$\|N_d\|$	$\|N_s\|$	$\|V\|$	Q	City Shape
L1	1000	1	50	22	Circular
L2	1000	2	50	22	Circular
L3	1000	3	50	22	Circular
L4	1000	4	48	22	Circular
L5	1000	4	49	22	Circular
L6	1000	4	50	20	Circular
L7	1000	4	50	21	Circular
L8	1000	4	50	22	Circular
L9	1000	4	50	23	Circular
L10	1000	4	50	24	Circular
L11	1000	4	51	22	Circular
L12	1000	4	52	22	Circular
L13	1000	5	50	22	Circular
L19	1000	1	50	22	Semi Circular
L20	1000	2	50	22	Semi Circular
L21	1000	3	50	22	Semi Circular
L22	1000	4	48	22	Semi Circular
L23	1000	4	49	22	Semi Circular
L24	1000	4	50	20	Semi Circular
L25	1000	4	50	21	Semi Circular
L26	1000	4	50	22	Semi Circular
L27	1000	4	50	23	Semi Circular
L28	1000	4	50	24	Semi Circular
L29	1000	4	51	22	Semi Circular
L30	1000	4	52	22	Semi Circular
L31	1000	5	50	22	Semi Circular

Table 2.4: Large Instances - Unplanned Supplier

ID	$\|N_d\|$	$\|N_s\|$	$\|V\|$	Q	City Shape
L14	1000	1	50	22	Circular
L15	1000	2	50	22	Circular
L16	1000	3	50	22	Circular
L17	1000	4	50	22	Circular
L18	1000	5	50	22	Circular
L32	1000	1	50	22	Semi Circular
L33	1000	2	50	22	Semi Circular
L34	1000	3	50	22	Semi Circular
L35	1000	4	50	22	Semi Circular
L36	1000	5	50	22	Semi Circular

Supplier Location Planning We first sorted the demand points with respect to the x coordinates. Then 20^{th}, 50^{th} and 80^{th} percentile x coordinates are identified. Similar treatment is done with y coordinates. Consequently, we have a 3×3 grid on the geography that lies on the pair-wise intersection of these coordinates. Based on the number of suppliers, we pick the points on the grid for suppliers in such a way that a symmetry is maintained as much as possible. For instance, if there is only one supplier, we pick only the central point. If there are 5 suppliers, we pick the central point and four corner points. Figure 2.4 depicts the chosen location planning for circular and semi-circular instances, respectively. The numbers close to each node indicate the number of planned supply points for which the corresponding node is chosen. For example, in Figure 2.4(a), the node on the top-left corner is chosen in instances with 4 and 5 supply points, and not in instances with 1, 2 and 3 supply



(a) Population heat-map- Paris & London

(b) Circular Instance

(c) Population heat-map- Toronto & Sydney

(d) Semi-circular Instance

Figure 2.3: Creation of realistic large instances

points.

2.5.2 Comparison of objective functions

A comparison of performance of different objective functions gives some interesting insights. The aim of this section is to analyse what happens to the solution value in case a different objective function is used instead of the min-max objective function proposed. Indeed, one might argue that, in the specific context considered, it would be better to optimize over a different function. Thus, we compare the following four key performance indicators (KPI):

$$\textbf{KPI 1:} \qquad minimize \left[\max_{v \in V} \sum_{i \in N} \sum_{j \in N'} x_{ij}^{v} t_{ij} \right] \tag{2.26}$$

$$\textbf{KPI 2:} \qquad minimize \left[\sum_{v \in V} \sum_{i \in N} \sum_{j \in N'} x_{ij}^{v} t_{ij} \right] \tag{2.27}$$



(a) Circular city **(b)** Semi-circular city

Figure 2.4: Supplier Location Planning

$$\textbf{KPI 3:} \qquad \textit{minimize} \quad \frac{1}{|N_d|} \times \left[\sum_{m \in N_d} \sum_{v \in V'} \sum_{i \in N} \sum_{j \in N} f_{ij}^{mv} t_{ij} \right] \tag{2.28}$$

$$\textbf{KPI 4:} \qquad \textit{minimize} \left[\max_{v \in V} \sum_{i \in N} \sum_{j \in N'} x_{ij}^{v} t_{ij} - \min_{v \in V} \sum_{i \in d(v) \cup N_s} \sum_{j \in N'} x_{ij}^{v} t_{ij} \right] \tag{2.29}$$

KPI 1 is the objective function we adopted in the model. **KPI 2** on the other hand minimizes the sum of all travel times for each vehicle. This is also equivalent to minimizing the average trip length over all vehicles and is a standard measure in routing problems. **KPI 3** minimizes the average rescue time over all people. **KPI 4** represents an objective function that puts emphasis on the fairness of the rescue effort. This objective function minimizes the difference between rescue times of the last person and the first person to be rescued. The domain of summation of the second term in **KPI 4** is defined to take into account only two sets of arcs, between the depots and the supply point, and between the supply points and first customer. The tests are done by modifying the objective function of the formulation presented in Section 2.3. Results are summarized in Table 2.5 where the percentage loss of each KPI (with respect to the best KPI value) is reported, when optimizing over each KPI. The experiments were made on small instances 1-10 as **KPI 3** could not be solved to optimality for the other instances. It can be clearly seen that when optimizing for **KPI 2** and **KPI 3**, the losses in terms of **KPI 1** and **KPI 4** are quite large. Also, not only does **KPI 1** offer a better balance in terms of performance

on other KPIs as compared to **KPI 4**, a closer comparison between these two KPIs reveals the clear advantage that **KPI 1** has. Figure 2.5 shows the comparative performance between the two KPIs on all the small instances. It can be clearly seen that though optimizing over **KPI 4** ensures more equality (shorter time difference between first and the last person to be saved), **KPI 1** provides a much better overall performance in terms of lower time to reach out to both the first and last person to be rescued. This ensures that majority of victims will be better off when optimizing over **KPI 1**.

Table 2.5: Comparison of performance of different KPIs

	Optimizing KPI 1				Optimizing KPI 2			
Instance No	KPI 1	KPI 2	KPI 3	KPI 4	KPI 1	KPI 2	KPI 3	KPI 4
S1	0.0%	2.8%	11.1%	9.0%	15.4%	0.0%	13.4%	41.5%
S2	0.0%	0.0%	1.1%	42.1%	0.0%	0.0%	1.1%	42.1%
S3	0.0%	8.5%	12.5%	12.9%	6.1%	0.0%	0.0%	51.8%
S4	0.0%	0.3%	2.7%	20.4%	0.0%	0.0%	0.6%	20.4%
S5	0.0%	3.4%	18.0%	29.9%	45.2%	0.0%	14.7%	155.5%
S6	0.0%	8.2%	13.5%	48.9%	3.5%	0.0%	9.0%	33.8%
S7	0.0%	12.8%	16.2%	56.4%	0.9%	0.0%	0.0%	58.1%
S8	0.0%	0.0%	3.2%	35.7%	0.0%	0.0%	3.2%	35.7%
S9	0.0%	5.7%	10.1%	29.1%	23.2%	0.0%	7.6%	94.6%
S10	0.0%	0.0%	4.0%	51.0%	0.0%	0.0%	4.0%	51.0%
Avg Deviation		4.2%	9.2%	33.5%	9.4%		5.4%	58.5%
Max Deviation		12.8%	18.0%	56.4%	45.2%		14.7%	155.5%

	Optimizing KPI 3				Optimizing KPI 4			
Instance No	KPI 1	KPI 2	KPI 3	KPI 4	KPI 1	KPI 2	KPI 3	KPI 4
S1	32.2%	2.8%	0.0%	149.3%	41.8%	49.7%	76.2%	0.0%
S2	3.8%	1.4%	0.0%	49.5%	14.0%	20.2%	39.7%	0.0%
S3	6.1%	0.0%	0.0%	51.8%	26.9%	29.9%	36.9%	0.0%
S4	29.7%	13.4%	0.0%	86.6%	18.2%	22.8%	26.2%	0.0%
S5	18.6%	12.9%	0.0%	98.3%	26.9%	44.9%	69.0%	0.0%
S6	25.0%	8.0%	0.0%	125.3%	12.9%	28.5%	41.9%	0.0%
S7	0.9%	0.0%	0.0%	58.1%	40.8%	53.6%	71.9%	0.0%
S8	14.9%	6.3%	0.0%	60.8%	16.3%	25.6%	37.2%	0.0%
S9	24.0%	3.3%	0.0%	69.6%	35.8%	44.7%	60.1%	0.0%
S10	23.2%	1.5%	0.0%	65.8%	33.7%	35.5%	60.2%	0.0%
Avg Deviation	17.8%	5.0%		81.5%	26.7%	35.5%	51.9%	
Max Deviation	32.2%	13.4%		149.3%	41.8%	53.6%	76.2%	

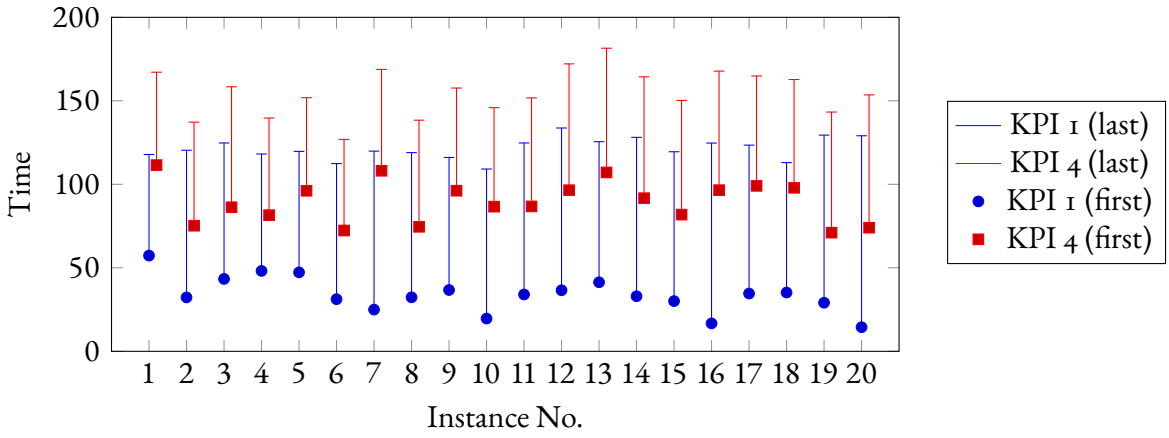


Figure 2.5: First and last person to be rescued **KPI 1** vs **KPI 4**

2.5.3 ILS Effectiveness

This section is devoted to compare the solutions provided by ILS with the ones obtained by solving the MILP formulation proposed in Section 2.3, called MILP from now on. MILP was solved through `Cplex 12.10` with a maximum computing time of one hour. Table 2.6 reports the results. The table reports in the last column the gap between the MILP and the ILS solution. Tests were conducted on small instances as MILP is not able to solve instances with more than 20 demand nodes. Computational times are reported in seconds. All instances with 10 demand nodes and one instance with 20 demand nodes were solved to optimality. For the remainder of instances with 20 demand nodes, the solution obtained after reaching the time limit of one hour is reported. It can be observed that, in most cases, ILS provides high quality solutions in a negligible computing time. There are certain instances where the deviation goes as high as 6%. However, this can be attributed to the fact that for smaller instances, the solution space is narrow and ILS may get trapped in local optima. Moreover, ILS is capable of substantially improving MILP solutions on some instances.

Table 2.6: Comparison of ILS vs MILP

ID	Best Run	Time to best ILS	Total Time ILS	Total time MILP	Best Result ILS	z value MILP	ILS Gap (%)	KPI 2 ILS	KPI 2 MILP	KPI 2 Gap (%)
S1	8	1.41	3.02	3.45	117.89	117.89	0.0%	338.92	338.92	0.0%
S2	1	1.06	3.40	3.99	122.06	120.42	1.4%	330.18	328.55	0.5%
S3	1	1.06	3.05	3.58	124.78	124.78	0.0%	356.30	356.30	0.0%
S4	1	1.07	3.08	3.77	118.17	118.17	0.0%	332.92	332.92	0.0%
S5	8	1.41	3.03	2.85	127.14	119.72	6.2%	343.20	322.24	6.5%
S6	13	2.80	3.27	3.22	114.52	112.44	1.8%	313.69	325.76	-3.7%
S7	1	1.06	3.19	3.49	120.95	119.90	0.9%	314.05	354.18	-11.3%
S8	1	1.06	3.21	3.80	118.98	118.98	0.0%	327.46	325.22	0.7%
S9	8	1.44	3.08	2.62	116.10	116.10	0.0%	339.36	339.36	0.0%
S10	1	1.08	3.24	2.96	109.16	109.16	0.0%	315.13	319.17	-1.3%
S11	1	1.13	4.39	3600	134.88	127.84*	5.5%	388.93	374.72	3.8%
S12	1	1.13	4.48	3600	133.73	134.15*	-0.3%	394.28	398.46	-1.0%
S13	8	1.97	4.50	3600	139.15	134.96*	3.1%	403.20	401.21	0.5%
S14	1	1.12	4.51	3600	128.13	145.01*	-11.6%	363.62	406.33	-10.5%
S15	11	3.32	4.43	3115	124.42	119.51	4.1%	352.19	338.05	4.2%
S16	3	1.39	4.80	3600	138.87	133.95*	3.7%	394.05	378.49	4.1%
S17	1	1.14	4.63	3600	129.04	140.23*	-8.0%	376.82	403.69	-6.7%
S18	1	1.11	4.37	3600	113.00	113.07*	-0.1%	334.20	318.05	5.1%
S19	8	1.96	4.48	3600	144.61	151.35*	-4.5%	417.70	437.10	-4.4%
S20	3	1.40	4.73	3600	142.15	134.08*	6.0%	420.70	374.76	12.3%
						Average	0.4%			-0.1%

* MILP did not solve to optimality within the time limit

2.5.4 Analysis of ILS performance

We now analyze statistics related to the performance of the ILS. The analysis is divided in two parts. We first focus on the efficiency of the operators used in the local search phase in Section 2.5.4. Then, we move to the analysis of the performance of the full ILS by providing statistics related to the effectiveness in improving initial solutions.

Operator Efficiency

We now study the operator efficiency, i.e., what is the benefit of using each operator described in Section 2.4.2. In particular, the operators analyzed are `CRR`, `2-opt`, `Swap` and `S-Assign`. Tests are made on medium instances and results are reported in Appendix Table A.1. For each operator, we report the

solution obtained when running ILS without it and the percentage gap with respect to the solution obtained by the full ILS. It is also worth noting that the operator that influences the solution time the most is CRR. Also, removing Swap offers the most savings without compromising on solution quality too much, if solution time needs to be cut down, as can be inferred from Appendix Table A.2.

We first note that one of the parameters of the algorithm is the closeness threshold used in CRR. The value of the closeness threshold is calculated as the average of distances to the closest neighbor for each demand node. This value has been chosen as this is a good indicator of the density of the instance. For a very dense instance, the nearest neighbor will be very close, leading to a low value of closeness threshold. Conversely for a sparse instance, the nearest neighbor for most demand nodes will be quite far away, leading to a higher value of closeness threshold. The calculated value ensures that a significant number of nodes will have at least 2 nodes in a cluster.

It can be seen that for a majority of instances, 2-OPT and CRR operators were the most effective. On an average, for 18 instances studied, CRR had an average improvement percentage of 8.5% and 2-OPT had an average improvement of 9.1%. It can be seen that in certain instances where the demand is exactly equal to the capacity available, CRR is not useful at all. In such cases, all routes will be at full capacity and the reinsertion in CRR will not work because of capacity restrictions. It is also worth noting that if the vehicle number is increased to the point of having high amount of excess capacity, the problem becomes simpler as a whole (with routes becoming shorter and with fewer stops) and the efficiency of the operators decreases. The Swap operator had an average improvement close to 0%. However, in certain instances, (eg. M8 and M12), Swap gives a considerable boost to the quality of solution obtained. Moreover, in some cases, the use of Swap results in the heuristic getting trapped in a local optima (M3- M6). However, as a whole, the benefits of including Swap outweighs any potential losses, as indicated by the maximum savings and losses, hence it was decided to retain the operator.

The S-assign operator showed an average efficiency of 1%. However, as the operator did not negatively affect any solution and even showed improvement as high as 6.8% in certain instances, it was retained. The effectiveness of all operators means that they all contribute to the effectiveness of the ILS.

ILS efficiency

We now move to the analysis of the ability of ILS to improve upon initial solutions. For evaluation of the heuristic, the best run start value given in Tables 2.7 and 2.8 can be considered as a benchmark for comparison. The best value obtained is also reported for both medium as well as large instances. In particular, we focus on the number of runs needed to find the best solution and on the average starting value of each run when compared with the starting value of the best run. This helps us understand if the local search efficiency is heavily dependent on the quality of initial solution. Tests are made on medium and large instances.

Number of runs: For medium instances, as evident from Table 2.7, 8 out of 18 instances had the best run later than 10. Also, for large instances, as evident from Table 2.8, the best run is well distributed between 1 and 20. For roughly half of the instances, the best result is achieved in later than 10 runs. There is also a significant proportion of instances where the best result has been achieved later than 15 runs (7 out of 36). This justifies preserving the current number of runs as given in the heuristic given that the computational time is reasonable. Additionally, for large instances, we also test the difference in final result due to reduction in total runs from 20 to 10. Table 2.8 columns 4 and 5 report the best result generated using 20 runs and 10 runs respectively. It can be seen that on an average the improvement due to having 20 runs is around 1.2%. In cases where it is needed to reduce the computational time, especially for large instances, reducing the total runs to 10 is a pragmatic op-

tion, drastically reducing the computational time.

Impact of initial solutions: While looking at the improvements (over all 20 runs), two different pictures emerge from studying large and small instances. For large instances, it can be seen that in approximately 30% of cases, the best run had a worse starting point than the average and in 70% of cases had a better starting point than the average. For medium instances, the best run had a better starting solution than average in 15 out of 18 instances. Two conclusions can be derived from this observation. First, because there is a significant proportion of instances where the best run got a better solution despite starting from a worse initial solution as compared to the average starting point, we can conclude that the initial solution is not the only factor that determines the quality of the final solution. Second, because there is also a significantly high proportion of instances where the best run started from a better initial solution than the average starting point, the value of initial solution does have an effect on the quality of the final best solution, especially in smaller instances. The observed numerical results make a strong case for retaining the current structure of the heuristic.

2.5.5 Managerial levers: Impact of problem parameters

When it comes to deriving managerial insights, we analyse the effectiveness of different levers at the disposal of the crisis manager. In the following, we study the impact of vehicle number, vehicle capacity and supply point number for both random instances and realistic instances. The first two parameters are out of the control of the crisis manager. Thus, the aim here is to see their effect on solution values in order to compare the effect of having a higher number of vehicles or larger vehicles, which can reach out to more people. This might help crisis manager in detecting actions to increase availability of drivers or search for larger capacity vehicles. The last parameter instead, the number of suppliers, is definitely a lever on which crisis manager might have some sort of control. In addition, for realistic

Table 2.7: Statistics on best run - Medium instances

ID	Best Run	Best Result	Time to Best	Total Time	Average Start Value	Best Run Start Value	Av Start > Best Start
M1	13	184.01	46.63	71.42	232.50	219.33	Yes
M2	13	173.50	53.05	91.55	232.50	219.33	Yes
M3	13	172.71	53.55	83.28	227.60	213.15	Yes
M4	13	172.71	55.50	88.10	227.60	213.15	Yes
M5	13	172.71	56.07	89.32	227.60	213.15	Yes
M6	13	172.71	57.77	92.10	227.60	213.15	Yes
M7	8	195.02	31.43	85.37	242.48	238.15	Yes
M8	2	164.15	8.50	75.27	213.12	220.93	No
M9	2	172.40	8.66	78.33	193.39	193.39	No
M10	3	176.28	11.19	71.68	190.57	189.83	Yes
M11	11	214.61	22.50	39.78	245.02	245.81	No
M12	2	147.68	15.62	74.95	209.76	185.27	Yes
M13	10	147.30	33.52	59.64	218.05	187.70	Yes
M14	4	139.90	9.74	50.38	213.15	185.21	Yes
M15	7	147.36	39.22	124.16	181.51	174.09	Yes
M16	10	135.10	66.91	143.73	182.02	171.13	Yes
M17	4	139.33	31.76	188.47	171.14	167.27	Yes
M18	19	126.35	236.76	247.16	174.70	172.31	Yes

instances, we also consider the lever related to the possibility of planning the location of suppliers.

Increasing the number of crowdsourced vehicles: Figure 2.6(a) and Figure 2.7(a) show the variation of objective function over the number of crowdsourced vehicles for both medium and large instances respectively. It can be seen that in terms of magnitude, increasing the vehicle number has the biggest effect on the value of objective function for random instances. In case of large instances (realistic instances), the effect is a bit less pronounced. This might be due to the increased magnitude of the instances. As expected, the number of vehicles play an important role when the number of crowdsourced drivers is close to the minimum needed to serve the demand points. If disaster relief is done in multiple successive cycles based on the amount of calls received over time, in a more practical sense, this gives the crisis manager some idea about the ideal number of vehicles to be allocated to one single rescue cycle. If more vehicles are available at his/her disposal, the manager can decide to hold deployment of some vehicles to a later cycle where the demand might be more. Additionally, when a

sufficient number of vehicles are available for the disaster relief, any extra vehicle can be deployed in the mobilization phase where commodities are to be gathered at the supply points, ready to be distributed in the next rescue cycles.

Increasing the vehicle capacity: The effect of vehicle capacity on the value of objective function can be seen in Figure 2.6(b) and Figure 2.7(b) for random and realistic instances, respectively. The benefits of increasing vehicle capacity is quite significant at the minimum possible capacity. However, this effect plateaus off very soon with almost negligible benefits after a certain point. The observed effect is same for both random and realistic city instances. This means that vehicle capacity is not a significant lever once the capacity is slightly above the minimum needed. This is intuitive when considering the objective function: a larger capacity is not exploited when the objective is to minimize the maximum time for reaching the persons in need, as routes tend to be short (thus including few stops).

Increasing the number of supply points: Figure 2.6(c) and Figure 2.7(c) show the variation of objective function over the number of supply points for both random and realistic instances, respectively. It can be noted that increasing the number of supply points has a very reliable effect in improving the value of the objective function for both types of instances. A comment has to be made on random instances. In fact, the average value associated with 3 supply points is lower than the one associated with 4 and 5 supply points, even if by a small amount. This is due to the heuristic nature of the solution approach proposed. Anyway, the effect is not as high as increasing vehicle number in random instances. For realistic instances however, increasing the number of supply points holds an edge over increasing number of vehicles. As expected, establishing multiple supply points has a clear advantage over having a single central supply point. This can also act as a risk mitigation strategy in case one or more supply points get disrupted during the crisis. This lever is all the more interesting, as during the mobilization phase, increasing the number of supply points can ensure that the collection

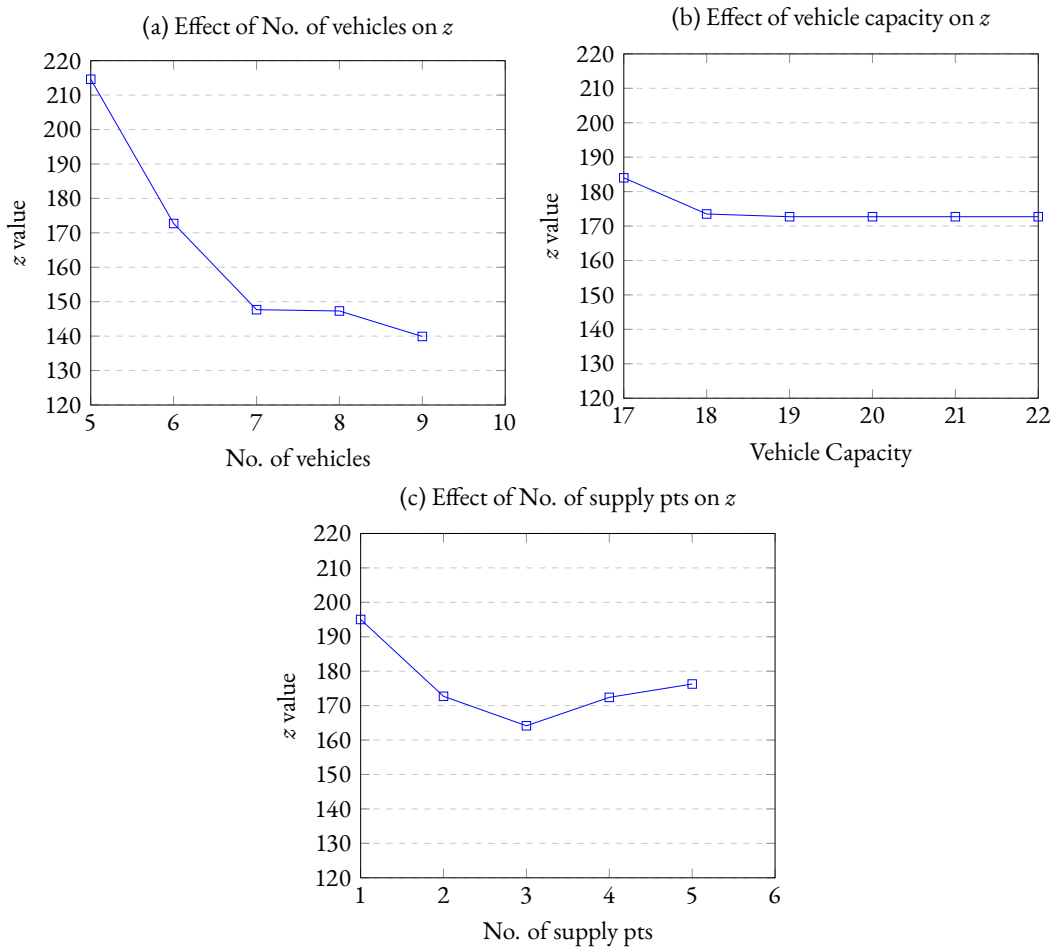


Figure 2.6: Variation of z on random instances

of resources to supply points can be done more efficiently. When combined with the advantages that an extra supply point provides during the rescue phase, focusing on increasing supply points is an effective measure to be considered.

Planning supply point location: In case of realistic instances, Figure 2.7 (d) compares the value of objective function when the location of supply point is planned to be geographically distributed with the value of objective function when the location of supply point is randomized. However, ran-

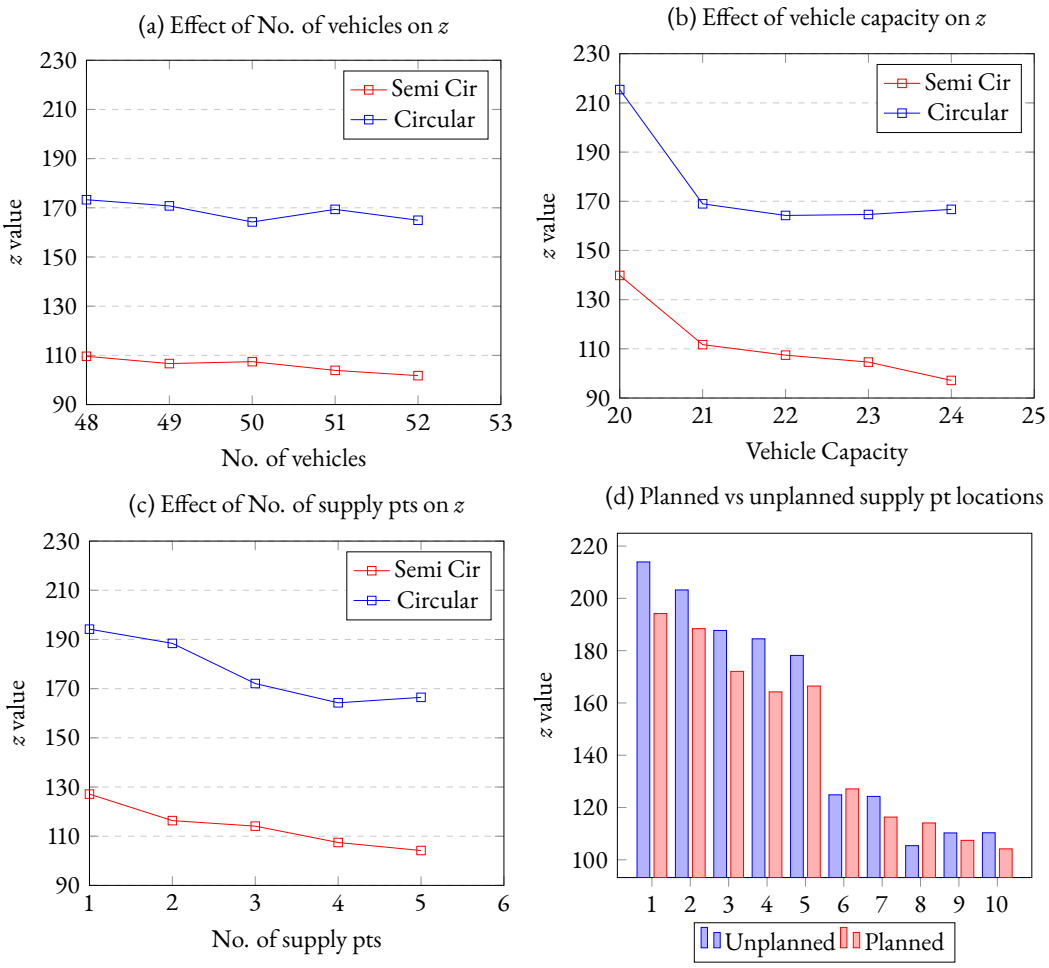


Figure 2.7: Variation of z on realistic instances

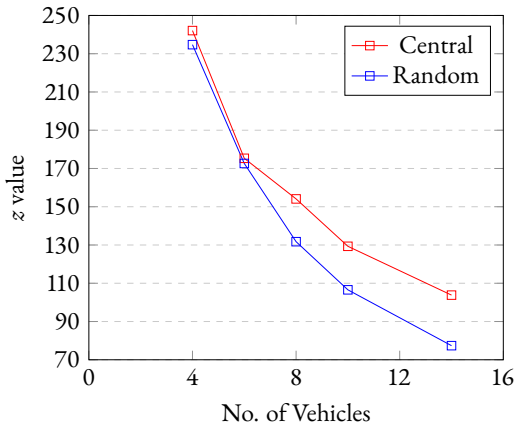


Figure 2.8: Variation of z with Vehicle count- `Latet` instances

domization is subject to the condition that no two supply points are within a minimum distance from each other, which is a realistic assumption. In most of the instances, we can see that planning supply location holds a considerable advantage over having random supply locations. However, this effect is more prominent for circular instances than semi circular instances. This can be attributed to the fact that the planning strategy suggested in previous section is inherently advantageous for cities that are symmetrical in both directions. For a semi circular city, there is a very high concentration of demand points along the edge (or a coastline). It is quite possible that a better optimized supplier planning strategy might offer slight improvements in rescue times.

2.5.6 Advantage of crowdsourcing over centralized depot

A crucial element of the study is to understand the impact of crowdsourcing over a centralized depot system. For this purpose, experiments were carried out on the medium instances as well as the `Latet` instance. For representing a centralized scenario, for the medium instances, we assume that each of the supply points also act as vehicle depots. In this case, the number of available vehicles will be lower, al-

beit with a higher capacity. The details of instances are given in Table 2.9. It can be clearly seen that in this comparison, crowdsourcing has a clear advantage over a centralized scenario. A critical feature is the number of vehicles available. Clearly, the picture would have been different with a larger number of vehicles available in each supply point. However, it is quite reasonable to assume that, on practical scenarios, we have a number of crowdsourced drivers which is larger than the number of dedicated vehicles available in each supply point, although having a smaller capacity.

Though the original `Latet` instance just has a central depot, for experiments with crowdsourcing setting, we generate random locations which act as vehicle depots and compare the two. In the centralized setting, all vehicles start from a central depot as given in the original instance. In a crowdsourced scenario, each vehicle starts from a random point across the geography. Table 2.10 gives the variation of the objective function on changing the count of vehicles while keeping the overall capacity to be fairly constant for the `Latet` instance. It also gives the improvement in percentage while using a crowdsourced system. It can be seen that for very low vehicle counts, the effects of crowdsourcing are not that evident. However, as the vehicle count increases the effect of crowdsourcing is more and more prominent. This further strengthens the argument for utilizing crowdsourcing for large scale humanitarian operations.

2.6 Conclusions

Public involvement in rescue efforts during crises is the next logical step in improving response times in such situations. Crowdsourcing of rescue efforts has been shown as an effective tool for implementing this in practice. This chapter establishes a simple, workable model in this context and suggests an efficient heuristic that can solve instances in a reasonable time. An extensive computational study is

performed to verify the effectiveness of the heuristic and to identify managerial levers at the disposal of the crisis manager.

It can be argued that the assumption that all demand can be fulfilled, that is considered in this model, is a simplifying assumption. However, here we assume that the planning is done in multiple rounds and in each round of rescue efforts, we take into account only as many demand points as there is available capacity, on a first come first served basis. An interesting future research direction as a result of this assumption would be how we could prioritize over all the demand points. In most of the rescue efforts, specific vulnerable groups can be identified (for instance, children and senior citizens). In such a case, the planning of rescue efforts should include priorities and it should be reflected in the objective function. Another natural extension of this idea would be when we consider a mixed model, where crowdsourcing and centralized distribution are simultaneously present. In this scenario, an additional decision of which demand points need to be served by which mode, also comes into play, which makes the problem complicated, yet interesting. Also, a further interesting avenue of development would be to explore the model from a dynamic and stochastic stand point. It is possible that more emergency calls might come in during a rescue run and it would be interesting to incorporate this into the system.

