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INFORMATION DIFFUSION IN FINANCIAL MARKETS
An Agent-Based approach to test the fundamental value discovery in different market structures

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Résumé

L'objectif des travaux présentés dans cette thèse est d'étudier la diffusion de l'information dans les marchés financiers. Considérant comme établi que les individus sont hétérogènes et à rationalité limitée, nous avons fondé nos travaux sur une catégorie de modèles computationnels dans le but de simuler les actions et les interactions des agents autonomes. Cette catégorie est communément nommée modélisation agent (ABM).

Plus concrètement, cette recherche se concentre sur le rôle de l'hétérogénéité des agents dans la diffusion et l'utilisation de l'information. À cet effet, nous avons développé deux structures de marché, qui diffèrent par leur transparence. Dans les chapitres 1 et 2, nous introduisons un marché centralisé, où une partie du carnet d'ordre est accessible (information publique). Dans le chapitre 3, nous développons un marché de gré à gré dans lequel les agents négocient et échangent avec leurs relations.

Le premier chapitre examine si l'efficience du marché et le volume des transactions peuvent être expliqués par la rationalité limitée des investisseurs. Dans un marché dirigé par les ordres, où deux actifs risqués sont échangés, chaque intervenant se caractérise par une part de chartisme et une part de fondamentalisme. De surcroit, dans ce marché, la valeur fondamentale évolue au cours du temps. Les agents de par leur persévérance des croyances sont réticents à modifier leurs estimations. Ainsi, des bulles financières émergent suite aux attentes des chartistes ou à la méprise des variations des fondamentaux par les fondamentalistes. À cela s'ajoutent des comportements stratégiques dans le processus de soumission des ordres. Par conséquent, l'efficience du marché est tributaire des croyances des agents, du mécanisme de correction des croyances, de la variance de la vraie valeur fondamentale et des stratégies de soumission. La modélisation des stratégies de placement engendre un marché plus profond et liquide, mais moins informatif.

Le second chapitre considère si l'efficience de marché peut être atteinte grâce à un processus d'apprentissage. En effet, les agents, sont-ils capables de conjecturer les flux de trésorerie et ainsi d'estimer correctement la valeur fondamentale ? Nous testons si cela est possible, et pour cela, nous utilisons des mécanismes d'apprentissage différents afin de confirmer la généralité de ce résultat. Nous nous concentrons également sur la façon dont l'information publique peut améliorer le processus d'apprentissage des fondamentalistes et donc la précision de leurs prévisions. Notre analyse aboutit à la conclusion que les agents sont tous capables de conjecturer de manière plus ou moins fiable la valeur fondamentale. Les apprentissages par pondération de l'expérience (EWA) et par jeu fic-

tif (FP) surpassent l'apprentissage par renforcement (RL). Indépendamment du type d'apprentissage, la diffusion d'une information publique exacte améliore la découverte de la valeur fondamentale et l'émergence d'un consensus.

Le troisième chapitre examine si la diffusion de l'information est effective dans une structure de marché plus opaque que celle développée précédemment. Dans un marché de gré à gré, les agents sont-ils capables d'extraire correctement les informations de leur négociation afin de partager les mêmes prévisions de la valeur fondamentale ? Nous démontrons que la diffusion de l'information est effective, mais pas efficiente dans le réseau complément connecté, le réseau aléatoire $1/2$ et le réseau en étoile. Dans le réseau en étoile, quelle que soit l'information des agents, le noeud central dirige le marché (estimation de la valeur fondamentale et prix d'échange). Dans le réseau petit monde, l'introduction de gourous génère deux dynamiques différentes. Lorsque l'information des gourous est proche des attentes des non-gourous, la volatilité du marché est faible et le prix correctement estimé. En revanche, lorsque l'information des gourous est trop éloignée des attentes des non-gourous, les prix de négociation collectés par les non-gourous sont trop altérés pour être efficacement utilisés. Nous confirmons ici que la vitesse de convergence et la stabilité de la valeur d'échange sont liées à l'opacité du marché. Enfin, la précision de l'estimation de la valeur fondamentale dépend de la structure du marché et de la position de l'agent dans le réseau.

Abstract

The piece of work's aim is to understand information diffusion in financial markets. Starting from the empirical evidences that agents are heterogeneous and bounded rational, we based our investigations on a class of computational models for simulating the actions and interactions of autonomous agents: the agent-based model (ABM).

More precisely, this research focuses on the impacts of agents heterogeneity in diffusion and use of information. For this purpose, we developed two market structures, in which the market transparency varies. In the chapters 1 and 2, we introduce a centralised market, where a part of the order-book is available as a public information. In the chapter 3, we build an Over-The-Counter market, where agents bargains with their trading contacts.

The first chapter investigates whether trading volume and price distortion can be explained by the investor's bounded rationality. In an order-driven market, where two risky assets are traded, the traders are defined as a mix between fundamentalist and chartist outlooks. We place agents in a context of evolving fundamental values and order-placement strategy; they perceive the fundamental but they also have some heterogeneous belief perseverance; and they adapt their orders to the market depth so as to maximise their execution probability and their profit. Adding to the usual bubbles that are created by chartists, we could produce bubbles due to misunderstanding of the fundamentals change. By adding bounded rationality in their information processing, we show that market efficiency depends on the agents beliefs, the correction mechanism of the adaptive learning, the volatility of the true fundamental value and the order-placement strategy.

The second chapter explores whether market efficiency can be explained by investor's learning process in an agent-based model. Are agents able to discover the right cash flow process and hence correctly compute the fundamental value? We test if this is indeed possible, and for this we used different learning mechanisms to test the generality of this result. We also focus on how public information about fundamental correlation can improve the fundamentalist's process of expectation formation. Our main result is that agents are able to compute the fundamental value regardless the learning mechanism. Experience-Weighted Attraction (EWA) and Fictitious Play (FP) algorithms outperform Reinforcement Learning (RL). Regardless of the learning type, a proper public information improves discovery of the fundamental value and emergence of a consensus.

The third chapter considers whether information extraction through bargaining process can be explained in a market, in which transactions are not made public and where no prices are posted. In Over The Counter markets, are agents able to extract information through bargaining process so as to properly forecast the fundamental value? We show that the information diffusion is effective in the complete, random and star network, but not informationally efficient. In the star network, the central agent drives the market, no matter the expectations of the other agents. In the small world network, adding gurus engenders two main dynamics. When gurus' information is close to others' expectations, the market volatility is low and the price is properly forecasted. When gurus' information is too far from the expectation of the others, the prices collected by the others are too noisy to be properly used. By studying the difficulties that parties have in reaching mutually beneficial agreement, this paper confirms that the price stability is linked to the market opacity. Moreover, the accuracy of the forecasted fundamental value depends on the market structure and the agent's network.

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Mnemonic and Notation

The following notations are used throughout the thesis.

$\mathcal{U}(a;b)$	Uniform distribution on the interval $[a,b]$.
$\mathcal{N}(0,r)$	Normal distribution with 0 mean and r variance.
$\mathcal{W}(r)$	Geometrical Brownian movement with r variance.
$\sim$	Follow.
CARA	Constant Absolute Risk Aversion.
FV	Fundamental Value.
ABM	Agent-Based Model. Modélisation agent.
EWA	Experience-Weighted Attraction. Apprentissage par pondération de l'expérience.
FP	Fictitious play. Apprentissage par jeu fictif.
HF	High Frequency. Haute Fréquence.
LO	Limit Order. Ordre à court limité.
MO	Market Order. Ordre au marché.
OTC	Over The Counter market. Marché de gré à gré.
RL	Reinforcement Learning. Apprentissage par renforcement.

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Introduction Générale

0.1. Avant propos

Selon la théorie financière classique, la recherche du profit individuel (microéconomique) bénéficie à la société dans son ensemble (macroéconomique). La main invisible d'Adam Smith (1776) stipule donc que même si les agents pris séparément poursuivent des intérêts idiosyncratiques, les marchés financiers tendent naturellement vers la stabilité et l'équilibre. Alan Greenspan[a], fervent défenseur de la déréglementation, déclarait à ce propos dans une tribune du Financial Times le 29 mars 2011 :

> L'ennui est que les régulateurs (...) ne pourront jamais comprendre qu'une partie infime du fonctionnement interne du plus simple des systèmes financiers modernes.(...) Qu'on le veuille ou non, les marchés concurrentiels d'aujourd'hui sont mus par une version internationale et sacrément opaque de la "main invisible" d'Adam Smith.(...) À de très rares exceptions près (en 2008, par exemple), l'action de la "main invisible" mondiale a débouché sur une relative stabilité des taux de change, des taux d'intérêt, des prix et des salaires.

À demi-mot, l'ancien président de la Banque Centrale Américaine (FED) reconnaît qu'une intervention du régulateur peut dans certains cas être nécessaire. Cependant, cette intervention n'est souhaitable qu'à la condition qu'il y ait compréhension des marchés financiers ainsi que de l'impact des décisions du régulateur. Or, les marchés financiers demeurent relativement obscurs.

La répétition des crises financières à développement endogène est un phénomène préoccupant pour la stabilité du système financier international. En seulement vingt ans, il est possible d'énumérer quatre crises majeures : le krach d'octobre 1987, la quasi-faillite du fond d'investissement Long Term Capital Management en 1998, l'éclatement de la bulle internet entre septembre 2000 et mars 2003, et la crise des prêts à haut risque (*subprimes*) de 2008. Cette dernière crise, dont nous ne sommes pas encore sortis totalement, a ravivé une nouvelle fois le débat de l'intérêt de la modélisation de l'instabilité financière et de la capacité des sciences économiques à aider à la gouvernance internationale dans un monde où la finance se complexifie et les marchés s'interconnectent. Car, pour le moment, aucun courant de pensée n'a réussi à délivrer un modèle capable de reproduire et de comprendre les plus graves crises financières, ni même de les anticiper.

[a] Président de la Banque Centrale Américaine (FED) de 1987 à 2006.

L'objet de cette thèse est l'étude de l'agrégation et de la dissémination de l'information dans les marchés financiers.

Partant de la structure de marché la plus transparente, un marché dirigé par les ordres avec observation partielle du carnet d'ordre, pour terminer par un marché totalement opaque, le marché de gré à gré, il convient de s'intéresser à la découverte de la valeur fondamentale par les agents. Les principales questions sont :

- Quelles sont les raisons qui peuvent expliquer la déconnexion entre la valeur fondamentale et le prix de marché dans une structure sans coût d'information ni coût de transaction ?
- Les agents sont-ils capables de conjecturer une valeur fondamentale à partir d'informations extraites du marché ?
- Dans quelle proportion la structure du marché est-elle responsable d'une déconnexion entre les fondamentaux et le prix de certains actifs ?

À travers les différents chapitres de cette thèse, des éléments de réponses seront apportés à chacune de ces interrogations. Avant d'entrer dans le détail, il convient de présenter les principales notions qui seront abordées et les hypothèses qui en découlent. Il parait également pertinent d'introduire rapidement les travaux en finance de marché afin de motiver le recours aux modèles agents. Cette introduction s'organise de la manière suivante : une brève description du cheminement de ces travaux de recherche, puis quatre parties qui reviennent successivement sur les concepts économiques employés, les limites de la théorie classique, les principaux travaux de la modélisation agent appliqués aux marchés financiers afin de situer plus précisément les apports de cette thèse, et enfin un résumé des contributions.

0.2. Cheminement

L'idée de cette recherche a émergé du mémoire de master 2 intitulé : « Impacts of behaviour on dynamics price on financial market ». J'y ai découvert la modélisation agent avec un modèle de marché dirigé par les prix fondé sur les travaux de Brock and Hommes (1998). Dans ce travail, j'ai étudié l'impact de chaque type d'agent sur les marchés financiers : un phénomène d'amplification de la tendance pour les chartistes, un phénomène de convergence vers la valeur fondamentale pour les fondamentalistes et un effet de « bruits parasites » pour les agents aléatoires (*Noise Trader Agents*). J'ai aussi redécouvert des résultats évidents (LeBaron, 2006) tel que : l'inutilité de s'informer lorsque le marché est efficient et l'information couteuse. Ma thèse s'est alors articulée autour d'une structure de marché plus complexe : un marché dirigé par les ordres, avec deux actifs risqués échangés. Le choix de la modélisation agent s'est imposé par les limites de la théorie classique, qui seront développées ultérieurement, puis confirmé par les interactions que j'ai eu avec les chercheurs de mon laboratoire de recherche, le GREQAM (Aix-Marseille School of Economics).

Durant mon parcours de doctorat, le Groupe de Travail Interaction (GTI) organisé par Nobuyuki Hanaki ainsi que le séminaire « du marché aux marchés » organisé par Juliette Rouchier et Alan Kirman, m'ont permis d'acquérir une connaissance plus vaste des travaux en modélisation agents. Ce fut l'occasion pour moi de m'ouvrir à d'autres disciplines utilisant cet outil. J'ai par exemple rencontré Murat Yildizoglu, et assisté au premier « Workshop on Agent-Based Macroeconomics » organisé par le GRETHA de Bordeaux (2013). Les différents séminaires de mon laboratoire (le séminaire du lundi et le séminaire doctorants) m'ont permis d'approfondir mes connaissances en microéconomie et en macroéconomie, que ce soit par des travaux classiques, computationnels ou expérimentaux.

Avant d'aborder l'objet de ce travail, il me parait important de revenir sur deux points. Le premier est la « boîte noire » que représentent les modèles agents. Nombre d'auteurs ne partagent pas leurs algorithmes. Les requêtes d'accès aux codes restent souvent muettes. Heureusement, cela est en train de changer ; les articles les plus récents commencent à être publiés avec leur code (ou algorithme) en appendice. Ce libre accès est une nécessité à la fois pour la recherche et les futurs doctorants qui auront une base de données solide pour entreprendre leurs projets. Ce libre accès permet aussi une double vérification du code et de démystifier cette « boîte noire ». Le second point découle du premier. Gardant à l'esprit que les programmes (codes) ne sont pas accessibles, ils doivent être intégralement développés. Il est important de revenir sur le travail d'élaboration de ces derniers. Cela ressemble à un parcours du combattant où le temps est le principal ennemi. Un des défis fut pour moi de créer un modèle dont la structure soit aussi simple que possible. Le modèle développé dans les deux premiers chapitres a pris plus d'un an avant d'arriver à « maturité ». Le deuxième modèle qui n'en est encore qu'à ses balbutiements représente huit mois de travail. Le passage d'un modèle théorique à un modèle computationnel n'est pas des plus aisé. Les résultats théorétiques ne se répliquent que dans des cas particuliers qu'il est nécessaire de définir. À cela s'ajoutent les processus de débogage et de vérification qui sont chronophages. En effet, beaucoup de choses doivent être contrôlées, telles que le fonctionnement du programme pour des paramètres différents, la stabilité des résultats ou encore les limites du modèle proposé.

0.3. Concepts et définitions

Alors que certaines limites sont habituellement mesurables précisément par des outils, les limites cognitives sont définies par l'imparfaite perception du cerveau humain et l'influence des émotions dans la prise de décision. Partant du postulat que les agents sont à rationalité limitée et qu'ils interagissent sur les marchés financiers (Vriend, 2006), il convient de définir les marchés considérés, le concept de valeur fondamentale, ainsi que les limites qui caractérisent les agents.

0.3.1. Les structures de marché

Pour pouvoir échanger, il faut une place, aussi appelée marché (pas nécessairement physique), où les agents se rencontrent et négocient. Lorsque deux parties, un acheteur et un vendeur, souhaitent conclure une transaction, elles peuvent le faire sur deux types de marchés : (i) un marché organisé ou (ii) un marche de gré à gré. Sur le marché de gré à gré, la transaction est conclue bilatéralement entre les deux parties, tandis que sur le marché organisé, les contreparties placent des ordres d'achat et de vente, via une société de bourse du type NYSE Euronext. Cette différence à de nombreuses conséquences sur les risques et la transparence des transactions réalisées.

La principale caractéristique du marché organisé réside dans la centralisation des offres d'achat et de vente. Une chambre de compensation joue l'intermédiaire entre les acheteurs et vendeurs. Ces derniers n'entrent pas directement en contact. Cela aboutit à la formation d'un prix unique : le prix d'équilibre entre l'offre et la demande. Aucun intervenant n'est privilégié par rapport à un autre, quelle que soit sa taille.

Le marché organisé se caractérise aussi par une liquidité relativement importante. Tous les ordres étant regroupés par la chambre de compensation, la probabilité d'exécution d'un ordre est élevée. En plus de cette liquidité, tous les ordres sont collectés dans un carnet d'ordre public et spécifique à chaque produit. L'investisseur peut extraire certaines informations de ce carnet d'ordre afin de définir sa position (sens de l'ordre, type d'ordre, prix...).

Concrètement, le carnet d'ordre récapitule à un instant donné l'état de l'offre et de la demande sur une valeur. Il se divise en deux colonnes : achat et vente. À l'intérieur de chacune d'elles se trouve les meilleures offres et les meilleures demandes, définies par leur cour, le nombre de titres ainsi que le nombre d'ordres. Le nombre d'offres (ou demandes) affichées dépend de la place de cotation. Généralement, le carnet d'ordre contient les cinq ou dix meilleurs offres. Ainsi, l'investisseur connaît le prix auquel il peut vendre ou acheter immédiatement. Les ordres centralisés dans le carnet d'ordre respectent une priorité de prix ainsi qu'une priorité temporelle. Le marché organisé est doté d'une grande transparence. Nous retrouverons ce marché dans les chapitres 1 et 2.

Le marché de gré à gré se distingue du marché organisé par son absence de chambre de compensation. Cela ne signifie pas nécessairement que les acheteurs et les vendeurs sont en relation directe. Un ou plusieurs courtiers peuvent servir d'intermédiaires. Leur rôle est alors de faire se rencontrer les demandes des acheteurs et les offres des vendeurs. Toutefois, dans la majorité des cas (en dehors des particuliers), acheteurs et vendeurs se rencontrent pour déterminer ensemble les conditions de la transaction.

Le marché de gré à gré par opposition au marché organisé présente des coûts de transactions réduits et une souplesse dans la transaction. Il est en effet possible de passer une transaction sur mesure afin de répondre parfaitement aux besoins de chacune des deux parties. Il y a négociation entre l'acheteur et le vendeur pour s'entendre sur le prix, la quantité, les différentes clauses du contrat, *etc*.

Du fait de ces contrats développés sur mesure, le marché de gré à gré se caractérise par une liquidité très faible. À cela s'ajoute un certain manque de transparence : les prix et les quantités ne sont pas centralisés, aucun carnet d'ordre n'est accessible (sous forme d'information publique). Différents agents à l'intérieur d'un même marché peuvent avoir accès à des informations et des opportunités d'échange différentes. Au même instant, un même type d'échange peut être réalisé à différents prix. Un rapport de force dans la négociation du contrat s'installe. Cette structure sera utilisée dans le chapitre 3.

0.3.2. La valeur fondamentale

La valeur fondamentale d'une firme est un concept qui est défini par l'actualisation des flux futurs de revenus en fonction des perspectives financières de cette dernière. À partir de cette approche qui consiste effectivement à déterminer la valeur la plus probable qui serait à prendre en considération dans une opération réelle, l'importance des paramètres de marché est mise en évidence. En introduisant un lien direct entre valeur et rentabilité d'un actif, la valeur fondamentale (f), dans sa forme la plus simple, se définit comme la somme actualisée des flux de trésorerie futurs (y) corrigés d'une prime de risque pour la détention de l'actif ($\alpha\sigma^2 Z$).

$$f = \sum_{k=1}^{\infty} \frac{\mathrm{E}_t(y_{t+k}) - \alpha\sigma^2 Z}{R^k}$$

Dans le cadre classique, où les agents sont parfaitement informés et rationnels, cette valeur est supposée unique et connue de tous. La notion de valeur fondamentale d'une action et le concept d'efficacité informationnelle d'un marché sont alors étroitement associés : en l'absence de bulle rationnelle ou de parasitage des cours par des bruits divers, l'hypothèse d'efficacité informationnelle garantit que la valeur fondamentale d'une action et son cours de bourse coïncident. Édouard Challe propose d'ailleurs de traduire le terme anglais *efficient* par « efficace » et non par « efficience » ; cela afin de qualifier un *efficient market* de marché efficace. Dès lors, il apparaît plus clairement que le but de valeur fondamentale est de représenter la capacité du marché à « correctement » valoriser les titres. La valeur fondamentale devient un outil. Néanmoins, le marché est par définition aléatoire. Les flux futurs reposent sur un calcul d'espérance qui peut s'avérer erroné.

0.3.3. Les agents

Les modèles orientés sur l'hétérogénéité des agents distinguent habituellement deux ou trois types d'agents : les fondamentalistes, les chartistes et parfois les agents aléatoires.
– Les fondamentalistes sont le pendant de l'analyse technique. Ils étudient le chiffre d'affaires, les compétences des dirigeants ou encore les opportunités de croissance du secteur, et ne s'intéressent pas aux évolutions du cours de bourse. Concrètement, ils estiment la valeur fondamentale de l'entreprise à partir des « fondamentaux » et soumettent des ordres d'achat ou de vente en fonction de l'inefficience du marché. En accord avec les travaux de Gordon and Shapiro (1956) et Modigliani and Miller (1958), les fondamentalistes achètent des actifs sous-évalués et vendent des actifs sur-évalués. De par leur comportement, ils contribuent à rendre le marché informatif en faisant en sorte que le prix du marché soit égal à la valeur fondamentale.
– Les chartistes, à l'inverse des fondamentalistes, utilisent les tendances passées afin d'anticiper les futures évolutions du marché. Peu importe l'écart entre la valorisation actuelle et la valeur fondamentale, les chartistes surfent sur les bulles. Ils sont le pendant de l'analyse graphique. Les « suiveurs de tendance » amplifient la tendance haussière ou baissière du court. Les « contrariants » quant à eux prennent position contre le marché, en espérant être les premiers dans la zone d'inversement de la tendance. Les chartistes ont un effet déstabilisateur. Ils génèrent une forte volatilité du prix, et rendent le marché moins informatif.
– Les agents aléatoires (NTA) sont appelés métaphoriquement des *Noise Trader Agents*, parce que leurs anticipations sont biaisées par des considérations qui ont un effet similaire à celui des « bruits parasites » dans un phénomène de communication radioélectrique. Ces investisseurs ne sont pas pleinement rationnels. Leur demande d'actifs à risque est affectée par leurs croyances ou leurs émotions, lesquelles ne sont évidemment pas pleinement justifiées par les fondamentaux économiques.

0.4. La théorie classique

De manière empirique, une relation forte entre les indicateurs macroéconomiques et le cours des actifs est observable. Lucas, en 1978, l'avait déjà mis en lumière. Il est donc attendu que des annonces publiques puissent générer une réaction des marchés. Cependant, le nouveau vent de panique qui a suivi les annonces de la Banque Centrale Européenne (BCE), le 10 mars 2016, ne semble pas reposer uniquement sur l'intégration de l'information dans le prix. En effet, après une ouverture dans le rouge, les marchés européens rebondissent favorablement aux annonces de la BCE. À 13h55, le CAC 40 bondit de 3.3%, tout comme les autres places européennes qui évoluent alors dans le vert. Quelques heures plus tard, le CAC 40 ferme en repli de 1.7% ; le Footsie britannique perd $1,78\%$ et le Dax allemand $2,31\%$[b]. Le lendemain, à 12h40, le CAC 40 gagne 2.60% et le SBF 120 avance de 2.5%, tous deux dopés par les valeurs financières, alors que les *futures* tablaient sur un rebond de 1.8%.

La théorie classique propose quelques pistes afin de justifier cette forte volatilité. Ainsi, l'hypothèse d'incomplétude des marchés est un facteur explicatif de crise et d'instabilité. À cause de cela, les investisseurs ne sont pas en mesure de se couvrir totalement du risque et d'anticiper tous les événements futurs (Beker and Chattopadhyay, 2006; Blume and Easley, 2008). La théorie économique met en avant d'autres imperfections de marché telles que les frictions qui génèrent une baisse de la liquidité, et qui peuvent être expliquées par les coûts de transactions, le nombre fini d'animateurs de marché, ou encore le fait que les marchés ne soient pas accessibles à tous les investisseurs. Ces frictions empêchent certains arbitrages. Le résultat est donc un marché pas totalement efficient et potentiellement volatil. Cependant, la théorie classique se heurte à des faits observés (appelés faits stylisés) qu'elle ne peut expliquer ni répliquer (Campbell and Cochrane, 1999; Campbell et al., 2001; Schwert, 2003) tels que le mystère de la volatilité (*volatility puzzle*)[c] ou encore le mystère de la prévisibilité (*predictability puzzle*)[d]. Que ce soit la crise de 1987 ou celle de 2008, il existe des facteurs qui ont causé une inefficience des marchés. Cela repose sur des biais générés par la nature humaine et la structure des marchés.

Cohen et al. établissent dès 1980 une relation entre le prix des actifs et le comportement des agents. De Long et al. (1990) mettent en avant l'importance d'agents non informés au comportement erratique (NTA) pour rendre le marché plus liquide. Ce domaine de recherche, connu sous le nom de microstructure, se définit comme l'étude des processus et des résultats de l'échange d'actifs en fonction des caractéristiques des marchés (O'Hara and Easley, 1995). Étant donné sa

[b]Lien : http ://www.zonebourse.com/actualite-bourse/Cloture-en-recul-des-marches-en-Europe-apres-une-seance-volatile–21994032/

[c]Une attente raisonnable des investisseurs est un rendement à la hauteur de leur prise de risque. Plus un actif est volatil, plus il est risqué. Cependant, plusieurs analyses empiriques tendent à démontrer que des actifs à forte volatilité sont caractérisés par des rendements faibles (Yamada and Nagawatari, 2011). De plus, il est couramment admis que lorsque le prix d'un actif chute, sa volatilité est plus élevée que lorsque ce même actif s'apprécie.

[d]Les rendements des actifs sont prévisibles, la persistance des profits n'est pas érodée par l'arbitrage des marchés.

définition, celle-ci semble plus à même de traiter la question de la dissémination de l'information sur un marché financier. Cette littérature accorde de l'importance aux agents non informés qui permettent de cacher l'information des agents informés (Kyle, 1985), mais qui rendent aussi les prix plus volatils et instables (De Long et al., 1990; Haruvy and Noussair, 2006). Les agents sont hétérogènes et contribuent chacun à la formation d'un prix agrégé qui représente la somme des croyances des intervenants du marché. Les déviations à court terme peuvent donc s'expliquer par les coûts de transactions, les coûts de détention, l'asymétrie de l'information ou encore les comportements stratégiques (Abel et al., 2013). Madhavan and Smidt (1991) mettent même en avant l'importance de la structure de marché. Ils établissent que l'animateur de marché joue un rôle dans la formation du prix ainsi que dans la soumission des ordres ou la détention des actifs. La microstructure s'intéresse aussi à l'importance de la structure de marché. Foucault et al. (2005) ont mis en avant, dans un modèle simple, les relations existantes entre la liquidité, la profondeur de marché, le temps d'execution d'un ordre et l'écart acheteur-vendeur (*bid-ask spread*).

Cependant, les marchés ne sont pas déterministes mais stochastiques (Black and Scholes, 1973). Les différents états du monde doivent être anticipés et pondérés en fonction de leur vraisemblance d'occurrence. Les individus sont limités dans la connaissance de leur environnement ainsi que dans leur capacité à accéder aux informations puis à les traiter. Partant de l'idée que l'information est coûteuse (d'un point de vue temporel comme financier), et que sans information, l'individu ne peut agir rationnellement (prendre une décision optimale au sens de la théorie classique), Simon (1982) estime que la modélisation d'un agent via des règles basées sur des observations empiriques parait plus pertinente et réaliste que l'utilisation d'un agent parfaitement informé et rationnel. Les agents sont alors qualifiés d'agents à rationalité limitée et les déviations de court terme peuvent persister à long terme.

Si l'on prend un des produits les plus simples : une action d'entreprise qui est cotée tous les jours sur un marché, on observe des transactions et le prix varie. Deux raisons justifient ces échanges et rééchanges : soit une situation de l'investisseur différente soit une évolution des anticipations du futur de cette entreprise. Cependant, en se concentrant sur les positions intrajournalières[e] du CAC 40 entre le 10 mars 2016 et le 16 mars 2016, on peut observer une forte variabilité, voire des discontinuités dans le comportement global du marché. Partant du constat empirique que les flux d'ordre sont informatifs (Karolyi et al., 2012) et génèrent des réactions de la part des investisseurs (Lin and Rassenti, 2012) et que ces investisseurs ont une rationalité limitée, il semble pertinent d'utiliser les outils de la modélisation agents afin de traiter de la découverte de la valeur fondamentale sur les marchés financiers.

[e]Opérer en intrajournalier revient à prendre et à solder ses positions dans la même journée de cotation.

0.5. Les modèles basés agents

La modélisation basée agent en finance de marché est définie par Markose et al. (2005) comme « l'étude des décisions d'investissement, des stratégies d'échange et de la gestion du risque par le biais de simulations numériques (computationnelles).» Les marchés financiers artificiels permettent d'étudier le lien entre le comportement de l'investisseur individuel et la dynamique du marché financier. Cette dynamique complexe, qui cherche à mettre en relation les décisions microéconomiques et leurs conséquences macroéconomiques, est à la base de nombreux travaux de recherche en sciences économiques. Les défis de ce champ de recherche se concentrent sur :

- la notion de valeur fondamentale en économie et en finance : rationalité et hypothèse d'efficience dans les marchés qui restent sans réponse (LeBaron, 2006; Orléan, 2012) ;
- la compréhension des dynamiques des marchés financiers : impact des structures de marché (Chiarella and Iori, 2002; Babus and Kondor, 2013; De Kamps et al., 2013) et explication/réplication des faits stylisés (LeBaron, 2006; Lux, 2008) ;
- l'anticipation ou la prédiction : explication/anticipation des crises financières (DeBondt et al., 2008), impact des transactions à haute fréquence (Jacob-Leal et al., 2016; Yamamoto, 2015).

Afin de tenter de répondre à ces problématiques, les modèles basés agents se concentrent sur :

- l'étude comportementale de l'individu (Fischoff et al., 1977; Kahneman and Tversky, 1979; Beja and Goldman, 1980; Simon, 1982; Barberis and Thaler, 2003) ;
- la microstructure, telle que les mécanismes de découverte du prix et la structure de marché : les marchés dirigés par les ordres (Chiarella et al., 2009), l'opacité du carnet d'ordre (Kovaleva and Iori, 2015), le tâtonnement Walrassien (LeBaron, 2006), ou encore l'efficience allocative induite par la structure du marché (Gode and Sunder, 1993).

L'expansion de la modélisation agent s'est faite de concert avec le développement des systèmes complexes et l'intégration de l'approche computationnelle dans les modèles économiques (tel que Arthur et al., 1997). Le modèle de ségrégation de Schelling (1971) pourrait être considéré comme l'un des premiers travaux en modélisation agent. Schelling y étudie l'émergence de phénomènes de ségrégation dans une structure résidentielle où les agents n'ont pas de préférences fortes pour l'homogénéité. En effet, chaque agent accepte un voisinage majoritairement « différent » à la condition qu'au moins un tiers lui soit semblable. Les interactions locales des agents suffisent alors à faire émerger une configuration globale fortement homogène. Même si initialement Schelling n'utilise pas d'ordinateur, son modèle incarne les fondements de la modélisation agents : à savoir des agents autonomes qui interagissent dans un environnent commun d'où émerge une dynamique.

Attardons-nous maintenant sur les contributions en modélisation agents qui ont motivé la rédaction de ce projet de recherche. Chiarella (1992), Lux (1995), et Brock and Hommes (1998) ont proposé chacun une étude rigoureuse de l'impact des croyances hétérogènes dans la dynamique des marchés. Pour ce faire, chacun a développé un modèle de marché basique (un marché Walrassien ou un marché dirigé par les prix), où différents types d'agents interagissent afin d'échanger un actif risqué. Les fondamentalistes connaissent la valeur fondamentale et tentent de faire converger le marché vers cette dernière. Les chartistes se concentrent sur l'analyse graphique ; ils dégagent du profit en exploitant les tendances du marché et ne se soucient nullement de l'efficience[f] de ce dernier. Enfin, les NTA sont des agents aux comportements irrationnels et dépourvus d'information. Il ressort des travaux de Brock and Hommes (1998) que l'efficience de marché est assurée à partir du moment où tous les agents sont rationnels et identiques. En revanche, dès lors que les agents ne sont plus homogènes, et que ces derniers cherchent le comportement dont la rentabilité observée est la plus forte, les marchés deviennent moins informatifs et plus volatils. La dynamique du prix se complique. La convergence générée par les fondamentalistes (agents rationnels) est altérée par des agents ne souhaitant pas s'informer (NTA) ou préférant une analyse graphique (chartistes). Cela génère des comportements mimétiques qui justifient la formation de bulles spéculatives (Lux, 1995). La différence entre prix de marché et valeur fondamentale s'explique alors par un mécanisme endogène définissant le poids de chaque type d'agents dans le marché. Ce mécanisme est responsable de plusieurs dynamiques allant de la simple oscillation du prix de marché autour de la valeur fondamentale (volatilité de marché) à la déconnexion entre la valeur réelle et la capitalisation boursière (route vers le chaos).

Par la suite, ces trois types d'agents (fondamentalistes, chartistes et NTA) interagissent dans une structure de marché dirigée par les ordres (Chiarella and Iori, 2002). Cette structure, plus complexe, permet une étude de la liquidité du marché. En effet, jusqu'à maintenant, l'animateur de marché était garant de la liquidité. Ici, les agents peuvent observer une partie du carnet d'ordre (information publique) et soumettre différents types d'ordres (ordres à court limité et

[f]L'efficience des marchés (EMH) se décline sous trois formes. La première, l'efficience faible, admet que l'information contenue dans les prix de marché passés est complètement intégrée dans le prix des actifs. La seconde, l'efficience semi-forte, suppose que le prix des actifs intègre toutes les informations publiques disponibles. Enfin, la dernière, l'efficience forte, supporte l'idée selon laquelle toutes les informations disponibles, publiques et privées, sont intégrées dans le prix des actifs (Fama, 1970).

ordres au marché). Il ressort des travaux de Chiarella and Iori que la liquidité du marché dépend du pas de cotation, du type d'ordre ainsi que de sa durée de validité. Cette structure ouvre de nouvelles perspectives de recherche telles que l'importance de la transparence du marché dans la dynamique de ce dernier (Chiarella et al., 2009; Yamamoto and LeBaron, 2010; Yamamoto, 2011; Chiarella and Iori, 2002; Kovaleva and Iori, 2015). Ainsi, Yamamoto (2011) explique le fait stylisé de mémoire longue par la prise en compte, dans le comportement des traders des déséquilibres observés dans le carnet d'ordre. Chiarella and Iori (2002) et Kovaleva and Iori (2015), quant à eux, insistent sur le fait que l'accès complet au carnet d'ordre n'est pas optimal. En effet, cela génère des coûts de transactions élevés et fait chuter drastiquement la liquidité du marché. A contrario, un accès partiel au carnet d'ordre améliore la qualité du marché : cela réduit les coûts de transactions, maintient une plus grande liquidité et une volatilité modérée, rend le marché équilibré (autant d'ordres d'achat que de vente) et améliore la découverte du prix. Néanmoins, aucun modèle ne s'intéresse à un marché à plusieurs actifs risqués. Or, un marché multi-actifs permet entre autres de se concentrer sur les volumes d'échanges afin d'étudier la corrélation croisée entre volatilité et volume d'échange (LeBaron, 2006), ou de comprendre l'émergence de corrélation entre plusieurs actifs *a priori* indépendants (Westerhoff, 2004).

L'information accessible aux agents, qu'elle soit publique ou privée, joue un rôle prépondérant dans la convergence du marché. Le marché de gré à gré, par opposition au marché organisé, présente des coûts de transactions réduits et une souplesse dans les échanges. Il est en effet possible de passer une transaction sur mesure afin de répondre parfaitement aux besoins de chacune des deux parties. Du fait de ces contrats développés sur mesure, le marché de gré à gré se caractérise par une liquidité très faible et un manque de transparence. Les prix et les quantités ne sont pas centralisés, aucun carnet d'ordre n'est accessible (sous forme d'information publique). Ainsi, différents agents, à l'intérieur d'un même marché, peuvent avoir accès à des informations et des opportunités d'échanges différentes. Néanmoins, la littérature multi-agents fait face à un manque de travaux dans le domaine des marchés de gré à gré. À ce jour, seuls Babus and Kondor (2013) et De Kamps et al. (2013) tentent d'apporter une réponse à la dissémination de l'information dans des marchés opaques. Babus and Kondor (2013) mettent en avant dans un jeu statique (*one-shot game*) la capacité des agents à extraire et partager des informations au sein de leur réseau. Ils concluent que la diffusion d'information n'est pas efficiente. De Kamps et al. (2013), quant à eux, démontrent l'importance de la structure du réseau des agents dans un modèle où les agents sont capables d'estimer correctement la valeur fondamentale. Aucun de ces modèles ne combine à la fois une information incomplète des deux parties, un jeu répété et un apprentissage asynchrone.

La multiplication des échanges électroniques, l'évolution technologique et l'accroissement des interactions entre les acteurs de marché font partie des éléments qui agissent sur le processus de formation des prix des actifs financiers. La modélisation agents vise à concevoir des programmes permettant de répliquer et résoudre des phénomènes complexes afin d'assister le régulateur dans ses choix. Avec le développement tous azimuts des transactions à Haute Fréquence (HF), de nouveaux travaux commencent à apparaître sur les avantages et inconvénients de ce dernier dans notre système financier. Jacob-Leal et al. (2016) proposent une analyse de la dynamique du marché avec des traders HF. Ils mettent en avant un pouvoir déstabilisateur de ces derniers : les traders HF amplifient la volatilité du marché et génèrent des crises éclair. Jacob-Leal et al. soulignent d'ailleurs que l'annulation d'un grand nombre d'ordres de la part des traders HF augmente significativement l'impact des crises éclair tout en réduisant significativement leurs durées. Yamamoto (2015) propose un modèle dans lequel il compare les profits des traders non-HF avec ceux des HF. Il démontre que l'introduction d'une taxe sur les transactions HF améliore la qualité du marché. En revanche, l'instauration de frais d'annulations des ordres HF amplifie le profit des traders HF au détriment des traders plus classiques (non-HF). Enfin, Bongaerts and Van Achter (2016) mettent en avant la capacité des traders HF à apporter de la liquidité plus rapidement que les traders non-HF, et cela lorsque le marché en a le plus besoin. Ainsi, les traders HF participent à la stabilité du marché.

