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Three Essays on Information Efficiency in Financial Markets and Product Market Interaction

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ABSTRACT


This dissertation contains three independent essays. The first two essays look at the informational role of stock prices and its impact on the real economy. The last one explores the relationship between managerial incentive and product market competition.

In the first essay, two firms compete in a product market and have an opportunity to invest in a risky technology either early on as a leader or later once stock prices reveal the value of the technology. Information leakage thus introduces an option of waiting, which enhances production efficiency. A potential leader may nevertheless be discouraged from investing upfront, when anticipating its competitor to invest later in response to good news. I show that an increase in product market competition increases the option value of waiting but has an ambiguous effect on information production. It may thus be the case that intense competition leads to more leakage such that no firm would invest, especially so in a smaller market. Given a moderate level of competition, price informativeness may improve investment outcome when investment profitability and the market size are relatively large.

The second essay examines the feedback effects of certifications in financial markets. A firm has to decide whether to monitor (or to ascertain) internally the prospect of a potential investment or to delegate this task to a certifier who reveals his evaluations to the outsiders. The investment decision is then taken based on all of the information available in the market. The information asymmetry between the firm and lenders is alleviated under delegation, and hence the firm enjoys a lower cost of capital at the financing stage. Delegation however reduces the information advantage of
speculators who then make less effort to acquire information. This results in a potential information crowding-out effect. We show that the firm may prefer to delegate when the prior belief about the investment prospect is relatively high, and to choose in-house information production when its own signal is more precise and when its current assets in place generate a higher expected payoff.

The third essay considers a spatial competition model with horizontal and vertical differentiation. Two firms are assigned to exogenous locations on a circular city. Consumers, distributed on the circle, need to pay a transportation cost for purchasing. Anticipating a future uncertainty in product quality, firms simultaneously offer incentive contracts to managers to induce an optimal effort level. I show that competition may adversely affects incentives, as a lower transportation cost impairs a firm’s local market power and consequently reduces a firm’s marginal benefit from producing a high quality product, particularly when its competitor also produces a high quality product. On the other hand, greater competition reduces a firm’s profit if it fails to improve product quality. This effect increases the optimal effort level and becomes dominant if the quality improvement is relatively large compared to the effort cost. Moreover, a large decrease in the transportation cost may change the market structure, such that the firm with better quality goods attracts all the demand, and thus the positive effect of competition on managerial effort becomes more significant.
DEDICATION

I humbly dedicate this work to my parents, Bingyu Chen and Qiji Ding, for their everlasting love and unconditional support, and for all the sacrifices they have made when hoping to give me a better future.
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CHAPTER 1.

INTRODUCTION

Proposed independently by Eugene F. Fama and Paul A. Samuelson in the 1960’s, the efficient market hypothesis (EMH) has been one of the main cornerstones in the finance theory. The general concept of the EMH is that financial markets are efficient in the sense that asset prices reflect all the relevant information about an asset. Researchers and practitioners have since applied extensively this idea to theoretical models and empirical tests of securities prices in financial markets. A growing literature in the extension of the EMH suggests that stock market can provide a useful source of information.\(^1\) When one agrees, or at least partially, with the EMH, he may believe that stock prices are efficient in reflecting the consequence of corporate decisions in the expected future cash flows of a firm, which make stock prices rather like a side show\(^2\). The newly developed theory argues that stock prices can take a more active role in aggregating diverse pieces of information from the big pool of informed outsiders who hope to profit from trading on their information in the stock market. Information transmitted through share prices may then be incorporated into corporate decisions, such that stock market efficiency has a real impact on the economy.

The active informational role of stock prices has been explored by Dow and Gorton (1997), and Subrahmanyam and Titman (1999) among others,\(^3\) and it is also well supported by recent empirical evidence. Edmans, Goldstein and Jiang (2012) show the stock prices can discipline managers by triggering takeover activities. Fresard (2012) finds that a firm’s cash

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\(^1\)The origin of this idea can be traced back to Hayek (1945).

\(^2\)See, for example, Morck, Shleifer and Vishny (1990).

\(^3\)See Bond, Edmans and Goldstein (2012) for a detailed review on this subject.
saving is sensitive to stock price typically when the price contains more new information previously unpossessed by managers. Zuo (2013) studies whether managers use the information contained in stock prices when forecasting future earnings, and the author confirms the hypothesis. More studies focus on the impact of stock price efficiency on decisions in investment and production, and find that the level of price informativeness affects positively the sensitivity of corporate investment to stock price.\textsuperscript{4} It is only natural for one to think that managers may use the information produced by outside investors into investment decisions, since those are possibly most complex tasks for managers in terms of understanding the investment prospect with various future uncertainties. Those uncertainties may include the changes in market regulations, in demands and supplies from upstream and downstream clients, as well as the changes in market structure and the industrial evolution, among which the production market interaction should obviously play an important role.

For example, Foucault and Frésard (2012) find a positive relationship between a firm’s investment and the market valuation of its peers selling related products. The significance of this link increases with the stock price informativeness of the peers and the demand correlation between products. Ozoguz and Rebello (2013) document similar results and further show that the investment sensitivity to peers’ share prices is stronger in an industry with faster growth, higher competition and greater dependence on capital. Relating to the empirical findings, we may wonder about the potential underlying mechanisms for one firm’s investment decisions being affected by stock price movements of its peers or competitors. The first essay (Chapter 2) in my thesis directly concerns this research question.

\textsuperscript{4}See for example Chen, Goldstein and Jiang (2007), Bakke and Whited (2010), Foucault and Fresard (2012, 2013) among others.
Chapter 2 studies theoretically how information leakage due to stock price movements affects firms’ innovation incentives. I consider information leakage of a public firm that invests in an innovation. The actual result or realization of this innovation is the proprietary knowledge to this firm. However, speculators in the stock market may exert effort and use their expertise to acquire this information. Once speculators participate in trading, the value of this innovation investment can be transmitted by share prices and observed by other firms. Share prices thus become the channel of information leakage, revealing one firm’s private knowledge to the others. In an industry where firms watch closely their rivals’ actions while striving to protect their own secrets, it is rather plausible that such a leakage about one firm’s prospect will benefit its competitors in their decisions to make similar investments. Consequently, good news about one firm’s innovation makes its rival more optimistic about their own opportunities and thus more incentivized to invest.

More specifically, I consider two firms competing in a duopoly industry. Firms are identical to each other except that they produce substitute goods. The degree of product substitution then determines the level of competition in the product market. Firms need to decide simultaneously whether to invest upfront in a technology that may reduce the production cost. I assume that this technology is the same to both firms, and thus success or failure of the investment is perfectly correlated across firms. Both firms are publicly listed. If one firm chooses to invest in the technology early on as a leader, it learns privately the value of technology at the intermediate stage. This information can then be acquired by speculators at a cost and partially revealed by share prices after trading takes place in the stock market. Firms therefore have an option of waiting for additional information before
making investment decisions. If the technology succeeds, waiting would however impose the cost disadvantage to the follower until it also invests in the technology.

I show that information leakage can have both positive and negative impact on resource allocation. On the one hand, when expecting information leakage about an innovation investment taken by one firm in the industry, other firms can wait and delay their own investment decisions until learning about the value of this innovation. This would reduce investment in something eventually proved to be useless. Information leakage also reduces the uncertainty of the investment outcome to potential followers and may thus encourage their investment, which would not take place otherwise. On the other hand, information leakage will induce more incentives for firms to free ride on the leader firm who invests up-front, such that the potential leader may be deterred from investing due to rent dissipation. As a consequence, no firm would invest in equilibrium and it becomes impossible to learn the actual value of the technology. In this case, leakage reduces the efficiency in resource allocation.

I also show that the aforementioned effects can be either alleviated or amplified depending on the industry characteristics. More particularly, I show that when there is intense competition in a relatively small product market (i.e., with relatively low demand), the leakage through stock prices would exacerbate the negative effect and lead to a lower production efficiency, i.e., it becomes more likely to reach the equilibrium with no firm investing in the technology. This is because the amount of information leakage is endogenized and dependent on the characteristics of the innovation and the industry, since those characteristics such as market competition or the market size of the industry determine the variance of firms’ final
payoffs and hence the incentive of speculators to acquire information. As a consequence, the fundamentals in the economy have a direct impact on the value of the option of waiting, and they also affect the option value indirectly by changing the amount of information production in the stock market. Two effects may work to mitigate each other or to amplify the result. While the model employs the logic of real options, the information leakage is endogenized as information becomes available through share prices. It thus differentiates the model from the standard industry organization literature, which usually assumes that either leakage (or information spillover) occurs when firms take certain actions or leakage arrives with an exogenous probability.\footnote{See for example Thijssen and Huisman (2001).}

I find that an increase in the level of product market competition increases the option value of waiting, but it has an ambiguous effect on the information production in the stock market. It may thus be the case that more competition increases the variance of firms’ future payoffs such that it would potentially lead to more information leakage available to the follower. This effect further increases the option value, and amplifies the free-rider problem. The rent dissipation becomes more severe in a smaller product market such that the potential leader is deterred, no firm would invest in equilibrium, and the production efficiency is lowered.

By endogenizing many parameters that characterize the innovation and the industry, as well as the process of information transmission in the stock market, the model is able to provide cross-sectional implications. We shall expect that the follower firm’s investment is more sensitive to its competitor’s share price movements, given a relatively higher competi-
tion level, a larger market size and a higher profitability of the innovation investment, which is largely consistent with empirical findings, such as in Ozoguz and Rebello (2013). This relationship is however not monotonic due to the endogeneity of information leakage. We should thus observe that in a relatively small market, intense competition leads to weaker investment sensitivity to share prices. In this case, the correlation between firms’ specific returns as well as the total amount of R&D investment should also be lower. The model thus provides new empirical implications that wait to be tested in the future.

Chapter 3 is a co-authored paper with Alexander Guembel. This chapter is related to the previous one in the sense that it is based on the same theoretical background about the impact of information efficiency in the stock market on the real economy, but Chapter 3 takes a different perspective. In Chapter 2, information acquisition of speculators is affected purely by firms’ investment decisions. Firms do not, and they cannot, credibly reveal their private information concerning the investment, and thus there are no direct interactions between speculators’ activities with other informed agents. In Chapter 3, however, we take into account such an interaction and model the consequent feedback effect on firms’ decisions.

More specifically, we consider the application of the informational role of certification intermediaries in a market with asymmetric information between buyers and sellers whereas either side cannot credibly disseminate their private information. These intermediaries are designed to acquire the signals about the privately informed parties and then to reveal to uninformed parties. Their credibility can be endorsed by laws and regulations, and/or determined by various mechanisms in different markets. Examples of certification intermediaries include auditors, industrial certification systems, credit rating agencies, and investment
banks that evaluate the quality of firms that want to raise capital. The literature related to certification intermediaries focuses either on their strategies of information disclosure due to conflicts of interests between the users of the information and the intermediaries\(^6\), or on the functionality of certifiers as a device for inspection or signalling\(^7\).

The chapter may thus concern the certifiers in financial market, such as a credit rating agency who is perceived to take the role of providing an independent opinion on the credit quality of firms. Moreover, if ratings contain information, they may alter the expectation of market participants about the overall quality of a firm, and thus influence a firm’s financing cost and subsequent investment decisions\(^8\). This then raises the question how a certifier, such as rating agencies, affects the information production by speculators in the financial markets, whose payoffs are directly related to firms’ investment decisions. Being outsiders of a firm, speculators can actively acquire information on firm value and profit from trading. When the private information possessed by speculators is revealed via share prices, it may then improve the decision taken by the firm. As a result, by changing the firm’s cost of capital and subsequent investment actions, the announcement made by certification intermediaries influences speculators’ incentive of information acquisition and ultimately the total amount of information available for guiding resource allocation. Despite its importance to market efficiency, the interaction between information production by certification agencies and private speculators has not been analyzed.

In the model, a firm has to decide whether to monitor (or to ascertain) internally

\(^7\)See for example Fasten and Hofmann (2010), Stahl and Strausz (2011).
the prospect of a potential investment or to delegate this task to a certifier who reveals his evaluations to the outsiders. In our model, the difference between delegating and in-house (internal) information production only lies in whether this piece of information is publicized by the certifier or remains private to the firm itself. The firm needs to decide, after updating its belief by using all available information in the market, whether to make the investment. Under delegation, lenders have access to the certifier’s evaluation, and hence the information asymmetry between the firm and lenders is alleviated. On the other hand, while delegation increases the transparency of a firm’s prospects, it may reduce the expected trading gain to the speculators, who now have less information advantage. As a result, speculators may make less effort to acquire information, which leads to a potential information crowding-out. The firm thus faces the following trade-off. Delegation avoids the adverse selection problem at the financing stage, but possibly reduces information available from stock prices, and conversely for the in-house information production.