Table 2.8: Statistics on best run - Large Instances

City Shape	Supplier Planning	Instance ID	Best in 20 runs	Best in 10 runs	Time for 20 runs	Time for 10 runs	Best Run	Average Start Value	Best Run Start Value	Avg start > Best start	Gap 10 vs 20 runs
Circular	Planned	L1	194.19	194.19	8734.18	4581.28	4	403.56	438.82	No	0.00%
		L2	188.41	189.91	6855.8	3483.51	20	377.59	403.17	No	0.80%
		L3	172.08	172.08	7203.36	3471.78	13	331.54	308.54	Yes	0.00%
		L4	173.28	177.48	8626.21	4442.3	19	338.8	378.8	No	2.40%
		L5	170.78	172.55	9847.15	4451.05	20	349.85	381.49	No	1.00%
		L6	215.42	215.42	315.49	157.37	3	308.72	298.3	Yes	0.00%
		L7	168.97	168.97	7734.04	3995.97	5	308.72	315.88	No	0.00%
		L8	164.24	164.24	8088.67	3944.57	11	335.67	308.45	Yes	0.00%
		L9	164.63	167.78	7963.95	3586.95	7	335.67	290.42	Yes	1.90%
		L10	166.71	167.46	8151.17	3755.77	7	335.67	290.42	Yes	0.40%
		L11	169.37	169.37	7621.01	3881.4	11	334.82	306.56	Yes	0.00%
		L12	164.95	169.76	8601.75	4338.88	10	334.02	297.7	Yes	2.90%
		L13	166.46	166.46	10069.35	5399.14	3	358.77	331.52	Yes	0.00%
	Unplanned	L14	213.93	218.43	8683.06	4426.95	6	447.54	403.24	Yes	2.10%
		L15	203.22	203.22	7195.26	3409.05	1	455.86	316.18	Yes	0.00%
		L16	187.73	187.73	7445.48	3674.98	5	397.83	425.28	No	0.00%
		L17	184.52	186.72	7190.32	3916.85	18	397.77	427.02	No	1.20%
		L18	178.17	180.95	7250.5	3345.43	6	353.62	307.2	Yes	1.60%
Semi	Planned	L19	127.12	129.86	7898.2	3811.98	6	230.21	276.23	No	2.20%
		L20	116.34	119.18	6049.27	2793.29	7	202.24	202.52	No	2.40%
		L21	114.1	115.8	6381.12	3109.68	9	200.62	181.76	Yes	1.50%
		L22	109.63	109.63	7395.78	3683.2	13	180.1	170.95	Yes	0.00%
		L23	106.66	107.94	6814.44	3273.97	16	191.06	194.63	No	1.20%
		L24	139.85	139.85	309.73	141.62	1	189.63	184.19	Yes	0.00%
		L25	111.72	111.72	7942.26	3760.22	12	189.63	181.62	Yes	0.00%
		L26	107.44	107.44	6496.74	3381.4	4	189.52	188.8	Yes	0.00%
		L27	104.62	104.62	5889.21	3209.69	4	189.52	188.8	Yes	0.00%
		L28	97.16	102.1	5948.55	3003.45	8	189.52	184.79	Yes	5.10%
		L29	103.9	110.65	6649.64	3288.27	16	192.86	185.88	Yes	6.50%
		L30	101.76	105.04	6886.62	3748.75	17	194.52	189.58	Yes	3.20%
		L31	104.18	109.54	6677.72	3155.83	10	186.63	199.76	No	5.10%
	Unplanned	L32	124.86	124.86	7700.52	3789.35	4	226.93	223.31	Yes	0.00%
		L33	124.25	126.34	6534.71	3541.41	9	224.28	222.2	Yes	1.70%
		L34	105.41	105.41	7305.52	3374.58	14	196.51	178.88	Yes	0.00%
		L35	110.28	110.28	8205.16	4393.43	14	186.81	178.88	Yes	0.00%
		L36	110.34	110.34	7017.86	3168.25	1	184.47	172.41	Yes	0.00%

Table 2.9: Comparison of Centralized and Crowdsourced systems

		Centralized			Crowdsourced		
Instance No	Customers	Vehicles	Capacity	z value	Vehicles	Capacity	z value
M1	100	2	51	428.92	6	17	184.01
M2	100	2	52	425.48	6	18	173.50
M3	100	2	53	421.41	6	19	172.71
M4	100	2	54	414.76	6	20	172.71
M5	100	2	55	407.89	6	21	172.71
M6	100	2	56	400.23	6	22	172.71
M8	100	3	35	276.80	6	20	164.15
M9	100	4	30	202.58	6	20	172.40
M10	100	5	25	182.72	6	20	176.28
M11	100	3	35	268.40	5	20	214.61
M12	100	4	30	221.06	7	20	147.68
M13	100	5	25	176.81	8	20	147.30
M14	100	6	20	160.92	9	20	139.90
M15	140	3	50	316.07	9	20	147.36
M16	160	4	45	259.75	10	20	135.10
M17	180	4	50	258.97	11	20	139.33
M18	200	5	45	229.26	12	20	126.35

Table 2.10: Results from `Latet` instance

Instance No	Demand	Suppliers	Vehicle count	Capacity	Central Depot Obj Value	Crowdsourcing Obj Value	Gap
LATET_2808_1	62	8	4	20	242.09	234.73	3.04
LATET_2808_2	62	8	6	14	175.32	172.62	1.54
LATET_2808_3	62	8	8	10	154.09	131.73	14.51
LATET_2808_4	62	8	10	8	129.32	106.56	17.60
LATET_2808_5	62	8	14	6	103.8	77.37	25.46

3

Exact solution approaches to the crowdsourced humanitarian relief vehicle routing problem

joint work with **Claudia Archetti** and **Ivana Ljubić**

3.1 Introduction

We further study the same problem that we had introduced in Chapter 2. For the sake of completeness, we provide a brief description of the circumstances leading to the problem at hand. In a post-disaster scenario, a set of volunteers with vehicles are ready and waiting for instructions. The volunteers would start from their location, visit a supply point to pick up relief kits and then visit the people in need one after the other. The people in need of the supplies are dispersed over a geography. It is assumed that in a given cycle (volunteers making one trip of rescue), the volunteers cater to only as many people as there is capacity. We also assume that there is enough supply of relief kits to meet all demand. Each relief kit may contain multiple items like drinking water, medicines, food, batteries etc. This helps simplify the problem to have just one commodity in consideration. For tractability, it is assumed that the vehicles are homogeneous and have a finite capacity. The success of the relief effort is expressed using the objective function which we term as *min-max rescue time* - in other words, minimizing the time required to reach out to the last person in need. An alternative objective function suggested in the previous chapter, average rescue time, is also considered - where the average of the times required to reach out to each person in need is minimized. In this chapter, we extend the study of the problem by suggesting multiple possible formulations as well as exact solution approaches for the CHR-VRP using both objective functions and comparing performance of different models.

The contributions of the chapter are the following. First we develop MILP formulations for the CHR-VRP. We suggest two separate variants of the min-max rescue time model (disaggregated and aggregated) and study their features and performance to reveal some interesting insights. We also suggest two alternative ways of expressing the objective function for the disaggregated model. The suggested disaggregated model makes it possible to study the effects of a Benders decomposition method as well. Second, we introduce global lower bounds and related valid inequalities and study their ef-

fect on the performance of the models. Third, we study and compare three separate exact solution methods; solving the compact formulation of the problem, a branch-and-cut method using Generalized Subtour Elimination Constraints (GSECs) and finally the decomposition-based method. We also study the effects of tightening some of the coefficients of the Benders cuts that we derive. We further study the problem using a different objective function based on the average rescue time. For this variant of the problem as well, we study the development of tailored Benders decomposition and we suggest valid inequalities. We compare and comment on the best performing method in both cases.

The rest of the chapter is organized as follows. Section 2 presents a review of the relevant literature pertaining to the problem. Section 3 delves deep into the different models used in each of the approaches and also establishes some valid bounds to help solve the problem. Section 4 lists down the computational results from the study and examines the computational insights. Section 5 summarizes the findings and concludes the chapter.

3.2 Literature Review

As mentioned above, the CHR-VRP has been recently introduced in the literature by Parappathodi and Archetti (2022). The authors propose a mathematical formulation of the problem and a heuristic algorithm, namely an iterated local search. They also propose a set of benchmark instances on which they perform tests to gather managerial insights. A review of problems that share similarities with the CHR-VRP is presented below.

CHR-VRP belongs to the family of vehicle routing problems (VRP). Toth and Vigo (2014) provides a summary of the different classifications of VRP. The review of relevant literature can be broadly

classified into four distinct categories. The first stream of research deals with humanitarian logistics. With the increased frequency and severity of natural and man made disasters, humanitarian logistics has been an intensely researched topic. Özdamar and Ertem (2015) provides a thorough survey of literature pertaining to humanitarian logistics. For a broader study on humanitarian supply chain in general, including disaster preparedness and recovery, we refer to Behl and Dutta (2019). One of the recent examples of research in the area of humanitarian logistics is Eisenhandler and Tzur (2019b) where the authors consider the operations of a food distribution system based out of Tel Aviv and Houston.

The second stream of research deals with crowdsourcing and its application in vehicle routing problems. Crowdsourcing as a business concept has been gaining momentum after the turn of the century with the advancements in communication and popularity of gig economy. Crowdsourcing has particularly been effective in last mile delivery systems. Archetti et al. (2016a) introduced crowdsourcing to routing problems in the form of occasional drivers. Alnaggar et al. (2021) propose a recent survey of literature on the use of crowdsourcing in last mile delivery operations. Another application of crowdsourcing has been with the dynamic ride sharing problem. Lee and Savelsbergh (2015) examines this situation and uses a sequential insertion heuristic for computational study. One of the more recent application of crowdsourcing in the context of routing problems is in Santini et al. (2022), where the authors study a probabilistic travelling salesman problem with crowdsourcing.

The third stream of research deals with multi-depot vehicle routing problems (MDVRP) and its variations. MDVRP typically has multiple identical depots from which vehicles start and then proceeds to demand nodes one after the other. Montoya-Torres et al. (2015) provides a comprehensive review of multi depot vehicle routing problem (MDVRP). An interesting variation of MDVRP is the multi depot open vehicle routing problem (MDOVRP). In this problem variation, the vehicles

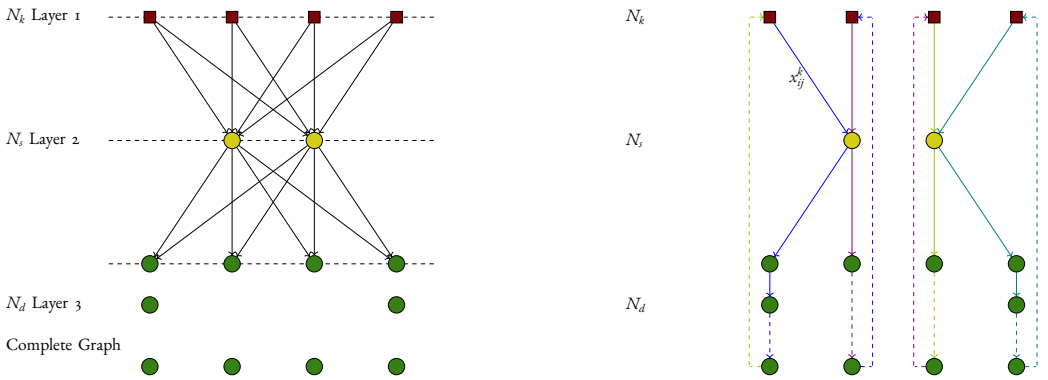
do not have to end in the depot itself. The CHR-VRP differs from the MDOVRP in two key aspects. First, the existence of supply points (which can be visited by multiple vehicles) as the first stop right after the depot makes the routing decision quite complex. Second, the objective function is a min-max objective function.

One of the exact solution methods explored in this chapter is based on Benders Decomposition. The method introduced by Benders (1962) has been extensively applied in network based problems. Rahmaniani et al. (2017) provides a literature survey on the applications of Benders Decomposition in various contexts. A recent work on routing problems involving drones has been done by Alfandari et al. (2022). In the aforementioned paper, the authors explore a customized Benders decomposition method based on the properties of the problem. The origins of the decomposition method used in our paper is similar to Alfandari et al. (2022) in that we use the properties of the problem to entirely skip the need to compute the coefficients of optimality cuts using a linear program - instead, we provide a combinatorial procedure for it. More details about this approach are given in the relevant section.

3.3 Problem Formulation

The CHR-VRP is defined on a complete graph $G = (N, A)$. K denotes the set of all the vehicles. The node set N comprises of three subsets of nodes represented by the three layers as given in depicted in Figures 3.1a and 3.1b. N_k denotes the set of all depot nodes (Layer 1), N_s (Layer 2) denotes the set of all supplier nodes and N_d denotes the set of all demand nodes (customers) (Layer 3). Each vehicle is denoted by the index $k \in K$, and the depot of vehicle k is denoted by $d(k)$, for all $k \in K$. There are people in need of supplies at each of the demand nodes N_d, who can be served using the materials that can be picked up at the supply points at the nodes N_s. For convenience, we also define a subset

$N_{sd} = N_s \cup N_d$. There are vehicles belonging to volunteers located at $d(k)$, ready to participate in rescue efforts, and waiting for instructions (route plan). The volunteers start from their depot, visit one of the supply points (given by the arc from Layer 1 to Layer 2 in Fig 3.1b) and then visit the victims located at N_d one by one, dropping off one unit of material at each demand point. We assume that each supply point has enough materials to serve all customers. We also assume that each of the vehicles have a finite known capacity Q. The time required to travel from node i to node j is denoted by $t_{ij} > 0$ which is known apriori. There are two objective functions explored in this chapter based on previous work. The first objective function, that we call `min-max rescue time` minimizes the time required to reach out to the last person in need. The second objective function `average rescue time` minimizes the average of all the times required to reach out to the demand nodes. The key difference between the objective functions is the fact that for `average rescue time`, all the demand nodes are taken into account when calculating the performance metric as opposed to the `min-max rescue time` which has only the worst rescue time being included in the objective function. The comparison of effectiveness of these objective functions has already been discussed in the previous chapter.



(a) Input Graph **(b)** Feasible Solution

Figure 3.1: Model Configuration

A summary of the used notations is given below:

- N_k: Set of depot nodes (Layer 1)
- N_s: Set of supplier nodes (Layer 2)
- N_d: Set of demand nodes (Layer 3)
- K: Set of vehicles
- $d(k)$: Depot of vehicle $k \in K$
- Q: Vehicle capacity
- N_{sd}: $N_s \cup N_d$
- N: $N_s \cup N_d \cup N_k$
- $A = \{(i,j) : i,j \in N\}$: Set of all arcs between Layer 1 & Layer 2, between Layer 2 & Layer 3 and complete graph within Layer 3 as per Fig 3.1a
- $G = (N, A)$: The graph on which the problem is defined

We consider the objective functions one after the other. First we explore different possible model formulations using the `min-max rescue time` objective function. Within the `min-max rescue time` model, we develop two variants of formulations. The first variant is based on a disaggregated approach where a vehicle index is associated with arc variables. The second variant is based on an aggregated approach where the vehicle index is absent. We now present these two modelling approaches.

3.3.1 The min-max rescue time problem

The objective of the min-max rescue time problem as stated above is to minimize the time required to reach out to the last person to be rescued. Hence we have the first variant of the model:

Disaggregated *z-x* model ($\mathtt{D}_{z-x}$)

The mathematical formulation of the disaggregated version for CHR-VRP makes use of the following decision variables:

$$x_{ij}^k = \begin{cases} 1 & \text{if arc } (i,j) \text{ is traversed by vehicle } k \\ 0 & \text{otherwise} \end{cases} \qquad \forall (i,j) \in A, \forall k \in K \quad (3.1a)$$

$$w_i^k = \begin{cases} 1 & \text{if node } i \text{ is visited by vehicle } k \\ 0 & \text{otherwise} \end{cases} \qquad \forall i \in N, \forall k \in K \quad (3.1b)$$

$$f_{ij}^{mk} = \begin{cases} 1 & \text{if vehicle } k \text{ traverses } (i,j) \text{ to serve node } m \\ 0 & \text{otherwise} \end{cases} \qquad \forall (i,j) \in A, \forall m \in N_{sd}, \forall k \in K \quad (3.1c)$$

$$z = \text{the time required to reach out to the last person in need} \quad (3.1d)$$

The problem is then formulated as the following MIP, which we will refer to as ($\mathtt{D}_{z-x}$):

$$\text{minimize} \qquad z \quad (3.2a)$$

subject to

$$\sum_{i \in N} x_{ij}^k - \sum_{l \in N} x_{jl}^k = 0 \qquad \forall j \in N, \forall k \in K \quad (3.2b)$$

$$\sum_{i \in N} x_{ij}^k = w_j^k \qquad \forall j \in N, \forall k \in K \quad (3.2c)$$

$$\sum_{k \in K} w_j^k = 1 \qquad \forall j \in N_d \qquad (3.2d)$$

$$\sum_{j \in N_s} x_{d(k)j}^k \leq 1 \qquad \forall k \in K \qquad (3.2e)$$

$$w_i^k \leq w_{d(k)}^k \qquad \forall i \in N_{sd}, \forall k \in K \qquad (3.2f)$$

$$x_{ij}^k + x_{ji}^k \leq w_i^k \qquad \forall i \in N, \forall j \in N, \forall k \in K \qquad (3.2g)$$

$$\sum_{i \in N} w_i^k \leq Q + 2 \qquad \forall k \in K \qquad (3.2h)$$

$$\sum_{(i,j) \in A} f_{ji}^{mk} - \sum_{(j,i) \in A} f_{ij}^{mk} = \begin{cases} -w_m^k & \text{if } i = d(k) \\ w_m^k & \text{if } i = m \\ 0 & \text{otherwise} \end{cases} \qquad \forall k \in K, \forall m \in N_{sd}, \forall i \in N \qquad (3.2i)$$

$$0 \leq f_{ij}^{mk} \leq x_{ij}^k \qquad \forall (i,j) \in A, \forall m \in N_{sd}, \forall k \in K \qquad (3.2j)$$

$$\sum_{i \in N, j \in N_{sd}} x_{ij}^k t_{ij} \leq z \qquad \forall k \in K \qquad (3.2k)$$

$$x_{ij}^k \in \{0,1\} \qquad \forall (i,j) \in A, \forall k \in K \qquad (3.2l)$$

$$w_i^k \in \{0,1\} \qquad \forall i \in N, \forall k \in K \qquad (3.2m)$$

The mathematical model is defined on the basis of having routes for each vehicle start and end at the depot nodes. However, when the objective function is defined as given in (3.2k), we exclude the arc from the last customer back to the vehicle depot, to ensure we only calculate the time until reaching out to the last person in need. Constraint (3.2b) ensures balance between incoming and outgoing arcs. Constraints (3.2c) link the visit and the arc variables. Constraints (3.2d) ensure that each demand point will have only one incoming arc and that each demand point will be visited exactly once by each vehicle. Constraints (3.2e) ensure that each vehicle only visits one supply point. To ensure that if a

vehicle visits any node, then it has to start from its depot, we add constraints (3.2f). Constraints (3.2g) ensure that we avoid sub-tours involving just two nodes. Indeed, since each tour has to leave the depot, pass through a supply node and service at least one customer, tours with just two nodes are not allowed. While these constraints are not needed to define the model, they are valid inequalities that strengthen the formulation. Constraints (3.2h) are capacity constraints. The capacity Q given in the problem definition is the number of demand points each vehicle can serve. When we include the visit variables of depot and supplier nodes, we add 2 to the given value of Q to account for these additional nodes. Constraints (3.2i) are flow preservation constraints that guarantee that one unit of flow is sent from the depot of the vehicle k to each demand node visited by that vehicle. Constraints (3.2j) link arc variables and flow variables. Constraints (3.3) define the objective function. Constraints (3.2l) and (3.2m) are domain definition constraints. We define the set A based on the configuration of arcs as given in Figure 3.1a. Consequently, we no not need any additional constraints that restrict the infeasible arcs based on the definition of the problem as given in the previous chapter (See constraints (2.15)).

We call this model the 'disaggregated $z - x$' model denoted as $\mathtt{D}_{z-x}$ due to two reasons. First, the objective function as given in constraints (3.2k) is defined using the link between the variables z and x. Second, the arc variable between any node i and j is defined for each vehicle k separately using the variable x_{ij}^k. We now explore an alternate model where the objective function is instead expressed using a relationship between the variables z and f.

Disaggregated *z-f* model ($\mathtt{D}_{z-f}$)

The disaggregated formulation can be defined in an alternative way by changing how the objective function is defined. The first method of expressing the objective function is given in (3.2k). An alternative method of expressing the objective function is by defining it as given in (3.3). In this form, for a given vehicle k and a given demand node m, the LHS represents the time needed to reach out to

the demand node m by vehicle k. Finding the maximum of this value over all vehicles and all demand nodes will give us the value of the `min-max rescue time` objective function. This is due to the fact that for a given integer values of x and w, the constraint matrix determined by constraints (3.2i)-(3.2j) is totally unimodular. Hence, even though variables f_{ij}^{mk} are not restricted to be integers, they will ensure a unique path from the depot $d(k)$ to the demand point $m \in N_{sd}$. Therefore, the LHS will measure the length of this path in terms of t_{ij} values. What this means is that we can still use the basic graph defined by the model (3.2) with (3.3) in place of (3.2k). We will refer to this model as $\mathtt{D}_{z-f}$ as the objective function is expressed using relationship between z and f variables, while maintaining the disaggregated nature of arc variables x.

$$\sum_{i,j \in N} f_{ij}^{mk} t_{ij} \leq z \qquad \forall k \in K, m \in N_{sd} \tag{3.3}$$

Aggregated model ($\mathtt{A}_{z-f}$)

We call the following model the aggregated model as there is a key difference in how the arc variables are defined. In this case, for the arc between nodes i and j, we just define a variable x_{ij} irrespective of which vehicle traverses the arc. To keep track of which vehicle traverses the arc, we use a combination of constraints using visit variables w and flow variables f. The mathematical formulation of the aggregated CHR-VRP model uses the decision variables w_i^k and f_{ij}^{mk} as defined in (3.1b) and (3.1c) respectively. In addition, we also make use of the following definition for the arc variables:

$$x_{ij} = \begin{cases} 1 & \text{if arc } (i,j) \text{ is traversed by any vehicle} \\ 0 & \text{otherwise} \end{cases} \qquad \forall i,j \in N \tag{3.4}$$

The formulation is the following, which we refer to as $\mathtt{A}_{z-f}$:

$$\text{minimize} \qquad z \tag{3.5}$$

subject to

$$\sum_{(i,j)\in A} x_{ij} - \sum_{(j,n)\in A} x_{jn} = 0 \qquad \forall j \in N \tag{3.6a}$$

$$\sum_{(i,j)\in A} x_{ij} = \sum_{k\in K} w_j^k \qquad \forall j \in N \tag{3.6b}$$

$$\sum_{j\in N_s} x_{d(k)j} \leq 1 \qquad \forall k \in K \tag{3.6c}$$

$$x_{ij} + x_{ji} \leq 1 \qquad \forall (i,j) \in A \tag{3.6d}$$

$$\sum_{k\in K} w_i^k = 1 \qquad \forall i \in N_d \tag{3.6e}$$

$$\sum_{i\in N} w_i^k \leq Q + 2 \qquad \forall k \in K \tag{3.6f}$$

$$w_i^k \leq w_{d(k)}^k \qquad \forall i \in N, \forall k \in K \tag{3.6g}$$

$$x_{ij} + w_i^k - w_j^k \leq 1 \qquad \forall k \in K, \forall i \in N_d \cup d(k), \forall j \in N \tag{3.6h}$$

$$x_{ij} + w_j^k - w_i^k \leq 1 \qquad \forall k \in K, \forall i \in N_s, \forall j \in N_d \tag{3.6i}$$

$$\sum_{(i,j)\in A} f_{ji}^{mk} - \sum_{(i,j)\in A} f_{ij}^{mk} = \begin{cases} -w_m^k & \text{if } i = d(k) \\ w_m^k & \text{if } i = m \\ 0 & \text{otherwise} \end{cases} \qquad \forall k \in K, \forall m \in N_{sd}, \forall i \in N \tag{3.6j}$$

$$0 \leq f_{ij}^{mk} \leq x_{ij} \qquad \forall (i,j) \in A, \forall k \in K, \forall m \in N_{sd} \tag{3.6k}$$

$$z \geq \sum_{i,j\in N} f_{ij}^{mk} t_{ij} \qquad \forall k \in K, \forall m \in N_{sd} \tag{3.6l}$$

$$x_{ij} \in \{0,1\} \qquad \forall (i,j) \in A \qquad (3.6m)$$

$$w_i^k \in \{0,1\} \qquad \forall i \in N, \forall k \in K \qquad (3.6n)$$

A clear parallel can be drawn between the constraints in the disaggregated formulation and the constraints in the aggregated formulation. Constraints (3.6a) - (3.6c) correspond to constraints (3.2b) - (3.2e) from the disaggregated model. Same can be said about constraints (3.6d), (3.6f), (3.6g), (3.6m), (3.6n) and (3.2g),(3.2h), (3.2f), (3.2l), (3.2m). Constraints (3.6j) - (3.6l) are counterparts of constraints (3.2i) - (3.2k). It is worth noting that constraints (3.6h) and (3.6i) are needed for the validity of the aggregated model. Figure 3.2 shows how a feasible solution of $\mathtt{A}_{z-f}$ would look like. It is important to note that the aggregated nature of the arc variables ensures that the out-degree of arcs at supplier nodes are no longer limited to 1 (as is the case in disaggregated models). This is ensured by constraints (3.6a) and (3.6b). However, the out degree at the N_d layer continues to be limited to 1 even in this case.

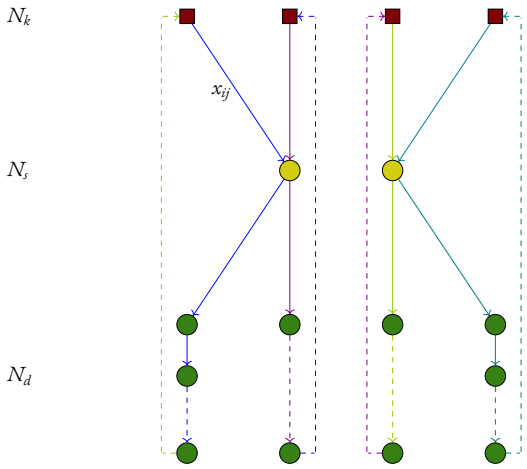


Figure 3.2: Feasible solution- Aggregated model

One of the major challenges of moving towards an aggregated model is keeping track of the vehicle routes after a supplier which serves multiple vehicles. The lack of vehicle index while expressing the

arc variable makes this particularly challenging. Constraints (3.6h) and constraints (3.6i) ensure that if an arc exists in the solution, then both its origin and destination nodes have to be visited by the same vehicle. Additionally, constraints (3.6e) ensure that each demand point is visited by exactly 1 vehicle. It can be seen that in case of aggregated formulation, the alternative way of expressing the objective function as in (3.3) and (3.2k) is not possible, as we do not differentiate between the arc traversed by different vehicles.

3.3.2 The average rescue time problem

Next, we look at the `average rescue time` problem. For the `average rescue time` problem, the model is presented based on a disaggregated formulation. The objective function is defined in such a way that the rescue times of every single person who receives the relief effort is represented. As mentioned earlier when discussing $\mathtt{D}_{z-f}$, the LHS of (3.3) denotes the rescue time to reach out to demand node m by vehicle k. This is due to the fact that the flow variables ensure that one unit of flow is sent over the network to reach the node m using vehicle k. Multiplying the flow variable with the time required to traverse each arc ensures that the time required to reach the destination is taken into account starting from the origin depot $d(k)$ to destination node m. Summing this term up over all vehicles does not change the value as only one vehicle will serve each demand node as per problem definition. Summing this term up over all nodes and taking the average with respect to the total number of demand nodes will give us the value of the `average rescue time` objective function. Moreover, we can also see that using an objective function resembling that of the $\mathtt{D}_{z-x}$ using only x variables, cannot express the individual rescue times for calculating the objective function. The rest of the constraints and the structure of the graph remains the same. This means that the following objective function,

$$\text{minimize} \qquad \frac{1}{|N_d|}\left[\sum_{m\in N_d}\sum_{k\in K}\sum_{i\in N}\sum_{j\in N} f_{ij}^{mk} t_{ij}\right] \tag{3.7}$$

subject to constraints (3.2b) - (3.2m) defines the `average rescue time` problem.

We now discuss how the prescribed formulations can be strengthened by using lower bounds and valid inequalities based on the properties of the problem. We develop lower bounds for both `min-max rescue time` as well as the `average rescue time` problem.

3.4 Lower Bounds

3.4.1 Lower Bounds for the min-max rescue time problem

In order to improve the performance of the solution approach presented in the following, we develop four valid lower bounds. Two of the lower bounds are expressed as valid inequalities and two lower bounds are given as constant values.

Proposition 1. ***[LB1]:*** *The constraints-*

$$z \geq [w_s^k \cdot t_{d(k)s} + (w_m^k + w_s^k - 1) \cdot t_{sm}] \quad \forall k \in K, \forall m \in N_d, \forall s \in N_s \tag{3.8}$$

are valid inequalities for models $\mathtt{D}_{z-x}$, $\mathtt{D}_{z-f}$ *and* $\mathtt{A}_{z-f}$.

Proof. We can look at any route as a combination of two legs of travel, first leg being the travel from depot to supplier and second leg being travel from supplier to demand nodes. For a given combination of vehicle, supplier and demand node, if the vehicle k visits the supplier s on the way to the demand node m, the visit variable $w_s^k = 1$. This makes sure that the first leg of the journey $t_{d(k)s}$ is included in

the RHS. The validity of these constraints are ensured because we have assumed that all the distances satisfy the triangle inequality. Also, the term $(w_m^k + w_s^k - 1)$ ensures that in case the given vehicle visits both the supplier and demand node, then t_{sm} is included in the RHS. No matter what actual route is, if this demand node is visited by the vehicle, it is not possible to have a shorter route than when the vehicle visits the demand node directly after the supplier. This also implies that this is a valid inequality giving a lower bound on the objective function. □

Definition 1. *Let $\hat{\ell}_m^k$ be the minimum time required to reach out to any demand node m by a given vehicle k after passing through a supplier, i.e.,*

$$\hat{\ell}_m^k = \min_{s \in N_s} \left[t_{d(k)s} + t_{sm} \right] \qquad \forall k \in K, \forall m \in N_d$$

Proposition 2. ***[LB2]**: The value*

$$\max_{m \in N_d} \min_{k \in K} \hat{\ell}_m^k \tag{3.9}$$

is a valid lower bound for the `min-max rescue time` *problem.*

Proof. By virtue of the problem setting, all customers must be served by exactly one vehicle. For each vehicle-demand node combination, we calculate $\hat{\ell}_m^k$, which is the quickest way to get to the demand node by the vehicle after passing through any of the supplier nodes. For each demand node, the vehicle that takes the least time to serve the node is calculated ($\min_{k \in K} \hat{\ell}_m^k$). This value gives a lower bound on the minimum service time for a demand node $m \in N_d$. Then the demand node with the largest minimum serving time is identified for the entire graph. Due to the min-max nature of the objective function, it can be seen that the objective function will be greater than or equal to the largest minimum serving time. □

We define two parameters μ and κ in the following manner.

Definition 2. *Let μ be the minimum number of demand nodes that should be served by any vehicle, i.e.,*

$$\mu = max(|N_d| - (|K| - 1) * Q, 0) \tag{3.10}$$

Definition 3. *Let κ be the the minimum number of vehicles needed to serve all demand points, i.e.,*

$$\kappa = \lceil |N_d|/Q \rceil \tag{3.11}$$

It can be noted that $\mu \geq 1 \implies \kappa = |K|$. We utilize these parameters for generating the following lower bound.

Definition 4. *Let $\overline{\ell}^k_\mu$ be the minimum time taken by a vehicle k to reach the μ demand nodes after passing through a supplier. We also define the notation $low(\overline{\ell}^k_1, \kappa)$ to represent the κ^{th} lowest value of $\overline{\ell}^k_1$ for all $k \in K$.*

Proposition 3 (LB3). *: The value*

$$\max(\max_{k \in K} \overline{\ell}^k_\mu, low(\overline{\ell}^k_1, \kappa)) \tag{3.12}$$

is a valid lower bound for the `min-max rescue time` *problem.*

Proof. We derive this lower bound in case of two possible scenarios. In the first scenario, if $\mu \geq 1$, this means that all vehicles are utilized. Here, for each vehicle we take the minimum time to reach the μ^{th} demand node. Then among all vehicles, we take the value corresponding to the vehicle that has the maximum time from the above list. Indeed, given that we know each vehicle is going to visit μ demand points, the vehicle that takes the longest to visit the minimum number of demand points will be a lower bound. This corresponds to the value $\max_{k \in K} \overline{\ell}^k_m$ as given in (3.12). In the second scenario, $\mu = 0$. In this situation, we know for a fact that a minimum of κ vehicles have to be used.

Which in turn means that at least κ vehicles will visit at least one demand point. For all vehicles, we list down the minimum time required to serve one demand point. We then pick the κ^{th} lowest value as the lower bound, attributing each lower value to each of the possible vehicle being used. This value is represented by $low(\bar{\ell}_1^k, \kappa)$ as given in (3.12). □

Proposition 4. ***[LB4]*** *Let $\mu > 0$ and let $t_{s\mu}$ be the minimum time taken by a path that connects μ demand nodes starting from supplier s. Then*

$$z \geq w_s^k \cdot t_{d(k)s} + t_{s\mu} \cdot w_s^k \quad \forall k \in K, \forall s \in N_s \tag{3.13}$$

are valid inequalities for $\mathtt{D}_{z-x}$, $\mathtt{D}_{z-f}$ *and* $\mathtt{A}_{z-f}$ *models.*

Proof. Instead of considering a combination of vehicle, supplier and demand node as in Proposition 1, we use the knowledge that each vehicle has to serve at least μ demand points. For each vehicle and supplier combination, we calculate the time required in the two legs as given in $(LB1)$, with the only change that the second leg now consists of time required to serve μ demand points. □

3.4.2 Lower Bounds for the average rescue time problem

We develop an argument for a lower bound for average rescue time model in similar lines of LB1 we defined earlier. In this case, the lower bound is defined for the term $\sum_{i\in N}\sum_{j\in N} f_{ij}^{mk}$.

Proposition 5. ***[LB5]****: The inequalities*

$$\sum_{i\in N}\sum_{j\in N} f_{ij}^{mk} \geq [(w_m^k + w_s^k - 1) \cdot (t_{d(k)s} + t_{sm})] \quad \forall k \in K, m \in N_d, s \in N_s \tag{3.14}$$

are valid for the $\mathtt{D}_{z-f}$ *model of the* `average rescue time` *problem. .*

Proof. For a given node m and vehicle k the RHS is a valid lower bound for the θ_m^k as it is impossible to visit demand node m any quicker than when visiting the node right after the supply node s. The

addition of visit variables ensures that the lower bound is binding only when the supply node and demand node are visited by the same vehicle.

