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\[ LPV/\mathcal{H}_\infty \] Contrôle utilisé à
crecevoir des gestion
énergétique à bord des
véhicules électriques

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LPV/$\mathcal{H}_\infty$ control design of on-board energy management systems for electric vehicles
Remerciements

To my supervisors, family, friends, colleague...
Pour mes directeurs de thèse, famille, amis, collègues ...

إلى المشرفين، أسرتي، أصدقائي، زملائي...

Sincerely, increment, بإخلاص
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<th>Description</th>
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<tbody>
<tr>
<td>EMS</td>
<td>Energy Management System</td>
</tr>
<tr>
<td>NEDC</td>
<td>New European Driving Cycle</td>
</tr>
<tr>
<td>LMI</td>
<td>Linear Matrix Inequality</td>
</tr>
<tr>
<td>NLMIs</td>
<td>Non-Linear Matrix Inequalities</td>
</tr>
<tr>
<td>LTI</td>
<td>Linear Time Invariant</td>
</tr>
<tr>
<td>LPV</td>
<td>Linear Parameter Varying</td>
</tr>
<tr>
<td>LFT</td>
<td>Linear Fractional Transformation</td>
</tr>
<tr>
<td>$I_n$</td>
<td>Identity matrix of dimension $n$</td>
</tr>
<tr>
<td>SC</td>
<td>Supercapacitor</td>
</tr>
<tr>
<td>FC</td>
<td>Fuel Cell</td>
</tr>
<tr>
<td>GA</td>
<td>Genetic algorithm</td>
</tr>
<tr>
<td>MEA</td>
<td>Membrane Electrolyte Assembly</td>
</tr>
<tr>
<td>MTBF</td>
<td>Mean Time Between Failure</td>
</tr>
<tr>
<td>SOC</td>
<td>State Of Charge</td>
</tr>
<tr>
<td>PWM</td>
<td>Pulse Width Modulation</td>
</tr>
<tr>
<td>DARPO</td>
<td>Descent Algorithm for Residues and Poles Optimization</td>
</tr>
<tr>
<td>ISTIA</td>
<td>Iterative SVD Tangential Krylov Algorithm</td>
</tr>
<tr>
<td>IRTES-SET</td>
<td>Institut de Recherche sur les Transports, l’Energie et la société - Systèmes et Transports laboratory</td>
</tr>
<tr>
<td>HMI</td>
<td>Human Machine Interface</td>
</tr>
<tr>
<td>PSD</td>
<td>Power Spectral Density</td>
</tr>
<tr>
<td>NPSD</td>
<td>Normalized Power Spectral Density</td>
</tr>
<tr>
<td>LPV</td>
<td>Linear Parameter Varying</td>
</tr>
<tr>
<td>SISO</td>
<td>Single Input Single Output</td>
</tr>
<tr>
<td>MIMO</td>
<td>Multi Inputs Multi Outputs</td>
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General introduction

The importance of the electric vehicles comes from the growing prices of petrol products besides the increasing in the atmosphere pollution [1]. Car manufacturers try to find clean power sources (e.g., fuel cells and solar panels) and also develop innovative way to manage the power flow inside the vehicle in order to minimize the global consumption [2]. Alternately, auxiliary power sources are added in order to increase the efficiency by collecting the reversed power (power generated during the braking phase) and store it.

As in any micro-grid, the electric system of a vehicle can be divided into three main parts:

- Input stage: this stage contains all different AC or DC power sources in the system. Depending on their role within the on-board power supply system, sources can be classified into two types:
  - primary power source: which can transform the energy from different physical forms into electrical one. One directional power flow is usually allowed for such power sources to feed the electrical system. Fuel cells (change hydrogen power into electrical one), wind turbines (change wind power into electrical one), solar panels (change solar power into electrical one), diesel/thermal engines (change fossil fuel power into electrical one) are good examples of primary power sources.
  - auxiliary power source: this kind of sources is usually used to stock electrical energy. Bidirectional power flow is needed in order to charge/discharge such power sources. Battery and supercapacitor are good examples of auxiliary power sources.

- Output stage: this stage is usually represented by the DC-bus which stands for the common connection between system’s sources and loads. The DC-bus is in charge with feeding the system load regardless of its type.

- Conversion stage: this stage includes all needed DC-DC, DC-AC, and AC-DC power converters that adapt voltage and current levels of the different sources and loads connected to the common DC-bus. Converters can be 1-quadrant type that allows power flow in one direction, or they can be 2-quadrant type when bidirectional power flow is needed.

Fig. 1 shows the different possible devices that can be used in a micro-grid configuration. Energy management system (EMS) is then required to operate the different devices and to ensure a desired power sharing between sources to satisfy the load demands. It acts on the conversion stage by determining each power converter duty cycle. The EMS specifications and requirements are discussed later in this thesis.

In the literature, the considered electrical systems use two, three, or even more power sources in different combinations between primary and auxiliary sources, where the use of
battery and supercapacitor can be managed to reduce the energy loss by collecting the reversed power from the load and consequently increasing the system efficiency [3].

Besides, EMS can be designed to manage the power flow such that to respect operating conditions for each device in order to preserve its reliability and extend its life. In other words, each power source must be operated to be as efficient as possible, all by ensuring exploitation conditions that guarantee its reliability. [4]. For example, avoid changing the battery load (the current) very quickly and respect the operating charging and discharging curves (depending on the battery type) will lead to improve exploitation conditions and extend the battery’s life.

During the thesis, several methodologies have been developed as in [5, 6, 7]. In this dissertation, we have chosen to detail only the $LPV/H_{\infty}$ approach as energy management system. Also it is a generic solution that we will detail starting from the choice of the power supply system until the real-time application which had taken place in collaboration with IRTES-Set laboratory in Belfort-France.

This thesis consists of four main chapters, which are described as follows:

* **Background on control theory and optimization**: in this chapter, the different mathematical tools used in synthesizing an $LPV/H_{\infty}$ as an EMS for electric vehicle are presented.

* **On-board power supply system configuration and EMS requirements**: this chapter illustrates the considered electrical configuration of the power supply system used in electric vehicles. Models of different elements are presented and detailed. Also,
the EMS requirements and specifications are listed.

* Full- and reduced-order LPV controller as EMS: this chapter details the different steps used in synthesizing the $LPV/\mathcal{H}_\infty$ in a generic way. Then later, a reduced-order version of the controller is obtained in order to decrease its complexity and to be used in a real-time application.

* Real-time validation using a rapid-prototyping platform test bench: this chapter illustrates the effectiveness of the considered EMS in a real-time application using a dedicated test bench. The results and the advantages of the proposed power sharing strategy are discussed and evaluated against the requirements and control objectives.

Finally, a general conclusion with both short-term and long-term further investigation issues of the work are presented at the end of this thesis.

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CHAPTER 1

Background on control theory and optimization

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This chapter is devoted to briefly present the different tools used during the thesis in optimization and robust control synthesis. These tools are widely used in control theory nowadays and can be easily found in the literature. Linear matrix inequality (LMI), LPV/\( \mathcal{H}_\infty \), and genetic algorithm are the main keys to what is coming next in this thesis, so some basics related to these concepts are recalled herein. To get more detailed developments the interested reader may study, among others, the references [9, 10].
1.1 Convex set

In optimization process there are two important issues, the first one concerns the existence of a solution for the proposed problem, whereas the second issue is related to the solution optimality whether it is local or global.

The convex property plays a major role in optimization process. If a bounded set is convex, then the solution exists and, moreover, it is global. Many tools are developed in order to solve convex problems efficiently regarding the optimization time such that CVX tool and Sedumi/Yalmip ([11, 12]). Some useful definitions in convex optimization are listed next.

- **Convex set**: A set $\mathcal{X}$ in linear space is convex if

$$\{x_1, x_2 \in \mathcal{X}\} \Rightarrow \{x = \alpha x_1 + (1 - \alpha) x_2 \in \mathcal{X}, \hspace{1cm} \forall \alpha \in [0, 1]\}$$

- **Convex combination**: if $\mathcal{X}$ is convex and $(x_1, \ldots, x_n) \in \mathcal{X}$, then the point $x$ where

$$x = \sum_{i=1}^{n} \alpha_i x_i$$

is called convex combination of $(x_1, \ldots, x_n)$ if $\alpha_i \geq 0$ for $1 \leq i \leq n$ and $\sum_{i=1}^{n} \alpha_i = 1$.

- **Convex hull**: The convex hull $conv \kappa$ of any subset $\kappa \subset \mathcal{X}$ is the intersection of all convex sets containing $\kappa$. If $\kappa$ consists of a finite number of elements, then these elements are referred to as the vertices of $conv \kappa$. It is easily seen that the convex hull of a finite set is a polytope. The converse is also true: any polytope is the convex hull of a finite set.

- **Convex function**: A function $f : \mathcal{X} \rightarrow \mathbb{R}$ is convex if $\mathcal{X} \neq \phi$ and

$$f(\alpha x_1 + (1 - \alpha) x_2) \leq \alpha f(x_1) + (1 - \alpha) f(x_2), \hspace{1cm} \forall x_1, x_2 \in \mathcal{X}, \hspace{1cm} \forall \alpha \in [0, 1]. \hspace{1cm} (1.1)$$

$f$ is strictly convex if the previous inequality is strict $\forall x_1 \neq x_2$.

- **affine function**: A function $f : \mathcal{X} \rightarrow \mathbb{R}$ is affine if

$$f(\alpha x_1 + (1 - \alpha) x_2) = \alpha f(x_1) + (1 - \alpha) f(x_2), \hspace{1cm} \forall x_1, x_2 \in \mathcal{X}, \hspace{1cm} \forall \alpha \in \mathbb{R}. \hspace{1cm} (1.2)$$

One can notice that all affine functions are convex by definition. This class of functions is widely used in LMI formulations.

1.2 Linear Matrix Inequality (LMI)

Some issues related to LMIs are listed here below, for further information about LMIs and their applications in control system please refer to [13].
1.2. Linear Matrix Inequality (LMI)

1.2.1 LMI definition

The LMI is an efficient tool for many convex optimization problems in the field of automatic control. An LMI is an expression of the form [9]:

\[ F(x) := F_0 + x_1 F_1 \cdots + x_n F_n < 0. \]  

(1.3)

where \( x = (x_1, \ldots, x_n) \) is a vector of \( n \) real numbers, \( \{F_i : 0 \leq i \leq n\} \) is a real symmetric matrix \( F_i^T = F_i \). This is equivalent to \( u^T F(x) u < 0 \) for all real vector \( u \) different from zero.

1.2.2 LMI & subLMI

A finite set of LMIs:

\[ F_1(x) < 0, \ldots, F_n(x) < 0. \]  

(1.4)

is equivalent to the following LMI:

\[
F(x) := 
\begin{pmatrix}
F_1(x) & 0 & \cdots & 0 \\
0 & F_2(x) & \cdots & 0 \\
\vdots & \ddots & \ddots & \vdots \\
0 & 0 & \cdots & F_n(x)
\end{pmatrix} < 0.
\]  

(1.5)

\( F(x) \) in the last equation is symmetric. The subLMIs defined in (1.4) can be grouped in one LMI as in (1.5), where both formulations are equivalent taking into consideration the fact that the eigenvalues of \( F(x) \) are the union of eigenvalues of the set \( \{F_i(x) : 1 \leq i \leq n\} \).

1.2.3 LMI advantages

Two major problems can be expressed by using LMI:

- Feasibility problem: the existence of an element \( x \) that satisfies \( F(x) < 0 \) makes the LMI feasible, otherwise the LMI is said to be infeasible.

A very important example of such a problem is related to Lyapunov stability. According to Lyapunov, the linear system

\[
\dot{x} = A \cdot x, \quad x(t_0) = x_0, \quad x \in \mathbb{R}^n, \quad A \in \mathbb{R}^{n \times n}.
\]  

(1.6)

is stable if there exists a symmetric matrix \( P > 0 \): \( A^T P + PA < 0 \).

This is equivalent to the feasibility of the following LMI:

\[
\begin{pmatrix}
-P & 0 \\
0 & A^T + PA
\end{pmatrix} < 0.
\]  

(1.7)
• Optimization problem: let us define the set $\mathcal{X} = \{x\mid F(x) < 0\}$, and the objective function $f : \mathcal{X} \to \mathbb{R}$. The problem to determine $x$ that satisfies

$$V_{opt} = \inf_{x \in \mathcal{X}} f(x)$$

is called an optimization problem with an LMI constraint.

A good example of such problem is the $\mathcal{H}_\infty$ optimization under LMI formulation as it will be explained later.

### 1.2.4 Some useful tools related to LMI formulation

The following methods are useful to convert a nonlinear optimization problem (for stability or controller design) into a convex linear one.

• **Schur’s lemma**: the LMI

$$\begin{pmatrix} Q(x) & S(x) \\ S(x)^T & R(x) \end{pmatrix} < 0$$

is equivalent to

$$\begin{cases} Q(x) < 0 \\ R(x) - S(x)^T Q(x)^{-1} S(x) < 0, \end{cases}$$

and to

$$\begin{cases} R(x) < 0 \\ Q(x) - S(x)^T R(x)^{-1} S(x) < 0. \end{cases}$$

The second and third inequalities (1.10) and (1.11) are non-linear constraints in $x$, while the first LMI (1.9) is linear. Thus, it is possible to reformulate either of the non-linear matrix inequalities (NLMI) (1.10) or (1.11) into a linear one in the form (1.9). Besides, NLMI of the form (1.10) and (1.11) define convex constraints on the variable $x$ in the sense that the solution set of these inequalities is convex.

• **Projection lemma**: for arbitrary matrices $A, B$ and symmetric $P$, the following LMI

$$A^TXB + B^TX^TA + P < 0$$

has a solution if and only if

$$Ax = 0 \quad \text{or} \quad Bx = 0 \quad \Rightarrow \quad x^TPx < 0 \quad \text{or} \quad x = 0.$$  

which is equivalent to

$$A^T_P A \perp < 0 \quad \text{and} \quad B^T_P B \perp < 0.$$  

where $A \perp$ and $B \perp$ are arbitrary matrices whose columns form a basis of $\ker(A)$ and $\ker(B)$ respectively.

As it is mentioned before, the issues related to LMIs are quite common. For further information a detailed survey on LMI theory can be found in [9, 10].
1.3 Signal and system norms

1.3.1 LTI system definition

Linear time invariant (LTI) system is represented generally by the following dynamic differential equation (state space representation):

\[
\sum_{LTI} \begin{cases} \dot{x} = Ax(t) + B\omega(t) \\ z = Cx(t) + D\omega(t) \end{cases},
\]

where \( x(t) \in \mathbb{R}^n \) is the state vector, \( \omega(t) \in \mathbb{R}^r \) is the input vector, \( z(t) \in \mathbb{R}^q \) is the output vector. \( A \in \mathbb{R}^{n \times n}, B \in \mathbb{R}^{n \times r}, C \in \mathbb{R}^{q \times n} \) and \( D \in \mathbb{R}^{q \times r} \) are the system matrices.

1.3.2 Vector norms

Given a vector space \( \mathcal{X} \) of \( n \) dimensions, the \( p \)-norm of vector \( x \in \mathcal{X} \) is defined as:

\[
\|x\|_p = \left( \sum_{i=1}^{n} |x_i|^p \right)^{\frac{1}{p}}, \quad \forall p \in [1, +\infty].
\]

In particular case of \( p \in \{1, 2, \infty\} \), the following well known vector norms can be defined:

\[
p = 1 \quad \rightarrow \quad \|x\|_1 = \sum_{i=1}^{n} |x_i|
\]

\[
p = 2 \quad \rightarrow \quad \|x\|_2 = \left( \sum_{i=1}^{n} |x_i|^2 \right)^{\frac{1}{2}}
\]

\[
p = \infty \quad \rightarrow \quad \|x\|_\infty = \max_{1 \leq i \leq n} |x_i|
\]

1.3.3 Signal norms

Assuming \( x(t) \) is a function in the complex space where \( x(t) \in \mathbb{C} \), the signal norms are defined as follows:

- The 1-norm of \( x(t) \) is:

\[
\|x(t)\|_1 = \int_0^{+\infty} |x(t)| \, dt
\]
Chapter 1. Background on control theory and optimization

- The 2-norm of $x(t)$ is:
  $$\|x(t)\|_2 = \sqrt{\int_0^{+\infty} x^*(t)x(t) \, dt}$$
  where $x^*(t)$ is the conjugate of the signal $x(t)$. This is an important norm since it indicates the signal power.

- The $\infty$-norm of $x(t)$ is:
  $$\|x(t)\|_\infty = \text{sup}_t |x(t)|$$
equivalently in Laplace space,
  $$\|X\|_\infty = \text{sup}_{\text{Re}(s)>0} \|X(s)\| = \text{sup}_\omega \|X(j\omega)\|$$

1.3.4 $H_\infty/H_2$ system norms

- $H_2$ norm: the $H_2$ norm of a strictly proper LTI system (1.12) (with matrix $D = 0$) from input $\omega(t)$ to output $z(t)$ is the energy of the impulse input $g(t) = \frac{z(t)}{\omega(t)} : \omega(t) = \delta(t)$ where $\delta(t)$ is the Dirac signal) defined as:
  $$\|G(\omega j)\|_2 = \sqrt{\int_{-\infty}^{+\infty} g^*(t)g(t)dt} = \sqrt{\frac{1}{2\pi} \int_{-\infty}^{+\infty} \text{Tr}[G^*(j\omega)G(j\omega)]d\omega}$$ (1.14)
  The $H_2$ is finite if and only if the system is strictly proper and stable, i.e., $G(s) \in \mathcal{RH}_2$.
  - This norm for SISO systems represents the area located below the Bode diagram.
  - For MIMO systems, this norm corresponds to impulse-to-energy gain of the output $z(t)$.
  - The $H_2$ norm can be computed either analytically (in the case of controllability and observability grammians), or numerically using LMIs.