0.6. Contributions

Suite à cette rapide revue de la littérature, plusieurs questions intéressantes sont susceptibles d'émerger :

- Comment les caractéristiques des agents (connaissance de la valeur fondamentale et stratégie de soumission des ordres) affectent-elles les indicateurs boursiers (volume des transactions et efficience du marché) ?
- Est-ce que les agents sont capables de conjecturer le flux de trésorerie futur et d'estimer la corrélation entre des actifs dans un marché dirigé par les ordres ?
- Est-ce que les agents sont capables d'extraire, traiter et diffuser des informations privées dans un marché de gré à gré ?

Afin de répondre à ces questions, cette thèse s'articule autour de trois chapitres.

Le premier chapitre examine si l'efficience du marché et le volume des transactions peuvent être expliqués par la rationalité limitée des investisseurs. Pour ce faire, nous développons un marché dirigé par les ordres où deux actifs risqués sont échangés. Chaque agent intervenant sur le marché se caractérise par une part de chartisme et une part de fondamentalisme. Cette moyenne pondérée entre fondamentalisme et chartisme permet aussi de définir le comportement, l'humeur, la connaissance, l'aversion au risque et l'horizon d'investissement de chaque agent. Dans ce marché, la valeur fondamentale évolue au cours du temps. En revanche, à cause de la persévérance des croyances, les agents sont réticents à modifier leurs estimations. Cela conduit à une interprétation idiosyncratique de la valeur fondamentale. Cette valeur fondamentale idiosyncratique est alors utilisée dans la soumission des ordres d'achat et de vente. Elle permet de générer de manière endogène des bulles financières persistantes dues aux attentes des chartistes ou aux méprises des variations des fondamentaux par les fondamentalistes. À cela s'ajoutent des comportements stratégiques dans le processus de soumission des ordres : les investisseurs prennent en compte la profondeur du marché pour adapter leurs ordres afin de maximiser la probabilité d'exécution et l'espérance de gain. Par conséquent, l'efficience du marché est tributaire des croyances des agents, du mécanisme de correction des croyances, de la variance de la vraie valeur fondamentale et des stratégies de soumission. La modélisation des stratégies de placement engendre un marché plus profond et liquide, mais moins informatif.

Le second chapitre examine si l'efficience de marché peut être atteinte grâce à un processus d'apprentissage. La structure de marché demeure identique au premier chapitre à une exception près : les deux actifs risqués ne sont plus indépendants. Les agents, sont-ils capables de conjecturer les flux de trésorerie et ainsi d'estimer correctement la valeur fondamentale ? Nous testons si cela est en effet possible, et pour cela, nous utilisons des mécanismes d'apprentissage différents afin de confirmer la généralité de ce résultat. Nous nous concentrons également sur la façon dont l'information publique peut améliorer le processus d'apprentissage des fondamentalistes et donc la précision de leurs prévisions. Notre analyse aboutit à la conclusion que les agents sont tous capables de conjecturer de manière plus ou moins fiable la valeur fondamentale. Les apprentissages par pondération de l'expérience (EWA) et par jeu fictif (FP) surpassent l'apprentissage par renforcement (RL). Indépendamment du type d'apprentissage, la diffusion d'une information publique exacte améliore la découverte de la valeur fondamentale et l'émergence d'un consensus. Il est à noter que EWA est l'apprentissage le plus sensible à la véracité de l'information. Cela s'explique principalement par la persévérance des croyances présente dans ce dernier, et absente de l'apprentissage par jeu fictif.

Le troisième chapitre examine si la dissémination de l'information est effective dans une structure de marché plus opaque que celle développée précédemment. Nous considérons donc un marché de gré à gré où un actif risqué est échangé et où aucune information publique telle que le cours de l'actif n'est disponible. Nous limitons notre attention aux situations dans lesquelles les agents doivent uniquement compter sur leurs propres expériences. Les agents sont-ils capables d'extraire correctement les informations de leur négociation afin de partager les mêmes prévisions de la valeur fondamentale ? La présence de gourous permet-elle d'améliorer l'efficience du marché ou de manipuler les attentes des acteurs du marché ? Nous montrons que la diffusion de l'information est effective mais pas efficiente dans le réseau complément connecté, le réseau aléatoire $1/2$ et le réseau en étoile. Dans le réseau en étoile, quelle que soit l'information des agents, le noeud central dirige le marché (estimation de la valeur fondamentale et prix d'échange). Dans cette structure, les gourous ne sont pas en mesure d'influencer la croyance du noeud central. Dans le réseau petit monde, deux dynamiques émergent. Lorsque l'information des gourous est proche des attentes des non-gourous, la volatilité du marché est faible et le prix correctement estimé. Les agents considèrent le marché comme une source fiable d'information, l'estimation de la valeur fondamentale est exacte. En revanche, lorsque l'information des gourous est trop éloignée des attentes des non-gourous, les prix de négociation collectés par les non-gourous sont trop altérés pour être efficacement utilisés. Par conséquent, les non-gourous n'accordent aucune confiance au marché (ni aux gourous) et préfèrent utiliser leur propre signal. Dans ce chapitre, nous confirmons que la vitesse de convergence et la stabilité de la valeur d'échange sont liées à l'opacité du marché. Enfin, la précision de l'estimation de la valeur fondamentale dépend de la structure du marché et de la position de l'agent dans le réseau.

Chapitre 1 : Hétérogénéité des agents et de la valeur fondamentale

Motivation

Selon l'adaptation de Fama (1970) par Pouget (2000), lorsque les fondamentalistes sont en nombre conséquent, ils imposent leur vision du marché et génèrent une dynamique de prix autour de la valeur fondamentale. En revanche, lorsque la dynamique imposée par les fondamentalistes est trop faible, le prix évolue indépendamment de la valeur fondamentale. Les conditions à l'efficience de marché selon Pouget sont au nombre de trois :

- des fondamentalistes caractérisés par un horizon d'investissement à long terme ;
- des fondamentalistes qui interagissent tous en même temps afin de recentrer le prix de marché ;
- une information foisonnante et fréquente à propos des fondamentaux dans le but de justifier les interventions des fondamentalistes.

L'efficience informationnelle, quant à elle, dépend du pouvoir de marché ainsi que de l'hétérogénéité (type, motivation) de chaque agent.

Dans ce contexte, comment expliquer la chute de plus de 22% du CAC40 en seulement une semaine (entre le 6 et le 10 octobre 2008[g], avec une correction de 9.04% pour le lundi uniquement[h]) durant la crise des *subprimes* ? La découverte de titres pourris a généré potentiellement une incitation à s'informer de la part des investisseurs. Le marché s'est corrigé et il est redevenu efficient ! Si tel fut le cas, comment expliquer que certains indices (tel que le DOW JONES) reviennent actuellement à leur plus haut de 2008[i] ? Une partie du rebond (mais pas son intégralité) se justifie théoriquement et empiriquement par la tendance des investisseurs à sur-réagir face à une mauvaise nouvelle (Lin and Rassenti,

[g] http ://www.lefigaro.fr/marches/2008/10/11/04003-20081011ARTFIG00190-le-cac-perd-et-acheve-la-pire-semaine-de-son-histoire-.php
http ://www.develop-patrimoine.fr/actualite/6122/il-n-y-a-pas-que-les-feuilles-qui-chutent-en-novembre

[h] http ://www.lefigaro.fr/marches/2008/10/06/04003-20081006ARTFIG00305-la-bourse-de-paris-devisse-.php
http ://archives.investir.fr/2008/jdf/20081006ARTJDF00009-cac-en-sursis-au-dessus-de-points-.php

[i] http ://www.lemonde.fr/economie/article/2013/02/01/wall-street-depasse-les-14-000-points-pour-la-premiere-fois-depuis-2007_1826178_3234.html

2012). Ainsi la correction appliquée fut trop forte, ce qui justifie que les cours remontent légèrement. Selon Pouget (2002), l'incitation à s'informer ne remet pas en cause l'efficience informationnelle. Cette chute n'est donc pas uniquement due à une correction de la valeur fondamentale ! Pourquoi le marché a-t-il mis autant de temps à retrouver ses plus hauts niveaux ?

La correction des prix et la chute de la liquidité observées sur les marchés financiers mondiaux traduisent une défiance des investisseurs. Un État ou une entité supra nationale telle qu'une Banque centrale n'ont que peu de moyens pour tenter de rassurer les investisseurs :

- Contrôler la liquidité du marché (Brunnermeier and Pedersen, 2009), par exemple en injectant des ordres qui se compensent mutuellement.
- Diffuser une information publique portant sur le prix en injectant des ordres d'achat/vente dans le carnet d'ordre.
- Informer certains agents uniquement par le biais d'informations privées (sur la valeur fondamentale).

Est-ce que ces outils permettent réellement un retour à la « normale » (une valeur boursière au moins égale à la valeur comptable de l'entreprise) ?

Méthode

Par le biais d'un modèle agents, nous proposons d'étudier les liens qui existent entre l'information privée (l'estimation de la valeur fondamentale et les erreurs qui sont faites dans cette estimation), l'extraction d'information publique (les prix du carnet d'ordre) et la prise en compte de la liquidité du marché (profondeur du carnet d'ordre et peur de non exécution) afin de discuter des possibilités précédemment citées pour rassurer les investisseurs.

Gode and Sunder (1993) ont mis en évidence les paramètres pouvant jouer sur l'efficience d'un marché : les agents et la structure. Les agents de par leur autonomie ne peuvent être contrôlés directement par une entité gouvernementale. En outre, la crise a touché toutes les structures de marché[j]. Que ce soient les marchés dirigés par les prix, les marchés dirigés par les ordres ou les marchés de gré à gré, ils ont tous subi une chute des prix et une fuite des investisseurs (baisse de la liquidité). Partant de ce constat, et gardant à l'esprit que les sources d'informations jouent un rôle crucial sur la perception du marché (ce que nous testerons via l'information privée de la valeur fondamentale ou l'information publique du carnet d'ordre), nous choisissons de nous concentrer sur l'interaction

[j] La crise des *subprimes* (ou prêts à haut risque) voit son origine dans les marchés de gré à gré où ces titres « obscurs » étaient initialement échangés. La contagion aux marchés organisés s'est faite par la défiance aux systèmes bancaire puis financier susceptibles de détenir ces dérivés de crédit.

des agents dans le marché le plus transparent possible, un marché dirigé par les prix.

Le développement de ce modèle vise à proposer une meilleure compréhension du fonctionnement du marché, afin de répondre tout particulièrement aux questions suivantes :

- Comment les agents réagissent-ils à la profondeur du carnet d'ordre ?
- Quel a été le rôle de la ruée vers les titres de qualité durant la crise ?
- Pouvons-nous établir un lien entre le déséquilibre relatif aux ordres et l'efficience du marché ?

Pour répondre à ces diverses questions, nous avons développé un modèle à 2 actifs risqués où les agents ne sont pas parfaitement informés (information privée de la valeur fondamentale) et prennent en compte la « température du marché » (information publique : déséquilibre relatif aux ordres).

- L'ajout d'un deuxième actif risqué permet de se concentrer sur la liquidité du marché (LeBaron, 2006).
- L'hétérogénéité de la connaissance de la valeur fondamentale permet d'approfondir la question de l'information privée et publique dans le marché. Il devient possible de quantifier le temps dont le marché a besoin pour assimiler/intégrer une information et la rendre publique (lecture du carnet d'ordre). À l'inverse, nous pouvons aussi nous intéresser au temps nécessaire à un trader pour intégrer les informations publiques (estimation de la valeur fondamentale).
- La prise en compte de la profondeur de marché (modélisation inspirée de Parlour, 1998) permet de voir l'impact du déséquilibre des carnets d'ordre sur la soumission des ordres et donc sur le prix du marché. Cela illustre la réaction des agents en cas de stress. Le phénomène d'impatience à la vente peut entraîner une chute rapide d'un cours de bourse.
- L'analyse des volumes de marchés permet de confirmer un phénomène de ruée vers les titres de qualité : le prix chute et la volatilité augmente, cela engendre une baisse de la liquidité, car tous les agents veulent se séparer du titre.

Discussion

– L'hétérogénéité de l'information à propos de la valeur fondamentale a permis de mettre en avant une déconnexion entre la valeur fondamentale estimée et la vraie valeur fondamentale. Par rapport aux bulles classiques (bulles chartistes), qui illustrent le temps nécessaire au marché pour traiter l'information donnée par les fondamentalistes, nous mettons en avant le temps nécessaire aux fondamentalistes pour intégrer une information privée. Cette correction prône l'intervention d'une entité ayant assez de poids pour être prise au sérieux et ainsi donner aux agents qui le souhaitent une information privée valable. Si les agents informés n'accordent pas suffisamment de crédit à l'information, nous avons vu dans notre modèle que la valeur fondamentale estimée n'est pas corrigée. L'information n'est pas traitée, elle disparaît. A contrario, si l'information est perçue comme juste, le marché converge relativement vite vers cette information (paramètre ψ, correction/prise en compte de la nouvelle information). Ce comportement peut aussi expliquer une partie de la correction des marchés.

– L'hétérogénéité de la connaissance de la valeur fondamentale avec des agents qui ont tendance à s'obstiner permet de stabiliser le marché. Les erreurs d'estimation de chacun ne se compensent pas nécessairement en moyenne. Le marché n'intègre donc pas toutes les informations, mais uniquement les plus « importantes » (récurrentes). Dans notre modèle, nous avons mis en lumière le fait que la volatilité du marché domine l'espérance de rendement futur. Ainsi, si le prix de marché est continuellement corrigé par les fondamentalistes, il y a une augmentation de la volatilité qui génère une chute des volumes échangés (dans un marché stable). Nous confirmons la corrélation négative (établie par la théorie classique) entre volume de change et volatilité de l'actif. Pendant la crise financière, on peut estimer que l'incitation à s'informer était forte. Cependant, d'un point de vue purement fondamentaliste, cela n'explique pas tout. Les fondamentaux étaient bien supérieurs à la valeur des cours des actifs, et cela était connu de tous (information publique).

– Dans un marché avec carnet d'ordre, la profondeur de marché (ou déséquilibre relatif aux ordres) est une information publique. Les agents l'utilisent pour définir leur manière d'échanger (plus ou moins agressive). Elle peut aussi s'apparenter à un thermomètre de l'impatience d'exécution ou de la peur de non exécution (perte de bénéfice à la vente). Notre modèle permet de justifier un lien entre la valeur fondamentale et la valeur de marché, avec des agents parfaitement informés. Lorsque les fondamentalistes ont corrigé les prix, le déséquilibre de marché était en faveur d'une baisse du cours de l'actif. Les positions étaient clairement à la vente. Notre modèle confirme que les agents sont plus impatients à la vente qu'à l'achat. Cela se caractérise par des vendeurs plus enclin à soumettre des ordres au marché, et des acheteurs préférant des ordres à court limité. De plus, lorsque les agents informés prennent en compte le déséquilibre du carnet d'ordre le marché perd grandement en efficience. Même si les agents estiment correctement la valeur fondamentale (par leur information privée), le prix de leur soumission ne reflète plus la valeur fondamentale mais la peur de non exécution (ou le prix consenti pour trouver une contrepartie). L'information dispensée par le carnet d'ordre (prix de marché) est donc erronée

du point de vue de la valeur de l'entreprise. En revanche, ce prix indique une crainte contagieuse dans un marché peu liquide. Pour vendre, les agents génèrent une prophétie auto-réalisatrice de baisse des prix par le biais de l'information publique qu'ils véhiculent jusqu'à atteindre un seuil minimal. Puis, vient une correction du marché qui peut, à nouveau, générer une amplification de la tendance (bulle haussière ou baissière en fonction du déséquilibre du carnet d'ordre). Les chartistes, qui ne se basent que sur l'observation des prix passés pour échanger, exagèrent la tendance et ralentissent le retour à la vraie valeur. La prophétie auto-réalisatrice continue. Les agents non informés ont donc accentué la baisse. Par ce comportement, ils ont augmenté la volatilité de l'actif et aggravé le phénomène de panique. Dans le modèle, la prise en compte des déséquilibres de marché génère des périodes de forte volatilité avec une importante décorrélation entre la valeur fondamentale et la valeur de marché, ainsi qu'un gain de liquidité sur la période concernée.
– Une information privée dispensée par l'État aux agents informés (ou une information des agences de notation) n'aura pas d'impact sur le cours de bourse si la crainte du manque de liquidité empêche l'information de se diffuser dans le carnet d'ordre. L'information doit donc être directement implémentée dans le carnet d'ordre. Pour ce faire, l'entité peut devenir elle-même actionnaire. Le résultat dans notre modèle est le suivant : si l'État se porte contrepartie pour une assez grande quantité d'actifs, le déséquilibre du carnet d'ordre disparait, et le marché peut retourner à des valeurs plus représentatives. En revanche cette méthode demande beaucoup d'investissements et donc de liquidités disponibles. De plus, la théorie classique a déjà mis en avant que les investisseurs n'apprécient pas vraiment que l'État soit actionnaire. Il ne doit donc pas le rester longtemps. Sinon, cette prise de pouvoir se traduit souvent par une légère baisse de la valeur de l'action. Or, la revente des titres peut à nouveau générer un déséquilibre si le marché ne s'est pas réorienté à la hausse. Et durant la crise, le marché est resté longtemps atone ou orienté à la baisse.

Le chapitre 1 tel qu'il est présenté dans ce document de thèse a été soumis à la revue *Computational Economics* le 22/07/2015 et révisé le 04/07/2016. Le processus de publication est donc actuellement engagé. Il est à noter qu'une version se concentrant plus particulièrement sur la chute du volume de change est déjà publiée dans la revue *Advances in Complex Systems* (Lespagnol and Rouchier, 2015). Le lecteur intéressé pourra se référer à la *section A.3* de l'annexe (voir page 126 et suivantes) tout en gardant à l'esprit que le modèle développé est strictement identique à celui du chapitre 1.

1. Trading volume and market efficiency : an Agent-Based Model with heterogenous knowledge of fundamentals

1.1. Introduction

The main goal of financial markets is, according to Fama (1970), to guarantee an optimal transfer of resources from supply to demand. This aim can be attained only if exchanges do actually occur in a given period, i.e. the market is liquid. Amihud et al. (2005) state that "liquidity is a complex concept. Stated simply, liquidity is the ease of trading a security." It is an important feature which ensures the functioning of the financial market.

Indeed, during the last financial crisis, there was no way analysts could anticipate the falls in liquidity and price that took place. Moreover, to the best of our knowledge, there is no known means of impacting market liquidity and we are left simply with *ex post* observations and attempts to understand the data. As an example: Air France -KLM was valued under 5€ on the CAC40, even though the consensus among analysts was that the book value was at least six times higher. However, nobody wanted to hold this asset so it was undervalued from the start, which was neither predictable nor rational. It has been shown that agents' behaviour is driven by asset exposure to liquidity risk (Amihud, 2002), which is consistent with "flight to liquidity" in times of market stress (Acharyaa and Pedersen, 2005). This makes liquidity an interesting topic, which mechanism is worth studying.

The field of microstructure focuses on the concept of liquidity, impact of trader type or information revealed. Kyle (1985) and Grossman and Miller (1988) incontestably provide the reference microstructure models. Theoretically, they investigate the importance of market liquidity for the optimal behaviour of discretionary traders and their effect on patterns of trade. Traditionally, measures of market liquidity include indicators of market impact, market resilience and market depth (Muranaga and Shimizu, 1999). Numerous indicators are in fact relevant to liquidity: trade frequency, trading volume, price volatility, bid-ask spread, gross order-book volume (buying order volume plus selling order volume) and net order-book volume (buying order volume minus selling order volume). Despite the precision of the analysis, microstructure theory does not study liquidity as an endogenous characteristic of markets, but as an exogenous parameter that influences agents' behaviours.

The classical efficient market hypothesis assumes that prices move up or down because investors react rationally and immediately to news (Fama, 1970). Over the last three decades with financial data becoming available, academics and practitioners have discovered that financial markets are more complicated than the theory was claiming. This leaves researchers with the problem of developing the right tools and models to analyse financial markets.

In the description of ABM, what are the elements of bounded rationality which have an impact on trading volume and price distortion?

It makes sense that a link exists between agent's rationality and market indicators (liquidity and mispricing). The contribution of Agent-based Computational Economics is to produce models that cover agents' bounded rationality as well as their heterogeneity in terms of information and cognition. Several authors have already proven that this assumption of heterogeneity is necessary if we want to reproduce the dynamics of actual behaviours (Bao et al., 2012).

When agents are designed with adaptive learning of fundamentals, different sources of market inefficiency appear: some attributed to the fundamentalist outlook, based on misunderstanding (Westerhoff, 2004), and some generated by the trend-follower outlook, based on trend extrapolation (Hommes, 2006). Be aware that increasing the fundamentalist outlook in a previous stable system may make it less stable (Beja and Goldman, 1980). Agents also differ by their sensitivity to order-book depth: the observed number of open buy and sell positions for a security at different prices. They adapt their orders to the market depth and thus influence the market liquidity and efficiency (Order-placement strategy - Parlour, 1998).

Gode and Sunder (1993) established that two components impact on the market dynamics: the market structure and the agent's behaviour. Agent-Based Modelling (ABM) enables us to generate a real market composed of artificial agents and to observe it. In a given exchange structure, it is possible to test the influence of the decisional and behavioural characteristics of agents on dynamics (individual and collective).

In this paper, we decided to work on the impact of some elements of non-optimising behaviour on the market liquidity and efficiency. In an ABM with a call auction market structure, where two risky assets are traded, we consider one dimension of liquidity that has received little attention: trading volume. The trading volume and the price distortion both depend on information processing. What we call information processing of the agent is the learning fundamental value, the agent's mood, and the order-placement strategy.

We examine the link between market indicators (trading volume and market efficiency) and information processing by wondering how the rationality of agents is bounded.

• As noted in many works (Gode and Sunder, 1993; Hommes, 2006; LeBaron, 2006; O'Hara and Easley, 1995; Vriend, 2006), the market structure impacts on market efficiency, and the trend-follower outlook destabilises the market by disturbing the trading price and bringing up the number of quoted orders. Unexpectedly, in our model, the remaining quoted orders are not consumed by the fundamentalist outlook due to the volatility rise, generated by the trend-follower outlook. The trading volume decreases with the trend-follower outlook, and the order-book stays filled up of non executed orders. This is in line with the already established link between trading volume and market volatility (Amihud et al., 2005).
• Including adaptive learning of fundamentals enables to identify market inefficiency attributed to fundamentalist outlook and based on misunderstanding of fundamental changes. Trend-follower outlook is not the only one which generates mispricing. The adaptive learning makes the market price and the trading volume more stable. The market efficiency thus depends on the variance of the fundamental, the memory of the correction mechanism and the anchor.
• The agent's mood by preventing illiquid market and adding noise replaces the usual noise trader outlook.
• By making agents responsive to the order-book depth, the order-placement strategy produces long-term bubbles, high trading volume per simulation and largest exchange volume realised in a round.
• All these features about information access and information processing keep the impatience to sell of the traders. What we call impatience to sell is the fact that market sell orders are always more numerous than the market buy orders. However, the order-placement strategy reduces the ratio of market orders to limit orders by increasing the market liquidity. The impatience to sell is less visible.

The paper is organised as follows: Section 2 reviews the existing literature. Section 3 outlines the structure of our model. Section 4 discusses results and Section 5 concludes.

1.2. Literature

An ABM market relies on two components: the market structure which includes the temporal organisation, the exchange mode and the information tools, and the agent rationality which includes the agent's features and their learning.

1.2.1. Market structure

Gode and Sunder (1993) proved with their Zero-Intelligence traders that computational market structure plays an important role in market efficiency and price convergence. They argue that it is very important to fix a structure once and for all, so as to be able to compare while testing different elements of rationality, which is our goal. What we are calling here structure consists of the price formation mechanism and the quantity of risky assets.

The first agent-based financial market platform used to study the impact of agent interactions and group learning dynamics in a financial setting was the Santa Fe Institute market (SFI market - LeBaron et al., 1999). This market relies on a simple market clearing mechanism and was followed by many studies (Arthur et al., 1997; Brock and Hommes, 1998). The other widely used mechanism is the market maker (Lux, 1995; Iori, 2002; Hommes et al., 2005a; Harras and Sornette, 2011). However, Domowitz (1993) demonstrated that in the early 90's, over thirty major financial markets worldwide had order-driven market features in their design. While some markets in the recent past were driven by prices (the NASDAQ until 2002), today most stock exchanges operate on an order-book. Foucault et al. (2005) have written, "a trader who monitors the market and occasionally competes with the patient traders by submitting limit orders, can significantly alter the equilibrium. His intervention forces patient traders to submit aggressive limit orders and hence narrows the spreads. This feature may provide important guidance for market design." The most realistic modelling choice is thus an order-driven market. In this market structure, trading volume becomes endogenous to the model, generated only by the execution of quoted orders. This structure is the most difficult to compute, but is clearly essential here given our focus on trading volume (LeBaron, 2006).

The original SFI market consists of learning agents who trade two assets, a risky and a non-risky one. To the best of our knowledge, the main ABMs in finance follow the same structure. In Westerhoff (2004) and Chiarella et al. (2007), two risky assets are traded but with the intervention of a specialist (a market maker). The novelty of our model lies in the aggregation of an order-book structure where two risky assets are traded. Agents have the choice between the risky assets and a risk-free bond, which pays zero interest. The decision process determines the amount of wealth the trader would like to invest in the risky assets for some transaction price. The residual wealth is necessarily invested in the bond. The risk-free bond is not further considered afterwards because agents are not monetary constrained. As Chowdry and Nanda (1991) state, multi assets enable us to study liquidity, which is essential for both the viability and the dynamics of a market. Moreover, according to portfolio theory, the mean-variance criteria reveals that it is possible to minimise portfolio risk when there are multi-risky assets (if $\rho < 1$, the correlation coefficient between the assets). They would better fit the traders' needs by avoiding the "take it or leave it" case that traders face with a single risky asset. Multi-risky assets allow to focus on the optimisation techniques, and the application of proper volatility and factor models. In addition, they provide insight into the evaluation of traditional and nontraditional sources of risk. The contagion effect from an asset to the whole market, or the flight to quality might be highlighted in this specific structure. Thus, an order-driven market where two risky assets (stocks) are traded seems more realistic than a single risky asset, and can be extended to n risky assets.

The order-driven market is usually modelled according to two types of orders: the market order (MO) and the limit order (LO). The order type is determined by the submission price and the bid-ask spread. An agent who submits a MO can soon execute it, ensuring positive profits, but she will then lose the opportunity to execute her order at a more favourable price. Conversely, the LO insures the trading price but not its execution. The ratio of submitted LO and MO is a measure of impatience. According to O'Hara and Easley (1995), traders are more impatient to sell than to buy, which means they submit more MO on short stock positions (to sell) than on long stock positions (to buy).

With an order-book – where at least a part of the book is observable – Parlour (1998) shows how the order-placement decision is influenced by the state of the book. Particularly influential is the depth available at the inside quotes. Empirical research reveals that investors place "more aggressive bids (asks)" when the corresponding side of the order-book is thicker, and less aggressive orders when it is thinner (Handa et al., 2003). Chiarella and Iori (2002) introduce the first order-driven market model with heterogeneous agents trading via a central order mechanism. From this market model, Chiarella et al. (2009) provide further evidence that large price changes are likely to be generated by the presence of large gaps in the book. Yamamoto and LeBaron (2010) explain the long memory effect by the order-placement strategy. Tedeschi et al. (2012) study the herding effect through a dynamic network structure: the communication structure allows rises and falls of Gurus over time. Kovaleva and Iori (2015) highlight that endogenous restriction of displayed quote depth, by means of iceberg orders, improve liquidity, moderate volatility, balance the limit order-book and enhance price discovery.

1.2.2. Agent's rationality

In 1980, Beja and Goldman showed that agent type affects the quality of the price signal. In general, modellers distinguish two (Chiarella et al., 2007; Jacob-Leal, 2014) or three (Chiarella et al., 2009; Hommes, 2006) types of agent: fundamentalists, chartists and noise traders.

According to the financial theory, the fundamentalists base their forecasts of future prices and returns on economic fundamentals, such as cash flows, dividends, interest rates, price-earning ratios, and so on. Following discounted earnings models (Gordon and Shapiro, 1956; Modigliani and Miller, 1958), which express stock prices as the present value of the stream of future cash flows, fundamentalists buy the asset when it is undervalued, and sell it when it is overvalued. They are considered as informed traders, their expectation on market price being equal to the fundamental price. They are assumed to be the most risk-averse agents and have long investment horizons. These features suggest that a pure fundamentalist market should be efficient in the sense of Fama (1970); the market price should reflect the fundamental trend. However, without the emergence of bubbles (spread between market and fundamental prices), fundamentalists have no incentive to trade and the market becomes illiquid.

Chartists increase the market depth because of their short horizon and lack of information (O'Hara and Easley, 1995). It is generally accepted in economic works that "positive feedback" traders prevail in financial markets. We limit chartist to trend-follower aspect. Theoretical models and laboratory experiments (De Long et al., 1990; Hommes et al., 2005b, 2007) show that positive feedback may destabilise prices and earn larger returns. The trend-followers are described as speculators; they have a destabilising impact. They try to predict price evolution so as to surf on the bubbles and hence exploit market trends to make a profit. As a consequence they revise their expectations frequently and prefer short-term investments. Trend-followers do not mind about changes in fundamentals and bring incentive to trade to fundamentalists: fundamentalists trade so as to burst trend-followers' bubble. The market remains liquid.

Harras and Sornette (2011) state that the "agent forms her opinion based on a combination of different sources". To get away from clear-cut rationality, Yamamoto (2011) combines fundamentalist and trend-follower features. Actually we took the option to follow this view of mixed agent. The fundamentalist outlook of an agent expects the forward price to converge to its fundamental, while the trend-follower outlook assumes that future prices follow past trends (Hommes, 2006).

Lux et al. (2001) introduce the notion of mood. They observe pessimistic phases in which the asset is undervalued (compared to the fundamental value), and switching to optimistic phases in which it is overvalued. In the first case the majority of noise traders is in the "pessimistic mood", while the second case most of them are in an "optimistic mood". In their paper, the switching mechanism is influenced by two factors: the opinion of others and the local dynamics of the price. The mood parameter, fixed (Yamamoto, 2011) or evolutive (Lux, 1995), allows to replicate the long memory stylised fact. In our setup, the agent's mood is more basic. It is a randomly assigned value from a uniform distribution. This setup allows agents to place bids and asks that are lower or higher than their expectation. The introduction of noise and heterogeneity in the submitted price should make the market more liquid and weakly less informative. In some way, our agent's mood allows to replace the classic noise trader in order to prevent illiquid fundamentalist market. Indeed, the noise traders are uniformed agents who trade randomly. Thus, they add noise and liquidity which is caught by more informed traders (Bloomfield et al., 2009). The market liquidity should be higher with our mood parameter.

1.2.3. Agent's learning

In our model, the fundamentalist outlook has to learn the fundamental value. It is usually defined as: the expected cash flow of the firm ($\mathrm{E}_t[y_{t+k}]$) corrected by the risk premium required for a risky asset ($\alpha\sigma^2 Z$) divided by the actualisation rate (R). This value is assumed to be unique, known by all fundamentalist outlooks and equal to:

$$fv = \sum_{k=1}^{\infty} \frac{\mathrm{E}_t(y_{t+k}) - \alpha\sigma^2 Z}{R^k}$$

Common criticisms of this concept (Walter and Brian, 2008) include:

- The heterogeneity in estimation of the actualisation rates raises a selection issue. Should it be constant or time evolutive? How should it be fixed?
- The forward dividend raises the question of which economic variable to base our dividend expectation on? Usually, dividend expectations come from the market, rendering the fundamental value intrinsic to the market, whereas it should be intrinsic to the firm but exogenous to the market.

Tversky and Kahneman (1974) showed that agents in an unexpected context are reluctant to change their beliefs and are influenced by their original belief for a long time. Barberis and Thaler (2003) demonstrate that traders make errors in their expectations and are mainly overconfident. Orléan (2011) reveals that traders use the market as a source of information, hence the fundamental value is neither unique nor exogenous to the market. To be able to use the concept of fundamental value in our context of bounded rationality, we need to base the learning of our adaptive agents on these observations and the work of Westerhoff (2004). In our definition of bounded rationality, agents are limited to their information. The fundamental value becomes idiosyncratic.

1.3. Model

The agents are defined by their ratio of fundamentalist outlook to trend-follower outlook, their mood and their sensitivity to order-book depth (order-placement strategy - Parlour, 1998). These agents' characteristics influence the information processing and investment decisions.

1.3.1. Market and agent characteristics

1.3.1.1. Market structure

The investment choice is made in a call auction market which is composed of two independent risky assets; if agents do not want to invest, they can keep their wealth. The market structure is such that each asset price is given by the price at which a transaction, if any, occurs. If no new transaction occurs, a proxy for the price is given by the average of the quoted ask a_t^q (the lowest ask listed in the book) and the quoted bid b_t^q (the highest bid listed in the book), so that $p_t = (a_t^q + b_t^q)/2$. If no bids or asks are listed in the book, a proxy for the price is given by the previous traded or quoted price. Once submitted, the orders can't be modified. Either it is removed when it is executed or it remains unexecuted for its lifetime. Bid, ask and market price need to be positive and are defined on a pricing grid of width Δ, also called tick size. Finally, agents are not allowed to engage in short selling and are not monetary constrained.

A round in our model is such that:

1. an agent is randomly chosen (following a uniform law on the population);
2. for each asset available on the market, she formulates expectations on the forward price according to her particular features;
3. she defines her positions (buy or sell, and quantity) with a CARA utility function;
4. she defines her submission price according to her future expected price and her mood (simple rule-of-thumb);
5. she adapts it according to her sensitivity to order-book depth;
6. the submitted orders are interpreted by the market, leading to new asset prices.

1.3.1.2. Agent definition

The agents are defined by four traits:

- their fundamentalist outlook (g_f^i);
- their trend-follower outlook (g_c^i);
- their mood ($M_t^{i,j}$);
- their sensitivity to order-book depth ($\beta^{i,j}$).

All these parameters are fixed in time, except mood. For agent (i), the proportion of g_f^i and g_c^i are according to a set of exponential distributions of variance σ^2. For agent (i) and asset (j), β^{ij} and $M_t^{i,j}$ are generated with uniform laws on the interval $[0; \beta_{max}]$ and $[-0.01; 0.01]$, respectively. These agents are assigned risk aversion and investment horizon[a] that depend on this g_f^i, g_c^i ratio, following:

$$\alpha^i = \alpha \, \frac{1+g_f^i}{1+g_c^i} \tag{1.1}$$

$$\tau^i = \tau \, \frac{1+g_f^i}{1+g_c^i} \tag{1.2}$$

where τ and α are the parameters of the system to define the time horizon and the aversion to risk, respectively a reference time horizon and a reference degree of aversion to risk. The reference time horizon is built so that traders take intraday positions. Further investigations are needed to have a precise scale of the day length.

Decision-making occurs in three consecutive steps:

1. estimating the future prices;
2. calculating the optimal demand for each asset according to the estimated prices;
3. correcting the bid(s) and/or ask(s) in accordance with the state of the order-book.

1.3.2. Agent's expectation of future asset price

As stated above, agents are a mixture of fundamentalists (g_f^i) and trend-followers (g_c^i). The asset's future returns are thus computed from the weighted expectation of each outlook.

1.3.2.1. Trend-follower expectation

This expectation is what the agent infers from past returns and information received. The return at time t is defined as:

$$r_t = ln\left(\frac{p_t}{p_{t-1}}\right) \tag{1.3}$$

The average spot return ($\bar{r}_t^i$), used by the trend-follower outlook of our agent, is defined as the expected trend based on observations of spot returns over the last τ^i rounds.

$$\bar{r}_t^i = \frac{1}{\tau^i}\sum_{k=1}^{\tau^i} r_{t-k} = \frac{1}{\tau^i}\sum_{k=1}^{\tau^i} \ln\left(\frac{p_{t-k}}{p_{t-k-1}}\right) \tag{1.4}$$

[a]The investment horizon is rounded to the nearest integer, $\tau^i \in \mathbb{N}^*$.

1.3.2.2. Fundamentalist expectation

There are two means of estimating the fundamental value ($\hat{f}_t^i$) used by the fundamentalist outlook of our agent: with and without perfect knowledge, here called "perfect knowledge", and "adaptive learning".
– With perfect knowledge of the fundamental value, we assume that the estimation of the fundamental value is unique and right. The fundamentalist expectation of agent (i) is mathematically expressed as:

$$\widehat{fv}_t^i = \widehat{fv}_t = fv_t \quad , \forall i \tag{1.5}$$

– With adaptive learning, the fundamentalist expectation becomes idiosyncratic. We use Westerhoff's formalism (2004). The belief perseverance is defined by the last observed price (p_{t-1}), the previous estimation of the fundamental value ($\widehat{fv}_{t-1}^i$) and the original one ($\widehat{fv}_0^i$). γ_1, γ_2 and γ_3 are the respective weights given to these components and add up to 1.
Information processing consists of the arrival of news (N_t) – which is common to all –, plus a data processing: ω is the degree of misperception due to the limited time needed to process information, while the components in the parenthesis represent the degree of faith the agent places in the news. As an example, if the estimated fundamental value has recently been updated excessively with respect to news ($\widehat{fv}_{t-1}^i - \widehat{fv}_{t-2}^i \neq N_t$), the fundamentalist outlook tends to overreact to current news.
The correction mechanism relies on ψ. When ψ tends to 0, older and more recent observations tend to be weighted in the same way $(1/t)$. The correction mechanism is slow. When ψ tends to 1, more importance is attached to the recent observation than to the old one. The correction mechanism is fast.

Unfortunately, empirical guidance on how to pick the learning parameters is limited. All parameters (ω, ψ, γ_1, γ_2, γ_3) are equal among traders. The choices are summarised in Table 1.4, rely on further results.