□

We use the definition of $\hat{\ell}_m^k$ introduced in Definition 1 to derive another lower bound as given below:

Proposition 6. ***[LB6]****: The constraints*

$$\sum_{i \in N} \sum_{j \in N} f_{ij}^{mk} \geq w_m^k \hat{\ell}_m^k \quad \forall k \in K, m \in N_d \tag{3.15}$$

are valid inequalities for the $\mathtt{D}_{z-f}$ *model of* `average rescue time` *problem.*

Proof. If a vehicle k is visiting a node m, then the lower bound can be defined as the quickest time that the vehicle k needs to reach demand node m by passing through a supplier. □

Proposition 7. ***[LB7]****: The constraints*

$$\sum_{m \in N_d} \sum_{i \in N} \sum_{j \in N} f_{ij}^{mk} t_{ij} \geq \mu \times \bar{\ell}_1^k \quad \forall k \in K \tag{3.16}$$

are valid inequalities for the $\mathtt{D}_{z-f}$ *model of* `average rescue time` *problem.*

Proof. Given that any vehicle will visit at least μ demand points, for a given vehicle k, the sum of all rescue times (represented for each demand node m by the sum ($\sum_{i \in N} \sum_{j \in N} f_{ij}^{mk} t_{ij}$) should be greater than the minimum number of demand points per vehicle μ, times the time taken to the closest demand point from depot of vehicle k after passing through a supplier. It has to be noted that this lower bound will not be effective when $\mu = 0$. □

3.5 Solution Approaches

We study and compare three methods for solving the `min-max rescue time` problem and two methods for solving the `average rescue time` problem. For the `min-max rescue time` problem, the first method uses the default solver within `Cplex` to solve the $\mathtt{D}_{z-x}$, $\mathtt{D}_{z-f}$ and $\mathtt{A}_{z-f}$ models. The second and the third method project out the flow variables from the models $\mathtt{D}_{z-x}$ and $\mathtt{D}_{z-f}$ respectively. They are both implemented using a branch-and-cut procedure. By projecting out flow variables, we do not need to solve an auxiliary LP to derive a violated feasibility (*i.e.*, GSEC) cut. Instead, a max-flow based separation procedure can be employed. A similar comparison of formulations has been done for the inventory routing problem by Archetti and Ljubić (2022).

As per the seminal work of Benders (1962), we decompose a problem into two parts. The complicating binary variables are retained and solved in the relaxed master problem. The continuous variables and their corresponding constraints are projected out and replaced with two families of cuts, feasibility cuts and optimality cuts (which are only needed if the objective function value depends on the flow variables). Feasibility cuts are generated when the solution to the relaxed master problem is not feasible when it enters the sub-problem. Once we ensure that the solution generated after solving the relaxed master problem is always feasible, then we need to be concerned about optimality cuts. The methods are explained in the following sections.

3.5.1 Branch and Cut with GSEC

In this approach when solving the model $\mathtt{D}_{z-x}$, instead of the constraints (3.2i) and (3.2j), we project out the flow variables. By doing so, we obtain the so-called Generalized Sub-tour Elimination Constraints (GSECs):

$$\sum_{i,j \in S} x_{ij}^k \leq \sum_{i \in S} w_i^k - w_m^k \qquad \forall S \subset N, \forall k \in K, \forall m \in S \tag{3.17}$$

Because these constraints ensure that there are no sub-tours in the solution, the model $\mathtt{D}_{z-x}$ can be redefined with (3.2b)-(3.2h), (3.2l), (3.2m) in addition to (3.17) above.

3.5.2 Tailored Benders Decomposition

In this method also we project out flow variables, but this time, we derive *optimality cuts* due to the link between z and f variables. Moreover here, instead of solving the sub-problem using a linear program, we compute the values of the coefficients of the *optimality cuts* using a method similar to the one suggested by Magnanti et al. (1986). This is the main tenet of the tailored Benders decomposition devised for this problem. In this approach, we start with the disaggregated model by looking at the following sub-problem. Constraint (3.2i) to (3.2k) can be restated as

$$z \geq \max_{m \in N_d, k \in K} \{ z_k(\bar{x}, \bar{w}, m) : \quad (\bar{x}, \bar{w}) \text{ satisfy (3.2b)-(3.2h), (3.2l), (3.2m)} \}$$

For a given $k \in K$, $m \in N_d$, the value of z_k corresponds to the optimal value of the following LP

$$z_k(\bar{x}, \bar{w}, m) \quad = \text{minimize} \quad \sum_{i,j \in N} f_{ij}^{mk} t_{ij} \tag{3.18a}$$

subject to

$$\sum_{(i,j) \in A} f_{ji}^{mk} - \sum_{(j,i) \in A} f_{ij}^{mk} = \begin{cases} -\overline{w}_m^k & \text{if } i = d(k) \\ \overline{w}_m^k & \text{if } i = m \\ 0 & \text{otherwise} \end{cases} \qquad \forall i \in N \tag{3.18b}$$

$$0 \leq f_{ij}^{mk} \leq \bar{x}_{ij}^k \qquad \forall i, j \in N \tag{3.18c}$$

The dual formulation for the sub-problem given by (3.18a) to (3.18c) can be written as:

$$\text{maximize} \qquad -\alpha_{d(k)}\overline{w}^k_m + \alpha_m \overline{w}^k_m - \sum_{(i,j)\in A} \beta_{ij}\overline{x}^k_{ij} \tag{3.19a}$$

subject to

$$\alpha_j - \alpha_i - \beta_{ij} \leq t_{ij} \qquad \forall i \in N, \forall j \in N \tag{3.19b}$$

$$\alpha_i \in \mathbb{R}, \beta_{ij} \geq 0 \qquad \forall i \in N, \forall j \in N \tag{3.19c}$$

We can readily make the observation that the primal formulation given in (3.18) represents the shortest path problem (SPP), where for each vehicle k and destination m, the shortest path to reach the destination such that $w^k_m = 1$ is calculated, given the integer values of x and w variables. Let us focus on a given vehicle k and drop the index k in the following. Within this context, we denote the set of all feasible tours by set X. We start the procedure for adding optimality cuts only after we ensure that we add all the feasibility cuts using the GSECs defined above. The GSECs along with the other constraints in the master problem helps us get T, which denotes the tour of a vehicle k as per a given solution of the master problem. This is represented by binary vector $\bar{x} \subset X$. Let N_T denote the nodes present in the tour T, and arc set $A_T \subset \{N_T \times N_T\}$. The depot node of the vehicle under consideration is denoted as 0 for convenience. Hence, the sub-problem (3.18) is a shortest path problem (SPP) defined on subgraph $G_T = (N_T, A_T)$ from 0 to m for a given vehicle k. The inclusion of (3.18c) ensures that the flow is directed only through the arcs present in the solution to the relaxed master problem, in other words, through the tour T.

It can be observed that the polytope associated with the dual, $\mathscr{P}_{mk} = \{(\alpha, \beta) : (\alpha, \beta)$ satisfy

(3.19b) - (3.19c) } does not depend on $\bar{x}$ and $\bar{w}$. Let ext $(\mathscr{P}_{mk})$ denote all extreme points (α^*, β^*) of the polytope $\mathscr{P}_{mk}$. By enumerating all extreme points, we can replace (3.3) with the following family of cuts for each $k \in K$ and $m \in N_d$:

$$z \geq \alpha^*_m w^k_m - \alpha^*_{d(k)} w^k_m - \sum_{(i,j)\in A} \beta^*_{ij} x^k_{ij} \qquad (\alpha^*, \beta^*) \in \text{ext}\,(\mathscr{P}_{mk}) \quad \forall m \in N_d, \forall k \in K \qquad (3.20)$$

For a given solution of the relaxed master problem $(\bar{x}, \bar{w})$, we denote the incumbent value of z as $\bar{z}$. For a given vehicle k and demand node m, the shortest path between $d(k)$ to m denoted as $P_T(d(k), m)$ has a length (duration) of $t_T(d(k), m)$. If $t_T(d(k), m) \geq \bar{z}$, then (3.20) is a violated Benders optimality cut. Without losing any generality, we can set $\alpha^*_{d(k)} = 0$ as k is the depot node, which further simplifies the cut. Given the expression for the cut, if we prove that the product $\beta^*_{ij}\bar{x}^k_{ij} = 0$, the value of the cut is just the length of the shortest path to the given demand point m. The fact that the Benders sub-problem is a SPP enables us to use the results derived by Magnanti et al. (1986) and Alfandari et al. (2022) where the coefficients of Benders cuts are computed using Proposition 8.

Proposition 8. *For a binary $(\bar{x}, \bar{w})$ of the relaxed master problem and node m such that $\bar{w}^k_m = 1$, an optimal $(\alpha^*, \beta^*) \in$ ext $(\mathscr{P}_{mk})$ solution can be calculated as*

1. $\alpha^*_i = t_T(0, i)$ `if` $i \in N_T, i \in P_T(0, m)$
2. $\alpha^*_i = min(t_T(0, i), \alpha^*_m - t_G(i, m))$ `if` $i \in N_T, i \notin P_T(0, m)$
3. $\alpha^*_i = \alpha^*_m - t_G(i, m)$ `if` $i \notin N_T$
4. $\beta^*_{ij} = max(0, \alpha^*_j - \alpha^*_i - t_{ij})$ $\forall i \in N, j \in N$

A detailed proof of this proposition is given in Appendix A.2.1.

To proceed with developing tailored Benders decomposition method for the `average rescue time` problem, we split the MIP model we formulated in (3.7) into the master and sub problem as given

below. We also define a new variable θ_m^k defined as the time required to reach out to demand point m using vehicle k. The relaxed master problem will be:

$$\text{minimize} \qquad \frac{1}{|N_d|}\left[\sum_{m\in N_d}\sum_{k\in K}\theta_m^k\right] \tag{3.21a}$$

subject to

$$\sum_{(i,j)\in A} x_{ij}^k - \sum_{(j,l)\in A} x_{jl}^k = 0 \qquad \forall j \in N, \forall k \in K \tag{3.21b}$$

$$\sum_{i\in N} x_{ij}^k = w_j^k \qquad \forall j \in N, \forall k \in K \tag{3.21c}$$

$$\sum_{k\in K} w_i^k = 1 \qquad \forall j \in N_d \tag{3.21d}$$

$$\sum_{j\in N_s} x_{d(k)j}^k \leq 1 \qquad \forall k \in K \tag{3.21e}$$

$$w_i^k \leq w_{d(k)}^k \qquad \forall i \in N, k \in K \tag{3.21f}$$

$$x_{ij}^k + x_{ji}^k \leq w_i^k \qquad \forall i,j \in N, \forall k \in K \tag{3.21g}$$

$$\sum_{i\in N} w_i^k \leq Q + 2 \qquad \forall k \in K \tag{3.21h}$$

$$\sum_{i,j\in S} x_{ij}^k \leq \sum_{i\in S} w_i^k - w_m^k \qquad \forall S \subset N, \forall k \in K, \forall m \in S \tag{3.21i}$$

$$x_{ij}^k \in \{0,1\} \tag{3.21j}$$

$$w_i^k \in \{0,1\} \tag{3.21k}$$

The sub-problem associated with vehicle $k \in K$ and demand point $m \in N_{sd}$ will be:

$$\theta_m^k(\bar{x},\bar{w}) = \text{minimize} \qquad \sum_{i,j\in N} f_{ij}^{mk} t_{ij} \tag{3.22a}$$

subject to:

$$\sum_{(i,j)\in A} f_{ji}^{mk} - \sum_{(j,i)\in A} f_{ij}^{mk} = \begin{cases} -\bar{w}_m^k & \text{if } i = d(k) \\ \bar{w}_m^k & \text{if } i = m \\ 0 & \text{otherwise} \end{cases} \qquad \forall i \in N \tag{3.22b}$$

$$0 \leq f_{ij}^{mk} \leq \bar{x}_{ij}^k \qquad \forall i,j \in N \tag{3.22c}$$

The optimally cuts are derived using a similar reasoning. This means that the separation method will be to check if $\hat{\theta}_m^k < \theta_m^k(\bar{x},\bar{w})$ for a given $m \in N_d, k \in K$, where $\hat{\theta}_m^k$ is the current optimal value of θ_m^k in the relaxed master problem. In case this condition is violated, we add the following cut:

$$\theta_m^k \geq \alpha_m^* w_m^k - \alpha_{d(k)}^* w_m^k - \sum_{(i,j)\in A} \beta_{ij}^* x_{ij}^k \quad (\alpha^*,\beta^*) \in \text{ext}\,(\mathscr{P}_{mk}) \quad \forall m \in N_{sd}, k \in K \tag{3.23}$$

We point out that the tailored Benders cuts derived for Proposition 8 are imposed for binary points $(\bar{x},\bar{w})$ only.

3.5.3 Tightening the cuts

The cuts derived in the previous section can be improved by using the lower bounds we have established in Section 3.4.1. So, we have the following proposition.

Proposition 9. *Benders cuts established in* (3.20) *can be tightened in the following manner:*

$$z \geq \hat{\ell}_m^k + (\alpha_m^* - \hat{\ell}_m^k) w_m^k - \alpha_{d(k)}^* w_m^k - \sum_{(i,j)\in A} \beta_{ij}^* x_{ij}^k$$

$$(\alpha^*,\beta^*) \in ext\,(\mathscr{P}_{mk}) \quad \forall m \in N_d, k \in K \tag{3.24}$$

where $\hat{\ell}^k_m$ denotes the minimum time required to reach out to any demand node m by a given vehicle k after passing through a supplier as given in Definition 1.

Proof. Consider two cases. In the first case, when $w^k_m = 1$, (3.24) reduces to (3.20). When $w^k_m = 0$ the RHS becomes $\hat{\ell}^k_m - \sum_{(i,j)\in A} \beta^*_{ij} x^k_{ij}$. We have already proved that the value $\hat{\ell}^k_m$ is a valid lower bound in Proposition 2. This means that the constraint (3.24) is still valid.

□

Proposition 10. *We can further tighten the values of β as given in* (3.20) *and* (3.24) *to*

$$\overline{\beta}^*_{ij} = \min\{\beta^*_{ij}, \alpha^*_m - \hat{\ell}^k_m\} \quad \forall (i,j) \in A \tag{3.25}$$

Proof. First we note that $\alpha^*_m - \hat{\ell}^k_m \geq 0$ as α^*_m is the distance to the last customer m in the path of vehicle k given by the current choice of $\bar{x}$, which will be greater than the already established lower bound given in Proposition 2. Let $(\bar{x}, \bar{w})$ be a feasible solution. Recall that Benders cuts (3.20) will be potentially violated only if $\bar{w}^k_m = 1$. So, for the purposes of this proof, we assume $\bar{w}^k_m = 1$. We only need to prove cases where $\overline{\beta}^*_{ij} = \alpha^*_m - \hat{\ell}^k_m$. We prove the validity of the tightening in three steps. First, in the case where $\bar{x}^k_{ij} = 0$ for all $(i,j) \in A$, the tightening is not active at all. Second, we take the case where exactly one $\bar{x}^k_{ij} = 1$. Consider the inequality given in (3.24). There are two cases here. Since $\overline{w}^k_m = 1$, the inequality reduces to $z \geq \hat{\ell}^k_m + (\alpha^*_m - \hat{\ell}^k_m) - (\alpha^*_m - \hat{\ell}^k_m)$. This further reduces to $z \geq \hat{\ell}^k_m$, which is true by defition of the lower bound. Finally, when more than one $\bar{x}^k_{ij} = 1$, we keep subtracting the RHS by a positive value equivalent to $\alpha^*_m - \hat{\ell}^k_m$ for each pair of $\bar{x}^k_{ij} = 1$, making it valid for all cases. This proves the proposition. □

In the following section, we compare the performance of the different methods. We also compare the effectiveness of the various lower bounds discussed earlier.

3.6 Computational Results

The experiments were conducted on the small instances (10 demand points and 20 demand points with 3 and 4 depots respectively) generated by Parappathodi and Archetti (2022). Four aspects were investigated using computational results. First, different methods developed in Section 3.5 were compared and the difference in performance studied. This included the difference between aggregated and disaggregated formulation as well as the difference when the objective function is expressed in terms of arc and flow variables. Second, the effectiveness of the lower bounds developed in the Section 3.4.1 were studied. Third, the effect of β-tightening was studied on the basis of the number of nodes explored. For the `average rescue time` problem, we compare the performance of tailored Benders decomposition and default solver for the $\mathtt{D}_{z-f}$ model. We also study the effectiveness of the different valid inequalities presented in Section 3.4.2.

3.6.1 Performance comparison - methods

Table 3.1 and Table 3.2 summarize the performance of different solution strategies tested for the `min-max rescue time` problem and the `average rescue time` problem respectively. The column 'B&C $\mathtt{D}_{z-x}$' lists the solution times when $\mathtt{D}_{z-x}$ model is implemented using a Branch-and-Cut strategy, as explained in Section 3.5.1. '$\mathtt{D}_{z-x}$' lists the performance values when $\mathtt{D}_{z-x}$ model is solved using the default `Cplex` solver. 'B&C $\mathtt{D}_{z-f}$(TB)' lists the solution times when Branch-and-Cut strategy (that includes the tailored Benders cuts) is applied to the $\mathtt{D}_{z-f}$ model as explained in Section 3.5.2. The column '$\mathtt{D}_{z-f}$' lists the solution times when the $\mathtt{D}_{z-f}$ is solved using the default `Cplex` solver. Finally, the column'$\mathtt{A}_{z-f}$' gives the performance values of the aggregated model solved using the default `Cplex` solver. When comparing the different solution methods, the results of which are given in Table 3.1, it can be seen that the best performing method is the Branch-and-Cut for the $\mathtt{D}_{z-x}$ model. All small sized instances (S1-S20) were solved to optimality. While the same instances were solved to optimality

by the default solver using the flow-based formulation, the performance was not as impressive as that of the Branch-and-Cut method. The Branch-and-Cut for the $\mathtt{D}_{z-f}$ model solved the smaller 10 customer instances, but failed to solve any of the 20 customer instances within the stipulated time limit. This can be explained by weaker lower bounds obtained for the LP-relaxation of the the Branch-and-Cut for the $\mathtt{D}_{z-f}$ model and the fact that $\mathtt{D}_{z-x}$ model had extremely good LP-relaxation values as seen in Figure 3.4. Using the default solver on $\mathtt{D}_{z-f}$ model performed the worst among all disaggregated formulations. The aggregated formulation $\mathtt{A}_{z-f}$ performed even worse, with the solution times being significantly higher than any of the disaggregated methods as can be seen from Table 3.1. None of the larger instances could be solved within the time limit by the $\mathtt{A}_{z-f}$ model. This was expected, as the LP-relaxation values are the worst for the $\mathtt{A}_{z-f}$ model.

Similar tests are run for the `average rescue time` problem with the performance of the flow-based formulation and Branch-and-Cut (with tailored Benders cuts) method. Table 3.2 captures the performance of the two solution methods when applied to the `average rescue time` problem. Column '$\mathtt{D}_{z-f}$' lists the performance of the default solver when solving the $\mathtt{D}_{z-f}$ model. Columns '$\mathtt{D}_{z-f}$ (TB + LB)' and '$\mathtt{D}_{z-f}$ (TB)' list the performance of the Branch-and-Cut implementation for the $\mathtt{D}_{z-f}$ model with and without the lower bounds described in Section 3.4.2, respectively. Column '$\mathtt{A}_{z-f}$' lists the performance of the default solver when solving the corresponding model. Interestingly, the performance gap between aggregated and non-aggregated models is not as high as for the `min-max rescue time` problem. This is probably due to the aggregated nature of the objective function itself which helps the performance of the aggregated model. However, the aggregated model continues to be the worst performing model even in case of the `average rescue time` problem. The column 'Optimal Sol' lists the value of the objective function at optimality for all the instances and column 'LP Rel' lists the LP-relaxation values of the model. The performance of B&C for $\mathtt{D}_{z-f}$ model is definitely improved by the suggested lower bounds. However, overall, the decomposition-based method does not

Table 3.1: Relative Performance- computational time (s) -`min-max rescue time`

Instance No	B&C $\mathtt{D}_{z-x}$	$\mathtt{D}_{z-x}$	B&C $\mathtt{D}_{z-f}$ (TB)	$\mathtt{D}_{z-f}$	$\mathtt{A}_{z-f}$
S1	0.64	3.78	7.31	13.21	366.52
S2	0.56	1.66	8.27	10.33	257.69
S3	0.48	2.74	8.34	17.46	203.76
S4	0.42	2.31	5.52	16.21	198.65
S5	0.75	4.47	16.75	17.7	530.16
S6	0.67	3.12	10.34	10.94	529.52
S7	0.42	2.72	16.83	8.10	603.06
S8	0.95	3.59	7.31	17.63	489.05
S9	0.33	1.07	5.89	10.33	86.07
S10	0.36	3.16	12.72	16.99	325.24
S11	33.57	2615.41	3600* (59%)	3600*	3600*
S12	32.43	1068.58	3600* (61%)	3600*	3600*
S13	2.67	100.9	3600* (57%)	3600*	3600*
S14	10.74	2109.44	3600* (62%)	3600*	3600*
S15	97.74	3600*(6%)	3600* (58%)	3600*	3600*
S16	112.4	3600*(7%)	3600* (54%)	3600*	3600*
S17	5.83	494.66	3600* (58%)	3600*	3600*
S18	18.71	2504.57	3600* (59%)	3600*	3600*
S19	23.92	2497.65	3600* (56%)	3600*	3600*
S20	66.73	3600*(7%)	3600* (52%)	3600*	3600*

outperform the default solver in contrast to the case of `min-max rescue time` problem when solving $\mathtt{D}_{z-f}$ model. One of the major reasons behind this performance of the $\mathtt{D}_{z-f}$ model is how close the LP-relaxation values are to the value of the objective function at optimality for the base model, as can be seen from the last two columns of the table.

3.6.2 Lower bounds and formulations

Figure 3.3 can be used to compare the relative performance of the lower bounds for the disaggregated and aggregated formulations. Figure 3.3(a) and 3.3(b) denote the percentage improvement in the LP relaxation lower bounds for disaggregated and aggregated formulations respectively. For disaggregated formulation, it can be seen that LB1 offers the lowest improvement while LB2 and LB3 offer the highest improvement in lower bound. For aggregated formulation as well, the best performing lower bounds are LB2 and LB3.

Table 3.2: Computational time (s) and LP Relaxation- `average rescue time`

Instance No	$\mathtt{D}_{z-f}$	$\mathtt{D}_{z-f}$ (TB + LB)	$\mathtt{D}_{z-f}$ (TB)	$\mathtt{A}_{z-f}$	Optimal Sol	LP Rel
S1	0.34	2.85	2.18	3.18	79.15	73.21
S2	0.25	1.21	2.7	1.86	77.29	75.77
S3	0.28	1.82	3.37	4.12	83.17	80.61
S4	0.31	1.57	2.9	3.37	86.37	79.73
S5	0.28	1.46	4.81	1.68	74.27	69.98
S6	0.36	1.47	3.23	4.00	71.41	66.98
S7	0.25	1.85	3.00	2.12	77.78	76.05
S8	0.33	3.18	2.21	2.36	79.53	74.85
S9	0.27	1.02	2.49	2.2	71.83	68.81
S10	0.22	1.13	3.23	1.50	74.89	71.42
S11	59.56	3600*	3600*	3600*	78.09	72.79
S12	47.31	3600*	3600*	3600*	78.44	74.10
S13	26.47	3600*	3600*	3600*	85.68	81.65
S14	454.67	3600*	3600*	3600*	80.59	74.69
S15	325.11	3600*	3600*	3600*	74.25	69.75
S16	490.58	3600*	3600*	3600*	80.39	73.91
S17	57.34	3600*	3600*	3600*	81.79	75.51
S18	50.24	3600*	3600*	3600*	76.02	72.16
S19	196.19	3600*	3600*	3600*	90.62	83.04
S20	664.33	3600*	3600*	3600*	81.88	74.26

Table 3.3: Number of nodes

	`min-max rescue time`					`average rescue time`			
Instance No	B&C $\mathtt{D}_{z-x}$	$\mathtt{D}_{z-x}$	B&C $\mathtt{D}_{z-f}$ (TB)	$\mathtt{D}_{z-f}$	$\mathtt{A}_{z-f}$	$\mathtt{D}_{z-f}$	$\mathtt{D}_{z-f}$ (TB + LB)	$\mathtt{D}_{z-f}$ (TB)	$\mathtt{A}_{z-f}$
S1	110	73	13117	3200	23765	0	3409	5366	1164
S2	0	4	23452	2137	13285	0	791	5050	406
S3	140	122	30234	2728	18855	0	1528	5015	1262
S4	47	15	18568	1934	17950	0	1487	5933	1672
S5	70	283	50994	5025	40447	0	1860	2025	613
S6	163	192	34163	5609	46974	0	1812	12432	2409
S7	217	80	54432	3350	44987	0	11420	10374	628
S8	50	91	27714	2811	49487	0	3592	3297	893
S9	10	0	13117	1634	7521	0	1030	2330	861
S10	0	130	44076	6720	29321	0	620	2689	491

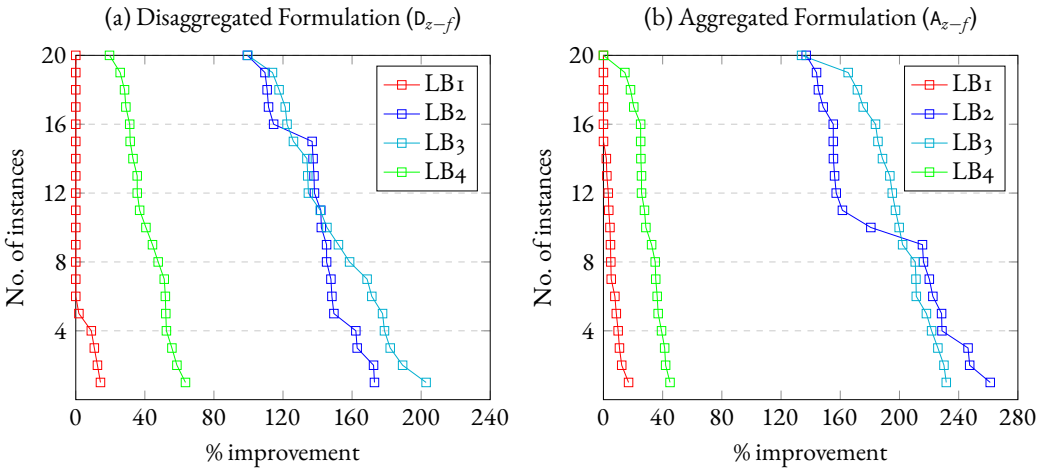


Figure 3.3: Relative improvement of lower bounds for min-max rescue time vs. no lower bounds

Table 3.3 summarizes the number of nodes evaluated for each of the solution methods for both the problems. The performance difference between the B&C D_{z-x} and other methods are highlighted by the difference in number of nodes evaluated. In case of the `average rescue time` problem, it is interesting to note that the D_{z-f} model using default solver gets solved at the root node. This could be due to the additional cuts that `Cplex` adds in its default method in conjunction with the fact that the problem has a very high LP relaxation value. Moreover, the developed valid inequalities in 3.4.2 improve the performance of the decomposition approach since not all flow constraints are included in the starting model. Their inclusion improves the convergence by reducing the number of nodes evaluated. This can be seen in the Table 3.3 between column D_{z-f}(TB + LB), which lists the number of nodes when all lower bounds are active; and column D_{z-f}(TB) which lists the number of nodes when no lower bounds are active. However, it has to be noted that the LP-relaxation values are not affected by the proposed lower bounds for the `average rescue time` problem. This points to the possibility that the valid inequalities could be implied by the formulation.

Figure 3.4 is used to compare the effectiveness of different models. We plot how close each model's

LP relaxation is, to the optimal solution for each instances. For each model, we plot the % gap of LP relaxation value to optimal solution, calculated as $(OptimalSolution - LPRelaxation)/OptimalSolution$. The closer the LP relaxation is to the optimal solution, the better the model is. The figure shows the number of instances that have at least a given percentage gap between the LP relaxation lower bound and the optimal solution. From this chart, we can observe that the $\mathtt{D}_{z-x}$ formulation is by far the best performing one with the instances recording much smaller gap to the optimal solution. When all the lower bounds are applied on the $\mathtt{A}_{z-f}$ and the $\mathtt{D}_{z-f}$ formulation, the LP relaxation lower bound is the same. This is denoted by the green line in the figure. This is not surprising, given the fact that in both aggregated as well as disaggregated formulations, the best performing lower bounds are the numerical bounds LB2 and LB3. Regardless of the model used, the numerical bounds would remain the same for either model.

Comparing the aggregated and disaggregated formulations between themselves without any of the applied lower bounds, it can be seen that the $\mathtt{D}_{z-f}$ formulation performs better than the $\mathtt{A}_{z-f}$ formulation. When comparing the size of the problems in both cases, disaggregated formulation has the same number of visit and flow variables ($|N| \times |K|$, $|N|^2 \times |N_{sd} \times |K|$ respectively). While disaggregated formulation has $|N|^2 \times |K|$ arc variables; aggregated formulation only has $|N|^2$ arc variables. Despite having more variables, disaggregated model wins out due to the additional constraints (3.6h) and (3.6i) that have to be placed to keep track of the vehicle serving different arcs.

3.6.3 Effect of β tightening

Table 3.4 lists the number of nodes explored by the branch-and-cut procedure before and after implementing the β tightening procedure as given in Section 3.5 when using the Branch-and-Cut with tailored Benders cuts as the solution method. The instances with 10 demand points were solved to optimality (S1-S10) and the rest were exceeding the time limit of one hour. It can be seen that β tight-

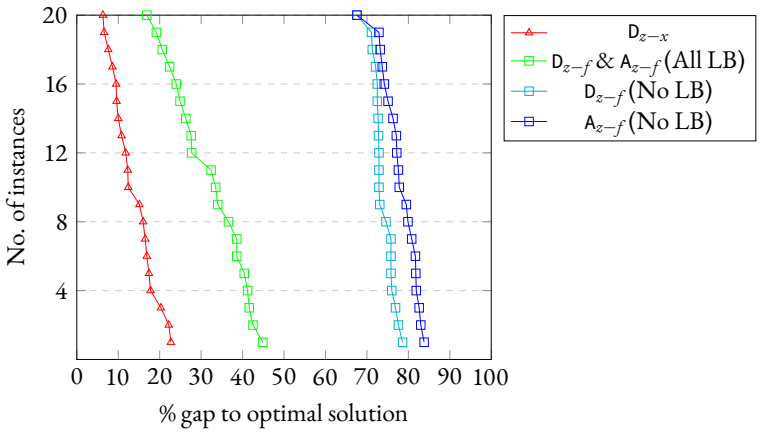


Figure 3.4: LP relaxation comparison - min-max rescue time

ening has a significant effect on the number of nodes for most of the instances. Tightening of β values in this manner ensures, for the problems solved to optimality (where the effect is more consistent), that the solution is reached with fewer nodes explored. For the instances where the time limit was reached, the optimality gap was lower, pointing to the fact that the search process was more efficient with β tightening in place.

3.7 Conclusion

Through this study we have explored exact solution approaches to the CHR-VRP. For the `min-max rescue time` problem, when comparing between D_{z-f} and A_{z-f}, despite having higher number of variables, it can be seen that the disaggregated formulation consistently outperforms its rival. This is primarily due to much better LP relaxation value of the disaggregated models compared to its aggregated counterpart. The additional constraints that have to be placed on A_{z-f}, (3.6h) and (3.6i) also contribute to the dip in performance. Second, within the disaggregated model itself, the way the objective function is defined can have huge implications when it comes to performance of the model. The change of objective function from (3.3) to (3.2k), i.e., moving from D_{z-f} to D_{z-x} brings about a marked

Table 3.4: β tightening: Effect on node count- Tailored Benders

Instance No	Nodes Before	Nodes After	Reduction
S1	27137	20542	24.3%
S2	16749	15345	8.4%
S3	45182	15009	66.8%
S4	19509	13257	32.0%
S5	49514	20094	59.4%
S6	44169	40839	7.5%
S7	78664	35954	54.3%
S8	36521	28621	21.6%
S9	7841	7416	5.4%
S10	65481	55133	15.8%
S11	150450	189402	-25.9%
S12	179296	112134	37.5%
S13	174481	102771	41.1%
S14	128654	102920	20.0%
S15	117709	103930	11.7%
S16	134406	105224	21.1%
S17	233956	113756	51.4%
S18	172866	155043	10.3%
S19	135172	153362	-13.5%
S20	102678	116994	-13.9%

improvement in terms of performance. With the better LP relaxation values, $\mathtt{D}_{z-x}$ outperforms $\mathtt{D}_{z-f}$ across all 20 instances. Third, the tailored Benders decomposition applied to $\mathtt{D}_{z-f}$ outperforms the default solver, though the performance is not good enough to beat the $\mathtt{D}_{z-x}$ model. Among the lower bounds suggested in 3.4.1, ***LB2*** and ***LB3*** are the most effective. Finally, it can be seen that across all the alternatives considered, branch-and-cut applied to the $\mathtt{D}_{z-x}$ model is the best performing method.

As for the `average rescue time` problem, $\mathtt{D}_{z-f}$ model when solved with the default solver performed better than the $\mathtt{D}_{z-f}$ model with tailored Benders cuts. The lower bounds developed for this problem did not improve upon the LP relaxation lower bound, which was extremely close to the optimal solution value for most instances. However, they were useful in reducing the number of nodes evaluated for the $\mathtt{D}_{z-f}$ model with tailored Benders cuts. The aggregated model $\mathtt{A}_{z-f}$ continued to be the worst performing model, but the performance difference between aggregated and disaggregated models was not as stark as the `min-max rescue time` problem.

There are quite a few possible research extensions to this problem. First and foremost, we have assumed that the location of the crowdsourced volunteers are known apriori and that they are deterministic. A relaxation of this constraint will introduce some stochasticity and the possibility of a few higher level strategic decisions to be taken in this context. Yet another interesting avenue of exploration would be to have more demand than the available supply and establishment of a priority system. This would introduce an interesting tussle between equity and equality between different solutions.

4

Forced Labor in Labor supply chains- contracting and information asymmetry

joint work with **Felix Papier** and **Christopher S. Tang**

4.1 Introduction

Forced labor is present in many sectors ranging from agriculture to construction, and from food processing to hospitality services.[1] Poor countries are prone to forced labor, but wealthy countries such as the United States, Kuwait, and Qatar are not immune. In 2022, the United States military has been accused of working with several labor contractors (e.g., Saudi Arabia-based Tamimi Global Company) who massively used forced labor to perform basic services on U.S. military sites located in Kuwait.[2] Forced labor is a global issue affecting 23.6 million people worldwide.[3]

The trend of forced labor issues has not abated in the US. Since 2007, the U.S. National Human Trafficking Hotline identified more than 164,000 victims of human trafficking.[4] In 2021, a former employee of a U.S.-based labor contractor pleaded guilty to coercing and forcing hundreds of Mexican migrants to perform physically-demanding agricultural work under inhumane working conditions that are deemed illegal.[5]

Forced labor is particularly prevalent when companies rely on *recruitment agencies* (also known as labor contractors or agents) to recruit workers. There are 77,000 agents globally providing services ranging from recruitment to on-site labor management. However, poor accountability and standards

[1] The ILO defines forced labor as *"all work or service which is exacted from any person under the menace of any penalty and for which the said person has not offered himself voluntarily"*. See: https://www.ilo.org/wcmsp5/groups/public/—ed_norm/—declaration/documents/publication/wcms_203832.pdf [31 Oct 2022].