We show that, for some parameter regions, if the firm chooses in-house information production there is a separating equilibrium at the financing stage such that the borrowing cost is higher than under delegation, in which case the low-type borrower can be observed instead of screened by a higher interest rate. When a priori it is more likely for the investment to realize a high payoff in the future, it is preferable for a firm to choose delegation except when the prior belief about the investment is very high. The causes are twofold. Firstly, a higher prior makes the low type borrower more inclined to mimic under the regime of in-house production, while it reduces the lending interest rate under the regime of delegation. Two effects combined enlarge the difference in the financing cost between two regimes. Moreover,
with in-house information production, a higher prior reduces the variance of payoff realizations of the investment and hence speculators’ incentive of information acquisition, which is again in the opposite direction compared to the regime of delegation. The crowding-out effect thus becomes less severe, and the firm more likely chooses to delegate.

Furthermore, if the signal obtained and kept private by the firm predicts better the state of the world, the rent from mimicking falls as well as the interest demanded by the lenders. This increases the payoff variance and thus the information acquired by speculators. As a comparison, when the information prevailing in the market is more precise under delegation, speculators find it less profitable to acquire additional information, which reduces information available through stock price. As a consequence, when the firm expects to get a private signal with a higher precision, the advantage of delegation in having a lower financing cost is reduced while information crowding-out becomes more severe and dominant. The firm thus more likely chooses not to reveal its private signal through delegation. Under a similar reasoning, we show in addition that the firm prefers not to delegate when the expected payoff of the firm’s current assets-in-place is higher.

The last chapter looks at a somewhat different topic - the relationship between product market competition and incentives. It has long been a popular research topic, not only because its ambiguity and complexity provide a fruitful area for exploration, but because the great relevance of the subject to the real world. Shareholders want to know whether market competition can discipline managers and thus substitute for the incentive scheme. Regulators need to design policies of shareholder protection or corporate governance within the environment where the degree of market competition or entry and exit rules should
be taken into consideration. There has been an on-going debate whether stronger market competition would increase the incentives of managers, while empirical evidence is limited but not conclusive.

This paper is another attempt to explore this research area. I consider a spatial competition model with both horizontal and vertical differentiation. Two firms are assigned to exogenous locations on a circular city. Consumers are uniformly distributed on the circle, and they need to pay a transportation cost for making a purchase. The firms therefore enjoy a local market power. The level of competition between two firms is represented by the transportation cost. At the beginning of the game, both firms anticipate a future uncertainty in their product qualities. They simultaneously offer incentive contracts to the managers in order to induce an effort level such that the expected firm profit is maximized. Such a model setup may fit better the applications in service sector than in manufacturing sector, as it links directly the effort choice of managers to the product quality. In addition, it is usually more difficult to verify the quality of services, which justifies the assumption that firms cannot write a contract directly on the quality of the output. The effect of firms’ locations may also give relevant interpretations for the service sector. For example, it is of interest to understand how the managerial effort in the financial sector is affected by the product differentiation, which includes not only the conventional banks’ geographical location choice but also the designs in credit products, for instance.

I show that competition has two opposite effects on equilibrium effort level. A lower transportation cost impairs a firm’s local market power, which consequently reduces the marginal benefit that a firm may enjoy from producing a high quality product, particularly
when expecting its competitor’s product to be of a high quality. Competition may thus affect adversely incentives. On the other hand, greater competition reduces a firm’s profit if it fails to improve product quality. The effect increases the optimal effort level and becomes dominant if the magnitude of quality improvement is relatively large compared to the cost to exert effort. Both effects are less significant when firms are located further away from each other and thus more differentiated horizontally. Moreover, I show that a large decrease in the transportation cost may change the market structure, such that the firm with better quality goods succeeds to attract all the demand. The positive effect of competition on effort level becomes more significant. The results seem to suggest that the relationship between competition and incentive depends on the absolute level of competition on top of the size of vertical differentiation and effort cost, which is partially consistent with the findings of Beiner, Schmid and Wanzenried (2011).

To summarize, the main contributions of this thesis are the followings. Chapter 2 studies the active role of stock prices in transmitting one firm’s proprietary information to its competing firm, and it applies the feedback mechanism to feedback across firms, which is rather under-investigated in the literature. This chapter also provides a framework that can be applied to a wide array of corporate decisions in practice, where the payoff of one firm’s action is strategically affected by similar actions taken by its competitors or industry peers. Examples are, but not limited to, investments in enlarging production capacities, vertical integrations for the purpose of reducing input price or operating cost, and outsourcing strategies. Chapter 3 helps to better understand the interaction between information production by private speculators and other informed outsiders and its impact.
on the information efficiency, which is again an important question that has been paid little attention in the past. The model also answers the question when firms prefer to delegate monitoring (information production) to a certifier and when they prefer an internal monitoring. Chapter 3 thus contributes to the literature of certification intermediaries, and may be used to derive implications on information disclosure policies for both firms and regulators. Chapter 4 provides a new explanation for the ambiguous relationship between product market competition and managerial incentive, with an application of the optimal contracting framework in a multiple agent setting.
CHAPTER 2.

INNOVATION STRATEGIES AND STOCK PRICE INFORMATIVENESS

2.1 Introduction

Innovations are regarded as the mainspring of economic dynamics. While providing firms with advantages in competition, innovations are mostly subject to a large irreversible investment, uncertainty, and potentially asymmetric information. In this regard, the financial market contributes to technological evolution by facilitating resource allocation, by financing and evaluating R&D investments, and by providing channels of risk sharing and diversification (Levine, 2005). It nevertheless exposes listed firms to the risk of leaking their proprietary information related to R&D progress, which is one of the main concerns in the IPO decisions of innovation-intensive firms.\(^1\) Information leakage changes the market positions of leaders and the innovation rent they can seize, which thus affects their incentives to invest. It consequently influences competition in an industry and social welfare.

The direct and mostly discussed cause of information leakage are mandatory disclosure requirements for public firms. Little attention is paid to an indirect leakage via stock price movements related to R&D investments. Recent literature, however, argues that prices in financial markets often take a more active role in providing to managers a source of information\(^2\). Empirical studies also find strong evidence that firms use the information contained in their stock prices when making decisions on corporate disclosure, cash savings, 

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\(^1\)Brau and Fawcett (2006) report in a survey of CFOs that "Disclosing information to competitors" and "SEC reporting requirements" are ranked the fourth and fifth factors in firms’ decisions to go public.

\(^2\)The origin of the idea goes back to Hayek (1945), and has been explored in Dow and Gorton (1997), and Subrahmanyam and Titman (1999) among others. See Bond, Edmans and Goldstein (2012) for a survey on the active informational role of prices.
investment and takeovers\textsuperscript{3}. In an industry where firms watch closely their rivals’ actions while striving to protect their own secrets, it is thus plausible that once stock prices reveal one firm’s private information about its innovation progress, this information will be employed by its competitors in their decisions to make similar investments. Consequently, good news about one firm’s innovation makes its rival more optimistic about their own opportunities and thus more incentivized to invest.\textsuperscript{4}

To address this indirect information leakage, I propose a simple model in which two firms produce differentiated products and compete in a duopoly market. Both firms have an opportunity to invest in a risky innovation technology which may reduce production cost. If a firm makes an investment up front, i.e., before learning about the technology, it is informed privately at the intermediate stage whether the innovation succeeds. Meanwhile, the same information can be acquired at a cost and traded on by some investors (speculators) in the stock market. The second firm can then decide whether to invest in the same technology after observing the leader’s and its own share prices. The innovation progress of one firm may consequently be leaked via share price movements to its competitor. When this leakage is factored in firms’ decisions ex ante, it provides firms with an option to delay the investment decision till learning more about the innovation prospect from share prices.

This channel of information leakage is distinct from the one via mandatory disclosures in two ways. Firstly, the extent of the latter is limited to R&D expenses, R&D acquisition


\textsuperscript{4}Choi (1991) uses an example of the break-through of cold super-conductivity in 1986 by IBM. IBM’s intermediate success made other firms more optimistic about this technology and increase their investment intensity. Similarly, Austin (1993) observes in the biotech industry that an intermediate success in R&D of one firm leads to an increase in the valuation of its competitors. Shi and Du (2012) document similar results by investigating knowledge spillovers among publicly listed firms.
and contracting,\textsuperscript{5} while stock prices aggregate private information from various sources and may thus serve better to reveal the true investment prospect. Secondly and more fundamentally, industry characteristics affect both the option value of delay and speculators’ trading incentives. This determines the probability and thus the consequence of the leakage via share price movements. The indirect information leakage has a real impact on the investment outcome when the option of delay is useful to the follower firm and at the same time speculators have sufficient incentives to trade.

More specifically, I show that the option value increases in the probability of information leakage and the degree of competition in the product market (characterized by the degree of product substitution). It however decreases in the market size as well as the profitability of the investment. When the values of corresponding parameters are moderate, the option helps to reduce the resources allocated on unproductive innovations and to encourage effective investments made by the follower firm. Information leakage is not useful if the option value is too low since both firms would prefer to invest up front. It can even be harmful. This is the case if a potential leader stops investing up front, because he anticipates imitation by a follower typically when the option is very valuable. The resulting competition reduces the benefit from innovating so much as to get no firm to be willing to invest up front. In this case learning is impossible and the technology is never adopted.

These effects of information leakage may become part of the equilibrium if information production in the stock market is achievable. That is, if the cost of information acquisition to speculators is comparatively smaller than their expected trading profit. When this is not the

\textsuperscript{5}See Statement of Financial Accounting Standards (SFAS) 2, 68, 141 and 142.
case, the exit of speculators reduces stock price informativeness and consequently the option value. I show that an increase in market competition augments speculators’ trading profit by enlarging the advantage to the leader firm thanks to a successful innovation as well as the disadvantage to the follower. Speculators are also more incentivized to participate given an increase in either investment profitability or market size of the industry. As a consequence, information production in the stock market and improved efficiency of investment may be both achieved when the investment is relatively profitable and it takes place in a sufficiently large market where the level of competition is moderate. When a strong trading incentive of speculators coincides with a high option value, the leader firm is deterred from investing up front. This is the case if the competition is intense, especially so in a small market. Having both the option value and information leakage endogenized in a model thus helps to reveal the real impact of price efficiency in the financial market.

These results show that the indirect information leakage may be most beneficial in an industry during the growth phase, where the investment return, the market size and competition level are comparably larger (higher) than at the introduction stage and lower than the maturity stage. Under those conditions, we should observe empirically that the investments of follower firms are more sensitive to share price movements of industry leaders. Moreover, there should be a higher correlation of specific stock returns between leader firms and followers. While providing cross-sectional characteristics, these implications are mostly consistent with the empirical evidence uncovered in recent studies. Foucault and Frésard (2012) find a positive relationship between a firm’s investment and the market valuation of its peers selling related products, the significance of which increases in the stock price
informativeness of the peers and the correlation of product demand. These authors however
do not consider the level of competition in the industry. Ozoguz and Rebello (2013) document
similar results and further show that this link is stronger in an industry with faster growth
and greater dependence on capital.

This paper also provides additional implications by modelling the two-way causality
between product market interactions and share price informativeness, which is little men-
tioned in the literature. For example, the theoretical model of Foucault and Frésard (2012)
allows only one duopoly firm the opportunity to expand the production capacity based on
the information revealed from share prices, while the competitor of this firm cannot react. 6
As pointed out previously, however, when information production in the stock market is
feasible, the leader may be deterred from investing up front in anticipation of an imitation
by the follower. This is the case given a high level of competition especially in a relatively
small market. The model thus predicts that, under this condition, there is fewer R&D in-
vestments in the industry and the link becomes weaker between one firm’s investment and
price movements of its competitors’ and its own stock. The correlation of firms’ specific
stock returns is also lower. These conjectures can be easily overlooked when one neglects
the feedback from share prices on firms’ ex-ante decisions.

This paper relates to the research on the interaction between product market compet-
tition and firms’ financing decision. An exogenous cost (probability) of information leakage
is usually imposed, typically in the studies focused on the trade-off between a cheaper capital

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6 Similarly, in the paper of Spiegel and Tookes (2008) who investigate firms’ financing choices for innovation
investment in a dynamic duopoly, only one firm can invest in the technology up front. The impact of the
waiting option on the ex-ante financing decision is not considered.
raised from the equity market and more intensive competition caused by information disclosures. Although this paper stays away from firms’ financing problem, whether stock price movements reveal innovation-related information and its extent, depending on industrial characteristics, may add extra concerns to firms’ financing decisions.