- Agents in an unexpected context are reluctant to change their beliefs and are influenced by their original belief for a long time (Tversky and Kahneman, 1974).
- Agents are reluctant to search for evidence that contradicts their belief (Lord et al., 1979).
- The correction mechanism is slow over time and small in magnitude (Fischoff et al., 1977).
- Many traders assume that asset prices themselves reflect relevant information (Murphy, 1999).

It is structured as:

$$\begin{aligned}
\widehat{fv}_t^i &= \gamma_1\, p_{t-1} + \gamma_2\, \widehat{fv}_{t-1}^i + \gamma_3\, \widehat{fv}_0^i && \leftarrow \text{belief perseverance} \\
&\quad + N_t + \omega(\widehat{fv}_{t-1}^i - \widehat{fv}_{t-2}^i - N_t) && \leftarrow \text{information processing} \\
&\quad + \psi(fv_{t-1} - \widehat{fv}_{t-1}^i) && \leftarrow \text{correction mechanism} \\
\widehat{fv}_t^i &\neq \widehat{fv}_t^j && ,\forall i \neq j
\end{aligned} \tag{1.6}$$

1.3.2.3. Expected return

Our agents take the current asset price, the fundamentalist expectation and the trend-follower expectation into account to form their weighted forecast of the asset's future return in the interval $(t + \tau^i)$:

$$\hat{r}^i_{t,t+\tau^i} \quad = \quad \frac{1}{g^i_f + g^i_c} \left[g^i_f . \ln\left(\frac{\widehat{fv}_t}{p_t}\right) + g^i_c . \bar{r}^i_t \right] \tag{1.7}$$

The weighted forecasted return $(\hat{r}^i_{t,t+\tau^i})$ allows the future expected price to be formulated as follows:

$$\hat{p}_{t+\tau^i} = p_t \exp(\hat{r}^i_{t,t+\tau^i}) \tag{1.8}$$

1.3.3. Asset demand

1.3.3.1. Optimal demand

Once the expected prices $(\hat{p}^1_{t+\tau^i},\ \hat{p}^2_{t+\tau^i})$ are computed, the optimal demand of assets is defined by the maximisation of a constant absolute risk aversion (CARA) utility function under a Gaussian return on assets[b]. The optimal demand can be expressed for asset 1 as:

$$\pi_t^{i,1}(\hat{p}^1_{t+\tau^i}\ , \hat{p}^2_{t+\tau^i}) = \frac{\ln\left(\dfrac{\hat{p}^1_{t+\tau^i}}{p^1_t}\right)}{\alpha^i\ p^1_t\ Var^i_1} \tag{1.9}$$

and for asset 2 as:

$$\pi_t^{i,2}(\hat{p}^1_{t+\tau^i}\ , \hat{p}^2_{t+\tau^i}) = \frac{\ln\left(\dfrac{\hat{p}^2_{t+\tau^i}}{p^2_t}\right)}{\alpha^i\ p^2_t\ Var^i_2} \tag{1.10}$$

which is also the computation of the optimal demand in Yamamoto's paper (2011). The variance (Var^i_j) denotes the investment risk of the asset. It is

[b] See appendix for more details (*section 1.6.1*)

assumed to be equal to the variance of the logarithm of the return rate.

$$Var_j^i = \frac{1}{\tau^i} \sum_{k=1}^{\tau^i} [r_{t-k}^j - \bar{r}_t^{i,j}]^2 \tag{1.11}$$

1.3.3.2. Submission quantity

The amount of assets ($s_t^{i,j}$) the agent is willing to trade is determined by the absolute difference in the optimal demand for asset j at time t and $t-1$ as:

$$s_t^{i,j} = |\pi_t^{i,j} - \pi_{t-1}^{i,j}| \tag{1.12}$$

The sign of this difference ($\pi_t^{i,j} - \pi_{t-1}^{i,j}$) gives the agent's position: buying if it is positive, selling if it is negative, otherwise doing nothing.

1.3.3.3. Mood and submission price

The agent's mood is used in the definition of the submitted bid ($b_t^{i,j}$) and ask ($a_t^{i,j}$) which are different from the expected future price ($\hat{p}_{t+\tau^i}^{i,j}$).

$$b_t^{i,j} = \hat{p}_{t+\tau^i}^{i,j}(1 + M_t^{i,j}) \tag{1.13}$$
$$a_t^{i,j} = \hat{p}_{t+\tau^i}^{i,j}(1 + M_t^{i,j}) \tag{1.14}$$

In an optimistic mood ($M_t^{i,j} > 0$), the agent agrees to buy ($b_t^i > \hat{p}_t^i$) or sell ($a_t^i > \hat{p}_t^i$) at a higher price. She believes that the asset price will continue to increase. The buyer expects to make a profit on the future sell and the seller expects to earn an additional premium ($a_t^i - \hat{p}_{t+\tau^i}^i$). Conversely, when a trader is in a pessimistic mood ($M_t^{i,j} < 0$), she submits a lower bid (ask) than she predicted. Prices should fall, thus she is trying to reduce her loss.

1.3.4. Order-placement strategy and state of the order-book

Once the submission price and quantity are defined, the trader needs to choose a type of order. She chooses a market order (MO) when her defined bid (ask) is above (below) the best quoted ask (bid). Otherwise, she chooses a limit order (LO).

At this stage, traders have two means of submitting an order, with and without order-placement strategy. Without order-placement strategy, the trader submits her order at the previously defined bid or ask (Table 1.1). With order-placement strategy, using the visible part of the order-book, the trader solve the trade-off between accepting non-execution risk and paying a bid-ask spread. In this

Table 1.1.: Order definition before/without placement strategy.

Volume	Position	Order price	Order type[1]	
$\pi_t^{i,j} - \pi_{t-1}^{i,j} > 0$	buy	$b_t = \hat{p}_{t+\tau^i}(1 + M_t^{i,j})$	$b_t > a^q$	$\Leftrightarrow$ MO
			$b_t \leq a^q$	$\Leftrightarrow$ LO
$\pi_t^{i,j} - \pi_{t-1}^{i,j} < 0$	sell	$a_t = \hat{p}_{t+\tau^i}(1 + M_t^{i,j})$	$a_t > b^q$	$\Leftrightarrow$ MO
			$a_t \leq b^q$	$\Leftrightarrow$ LO
$\pi_t^{i,j} - \pi_{t-1}^{i,j} = 0$	do nothing	$\varnothing$	$\varnothing$	$\varnothing$

[1] b^q and a^q are the best quoted bid and ask at time t.

market, the submitters have access to the five best quoted bids and asks[c]. She computes the order imbalance x_t^{ob} where $ob = \{bid; ask\}$ is the trader's position and x^{bid} (x^{ask}) is the log difference of the current depth of the five best bids (asks) from the current depth of the best asks (bids). It is calculated by the buyer (seller) for each asset.

$$Prob_t^{i,j} = \tanh(\beta^{i,j} * |x_t^{ob,j}|) \tag{1.15}$$

With probability $Prob_t^{i,j}$, a trader plans to place a more aggressive order when the book becomes thicker on the corresponding side than on the other side. More specifically, if b_t^i (a_t^i), defined by Eq. (1.13) (Eq. (1.14)), had suggested that a trader was submitting a LO below (above) the best quoted bid (ask), then she would have changed it for an order at the best quoted bid (ask) price plus (minus) a tick size. The order would have taken first place in the order-book. Similarly, if she had decided to submit a limit bid (ask) order in the bid-ask spread, then she would have chosen to submit a MO (Table 1.2 - Behaviour: More aggressive).

In the same way, a trader plans to place a less aggressive order when the book becomes thinner on the corresponding side than on the other side. If she had chosen to submit a limit buy (sell) order in the bid-ask spread, then she would have decreased (increased) her price submission to the best quoted bid (ask). The non-execution risk is smaller, so she can wait and expect to earn more. If she had decided to submit a market buy (sell) order, then she would have chosen to submit a LO at the best quoted ask (bid) minus (plus) a tick size (Table 1.2 - Behaviour: Less aggressive).

With probability $1 - Prob_t^{i,j}$, the trader will not undercut the best price and will place the order just as in the case without order-placement strategy (Table 1.1).

Table 1.1 summarises the decision-making without (or before) placement

[c] Same amount of observable orders as in Yamamoto (2011). Be aware that each real-life stock exchange has its own degree of transparency.

Table 1.2.: Impact of placement strategy on order definition corps.

Position	Behaviour	Cases[1,2]	New order price[1]	Order type switching[3]
buy	More aggressive	$b_b < b^q$	$b_t = b^q + \Delta$	LO $\rightarrow$ LO
		$b^q < b_b < a^q$	$b_t = a^q + \Delta$	LO $\rightarrow$ MO
	Less aggressive	$b^q < b_b < a^q$	$b_t = b^q$	LO $\rightarrow$ LO
		$a_b > a^q$	$b_t = a^q - \Delta$	MO $\rightarrow$ LO
sell	More aggressive	$a_b > a^q$	$a_t = a^q - \Delta$	LO $\rightarrow$ LO
		$b^q < a_b < a^q$	$a_t = b^q - \Delta$	LO $\rightarrow$ MO
	Less aggressive	$b^q < a_t^b < a^q$	$a_t = a^q$	LO $\rightarrow$ LO
		$a^b > a^q$	$b_t = b^q + \Delta$	MO $\rightarrow$ LO

[1] b^q and a^q are the best quoted bid and ask at time t.
[2] b_b and a_b are the orders defined before/without placement strategy.
[3] [order type before placement strategy] $\rightarrow$ [order type after placement strategy].

Table 1.3.: Parameters.

Number of rounds	$8,000$
Number of runs	200
Number of agents: n	$1,000$
Original Fundamental Values: fv_0^1 and fv_0^2	300
Fundamentalist outlook - exponential law	σ_f^2
Trend-follower outlook - exponential law	σ_c^2
Agent's mood: $M_t^{i,j}$	$\mathcal{U}(-1\%;+1\%)$
Sensitivity to order-book depth: $\beta^{i,j}$	$\mathcal{U}(0;2)$
Reference order life: τ	300
Reference risk aversion: α	0.1 or 0.3
Pricing grid width: Δ	10^{-3}

strategy. Table 1.2 shows the choices available to the agent with placement strategy.

1.3.5. Order submission

Once the orders are fully defined (by a submission price, a quantity and a type), they are submitted and quoted in the order-books. The orders are executed following price/time priority. Let's not forget that a submitting order which enables an immediate exchange is necessarily a LO equal to the best counterpart or a MO (see Table 1.1). As a result, the exchange occurs at the already best quoted price. As in reality, if the order submitted is longer than the best counterpart, the remaining volume is executed against other quoted LOs. In the case of MO submissions, if there are not enough quoted counterparts, the remaining volume

Table 1.4.: The 4 types of market compared in simulation (parameters that differ are indicated).

<table>
<tr><th></th><th>Without order-placement strategy</th><th>With order-placement strategy</th></tr>
<tr><td rowspan="2">Perfect knowledge</td><td colspan="2">$$\widehat{fv}_t^i = \widehat{fv}_t = f_t \quad ,\forall i$$</td></tr>
<tr><td>• $\beta^{i,j} = 0$
• Submitting price at the expected forward price ($\hat{p}^{i,j}_{t+\tau^i}$)</td><td>• $\beta^{i,j} = \mathcal{U}(0; \beta_{max})$
• Submitting price according to order-book depth: more or less aggressive order</td></tr>
<tr><td rowspan="2">Adaptive learning</td><td colspan="2">$$\begin{aligned}\widehat{fv}_t^i &= 0.58 p_{t-1} + 0.4 \widehat{fv}_{t-1}^i + 0.02 \widehat{fv}_0^i \\ &\quad + N_t + 0.98(\widehat{fv}_{t-1}^i - \widehat{fv}_{t-2}^i - N_t) \\ &\quad + 0.005\ (fv_{t-1} - \widehat{fv}_{t-1}^i) \\ \widehat{fv}_t^i &\neq \widehat{fv}_t^j\end{aligned}$$</td></tr>
<tr><td>• $\beta^{i,j} = 0$
• Submitting price at the expected forward price ($\hat{p}^{i,j}_{t+\tau^i}$)</td><td>• $\beta^{i,j} = \mathcal{U}(0; \beta_{max})$
• Submitting price according to order-book depth: more or less aggressive order</td></tr>
</table>

is executed as new orders are submitted. If the submitted bid (ask) is smaller than the best quoted ask (bid), then the order remains in the order-book as a LO at its limit price. Once the order is quoted in the order-book, it cannot be modified. Either it is removed when it is executed or it remains unexecuted for its lifetime (τ^i).

1.4. Simulation and Results

In order to study the model, we use computer-based simulations. We simulate the model using Monte Carlo simulations (Salle and Yildizoglu, 2012). While we report results only for a specific set of parameters (Table 1.3), we run the model for a wide range of values of σ_f^2, σ_c^2 and $M_t^{i,j}$ in order to study the influence of different ratios between fundamentalist and trend-follower outlooks, and the mood parameter. Each simulation run consists of $9,000$ rounds. Since every market simulation begins with an empty limit order-book, we discard the first $1,000$ rounds and analyse the data from the remaining $8,000$ rounds. In order to gauge how the bounded rationality affects the collective behaviour of heterogeneous traders and what emergent market properties they generate, we replicate each simulation 200 times and compute average estimates of the key market statistics.

We analyse exchange and price dynamics as a function of agents' (σ_f^2, σ_c^2) and FV's characteristics ($\sigma^2_{fv_1}$,$\sigma^2_{fv_2}$). We consider two indicators of exchange dynamics:

a) exchange volume per simulation;

b) the largest exchange volume realised in one round;

and four indicators of price dynamics:

a) trading price;

b) asset volatility (price variance);

c) "fundamental spread", *i.e.* the spread between the fundamental and the trading price;

d) "learning spread", *i.e.* the spread between the true and the estimated fundamental value.

The fundamental spread reflects the time that the market structure takes to factor in traders' information, while the learning spread reflects the time that the fundamentalist outlook takes to factor information on the fundamental into its expectation.

1.4.1. Perfect knowledge market

In this subsection, we show that our model reproduces stylised facts of classical ABM literature (Lux, 1995; Brock and Hommes, 1998; Westerhoff, 2004; Chiarella et al., 2007) in a setup with fundamentalist and trend-follower outlook and perfect knowledge of fundamentals.

1.4.1.1. Bubbles boom and burst

When agents have only fundamentalist outlook, strong market efficiency is never verified. The fundamental spread tends to zero, but it is still weakly positive. This weak efficiency might be easily explained by the heterogeneous investment time horizon of the agents, even though they have the same outlook. Submitted in the past and quoted in the order-book until its execution or the end of its lifetime, the order-book might contain orders which are not equal to the actual fundamental value and generate noise in the trading price. The more the reference time horizon is small, the more the market becomes strongly efficient (the trading price equals the fundamental value). The market structure, by the absence of market clearing mechanism, is responsible for market mispricing. After a while, the market becomes illiquid when it is made of only fundamentalist outlook. All agents converge to similar expectations and stick to the same way of the order-book.

The same simulations with agent's mood yield a more volatile and far less efficient market, which never becomes illiquid. We observe a wide oscillation of the trading price around its fundamental. Whatever the original fundamental value, the fundamental spread evolves along a path of $\pm 20\%$[d]. This inefficiency is strongly linked to the agent's mood, which directly impacts the submitted price. However, the market price reflects fundamental trends. The fundamentalists perceive this movement and update their expectations according to the news. We assume this market at least weakly efficient.

Even though trend-follower outlook has a destabilising impact on the market, the market never diverges when a fundamentalist trend exists. Endogenous bubbles boom and burst, glaring disconnections between market and fundamental prices happen. The fundamental spread becomes larger and asset volatility increases as trend-follower outlook rises. Although the trading price less frequently exceeds the fundamental value, it is on average more overvalued! This can be explained by the positive (fundamental) spreads being much larger than the negative spreads, and by extreme cases which do not counterbalance each other. Furthermore, the qualitative patterns emerging from the simulation differ, with much less predictability than when there is no trend-follower outlook ($\sigma_c^2 = 0$).

[d] 95% confidence interval

Table 1.5.: Volume and volatility ($\sigma^2_{fv} = 0.2$) with fundamentalist outlook (indicated for different degrees: $\sigma^2_f = 0.1, 0.6, 1, 10$). The mean and the standard deviation (in brackets) are reported.

	$\sigma^2_f = 0.1$	$\sigma^2_f = 0.6$	$\sigma^2_f = 1$	$\sigma^2_f = 10$
Vol. per simu.	710	592	554	359
	(56)	(92)	(70)	(46)
Max. vol./step	11.3604	11.0281	10.9476	9.6282
	(6.23)	(9.17)	(6.25)	(6.98)
Price variance	669	695	710	754
	(205)	(227)	(245)	(245)

Market prices follow a cyclical process, bubbles have greater amplitude and take more time to burst as the trend-follower outlook increases. Fundamentalist outlook makes numerous price adjustments so as to trade at the fundamental value and trend-follower outlook easily observes the most persistent trends, which it amplifies. This is in line with the reinforcement dynamics of trend-follower outlook and its destabilising impact (Hommes et al., 2005b, 2007; O'Hara and Easley, 1995; Pouget, 2000).

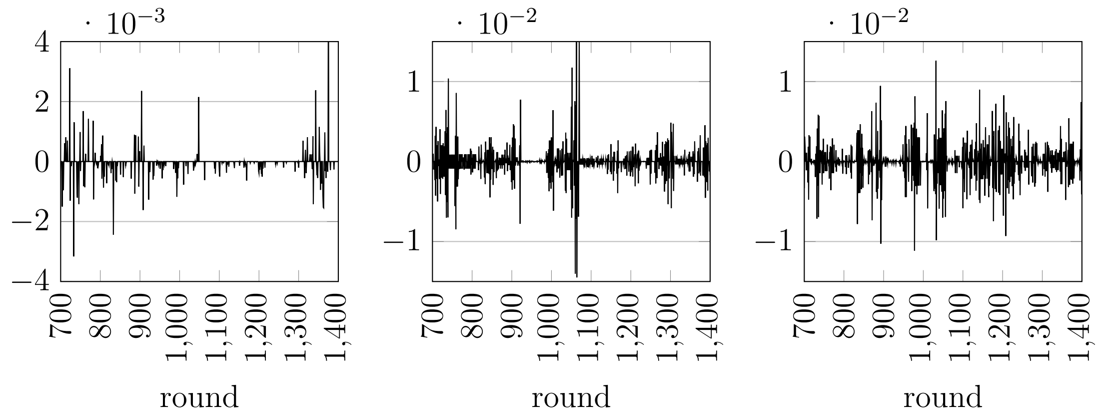
1.4.1.2. Impatience to sell and order types

Whatever the weight given to fundamentalist and trend-follower outlook, "impatience" to sell is still present. Indeed, the frequency of MO submissions – *i.e.* the ratio of submitted MOs to total submitted orders – is greater on the ask side than on the bid side. When the weight of trend-follower outlook increases, the frequency of MO submissions decreases, converging at 22% to sell and 20% to buy. These decreasing frequencies are consistent with O'Hara and Easley (1995), who point out that the trend-followers are the liquidity suppliers, submitting LOs, and that the fundamentalists, due to their informational advantage, capture this liquidity by submitting MOs.

1.4.1.3. Link between volume and volatility

Increasing the weight of fundamentalist outlook (σ^2_f) makes traders "greater fundamentalists" in the sense that, on average, they become more risk-averse and longer-term investors. We simulate trading runs for increasing value of σ^2_f, and observe significant decreasing exchange volume and non-significant increasing market variance (Table 1.5). This model is able to replicate the finding of Beja and Goldman (1980): increasing the fundamentalist outlook in a previously stable system makes it less stable.

When agents have only trend-follower outlook, increasing σ^2_c from 0.1 to 10 makes the traders less risk-averse and shorter-term investors. As a result, the



(a) $\sigma_f^2 = 0.6$ (b) $+$ Mood (c) $+$ Mood $+ \sigma_c^2 = 10$

Figure 1.1.: Return time series ($\sigma_{fv}^2 = 0.2$) with (a) fundamentalist outlook, (b) fundamentalist outlook with mood and (c) fundamentalist and chartist outlook with mood.

trading volume rises from 278 to 908. The largest exchange volume realised in a round is at least twice higher than with only fundamentalist outlook (26 *versus* 11). Be aware that when traders are defined as a mix between fundamentalist and trend-follower outlooks, the trading volume is around 330 (setup: $\sigma_f^2 = 0.6/\sigma_c^2 = 1/\sigma_{fv}^2 = 0.2$).

Our simulation is also consistent with empirical data (Amihud et al., 2005), with decreasing trading volume implying increased market price volatility: fewer assets are quoted and traded on the market, the bid-ask spread rises, so that price movements are larger. In a setup with fundamentalist and trend-follower outlook, an increase in the weight of trend-follower outlook (from $\sigma_c^2 = 0.6$ to 1, with $\sigma_f^2 = 0.6$) doubles price variance and decreases trading volume by 10%.

Concerning volatility, our model generates two main dynamics that alternate in a cyclical manner. We distinguish a short period of strong price variation and a longer period of weak variation. This is consistent with the volatility clustering stylised fact[e]. Fig. 1.1 displays the snapshots of the return series in one of the runs for fundamentalist outlook without mood parameter, fundamentalist outlook with mood parameter, and fundamentalist and chartist outlook with mood parameter, respectively. The changes in returns look clustered in each market.

1.4.1.4. A convincing model

According to earlier works on the effect of trend-followers – and more broadly chartists – the presence of both fundamentalist and trend-follower outlook is

[e]This stylised fact was identified by Chiarella et al.'s and Yamamoto's models and explained by the interaction between fundamentalist and chartist analysts.

crucial for generating excess volatility of asset prices (Beja and Goldman, 1980; Brock and Hommes, 1998; Chiarella, 1992; Hommes et al., 2005a; Lux, 1995). In our model, the presence of bounds in agent's rationality such as trend-follower outlook and mood is crucial for generating long-term price distortion and increasing the asset volatility. In addition, increasing the fundamentalist outlook may make the market efficient while the trend-follower outlook has a destabilising impact, as noted in Hommes (2006). The trend-follower outlook increases the maximum amount of assets exchanged in a round. However, it unexpectedly decreases the trading volume per simulation. The fall in liquidity seems counter-intuitive to usual findings (Barber and Odean, 2000; Shiller, 2003) but might be explained by the rise in volatility. The model is able to produce bubbles boom and burst. The mood parameter noises the market price, while maintaining the main fundamental trends and bringing up the market liquidity. In some way, our agent's mood replaces the classic noise trader in order to prevent illiquid markets by submitting orders different from the fundamental expectation (Bloomfield et al., 2009). Increasing the fundamentalist outlook may destabilise an already stable market (Beja and Goldman, 1980).

In the absence of market clearing mechanism, the already quoted prices might add noise in the new market price. The market structure is responsible for the lack of strong efficiency, even when agents have only fundamentalist outlook. However it allows to track the impatience to sell of traders (O'Hara and Easley, 1995) and to work on liquidity. Our model also replicates the link between trading volume and market volatility (Amihud et al., 2005).

We pointed out the similarities between our model and the classical ABM results. From now, we have an interesting market structure with which to work on the impact of agent's learning and order-placement strategy on trading volume and price distortion. We already highlight that 1/the market distortion is impacted by the trend-follower outlook and the investment horizon due to the absence of clearing market mechanism; 2/ the trend-follower outlook brings the liquidity by quoting orders; 3/ the agent's mood adds noise in the trading price without obscuring the main fundamental trends and brings up liquidity.

1.4.2. Adaptive learning market

In this subsection, we focus on changes in price and exchange dynamics when the assumption of perfect knowledge of the fundamental value is relaxed. We study the impact of volatility of fundamentals and memory of the correction mechanism on the fundamentalist's expectation, and then focus on the impact of trader's order-placement strategy.

Table 1.6.: Comparison among perfect knowledge of the fundamental value, adaptive learning and adaptive learning with correction mechanism in a fundamentalist market ($\sigma_f^2 = 0.6$). The mean and the standard deviation (in brackets) are reported.

		Perfect knowledge	Adapt. learning	Adapt. learning + corr. mechanism
Asset 1 $\sigma_{fv}^2 = 0.2$	Fundamental spread	2.290	2.833	1.183
		(1.805)	(10.531)	(4.579)
	Var. funda. spread	103.421	130.453	88.915
		(29.342)	(43.556)	(13.132)
	Learning spread	0	1.082	0.026
		(xxx)	(10.391)	(4.324)
	Var. learn. spread	0	43.793	7.269
		(xxx)	(37.735)	(4.780)
	Vol. per simu.	470	463	457
		(30)	(25)	(22)
	Price volatility	145.806	89.767	99.111
		(58.935)	(17.043)	(21.385)
Asset 2 $\sigma_{fv}^2 = 1$	Fundamental spread	1.867	2.196	1.673
		(2.647)	(62.515)	(23.140)
	Var. funda. spread	144.189	1284.129	295.332
		(68.039)	(971.632)	(174.858)
	Learning spread	0	0.083	0.761
		(xxx)	(62.598)	(22.341)
	Var. learn. spread	0	1190.739	199.269
		(xxx)	(968.414)	(160.356)
	Vol. per simu.	747	467	555
		(200)	(77)	(115)
	Price volatility	1252.783	90.918	572.295
		(1114.444)	(19.565)	(473.826)

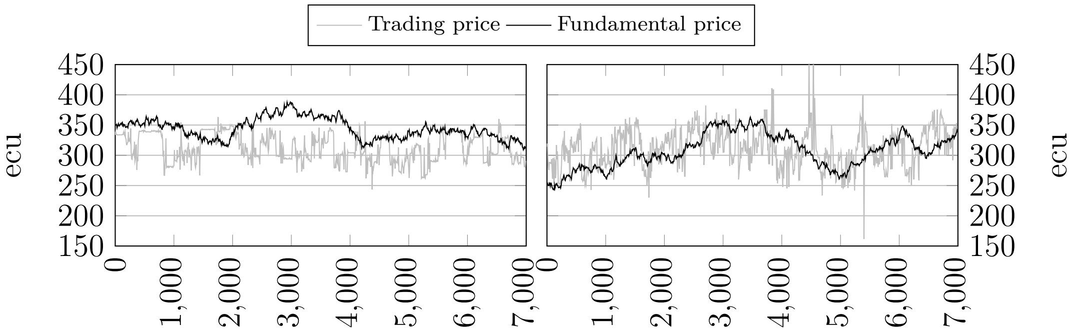
1.4.2.1. Volatility of fundamentals

If each original anchor is generated close to the initial true fundamental value and the true fundamental value has a low variance ($\sigma^2_{fv} = 0.2$; Table 1.6 - asset 1), the introduction of adaptive learning does not decrease market efficiency so greatly. The market price is more stable (smoother oscillation, lower volatility) and the runs are more similar (decreasing standard error of trading price and price volatility). However, the fundamental value is misestimated (positive learning spread and huge standard deviation[f]). The learning spread evolves in the interval $[-32; 26]$ for a fundamental value around 300 economic currency units. As a result, even though on average the learning spread is 1.082, its standard error is ten times higher. Traders are reluctant to change their minds, resulting in a loss of efficiency and a gain in market stability. The trading volume stays stable, around 463.

If each original anchor is generated close to the initial true fundamental value and the true fundamental value has a high variance ($\sigma^2_{fv} = 1$; Table 1.6 - asset 2) or if the original anchors are not consistent with the original fundamental value, the market price will not reflect the fundamental at all (see Var. funda. spread and Var. learn. spread). There is less price adjustment than in perfect knowledge case. The trading volume and the price volatility fall down, from 747 to 467 and from 1252 to 90, respectively. Both market indicators are independent of the variance of the true fundamental value in the adaptive learning setup. The result is an inefficient market with a stable market price which evolves around the original aggregate fundamental expectation ($\sum_{i=1}^{n} \widehat{fv}_0^i / n$).

In this market, where the outlook is only fundamentalist, we showed that persistent bubbles can be generated provided knowledge of the fundamentals is not perfect. Adding trend-follower outlook to our previous setup (see table 1.9 in *appendix*) increases price volatility and the variance of the fundamental spread. The average price and the learning spread keep the same ratio of standard error. With adaptive learning, therefore, market efficiency is dependent on the agent's beliefs, the volatility of the true fundamental value and the anchor. In any case, our adaptive learning makes runs more similar, and the standard deviation of the average price is at least twice as low as in the same setup with perfect knowledge. With the adaptive learning setup, the market volatility converges towards a case with perfect knowledge of the fundamental value and with low variance ($\sigma^2_{fv} = 0.2$). As a consequence, the market liquidity becomes more stable. In the specific case of adaptive learning without order-placement strategy, the ratio of submitted MOs is 20% to buy and 22% to sell.

[f] In the case of perfect knowledge, we assume that traders are perfectly informed: the estimated fundamental value is equal to the true one, see Eq. (1.5). Thus, by assumptions the learning spread is null.



(a) $\psi = 0.005$ (b) $\psi = 1$

Figure 1.2.: Impact of the adaptive learning on the market efficiency with (a) slow and (b) fast correction mechanism ($\sigma_f^2 = 0.6$, $\sigma_{fv}^2 = 1$).

1.4.2.2. Memory of the correction mechanism

One way to correct misunderstanding of changes in the fundamentals is to adjust parameter ψ, the memory of the correction mechanism defined in Eq. (1.6). Originally set at 0.005, increasing ψ to 1 makes traders update their expectations quickly. The result is a more informative and volatile price (see Table 1.6, adapt. learning + corr. mechanism). The learning spread and the fundamental spread are not necessarily lower than in the classic adaptive learning case. However, the standard deviation is far smaller. The trading volume rises from 467 to 555, in case of adaptive learning with high volatility of fundamentals, and stays stable in case of low volatility of fundamentals. The effect of the anchor is weaker, but still present (Fig. 1.2). When the true fundamental value falls, the trading price also decreases, but with a lag.

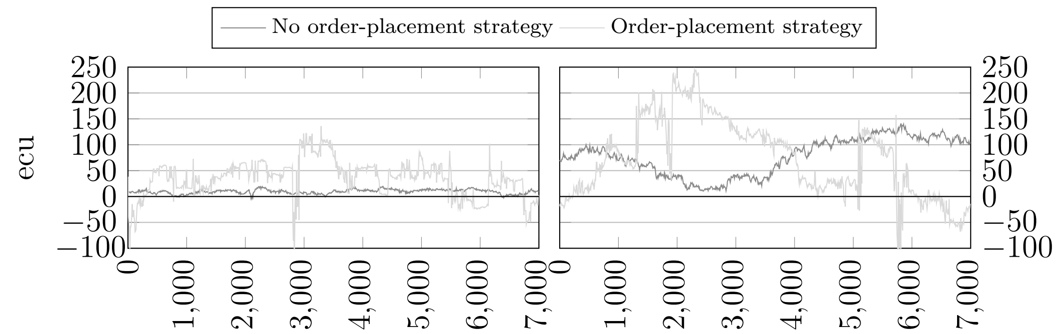
1.4.2.3. Order-placement strategy

In the real world, traders take into account market depth and adapt their orders so as to maximise their execution probability and their profit.

With $\beta = 2$, agents are strongly influenced by order-book depth. The result is a deeper market. The exchange volume per simulation is (on average) multiplied by 1.67 compared to the same market without an order-placement strategy. The extreme values of the largest exchange volume realised in one round are also hugely impacted (at least doubled). However, market efficiency is negatively impacted: the fundamental spread increases (Table 1.7). It is multiplied by a factor of 1.5 in the best case and 3.5 in the worst case. The price remains far from the fundamental value for a relatively long period (Fig. 1.3a). The bubbles take at least twice as long to burst as when there is perfect knowledge. In the specific case of adaptive learning with order-placement strategy, on average 16% of MOs are submitted to buy and 19% to sell – no matter what the proportion of fundamentalist to trend-follower outlook.

Table 1.7.: Comparison among adaptive learning and adaptive learning with order-placement strategy in a market with fundamentalist and chartist outlook ($\sigma_f^2 = 0.6$ / $\sigma_c^2 = 1$). The mean and the standard deviation (in brackets) are reported.

		Adapt. learning	Adapt. learning + order plac. strat.
Asset 1 $\sigma_{fv}^2 = 0.2$	Vol. per simu.	319.487 (17.494)	535.259 (66.479)
	Max. vol./step	5.399 (4.269)	17.430 (14.185)
	Fundamental spread	1.599 (11.341)	3.338 (11.773)
	Var. funda. spread	163.434 (49.634)	188.176 (89.354)
	Price volatility	124.306 (29.038)	146.972 (67.066)
Asset 2 $\sigma_{fv}^2 = 1$	Vol. per simu.	328.571 (55.310)	668.641 (224.133)
	Max. vol./step	8.633 (5.593)	28.14837 (17.18333)
	Fundamental spread	3.781 (59.748)	11.2396 (60.71573)
	Var. funda. spread	1266.342 (913.671)	11471.484 (1280.870)
	Price volatility	135.575 (37.649)	228.022 (129.310)



(a) Asset 1: $\sigma_{fv_1}^2 = 0.2$ (b) Asset 2: $\sigma_{fv_2}^2 = 1$

Figure 1.3.: Impact of order-placement strategy on fundamental spread with adaptive learning ($\sigma_f^2 = 0.6$ and $\sigma_c^2 = 1$).

1.4.2.4. Discussion

To assign bounded rationality to our agents, we relaxed the strong assumption of perfect knowledge of the fundamentals. Now, the trend-follower outlook is not crucial for generating long-term price distortion. The information processing (learning fundamental value) enables belief perseverance and persistent bubbles to be produced even with only fundamentalist outlook. The misunderstanding of changes in the fundamental can also lead the market with more information. When the true fundamental value has a low variance and the anchor is generated close to the initial true fundamental value, the lack of knowledge has a stabilising impact on price dynamics. Some shocks in fundamentals are unperceived, so the trading price oscillates less (decreasing volatility) and over a smaller fundamental spread. The trading price reflects the main changes in the fundamental, with less noise. The adaptive learning stabilises the market volatility and thus the market liquidity. The volumes converge towards the case with perfect knowledge of the fundamental value with low variance.

As seen in the description of the model, we do not adjust the memory of the correction mechanism ψ (see Eq. 1.6) so as to keep in line with bounded rationality of agents. People begin with a starting value, either supplied to them or generated by them, and insufficiently adjust their estimates around this anchor (Barberis and Thaler, 2003; Fischoff et al., 1977; Simon, 1982; Tversky and Kahneman, 1974). Market efficiency is thus strongly linked to trend-follower outlook, learning FV, agents' anchor and the volatility of the true fundamental value.

The information processing of the agent is also composed by the state of the order-book and the order-placement strategy. The arbitrage that each agent makes: paying for a higher bid-ask spread or taking the risk of non-execution, produces bubbles more persistent. When agents are sensitive to the order-book depth, the market price never reflects its fundamentals. The standard deviation of volatility increases three-fold (Table 1.7), and the standard deviation of the trading price doubles, indicating that different patterns are emerging from the simulations. This is no doubt due to the impact of market depth on price valuation, which evolves at each round, and which is enhanced by trend-follower outlook. In this context, the trend-follower's destabilising capacity is less marked, but still present. See the volatility in cases of $\sigma_c^2 = 1$ (Table 1.7) and of $\sigma_c^2 = 10$ (Table 1.10 in *appendix*, page 48 and following). In addition to this fall in market efficiency, there is a huge improvement in trading volume. The extreme values of the largest exchange volume realised in one round are more than double compared to the same simulation without order-placement strategy. From a certain point of view, this inefficiency (fundamental spread higher and bubbles duration longer with no change in the learning spread) corresponds to the cost of the "ease of finding a counterpart" which is not taken into account in our definition of the fundamental value.

The frequency of MO submissions is not impacted by the learning fundamental. This result is consistent with the literature: the ratio of submitted MOs is dependent on the informational advantage, not on the information *per se*. There is no informational advantage either in perfect knowledge or in adaptive learning. In the first case everybody knows the true fundamental value, while in the second case nobody knows it. In the specific case of adaptive learning with order-placement strategy, the impatience to sell is still observed, but the ratio of submitted MOs is lower. This fall is explained by agents taking into account market depth, and adapting their orders: the more liquid the market, the less risk they take and the fewer MOs are submitted. In reality, the frequency of MO submission is between 28% and 53%. Since we do not wish to reproduce the stylised fact of submission frequency, the fact that real investors submit more aggressive orders than our agents do, is not an issue.

By adding bounded rationality through the information processing, we showed that 1/the learning fundamental value stabilises the market price and the trading volume; 2/ the order-placement strategy hugely increases the market liquidity and makes the market inefficient. The difference between the fundamental spread and the learning spread might include the price of liquidity. Table 1.8 summarises the impact of information processing on the market indicators.

The order-placement strategy raises a question about providing public or private information. Even though traders compute the fundamental value properly, the submitting order reflects a part of the public information driven by the observable part of the order-book. This collected information is not necessarily relevant in the sense that it does not reflect the agent's expectation. However it may drive fear. Thus we may assume that the incentive to find private information will be offset because it will not appear in the trading price. So as to reassure the market and correct its misunderstanding, the best way seems to provide information from the order-book.

1.5. Conclusion

We built a call auction market, where two independent risky assets are traded, in order to study how is the rationality of agents bounded by their information access and their information processing? It was made clear that agents type, learning fundamental value and collected information from the order-book such as best quoted prices and depth increase the market distortion. Indeed, fundamentalist outlook makes the market converge to its computed fundamental price, while trend-follower outlook makes it diverge. The trend-follower outlook amplifies the fundamentalists' trend and makes long life-duration bubbles. It is also responsible for a fall in liquidity due to its impact on the price variance. The mood parameter slightly impacts the price oscillation and brings up the

Table 1.8.: Impact of each bounded rational feature.