[2] https://www.nbcnews.com/news/world/private-contractors-are-accused-abusive-labor-practices-us-military-ba-rcna52110 [31 Oct 2022].

[3] Vulnerable people such as women, migrants and children are prime target for forced labor. Among those forced labors identified by the International Labour Organization (ILO), 12% of them are children, 46% are women, and 15% are migrants. See: https://www.ilo.org/wcmsp5/groups/public/—ed_norm/—ipec/documents/publication/wcms_854733.pdf [31 Oct 2022].

[4] See: https://humantraffickinghotline.org/en/statistics [5 Jul 2023].

[5] https://fruitgrowersnews.com/news/forced-labor-of-mexican-agricultural-h-2a-workers-brings-charges/ [3/1/2023].

for these agencies increase the risk of coercion in the 'labor supply chain' especially when companies do not establish on-site direct oversight or conduct random independent audits. To address the rising concern of forced labor, we present a model of a labor supply chain and analyze mechanisms to deter recruitment agencies from coercing workers into forced labor. We consider a buyer who outsources its recruiting activities to a recruitment agency. (The terms recruitment agencies, labor contractors, and labor agents are interchangeable in this paper.)

Outsourcing labor-related activities to agencies is commonly observed especially when the buyer needs to source labor from a foreign country to perform production-, agricultural-, or construction-related tasks. Due to the lack of knowledge (e.g., about labor laws) and experience (in recruitment), the buyer usually contracts with recruitment agencies to provide a turnkey solution that includes sourcing, hiring, and training workers, as well as managing the workers on site. For example, when a food company needs temporary foreign workers to work on certain farms during the harvest season, the contracted agency often provides such a turnkey solution. Unless the buyer establishes an effective mechanism, recruitment agencies have financial incentives to coerce workers into forced labor.[6] In Section 4.2, we illustrate the scale and scope of recruitment agencies and the risk of forced labor with the example of foreign agricultural workers in the United States.

Our model captures the interplay between rational workers, a profit-maximizing agency, and a socially-responsible buyer. Our analytical results enable us to pinpoint how the market conditions (the size of the labor pool accessible by the agency, and the worker's reservation wage) and the economic conditions (the agency's cost of coercion, the buyer's audit cost, and the buyer's contract price) create an incentive for the agency to coerce. By examining the dynamics between the buyer and the agent in a Stackelberg game context, we find that, when the buyer has complete information about the

[6]For example, see https://www.seattletimes.com/business/slavery-forced-labor-a-florida-man-did-it-to-seasonal-foreign-workers/ [1 May 2023], or https://fruitgrowersnews.com/news/forced-labor-of-mexican-agricultural-h-2a-workers-brings-charges/ [1 May 2023].

agent, there exists a unique optimal contract (contract price and audit level) that deters the agent from coercion. Also, we investigate how the size of the labor pool accessible by the agent and the underlying labor demand affect the optimal coercion-free contract.

Using the results of the base model, we investigate the case when the actual size of the labor pool accessible by a single agent is private information even though the buyer knows its probability distribution. By analyzing a menu of contracts that is intended to entice the agent to select the contract according to his true type (i.e., high or low labor pool size), we find that the optimal menu of contracts possesses certain characteristics: the menu has to be designed to allow a high type agent to obtain a higher revenue. However, interestingly, we find that it is not necessarily optimal to offer a higher contract price to a high-type agent. Armed with the insights from the menu of contracts for the single-agent case, we analyze the agent selection problem of the multi-agent case, where each agent belongs to one of the multiple types (i.e., the agent's accessible labor pool can take on different discrete values), and each agent's actual labor pool size is private information. We develop a 'sequential' mechanism that mimics the *Dutch reverse auction* and that enables the buyer to select the agent with the highest labor pool size. We show how this sequential mechanism is coercion-free and profit-maximizing for the buyer.

We examine the results of our multi-agent model with private information by using a data set about labor agents and foreign agricultural workers in the US. Specifically, we examine how the composition of the agents based on different labor pool sizes affects the division of the total revenue among the buyer, the agent, and the workers. Based on our numerical analysis, we find that our optimal contract enables the buyer to capture 84% of the entire supply chain profit, while the agent only gets 3%, and the workers 13%. An argument can be made that this split-up of the profit is not fair. However, the observation of contracts in practice reveals that this is not too far from reality. Moreover, Stackelberg games usually result in the buyer maximizing their profit by ensuring that the agent just breaks even. From this angle, the mechanism suggested in this chapter is more equitable.

Our contribution is threefold. First, our paper represents an initial attempt to show how a 'well-balanced' contract that offers the right price and sets the right audit intensity level can indeed deter forced labor. Second, when the size of the labor pool is not observable or verifiable by the buyer, we show how an optimal contract of menus can entice the agent to report his true labor pool size and enable the buyer to deter the agent from coercing workers. Third, when there are multiple agents with different labor pool sizes that are not known to the buyer, we present a sequential contracting process that can enable the buyer to select the agent with the largest labor pool size while deterring this agent from coercion.

4.2 Background: Forced Labor in the U.S. Agricultural Sector

The number of temporary and seasonal workers working in the US is large. In the fiscal year 2020-21, more than 500,000 temporary and seasonal workers were recruited to work in the US with 310,000 working in the agricultural sector. In 2023, *The New York Times* investigated massive exploitation of the 130,000 unaccompanied minors migrating annually to the US by recruitment agencies. It was estimated that two-thirds of these minors illegally work full time.[7]

To understand the scope and scale of the use of labor contractors (LCs) (i.e., recruitment agencies) in recruiting foreign temporary workers in the United States, we examine the number of H2A temporary agricultural work visas issued by the U.S. Department of Labor between 2017 and 2022. Observe from Figure 4.1(a) that the percentage of these H2A visas that are filed by labor contractors has increased from 28% to 44% over the years. Also, Figure 4.1(b) reveals that the number of involved LCs has increased from below 500 to over 1,100 within the 6-year time window, and the average annual number of H2A visas filed by a labor contractor has increased from 80 to 150 over the years. While the number of visas filed by LCs has increased, the number of H2A visas filed by non-labor con-

[7] https://www.nytimes.com/2023/02/25/us/unaccompanied-migrant-child-workers-exploitation.html [28 Feb 2023].

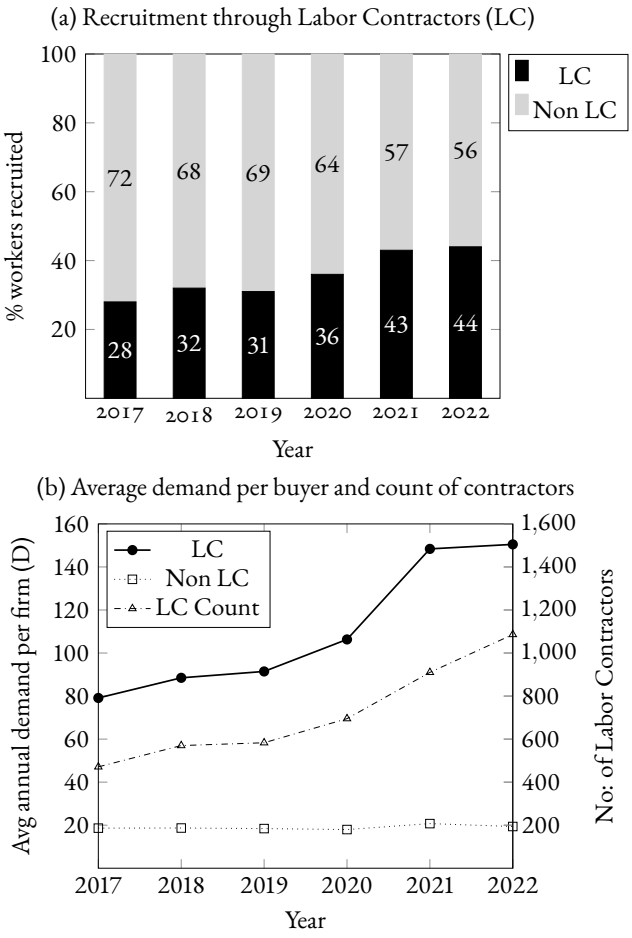


Figure 4.1: Statistics on labor contractors of foreign agricultural workers in the U.S., 2017 to 2022.

tractors (e.g., firms that do not use LCs and directly employ workers) remained flat. As US buying firms are relying heavily on labor contractors to provide services ranging from recruitment and labor management, these buyers do not have direct control of the recruitment process nor full visibility of the on-site working conditions. With hidden action and hidden information, there are incentives for some unethical contractors to coerce workers into forced labor. In the United States, 70% of the labor-related human trafficking alerts received by the US Hotline of Human Trafficking concern victims on H2A/H2B temporary visas.[8] Based on our analysis, there is a correlation of 0.424 between the num-

[8] https://polarisproject.org/wp-content/uploads/2022/07/Labor-Trafficking-on-Specific-Temporary-

ber of human-trafficking alerts received by the U.S. National Human Trafficking Hotline (per 1,000 workers with H2A visa) in each state and year, and the share of recruitment through labor contractors.[9] This correlation reveals that buyers are exposed to the risk of involving forced labor when they rely on agencies to recruit. As such, there is a need to develop mechanisms to deter these agencies from coercing workers into forced labor.

4.3 Literature Review

Forced labor in operations has received considerable attention in recent years. In this section, we review three related streams of research: (a) Studies of modern slavery from an economic theory perspective, (b) studies of modern slavery in supply chains, and (c) studies of buyer-supplier-interaction that entail unethical activities. In the economics literature, the seminal paper Basu and Van (1998) models child and adult labor markets and studies the implications of stricter labor laws implemented by first-world countries. The authors conclude that such legal measures have unintended consequences such as child laborers moving from export-oriented industries to domestic industries. Basu and Chau (2004) study the use of child labor in case of debt bondage and conclude that trade sanctions as a mitigation strategy can affect vulnerable families negatively, Acemoglu and Wolitzky (2011) model the concept of 'guns' that the employer can coerce workers. The authors observe that if these 'guns' are sufficiently cheap and effective, then coerced workers will stay with abusive employer. Beber and Blattman (2013) develop a model for studying the recruitment of child soldiers. They find that misinformation and indoctrination are effective for rebel leaders to exploit child soldiers. Crane (2013) considers modern slavery as a management practice and shows how micro-level organizational capabilities allow firms to exploit macro-institutional conditions to install. Overall, this stream of research

Work-Visas-by-Polaris.pdf [31 Oct 2022].

[9] Details can be provided by the authors upon request.

does not take the supply chain perspective into consideration.

Several papers analyze modern slavery and human trafficking in supply chains. Gold et al. (2015) examined various mitigation strategies against modern slavery in supply chains. Stevenson and Cole (2018) critically examine and classify companies' release statements in response to the UK Modern Slavery Act and find that many firms use the same approaches for tackling modern slavery but have to be more innovative in particular with respect to handling labor contractors. Cousins et al. (2020) analyze the effect of the UK Modern Slavery Act on the shareholder wealth of affected companies. They find that the introduction of this Act had a positive wealth effect only for firms that have a proven track record of fighting modern slavery. Stevenson (2021) investigates how the bystander effect reduces the collective propensity of whistle-blowing in a society where exploitation and forced labor are rampant. Pinnington et al. (2022) use the ETI Framework to assess modern slavery statements of 95 suppliers of the UK government. Konrad et al. (2023) identify key features of human trafficking activity and argue that researchers in the field of forced labor have to consider these, sometimes subtle, features in their work. LeBaron (2021) argues that supply chains provide the right context for examining forced labor, which justifies the context of this paper. While this stream advocates the supply chain context for studying forced labor, it does not delve into the interaction and the contracting issues between buyers and agents in the 'labor' supply chain.

There is an emerging stream in which researchers examined unethical activities arising from buyer-supplier interaction and contracting. Babich and Tang (2012) analyze hidden product adulteration by suppliers in a two-tier supply chain and propose a deferred-payment mechanism and an inspection mechanism (as well as a combined approach) to prevent suppliers from providing low-quality supplies. The authors identify conditions under which a certain approach is dominant. Rui and Lai (2015) extend the work of Babich and Tang (2012) by endogenizing the buyer's quantity decision into the model. Plambeck and Taylor (2016) study the effect of the buyer imposing penalties on suppliers for unethical business practices. The authors show that, under larger penalties, the supplier uses more

effort on hiding information (termed 'backfiring') than on avoiding harm. Guo et al. (2016) consider a scenario with one buyer and two suppliers, a responsible and a risky one. The buyer sells to a market in which part of the market is ethically sensitive. The buyer has to decide between single sourcing from either supplier, sourcing responsibly only for the ethically sensitive part of the market, and sourcing responsibly for the whole market. Chen and Lee (2017) analyze the effectiveness of process audits, contingency payments, and supplier certification in controlling the risk of unethical supplier behavior. They find that joint usage of all three strategies makes supplier screening more effective and lowers buyer costs. Caro et al. (2018) explore a setting with two buyers who each audit the same supplier. The authors compare independent, joint, and shared auditing between the two buyers and find that joint auditing creates supply chain value under certain conditions.

Cho et al. (2019) consider potential child labor in a two-tier supply chain. The authors observe that the optimal supplier strategy depends on the inspection cost. For low inspection cost, the buyer chooses a high inspection level, for high inspection cost a medium level combined with a sufficiently high wholesale price. Kalkanci and Plambeck (2020b) explore the conditions under which a buyer should publish the list of suppliers who violated standards (known as 'name and shame' approach). Caro et al. (2020) analyze data of subcontracting in the apparel industry and find that more than one third of subcontracting in their data set has been performed illegally. The authors identify, among others, price pressure as a major factor for illegal subcontracting. Lu and Tomlin (2022) explore communication of a supplier's social responsibility through self assessments and identify an equilibrium of truthful information sharing, in which the buyer differentiates quantity and audit level between supplier types. Huang et al. (2022) consider a 3-tier supply chain and identify three strategies at the disposal of the buyer (direct control, total delegation, and no effort), which are each optimal under certain conditions. Ramchandani et al. (2021) use machine learning tools to detect human trafficking in commercial sex supply chains, and Li et al. (2023) propose classification tools to detect illegal massage businesses from online customer reviews. Kunz et al. (2023) analyze the case of a success-

ful program to combat modern slavery in the agricultural industry in Florida. The authors develop a model to analyze the adoption time of the program after market shocks and identify factors that determine the adoption time. Our work complements this stream of research work by considering the worker's reservation price to model the interplay between wage and coercion. In doing so, it enables us to examine the market conditions that induce recruitment agents to use coercion, and how the buyer can use pricing and auditing mechanism to curb forced labor.

4.4 Base Model Description: A Single Agent

We consider a 'labor supply chain' comprising a single *buyer* and a single *agent*. The buyer intends to produce products (or provide services) that require D units of labor from a recruitment agency to complete a project (e.g., a construction project, a project to harvest a certain crop). For each unit of labor, the buyer obtains revenue r (derived from its contract with its direct customer) that has been established in advance. We consider the case when D and r are exogenous parameters, which allows us to focus on the contract between the buyer and the agent.

The buyer chooses and offers a contract price p per worker to the agent. In return, the agent is responsible to recruit and manage workers on behalf of the buyer. The agent has to hire a sufficient number of workers to fulfill demand (or the contract quantity) D. The agent may incur cost for recruiting and managing workers like cost of organizing the recruitment event and other administrative costs to get them to the destination, but we normalize these costs to zero.

Worker pool size K. The buyer sources labor from the agent because of the agent's knowledge about labor laws in different countries and its access to a workforce pool of size K. (In the base model we assume K is known to both parties, but we shall extend our analysis to the case when K is private information that is known only to the agent in Section 4.6.) To ensure that the contract between the buyer and the agent is feasible, we assume that the size of the labor market $K \geq D$.

Worker's reservation value v. In the labor market pool of size K, each worker corresponds to one unit of labor (without loss of generality). Each worker has a reservation value $v \sim U[0,1]$, where v represents the outside option value (this could be an alternative job or unemployment benefits). For any given wage $w \in [0,1]$ offered by the agent, workers with reservation value $v \leq w$ will agree to work *willingly* without the need to exert coercion. Because $v \sim U[0,1]$ and the labor market size is K, $w \cdot K$ workers will agree to work for the buyer (through the agent) at wage w, costing the agent $w^2 \cdot K$ in wages.[10] To meet demand D without coercion, the agent has to set $w = D/K$ to recruit $w \cdot K = D$ workers. In doing so, the agent's cost is $w^2 \cdot K = \frac{D^2}{K}$, which is convex-increasing in the buyer's demand D.

Agent's coercion level g. Observe that the agent's labor cost $\frac{D^2}{K}$ is convex-increasing and yet his revenue pD (to be collected from the buyer) is linear in D. Hence, when D is high, the agent can incur a loss unless p is sufficiently high. However, the agent can cut labor cost to ensure profitability by recruiting 'additional' workers through *coercion* and retaining its existing workers by keeping the original low wage (see, also Acemoglu and Wolitzky, 2011). In our model, the agent can exert costly *coercion level* $g \geq 0$ to force additional workers to work 'unwillingly'.[11] Essentially, the agent can use coercion g as a *'low cost' substitute* for wage w. For any given coercion level g, the corresponding coercion cost is $c_g \cdot g$, and it will generate a 'burden' $\gamma \cdot g$ for an affected worker to refuse to work for the agent, where γ represents the 'effectiveness' of coercion,[12] where this burden γ will effectively reduce a worker's reservation value from v to $(v - \gamma g)$.

Two groups of workers: wK coercion-free workers and $\gamma g \cdot K$ coerced workers. Given wage

[10] We assume that the agent offers the same wage to all workers and cannot observe the reservation value v of each worker and cannot pay workers at different wages for the same tasks. Paying workers at different wages will violate the labor law and would not be tolerated neither by the buyer nor the local government.

[11] Coercion level g is a measure of the pressure on workers that can be created through threats, withholding pay, retaining official documents, debt bondage, etc.

[12] The burden generated by coercion includes the mental, legal, and physical harm inflicted on the workers by the agent (Acemoglu and Wolitzky, 2011).

w and coercion level g, wK workers (with $v \leq w$) will work willingly without the need for the agent to exert costly coercion. However, in order to get additional workers (from the remaining labor pool that has $v > w$) by paying the same wage w, the agent has to exert coercion effort g on top of the wage w to force those workers with $v \in (w, w + \gamma g]$ to work for the agent (who take the aforementioned burden γg into consideration).

Therefore, the agent can recruit two groups of workers by paying w to each worker in 'real cost': wK workers to work willingly without coercion, and additional $\gamma g \cdot K$ forced labor workers by exerting costly coercion g.[13] Hence, in the spirit of Caro et al. (2018) and Plambeck and Taylor (2016), the agent's 'non-compliance' labor practice level can be measured in terms of the percentage of workers who have been coerced to work; i.e., $\frac{\gamma g}{w+\gamma g}$. Consequently, the agent has a *mixed pool* of labor. The regular wage for the entire pool $[0, w + \gamma g]$ is $w \cdot (w + \gamma g)K$. Also, given the coercion cost $c_g \cdot g$ per coerced worker, the cost of coercion for those $\gamma g \cdot K$ forced labor workers is $c_g \cdot g \cdot \gamma g K = c_g \gamma g^2 K$.

In order to recruit D workers, the agent has to set the coercion level g and wage w such that $(w + \gamma g)K = D$. This linear relationship between w and g implies that it suffices for the agent to decide on g "only" and retrieve w by considering:

$$w = \frac{D}{K} - \gamma g. \tag{4.1}$$

To ensure $w \geq 0$, we require $g \leq \frac{D}{K\gamma}$. Consequently, the total cost incurred by the agent can be written as a function of coercion g succinctly as $wD + c_g \gamma g^2 K = \left(\frac{D}{K} - \gamma g\right) D + c_g \gamma g^2 K$.

Buyer's audit effort a. Because the buyer is effectively delegating the recruitment and the management of the workers to the agent, it is prudent for the buyer to conduct audits to discourage the agent from exerting coercion. Doing so will also reduce potential legal liability and collateral damage (that

[13] We assume that the agent can first identify workers who decline to work willingly at wage w and then apply coercion to get these workers to work unwillingly.

we shall explain later). To discourage potential coercion, consider the case when the buyer conducts audit with probability a in the same spirit as Caro et al. (2018). For any given audit effort level a, the buyer incurs an audit cost $c_a \cdot a$. Specifically, given the buyer's audit level a and the agent's coercion level g, we assume that the audit detection probability is $f(a,g) = a\frac{\gamma g}{w+\gamma g} = a\frac{\gamma g K}{D}$.[14] Note that $f(a,g)$ is increasing in a and in g.

Agent's loss m and buyer's reputation damage Ψ when forced labor is involved. When the buyer's audit detects coercion, the buyer cancels the agent's contract without paying pD to the agent. The agent, however, has already hired the workers and thus faces labor cost wD and coercion cost $c_g\gamma g^2 K$ regardless of the audit result. In addition to contract cancellation, the agent may face legal fines and reputation damage m that result from coercing workers.

If the buyer's audit failed to detect forced labor, then the buyer suffers reputation damage and legal liabilities Ψ that result from the presence of forced labor in the buyer's supply chain. We assume that damage Ψ is sufficiently large such that the buyer prefers sourcing forced-labor-free work, which is the case for $\Psi > rD$. It is reasonable to assume for major multinational firms to prefer labor-free work to avoid long-term damage and liabilities.

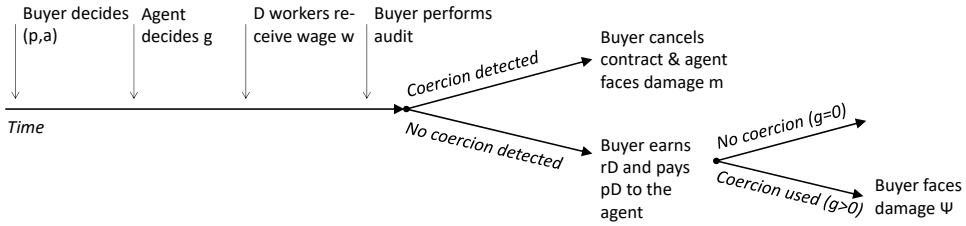


Figure 4.2: Timeline of decisions and payments.

Agent's profit $\pi(\cdot)$ and buyer's profit $\Pi(\cdot)$. The above model allows us to specify the buyer's and the agent's profits when the buyer sets strategy (p,a) and the agent sets strategy g (where the wage w is

[14] We consider this particular form because $\frac{\gamma g}{w+\gamma g}$ represents the proportion of the workers recruited by the agent is subjected to coercion, and a represents the audit level.

a linear function of g as stated in (4.1)). By considering the audit-detection-probability $f(a,g)$ along with payments, the agent's expected profit is given by:

$$\pi(p,a,g) = pD \cdot (1 - f(a,g)) - wD - c_g \cdot \gamma g^2 K - f(a,g) \cdot m \cdot 1_{\{g>0\}}, \tag{4.2}$$

where $f(a,g) = a\frac{\gamma g K}{D}$ and 1 is the indicator function. Here, the first term represents the income from the buyer (pD) if the contract concludes (i.e., if the audit does *not* detect coercion, which happens with probability $(1-f(a,g))$), the second term are the wages that are paid to the workers before audit (wD), the third term represents the coercion cost ($c_g \cdot \gamma g^2 K$), and the last term the loss (m) in the case that coercion is detected (which happens with probability $f(a,g)$). As a consequence, the agent has to carefully choose g: a higher g leads to a lower wage w but increases the detection probability $f(a,g) = a\frac{\gamma g K}{D}$ of getting caught, losing the contract, and facing the loss m. Similarly, the buyer's expected profit is given by:

$$\Pi(p,a,g) = (r-p)D \cdot (1 - f(a,g)) - c_a \cdot a - (1 - f(a,g)) \cdot \Upsilon \cdot 1_{\{g>0\}}. \tag{4.3}$$

Here, the first term represents the buyer's net margin on the work ($(r-p)D$) if the contract is not cancelled (when no coercion has been found through audit), the second term is the audit cost ($c_a \cdot a$), and the third term represents the reputation or remediation cost Υ detected by the audit (with probability $1 - f(a,g) = 1 - a\frac{\gamma g K}{D}$). We assume that the parameters allow for non-negative profit for the buyer.

A sequential game. The dynamic interaction between the buyer and the agent can be captured in a Stackelberg game as depicted in Figure 4.2. In this game, the buyer acts as the leader who proposes the contract (p,a) to the agent, and the agent acts as the follower by deciding its best profit-maximizing-response strategy by choosing $g^*(p,a)$ as a function of contract (p,a). The agent's participation con-

straint is based on the assumption that he will accept the contract only if he can achieve a non-negative profit. We assume that the model parameters are common knowledge. As such, we can solve this sequential game depicted in Figure 4.2 via backward induction.

4.5 Base Model with a Single Agent and Full Information

In this section, we first establish a benchmark case in which the buyer does not rely on the agent. Instead, the buyer uses its costly internal resources to ensure that no forced labor is involved. Then we analyze the case when the buyer outsources its recruitment to an external agent who may exert coercion due to self-interest. By comparing these two mechanisms, we examine if there exists a contract (p, a) that can enable the buyer to induce the agent to refrain from coercing workers.

4.5.1 No coercion through in-house recruitment: a benchmark

In the benchmark case, the buyer uses its internal recruiters to find workers to ensure no forced labor is involved. However, without experience, the buyer's recruitment cost u per worker is greater than the agent's recruitment cost (normalized to zero).[15] Therefore, if the buyer offers a wage w to each worker directly (instead of going through an agent), wK workers will work willingly because their reservation value $v \sim U[0,1]$. Hence, the buyer will need to offer a 'fair wage' $w^0 = D/K$ in order to recruit $w^0 K = D$ workers to meet the demand D.[16] Consequently, the buyer will incur $w^0 \cdot w^0 K = D^2/K$ in wages (which is the same cost incurred by the agent as explained in Section 4.4). Hence, when the

[15] If the buyer and the agent were a centralized entity (for example, if the buyer acquired the agent), the centralized supply chain solution would be reduced to the benchmark case with $u = 0$.

[16] The notion of fairness is not well-defined (Chen et al., 2022). However, we define fair wage to be the wage at which workers work willingly without coercion.

buyer recruits workers directly, the buyer's benchmark profit is Π^0, where:

$$\Pi^0 = (r - w^0 - u)D = \left(r - \frac{D}{K} - u\right) D. \tag{4.4}$$

Given the fair wage $w^0 = D/K$, we can observe from (4.1) that the agent's wage $w = D/K - \gamma g$ (in the case of outsourcing) is always strictly lower than the fair wage w^0 if the coercion level $g > 0$. Therefore, if there is transparency in the wage w, the buyer can detect coercion if $w < w^0$. (Of course, coercion can be present even when $w \geq w^0$.) As commonly observed in labor supply chains, we shall assume that wage is opaque and the buyer can learn about the real wage through auditing as explained in the next section.[17]

4.5.2 Potential coercion in outsourced recruitment

We now analyze the case when the buyer outsources the labor management to the agent by contracting labor quantity D.[18] We begin by deriving the agent's profit-maximizing response level $g^*(p, a)$ to contract (p, a).

Agent's Best Profit-Maximizing Response: $g^*(p, a)$.

For any given buyer contract (p, a), we examine the agent's profit-maximizing coercion response $g^*(p, a)$ by solving the agent's problem: $\max_{\frac{D}{K\gamma} \geq g \geq 0} \pi(p, a, g)$, where the agent's expected profit function $\pi(p, a, g)$ is given in (4.2), and the upper bound on g is intended to ensure the corresponding wage w given in (4.1) is non-negative. By considering $f(a, g) = a\frac{\gamma g K}{D}$ and the first order condition with respect to g, we get:

[17] Chen et al. (2022) discussed that some agents can ask for 'kickback' so that the real wage paid to the workers is difficult to verify even with audits.

[18] When we consider the case when the size of labor pool is private information in Section 4.6, the contract labor quantity will depend on the contract.

Proposition 1. *For any given buyer contract (p, a), the agent's best profit-maximizing coercion response $g^*(p, a)$ satisfies:*

$$g^*(p, a) = \left[min \left(\frac{D}{2c_g K} - \frac{(p + \frac{m}{D})a}{2c_g}, \frac{D}{K\gamma}. \right) \right]^+, \tag{4.5}$$

where coercion level $g^*(p, a)$ is decreasing in price p, audit effort a, and coercion cost c_g. The corresponding wage $w^*(p, a) = D/K - \gamma g^*$. Also, if $\gamma < 2c_g$, then $g^*(p, a) = [\frac{D}{2c_g K} - \frac{(p+\frac{m}{D})a}{2c_g}]^+$.

All proofs are provided in the Appendix. For any given buyer contract (p, a), Proposition 1 specifies the agent's coercion level $g^*(p, a)$ that possesses various expected characteristics (e.g., decreasing in the price p and audit level a). More importantly, it is easy to observe from (4.5) that the agent's optimal coercion level $g^*(p, a) > 0$ if $(D/K) > (p + m/D)a$, or equivalently, $a < \frac{D/K}{(p+m/D)}$. Note that this condition for the agent to exert coercion holds when: (1) the buyer's demand D is high; (2) the buyer offers a low price p to the agent; or (3) the buyer's audit effort a is low. Hence, when the buyer's demand D is high, the agent has strong incentive to use coercion to reduce wages as explained in Section 4.4. Therefore, to curb coercion, the buyer must offer the agent a decent price p and exert commensurable audit effort a. It is interesting that the coercion level $g^*(p, a)$ does not depend on the effectiveness of coercion, γ and that the condition $g^*(p, a) > 0$ is independent of the coercion cost rate c_g.

To ensure a coercion-free-setting so that $g^* = 0$, (4.5) implies that the buyer needs to set (p, a) that satisfies: $(p + m/D) \cdot a = D/K \equiv w^0$. This observation reveals that p and a are 'substitutes' for the buyer, which is analogous to w and g that are substitutes for the agent. Specifically, the buyer can deter the agent from coercing its workers by offering a price $p = \frac{D/K}{a} - \frac{m}{D}$.

Buyer's optimal profit-maximizing contract: (p^*, a^*).

Anticipating the agent's profit-maximizing response $g^*(p,a)$ as stated in Proposition 1, we now determine the buyer's profit-maximizing contract (p^*, a^*) by solving the buyer's problem:

$$\max_{\pi(p,a,g^*(p,a))\geq 0} \Pi(p,a,g^*(p,a)), \tag{4.6}$$

where the buyer's profit $\Pi(p,a,g)$ is given in (4.3), and $\pi(p,a,g^*(p,a)) \geq 0$ is the agent's participation constraint with $\pi(p,a,g)$ given in (4.2) and $g^*(p,a)$ given in Proposition 1.

By considering the model assumption that $\Psi > rD$ as stated in Section 4.4, we know that the buyer will always prefer a contract that can deter the agent from coercing workers over a contract that induces worker coercion, which we formally establish in the following lemma:

Lemma 1. *Suppose the reputation damage $\Psi > rD$. Then it is optimal for the buyer to choose a profit-maximizing contract (p^*, a^*) that deters the agent from coercing, with $g^*(p^*, a^*) = 0$.*

To deter the agent from coercing workers so that $g^*(p,a) = 0$, we can apply (4.5) to show that, for given a, the buyer has to set: $g^*(p,a) = 0 \Leftrightarrow p = \frac{D}{aK} - \frac{m}{D}$. Under such a coercion-free contract, we can apply (4.1) to show that the agent will offer the worker a wage $w^*(p,a) = \frac{D}{K} - \gamma g^*(p,a) = \frac{D}{K} = w^0$, which is the 'fair' wage as defined in Section 4.5.1.

To ensure the agent will participate in such a coercion free contract (p,a) that has $g^* = 0$, it is easy to check from (4.2) that the agent's participation constraint can be simplified as: $\pi(p,a,g^* = 0) = pD - \frac{D}{K}D \geq 0$, which holds if and only if $p \geq D/K$. This implies that the buyer must pay the agent above the fair wage $p \geq D/K = w^0$ to ensure participation from the agent. It shows what experts claim: Paying below the 'fair' wage cannot be profitable for the agent without coercion, and it demonstrates why forced labor arises in such settings.

By considering Lemma 1, we can restrict our attention to coercion-free contracts that have $g^*(p,a) =$

0 and that ensure the agent's participation constraint $p \geq D/K$. The buyer can determine his optimal coercion free contract by solving:

$$\max_{p \geq D/K} \Pi(p, a, g^*(p, a) = 0), \tag{4.7}$$

where the buyer's profit $\Pi(p, a, g)$ is given in (4.3). By considering the first order condition along with the lower bound for p, we get:

Proposition 2. *Suppose $\Psi > rD$. Then the buyer's optimal contract (p^*, a^*) satisfies:*

1. **High audit cost.** *When $c_a \geq K(\frac{D}{K} + \frac{m}{D})^2$, $(p^*, a^*) = (\sqrt{\frac{c_a}{K}} - \frac{m}{D}, \frac{D}{\sqrt{c_a K}})$, where $p^* \geq (D/K)$, and the buyer's profit $\Pi(p^*, a^*, g^*(p^*, a^*)) = D\left(r - 2\sqrt{\frac{c_a}{K}}\right) + m$.*

2. **Low audit cost.** *When $c_a < K(\frac{D}{K} + \frac{m}{D})^2$, $(p^*, a^*) = \left(\frac{D}{K}, \frac{D^2}{D^2 + mK}\right)$ and the buyer's profit $\Pi(p^*, a^*, g^*(p^*, a^*)) = rD - \frac{D^2}{K} - \frac{c_a D^2}{D^2 + mK}$.*

The buyer's profits $\Pi(p^*, a^*, g^*(p^*, a^*))$ in both cases are increasing in K.

Observe from Proposition 2 that the optimal price p^* (and consequently the agent's 'earning potential' $p^* \cdot D$) as well as the optimal audit level a^*, are increasing in the demand D. This is because, when D increases, the buyer has to offer a higher unit price p^* because the agent has to pay a higher wage to 'dig deeper into the labor pool' to recruit workers with higher reservation value v as explained in Section 4.4. As the agent's wage cost increases, the agent has a stronger incentive to increase his coercion level g^* to reduce the wage cost. To mitigate this increased coercion risk, the buyer has to increase its audit level a^* as a countermeasure. The exact reverse effects occur when labor pool size K increases instead of demand D.