- $H_\infty$ norm: the $H_\infty$ norm of a proper LTI system (1.12) from input $\omega(t)$ to output $z(t)$, is the energy-to-energy gain which is defined as:
  $$\|G(\omega j)\|_\infty = \text{sup}_{\omega \in \mathbb{R}} \sigma(G(j\omega)) = \text{sup}_{\omega(t) \neq 0} \frac{\|\dot{z}\|_2}{\|\omega\|_2}$$ (1.15)
  where $\sigma$ is the maximum singular value of the matrix $G(j\omega)$.
  - The $H_\infty$ for SISO (resp MIMO) systems is the maximum peak of the Bode plot (resp of the singular value plot versus the frequency range), which represents the largest gain or amplification of the input signal.
  - Different from $H_2$, $H_\infty$ is obtained only through numerical solution (LMI resolution for example).
1.4 LPV/$\mathcal{H}_\infty$ control

Linear Parameter Varying (LPV) is a powerful tool used in modeling and control of a large class of systems. Venn diagram appearing in Fig.1.1 [14] shows the importance of LPV systems as a bridge between the non-linear and well known LTI systems. The theory of LPV systems offers great advantages in term of robust stability and performance compared to classical gain-scheduled control (interpolation of LTI controllers). LPV systems are more representative for real systems taking into consideration more dynamics and more information on scheduling parameters (i.e. the parameters bounds and rate bounds if exist) [15]. Besides, LPV controller synthesis results are automatically gain-scheduled without any need for extra methods as in classical methodology [16].

1.4.1 LPV system

An LPV system can be formulated by the following state-space representation:

$$\sum_{LPV} \begin{cases} \dot{x}(t) = A(\rho)x(t) + B(\rho)\omega(t) \\ z(t) = C(\rho)x(t) + D(\rho)\omega(t) \end{cases},$$ (1.16)

where at least one of the matrices $A(\rho), B(\rho), C(\rho), D(\rho)$ depends on the parameter vector $\rho = [\rho_1, \ldots, \rho_N]^T \in \mathbb{R}^N$. Systems in 1.16 are then classified into four different types [10]:

- if $\rho(.) = \rho = ct$ is a constant value, then the system 1.16 is a Linear Time Invariant (LTI) system;
- if $\rho(.) = \rho(t)$, where the time depended is explicit, then the system 1.16 is a Linear Time Variant (LTV) system;
- if $\rho(.) = \theta(t)$ with $\theta(t)$ being an external parameter, then the system 1.16 is an LPV system;
- if $\rho(.) = \rho(x(t))$ depends on the state vector, then the system 1.16 is a quasi-Linear Parameter Varying (qLPV) system.

An important remark:

In control design the parameter vector $(\rho(t) \in \mathbb{R}^N)$ is assumed to be measured (or estimated) and bounded $(\rho \in \mathcal{P})$ for all time instances. $\rho(t)$ is denoted as $\rho$ in the sequel for sake of simplicity. Indeed, system in 1.16 can also be represented by:

$$S(\rho) = \begin{bmatrix} A(\rho) & B(\rho) \\ C(\rho) & D(\rho) \end{bmatrix}$$ (1.17)

Based on the dependence of the system matrices on the scheduling parameters, we consider in the sequel two types of LPV systems: affine and polytopic systems.
• **Affine systems**: In this case, all matrices $A$, $B$, $C$, and $D$ are affine regarding the parameter vector $\rho$:

$$
A(\rho) = A_0 + \sum_{i=1}^{N} \rho_i A_i \\
B(\rho) = B_0 + \sum_{i=1}^{N} \rho_i B_i \\
C(\rho) = C_0 + \sum_{i=1}^{N} \rho_i C_i \\
D(\rho) = D_0 + \sum_{i=1}^{N} \rho_i D_i
$$

(1.18)

where $\rho_i$ is the $i$th element of $\rho$. $A_i$, $B_i$, $C_i$, $D_i$ are constants matrices $\forall \ 0 \leq i \leq N$.

• **Polytopic systems**: this is a special case of affine systems where the matrices are represented by:

$$
A(\rho) = \sum_{i=1}^{N} \rho_i A_i \\
B(\rho) = \sum_{i=1}^{N} \rho_i B_i \\
C(\rho) = \sum_{i=1}^{N} \rho_i C_i \\
D(\rho) = \sum_{i=1}^{N} \rho_i D_i
$$

(1.19)

with $\sum_{i=1}^{N} \rho_i = 1$ and $\rho_i \geq 0$.

The polytopic systems are of a great interest in controller design and implementation. As, in this case, the LPV system is a convex hull of a finite number of LTI systems, it allows to solve a finite number of LMI problems ([17, 18, 19]) to find a global LPV controller (which is also a convex hull of a finite number of local LTI controllers).

![Figure 1.1: Relation between the different classes of systems.](image)

**Remark:** Note that there exist other representations of LPV systems as polynomial systems ([20]) and LFT representation ([21]).
1.4. LPV/$\mathcal{H}_\infty$ control

1.4.2 LPV system stability

The Lyapunov approach for LPV systems needs to consider the two main cases: constant Lyapunov functions and parameter depended ones.

- Quadratic stability
  
  The LPV system in 1.16 is quadratically stable if there exists a quadratic Lyapunov function $V(x(t)) = x(t)^T P x(t) > 0$, $P = P^T > 0$, $V(0) = 0$, $\forall x \neq 0$ satisfying:
  
  $$\dot{V}(t) = x(t)^T [A(\rho)^T P + P A(\rho)] x(t) < 0, \; \forall \rho \in \mathbb{R}^N \quad (1.20)$$

- Robust stability
  
  The LPV system in 1.16 is robustly stable if there exist a parameter-dependent Lyapunov function $V(x(t)) = x(t)^T P(\rho) x(t) > 0$, $P(\rho) = P(\rho)^T > 0$, $V(0) = 0$, $\forall x \neq 0$ satisfying:
  
  $$\dot{V}(t) = x(t)^T [A(\rho)^T P(\rho) + P(\rho) A(\rho) + \rho \frac{\partial P}{\partial \rho^T}] x(t) < 0, \; \forall \rho \in \mathbb{R}^N \quad (1.21)$$

One can notice that (1.20) and (1.21) are infinite dimension problems since $\rho$ can take any value in $\mathcal{P}$.

1.4.3 LPV control synthesis

The LPV system in 1.16 is here reformulated into a more general form as shown in Fig.1.2:

![System Diagram](image)

**Figure 1.2:** Inputs and outputs for an LPV system in general form.

$$\Sigma(\rho) : \begin{bmatrix} \dot{x} \\ z \\ y \end{bmatrix} = \begin{bmatrix} A(\rho(t)) & B_1(\rho(t)) & B_2(\rho(t)) \\ C_1(\rho(t)) & D_{11}(\rho(t)) & D_{12}(\rho(t)) \\ C_2(\rho(t)) & D_{21}(\rho(t)) & D_{22}(\rho(t)) \end{bmatrix} \begin{bmatrix} x \\ \omega \\ u \end{bmatrix} \quad (1.22)$$

where $x \in \mathbb{R}^n$ is the state vector of the system, $u \in \mathbb{R}^{n \times n_u}$ is the control input vector, $w \in \mathbb{R}^{n \times n_w}$ is the exogenous input vector, $z \in \mathbb{R}^{n \times n_z}$ is the controlled output vector. $y \in \mathbb{R}^{n \times n_y}$ is
the measured output. \( \rho(t) = [\rho_1(t), \ldots, \rho_N(t)]^T \in \mathbb{R}^N \) is the vector of scheduling parameters and is assumed to be known (measured or estimated) and bounded where \( \rho(t) \in \mathcal{P} \). In the sequel, \( \rho(t) = \rho \) for sake of simplicity.

The \( \text{LPV}/\mathcal{H}_\infty \) control problem is to find a controller \( K(\rho) \) associated to the LPV system (1.22) that guarantees not only the stability in closed-loop but also satisfies the performance \( \sup_{\|\omega\|_2} \|\frac{z}{\omega}\|_2 < \gamma_\infty \), we consider here dynamic output feedback controller that can be expressed by the following state-space representation:

\[
K(\rho) : \begin{pmatrix}
\dot{x}_c \\
u
\end{pmatrix} = \begin{pmatrix}
A_c(\rho) & B_c(\rho) \\
C_c(\rho) & D_c(\rho)
\end{pmatrix} \begin{pmatrix}
x_c \\
y
\end{pmatrix}
\]  

(1.23)

where \( x_c \in \mathbb{R}^{n_c} \) is the controller state vector. \( y \) and \( u \) have the same notation in (1.22). \( A_c \in \mathbb{R}^{n_c \times n_c} \), \( B_c \in \mathbb{R}^{n_c \times n_y} \), \( C_c \in \mathbb{R}^{n_u \times n_c} \) and \( D_c \in \mathbb{R}^{n_u \times n_y} \).

As seen in Section 1.4.2, finding a Lyapunov function that ensures the stability of the parameter-dependent closed-loop system results in an infinite set of LMIs (due to the infinite number of parameter values). Different approaches can be used in order to relax this problem. In other words, to reduce this problem into a finite number of LMIs ([22]):

- gridding parameter space ([23]);
- transforming the parameter dependent system into an uncertain system using the Linear Fractional Transformation (LFT) ([24]),
- polytopic approach ([17]); this technique is used in this thesis due to the finite number of bounded varying parameters \( \rho = [\rho_1, \rho_2, \rho_3]^T \in [0, 1]^3 \) as shown later.

### 1.4.3.1 \( \text{LPV}/\mathcal{H}_\infty \) solution based on LMI

\( \mathcal{H}_\infty \) control problem definition: Considering the system in 1.22, the \( \mathcal{H}_\infty \) control problem is disturbance attenuation problem, which means finding a controller of the form 1.23 that minimizes (in closed-loop) the impact of the exogenous input vector \( \omega(t) \) on the controlled output vector \( z(t) \). For a given real number \( \gamma \) and by using the \( \mathcal{L}_2 \) norm, the \( \mathcal{H}_\infty \) controller in its LPV form satisfies:

\[
\sup_{\omega(t) \neq 0} \frac{\|z\|_2}{\|\omega\|_2} \leq \gamma \quad \forall \rho \in \mathcal{P}
\]  

(1.24)

which is equivalent to:

\[
\|T_{zw}(s)\|_\infty = \|C(sI - A)^{-1}B + D\|_\infty \leq \gamma
\]  

(1.25)
where $I$ is the identity matrix, the matrices $A, B, C, D$ are the state-space matrices for the closed-loop system where:

$$
A = 
\begin{bmatrix}
A + B_2(I - D_cD_{22})^{-1}D_cC_2 & B_2(I - D_cD_{22})^{-1}C_c \\
B_c(I - D_cD_{22})^{-1}C_2 & A_c + B_c(I - D_cD_{22})^{-1}D_{22}C_c
\end{bmatrix}
$$

$$
B = 
\begin{bmatrix}
B_1 + B_2(I - D_cD_{22})^{-1}D_cD_{21} \\
B_c(I - D_cD_{22})^{-1}D_{21}
\end{bmatrix}
$$

$$
C = 
\begin{bmatrix}
C_1 + D_{12}(I - D_cD_{22})^{-1}D_cC_2 & D_{12}(I - D_cD_{22})^{-1}C_c
\end{bmatrix}
$$

$$
D = 
\begin{bmatrix}
D_{11} + D_{12}(I - D_cD_{22})^{-1}D_cD_{21}
\end{bmatrix}
$$

Then the optimal $\mathcal{H}_\infty$ gain is denoted as $\gamma^*$ which is the smallest gain for all existent controller $K(\rho)$, which can be expressed as:

$$
\gamma^* = \min_{\forall K(\rho)} \|T_{zw}(s)\|_\infty
$$

**LPV controller design** According to [25, 17], a dynamical output feedback controller of the form presented in (1.23) (with $n_c = n$) that solves the $\mathcal{H}_\infty$ control problem is obtained by solving the following LMIs in $(X(\rho), Y(\rho), \hat{A}, \hat{B}, \hat{C}, \hat{D})$ while minimizing $\gamma$,

$$
\begin{bmatrix}
M_{11}(\rho) & * & * & * \\
M_{21}(\rho) & M_{22}(\rho) & * & * \\
M_{31}(\rho) & M_{32}(\rho) & -\gamma I_{n_u} & * \\
M_{41}(\rho) & M_{42}(\rho) & M_{43}(\rho) & -\gamma I_{n_y}
\end{bmatrix} < 0
$$

$$
\begin{bmatrix}
X(\rho) & I_n \\
I_n & Y(\rho)
\end{bmatrix} > 0
$$

where

$$
M_{11}(\rho) = A(\rho)X(\rho) + X(\rho)A(\rho)^T + \frac{\partial X(\rho)}{\partial \rho} \hat{\rho} + B_2\hat{C}(\rho) + \hat{C}(\rho)^T B_2^T
$$

$$
M_{21}(\rho) = \hat{A}(\rho) + A(\rho)^T + C_2^T D(\rho)^T B_2^T
$$

$$
M_{22}(\rho) = Y(\rho)A(\rho) + A(\rho)^TY(\rho) + \frac{\partial Y(\rho)}{\partial \rho} \hat{\rho} + \hat{B}(\rho)C_2 + C_2^T \hat{B}(\rho)^T
$$

$$
M_{31}(\rho) = B_1(\rho)^T + D_{21}(\rho)\hat{D}(\rho)^T B_2^T
$$

$$
M_{32}(\rho) = B_1(\rho)^TY(\rho) + D_{21}(\rho)\hat{B}(\rho)^T
$$

$$
M_{41}(\rho) = C_1(\rho)X(\rho) + D_{12}(\rho)\hat{C}(\rho)
$$

$$
M_{42}(\rho) = C_1(\rho) + D_{12}(\rho)\hat{D}(\rho)C_2
$$

$$
M_{43}(\rho) = D_{11}(\rho) + D_{12}(\rho)\hat{D}(\rho)D_{21}(\rho)
$$

Then the vertices of the controller $K(\rho)$ are obtained by the following calculations (taking
into consideration \( \frac{\partial X(\rho)}{\partial \rho} = \frac{\partial Y(\rho)}{\partial \rho} = 0 \):

\[
D_c(\rho) = \hat{D}(\rho) \\
C_c(\rho) = \left( \hat{C}(\rho) - D_c(\rho)C_2X \right) M^{-T} \\
B_c(\rho) = N^{-1} \left( \hat{B}(\rho) - YB_2D_c(\rho) \right) \\
A_c(\rho) = N^{-1} \left( \hat{A}(\rho) - YA(\rho)X - YB_2D_c(\rho)C_2X \right) M^{-T} - B_c(\rho)C_2XM^{-T} - N^{-1}YB_2C_2(\rho)
\]

where \( M(\rho), N(\rho) \) are defined such that \( MN_T = I_n - XY \) which can be solved through a singular value decomposition and a Cholesky factorization. The global \( LPV/H_\infty \) controller is then the convex combination of these local controllers as in (1.30). Gain-scheduled controller is obtained as a result of this polytopic approach as seen in Fig.1.3. A complete review concerning this approach can be found in [10].

![Figure 1.3: Polytopic approach leads to a gain-scheduled controller.](image)

### 1.4.3.2 Polytopic LPV dynamical system

A polytopic LPV system is a convex combination of multiple LTI systems as follows:

\[
\begin{bmatrix}
A(\rho) & B(\rho) \\
C(\rho) & D(\rho)
\end{bmatrix} = \sum_{i=1}^{2^N} \alpha_i(\rho) \begin{bmatrix}
A(\omega_i) & B(\omega_i) \\
C(\omega_i) & D(\omega_i)
\end{bmatrix} \in \mathbb{C} \left\{ \begin{bmatrix}
A_i & B_i \\
C_i & D_i
\end{bmatrix} : \quad 1 \leq i \leq N \right\} \tag{1.30}
\]

\[
\alpha_i(\rho) = \frac{\prod_{j=1}^{N} |\rho_j - \zeta(\omega_i)_j|}{\prod_{j=1}^{N} |\tilde{\rho}_j - \rho_j|} \leq 0, \quad \sum_{i=1}^{2^N} \alpha_i = 1 \tag{1.31}
\]

where \( N \) is the number of varying parameters \( \rho = [\rho_1, \rho_2, \ldots, \rho_N]^T \), which in turn leads to \( 2^N \) vertices of the polytope corresponding to the extreme values of the parameter vector \( \rho \). For each component \( \rho_j \) we define \( \tilde{\rho}_j \) and \( \rho_j \) its maximum and minimum values, respectively. \( \zeta(\omega_i)_j \) is the \( j^{th} \) component of the vector \( \zeta(\omega_i) \) defined as:
\[ \zeta(w_i)_j = \begin{cases} \overline{p}_j & \text{if } \omega_i = \rho_j \\ \rho_j & \text{otherwise} \end{cases} \]

the LTI system \( \begin{bmatrix} A_i & B_i \\ C_i & D_i \end{bmatrix} \) represents the system evaluated at each vertex \( \omega_i \) of the polytope.