Bounded rationality elements	Parameter	Impact
Trend-follower outlook	$\nearrow \sigma_c^2$	• $\searrow$ market efficiency • has destabilising impact • $\nearrow$ trading volume without funda. outlook • $\searrow$ trading volume with funda. outlook • generates bubbles burst and boom • $\nearrow$ max. volume per step
Agent's mood	$+ M_t^{i,j}$	• prevents illiquid market • weakly $\nearrow$ market volatility • no impact on impatience to sell • keeps the fundamental trend
Investment horizon	$\nearrow \tau^i$	• avoids empty order-book • avoids strong market efficiency
Agent's learning	$+ \widehat{fv}_t^i$	• generates misestimation in the fundamental value • generates bubbles boom without trend-follower outlook • stabilises price volatility and thus trading volume • no impact on ratio of submitted MOs
Memory of the correction mechanism	$\nearrow \psi$	• fixes the misunderstanding change in fundamentals • $\searrow$ learning spread • $\nearrow$ market efficiency • $\nearrow$ trading volume for price adjustment • goes against bounded rational evidences
Order placement strategy	$\nearrow \beta^{i,j}$	• $\searrow$ market efficiency • $\nearrow$ bubble duration • $\nearrow$ fundamental spread • $\nearrow$ extreme values of the largest exchange volume realised in one round • $\searrow$ ratios of submitted MOs • impatience to sell still present

$\nearrow$: increasing | $\searrow$: decreasing | $+$: adding market liquidity. In some way, our agent's mood replaces the classic noise trader in order to prevent illiquid markets. Relaxing the strong assumption of perfect knowledge of fundamentals enables belief perseverance and persistent bubbles to be produced even with only fundamentalist outlook. The analysis of adaptive learning leads to identification of main sources of bubbles: some attributed to the fundamentalist outlook, based on misunderstanding change in fundamentals and/or some generated by the trend-follower outlook, based on trend-extrapolation. The learning fundamental value makes the market more stable. Its efficiency depends on variation of fundamentals. The order-placement strategy makes bubbles more persistent in return for its rise in liquidity. This parameter should allow the asset's liquidity to be priced. At this stage, we do not perform econometric tests. We simply observe stylised facts and focus exclusively on trading volume, a component of the liquidity.

The contribution of our model lies in this combination of features that makes it applicable to trading volume and market efficiency in an artificial market. We have added several complex elements to the usual framework. In particular, imperfect knowledge of the fundamental value appears to produce the dynamics that we were expecting, which means that this framework can be used for further investigations.

From now, we have an interesting market structure with which to work on numerous indicators of liquidity (indicators of market impact and market resilience - Acharyaa and Pedersen, 2005). The development of this multi-risky assets structure is a part of a larger aim which is to study the impact of estimated comovement of fundamentals on the spot prices and the effectiveness of portfolio allocation. Actually, some evidence suggests that the gain by diversification is more than offset by estimation errors (DeMiguel et al., 2009). We also intend to explore new ways for the agent to obtain information, and in particular to examine the impact of an Experience-Weighted Attraction learning (Camerer and Ho, 1999) on the expected fundamental value and the impact of a genetic algorithm on the trend-follower outlook. We also wish to explore the importance of belief and the potential for the self-fulfilling prophecy, for example by identifying a correlation between the two asset returns when the fundamentals are independent and *vice versa*.

1.6. Appendix

1.6.1. Demand function

Each trader has its own demand function and decides the amount of wealth she would like to invest in the risky assets. The residual wealth is invested in a bond with zero interest rate. To facilitate reading of the appendix, we drop the agent

superscript i from all variables.
The utility function is defined as a constant absolute risk-aversion class, where α is the risk aversion coefficient of the trader, and W_t the trader wealth at time t.

$$U(W, \alpha) = -\exp(-\alpha W) \tag{1.16}$$

The wealth is given by a cash amount C_t and the quantity of the risky assets z_j^t held times their spot prices P_j^t.

$$W_t = z_1^t.P_1^t + z_2^t.P_2^t + C_t \tag{1.17}$$

The wealth at time $t + \tau$ (assuming the order is executed at a price P_j before $t + \tau$) is given by:

$$\begin{aligned} W_{t+\tau} &= z_1^{t+\tau}.P_1 + z_2^{t+\tau 2}.P_2 + C_{t+\tau} && (1.18) \\ &= W_t + s_1^t.P_1.r_1^{t+\tau} + s_2^t.P_2.r_2^{t+\tau} && (1.19) \end{aligned}$$

where $r_{t+\tau} = p_{t+\tau}/p_t - 1$ is the return from t to $t + \tau$. As a zero order approximation, the agent's expectations on future returns are taken to be Gaussian where future return is assumed to be $r_{t+\tau} = p_{t+\tau}/p_t - 1 \simeq ln(p_{t+\tau}/p_t)$. Based on her knowledge, the trader tries to maximise at time t her forward utility ($U_{t+\tau}$):

$$\max_{W_{t+\tau}} \mathbb{E}_t[U(W_{t+\tau}, \alpha)] = \max_{W_{t+\tau}} \mathbb{E}_t[-exp(-\alpha.W_{t+\tau})] \tag{1.20}$$

Because the utility is exponential and the returns are assumed to be Gaussian, the utility may be expressed as:

$$\hat{U}(W_{t+\tau}, \alpha) = -\exp\Big(-\alpha.\mathbb{E}_t[W_{t+\tau}] + \alpha^2.\sigma_t^2[W_{t+\tau}]/2\Big) \tag{1.21}$$

where the risk of investment is assigned its variance. The expected wealth and the variance are:

$$\mathbb{E}_t[W_{t+\tau}] = W_t + s_1^t.p_1\mathbb{E}_t[r_1] + s_2^t.p_2\mathbb{E}_t[r_2] \tag{1.22}$$

$$\sigma_t^2[W_{t+\tau}] = s_1^t.{p_1}^2.\sigma_t^2[r_1] + s_2^t.{p_2}^2.\sigma_t^2[r_2] + 2.s_1^t.s_2^t.\sigma_t[r_1; r_2] \tag{1.23}$$

Eq. (1.21) could be rewritten as:

$$\begin{aligned} \hat{U}_{t+\tau} = \hat{U}_t \quad &\times \quad \exp\Big(- \alpha\Big[s_1 p_1 \mathbb{E}_t[r_1] + s_2 p_2 \mathbb{E}_t[r_2]\Big] \\ &+ \quad \frac{\alpha^2}{2}\Big[{s_1}^2 {p_1}^2 \sigma_t^2[r_1] + {s_2}^2 {p_2}^2 \sigma_t^2[r_2] + 2.s_1^t.s_2^t.\sigma_t[r_1; r_2]\Big]\Big) \qquad (1.24) \end{aligned}$$

Differentiating the expected utility function (1.24) with respect to s_j^t gives:

$$\begin{cases} \frac{d}{ds_1^t}\hat{U}_{t+\tau} = -\hat{U}_{t+\tau}\left[\alpha p_1^t \ln\left(\frac{p_1^{t+\tau}}{p_1}\right) - \alpha^2 s_1^t (p_1)^2 \sigma_t^2[r_1] - 2s_2^t \sigma_t[r_1;r_2]\right] \\ \frac{d}{ds_2^t}\hat{U}_{t+\tau} = -\hat{U}_{t+\tau}\left[\alpha p_2^t \ln\left(\frac{p_2^{t+\tau}}{p_2}\right) - \alpha^2 s_2^t (p_2)^2 \sigma_t^2[r_2] - 2s_1^t \sigma_t[r_1;r_2]\right] \end{cases} \tag{1.25}$$

Setting the expression to zero, we determine the optimal number of stocks ($S_t^* = \pi(p)$) that the agent wishes to hold in her portfolio for a given price level p.

$$\begin{cases} \pi_t^1(\hat{p}_1,\hat{p}_2) = \dfrac{\alpha.p_2\Big[\ln\left(\dfrac{\hat{p}_1}{p_1}\right)\alpha^2.p_1.p_2.\sigma^2[r_2] - 2\ln\left(\dfrac{\hat{p}_2}{p_2}\right)\sigma[r_1;r_2]\Big]}{\alpha^4.p_1^2.p_2^2.\sigma^2[r_1].\sigma^2[r_2] - 4\sigma^2[r_1;r_2]} \\ \pi_t^2(\hat{p}_1,\hat{p}_2) = \dfrac{\alpha.p_1\Big[\ln\left(\dfrac{\hat{p}_2}{p_2}\right)\alpha^2.p_1 p_2.\sigma^2[r_1] - 2\ln\left(\dfrac{\hat{p}_1}{p_1}\right)\sigma[r_1;r_2]\Big]}{\alpha^4.p_1^2.p_2^2.\sigma^2[r_1].\sigma^2[r_2] - 4\sigma^2[r_1;r_2]} \end{cases} \tag{1.26}$$

Notice that in Eq. (1.26), the parameters are time- and agent-dependent. If we assume the assets to be independent ($\sigma[r_1;r_2] = 0$) and replace the variance notation $\sigma^2[x]$ by Var_x, it corresponds to Eq. (1.9) and (1.10).

1.6.2. Tables

Table 1.9.: Comparison among perfect knowledge of the fundamental value, adaptive learning and adaptive learning with order-placement strategy in a market with fundamentalist and chartist outlook ($\sigma_f^2 = 0.6$ / $\sigma_c^2 = 1$). The mean and the standard deviation (in brackets) are reported.

		Perfect knowledge	Adapt. learning	Adapt. learning + order plac. strat.
Asset 1 $\sigma_{fv}^2 = 0.2$	Fundamental spread	2.858	1.599	2.060
		(2.437)	(11.341)	(4.627)
	Var. funda. spread	133.989	163.434	112.433
		(33.507)	(49.634)	(20.116)
	Learning spread	0	0.807	0.267
		(xxx)	(11.065)	(4.181)
	Var. learn. spread	0	44.499	7.658
		(xxx)	(36.449)	(6.284)
	Price volatility	176.144	124.306	119.568
		(66.223)	(29.038)	(30.619)
Asset 2 $\sigma_{fv}^2 = 1$	Fundamental spread	2.202	3.781	1.284
		(3.739)	(59.748)	(22.142)
	Var. funda. spread	225.279	1266.342	321.797
		(80.480)	(913.671)	(142.248)
	Learning spread	0	1.200	2.546
		(xxx)	(59.434)	(21.075)
	Var. learn. spread	0	1132.554	171.260
		(xxx)	(921.583)	(121.041)
	Price volatility	1219.996	135.575	498.891
		(1037.886)	(37.649)	(363.781)

Table 1.10.: Comparison among adaptive learning and adaptive learning with order-placement strategy in a market with fundamentalist and chartist outlook ($\sigma_f^2 = 0.6$ / $\sigma_c^2 = 10$). The mean and the standard deviation (in brackets) are reported.

		Adapt. learning	Adapt. learning + order plac. strat.
Asset 1 $\sigma_{fv}^2 = 0.2$	Vol. per simu.	292.164	396.817
		(15.228)	(40.867)
	Fundamental spread	2.117	6.790
		(13.016)	(12.384)
	Var. funda. spread	338.683	461.141
		(106.132)	(192.081)
	Price volatility	295.130	419.112
		(87.853)	(156.617)
Asset 2 $\sigma_{fv}^2 = 1$	Vol. per simu.	302.583	464.508
		(44.904)	(117.173)
	Fundamental spread	1.886	4.525
		(54.386)	(58.760)
	Var. funda. spread	1139.430	1234.822
		(979.643)	(1109.063)
	Price volatility	351.034	589.945
		(119.711)	(244.081)

Chapitre 2 : Consensus sur l'anticipation des flux de trésorerie

Motivation

Timmermann (1996) propose un modèle où les investisseurs sont capables d'estimer la valeur fondamentale d'un actif par une régression linéaire sur le payement des dividendes (Moindres carrés ordinaires). Cette hypothèse peut paraître discutable du fait des capacités qui sont attribuées aux agents (agents à rationalité limitée). Nous proposons ici de nous concentrer sur la capacité d'apprentissage de la valeur fondamentale au travers d'un flux de trésorerie exogène[g]. La principale différence avec les travaux de Timmermann réside dans la manière de calculer la valeur fondamentale. Les agents de ce modèle ne doivent pas découvrir la régression, mais ont plusieurs stratégies déjà définies et doivent choisir la meilleure. Fixer *ex ante*, les stratégies permettent de modéliser la réticence des agents à adapter leurs espérances et leurs comportements suite à un choc ou à l'apparition brutale d'une information contradictoire (confiance excessive, voeu pieux et persévérance des croyances - Lord et al., 1979).

Ce travail traite de l'apport de l'information publique dans le processus d'apprentissage de la valeur fondamentale des individus. La principale question est : les agents sont-ils capables d'apprendre le processus de flux de trésorerie afin d'estimer la valeur fondamentale ? Concrètement cela s'articule autour de trois sous questions :

- Les apprentissages par renforcement (RL), par jeu fictif (FG) et par pondération de l'expérience (EWA), permettent-ils tous d'estimer correctement la valeur fondamentale ?
- Existe-t-il une justification à la dispense d'information publique ?
- À partir du moment où nous sommes conscients que nous sommes dans un cadre d'information imparfaite, quelle méthode d'optimisation du portefeuille choisir (entre une allocation naïve et une gestion plus complexe, telle que le critère de rendement moyen, le critère moyenne-variance, *etc.*) ?

[g] Dû à l'échelle de temps du modèle (intrajournalière), le terme flux de trésorerie semble plus approprié que celui de dividende. En effet, le détachement des dividendes est une opération qui se veut annuelle, bi-annuelle ou trimestrielle alors que le flux de trésorerie – qui est la capacité d'une entreprise à générer des ressources supplémentaires – peut être constamment réévalué.

Méthode

Pour comprendre ce problème d'apprentissage des agents et leurs capacités à inclure une information (plus ou moins correcte), nous avons modélisé un marché dirigé par les prix, où chaque agent est l'agrégation d'un comportement fondamentaliste, chartiste et aléatoire. L'apprentissage des agents se veut simple (RL, FP et EWA), mais basé sur la réalité. En effet, la calibration de l'apprentissage est extraite de tests empiriques réalisés par Camerer and Ho (1999). Le marché se compose de deux actifs risqués (comme dans le précèdent modèle) à la différence près que les deux actifs sont corrélés (indépendants dans le premier modèle). Cette corrélation fixée *ex ante* et de manière exogène est constante au cours du temps et s'applique au flux de trésorerie.

Ce papier permet de :

- modéliser des apprentissages cohérents et basiques (simples à reproduire et à interpréter) ;
- justifier l'émergence de tendances (non fondées) entre deux actifs ;
- identifier l'impact de l'information publique sur le cours d'un actif (information vraie ou fausse) ;
- mettre en avant la capacité d'extraction de l'information des agents (à partir du carnet d'ordre ou d'une information disponible sur le marché) et d'identifier si l'apprentissage à partir du marché permet de corriger une information privée ou publique partiellement erronée.

Discussion

– Dans le cas d'un apprentissage par renforcement (RL), l'introduction d'une information publique correcte peut prendre beaucoup de temps à être intégrée par les agents puis par le marché. En revanche, dans le cas d'un apprentissage de type FG ou EWA, les résultats sont plus immédiats. Dans tous les cas, à plus ou moins long terme, l'information est intégrée sur le marché.

– L'information publique, si elle est erronée, induit une tendance qui est partiellement corrigée par l'apprentissage. La correction est loin d'être parfaite, mais existe. Cela peut s'expliquer d'une part par le fait que la bonne stratégie à jouer, en tenant compte de l'erreur d'information, n'est peut être pas dans l'ensemble des stratégies possibles. Et d'autre part que cette stratégie n'est pas stable au cours du temps. Dans notre modèle, la bonne stratégie en cas d'information erronée n'existe pas. Nous pouvons conclure que l'apprentissage permet en partie de compenser une information erronée.

Cela amène à se questionner sur le pouvoir informationnel du carnet d'ordre lorsque le marché est altéré par différents comportements d'agents. Dans notre

modèle, l'information extraite du carnet d'ordre ne permet pas d'en déduire la corrélation entre les actifs. En revanche l'observation des flux de trésorerie passés, de par leur neutralité (par opposition au carnet d'ordre), permet une bonne approximation de la corrélation des actifs par les fondamentalistes.
– La prise en compte de l'information publique, en plus de l'information privée, permet une meilleure estimation de la valeur fondamentale. Cependant, l'intérêt de dispenser de l'information publique varie en fonction de la précision de l'information privée.

- Information privée erronée : gain important en efficience.
- Information privée cohérente, mais légèrement erronée : plus l'information privée est bonne et moins l'information publique aura d'impact. Gain moyen.
- Information privée cohérente : l'information publique n'a pas d'intérêt sauf à enlever la position privilégiée des fondamentalistes.

Dans un modèle précèdent (Mémoire : Impacts of behaviour on dynamics price on financial market - Section : Switching behaviour) basé sur les travaux de Brock and Hommes (1998), le comportement opportuniste des agents a été mis en évidence lorsque le marché est relativement efficient et que l'information est coûteuse. Dans ce cas précis, les fondamentalistes ont intérêt à se comporter comme des chartistes. En effet, si le cours de l'actif est informatif, pourquoi payer pour quelque chose de disponible sous forme d'information publique et donc gratuite ! Les fondamentalistes tendent alors à s'informer par intermittence afin de vérifier que le marché conserve une certaine efficience. Si ce dernier se décorrèle trop, l'incitation à s'informer retrouve son importance. Le rendement des fondamentalistes qui s'informent dépasse celui des fondamentalistes non informés jusqu'à ce que le marché redevienne efficient.
Dispenser une information publique dans un marché où l'information privée est correcte n'est pas souhaitable. En effet, cela est coûteux pour les pouvoirs publics et n'est pas cohérent pour les fondamentalistes qui perdront leur avantage informationnel et disparaîtront (ils arrêteront de s'informer ou quitteront le marché).
– Les marchés ne sont jamais parfaitement informationnels, les agents font des erreurs dans l'estimation des corrélations, et ces erreurs persistent. C'est dans ces conditions que DeMiguel et al. (2009) ont démontré qu'une allocation naïve du portefeuille peut être plus efficace que des constructions plus complexes basées par exemple sur le critère moyenne-variance. En effet, optimiser un portefeuille sous une contrainte erronée entraîne une sous-performance de ce dernier.

Application à la réalité :

- Transposer la notion de valeur fondamentale à l'observation d'un index permet d'expliquer l'émergence d'une corrélation entre actifs sous la forme de prophétie auto-réalisatrice. Celle-ci est basée sur la corrélation accidentelle des flux de trésorerie sur une période donnée.

- Transférer la dispense d'information publique à une entité telle que les agences de notation indique le poids de la source publique dans le marché.

- Admettre que l'information publique soit une estimation erronée et partagée par tout le monde permet de décorréler les actifs.

Chaque apprentissage implique des hypothèses (plus ou moins fortes) sur les agents. Un raffinement de ce modèle est prévu avant de le soumettre à publication. Il consiste à faire interagir deux apprentissages (RL et EWA) dans un marché, afin d'étudier la dynamique de la corrélation des actifs entre eux et avec leurs fondamentaux. Les poids seront adossés en fonction des caractéristiques des agents. EWA demandant beaucoup plus d'informations et de traitements que RL, il caractérisera les fondamentalistes. Les agents moins informés se baseront sur l'apprentissage par renforcement (RL).

2. Learning of unknown cash flow process : focus on fundamental comovements in an artificial stock market

2.1. Introduction

A key assumption in financial theory is the existence of a unique exogenous fundamental value (FV - Fama, 1970). Empirically, it is often assumed that the markets have absurdly overvalued some prices of financial assets and then precipitated them, when they realised the glaring disconnection between the economic reality and the valuation of financial assets in question.

In this paper, the assumption of "perfect knowledge of fundamentals" is relaxed (Orléan, 2011) and the agents are situated in a context of evolving cash flows (Timmermann, 1996). Each trader receives and processes information about the cash flow mechanism, then deduces the fundamental value to be used in her orders. Different learning types about the cash flow mechanism will be tested – Reinforcement Learning (RL), Fictitious Play (FP) and Experience-Weighted Attraction Learning (EWA) – so as to highlight in which situation may agents learn the fundamental values and compute the correlation among fundamentals?

Pastor and Veronesi (2009) explain some financial puzzling facts through agents' learning in a context of uncertainty[a]. Parameter uncertainty is ubiquitous in finance. Agents are uncertain about many of the parameters characterising financial markets, and they learn about these parameters by observing data. This paper focuses on a small piece of fundamental value learning: the learning of the cash flow mechanism and the estimation of the asset correlation (comovement). A J.P.Morgan's analyst[b] highlights that cross-asset correlations roughly doubled over the past ten years (2000-2010). In practice, the covariance matrix is estimated from historical data available up to a given date or given by experts (rating agencies, financial analysts). The portfolio weights are computed from this estimate, then the portfolio is formed on that date and held until the next rebalancing occurs. However, the estimate is based on the past, and does not

[a]The use of "uncertainty" might be controversial. Indeed, according to the risk theory of Knight (1921), "uncertainty" defines a risk that is immeasurable, not possible to calculate; while "risk" means a quantity susceptible of measurement and thus insurable. However, according to the authors, the learning is hampered by a large amount of randomness pervading financial assets.

[b]Marko Kolanovic et al., Rise of Cross-Asset Correlations - May 16, 2011

warrant that it will be the same for the future. As an example, Yvan Berthoux mentioned on his wordpress blog[c] about EURUSD and VIX Index over the past six months: "the 20-day correlation between the two underlying assets has switched from a negative 80 in Mid-March to a positive 80.6% today [09/17/2015]". Perfectly aware of this weakness, academics developed new statistical tools so as to improve the forecast. The L-moment statistics (Hosking and Wallis, 2005) and the shrinkage (Ledoit and Wolf, 2003) are two examples of this improved tools. The first one is well suited for characterising data that commonly exhibit high skewness. The second one can be seen as a way to account for extra-market covariances by mixing two existing estimators: the sample covariance matrix and the single-index covariance matrix.

Knowing that the correlation matrix is at the heart of the "Portfolio Selection" of Markowitz (1952), the purpose of this work is to testify the agents' ability to learn the right cash flow process and compute the fundamental values properly in a context of correlated fundamentals. Comparisons of learning models have been a central preoccupation of experimental and behavioural economics in the early 2000's. Empirical evidences already suggest that agents do not respond only to their own previous experience; they take into account others' payoff information. The behaviour is sensitive to the way in which agents match (Camerer et al., 2002). Jannsen and Ahn (2006) compare the empirical performance of several alternative learnings models based on social preferences models so as to highlight the rise of equilibrium in non-cooperative game. Rouchier and Robin (2006) combine experimental economics and ABM to formulate assumptions about procedural rationality of individuals. Wilcox (2006) studies the impact of heterogeneity bias in empirical comparison of reinforcement and belief learning models. More recently, Noe et al. (2012) employ artificial agents to examine different learnings in an auction market. Our study among learning processes (RL, FP and EWA) is a part of a larger aim which is to investigate the impacts of the learning on the stock prices in a context of correlated risky assets. So as to keep focus on the learning, the covariance matrix is basically estimated from historical data available in the market.

In a call auction market, are agents able to properly learn an unknown cash flow process and estimate fundamental comovements?

By understanding two separates learning components – belief and reinforcement learning components –, this paper shows that the efficiency of each learning depends on the accuracy of the informational source.
• The current results are that RL underperforms FP and EWA regardless of setups, and EWA does at least as well as FP. Even if EWA allows the most

[c] Article posted in September 17^{th}, 2015; yvanberthoux.wordpress.com

accurate forecast of fundamentals, it also requires the biggest amount of information used in the learning process. When agents know about the correlation of the cash flow, RL proves that with a minimum of effort and information (reinforcement strategy), agents are able to compute the fundamental values properly. A consensus always rises on the right strategy to play.

• In case the information about the correlation is wrong, no trader is able to compute the fundamentals properly. The comovement of the estimated fundamentals is neither equal to the fundamental comovement nor to the given wrong public information. It is mainly explained by the learning process which tries to correct the forecast by updating the probability to play each strategy according to its return. Even though the forecast is wrong, the traders try to play the strategy which is the less inaccurate. Traders are unable to estimate the fundamentals properly. This misestimation goes in the sense of DeMiguel et al. (2009) who prove that the gain by diversification is more than offset by estimation errors.

• By making agents backward looking on the market price, their herding behaviour (Banerjee, 1992) confirms that the market disseminates information better than it aggregates (Veiga and Vorsatz, 2008).

• In the case of wrong private information, the average bias of the fundamental spread (error between the computed fundamental value and the right one), and the accuracy of the correlation of the computed fundamental values are driven by the weight given to the right public information. The more the private information is inaccurate, the more the public source quickly minimises the error. RL is the learning which provides the greatest benefit from the public information.

• Finally, increasing the amount of information (extended the backward looking) means more information to collect and process but not necessarily better forecast. In the same way, it is totally pointless to share public information in an already informative market. The market should be monitored, and public information revealed when the market fails in its informative purpose.

The paper is organised as follows: Section 2 reviews the existing literature. Section 3 outlines the structure of our model. Section 4 discusses results and Section 5 concludes.

2.2. Literature

2.2.1. Stock prices

It would be wonderful to be able to perfectly foretell what the stock prices will be like, because in that case one could really strike optimal investment decisions. Unfortunately, this is not possible due to manifold influences which determine the layout of a stock price (such as value of the future prospects of the company, general economic situation, political decisions, consumer behaviour, *etc.*). The first indications of the future development of a stock price can provide us with estimations for the expected value and the variance of the rate of return.

The effect of a firm's dividend policy on the current price of its shares is a matter of considerable importance, not only to the corporate officials who must set the policy, but to investors planning portfolios and to economists seeking to understand and appraise the functioning of the capital market. The dividend discount model (Gordon and Shapiro, 1956), which expresses stock prices as the present value of the stream of future dividends, is a key component of fundamentalist investors. However, empirical evidences prove that dividends are artificially maintained stable (Lintner, 1956; Mantripragada, 1972); this signal reassures the investors. Modigliani and Miller (1958) by assuming dividend irrelevance, provide a more general version of the discounted dividend model: the value of a company is based on how much money is earned by the company (discounted earnings and discounted cash flow models). Stock price valuation models imply that price changes are driven by expectations of future cash flows. Forward dividend payments are definitely less important (Modigliani and Miller, 1961). According to these discounting models, fundamentalist investors are assumed to buy when the asset is undervalued. Indeed, the future expected returns allow them to expect an additional payoff. Conversely, they are assumed to sell when the asset is overvalued. The future expected returns are lower than the present value; holding the asset implies a future loss.

Originally, the cash flow forecast is based on economic variables, independent from the market. The computed fundamental value is thus intrinsic to the firm and exogenous to the market. However, in a context of bounded rational agents, the investors might consider the others as better informed. The estimate of the asset correlation is based on the past, and does not warrant that it will be the same in the future. So, the trading price might be assumed to be a better information source because it resumes the interaction of supply and demand of well-informed agents. Psychologic analyses show that if an agent is not confident in her own signal (*e.g.* sceptical about her computed fundamental value) the temptation is high for her to look after some exogenous valuation (*e.g.* the market price). Banerjee (1992) in a simple model formalises this type of herding behaviour with the famous case of the restaurant selection. He studies how agents revise the selection of an unknown restaurant according to its crowding density. Thus, in a context of bounded rational agents, it seems legitimate to testify the herding behaviour and observe if the market price allows agents to get better estimate.

In practice, a model called the geometrical Brownian movement is often used to model stock prices. This model considers continuous stock price development

(Black and Scholes model, 1973). The geometrical Brownian movement may be intuitively thought as the continuous-time limit of a random walk process in which the integer time index ($t = 1, 2, ...$) has been rescaled to the continuous time index ($r = 0, ..., 1$). The geometrical Brownian movement may be shown to have the following property: $\mathcal{W}(r) \sim \mathcal{N}(0, r)$. Based on the results of works on earnings per share, including those on individual stocks (Watts and Leftwich, 1977; Stambaugh, 1999), a simple autoregressive model allows to replicate cash flow (or dividend) process with realism. In the forward parts, let us assume that the cash flow defines a dividend payment which is paid in a constant flux (*e.g.* per round). Thus, each stock is characterised by its cash flow which is modelled through a trend-stationary process (Barucci et al., 2004; Timmermann, 1996).

2.2.2. Learning

In his model, Timmermann (1996) proposes that investors would be able to estimate the fundamental value by ordinary least square, assuming an exogenous dividend process. Due to the time scale of the current model (intraday) and the assumption of continuous dividend payment, the term cash flow process seems more appropriate. In every instance, assuming bounded rational traders (Simon, 1982) make them reluctant to this kind of estimation: it is too costly and takes too much time. Instead, they learn so as to discover the parameter of the cash flow process and compute the FV.

Experienced-Weighted Attraction, introduced by Camerer and Ho (1999), is a general model that tries to capture and isolate as many psychological features of statistical adaptive behaviour. It is shaped by a combination of Reinforcement Learning (Roth and Erev, 1995) and Fictitious Play (Brown, 1951). We proceed to present the general EWA model and the particular cases that we use.

EWA model, as any adaptive learning model, requires specifying three elements: initial values, an updating rule and a decision rule. Since learning models are individual learning models, EWA will be introduced for player i. The core of the EWA model is made up of two variables which are updated after each round. The first variable is $N(t)$, which is interpreted by the authors' model as the number of "observations-equivalents" of past experience. The second variable is $A_i^k(t)$, player i's attraction of strategy k, or attraction associated with strategy s_i^k at time t.

Initial Values: the two variables start with some initial values, $N(0)$ and $A_i^k(0)$, that are experience measure and attraction at $t = 0$. These initial values reflect pre-game experience.

Updating Rules:

$$N(t) = \rho N(t-1) + 1 \text{ , where } t \geq 1 \tag{2.1}$$

$$A_i^k(t) = \frac{\phi N(t-1) A_i^k(t-1) + [\delta + (1-\delta) I(s_i^k, s_i(t))] V_i(s_i^k, s_{-i}(t))}{N(t)} \tag{2.2}$$

The parameter ρ depreciates the experience measure, $N(t)$. The parameter ϕ depreciates the past attraction, $A_i^k(t)$. Let's call V the payoff function, thus δ measures the relative weight given to foregone payoffs, compared to actual payoffs, in updating attractions. $I(x, y)$ is an indicator function which equals 1 if $x = y$ and 0 otherwise (Camerer and Ho, 1999).

Decision Rules:

$$Prob_i^k(t+1) = \frac{e^{\lambda A_i^k(t)}}{\sum_{l=1}^{m_i} e^{\lambda A^l(t)}} \tag{2.3}$$

The parameter λ measures sensitivity of players to attraction. Notice that the probability of choosing strategies is modelled by a logit function[d].

Basically, there are three main EWA parameters (without the logit parameter). The discount factors ϕ and ρ depreciate past attractions and experience separately. The parameter δ measures the relative weight given to foregone payoffs, compared to actual payoffs, in updating attractions. Imposing constraints on these three parameters allows to identify the RL of Roth and Erev ($\rho = 0$ and $\delta = 0$) or the FP of Brown ($\phi = \rho$ and $\delta = 1$). Specifically, in RL: agents tend to play strategies that paid off relatively well in the past. While, in FP: agents form beliefs on the basis of opponents' past decisions and tend to play strategies today that could have had high payoffs in the past. Actually, EWA is a hybrid model which tries to combine the advantages of both. The objective of these learning models is to learn about the cash flow mechanism and accordingly to learn to play optimally.

As written above the agents are not able to foretell the forward stock prices. The same problem happens when heterogeneous bounded rational agents forecast the fundamental value. Many parameters in the definition of the fundamental value are subject to personal beliefs (such as forward dividends, cash flows, actualisation rates, *etc.*). There are as many forward of the fundamental value as heterogeneous investors (Walter and Brian, 2008). However, *ex post*, the investors can judge perfectly well whether or not their predictions (played strategy) have proved to be accurate. There is no hazard on what was the cash flow and the actualisation rate to apply. By using backward induction processes, agents are able to price with no uncertainty an option (Black and Scholes, 1973). This is an important fact, which limits the *ex post* fundamental value to a unique value; while the *ex ante* fundamental value might be idiosyncratic (Orléan, 2012). The interpretation of the future adopted by the learning strategy might be backed up, if not by full verification, at least by an absence of contra-

[d] Camerer and Ho (1999) propose in their original paper three different probability choices: logit, power and probit choice probabilities. The logit is commonly used in studies of choice under risk and uncertainty.

diction in the subsequent observed value. If *ex ante* we simply do not know, *ex post* we can judge the accuracy of valuations and thus of played strategies. For a strategy to endure, the observed facts must be in keeping with the predicted facts.

2.3. Model

The market structure is based on a previous work entitled "Fair price and trading price: an ABM approach with order-placement strategy and misunderstanding of fundamental value" (Lespagnol and Rouchier, 2015). This previous paper was a generalised version of Chiarella et al. (2009) and Yamamoto (2011). The new model lies in the forecasting of future cash flows through information extraction and focuses on fundamental comovements.

2.3.1. Market and agent characteristics

2.3.1.1. Market structure

The investment choice is made in a double auction market (Madhavan, 2000) composed of two correlated risky assets (stocks). Each agent's decision process determines the amount of wealth the trader would like to invest in the risky assets for some transaction price. Be aware that traders do not have to invest all their wealth in the risky assets. Let us assume that the residual wealth is invested in a bond, which is not further considered afterwards because agents are not monetary constrained. The market structure is such that each asset price is given by the price at which a transaction, if any, occurs. If no new transaction occurs, a proxy for the price is given by the average of the quoted ask a_t^q (the lowest ask listed in the book) and the quoted bid b_t^q (the highest bid listed in the book), so that $p_t = (a_t^q + b_t^q)/2$. If no bids or asks are listed in the book, a proxy for the price is given by the previous traded or quoted price. Once submitted, the order cannot be modified. It is removed from the order-book when it is executed or it remains unexecuted for its lifetime (τ^i). Be aware that the best quoted bid and ask are observable. All bids, asks and market prices need to be positive and are defined on a pricing grid of width $\Delta = 10^{-3}$. Finally, agents are not allowed to engage in short selling.

A round in our model is such that:

1. All agents compute their expectations about the cash flow processes.
2. 5 agents are randomly chosen to trade[e].

[e]The simulation was run for 1, 2 and 5 randomly chosen agents. 5 is the maximum number of agents which does not create memory optimisation problems. From now, limiting the number of agents involved is the only way to avoid "java heap space out of memory".

3. For each asset available on the market, each trader formulates expectations on the forward price according to her features.
4. The 5 agents define simultaneously their positions (buy or sell, quantity and price) by maximising a constant absolute risk aversion utility function and adapt their order type to the visible part of the order book.
5. The 5 agents submit their orders according to time preference (the first randomly chosen agent is the first to submit).
6. All submitted orders are interpreted by the market to produce the new asset prices.

2.3.1.2. Fundamental value process

According to Timmermann (1996), each stock is characterised by a cash flow d_t that is modelled through a trend-stationary AR(1) process.

$$d_{t+1} = \alpha + \gamma(t+1) + \beta d_t + N^d_{t+1} \tag{2.4}$$

α denotes the drift, β the time trend and γ the persistence of cash flow. All these parameters are fixed. When β tends to 1, the cash flow tends to be highly persistent[f], which is an empirical fact (Dechow et al., 2008). Ω_t is the information set of the agent and N^d_t is the arrival of new information:

- for asset 1, $N^{d_1}_{t+1} = \epsilon^{d_1}_{t+1}$, such that $\epsilon^{d_1}_{t+1} \sim \mathcal{N}(0, \sigma^2_\epsilon)$, iid;
- for asset 2, $N^{d_2}_{t+1} = \epsilon^{d_1}_{t+1}.Corr + \epsilon^{d_2}_{t+1}.\sqrt{1 - Corr^2}$, such that $\epsilon^{d_2}_{t+1} \sim \mathcal{N}(0, \sigma^2_\epsilon)$, iid.

According to Cholesky formula, the cash flow returns are correlated by $Corr$ and follow two independent and identically distributed normal random variables. As in Barucci et al. (2004), γ is set to zero in order to have no time trend in time series. The formulation is also similar to Lintner (1956). So as to keep the persistence in the cash flow, β is set to 0.9. The no arbitrage condition implies that the today price is equal to the forward price plus cash flow, all discounted by the actualisation rate[g]:

$$f_t = \frac{1}{1 + r_f} \mathbb{E}(f_{t+1} + d_{t+1} |_{\Omega_t}) \tag{2.5}$$

Assuming that the form and parameters of Eq. (2.4) are known, $\beta < 1 + r_f$ and $d_{t-j} \in \Omega_t$, $\forall j \geq 0$, agents can forecast future cash flows (Jacob-Leal, 2014).

[f] Assuming the definition of cash flow which is the dividend payment paid in a constant flux.

[g] For simplicity, the actualisation rate is assumed to equal the risk free rate (r_f) which is constant over time (Barucci et al., 2004; Jacob-Leal, 2014).

Moreover, under the assumption of rational expectation hypothesis (REH - Muth, 1961), the present-value stock price (also known as fundamental value) could be rewritten as:

$$f_t = \frac{\beta}{1 + r_f - \beta} d_t + \frac{1 + r_f}{r_f(1 + r_f - \beta)} \alpha \tag{2.6}$$

This formulation of the fundamental value is the most basic one. It does not even price the risk of the asset. In so doing, the uniqueness of the fundamental value is ensured.

In what follows, agents are not assumed fully rational. They compute the fundamental value from a learning mechanism. We propose to study the agent's ability to learn the cash flow process in a context of uncertainty; especially, monitoring if a consensus rises on some values when the accuracy of the informational source varies.

2.3.1.3. Agent definition

The agents are defined by three traits, fixed in time:

- their fundamentalist component (g_f^i);
- their trend-follower component (g_c^i);
- their Noise Trader Agent (NTA) component (g_n^i).

For agent i, the proportion of g_f^i, g_c^i and g_n^i are according to a set of exponential distributions. The agents are risk-averse and are assigned investment horizons[h] that depend on this g_f^i, g_c^i ratio, following:

$$\tau^i = \tau \, \frac{1 + g_f^i}{1 + g_c^i} \tag{2.7}$$

where τ is a parameter of the system to define the time horizon.

The decision-making is a succession of three steps :

1. the estimation of the cash flows and the computation of the fundamental values;
2. the estimation of the future prices;
3. the computation of the optimal demand of each asset according to the estimated prices.

[h] The investment horizon is rounded to the nearest integer, $\tau^i \in \mathbb{N}^*$.