Next, it is interesting to observe from statement 2 of Proposition 2 that, when the audit cost c_a is low, the buyer can always set $p^* = D/K$ (which is the same as the fair wage $w^0 = D/K$) and set

the corresponding audit level $a^* = D^2/(D^2 + mK)$. Doing so will eliminate coercion and leaves no surplus, $(p - D/K)D$, to the agent. However, when the audit cost c_a is sufficiently high, statement 1 of Proposition 2 reveals that it is more cost effective to reduce the audit level a to a minimum level to keep the agent 'at bay' (to ensure $g^* = 0$) and yet increase the price p^* above the fair wage $w^0 = D/K$ without extracting the entire surplus from the agent.

To elaborate, it is easy to check from statement 1 that, when the audit cost c_a is high, the agent has a stronger incentive to coerce because he knows the buyer cannot afford to increase its audit level. To deter the agent from coercion, statement 1 reveals that the buyer has to offer a unit price above the fair wage $p^* \geq (D/K)$ so that the agent can enjoy a higher surplus without the need to coerce. However, when the audit cost c_a is low, the buyer can afford to exert a higher level of audit. As such, it is adequate for the buyer to offer the fair wage $p^* = D/K$ to deter the agent from coercing. In summary, to deter agent from coercing, Proposition 2 asserts that the buyer must offer the agent a unit price that is at least the fair wage $w^0 = D/K$.

If in-house recruitment is possible, under which conditions is outsourcing preferable to insourcing? In the following corollary, we show that if the buyer's internal recruitment cost u is above a certain threshold, sourcing through labor contractors is preferable.

Corollary 1. *When the buyer's internal recruitment cost u is high so that $u + \frac{D}{K} > p^* + \frac{c_a a^*}{D}$, then it is beneficial for the buyer to source from the agent by offering the optimal, coercion-free contract (p^*, a^*) as stated in Proposition 2.*

Even though Corollary 1 is intuitive, it provides a specific condition under which it is beneficial for the buyer to source his labor from the agent by offering an optimal contract (p^*, a^*) (as stated in Proposition 2) that can entice the agent to recruit workers without coercion. Recognizing this optimal contract relies on the assumption that the buyer has full information about the agent, we shall extend our analysis to the case when size of the labor pool accessible by the agent is private information in the

next section.

4.6 Single Agent with Private Information

In the base model as presented in Section 4.4 and analyzed in Section 4.5, the agent's labor pool size K is common knowledge. However, the exact value of this parameter may not be observable or verifiable by the buyer especially when agents are small or when the recruitment is seasonal and conducted in a foreign country. In addition, observe from Proposition 2 that the buyer's optimal profit $\Pi(p^*, a^*, g^*)$ is increasing in the size of labor pool K, which implies that the agent has the incentive to deceive the buyer by reporting a greater labor pool size K to be more attractive to the buyer. This observation motivates us to develop a truth-telling contract for the case when the labor pool size K is privately known to the agent and yet the buyer knows only its distribution.

4.6.1 Model Preliminaries

Information Asymmetry. Contracts under information asymmetry have been examined in the operations management literature (e.g., Ha and Tong, 2008; Yang et al., 2009; Shen et al., 2019). For simplicity, we assume that the size of the labor market pool accessible by the agent is either low with size K_l (i.e., the agent is of 'type l') or high with size K_h (i.e., the agent is of 'type h'), where $K_l < K_h$. The actual labor pool size is private information that is only known to the agent; however, the buyer knows that K_l occurs with probability $\alpha_l = \alpha$, and K_h with probability $\alpha_h = 1 - \alpha$, where where $0 < \alpha < 1$.

Menu of Contracts. Instead of proposing a single contract price p as in the base model (Section 4.4), the buyer needs to offer a *menu of contracts* to induce the agent to select the contract that reveals his true type. Given there are only two types of agents, we consider a menu of two contracts (p_i, D_i, a_i) for $i = l, h$ for the agent to choose from, where p_i is the price of contract i, a_i the audit level, and D_i

the quantity to contract to the agent.[19]

We assume that the buyer will source the remaining $(D - D_i)$ workers internally as described in Section 4.5.1 by incurring wage $(D - D_i)/K$ and management fee u per worker, where K is the labor pool size accessible by the buyer, such that the buyer can earn additional profit $\Pi^c(D_i)$ as explained in Section 4.5.1, where:

$$\Pi^c(D_i) = (r - u - (D - D_i)/K)(D - D_i). \tag{4.8}$$

Furthermore, we assume that the damage m to the agent in case of coercion is proportional to the size of the contract, i.e., damage under contract i is $m\frac{D_i}{D}$. This construct is consistent as the damage equals m when $D_i = D$ as in the base model.

Agent's profit. For an agent of type t (with accessible workforce of size K_t) to opt for contract i, his expected profit is $\pi_t(p_i, D_i, a_i, g_t^*(p_i, D_i, a_i))$. By applying (4.2) and (4.1), we get:

$$\begin{aligned}
&\pi_t(p_i, D_i, a_i, g_t^*(p_i, D_i, a_i)) = \\
&= p_i D_i (1 - f(a_i, g_t^*(p_i, D_i, a_i))) - \left(\frac{D_i}{K_t} - \gamma g_t^*(p_i, D_i, a_i)\right) D_i \\
&- c_g \gamma g_t^*(p_i, D_i, a_i)^2 K_t - f(a_i, g_t^*(p_i, D_i, a_i)) m \frac{D_i}{D} 1_{\{g_t^*(p_i, D_i, a_i) > 0\}},
\end{aligned} \tag{4.9}$$

where the coercion-detection probability is $f(a_i, g_t) = a_i \frac{\gamma g_t K_t}{D_i}$. By considering the first-order condition, a type t agent who chooses contract i will exert coercion level $g_t^*(p_i, D_i, a_i)$, where:

$$g_t^*(p_i, D_i, a_i) = \left[\frac{D_i}{2c_g K_t} - \frac{(p_i + \frac{1}{D_i}(m\frac{D_i}{D}))a_i}{2c_g}\right]^+, \; t = h, l \text{ and } i = h, l. \tag{4.10}$$

[19] It is easy to check that truth-telling contracts do not exist for the contract labor quantities $D_h = D_l = D$. For this reason, we shall consider the case when the contracted labor quantity is type-specific.

Because of the assumption $\gamma < 2c_g$, it is easy to check that $g_t^* < \frac{D_i}{K_t\gamma}$ (see Proposition 1). Then, with this level of coercion $g_t^*(p_i, D_i, a_i)$, an agent of type t who chooses contract i earns:

$$\pi_t(p_i, D_i, a_i, g_t^*(p_i, D_i, a_i)) \tag{4.11}$$

$$= \left(p_i D_i - \frac{D_i^2}{K_t}\right) + \gamma g_t^*(p_i, D_i, a_i)\left(D_i - c_g K_t g_t^*(p_i, D_i, a_i) - \frac{a_i K_t}{D_i}(p_i D_i + m\frac{D_i}{D})\right)$$

$$= \left(p_i D_i - \frac{D_i^2}{K_t}\right) + \frac{\gamma}{4c_g K_t}\left(\left[D_i - K_t\left(p_i + \frac{m}{D}\right)a_i\right]^+\right)^2, \tag{4.12}$$

where the second term corresponds to additional profit obtained by the agent from using coercion.

Incentive Compatibility (IC) and Individual Rational (IR) constraints. To ensure that the proposed menu of contracts (p_i, D_i, a_i) entices the agent to participate and reveal his true type, the menu of contracts must satisfy the IC constraints $\pi_t(p_t, D_t, a_t, g_t^*(p_t, D_t, a_t))) \geq \pi_t(p_i, D_i, a_i, g_t^*(p_i, D_i, a_i)))$ and the IR constraints $\pi_t(p_t, D_t, a_t, g_t^*(p_t, D_t, a_t))) \geq 0$. The IC constraints ensure an agent of type t is always better off by selecting the contract according to his true type, whereas the IR constraints ensure the agent's participation.

4.6.2 The buyer's problem: Optimal menu of contracts

The contract menu (p_i, D_i, a_i) involves the audit level a_i to be chosen by the buyer. Because $\Psi > rD \geq rD_i$, the buyer prefers to choose a contract menu that is coercion-free. With the IC constraint, an agent of type t chooses the contract t according to his true type t. To enforce zero coercion at minimal audit cost, we apply (4.10) to determine the corresponding audit level so that:

$$g_i^*(p_i, D_i, a_i) = 0 \;\Leftrightarrow\; a_i^*(p_i, D_i) = \frac{D_i}{K_i(p_i + m/D)}, i = l, h. \tag{4.13}$$

By applying the audit level $a_i^*(p_i, D_i)$ given in (4.13), it is easy to check from (4.12) that, when the agent of type i, $i = l, h$, selects the contract truthfully, he earns $\pi_i(p_i, D_i, a_i^*(p_i, D_i)) = p_i D_i - \frac{D_i^2}{K_i}$.

[20] However, when the agent of type i selects the contract untruthfully, we have two cases to consider. First, when the agent of type h chooses contract l, he earns $\pi_h(p_l, D_l, a_l^*(p_l, D_l) = p_l D_l - \frac{D_l^2}{K_h}$. Second, when the agent of type l chooses contract h, he has an incentive to coerce and earns:

$$\begin{aligned}\pi_l(p_h, D_h, a_h^*(p_h, D_h)) &= \left(p_h D_h - \frac{D_h^2}{K_l}\right) + \frac{\gamma}{4c_g K_l}\left(\left[D_h - K_l\left(p_h + \frac{m}{D}\right)a_h^*(p_h, D_h)\right]^+\right)^2 \\ &= \left(p_h D_h - \frac{D_h^2}{K_l}\right) + \frac{\gamma}{4c_g K_l}\left(1 - \frac{K_l}{K_h}\right)^2 D_h^2 = p_h D_h - \delta\frac{D_h^2}{K_l}, \qquad (4.14)\end{aligned}$$

where we let $\delta \equiv 1 - \frac{\gamma}{4c_g}\left(1 - \frac{K_l}{K_h}\right)^2$, with $0 < \delta < 1$ due to assumption $\gamma < 2c_g$ (see Proposition 1).

As the buyer relies on his internal resources to recruit the remaining $(D - D_i)$ workers, he can use (4.3) along with the additional profit function $\Pi^c(D_i)$ given in (4.8) to determine his expected total profit, $\widehat{\Pi}(p_l, D_l, p_h, D_h)$, where:

$$\begin{aligned}&\widehat{\Pi}(p_l, D_l, p_h, D_h) \\ &= \alpha_l\left((r - p_l)D_l - c_a a_l^*(p_l, D_l) + \Pi^c(D_l)\right) + \alpha_h\left((r - p_h)D_h - c_a a_h^*(p_h, D_h) + \Pi^c(D_h)\right) \\ &= \alpha\left((r - p_l)D_l - \frac{c_a D_l}{K_l(p_l + m/D)} + \left(r - u - \frac{D - D_l}{K}\right)(D - D_l)\right) \\ &+ (1 - \alpha)\left((r - p_h)D_h - \frac{c_a D_h}{K_h(p_h + m/D)} + \left(r - u - \frac{D - D_h}{K}\right)(D - D_h)\right). \qquad (4.15)\end{aligned}$$

By considering the buyer's profit function $\widehat{\Pi}(p_l, D_l, p_h, D_h)$ given in (4.15) along with the agent's profit $\pi_l(p_h, D_h, a_h^*)$ given in (4.14), we can formulate the buyer's 'menu contracting' problem that involves the IC and IR constraints as follows:

[20] Note that we suppress the argument $g_t^*(p_i, D_i, a_i)$ for notation convenience.

$$\max_{(p_l, D_l, p_h, D_h)} \widehat{\Pi}(p_l, D_l, p_h, D_h) \tag{4.16}$$

subject to

$$ICh: \ \pi_h(p_h, D_h, a_h^*(p_h, D_h)) \geq \pi_h(p_l, D_l, a_l^*(p_l, D_l))$$
$$\implies p_h D_h - \frac{D_h^2}{K_h} \geq p_l D_l - \frac{D_l^2}{K_h} \tag{4.17}$$

$$ICl: \ \pi_l(p_l, D_l, a_l^*(p_l, D_l)) \geq \pi_l(p_h, D_h, a_h^*(p_h, D_h))$$
$$\implies p_l D_l - \frac{D_l^2}{K_l} \geq p_h D_h - \delta\frac{D_h^2}{K_l} \tag{4.18}$$

$$IRh: \pi_h(p_h, D_h, a_h^*(p_h, D_h)) = p_h D_h - \frac{D_h^2}{K_h} \geq 0, \tag{4.19}$$

$$IRl: \pi_l(p_l, D_l, a_l^*(p_l, D_l)) = p_l D_l - \frac{D_l^2}{K_l} \geq 0, \tag{4.20}$$

$$Other: \max(0, D - K) \leq D_j \leq \min(D, K_j), j = l, h, \tag{4.21}$$

where the constraints in the last line ensure non-negative and feasible solutions. In particular, constraint $D - D_j \leq K$ ensures that the buyer can source the 'shortfall' number of workers $(D - D_j)$ internally. In the case with private information, the IR constraints ensure that the buyer pays at least the 'fair wage' $w_j = D_j/K_j$, which is specific to each agent type.

4.6.3 Solving the Buyer's Problem

Because the buyer's profit function $\widehat{\Pi}(p_l, D_l, p_h, D_h)$ stated in (4.15) is a complex function that involves four decision variables, solving the buyer's problem (4.16) directly is intractable. For this reason, we examine the solution to the buyer's problem indirectly in two steps as follows.

First, in Subsection 4.6.3, we consider the 'unconstrained version' of the buyer's problem (4.16) by ignoring all constraints and obtain closed-form expressions for the unconstrained optimal solution, $(p_l^*, D_l^*, p_h^*, D_h^*)$. If this unconstrained optimal solution satisfies all constraints, then this solution will

be the optimal menu of contracts for the constrained buyer's problem. If this unconstrained optimal solution violates some of the constraints, then we need to develop a feasible solution by perturbing this solution.

Second, in Subsection 4.6.3, we consider a sub-problem of the buyer's problem (4.16) by focusing on a given feasible pair of demand (D_l, D_h) that satisfies all constraints. We use Lagrangian relaxation to solve this sub-problem to determine the optimal pricing solution to this sub-problem $(\hat{p}_l^* .\hat{p}_h^*)$ that depends on (D_l, D_h). Then by leveraging the properties of the unconstrained optimal solution $(p_l^*, D_l^*, p_h^*, D_h^*)$, we can develop a perturbation strategy that will enable us to determine the optimal $(\hat{D}_l^*, \hat{D}_h^*)$ and, consequently, retrieve the corresponding optimal pricing decisions.

Unconstrained Optimal Solution of the Buyer's Problem: (p_i^*, D_i^*).

We first consider the unconstrained version of the buyer's problem (4.16) by ignoring the IC, IR, and other constraints. By considering the first order conditions with respect to the buyer's profit function $\widehat{\Pi}(p_l, D_l, p_h, D_h)$ given in (4.16) with respect to p_i and D_i, we obtain the unconstrained optimal solution $(p_l^*, D_l^*, p_h^*, D_h^*)$ as follows:

Lemma 2. *By considering the unconstrained version of the buyer's problem* (4.16)*, the unconstrained optimal solution* (p_i^*, D_i^*)*, for* $i = l, h$*, satisfies:*

$$p_i^* = \sqrt{\frac{c_a}{K_i}} - \frac{m}{D}, \;\; D_i^* = D - K\left(\sqrt{\frac{c_a}{K_i}} - \frac{m/D + u}{2}\right). \tag{4.22}$$

In the following, we focus on situations in which p_i^* and D_i^*, $i = l, h$ are non-negative. It is easy to check from (4.22) that the unconstrained optimal solution are non-negative when D is sufficiently high. If the optimal unconstrained solution (p_i^*, D_i^*) given in (4.22) satisfies the constraints (4.17)-(4.21) associated with the buyer's problem (4.16), then the contract menu (p_i^*, D_i^*, a_i^*) with a_i^* given in (4.13) constitutes the optimal menu of contracts for the constrained buyer's problem. (4.22) indi-

cates that $p_l^* > p_h^*$ and $D_l^* < D_h^*$, i.e., that the price of the contract for the agent of type l is greater than the price of the contract for agent of type h, while the allocated demand quantity is less in the contract of agent of type l than in the contract of agent of type h.

However, in the event when (p_i^*, D_i^*) given in (4.22) do not satisfy the problem constraints, then one needs to perturb the optimal unconstrained solution to the *constrained* optimal solution $(\hat{p}_i^*, \hat{D}_i^*)$ that satisfies constraints (4.17)-(4.21). How should one perturb the unconstrained solution (p_i^*, D_i^*)? The buyer's profit function $\widehat{\Pi}(p_l, D_l, p_h, D_h)$ as stated in (4.16) is easily seen to be concave in each variable, so that we can project the unconstrained solution (p_i^*, D_i^*) onto the feasible region of the buyer's problem by sacrificing the buyer's profit as little as possible.

To illustrate how we perturb the optimal unconstrained solution, we present a numerical example based on the following parameter values: $r = 3, m = 2, u = 0.1, c_a = 12, D = K = 32, K_l = 45, K_h = 60, \alpha = 0.5, \delta = 0.97$. The optimal unconstrained solution is $(p_l^*, D_l^*), (p_h^*, D_h^*) = (0.45, 18.1), (0.38, 20.3)$. However, it is easy to check that this solution violates the ICh constraint (4.17). In this case, we perturb our infeasible solution $(p_l^*, D_l^*), (p_h^*, D_h^*)$ to a feasible solution that satisfies both IC constraints (4.17) and (4.18). The optimal feasible solution is $(\hat{p}_l^*, \hat{D}_l^*), (\hat{p}_h^*, \hat{D}_h^*) = (0.41, 18.5), (0.43, 19.7)$, which satisfies all constraints. Through this perturbation, the buyer's profit reduced from $\widehat{\Pi} = 72.34$ (based on the infeasible unconstrained solution $(p_l^*, D_l^*), (p_h^*, D_h^*)$) to $\widehat{\Pi} = 72.26$ (based on the feasible perturbed solution $(\hat{p}_l^*, \hat{D}_l^*), (\hat{p}_h^*, \hat{D}_h^*)$.

Optimal Menu of Contracts: $(\hat{p}_i^*.\hat{D}_i^*)$.

We now formalize the above perturbation process that enables us to solve the buyer's problem (4.16). To do so, we first consider the 'pricing' sub-problem of the buyer's problem (4.16) when demand quantities (D_l, D_h) are given and feasible. Specifically, we use Lagrangian relaxation to determine the optimal pricing solution to this sub-problem $(\hat{p}_l^*, \hat{p}_h^*)$ that depends on any given demand (D_l, D_h). Then, by leveraging the properties of the unconstrained optimal solution $(p_l^*, D_l^*, p_h^*, D_h^*)$, we de-

velop a "perturbation strategy" for determining the optimal $(\hat{D}_l^*, \hat{D}_h^*)$ so that we can retrieve the corresponding optimal pricing decisions.

Before we solve the sub-problem of the buyer's problem, we make three observations. First, let us simplify the buyer problem (4.16) by eliminating a redundant constraint by using the following observation. Because $K_l < K_h$, the IRh constraint (4.19) holds when both ICh and IRl constraints (4.17) and (4.20) hold. Hence, the IRh constraint (4.19) is redundant and can be removed. This observation is interesting, because it implies that in the case with information asymmetry, contrary to Proposition 2, the agent of type h under private information can always extract a positive surplus from the contract, independent of audit cost c_a. Second, it is easy to check from both ICh and IRl constraints (4.17) and (4.20) that the buyer's problem (4.16) has a feasible solution if and only if $\frac{\delta}{K_l} > \frac{1}{K_h}$. Fortunately, by using the definition of $\delta = 1 - \frac{\gamma}{4c_g}\left(1 - \frac{K_l}{K_h}\right)^2$ and our assumption that $\gamma < 2c_g$ (see Proposition 1), it is easy to show that $\frac{\delta}{K_l} > \frac{1}{K_h}$ will always hold for any $K_l < K_h$. Third, by rearranging the ICh constraint (4.17) and the ICl constraint (4.18), we observe that a feasible solution $(p_l, D_l), (p_h, D_h)$ must satisfy: $\frac{1}{K_h}((D_h)^2 - (D_l)^2) \leq p_h D_h - p_l D_l \leq \frac{1}{K_l}(\delta(D_h)^2 - (D_l)^2)$. Hence, for any feasible demand (D_l, D_h) to the buyer's problem, it must satisfy: $\frac{1}{K_h}((D_h)^2 - (D_l)^2) \leq \frac{1}{K_l}(\delta(D_h)^2 - (D_l)^2) \Leftrightarrow D_h \geq \sqrt{\left(\frac{1}{K_l} - \frac{1}{K_h}\right)\left(\frac{\delta}{K_l} - \frac{1}{K_h}\right)^{-1}} \cdot D_l \ (> D_l)$, and we can restrict our attention to this region of demand demand quantities.

Determining the Buyer's Optimal Prices $(\hat{p}_l^*, \hat{p}_h^*)$. We now solve the buyer's 'pricing problem' by determining the optimal constrained prices $(\hat{p}_l^*, \hat{p}_h^*)$ for any feasible demand quantities (D_l, D_h). By considering the Lagrangian relaxation of the buyer's 'pricing problem' and by letting λ_h and λ_l be the Lagrangian multipliers associated with the ICh constraint (4.17) and ICl constraint (4.18); respectively, the optimal pricing solution $(\hat{p}_l^*, \hat{p}_h^*)$ is as follows:

Proposition 3. *For given feasible demand quantities* (D_l, D_h) *that satisfy* (4.21) *and*

$$D_h \geq \sqrt{\left(\frac{1}{K_l} - \frac{1}{K_h}\right)\left(\frac{\delta}{K_l} - \frac{1}{K_h}\right)^{-1}} \cdot D_l \ (> D_l), \tag{4.23}$$

the optimal prices $(\hat{p}_l^*, \hat{p}_h^*)$ *for the buyer's sub-problem (4.16)-(4.21) possess the following properties:*

(1) **High audit cost.** *If* $c_a \geq K_l \left(\frac{D_l}{K_l} + \frac{m}{D}\right)^2$ *so that* $p_l^* \geq \frac{D_l}{K_l}$*, then:*

(1A) If $p_h^* D_h - p_l^* D_l < \frac{D_h^2}{K_h} - \frac{D_l^2}{K_h}$*, then the ICh constraint* (4.17) *is binding so that:*[21]

$$\hat{p}_l^* = \max\left\{\sqrt{\frac{\alpha c_a}{K_l(\alpha + \lambda_h)}} - \frac{m}{D}, \frac{D_l}{K_l}\right\}, \text{and } \hat{p}_h^* = \frac{1}{D_h K_h}\left(D_h^2 - D_l^2\right) + \hat{p}_l^* \frac{D_l}{D_h}, \tag{4.24}$$

(1B) If $\frac{D_h^2}{K_h} - \frac{D_l^2}{K_h} \leq p_h^* D_h - p_l^* D_l \leq \delta \frac{D_h^2}{K_l} - \frac{D_l^2}{K_l}$*, then none of the IC constraints are binding so that:*

$$\hat{p}_l^* = \sqrt{\frac{c_a}{K_l}} - \frac{m}{D}, \text{and } \hat{p}_h^* = \sqrt{\frac{c_a}{K_h}} - \frac{m}{D}. \tag{4.25}$$

(1C) If $\delta \frac{D_h^2}{K_l} - \frac{D_l^2}{K_l} < p_h^* D_h - p_l^* D_l$*, then the ICl constraint* (4.18) *is binding so that:*[22]

$$\hat{p}_l^* = \sqrt{\frac{\alpha c_a}{K_l(\alpha - \lambda_l)}} - \frac{m}{D}, \text{and } \hat{p}_h^* = \frac{1}{D_h K_l}\left(\delta D_h^2 - D_l^2\right) + p_l^* \frac{D_l}{D_h}. \tag{4.26}$$

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[21] The Lagrangian multiplier $\lambda_h \geq 0$ can be derived from solving: $\frac{b_h}{\sqrt{1-\alpha-\lambda_h}} - \frac{b_l}{\sqrt{\alpha+\lambda_h}} = b_0$, where $b_h = \sqrt{\frac{(1-\alpha)c_a}{K_h}} D_h$, $b_l = \sqrt{\frac{\alpha c_a}{K_l}} D_l$, and $b_0 = \frac{1}{K_h}\left(D_h^2 - D_l^2\right) + \frac{m}{D}\left(D_h - D_l\right)$. Also, the Lagrangian multiplier λ_h can be derived in closed-form from the Quartic Formula (details available upon request), and it can be approximated by: $\lambda_h \approx 1 - 2\alpha - \frac{b_l}{b_0}\sqrt{\alpha} - \frac{b_h}{b_0}\sqrt{1-\alpha} + \sqrt{\left(1 - 2\alpha - \frac{b_l}{b_0}\sqrt{\alpha} - \frac{b_h}{b_0}\sqrt{1-\alpha}\right)^2 - 4\frac{\alpha(1-\alpha)}{b_0}\left(\frac{b_h}{\sqrt{1-\alpha}} - \frac{b_l}{\sqrt{\alpha}} - b_0\right)}$.

[22] The Lagrangian multiplier $\lambda_l \geq 0$ can be derived by solving: $\frac{b_h}{\sqrt{1-\alpha+\lambda_l}} - \frac{b_l}{\sqrt{\alpha-\lambda_l}} = b_0'$, where $b_0' = \frac{1}{K_l}\left(\delta D_h^2 - D_l^2\right) + \frac{m}{D}\left(D_h - D_l\right)$. Also, the Lagrangian multiplier λ_l can be approximated by: $\lambda_l \approx 2\alpha - 1 + \frac{b_l}{b_0'}\sqrt{\alpha} + \frac{b_h}{b_0'}\sqrt{1-\alpha} - \sqrt{\left(1 - 2\alpha - \frac{b_l}{b_0'}\sqrt{\alpha} - \frac{b_h}{b_0'}\sqrt{1-\alpha}\right)^2 - 4\frac{\alpha(1-\alpha)}{b_0'}\left(\frac{b_h}{\sqrt{1-\alpha}} - \frac{b_l}{\sqrt{\alpha}} - b_0'\right)}$.

(2) **Low audit cost.** *If* $c_a < K_l \left(\frac{D_l}{K_l} + \frac{m}{D}\right)^2$, *then*

$$\hat{p}_l^* = \frac{D_l}{K_l}, \text{ and } \hat{p}_h^* = \max\left\{\frac{D_h}{K_h} + \frac{D_l^2}{D_h}\left(\frac{1}{K_l} - \frac{1}{K_h}\right), p_h^*\right\}. \tag{4.27}$$

Akin to Proposition 2, Proposition 3 exhibits a similar effect of the audit cost c_a on optimal prices $(\hat{p}_l^*, \hat{p}_h^*)$ even when the labor pool size K is not known to the buyer. However, unlike Proposition 2, the buyer cannot extract the entire surplus from the agent of type h, because of the IC constraints (see discussion above). For the agent of type l, however, when the audit cost c_a is high, statement (1) of Proposition 3 informs us about the conditions when the buyer is unable to use these optimal prices $(\hat{p}_l^*, \hat{p}_h^*)$ to extract the entire surplus from this agent. When the audit cost c_a is low, due to information asymmetry, statement (2) reveals that the buyer can use optimal prices to always extract the entire surplus from the agent of type l.

Statement 1 can be interpreted along with the perturbation idea as explained in Subsection 4.6.3 as follows. First recall from the ICh constraint (4.17) and the ICl constraint (4.18) that a feasible solution $(p_l, D_l), (p_h, D_h)$ must satisfy: $\frac{1}{K_h}((D_h)^2 - (D_l)^2) \leq p_h D_h - p_l D_l \leq \frac{1}{K_l}(\delta(D_h)^2 - (D_l)^2)$. Hence, if the unconstrained optimal prices (p_l^*, p_h^*) as stated in Lemma 2 satisfy this condition so that the condition for statement (1B) holds, then the unconstrained optimal prices (p_l^*, p_h^*) are also optimal for the buyer's sub-problem. This explains statement (1B), which is depicted as Case (B) in Figure 4.3.

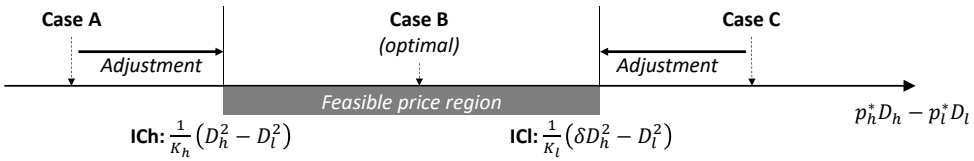


Figure 4.3: Illustration of cases of Proposition 3.

Next, when the unconstrained optimal prices (p_l^*, p_h^*) violate the condition $\frac{1}{K_h}((D_h)^2 - (D_l)^2) \leq p_h^* D_h - p_l^* D_l \leq \frac{1}{K_l}(\delta(D_h)^2 - (D_l)^2)$, we need to consider two cases as shown in Figure 4.3. Case (A) in Figure 4.3 corresponds to statement (1A) in Proposition 3; i.e., when $\frac{1}{K_h}((D_h)^2 - (D_l)^2) >$

$p_h^* D_h - p_l^* D_l$. Case (C) in Figure 4.3 corresponds to statement (1C) in Proposition 3; i.e. when $p_h^* D_h - p_l^* D_l > \frac{1}{K_l}(\delta(D_h)^2 - (D_l)^2)$. Specifically, for statement (1A) in Figure 4.3, the ICh constraint (4.17) is violated so that $p_h^* D_h - \frac{1}{K_h}(D_h)^2 < p_l^* D_l - \frac{1}{K_h}(D_l)^2$. Hence, a type h agent will be better off by choosing contract (p_l^*, D_l). To circumvent this situation, we have to perturb the unconstrained optimal solution (p_l^*, p_h^*) to the 'nearest' feasible solution that has ICh constraint (4.17) binding, as shown in Figure 4.3. In doing so, an agent of type h will be content with the contract (p_h^*, D_h) as intended. We can interpret statement (1C) in the same manner and we omit the details.

The statements (1A)-(1C) in Proposition 3 depend on the *difference in earnings* between both contracts, $p_h D_h - p_l D_l$. In (1A), the difference in earnings is too small, such that constraint ICh is binding. In this case, earnings $\hat{p}_l^* D_l$ have to be adjusted downwards, while the earning $\hat{p}_h^* D_h$ have to be adjusted upwards, such that the difference in earnings increases. The opposite arises under (1C) under which the difference between $p_h^* D_h$ and $p_l^* D_l$ is too large and constraint ICl is violated. In this case, the difference in earnings has to decrease by adjusting the earnings from contract h downwards and those of contract l upwards. It is interesting to note that in our setting, the *earnings* are the main factor that determines which agent accepts which contract, not the *prices*. It can even be optimal to have $\hat{p}_l^* \leq \hat{p}_h^*$, which is the case in the numerical example that we cited above, in which we obtained $\hat{p}_l^* = 0.41 < \hat{p}_h^* = 0.43$.

Characterizing the Buyer's Optimal Demands $(\hat{D}_l^*, \hat{D}_h^*)$. Proposition 3 specifies the optimal prices $(\hat{p}_l^*, \hat{p}_h^*)$ for any given demands (D_l, D_h) that satisfy (4.21) and (4.23). We now characterize the optimal demands $(\hat{D}_l^*, \hat{D}_h^*)$ for the buyer's problem (4.16)-(4.21). In doing so, it enables to use a simple search to find their optimal values so that we can use Proposition 3 to find the corresponding optimal prices $(\hat{p}_l^*, \hat{p}_h^*)$. In the following proposition, we show how to leverage the *unconstrained* solution $(p_l^*, D_l^*, p_h^*, D_h^*)$ by considering whether or not the condition $\frac{1}{K_h}((D_h^*)^2 - (D_l^*)^2) \leq p_h^* D_h^* - p_l^* D_l^* \leq \frac{1}{K_l}(\delta(D_h^*)^2 - (D_l^*)^2)$ holds. If this condition holds, then the unconstrained solution $(p_l^*, D_l^*), (p_h^*, D_h^*)$ is the optimal solution to the buyer's problem. Otherwise, under the optimal constrained demand

quantities $(\hat{D}_l^*, \hat{D}_h^*)$, we have to perturb the prices in the same direction as we would have to perturb them under the unconstrained demand quantities (D_l^*, D_h^*).

Proposition 4. *The optimal solution $(\hat{p}_l^*, \hat{D}_l^*, \hat{p}_h^*, \hat{D}_h^*)$ to the buyer's problem (4.16) satisfies: $\hat{D}_h^* > \hat{D}_l^*$ and $\hat{p}_h^* \cdot \hat{D}_h^* \geq \hat{p}_l^* \cdot \hat{D}_l^*$. Also, if (D_l^*, D_h^*) satisfy (4.23), the optimal solution has the partial structure: For **high audit cost**, such that $p_l^* \geq \frac{D_l^*}{K_l}$ (constraint IRl holds at the unconstrained solution), the following can be considered valid:*

1. *If for the unconstrained solution $(p_l^*, D_l^*, p_h^*, D_h^*)$ the condition $p_h^* D_h^* - p_l^* D_l^* < \frac{1}{K_h}((D_h^*)^2 - (D_l^*)^2)$ is true, i.e., it violates ICh, then at the optimal demand quantities $(\hat{D}_l^*, \hat{D}_h^*)$, Cases (1A) or (2) of Proposition 3 apply to the corresponding optimal prices $(\hat{p}_l^*, \hat{p}_h^*)$.*

2. *If for the unconstrained solution $(p_l^*, D_l^*, p_h^*, D_h^*)$ the condition $\frac{1}{K_h}((D_h^*)^2 - (D_l^*)^2) \leq p_h^* D_h^* - p_l^* D_l^* \leq \frac{1}{K_l}(\delta(D_h^*)^2 - (D_l^*)^2)$ is true, then the optimal solution to the buyer's problem is: $\left(\hat{p}_i^*, \hat{D}_i^*\right) = (p_i^*, D_i^*)$.*

3. *If for the unconstrained solution $(p_l^*, D_l^*, p_h^*, D_h^*)$ the condition $\frac{1}{K_l}(\delta(D_h^*)^2 - (D_l^*)^2) < p_h^* D_h^* - p_l^* D_l^*$ is true, i.e., it violates ICl, then at the optimal demand quantities $(\hat{D}_l^*, \hat{D}_h^*)$, Cases (1C) or (2) of Proposition 3 apply to the corresponding optimal prices $(\hat{p}_l^*, \hat{p}_h^*)$.*

Statements (1)-(3) indicate which case of Proposition 3 is applicable based on the conditions of the unconstrained optimal solution (p_i^*, D_i^*) for $i = l, h$: Either the optimal unconstrained solution has to be perturbed to make agent l prefer its corresponding contract or the solution has to be adapted to make the corresponding contract attractive to agent h.