In case of using the polytopic approach, the dynamical output feedback controller of the form presented previously in Section 1.4.3.1 is found by solving the LMI in (1.28) taking into consideration that \( X, Y \) are constants \( \forall \rho \).

1.5 Genetic algorithm

Genetic algorithm (GA) is one of the most popular multi-objective optimization methods being able to develop generations of parameters (members) in a way that satisfies multi-objective functions [26, 27]. Members in the decision space are classified regarding some fitness functions (objective functions) into the decision space (as in Fig. 1.4)[28]. GA is very effective into finding a convenient solution where there are large set of parameters to be determined using an evolutionary process. The idea is to use probabilistic, multi-point search, random combination (mutation and crossover), and data from the previous generation in order to improve the evaluation of current population.

Some genetic idioms are kept and widely used in the description of the genetic algorithm different parts, some of them are listed next:

- individual or member: it refers to any possible solution in decision space where any acceptable combination of the parameters can form an individuals.
population or generation: it refers to group of individuals.

- chromosome: it refers to one parameter among the individual parameters.

- genome: it refers to group of chromosomes in the individual.

The GA consists of different steps as described in Fig. 1.5. The algorithm starts by choosing randomly the first generation, then generations are developed using four main steps that are evaluated sequentially in a continuous loop. The process is terminated when the desired number of generation is achieved. The main four steps are:

- **fitness function**: it is the measure that evaluates each individual. Then, an individual with a high fitness is more probable to be selected;

- **selection**: this step sorts and copies individuals by order of satisfaction of the fitness function. The most popular methods in selection are *proportionate selection* ([29]) and *tournament selection* ([27]);

- **crossover**: (Fig. 1.6.a) this is the main operation acting on the population of parents. It is based on exchanging parts between two selected individuals (parents) to form two new individuals (children). This exchange process is applied either to a single or multiple chromosomes;

- **mutation**: (Fig. 1.6.b) this is applied by changing randomly either a chromosome or a genome in the individual (child). The mutation is an important operation since it creates new members different from all developed generations.

![Genetic algorithm steps](image)
The GA is used in this thesis in the selections of weighting functions used for $LPV/H_\infty$ controller synthesis, where there exists a large set of parameters (19 parameters) to be determined regarding some objective functions as described later in Section 3.4.3.

![Genetic operations](image)

Figure 1.6: Genetic operations a) Crossover, b) Mutation, c) Reproducing new child.

1.6 Conclusion

In this chapter, different mathematical tools used in synthesizing $LPV/H_\infty$ controller is introduced. These tools are basically used by Chapter 3, where the power sharing strategy based on multi-variable LPV controller in its polytopic form is formulated to meet the requirements of energy management systems on-board within electric vehicles. Genetic algorithm is used to choose the weighting functions’ parameters associated to $H_\infty$ control problem in automatic way, and to emphasize the frequency separation requirement between power sources while supplying the load power demands in the electric vehicle.
CHAPTER 2

On-board power supply system configuration and EMS requirements

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This chapter illustrates the different parts of the considered on-board power supply system used in electric vehicle. First, power sources with different dynamic behavior are chosen, after that the connection between these parts will be clearly defined with its advantages, then modeling of different components is introduced. Finally, the common requirements of an energy management system are presented as well as the advantages of the proposed power sharing strategy.

2.1 Introduction

Lots of studies are concerned today with the combination different electric power sources within the vehicles such as fuel cell, battery, photovoltaic sources and supercapacitors, etc., maximizing the energy efficiency. Most of works reported in the literature focus on two-power-source systems consisting, for example, of a fuel cell and either a battery or a supercapacitor as auxiliary source. In this thesis, three different power sources are considered: fuel cell (as primary power source), battery and supercapacitor (as auxiliary power sources). The reason of this choice is to deal with general power sharing problem including sources with different dynamic behavior that can cover most known power sources. Moreover, fuel cell combined
with battery and supercapacitor are able to achieve the same power and energy density similar to an ordinary internal combustion engine ([30]).

**Energy-power plane:**

There are two terms to describe a power source according to power supply ability [31]:

- Source with high *power density*, which is able to provide high power for short period of time, thus it has high frequency characteristic defined as $\frac{dP}{dW}$ where $P$ is the power and $W$ is the energy. The supercapacitors are most considered in such type of sources.

- Source with high *energy density*, which is able to provide power during long period of time, thus it has smaller frequency characteristic. The fuel cells are most considered as primary source with high energy density.

According to Ragone’s classification illustrated in Fig. 2.1 [31], the supercapacitors have high power density since they can provide several kilowatts in less than a second. This property makes the supercapacitor to correspond to the needs of the load when changing very fast (high frequency). Meanwhile, fuel cells can provide power for several hours when the load is in steady state or for charging the auxiliary sources like the supercapacitor and the battery (very slow variable mean power demand).

The battery plays a moderate role between the supercapacitor and the fuel cell, that is fulfilled when the power demand is changing slowly (the term "slowly" can be explained with respect to the battery type and depending on charging and discharging characteristics).

![Figure 2.1: Fuel cells energy density compared with the power density of the supercapacitor and the battery in between them [31].](image)
2.2 Power sources configuration within electric vehicle

There exist two main classifications for the connection between power sources:

- **passive/active connection**: the use of DC-DC converter decides the connection type between two power sources as shown in Fig. 2.2 [32].
  
  - passive connection: the two sources are connected directly to the common DC-bus (Fig.2.2.a);
  - semi-active connection: at least one DC-DC converter is used to connect the power source to the common DC-bus (Fig.2.2.b);
  - active connection: each power source is connected to the common DC-bus through a DC-DC converter(Fig.2.2.c)

All passive/active connections are possible in order to connect the different power sources in electric vehicle [32]. An active connection gives more degrees of freedom and allows controlling some desired variables within the associated source, but makes the control scheme more complicated. Therefore, the chosen connection presents a trade-off between flexibility and control complexity.

- **parallel/series connection**: this classification depends on the relative connection between the DC-DC converters. In parallel connection (Fig.2.2.d), each power source is connected to a DC-DC converter then to the common DC-bus. Whereas in series connection (Fig.2.2.e), DC-DC converters can be cascaded.

Parallel connection is widely used in most on-board power system configuration. However, the advantage of series connection is to boost low voltage source to fit DC-bus one by higher boost factor since converters are cascaded.
Figure 2.2: Different possible passive/active and parallel/series connections between power sources [32]. (a) passive connection for power sources. (b) semi-active connection for power sources. (c) Active connection for power sources. (d) parallel connection for power sources. (e) series connection for power sources.
2.3. On-board electrical system modeling

In this thesis, the parallel/active configuration is used in order to formulate the problem in the more generalized form. Thus, three DC/DC converters are used in order to connect in parallel each power source to the common DC bus in the vehicle. 1-quadrant boost DC/DC converter is used for the fuel cell, this converter allows the power flow in one direction toward the DC bus. 2-quadrant buck-boost DC/DC converters are used with the battery and the supercapacitor, these converters allow the power flow in both directions (charge and discharge the source). Fig.2.3 shows the considered configuration.

![Diagram showing the parallel/active configuration](image)

Figure 2.3: The parallel/active configuration considered for the on-board power supply system, which consists of three power sources paralleled with their associated DC-DC converters on a common DC-bus.

All power sources are treated as current sources in supplying the load. To that end, local current control loops are used to serve each source current reference generated by upper-level control loop as described in Section 3.2. This hierarchical strategy allows to impose more constrains from exceeding the current limits for each source separately.

2.3 On-board electrical system modeling

In this section, different elements in the considered electrical system (Fig. 2.3) are modeled using their equivalent electrical scheme. Detailed models for power sources are presented and used in order to obtain more realistic behavior of these sources. Other system components such that DC-DC converters and DC-bus are also modeled in this section. The complete model parameters with their values are listed in appendix (A).

2.3.1 Battery model

For control design, electrical equivalent circuit model mainly considered. However, there exist several models for different types of batteries with different level of complexity. The simplest one is represented by a voltage source connected with series resistance (Fig.2.4), this resistance has two different values for the charging and discharging process, respectively. [33].
Adding more dynamics to the battery model can be achieved by including capacitors effect and also varying the voltage source with respect to the battery’s state of charge (SOC) [1]. Fig. 2.5 shows a more complex dynamic model for the battery, where $Q_n$ is the battery’s initial SOC, and $C_1, R_1, C_2, R_2$ are constant parameters depend on the battery type whose values are identified in previous work (for more details please refer to [1]). These values are listed in the appendix(A).

The equivalent circuit of the battery is shown in Fig. 2.4, where $E_{eq}$ is the equivalent voltage and $V_{bat}$ is the battery voltage. The current $I_{bat}$ is the output of the battery model.

The SOCs of the battery are calculated as follows:

$$SOC(t) = 1 - \frac{Q_n}{Q(t)} \times 100\%, \quad Q_n = Q(t_0)$$  \hspace{1cm} (2.1)

where

$$Q(t) = Q(t_0) - \int_{t_0}^{t} \rho I_{bat} dt$$

$$\rho = \begin{cases} 1 & I_{bat} \geq 0 \\ 0.95 & I_{bat} < 0 \end{cases}$$

Temperature, battery aging and other parameters could be considered for further more accurate modeling. In addition, adaptive control and RLS (Recursive Least Squares) algorithm can be used in order to perform on-line identification for different battery’s parameters. For more detailed battery models, please refer to [34, 35, 36].
2.3. On-board electrical system modeling

2.3.2 Supercapacitor model

Like in the battery case, there exist several equivalent electrical models for the supercapacitor (SC). Detailed model for the SC is considered here in order to obtain further more accurate representation for supercapacitor behavior. Fig. 2.6 shows the considered supercapacitor model, where $C_0, R_0, C_1, R_1, C_2$ and $R_2$ are constant values identified in previous work (as in [1]). These values are listed in appendix (A).

![Supercapacitor dynamic equivalent electrical scheme](image)

Figure 2.6: Supercapacitor dynamic equivalent electrical scheme.

By applying the Kirchhoff’s law, one can find:

$$V_{sc} = I_{sc}(R_0 + \frac{R_1}{1+R_1C_1s} + \frac{R_2}{1+R_2C_2s})$$  \hspace{1cm} (2.2)

2.3.3 Fuel cell model

The fuel cell (FC) is composed of a membrane surrounded by two electrodes, the cathode and the anode. The combination of oxygen and hydrogen within this membrane, along with a catalyst element distributed in the cathode and anode surface (normally platinum), produces a chemical reaction with electron liberation (electrical current), heat and water as sub-products. The three components, cathode, anode and electrolyte are arranged together to form a single Membrane Electrolyte Assembly (MEA) [37]. There are different models representing the electro-chemical behavior inside the FC with different degrees of complexity. For a control-oriented model, we can consider the electrical equivalent representation of FC, more precisely the electrical model of the capacitive behavior of the layer of charge near the electrode/electrolyte membrane. Fig. 2.7 shows the electrical model of a fuel cell [37]:

In this model $E_0$ is the open-loop voltage, $R_m$ is the FC membrane resistance, $R_{ta}$ and $R_{tc}$ are the anode and cathode transfer resistances, $C_a$ and $C_c$ are the anode and cathode double layer capacitances. These are constant values found by previous work (as in [37]) and they are listed in appendix (A).
2.3.4 DC/DC converter models

Two types of DC/DC converters are used to connect the sources to the common DC bus. Averaged model is considered for the choppers, with no loss in the electrical elements (ideal components).

1-quadrant boost converter model This converter allows the power to flow in one direction, thus it is used in order to prevent the harmful effect of the reversed current back to FC [37]. Fig. 2.8 shows the electrical scheme of such type of converters.

2-quadrant buck/boost converter model This converter allows the power to flow in both directions, i.e., it can charge or discharge the auxiliary power sources like the battery and the supercapacitor [1]. Fig. 2.9 shows the electrical scheme of such type of converters.
2.4. Energy management system specifications

Considering the averaged model of each converter in the complete scheme in Fig. 2.10, the global dynamical model for sources currents is derived as follows:

\[
\begin{align*}
\frac{dI_{fc}}{dt} &= \frac{1}{L_{fc}}[V_{fc} - R_{fc}I_{fc} - V_{DC}(1 - \alpha_{fc})] \\
\frac{dI_{sc}}{dt} &= \frac{1}{L_{sc}}[V_{sc} - R_{sc}I_{sc} - V_{DC}\alpha_{sc}] \\
\frac{dI_{bat}}{dt} &= \frac{1}{L_{bat}}[V_{bat} - R_{bat}I_{bat} - V_{DC}\alpha_{bat}]
\end{align*}
\] (2.3)

2.3.5 DC-bus model

This is the output stage that supplies power to load (vehicle’s DC or AC motor with its associated power converter whose model is out of scope of this paper). Without any loss of generality, the DC-bus dynamic is investigated by using an output capacitor which leads to the following equation:

\[
\frac{dV_{DC}}{dt} = \frac{1}{C_{DC}}[\frac{1}{R_{DC}}V_{DC} - I_{load} + (1 - \alpha_{fc})I_{fc} + \alpha_{sc}I_{sc} + \alpha_{bat}I_{bat}]
\] (2.4)

2.4 Energy management system specifications

In micro-grid applications, where there exists multiple power source inputs driven to supply a desired load, the energy management system (EMS) is needed. EMSs are widely spread all over the world for applications such that two, three, or even more power sources are used, where hybridization of all types exists such that fuel cell/battery [38, 39], fuel cell/supercapacitor [37, 40], battery/supercapacitor [1, 31], etc.

The main goals of an energy management system (EMS) can be summarized by the following points, where basically, some electrical constrains should be satisfied with respect to power sources characteristics:

1. maintain the DC-bus voltage around a desired value within an accepted margin of error regardless of the load current variations;

2. respect operating conditions for all power sources specially concerning maximum current values and its maximum variation allowed to be handled. Which results in prolonging the power sources MTBF (Mean Time Between Failure). This requirement is achieved by applying frequency separation between power sources while satisfying load variation;

3. impose a desired steady-state behavior for each one of the power sources that corresponds to some desired power sharing strategy between sources for long term.

In the literature, many works are dedicated to design high-performance and efficient energy management systems for a variety of application with two or three different power sources. A significant number of strategies can be found such that:
Chapter 2. System configuration and requirements

- Proportional-integral controllers [41],
- fuzzy logic control [42, 39],
- filtering strategy [43, 44, 45],
- lqg optimal control [46],
- PI combined with lqr or $H_\infty$ [6].

A common advantage of the previous power sharing strategies is related to the complexity of the implementation on-board where it is considered relatively simple. While in the other hand, the drawbacks of such strategies that they do not consider the stability of closed-loop system, and the significant number of parameters to be tuned experimentally. A very few works take into consideration the system stability when designing the energy management system as in [47].

In this thesis, a generic power sharing strategy based on $LPV/H_\infty$ control in polytopic form is considered, which has the following advantages:

- The closed-loop system stability is guaranteed.
- Can be applied potentially to coordinate any number of power sources.
- The controller parameters are chosen in automatic manner grace to genetic algorithm and depending on some predefined requirements.
- The power sources are treated as current sources as seen in most works in the literature.
- No pre-require knowledge for the load demand.
- Steady-state behavior for all power sources can be chosen grace to external inputs. This makes the proposed energy management system flexible to be used in different applications.

The drawback of the proposed control strategy is related to the complexity of the on-board implementation especially when increasing the number of used power sources since polytopic approach is used ($2^n$ vertex controller where $n$ is the number of power sources).

2.5 Conclusion

In this chapter, a general on-board power supply system is detailed and modeled. This system consists of fuel cell, battery and supercapacitor whose power and energy density can be similar to an ordinary internal combustion engine to be used in electric vehicle. Besides, these power sources have different dynamic behaviors according to Ragone’s classification. Parallel/active configuration is chosen as general formulation in order to connect the different power sources
to a common DC-bus that supplies the system load. Thus, three DC-DC converters are used. Detailed equivalent electrical schemes are used for modeling purpose in order to have more realistic behaviors of the system’s different parts. Different from most used EMS strategies, the considered methodology in synthesizing the power sharing strategy in this thesis takes into consideration the system stability in closed-loop as well as the robust performance in satisfying multiple EMS requirements and specifications.
Figure 2.10: Complete electrical schematic of the considered on-board power supply system including power sources equivalent electrical models.
This chapter is devoted to illustrate the methodology used in controller synthesis. The control problem is formulated in a general manner that can be applied for any energy management system (EMS) used in any multi-source of micro-grid, potentially with any number of power sources. The power supply system on-board of the electric vehicle is here considered as a sufficiently general example, where variation of the load represented by the driving cycle is unknown a priori. The result of the approach detailed in this section is a generic robust $LPV/\mathcal{H}_\infty$ controller that guarantees not only the stability of the closed-loop system, but also the desired performance of the considered EMS.
Chapter 3. LPV control synthesis

This chapter is structured as follows: the control objectives are presented in Section 3.1. Section 3.2 details the control scheme used in this thesis. The system LPV model is presented regarding the desired objectives in Section 3.3. Section 3.4 presents the methodology followed in synthesising the full-order LPV/$\mathcal{H}_\infty$ controller. A reduced-order version of the obtained controller is formulated in Section 3.5. Finally, the control solution is assessed through to some illustrative numerical simulation scenarios in Section 3.6.