This paper contributes to the literature of firms’ strategies in industries with weak intellectual property protection, and particularly to process innovation that attracts relatively less attention compared to product innovation. Often related to cost reduction, process innovation is on average more difficult to be patented and less costly to copy compared to product innovation. Good examples include the "no frills" revolution in air travel started by South West Airlines, computerized reservation initiated by IBM and American Airlines, and the implementation of radio frequency identification system by Wal-Mart and Metro AG. These pioneering firms could hardly prevent their competitors from adopting a similar technology, while the second-mover advantage to the followers may be prominent. The general features of a process innovation are thus contained in the model proposed in this paper.

This paper also complements the innovation literature on knowledge spillover that is mostly related to voluntary or strategic disclosures. As shown by Gal-Or (1986) and Raith (1996), voluntary disclosure is not optimal to firms in the context of price competition when there exists an ex-ante uncertainty in the production cost. The model of indirect information leakage via share prices may provide a better framework to capture the knowledge spillover.

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7 See Brander and Lewis, 1986; Maksimovic, 1988; Chemmanur and Fulghieri, 1999; Spiegel and Tookes, 2008; Chod and Lyandres, 2011, among others, for discussions about how different sources of financing, private debt or equity, affect firms’ innovation strategies, and the intensity of product market competition.

8 See Jansen (2005 and 2008), Magazzini, Pammolli, Riccaboni and Rossi (2009), among others, who investigate firms’ disclosure strategies regarding their innovations given the presence of product market competition.
among firms that compete in price.

The paper proceeds as follows. Section 2.2 presents the setup of the model. Firms’ equilibrium strategies are computed in Section 2.3, and speculators’ participation and the endogenized information leakage are discussed in Section 2.4. Empirical implications are explained in Section 2.5. An extension is put in Section 2.6 regarding the welfare in the product market and the participation of noise traders. Section 2.7 concludes. Proofs are relegated to the Appendix.

2.2 The Model

The timeline The timing of the model is described in Figure 2.1. There are four dates. Both firms have an opportunity to invest in a risky innovation at either date 0 or date 1. If one firm invests in this innovation at date 0, it will know privately at the next date whether this innovation succeeds. Once an investment takes place in one firm, speculators can acquire private information about the success of the innovation and trade on this information if it is profitable. If the other firm decides not to invest at date 0, it can choose whether or not to do so at date 1 after observing the share prices at date 1. Firms then compete in the product market at both dates 2 and 3, and they liquidate at the end of date 3. Next, I explain the set-up in detail.

The product market The duopoly firm $i$ and $j$ produce differentiated products without capacity constraints. They produce and sell at dates 2 and 3. At date 0, the firms possess the same production technology and face an innovation decision that requires an investment $I$. This innovation will either decrease a firm’s marginal production cost by $\delta$ with probability...
\( \theta \) or make no change with probability \((1 - \theta)\), \( \theta \in (0, 1) \) and \(0 < \delta < c\). The success of the innovation is assumed to be perfectly correlated across firms regardless of the timing of innovation, and this is common knowledge.\(^9\)

For simplicity, I assume that the investment cost, \(I\), remains unchanged from date 0 to date 1. I also assume that it takes two periods for the invested innovation to exert influence on cost reduction. More specifically, if one firm invests at date 0, and the innovation succeeds, production costs at date 2 and 3 are \((c - \delta)\). If the firm invests at date 1 instead, its production cost at date 2 stays at \(c\), and if the innovation succeeds, the cost changes to \(c - \delta\) at date 3 only. If the innovation of the leader firm is found to be effective, the follower innovating at date 1 may thus be disadvantaged in the first-stage product market competition at date 2. This captures a cost of waiting to innovate. The opportunity to

\(^9\)This assumption is plausible given that innovation depends on technological feasibility which is fundamental and largely comparable among firms in the same industry. It can be relaxed by having an exogenous correlation between the success of the innovation of each firm, which would still make information leakage a problem. Therefore, it would not change the qualitative result in this paper.
invest in this innovation is no longer available after the end of date 1. Firms’ decision to
invest in innovation is also assumed to be publicly observable.\textsuperscript{10} To make the computations
more tractable, I follow most of the literature by assuming that the duopolists share the
information about production cost just before setting prices.\textsuperscript{11}

Following Singh and Vives (1984), I assume that there exists a representative con-
sumer in the economy, who maximizes at both date 2 and 3 his utility function $U(q_i, q_j) – \sum_{i=1}^{2} p_i q_i$, when consuming a quantity $q_i$ and $q_j$ of goods respectively from firm $i$ and $j$ at price
$p_i$ and $p_j$. $U(q_i, q_j)$ is quadratic, strictly concave and symmetric in $q_i$ and $q_j$,

$$U(q_i, q_j) = \alpha(q_i + q_j) - \frac{1}{2} \left(q_i^2 + 2\gamma q_i q_j + q_j^2\right), \quad (2.1)$$

where $\alpha > 0$ and $0 < \gamma < 1$. The parameter $\gamma$ measures the substitutability between the
goods produced by two firms\textsuperscript{12}. The higher is $\gamma$, the closer substitutes firms’ products are
and thus the fiercer their competition is. The following demand function for the goods of
firm $i$ maximizes the utility of the representative consumer,

$$q_i = \frac{(\alpha - p_i) - \gamma(\alpha - p_j)}{1 - \gamma^2} \quad (2.2)$$

provided that quantities are positive. Consequently, firm $i$ sets price $p_i$ to maximize its profit

\textsuperscript{10}This may be obligatory for the firms due to disclosure requirement, particularly when the innovation
investment is financed by the issuance of equity. This assumption also allows me to focus on the pure
equilibrium strategy.

\textsuperscript{11}More drastic restrictions on the communication about production cost may not only lead to a convolution
in results due to the effects from different sources, but also yield additional welfare losses since communication
between competing firms enables more efficient decision making in product market (Kuhn and Vives, 1995).
By simplifying the information structure that is less relevant to firms’ innovation decisions, I can draw clearer
inferences about the impact of the feedback effect regarding the innovation progress.

\textsuperscript{12}The qualitative results of this paper hold if $\gamma \in (-1, 0)$, that is, if the products are complements.
However, if firms produce complementary goods, it is optimal for the leader to communicate the innovation
progress when the innovation succeeds. The leakage via share prices becomes superfluous. I therefore neglect
the discussion for $\gamma \in (-1, 0)$.
\[ \pi_i, \]

\[ \pi_i = (p_i - c_i) \frac{(\alpha - p_i) - \gamma (\alpha - p_j)}{1 - \gamma^2}. \]  \hspace{1cm} (2.3)

\( q_j \) and \( \pi_j \) of firm \( j \) are symmetric to (2.2) and (2.3).

**The stock market** Three types of agents exist in the stock market: a noise trader, two speculators and a market maker. The noise trader buys or sells 1 unit of each listed firm for liquidity reason. Trading of the noise trader is uncorrelated across stocks. I endogenize the trading incentive of the noise trader in Section 2.6 (Extension). Two speculators can acquire at date 1 the private knowledge regarding firms’ innovation progress and trade on this information if profitable. The speculators can only submit market orders. Finally, the market maker is assumed to be competitive and provide liquidity by setting the share prices based on his rational expectation of a firm’s value when observing the submitted orders. The market maker earns zero profit in expectation.

Share trading is assumed to occur at date 1 after innovating firms learns the true prospect of the technology. Order flows in the stock market are publicly observable. When only one firm innovates at date 0, this information can be used by their competitor to decide whether to innovate at date 1. Speculators reap their trading profits at date 2 when the effectiveness of the innovation is observed and firms produce and sell. Note that I assume no other information leakage or spillover in this economy. Consequently no private knowledge about innovation progress will be revealed without informed trading in the stock market. Also, if no investment is made at date 0, speculators cannot know whether this innovation will be successful, and hence they will not trade.
2.3 Firms’ Equilibrium Strategies

A benchmark model with no feedback I consider first the case in which there is no stock market. As previously specified, firms know their rivals’ marginal production cost just before they enter price competition. The representative consumer chooses quantities of goods \((q_i, q_j)\) to maximize the utility function given in (2..1), and each firm maximizes its profit given in (2..3). By deriving the first order condition of the profit function with respect to \(p_i\), firm \(i\)’s best response function of price can be obtained as below:\(^{13}\)

\[
p_i = \frac{1}{2} \left[ \alpha (1 - \gamma) + \gamma p_j + c_i \right]. \tag{2..4}
\]

Solving the system of best response functions of firm \(i\), we can obtain the equilibrium price \(p_i^*\) for firm \(i\),

\[
p_i^* = \frac{\alpha (1 - \gamma)}{2 - \gamma} + \frac{2c_i + \gamma c_j}{4 - \gamma^2}. \tag{2..5}
\]

The expression of \(p_j^*\) is symmetric to (2..5). For simplification, I assume \(\alpha > c + \delta \frac{\gamma}{2 - \gamma - \gamma^2}\) such that \(q_i\) and \(q_j\) are positive \(\forall c_i, c_j \in \{c, c - \delta\}\). Using the equilibrium price \(p_i^*\) and \(p_j^*\), and the demand function \(q_i\) established in (2..2), I can then state firm \(i\)’s profit in equilibrium as a function of \(c_i\) and \(c_j\),

\[
\pi_{c_i,c_j} = \frac{1 - \gamma}{(1 + \gamma)(2 - \gamma)^2} \left( (\alpha - c_i) + \frac{\gamma (c_j - c_i)}{(2 + \gamma)(1 - \gamma)} \right)^2. \tag{2..6}
\]

Firm \(j\)’s profit \(\pi_{c_j,c_i}\) is symmetric to (2..6).

Formula (2..6) shows that firm \(i\)’s profit increases in its competitor’s cost \(c_j\). It is thus not optimal for firms to reveal voluntarily their innovation progress before the product

\(^{13}\)This is the Bertrand reaction function of firm \(i\), provided \(q_j\) is positive.
market competition. Due to the absence of information leakage in the benchmark case, it is never optimal to invest at date 1 if no firm invests at date 0. This therefore leaves two pure strategies to each firm, either to "invest in innovation at date 0", denoted by $L$, or "not to invest at all", denoted by $N$.

Strategy $L$ and $N$ complete firms’ action space $\Omega$ in the benchmark case, $\Omega = \{L, N\}$. $\Omega$ provides four possible combinations of strategies $(A_i, A_j)$ chosen by firm $i$ and its competitor $j$, and each combination leads to a different expected profit for both firms at either date 2 or 3. Since firms have the same action space and symmetric payoffs, the discussion of mixed strategies does not render additional insights and is therefore skipped. I restrict attention to pure strategy equilibrium in this paper.

To facilitate the illustration hereafter, I first compute and compare firms’ profit $\pi_{c_i,c_j}$ under each realization of their production cost, $c_i, c_j \in \{c, c - \delta\}$.

**Lemma 1** The size of firm $i$’s profit $\pi_{c_i,c_j}$ is ranked as follows: $\pi_{c, c - \delta} > \pi_{c - \delta, c - \delta} > \pi_{c,c} > \pi_{c,c - \delta}$.

Given the success rate of the innovation $\theta$, we can then compute the expectation of firm $i$’s payoff, denoted by $\Pi_i$, under each strategy pair $(A_i, A_j)$ chosen from $\Omega$. $\Pi_i (A_i, A_j)$ consists of firm $i$’s profit at both date 2 and 3 as well as the cost of innovation if the investment is to take place. As a result, the expected net profit of firm $i$ is $2\theta \pi_{c-c, c-\delta} + 2 (1 - \theta) \pi_{c,c} - I$ if both firms choose $L$, and $2\pi_{c,c}$ if both choose $N$. If, however, only firm $i$ invests in the innovation, $\Pi_i (L, N) = 2\theta \pi_{c-c, c-\delta} + 2 (1 - \theta) \pi_{c,c} - I$ and $\Pi_j (N, L) = 2\theta \pi_{c,c} - \delta + 2 (1 - \theta) \pi_{c,c}$.

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14Firm $i$ has no incentive to reveal a good progress of its innovation. Neither would it reveal bad news, since otherwise its competitor could perfectly infer the incidence of a successful innovation.
Assume that a firm chooses not to invest if \( \Pi(A_i, A_j) = 0 \). I derive the Nash equilibria and present the equilibrium conditions in Proposition 1.

**Proposition 1** If \( L \geq \theta I \), \((N, N)\) is the unique Nash equilibrium; if \( I < \theta L \), \((L, L)\) is the unique Nash equilibrium; and if \( \theta I > I > \theta L \), there are two equilibria: \((N, L) \) & \((L, N)\), where \( I = 2(\pi_{c-\delta,c-\delta} - \pi_{c,c}) \), and \( \bar{I} = 2(\pi_{c-\delta,c} - \pi_{c,c}) \).