Armed with the characterization of the optimal demands $(\hat{D}_l^*, \hat{D}_h^*)$ for the buyer's problem as stated in Proposition 4, one can conduct a simple search for the optimal values for $(\hat{D}_l^*, \hat{D}_h^*)$ and then apply Proposition 3 to retrieve the corresponding optimal prices $(\hat{p}_l^*, \hat{p}_h^*)$ for the buyer's problem.

4.7 Selection from Multiple Agents with Private Information

In Section 4.6, we considered the case when a single agent has private information about his labor pool size, and we solved the buyer's problem by identifying the optimal menu of contracts that is truth-telling. We now use the insights of Section 4.6 to analyze the case when there are $n(\geq 2)$ agents, where each agent belongs to one of $q(\geq 2)$ types. Each type i agent has a labor pool size of $K_i,\ i = 1, ..., q$. The true type of each agent is private information, and yet the buyer and other agents know an agent belongs to type i with probability $\alpha_i > 0$ so that $\sum_{i=1}^{q} \alpha_i = 1$. Without loss of generality, we assume that the K_i are ordered in descending order so that $K_i > K_{i+1}$.

In the presence of private information, we consider the case when the buyer has to contract with exactly one agent from a pool of n agents of q types. The buyer's problem is much more complex that the one examined in Section 4.6 that deals with one agent with two types. Even so, the following corollary helps us to develop a contracting mechanism for the buyer to select the optimal agent.

Corollary 2. *For any contract $(p, D, a_j^*(p, D))$ that has audit level a_j^* set for an agent of target type j,*[23] *the buyer can always obtain a higher profit with an agent of type j than with an agent of type i (where $j < i$) that has a smaller labor pool size, because:*

$$(r-p)D - \frac{c_a D}{K_j(p + \frac{m}{D})} > (r-p)D - \frac{c_a D}{K_i(p + \frac{m}{D})}. \tag{4.28}$$

Effectively, Corollary 2 reveals that the buyer is always better off to choose an agent of the type with the largest labor pool size. This insight motivates us to develop a menu of contracts that enables the buyer to select the agent with the largest labor pool size under information asymmetry.

To choose an agent, it is certainly possible for the buyer to offer a menu of contracts (similar to

[23] Throughout this section, we shall set the audit level according to the minimum audit level for an agent of type j, i.e., $a_j^*(p, D) = D/(K_j(p + m/D))$ as stated in (4.13) to ensure coercion free for agents of type j.

the case presented in Section 4.6) to n agents simultaneously. However, it can be awkward when all n agents accept their contracts $(\hat{p}_i^*, \hat{D}_i^*)$ simultaneously according to their true type i. Then, the buyer would have to first choose one particular type of agents to contract with and reject all other types. [24] This process is doable, but it can alienate agents who do not belong to the chosen type. To avoid the unpleasant feeling about the simultaneous process, without loss of generality[25], we propose a 'sequential' mechanism that is based on a multiple-round reverse auction to operationalize our menu of contracts in order for the buyer to contract with the agent with the largest pool of labor among all n agents.

Our sequential mechanism is intended to entice agents to accept a certain contract in each round according to their true type. During the round that has at least one agent accepting the proposed contract for the first time (i.e., no agent accepted any of the proposed contracts in earlier rounds), the buyer selects randomly among the agents who accept the proposed contract in this round and the process ends. Unlike the simultaneous process that requires the buyer to reject all $(n-1)$ agents, this sequential process is less aggravating because it was the agent who rejects the buyer's proposed contract in earlier rounds. Essentially, our sequential contracting mechanism is similar to a quantity-price *Dutch reverse auction*[26] in which the buyer successively proposes price-quantity offers to a pool of suppliers until a supplier accepts an offer.[27]

Sequential menu of contracts: $(\hat{p}_i^*, \hat{D}_i^*, a_i^*(\hat{p}_i^*, \hat{D}_i^*))$**.** The buyer designs the sequential menu of q contracts $(\hat{p}_i^*, \hat{D}_i^*, a_i^*(\hat{p}_i^*, \hat{D}_i^*))$, $i = 1, \ldots, q$, where the audit level $a_i^*(\hat{p}_i^*, \hat{D}_i^*)$ is given in (4.13) to

[24] Then, the buyer has to select one agent randomly among agents of the chosen type.

[25] The optimal menus of contracts for our sequential mechanism and for a simultaneous mechanism are identical.

[26] https://www.ibm.com/docs/en/emptoris-sourcing/10.1.3?topic=rt-dutch-reverse-auction [24 March 2023].

[27] Price-quantity auctions have been used in practice and have been subject of research as a mechanism for awarding a contract to a single supplier from a pool of potential suppliers (see, e.g., Chen, 2007).

ensure coercion-free. The menu of contracts is known to all agents in advance.[28]

Instead of offering the entire menu of contracts all at once, the buyer offers a contract $(\hat{p}_i^*, \hat{D}_i^*, a_i^*(\hat{p}_i^*, \hat{D}_i^*))$ in round i, with $i = 1, 2, \cdots, q$ with the intention to entice only agents with labor pool size K_i (if available) to accept the contract in round i. For example, because $K_1 > K_2 > \cdots > K_q$, Corollary 2 implies that it is optimal for the buyer to offer a contract $(\hat{p}_1^*, \hat{D}_1^*)$ in round 1 so that only type 1 agent(s) (if available) will accept. If type 1 agents exist (in the pool of n agents) who accept the contract, then the buyer will select one of the accepting type 1 agents using a random lottery. If no agent accepts the contract $(\hat{p}_1^*, \hat{D}_1^*)$, then the buyer and all agents will know that no type 1 agent is in the pool, and the buyer proceeds to round 2. This sequential process continues until at least one agent accepts the proposed contract.

Our sequential menu of contracts serves two purposes: (a) *truth telling* – each agent accepts only the contract corresponding to its true type; and (b) *sequential compatible* – only the agents with the largest labor pool size will accept the proposed contract in an earlier round.

To ensure each agent will only accept the buyer's contract according to his true type in the sequential process as described above, our sequential menu of contracts is based on the *Perfect Bayesian Equilibrium (PBE)* associated with the sequential game between the agents (Fudenberg and Tirole, 1991). Because of private information, the outcome in each round reveals some information about the composition of the agent types in the pool as explained above, and all agents can utilize this information to update their belief. We shall explain this next.

[28] If a contract specifies $D_i < D$ units, we assume, as before, that the remaining units $(D - D_i)$ are sourced internally at management cost u and wage $(D - D_i)/K$ as explained in Section 4.6.

4.7.1 Sequential menu of contracts (p_h, D_h) and (p_l, D_l) for two agent types $q = 2$.

We now explain how agents update their beliefs upon observing the outcome of each round of our sequential contracting process. To ease our exposition, we shall focus our discussion in the remainder of this section on the contracts (p_h, D_h) and (p_l, D_l) for the case of two agent types ($q = 2$) so that $K_1 \equiv K_h > K_l \equiv K_2$. (See Appendix 4.7.2 for the general case of $q(\geq 2)$.)

Agents' rational beliefs under the sequential menu of contracts: (p_h, D_h) and (p_l, D_l). Consider the case when the buyer offers a sequential menu of contracts so that (p_h, D_h) is offered in round 1. If no agent accepts the contract (p_h, D_h) in round 1, then the buyer proceeds to round 2 by offering (p_l, D_l).[29] Let us determine the belief of an agent of type h in round 1. Because the contract is intended for type h (or type 1) agents to accept in round 1, let us first examine the belief of an agent of type h. This agent believes that there are $j, j = 0, \ldots, (n-1)$, agents of type h in the pool (excluding himself) with probability $\binom{n-1}{j}(1-\alpha)^j \alpha^{(n-1)-j}$, where each agent belongs to type h with probability $(1-\alpha)$. Also, this type h agent knows that, when there are other j agents of type h, his chance of winning the contract is $1/(j+1)$ due to random lottery. Hence, each agent of type h believes that his (ex-ante) probability of winning the contract in the first round is:

$$\sum_{j=0}^{n-1}(1-\alpha)^j \alpha^{n-1-j}\binom{n-1}{j}\cdot\frac{1}{j+1} = \sum_{j=1}^{n}(1-\alpha)^j \alpha^{n-j}\binom{n}{j}\cdot\frac{1}{n(1-\alpha)}$$
$$= \frac{1-\alpha^n}{n(1-\alpha)}. \tag{4.29}$$

Next, we determine the belief of an agent of type l in round 1. As explained above, the sequential menu of contracts is designed to entice only type h agent(s) to accept the contract in round 1. Hence,

[29] If at least one agent accepts (p_h, D_h), then the accepting agents must belong to type 1 (type h). If more than one agent accepts the contract, the buyer will select one of the accepting agents of type 1 via lottery.

this agent of type l knows that only the contract in round 2 is accepted by him and other type l agents. However, all agents know that the process will end after round 1 if there is at least one agent of type h in the pool. Hence, in the event when the buyer proceeds to round 2, each of the n agents will know with certainty that all other agents are of type l. Due to random lottery, an agent with type l will know his winning probability is $1/n$ in round 2. Therefore, prior to the first round, each agent of type l knows that his (ex-ante) winning probability for the contract in the second round is $\alpha^{n-1} \cdot (1/n)$, where $\alpha^{(n-1)}$ is the probability that all other $(n-1)$ agents are of type l. However, if no agent accepts the contract in round 1, it reveals that all agents are of type l. Hence, when second round takes place, each agent knows his (ex-post) winning probability is $1/n$.

The buyer's profit under the sequential menu of contracts: (p_h, D_h) and (p_l, D_l). The buyer's expected profit $\widehat{\Pi}(p_l, D_l, p_h, D_h)$ can be expressed as:

$$\begin{aligned}&\widehat{\Pi}(p_l, D_l, p_h, D_h)\\ &= \alpha^n \left((r - p_l)D_l - c_a a_l^*(p_l, D_l) + \Pi^c(D_l)\right) + (1 - \alpha^n)\left((r - p_h)D_h - c_a a_h^*(p_h, D_h) + \Pi^c(D_h)\right) \quad (4.30)\end{aligned}$$

The expression for the buyer's profit stated above is based on two observations. First, observe that the probability of having the first round contract (p_h, D_h) accepted by at least one agent of type h is $(1 - \alpha^n)$.[30] Second, as explained earlier, the process will end if at least one agent of type h accepts the contract in the first round. Therefore, the process will continue in the second round only when all agents belong to type l with probability α^n so that the second round contract (p_l, D_l) will accepted by an agent of type l. Combining these observations, accounting for the auditing cost, and including the additional profit when the buyer needs to source $(D - D_i)$ internally and earn additional profit $\Pi^c(D_i)$ given in (4.8) as explained in Section 4.6, we can determine the buyer's expected profit $\widehat{\Pi}(p_l, D_l, p_h, D_h)$ as stated in (4.30).

[30] Because α is the probability of an agent of type l, the probability of having *at least one* agent of type h in the pool of agents is $(1 - \alpha^n)$.

Incentive compatibility and rational constraints under the sequential menu of contracts: (p_h, D_h) and (p_l, D_l). To ensure the contracts (p_h, D_h) and (p_l, D_l) possess the truth telling capabilities, we need to establish IC and IR constraints in the same manner as described in Section 4.6. In preparation, let us define a new term: $\widehat{\alpha}(n) \equiv \frac{(1-\alpha)\alpha^n}{\alpha(1-\alpha^n)}$. By using the ex-ante winning probabilities associated with type h and type l agents as explained above and by noting that the contract has an embedded audit level a_j^* that ensures coercion-free, we can specify the IC and IR constraints associated with the sequential menu of contracts as follows:

$$\begin{aligned}
ICh: \quad & \left(\frac{1-\alpha^n}{n(1-\alpha)}\right)\pi_h(p_h, D_h, a_h^*(p_h, D_h)) \geq \left(\frac{\alpha^{n-1}}{n}\right)\pi_h(p_l, D_l, a_l^*(p_l, D_l)) \\
& \Leftrightarrow \left(p_h D_h - \frac{D_h^2}{K_h}\right) \geq \widehat{\alpha}(n)\left(p_l D_l - \frac{D_l^2}{K_h}\right), \qquad (4.31) \\
ICl: \quad & \left(\frac{\alpha^{n-1}}{n}\right)\pi_l(p_l, D_l, a_l^*(p_l, D_l)) \geq \left(\frac{1-\alpha^n}{n(1-\alpha)}\right)\pi_l(p_h, D_h, a_h^*(p_h, D_h)) \\
& \Leftrightarrow \widehat{\alpha}(n)\left(p_l D_l - \frac{D_l^2}{K_l}\right) \geq \left(p_h D_h - \delta\frac{D_h^2}{K_l}\right), \qquad (4.32) \\
IRh: \quad & \pi_h(p_h, D_h, a_h^*(p_h, D_h)) = p_h D_h - \frac{D_h^2}{K_h} \geq 0, \qquad (4.33) \\
IRl: \quad & \pi_l(p_l, D_l, a_l^*(p_l, D_l)) = p_l D_l - \frac{D_l^2}{K_l} \geq 0, \qquad (4.34) \\
Other: \quad & \max(0, D-K) \leq D_j \leq \min(D, K_j), j = l, h, \qquad (4.35)
\end{aligned}$$

where the last constraint is based on the boundary conditions as explained in Section 4.6.2. Akin to the construct established in Section 4.6, the ICh constraint ensures that an agent of type h prefers the first round contract (p_h, D_h) based on its ex-ante belief. The ICl constraint ensures that an agent of type l, based on its updated belief, prefers the second round contract (p_l, D_l) as intended.

The buyer's problem under the sequential menu of contracts: (p_h, D_h) and (p_l, D_l). By considering the buyer's profit function $\widehat{\Pi}(p_l, D_l, p_h, D_h)$ given in (4.30), the buyer's problem can be formu-

lated as:

$$\max_{(p_l,D_l,p_h,D_h)} \widehat{\Pi}(p_l, D_l, p_h, D_h), \text{ subject to } (4.31) - (4.35). \tag{4.36}$$

Note that when there is only a single agent ($n = 1$), problem (4.36) reduces to problem (4.16)-(4.21) as examined in Section 4.6. Unlike problem (4.16)-(4.21) that considers a single agent, our problem (4.36) intends to find the optimal sequential menu of contracts that involves ex-ante winning probabilities of each type of agent as captured by the term $\widehat{\alpha}(n) \equiv \frac{(1-\alpha)\alpha^n}{\alpha(1-\alpha^n)}$. However, besides the existence of the term $\widehat{\alpha}(n)$, problem (4.36) resembles problem (4.16). For this reason, the optimal sequential menu of contracts possesses similar characteristics as the optimal menu of contracts in the case of a single agent. Specifically, constraint IRh is redundant with ICh and IRl such that agents of type h make a positive profit, which is also true for the case of multiple agents. Also, the demand quantities and earnings of the optimal contracts are ordered, as stated in the following Corollary 3.[31]

Corollary 3. *The optimal sequential menu of contracts* $(\hat{p}_h^*, \hat{D}_h^*)$ *and* $(\hat{p}_l^*, \hat{D}_l^*)$ *that solves the buyer's problem* (4.36) *possesses the following properties:*

$$\hat{D}_h^* \geq \sqrt{\hat{\alpha}(n)} \cdot \sqrt{\frac{\frac{1}{K_l} - \frac{1}{K_h}}{\frac{\delta}{K_l} - \frac{1}{K_h}}} \cdot \hat{D}_l^* > \sqrt{\hat{\alpha}(n)} \cdot \hat{D}_l^*, \text{ and } \hat{p}_h^* \hat{D}_h^* \geq \hat{\alpha}(n) \cdot \hat{p}_l^* \hat{D}_l^*. \tag{4.37}$$

The characteristics of the optimal sequential menu of contracts $(\hat{p}_h^*, \hat{D}_h^*)$ and $(\hat{p}_l^*, \hat{D}_l^*)$ can be interpreted in the same manner as Proposition 4. However, observe that, when $n \geq 2$, the term $\hat{\alpha}_n \equiv \frac{(1-\alpha)\alpha^n}{\alpha(1-\alpha^n)}$ is less than 1 and it is decreasing in n. Hence, when $n \geq 2$, the lower bound on the quantity D_h is less than D_l and the lower bound on earnings $\hat{p}_h^* \hat{D}_h^*$ less than $\hat{p}_l^* \hat{D}_l^*$. Corollary 3 enables us to develop a simple search algorithm to identify the optimal sequential menu of contracts in a similar manner as described in Section 4.6.

[31] For the proof of Corollary 3, refer to Proposition 6 for the general case of $q(\geq 2)$, in Appendix 4.7.2.

4.7.2 Agent selection in the general case of $q \geq 2$ types.

In this section, we analyze the general case for arbitrary $q \geq 2$. We define the optimal menu of contracts and show that it leads the agent strategy $s = (e_1, ...e_q)$ (e_i being the i-th unit row vector) to be a PBE. We also extend the results of Corollary 3 to the general case.

Belief updating: In any round i (i.e., when the i-th contract is proposed), an agent knows that none of the other agents has a type $j < i$ if they play strategy s, because then a contract would already have been awarded to some other agent. Thus, at the beginning of round i, an agent believes that the types of the other $n-1$ agents follow a multinomial distribution with $n-1$ trials, $q-i$ types (all types $j < i$ have zero probability), and type j having success probability of

$$\alpha_j^{(i)} = \frac{\alpha_j}{1 - \sum_{l=1}^{i-1} \alpha_l}. \tag{4.38}$$

If j other agents choose the contract in the same round as the focal agent, his probability of winning the contract through the random lottery is $1/(j+1)$. Hence, the probability for an agent of winning contract i, at the beginning of round i, is given by

$$\begin{aligned}
&\sum_{j=0}^{n-1} \left(\alpha_i^{(i)}\right)^j (1-\alpha_i^{(i)})^{n-1-j} \binom{n-1}{j} \cdot \frac{1}{j+1} \\
&= \sum_{j=1}^{n} (\alpha_i^{(i)})^j \left(1-\alpha_i^{(i)}\right)^{n-j} \binom{n}{j} \cdot \frac{1}{n\alpha_i^{(i)}} \\
&= \frac{1-(1-\alpha_i^{(i)})^n}{n\alpha_i^{(i)}}.
\end{aligned} \tag{4.39}$$

Similarly, if the same agent would wait until round $j > i$ to accept the contract, his success probability

of winning the contract would be

$$\left(1-\sum_{l=i}^{j-1}\alpha_l^{(i)}\right)^{n-1}\cdot\left(\sum_{j=0}^{n-1}\alpha_j^{(i)}(1-\alpha_j^{(i)})^{n-1-j}\binom{n-1}{j}\cdot\frac{1}{j+1}\right)$$
$$=\frac{1-(1-\alpha_j^{(i')})^n}{n\alpha_j^{(j)}}\cdot\left(1-\sum_{l=i}^{j-1}\alpha_l^{(i)}\right)^{n-1}=\tilde{\alpha}_n(j)\cdot\beta_n(i,j), \tag{4.40}$$

where $\tilde{\alpha}_n(j)\equiv\frac{1-(1-\alpha_j^{(i')})^n}{n\alpha_j^{(j)}}$ represents the probability, at the beginning of round j, that an agent that accepts a contract j will receive it, while term $\beta_n(i,j)\equiv\left(1-\sum_{l=i}^{j-1}\alpha_l^{(i)}\right)^{n-1}$ represents the probability that no other agent that plays strategy s will be awarded the contract between rounds i and j (excluding round j).

Necessary constraints for a PBE: The profit of an agent of type j with contract (p,D) and audit level $a_i^*(p,D)$ is given by

$$\pi_j(p,D,a_i^*(p,D))=pD-\delta_{i,j}\frac{D^2}{K_j}, \tag{4.41}$$

where $\delta_{i,j}=1-\frac{\gamma}{4c_g}\left(1-\frac{K_j}{K_i}\right)^2$ if $i<j$ and $\delta_{i,j}=1$ else. Term $\delta_{i,j}$ represents the cost reduction that an agent of type j can achieve from using coercion if the buyer uses the audit level that is optimal for an agent of type i. In order for strategy s to constitute a PBE, an agent should maximize his expected profit by playing strategy s compared to deviating from that strategy. That means that at the beginning of any round i, it should hold for an agent of any type j with $i<j$ that

$$\tilde{\alpha}_n(i)\cdot\pi_j(p_i,D_i,a_i^*(p_i,D_i))\leq\max_{(i'|i<i'\leq q)}\tilde{\alpha}_n(i')\cdot\beta_n(i,i')\cdot\pi_j(p_{i'},D_{i'},a_{i'}^*(p_{i'},D_{i'})) \tag{4.42}$$

and for an agent of any type j with $j = i$

$$\tilde{\alpha}_n(j) \cdot \pi_j(p_j, D_j, a_j^*(p_j, D_j)) \geq \max_{(i'|j<i'\leq q)} \tilde{\alpha}_n(i') \cdot \beta_n(j, i') \cdot \pi_j(p_{i'}, D_{i'}, a_{i'}^*(p_{i'}, D_{i'})), \tag{4.43}$$

where the first set of constraints prevents agents from accepting a contract 'too early' and the second set of constraints prevents agents from accepting contracts 'too late' for their type. Next, we use Lemma 3 to substantially reduce the number of constraints, from $(1/2)q^3$ IC constraints to $q(q-1)$ IC constraints.

Lemma 3. *The IC constraints of* (4.42) *and* (4.43) *can be simplified to*

$$IC(i,j): \ \tilde{\alpha}_n(i)\beta_n(j,i)\pi_j(p_i, D_i, a_i^*(p_i, D_i)) \leq \ \tilde{\alpha}_n(j)\pi_j(p_j, D_j, a_j^*(p_j, D_j)), \forall 1 \leq i, j \leq q \tag{4.44}$$

with $\beta_n(j,i) = \beta_n(i,j)^{-1}$.

Expected buyer profit under strategy s**.** The buyer awards contract $j = 1$ only if there is at least one agent of type $j = 1$ in the pool, which occurs with probability $1 - \alpha_1^n$. If no agent of type $j = 1$, the buyer proceeds to the next round, proposing the contract $j = 2$, and so on. Hence, the probability that there is at least one agent of type j (given that no agent has type $i < j$) is given by

$$\left(1 - \left(1 - \frac{\alpha_j}{1 - \sum_{l=1}^{j-1} \alpha_l}\right)^n\right), \tag{4.45}$$

while the probability of 'reaching' round j is given by

$$\left(1 - \sum_{l=1}^{j-1} \alpha_l\right)^n. \tag{4.46}$$

Let us write the vector of contracts as

$$(\mathbf{p}, \mathbf{D}, \mathbf{a}^*(\mathbf{p}, \mathbf{D})) = (p_1, D_1, a_1^*(p_1, D_1), ..., p_q, D_q, a_q^*(p_q, D_q))$$

Then, we can express expected buyer profit as

$$\widehat{\Pi}(\mathbf{p}, \mathbf{D}, \mathbf{a}^*(\mathbf{p}, \mathbf{D})) \\ \sum_{j=1}^{q} \left(1 - \sum_{l=1}^{j-1} \alpha_l\right)^n \left(1 - \left(1 - \frac{\alpha_j}{1 - \sum_{l=1}^{j-1} \alpha_l}\right)^n\right) \left((r - p_j)D_j - c_a a_j^*(p_j, D_j) + \Pi^c(D_j)\right) \tag{4.47}$$

The buyer's sequential contract design problem.

The solution to the following optimization problem

$$\max_{(\mathbf{p},\mathbf{D})} \widehat{\Pi}(\mathbf{p}, \mathbf{D}, \mathbf{a}^*(\mathbf{p}, \mathbf{D})), \tag{4.48}$$

subject to constraints

$$\text{IC}(i,j): \quad \tilde{\alpha}_n(i)\beta_n(j,i)\left(p_i D_i - \delta_{i,j}\frac{D_i^2}{K_j}\right) \leq \tilde{\alpha}_n(j)\left(p_j D_j - \frac{D_j^2}{K_j}\right) \forall 1 \leq i,j \leq q, \tag{4.49}$$

$$\text{IR}(j): \quad p_j \geq \frac{D_j}{K_j} \qquad \forall 1 \leq j \leq q, \tag{4.50}$$

$$\text{Other:} \quad \max(0, D - K) \leq D_j \leq \min(D, K_j) \qquad \forall 1 \leq j \leq q, \tag{4.51}$$

corresponds to the profit-maximizing sequential menu of contracts, which we formalize in the following proposition.

Proposition 5. *The optimal solution* $(\hat{\mathbf{p}}^*, \hat{\mathbf{D}}^*, \mathbf{a}^*(\hat{\mathbf{p}}^*, \hat{\mathbf{D}}^*))$ *to problem* (4.48)-(4.51) *guarantees the selection of the agent with the lowest type j in the agent pool and maximizes expected buyer profit* $\widehat{\Pi}$ *among*

the feasible menus of contracts that induce a PBE for the game between the n agents in the pool under agent strategy s.

In the following proposition, we extend some of the results of Corollary 3 of the case with $q = 2$ to the general case.

Proposition 6. *Problem* (4.48)-(4.51) *has the following characteristics:*

1. *Constraint IR*(j) *for any* $j < m$ *is redundant with other IC and IR constraints, such that the only IR constraint relevant to the problem is IR*(q).

2. *For every feasible solution, it holds for any pair* $i < j$ *that*

$$D_i \geq \sqrt{\frac{\tilde{\alpha}_n(j)}{\tilde{\alpha}_n(i)}\beta_n(i,j)} \cdot \sqrt{\frac{\frac{1}{K_j} - \frac{1}{K_i}}{\frac{\partial_{i,j}}{K_j} - \frac{1}{K_i}}} \cdot D_j > \sqrt{\frac{\tilde{\alpha}_n(j)}{\tilde{\alpha}_n(i)}\beta_n(i,j)} \cdot D_j, \tag{4.52}$$

and

$$p_i D_i \geq \frac{\tilde{\alpha}_n(j)}{\tilde{\alpha}_n(i)}\beta_n(i,j) \cdot p_j D_j. \tag{4.53}$$

Part (1) of Proposition 6 shows that the corresponding result of the case with two agent types applies to all agents except for those of the type with the smallest labor pool size. Part (2) allows us to restrict the search space of the optimal solution in a similar way as we could for the model with $q = 2$ types.

4.8 Numerical Experiments

Armed with the results obtained in Section 4.7, we now utilize the H2A data set obtained from the U.S. Department of Labor as described in Section 4.2 to illustrate our results. Specifically, our data

set contains the wage w and the number of workers D associated with each registered H2A labor contractor (agent) in the United States between 2017 and 2022.[32]

As our analytical model assumes that wages lie between zero and one, for each year, we perform a 90% winsorization (we only retain values between the 5%-percentile and the 95%-percentile) and normalize the resulting values to the interval from zero to one. Because the data set does not provide information about the labor pool size of each agent, we estimate the labor pool size K_{it} for each agent i in year t by $K_{it} \approx D_{it}/w_{it}$. Among all values in our data set, we restrict our attention to those with an estimated labor pool size $K_{it} \in [100, 1300]$.[33] For the remaining 1,452 values, the average annual demand is $\bar{D} = 189$, and the average normalized wage is $\bar{w} = 0.65$.

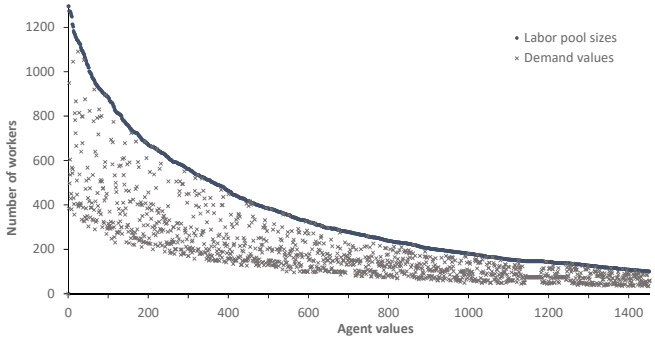


Figure 4.4: Distribution of estimated labor pool (K) and annual demand (D) values in the data set.

Figure 4.4 displays the 1,452 labor pool sizes in decreasing order (blue dots) and their corresponding annual demand values (gray crosses). For illustrative purposes, we classify these 1,452 values into four different agent types (so that $q = 4$) as summarized in Table 4.1. Each type of agent has a 300-worker labor pool size range; i.e., $[100, 400]$, $[400, 700]$, etc. We provide the average labor pool size, K_j, and the probability of occurrence α_j for each type of agent. For example, $\alpha_1 = 54/1452 = 0.04$.

[32] Labor contractors must satisfy various requirements set by the US Department of Labor (e.g., regarding location and duration of work or financial stability). See: https://www.ecfr.gov/current/title-20/chapter-V/part-655/subpart-B/section-655.132 [1 Jun 2023].

[33] We use this reduced data set to eliminate agents with a very small or very large labor pool so that we can focus on the characteristics of a typical agent for a mid-sized buyer.

j	Interval of K	No. of agents	α_j	Average D	Average w	K_j
1	[1000,1300]	54	0.04	554.5	0.489	1136
2	[700,1000)	128	0.09	420.0	0.503	835
3	[400,700)	291	0.20	271.3	0.504	537
4	[100,400)	979	0.67	113.4	0.547	210

Table 4.1: Four types of labor-agents.

To calibrate other agent-specific parameters, we use $\gamma = 1.96$ and $c_g = 1.0$ to ensure that our modeling assumption $\gamma < 2c_g$ holds. Also, we set the loss m to 10% of the average annual labor costs of an agent[34] ($\bar{w} \cdot \bar{D} = 0.65 \cdot 189 = 122.9$), such that $m = 0.1 \cdot 122.9 = 12.29$.

For buyer-specific parameters, we assume the in-house recruitment cost u to be 25% of the average wages so that $u = 0.25 \cdot \bar{w} = 0.25 \cdot 0.65 = 0.16$, and that the labor pool size for internal recruitment is $K = 120$ workers. We further assume that the worker-related costs in seasonal agriculture are 40% of total revenues [35], i.e., $(\bar{w} + u) = 0.4 \cdot r$, such that $r = (0.65 + 0.16)/0.4 = 2.03$. We also assume that audit cost are 5% of wage[36] such that at an average audit level of 50%, we have $0.5 \cdot c_a = 0.05 \cdot \bar{w} \cdot \bar{D} \Rightarrow c_a = 0.05 \cdot 0.65 \cdot 189/0.5 = 12.29$.

j	Price $\hat{p}_j^*$	Quantity $\hat{D}_j^*$ (%)	Audit a_j^*	Wage w_j	Agent markup	Prob.	Buyer profit Π_j	Agent profit π_j	Wages $w_j \cdot D_j$	**SC profit**
1	0.29	178.1 (94%)	0.44	0.16	81%	0.11	323.7	23.9	28.5	**376.1**
2	0.31	169.8 (90%)	0.54	0.20	55%	0.23	318.1	18.2	34.0	**370.3**
3	0.35	155.2 (82%)	0.69	0.29	21%	0.36	305.7	9.6	45.0	**360.4**
4	0.55	114.9 (61%)	0.89	0.55	–	0.30	252.2	0.0	63.2	**315.4**
$\mathbb{E}$	–	149.0 (79%)	0.69	0.33	18%	0.39	294.5	10.3	46.1	**350.9**

Agent markup defined as $(\hat{p}_j^* - w_j)/w_j$. Prob = selection probability. SC profit = Supply chain profit.

Table 4.2: Optimal contracts, wages, agent markup, selection probability, and profits ($n = 3$ agents).

We illustrate the analysis with a situation in which the buyer has to select one among three agents ($n = 3$), where each agent belongs to one of the $q = 4$ types with probability α_j estimated in Table 4.1. We apply the procedure introduced in Section 4.7 and Appendix 4.7.2. The left-hand side of Table

[34] See, e.g., https://www.dol.gov/newsroom/releases/whd/whd20220510 [1 Jun 2023].

[35] Specialty crops in the US have labor expenses at approx. 40% of revenue. See, e.g., https://www.ers.usda.gov/data-products/chart-gallery/gallery/chart-detail/?chartId=98569 [1 Jun 2023].

[36] For a mid-sized farm with quarterly audits, audit cost are approx. 5% of labor costs. See, e.g., https://migration.ucdavis.edu/rmn/blog/post/?id=2440 [1 Jun 2023].

4.2 provides the optimal contracts $(\hat{p}_j^*, \hat{D}_j^*, \hat{a}_j^*)$ to the buyer's problem under information asymmetry to be executed in a sequential manner, as well as the resulting wage w_j, agent mark-up $\left(\hat{p}_j^* - w_j\right)/w_j$, and the selection probability.[37] In the last line of the table (indicated $\mathbb{E}$), we provide the *expected* quantities.[38] The expected outsourced quantity is $\mathbb{E}D_j = 149.0$, which corresponds to 79% of the total sourcing quantity $D = 189$, indicating that the large majority is, on average, outsourced under the optimal contracts. The demand D_j (and the audit level a_j^*) increases from the first to the fourth contract, because agents with a smaller labor pool size can serve less demand and require more control. The expected agent markup that the agent earns is 18% on average. However, the markup depends strongly on the type of the agent: Agents of type 1 earn the highest mark-up, 81%, while agents of type 4 have zero markup, confirming the results of part (1) of Proposition 6.

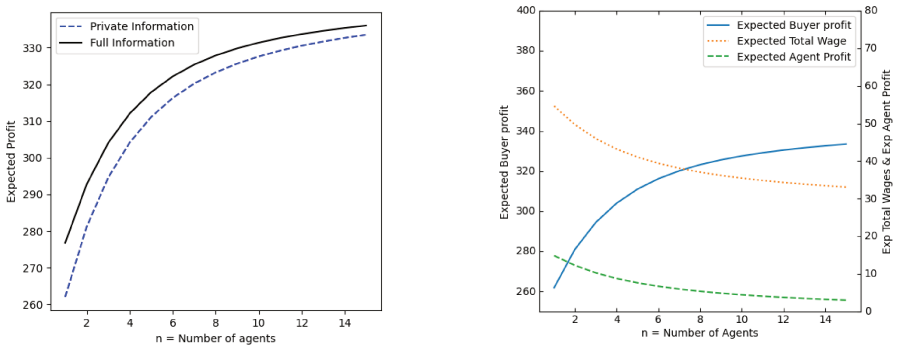
The right-hand-side of Table 4.2 indicates the profit of each actor (buyer, agent, and workers) for each contract and in expectation. We also provide the profit of the labor supply chain (the sum of the profit of the buyer, the profit of the agent, and the worker wages). The buyer earns, in expectation, $294.5/350.9 = 83.9\%$ of the total supply chain profit. Also, in expectation, the agent only earns 2.9% and the workers 13.2%, illustrating the difficulty of the agent as an intermediary in the supply chain to extract profit, as we have observed throughout this paper. Earnings vary with the contract that is selected. While the buyer and the agent earn less with higher contract types, the workers earn higher wages. This is a consequence of the smaller labor pool size, which is economically less advantageous for the buyer and the agent but which leads to higher wages for the concerned workers.