2.3.2. Agent's expectation of future asset price

2.3.2.1. Fundamentalist outlook

Since the traders are bounded rational and have access to the sample of information on current and past cash flows, they know the form of the cash flow process defined by Eq. (2.4) but not the value of the parameters (α, β). While the correlation of the cash flow returns is estimated by the agents ($\hat{Corr}_i$), we distinguish four cases:

- Right public information: $\hat{Corr}_i^{r.pub} = Corr$, the right signal common to all agents.
- Wrong public information: $\hat{Corr}_i^{w.pub} = -Corr$, the wrong signal common to all agents.
- Private information: $\hat{Corr}_i^{priv} = \frac{\hat{\sigma}^i_{\Omega_1,\Omega_2}}{\hat{\sigma}^i_{\Omega_1} \cdot \hat{\sigma}^i_{\Omega_2}}$, where $\hat{\sigma}^i_{X,Y}$ is the estimated covariance among X and Y, $\hat{\sigma}^i_X$ is the computed standard error of asset X and Ω_1, Ω_2 are the sets of information about assets 1 and 2, respectively.
- Mixed information: $\hat{Corr}_i^{mix} = w.Corr + (1-w).\hat{Corr}_i^{priv}$, weighted average of the right public information and the private information. In this setup, the public information is used to improve the accuracy of the agents' forecast, thus it is always assumed right.

Note that the horizon of backward looking, for the estimation of the correlation of the cash flow returns, is defined by the same formula as Eq. (2.7)[i].

The agents have to discover the unknown parameters according to a learning. They have a bundle of 25 fixed strategies (including the right one) so as to be able to formulate their expectation on the forward cash flow ($\hat{d}^i_{t+1}$) and compute their fundamental value ($\hat{f}^i_{t+1}$), defined by Eq. (2.4) and (2.6). The bundle of strategies is homogeneous and each strategy is a couple (α, β)[j]. The learning payoff function (V) is generated as:

$$V(t) = 30 * \frac{1}{\sqrt{5}\sqrt{2\pi}} \exp^{-\frac{1}{2}\left(\frac{f_{t-1} - \hat{f}_{t-1}}{\sqrt{5}}\right)^2} \tag{2.8}$$

[i]Based on interviews of Morningstar employees: risk and valuation models are broadly aligned with 1-5 years horizon. Most analysts only perform forward looking analysis on this time frame and they simply extrapolate current trends, not allowing for serious consideration of disruptive changes likely to materialise in more than 5-10 years. In addition, other practitioners declare that it is a waste of time to be backward looking on longer period than their investment horizon, especially because of information costs and speed needed.

[j]The 25 strategies are all possible pairs of α and β such that: $\alpha = \{0.45, 0.46, 0.47, 0.48, 0.49\}$ and $\beta = \{0.875, 0.8875, 0.9, 0.9125, 0.925\}$. The right strategy is $(\alpha, \beta) = (0.9, 0.47)$ for all assets.

Table 2.1.: Learning parameters setup according to learning model by Camerer and Ho (1999).

	ρ	ϕ	δ	λ
Reinforcement Learning (RL)	**0.000**	0.960	**0.000**	0.098
Fictitious Play (FP)	**0.989**	0.989	**1.000**	1.812
Experience Weighted Attraction (EWA)	0.986	0.935	0.413	0.646

Values in bold: Imposed by the learning type

At the initialisation, the attraction $A^k(0)$ is arbitrarily fixed to 1, as $N(0)$ (regardless of the agent i). The strategies are equi-attractive. Regarding the EWA parameters setup $(\rho, \phi, \delta, \lambda)$, they are summarised in Table 2.1. These values come from the Camerer and Ho's empirical work (1999) so as to fit human behaviours. Keep in mind that learning parameter values vary widely across games because they capture the game features. To the best of our knowledge, there is no theory of how parameter values depend on the game structure. The strength of the Camerer and Ho's paper is that it offers parameter values for all learning used in our model. Moreover, these learning parameter values are all calibrated on a unique game. Thus, even if the parameters are not accurately calibrated for our model (different structures, number of agents, number of strategies, *etc.*), at least all learnings suffer the same misestimation. The basic elements of each learning are preserved, even if a more accurate way would have been to run experimental calibrations so as to implement the results in our artificial stock market.

2.3.2.2. Trend-follower outlook

This expectation is what the agent infers from past returns. The return at time t is defined as:

$$r_t = ln\left(\frac{p_t}{p_{t-1}}\right) \tag{2.9}$$

The average spot return ($\bar{r}_t^i$), used by the trend-follower outlook of agent, is defined as the expected trend based on observations of spot returns over the last τ^i rounds.

$$\bar{r}_t^i = \frac{1}{\tau^i}\sum_{k=1}^{\tau^i} r_{t-k} = \frac{1}{\tau^i}\sum_{k=1}^{\tau^i} \ln\left(\frac{p_{t-k}}{p_{t-k-1}}\right) \tag{2.10}$$

2.3.2.3. Expected return

The agents take the price information, the fundamentalist outlook and the trend-follower outlook into account to form their weighted forecast of the asset's future returns that will prevail in the interval $(t + \tau^i)$. In addition, a noise induced component (which is the NTA outlook) is added, $\epsilon_t \sim \mathcal{N}(0; \sigma_\epsilon^2)$:

$$\hat{r}^i_{t,t+\tau^i} = \frac{1}{g^i_f + g^i_c + g^i_n} \left[g^i_f . \ln\left(\frac{\hat{f}_t}{p_t}\right) + g^i_c . \bar{r}^i_t + g^i_n . \epsilon_t \right] \tag{2.11}$$

The outlook weights of each agent g^i_f, g^i_c, g^i_n are generated following an exponential law of mean $\bar{g}_f$, $\bar{g}_c$, $\bar{g}_n$ respectively. All settings are summarised in Table 2.3. The weighted forecasted return $(\hat{r}^i_{t,t+\tau^i})$ allows the future expected price to be formulated as follows:

$$\hat{p}_{t+\tau^i} = p_t \exp(\hat{r}^i_{t,t+\tau^i}) \tag{2.12}$$

2.3.3. Asset demand

Once the expected prices $(\hat{p}^1_{t+\tau^i}, \hat{p}^2_{t+\tau^i})$ are computed, the optimal composition of the agent's portfolio is determined in the usual way by trading-off expected return against expected risk. The optimal portfolio allocation is defined by the maximisation of a constant absolute risk aversion (CARA) utility function under a Gaussian return on assets. Regarding to this maximisation, the optimal holding can be expressed for asset 1 as:

$$\pi^{i,1}_t(\hat{p}^1_{t+\tau^i}, \hat{p}^2_{t+\tau^i}) = \frac{p^2_t \Big[\ln\Big(\frac{\hat{p}^1_{t+\tau^i}}{p^1_t}\Big) p^1_t . p^2_t . \sigma^i_{2,2} - 2\sigma^i_{1,2} . \ln\Big(\frac{\hat{p}^2_{t+\tau^i}}{p^2_t}\Big)\Big]}{(p^1_t)^2\, \sigma^i_{1,1}\, (p^2_t)^2\, \sigma^i_{2,2} - 4(\sigma^i_{1,2})^2} \tag{2.13}$$

and for asset 2 as:

$$\pi^{i,2}_t(\hat{p}^1_{t+\tau^i}, \hat{p}^2_{t+\tau^i}) = \frac{p^1_t \Big[\ln\Big(\frac{\hat{p}^2_{t+\tau^i}}{p^2_t}\Big) p^1_t p^2_t \sigma^i_{1,1} - 2\sigma^i_{1,2} . \ln\Big(\frac{\hat{p}^1_{t+\tau^i}}{p^1_t}\Big)\Big]}{(p^1_t)^2\, \sigma^i_{1,1}\, (p^2_t)^2\, \sigma^i_{2,2} - 4(\sigma^i_{1,2})^2} \tag{2.14}$$

The variance $(\sigma^i_{j,j})$ denotes the investment risk of the asset j. It is assumed to equal the variance of the logarithmic rate of return.

$$\sigma^i_{j,j} = \frac{1}{\tau^i} \sum_{k=1}^{\tau^i} [r^j_{t-k} - \bar{r}^{i,j}_t]^2 \tag{2.15}$$

Table 2.2.: Decision-making and order definition.

Volume	Position	Order price	Order type	
$\pi_t^{i,j} - \pi_{t-1}^{i,j} > 0$	buy	$b_t = \hat{p}_{t+\tau^i}$	$b_t > a^q$ $b_t \leq a^q$	$\Leftrightarrow$ MO $\Leftrightarrow$ LO
$\pi_t^{i,j} - \pi_{t-1}^{i,j} < 0$	sell	$a_t = \hat{p}_{t+\tau^i}$	$a_t > b^q$ $a_t \leq b^q$	$\Leftrightarrow$ MO $\Leftrightarrow$ LO
$\pi_t^{i,j} - \pi_{t-1}^{i,j} = 0$	do nothing	$\varnothing$	$\varnothing$	$\varnothing$

b_t**:** bid, buying order; a_t**:** ask, selling order
b^q**:** best bid quoted; a^q**:** best ask quoted

$\sigma_{j,l}^i$ is the computed covariance of assets j and l:

$$\sigma_{j,l}^i = \hat{Corr}_i * \sigma_j^i \sigma_l^i \tag{2.16}$$

The amount of assets ($s_t^{i,j}$) the agent is willing to trade is determined by the absolute difference in the optimal demand for asset j at time t and $t-1$ as:

$$s_t^{i,j} = |\pi_t^{i,j} - \pi_{t-1}^{i,j}| \tag{2.17}$$

The sign of this difference ($\pi_t^{i,j} - \pi_{t-1}^{i,j}$) gives the agent's position: buying if it is positive, selling if it is negative, otherwise doing nothing.

2.3.4. Order submission

Once the orders are completely defined (by a submission price, a quantity and a type), they are submitted and quoted in the order-books. The orders are executed following price/time priority. As in reality, when the length of the submitted order is larger than the best counter part, the remaining volume is executed against other quoted limit order (LO). In case of market order submission (MO), when there is not enough quoted counter part, the remaining volume is executed as new orders are submitted. When the submitted bid (ask) is smaller than the best quoted ask (bid), then the order remains in the order-book as a LO at its limit price. Table 2.2 summarises the decision-making. Keep in mind that the best quoted bid and ask are observable for each agent: they impact the type of submitting order.

2.4. Simulations and results

In order to study the model, we use computer-based simulations. We simulate the model using Monte Carlo simulations. Each simulation run consists of $3,500$ rounds. Since every market simulation begins with an empty limit order-book, the first $1,000$ rounds are discarded. The analyses are performed from the

Table 2.3.: Parameters.

Number of rounds:	$2,500$
Number of runs:	25
Number of agents:	$1,000$
Fundamentalist component - exponential law: $\bar{g}_f$	10×10^{-3}
Trend-follower component - exponential law: $\bar{g}_c$	1.2×10^{-3}
NTA component - exponential law: $\bar{g}_n$	1×10^{-3}
Reference order life: τ	10
Pricing grid width: Δ	10^{-3}
Cash flows parameters: α / β / γ	0.47 / 0.9 / 0
True fundamental correlation: $Corr$	0.8
Risk free rate: r_f	5%
Initial experience measure: $N(0)$	1
Initial attraction measure: $A_i^k(0)$	$1 \quad \forall i, k$
Initial Strategies attractiveness: $Prob_i^k(0)$	$1/S \quad \forall i, k$
Number of strategies: S	25

remaining $2,500$ rounds. In order to track the learning process, all learnings parameters are reseted after the first $1,000$ rounds. Each simulation is replicated 25 times. The small quantity of rounds and simulations is a matter of software optimisation and available computing power. Nonetheless, no significative change appears when longer set of rounds are run. The parameter values used in the simulation are summarised in Table 2.3.

2.4.1. Accurate public information

In all setups, the agents estimate the fundamental value of asset 1 (FV-1), and hence compute FV-2 based on the assumption that the cash flow payment of asset 2 is correlated to the cash flow payment of asset 1 (see the definition of $N_{t+1}^{d_2}$ in Eq. (2.4)).

After a "learning period" of 500 rounds, the fundamental trends are well estimated. A consensus rises on the right strategy regardless of learning components. The computed fundamental values evolve around the right fundamental values as a white noise (heteroscedasticity test rejected for each fundamental spread - Engle, 1982). They are properly estimated, and thus the comovement of the computed FVs is similar to the comovement of the right FVs (see Table 2.4).

However, all learning types do not have the same features and thus the same accuracy. RL, due to its reinforcement learning component, needs the longest learning period (around 500 rounds). FP, based on belief learning component, requires no learning period. EWA, which is a mixture of the reinforcement and the belief learning component, needs a learning period which is five times shorter

Table 2.4.: Right Public Information – Correlation of the computed fundamentals during the learning period (1-500) and after (501-2500).

	Bench	RL	FP	EWA
1-500	**0.84**	0.45	0.82	0.75
501-2500	**0.80**	0.77	0.79	0.80

Bench: the comovement of the right fundamentals

Table 2.5.: Right Public Information – Average Bias of the fundamental spread during the learning period (1-500) and after (501-2500).

		RL	FP	EWA
Asset 1	1-500	2.14	0.28	0.35
	(std. dev.)	(0.01)	(0.002)	(0.004)
	501-2500	0.48	0.26	0.03
	(std. dev.)	(0.002)	(0.001)	(0.001)
Asset 2	1-500	2.17	0.28	0.37
	(std. dev.)	(0.01)	(0.002)	(0.004)
	501-2500	0.48	0.26	0.03
	(std. dev.)	(0.002)	(0.001)	(0.001)

than RL (Figure 2.1). In addition, a quick look to the "average bias of the fundamental spread" (Table 2.5) highlights a strong domination of EWA. Indeed, the average bias in case of EWA is ten times lower than in case of FP which is already twice lower than in case of RL. The average bias in a perfect knowledge setup is about 0.48 for RL, 0.26 for FP and 0.03 for EWA[k]. These results are in line with the attractiveness of the best strategy: the probability to play the right strategy is around 0.6 for RL; 0.75 for FP and close to 1 for EWA. Due to the perfect knowledge case (no misestimation of the correlation among the assets), the results are similar for the two assets. The comovement among the computed FVs is close to the right one. Traders are able to use the public information and properly forecast both fundamental values regardless of information required in the different learning processes.

[k]The average bias value is expressed in Economic Currency Unit.

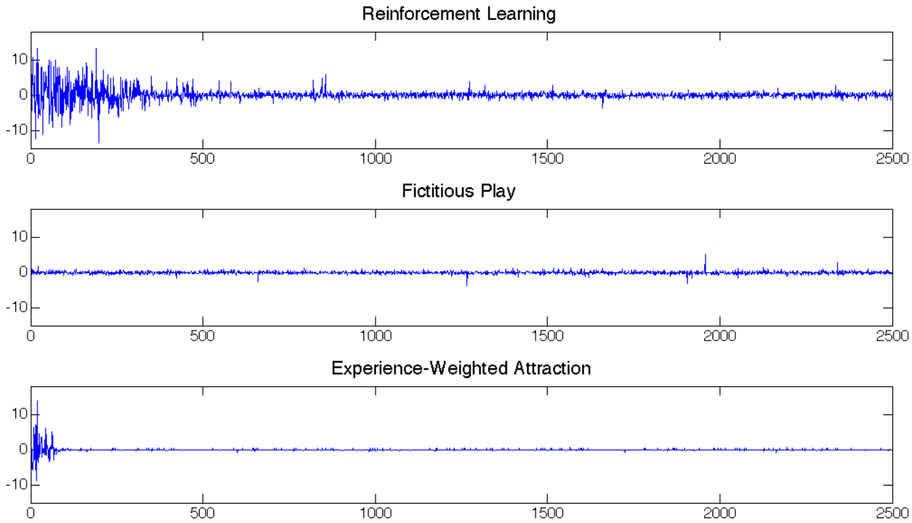


Figure 2.1.: Spread between the computed fundamental value and the true one.

2.4.2. Inaccurate public information

When the public information is biased and agents believe in it, the forecast of FV-2 is inaccurate. All learnings fail to estimate properly FV-2. The average bias of the fundamental spread of FV-2 (Table 2.6) is at least thirteen times higher than in the case of accurate public information[1]. RL is as usual the worst learning (6.31), however, FP weakly outperforms EWA (4.89 versus 5.02).

The weak outperformance of FP can be easily explained: traders try to play the best response according to their given biased information. As a result, the agents choose each time the strategy which gives the best forecast (the highest payoff). However, the best strategy is not necessarily constant across time. In the previous case (accurate public information), three strategies were played, the consensus was "strong". In this case, at least 8 strategies are played (all in all, 25 strategies are available). In addition, no strategy is avoided, while it was the case in right public information, and none of them is dominant. Traders are able to choose the strategy that minimises their error, but have no opportunity to play the strategy that allows us the right forecast. Because the best strategy is not constant across time, it is easily understandable that a learning based on reinforcement learning component (as RL or EWA) underperforms the fictitious game learning. In RL, all strategies are equi-probably chosen. No consensus rises or it quickly disappears. In EWA, traders update less quickly their strategy than in FP, due to the reinforcement learning component. As a result, the best

[1]The average bias of the fundamental spread of FV-1 stays unchanged, due to the model setup.

Table 2.6.: Wrong Public Information – Average Bias of the fundamental spread during the learning period (1-500) and after (501-2500).
$\hat{Corr}_i^{w.pub} = -Corr = -0.8$

		RL	FP	EWA
Asset 2	1-500	5.95	4.20	4.54
	(std. dev.)	(0.02)	(0.01)	(0.01)
	501-2500	6.31	4.89	5.02
	(std. dev.)	(0.01)	(0.005)	(0.005)

Table 2.7.: Wrong Public Information - Correlation of the computed fundamentals.
$\hat{Corr}_i^{w.pub} = -Corr = -0.8$

	Bench	RL	FP	EWA
1-500	**0.84**	-0.21	-0.30	-0.36
501-2500	**0.80**	-0.44	-0.54	-0.53

Bench: the comovement of the right fundamentals

strategy is not always chosen.

Unsurprisingly, the comovement of the computed FVs does not reflect the reality (Table 2.7). However, the computed comovement is also different from the public information. Because agents compute the return (payoff) of their played strategy by comparing the *ex post* FV, they tend to correct a part of this wrong information in their future forecast[m]. Unfortunately, the fixed strategies (α, β) do not allow the agents to properly correlate their computed fundamentals (Table 2.7). The forecast cannot be accurate because of the given fixed correlation parameter. When the focus is on the correlation of the computed fundamentals, the reinforcement learning outperforms the learnings based on belief components. Indeed, RL has the nearest correlation coefficient from the benchmark. Keep in mind that RL is the less costly learning from the agent point of view. It only requires exploration ability.

2.4.3. Private information

When traders are backward looking on the past price to estimate the correlation of the cash flow payments, all learnings fail in their purpose. Traders are unable to compute properly FV-2. No consensus rises, or not on the right strategy. The probability to play the right strategy is equal to 0.13 for RL, while for FP and EWA the learning is not stable. Unsurprisingly, distinctive upward and downward trends that alternate on short-term are observed in the fundamental spread of asset 2. The heteroscedasticity test is never rejected. Because traders

[m]The *ex post* FVs are properly estimated by all agents (Orléan, 2012).

make reasonably accurate estimates of FV-1, the error in the computation of FV-2 comes from the misestimation of the correlation coefficient ($\hat{Corr}^i$) and not from the learning by itself (already observed in section 2.4.2, inaccurate public information). The correlation between the computed fundamentals is close to zero (Table 2.8). Two factors might be responsible for the misleading:

- A too short backward looking: it reduces the number of observations and makes the estimate unstable and irrelevant.
- A wrong strategy: the herding behaviour does not ensure an accurate forecast, the intrinsic valuation of the firm is the most appropriate valuation.

These two factors allow to make a comparison among four different cases. There are the original case: classical backward looking on the past price, and three other cases: longer backward looking on the past price, classical backward looking on the *ex post* FV, and longer backward looking on the *ex post* FV. In case of accurate public information, agents are able to use the given comovement so as to compute the FVs properly, regardless of leaning components. This subsection focuses on fundamental value learnings when the comovement is estimated by each individual agent. The four different cases represent four levels of information accuracy: from the less to the more accurate, and so from the weakest to the strongest investigation seeking information. Unsurprisingly, the learning is sensitive to the accuracy of the information (Table 2.8 - Past price vs. *ex post* FV).

In the case of backward looking on the past price, the correlation between the computed FVs is close to zero, regardless of the learning type and the window of the backward looking. The comovement of the right fundamentals vanishes in the computed FVs. The increasing window of the backward looking which should stabilise the estimated correlation coefficient and thus avoid the misestimation of the fundamental trends fails in its purpose. Since the past price never reflects the fundamental value, increasing its backward looking can't improve its accuracy, especially when the observed data are noisy by trend-followers and/or NTA outlooks. This tends to confirm that "it is a waste of time to be backward looking on longer periods than the investment horizon" (footnote *i* in section 2.3.2.1). In case of inaccurate public information or backward looking on the past price, the traders are not able to use an accurate estimation of the fundamental comovements. The herding behaviour of agents has a destabilising impact on the estimated fundamentals instead of warranty a better accuracy. However, do this allow them to earn more profit? The first results seem to go in that way, but further analyses need to be done. Be aware that the payoff does not seem to be improved by a longer backward looking.

The backward looking on the *ex post* FV produces better estimates and results. The correlation of the computed FVs is at least greater than 0.6. The probability to choose the right strategy is stable and equal to 0.53 for RL, 0.64 for FP and

0.93 for EWA. A consensus on the right strategy rises. In addition, increasing the width of the backward looking improves the agent's estimates of FV-2. As a result, the comovement of the computed FVs tends to the Bench (Table 2.8). In this case, the longer backward looking is profitable. However, keep in mind that there are no information costs[n].

The average bias of the fundamental spread (of asset 2) confirms the previous assessment: when agents focus on spot price (herding behaviour), they are not able to estimate properly FV-2. The average bias stays large after the learning period, no matter the learning type or the window of backward looking. The information is too noisy. When agents focus on *ex post* FV, the error is hugely minimised. The information source is less noisy, more proper: the average bias falls from 4.35 to 0.98 for RL, from 2.72 to 0.66 for FP and from 2.66 to 0.57 for EWA. Increasing the window of backward looking (with focus on *ex post* FV) continues to bring down the error (Table 2.9 - *ex post* FV vs. *ex post* FV + L.b.l). A wider window of backward looking makes the estimation more accurate, when the source of the information is relevant. For all learnings, an increase of the reference backward looking from 10 to 200 allows a gain in precision of 0.1 in the correlation between computed fundamentals. In the same way, the average bias is divided by 2.

When agents are not confident in their own signal and prefer to look after some endogenous valuation (the market price), no one is able to compute properly FV-2. After the learning period, the herding behaviour makes the evolution of the two fundamentals independent. It is worth noting that on the learning period, FP highlights a correlation of the computed fundamentals equals 0.50 for the longer backward looking.

When agents are backward looking on the *ex post* FV, RL is the less efficient learning (the slowest learning speed, the highest bias in the forecast, the lowest correlation coefficient and the lowest probability to play the right strategy). However, the difference in learnings does not seem so important compared to the amount of information required by the belief component learning.

2.4.4. Mixed information

In practice, traders use to process different types or sources of information. As seen in the previous sections, the accuracy of this information plays an important role on the learning and so on the forecast of FV-2. In purpose to better forecast the fundamental value, this subsection focuses on the impact of public

[n]If the information costs are null (in time, effort and price), increasing the backward looking, and thus the information set, improves the accuracy of the forecast. It is beneficial to have more information. However, if there are information costs, the gain in accuracy is not obvious. The current model let the issue unsettled (further investigations should be done on the calibration of information costs).

Table 2.8.: Learning with private information – Correlation of the computed fundamentals.

		Past price			+ L.b.l.			*ex post* FV			+ L.b.l.		
	Bench	RL	FP	EWA	RL	FP	EWA	RL	FP	EWA	RL	FP	EWA
1-500	**0.84**	0.09	0.38	0.23	0.13	0.50	0.33	0.38	0.75	0.65	0.44	0.82	0.75
501-2500	**0.80**	-0.04	-0.02	0.06	-0.05	-0.02	-0.1	0.62	0.67	0.67	0.74	0.76	0.77

L.b.l. Longer Backward Looking – **Bench:** the comovement of the right fundamentals

Table 2.9.: Learning with private information – Average Bias of the fundamental spread.

		Past price			+ L.b.l.			*ex post* FV			+ L.b.l.		
		RL	FP	EWA	RL	FP	EWA	RL	FP	EWA	RL	FP	EWA
Asset 2	1-500	4.37	2.00	2.39	4.33	1.53	2.01	2.66	0.59	0.86	2.28	0.33	0.47
	(std. dev.)	(0.04)	(0.02)	(0.03)	(0.05)	(0.04)	(0.05)	(0.03)	(0.01)	(0.01)	(0.02)	(0.00)	(0.00)
	501-2500	4.35	2.72	2.66	4.40	2.71	2.75	0.98	0.66	0.57	0.57	0.38	0.25
	(std. dev.)	(0.02)	(0.02)	(0.01)	(0.06)	(0.03)	(0.05)	(0.01)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)

L.b.l. Longer Backward Looking

information sharing[o]. As an example, to have the same correlation (between computed fundamentals) as in the case of classical backward looking on the *ex post* FV, the weight of public information must be about 80% in the case of backward looking on the past price. Thus, the public information partially reviewed the misunderstanding due to the inaccuracy of the private source. In the same spirit, increasing the backward looking may also impact the forecast accuracy. As an example, to reach the precision of longer backward looking on the *ex post* FV with no public information, the weight of the public information must be at least equal to 90% in the case of longer backward looking on the past price.

So as to treat the impact of public information on the FV forecast, the case of backward looking on the past price will be studied first and then, the case of *ex post* FV.

When agents focus on the past price so as to compute the correlation among the assets, the FV-2 is badly forecasted. In the case of classical backward looking with 30% of public information, the correlation of the fundamentals is around 0.16 for RL, and 0.26 for FP & EWA. An increase of the backward looking reference from 10 (classical) to 200 (longer) makes the correlation for RL equals to 0.18 (+0.02) and for FP & EWA equal to 0.35 (+0.09). On average, from 10% to 70% of public information, the switch from classical to longer backward looking makes a gain in precision of 0.02 for RL and of 0.1 for FP & EWA. From 70% of public information, the correlation is higher than 0.6 and the average bias is lower than 1 (for all learnings). In the case of classical backward looking with RL, reducing the weight of public information from 100% to 70% makes the average bias twice higher. A fall to 30% makes the average bias five times higher. The same kind of impact is observed for FP. Notice that average bias in the case of RL with 30-40% of public information is similar to the average bias in the case of FP with 10-20% of public information. Increasing the weight of public information makes the average bias exponentially decreasing. However, the impact of the public information on EWA is more present. Indeed, a switch from 100% to 90% of public information makes the average bias ten times higher. This result needs to be put into perspective: EWA with 90% of public information and FP with 100% of public information have both of them the same average bias. The switch from classical to longer backward looking on past price makes one more time the accuracy of the forecast not so much better.

In backward looking on the past price, the public source allows to hugely improve the forecast. Indeed, from 10% of public information to 90%, the gain of precision in correlation of the computed FV is around 0.6 or 0.7 for all setup (Fig. 2.4). The average bias decrease by 3.4 for RL, 1.9 for FP and 2.1 for EWA

[o]The public information is assumed to be right, the government has no incentive to lie. Lyons (2001) already proved that if traders realised *ex post* that government cheats, lies or manipulates the information, they will never trust anymore in the public information and the correction power vanishes.

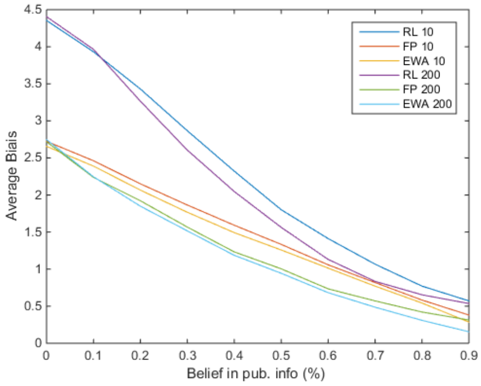


Figure 2.2.: Past price - Average Bias of asset 2 between the computed FV and the true one.

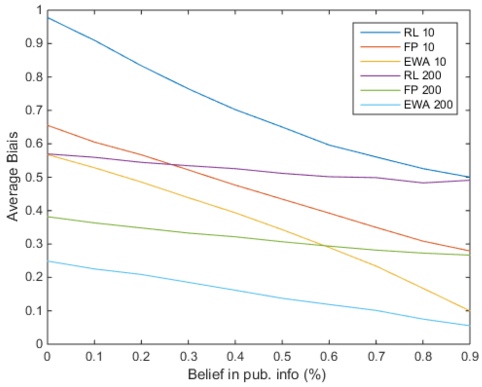


Figure 2.3.: *Ex post* FV - Average Bias of the fundamental spread of asset 2.

(longer backward looking setup – Fig. 2.2). Let us remind that in the case of only private information, the average biases are 4.40, 2.71 and 2.75 respectively for RL, FP and EWA. Without or for weak public information (from 0% to 10%) the average bias of EWA is weakly higher than the average bias of FP. The learning which provides the greatest benefit from the public information is RL (especially between 0% and 60% of public information). However, it is also the worst accurate forecast. In this setup, EWA does not outperform FP.

When agents focus on the *ex post* FV, the private source is already accurate. Thus, the impact of the public information becomes weaker, due to the accuracy of the private source.

In the case of longer backward looking with 30% of public information, the correlation is at least equal to 0.75 for RL. FP & EWA do not even require a public source to equalise this value (Fig. 2.5). In the case of backward looking on the *ex post* FV, the average bias never reaches 1. For 70% of public information, the average bias is around 0.6 for RL, 0.4 for FP and 0.25 for EWA. The average bias slightly decreases; the gain in precision is small (Fig. 2.3) compared to the backward looking on the spot price.

With RL, the average bias is double for a switch from public to private information. With FP, a switch from 100% to 40% of public information is enough to observe the same result. Once again, EWA is really sensitive to the information accuracy: a switch from 100% to 70% of public information makes the average bias ten times higher. However, this result needs to be put into perspective: the same average bias is observed for EWA with 60-70% of public information and for FP with 100% of public information.

No matter the learning, the average bias is lower than 1 with classical backward looking and never exceeds 0.56 with longer backward looking. The

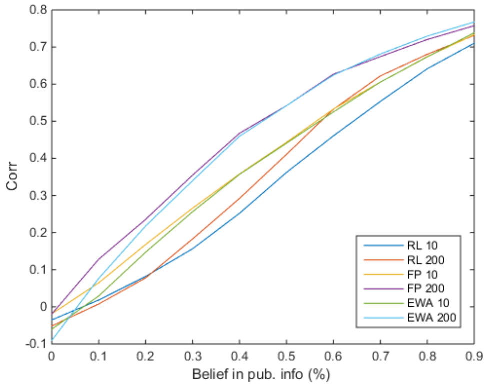


Figure 2.4.: Past price - Correlation between computed fundamentals.

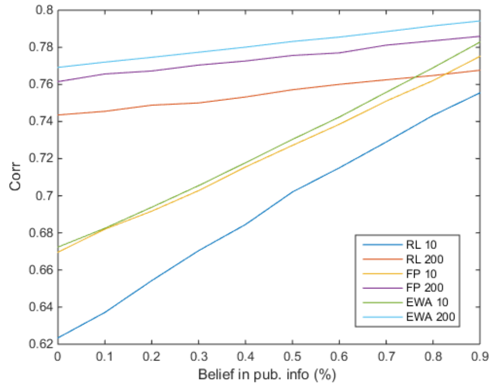


Figure 2.5.: *Ex post* FV - Correlation between computed fundamentals.

gain in FV-2 forecast is smaller than in the case of backward looking on the past price. Indeed, from 10% of public information to 90%, the gain of precision in correlation of the computed FV is around 0.1 for all setup. The average bias decreases by 0.12 for RL & FP, and 0.17 for EWA (longer backward looking setup). The impact of public information is relatively weak when the private source is already accurate.

From 80% of public information, the correlation between the computed fundamentals stays relatively stable, no matter the backward looking, the learning and the information source. The strong weight of public information makes the private cognition non-significant (learning and backward looking). The more the traders weight the public information in their estimation, the more accurate the computed FV is. The misunderstanding is reviewed. The comovement of the computed FVs converge to the benchmark, and the average bias of the fundamental spread decreases.

The impact of the public information is thus less observable when the private source is already accurate. This is definitely confirmed by a comparison among the extreme cases. A switch from 10% to 90% of public information brings down the average from 4 to 0.15 in the case of past price and from 1 to 0.05 in the case of *ex post* FV. The correlation between computed FV follows the same way, in the case of past price it rises from 0 to 0.77 while in the case of *ex post* FV it starts from 0.64 until 0.8.

2.5. Conclusion

The paper investigates whether fundamental misestimations can be explained by investor's learning process in an artificial double auction market. The ability of agents to learn the cash flow process and so compute the fundamental value was tested through the three most basic types of learning: Reinforcement learning of Roth and Erev, based on the reinforcement learning component; Fictitious Play of Brown, based on the belief learning component; and Experience-Weighted Attraction of Camerer et al., which is a mixture of both previous learning components.

The simulation results show that when agents know about the correct correlation of the cash flow between the two assets, a consensus always reaches on the right strategy. Even if EWA allows the most accurate forecast of fundamentals, it also required the biggest amount of information used in the learning process. The weak underperformance of RL proves that with a minimum of effort and information (reinforcement strategy), agents are able to compute the fundamental values properly.

In case the information about the correlation is wrong (inaccurate public information or backward looking on the past price), all three learnings fail in their purpose. A simple way to replicate the herding behaviour (Banerjee, 1992) in financial markets is to make agents backward looking on the trading price. The result is that market disseminates information better than it aggregates (Veiga and Vorsatz, 2008). Traders are unable to estimate the fundamentals properly. This misestimation comes only from the difficulty for the agents to estimate $\hat{Corr}$ and goes in the sense of DeMiguel et al. (2009), who prove that the gain by diversification is more than offset by estimation errors. However, all learnings tend to correct a part of the inaccurate information. As a consequence, the comovement of the computed FVs does not reflect the real comovement nor the public information. Due to its longer time to learn and its absence of belief learning component, RL highlights the less wrong correlation of the computed fundamentals. With FP and EWA, traders tend to play the best strategy and not necessarily the right one; meaning the one which allows to make the best forecast according to the information set. This might explain the absence of consensus in this setup. Unsurprisingly, the accuracy of expectations depends on the accuracy of the information source.

In the case of wrong private information, a partial access to an accurate public source allows agents to review their misunderstanding. When traders are backward looking on the trading price, RL is the learning which provides the greatest benefit from the public information. Based on market indicators, the fundamental value becomes intrinsic to the market whereas it must be intrinsic to the firm but exogenous to the market. The mixture of information makes the trading price more informative. The fundamentals comovements are better tracked. It also seems that traders earn more profit when they focus on the trading past price comovement even if their forecast is less accurate; further analyses need to be done.

In the case of accurate private information, sharing public information is useless. It is a waste of time and effort for the public source due to the small gain in precision.

The contribution of this model is to study learning ability of agents in different information accuracy setups. Since information is correct, agents are able to learn and properly forecast the fundamental values. Agents following RL require a much longer time to learn than agents following learning based on belief component. However, simple reinforcement strategy allows to forecast fundamental values with few efforts and learning information compared to EWA. When the private information source is less accurate, a public information sharing might be useful. Indeed, RL is the learning which provides the greatest benefit from the public information when the private one is inaccurate.

From this work, some recommendation may rise. Starting from the *ex post* fundamental value, much effort should be made in the direction of market transparency. Indeed, if agents are able to process information about firms, the market earns in efficiency. A rate agency is a possible example of an easy way to have accurate piece of information about them. However, it should be monitored so as to warranty its objectivity. Conversely, it is totally pointless to share information in an already informative market. So, when and how many pieces of information should be shared? Even if the quote "the backward looking does not need to be longer than the investment horizon" is awkward, it is relevant. Indeed, extending the backward looking means more information to collect and process, thus more cost (time and money) for a small gain in accuracy. Traders do not need to have access to the whole limit order book. In addition, too much information may make it difficult to compute. Public information should be revealed when the market shares wrong information. In this paper, it was assumed that agents believe in the public information, however, it is not necessarily the case. How to ensure the public information sharing? One way is thus to use the herding behaviour of traders. If an agency is strong enough to impact the market price, and if the agents are backward looking on the market price, the agent's belief might be partially controlled. The state may have incentive to take part in a firm, to avoid its bankruptcy. The signal will be observable in the trading price and used by the herding agents. However the nationalisation has also some perverse effect.

From a modeller point of view, RL mimics – with few freedom degrees – the belief perseverance and the reluctance to discover/try new strategies. RL may easily explain glaring disconnection between true and estimated fundamentals in the case of wrong information source. FP highlights the ability of agents to adapt their behaviour when they face inaccurate information. However, this learning – in which agents form beliefs on the basis of opponents' past decisions and tend to play strategies today that could have had high payoffs in the past – makes agents less bounded rational than the EWA learning. EWA tries to capture and isolate as many psychological features of statistical adaptive behaviour. It offers the most realistic basic learning type. It shows good results such as the ability of agents to learn and adapt their forecast with some belief perseverance. However, it is characterised by more degrees of freedom than RL or FG.

From now, we wish to explore the impact of this leanings on the trading price by making agents heterogeneous in their information access (private, public and mixed information) and processing (RL, FP and EWA).

Chapitre 3 : Un marché opaque

Motivation

Le marché de gré à gré est le marché à partir duquel la dernière crise financière (crise des *subprimes*) est née. Ce marché sert en effet à l'échange des dérivés de crédit. Or, la contagion aux marchés organisés s'est faite par la défiance aux systèmes bancaire puis financier susceptibles de détenir ces dérivés.

Jusqu'à maintenant, mon travail de recherche s'est orienté sur un marché dirigé par les ordres, où une partie de l'information est disponible de manière publique. En effet, les deux précédents chapitres se concentrent sur un marché où au moins une partie du carnet d'ordre est accessible à tout le monde et sans coût. Dans ces précédents modèles, le prix du marché dépend à la fois des croyances privées et de l'information publique disponible. En gardant à l'esprit que l'information privée joue un rôle crucial dans la perception du marché (Duffie et al., 2005, 2007; Duffie, 2012), le passage à une structure plus opaque s'inscrit dans la continuité. Dans cette structure de marché, nous admettrons que les prix d'échanges restent privés et peuvent être très volatils.