3.1 Control objectives

This section summarizes the main goals of the proposed EMS. Basically, some electrical constraints should be achieved with respect to power sources characteristics. The objectives are:

1. Keep the DC-bus voltage (as shown in Fig. 2.10) around $V_{DC}^* = 150$ V within an error of $\pm 10\%$ regardless of the load current variations.
2. Control the current of each power source in the system by using local closed-loops. This is feasible by using local PI controllers since the DC-bus voltage is controlled around a desired contestant value.
3. Apply frequency separation to power sources, *i.e.*, each power source supplies power with respect to its frequency characteristic according to Ragone’s classification (Fig. 2.1). This helps to protect the fuel cell and battery from harmful fast changes of load current. Frequency separation is achieved due to some suitable choice of weighting functions associated to $\mathcal{H}_\infty$ control design.
4. Maintain the supercapacitor state of charge (SOC) slowly variable around 50 $\%$, which allows to absorb/provide power to fulfill instantaneous load power demand.
5. Impose a desired long-term behavior (as external reference inputs of the controlled system) for the fuel cell and the battery that corresponds to some desired power sharing strategy between these sources in steady state. This specification gives an advantage for the proposed EMS to be more general from the application point of view, and to be used by different applications such that:
   - fuel cell can be operated at a desired working point, *e.g.*, which corresponds to its maximum efficiency;
   - steady-state behavior can be used to determine battery charging cycle for long term with respect to its technology;
   - Battery charging can be achieved by using the main power source (fuel cell). Even more, battery can easily be operated as the main power source of the system by replacing the fuel cell when the hydrogen is running out.

In order to satisfy these requirements we have chosen an hierarchical global control structure at two levels: low-level in charge with current control for the three power sources, and upper-level in charge with computing currents references for the low-level control loops.
3.2 Control scheme

This section presents the control scheme used in this thesis with different hierarchical control levels. As it is seen in most EMSs and according to control objectives, it is preferred—from the application point of view—to consider the power sources as current sources. Therefore, two control levels are used as explained next (Fig.3.1).

![Diagram of control scheme]

Figure 3.1: Global control block diagram based on controlling the currents of power sources by using low-level control which serves the references provided by the upper-level control.

3.2.1 Low-level control

At this level, the power converters are controlled independently from the main strategy (upper-level). The idea is to use a local controller for each DC-DC converter current. To this aim, a PI controller is designed (as in [48]) for each converter to generate its PWM duty-cycle satisfying tracking of a desired current reference, the last is generated by the upper-level control strategy. Fig.3.2 shows the local closed-loop attached to each DC-DC power converter.

Supposing that the DC-link voltage is kept constant by the upper-level control, local
current control loops can be implemented based on PI classical controllers. The PI controllers are designed in this low-level control in order to have faster dynamics compared to those expressed at the upper-level control. That allows to consider this control layer as transparent to the upper one. Therefore, we will consider in the sequel $I_{fc}^* = I_{fc}$, $I_{bat}^* = I_{bat}$ and $I_{sc}^* = I_{sc}$.

3.2.2 Upper-level control

The main contribution of the this thesis is represented by the design of this control level. As shown in Fig. 3.1, an LPV controller is synthesized at this level to implement a desired power sharing strategy and to coordinate the power sources such that to manage the power flow toward the vehicle’s load. Power sources are treated as current sources whose references are generated by the LPV controller.

3.3 System’s LPV model

In this section the LPV model of the electrical system is considered based on the control objectives listed in (Section 3.1) and the hierarchical control scheme illustrated in (Section 3.2). According to the control objectives, the supercapacitor voltage should be controlled as well as the DC-bus voltage dynamic. On the other hand, the power converters dynamics represented in (2.3) are compensated by the low-level control consisting of the three suitably tuned PI controllers.

Taking into consideration the dynamic differential equations in (2.2) and (2.4), the LPV
model of the electrical system can be reformulated in the following form:

\[
G(\rho) : \begin{cases}
\dot{x} = A \cdot x + B_1 \cdot \omega + B_2(\rho) \cdot u \\
y = C \cdot x + D \cdot u
\end{cases}
\] (3.1)

where the state vector is \(x = [V_{DC} \quad V_1 \quad V_2 \quad V_0]^T\), \(\omega = I_{load}\) is the current drawn/injected from/into the DC-bus being thus an image of the driving cycle which acts as a the disturbance input, \(u = [I_{fc} \quad I_{bat} \quad I_{sc}]^T\) is the control input vector composed of fuel cell, battery and supercapacitor currents, respectively, and \(\rho = [\rho_1 \quad \rho_2 \quad \rho_3]^T = [\alpha_{fc} \quad \alpha_{bat} \quad \alpha_{sc}]^T\) is the varying parameter vector. Matrices in (3.1) are:

\[
A = \begin{bmatrix}
-1/e_{dc}R_{dc} & 0 & 0 & 0 \\
0 & -1/e_{1}R_{1} & 0 & 0 \\
0 & 0 & -1/e_{2}R_{2} & 0 \\
0 & 0 & 0 & 0
\end{bmatrix}, \\
B_1 = \begin{bmatrix}
-1/e_{dc} \\
0 \\
0 \\
0
\end{bmatrix}, \\
B_2 = \begin{bmatrix}
1-\rho_1/e_{dc} & 0 & 0 \\
0 & 1-\rho_2/e_{dc} & 0 \\
0 & 0 & 1-\rho_3/e_{dc}
\end{bmatrix}
\]

\[
C = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 1 & 1
\end{bmatrix}, \\
D = \begin{bmatrix}
0 & 0 & 0 \\
0 & 0 & -R_0
\end{bmatrix}
\]

**Remarks:**

- one can notice that the system is not asymptotically stable in open-loop because of the existence of an integrator in the supercapacitor electrical model in (2.2), or in other words, there is a zero eigenvalue in matrix \(A\) of the system (3.1).

- matrix \(B_2\) is parameter depended which leads to use an extra filter should be used to get a simple matrix parameter-independent as described in [10].

- parameter vector \(\rho\) is bounded where \(\rho_i \in [0,1]\) and the number of parameters is limited (three parameters). Therefore, the polytopic approach can be used in the LPV control design.

### 3.4 LPV controller synthesis

This section details the procedure to find an LPV controller that coordinates the power sources in the vehicle. This controller is placed in the upper-level control which generates the sources’ current references in order to satisfy the control objectives presented in Section 3.1.

The control problem is to find an LPV controller \(K(\rho)\) that ensures the quadratic stability of the system in (3.1) by the existence of a single Lyapunov function \(P\) which satisfies:
\[ P = P^T > 0, \quad A_c(\rho)^T P + PA_c(\rho) < 0, \quad \forall \rho \in \mathcal{P}, \]  

(3.2)

where \( A_c(\rho) \) is the \( A \) matrix in the state-space representation of the closed-loop system, and \( \mathcal{P} \) is the set in which parameters take their values (in this case \( \mathcal{P} = [0, 1] \times [0, 1] \times [0, 1] \)).

The \( H_\infty \) control is quite justified to be used in control formulation with respect to the control objectives in (3.1) since:

- the first control objective is to minimize the error in the DC-bus voltage, \( V_{DC} \), regardless of the perturbation input \( I_{load} \);
- the weighting functions associated to the \( H_\infty \) control formulation play the major role in frequency separation demanded by the second control objective.

The polytopic approach is recommended to be used in this context since there exist three bounded varying parameters in the LPV representation of the system (3.1). That will lead to a polytope with \( 2^3 = 8 \) vertices.

For all previous reasons, an \( LPV/H_\infty \) controller in polytopic form will be considered to solve the control problem of the system in (3.1) as explained in Section 1.4.3.2.

![Diagram](image)

Figure 3.3: \( H_\infty \) Robust control design block diagram.

### 3.4.1 Weighting functions signification

In \( H_\infty \) robust control design, a group of weighting function are used to bound either the error vector or the control input vector of the control system as presented in Fig.3.3. These weighting functions are designed with respect to control objectives as follows:
1. regulation of DC-bus voltage $V_{DC}$: this is achieved using $W_{eV_{dc}}$, which determines both time response and acceptable tracking error range;

2. power sources frequency splitting: this is achieved thanks to $W_{uI_{fc}}$, $W_{uI_{bat}}$ and $W_{uI_{uc}}$ that impose the dynamic current supply (control input vector) of the fuel cell, the battery and the supercapacitor, respectively, according to the user's pre-specified frequency ranges;

3. tracking of supercapacitor state of charge (SOC): this is achieved using $W_{eV_{uc}}$ to maintain its SOC around 50% to be ready to supply/take current corresponding to load variation;

4. impose steady-state behavior for the fuel cell and the battery: this is achieved using $W_{eI_{fc}}$ and $W_{eI_{bat}}$, respectively. This is useful for imposing a desired steady-state power sharing using $I_{fc,steady\text{state}}$ and $I_{bat,steady\text{state}}$ reference inputs.

The usual form for previous weighting functions is considered as described in [49]. The weighting functions’ orders also should be chosen to achieve frequency separation with respect to the power sources dynamic behaviors. Thus, it is considered that:

- DC-bus voltage, supercapacitor voltage (state of charge) and fuel cell current are bounded by first-order weighting functions $W_{eV_{dc}}$, $W_{eV_{uc}}$ and $W_{uI_{fc}}$, respectively. For each weighting function there are 3 parameters to be determined later (9 parameters in total);

- battery and supercapacitor currents are bounded by fourth-order band-pass weighting function $W_{uI_{bat}}$, $W_{uI_{uc}}$, respectively, this fourth order choice is to ensure sharp separation within narrow frequency interval. For each weighting function there are 4 parameters to be determined for the band-pass filter (8 parameters in total);

- steady-state fuel cell and battery currents are bounded by constants weighting functions $W_{eI_{fc}}$ and $W_{eI_{bat}}$, respectively. 2 parameters should be determined for these weighting functions.
For all previous reasons, the mathematical forms of the considered weighting functions are chosen to be as follows:

\[
\begin{align*}
\frac{1}{W_{eV_{de}}} &= \frac{s + \omega_{b1} \cdot \epsilon_1}{s/M_{s1} + \omega_{b1}} \\
\frac{1}{W_{eI_{fc}}} &= \delta_1 \\
\frac{1}{W_{eI_{bat}}} &= \delta_2 \\
\frac{1}{W_{eV_{ac}}} &= \frac{s + \omega_{b2} \cdot \epsilon_2}{s/M_{s2} + \omega_{b2}} \\
\frac{1}{W_{uI_{fc}}} &= \frac{s + \omega_{BC1}}{s + \omega_{BC1}/M_{u1}} \\
\frac{1}{W_{uI_{bat}}} &= \left(\frac{s + \omega_{BC2} \cdot \epsilon_3 \cdot s + \omega_{BC3} \cdot \epsilon_4}{s/M_{u2} + \omega_{b2}}\right)^2 \\
\frac{1}{W_{uI_{ac}}} &= \left(\frac{s + \omega_{BC3} \cdot \epsilon_5 \cdot s + \omega_{BC3} \cdot \epsilon_6}{s/M_{u3} + \omega_{b3}}\right)^2
\end{align*}
\]

(3.3)

where \( \delta_1, \delta_2, \epsilon_1, \epsilon_2, \epsilon_3, \epsilon_4, \epsilon_5, \omega_{b1}, \omega_{b2}, \omega_{b3}, \omega_{b4}, M_{s1}, M_{s2}, M_{u1}, M_{u2}, M_{u3} \) are 19 constant parameters to be determined (they represent the chromosomes of each individual used by the genetic algorithm as explained in Section 1.5).

After specifying the shape and the order of the weighting functions, their parameters should be selected in a decent way since this selection is the key issue for the \( \mathcal{H}_\infty \) control design and it is quite complex. In order to propose an efficient and repeatable procedure of control design, an optimization process based on genetic algorithms (GAs) is considered.

According to Fig. 3.3, the considered control problem is proposed as finding an LPV controller \( K(\rho) \) that ensures the closed-loop stability for all parameter variations and satisfies

\[
\|e\|_2 < \gamma_{\infty}, \quad \text{where } \omega = [V_{dc,ref}, I_{fc,steadystate}, I_{bat,steadystate}, V_{sc,ref}, I_{load}], \quad \text{and } e \text{ is the controlled output vector } e = [e_1, e_2, ..., e_7].
\]

### 3.4.2 LPV/\( \mathcal{H}_\infty \) polytopic controller

Let us consider here that the 19 parameters of the weighting functions associated to \( \mathcal{H}_\infty \) control design are chosen. This choice will be explained in the following section where the 19 parameters are found in automatic way by using genetic algorithm.

The polytopic approach is used to find the desired LPV/\( \mathcal{H}_\infty \) controller. According to the methodology in the framework of quadratic stabilization described in [17, 16], the problem is treated off line by solving a set of LMIs using Yalmip/Sedumi solver (convex optimization using single Lyapunov function, i.e., quadratic stabilization) at each vertex of the polytope, which leads to vertex controllers \( K_i = \begin{bmatrix} A_i & B_i \\ C_i & D_i \end{bmatrix} \) with \( 1 \leq i \leq 2^3 = 8. \)
Then the LPV controller $K(\rho)$ is computed on line as a convex combination of the vertex controllers $K_i$ as follows:

$$K(\rho) = \sum_{i=1}^{8} \alpha_i(\rho) K_i$$  \quad (3.4)

with:

$$\alpha_i(\rho) = \frac{\prod_{j=1}^{3} |\rho_j - \zeta(w_{1j})|}{\prod_{j=1}^{3} |\bar{\rho}_j - \rho_j|} > 0, \quad \sum_{i=1}^{8} \alpha_i = 1,$$

where $\omega_i$ are the vertices of the polytope corresponding to the extreme values of the parameter vector $\rho$. $\zeta(w_{1j})$ is the $j^{th}$ component of the vector $\zeta(w_i)$ defined as:

$$\zeta(w_{1j}) = \begin{cases} 
\bar{\rho}_j & \text{if } \omega_i = \rho_i \\
\rho_j & \text{otherwise}, 
\end{cases}$$

where in this application

$$\bar{\rho}_j = \max(\rho_j) = 1, \quad \rho_j = \min(\rho_j) = 0.$$

### 3.4.3 Weighting functions selection using genetic algorithm

Genetic algorithm (GA) is used to find weighting functions’ parameters associated to LPV/$H_\infty$ synthesis. As shown in Fig. 3.7.b, this method develops generations of parameters to satisfy desired criteria, where objective (cost) functions are required to be minimized in order to meet certain desired optimal performance [26, 50, 16]. In our case, the genetic algorithm is designed to minimize two objective functions:

- **Objective function 1** (closed-loop stability): certain combinations of weighting functions’ parameters may lead to no solution during $H_\infty$ control optimization, this is considered as unstable generation (a high value will be assigned to $J_1$). On the contrary, if suitable combination of parameters is found, then the solution of $H_\infty$ optimal process necessarily stabilize the closed-loop system. And this cost function allows to search for further stable solution in the sense that the closed-loop eigenvalues are smaller than a certain desired degree $\delta$.

  Objective function 1 is expressed as:

  $$\min \left\{ f_1 = \max_i \{\Re(\lambda_i)\} < -\delta : \delta > 0 \right\},$$  \quad (3.5)

  where $\Re(\lambda_i)$ is the real part of the eigenvalue $\lambda_i$ of the closed-loop system.

- **Objective function 2**: this is used to ensure frequency splitting ability of weighting functions. It is based on minimizing the following criterion:

  $$\min \left\{ f_2 = \frac{J_1 + J_2 + J_3}{3} \right\},$$  \quad (3.6)
with

$$\begin{align*}
J_1 &= \frac{\|I_{fe}/I_{load}\|_{\infty,(\Omega_1,\Omega_2)}}{\|I_{fe}/I_{load}\|_{\infty}} \\
J_2 &= \frac{1}{2} \frac{\|I_{bat}/I_{load}\|_{\infty,(\Omega_3,\Omega_4)}}{\|I_{load}\|_{\infty}} + \frac{1}{2} \frac{\|I_{bat}/I_{load}\|_{\infty,(\Omega_5,\Omega_6)}}{\|I_{load}\|_{\infty}} \\
J_3 &= \frac{\|I_{sc}/I_{load}\|_{\infty,(\Omega_7,\Omega_8)}}{\|I_{load}\|_{\infty}}
\end{align*}$$

where $\|\cdot\|_{\infty,(\Omega_0,\Omega_j)}$ is the $\mathcal{H}_\infty$ norm calculated within a pre-specified $[\Omega_0, \Omega_j]$ frequency interval. Frequency values $\{\Omega_i, i = 1, 2, \ldots, 8\}$ are totally ordered like shown in Fig. 3.4 to correspond to the user-imposed relative frequency separation between power sources. These criteria allows to minimize the $\mathcal{H}_\infty$-norm for each power source outside its desired working frequency interval (as shown in Fig. 3.4).

The criteria (3.5),(3.6) guarantees an arbitrary imposed degree of closed-loop stability and satisfy the choice of a desired frequency separation between power sources according following the requirements of each different application.

![Diagram](image)

**Figure 3.4:** The objective function $f_2$ of the genetic algorithm with corresponding frequency intervals.

**Genetic algorithm steps:** The steps that are followed during the GA process are (Fig.3.7.b):

- find the first generation of $LPV/\mathcal{H}_\infty$ controllers that are able to stabilize the system in closed-loop. Besides, associate the values of the objective functions $f_1$ and $f_2$ to each member (controller) in the generation.
• develop the members into generations in iterative way until the desired number of generations is achieved (as explained in Fig. 1.5);

• select the member which has 19 parameters combination of weighting functions that minimizes $f_2$. This controller necessarily stabilizes the system in closed-loop and $f_1$ is satisfied.

GA provides the needed weighting functions that will be used in $LPV/H_\infty$ controller synthesis. Fig.3.5 shows how the frequency performance of the vertex controllers is shaped by the resulted templates of weighting functions.