Observe from Table 4.2 that the expected buyer profit $\mathbb{E}(\Pi_j) = 294.5$. Now, suppose that the buyer has full information about the type of each of the $n = 3$ agents and directly chooses the one

[37] The selection probability indicates the probability that a certain type is the best agent type in the pool of $n = 3$ agents, given by the product of (4.45) and (4.46).

[38] The expected quantities are obtained from averaging the quantities of each of the four contracts, weighted with the selection probabilities.

with the largest labor pool.[39] Then, the buyer can use Proposition 2 to determine the optimal contract and his corresponding expected profit, which is 304.0 in this case. Hence, we can conclude that the *loss from information asymmetry* values $304.0 - 294.5 = 9.5$, which is $9.5/294.5 = 3.2\%$ of buyer profit.



(a) Buyer profit with private or full information. **(b)** Buyer profit, agent profit, and workers' wages.

Figure 4.5: Sensitivity of profits as a function of the number of agents n.

Next, we perform sensitivity analyses by varying the number of agents from which the buyer can choose (n) from 1 to 15. On the left panel (Figure 4.5a), we provide the buyer profit in the case of private information (information asymmetry) and in the case with full information. We observe that buyer profit increases as expected with a larger number of agents to choose from. However, we also observe that the loss from information asymmetry decreases in n. This is because, as n increases, the chance of contracting with an agent with a large labor pool in the earlier round of our sequential contracting process increases. Consequently, information asymmetry is less penalizing, and the loss from information asymmetry decreases. Next, consider Figure 4.5b, in which we plot the buyer profit, agent profit, and workers' wages as a function of n. While buyer profit increases, worker wage decreases in n, which is because the probability of selecting an agent with a greater labor pool size increases as ex-

[39] By considering the probability of all combinations of the types of $n = 3$ agents, we can determine the probability of the buyer ending up contracting with a type j agent (given by the product of (4.45) and (4.46)) and use these probabilities to determine the buyer's expected profit.

plained above. As a result of increasing the odds of contracting with an agent with a larger labor pool, the buyer can afford to offer a lower wage to recruit workers. Agent profit also decreases because the buyer can use the higher probability of selecting an agent with a larger labor pool to extract information rent from the agent.

4.9 Conclusion

In this paper, we have examined the interaction between a buyer who cares about social responsibility, profit-seeking labor agents, and a group of workers with heterogeneous reservation wage in a typical labor supply chain, such as can be found in the agricultural sector in the U.S. We explored how the market conditions (size of the labor pool and the workers' reservation wage) and the economic conditions (coercion cost for the agent, audit cost, and contract price) affect the occurrence of forced labor in such labor supply chains. For this purpose, we presented a game-theoretic model of a buyer as leader and an agent as follower. We considered wage and coercion as tools for the agent to recruit workers, and price and auditing as tools for the buyer to contract with the agent and to prevent coercion.

We considered the case with full information and a single agent in the base model, and we derived the optimal buyer contract and the agent's profit-maximizing response. We showed that the optimal contract price and audit level increase with demand but decrease with the agent's labor pool size. We showed that for low audit cost, the buyer pays the minimum price that the agent needs to pay as wages to recruit workers, and audits extensively to deter coercion. For high audit cost, however, the buyer prefers to pay a higher-than-minimum contract price and audit less. Then, we extended our base model to the case where the agent's labor pool size (low or high) is private information. To prevent forced labor, the buyer offers a menu of two contracts to elicit the agent's true type. We characterized the optimal menu of contracts, and found that the difference in the agent's earnings between

two contracts determines how they have to be adjusted to ensure truthful revelation. Interestingly, we found that the optimal contract price is not always higher for the agent with the large labor pool. We also found that truthful revelation always gives a strictly positive profit to the agent with the large labor pool, while the profit of the other agent depends on the audit cost. We also considered the case when the buyer chooses an agent from several candidates with private information. We used the insights from the single-agent case to develop a reverse-auction-like sequential menu of contracts for the multi-agent case that enables the buyer to pick the agent with the largest labor pool, ensuring no coercion and maximum buyer profit. We applied our model to a data set of labor agents for agricultural work in the US, and found that the buyer keeps 84% of the supply chain's profit, while the agent only gets 3% and the workers 13%. Agent profit, however, varies strongly by type, from a 81% markup on wages for the largest labor pool to zero profit for the smallest labor pool.

Our results provide insights to decision makers in socially-responsible buying firms on how to reduce the risk of coercion in the labor supply chain. This can be achieved through effective contract sizing, pricing, and auditing. Labor agents, who often recruit workers abroad in an opaque manner, should be paid above the minimum fair wage to reduce their incentives to coerce workers. Additionally, designing incentive-compatible menus of contracts can induce the truth about the agent. However, addressing information asymmetry comes at a cost and may prohibit the buyer from extracting the entire surplus from the agent, which is typically possible in traditional supply chain settings.

Our model has several limitations. For example, we assume that the worker's reservation wage is uniformly distributed and that the audit detection probability is the product of audit level and share of coercion. As an initial attempt to examine coercion-free contracts, we made these assumptions for tractability. Our work can be extended in several ways. We focus our analysis on the case when the buyer selects exactly one agent from several candidates. It may be of interest to extend our setting to

a more general setting in which the buyer has to select multiple agents. Future research could also go beyond our paper and establish our findings empirically from field data. Finally, it may be worthwhile to investigate how competition among buyers and competition among agents on the labor market moderate our findings.

5

Conclusion

Chapters 2 and 3 study the crowdsourced humanitarian relief vehicle routing problem. In the current ecological scenario, it makes sense to use crowdsourcing to optimize rescue and relief efforts in a humanitarian crisis. This research explores heuristics as well as a few exact solution methods. While the heuristic was able to solve large instances quite effectively, the branch-and-cut method with tailored Benders cuts could not outperform the branch-and-cut with generalized sub-tour elimination constraints. An exploration of other exact methods like branch and price might bring better results. It has to be noted that for the min-max rescue time models, there is a high amount of symmetry. To break symmetry we can add a small component that can differentiate between solutions with same maximum rescue time. The research also explores different objective functions and studies the min-max rescue time and average rescue time objective functions. The choice of the objective function is a very relevant topic of research in humanitarian applications. A wider discussion on objective functions for routing problems in humanitarian applications will also add more value to the existing body of literature.

Chapter 4 explores the contract mechanism between a buyer and labor agent in labor supply chains as a Stackelberg game between buyer (leader) and the agent (follower). The relationship between optimum contract price and audit level and demand shows that increasing demand correspondingly increases the price and audit level. The optimal menu of contracts to maximize buyer's profit is suggested. The impact of information asymmetry can be minimized by focusing on increasing the number (and variety) of agents available to choose from. For high audit cost situations, the buyer would prefer offering higher than the minimum wage with low level of audit. For situations with low audit cost, the buyer will choose to drive down the price while focusing on higher level of audits.

With the establishment of multiple modern slavery prevention acts being implemented by different governments across the world, it is imperative for business to shield themselves against risks from

modern slavery being present in lower levels of the supply chain. History has shown that typically companies cannot hide behind saying they do not directly deal with unethical suppliers. The public push back is tier-agnostic. With the newer modern slavery regulations not restricting themselves merely as a 'reporting' act, any slip ups by the buying companies will quite significantly affect their bottom lines. There has to be more concerted efforts by everyone, including research community to come up with ethical, sustainable and economically viable solutions to ensure exploitation does not happen. One of the interesting areas of research in this field would be study how to bridge the reach-accountability incongruence. This term is used to denote the situation when a customer-facing buying company is held accountable for the actions of a supplier quite lower down the supply chain. An interesting question to explore would be how to design supply chain contracts to ensure ethical behavior in entire supply chain. Yet another interesting, but tricky question to explore would be the effect of modern slavery acts themselves on the workers. An interesting example was observed in Sri Lanka. A high proportion of female tea plantation workers in Sri Lanka, out of desperation and extreme poverty, were relying on prostitution as an alternate source of income. When the UK Modern Slavery act was introduced, the buying companies insisted the suppliers in Sri Lanka that having workers who indulged in activities that fall under modern slavery (sexual exploitation) would lead to their contracts being terminated. In this situation, the plantation managers had no choice but to give an ultimatum to the female workers to make a choice between the job at the plantation and sex-work. Superficially this looks like a win for the modern slavery act. But upon closer inspection, these women have been denied a significant chunk of their monthly income and without any formal education and skills they are pushed more into poverty. The solution is not simple. Modern slavery regulations have to involve rehabilitation and support for the workers who have been taken out of the oppressive system. But how much of the effort will be put in by the buyer and how much of the effort will be put in by the supplier is an interesting question to study.

At the end of the day, there still are a lot of problems that persist in the society. They deserve attention from all stakeholders and especially the academic community. And rightly so, as our very existence is contingent on finding workable solutions to these problems to create a safer tomorrow. This thesis ends on a hopeful note that the pace of 'true' progress will increase with each passing day.

A

A.1 Appendix- Chapter 2

Table A.1: Operator Efficiency

ID	Best Result	Best Result w/o CRR	Best Result w/o 2-opt	Best Result w/o Swap	Best Result w/o S-Assign	% Gap w/o CRR	% Gap w/o 2OPT	% Gap w/o SWAP	% Gap w/o S-Assign
M1	184.01	184.01	216.68	184.01	184.01	0.0%	17.8%	0.0%	0.0%
M2	173.50	184.01	195.07	173.50	174.83	6.1%	12.4%	0.0%	0.8%
M3	172.71	192.24	196.94	170.35	175.30	11.3%	14.0%	-1.4%	1.5%
M4	172.71	192.24	191.38	170.35	175.30	11.3%	10.8%	-1.4%	1.5%
M5	172.71	192.24	189.18	170.35	175.30	11.3%	9.5%	-1.4%	1.5%
M6	172.71	192.24	189.18	170.35	175.30	11.3%	9.5%	-1.4%	1.5%
M7	195.02	216.13	199.01	195.02	195.02	10.8%	2.0%	0.0%	0.0%
M8	164.15	171.20	182.86	170.59	164.15	4.3%	11.4%	3.9%	0.0%
M9	172.40	188.64	180.50	172.24	172.40	9.4%	4.7%	-0.1%	0.0%
M10	176.28	186.56	175.84	170.38	176.28	5.8%	-0.2%	-3.3%	0.0%
M11	214.61	214.61	223.47	214.61	214.61	0.0%	4.1%	0.0%	0.0%
M12	147.68	170.37	161.58	155.10	147.68	15.4%	9.4%	5.0%	0.0%
M13	147.30	156.67	162.40	149.25	147.30	6.4%	10.3%	1.3%	0.0%
M14	139.90	158.70	149.10	143.49	149.36	13.4%	6.6%	2.6%	6.8%
M15	147.36	156.24	159.15	143.43	147.36	6.0%	8.0%	-2.7%	0.0%
M16	135.10	151.27	149.16	136.25	135.10	12.0%	10.4%	0.9%	0.0%
M17	139.33	150.32	147.03	135.55	140.17	7.9%	5.5%	-2.7%	0.6%
M18	126.35	138.44	148.55	126.35	130.65	9.6%	17.6%	0.0%	3.4%
Average						8.5%	9.1%	0.0%	1.0%

Table A.2: Operator Efficiency- Time gains

ID	Base Time (secs)	% Time saved w/o S-Assign	% Time saved w/o 2-opt	% Time saved w/o CRR	% Time saved w/o Swap
M1	72.09	-1.3%	3.5%	-42.5%	-3.8%
M2	91.85	-22.5%	-20.8%	-54.8%	-24.2%
M3	83.76	-17.8%	-18.7%	-50.0%	-21.9%
M4	88.51	-17.6%	-14.8%	-52.5%	-20.9%
M5	90.27	-20.8%	-21.9%	-53.6%	-23.6%
M6	89.41	-19.0%	-17.8%	-50.5%	-22.5%
M7	83.34	1.1%	-25.9%	-41.7%	-28.1%
M8	73.28	3.2%	-2.6%	-38.0%	-9.2%
M9	77.69	1.2%	-5.2%	-52.5%	-8.9%
M10	71.02	2.8%	-16.9%	-50.6%	-18.3%
M11	37.97	-3.2%	-79.4%	51.2%	-79.1%
M12	73.89	-8.5%	-19.5%	-51.0%	-20.0%
M13	57.66	-5.8%	-20.6%	-47.4%	-22.4%
M14	49.47	-10.1%	10.1%	-46.9%	8.2%
M15	125.31	-10.2%	-0.5%	-50.7%	-0.8%
M16	142.9	-0.6%	-5.3%	-53.4%	16.9%
M17	192.24	-3.8%	-4.6%	-56.7%	-5.1%
M18	246.88	16.8%	4.3%	-60.7%	4.0%
Average		-6.5%	-14.3%	-44.6%	-15.5%

Table A.3: Comparison of performance of **KPI 1** vs **KPI 2**

	Optimizing KPI 1		Optimizing KPI 2	
Instance No	KPI 1	KPI 2	KPI 1	KPI 2
S11	0.0%	8.1%	12.1%	0.0%
S12	0.0%	3.9%	17.4%	0.0%
S13	0.0%	0.9%	13.6%	0.0%
S14	0.0%	0.0%	0.0%	0.0%
S15	0.0%	2.4%	20.1%	0.0%
S16	0.0%	3.1%	21.4%	0.0%
S17	0.0%	9.1%	24.3%	0.0%
S18	0.0%	3.1%	25.5%	0.0%
S19	0.0%	4.4%	22.2%	0.0%
S20	0.0%	5.8%	10.1%	0.0%
Avg Deviation		4.1%	16.7%	
Max Deviation		9.1%	25.5%	

A.2 Appendix- Chapter 3

A.2.1 Proof of tailored benders cuts

In this formulation, we look at the term β^*_{ij} as the potential reduction on the shortest path distance when setting the arc $x_{ij} = 1$ in the tour T. When looking at the visit variable, we have two options. $\overline{w}^k_m = 0, \overline{w}^k_m = 1$. In the first case, the constraint 3.20 is not imposed on the problem as $\beta^*_{ij} \geq 0, -\beta^*_{ij}\overline{x}^k_{ij} \leq 0$. When $\overline{w}^k_m = 1$ for a given vehicle k and customer m, for ease of representation, we re-write the constraint 3.20 as the following.

$$\alpha^*_m - \alpha^*_{d(k)} - \sum_{(i,j)\in A} \beta^*_{ij}\overline{x}^k_{ij} \tag{A.1}$$

We define the following terms:

- T the tour of a vehicle k as per a given solution of the master problem
- N_T set of nodes in the tour T
- $P_T(0, m)$ path from node 0 (here we define the vehicle depot node as node 0) to node m within tour T
- $t_T(i, j)$ the distance(time) traversed along the path from node i to node j within tour T
- $t_G(i, j)$ the distance(time) from node i to node j in the original graph G

When $\overline{w}^k_m = 1$, the dual formulation reduces to SPP when $\sum_{(i,j)\in A} \beta^*_{ij}\overline{x}^k_{ij} = 0$. As the dual formulation closely resembles the classical shortest path problem on a subgraph $G_T \subset G$ defined by the binary vector $\bar{x} \in X$, we can simplify the expression by setting $\alpha^*_{d(k)} = 0$. From the system (3.18a) - (3.18c), we can see that the subproblem is a SPP on the subgraph G_T. Now we use the following

proposition:

Proposition 11. *For a master solution $\bar{x} \in \{0,1\}^{|A|}$ and node m, an optimal (α^*, β^*) solution can be calculated as*

1. $\alpha_i^* = t_T(0,i)$ `if` $i \in N_T, i \in P_T(0,m)$
2. $\alpha_i^* = min(t_T(0,i), \alpha_m^* - t_G(i,m))$ `if` $i \in N_T, i \notin P_T(0,m)$
3. $\alpha_i^* = \alpha_m^* - t_G(i,m)$ `if` $i \notin N_T$
4. $\beta_{ij}^* = max(0, \alpha_j^* - \alpha_i^* - t_{ij})$ $\forall$ $i,j \in A$

Our next step is to prove that the expressions from Proposition 1 is a feasible solution for the dual problem denoted above and that the value of the objective function equals the length of the shortest path. This will prove that α^*, β^* are the optimal solution for the dual. The proof has two parts. First part is showing the feasibility. Clearly from (iv) above, $\beta_{ij}^* \geq \alpha_j^* - \alpha_i^* - t_{ij}$ and $\beta_{ij}^* \geq 0$. this satisfies both constraints 3.19b and 3.19c. For proving the value of the objective function after substituting these values is indeed the length of the shortest path, $\alpha_m^* = t_T(0,m)$, we just need to prove that $\beta_{ij}^* \bar{x}_{ij}^k = 0$. For this, it is sufficient to prove that when $\bar{x}_{ij} = 1, \beta_{ij}^* = 0$. As the arc $x_{ij}^* = 1$, we know that in the tour T, i,j nodes are connected. Which implies that $t_T(0,j) - t_T(0,i) = t_{ij}$.

We have the following situations for arc (i,j). For the given k, m we assume that $P_T^*(d(k),m)$ is the shortest path from the depot of vehicle k to the node m.

1. Arc (i,j) lies on the $P_T^*(d(k),m)$: Arc (i,j) lies on shortest path. Hence $\alpha_j^* - \alpha_i^* = t_{ij}$. Hence $\beta_{ij}^* = 0$
2. Arc (i,j) does not lie on $P_T^*(d(k),m)$, but node i does. This means that $\alpha_i^* = t_T(0,i)$ and $\alpha_j^* = min(t_T(0,j), \alpha_m^* - t_G(j,m)) \implies \alpha_j^* \leq t_T(0,j) \implies \alpha_j^* - \alpha_i^* - t_{ij} \leq t_T(0,j) -$

$t_T(0,i) - t_{ij} = 0$. So we have $\alpha_j^* - \alpha_i^* - t_{ij} = \beta_{ij}^* \leq 0$ and $\alpha_j^* - \alpha_i^* - t_{ij} = \beta_{ij}^* \geq 0$. This means that $\beta_{ij}^* = 0$

3. Arc (i,j) does not lie on $P_T^*(d(k),m)$, and neither does nodes i,j. This means that $\alpha_i^* = min(t_T(0,i), \alpha_m^* - t_G(i,m))$ and $\alpha_j^* = min(t_T(0,j), \alpha_m^* - t_G(j,m))$. Here there are 4 subcases.

   # $\alpha_i^* = t_T(0,i), \alpha_j^* = t_T(0,j)$: $\alpha_j^* - \alpha_i^* - t_{ij} = 0 \implies \beta_{ij}^* = 0$

   # $\alpha_i^* = t_T(0,i), \alpha_j^* = \alpha_m^* - t_G(j,m)$: $\alpha_j^* - \alpha_i^* - t_{ij} = \alpha_m^* - t_G(j,m) - t_T(0,i)$. We also observe that $\alpha_j^* \leq t_T(0,j) \implies \alpha_j^* - \alpha_i^* - t_{ij} \leq t_T(0,j) - t_T(0,i) - t_{ij} = 0$. So we have $\beta_{ij}^* = 0$

   # $\alpha_i^* = \alpha_m^* - t_G(i,m), \alpha_j^* = t_T(0,j) \implies \alpha_j^* - \alpha_i^* = t_T(0,j) - \alpha_m^* + t_G(i,m)$. Also we have $t_T(0,j) \leq \alpha_m^* - t_G(j,m) \implies \alpha_j^* - \alpha_i^* \leq \alpha_m^* - t_G(j,m) - \alpha_m^* + t_G(i,m) = t_G(i,m) - t_G(j,m)$. From the original graph, we can observe that $t_G(i,m) \leq t_G(j,m) + t_{ij}$. Otherwise, it would not be the shortest path. So, $t_G(i,m) - t_G(jm) \leq t_{ij} \implies \alpha_j^* - \alpha_i^* - t_{ij} \leq 0 \implies \beta_{ij}^* \leq 0$. Using this result and 3.19c we have $\beta_{ij}^* = 0$

   # $\alpha_i^* = \alpha_m^* - t_G(i,m), \alpha_j^* = \alpha_m^* - t_G(j,m)$: here we have $\alpha_j^* - \alpha_i^* - t_{ij} = t_G(i,m) - t_G(j,m) - t_{ij}$. However, from geometry we have the triangular inequality $t_G(i,m) \leq t_G(j,m) + t_{ij} \implies t_G(i,m) - t_G(j,m) - t_{ij} \leq 0 \implies \alpha_j^* - \alpha_i^* - t_{ij} = \beta_{ij}^* \leq 0$. So we have $\beta_{ij}^* \leq 0, \beta_{ij}^* \geq 0$. Hence $\beta_{ij}^* = 0$

A.2.2 Problem with tailored benders cuts- Aggregated formulation

When $x_{ij}^k = 1$ in the disaggregated formulation for a specific vehicle index k, we know for sure that arc ij is a part of the tour of the vehicle under consideration. However, for aggregated formulation, if $x_{ij} = 1$, it may not necessarily be a part of the tour of the vehicle under consideration. It could also be that there is an arc that extends from the same supplier present in the tour, but traversed by

another vehicle towards another node. In this case, statement number 2 is not valid anymore, as $\alpha_j^* = \alpha_m^* - t_G(i,m)$. This implies that $\alpha_j^* - \alpha_i^* - t_{ij} = \beta_{ij}^* = \alpha_m^* - t_G(j,m) - t_T(0,i) - t_{ij}$ need not be less than or equal to 0 as the proof concludes. When both i, j are not in tour and $x_{ij} = 1$, however, the proof is still valid. This is why the problem only comes up when either i is in the tour and j is not, or vice versa. Nevertheless, we can observe that constraints (3.6j) to (3.6l) can be expressed in the following form as the following sub-problem:

$$z_k(\bar{x}, \bar{w}, m, k) \qquad = \text{minimize} \qquad \sum_{i,j \in N} f_{ij}^{mk} t_{ij} \tag{A.2a}$$

subject to

$$\sum_{j \in N} f_{ji}^{mk} - \sum_{j \in N} f_{ij}^{mk} = \begin{cases} -w_m^k & \text{if } i = d(k) \\ w_m^k & \text{if } i = m \\ 0 & \text{otherwise} \end{cases} \qquad \forall k \in K, \forall m \in N_{sd}, i \in N \tag{A.2b}$$

$$f_{ij}^{mk} \leq x_{ij} \qquad \forall i, j \in N, k \in K, m \in N_{sd} \tag{A.2c}$$

$$f_{ij}^{mk} \geq 0 \qquad \forall i, j, m, k \tag{A.2d}$$

This is the starting point of derivation of Tailored Benders decomposition for the aggregated model.

A.3 Appendix- Chapter 4

A.3.1 Summary of Notation

Symbol	Description
a	Audit effort level of buyer
a_i	Audit effort level under contract i
c_g	Coercion cost incurred by the agent per unit coercion level
c_a	Audit cost for each audit level unit
γ	Effectiveness of coercion
D	Labor demand of the buyer
D_i	Labor quantity of contract i
$f(a,g)$	Probability of detecting coercion during audit for an audit effort level a and coercion level g
g	Coercion level of the agent
g_i	Coercion level of the agent of type i
K	Labor pool size
K_i	Size of labor pool available to agent of type i
m	Loss incurred by the agent in case coercion is detected during audits
n	Number of agents
p	Price paid to the agent per worker
p_i	Price per worker paid to the agent under contract i
Ψ	Reputation damage suffered by the buyer (in the event when forced labor is present)
$\pi(\cdot)$	Agent's profit
$\Pi(\cdot)$	Buyer's profit
q	Number of types of agents
r	Revenue obtained by the buyer for each worker
u	Inhouse recruitment cost per worker
v	Reservation value of workers, $v \sim U[0,1]$
w	Wage paid to the workers
w_i	Wage paid to the workers by agent of type i
w^0	Fair wage to be paid
$(\cdot)^0$	Benchmark values for in-house recruitment

Table A.4: Definition of Notation.

A.3.2 Proofs

Proof. **Proof of Proposition 1:** Taking the derivative of the agent's profit (4.2) with respect to g and using (4.1),

$$\frac{d}{dg}\pi(p,a,g) = -(pD+m)\frac{d}{dg}f(a,g) + \gamma D - 2c_g\gamma K g. \tag{A.3}$$

Further, using the form of detection probability $f(a,g) = a\frac{\gamma g K}{D}$, we obtain

$$\frac{d}{dg}\pi(p,a,g) = -(pD+m)a\frac{K\gamma}{D} + \gamma D - 2c_g\gamma K g, \tag{A.4}$$

which leads to

$$\begin{aligned} \frac{d}{dg}\pi(p,a,g) &= 0 \\ \Leftrightarrow -(pD+m)a\frac{K\gamma}{D} + \gamma D &= 2c_g\gamma K g \\ \Leftrightarrow g^* &= \frac{D}{2c_g \cdot K} - \frac{a}{2c_g}\left(p + \frac{m}{D}\right). \end{aligned} \tag{A.5}$$

We observe that $\pi(p,a,g)$ is concave in g for given p and a. Level g is limited by the upper bound that ensures non-negative wage w, see discussion in Section 4.4. From (4.1), we can retrieve w^*. □ □

Proof. **Proof of Lemma 1:** The lemma can be shown easily as follows. Recall from Section 4.4 that the assumption $\Psi > rD$ implies that any contract (p', a') that entices the agent to coerce $g^*(p', a') > 0$ cannot be optimal because the buyer profit $\Pi(p', a', g^*(p', a')) < 0$, while $\Pi(p^*, a^*, g^*(p^*, a^*)) \geq 0$ by assumption. Hence, under the assumption $\Psi > rD$, all optimal contracts (p^*, a^*) must be coercion free so that $g^*(p^*, a^*) = 0$. □ □

Proof. **Proof of Proposition 2** When $g^* = 0$, observe from (4.5) that we can express the audit level a in terms of price p so that $a = \frac{\frac{D}{K}}{p+\frac{m}{D}}$. Combine this observation with (4.3) and (4.7) for the case when $g^* = 0$, the buyer's problem can be rewritten as:

$$\max_{p \geq D/K}[(r-p)D - c_a a] = \max_{p \geq D/K}\left[(r-p)D - c_a\left(\frac{\frac{D}{K}}{p+\frac{m}{D}}\right)\right].$$

We obtain as derivative of the buyer profit with respect to p

$$\frac{d}{dp}\left((r-p)D - c_a\frac{D}{K}\left(p+\frac{m}{D}\right)^{-1}\right) = -D + c_a\frac{D}{K}\left(p+\frac{m}{D}\right)^{-2}, \tag{A.6}$$

which decreases in p, such that the first-order condition yields:

$$p^* = \max\left\{\frac{D}{K}, \sqrt{\frac{c_a}{K}} - \frac{m}{D}\right\}. \tag{A.7}$$

Optimal audit level a^* follows from

$$g^*(p^*, a^*) = 0 \Leftrightarrow p = \frac{D}{aK} - \frac{m}{D} \Leftrightarrow a^* = \frac{D}{K(p^* + \frac{m}{D})}.$$

For $c_a \geq K(\frac{D}{K} + \frac{m}{D})^2$, it follows consequently $p^* = \sqrt{\frac{c_a}{K}} - \frac{m}{D}$ and, hence, $a^* = \frac{D}{\sqrt{Kc_a}}$. For $c_a < K(\frac{D}{K} + \frac{m}{D})^2, p^* = \frac{D}{K}$ and, consequently, $a^* = \frac{D^2}{D^2+mK}$. The optimal buyer profit follows from $(r - p^*)D - c_a a$ according to (4.3) with $g^*(p^*, a^*) = 0$. □ □

Proof. **Proof of Corollary 1** We know from (4.4) that the buyer profit when recruiting directly is $\Pi^0 = rD - \frac{D^2}{K} - uD$. It makes sense for the buyer to outsource when the profit from the optimal coercion-free-contract (p^*, a^*), which is given by $\Pi = (r - p^*)D - c_a a^*$, is greater than benchmark profit Π^0. This implies

$$\begin{aligned}
(r - p^*)D - c_a a^* &> rD - \frac{D^2}{K} - uD \\
\Leftrightarrow rD - p^*D - c_a a^* &> rD - \frac{D^2}{K} - uD \\
\Leftrightarrow uD &> p^*D + c_a a^* - \frac{D^2}{K} \\
\Leftrightarrow u &> p^* - \frac{D}{K} + \frac{c_a a^*}{D}
\end{aligned} \tag{A.8}$$

which can also be expressed as

$$u > p^* - w_0 + \frac{c_a a^*}{D} \tag{A.9}$$

The additional conditions are easily obtained when substituting (p^*, a^*) in (A.8) with the optimal contracts derived in Proposition 2. □ □

Proof. **Proof of Lemma 2.** For any given (D_l, D_h), we can derive the optimal price p_i^* by considering the first order condition of $\widehat{\Pi}(p_l, D_l, p_h, D_h)$ given in (4.16) (with respect to p_i) so that:

$$\frac{\partial}{\partial p_i}\widehat{\Pi}(p_l, D_l, p_h, D_h) = -\alpha_i D_i + \alpha_i \frac{c_a D_i}{K_i(p_i + m/D)^2} = 0 \Leftrightarrow p_i^* = \sqrt{\frac{c_a}{K_i}} - \frac{m}{D}. \tag{A.10}$$

Note that the second derivative of the profit function with respect to p_i is strictly negative, i.e., buyer profit is concave in price p_i. Observe from (4.22) that p_i^* is independent of D_i. By substituting the expression for p_i^* into the profit function of the buyer given in (4.15) and by setting the derivative with respect to D_i to zero, we obtain:

$$\begin{aligned}
&\frac{\partial}{\partial D_i}\widehat{\Pi}(p_l^*, D_l, p_h^*, D_h) = -\alpha_i(p_i^* - u) - \alpha_i \frac{c_a}{K_i(p_i^* + m/D)} + \alpha_i \frac{2(D - D_i)}{K} = 0 \\
&\Leftrightarrow D_i^* = D - \frac{K}{2}\left(\sqrt{\frac{c_a}{K_i}} - \frac{m}{D} - u + \frac{c_a}{K_i(\sqrt{\frac{c_a}{K_i}})}\right) \\
&= D - K\left(\sqrt{\frac{c_a}{K_i}} - \frac{m/D + u}{2}\right),
\end{aligned} \tag{A.11}$$

The second derivative of buyer profit with respect to D_i is $-2\alpha_i/K < 0$. Consequently, buyer profit under the optimal price (under any given price p_i) is strictly concave in D_i. □ □

Proof. **Proof of Proposition 3.**

Constraints ICl and ICh are both satisfied for a given solution if and only if it holds that

$$\frac{1}{K_h}((D_h)^2 - (D_l)^2) \leq p_h D_h - p_l D_l \leq \frac{1}{K_l}(\delta(D_h)^2 - (D_l)^2). \tag{A.12}$$

Note from (A.12) that, if either ICl or ICh is tight at a feasible point, the condition that the second

constraint is satisfied can be written as

$$\frac{1}{K_h}((D_h)^2-(D_l)^2)\leq\frac{1}{K_l}(\delta(D_h)^2-(D_l)^2)\Leftrightarrow\left(\frac{\delta}{K_l}-\frac{1}{K_h}\right)(D_h)^2\geq\left(\frac{1}{K_l}-\frac{1}{K_h}\right)(D_l)^2$$

$$\Leftrightarrow D_h\geq\sqrt{\left(\frac{1}{K_l}-\frac{1}{K_h}\right)\left(\frac{\delta}{K_l}-\frac{1}{K_h}\right)^{-1}}\cdot D_l>D_l, \tag{A.13}$$

Thus, D_h has to be greater than the value given in (A.13) to allow both IC constraints to be respected (except for the case $D_h=D_l=0$ in which equality holds). Note that the term under the square root is strictly greater than one, because $K_l<K_h$ and $(K_l/K_h)<\delta<1$.

Under the assumptions of the proposition, we can rewrite problem (4.16)-(4.21) as

$$\max_{(y_l,y_h)}\alpha\left(-y_l-\frac{c_aD_l^2}{K_l}\left(y_l+\frac{mD_l}{D}\right)^{-1}\right)+(1-\alpha)\left(-y_h-\frac{c_aD_h^2}{K_h}\left(y_h+\frac{mD_h}{D}\right)^{-1}\right) \tag{A.14}$$

subject to

$$ICh: y_h-y_l\geq\frac{D_h^2}{K_h}-\frac{D_l^2}{K_h}, \tag{A.15}$$

$$ICl: y_l-y_h\geq\frac{D_l^2}{K_l}-\delta\frac{D_h^2}{K_l}, \tag{A.16}$$

$$IRl: y_l\geq\frac{D_l^2}{K_l}, \tag{A.17}$$

where we substitute $y_i=p_iD_i, i=l,h$. The term $\alpha(rD_l+\Pi^c(D_l))+(1-\alpha)(rD_h+\Pi^c(D_h))$ in the objective function is constant in p_i and constraints (4.21) on demand quantities D_i hold by assumption for (D_l,D_h). Note that the objective function is easily seen to be concave in y_l and in y_h and, because it is separable, it is also jointly concave in (y_l,y_h), and that the constraints are affine-linear in y_l,y_h, such that the optimization problem is concave.

First, let us assume that $y_l^*\geq D_l^2/K_l$**, i.e., that IRl is satisfied at the unconstrained optimal prices.** It is easy to observe from (4.22) that condition $y_l^*\geq D_l^2/K_l$, i.e., that IRl is satisfied, can be

expressed as

$$\left(\sqrt{c_a/K_m} - (m/D)\right) D_l \geq D_l^2/K_l \Leftrightarrow c_a \geq K_m((D_l/K_l) + (m/D))^2.$$

We can write the Lagrangian function as

$$\begin{aligned} L(y_l, y_h, \lambda_h, \lambda_l) = \alpha \left(-y_l - \frac{c_a D_l^2}{K_l}\left(y_l + \frac{mD_l}{D}\right)^{-1}\right) + (1-\alpha)\left(-y_h - \frac{c_a D_h^2}{K_h}\left(y_h + \frac{mD_h}{D}\right)^{-1}\right) \\ + \lambda_l \left(y_l - y_h - \frac{D_l^2}{K_l} + \delta\frac{D_h^2}{K_l}\right) + \lambda_h \left(y_h - y_l - \frac{D_h^2}{K_h} + \frac{D_l^2}{K_h}\right), \end{aligned} \tag{A.18}$$

where $\lambda_l, \lambda_h \geq 0$ are the Lagrangian multipliers of the IC constraints. We derive a saddle point for $L(\cdot)$, by solving

$$\begin{aligned} &\frac{\partial L}{\partial y_l} = -\alpha + \frac{\alpha c_a D_l^2}{K_l}(y_l + \frac{mD_l}{D})^{-2} + \lambda_l - \lambda_h = 0 \\ &\Leftrightarrow \hat{y}_l^* = \sqrt{\frac{\alpha c_a}{K_l(\alpha - \lambda_l + \lambda_h)}} D_l - \frac{mD_l}{D}, \end{aligned} \tag{A.19}$$

$$\begin{aligned} &\frac{\partial L}{\partial y_h} = -(1-\alpha) + \frac{(1-\alpha) c_a D_h^2}{K_h}(y_h + \frac{mD_h}{D})^{-2} - \lambda_l + \lambda_h = 0 \\ &\Leftrightarrow \hat{y}_h^* = \sqrt{\frac{(1-\alpha) c_a}{K_h(1 - \alpha + \lambda_l - \lambda_h)}} D_h - \frac{mD_h}{D}. \end{aligned} \tag{A.20}$$

Note that $0 < \alpha - \lambda_l + \lambda_h < 1$ must hold for the optimal constrained values $(\hat{y}_l^*, \hat{y}_h^*)$ to have a solution.