Actuellement, le comportement et la stabilité des marchés de gré à gré sont une problématique centrale des régulateurs. Cependant, peu de recherches expliquent comment la structure de ce type de marché détermine ses propriétés. La genèse de ce chapitre prend corps dans l'absence de travaux en modélisation agents sur les marchés de gré à gré. En effet, à ce jour, seuls les travaux de De Kamps et al. (2013) et Babus and Kondor (2013) se concentrent sur la diffusion de l'information dans cette structure de marché. Cela est d'autant plus surprenant que la littérature expérimentale s'est intéressée au marché de gré à gré de longue date (Chamberlin, 1948) et encore récemment (Duffie et al., 2005, 2007; Duffie, 2012; Attanasia et al., 2016). Cette littérature permet par le biais des données recueillies de calibrer et vérifier les comportements des futurs modèles agents.

Méthode

La contribution de cette étude réside dans la proposition d'une modélisation où chaque agent fait face à une information incomplète. Essentiellement, les modèles de négociations partent de l'hypothèse qu'au moins un des deux partis est parfaitement informé (Ausubel et al., 2002). Or, la valeur fondamentale est nécessairement idiosyncratique dans un monde peuplé d'agents à rationalité limitée (Walter and Brian, 2008). Cette hypothèse forte (parfaite information d'un des deux partis) va à l'encontre même de la perception du marché de gré à gré :

un marché à forte opacité, où se réalisent au même instant des transactions à prix différents. Nous nous proposons donc de traiter de la diffusion de l'information par le biais de la modélisation d'un marché de gré à gré avec un processus de négociation entre les agents où un seul actif risqué est échangé.

La structure d'échange est définie par le réseau des agents : les noeuds sont les agents et les liens représentent les opportunités d'interaction entre ces derniers (structure classique - Braun and Gautschi, 2006). Gardons à l'esprit que le marché de gré à gré est un marché décentralisé où les informations dispensées de manière publique sont faibles ou inexistantes. Dans le cadre du marché des contrats d'échange de taux d'intérêt, les prix d'échanges sont annoncés de manière publique et gratuite par une moyenne des cotations quotidiennes. Des services d'information spécialisés tels que Bloomberg L.P. ou Reuters proposent, via abonnement, de consulter de manière plus affinée le prix de ces contrats (information privée). Dans le modèle développé ci-après, l'information collectée par les agents est uniquement privée : aucune fixation de prix (*fixing*) n'est disponible sous forme d'information publique (De Kamps et al., 2013). Bien que discutable, ce choix de modélisation provient de l'objectif de ce chapitre qui est l'étude de l'extraction et de la diffusion de l'information résultant d'un processus de négociation lors d'interactions entre les agents. Ces interactions ont pour but de permettre à chaque agent de découvrir la valeur fondamentale et d'estimer le prix de marché. Cette négociation se justifie par le désir d'échanger au prix le plus avantageux si les agents parviennent à un accord, ou par celui d'extraire l'information la plus correcte si aucun accord n'émerge. Chacun tente donc d'exploiter la mauvaise estimation de la contrepartie ou de corriger sa propre information. La négociation se justifie aussi par le fait que tous les agents n'ont pas accès aux mêmes opportunités d'échange. Ils ne sont donc pas tous prêts à consentir aux mêmes efforts dans le processus de négociation (prise en compte du contrôle du réseau et du pouvoir de négociation).

En l'état actuel, ce modèle permet de :

- mettre en avant l'importance de la structure du réseau dans la propagation de l'information ;
- confirmer le manque d'efficience lorsque le marché autorise la diffusion de l'information ;
- proposer une modélisation arbitraire de la croyance dans le marché fondée sur le critère moyenne-variance.

Le travail entamé vise à estimer :

- Comment la position de l'agent (son contrôle du réseau) affecte-t'elle son comportement (émergence de *cluster*, position de gourou, probabilité d'échange) ?

- Comment le profit de l'agent peut-il être expliqué par sa position dans le réseau et sa connaissance des fondamentaux ?
- Comment les négociations passées influencent-elles les connexions entre agents dans un réseau dynamique ?

Discussion

L'objectif de cette recherche est de proposer une meilleure compréhension du marché de gré à gré afin d'aider le régulateur dans ses décisions futures.

Comme attendu, plus le marché est opaque et plus l'information aura du mal à se généraliser. Le réseau en anneau et le réseau avec attachement préférentiel génèrent un marché où les prix sont fortement volatils et où l'information ne se propage pas. En revanche, si l'on pousse l'opacité à son paroxysme et que le marché se compose d'un agent central avec ses satellites, réseau en étoile, le prix se stabilise, et l'agent central impose sa croyance.

En plus de la structure globale du réseau, il est important de considérer le comportement des individus à l'intérieur de leur propre *cluster* (relation de degré un). Plus le contrôle de l'agent est élevé, plus l'agent est central dans le réseau, et plus il pourra influencer ses contreparties. Les agents dont le contrôle est le plus élevé dirigent le marché, ils peuvent manipuler l'information. Cependant l'impact est relatif à la structure du réseau.

L'analyse de réseaux moins opaques, tels que le réseau complet ou le réseau aléatoire 1/2, met l'accent sur la capacité des agents à converger vers une valeur unique. Dans ces deux réseaux, l'information extraite se définit comme la moyenne de toutes les informations dispensées. Ainsi, même si des agents centraux et parfaitement informés (gourous avec information correcte) interagissent, leur information est pondérée par le poids qu'ils ont dans le réseau et les négociations. Or, dans ces deux réseaux, tous les agents ont un pouvoir de négociation semblable. Pour faire tendre le cours d'un titre vers sa valeur fondamentale par exemple, le prix d'échange du gourou ne doit pas être sa valeur fondamentale, mais une valeur modifiée qui tient compte de la perte d'information au cours des différents processus de négociation. Pour faciliter cette convergence du prix, il peut être judicieux d'avoir recours à plusieurs gourous à la fois tout en gardant à l'esprit qu'un bon niveau de contrôle du réseau assure une large audience. Cela confirme la capacité des agents à assimiler une information tout en l'altérant.

Enfin, l'étude du réseau petit monde montre les limites de l'intervention des gourous dans un marché. Le petit monde est un réseau dont l'opacité se situe entre le réseau aléatoire 1/2 et le réseau par attachement préférentiel. Il confirme l'augmentation du temps nécessaire à la diffusion de l'information dans le marché. Ainsi, sans gourou, ou lorsque les gourous ont une information qui n'est pas trop éloignée des autres agents, une convergence est possible. En revanche, ce type de réseau démontre qu'il ne suffit pas de diffuser une information au plus grand nombre pour qu'elle soit prise en compte par le marché. En effet, lorsque les gourous ont une estimation qui va à l'encontre des autres agents, les prix échangés deviennent très volatils. Les agents (non-gourous) préfèrent abandonner toute confiance dans le marché et rester sur leurs croyances initiales. Le partage de l'information échoue, et génère une augmentation de la volatilité du marché.

3. Diffusion of information in Over The Counter markets : an ABM model with bargaining on networks (WP)

3.1. Introduction

The idea of changing prices depending on the situation makes dynamic pricing superior to fixed pricing. Many forms of dynamic pricing exist; however the present work is focused on Over The Counter markets with bargaining on networks. Neoclassical economics boast of many bargaining models (Nash, 1950; Rubinstein and Wolinsky, 1985; Coase, 1972). One basic question is to know whether private information prevents the bargainers from reaping all possible gains from trade. However, the underlying assumptions of all these models render them impractical for real-world applications. Indeed, classical bargaining models assume the availability of complete information about negotiators, unlimited computational resources and mutual incentive/benefit to reach an agreement. In the real world, these assumptions do not apply because bidding strategy of buyers is not known by the sellers, even the utility value of a buyer cannot be estimated.

To the best of our knowledge, no or few models treat a two-sided incomplete information in an Over The Counter market (Ausubel et al., 2002). Lau et al.'s idea (2004) of adaptive negotiation agents does not allow traders to specify the walk-away price from which they prefer to leave the bargaining table. Babus and Kondor (2013) do not deal with heterogeneous information, while De Kamps et al. (2013) define the exchanged quantity and the trading price by the intersection of the demand function of the traders involved.

Over The Counter markets allow agents to trade assets directly between each other, rather than through centralised exchanges. In these markets, investors buy or sell directly from dealers. However, each customer may only know a subset of the dealers within the market, limiting their ability to observe the best price. The dealers themselves may only interact with a subset of the other dealers. In the most liquid Over The Counter markets (*e.g.* currency exchange markets and swap markets), a daily fixing is provided. The market considered in this paper contains heterogeneous traders who base their trading decision on their valuation of the asset. As in De Kamps et al. (2013), we assume that there is centralised information neither on the trading price nor the market price (no daily fixing). All agents face incomplete information; thus the walk-away price is not the fundamental value for rational agents and a random price for irrational agents as in Abreu and Gul (2000). The acquisition of information is a step-by-step process of inference from private information (collected during the bargaining process). Based on the idea that the relative power of the agent's network matters for bargaining (Easley and Kleinberg, 2010; Braun and Gautschi, 2006), we offer a model in which the walk-away price is not only defined by the current market price and the computed fundamental value, but also defined by the agent's network control. Indeed, agents prefer to sell (buy) at a reasonable lower (higher) price than their desired price, rather than not to trade. As it is commonly observed in several double auctions markets structures, customers that refuse a deal with an agent, usually shop around to find other offers. Indeed this is regarded as the main motivation for refusal in standard search models. Weisbuch et al. (2000) introduce an alternative explanation which is that customers refuse deals now in order to induce better offers in the future. In our model, the randomly chosen trader bargains with her trading contacts in turn and when an agreement is reached, exchanges (as in De Kamps et al., 2013).

At the initialisation, all traders start with private information about the fundamental value and some trust in it. Then, the trust parameter is revised across time and adds heterogeneity in the computed fundamental value. Informational differences provide an appealing explanation for bargaining inefficiency.

Knowing that information extraction during the bargaining process plays a key role (Hommes, 2006), traders face a trade-off between the exploitation of old knowledge (their past experiences) and the acquisition of new knowledge (information collected during the bargaining). In this work, two agents type are distinguished: gurus and non-gurus, also called others. Let us assume that gurus have homogeneous information that allows them to compute with confidence the fundamental value while others deduce the fundamental value from their bargaining.

In real Over The Counter markets, traders could trade multiple units at different prices based on their preferences and bargaining power. In addition, the offers are limited in time and the dealers may change their mind. The model is designed to be as simple as it can get and focuses only on information diffusion. It does not consider wealth or net positions (like Chiarella and Iori, 2002; De Kamps et al., 2013). Modelling the limit life time and the wealth would require further assumptions regarding individual preferences which would complicate it. It is also well known that limited arbitrage opportunities can exist in competitive markets due to market frictions (Duffie, 2001; Lamont and Thaler, 2003; Xiong, 2001), but they can't happen in this model.

We shall be interested here in information extraction during bargaining process, in particular, in markets in which transactions are not made public, and where no prices are posted. In such markets agents have to rely on their own information. We limit our attention to situations in which individuals have to rely on their own experience and do not observe that of others directly.

In Over The Counter markets, how does the network structure affect the information propagation?

By studying the difficulties that parties have in reaching mutually beneficial agreement, this paper confirms that the price stability is linked to the market opacity.
• The information diffusion is effective in the complete network, random network, small world network and star network but not informationally efficient, as Babus and Kondor (2013).
• In the complete and the random networks, the information diffusion is quick and the convergence emerges on the average initial expectation. When gurus are present in a relevant amount, they may influence the market in their way.
• In the small world network, the market is more opaque. The time needed by the market to process the information is definitely longer. In addition, when gurus' information is too far from the expectation of the others, the prices collected by the others are too noisy to be properly used. As a result, the others prefer to stick to their initial signal, they do not trust in the market.
• In the star network, the central agent drives the market (expected fundamental value and trading prices), no matter the expectations of the agents. The gurus are not able to influence the belief of the central agent.

The paper is organised as follows: Section 2 reviews the existing literature. Section 3 outlines the structure of our model. Section 4 brings and discusses results and Section 5 concludes.

3.2. Related Literature

Since the first models (Nash, 1950; Rubinstein and Wolinsky, 1985; Coase, 1972), the bargaining literature has improved to be more realistic. Some studies already highlight the importance of time horizon in the strategic behaviour and the efforts to reach an agreement. Hörner and Samuelson (2011) set with finite horizon and no discount rate that a seller makes unacceptable offers until the last moment and then makes the monopolist offer. Fuchs and Skrzypacz (2013) deal with the discount rate: delay is costly, all offers are serious and trade takes place before the deadline with positive probability.

In relation to double auction market, some researchers have explored the empirical puzzles of stylised market facts. Rouchier and Robin (2006) create and analyse a model of adaptive learning and demonstrate that such a model can capture the bidding patterns among human subjects in experimental auctions. LeBaron (2006) replicates empirical facts from financial data. He argues that

Table 3.1.: Model comparison - Babus and Kondor (2013).

Them	Us
- many bilateral transactions	- sequential trade of a single unit
- information extraction over time	- information extraction from bargaining
- bargaining on price and quantity	- bargaining on price
- one shot game	- discrete game
- wealth	- trust in private information (time evolutive)
- heterogeneous information - focus on information aggregation - agents know and sufficiently trust each other to trade in case they reach a mutual agreement	

agent-based model approaches make more sense economically than their representative agent model counterparts. Kirman and Vriend (2001), Weisbuch et al. (2000) and Moulet and Rouchier (2008) have worked on reproducing the behaviour of agents in Marseille wholesale markets and highlight a notion of loyalty to the seller[a]. They build an Agent-based Computational Economics model and compare the macro-system with the stylised facts derived from empirical data. Duffie et al. (2007) provide the impact on asset prices of search-and-bargaining frictions with 1-sided incomplete information based on Rubinstein and Wolinsky's model (1985).

To the best of our knowledge, no or few models treat a two-sided incomplete information in an Over The Counter market (Ausubel et al., 2002). Related to the OTC market structure, we can mention the working paper of De Kamps et al. (2013), in which they consider a market populated by heterogeneous boundedly rational agents. In this few-types market (chartist and fundamentalist) with one-sided incomplete information, the volume of exchange and the trading price are defined by the intersection of the demand function of the traders involved. Similar to our work, with two-sided incomplete information, Babus and Kondor (2013) deal with information diffusion. In their model, agents with private information can engage in many bilateral transactions at a time (see Table 3.1 for a quick comparison between their model and ours).

A central question in economics is understanding the difficulties that parties have in reaching mutually beneficial agreements. Myerson and Satterthwaite (1983) find that *ex post* efficiency is attainable if and only if it is common knowledge that gains from trade exist; that is, uncertainty about whether gains are

[a]In these models, the sellers are completely informed about their asset valuation or quality while the buyers are incompletely informed (one-sided incomplete information model).

possible necessarily prevents full efficiency. The mutual gain from the trade is based in our case on the computed fundamental value. Traders start with a private signal and adapt their trust in it according to the market behaviour based on a mean-variance criteria (Markowitz, 1952). To the best of our knowledge, no literature focuses on the trust of an agent on some information without backward looking. Be aware that many definitions of trust exist. According to Gambetta (2000), it is "the subjective probability with which an agent assesses that another agent or group of agents will perform a particular action". Dasgupta (2002) assumes that the trust plays a role in all the situations when "others know something about themselves or the world, which the person in question does not, and when what that person ought to do depends on the extent of his ignorance of these matters". In this work, we refer to our definition: the trust is the agent's belief about the accuracy of her initial private information. Usually, the main known rule-of-thumb type of social leaning process is DeGroot (1974). Golub and Jackson (2012) examined how the speed of learning and best-response processes depend on homophily. However, in their setup, a mutual agreement is reached for groups of agents. In the same spirit, the relative agreement interaction model (Deffuant et al., 2002) which is an extension of the Bounded Confidence model (Deffuant et al., 2000) may not be applied. In our model, we are not looking for a consensus, each agent should decide in which value she prefers to trust without revealing her own private information (Kyle, 1985). The learning based on past experience such as Reinforcement Learning (Roth and Erev, 1995), Fictitious Play (Brown, 1951) or Experience-Weighted Attraction Learning (Camerer et al., 2002) also seems useless; our agents are not able to compare their value to some past observable value (such as daily fixing).

3.3. Model

We analyse the diffusion of information through a network in Over The Counter markets where a unique risky asset is traded. This market type is therefore less transparent than usual double auction market. Dealers, who are bounded rational, have no access to an order book in which prices are quoted. In this opaque market structure, the computed prices are mainly based on information collected during the recent bargaining, the expectation of the forward fundamental value ($\hat{f}^i_{t+1}$), the network control (c_i) and the confidence in an initial source of information (α_i). Furthermore, the trade can be executed between two participants without others being aware of the price at which the transaction was effected. Thus, different traders within the market may have access to different trading opportunities and information sets, and hence may have different valuations.

In each round, the model proceeds as follows:

1. An agent is randomly picked up as the "active trader".

2. The active trader updates her expectations: the market prices (all the collected bids and asks since she was lastly selected as an active trader), the current fundamental value, her limit prices[b] and her trust in her initial source of information. The market bid and the market ask are defined by the previous bargaining of each agent and are not quoted in any order book. No daily fixing is provided, thus all the values are private information.

3. The active trader bargains sequentially with each of her counterparts in turn (direct neighbours in the network). Their bilateral relationship is strategic. Be aware there are two gathering of opinions, one to buy a unit of the asset and another to sell it.

4. Each agent who interacts (the active trader and her counterparts) extracts some private information from the bargaining. When an agreement is reached, the information is the trading price. When no agreement is reached, each agent collects the limit price of her bargainer.

By definition, each agent offers in any time an ask price weakly greater than her bid price. Thus, the counterparts cannot face to arbitrage opportunity and exploit it (see section 3.3.2 for more details).

3.3.1. Network structure and agent's power

The Over The Counter market is represented as an undirected graph where traders are nodes and trading connections are edges. Let $n \times n$ adjacency matrix A with main diagonal elements $a_{ii} = 0$ for all i and off-diagonal elements $a_{ij} = \{0; 1\}$ represent the exogenously given and symmetric bargaining relations. More specifically, $a_{ij} = a_{ji} = 1$ in presence of a mutual edge between the agents i and j otherwise it equals 0. These symmetric relations limit the matches of potential partners for negotiations and exchanges. The centrality measure of agent i is deduced from the A matrix such that:

$$cm_i = \sum_{k=1}^{n} \frac{a_{ik}}{n} \tag{3.1}$$

When $cm_i = 1$, agent i is connected to the whole network. Conversely, when $cm_i \to 0$, the agent tends to have no connection, thus few neighbours. Without loss of generality, the A matrix should be standardised. Let D be the $n \times n$ matrix of standardised agent relations such that $d_{ii} = 0$ for all i, $d_{ij} = a_{ij}/\sum_{k=1}^{n} a_{kj}$ and $\sum_{k=1}^{n} d_{kj} = 1$ for all j. The off-diagonal element d_{ij} represents i's degree of "control" over the relations to j in the system. Specifically, it holds $d_{ij} \in [0, 1]$,

[b] The limit prices are: the maximum price she agrees to buy one unit of the asset ($\hat{bid}^i_{t+1}$) and the minimum price she agrees to sell it ($\hat{ask}^i_{t+1}$).

where $d_{ij} = 0$ indicates that i has no link with j and so no control, and $d_{ij} = 1$ indicates that i is the only bargainer of j and so i has complete control over the$j$.

Following the formalism of Braun and Gautschi (2006), the i-th row of the matrix D informs about i's control over each other agent in the system, whereas the i-th column of D informs about each other's control over i. Adding up the relevant pairwise elements of D defines the "network control" of agent i.

$$c_i = \frac{1}{n_i} \sum_{k=1}^{n} d_{ik} = \frac{1}{n_i} \sum_{k \in S_i} \frac{1}{n_k} \tag{3.2}$$

Where n_i denotes the number of i's counterparts and S_i denotes the agent set of direct bargaining relations of agent i. Put it in words: i's network control c_i reflects the degree to which i controls the relations to him by his relations to others. Concretely, $c_i = \frac{3}{4}$ expresses that the average number of counterparts of i's partners is $4/3$. When $c_i = 0$, i has no counterparts. When $c_i = 1$, i is the central agent of a star network, meaning i is connected to everybody and i's partners have no other counterparts. When the assumption is made that i knows her network control, it is also postulated, that i takes account of the mean number of counterparts of her counterparts (Braun and Gautschi, 2006). This assumption does not imply a perfect knowledge of the network structure by each agent. Agent i only knows the number of counterparts of her own counterparts.

As Braun and Gautschi (2006) about bargaining power, we define individual bargaining power as the negative reciprocal of a logarithmic expression.

$$b_i = -1/\ln(c_i.w) \quad \text{, with} \qquad w = \frac{n+m}{n+m+1} \tag{3.3}$$

w is a network-specific fraction which rises with the number of mutual edges in the network, m, and the number of network members, n. The weight w scales the degrees of network control such that b_i is always a positive number.

Five types of networks are considered in this paper:

- a random network with an average density of $1/2$;
- a complete network;
- a small world network;
- a preferential attachment network;
- a star network.

The random network $1/2$ is a network in which all agents have a connection probability of being connected to each other equal to 0.5. The random network is generated by using the $G(n, p)$ variant of Erdős-Rényi random graph model

(Erdős and Rényi, 1959). The complete network is generated according to the previous Erdős-Rényi model with a probability of being connected to each other equal to 1. The small world network introduced by Watts and Strogatz (1998) is created by adding additional links between the nodes in a lattice network. The algorithm provided in this paper is an adaptation of Watts-Strogatz model by Kleinberg (2010), who "introduces one extra quantity that controls the scales spanned by the long-range weak ties". Loosely speaking, the basement of our small world is a lattice network (as in Watts and Strogatz, 1998), in which each of the additional random edges is generated in a way that decays with distance. This distance is controlled by a "clustering exponent" usually set up to 2, as in this paper. The preferential attachment is generated using the Barabási-Albert algorithm (Albert and Barabási, 2002). Each node is built with one edge; it means that each node is at least linked with another one. The more edges a node has, the greater the probability that this node attracts new edges, when they are added. The star network might be considered as an extreme case of the preferential attachment network in which a central agent attracts all edges.

3.3.2. Prices

The market prices are not public values given by a specialist or an electronic limit order book. When the agent is picked up, she computes her own valuation of the current market bid ($\hat{p}_t^{i,bid}$) and ask ($\hat{p}_t^{i,ask}$), the fundamental value ($\hat{f}_t^i$) and the limit prices she prefers to cease negotiation, so called walk-away prices ($\hat{bid}_t^i$ and $\hat{ask}_t^i$).

Current market prices

Once the active trader has established her valuations, she bargains with each of her trading contacts in turn. When the active trader meets a counterpart, their relation is strategic. Each agent extracts from the others some private information: the walk-away price ($\hat{bid}$ or $\hat{ask}$) in case of negotiation failure and the trading price ($p_{i,j,t}$) otherwise[c].

Thus, the computed current market prices ($\hat{p}_t^{i,bid}$ and $\hat{p}_t^{i,ask}$) depend on the features of the active trader and her counterparts.

$$\begin{cases} \hat{p}_t^{i,bid} = \hat{p}_{t-\delta}^{i,bid} + (1-c_i)\left[\bar{p}_{t-\delta,t}^{i,ask} - \hat{p}_{t-\delta}^{i,bid}\right] \\ \hat{p}_t^{i,ask} = \hat{p}_{t-\delta}^{i,ask} + (1-c_i)\left[\bar{p}_{t-\delta,t}^{i,bid} - \hat{p}_{t-\delta}^{i,ask}\right] \end{cases} \tag{3.4}$$

Let assume that δ is the previous period when agent i was picked up as an active trader; $\bar{p}_{t-\delta,t}^{i,bid}$ ($\bar{p}_{t-\delta,t}^{i,ask}$) is the average of all collected bids (asks) in the interval $[t-\delta; t]$. The current market prices are thus defined as an exponential moving

[c]Both of them depends on the agent's network control, defined in Eq. (3.2).

average. When $c_i \to 1$, the active trader is central. She assumes the market price is mainly driven by her trades. She weakly takes into account the counterparts' market price. When $c_i \to 0$, the active trader is peripheral. She is weakly connected, or her neighbours are as strongly connected as her. As a result, the market is not under the influence of agent i. She strongly revises her market valuation so as to trade. Be aware that the introduction of the bargaining makes the collected price noisy, meaning it reflects neither the computed market price nor the expected fundamental value, but something in between.

To avoid arbitrage opportunity, traders always offer a buying walk-away price lower than their selling walk-away price. Thus, when the active trader faces a computed current market bid price higher than her computed current market ask price, she assumes that her collected prices are biased. In order to fix it, when she reaches more agreement in the buying way, she assumes that her buying price is too high and replaces her computed current market bid price by $\hat{p}_t^{i,bid} = \hat{p}_t^{i,ask} - |\mathcal{N} \sim (0,1)|$. When she reaches more agreement in the selling way, she assumes that she sells too low and replaces her computed current market ask price by $\hat{p}_t^{i,ask} = \hat{p}_t^{i,bid} + |\mathcal{N} \sim (0,1)|$.

Fundamental price

The fundamental value f is constant across time. The agents are imperfectly informed and revise their estimation of the fundamental value across time. Each trader starts with a private signal (f_0^i) and some trust in it (α_0^i). At the initialisation, the fundamental value is set up according to a Normal Inverse Gaussian distribution following the Rydberg's algorithm[d] (1997). α_0^i is a given value normally distributed. Later, the agent updates her trust in the initial private signal according to a mean-variance criteria.

$$MV_t^i = \frac{|\bar{p}_{t-\delta,t}^i - f_0^i|}{\sigma_{p_{t-\delta,t}^i}} \tag{3.5}$$

Where $\bar{p}_{t-\delta,t}^i$ and $\sigma_{p_{t-\delta,t}^i}$ are respectively the mean and the variance of all collected bids and asks since agent i was lastly chosen as an active trader. Thus, the mean-variance criteria is defined by the absolute difference between the mean of the collected prices and the initial private signal divided by the variance of the collected prices. The absolute difference makes this parameter never negative.

[d]The NIG process is a pure-jump Levy process with infinite variation, which has been used successfully in modelling the distribution of stock returns on the German and Danish exchanges. In the limiting case when the variance of the subordinator is set to zero, the NIG process coincides with Brownian motion and the probability density is normal.

$$\alpha_t^i = \begin{cases} 1 & \text{, if } \quad |\overline{p}_{t-\delta,t}^i - f_0^i| = 0 \\ 1 & \text{, if } \quad \sigma_{p_{t-\delta,t}^i} = 0 \text{ and } |\overline{p}_{t-\delta,t}^i - f_0^i| < 5 \\ 0 & \text{, if } \quad \sigma_{p_{t-\delta,t}^i} = 0 \text{ and } |\overline{p}_{t-\delta,t}^i - f_0^i| \geq 5 \\ \dfrac{ln(1+MV_t^i)}{MV_t^i} & \text{, otherwise.} \end{cases} \tag{3.6}$$

The trust α is defined on $[0;1]$. When $\alpha_t^i \to 1$, agent i strongly trust in her private information. This happens for two reasons, either the market is highly volatile (high variance) and so she assumes that the market is not informative, or the spread between the computed market price and her initial information is low and thus she assumes that her private signal is right. When $\alpha_t^i \to 0$, the trader prefers to use the market as an information source, she does not trust in her private signal. This happens when the market has integrated some information that all agents agree on (low variance) and this information is not in her private signal (high spread). Thus, she assumes that the market is more informative than her own private signal (f_0^i). Be aware that if MV_t^i is not defined or null, α is also not defined. In this situation, three cases that keep the previous logic should be distinguished. When MV_t^i is null (first row of Eq. 3.6), it means that the private signal (f_0^i) is equal to the information extracted from the market. So, the trader strongly trusts in her private information, $\alpha_t^i = 1$. When MV_t^i is not defined (second and third rows of Eq. 3.6), the variance of the market is null. If the spread is lower than 5 (second row), the trader sticks to her private signal, $\alpha_t^i = 1$; otherwise $\alpha_t^i = 0$, she assumes that her private signal was wrong[e].

The fundamental value is a weighted average between the private signal and the collected market prices according to the trust parameter:

$$\hat{f}_{t+1}^i = f_0^i + (1-\alpha_t^i).[p_{t-\delta,t}^i - f_0^i] \tag{3.7}$$

walk-away prices

The agents define their limit prices above and below they prefer ceasing negotiations and walking away from the bargaining table. These limit prices are respectively the bid and ask of each agent. They are updated per round.

$$\begin{cases} \hat{ask}_{t+1}^i = \hat{p}_{t+1}^{i,bid} + (1-c_i).(\hat{f}_{t+1}^i - \hat{p}_{t+1}^{i,bid}) \\ \hat{bid}_{t+1}^i = \hat{p}_{t+1}^{i,ask} + (1-c_i).(\hat{f}_{t+1}^i - \hat{p}_{t+1}^{i,ask}) \end{cases} \tag{3.8}$$

The more the agent has a strong network control ($c_i \to 1$), the less she agrees to

[e]The value is arbitrarily fixed. The idea is to respect the belief perseverance of agents (Lord et al., 1979).

Table 3.2.: Parameters.

Number of rounds:	$5,000$
Number of runs:	50
Number of agents:	225
Pricing grid width:	10^{-3}
Network matrix:	A
Matrix of standardised actor relations:	D
Centrality Measure:	cm_i
Bargaining power:	b_i
Relative willingness to concede:	$B_{i,j}$
Trust parameter:	$\alpha_0^i \sim \lvert\mathcal{N}(0.5; 0.1)\rvert$
Private information:	$f_0^i \sim NIG(f; 0.1; 0; 300)$
NIG mean	$\bar{f}_0 = f$
NIG standard deviation	$\sigma_{f_0} = 55$
True fundamental value:	$f = 1000$
Computed current market bid:	$\hat{p}_0^{i,bid} = f_0^i - \lvert\mathcal{N}(0; 10)\rvert$
Computed current market ask:	$\hat{p}_0^{i,ask} = f_0^i + \lvert\mathcal{N}(0; 10)\rvert$
Walk-away bid:	$\hat{bid}_{t+1}^i$
Walk-away ask:	$\hat{ask}_{t+1}^i$
Expected fundamental value:	$\hat{f}_t^i$
Trading price:	$p_{i,j,t}$

move away from her computed fundamental value ($\hat{f}_{t+1}$). The opposite is also true, the more the agent has a weak network control ($c_i \to 0$), the more she agrees to trade at her valuation of the market price ($\hat{p}_{t+1}^{bid}$ or $\hat{p}_{t+1}^{ask}$). The lower is the network control of the agent, the less she has opportunity to trade. Be aware that the current market prices, defined in Eq. (3.4), could be compared to a kind of fixing price at which traders may quickly find a counterpart in their network. Table 3.2 summarises the main parameters and their initial value.

3.3.3. Meeting and bargaining

Once all prices are defined, the active trader bargains sequentially with all of her counterparts in turn. The active trader offers to each counterpart to buy a unit of the asset and to sell another one. The beliefs stay unchanged until the end of the round; the active trader does not update her expectation when she switches from one bargainer to another. When the agents meet, they face one of the two cases (see also Figure 3.1):

1. The bid (ask) of the active trader is lower (higher) than the ask (bid) of the counterpart: $\hat{bid}_{t+1}^i < \hat{ask}_{t+1}^j$ ($\hat{ask}_{t+1}^i > \hat{bid}_{t+1}^j$), the walk-away prices do

not match. No trade occurs. Each agent collects the walk-away price of her counterpart.

2. The bid (ask) of the active trader is weakly higher (lower) than the ask (bid) of the counterpart: $\hat{bid}^i_{t+1} \geq \hat{ask}^j_{t+1}$ ($\hat{ask}^i_{t+1} \leq \hat{bid}^j_{t+1}$), a trade is feasable. A bargaining process runs until an agreement is reached between the two agents. The bargaining price ($p_{i,j,t}$) depends on the bargaining power of both agents.

$$p_{i,j,t} = \begin{cases} \hat{ask}^j_{t+1} + B_{j,i}|\hat{bid}^i_{t+1} - \hat{ask}^j_{t+1}| & \text{, for i buyer.} \\ \hat{ask}^i_{t+1} + B_{i,j}|\hat{bid}^j_{t+1} - \hat{ask}^i_{t+1}| & \text{, for i seller.} \end{cases} \tag{3.9}$$

Let's call $B_{i,j} = b_i/(b_i + b_j)$, the relative bargaining power of the agent i. When each agent has the same bargaining power ($b_i = b_j$), they make both the same effort ($B_{i,j} = B_{j,i}$). When $b_i \neq b_j$, the stronger is the network control, the stronger is the bargaining power, the more the agent would trade at a price close to the walk-away of her counterpart (see Eq. (3.3) for bargaining power expression). In some way, the information extracted by the stronger agent is more accurate than the information extracted by the weaker agent, because it better reflects the walk-away price of her counterpart.

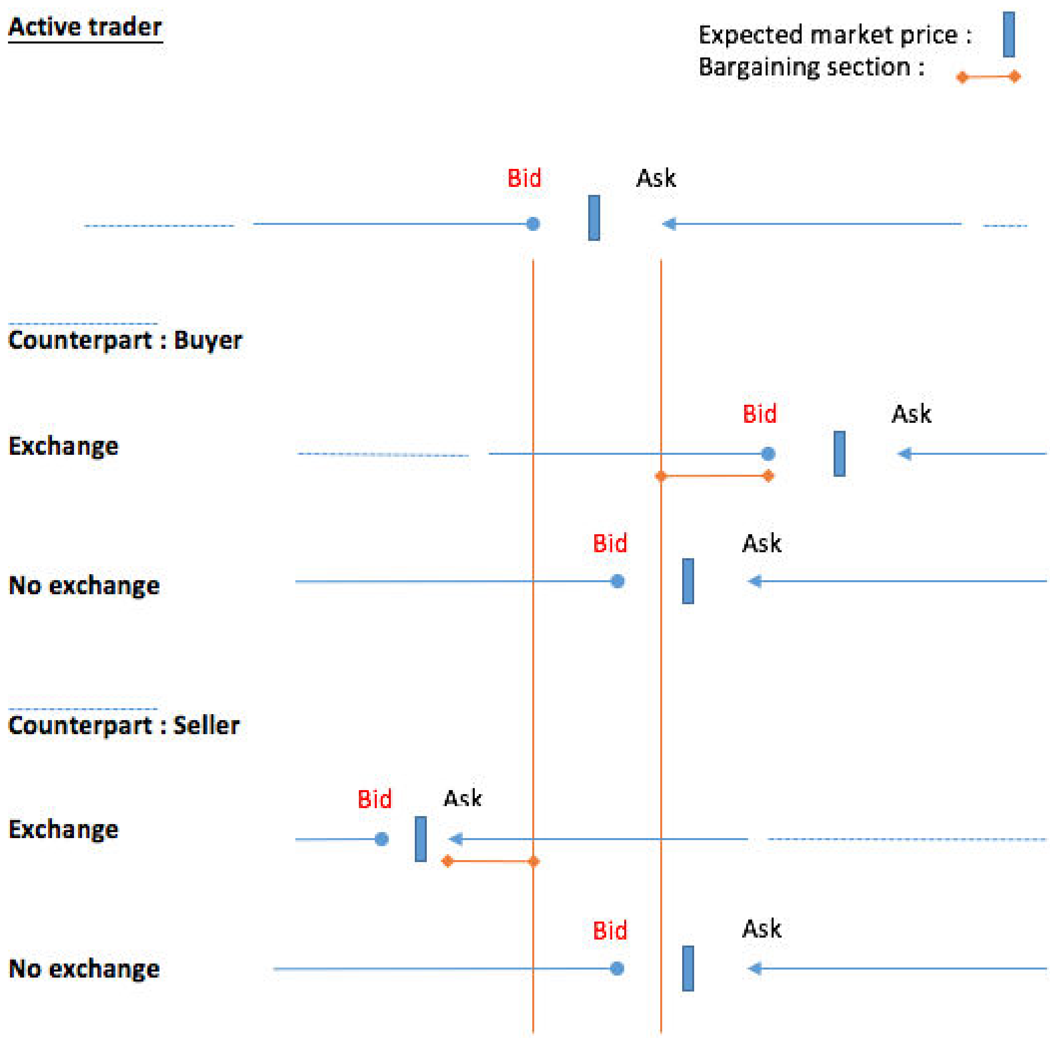


Figure 3.1.: Feasable cases according to the position of the active trader.

3.4. Simulations and discussion

This section focuses on descriptive statistics so as to understand the market behaviour in different network setups. We distinguish the complete network, the random network with the average density of $1/2$, the star network, the small world network and the preferential attachment network (see Table 3.3). Moreover, we draw a distinction between market within and without gurus. Let us assume that the gurus have homogeneous information that allows them to compute the fundamental value with confidence. All gurus share the same private information, $\hat{f}^{gu} = \{f; 2f\}$ and fully trust in ($\alpha_0^{gu} = \alpha^{gu} = 1$). This work is the first step in which we tackle: how do the type of network and the network

Table 3.3.: Network setup.

Network types:	• Complete network and random network 1/2 *based on* Erdős and Rényi (1959) • Small world network *based on* Kleinberg (2010) • Star network *no specific algorithm* • Preferential attachment network *based on* Albert and Barabási (2002)
Number of gurus:	$\{0; 1; 5; 50; 113\}$
Gurus' private information:	$f^{gu} = \{f; 2f\}$
Gurus' trust parameter:	$\alpha^{gu} = 1$

control affect the information propagation? The next step will be to perform econometrical regression in order to explain the trader's profit by a mixture of network control (c_i), computed current price ($\hat{p}$) and fundamental information ($\alpha^i; f_0^i; \hat{f}_t^i$), and identify the impact of each component.

3.4.1. Complete network

In a complete network, all agents are connected to each other. Thus, they are all involved in a bargaining process per round and they all share the same bargaining power (see Figure 3.3). This allows them to collect a huge amount of information.