This figure shows firms’ equilibrium strategies without information leakage in a numerical example with \( \alpha = 6, c = 3, \delta = 2 \) and \( \gamma = \frac{3}{4} \). \((N, N)\) marks the parameter region of no firm investing in equilibrium. \((L, L)\) marks the region of both firm investing, and \((L, N) \) & \((N, L)\) only one firm investing in equilibrium.

The black lines are the thresholds, \( \theta I \) and \( \theta \bar{I} \), defined in Proposition 1.

I plot in Figure 2.2 the equilibrium strategies for a numerical example, in which \( \gamma = \frac{3}{4} \).
\( c = 3, \delta = 2 \) and the demand parameter \( \alpha = 6 \). The parameter values remain unchanged for the illustrations throughout the paper, unless indicated differently. The required investment \( I \) for the innovation is scaled on the vertical axis and the success rate \( \theta \) is on the horizontal axis. The thresholds in the scale of required investment, \( \theta I_1^{15} \) and \( \theta \bar{I} \), separate three regions that represent firms’ strategies in different equilibria. Notice that both thresholds increase in the success rate (\( \theta \)) as well as the magnitude of the cost reduction (\( \delta \)). Intuitively, the investment in an innovation technology is more likely to be taken when the innovation has a high probability to succeed and brings a bigger advantage in product market competition.

**Equilibrium in a model with feedback** I now introduce the stock market to the economy, where speculators acquire and trade on their private information about firms’ investment prospect. I assume that with probability \( \lambda, \lambda \in (0,1) \), share prices are fully informative about the value of the innovation invested at date 0. With probability \( (1 - \lambda) \), share prices reveal no private information. \( \lambda \) is endogenized in Section 2.4. All other assumptions regarding the competition in the product market remain as previously stated. The equilibrium is now defined as, for a given \( \lambda \), the investment strategies chosen by firms that maximize expected firm value.

Compared to the benchmark which is a special case with \( \lambda = 0 \), the private information about one firm’s innovation progress is leaked to its competitor via share price. This additional ingredient introduces an option: a firm can now choose to wait and make the decision at the intermediate stage (date 1) after observing share prices. If no firm invests in the innovation at date 0, there will be no private information for the speculator to acquire.

\(^{15}\)The lower threshold \( I \) is zero when the degree of substitution converges to 1 (i.e., the perfect substitution).
and trade on, and consequently prices will contain no relevant information. Product market
competition still takes place at date 2 and 3.

When prices reveal bad news, it is obvious that a follower would never invest since the
investment would be a pure waste. When prices are not informative, a firm choosing not to
invest upfront has to decide whether to follow based on its prior belief. Continuing with the
notation "L" and "N" as in the benchmark case, I add two others for the strategies of the
follower firm: "F" denoting the strategy "to invest at date 1 only when share prices reveal
good news about the innovation", and "F" denoting "to invest at date 1 when share prices
reveal good news or no private information". The action space for each firm now consists of
four pure strategies, \( \Omega = \{L, F, \tilde{F}, N\} \). Lemma 2 points out that \( \tilde{F} \) cannot be an equilibrium
strategy, however.

**Lemma 2**  It is a strictly dominated strategy to invest in the innovation at date 1 with no
additional information from the stock market, i.e., \( \tilde{F} \) is a strictly dominated strategy.

The other strategies \( \{L, F, N\} \) survive in equilibrium. For a given \( \lambda \) (the probability
of information leakage), Proposition 3 summarize firms' strategies in equilibrium.

**Proposition 2**  If \( \theta > \frac{1}{2} \), strategy \( F \) cannot be sustained in equilibrium, and thus the equi-
librium remains as in the description of Proposition 1. If \( \theta \leq \frac{1}{2} \), the equilibrium strategies
are as follows.

\[
\begin{align*}
\text{If } I < \frac{(2-\lambda)\theta}{2(1-\lambda\theta)} I, \ (L, L) \text{ is the unique Nash equilibrium;} \\
\text{if } \frac{(2-\lambda)\theta}{2(1-\lambda\theta)} I \leq I < \min \left\{ \theta I, \frac{1}{2} I \right\}, \text{ there are two equilibria: } (L, F) \ \& \ (F, L); \\
\text{if } \frac{1}{2} I \leq I < \theta I, \text{ there are two equilibria: } (L, N) \ \& \ (N, L); \\
\end{align*}
\]
and if $\theta \bar{I} \leq I < \min \left\{ \frac{1}{2} \bar{I}, \theta \bar{I} \right\}$ or if $I \geq \theta \bar{I}$, $(N, N)$ is the unique equilibrium.

$\bar{I}$ and $I$ are defined as in Proposition 1, and $\bar{I} = (2 - \lambda) \pi_{c-\delta,c} + \lambda \pi_{c-\delta,c-\delta} - 2\pi_{c,c}$.

Figure 2.3: Firms’ Equilibrium Strategies - With Leakage

This figure shows firms’ equilibrium strategies with information leakage in an example with $\lambda = \frac{3}{4}$ (other parameters taking the same values as in Figure 2). $(L, F) \& (F, L)$ marks the region in which given $\lambda = \frac{3}{4}$ one firm chooses to lead and the other firm chooses to follow after learning good news from share prices. When $\lambda$ increases from 0 to $\frac{3}{4}$, the thresholds of strategy $F$, $\theta \bar{I}$ and $\frac{(2-\lambda)\theta}{2(1-\lambda \theta)} \bar{I}$, are shifted rightwards from the dotted lines to the solid lines.

Note that to assure the existence of a pure-strategy equilibrium, I assume that the value of parameter $\alpha$ is not too high such that $\theta \bar{I} > \frac{(2-\lambda)\theta}{2(1-\lambda \theta)} \bar{I}$ for $\theta = \frac{1}{2}$, i.e., $\alpha < c + \delta \frac{8-4\lambda+\gamma(2\gamma-2+\lambda)+\lambda}{2(2+\gamma)(1-\gamma)\lambda}$. 

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I plot in Figure 2.3 the thresholds of equilibrium strategies in Proposition 2 in solid lines in contrast to the dotted ones from Proposition 1. Proposition 2 shows that firms’ strategies in equilibrium remains unchanged from the benchmark case if the investment costs more than $\frac{1}{2}I$. This is the condition for the follower firm not to invest at date 1 even if share prices reveal good news. In addition, choosing $F$ is no longer optimal for the follower firm when the success probability $\theta$ is above $\frac{1}{2}$. The intuition is that strategy $F$ is preferable only if the wasteful investment $\lambda(1 - \theta)I$ avoided by using the option of waiting outweighs the expected benefit $\lambda\theta(\pi_{c-\delta,c-\delta} - \pi_{c,c-\delta})$ during the market competition at date 3, i.e., $I > \frac{\theta}{2(1-\theta^2)}I$. If $\theta > \frac{1}{2}$, this condition contradicts the threshold for investing upon good news at date 1 (i.e., $I < \frac{1}{2}I$).

Moreover, the new strategy $F$ and thus the option of waiting lead to fundamental changes in Proposition 2 compared to the benchmark case. To illustrate, I first define the option value of waiting.

**Lemma 3** The option value of waiting is the benefit to a firm from choosing the strategy $F$ over $L$ given its competitor chooses $L$, that is, $(2 - \lambda)\theta(\pi_{c,c-\delta} - \pi_{c-\delta,c-\delta}) + (1 - \theta\lambda)I$.

The option value consists of two parts. The first part is the sum of the potential loss in the competition at both date 2 and date 3 if the innovation succeeds, which are respectively $\theta(\pi_{c,c-\delta} - \pi_{c-\delta,c-\delta})$ and $(1 - \lambda)\theta(\pi_{c,c-\delta} - \pi_{c-\delta,c-\delta})$. The second part is the amount of investment saved from waiting, $(1 - \theta\lambda)I$, where $\theta\lambda$ is the joint probability of good news being revealed. In other words, the option value is the difference between $\Pi(F, L)$ and $\Pi(L, L)$.

The option value equals zero at the lower threshold of $(L, F) \& (F, L)$ in the investment cost, $\frac{(2-\lambda)\theta}{2(1-\lambda\theta)}I$. When $I$ is above this threshold, $(L, L)$ is replaced by $(L, F) \& (F, L)$ since it
is optimal for one firm to take advantage of the option and act as the follower. The option value is even higher in the parameter region where a firm switches its strategy from $N$ to $F$ and invests upon good news at date 1. The efficiency in the product market is improved in $(L, F) \& (F, L)$ due to either a more effective investment at the intermediate stage or a reduced wasteful investment, since the follower firm can now invest with a better knowledge about the innovation.

On the other hand, $(L, N) \& (N, L)$ are replaced by $(N, N)$ in the region where strategy $F$ reduces the innovation rent of the potential leader to the extent that he no longer profits from investing at date 0. In this scenario, the technology is never adopted and learning about its value is impossible. The information leakage leads to a lower efficiency in production. We thus observe a new threshold $\theta \bar{I}$ between $(N, N)$ and $(L, F) \& (F, L)$, that is below $\theta \bar{I}$.

We now take a look at how the option value varies with stock price informativeness as well as the characteristics of the innovation and the product market.

**Proposition 3** The option value of waiting increases in the probability of information leakage $\lambda$, the degree of competition $\gamma$ and the investment cost $I$. It decreases in the success rate $\theta$, the size of cost reduction $\delta$ and the demand parameter $\alpha$.

Firstly, it is intuitive that the option is more valuable when share prices are informative with a higher probability ($\lambda$), since it becomes more likely for the follower to learn at date 1 the true prospect. Meanwhile, a higher $\lambda$ also imposes a larger cost of information leakage to the leader firm such that the up-front investment is more likely to be deterred.
The consequence is that both thresholds \( \frac{(2-\lambda)\theta}{2(1-\lambda)} I \) and \( \theta \bar{I} \) shift towards the right in Figure 2.3.

The option value decreases when the profitability of the innovation investment increases due to a higher benefit from investing up front. Investment profitability is characterized by the parameters \( \theta, I \) and \( \delta \). Let us look at Figure 2.3. When the success rate is small (e.g., at point A) for a given \( I \), the option prevents the follower from wasting its investment with a high probability. The profitability at this point is however sufficiently low to the potential leader given that its competitor is likely to follow. In contrast, the option value becomes so small when \( I < \frac{(2-\lambda)\theta}{2(1-\lambda)} I \) such that \( F \) is no longer optimal, whilst at point B, the size of profitability and option value suffice to accommodate the incentive to both the leader and the follower. The same reasoning can be applied regarding the required investment \( I \) and the size of cost reduction \( \delta \).

Figure 2.4 shows the impact of industry competition and the market size on equilibrium strategies, with \( \theta = 0.4 \) and other parameters remaining unchanged. When \( \gamma \) increases, products of firm \( i \) and \( j \) become closer substitutes, and the competition level in the industry increases. While both \( \pi_{c-\delta,c-\delta} \) and \( \pi_{c,c-\delta} \) drop for a higher \( \gamma \), the decreases of \( \pi_{c-\delta,c-\delta} \) is more significant. This is because having the same production cost as its competitor, a firm is obliged to reduce more its product price under a higher level of competition in order to attract demand from the consumer. Expecting its competitor to invest at date 0, a firm thus has a lower incentive to invest at the same time. The option value increases in \( \gamma \). For example, at point A in Figure 2.4 the option of delay has a low value such that both firms

\[ ^{16} \text{See Figure A1 in the Appendix for firms’ equilibrium strategies when the size of cost reduction varies.} \]
This figure shows the impact of industry competition and the market size on firms’ strategies in equilibrium. The dotted lines represent the thresholds of the benchmark case. When the probability of information leakage increases, these thresholds are shifted upwards. The solid lines represent the thresholds for $\lambda = \frac{3}{4}$.

invest up front. When $\gamma$ increases to point $B$, one firm takes the option to wait. While at point $C$, the innovation rent to the leading firm becomes too low and $(N, N)$ emerges in equilibrium. Note that the information leakage does not affect the equilibrium outcome when $\gamma$ is approaching 1, i.e., products become perfect substitutes.

Figure 2.4 also shows the impact of the demand parameter $\alpha$ that is associated with the market size. Using the demand function in (2.2) and the equilibrium price in (2.5),
we can obtain the demand intercept and the price elasticity of demand\textsuperscript{17}. It is then easy to see that a higher $\alpha$ leads to a larger intercept of demand and a lower price elasticity, and therefore a larger market size in the industry. When the consumption expands given a higher $\alpha$, a successful innovation brings a more significant advantage in competition and thus a stronger incentive for firms to invest up front. The option value of delay drops as a consequence.

It is clear so far that, the information leakage with a given probability $\lambda$ is beneficial to an industry when the innovation is associated with a relatively high profitability and a sufficient market size and when the competition is not too intense. Informative prices encourage innovations and improve the efficiency of innovation investment. This may fit an industry at the growth stage of its life cycle, in which incremental innovations are frequently needed and often more profitable, and the competition is lower. The opposite can be said for industries at the stage of maturity, where the competition is intense and the improvements on the prevailing technologies carry small impact on production.