By assumption that the demand quantities are feasible, it holds that

$$\frac{D_h^2}{K_h} - \frac{D_l^2}{K_h} \leq \delta\frac{D_h^2}{K_l} - \frac{D_l^2}{K_l}.$$

Thus, only one of the two IC constraints (or none) can hold for given (D_l, D_h), leading to three

distinct cases, depending on the location of the difference of the unconstrained optimal values $y_h^* - y_l^*$:

$$(a)\, y_h^* - y_l^* < \frac{D_h^2}{K_h} - \frac{D_l^2}{K_h},\ (b)\, \frac{D_h^2}{K_h} - \frac{D_l^2}{K_h} \leq y_h^* - y_l^* \leq \delta \frac{D_h^2}{K_l} - \frac{D_l^2}{K_l},\ (c)\, \delta \frac{D_h^2}{K_l} - \frac{D_l^2}{K_l} < y_h^* - y_l^*. \quad \text{(A.21)}$$

Case (1A): For the first case, ICh is violated at the optimal unconstrained values (y_l^*, y_h^*). From the concavity of the buyer profit in (y_l, y_h), ICh will hold tight at its optimal constrained values, and we have $\lambda_h \geq 0, \lambda_l = 0$. We obtain

$$\begin{aligned} \hat{y}_h^* - \hat{y}_l^* = \frac{D_h^2}{K_h} - \frac{D_l^2}{K_h} &= \sqrt{\frac{(1-\alpha)c_a}{K_h\,(1-\alpha-\lambda_h)}} D_h - \frac{mD_h}{D} - \sqrt{\frac{\alpha c_a}{K_l\,(\alpha+\lambda_h)}} D_l + \frac{mD_l}{D} \\ \Leftrightarrow \frac{b_h}{\sqrt{1-\alpha-\lambda_h}} - \frac{b_l}{\sqrt{\alpha+\lambda_h}} &= b_0, \end{aligned} \quad \text{(A.22)}$$

with

$$b_h = \sqrt{\frac{(1-\alpha)c_a}{K_h}} D_h \geq 0,\ b_l = \sqrt{\frac{\alpha c_a}{K_l}} D_l \geq 0,\ b_0 = \frac{1}{K_h}\left(D_h^2 - D_l^2\right) + \frac{m}{D}\left(D_h - D_l\right). \quad \text{(A.23)}$$

Note that (A.22) must have exactly one root in the interval $[0, 1-\alpha[$, because the left-hand side is strictly increasing in λ_h and goes from a value less than b_0 (by the condition of case 1A) to $+\infty$.

We can approximate the solution to (A.22) by using the Babylonian Method[1] to approximate the

[1] see, e.g., https://www.sciencedirect.com/science/article/pii/S0315086098922091 [13 April 2023].

square roots, $\sqrt{\alpha+\lambda_h} \approx \sqrt{\alpha} + \frac{1}{2\sqrt{\alpha}} \cdot \lambda_h$ and $\sqrt{1-\alpha-\lambda_h} \approx \sqrt{1-\alpha} - \frac{1}{2\sqrt{1-\alpha}}\lambda_h$, which leads to:

$$
\begin{aligned}
&\frac{b_h}{\sqrt{1-\alpha-\lambda_h}} - \frac{b_l}{\sqrt{\alpha+\lambda_h}} - b_0 = 0 \\
\Leftrightarrow\ & b_h\sqrt{\alpha+\lambda_h} - b_l\sqrt{1-\alpha-\lambda_h} - b_0\sqrt{\alpha+\lambda_h}\sqrt{1-\alpha-\lambda_h} = 0 \\
\approx\ & b_h\left(\sqrt{\alpha} + \frac{1}{2\sqrt{\alpha}}\lambda_h\right) - b_l\left(\sqrt{1-\alpha} - \frac{1}{2\sqrt{1-\alpha}}\lambda_h\right) \\
-\ & b_0\left(\sqrt{\alpha} + \frac{1}{2\sqrt{\alpha}}\lambda_h\right)\left(\sqrt{1-\alpha} - \frac{1}{2\sqrt{1-\alpha}}\lambda_h\right) \\
\Leftrightarrow\ & \frac{b_0}{4\sqrt{\alpha(1-\alpha)}}\lambda_h^2 + \left(\frac{b_0\sqrt{\alpha}+b_l}{2\sqrt{1-\alpha}} - \frac{b_0\sqrt{1-\alpha}-b_h}{2\sqrt{\alpha}}\right)\lambda_h + \\
& (b_h\sqrt{\alpha} - b_l\sqrt{1-\alpha} - b_0\sqrt{\alpha(1-\alpha)}) = 0 \\
\Leftrightarrow\ & \lambda_h^2 + 2\left(2\alpha - 1 + \frac{b_l}{b_0}\sqrt{\alpha} + \frac{b_h}{b_0}\sqrt{(1-\alpha)}\right)\lambda_h + \\
& 4\left(\frac{b_h}{b_0}\alpha\sqrt{1-\alpha} - \frac{b_l}{b_0}(1-\alpha)\sqrt{\alpha} - \alpha(1-\alpha)\right) = 0 \\
\Rightarrow\ & \lambda_{h|1,2} \approx 1 - 2\alpha - \frac{b_l}{b_0}\sqrt{\alpha} - \frac{b_h}{b_0}\sqrt{1-\alpha} \\
& \pm\sqrt{\left(1-2\alpha-\frac{b_l}{b_0}\sqrt{\alpha}-\frac{b_h}{b_0}\sqrt{1-\alpha}\right)^2 - 4\frac{\alpha(1-\alpha)}{b_0}\left(\frac{b_h}{\sqrt{1-\alpha}} - \frac{b_l}{\sqrt{\alpha}} - b_0\right)}
\end{aligned}
\tag{A.24}
$$

where we retain the larger of the two roots (where $\pm$ is $+$) of the above quadratic function for λ_h, because the other root is non-positive (which follows from the condition that ICh is violated), which implies $b_h/\sqrt{1-\alpha} - b_l/\sqrt{\alpha} < b_0$. Because the optimization problem (A.14)-(A.16) is concave, the reduced problem with ICh enforced with equality is also concave (see Boyd and Vandenberghe, 2004, Section 4.2.4). Hence, if λ_h is such that $\hat{y}_l^* < D_l^2/K_l$, then the optimal solution is $\hat{y}_l^* = D_l^2/K_l$ and $\hat{y}_h^* = \frac{D_l^2}{K_l} + \frac{D_h^2}{K_h} - \frac{D_l^2}{K_h}$ such that ICh holds with equality.

Case (1B): For the second case, the IC constraints are not binding for the optimal unconstrained values (y_l^*, y_h^*), and we have $\lambda_h = \lambda_l = 0$. Constraint IRl is satisfied by assumption.

Case (1C): For the third case, ICl is violated at values (y_l^*, y_h^*). Therefore, ICl will be tight under

the constrained optimal prices and $\lambda_b = 0, \lambda_l \geq 0$. Condition $\lambda_b = 0, \lambda_l \geq 0$ leads to $\hat{y}_l^* > y_l^*$, such that assumption $y_l^* > D_l^2/K_l$ implies $\hat{y}_l^* > D_l^2/K_l$ so that IRl is satisfied. Then, we obtain

$$\begin{aligned}
&\hat{y}_b^* - \hat{y}_l^* = \frac{\delta D_b^2}{K_l} - \frac{D_l^2}{K_l} = \sqrt{\frac{(1-\alpha)c_a}{K_b\,(1-\alpha+\lambda_l)}}D_b - \frac{mD_b}{D} - \sqrt{\frac{\alpha c_a}{K_l\,(\alpha-\lambda_l)}}D_l + \frac{mD_l}{D} \\
\Leftrightarrow\ &\frac{b_b}{\sqrt{1-\alpha+\lambda_l}} - \frac{b_l}{\sqrt{\alpha-\lambda_l}} = b_0', \qquad \text{(A.25)}
\end{aligned}$$

with b_l, b_b as defined in (A.23) and

$$b_0' = \frac{1}{K_l}\left(\delta D_b^2 - D_l^2\right) + \frac{m}{D}\left(D_b - D_l\right). \qquad \text{(A.26)}$$

Note that (A.25) must have exactly one root in the interval $[0, \alpha[$, because the left-hand side is strictly decreasing in λ_l and goes from a value greater than b_0' (by the condition of case 1C) to $-\infty$. As with the first case, we can approximate the solution to (A.25) by using the Babylonian Method, $\sqrt{\alpha-\lambda_l} \approx \sqrt{\alpha} - \frac{1}{2\sqrt{\alpha}} \cdot \lambda_l$ and $\sqrt{1-\alpha+\lambda_l} \approx \sqrt{1-\alpha} + \frac{1}{2\sqrt{1-\alpha}}\lambda_l$, which leads to:

$$\begin{aligned}
&\frac{b_b}{\sqrt{1-\alpha+\lambda_l}} - \frac{b_l}{\sqrt{\alpha-\lambda_l}} - b_0' = 0 \\
&\Leftrightarrow b_b\sqrt{\alpha-\lambda_l}b_l\sqrt{1-\alpha+\lambda_l} - b_0'\sqrt{\alpha-\lambda_l}\sqrt{1-\alpha+\lambda_l} = 0 \\
&\approx b_b\left(\sqrt{\alpha} - \frac{1}{2\sqrt{\alpha}}\lambda_l\right) - b_l\left(\sqrt{1-\alpha} + \frac{1}{2\sqrt{1-\alpha}}\lambda_l\right) - b_0'\left(\sqrt{\alpha} - \frac{1}{2\sqrt{\alpha}}\lambda_l\right)\left(\sqrt{1-\alpha} + \frac{1}{2\sqrt{1-\alpha}}\lambda_l\right)
\end{aligned}$$

$$\begin{aligned}
&\Leftrightarrow \lambda_l^2 - 2\left(2\alpha - 1 + \frac{b_l}{b_0'}\sqrt{\alpha} + \frac{b_b}{b_0'}\sqrt{(1-\alpha)}\right)\lambda_l + 4\left(\frac{b_b}{b_0'}\alpha\sqrt{1-\alpha} - \frac{b_l}{b_0'}(1-\alpha)\sqrt{\alpha} - \alpha(1-\alpha)\right) = 0, \\
&\Rightarrow \lambda_{l|1,2} \approx 2\alpha - 1 + \frac{b_l}{b_0'}\sqrt{\alpha} + \frac{b_b}{b_0'}\sqrt{(1-\alpha)} \\
&\pm\sqrt{\left(1 - 2\alpha - \frac{b_l}{b_0'}\sqrt{\alpha} - \frac{b_b}{b_0'}\sqrt{1-\alpha}\right)^2 - 4\frac{\alpha(1-\alpha)}{b_0'}\left(\frac{b_b}{\sqrt{1-\alpha}} - \frac{b_l}{\sqrt{\alpha}} - b_0'\right)} \qquad \text{(A.27)}
\end{aligned}$$

where we retain the smaller of the two roots (where $\pm$ is $-$) of the above quadratic function for λ_l, which is positive because of the violation of ICl, which implies $b_h/\sqrt{1-\alpha} - b_l/\sqrt{\alpha} > b'_0$. Please note that if the first term in the expression for $\lambda_{l|1,2}$ is negative, both roots are negative, which is not possible.

Next, let us assume that $y_l^* < D_l^2/K_l$**, i.e., that IRl is not satisfied at optimal prices.**

If solution $y_l = D_l^2/K_l$ and $y_h = y_h^*$ satisfies ICl and ICh, it is the optimal solution, because the buyer profit is concave and 'separable' in y_l, y_h.

If solution $y_l = D_l^2/K_l$ and $y_h = y_h^*$ violates ICh, we know from Case (1A) that $\lambda_h \geq 0, \lambda_l = 0$ and hence $\hat{y}_l^*$ without IRl is less than or equal to y_l^* and, by consequence, violates IRl. Because concavity of the problem is preserved when ICh is enforced with equality and because $y_l^* < D_l^2/K_l$, the optimal solution is $\hat{y}_l^* = D_l^2/K_l$ and $\hat{y}_h^* = \frac{D_l^2}{K_l} + \frac{D_h^2}{K_h} - \frac{D_l^2}{K_h}$, such that ICh is enforced.

The solution $y_l = D_l^2/K_l$ and $y_h = y_h^*$ can never violate ICl, which is equivalent to

$$\hat{y}_l^* - \hat{y}_h^* = \frac{D_l^2}{K_l} - \left(\sqrt{\frac{c_a}{K_h}} - \frac{m}{D}\right) D_h < \frac{D_l^2}{K_l} - \delta\frac{D_h^2}{K_l} \Leftrightarrow \sqrt{\frac{c_a}{K_h}} > \delta\frac{D_h}{K_l} + \frac{m}{D}, \tag{A.28}$$

because assumption $c_a < K_l\left(\frac{D_l}{K_l} + \frac{m}{D}\right)^2$ implies

$$\begin{aligned}
&c_a < K_l\left(\frac{D_l}{K_l} + \frac{m}{D}\right)^2 \Leftrightarrow \sqrt{\frac{c_a}{K_l}} < \frac{D_l}{K_l} + \frac{m}{D} \\
&\Leftrightarrow \sqrt{\frac{c_a}{K_h}} < \sqrt{\frac{K_l}{K_h}}\left(\frac{D_l}{K_l} + \frac{m}{D}\right) \leq \sqrt{\frac{K_l}{K_h}} \cdot \sqrt{\frac{\frac{\delta}{K_l} - \frac{1}{K_h}}{\frac{1}{K_l} - \frac{1}{K_h}}} \cdot \frac{D_h}{K_l} + \sqrt{\frac{K_l}{K_h}} \cdot \frac{m}{D} \\
&\Leftrightarrow \sqrt{\frac{K_l}{K_h}} \cdot \sqrt{\frac{\frac{\delta}{K_l} - \frac{1}{K_h}}{\frac{1}{K_l} - \frac{1}{K_h}}} \cdot \frac{D_h}{K_l} + \sqrt{\frac{K_l}{K_h}} \cdot \frac{m}{D} \leq \delta \cdot \frac{D_h}{K_l} + \sqrt{\frac{K_l}{K_h}} \cdot \frac{m}{D} < \delta \cdot \frac{D_h}{K_l} + \frac{m}{D}
\end{aligned} \tag{A.29}$$

where the first inequality in the line above is a consequence of (4.23) and the second inequality is a

consequence of the following relation:

$$\sqrt{\frac{K_l}{K_h}} \cdot \sqrt{\frac{\frac{\delta}{K_l} - \frac{1}{K_h}}{\frac{1}{K_l} - \frac{1}{K_h}}} \leq \delta$$

$$\Leftrightarrow \frac{\delta}{K_l} - \frac{1}{K_h} \leq \delta^2 \frac{K_h}{K_l} \left(\frac{1}{K_l} - \frac{1}{K_h} \right) \Leftrightarrow \delta K_l - \frac{K_l^2}{K_h} \leq \delta^2 \left(K_h - K_l \right)$$

$$\Leftrightarrow \left(K_h - K_l \right) \delta^2 - K_l \delta + \frac{K_l^2}{K_h} \geq 0$$

$$\Leftrightarrow \left(1 - \frac{K_l}{K_h} \right) \delta^2 - \frac{K_l}{K_h} \delta + \left(\frac{K_l}{K_h} \right)^2 \geq 0$$

$$\Leftrightarrow \left(1 - \frac{K_l}{K_h} \right) \delta^2 - \frac{K_l}{K_h} \delta + \left(\frac{K_l}{K_h} \right)^2 \geq \left(1 - \frac{K_l}{K_h} \right) \frac{K_l}{K_h} \delta - \frac{K_l}{K_h} \delta + \left(\frac{K_l}{K_h} \right)^2$$

$$= -\frac{K_l^2}{K_h^2} \delta + \left(\frac{K_l}{K_h} \right)^2 \geq 0, \tag{A.30}$$

which follows from assumptions $\frac{K_l}{K_h} < \delta < 1$. □ □

Proof. **Proof of Proposition 4.**

The relation $D_h > D_l$ (except for $D_h = D_l = 0$ in which case equality holds) follows from Proposition 3. Note that this constraint is linear in D_l, D_h and independent of prices p_l, p_h. From the left-side inequality of (A.12) and $D_h \geq D_l$, it follows that $p_h D_h - p_l D_l \geq \frac{1}{K_h}(D_h^2 - D_l^2) \geq 0$, i.e., $p_h D_h \geq p_l D_l$ for any feasible solution. Next, we show statements (1)-(3) of the proposition.

Statement (1): ICh is violated at the optimal unconstrained solution: $p_h^* D_h^* - p_l^* D_l^* < (1/K_h)((D_h^*)^2 - (D_l^*)^2)$

We prove Statement (1) of the proposition, by showing that, if IRl and (4.23) hold for the optimal unconstrained solution $(p_l^*, D_l^*, p_h^*, D_h^*)$, for any feasible point (p_l', D_l', p_h', D_h') (i.e., a point that satisfies Eqs. (4.17)-(4.21)) for which $p_h^* D_h' - p_l^* D_l' > (1/K_h)((D_h')^2 - (D_l')^2)$, there exists another feasible point with demand quantities (D_l'', D_h'') with greater or equal profit for which either Case (1A) or Case (2) of Proposition 3 applies.

Let us label the directed line segment from (p'_l, D'_l, p'_h, D'_h) to $(p^*_l, D'_l, p^*_h, D'_h)$ by vector $\mathbf{v}_1$ and the vector from $(p^*_l, D'_l, p^*_h, D'_h)$ to $(p^*_l, D^*_l, p^*_h, D^*_h)$ by $\mathbf{v}_2$. Vectors $\mathbf{v}_1$ and $\mathbf{v}_2$ are 'profit-increasing', i.e., the closer a point is to the end point of the vector, the greater the buyer profit, and all points on vector $\mathbf{v}_2$ have higher buyer profit than all points on vector $\mathbf{v}_1$. This is obvious from the fact that $\hat{\Pi}(p_l, D_l, p_h, D_h)$ is concave in (p_l, p_h) for any given (D_l, D_h) with maximum value (p^*_l, p^*_h), and the fact $\hat{\Pi}(p^*_l, D_l, p^*_h, D_h)$ is concave in (D_l, D_h) at prices (p^*_l, p^*_h) with maximum value (D^*_l, D^*_h) (see proof of Lemma 2), and approaching the optimal value of a concave function on a direct line increases the objective function. Let us consider three distinct cases:

(i) **Point $(p^*_l, D'_l, p^*_h, D'_h)$ violates IRl, i.e., $p^*_l < D'_l/K_l$.** Then, Case (2) of Proposition 3 applies to demand quantities (D'_l, D'_h).

(ii) **Point $(p^*_l, D'_l, p^*_h, D'_h)$ satisfies IRl, i.e., $p^*_l \geq D'_l/K_l$, but violates ICh, i.e., $p^*_h D'_h - p^*_l D'_l < (1/K_h)((D'_h)^2 - (D'_l)^2)$.** Then, the optimal prices for demand quantities (D'_l, D'_h) are given by Case (1A) of Proposition 3.

(iii) **Point $(p^*_l, D'_l, p^*_h, D'_h)$ satisfies IRl, i.e., $p^*_l \geq D'_l/K_l$ and satisfies ICh, i.e., $p^*_h D'_h - p^*_l D'_l \geq (1/K_h)((D'_h)^2 - (D'_l)^2)$.** Then, we can derive demand quantities (D''_l, D''_h) that lie on vector $\mathbf{v}_2$ between (D'_l, D'_h) and (D^*_l, D^*_h).

By assumption, IRl holds at point $(p^*_l, D^*_l, p^*_h, D^*_h)$, i.e., $p^*_l \geq D^*_l/K_l$. Let us choose (D''_l, D''_h) such that $(p^*_l, D''_l, p^*_h, D''_h)$ is the point that lies closest to the end on $\mathbf{v}_2$ and for which holds

$$p^*_h D''_h - p^*_l D''_l = \frac{1}{K_h}((D''_h)^2 - (D''_l)^2).$$

There has to be at least one such point, because $p^*_h D'_h - p^*_l D'_l \geq (1/K_h)((D'_h)^2 - (D'_l)^2)$ and $p^*_h D^*_h - p^*_l D^*_l < (1/K_h)((D^*_h)^2 - (D^*_l)^2)$ by assumption, such that the Intermediate Value Theorem applies. Then, because both points $(p^*_l, D^*_l, p^*_h, D^*_h)$ and $(p^*_l, D'_l, p^*_h, D'_h)$ satisfy IRl and because the constraint IRl defines a half-space on (D_l, D_h) at given price p^*_l, any point on the direct line between these points

also lies in the same half-space, i.e., $(p_l^*, D_l'', p_h^*, D_h'')$ also satisfies IRl. Point $(p_l^*, D_l'', p_h^*, D_h'')$ also satisfies ICl, because ICh holds with equality (see definition of (D_l'', D_h'') above) and the region on (D_l, D_h) on which ICh and ICl cannot both be violated (the region defined by (4.23)) is a half-space. Demand quantity pairs (D_l^*, D_h^*) and (D_l', D_h') both lie in this half-space by assumption, such that pair (D_l'', D_h'') also lies in the same half-space because it lies on the line segment between (D_l^*, D_h^*) and (D_l', D_h'). We conclude that Case (1A) of Proposition 3 applies.

Statement (2): ICh and ICl are satisfied at the optimal solution. Statement (2) is obvious: If ICh, ICl, and IRl are satisfied at the optimal unconstrained solution, the optimal solutions of the constrained and the unconstrained problems coincide.

Statement (3): ICl is violated for the optimal unconstrained solution: $\frac{1}{K_l}(\delta(D_h^*)^2 - (D_l^*)^2) < p_h^* D_h^* - p_l^* D_l^*$**.** The proof of statement (3) follows the same line of argumentation as the one for statement (1), by using constraint ICl, $\frac{1}{K_l}(\delta D_h^2 - D_l^2)$, instead of ICh, $\frac{1}{K_h}(D_h^2 - D_l^2)$ (and reverse signs for the inequalities). We do not repeat the arguments here for space considerations. □ □

Proof. **Proof of Lemma 3.** Note that it easily follows from (4.40) that $\beta_n(i,i) = 1$ and $\beta_n(i,i') = \beta_n(i,i'') \cdot \beta_n(i'',i')$ for any $i \le i'' \le i'$. For the constraint set of (4.42), let us write the right-hand side as

$$
\begin{aligned}
&\max_{(i'|i<i'\le q)} \tilde{\alpha}_n(i') \cdot \beta_n(i,i') \cdot \pi_j(p_{i'}, D_{i'}, a_{i'}^*(p_{i'}, D_{i'})) \\
&= \max\{ \max_{(i'|i<i'<j)} \tilde{\alpha}_n(i') \beta_n(i,i') \pi_j(p_{i'}, D_{i'}, a_{i'}^*(p_{i'}, D_{i'})), \\
&\quad \beta_n(i,j) \max_{(i'|j\le i'\le q)} \tilde{\alpha}_n(i') \beta_n(j,i') \pi_j(p_{i'}, D_{i'}, a_{i'}^*(p_{i'}, D_{i'}))\} \\
&= \max\{ \max_{(i'|i<i'<j)} \tilde{\alpha}_n(i') \beta_n(i,i') \pi_j(p_{i'}, D_{i'}, a_{i'}^*(p_{i'}, D_{i'})), \\
&\quad \beta_n(i,j) \tilde{\alpha}_n(j) \pi_j(p_j, D_j, a_j^*(p_j, D_j))\}
\end{aligned} \tag{A.31}
$$

where the equality in the second line comes from the constraint set of (4.43). Next, let us consider $i = j - 1$. Then, with the above simplification, we can write the constraints as

$$\tilde{\alpha}_n(j-1)\pi_j(p_{j-1}, D_{j-1}, a^*_{j-1}(p_{j-1}, D_{j-1})) \leq \beta_n(j-1, j)\tilde{\alpha}_n(j)\pi_j(p_j, D_j, a^*_j(p_j, D_j)), \tag{A.32}$$

and with $i = j - 2$, as

$$\begin{aligned}
&\tilde{\alpha}_n(j-2)\pi_j(p_{j-2}, D_{j-2}, a^*_{j-2}(p_{j-2}, D_{j-2})) \\
&\leq \max\{\beta_n(j-2, j-1)\tilde{\alpha}_n(j-1)\pi_j(p_{j-1}, D_{j-1}, a^*_{j-1}(p_{j-1}, D_{j-1})), \\
&\qquad \beta_n(j-2, j)\tilde{\alpha}_n(j)\pi_j(p_j, D_j, a^*_j(p_j, D_j))\} \\
&= \beta_n(j-2, j-1) \cdot \max\{\tilde{\alpha}_n(j-1)\pi_j(p_{j-1}, D_{j-1}, a^*_{j-1}(p_{j-1}, D_{j-1})), \\
&\qquad \beta_n(j-1, j)\tilde{\alpha}_n(j)\pi_j(p_j, D_j, a^*_j(p_j, D_j))\} \\
&= \beta_n(j-2, j-1)\beta_n(j-1, j)\tilde{\alpha}_n(j)\pi_j(p_j, D_j, a^*_j(p_j, D_j)) \\
&= \beta_n(j-2, j)\tilde{\alpha}_n(j)\pi_j(p_j, D_j, a^*_j(p_j, D_j)),
\end{aligned} \tag{A.33}$$

where the equality in the last line follows from (A.32) for $i = j - 1$. We can continue the same reasoning to show that for (4.42),

$$\begin{aligned}
&\tilde{\alpha}_n(i) \cdot \pi_j(p_i, D_i, a^*_i(p_i, D_i)) \leq \max_{(i' | i < i' \leq q)} \tilde{\alpha}_n(i') \cdot \beta_n(i, i') \cdot \pi_j(p_{i'}, D_{i'}, a^*_{i'}(p_{i'}, D_{i'})), \forall i < j \\
&\Leftrightarrow \tilde{\alpha}_n(i) \cdot \pi_j(p_i, D_i, a^*_i(p_i, D_i)) \leq \tilde{\alpha}_n(j) \cdot \beta_n(i, j) \cdot \pi_j(p_j, D_j, a^*_j(p_j, D_j)), \forall 1 \leq i \leq j.
\end{aligned} \tag{A.34}$$

Then, by defining $\beta_n(i,j) = \beta_n(j,i)^{-1}$, and by rewriting (4.43) as

$$\begin{aligned}
&\tilde{\alpha}_n(j) \cdot \pi_j(p_j, D_j, a_j^*(p_j, D_j)) \geq \max_{(i'|j<i'\leq q)} \tilde{\alpha}_n(i') \cdot \beta_n(j,i') \cdot \pi_j(p_{i'}, D_{i'}, a_{i'}^*(p_{i'}, D_{i'})), \forall j < q, \\
\Leftrightarrow\ &\tilde{\alpha}_n(j) \cdot \pi_j(p_j, D_j, a_j^*(p_j, D_j)) \geq \tilde{\alpha}_n(i) \cdot \beta_n(j,i) \cdot \pi_j(p_i, D_i, a_i^*(p_i, D_i)), \forall j < i \leq q \\
\Leftrightarrow\ &\tilde{\alpha}_n(i) \cdot \pi_j(p_i, D_i, a_i^*(p_i, D_i)) \leq \tilde{\alpha}_n(j) \cdot \beta_n(i,j) \cdot \pi_j(p_j, D_j, a_j^*(p_j, D_j)), \forall j < i \leq q, \qquad \text{(A.35)}
\end{aligned}$$

both (4.42) and (4.43) can be written in the single equation (4.44), which finalizes the proof. □ □

Proof. **Proof of Proposition 5.** A strategy induces a PBE if it allows a player to update its belief about the types of the other players in each round, and that, in each round, each player's decisions are expected-profit-maximizing under the player's current belief (Fudenberg and Tirole, 1991, p. 326).

The first condition is met, because we have shown with the discussion around (4.38)-(4.40) that an agent at the beginning of round i can update the probability of winning the contract of round i with $\tilde{\alpha}_n(i)$, and the probability of winning the contract of round j with $j > i$ with $\tilde{\alpha}_n(j) \cdot \beta_n(i,j)$, exploiting the information that no agent in the pool has type $i' < i$ (because then the sequential process would have already been terminated). The second condition is also met, because (4.42) and (4.43) ensure that each agent of type $j = 1, \ldots, q$ maximizes profit when choosing contract j (and only contract j) under belief updating and strategy s, and those constraints are equivalent to (4.49) according to Lemma 3. Thus, (4.49) enforces the second condition of a PBE for strategy s.

Since each agent accepts the contract of its corresponding type, and the menu of contracts has a contract for each agent type, the procedure always terminates with the selection of an agent. Since strategy s implies that an agent with a lower type accepts a contract before an agent of a higher type, the selection of the agent with the lowest type (i.e., the one with the largest labor pool) is guaranteed.

□ □

Proof. **Proof of Proposition 6.** For part (1), observe that $\tilde{\alpha}_n(j), \beta_n(i,j) \geq 0$ for all i, j. Constraint

$IC(i,j)$ ensures that

$$\pi_j(p_j, D_j, a_j^*(p_j, D_j)) \geq (\tilde{\alpha}_n(i)\beta_n(j,i)/\tilde{\alpha}_n(j)) \left(p_i D_i - \delta_{i,j}\frac{D_i^2}{K_j}\right)$$

and $IR(i)$ ensures that $\left(p_i D_i - \frac{D_i^2}{K_i}\right) \geq 0$. For any $j < i$, it further holds $\left(p_i D_i - \delta_{i,j}\frac{D_i^2}{K_j}\right) \geq \left(p_i D_i - \frac{D_i^2}{K_i}\right)$, because $\delta_{i,j} = 1$ and $K_j > K_i$, such that $IC(i,j)$ and $IR(i)$ for any $i < j$ imply $IR(j)$. However, for $j = q$, we cannot find $j < i \leq q$, such that $IR(q)$ is the only IR constraint that is not redundant with other constraints.

For parts (2) and (3), note that for any pair $i < j$, Eq. (4.49) leads to two IC constraints

$$\begin{aligned}
&\tilde{\alpha}_n(i)\beta_n(j,i)\left(p_i D_i - \delta_{i,j}\frac{D_i^2}{K_j}\right) \leq \tilde{\alpha}_n(j)\left(p_j D_j - \frac{D_j^2}{K_j}\right), \\
\text{and}\quad &\tilde{\alpha}_n(i)\left(p_i D_i - \frac{D_i^2}{K_i}\right) \geq \tilde{\alpha}_n(j)\beta_n(i,j)\left(p_j D_j - \delta_{j,i}\frac{D_j^2}{K_i}\right) \\
&\Leftrightarrow \tilde{\alpha}_n(i)\beta_n(j,i)\left(p_i D_i - \frac{D_i^2}{K_i}\right) \geq \tilde{\alpha}_n(j)\left(p_j D_j - \delta_{j,i}\frac{D_j^2}{K_i}\right). \qquad \text{(A.36)}
\end{aligned}$$

Then, by combining both constraints, we obtain

$$\begin{aligned}
&\tilde{\alpha}_n(i)\beta_n(j,i)\frac{D_i^2}{K_i} - \tilde{\alpha}_n(j)\frac{\delta_{j,i}D_j^2}{K_i} \leq \tilde{\alpha}_n(i)\beta_n(j,i)p_i D_i - \tilde{\alpha}_n(j)p_j D_j \leq \tilde{\alpha}_n(i)\beta_n(j,i)\frac{\delta_{i,j}D_i^2}{K_j} - \tilde{\alpha}_n(j)\frac{D_j^2}{K_j} \\
&\Leftrightarrow \tilde{\alpha}_n(j)\left(\frac{1}{K_j} - \frac{\delta_{j,i}}{K_i}\right)D_j^2 \leq \tilde{\alpha}_n(i)\beta_n(j,i)\left(\frac{\delta_{i,j}}{K_j} - \frac{1}{K_i}\right)D_i^2 \\
&\Leftrightarrow D_i \geq \sqrt{\frac{\tilde{\alpha}_n(j)\left(\frac{1}{K_j} - \frac{\delta_{j,i}}{K_i}\right)}{\tilde{\alpha}_n(i)\beta_n(j,i)\left(\frac{\delta_{i,j}}{K_j} - \frac{1}{K_i}\right)}} \cdot D_j = \sqrt{\frac{\tilde{\alpha}_n(j)\beta_n(i,j)\left(\frac{1}{K_j} - \frac{1}{K_i}\right)}{\tilde{\alpha}_n(i)\left(\frac{\delta_{i,j}}{K_j} - \frac{1}{K_i}\right)}} \cdot D_j, \qquad \text{(A.37)}
\end{aligned}$$

because $\delta_{j,i} = 1$ and $K_i > K_j$ for $i \leq j$ by assumption and $D_i, D_j \geq 0$. Also, from (A.36) and $\delta_{j,i} = 1$ follows that it must hold $\frac{\delta_{i,j}}{K_j} - \frac{1}{K_i} > 0 \Leftrightarrow \delta_{i,j} > \frac{K_j}{K_i} \Leftrightarrow \frac{K_j}{K_i} > 1 - \frac{4c_g}{\gamma}$ for the constraints to admit a feasible solution, which is ensured by the assumption $\gamma < 2c_g$ (see Proposition 1). Finally, plugging

the last line of (A.37) into the left-hand-side of the first line of (A.37) shows that the left-hand side must be non-negative, which leads to

$$\begin{aligned} \tilde{\alpha}_n(i)\beta_n(j,i)p_iD_i - \tilde{\alpha}_n(j)p_jD_j \geq 0 \Leftrightarrow \quad & \tilde{\alpha}_n(i)\beta_n(j,i)p_iD_i \geq \tilde{\alpha}_n(j)p_jD_j \\ \Leftrightarrow \quad & p_iD_i \geq \frac{\tilde{\alpha}_n(j)}{\tilde{\alpha}_n(i)}\beta_n(i,j) \cdot p_jD_j. \quad \square \end{aligned} \tag{A.38}$$

□

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