The average deviation of the computed fundamental value highlights the information sharing process. It starts from 50 on the interval $[1; 1000]$ and converges to 3.29 on the last time interval $[4001; 5000]$. The standard deviation of the last interval is under 0.06, meaning that the deviation of the computed fundamental value is stable. The convergence is relatively quick. On the interval $[1001; 2000]$ the deviation is already divided by two and on the interval $[2001; 3000]$ the deviation is nine times lower than on the first time interval. On the interval $[3001; 4000]$, the deviation of the computed fundamental value is equal to 3.58. The information is mainly integrated by the traders, the deviation inside the interval does not rise (the standard deviation is around 0.29).

A consensus is reached on the market accuracy: more than 90% of traders strongly use their computed market price to estimate their fundamental value ($\alpha \leq 0.3$). On average, the trust parameter α is equal to 0.14 (std. dev 0.13). Only 2% of traders stick to their initial private information.

Traders agree to follow the market price. However, is the market price really accurate and similar for all? The standard deviation of the market price computed by the active trader is always lower than 1 (first time interval excluded). Thus, the computed market price is similar for all agents. The buying and selling prices stay in a small spread ($f \pm 5$). The computed price reflects the fundamental value. It needs a time interval to be stable (1000 rounds), and the computed fundamental value needs at least two more times. The convergence requires two steps: 1/traders make the market informative (convergence of the computed buying and selling prices); 2/traders trust in their computed price so as to define their fundamental value. As a consequence, the average of the distribution of the computed fundamental value is similar to the average of the distribution of the initial private signal. The deviation of the distribution of the computed fundamental value is equal to 3.273 on the last round of the simulation[f]. Be aware that when the spread between the computed fundamental value decreases, the probability to find a counterpart who agrees to deal with decreases from 100% to 54% (see Table 3.4). Buying and selling orders are similar, no disequilibrium appears in the market.

Adding to this market a guru does not change drastically the market dynamics. The deviation of the computed fundamental value on the last thousand rounds stays the same. The main change is the probability to find a counterpart who wants to buy. Indeed, it stays equal to 1 when the guru's private information is $f^{gu} = 2f$. By definition, the average of the others' initial private information (non-gurus) is set to 1000, so the guru always agrees to buy. All traders have the same bargaining power (Table 3.3), a unique agent has not enough weight to change the price dynamic. Thus, the computed market price and the computed fundamental value are similar to the case with no-guru.

Increasing the ratio of gurus from 0.5% (1 guru) to 2% (5 gurus), 22.2% (50 gurus) or 50.2% (113 gurus) impacts the market dynamics. When the guru's private information is accurate ($f^{gu} = f$): the higher the ratio of gurus is, the quicker the convergence among the computed fundamental value and the lower the final deviation of the others (2.21 for $22,2\%$ and 1.34 for $50,2\%$) are. The less noisy is the information extracted from the bargaining, the more accurate is the computed fundamental value. Gurus also make the probability to find a counterpart to trade higher: it rises from 0.55 to 0.8.

When gurus have private information that goes against the others (non-gurus), the others still believe that the market price is the most accurate source of information. The ratio of agent with $\alpha \leq 0.3$ is similar to the ratio with no-guru. However, the consensus does not emerge on the gurus' private information ($f^{gu} = 2f$). The computed market price becomes a weighted average between gurus' private information and others' initial private information. When the gurus are 50.2%, the computed market price should be around 1600. It exactly equals 1605.1 (std. dev. 1.85), and the consensus for the computed fundamental value is at 1596.6 (std. dev. 1.88). The standard deviation of the market price is small, the price fluctuates in a tiny spread. The deviation of the computed

[f]According to the NIG parameter setup, the standard deviation of the distribution of the initial private signal is 55.

Table 3.4.: No Guru - Probability to find a counterpart.

	Interval	Comp.	Rand.	Small	Star	Pref. Attach.
who buy	$[1;1000]$	1.00	1.00	0.95	0.47	0.56
	$[1001;2000]$	0.98	0.95	0.94	0.56	0.53
	$[2001;3000]$	0.70	0.64	0.93	0.47	0.50
	$[3001;4000]$	0.55	0.55	0.91	0.43	0.47
	$[40001;5000]$	0.54	0.55	0.90	0.55	0.48
who sell	$[1;1000]$	1.00	1.00	0.94	0.45	0.56
	$[1001;2000]$	0.94	0.94	0.94	0.55	0.51
	$[2001;3000]$	0.67	0.64	0.93	0.47	0.50
	$[3001;4000]$	0.55	0.56	0.92	0.44	0.46
	$[40001;5000]$	0.54	0.55	0.90	0.52	0.47

fundamental value is also small (2.05), there is a convergence of expectations. In the same spirit, when the gurus are 22.2%, the computed market price is around 1333.4 (std. dev. 1.31), and the consensus for the computed fundamental value is 1322.6 (std. dev. 1.35). The deviation of the computed fundamental value is 1.25. Be aware that the trading price does not converge to the same value each time, but to a spread. The impact on the computed fundamental value is $+4.44$ for a guru, however, the standard deviation makes it non-significant.

We run the same simulations with a random network with the average density of $1/2$. In no-guru setup, the values are similar to the complete network, the same dynamics arise. In guru setup, we distinguish two cases: "strong" and "weak". Strong means that the gurus are the agents who are the strongly connected, while weak means they are the weakly connected. Since the gurus are strongly connected, the computed market price is impacted as previously seen for the complete network, and the computed fundamental value follows it (α tends to 0). When the gurus are weakly connected, the average computed prices are similar, no matter the gurus' private information (f^{gu}). In this case, the gurus have not enough power to make the price deviate from its long-term path. However the volatility of the price is two times higher for 1 guru and three times higher for 5 gurus on the interval $[1;1000]$. Weakly connected gurus disturb the market on short-term. Notice that an increasing volatility is observed in all setups for both networks. On the first time interval, agents trust in their own initial signal to compute their fundamental value. Once they have collected enough information from the market, they start to use it as the accurate source of information.

3.4.2. Small world network

In a small world network, all agents are connected to each other through six degrees of separation. Thus the information needs time to be shared in the whole network. The histogram of the distribution of the bargaining power may be observed in Figure 3.3. On the interval $[4001; 5000]$, the deviation of the distribution of the computed fundamental value is equal to 22.4 (std. dev. 0.58). The deviation of the spot price is lower than 6. By increasing the duration of the simulation, we confirm that the small world network needs two more times to end up the information aggregation ($10,000$ rounds). At the end, on the interval $[10,001; 11,000]$, the deviation of the computed fundamental value is stable, around 8. The deviation of the spot price stays around 6. However, we perform our analyses on the time interval $[1; 5000]$. On average, the trust parameter is equal to 0.44 (std. dev 0.24). 34% of agents have a trust parameter lower or equal to 0.3 and 20% of agents have a trust parameter higher or equal to 0.7. On long-term, the agents tend to trust in the market, the ratio of $\alpha \leq 0.3$ increases.

The computed market price tends to 1001.7 and its standard deviation is equal to 4.8. As a comparison, this value is reached after the first time interval in the random network setup. The spot price is volatile, and there is no perfect convergence. However, the extreme cases always disappear, and the computed fundamental value fluctuates in a smaller path than the initialisation. The exchanges tend to happen around the mean of the initial private information.

The probabilities to find a counterpart to buy or to sell are balanced. They slowly decrease from 0.95 on the interval $[1; 1000]$ to 0.90 on the interval $[4001; 5000]$, see Table 3.4. There are many different valuations of the computed market price and the computed fundamental value. This heterogeneity comes from a slow sharing information process. If we extend the window of the study, the distribution of the computed fundamental value decreases as the volatility of computed market prices. Indeed, with $10,000$ rounds, the trading prices oscillate in a spread ± 7 around the mean of the computed fundamental value .

Adding gurus has different impacts on the deviation of the computed fundamental value. Increasing the number of gurus when their private information is accurate ($f^{gu} = f$) makes the deviation of others lower. It decreases until 18.6 for 22.2% of gurus and falls to 10 for 50.2% of gurus regardless of their position, strong or weak. When $f^{gu} = 2f$, increasing the number of gurus increases the deviation of the computed fundamental value. For 50.2% of strongly connected gurus, the deviation of the distribution is even higher than the deviation set up by the NIG process. Instead of aggregating the information, when the gurus' information goes against the market, the non-gurus are lost in their valuation.

The trust parameter and the computed price should help us to understand this increase in the deviation of the computed fundamental value. Concerning the trust parameter (see Eq. 3.6): when $f^{gu} = 2f$, the more are the gurus and the better they are connected, the more alpha tends to 0.98. From 5 gurus strongly connected, alpha is greater than 0.8. These values are confirmed by the ratio of agents who trust in the market price as an accurate source of information, which is lower than 2% and the ratio of $\alpha \geq 0.7$, which is higher than 74%. In addition, the exchange occurs in a wide spread across time: $[900; 2000]$ on the first time interval and $[1300; 1800]$ on the last time interval. The computed price is highly volatile and non-informative. The increase in the deviation of the computed fundamental value is explained by the few percent of traders who do not totally trust in their own initial information. Three clusters of information are distinguished: 1/the gurus, 2/the others who trust in their initial signal, 3/a small group of others with computed fundamental value around 1300. As a result, the right bound of the distribution of the computed fundamental values for the others is moved to the right until 1500, while the left bound stays unchanged. In the same situation with $f^{gu} = f$, there is more than 40% of agents who used the market to compute their fundamental value ($\alpha \leq 0.3$). Regardless of agent position, when there are gurus with $f^{gu} = f$, traders trust in the market. However when $f^{gu} = 2f$, the computed price volatility is too high and the agents prefer to stick to their initial private information.

The probability to find a counterpart stays the highest of all networks. Regardless the gurus position, the probability to find a counterpart is always higher than 0.9. The lack of convergence of the computed fundamental value seems to drive up the probability to find a counterpart.

Because a large number of agents trusts in their own initial private information, the gurus increase the market price volatility. As a result, in case of 50.2% of gurus with $f^{gu} = 2f$, the average of the computed fundamental value is around 1036.6 and the deviation equals 108. The average spot price is around 1643.2 with a standard deviation of 105. The market price is destabilised by the gurus who are not able to impact on the others' beliefs.

3.4.3. Star network

The star network is defined by a central agent who collects all the information. She has a strong bargaining power compared to the peripheral agents (see Figure 3.3). According to these features, the market highlights the quickest convergence and the smallest deviation of the computed fundamental value. It is the only market in which the deviation of the fundamental value is lower than 50 on the first time interval $[1; 1000]$. On the interval $[3001; 4000]$, it equals 0.67, ten times lower than the complete network. This quick learning is due to the central agent: all the traders have only one counterpart, her. The central agent shares all the time the same information and thus the counterparts assume that she is better informed than them. Be aware that they have no other way to collect information. The trust parameter is over 0.7 for less than 4% of the population.

The other agents are all under 0.3. The average of the trust parameter is equal to 0.03 (std. dev. 0.17). The probability to find a counterpart oscillates between 0.43 and 0.56 (Table 3.4). The values are similar in the buying and selling ways. The computed price and the computed fundamental value are equal to the computed fundamental value of the central agent. The standard deviation among the simulation is lower than 0.07.

The information of the central agent is quickly and properly given to the peripheral agents. Indeed, the central agent controls and shares the information. In our setup, the trust of the central agent stays in $[0.7; 1]$, she is always confident in her initial private information.

Making the central agent a guru ensures the convergence of the computed fundamental value to the central agent's expectation. No matter f^{gu}, the trading price converge to her value within a tiny standard deviation (0.3). The others converge to the guru's expectation and share similar computed fundamental values (std. dev 0.4). Indeed, the ratio of agents who trust in their initial private information is close to 0. Thus the probability for the guru to find a counterpart tends artificially to 1.

In case of peripheral gurus (weakly connected) and $f^{gu} = 2f$, the gurus have not enough weight to influence the central agent. Increasing the ratio of gurus makes the probability for the active trader to find a counterpart decreasing in the selling way and increasing in the buying way. The volatility generated by gurus explains the reluctance of the central agent to change her mind. As a result, the others' computed trading price sticks to the initial private information of the central agent. The information shared is still the central agent's computed fundamental value.

3.4.4. Preferential attachment network

In the preferential attachment network (Barabási-Albert algorithm, 2002), numerous nodes have a unique edge while few nodes have lots of them (see Figure 3.2). The histogram of the distribution of the bargaining power may be observed in Figure 3.3. In this network, the deviation of the computed fundamental value stays high. It oscillates around 48 since the time interval $[3001; 4000]$. There is no information aggregation. Let us remind that the deviation of the initial private information is 55. Thus, the deviation of the computed fundamental value never exceeds the initial one.

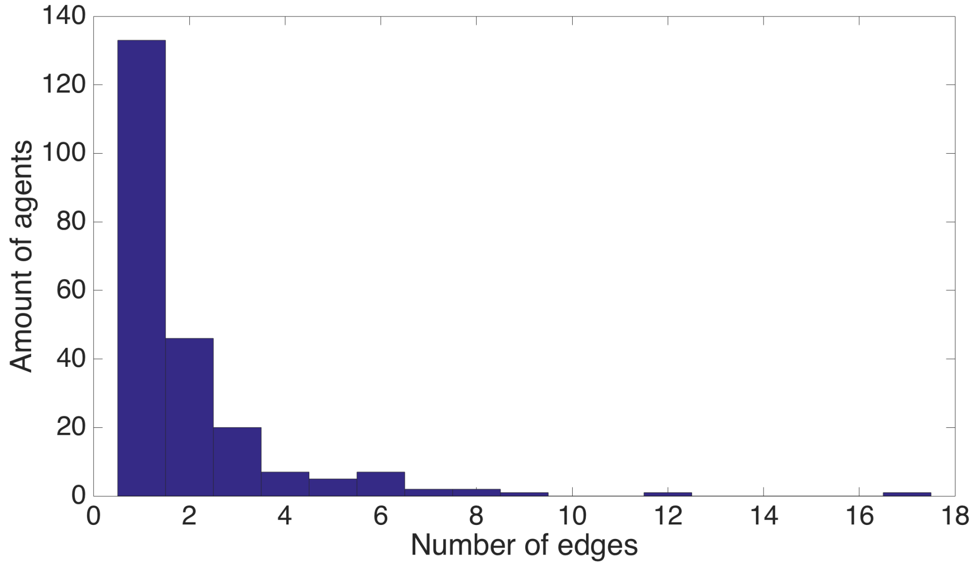


Figure 3.2.: Histogram of edges per node in preferential attachment network.

Two reasons might explain the information aggregation failure: 1/the agents are not able to communicate and extract accurate information 2/they neglect the information they extract. A part of the answer is in the trust parameter. The average of the trust parameter is 0.49 (std. dev. 0.42). This value is thus non-significant. When we focus on agents who hardly believe in their initial private information ($\alpha \geq 0.7$), there is a cluster of 44% of traders. The same ratio is observed for agents who think that the market price is more accurate than their own information ($\alpha \leq 0.3$). Two clusters are identified, one who highly trusts in her initial private information and a second who trusts in the computed market price. Unfortunately, the computed market price is highly volatile. The standard deviation is never lower than 37. Many trades occur at different price levels. The strong volatility in the computed fundamental comes from the volatility in the computed market price. The agents who believe in the market are not able to extract accurate information and they contribute to the instability. When we focus only on the last rounds, the mean and the standard deviation of the computed fundamental value are similar to the initial private information ones. Thus there is no information aggregation.

The probability to find a counterpart decreases from 56% to 50% on the interval $[1; 3000]$, and stays around 47% after that (Table 3.4). The huge difference in the computed fundamental value makes the probability stable.

Adding gurus in this market never contributes to stabilising the computed price nor the computed fundamental value. However, in the case with strongly connected gurus and $f^{gu} = 2f$, the average computed price seems to depend on the ratio of gurus. It oscillates around 1047 (std. dev. 172) for 1 guru, 1143 (std.

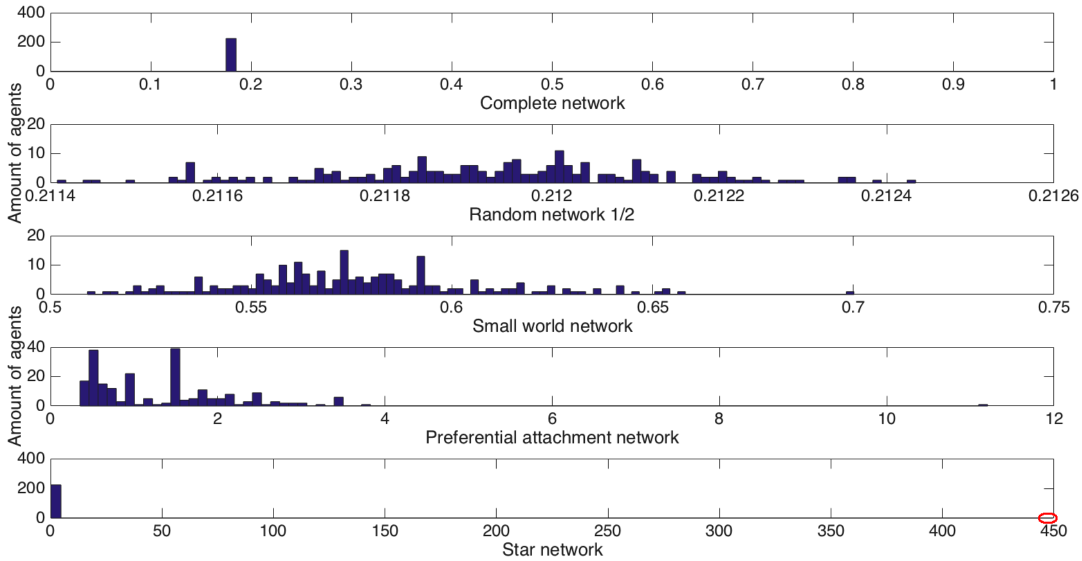


Figure 3.3.: Histograms of the distribution of the bargaining power of agents for each network. The X-axis is always sorted into 100 bins.

dev. 268) for 5 gurus, 1674.7 (std. dev. 297) for 50 gurus and 1910 (std. dev. 82) for 113 gurus. The average price increases and the standard deviation confirms that many trades happen at different prices.

These prices explain the rise of the deviation of the computed fundamental value to 149 for 1 guru, to 194 for 5 gurus, to 235 for 50 gurus and 248 for 113 gurus. In this network, gurus may have a negative impact, they contribute to making the market not efficient at all.

3.4.5. Discussion

We perform the model for different initial trust setups, α_0. After 1,000 rounds, the trust dynamic depends only on the network structure. This is due to the lack of anchoring in our formulation of the initial belief in the private information (see Eq. 3.6). Thus the initial distribution of the trust parameter does not matter.

In the complete network, all agents share their information at each round. Even if the structure is an OTC market, all agents can properly estimate the computed market price and deduce the fundamental value. Due to the strong trust in the market and the small volatility in the market price, the private information (expected fundamental value) becomes homogeneous. The strong trust in the market (or the weak trust in the initial private information) might also be interpreted as a rise of mimetic behaviours. Wisdom et al. (2008) already highlight in a simple puzzle game, where participants are allowed to view and imitate the guesses of others during the game, that agents tend to mimic each other when they are fully connected. The information aggregation and diffusion

Table 3.5.: No Guru - Deviation of the distribution of the expected fundamental.

Interval	Comp.	Rand.	Small	Star	Pref. Attach.
[1; 1000]	50.0307	49.2867	49.8197	35.6152	53.7084
std. dev	*3.9945*	*4.3834*	*3.0588*	*13.2506*	*1.8163*
[1001; 2000]	24.4200	22.3963	37.0053	6.6213	49.4277
std. dev	*8.4082*	*8.7390*	*3.1968*	*4.3964*	*0.5471*
[2001; 3000]	6.6683	5.1704	28.8036	0.6741	48.6262
std. dev	*2.1995*	*1.7073*	*1.6671*	*0.5067*	*0.1679*
[3001; 4000]	3.5856	3.3554	24.7470	0.3305	48.5851
std. dev	*0.2978*	*0.1265*	*0.8935*	*0.1207*	*0.1096*
[40001; 5000]	3.2979	3.2898	22.4137	0.3305	48.6242
std. dev	*0.0554*	*0.0582*	*0.5826*	*0.1063*	*0.1399*

is complete. The complete network is thus the less opaque OTC of our study. It is characterised by small deviations in the computed fundamental value and small deviations in the computed and traded prices. The deviations of the distribution of the expected fundamental are summarised in Table 3.5. The random network is one step more opaque than the complete network: agents are connected to each other with a probability of one half. Due to this strong connection, the information is still quickly shared. The market price stays an accurate source of information, the fundamental value is properly estimated by all agents. Assuming the trust in the market as the risk tolerance, we can also feet real data so as to test our model (Linciano and Soccorso, 2012). In these market structures, the computed market price is defined by the average of traders' initial information. Adding the relevant number of gurus and/or the relevant information allows to improve the accuracy of the computed fundamental value because agents have similar bargaining power.

Increasing the market opacity, such as the preferential attachment network or the small world network makes the talk completely different. The computed price is unstable, highly volatile, for the preferential attachment network. The information propagation takes a while for the small world network. Because of the lack of connection, many trades happen at different prices in a time interval (1000 rounds). In the complete network, the standard deviation of the spot price is lower than 2 while it rises to 8 for the small world and to 100 for the preferential attachment, all things otherwise being equal. As a result, there is only limited convergence (high deviation of the distribution of the expected fundamental value). This convergence disappears when guru's information goes against the market. Mainly, traders prefer to stick to their initial private information due to the high variance in their collected market prices. We already highlighted three clusters in the small world case: 1/the gurus, 2/the others who trust in their initial signal, 3/a small group of others which is partially influenced by their computed market price.

When agents receive a unique source of information, a consensus is reached quickly on the source. This is the case in the most opaque structure, the star network. The price is driven by the central agent, no matter the number of gurus. The central agent exploits the information of the gurus so as to make profit but does not take it into account. The volatility of the observed spot prices is high, around 246, and explains why the central agent stays stick to her initial private information. Two clusters appear: one which is influenced by the central agent, and another which is composed of the gurus. When we compute the spot prices by clusters, we observe two different trading prices: one around the central agent's expectation, where the probability to find a counterpart is one half for both sides (buy and sell) and one around the gurus' expectation, where the probability to find a counterpart is 1 on one side and 0 on the other. The spot price volatility of each cluster tends to 0.

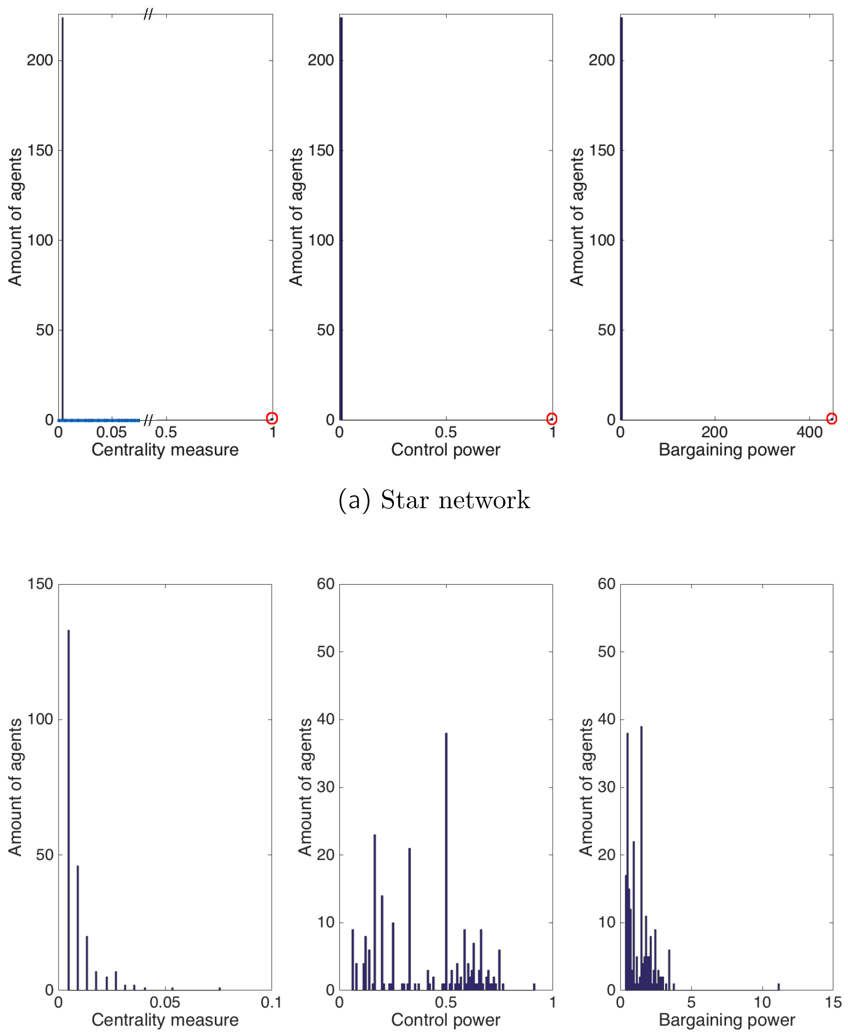
A star network is the extreme case of the preferential attachment. It is surprising that both networks show such different results. In the preferential attachment, the more edges a node has, the greater the probability that this node attracts new edges, when they are added. As a result, this network looks like numerous star networks of different sizes (quantity of nodes) interlinked together by an edge. In both networks, the local and the average clustering coefficients are equal to zero. As expected, in the star network, all peripheral agents (224 nodes) have a closeness centrality measure of 0.0045 while the central agent has a closeness centrality measure of 1. In the preferential attachment, there is no unique central agent as in the star network. However, 60% of agents have still only one connection (edge). Their centrality measure is equal to 0.0045 also. 90% of agents have a centrality measure lower or equal to 0.0179. The most well-connected agent has a centrality measure of 0.0759 and only 7 agents have a value higher than 0.03. The idea that there are many weakly connected agents and few relatively strongly connected agents is confirmed. The null values of the average and local clustering coefficients highlight that many nodes are connected to a central one. There are hardly any connections in the neighbourhood. According to the histogram of edges (Figure 3.2), 60% of agents have only an edge. 90% of agents have less than 4 connections. The most connected agents have 9, 12 and 17 edges respectively. Moreover, there is no clique. The average path length in case of star network is around 2, while in case of preferential attachment it is around 7. Be aware that a short average path length facilitates the quick transfer of information in the network.

All these differences in the network structure impact the control power of the agents and so their bargaining power. Remember that the i's network control reflects the degree to which i controls the relations to him by his relations to other. Figure 3.4 summarises the difference of the distribution on the centrality measure, the control power and the bargaining power with the star network and the preferential attachment network. In case of star network, the bargaining power of the central agent is equal to 449.5 while all peripheral agents have the same bargaining power close to zero (0.1847). In case of preferential attachment, the network control is distributed on the whole interval[g]; there is not a unique central agent. The average control power is equal to 0.3989 and the median to 0.4375. 16.8% of agents have a control power around 0.5. As a consequence, the bargaining power of the agents with preferential attachment network is distributed mainly on the interval $[0.3527; 3.7323]$. The mean is equal to 1.35 and the median to 1.2064. There is a unique agent out of this interval; the most connected one. She has a bargaining power equal to 11.2062. In case of star network, the bargaining power is equal to 0.1847 for all peripheral agents while it is equal to 449.49 for the central agent. The higher average path length explains the lack of information convergence while the distribution of the bargaining power justifies the high price volatility across rounds.

We confirm that in opaque market structures, the price is unstable. While all agents aggregate the information, there is a decreasing volatility. We show that the information diffusion is effective in the complete, random, small world and star network but not informationally efficient as Babus and Kondor (2013).

With no-guru, the market liquidity is balanced. The probability to find a counterpart to sell is similar to the probability to find a counterpart to buy, regardless of the network type. However, these probabilities depend on the market structure (see Table 3.4). The complete and the random networks have the biggest fall in liquidity, while the star network oscillates around 0.5. As expected, the stability of the probability to find a counterpart seems to be linked to the dispersion of the computed fundamental value and the gurus' expectation. Be aware that the rise in "liquidity" is artificially due to the absence of trading fees in case of complete and star networks. In these setups, when traders converge to a unique price, they agree to buy and sell a unit of the asset at the same price which improves the probability to find a counterpart. By adding trading fees, the probability to find a counterpart should fall to 0.

[g]According to the definition of the network control in section 3.3.1, the network control is defined on the interval $[0; 1]$.



(a) Star network

(b) Preferential attachment network

Figure 3.4.: Histograms of the centrality measure, the control power and the bargaining power for the star network and the preferential attachment network.

3.5. Conclusion

This paper investigates whether information extraction through bargaining process can be explained in a market, in which transactions are not made public and where no prices are posted. To deal with, we develop an Over The Counter market (OTC) where a unique risky asset is traded. To keep the model simple as it can get, we set up the market as an undirected graph where traders are nodes and trading connection are edges. From this adjacency matrix, the "network control" and the "bargaining power" of each agent are computed. In addition, each trader receives at the initialisation a private signal (her fundamental value) and some trust in (α). The relative willingness to adapt her trading prices and be influenced by others depends on her network control.

The impact of the opacity in the OTC market for the information aggregation and dissemination is confirmed. In the complete network and the random network, all agents have on average the same network control, and access to a huge amount of information. As a result the market integrates the main information: the trading price and the computed fundamental value are similar for all agents. In the small world, where the opacity is a bit deeper, the information aggregation is longer but still exists. The volatility of the trading prices and the deviation of the computed fundamental value are a bit more important. However, since gurus appear with counter information, the market becomes highly volatile and not informative. In the preferential attachment network, in which some agents are weakly connected while other agents are strongly connected (huge spread in the bargaining power), the dissemination of information fails. Traders prefer to stick to their own initial private information so as to exchange. As a result, trades happen in a wide spread and no consensus on the right fundamental value rises. In the star network, all agents have a small bargaining power except one: the central agent. The price is driven by her, regardless of gurus' information.

In this preliminary work, we proceed descriptive statistics to check the market structure behaviour. Now, we will perform econometrical regression in order to explain the trader's profit by a mixture of network control (c_i), computed current price ($\hat{p}$) and fundamental information ($\alpha^i; f_0^i; \hat{f}_t^i$), and identify the impact of each component. In addition, further tests will be driven on the small world network and the preferential attachment network to check the rise of little guru in the market with no initial guru.

3.6. Appendix: Network structure

- Network Matrix (undirected graph):

$$A = \begin{bmatrix} a_{1,1} & \cdots & a_{1,n} \\ \vdots & \ddots & \vdots \\ a_{n,1} & \cdots & a_{n,n} \end{bmatrix} \Rightarrow D = \begin{bmatrix} a_{1,1}/\sum_{k=1}^{n} a_{k,1} & \cdots & a_{1,n}/\sum_{k=1}^{n} a_{k,n} \\ \vdots & \ddots & \vdots \\ a_{n,1}/\sum_{k=1}^{n} a_{k,1} & \cdots & a_{n,n}/\sum_{k=1}^{n} a_{k,n} \end{bmatrix}$$

- Network control of agent i:

$$c_i = \frac{1}{n_i}\sum_{k=1}^{n} d_{ik} = \frac{1}{n_i}\sum_{k\in S_i}\frac{1}{n_k}$$
$$c_i \in [0;1]$$

Example 1: Star network

$$A = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \Rightarrow D = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1/3 & 0 & 0 & 0 \\ 1/3 & 0 & 0 & 0 \\ 1/3 & 0 & 0 & 0 \end{bmatrix} \Rightarrow C = \begin{bmatrix} 1 \\ 1/3 \\ 1/3 \\ 1/3 \end{bmatrix}$$

Example 2: Complete network

$$A = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix} \Rightarrow D = \begin{bmatrix} 0 & 1/3 & 1/3 & 1/3 \\ 1/3 & 0 & 1/3 & 1/3 \\ 1/3 & 1/3 & 0 & 1/3 \\ 1/3 & 1/3 & 1/3 & 0 \end{bmatrix} \Rightarrow C = \begin{bmatrix} 1/3 \\ 1/3 \\ 1/3 \\ 1/3 \end{bmatrix}$$

Example 3: Random network

$$A = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix} \Rightarrow D = \begin{bmatrix} 0 & 1/2 & 0 & 1/3 \\ 1/2 & 0 & 0 & 1/3 \\ 0 & 0 & 0 & 1/3 \\ 1/2 & 1/2 & 1 & 0 \end{bmatrix} \Rightarrow C = \begin{bmatrix} 5/12 \\ 5/12 \\ 1/3 \\ 2/3 \end{bmatrix}$$

Conclusion Générale

Le prix de marché se définit par la rencontre entre l'offre et la demande. Ce prix, auquel deux partis acceptent d'échanger à un instant t, représente la « juste » valorisation de l'actif concerné. Cependant, une rapide analyse empirique démontre que cela est plus complexe que la simple rencontre entre l'offre et la demande (Cohen et al., 1980; Hommes et al., 2007; Kandel and Pearson, 1995). Les marchés financiers sur-évaluent le prix de certains actifs, alors qu'ils en sous-évaluent d'autres. Cela génère des krachs et de violentes corrections (Brunnermeier and Oehmke, 2013).

Le thème de la déconnexion entre les fondamentaux et les marchés financiers n'est pas nouveau (Culter et al., 1989). L'Histoire a connu des marchés d'actifs qui montent, malgré des fondamentaux médiocres et des banques centrales un peu trop rassurantes. Il est aussi possible de mentionner des marchés qui dévissent malgré des fondamentaux *a priori* stables. Les marchés financiers, et donc leurs investisseurs, craignent, à tort ou à raison, un tarissement de la liquidité et de la confiance.

Dans cette thèse, nous avons exploré le « rôle de l'hétérogénéité des agents dans la diffusion et l'utilisation de l'information » sur un marché financier. De nombreuses études empiriques ont mis en lumière la faiblesse de la finance classique à caractériser les anomalies de marchés (Campbell and Cochrane, 1999; Campbell et al., 2001; Schwert, 2003). L'alternative utilisée dans les travaux présentés repose sur la prise en compte des caractéristiques individuelles des agents par le biais de la modélisation agent (Bao et al., 2012). La prise en compte des caractéristiques et des comportements individuels ainsi que des interactions entre les agents permet de répliquer et motiver les dynamiques de prix que la finance classique peine à considérer comme autre chose que des distorsions à court terme. Les agents n'agissent pas de manières irrationnelles, ils suivent simplement des voies différentes afin de maximiser leurs profits ou minimiser leurs pertes (Takahashi and Terano, 2003).

Hommes et Lux concluaient la $20^{ème}$ WEHIA[h] (Nice - 2015) en déclarant que des progrès (à minima pour les articles déjà publiés) devaient être apportés quant à l'accessibilité et la réplicabilité des structures en modèles agents. En gardant à l'esprit que ces algorithmes sont difficilement accessibles, la première contribution de cette thèse réside dans le développement de deux nouvelles structures de marché.

- La première structure de marché repose sur les travaux de Chiarella and

[h] The 20^{th} Workshop on the Economic Science with Heterogeneous Interaction Agents - Sophia Antipolis, du 21 au 23 mai 2015.

Iori (2002). Les chapitres 1 et 2 introduisent un marché dirigé par les prix, où une partie du carnet d'ordre est visible de tous et plusieurs actifs risqués sont échangés. Peu courant dans la littérature, l'ajout d'un deuxième actif risqué permet entre autres d'étudier la liquidité du marché, les évolutions corrélées entre les actifs ainsi que les techniques d'optimisation du portefeuille (Chowdry and Nanda, 1991).
• La deuxième structure de marché (chapitre 3) comble un vide de la modélisation agent. À ce jour, aucun autre modèle ne propose une structure décentralisée (marché de gré à gré) où les acteurs font tous face à une information incomplète de la valeur fondamentale et n'ont pas nécessairement d'intérêt à échanger (prix de réserve). Seuls les travaux de Babus and Kondor (2013) et de De Kamps et al. (2013) développent des structures similaires afin de traiter de la diffusion de l'information.