In the next section, I discuss speculators’ trading strategies and how the probability of information leakage is endogenized in this economy.

2.4 Participation of Speculators

Trading strategies Assume that both firms are publicly listed and each of the two speculators are assumed to trade only one firm’s shares, though they may have access to the private information about both firms. This assumption, simplifying the discussion of the trading

\textsuperscript{17}The demand intercept equals $\frac{\gamma}{1-\gamma}$, and the price elasticity of demand $E_{di}$ equals $-\frac{\gamma}{\gamma-1} p_i q_i$.  

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part of the game, can be justified by limits on exposure that a trader is willing to take. Let us denote the order submitted by the speculator of firm $i$ and $j$ by $x_i$ and $x_j$, respectively. Recall that if no investment takes place at date 0, speculators do not trade at the next date since no private information is there for acquiring, thus $x_i = x_j = 0$.

When at least one firm invests at date 0, speculators can acquire perfect information about the true state of the world $\omega$, $\omega \in \{s, f\}$. $\omega = s$ if the innovation is successful, and $\omega = f$ otherwise. Speculators’ orders are thus functions of $\omega$, i.e., $x_i(\omega)$ and $x_j(\omega)$. Although speculators are allowed to choose any order size to submit, they follow nevertheless the optimal trading strategy defined by the lemma below.

**Lemma 4** When both firms invest at date 0, if speculators learn $\omega = s$, i.e., the innovation will succeed, \[ x_i(s) = 1 \quad \text{and if they learn } \omega = f, \text{ i.e., the innovation will fail,} \]
\[
x_j(s) = 1
\]
\[
\begin{align*}
x_i(f) &= -1 \\
x_j(f) &= -1
\end{align*}
\]

When only firm $i$ invests at date 0, if speculators learn $\omega = s$, \[
\begin{align*}
x_i(s) &= 1 \\
x_j(s) &= -1
\end{align*}
\]

if they learn $\omega = f$ \[
\begin{align*}
x_i(f) &= -1 \\
x_j(f) &= 1
\end{align*}
\]. The strategies are symmetric when only firm $j$ invests at date 0.

Recall that the noise trader buys or sells 1 unit of both firms’ shares with equal probability and there is no correlation of their orders across firms. Let $X_i$ and $X_j$ denote the total order flow of firm $i$ and of firm $j$. It is straightforward to see that the trading direction
of speculators are hidden if $X_i = X_j = 0$, and their private information about $\omega$ is not revealed. If we assume that information acquisition incurs no cost, both speculators trade actively when at least one firm invests in the innovation. Lemma 5 follows immediately.

**Lemma 5** When both speculators are active, the probability of information leakage ($\lambda$) is $\frac{3}{4}$.

**Speculator’s profit** The probability of leakage $\lambda$ may however vary with the trading incentive of speculators once we impose an information cost. To understand this, I first compute speculators’ expected profit and show how the profitability of their information acquisition can be affected.

Recall that trading is profitable to speculators only when $X_i = X_j = 0$, which occurs with probability $\frac{1}{4}$. If firm $i$ invests as a leader at date 0, we know from Lemma 2 that firm $j$ does not invest if $X_i = X_j = 0$. Given that the profit functions of both firms are publicly known, the market maker is then able to anticipate the optimal strategy of firm $j$ and quotes the price $P_i$ and $P_j$ as exactly the expected firm values, if $X_i = X_j = 0$. We can obtain the expected trading profits of speculator $i$ and $j$, denoted by $\Psi_i (L, F)$ and $\Psi_j (F, L)$, which are respectively $\theta (1 - \theta) (\pi_{c-\delta,c} - \pi_{c,c})$ and $\theta (1 - \theta) (\pi_{c,c} - \pi_{c,c-\delta})$.\(^{18}\) It is easy to observe $\Psi_i (L, F) > \Psi_j (F, L)$. When both firms invest at date 0, private information contained in share prices is no longer used for firms’ decision making. In this case, the market maker quotes the same price for two firms, $P_i = P_j = \Pi_{i,j} (L, L)$, and both speculators expect to earn $\theta (1 - \theta) (\pi_{c-\delta,c-\delta} - \pi_{c,c})$.

\(^{18}\)Note that the expected trading profits of speculators in the parameter region $(L, N)$ & $(N, L)$ have the same expressions.
Speculators’ trading profits are not related to the investment cost $I$, due to the assumption that firms’ investment actions can be observed by all agents. The next proposition summarizes the impact of other parameters on speculators’ trading profits.

**Proposition 4** Regardless of firms’ strategies in equilibrium, speculators’ expected trading profit increases in both $\delta$ and $\alpha$.

*If $(L, L)$ is the equilibrium strategy, speculators’ profit decreases in $\gamma$;*

*If firm $i$ invests as the leader, speculator $i$’s profit $\Psi_i(L, F)$ increases in $\gamma$ for $\alpha < \bar{\alpha}$, and decreases in $\gamma$ otherwise; while speculator $j$’s profit $\Psi_j(F, L)$ increases in $\gamma$ for $\alpha > \bar{\alpha}$, and decreases in $\gamma$ otherwise, where $\bar{\alpha} = c + \frac{\gamma(8-8\gamma^2+4\gamma^4-\gamma^6)\delta}{(1-\gamma)^2(8-6\gamma^2+4\gamma^3+7\gamma^4+2\gamma^5)}$ and $\alpha = c + \frac{\gamma(4+\gamma^2-2\gamma^4)\delta}{(1-\gamma)^2(2+\gamma)(4+2\gamma+4\gamma^2+3\gamma^3)}$, $\bar{\alpha} > \alpha$.*

Intuitively, the size of cost reduction $\delta$ has a positive impact on the profitability of the investment and therefore the dispersion of firms’ payoffs, provided that at least one firm invests in the innovation. Similarly, a higher $\alpha$, associated with a bigger market size, enlarges the leader’s advantage as well as the follower’s disadvantage in competition.

The effect of the competition level ($\gamma$) is less straightforward. As shown previously, speculator $i$’s profit depends on the difference between $\pi_{e-\delta, c}$ and $\pi_{e, c}$. Assuming firm $i$ chooses to be the leader and its innovation succeeds, an increase of $\gamma$ has two effects: a negative impact on the product price and a positive impact on the demand. The net effect depends on the market size. For $\alpha < \bar{\alpha}$, a higher $\gamma$ and thus a higher competition level enables the leader firm to seize a higher market share that is sufficient to compensate the price impact, and thus $(\pi_{e-\delta, c} - \pi_{e, c})$ increases. The dominance of the demand impact becomes weaker when the market size increases, i.e., $\frac{\partial^2 \Psi_i(L, F)}{\partial \gamma \partial \alpha} < 0$, and it is eventually reversed when
\( \alpha > \bar{\alpha} \). Similarly, speculator \( j \)'s profit depends on the difference between \( \pi_{e,c} \) and \( \pi_{e,c-\delta} \). In a sufficiently large market, intense competition reduces the follower firm’s market share and pricing power more than when both firms have the same production cost. This effect goes down when \( \alpha \) becomes smaller, i.e., \( \frac{\partial^2 \Psi_j(F,L)}{\partial \gamma \partial \alpha} > 0 \), and it is reversed if \( \alpha < \bar{\alpha} \). At last, if \((L, L)\) is chosen in equilibrium, two firms are equally positioned in competition. An increase in \( \gamma \) reduces firms' payoff more significantly when the innovation succeeds (the production cost is lower) than otherwise, i.e., \( \frac{\partial \pi_{e,c_i,c_j}}{\partial \gamma \partial c_i} > 0 \) if \( c_i = c_j \). Consequently, speculators’ expected trading profit decreases in \( \gamma \).

**Endogenized information leakage** Let us now assume that it costs \( \epsilon \) for each speculator to acquire information about the innovation progress. Speculators will participate only when their net expected payoff is positive, i.e., \( \epsilon < \Psi \). As a result, three possible outcomes can arise corresponding to the size of \( \epsilon \) relative to other parameters: both speculators stop acquiring information (i.e. exit the market) and firms chooses strategies at date 0 as in the benchmark (Proposition 1); both speculators are active; the speculator earning a higher expected profit remains active while the other one quits. The third outcome can occur in equilibrium in which only one firm chooses the strategy \( L \) and the expected profit of the speculator of the follower firm is not sufficient to cover the information cost \( \epsilon \). Recall that the information acquisition of speculators take place after observing firms’ actions at date 0. In equilibrium, firms’ innovation strategies correspond to the number of active speculators in expectation.

The equilibrium is thus defined as follows: (i) A trading strategy for speculators that maximizes their expected payoffs, given the investment strategies of the firms, (ii) the investment strategies by the firms that maximize expected firm value given all other
strategies, (iii) a price-setting strategy by the market maker that allows him to break even in expectation, given the strategies taken by the speculators and firms.

We next have a look at the case that should both speculators trade actively, only firm $i$ invests at date 0 in equilibrium and speculator $i$ earns a higher expected profit than speculator $j$. If the parameter values are such that $\epsilon$ is between $\Psi_i(L, F)$ and $\Psi_j(F, L)$ and speculator $j$ exits the market. This leaves speculator $i$ the only informed trader in the stock market, thereafter called the monopoly speculator. Share prices become less informative with a monopoly speculator, since the market maker can no longer update his belief about the state of the world based on the order flows of both firms. See the Appendix for a complete proof for the following lemma.

**Lemma 6**  With a monopoly speculator, the probability of information leakage ($\lambda$) is $\frac{1}{2}$.

Relating to Proposition 2 and 3, we know that both firms are inclined to innovate at date 0 when the profitability of the innovation investment is particularly high (i.e., a large $\delta$). In this case, speculators have strong incentives to trade, but the information is less useful to firms in the product market. It is similar regarding the market size that a very high $\alpha$ provides speculators with a strong incentive to trade while the information leakage has little impact on the investment outcome. Under the opposite conditions (a very small $\delta$ or $\alpha$), speculators have a low incentive to acquire information, while the option is very valuable such that information leakage could deter the potential leader. Nevertheless, the lack of price informativeness may actually help to alleviate this problem.

We can now look at the equilibrium outcome with an endogenous information leakage. To visualize how it is different from having an exogenous probability of leakage, I present two
This figure shows the equilibrium outcome of a numerical example in which the information cost is sufficiently high ($\epsilon = 2.1$) such that there may be a monopoly speculator trading in the stock market. The grey solid line is the cutoff for the monopoly trading profit to be equal to the information cost. Below the grey line, the monopoly speculator does not acquire information and thus there is no informed trading.

Numerical examples separately in Figure 2.5 and 2.6, with respectively a high information acquisition cost ($\epsilon = 2.1$) and a low cost ($\epsilon = 0.2$). Assume again that firm $i$ is the leader. Let us first look at the example in Figure 2.5, in which the expected profit of speculator of the follower firm $j$ is not sufficient to cover the information cost and thus speculator $j$ leaves the market. Speculator $i$ may remain active depending on the values of parameters $\alpha$ and $\gamma$. The gray line represents the cutoff where the information cost is equal to the monopoly trading profit of speculator $i$, i.e., $\epsilon = \Psi_i(L, F) | \lambda = \frac{1}{2}$. Below this cutoff line, speculator $i$
also stops acquiring information and hence there is no informed trading.

First, observe at point A in Figure 2.5 both a small market size and low competition give sufficient disincentives to speculator i, such that the equilibrium goes back to \((L, L)\) in the benchmark case \((\lambda = 0)\). The outcome with an exogenous \(\lambda\) being \(\frac{1}{2}\) at point A would be \((L, F)\) & \((F, L)\). The expected profit to speculator i at point C is still not sufficient due to a small market size, and there is no information production in the stock market. As a comparison, information leakage has a real impact at point B with a higher value of \(\alpha\) by enabling the follower firm to choose strategy \(F\). Notice in the gridded region in Figure 2.5, where the competition is intense in the product market, the monopoly speculator has a strong incentive and the option is valuable to the follower. This alignment deters the leader from investing up front. Information leakage \((\lambda = \frac{1}{2})\) switches the equilibrium from \((L, N)\) & \((N, L)\) to \((N, N)\).