![Bode gain diagrams](image)

Figure 3.5: Bode gain diagrams for closed-loop transfer functions from $I_{load}$ disturbance input to controlled outputs ($\frac{V_{dc}}{I_{load}}$, $\frac{I_{fc}}{I_{load}}$, $\frac{I_{bat}}{I_{load}}$, and $\frac{V_{sc}}{I_{load}}$, respectively) with their corresponding weighting functions found by genetic algorithm.
Figure 3.6: Bode gain diagrams for closed-loop transfer functions from $I_{\text{load}}$ disturbance input to controlled outputs $\frac{I_{\text{fc}}}{I_{\text{load}}}$, $\frac{I_{\text{bat}}}{I_{\text{load}}}$, and $\frac{I_{\text{sc}}}{I_{\text{load}}}$, respectively) with their corresponding weighting functions found by genetic algorithm.

GA gives the following weighting functions further used in $LPV/H_{\infty}$ synthesis:

$$
\begin{cases}
    1 = \frac{1}{W_{eV_{\text{dc}}}} = s + 0.05 \\
    1 = \frac{1}{W_{eI_{\text{fc}}}} = 3 \\
    1 = \frac{1}{W_{eI_{\text{bat}}}} = 1.9 \\
    1 = \frac{1}{W_{eV_{\text{sc}}}} = s + 0.0005 \\
    1 = \frac{1}{W_{aI_{\text{fc}}}} = s + 0.0007 \\
    1 = \frac{1}{W_{aI_{\text{bat}}}} = s^4 + 57.14s^3 + 816.3s^2 + 0.005714s + 10^{-8} \\
    1 = \frac{1}{W_{aI_{\text{sc}}}} = s^4 + 395s^3 + 3.907 \cdot 10^4s^2 + 1.185 \cdot 10^4s + 900 \\
\end{cases}
$$

In order to show the value of GA objective function $f_2$ (represented in 3.6) corresponding to the GA solution, this function is evaluated for all controller vertices $K_i$ then its maximum value is represented in Table. 3.1.
Table 3.1: The maximum value of genetic algorithm objective function $f_2$ (represented by 3.6) evaluated at all controller vertices.

<table>
<thead>
<tr>
<th>$J_1$</th>
<th>$J_2$</th>
<th>$J_3$</th>
<th>$f_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.362</td>
<td>0.762</td>
<td>0.645</td>
<td>0.590</td>
</tr>
</tbody>
</table>

Figure 3.7: LPV/$H_\infty$ controller design procedure.

3.4.4 Steps towards the solution

The LPV/$H_\infty$ controller synthesis which is presented so far is a generic procedure, which can be applied for other electrical systems with different number of power sources. However, the complexity is related to the number of power sources by $2^n$ since we have used polytopic approach. A summary of this procedure is shown in Fig. 3.7a, and the steps are presented as follows:

1. Decide the reactivity frequency interval for each power source. This interval is related to the so-called frequency characteristics of the concerned power source. The union of those intervals gives the bandwidth of the closed-loop system.

2. Choose the order and structure of associated weighting functions.

3. Use the genetic algorithm to find suitable combination of parameters for the chosen weighting functions.
4. Design the $K_i$ vertex controllers (off-line).

5. Calculate the $LPV/H_\infty$ controller depending on the measured or estimated value of the parameter vector $\rho$ as in (3.4). This step is achieved on-line.

Controller reduction method can be applied in between step 4 and step 5 depending on the complexity of the vertex controllers $K_i$. In our case, each vertex controller has the order of $O(18)$. Therefore, all calculations and memory resources required for one vertex controller will be repeated eight times in order to obtain the final $LPV/H_\infty$ controller. Controller reduction method is proposed in the next section for such controller.

3.5 Reduced-order control

This section presents the method used to reduce the complexity of the LPV controller found previously. The reduced-order controller permits to avoid the computational burden from application point of view. General aspects used to obtain reduced-order controllers are presented next, where the proposed LPV controller reduction method is detailed. Moreover, frequency-domain comparison between full-order LPV controller and the reduced one is provided.

3.5.1 Introduction

The first step toward the application is to consider the complexity of the on-board system and its implementation. People and more especially the engineers who use the control theory in real application need controllers with the following specifications (according to [51]):

- Low complexity.
- Can be discretized to be implemented in numerical machines.
- Free of numerical problems.

For previous reasons, the simple linear time invariant controllers are always preferred rather than complex time variant ones. However, this is not the case of some complex plant where the compensator has to be more sophisticated in order to handle the closed-loop dynamics and to satisfy desired performances. In most cases, complex controllers designed by different approaches (such that $H_\infty$) are likely to have the same order of the original plant (open-loop system).

Two different cases should be considered clearly when order reduction is needed: model reduction and controller reduction. Model reduction considers the open-loop behavior of the system, whereas controller reduction takes into account all closed-loop issues including stability, performance and transfer function form ([52]). In other words, the controller reduction
process takes into account the existence of the plant when evaluating the closed-loop performance.

Different strategies can be followed in order to obtain low-order controllers designed for high-order plant models, these approaches can be summarized as following (Fig. 3.8 as in [52]):

- High-order plant model \(\rightarrow\) low-order plant model \(\rightarrow\) low-order controller: although this approach leads to simplify the design, it is more likely to fail in practice since the first step of this strategy is to approximate the original complex plant. The approximation could lead to unexpected dynamic closed-loop behavior.

- High-order plant model \(\rightarrow\) low-order controller: it is rare to have direct low-order controller design starting with high-order model, but it is still possible for some special plant models with dedicated softwares or solvers. Therefore, this is out of scope from the general practical consideration of the reduced controller synthesis.

- High-order plant model \(\rightarrow\) high-order controller \(\rightarrow\) low-order controller: This strategy is widely used in most applications. There exist different methods in the literature concerns controller reduction, like balanced truncation, Hankel norm approximation, etc. [53]. However, some points should be considered when applying controller reduction methods such that ([51])
  - Transfer function input/output matching,
  - Closed-loop stability and performance,
  - Signal frequency matching.

A practical controller order reduction is not straightforward process simple process. Also comparing the reduced-order controller to the full-order one should be held always with respect to some criteria calculated in the closed-loop. Some loss of performance is expected when reducing controller complexity, further more closed-loop stability is not guaranteed.
3.5.2 Reduced-order LPV controller

As seen in the previous section, an LPV controller is synthesized following polytopic approach. The final LPV controller is a convex combination of eight vertex controller as in (3.4), where each of which has 18 states.

The aim is to reduce the previously described LPV controller in order to have an as simple as possible controller that facilitates the on-board application of the proposed energy management system. However LPV controller reduction is a very complex problem due to the preservation of closed-loop stability. Today there does not exist a generic method for such a problem, and only some case studies where the stability is checked a posteriori can be found. In this thesis we wish to present and implement for the first time a technique to achieve "simple" reduced-order LPV controller.

The reduction technique is applied for each vertex of the full-order LPV controller in its polytopic form. Then, it is shown that the reduced-order LPV controller preserves global stability and performances of the closed-loop system.

Fig. 3.9 shows the transfer matrix structure of a MIMO vertex controller, with 4 inputs and 3 outputs. One can notice that each entry $K_{ih,j}$ (where $1 \leq i \leq 8$ is the number of vertex controllers, $1 \leq j \leq 4$ number of columns in the $K_i$ transfer matrix, and $1 \leq h \leq 3$ number of rows in the $K_i$ transfer matrix) of the matrix is a SISO system.

$$
\begin{pmatrix}
K_{11}(s) & K_{12}(s) & K_{13}(s) & K_{14}(s) \\
K_{21}(s) & K_{22}(s) & K_{23}(s) & K_{24}(s) \\
K_{31}(s) & K_{32}(s) & K_{33}(s) & K_{34}(s)
\end{pmatrix}
$$

Figure 3.9: Transfer matrix of one vertex controller.

Moreover, the $K_i$ vertex controller can also be expressed by the following state-space representation:

$$
\begin{align*}
K_i: \quad \dot{x}_c(t) &= A_i x_c(t) + B_i y(t) \\
u(t) &= C_i x_c(t) + D_i y(t),
\end{align*}
$$

where

$$
A_i \in \mathbb{R}^{n \times n}, B_i \in \mathbb{R}^{n \times 4}, C_i \in \mathbb{R}^{3 \times n}, D_i \in \mathbb{R}^{3 \times 4}
$$

with $n = 18$. 
3.5. Reduced-order control

Let us denote the desired reduced-order controller as:

\[
\hat{K}_i : \begin{cases}
\dot{x}_c(t) &= \hat{A}_i \dot{x}_c(t) + \hat{B}_i y(t) \\
u(t) &= \hat{C}_i \dot{x}_c(t) + \hat{D}_i y(t)
\end{cases},
\]

where

\[
\hat{A}_i \in \mathbb{R}^{r \times r}, \hat{B}_i \in \mathbb{R}^{r \times 4}, \hat{C}_i \in \mathbb{R}^{3 \times r}, \hat{D}_i \in \mathbb{R}^{3 \times 4}
\]

and \( r < n \) is the reduced controller order.

The problem under consideration is then: find an integer \( r \) and matrices \( \hat{A}_i, \hat{B}_i, \hat{C}_i, \hat{D}_i \) such that \( \|K_i(s) - \hat{K}_i(s)\|_{[0,\Omega]} \) is small with respect to a certain norm. More precisely we aim to find the reduced controller \( \hat{K}_i(s) \) that fits the original controller in predefined frequency range \([0,\Omega]\), where \([0,\Omega]\) is the bandwidth of the closed-loop system. For sake of simplicity, all vertex controllers are considered to have the same reduced order \( r \).

MORE toolbox is used for this purpose under MATLAB® environment (for more details please refer to [54]). We consider using MORE toolbox because of its two main properties:

- it can be used for MIMO systems,
- it allows reducing the model within predefined frequency range.

Different algorithms for order reduction are found in MORE toolbox. The Iterative SVD Tangential Krylov Algorithm (ISTIA) and Descent Algorithm for Residues and Poles Optimisation (DARPO) are used in this context because of their ability to handle MIMO problems and to preserve stability for reduced-order system as explained in ([55, 54]). Besides, DARPO allows to determine the optimal reduced-order \( r \) by defining an accepted threshold limit for the error \( |K_i - \hat{K}_i| \).

Furthermore, according to the considered control problem where the frequency separation of the power sources is required, it is important to evaluate the controller order reduction effect over some predefined frequency range. Therefore, \( \mathcal{H}_2([0,\Omega]) \) norm is the best choice in this case since it allows to preserve frequency characteristics found in Section 3.4.3 with respect to the bandwidth \([0,\Omega]\) of the closed-loop system. The \( \mathcal{H}_2([0,\Omega]) \) norm is calculated using the MORE toolbox while applying the controller reduction procedure explained next.

3.5.3 Reduced-order LPV controller solution

Corresponding to frequency intervals introduced in weighting function selection in Section 3.4.3, DARPO is applied on vertex controllers with respect to specified frequency ranges of power sources. To this end, each transfer matrix \( K_i \) is subdivided into three (number of power sources) transfer matrices as follows:
\[
\begin{bmatrix}
v \\
I_{fc} \\
I_{bat} \\
I_{sc}
\end{bmatrix} =
\begin{bmatrix}
K_{11}(s) & K_{12}(s) & K_{13}(s) & K_{14}(s) \\
K_{21}(s) & K_{22}(s) & K_{23}(s) & K_{24}(s) \\
K_{31}(s) & K_{32}(s) & K_{33}(s) & K_{34}(s)
\end{bmatrix}
\begin{bmatrix}
e_{V_{dc}} \\
e_{I_{fc}} \\
e_{I_{bat}} \\
e_{V_{sc}}
\end{bmatrix}
\]
\[= \begin{bmatrix}
M_{Fuelcell}(s) \\
M_{Battery}(s) \\
M_{Super capacitor}(s)
\end{bmatrix}
\begin{bmatrix}
e_{V_{dc}} \\
e_{I_{fc}} \\
e_{I_{bat}} \\
e_{V_{sc}}
\end{bmatrix} = K_i \times y,
\]

which represents the input/output relationship for each power source current. \( y = [e_{V_{dc}} \ e_{I_{fc}} \ e_{I_{bat}} \ e_{V_{sc}}]^T \) is the feedback error vector corresponding to DC-bus voltage, fuel cell steady-state current, battery steady-state current and supercapacitor SOC, respectively, \( u = [I_{fc} \ I_{bat} \ I_{sc}]^T \) is the control input containing sources currents (Fig. 3.9).

Both DARPO and ISTIA are applied for each subsystem \( M_{Fuelcell}(s), M_{Battery}(s), \) and \( M_{Super capacitor}(s) \) in the vertex controller and is associated \( \mathcal{H}_2(\omega, \omega_j) \) norm. where \( M_{Fuelcell}(s) \) is reduced within the frequency interval \( [0, \omega_1] \), \( M_{Battery}(s) \) is reduced in \( [\omega_4, \omega_5] \), and \( M_{Super capacitor}(s) \) is reduced in \( [\omega_8, \Omega] \). where \( \omega_1 = 0.1 \text{ rad/s}, \omega_4 = 0.1 \text{ rad/s}, \omega_5 = 0.25 \text{ rad/s}, \omega_8 = 4 \text{ rad/s} \) are the same values used in genetic algorithm in Fig. 3.4 Section 3.4.3.

Firstly, DARPO is used to determine the best reduced order \( r \), where vertex controllers are reduced for the bandwidth of the closed-loop system \( [0, \Omega] = [0,10] \text{ rad/sec} \) to order \( r = 11 \) with negligible maximum error \( \sup_i \left\| K_i(s) - \hat{K}_i(s) \right\|_{\mathcal{H}_2[0,\Omega]} \). Secondly, ISTIA is applied to check the validity of the reduced-order results.

At this stage, the transfer matrix of the reduced-order controller becomes:
\[
\begin{bmatrix}
\sim 0 & K_{12} & \sim 0 & \sim 0 \\
K_{21} & \sim 0 & K_{23} & \sim 0 \\
K_{31} & \sim 0 & \sim 0 & K_{34}
\end{bmatrix}
y
\]

According to (3.9) one can notice that some entries of the transfer matrix \( K_{ih,j} \) vanish (less than -200 dB). It is worth nothing that this new form of the transfer matrix can be explained regarding the physical properties of the studied system. Thus:

- supercapacitor influences its SOC steady-state error (represented directly by supercapacitor voltage \( e_{V_{sc}} \)) and the DC-bus voltage error \( e_{V_{dc}} \). That explains the existence of \( K_{31} = \frac{I_{sc}}{e_{V_{dc}}} \) and \( K_{34} = \frac{I_{sc}}{e_{V_{sc}}} \), respectively.

- also for the previous reasons, battery influences its steady-state current error \( e_{I_{bat}} \) and the DC-bus voltage error \( e_{V_{dc}} \). That explains the existence of \( K_{21} = \frac{I_{bat}}{e_{V_{dc}}} \) and \( K_{23} = \frac{I_{bat}}{e_{I_{bat}}} \), respectively.
3.5. Reduced-order control

- the fuel cell influences only its steady-state current error $e_{I_{fc}}$, but not $e_{V_{dc}}$, taking into consideration the fact that there exist a current constraint expressed by the equation (2.4). That explains the existence of just $K_{12} = \frac{I_{fc}}{e_{I_{fc}}}$ in the transfer matrix in (3.9).

**Important remark concerning closed-loop stability:**

There is no proof to guarantee the global LPV system stability by using neither ISTIA nor DARPO, but we have checked that there exists a single Lyapunov matrix $P^T = P > 0$ is found, where $A_{iclosedloop}^T P + P A_{iclosedloop} < 0$, $1 \leq i \leq 8$. This results allows to conclude about the quadratic-stability of the closed-loop system.

![Graphs showing frequency analysis comparison](image)

Figure 3.10: Frequency analysis comparison between full-order and reduced-order vertex controllers.
3.5.4 Reduced-order and full-order LPV controllers comparison

In order to achieve full comparison between the reduced-order and the full-order vertex controllers as described in [51], three main points should be considered, as follows.

- Transform matrix match: the form of reduced-order controller is preserved with respect to input/output relationship. The reduced LPV controller has the same MIMO definition as the original controller.

- Closed-loop stability and performance: the stability of the closed-loop system using the reduced-order LPV controller is checked by the existence of a single Lyapunov function as described in previous section.

Fig. 3.10 illustrates the frequency performances of the closed-loop systems (at vertices of the polytope) using both the reduced-order and the full-order vertex controllers. The reduced-order controllers match the full-order ones in the predefined frequency ranges corresponding to each power source characteristics as they are designed for. However, one can notice that $K_{23}$ could also be neglected for all vertex controllers with respect to the other elements of the transfer matrix.

- Frequency matching: this is obvious since it is the principle of the proposed reduction method, where the reduced LPV controller is sought by minimizing the different between the full-order and reduced-order controllers within the closed-loop bandwidth $[0, \Omega]$ of the system.

Further more, some norms are also provided in order to show the performance of the proposed solution with respect to the full-order LPV controller. Table 1 shows the maximum of both $\mathcal{H}_\infty$ and $\mathcal{H}_2$ norms for the closed-loop system using all vertex controllers (reduced-order and full-order). Besides, the criteria $J_1, J_2, J_3$ used in both weighting function optimization process (Section 3.4.3) and controller reduction method (Section 3.5.3) are also calculated.