Le développement de ces deux structures de marché financier a permis de mettre en lumière l'influence des caractéristiques comportementales et décisionnelles sur la diffusion de l'information et plus particulièrement la découverte de la valeur fondamentale. Ces travaux ont débouché sur cinq contributions dans le domaine de la finance de marché.
• La première contribution est une meilleure compréhension de la décision d'investissement des agents. Cette décision est influencée par quatre caractéristiques.
(i) L'hétérogénéité des agents à la fois au niveau individuel et collectif est responsable de perspectives d'échanges différentes.
(ii) Le processus de traitement de l'information ainsi que le système d'apprentissage de la valeur fondamentale génère pour une information publique des déductions et des anticipations hétérogènes.
(iii) La structure dans laquelle l'agent évolue (réseau) définit son accès à l'information, ses attentes et ses anticipations. Cela définit aussi la véracité de l'information qu'il transmettra ainsi que le rayon de diffusion de cette dernière.
(iv) La transparence du marché influence la stratégie de soumission des ordres (Parlour, 1998) ou la définition du prix de réserve.
• La seconde contribution est une mise en lumière de certains éléments de la rationalité limitée sur la dynamique du marché. Ces éléments sont essentiellement introduits à partir du chapitre 1.
(i) En renforçant des tendances observées, les chartistes (*trend-follower outlook*) déstabilisent le marché à court terme tout en le rendant plus liquide.
(ii) Les horizons d'investissement hétérogènes génèrent une faible distorsion entre le prix de marché et la valeur fondamentale. Cette inefficience faible contribue à maintenir un marché liquide, où des agents échangent constamment dans le but de corriger cette erreur d'évaluation.
(iii) La stratégie de soumission des ordres (Parlour, 1998) qui consiste à adapter un ordre à la profondeur du carnet d'ordre justifie des bulles persistantes. Le

marché devient très volatil, le cours de l'actif évolue alors indépendamment des fondamentaux. La résultante est un marché liquide où le prix d'échange traduit la facilité de trouver une contrepartie et non l'évaluation des fondamentaux.
(iv) Finalement, le processus d'apprentissage de la valeur fondamentale basé sur le concept de persévérance des croyances joue un effet stabilisateur sur le prix de marché. Il a aussi pour conséquence de stabiliser les volumes d'échange. Néanmoins, la mauvaise perception des variations des fondamentaux peut être responsable d'une perte d'efficience du marché, et cela sans l'intervention de spéculateurs.
• En testant différents processus d'apprentissage de la valeur fondamentale (chapitre 2), la troisième contribution de cette thèse réside dans la démonstration de la capacité des agents à prévoir les flux de trésorerie dans un marché relativement transparent (accès à une partie du carnet d'ordre). La condition primordiale et attendue est la fiabilité de la source d'information. À partir d'apprentissages basiques, cette recherche vise à justifier l'intervention ponctuelle de l'État, lorsque le marché ne permet pas aux agents d'être assez informés. Elle permet aussi de mieux comprendre l'émergence de croyances et leurs impacts sur la valorisation des actifs. Deux moyens de transmissions de l'information ont étés mis en avant : la dispense d'information par annonce et l'extraction d'information à partir du carnet d'ordre. Ce travail prône une entité capable de dispenser une information fiable aux investisseurs souhaitant s'informer. Lorsque les acheteurs se servent du marché pour définir une valeur extrinsèque au marché ou lorsque l'information est inexacte, les agents corrigent partiellement leurs mauvaises prévisions. Cependant cette correction est insuffisante pour leur permettre une bonne estimation des fondamentaux.
• La quatrième contribution réside dans l'étude de la dissémination de l'information dans un marché opaque et décentralisé (marché de gré à gré). Nous avons mis en avant l'importance de la structure de marché dans le processus de diffusion de l'information. Cette structure amène à se questionner sur le nombre idéal d'animateurs de marché. En associant les agents fortement connectés à des animateurs de marché, il est aisé d'observer le contrôle qu'ils opèrent sur les prix. Par le biais de différentes structures de réseau, cette recherche met aussi en évidence la difficulté de l'information à être véhiculée sans être altérée dans un marché opaque. Un fossé existe entre l'information diffusée et l'information perçue. Afin de permettre un gain en efficience, l'information dispensée doit être corrigée de sa future altération. Cette notion émerge dans le chapitre 2 et prend corps dans le chapitre 3 avec l'étude de l'impact des gourous.
• La cinquième et dernière contribution est une réflexion sur le degré de transparence des marchés financiers. Les caractères hétérogènes et la rationalité limitée des agents déstabilisent le marché à court terme. Cependant, nous avons aussi démontré que les agents sont capables d'extraire du carnet d'ordre certaines informations afin de découvrir de manière plus ou moins fiable la valeur fondamentale. Même si le marché n'est pas nécessairement efficient,

les agents peuvent prévoir correctement la valeur fondamentale s'ils ont accès à des informations fiables et non contradictoires. L'information est extraite correctement même si elle n'est pas par la suite divulguée. Un niveau optimal de transparence semble exister en deçà duquel il y a manipulation des prix et au-delà duquel le gain en efficience n'est plus significatif. Ceci a été démontré dans le chapitre 2 en variant l'horizon temporel de l'approche rétrograde et dans le chapitre 3 par l'utilisation de différentes structures de réseau.

Quelques limites à nos travaux apparaissent. Dans le chapitre 1, nous limitons l'étude de la liquidité au volume d'échange, à la volatilité de l'actif et à la profondeur de marché. De nombreux autres facteurs permettent d'étudier la liquidité telle que la fréquence d'échange, la résilience du marché ou encore l'écart entre les cours acheteur et vendeur.

Dans le chapitre 2, les apprentissages testés sont basiques. De plus, seulement un apprentissage à la fois est expérimenté. Deux évolutions majeures peuvent être signalées. La première est le développement des agents aux apprentissages hétérogènes au sein d'un même marché. La seconde est l'amélioration du processus d'apprentissage par l'utilisation d'un algorithme génétique. Concernant ce chapitre, il semble pertinent de rendre la corrélation entre les deux actifs évolutive en fonction du temps.

S'agissant du chapitre 3, il paraît intéressant de rendre la structure du réseau dynamique afin de mettre en avant la notion de loyauté entre les agents. Cela peut aussi être remplacé ou complété par une fonction d'avidité (*greediness function*). En effet, le processus de négociation est un processus long et coûteux, les agents font un arbitrage entre la recherche d'une nouvelle contrepartie et l'acceptation d'un échange défavorable. Une première étape peut être simplement de remplacer le graphe actuel (non orienté) par un graphe orienté. Les relations d'échange ne sont pas nécessairement symétriques. Un agent peut souhaiter échanger avec un autre, sans que cela soit réciproque.

Enfin, il est à noter que le concept de valeur fondamentale utilisé dans ces travaux est le plus basique. De nombreuses complexifications existent afin de rendre cette valeur dépendante des risques de liquidités... Plus généralement, une calibration précise des comportements des individus serait une vraie plus-value. Eyensenck (1991) propose dans ses travaux un questionnaire qui permet de classer les individus à partir de trois grandes dimensions psychologiques : le psychotisme, l'extraversion et le névrosisme. En reprenant les précédentes dimensions psychologiques et en les appliquant à la finance, le psychotisme permet de quantifier l'aversion au risque et la capacité de négociation des individus ; l'extraversion contribue à définir l'agressivité (type d'ordre, impatience) ainsi que la taille du réseau de l'agent ; enfin le névrosisme qualifie le comportement (plus ou moins chartiste) et l'horizon d'investissement. En couplant la calibration en laboratoire aux modèles agents présentés, ces structures de marché s'avéreraient alors incontestables dans leurs capacités à refléter la réalité et gagneraient grandement en précision.

De la même manière que les marchés peuvent être euphoriques en dépit des fondamentaux, ils peuvent causer de mini-krachs ponctuels à cause de craintes plus ou moins fondées. Cependant, ces krachs deviennent de plus en plus violents. Les nouveaux modes de fonctionnement des marchés financiers ne semblent pas y être étrangers. Cela amène à se questionner sur les effets des algorithmes automatiques d'échange sur le carnet d'ordre, et plus particulièrement sur l'information qui est véhiculée. Actuellement les avis divergent. Il semble intéressant de se concentrer sur la nécessité ou non d'encadrer les échanges HF.

A. Fair price and trading price : an ABM approach with order-placement strategy and misunderstanding of fundamental value

A.1. Introduction

The main goal of financial markets is to guarantee an optimal transfer of resources from supply to demand. This aim can be attained only if exchanges do actually occur in a considered period, *i.e.* the market is liquid. "Stated simply, liquidity is the ease of trading a security "Amihud et al. (2005). Hence it is a property of the system, which cannot be attained by just one agent with not enough influence on the market to create a context of easy trade. It impacts the price, the volatility, and the amount of quoted orders.

During the last crisis, there was no way analysts could anticipate the liquidity and price falls that took place. Furthermore, to the best of our knowledge, there is no known mean to impact on the market liquidity. We are just faced with *ex post* observations and attempt to understand the data. It has already been shown that agent's behaviour is drastically driven by asset exposure to liquidity risk Amihud (2002).

The field of microstructure focuses on the concept of liquidity, impact of traders type or information revealed. The importance of market liquidity for the optimal behaviour of discretionary traders and its effect on patterns of trade has already been studied (Kyle, 1985; Grossman and Miller, 1988). Traditionally, measures of market liquidity include indicators of market impact, market resilience and market depth (Muranaga and Shimizu, 1999). Specifying this analysis, micro structure theory does not study liquidity as an endogenous characteristics of markets, but as an exogenous parameter that influences agents' behaviours.

Several authors have already proven that the assumption of heterogeneity among bounded rational agents is necessary to reproduce, with models, results from actual behaviours (*e.g.* experimental data) (Bao et al., 2012). Financial Agent-Based Models with multi assets enable to study liquidity that is essential for both viability and dynamics of market Chowdry and Nanda (1991). A two-risky assets model allows traders to reduce their risk according to the portfolio theory. It also seems more realistic than a unique risky asset and can be extended to a n-risky one. The development of this model is a part of a larger aim which is to study the impact of estimated comovement of fundamentals on the spot prices and the effectiveness of the portfolio allocation. Actually, some research suggests that the gain by diversification is more than offset by estimation errors DeMiguel et al. (2009).

The current paper aims at providing some explanations to the disconnection between fair price and trading price. We define the efficiency as the ability of the market price to reflect the fundamental trends (Fama, 1998). We relax the assumption of perfect knowledge of the fundamentals and focus on one dimension of liquidity which is not studied much: the trading volume. The introduction of an adaptive learning in the estimation of the fundamental value enables us to identify different bubble types: some attributed to the fundamentalist behaviour, based on misunderstanding (Westerhoff, 2004), and some generated by chartist behaviour, based on trend extrapolation (Hommes, 2006). Note that the introduction of fundamentalists in a previous stable market makes the variance increase and the liquidity fall. We also track the link between the order imbalance and the market efficiency. The arbitrage solved by traders, between paying a higher price and taking a no-execution risk Parlour (1998) makes the market inefficient. The spot price reflects the consent to pay for a fast order execution. As a result, the market liquidity is hugely increased. We also check the ability of this model to behave as usual one-risky asset models (Chiarella et al., 2011; Hommes et al., 2005a; Lux and Marchesi, 2000). Especially, it is able to replicate some usual stylised facts as volatility clustering, bubble boom and burst.

A.2. Model

The model we base our call auction market upon is Yamamoto's model Yamamoto (2011), where agents place their oder in an electronic order book. We extend it by relaxing the assumption of perfect knowledge of the fundamentals and adding a second trading asset. These two risky assets ($j = \{1, 2\}$) are assumed to be independent. The market structure is such that the asset price of each asset is given by the price at which a transaction, if any, occurs. If no new transaction occurs, a proxy for the price is given by the average of the lowest quoted ask a_t^q and the highest quoted bid b_t^q, so that $p_t = (a_t^q + b_t^q)/2$. If no bids or asks are listed in the book, a proxy for the price is given by the previous traded or quoted price. Once the order is submitted, it can't be modified. It is removed from the order-book when it is executed or it remains unexecuted for its lifetime (τ^i). Note that bid, ask and market price need to be positive and are defined on a pricing grid of width Δ. Finally, agents are not allowed to engage in short selling and are not monetary constrained.

The agents are heterogenous in both their fundamentalist and their chartist ratios, their investment horizon and their risk aversion. Each agent i is defined by:

- her fundamentalist component (g_f^i)
- her chartist component (g_c^i)
- her mood ($M_t^{i,j}$)
- her sensitivity to the order-book depth ($\beta^{i,j}$)

All these parameters are fixed in time, except the mood. Each agent is assigned risk-aversion and investment horizon that depend on its g_f^i, g_c^i ratio:

$$\alpha^i = \alpha \, \frac{1+g_f^i}{1+g_c^i} \quad \text{and} \quad \tau^i = \tau \, \frac{1+g_f^i}{1+g_c^i}$$

where α is a reference time horizon and τ a reference degree of aversion to risk. The decision-making is a succession of three steps :

1. the estimation of the future price of each risky asset
2. the calculus of the optimal demand according to the forecast
3. the correction of the bid(s) and/or ask(s) to adapt to the order-book state

A.2.1. Estimation of the future price

The chartist's expectation is what the agent infers from the past returns and the received information. The average spot return ($\bar{r}_t^i$) is defined as the expected trend based on the observations of the spot returns over the last τ^i time steps.

$$\bar{r}_t^i = \frac{1}{\tau^i} \sum_{k=1}^{\tau^i} r_{t-k} = \frac{1}{\tau^i} \sum_{k=1}^{\tau^i} \ln\left(\frac{p_{t-k}}{p_{t-k-1}}\right) \tag{A.1}$$

Regarding the fundamentalist's expectation, we distinguish two means to compute the fundamental value ($\hat{f}_t^i$), with and without perfect knowledge, here called "perfect knowledge" and "adaptive learning". In both settings, we assume that the fair value (f_t) follows a random walk and agents have access to the entire history of asset prices.
– In the case of perfect knowledge of the fundamental value, we assume that the computed fundamental value is unique and right.

$$\hat{f}_t^i = \hat{f}_t = f_t \quad , \forall i \tag{A.2}$$

– In the case of adaptive learning, the fundamentalist expectation becomes idiosyncratic. Based on Westerhoff's formalism (Westerhoff, 2004), the perseverance belief is defined by the last observed price (p_{t-1}), the previous estimation

of the fundamental value ($\hat{f}^i_{t-1}$) and the original one ($\hat{f}^i_0$). γ_1, γ_2 and γ_3 are the weight given to each component and add up to 1. The information processing is made by the arrival of news (N_t) – common for all –, plus the processing: the ω parameter is the degree of misperception, due to the finite time needed to process information while the components in the parenthesis describe the faith of the agent relative to the news. As an example, if the recent update of the estimated fundamental value has been above the news impact ($\hat{f}^i_{t-1} - \hat{f}^i_{t-2} > N_t$), the fundamentalist outlook tends to overreact to news, and *vice-versa*. The correction mechanism relies on parameter ψ. When ψ tends to 0, remote and recent observations tend to be weighted in the same way ($1/t$). The correction mechanism is slow. When ψ tends to 1, more importance is given to the recent observation than to the old one. The correction mechanism is fast. All parameters (ω, ψ, γ_1, γ_2, γ_3) are equal among traders.
This adaptive learning is justified by the interpretation of Tversky and Kahneman's evidence Tversky and Kahneman (1974). Furthermore, many traders assume that asset prices themselves reflect relevant information Murphy (1999). The formulation of the belief perseverance is motivated by the reluctance of people to search for evidence that contradicts their belief Lord et al. (1979). This explains a correction mechanism slow over time and small in magnitude Fischoff et al. (1977).

$$\begin{aligned}
\hat{f}^i_t &= \gamma_1\, p_{t-1} + \gamma_2\, \hat{f}^i_{t-1} + \gamma_3\, \hat{f}^i_0 && \leftarrow \text{belief perseverance} \\
&\quad + N_t + \omega(\hat{f}^i_{t-1} - \hat{f}^i_{t-2} - N_t) && \leftarrow \text{information processing} \\
&\quad + \psi(f_{t-1} - \hat{f}^i_{t-1}) && \leftarrow \text{correction mechanism} \qquad (A.3) \\
\hat{f}^i_t &\neq \hat{f}^j_t && , \forall i \neq j
\end{aligned}$$

Our agents take the price information, the fundamentalist expectation and the chartist expectation into account to form their weighted forecast of the asset's future returns that will prevail in the interval ($t + \tau^i$):

$$\hat{r}^i_{t+\tau^i} = \frac{1}{g^i_f + g^i_c}\left[g^i_f . \ln\left(\frac{\hat{f}_t}{p_t}\right) + g^i_c . \bar{r}^i_t \right] \qquad (A.4)$$

The weighted forecasted return ($\hat{r}^i_{t+\tau^i}$) allows to formulate at time t the future expected price:

$$\hat{p}_{t+\tau^i} = p_t \exp(\hat{r}^i_{t,t+\tau^i}) \qquad (A.5)$$

Note that the estimation of the future price should be defined for each asset j.

A.2.2. Asset demand

The optimal demand of asset is defined by the maximisation of a constant absolute risk aversion (CARA) utility function under a gaussian return of assets. Regarding this maximisation, the optimal demand for asset j is:

$$\pi_t^{i,j}(\hat{p}^j_{t+\tau^i}) = \frac{\ln\left(\frac{\hat{p}^j_{t+\tau^i}}{p^j_t}\right)}{\alpha^i \; p^j_t \; Var^{i,j}} \tag{A.6}$$

The variance ($Var^{i,j}$) denotes the risk investment of the specific asset. It is assumed to be equal to the variance of the logarithm of the return rate, see Eq.(A.1).

$$Var^{i,j} = \frac{1}{\tau^i} \sum_{k=1}^{\tau^i} [r^j_{t-k} - \bar{r}^{i,j}_t]^2 \tag{A.7}$$

The amount of assets ($s_t^{i,j}$) the agent is willing to trade is determined by the absolute difference in the optimal demand for the risky asset at time t and $t-1$ as:

$$s_t^{i,j} = |\pi_t^{i,j} - \pi_{t-1}^{i,j}| \tag{A.8}$$

The sign of this difference ($\pi_t^{i,j} - \pi_{t-1}^{i,j}$), gives the agent's position: buying if it is positive, selling if it is negative, otherwise doing nothing.

A.2.3. Price correction and submission

The agent's mood implies that the submitted bid ($b_t^{i,j}$) or ask ($a_t^{i,j}$) is different from the expected future price ($\hat{p}^j_{t+\tau^i}$). For an easy reading, we voluntarily forget the index j in this subsection. In an optimistic mood ($M_t^i > 0$), the agent accepts to buy ($b_t^i > \hat{p}_t^i$) or sell ($a_t^i < \hat{p}_t^i$) at higher price. She believes that the asset price will continue to increase. On the contrary, when a trader is in a pessimistic mood ($M_t^i < 0$), she submits at a lower price than she predicted. She tries to reduce her loss in case of a fall. Once the submission price and the quantity are defined, the trader needs to choose a type of order. She chooses a market order (MO) when her defined bid (ask) is above (below) the best quoted ask (bid). Otherwise, she prefers a limit order (LO).

At this step, the trader has two means to submit an order. Without order-placement strategy, she submits her order at the previously defined bid or ask (Table A.1). With order-placement strategy, using the visible part of the order book, she resolves the trade-off between accepting non-execution risk and paying the bid-ask spread. In this market, the submitters have access to the

Table A.1.: Order definition before/without placement strategy.

Volume	Position	Order price	Order type[1]	
$\pi_t^{i,j} - \pi_{t-1}^{i,j} > 0$	buy	$b_t = \hat{p}^j_{t+\tau^i}(1+M_t^{i,j})$	$b_t > a^q$	$\Leftrightarrow$ MO
			$b_t \leq a^q$	$\Leftrightarrow$ LO
$\pi_t^{i,j} - \pi_{t-1}^{i,j} < 0$	sell	$a_t = \hat{p}^j_{t+\tau^i}(1+M_t^{i,j})$	$a_t > b^q$	$\Leftrightarrow$ MO
			$a_t \leq b^q$	$\Leftrightarrow$ LO
$\pi_t^{i,j} - \pi_{t-1}^{i,j} = 0$	do nothing	$\varnothing$	$\varnothing$	$\varnothing$

[1] b^q and a^q are the best quoted bid and ask at time t

Table A.2.: Impact of placement strategy on order definition.

Position	behaviour	Cases[1,2]	New order price[2]	Order type switching[2]
buy	More aggressive	$b_t < b^q$	$b_t = b^q + \Delta$	LO $\rightarrow$ LO
		$b^q < b_t < a^q$	$b_t = a^q + \Delta$	LO $\rightarrow$ MO
	Less aggressive	$b^q < b_t < a^q$	$b_t = b^q$	LO $\rightarrow$ LO
		$a_t > a^q$	$b_t = a^q - \Delta$	MO $\rightarrow$ LO
sell	More aggressive	$a_t > a^q$	$a_t = a^q - \Delta$	LO $\rightarrow$ LO
		$b^q < a_t < a^q$	$a_t = b^q - \Delta$	LO $\rightarrow$ MO
	Less aggressive	$b^q < a_t < a^q$	$a_t = a^q$	LO $\rightarrow$ LO
		$a_t > a^q$	$b_t = b^q + \Delta$	MO $\rightarrow$ LO

[1] b_t and a_t are the order defined before the implementation of the placement strategy
[2] b^q and a^q are the best quoted bid and ask at time t
[3] [order type before placement strategy] $\rightarrow$ [order type after placement strategy]

five best quoted bids and asks (each real-life stock exchange has its own degree of transparency). With probability $Prob_t^i$, the trader plans to place a more aggressive order when the visible part of the order book becomes thicker on the corresponding side of her position, and a less aggressive order when it becomes thinner (Table A.2).

$$Prob_t^i = \tanh(\beta^i * |x_t^{ob}|) \tag{A.9}$$

where x_t^{ob} is the order imbalance and $ob = \{bid, ask\}$ is the position of the trader. $x^{bid}(x^{ask})$ represents the log difference of the current depth of the five best bids (asks) from the current depth of the five best asks (bids).

A.3. Simulations analysis and discussion

In order to study the model, we use computer-based simulations. We simulate the model using the Monte Carlo protocol. The parameter values used in the simulation are given in Table A.3.

Table A.3.: Parameters.

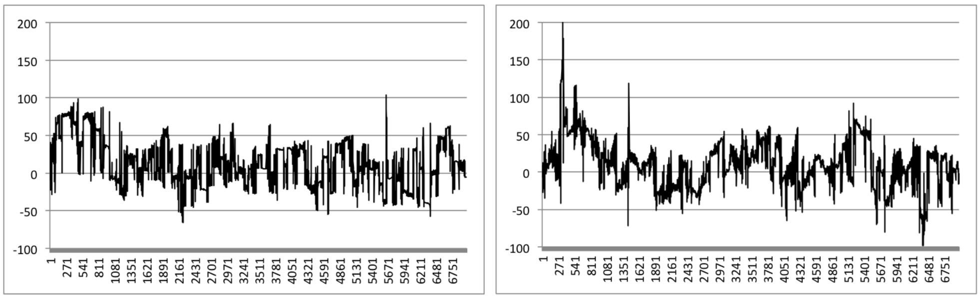
Number of periods	$8,000$
Number of runs	200
Number of agents	$1,000$
Pricing grid width: Δ	10^{-3}
Original Fundamental Value: fv_0	300
Fundamental value process, AR(1): fv_t	$fv_{t-1} + \epsilon_t$
Noise of the fundamental value: ϵ_t	$\mathcal{N}(0, \sigma^2_{fv})$
Fundamentalist component - exponential law	σ^2_f
Chartist component - exponential law	σ^2_c
Reference order life: τ	300
Reference risk aversion: α	0.1
Agent's mood: $M^{i,j}_t$	$\mathcal{U}(-5\%; +5\%)$
Sensitivity to order-book depth: $\beta^{i,j}$	$\mathcal{U}(0; \beta^{max})$
Belief perseverance: γ_1, γ_2, γ_3	0.58, 0.40, 0.02
Degree of misperception: ω	0.98
Correction mechanism: ψ	0.005

A.3.1. Perfect knowledge case

As we can expect – when a fundamentalist trend exists – the market never diverges even if chartists have a destabilising impact. In a market where agents are on average 19% fundamentalist and 81% chartist (a 19%/81% market is equivalent to $\sigma^2_f = 0.6$ and $\sigma^2_c = 10$), the trading price evolves in a wide interval around its fundamental value $[-150; +250]$ but stays in this path. Minimising the chartist component (43%/57% market) does not impact drastically the average trading price but makes its volatility significantly lower (849 vs. 200). In the same way, the trading price evolves in a smaller interval around its fundamental value.

Fundamentalists perceive the fundamental changes and update their expectations according to the news. Regardless of the variance of the white noise of the true fundamental value (σ^2_{fv}), the trading price oscillates around its fundamental. In the case of perfect knowledge, the price variance depends of the ratio of chartists and the σ^2_{fv} parameter.

The average trading price in a one-type market - populated by fundamentalist ($\sigma^2_f = 0.6$) - is 308.157 ECU. In the case of two-types market, the trading price is a little below 310 ECU, exactly 309.623 ECU in a 43%/57% market ($\sigma^2_f = 0.6$, $\sigma^2_c = 1$). The variance rises of 22%, from 695.665 to 849.258 (Table A.4 - first column of each market type). The destabilisation power of chartists increases the average price, its variance, the fundamental spread and its range of fluctuation. It also makes the liquidity falls of 33%, from 592.8899 to 391.1416.



(a) Stable fundamental value ($\sigma^2_{fv} = 0.2$) (b) Unstable fundamental value ($\sigma^2_{fv} = 1$)

Figure A.1.: Fundamental spread with perfect knowledge of fundamentals ($\sigma^2_f = 0.6$, and $\sigma^2_c = 1$).

A.3.2. Adaptive learning case

In our simulations, the original anchor is generated for each trader closely to the original fundamental value ($\hat{f}^i_{origin} \simeq f_0$). The anchor has a strong impact on the market. If it is not consistent with the fundamental value, and if all agents have approximately the same expectations, the market price will not reflect its fundamentals. This is due to a slow learning process and a strong anchoring.

When σ^2_{fv} is high, the fundamentalists do not perceive the fundamental movements, making the estimation of the fundamental value stable and wrong. As a result, the average trading price and its variance decreases, while the standard deviation of the fundamental spread increases (Table A.4 - two-types market "$+\hat{f}_i$"). The disconnection between the trading price and the true fundamental value is thus due to the fundamentalist's anchor and to the chartist's destabilising impact. In the case of a stable fundamental value ($\sigma^2_{fv} = 0.2$), the trading price evolves essentially around its fundamentals with a slightly overvalued trend (Fig. A.2a). Sometimes, a disconnection between the spot price and the fundamental may appear. Let's have a look at figures A.2a and A.2c, a bubble blooms around time step 2100, the fundamental value is overvalued by 30 ECU and the price by more than 100 ECU. The most destabilising impact is due to the fundamentalists. When the fundamental value moves ($\sigma^2_{fv} = 1$), the price still evolves around the anchor which is close to 300 ECU (Fig. A.2b). However, the anchor makes the fundamental expectation far from the true one (Fig. A.2b).

Unsurprisingly, the market liquidity is insensitive to the learning parameter. There is no informational advantage either in perfect knowledge or in adaptive learning which may explain a variation in the liquidity.

A.3.3. Adaptive learning case with order placement strategy

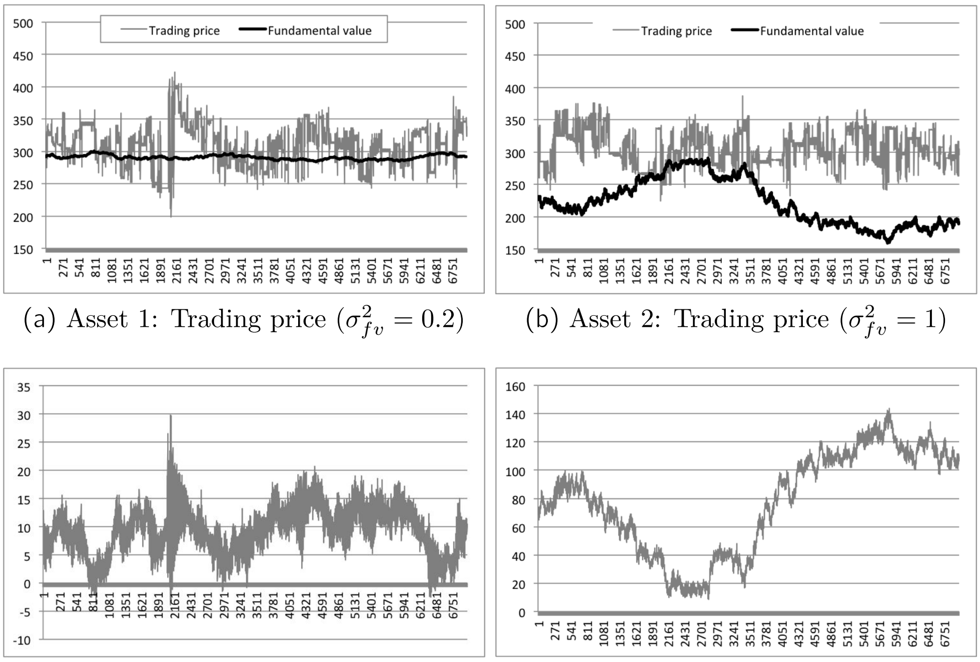
In addition to the belief perseverance, the traders are able to adapt their orders. After formulating their expectations and before the submission, they can change

Table A.4.: Impact of components (with $\sigma^2_{fv} = 0.2$).

	One-type market	Two-types market		
Parameter	$\sigma^2_f = 0.6$	$\sigma^2_f = 0.6$, $\sigma^2_c = 1$	$+ \hat{f}_i$	$+ \beta$
Average price (Std. Dev)	308.157 (12.33)	309.623 (13.34)	307.580 (6.04)	314.148 (9.11)
Price variance	695.665 (227.26)	849.258 (223.35)	726.803 (155.92)	801.683 (334.26)
Fundamental spread	7.79 (4.58)	9.21 (6.09)	8.16 (12.30)	14.70 (15.80)
Liquidity	592.8899 (92.22)	391.1416 (26.55)	392.1275 (21.38)	632.5406 (76.86)

the order type (MO or LO) and its limit price execution. This choice has a direct impact on the market efficiency. In this setup, agents are strongly influenced by the order book depth. The market liquidity is around 600, one and a half times higher than the same market without order book looking (Table A.4 - two-types market, "$+\hat{f}_i$" vs. "$+\beta$"). The average price increases by 6.568 ECU and the stabilisation impact of the adaptive learning tends to disappear (increasing standard deviation of the average price). Agents adapt their submissions in order to increase the probability of order execution. As a result, the maximum amount of assets traded in a time step is multiplied by three. The market price does not reflect the fundamentals, bubbles need time to burst (around 3000 time steps). Let's take a closer look at figures A.3b and A.3d, the market price follows a double rise between time steps 800 and 2500. The bubble occurs with a perceived fundamental value around 300 ECU and a true fundamental value around 200 ECU (Fig. A.3d). This misestimation is due to the belief perseverance of fundamentalists. In addition, the trading price evolves at least above 350 ECU. This overestimation is attributed to the chartist component and the order placement strategy Between time steps 2500 and 3500, the price is relatively constant but the true fundamental value increases (Fig. A.3b). Independent of agents' expectations, the bubble bursts partially. After time step 3500, the traders makes the trading price fall to the true fundamental value.

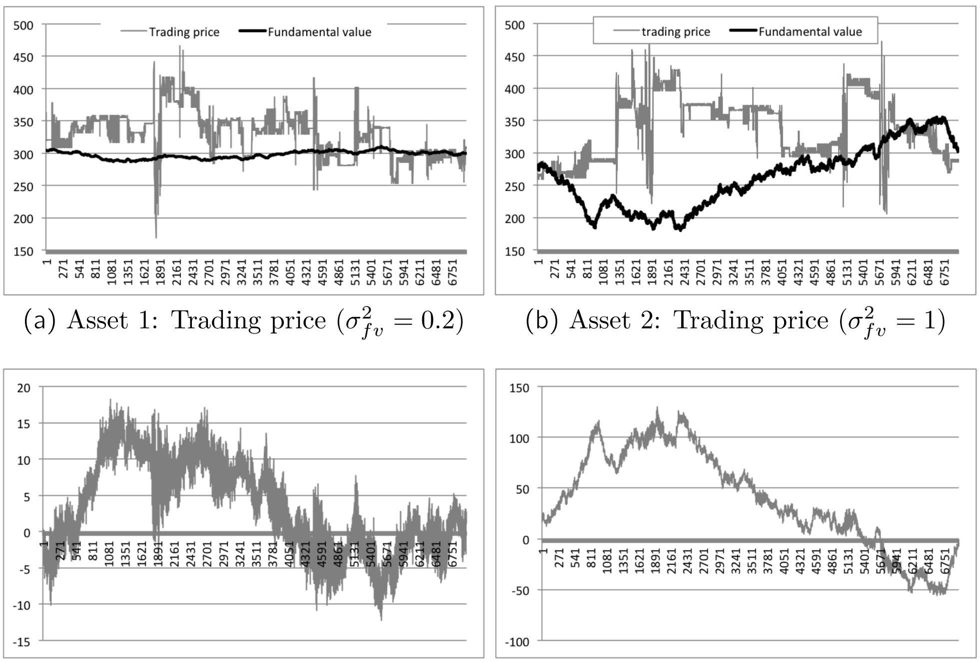
The agents' aggressiveness creates a change in the price dynamics. When agents are receptive to the order book statement , even if the fundamental value is stable (Fig. A.3a and A.3c), the market price can stay overvalued for a longer period. The trading price also evolves more brutally (huge gap between trading prices), and the market is less efficient. The order placement strategy, because of the arbitrage between higher price and no-execution risk, makes long-term trends easily identified by chartists, and causes more persistent bubbles. However, a rise in the chartist weight makes the liquidity fall one more time. A switch from 57% to 81% of chartists makes the volume decreases from 632 to 486.



(a) Asset 1: Trading price ($\sigma^2_{fv} = 0.2$) (b) Asset 2: Trading price ($\sigma^2_{fv} = 1$)

(c) Asset 1: Spread between the perceived fundamental value and the true one (d) Asset 2: Spread between the perceived fundamental value and the true one

Figure A.2.: Price dynamics without order-placement strategy in two-types market with belief perseverance ($\beta^{max} = 0$, $\sigma^2_f = 0.6$ and $\sigma^2_c = 1$).



(a) Asset 1: Trading price ($\sigma^2_{fv} = 0.2$) (b) Asset 2: Trading price ($\sigma^2_{fv} = 1$)

(c) Asset 1: Spread between the perceived fundamental value and the true one (d) Asset 2: Spread between the perceived fundamental value and the true one

Figure A.3.: Price dynamics with order-placement strategy in two-types market with belief perseverance ($\beta^{max} = 2$, $\sigma^2_f = 0.6$ and $\sigma^2_c = 1$).

A.3.4. Trading volume discussion

In one-type markets, the fundamentalists differ by their investment horizon and their risk aversion. When the fundamental value has a low variance and traders do not take into account their mood, the market price converges quickly to its fundamental value. Starting from the idea that fundamentalists make the market efficient, it is expected that the fundamental value reflects the spot price. However, when the efficiency is reached, the fundamentalists no longer need to trade. No new orders are quoted in the order book or all fundamentalists submit on the same side of the book. As a result, the market becomes illiquid.

Agent's mood allows agents who have the same expected forward price to submit at different prices. The market still reflects the fundamental price but with some noise. Moreover, because the market is never exactly equal to the fundamental value, the volume of exchange is higher. Note that the price fluctuation in the one-type market with perfect knowledge of the fundamentals is mainly due to the mood parameter. It impacts the market as noise trader agents and hence brings liquidity. In the case of two-types market, the oscillation generated by the mood tends to be hid by the destabilising impact of chartists.

The chartist's trend increases the maximum amount of assets exchanged in a time step. However, they unexpectedly decrease the exchange volume per simulation. This is counter-intuitive to usual findings (Barber and Odean, 2000; Shiller, 2003). The trading volumes fall from 600 for a one-type market ($\sigma^2_{g_1} = 0.6$) to 391 for a two-types market ($\sigma^2_f = 0.6$ and $\sigma^2_c = 1$). The liquidity of this two-types market is similar to a one-type market with strongly risk-averse fundamentalists ($\sigma^2_f = 10$). In a market populated only by chartists, the trading volume increases from 300 to 900, respectively for $\sigma^2_c = 0.6$ and $\sigma^2_{g_2} = 10$. Chartists should at least counter-balance the decreasing part of fundamentalists submissions, since we build them as risk-lovers.

This decreasing trading volumes in our two-types market is in opposition with empirical study and logically incoherent: adding non informed traders increase the market depth. However, the result can be explained by the rise in the price variance and our structure: the asset demand defined by Eq. (A.6). The first and second differentials in the case of two independent assets are:

$$\frac{\partial \pi_t}{\partial \hat{p}_{t+\tau}} = \frac{1}{\alpha\; p_t\; \hat{p}_{t+\tau} Var} > 0 \qquad \textit{and} \qquad \frac{\partial \pi_t}{\partial Var} = -\frac{\ln\left(\dfrac{\hat{p}_{t+\tau}}{p_t}\right)}{\alpha\; p_t\; Var^2} < 0$$

$$\frac{\partial \pi_t}{\partial^2 \hat{p}_{t+\tau}} = \frac{-1}{\alpha\; p_t\; \hat{p}^2_{t+\tau} Var} < 0 \qquad \textit{and} \qquad \frac{\partial \pi_t}{\partial^2 Var} = \frac{2\ln\left(\dfrac{\hat{p}_{t+\tau}}{p_t}\right)}{\alpha\; p_t Var^3} > 0$$

The positive impact of the expected price and the negative impact of the variance on the asset demand are verified. The second differential highlights a stronger effect of the price variance. The volatility rises by 22% in our case. Fundamentalists are risk-averse, due to the increasing price variance, they minimise their asset demand. The decrease of fundamentalists' demand is not totally counterbalanced by the chartists' demand.

Another explantation is that indeed agents want to trade more and they submit higher volume, but they do not meet counterparts. This idea is correlated with the increasing maximum amount of assets exchange in a time step due to chartists' behaviour and the decreasing asset demand of fundamentalists. The trading price is defined by a match in which a few amount of assets are traded, or if no trade occurs by a mid point between bid and ask. Thus, the price variance increases, and the trading volume stays small.

Nonetheless, with this fall in trading volume due to chartist component, we observe an increasing price variance as an increasing bid-ask spread. That is consistent with the portfolio allocation literature and the notion of flight to quality Pastor and Veronesi (2009).

A.4. Conclusion

In this paper we built an order driven market, where two independent risky assets are traded, in order to study the market depth and efficiency dynamics. We proved that agent types influence the market dynamics. Indeed, fundamentalists make the market converge to its fundamental price, while chartists make it diverge. The chartists amplify the fundamentalists' trend and make long life-duration bubbles. They are also responsible of a fall in liquidity due to their impact on the price variance. The mood parameter slightly impacts the price oscillation. In some way, our agent's mood replaces the classic Noise Trader Agent in order to prevent illiquid markets. In any cases, this model is able to generate endogenous bubbles boom and bust.

To assign bounded rationality to our agents, we relaxed the strong assumption of perfect knowledge of the fundamentals. Our adaptive agents estimate their own fundamental value according to the arrival of news (common knowledge), an anchor and a learning process. Due to the misperception of fundamental movements, the trading price becomes more stable. The trading volume is not impacted. Indeed, all traders misunderstand the fundamental changes. There is no informational advantage either in perfect knowledge or in adaptive learning which can be exploited by some agents so as to generate a rise in the trading volume. If we stick to our definition of market efficiency as market price being close to the true fundamental value, the market efficiency in case of adaptive learning is thus dependent on the agent's beliefs, the volatility of the true fundamental value and the anchor. Adding to the usual bubbles that are created by speculators, we could produce bubbles due to misunderstanding of the fundamentals change.

When traders place their orders strategically, the market price diverges more easily. The fundamentalists care less about fundamentals in their order submissions, and the market trends are easily identified and amplified by chartists. The result is a huge increase of the trading volume and a loss in market efficiency. From a certain point of view, this inefficiency corresponds to the cost of the "fear of no-execution risk" that is not taken into account in our definition of the fundamental value.

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