Next, Figure 2.6 shows an example with a sufficiently low cost of information acquisition such that the speculator of the follower firm may also have incentive to participate. The gray line here represents the cutoff of zero trading profit to the speculator of the follower \(j\), netting the information cost, i.e., \(\epsilon = \Psi_j (F, L) | \lambda = \frac{3}{4}\). Therefore, below this cutoff line, speculator \(j\) exits the market and leaves speculator \(i\) the monopoly trader. Consequently, \(\lambda\) becomes \(\frac{1}{2}\) below this cutoff. Again, in Figure 2.6 the information leakage does not affect firms’ strategies at point A. Were the information leakage exogenous \((\lambda = \frac{3}{4})\), the follower firm would find it optimal to use the option of waiting and choose \(F\). Nevertheless, speculator \(j\) does not trade at point A due to a low expectation of trading profit. As a comparison, we observe that when \(\gamma\) increases to point B, both speculators have incentive to trade while
This figure shows the equilibrium outcome of a numerical example with a low information cost ($\epsilon = 0.2$).

In this case, both speculator may be active in the market. The gray line represents the cutoff where the speculator of the follower firm earns zero expected profit netting the information cost and he stops acquiring information in the region below this cutoff. The probability of information leakage drops from to $\frac{1}{2}$ below the gray line.

the option value is sufficient for firm $j$ to act as a follower and not too high to deter firm $i$ from investing up front. Price informativeness has a positive impact on the investment outcome.

Now look at point $C$ which has the same location as in the previous figure. When the information is more expensive such that only speculator $i$ stays active in the stock market, as in Figure 2.5, the lack of trading incentive for the monopoly speculator at $C$ leads to the
equilibrium \((L,N)\) \& \((N,L)\). Given a much lower information cost in the example here, the equilibrium outcome becomes nevertheless \((N,N)\). A high option value is now accompanied by a strong trading incentive of the speculator of the follower firm. This deters the potential leader and exerts a negative impact on the investment outcome.

These examples show clearly the difference in the real impact of an endogenized information leakage compared to an exogenous leakage. Conclusion 1 and 2 summarize the discussions above.

**Conclusion 1**  *Stock price informativeness improves the investment outcome when the profitability of the investment and the market size are relatively large.*

**Conclusion 2** *When speculators’ trading incentive varies with product market competition, stock price informativeness worsens the investment outcome when the competition level is relatively high in a small market. It may improve the investment efficiency when the competition is not so intense.*

In addition, I show in Figure A2 in the Appendix a numerical example with a moderate information cost \((\epsilon = 1.05)\), in which the monopoly speculator \(i\) remains always active. At the same location of point \(C\), firms’ equilibrium strategies are \((L,F)\) \& \((F,L)\). Comparing it to Figure 2.5 and 2.6, we observe that the information cost has a non-monotonic effect on investment strategies in equilibrium.

Information cost depends on how difficult it is to understand the nature of an innovation technology and the true value of the technology to a certain industry. Cost of acquiring information and trading to speculators can also come from low analyst coverage, low trans-
parency of firms’ disclosure policies and restrictions on short selling, which are often subject to regulatory constraints. The regulatory concerns are particularly relevant to growing and innovation-intensive industries that rely heavily on equity financing due to volatile returns, inherent riskiness of investment, and limited collateral value of intangible assets.\textsuperscript{19} Conclusion 1 and 2 show that these industries may also benefit largely from investment efficiency that is promoted by price efficiency in the stock market. The non-monotonic impact of the cost parameter $\epsilon$ implies the intricacy in the related policies. A detailed discussion in this regard is nevertheless beyond the scope of this paper.

2.5 Empirical Implications

The model provides empirical implications from two aspects. First, when information leakage occurs via trading in the stock market, we expect to observe a link between the share price of one firm and the investment taken by its competitor. More specifically, discussions in the previous section conclude that price efficiency in the stock market enables firms to act as followers when the market size and the investment profitability are neither too small nor too large. There may not be sufficient incentive for speculators to acquire information if the parameter values are too small. Or in the opposite case, the option is not valuable and both firms invest up front. The model thus provides the first implication, which is a direct consequence of Conclusion 1. See Table A3 in the Appendix for possible empirical proxies for the model’s parameters.

\textbf{IMPLICATION 1: The investment of followers is more sensitive to share price move-}

ments of leading competitors in an industry with a relatively large market size and profitable investment opportunities than otherwise.

Additionally, Conclusion 2 says information efficiency in the stock market share prices can have a negative effect on the investment outcome depending on the competition intensity in the industry. Proposition 3 states that the option of waiting becomes more valuable for a higher $\gamma$. When the competition level rises, the alignment between speculators’ incentive and the option value makes it possible for one firm to act as a follower. When market competition is intense, it however drives out the up-front investment, especially in an industry with a relatively small market where the competition advantage to the leader is low. Implication 2 thus follows.

**IMPLICATION 2: The investment of followers is more sensitive to share price movements of leading competitors when the level of competition increases in the product market. This sensitivity is however weakened when competition becomes intense particularly in a smaller market.**

It is worth mentioning that one technology can be adopted at different timings and brings different benefits across industries, depending on the characteristics of each industry, the functionality of the technology itself, and the development of supporting technologies. For example, as a long-existed technology, the adoption timing of radio frequency identification (RFID) system varies largely from the early 1990s in factory automation to the mid-late 2000s in asset tracking in the retail and banking industry. Investment returns and implementation risks vary accordingly. As a consequence, the relationship between investments and share
prices should differ for a given technology adopted across industries and in different periods. This gives another interpretation of Implication 1 and 2.

While providing cross-sectional characteristics, Implication 1 and 2 are mostly consistent with the empirical evidence uncovered in recent studies. Foucault and Frésard (2012) find a positive relationship between a firm’s investment and the market valuation of its peers selling related products, the significance of which increases in the stock price informativeness of the peers and the correlation of product demand. These authors however do not consider the level of competition in the industry. Ozoguz and Rebello (2013) document similar results and further show that this link is stronger in an industry with faster growth, higher competition and greater dependence on capital.

Analogically, we should observe the difference in the correlation of firms’ specific returns and their investment behaviors. Since the market return is not modelled in this paper, the correlation of firms’ specific returns is equivalent to the price correlation. Consider that both speculators are active. Stock prices of firms are perfectly correlated if they both invest up front. In the parameter regions where \((L, L)\) is replaced by \((L, F)\) & \((F, L)\), price correlation obviously goes down. Empirically, it should also be similar in the region where \((L, L)\) is replaced by \((N, N)\) since the specific return related to this innovation investment no long exists if no firm invests in it. On the other hand, the information leakage increases the correlation in the parameter region where \((L, F)\) & \((F, L)\) replace \((L, N)\) & \((N, L)\). This is because with probability \((\lambda \theta)\) the follower firm invests in the innovation and makes the same profit as the leader during the product market competition at date 3, which reduces the variance of market maker prices. The amount of investment taken by firms also becomes larger.
in this region. In the parameter regions where \((L, L)\) is replaced by \((N, N)\), the correlation of firms’ specific returns is reduced and so is the amount of investment. Implication 3 follows.

**IMPLICATION 3:** *The model suggests that the correlation of firms’ specific returns is positively related the amount of R&D investment made by firms. This link is stronger when share prices are more informative.*

Another observation is that a higher probability \((\lambda)\) of information leakage (e.g., a second firm going public) may lead to a lower amount of investment in the industry. This happens in equilibrium when speculators’ incentives are aligned with the option value of waiting such that either one firm switches from \(L\) to \(F\) and invests only upon good news (e.g., point \(B\) in Figure 2.5) or the leader is deterred from investing up front (e.g., point \(C\) in Figure 2.6). A higher \(\lambda\) can also lead to more investment in the region where the non-leading firm switches from \(N\) to \(F\) and invests with a higher probability at date 1. This thus provides a cross-section implication regarding the amount of R&D investment.

**IMPLICATION 4:** *The amount of R&D investment may be lower in an industry in which share prices of competing firms are more informative, the market size and investment profitability are larger and when the level of competition higher. It can also occur when the competition is intense while the market size is relatively small.*\(^{20}\)

Foucault and Frésard (2012) find that the investments of private firms, after they go public, are less correlated to their peers’ share prices because these firms can thereafter

\(^{20}\)This may thus provide a partial explanation to the empirical evidence that public firms invest less and hoard more cash than private firms. For instance, Asker et al. (2011) find that compared to private firms, public firms take fewer investments and they are less responsive to investment opportunities, and associate their findings with agency costs.
learn from their own stock prices. The results in this paper suggest the cross-sectional difference in this aspect. The numbers of firms traded actively by speculators change the probability of information leakage and thus the option value of waiting. When this is taken into account in firms’ decisions ex-ante, as discussed previously, the characteristics of the product market and the R&D investment determines whether firms invest in the same (or a similar) innovation after the IPO of their rivals. It is summarized below.

**IMPLICATION 5:** *After a private firm goes public, the sensitivity of its investment to its competitors’ share prices increases if these firms are in an industry with a relatively high competition and large market size, and if their R&D investments are associated with a relatively high profitability.*

When managers can learn from the stock prices of other firms in the same industry, they share the aggregated belief about the prospect of a certain technology and possibly behave in a similar way. This indirect information leakage may thus contribute to explain why public firms may rationally herd in their investment decisions (See for example Scharfstein and Stein (1990)). That is, when price informativeness allows the follower firm to switch from strategy $N$ to $F$ in equilibrium, firms have more correlated investment.

**IMPLICATION 6:** *A higher correlation of R&D investments among publicly-listed competing firms may be found in an industry with a relatively large market where both the competition level and the investment profitability are moderate.*
2.6 Extension

Surplus in the product market Regulators pay much attention to innovation investment at firm level due to its vital impact on technological development in the economy. I therefore discuss briefly the changes in welfare due to the presence of the feedback from the stock market. First, let us denote the consumer surplus by \( CS \). Using the formula \( CS = U(q_i, q_j) - p_i q_i - p_j q_j \), with \( U(q_1, q_2) \) as the utility given in formula (2.1), it is straightforward to compute the expectation of consumer surplus for each strategy profile \((A_i, A_j)\), \( A_i, A_j \in \{L, F, N\} \).

By comparing the ex-ante expectation of consumer surplus in different equilibria, I obtain the proposition below.

**Proposition 5** The expected consumer surplus increases in the expected amount of innovation investment.

In other words, the expected consumer surplus descends by the order of \((L, L)\), \((L, F)\), \((L, N)\), and finally \((N, N)\). The information leakage via share prices is beneficial to the consumer when the non-leading firm choosing the strategy \( F \) over \( N \) compared to the benchmark case. It, however, has a negative impact either when the potential leader is deterred from investing at date 0 or when one firm switches its strategy from \( L \) to \( F \). As a result, whether consumers benefit from having more information revealed from the stock market depends on the parameter values in this economy.

Combining the consumer surplus and the expected firm profits, we can obtain the expected total surplus \((TS)\) in the product market. Using Proposition 2 and Proposition 5, we know that the total surplus increases when the non-leading firm choose the strategy \( F \)
over $N$, and it is reduced when firms’ strategies changes from $(L, N)$ to $(N, N)$.

**Corollary 1** The expected total surplus in the product market is higher with a leader and a follower firm than with only one firm investing. It is however reduced when the up-front investment is deterred such that firms’ strategies change from $(L, N)$ & $(N, L)$ to $(N, N)$.

Corollary 1 shows that the impact of information leakage on the total surplus in the product market has a similar pattern as on consumer welfare, except that it is ambiguous in the parameter region where $(L, L)$ is replaced by $(L, F)$ and $(F, L)$. However, it is certain that when information leakage deters the potential leader from investing upfront, it not only undermines production efficiency but further reduces the total surplus in the product market.

**Noise traders’ private benefit** In this subsection, I extend the analysis by endogenizing the participation of noise traders and explore the impact on the equilibrium outcome. The assumption that noise traders are completely unconcerned about their trading profit is more convenient rather than realistic. To relax this assumption, I assume that there exists for each firm a continuum of noise traders with measure 1, who trade for exogenous needs of liquidity. Noise traders are indexed by $k_i$ for firm $i$ (ergo $k_j$ for firm $j$), which distinguishes the magnitude of their private benefit of having a position in the stock. I denote this benefit by $b$, $b_{k_i} = (1 - k_i)\tau$, where $\tau$ signifies the common nature of the trading motive shared by noise traders, $\tau > 0$. Noise traders are thus heterogeneous only in the size of private benefit. I define the utility of noise trader $k_i$ as

$$u_{k_i} = \begin{cases} 
  b_{k_i}, & \text{if } X_{k_i} = z_i, \ z_i \in \{-1, 1\}, \\
  0, & \text{otherwise}
\end{cases} \quad (2.7)$$

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where \( z_i \) denotes the state of world and \( X_{k_i} \) is the trading order of the \( k^{th} \) noise trader of firm \( i \). Noise traders of each firm have the same preference for the size and sign of the orders to submit. For instance, if \( z_i = -1 \), the spectrum of noise traders of firm \( i \) are in need of liquidity and \( X_{k_i} \) equals \(-1\). The realizations of \( z \) are uncorrelated across firms, and noise traders’ preference between cash and share is decided by nature with equal probability.\(^{21}\)

The realization of \( z \) is private information to noise traders.