Table 2 illustrates the relative error between the two closed-loop systems using both $\mathcal{H}_\infty$ and $\mathcal{H}_2$ norms, which is quite small when DARPO is used in determining the reduced order $r$.

Table 3.2: $\mathcal{H}_\infty, \mathcal{H}_2$ Norms calculations for both reduced and full-order controllers.

<table>
<thead>
<tr>
<th>Controller</th>
<th>$J_1$</th>
<th>$J_2$</th>
<th>$J_3$</th>
<th>$H_\infty$</th>
<th>$H_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Full-order controller O(18)</td>
<td>0.362</td>
<td>0.762</td>
<td>0.645</td>
<td>59.555</td>
<td>0.090</td>
</tr>
<tr>
<td>Reduced-order controller O(10)</td>
<td>0.371</td>
<td>0.794</td>
<td>0.624</td>
<td>59.082</td>
<td>1.099</td>
</tr>
</tbody>
</table>
Table 3.3: Relative errors between reduced-order and full-order controllers using $H_\infty, H_2$ Norms.

<table>
<thead>
<tr>
<th>Relative error</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{\sup_i |K_i|<em>\infty - \sup_i | \hat{K}<em>i |</em>\infty}{\sup_i |K_i|</em>\infty}$</td>
<td>$7.942 \times 10^{-3}$</td>
</tr>
<tr>
<td>$\frac{\sup_i |K_i|_2 - \sup_i | \hat{K}_i |_2}{\sup_i |K_i|_2}$</td>
<td>$3.592 \times 10^{-2}$</td>
</tr>
</tbody>
</table>

### 3.6 Simulation results

Numerical simulation under MATLAB®/Simulink® shows the effectiveness of the proposed LPV/$H_\infty$ control approach in both full-order formulation and reduced-order one for the energy management system on board of electric vehicles. Numerical simulations are carried out using nonlinear electrical models (as described in Section 2.3) for different system parts shown in Fig. 2.10, where all dynamics of DC-DC converters, fuel cell and battery are taken into consideration. To that end, the detailed models of fuel cell and battery (as shown in Fig. 2.7 and Fig. 2.5, respectively) are considered to be as close as possible to the real behavior of these power sources. Standardized driving cycles are chosen to prove the closed-loop system capability to cope with various driving modes and satisfy the required control objectives. The driving cycles used here are New European Driving Cycle (NEDC) and IFSTTAR driving cycle (provided in Sections 3.6.1 and 3.6.2, respectively).

These two profiles represent various driving conditions, i.e., acceleration, deceleration, fixed speed and full brake, which allow assessing performance of DC-bus voltage regulation and the way how the three sources are coordinated to satisfy the power demand. For these scenarios, it is considered that load current is served exclusively by fuel cell in steady-state, hence no change demand for battery’s SOC. For this reason, exogenous input references are chosen to be $I_{fc,steady-state}(t) = \frac{\text{Average}(I_{load}(t))}{1-\alpha_{fc}(t)}$ and $I_{bat,steady-state} = 0$, respectively, where $\text{Average}(I_{load}(t))$ is the average value of load current obtained by low-pass filtering with 0.1 rad/s cut-off frequency. $\alpha_{fc}(t)$ is the duty cycle control input applied to the fuel cell’s DC-DC boost converter.

A third simulation scenario is tested (in Section 3.6.3), when the load current is constant in order to test the system steady-state behavior. This scenario illustrates how the two proposed exogenous input references determine the participations of fuel cell and battery while serving the constant load demand. Negative battery current value can also be used, which implies charging the battery in long term.

Next, the results are carried out using both full-order and reduced-order controllers as power sharing strategies. The system behavior is examined using NEDC and IFSTTAR driving cycles, where a summary of the presented results can be presented as follows:
3.6.1 New European Driving Cycle (NEDC) test

This load profile corresponds to a harsh environment driving cycle (e.g., suburban driving), where acceleration followed by full brake occurs frequently [56]. Fig. 3.11.a shows the DC-link current, which is an image of the vehicle speed. In the sequel, DC-link current is called load current for sake of simplicity.

The simulation results will be carried out regarding the control objectives presented in Section 3.1.

1. DC-bus voltage regulation: both full-and reduced-order controllers satisfy voltage regulation objective as shown in Fig. 3.12.a, where the DC-bus voltage is well controlled at voltage reference 150 V within the allowed error ±10%.

2. Power sources frequency separation: Fig. 3.13.a and Fig. 3.13.b show how the power sources’ currents are provided to the system using full-order and reduced-order controllers, respectively. The currents are demanded in a way such that fuel cell supplying average current and supercapacitor handles the peak variations, while the battery provides the midrange-frequency current. In order to complete the frequency analysis, the power spectral density (PSD) of each source current is computed, then it is normalized with respect to the maximum power delivered by each source, as following:

\[
PSD(x) = \frac{1}{N} \sqrt{\hat{X}(f)\hat{X}^*(f)}
\]

\[
NormalizedPSD = \frac{PSD}{\max(PSD)} \times 100% ,
\]

where \(\hat{X}(f)\) is the discrete Fourier transform of the discrete-time signal signal \(x(k)\) of length \(N\). \(\hat{X}^*(f)\) is the conjugated of \(\hat{X}(f)\).

The normalized-PSD allows to identify the frequency where the maximum power is provided by each source. According to Fig. 3.18.a, by using the full-order controller, fuel cell current variations are always placed in low-frequency range \([0,0.1]\) rad/s, while it is not easy to distinguish battery and supercapacitor behaviors for the NEDC scenario. The same fuel cell behavior is preserved in the case of using reduced-order controller, as shown in Fig. 3.18.b. Battery and supercapacitor supply their currents within the frequency intervals \([0.1,0.25]\) rad/s and \([0.25,0.6]\) rad/s, respectively. These frequency intervals
are used in reduction method employed to obtain the reduced-order controller, therefore better performance regarding the frequency separation for battery and supercapacitor is obtained in the case of using reduced-order controller rather than the full-order one. Fig.3.15.a, Fig.3.16.a and Fig.3.17.a show the current demands of fuel cell, battery and supercapacitor, respectively. All figures show that all sources’ currents are more filtered in the case of using reduced-order controller than the full-order one, which is a straightforward result of the reduction method since the behavior of power sources is confined within the corresponding frequency ranges.

3. Supercapacitor state of charge (SOC) behavior: the supercapacitor SOC is always kept within reasonable limits (avoiding extreme states, completely empty or completely full), as shown in Fig. 3.20.a.

4. Fuel cell and battery steady-state behavior: It is not easy to identify the steady-state behavior for this given scenario. However, the third test (described in Section 3.6.3) is dedicated to illustrate this control objective.

### 3.6.2 IFSTTAR cycle test

This load profile is rich in frequency content and challenges the vehicle’s power supply management system in a way corresponding to urban driving conditions [1]. Fig. 3.11.b shows this driving cycle. This profile is used in order to illustrate clearly the second control objective, where frequency separation is applied to power sources usage.

As in the previous driving cycle (NEDC), both full-order and reduced-order controllers are used in determining the power sharing strategy between the power sources in the system. These strategies are assessed with respect to the imposed control objectives as follows.

1. DC-bus voltage regulation: the voltage regulation is achieved by using both full-order and reduced-order controllers since the DC-bus voltage is well controlled at voltage reference 150 V within the allowed error ±10%, as seen in Fig. 3.12.b.

2. Power sources frequency separation: as shown in Fig. 3.14.a and Fig. 3.14.b, power sources’ currents are also supplied with respect to their decent frequency ranges by using full-order and reduced-order controllers, respectively. Fig.3.18.a and Fig.3.18.b show the normalized power spectral density of each source current corresponding to the use of full-order and reduced-order controllers, respectively. Different from the previous driving cycle scenario (NEDC), it is clear that the fuel cell and supercapacitor are used in the low-frequency and high-frequency domain, respectively, while the battery is used in the frequency range located in between the other two power sources.

3. Supercapacitor state of charge (SOC) behavior: as shown in Fig. 3.20.b, the supercapacitor SOC is always kept within reasonable limits for both cases, where full-order and reduced-order controllers are used. However, the full-order controller results in SOC mean-value different from zero, which implies the existence of a DC-component in the
supercapacitor SOC behavior. On the contrary, this phenomena disappears when using the reduced-order controller. This can also be seen in Fig. 3.19.a, where there exists a non negligible use of supercapacitor at the frequency 0.1 rad/s, but still the maximum performance of supercapacitor is placed within the frequency interval [0.25,0.6] rad/s, as it is designed in Section 3.4.

4. Fuel cell and battery steady-state behavior: as in the previous scenario, it is not easy to distinguish the steady-state behavior of the fuel cell and the battery.

Important remark:

Fig. 3.21.a and Fig. 3.21.b are used to illustrate the duty cycles applied to the power converters (\( \rho = [\alpha_{fc}, \alpha_{bat}, \alpha_{sc}] \)) in the case of full-order and reduced-order controller, respectively, and by using NEDC profile. These duty cycles corresponding to IFSTTAR scenario are provided in Fig. 3.21.c and Fig. 3.21.d and by using full-order and reduced-order controllers, respectively.

The duty cycles of the DC-DC converters represent the scheduling parameter vector \( \rho = [\rho_1, \rho_2, \rho_3]^T \). One can notice that this parameter vector is not changing to cover all the working set, and it is almost fixed around one working point. However, the initial state of the supercapacitor and the battery state of charge play the main role in varying this operating point, which means that practically, this operating point can be placed anywhere in the entire working space, as it is shown later in Section 4.3.4 where real-time validations are explained.

3.6.3 Fixed load test

This test illustrates the fourth control objective (Section 3.1) concerning the steady-state behavior of fuel cell and battery, respectively. A simple simulation scenario is given here, where the external reference inputs define the steady-state distribution of the considered power sources currents in order to satisfy the load demand. Therefore, a constant load current is applied which permits to reach steady-state equilibrium. Two different distributions are used to satisfy same constant load current (35 A):

- \( I_{fc,\text{steady state}} = \frac{0.70 \times 35}{1 - \alpha_{fc,\text{steady state}}} = 81 A \) and \( I_{bat,\text{steady state}} = \frac{0.30 \times 35}{\alpha_{bat,\text{steady state}}} = 30 A \);
  
  this corresponds to 70% \( I_{load} \) supplied by the fuel cell and 30% by the battery.

- \( I_{fc,\text{steady state}} = \frac{1.30 \times 35}{1 - \alpha_{fc,\text{steady state}}} = 190 A \) and \( I_{bat,\text{steady state}} = \frac{-0.30 \times 35}{\alpha_{bat,\text{steady state}}} = -24.5 A \);
  
  this corresponds to 130% \( I_{load} \) supplied by the fuel cell and -30% by the battery, meaning that the fuel cell is managed to supply load current and to charge the battery in the same time.

Fig. 3.22 shows the slowly varying \( I_{fc} \) and \( I_{bat} \) currents corresponding to the constant load.
3.7. Conclusion

From an application point of view, these two external inputs are useful to impose a steady-state operating point that corresponds to a user-defined long-term power sharing between the fuel cell and the battery. Note that there exists a slight tracking error which could be reduced by using more complex weighting functions $W_{e_{fc}}$ and $W_{e_{bat}}$.

3.7 Conclusion

In this chapter, the power sharing strategy used to control the power sources on board of multi-source electric vehicles is detailed. The stability and the robustness are the main advantages of the proposed on-board energy management system compared to other used strategies. A hierarchical control scheme is proposed including two control levels: low-level control and high-level one. The latter is represented by a multi-variable LPV/$\mathcal{H}_\infty$ controller which is synthesized to satisfy different specifications and control objectives, among which the frequency separation of sources' currents variations is imposed to ensure most reliable exploitation conditions for each source. The multi-variable controller can potentially deals with any number of power sources by virtue of the generic method of choosing the weighting functions. To that end, an optimization process based on genetic algorithm is applied. Reduced-order version of the synthesized controller is formulated in order to have less computational burden from real-time application point of view. The found reduced-order controller structure is consistent with the physical properties of the considered power supply system, as suggested by the quasi-diagonal shape of the controller transfer matrix.

The nonlinear electrical system is simulated using two standard driving cycles: NEDC and IFSTTAR in order to examine the behavior of the proposed strategy by using both reduced-order and full-order versions. Numerical simulation shows good performance in satisfying the control objectives, that is, represented by DC-bus voltage regulation, and frequency separation between power sources when supplying the load current. Moreover, steady-state behaviors of all power sources can be chosen by means of external inputs to comply with different application requirements.
Figure 3.11: Load current scenarios representing (a) NEDC and (b) IFSTTAR driving cycles.
Figure 3.12: Regulated DC-bus voltage corresponding to (a) NEDC and (b) IFSTTAR driving cycles.
Figure 3.13: The three power sources’ currents corresponding to NEDC load current profile and generated by (a) full-order controller and (b) reduced-order controller.
Figure 3.14: The three power sources’ currents corresponding to IFSTTAR load current profile and generated by (a) full-order controller and (b) reduced-order controller.
Figure 3.15: Fuel cell current generated by full-order and reduced-order controllers and corresponding to (a) NEDC and (b) IFSTTAR driving cycles.
Figure 3.16: Battery current generated by full-order and reduced-order controllers and corresponding to (a) NEDC and (b) IFSTTAR driving cycles.
Figure 3.17: Supercapacitor current generated by full-order and reduced-order controllers and corresponding to (a) NEDC and (b) IFSTTAR driving cycles.
Figure 3.18: Normalized power spectral density (NPSD) of the three sources' currents corresponding to NEDC load current profile generated by (a) full-order controller and (b) reduced-order controller.
Figure 3.19: Normalized power spectral density (NPSD) of the three sources currents corresponding to IFSTTAR load current profile generated by (a) full-order controller and (b) reduced-order controller.
3.7. Conclusion

Figure 3.20: Supercapacitor state of charge (SOC) regulated by full-order and reduced-order controllers and corresponding to (a) NEDC and (b) IFSTTAR driving cycles.
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3.7. Conclusion

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CHAPTER 4

Real-time validation using a rapid-prototyping platform test bench

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4.1 Introduction

This chapter is devoted to show the real-time validation of the proposed power sharing strategy on a rapid-prototyping platform. A dedicated test bench was designed and built in collaboration with IRTES-SET (Institut de Recherche sur les Transports, l’Energie et la Société - Systèmes et Transports laboratory) in Belfort-France in order to study the real-time behavior of the synthesized $LPV/H_{\infty}$ controller as energy management system. Precisely, I would like to thank here all the work-team represented by Associate Professor David BOUQUAIN, Associate Professor Alexandre RAVEY, Associate Professor Fei GAO, Mr. Patrice LUBERDA, and Mr. Pierre DIEBOLT for their efforts spent during the implementation process (two months in total) helping me to carry out the theoretical and simulation ideas of this thesis.
into real application. Fig. 4.1 shows the test bench designed and built in collaboration with IRTES-SET laboratory team.
Figure 4.1: Details of the real-time validation setup in IRTES-SET laboratory in Belfort, France.
The designed system composed of three power sources (fuel cell, battery, and supercapacitor) are connected in parallel to a DC-bus through DC-DC buck/boost converters, then the sources are managed to supply an active load whose variation is imposed to represent the desired driving cycle of the electric vehicle by means of a rapid-prototyping platform based on dSPACE MicroAutoBox system device. Fig. 4.2 shows the different parts used in the assembled test bench.

The test bench different elements are presented next. Then Section 4.3 analyses the behavior of the energy management system in response to different driving cycles scenarios and how the control objectives are satisfied in both time and frequency domains. Finally, a conclusion is provided at the end of this chapter.

Figure 4.2: Schematics of the rapid-prototyping test bench used to validate the energy management system.
4.2 Description of test bench different elements

According to the availability of the electronic parts, there exists a slight implementation difference in power level between the system configuration used in numerical simulation (see Section 3.6) and the one used for real-time application. However, this difference can be compensated by increasing the power capability of the electronic parts (such that power sources, converters, etc...) without any loss of generality from the control strategy point of view.

4.2.1 Power sources

This part is numbered as 1 in the global scheme of the test bench (shown in Fig. 4.2). As considered in all chapters of this thesis, the studied system consists of three power sources which are fuel cell, battery, and supercapacitor, respectively. The fuel cell is here emulated, whereas real-world battery and supercapacitor are used.

- **Supercapacitor**: this power source is used to provide/absorb the fast variations in load current. Two MAXWELL supercapacitors are used in series in order to cover the voltage interval $[0,32]$ V, each of which has the capacity of $56$ F with maximum voltage value of $16$ V. Fig. 4.3 shows the described supercapacitors. The supercapacitor voltage range is limited at $[15,30]$ V, which is considered equivalent linearly to $[0,100]$ % of state of charge (SOC). Thus, each 1% of the SOC is equivalent to $0.15$ V.

![Figure 4.3: The two supercapacitors used in the test bench in series connection.](image)

- **Battery**: this power source is used to handle the mid-range variations of the system’s load current. Furthermore, the battery could be managed to replace the fuel cell (as the main power source) in the case of running out of hydrogen (in the case of real vehicle). Besides, the battery can be used to feed all different electrical devices of the vehicle.
Three batteries of 12 V and 110 Ah are connected in series to be used in the considered system. Fig. 4.4 shows the described battery.

![Image of batteries in series](image)

Figure 4.4: The three batteries used in the test bench in series connection.