Each noise trader plays strategically and thus participates only when the net expected payoff is non-negative. As a result, there exists a \( k_i^{*th} \) noise trader of firm \( i \) who is indifferent between trading and otherwise, and all the others with \( k_i > k_i^{*} \) will quit the market. Based on the same argument as in Section 2.3, the threshold \( k_i^{*} \) determines the optimal trading size of speculator \( i \). By comparing speculator \( i \)'s expected profit to the \( k^{th} \) noise trader’s private benefit, we can find the threshold \( k_i^{*} \) for the indifferent noise trader. We can express \( k_i^{*} \) as

\[
k_i^{*} = \begin{cases} 
\max \left( 1 - \frac{\Psi_i}{\tau}, 0 \right), & \text{if } 1 - \frac{\Psi_i}{\tau} < 0 \\
\min \left( 1 - \frac{\Psi_i}{\tau}, 1 \right), & \text{if } 1 - \frac{\Psi_i}{\tau} > 0 
\end{cases} \tag{2.8}
\]

where \( \Pi_i^{S} \) is the expected trading profit of speculator \( i \). The result is summarized in the lemma below.\(^{22}\)

**Lemma 7** When firm \( i \) innovates at date 0, firm \( j \)'s decision will be changed by the size of private benefit. If \( \tau \leq 2\theta (1 - \theta) (\pi_{c,c} - \pi_{c,c-\delta}) \), the feedback effect no longer prevails and firms choose their optimal strategies as stated in Proposition 1. If \( \tau \in (2\theta (1 - \theta) (\pi_{c,c} - \pi_{c,c-\delta}) \),

\(^{21}\)If noise traders expect to have a liquidity shock with a positive probability, there will be a higher probability for them to prefer cash over equity. To simplify the illustration, I assume that there is no other shock to the liquidity need of noise traders.

\(^{22}\)For the purpose of presentation, I discuss the additional assumptions in the Proof of Lemma 6 in the Appendix.
\( \pi_{e, c - \delta}, \theta (1 - \theta) (\pi_{e - \delta, c} - \pi_{e, c}) \), speculator \( i \) leaves the market and firms’ optimal strategies are determined when speculator \( j \) trades as a monopolist, the feedback effect is weakened as described by Proposition 5. If \( \tau > \theta (1 - \theta) (\pi_{e - \delta, c} - \pi_{e, c}) \), both speculators trade actively, and firms’ equilibrium strategies follow Proposition 2.

2.7 Concluding remarks

The financial market plays an important role in allocating scarce resource via information exchange and revelation given that prices contain information that can improve capital allocation (Fama and Miller, 1972). The impact of information efficiency on the real economy starts to change when one takes into account the feedback effect from prices on corporate decisions, since the expected cash flows of the asset are endogenized in equilibrium. This paper is an attempt to investigate this process when share prices from the secondary market feed back to firms’ innovating strategies. Using a simple setup in a differentiated Bertrand duopoly, I model information leakage related to a risky process innovation, which induces an intra-industry knowledge spillover and alters firms’ ex-ante decisions in innovation investment. This information leakage then provides firms an option to invest as a follower with better knowledge. It may also discourage the up-front investment and leads to a lower efficiency in the product market. This is the case if the leader firm anticipates that its innovation rent becomes insufficient when being imitated by a follower firm. When it is costly for traders in the stock market to acquire private information, the amount of information leakage and hence its impact on the option value of waiting are both endogenized in equilibrium. I show that stock price informativeness may worsen the investment outcome when
there is intense competition in a relatively small market. The model therefore sheds light on the two-way causality between the amount of information produced in the stock market and the fundamentals in the real economy.

Even though this paper focuses on the context of innovation strategies, it provides a framework that can be applied to a wide array of corporate decisions in practice, where the payoff of one firm’s action is strategically affected by similar actions taken by its competitors or industry peers. Examples are, but not limited to, investments in enlarging production capacities, vertical integrations for the purpose of reducing input price or operating cost, and outsourcing strategies.

Finally, one relevant question to ask is that when firms’ pre-commitments or strategic disclosures already prevail, how stock trading contributes to technological advances by introducing additional information. It is interesting to explore whether share trading acts to verify or to obscure the information being revealed via other channels\textsuperscript{23}. It may also be interesting to consider a different design of information structure. For instance, if firms’ investment action can not be immediately observed, the information revealed via stock prices may become more obscure. The optimal strategy of both firms and stock market participants will change accordingly. The answers to these questions are beyond the scope of this paper, but they may provide policy makers with implications in practice, particularly when the characteristics of different industries are taken into account.

\textsuperscript{23} Amir Ziv (1993) proves that when the incentive for truthful information sharing is endogenized, firms no longer find it in their interest to honestly disclose production information, particularly in a one-stage game when information verification is not quite feasible.
REFERENCES CITED


This figure shows firms’ equilibrium strategies with information leakage in a numerical example with 

\[ \alpha = 6, c = 3, \theta = 0.4, \gamma = \frac{3}{4} \]  

\( (N, N) \) marks the parameter region of no firm investing in equilibrium.  
\( (L, L) \) marks the region of both firm investing, and \( (L, N) \) & \( (N, L) \) only one firm investing in equilibrium.
Figure A2: Equilibrium - Endogenous Leakage (Moderate $\epsilon$)

This figure shows the equilibrium outcome of a numerical example in which the information cost is moderate ($\epsilon = 1$). There may be a monopoly speculator staying active in the stock market. The grey solid line is the cutoff for the monopoly trading profit to be equal to the information cost. Below the grey line, the monopoly speculator does not acquire information and thus there is no informed trading. Point C in the figure is at the exactly the same location as in Figure 5 and 6. Observe that the equilibrium is now switched to $(L, F)$ & $(F, L)$ since given a lower information cost compared to Figure 5, the speculator of the leader firm now has incentive to acquire information and trade.
Appendix B: Proofs

Proof of Lemma 1. It is easy to compute firms’ profit under each realization of production cost \((c_i, c_j)\).

\[
\pi_{c,\delta,\gamma} = \frac{1}{1-\gamma^2} \left( \frac{1-\gamma}{2-\gamma} \right)^2 \left[ (\alpha - c) + \frac{(2-\gamma^2)\delta}{(2+\gamma)(1-\gamma)} \right]^2
\]

\[
\pi_{c,\delta,\gamma} = \frac{1}{1-\gamma^2} \left( \frac{1-\gamma}{2-\gamma} \right)^2 (\alpha - c + \delta)^2
\]

\[
\pi_{c,\gamma} = \frac{1}{1-\gamma^2} \left( \frac{1-\gamma}{2-\gamma} \right)^2 (\alpha - c)^2
\]

\[
\pi_{c,\delta} = \frac{1}{1-\gamma^2} \left( \frac{1-\gamma}{2-\gamma} \right)^2 \left[ (\alpha - c) - \frac{\gamma\delta}{(2+\gamma)(1-\gamma)} \right]^2
\]

Since \(\frac{(2-\gamma^2)}{(2+\gamma)(1-\gamma)} > 1\) and \(-\frac{\gamma}{(2+\gamma)(1-\gamma)} < 0, \forall \gamma \in (0, 1)\), it is evident that \(\pi_{c,\delta,\gamma} > \pi_{c,\delta,\gamma} \delta\) and \(\pi_{c,\gamma} > \pi_{c,\delta} \delta\). We therefore obtain \(\pi_{c,\delta,\gamma} > \pi_{c,\delta,\gamma} \delta > \pi_{c,\gamma} > \pi_{c,\delta} \delta\). ■

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Proof of Proposition 1. When there is no information leakage, we can use the proof of Lemma 1 to obtain the following; \( \Pi_i (L, L) = 2\theta \pi_{c-\delta,c-\delta} + 2(1-\theta)\pi_{c,c} - I \), \( \Pi_i (N, N) = 2\pi_{c,c} \), \( \Pi_i (L, N) = 2\theta \pi_{c-\delta,c} + 2(1-\theta)\pi_{c,c} - I \), and \( \Pi_i (N, L) = 2\theta \pi_{c,c-\delta} + 2(1-\theta)\pi_{c,c} \).

Therefore, for firm \( i \) to deviate from \( L \) to \( N \) given firm \( j \) chooses \( L \), it must be true that:

\[
\Pi_i (L, L) - \Pi_i (N, L) < 0 \quad \text{and thus} \quad I > 2\theta (\pi_{c-\delta,c-\delta} - \pi_{c,c-\delta}). \quad \text{Let} \quad \bar{I} = 2 (\pi_{c-\delta,c-\delta} - \pi_{c,c-\delta}).
\]

Similarly, for firm \( j \) to deviate from \( N \) to \( L \) given firm \( i \) chooses \( N \), it must be true that:

\[
\Pi_j (N, N) - \Pi_j (L, N) < 0 \quad \text{and thus} \quad I < 2\theta (\pi_{c-\delta,c} - \pi_{c,c}). \quad \text{Let} \quad \bar{I} = 2 (\pi_{c-\delta,c} - \pi_{c,c}).
\]

These two inequalities must be both satisfied for the strategy pairs \((N, L)\) & \((L, N)\) to be the equilibria, i.e., \( \theta \bar{I} > I > \theta \bar{I} \). Due to the symmetry of the payoff matrix, if \( I > \theta \bar{I} \), \((N, N)\) is the Nash equilibrium; and if \( I < \theta \bar{I} \), the equilibrium strategy pair is \((L, L)\). ■

Proof of Lemma 2. From intuition, given that share prices are not informative, the prior of the non-leading firm about the innovation remains unchanged. Were it optimal for this firm to invest at date 1, it must be better off to invest at the beginning of the game. It is because, based on the same prior, the strategy \( L \) guarantees that a firm does not lose in product market competition at either date 2 or 3, compared to a possible loss from competition at date 2 due to a late investment in innovation. Therefore, \( \tilde{F} \) is dominated by either \( L \) or \( F \).

Mathematically, assume firm \( i \) leads in innovation investment. Conditioning on \( X_i = X_j = 0 \), the difference in the expected profit between choosing \( \tilde{F} \) and \( N \), \( \theta \pi_{c-\delta,c-\delta} - I - \theta \pi_{c,c-\delta} \). Therefore firm \( j \) chooses \( \tilde{F} \) over \( N \) when \( I < \theta (\pi_{c-\delta,c-\delta} - \pi_{c,c-\delta}) \), i.e., \( I < \frac{1}{2} \theta \bar{I} \).
Next, given firm $i$ chooses $L$, for $\tilde{F}$ to be optimal to firm $j$ there needs to be a profitable deviation from the strategy $L$. For a given probability of information leakage, $\Pi_j(\tilde{F}, L) = \theta (\pi_{c,c-\delta} + \pi_{c-\delta,c-\delta}) + 2 (1 - \theta) \pi_{c,c} - (1 - \lambda(1 - \theta)) I$. To have $\Pi_j(L, L) - \Pi_j(\tilde{F}, L) < 0$, it must be $I > \frac{\theta}{2\lambda(1-\theta)} L$. The conditions $I < \frac{1}{2} \theta I$ and $I > \frac{\theta}{2\lambda(1-\theta)} L$ cannot be both satisfied at the same time, $\forall \theta \in (0, 1)$, $\lambda \in (0,1)$. $(L, \tilde{F})$ and $(\tilde{F}, L)$ thus cannot be Nash equilibria.

**Proof of Proposition 2.** First, I compute the equilibrium conditions for the strategy pairs $(L, F)$ and $(F, L)$. Given the probability of information leakage being $\lambda$, the expected payoff of firm $i$ choosing $F$ when firm $j$ chooses $L$, is $\Pi_i(F, L) = \theta ((2 - \lambda) \pi_{c,c-\delta} + \lambda \pi_{c-\delta,c-\delta}) + 2 (1 - \theta) \pi_{c,c} - \lambda \theta I$. For $(A_i, A_j) = (F, L)$ to be a Nash equilibrium, it has to be profitable for firm $i$ to deviate from the strategy $L$ to $F$ when firm $j$ chooses $L$, i.e., $\Pi_i(L, L) - \Pi_i(F, L) < 0$. This leads to $I > \frac{(2-\lambda)\theta}{2(1-\lambda\theta)} L$.

Similarly for firm $i$ to deviates from $N$ to $F$ given firm $j$ choosing $L$, it has to be $\Pi_i(N, L) - \Pi_i(F, L) < 0$, i.e., $I < \frac{1}{2} L$. Notice that when $\theta > \frac{1}{2}$, the inequality $\frac{(2-\lambda)\theta}{2(1-\lambda\theta)} L < I < \frac{1}{2} L$ does not hold. Thus, the strategy $(L, F)$ cannot be the equilibrium if $\theta > \frac{1}{2}$.