- **Fuel cell**: this is the main power source in the system. A 2 kW NEXA fuel cell with 50 V open-circuit voltage is employed. Research effort in IRTES-SET laboratory in Belfort-Francehas resulted in a fuel cell emulator [57], which is used in our test bench. This emulator allows to perform various tests with the same behavior of the considered fuel cell without hydrogen consumption. The emulator consists mainly of a programmable power supply driven by a real-time computer which handles the fuel cell model. The device includes also a human-machine interface (HMI), which allows to investigate the different parameters of the emulated fuel cell during an operation. Fig. 4.5 shows the NEXA fuel cell with its emulator used in the system. For further details please refer to [57, 58].

### 4.2.2 DC-DC converters

This part is numbered as 2 in the global scheme of the test bench (shown in Fig. 4.2). Each power source is connected to the DC-bus through a DC-DC converter depending on its type, where the fuel cell is connected to 1-quadrant DC-DC boost converter and the rest of sources are connected to 2-quadrant DC-DC buck/boost converters. A 2-quadrant converter permits the power flow in both directions which in turn allows to charge/discharge the connected power source.

Practically, a three-phase DC-AC inverter is transformed into three different DC-DC converters connected in parallel to a DC-bus. All inverter inputs/outputs can handle 25 A DC current and up to 350 V DC voltage. The electrical schematics of the three DC-DC converters are depicted in Fig. 4.6.
Each DC-DC converter consists of two IGBT transistors which are operated by two complementary pulse-width modulated (PWM) signals. Average duty cycle of each PWM signal is donated by $\alpha$ where $\alpha \in [0, 1]$. Practically, $\alpha \in [0.05, 0.95]$ in order to avoid applying continues signal to the duty cycle input of the converter.

### 4.2.3 DC-bus

This part is numbered as 3 in the global scheme of the test bench (shown in Fig. 4.2). The DC-bus represents the physical connection between all paralleled power converters and the load. The DC-bus dynamics can be investigated without loss of generality by adding a capacitor in parallel with the load. In almost all applications, the DC-bus voltage should be regulated around a desired value regardless of load current variations. In this application the DC-bus voltage reference value is chosen to be 60 V which can be ensured by all boost converters with proper duty cycle values in between admissible limits.
Figure 4.6: Schematics of a three-phase DC-AC inverter whose branches are used as three DC-DC converters connected in parallel to a DC-bus.
4.2. Description of test bench different elements

4.2.4 Active load

This part is numbered as 4 in the global scheme of the test bench (shown in Fig. 4.2). The active load is a dedicated device used in order to emulate a real driving cycle load current demand. This device is connected to the DC-bus and it can be programmed in a manner to ensure a desired current profile that corresponds to the driving cycle in our case. The active load consists of three main parts, as shown in Fig. 4.7:

- Programmable series of resistors: this part varies the current load dragged from the DC-bus, where in this case load current is considered positive.
- Programmable power supply: this part varies the current load injected into the DC-bus, which represents mainly the current generated during the breaking phase. The load current is considered negative in this case.
- PC: this is used to drive the other two parts in order to perform variation in load current corresponding to a desired driving cycle. The load current profile is provided as a table of values (Excel file), where each value corresponds to a DC current applied each second.

![Figure 4.7: Programmable active load device used to apply desired load current profile to DC-bus.](image)

4.2.5 Sensors

This part is numbered as 5 in the global scheme of the test bench (shown in Fig. 4.2). The sensors represent an important part in controlling the system since they convey the feedback
signals. Current and voltage sensors are deployed in the test bench to allow implementation of the low-level and high-level control loops as in Sections 3.2.1 and 3.2.2 respectively, where the following signals must be measured:

- for low-level control loops: the currents of all power sources (i.e., $I_{fc}$ for fuel cell current, $I_{bat}$ for battery current and $I_{sc}$ for supercapacitor current) are measured, where these measurements are used as feedback signals in current control loops.

- for high-level control: both DC-bus voltage $V_{dc}$ and supercapacitor voltage $V_{sc}$ are measured, where the last one represents directly the SOC of supercapacitor. Besides, the previous measurements $I_{fc}$ and $I_{bat}$ are used here in order to impose a desired steady-state power sharing between the fuel cell and the battery regarding the control objectives listed in Section 3.1.

All measured values are represented by voltage signals within $[0,5]$ V by using a dedicated conditioning circuit described in the appendix (B). This voltage interval is then mapped into $[0,1]$ by using 16-bit analog-to-digital converters (ADC).

Remark:

The load current information is not necessary for control. However, the local current is measured and averaged, then it is used for varying the fuel cell operating point in such a way that the fuel cell supplies the needed power in a long term (steady-state behavior of the main power source).

4.2.6 Real-time control system

This part is numbered as 6 in the global scheme of the test bench (shown in Fig. 4.2). This represents the on-board control system, which is used to implement the proposed EMS. MicroAutoBox II from dSPACE is used to that purpose because of its ability to be programmed by Matlab®/Simulink® directly. More precisely, there exists a dedicated compiler that converts automatically the continuous Simulink diagram into digital C code used by MicroAutoBox II. The use of such device allows to shorten the time of practical implementation and real-time validation (2 months in total).

The different tasks of the real-time control system are summarized as follows:

- Convert all sensor signals into digital form belonging to $[0,1]$, this is achieved using built-in 16-bit ADCs.

- Retrieve the actual sensor values by applying suitable mathematical operations (multiplying by appropriately adjusted gain corresponding to each measured signal).

- Implement both high-level and low-level control loops.
• Generate three 20-KHz PWM signals that correspond to control inputs of the three DC-DC converters, respectively.

• Do security check to prevent the measured values from exceeding admissible limits. For example, DC-bus voltage value should be kept within the acceptable limits of DC-bus capacitor voltage, otherwise the capacitor will explode.

All previous tasks are repeated iteratively and are performed within one sampling time. The sample time is chosen in relation to the fastest dynamic in the system, that is, generating the frequency of the PWM signals. Thus, the sampling period is 50 $\mu s$ corresponding to 20 KHz. However, multi-tasking with different sampling time periods can be programmed but this is not used in this validation phase.

### 4.2.7 Human machine interface (HMI)

This part is numbered as 7 in the global scheme of the test bench (shown in Fig. 4.2). This part allows communication with the real-time control system (MicroAutoBox II) in order to visualize the system state and the different measured variables, besides changing some desired reference values on-line.

Fig. 4.8 shows the HMI used to communicate with the MicroAutoBox II. The HMI is designed using the program ControlDesk 4.2, which allows exchange and monitoring the predefined variables in Matlab®/Simulink®. The main HMI tasks are summarized as follows:

• Read and visualize the actual values of all current and voltage sensors. The variables are visualized either as numerical value or plot versus time.

• Visualize the states of the three DC-DC converters with their actual duty cycle average values.

• Switch between auto and manual mode of each part of the system (high-level control loop, each low-level current control loop, and each DC-DC converter PWM signal). The manual mode allows modifying each variable in the concerned part.

• Tune in real-time the coefficients of the low-level PI control loops.

• Show the emergency stop state of the system in the case of exceeding admissible limits. Reinitialization procedure is also programmed to bring the system into normal operation.

• Record all real-time desired values in order to be analyzed later.

### 4.3 Real-time validation results

This section is dedicated to present and discuss the real-time validation results of the detailed test bench. The proposed energy management system is validated on two different driving
Figure 4.8: Human-machine interface built to communicate with the real-time control system.

cycle patterns and analyzed in both time-domain and frequency-domain.

4.3.1 Control synthesis

The real-time tests are carried out using the reduced-order controller strategy explained in Section 3.5. The obtained weighting functions used in controller synthesis have been modified practically by adding some proper gains to their formulas. This modification is necessary from application point of view in order to avoid sources’ currents saturation, and this is obvious since there exists some slight differences between the theoretical and practical model of the considered power supply system.

Simulation results are also provided here against the real-time performance results for sake of comparison. Overlapping both real-time and numerical simulation results is not a
trivial step for exact period of time, thus the measured load current samples \( I_{\text{load}} \) are used in numerical simulation, as it is shown in Fig.4.9.

### 4.3.2 Control references selection

There are four exogenous inputs representing the closed-loop references for the high-level control loop as shown in Fig. 3.1. These references are selected as follows:

- reference for DC-bus voltage \( V_{\text{dc,ref}} \): it is chosen to be 60 V depending on the characteristics of the DC-DC converters. These converters are able to boost their input voltages (power sources’ voltages) up to 60 V with a proper choice for their duty cycle values.

- Fuel cell steady-state behavior: fuel cell is the main power source in the system, therefore this reference input is chosen to make the fuel cell serve the average load current as follows:

\[
I_{\text{fc,ref}}(t) = \frac{\text{Average}(I_{\text{load}}(t))}{1 - \alpha_{\text{fc}}(t)}
\]

where \( \text{Average}(I_{\text{load}}(t)) \) is the average value of load current obtained by low-pass filtering with 0.1 [rad/s] cut-off frequency. \( \alpha_{\text{fc}}(t) \) is the real-time value of the duty cycle control input applied to the associated DC-DC boost converter.

However, this reference choice is not unique and the fuel cell can be managed to provide constant current that corresponds to maximum power efficiency with respect to the fuel cell characteristics.

- Battery steady-state behavior: the battery is managed to provide no current in long term, which leads to preserve its state of charge (SOC), therefore \( I_{\text{bat,ref}} = 0 \). However, the battery can be used as main power source in the system in case of fuel cell failure (e.g., hydrogen run out) with a suitable choice for this reference input.

- Supercapacitor state of charge (SOC): this SOC reference is chosen to be 50 % which makes the supercapacitor ready all the time to provide/absorb power in case of fast and important variation of load current demand (positive/negative, respectively).

### 4.3.3 Driving cycles description

The same driving cycles used for numerical simulation purpose (Section 3.6) are used here. The active load is programmed to perform both standard driving cycle tests, which are New European Driving Cycle (NEDC) and IFSTTAR driving cycles, respectively.

- The NEDC applies current load demand corresponding to urban drive where acceleration, deceleration and full stop are required periodically (Fig.4.9.(a)).

- The IFSTTAR driving cycle can refer to extra-urban drive, where acceleration, speed variation on high-way, deceleration and full break can happen sequentially (Fig.4.9.(b)).
4.3.4 Results analysis

4.3.4.1 Time-domain analysis

Both real-time and numerical simulation results are provided in next figures, in order to observe and explain the differences between the two. The active load is programmed to perform one driving cycle (either NEDC or IFSTTAR), then all interest variables are measured and recorded by using the HMI. Then, the measured load current $I_{load}$ is filtered and used for simulation purpose. To that end a low-pass filter is used whose cut-off frequency is $10\,[\text{rad/s}]$ to remove noise. Fig.4.9 shows load current variations for both driving cycles in both forms, i.e., measured and then filtered for simulation purpose.

Below, the time domain analysis is provided regarding the control objectives presented in Section 3.1.

- DC-bus voltage control: Fig. 4.10 shows the $V_{DC}$ behavior corresponding to driving cycle current load demands as in (a) NEDC real-time and simulated performances and (b) IFSTTAR real-time and simulated performances. Both real-time and simulation results are quite close (differences are due to filtering for noise reduction) and show $V_{DC}$ regulation within the admissible limits ($60\, \pm 10\,$).

- Power sources frequency separation: Recall that the three power sources, i.e., fuel cell, battery and supercapacitor should be managed to operate in low, midrange and high frequency, respectively. Fig. 4.11 shows their different currents provided to satisfy $I_{load}$ variation. One can notice that, fuel cell current is smooth compared to battery and supercapacitor ones. The fastest variation is obtained for the supercapacitor current to cover fast changes in load current. Further frequency analysis for these sources’ currents is provided later in Sec. 4.3.4.2.

For a detailed analysis, Fig. 4.12.(a) and Fig. 4.12.(b) show the time evolution of fuel cell current for the driving cycles NEDC and IFSTTAR, respectively. Simulation and real-time results are quite close even though the initial states are different. This can also be noticed for the battery current time evolution shown in Fig. 4.13.(a), Fig. 4.13.(b). Supercapacitor provided/absorbed current is shown in Fig. 4.14.(a), Fig. 4.14.(b) for both NEDC and IFSTTAR, respectively. There exists a slight difference between the simulation and the real-time performances, especially concerning variation amplitude. This can be explained by the existence of noise from different sources (sensors, converters, etc.) placed in relatively high frequency within the supercapacitor bandwidth.

- Supercapacitor state of charge (SOC) regulation: this SOC is shown in Fig. 4.15.(a), Fig. 4.15.(b) for both NEDC and IFSTTAR, respectively. Here it is required that supercapacitor SOC to be maintained around $50\,$ in order to make the supercapacitor ready to take/provide current instantaneously irrespective of the load current. The parameters of the supercapacitor model (described in Section 2.3.2) used for simulation purpose were not identified practically, and that leads to some differences between the
SOC simulation performance and the real-time one. Another reason for this difference between the two performances is related to the measurement noise, where variation of 1 % SOC is equivalent to 0.15 V for the used supercapacitor. However, the SOC regulation objective is satisfied in both cases (and for both tested driving cycles) where it is kept around 50 %.

One can notice that, supercapacitor SOC varies at maximum ±6% around the desired reference (50%), this suggests that the system is able to handle larger current variation in load demand or a smaller supercapacitor can be used instead.

- Steady state behavior of fuel cell and battery: this property is validated in real-time by changing the fuel cell and battery current desired references. However, the steady-state behaviors of these power sources are not trivial to be noticed from Fig. 4.11, and unfortunately no dedicated scenario is used in order to illustrate this property in real-time.

The duty cycles of the DC-DC converters are shown in Fig. 4.16, where these variables represent the scheduling parameter vector \( \rho = [\alpha_{fc}, \alpha_{bat}, \alpha_{sc}]^T \). One can notice that, although this parameter vector is changing significantly in the working set, the control objectives are still satisfied for both used scenarios (NEDC and IFSTTAR driving cycles). Moreover, the initial state of the supercapacitor and the battery state of charge play a main role in varying this operating point.

4.3.4.2 Frequency-domain analysis

This section is devoted to illustrate the frequency separation property of the proposed energy management system. To that end power spectral density (PSD) is calculated for all current signals \( I_{fc}, I_{bat}, I_{sc}, I_{load} \) by using the following the formula represented by Equation (3.10).

Fig. 4.17 illustrates the frequency content in the form of power spectral density calculated for the load current signal in both NEDC and IFSTTAR cases. It is clear that the DC average value is dominant since it has the highest value for all frequencies.

In order to illustrate clearly the frequency content of current signals, the normalized PSD (NPSD) is calculated for each signal by division by its maximum value, as described by Equation (3.11).

The normalized PSD allows to identify the frequency where the maximum power is provided by each source. Fig. 4.18(a), Fig. 4.18(b) show the normalized PSD for all currents \( I_{fc}, I_{bat}, I_{sc} \), for both NEDC and IFSTTAR scenarios, respectively. For both cases and, the fuel cell maximum power is provided at very low frequency within the interval \([0, \omega_1] = [0, 0.1] \text{ rad/s}\), which represents the average value (or DC value). The battery maximum power is delivered within the frequency range next to fuel cell i.e., \([\omega_4, \omega_5] = [0.1, 0.25] \text{ rad/s}\). Finally and as expected, the supercapacitor maximum use is within the frequency range \([0.0.25, 0.6] \text{ rad/s}\) which is considered relatively high compared to the other two sources. One can notice that, supercapacitor maximum power is provided depending on the load profile frequency.
content, which is clearly higher in the case of the IFSTTAR scenario (Fig. 4.18.(b)) than the NEDC one (Fig. 4.18.(a)).

4.4 Conclusion

In this chapter, the real-time validation of the multi-variable energy management system based on $LPV/H_\infty$ control strategy is presented. The reduced-order version of the previously designed $LPV/H_\infty$ controller is tested on a multi-source electrical system representing the power supply on board of an electric vehicle. A dedicated test bench was built to that purpose in collaboration with IRTES-SET laboratory in Belfort-France. The electrical system emulates the real behavior of vehicle equipped with three power sources (fuel cell, battery and supercapacitor). Each power source is attached to a DC-DC converter and then all converters are connected in parallel to a DC-bus, which is in charge to feed the load represented by the vehicle’s electrical motor. Standard driving cycles are used to assess the energy management performance, which are NEDC and IFSTTAR driving cycles. The proposed strategy shows promising results related to the imposed control objectives, that is, DC-bus regulation, frequency separation for power sources’ current variations, and imposing desired steady-state behavior for the three power sources.

The system operates without previous knowledge nor measure of the load demand, which is a notable advantage. The chosen driving cycles are considered sufficiently relevant to validate the proposed energy management strategy.

Practically, the reduced version of the $LPVH_\infty$ as energy management strategy is tested and applied in multi-source electric vehicle. Whereas, the full-order controller is not implemented because of the lack of time.
Figure 4.9: Load current scenarios representing (a) NEDC and (b) IFSTTAR driving cycles.
Figure 4.10: Regulated DC-bus voltage corresponding to (a) NEDC and (b) IFSTTAR driving cycles.
Figure 4.11: Three power sources currents corresponding to (a) NEDC and (b) IFSTTAR driving cycles.
Figure 4.12: Fuel cell current corresponding to (a) NEDC and (b) IFSTTAR driving cycles.
Figure 4.13. Battery current corresponding to (a) NEDC and (b) IFSTTAR driving cycles.
Figure 4.14: Supercapacitor current corresponding to (a) NEDC and (b) IFSTTAR driving cycles.
Figure 4.15: Supercapacitor state of charge (SOC) corresponding to (a) NEDC and (b) IFSTTAR driving cycles.
Figure 4.16: The varying parameter vector $\rho = [\alpha_{fc}, \alpha_{bat}, \alpha_{sc}]$ corresponding to (a) NEDC and (b) IFSTTAR driving cycles.
4.4. Conclusion

Figure 4.17: Power spectral density of load current corresponds to NEDC (in blue) and IFSTTAR (in red) driving cycles.
Figure 4.18: Normalized power spectral density of the three sources currents corresponding to (a) NEDC and (b) IFSTTAR driving cycles.
Conclusion and Perspectives

This thesis proposes a multi-variable $LPV/\mathcal{H}_\infty$ control design approach for energy management systems for multiple power sources coordination within micro-grids. An electric vehicle is considered a sufficiently representative example of such micro-grids, where there exists several power sources to be coordinated such as to satisfy the vehicle load demands. The considered problem is formulated and solved in a general manner, where the following points are taken into consideration:

- **Power sources choice**: the considered system is equipped with three different power sources such that fuel cell, battery, and supercapacitor, where the frequency characteristics of each source is different from the others according to Ragone’s taxonomy. Fuel cell represents the main energy source in the system that provides mean power demand for long term in very low-frequency domain. On the contrary, supercapacitor is an auxiliary power source that provides/absorbs load current variations in relatively high-frequency domain. Battery is also an auxiliary energy source, which plays its role in between the other two sources frequency domains. Moreover, battery can be managed to be the main energy source in the system in the case of hydrogen running out.

- **Electrical system choice**: a parallel/active configuration is considered, which supposes connecting each power source to the common DC-bus through a dedicated DC-DC converter. 1-quadrant DC-DC converter is used with the fuel cell connection which allows unidirectional power flow from the source. Whereas, 2-quadrant DC-DC converters are used to connect the auxiliary power sources to the DC-bus, where these converters allow bidirectional power flow in order to charge/discharge the battery and the supercapacitor, respectively.

- **Current-controlled sources**: treating the power sources as current sources is widely spread in most application concerning power sharing strategies. Low-level control loops are deployed not only to serve current references generated by high-level control, but also to prevent sources currents from exceeding admissible limits.

- **Automatic parameter selection**: the proposed power sharing strategy can be applied potentially to coordinate any number of power sources. The user decides the desired operating frequency interval for each source depending on its characteristics, then an automatic procedure based on genetic algorithm is used to find suitably tuned parameters of the weighting functions used in $LPV/\mathcal{H}_\infty$ control synthesis.

- **Standard tests**: both load profiles such that New European Driving Cycle (NEDC) and IFSTTAR driving cycles are used to assess the behavior of the proposed energy management system and to illustrate the control objectives satisfaction.

Different from most energy management systems, the proposed strategy based on $LPV/\mathcal{H}_\infty$ control guarantees not only the stability of the closed-loop system, but also satis-
ifies multiple requirement regarding the power sharing between sources on board of the electric vehicle such that the following objectives to be met:

- DC-bus voltage regulation: in most power supply systems, the DC-bus voltage must be controlled at a desired reference value within admissible error regardless of load variation. Neither pre-knowledge nor measures of load current are required, which is a notable advantage of the proposed power sharing strategy.

- Frequency separation: the power sources are managed to satisfy the load demands in a manner that suits best their characteristics as either high-energy-density or high-power-density source, according to Ragone’s taxonomy. Thus, fuel cell and battery are protected from sudden power variations in order to prolong their mean time between failures (MTBF), while supercapacitor is used to provide/absorb high variations of power demand.

- Long-term behavior of power sources: a desired steady state can be chosen for each power source separately by using external inputs for the LPV/H∞ controller. Supercapacitor state of charge is chosen to be around 50 %, which makes it ready all the time to provide/absorb current demands. Fuel cell is managed to supply average current in long term. Moreover, steady-state battery current can be chosen according to a desired charging or discharging profiles.

Detailed electrical equivalent schemes are used in modeling different devices in the electric vehicle. Then nonlinear electrical system is simulated using the standardized driving profiles, NEDC and IFSTTAR, whose frequency content is rich and allows to appropriately illustrate the frequency-separation ability of the proposed approach. Numerical simulations are carried out using MATLAB® /Simulink® and they show good performance in meeting the vehicle’s power demand according to the frequency-separation power sharing regime imposed by user, all by regulating the DC-bus voltage at a desired setpoint.

In order to decrease the complexity of the LPV/H∞ controller, a reduced-order version of the multi-variable controller is synthesized in order to alleviate the computational burden of real-time implementations. To that end, MORE toolbox is used under MATLAB® and applied directly to reduce the LPV controller vertices in its polytopic form. Results show that the reduced-order version is consistent with the physical properties of the studied system, and it satisfies (as well as in the case of the full-order version of the controller) all specified control objectives and the energy management requirements under the same simulation scenarii.

The reduced-order version of the multi-variable LPV/H∞ is evaluated in a real-time application using a dedicated rapid prototyping platform. This platform was designed and built in collaboration with IRITES-SET laboratory in Belfort-France. The electrical system emulates the real behavior of a vehicle equipped with three power sources (fuel cell, battery and supercapacitor). Each power source is attached to a DC-DC converter and then all converters are connected in parallel to a DC-bus, which is in charge to feed an active load representing the vehicle’s electrical motor. The NEDC and IFSTTAR driving cycles are also used to assess
the energy management performance. The proposed strategy shows promising results related to the imposed control objectives, that is, DC-bus regulation, frequency separation for power sources’ current variations, and imposing desired steady-state behavior for the three power sources.

**Future works:** In terms of perspectives, this work suggests different further investigation issues in both short-term and long term.

- **Short-term perspectives** are based on assessing the energy management system in some different real-time scenarios to improve its reliability.

  - **Steady-state test:** The steady-state behaviors of the fuel cell and battery can be tested using simple fixed load current demand. Some changes can be applied for the external references of the $LPV/H_\infty$ controller in order to examine sources’ dynamics in long term.

  - **Hydrogen running out:** An interesting scenario resembling the case of running out of hydrogen in reality can be tested by disconnecting the fuel cell from the system in order to examine the energy management behavior.

  - **Comparison with another energy management systems:** Experimental results are carried out in the same laboratory using an energy management system for two power sources, fuel cell and battery, as shown in [38]. The proposed algorithm can be applied for the same power system in order to compare the two algorithms in terms of effectiveness, flexibility and ease of implementation.

- **Long-term perspectives:**

  - **Structured $H_\infty$ optimization:** The reduced-order version of the multi-variable $LPV/H_\infty$ controller exhibits a structure consistent with the physical system properties, where power sources behave independently when connected to the DC-link. This feature is reflected in the quasi-diagonal form of the reduced-order controller transfer matrix. Therefore, a structured $H_\infty$ control design approach can be used for such type of multi-power-source system, where a certain desired shape of the controller can be imposed and optimized.

  - **Statistical or repetitive load variations:** The proposed energy management strategy based on $LPV/H_\infty$ is able to supply any driving cycle profile, where it does not require neither previous knowledge for the load current nor its real-time measurements. Therefore, the problem can be simplified significantly using less complex approaches to design the on-board energy management system depending on *a priori* statistical information about load current, or even its repetitive well-known variations like in the case of postman or garbage collector driving cycles.
This appendix is dedicated to illustrate the different electrical system parameters used for both simulation (Chapter. 3) and real-time validation (Chapter. 4). The following table shows the description and the value of each system parameter (shown in Fig. 2.10).
Table A.1: Electrical system parameters used for simulation and real-time validation.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Simulation value</th>
<th>Real-time validation value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_{DC}$</td>
<td>DC-bus voltage</td>
<td>150 V</td>
<td>60 V</td>
</tr>
<tr>
<td>$C_{DC}$</td>
<td>DC-bus capacitor</td>
<td>$2.0 \times 10^{-3}$ F</td>
<td>$6.6 \times 10^{-3}$ F</td>
</tr>
<tr>
<td>$R_{DC}$</td>
<td>DC-bus discharging resistor</td>
<td>$10.0 \times 10^3$ Ω</td>
<td>$10.0 \times 10^3$ Ω</td>
</tr>
<tr>
<td>Fuel cell</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$L_{fc}$</td>
<td>Fuel cell converter’s inductor</td>
<td>$2.0 \times 10^{-3}$ H</td>
<td>$2.0 \times 10^{-3}$ H</td>
</tr>
<tr>
<td>$E_0$</td>
<td>Fuel cell open-circuit voltage</td>
<td>42 V</td>
<td>50 V</td>
</tr>
<tr>
<td>$R_m$</td>
<td>Fuel cell model’s resistor</td>
<td>$7.63 \times 10^{-2}$ Ω</td>
<td>-</td>
</tr>
<tr>
<td>$R_{ta}$</td>
<td>Fuel cell model’s resistor</td>
<td>$2.0 \times 10^{-3}$ Ω</td>
<td>-</td>
</tr>
<tr>
<td>$R_{tc}$</td>
<td>Fuel cell model’s resistor</td>
<td>$4.72 \times 10^{-4}$ Ω</td>
<td>-</td>
</tr>
<tr>
<td>$C_a$</td>
<td>Fuel cell model’s capacitor</td>
<td>$2.12 \times 10^{-3}$ F</td>
<td>-</td>
</tr>
<tr>
<td>$C_c$</td>
<td>Fuel cell model’s capacitor</td>
<td>$2.12 \times 10^{-2}$ F</td>
<td>-</td>
</tr>
<tr>
<td>Battery</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$L_{bat}$</td>
<td>Battery converter’s inductor</td>
<td>$1.0 \times 10^{-5}$ H</td>
<td>$1.0 \times 10^{-5}$ H</td>
</tr>
<tr>
<td>$Q_n$</td>
<td>Initial SOC of battery</td>
<td>70 %</td>
<td></td>
</tr>
<tr>
<td>$C_1$</td>
<td>Battery model’s capacitor</td>
<td>$2.92 \times 10^{-4}$ F</td>
<td>-</td>
</tr>
<tr>
<td>$R_1$</td>
<td>Battery model’s resistor</td>
<td>$13.4 \times 10^{-3}$ Ω</td>
<td>-</td>
</tr>
<tr>
<td>$C_2$</td>
<td>Battery model’s capacitor</td>
<td>$2.92 \times 10^{-4}$ F</td>
<td>-</td>
</tr>
<tr>
<td>$R_2$</td>
<td>Battery model’s resistor</td>
<td>$13.4 \times 10^{-3}$ Ω</td>
<td>-</td>
</tr>
<tr>
<td>Supercapacitor</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$L_{sc}$</td>
<td>Supercapacitor converter’s inductor</td>
<td>$1.0 \times 10^{-5}$ H</td>
<td>$1.0 \times 10^{-5}$ H</td>
</tr>
<tr>
<td>$R_s$</td>
<td>Supercapacitor model’s resistor</td>
<td>$0.8 \times 10^{-3}$ Ω</td>
<td>-</td>
</tr>
<tr>
<td>$C_0$</td>
<td>Supercapacitor model’s capacitor</td>
<td>56.0 F</td>
<td>-</td>
</tr>
<tr>
<td>$C_1$</td>
<td>Supercapacitor model’s capacitor</td>
<td>1.0 F</td>
<td>-</td>
</tr>
<tr>
<td>$C_2$</td>
<td>Supercapacitor model’s capacitor</td>
<td>1.0 F</td>
<td>-</td>
</tr>
<tr>
<td>$R_1$</td>
<td>Supercapacitor model’s resistor</td>
<td>$6.0 \times 10^{-4}$ Ω</td>
<td>-</td>
</tr>
<tr>
<td>$R_2$</td>
<td>Supercapacitor model’s resistor</td>
<td>$4.5 \times 10^{-4}$ Ω</td>
<td>-</td>
</tr>
<tr>
<td>PI’s local control loops</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$K_{P_{fc}}$ ,</td>
<td>PI parameters used for fuel cell</td>
<td>0.5 , 0.1</td>
<td>0.5 , 0.1</td>
</tr>
<tr>
<td>$K_{I_{fc}}$</td>
<td>current control loop</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$K_{P_{bat}}$ ,</td>
<td>PI parameters used for battery</td>
<td>0.01 , 0.8</td>
<td>0.01 , 0.8</td>
</tr>
<tr>
<td>$K_{I_{bat}}$</td>
<td>current control loop</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$K_{P_{sc}}$ ,</td>
<td>PI parameters used for</td>
<td>0.01 , 0.4</td>
<td>0.01 , 0.4</td>
</tr>
<tr>
<td>$K_{I_{sc}}$</td>
<td>supercapacitor current control loop</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Appendix B

Sensors’ conditioning circuit

Most different parts used to build the electrical system exist for academic purpose in IRTES-Set laboratory, while a dedicated electronic circuit was made in order to adapt all the measured feedback signals to the MicroAutoBox analog voltage inputs, as illustrated in Fig. B.1; then the measured signals are mapped into the interval $[0,1]$ by using 16-bit analog-to-digital converters (ADC). This circuit was designed by M. Patrice Luberda. The electrical schematic of this circuit is presented in Fig. B.2 and Fig. B.3, respectively.

$\begin{align*}
I_{fc} & \quad [-10,+10] \text{ A} \\
I_{bat} & \quad [-10,+10] \text{ A} \\
I_{sc} & \quad [-10,+10] \text{ A} \\
I_{load} & \quad [-10,+10] \text{ A} \\
V_{sc} & \quad [0,+33] \text{ V} \\
V_{DC} & \quad [0,+100] \text{ V}
\end{align*}$

All measured signals are conveyed to $[0,+5] \text{ V}$ to fit the MicroAutoBox analog inputs.

Figure B.1: The possible strategies for controller reduction.
Figure B.2: The possible strategies for controller reduction.
Figure B.3: The possible strategies for controller reduction.


Bibliography


Abstract — In this thesis the problem of multi-source power sharing strategy within electric vehicles is considered. Three different kinds of power sources – fuel cell, battery and supercapacitor – compose the power supply system, where all sources are current-controlled and paralleled together with their associated DC-DC converters on a common DC-link. The DC-link voltage must be regulated regardless of load variations corresponding to the driving cycle. The proposed strategy is a robust control solution using a MIMO LPV/$\mathcal{H}_\infty$ controller which provides the three current references with respect to source frequency characteristics. The selection of the weighting functions is guided by a genetic algorithm whose optimization criterion expresses the frequency separation requirements. A reduced-order version of the LPV/$\mathcal{H}_\infty$ controller is also proposed to handle an embedded implementation with limited computational burden. The nonlinear multi-source system is simulated in MATLAB®/Simulink® using two standardized driving cycles: the New European Driving Cycle (NEDC), the driving cycle of IFSTTAR (Institut Français des Sciences et Technologies des Transports, de l’Aménagement et des Réseaux). Simulation results show good performance in supplying the load at constant DC-link voltage according to user-configured frequency-separation power sharing strategy. The proposed energy management system is evaluated in a real-time application using a dedicated rapid prototyping platform. This platform was designed and built in collaboration with IRTES-SET laboratory in Belfort-France. The proposed strategy shows promising results related to the imposed control objectives, that is, DC-bus regulation, frequency separation for power sources’ current variations, and imposing desired steady-state behavior for the three power sources.

Keywords: $\mathcal{H}_\infty$ control, LPV systems, power source coordination, reduced-order controller, frequency separation, electric vehicle.
La commande $LPV/H_\infty$ des systèmes de gestion de l'énergie à bord pour les véhicules électriques

Résumé — Dans cette thèse, le problème de gestion de l'énergie multi-source dans le cas du véhicule électrique est considéré. Le système d'alimentation est composé de différentes sources électriques : pile à combustible, batterie et super-condensateur. Chacune des sources fonctionne de façon efficace dans une zone fréquentielle spécifique, la pile à combustible fournit sa puissance en basse fréquence, tandis que le super-condensateur joue son rôle en haute fréquence, la batterie fournissant la partie moyenne fréquence. Le système est bilinéaire; il est linéarisé dans un système linéaire à des paramètres variants. Dans ce contexte, nous proposons les techniques de commande multi-variable robuste de type LPV (linear parameter varying)/$H_\infty$ afin de spécifier la dynamique du courant de chaque source dans sa zone fréquentielle, en contribuant ainsi à la prolongation de sa durée de vie. Chaque source électrique est couplée avec un convertisseur DC-DC, les trois convertisseurs étant connectés en parallèle à un bus DC commun qui alimente le moteur électrique du véhicule jouant le rôle de la charge. La tension de ce bus DC doit être maintenue autour d'une référence avec une marge d'erreur de 10% au maximum. Les trois sources sont coordonnées pour fournir la puissance demandée par la charge quel que soit le cycle de conduite. Nous proposons également une méthode de réduction de modèle pour simplifier le contrôleur $LPV/H_\infty$, qui sera adapté à l'implémentation pratique. Le système complet est simulé numériquement sur MATLAB®/Simulink® en utilisant deux cycles de conduite : NEDC (le nouveau cycle européen de conduite) et un cycle proposé par IFSTTAR (Institut Français des Sciences et Technologies des Transports, de l'Aménagement et des Réseaux). Les résultats sont validés expérimentalement sur un banc d'essai temps réel réalisé en collaboration avec le laboratoire IRTES-Set à Belfort-France. La stratégie de commande utilisée montre des résultats prometteurs par rapport aux objectifs qui sont : la régulation de tension du bus DC, la séparation fréquentielle des courants fournis par les sources de puissance et le comportement en régime continu des trois sources de puissance. **Mots clés** : contrôle $H_\infty$, LPV système, gestion d’énergie, contrôle d’ordre réduit, séparation fréquentielle, véhicules électriques.

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