On the other hand, for $F$ to be an equilibrium strategy it must be profitable for firm $j$ to choose $L$ over $N$, when expecting firm $i$ to follow when learning good news at date 1. This is true because if the leader firm does not invest at date 0, share prices no longer contain private information and the other firm cannot act a follower either. We therefore need $\Pi_j(L, F) > \Pi_j(N, N)$, which gives $I < \theta ((2 - \lambda) \pi_{c-\delta,c} + \lambda \pi_{c-\delta,c-\delta}) - 2 \theta \pi_{c,c}$. Let this expression be $\theta \tilde{I}$.

$(L, F)$ and $(F, L)$ are the equilibria when all three conditions above are satisfied, that is, $\frac{(2-\lambda)\theta}{2(1-\lambda\theta)} L < I < \min \left\{ \theta \tilde{I}, \frac{1}{2} I \right\}, \forall \theta \leq \frac{1}{2}$. Note that to ensure the existence of a pure-strategy
equilibrium, we need $\theta \tilde{I} > \frac{(2-\lambda)\theta}{2(1-\lambda\theta)} L$, $\forall \theta \in (0, \frac{1}{2})$ and $\lambda \in (0, 1)$. This can be guaranteed with $\theta \tilde{I} > \frac{(2-\lambda)\theta}{2(1-\lambda\theta)} I$ for $\theta = \frac{1}{2}$, or equivalently with $\alpha$ lower than $c + \delta^{8-4(2-\lambda)(1-\lambda)\gamma(1+\gamma)/(1-\lambda)\gamma}$.  

Next, it is similar to compute the equilibrium condition for $(L, N)$ and $(N, L)$. We already know that firm $j$ deviates from $L$ to $N$ when $I > \theta I$ if firm $i$ chooses $L$. Also from the proof of Proposition 1, we know that given $I < \theta I$ firm $i$ chooses $L$ over $N$ when firm $i$ chooses $N$. Combining the condition $I > \frac{1}{2} I$ for firm $j$ to deviate from $F$ to $N$, it is evident that for $\theta \geq \frac{1}{2}$ both inequalities are satisfied when $I > \theta I$, and for $\theta < \frac{1}{2}$, $I > \frac{1}{2} I$ suffices. Proposition 1 shows that given firm $j$ choosing $N$, firm $i$ prefers $L$ to $N$ if $I < \theta I$. The conditions for $(L, N)$ and $(N, L)$ to be equilibria are: $\theta I < I < \theta \tilde{I}$ if $\theta > \frac{1}{2}$; and $\frac{1}{2} I < I < \theta \tilde{I}$ if $\theta \leq \frac{1}{2}$.

The threshold $\theta \tilde{I}$ and $\theta I$ then define the equilibrium conditions for $(N, N)$. If $I > \frac{1}{2} I$, $(N, N)$ is the unique equilibrium when $I > \max \{\frac{1}{2} I, \theta \tilde{I}\}$; and if $I \leq \frac{1}{2} I$, $(N, N)$ is the unique equilibrium when $\frac{1}{2} I \geq I > \theta \tilde{I}$.

By the same algorithm, for $(L, L)$ to be a Nash equilibrium, we need to ensure when firm $j$ chooses $L$ and firm $i$ cannot profit from deviating to any other action than $L$. That is, $I < \frac{(2-\lambda)\theta}{2(1-\lambda\theta)} I$ and $I < \theta I$. Notice that $\frac{(2-\lambda)\theta}{2(1-\lambda\theta)}$ is lower than $\theta$ when $\theta < \frac{1}{2}$. Combining the conditions obtained previously, we know $(L, L)$ is the equilibrium when $I < \frac{(2-\lambda)\theta}{2(1-\lambda\theta)} I$ for $\theta < \frac{1}{2}$, and when $I < \theta I$ for $\theta > \frac{1}{2}$.

**Proof of Proposition 3.** The option value of waiting is the difference between $\Pi_i (F, L)$ and $\Pi_i (L, L)$ which is $(2 - \lambda) \theta (\pi_{c, c - \delta} - \pi_{c - \delta, c - \delta}) + (1 - \theta \lambda) I$.

The first order derivative of the option value with respect to $\theta$ and $\alpha$ are respectively, $(2 - \lambda) (\pi_{c, c - \delta} - \pi_{c - \delta, c - \delta}) - \lambda I$ and $- \frac{2(2-\gamma^2)\delta(2-\lambda)}{(2-\gamma)^2(1+\gamma)(2+\gamma)}$ both negative. Similarly, the first order
derivative with respect to $I$ is $(1 - \theta \lambda) > 0$, $\forall \theta \in (0, 1)$ and $\lambda \in (0, 1)$.

The impact of competition level $\gamma$ on the option value depends on the relative magnitude of $\frac{\partial}{\partial \gamma} \pi_{c,c-\delta}$ to $\frac{\partial}{\partial \gamma} \pi_{c-\delta,c-\delta}$. The difference is

$$2 \frac{(a-c)(1-\gamma)^2 \left(8+\gamma^2(-6+\gamma(4+\gamma(7+2\gamma)))\delta + 2 \left(8+\gamma(-2+\gamma^2)(4+\gamma(-1+\gamma(-2+\gamma(3+\gamma))))\delta^2\right)\right)}{(4-\gamma^2)^2(1-\gamma^2)},$$

which is positive $\forall \gamma \in (0, 1)$. ■

**Proof of Lemma 4.** There are two parts in this proof. The first is to show that it is optimal for the speculators to submit an order with a fixed size 1. Since the noise trader always submits an order of one unit for each firm, the expected order flow for a listed firm is zero. The market maker will then quote higher based on a total order flow greater than zero, or lower otherwise. The speculators would thus either easily expose their identities by submitting an order with a whole size larger than one, or make lower profit by trading fractional orders. The optimal way to hide his identity and obtain a favorable quote is to submit an order of the same size as the one from the noise trader, regardless of the trading direction.

Next, we consider the trading direction of the speculators. If both firms make an investment at date 0, both speculators buy if the innovation succeeds and both of them sell if otherwise. Now consider the case in which only firm $i$ invests at date 0 and learns at date 1 that its innovation will succeed in reducing its production cost $c_i$. Firm $j$ is then disadvantaged in price competition for at least one stage. Consequently, speculator $i$ buy one share of firm $i$ and speculator $j$ submits a sell order of firm $j$.

On the other hand, if only firm $i$ invests at date 0 but the innovation fails, firm $i$ incurs a loss $I$. A failed innovation does not change the price competition in the product
market, it however lowers the liquidation value of firm $i$. As a result, speculator $i$ sells. As for firm $j$, it will not invest at date 1 when bad news are revealed by the total order flow (share prices). Neither will it when share prices are not informative, because the strategy of investing at the intermediate stage without additional information from the stock market is strictly dominated by the strategy of investing up front.\textsuperscript{24} Since the market maker is uninformed when speculators’ orders are hidden in the total order flow, his quote of firm $j$ must be lower than the actual liquidation value. Consequently, speculator $j$ will submit a buy order of firm $j$. $\blacksquare$

\textbf{Proof of Lemma 5.} The noise trader buys or sells 1 unit of both firms’s shares with equal probability and there is no correlation in their orders across firms. Evidently, the total order flow of each firm belongs to the set $\{-2, 0, 2\}$. Suppose only firm $i$ invests at date 0 and its innovation succeeds. $x_i \in \{0, 2\}$ and $x_j \in \{-2, 0\}$ as a consequence. We observe immediately that there are four possible combinations of $x_i$ and $x_j$, each attached with the same conditional probability $\frac{1}{4}$. Given that firms’ innovating activities are publicly observable, the good news of firm $i$ can be inferred by the other agents except when the order flows of both firms are zero. More specifically, when $(x_i, x_j)$ belongs to the set $\{(2, -2), (2, 0), (0, -2)\}$, the private information $c_i = c - \delta$ is fully revealed by informed trading. Order flows thus reveal the private information with probability $\frac{3}{4}$ conditional on that the innovation succeeds, thus a total probability $\frac{3}{4} \theta$. Similarly, the probability of revealing the information that the innovation fails is $\frac{3}{4} (1 - \theta)$, and $\frac{1}{4} (1 - \theta)$ otherwise. Using

\textsuperscript{24}If share prices are not informative at date 1, the non-leading firm has the same prior about the innovation as before the game starts. Were it optimal for this firm to invest then, it must be better off to invest up-front, by which it can be assured not to lose in product market competition at either date 2 or 3. The proof of Lemma 3 formally shows this point.
the same algorithm, we conclude that the probability of information revelation is the same for the case where both firms invests at date 0. ■

**Proof of Proposition 4.** When both firms choose strategy $L$, $\Psi(L, L) = \theta (1 - \theta) (\pi_{c-\delta, c-\delta} - \pi_{c,c})$.

When firm $i$ chooses to invest at date 0, and firm $j$ is the non-leading firm, we have $\Psi_i(L, N) = \theta (1 - \theta) (\pi_{c-\delta,c} - \pi_{c,c})$ and $\Psi_j(N, L) = \theta (1 - \theta) (\pi_{c,c} - \pi_{c,c-\delta})$.

$\Psi_i(A_i, A_j)$ for each $(A_i, A_j)$ above is concave in $\theta$ and linear in $\delta$ and $\alpha$. By taking the first order derivative of $\Psi_i(A_i, A_j)$, with respect to $\theta$, we see that all derivatives are negative when $\theta > \frac{1}{2}$ and positive otherwise. Similarly, the first order derivatives of $\Psi_i(A_i, A_j)$ for each strategy profile is positive respect to both both $\delta$ and $\alpha$. Similarly, we can compute the first order derivative of $\Psi_i(L, N)$ and $\Psi_j(N, L)$ with respect to $\gamma$, and obtain the parameter region of $\alpha$ in Proposition 4. ■

**Proof of Lemma 6.** This lemma concerns the case where speculator $j$ exits the stock market while speculator $i$ continues to acquire information and trade in firm $i$. The feasible set of order flow is $\{-2, 0, 2\}$ for firm $i$, and $\{-1, 1\}$ for firm $j$. So the possible combinations are $\{2, 1\}$, $\{2, -1\}$, $\{0, 1\}$, and $\{0, -1\}$ when the innovation is successful, and $\{-2, 1\}$, $\{+2, -1\}$, $\{0, 1\}$, and $\{0, -1\}$ when the innovation fails. Evidently, the order submitted by speculator $i$ is hidden when the set $(x_i, x_j) \in \{(0, -1), (0, +1)\}$, which occurs with probability $\frac{1}{2}$. The share price $P_i$ is thus informative with probability $\frac{1}{2}$. ■

**Proof of Proposition 5.** Let $CS_{A_i,A_j}^t$ denote the sum of consumer surplus at date $t$ in the equilibrium where firms choose the action $(A_i, A_j)$, and let $c_i^t$, $p_i^t$, and $q_i^t$ denote the production cost, price and the output for firm $i$ at date $t$, $t = 2, 3$. The innovating

\[25\text{Even without limits of exposure, it can be shown easily that trading only firm } i \text{'s shares is more profitable than trading in both firms which would reveal private information with probability } \frac{3}{4}.\]
firm will have the production cost $c - \delta$ with probability $\theta$, or $c$ otherwise. For example, when $(A_i, A_j) = (L, L)$ and the innovation is successful, product prices and demands can be computed: $p_{i,j}^2 = p_{i,j}^3 = \frac{(1-\gamma)\alpha+c-\delta}{2-\gamma}, q_j^2 = q_j^3 = \frac{\alpha-c+\delta}{(2-\gamma)(1+\gamma)}$.

The total consumer surplus over two stages is, conditional on that the innovation succeeds, the sum of $CS_{L,L\theta}^2$ and $CS_{L,L\theta}^3$, which equals $\frac{2(\alpha-c+\delta)^2}{(2-\gamma)^2(1+\gamma)}$. This expression can be simplified to $\frac{2}{1-\gamma} \pi_{c-\delta,c-\delta}$, using the notation defined in (2.8). Similarly, if the innovation fails, the consumer surplus over date 2 and 3 is $\frac{2(\alpha-c)^2}{(2-\gamma)^2(1+\gamma)}$, expressed by $\frac{2}{1-\gamma} \pi_{c,c}$ by the notation in (2.9). The ex-ante expected consumer surplus is therefore $\frac{2}{1-\gamma} (\theta \pi_{c-\delta,c-\delta} + (1 - \theta) \pi_{c,c})$ if both firms innovate at date 0, and $\frac{2}{1-\gamma} \pi_{c,c}$ if no firm invests.

Using the same method, I compute the expected consumer surplus for the equilibrium $(L, N)$. $CS_{L,N}$ equals $2\theta CS_{L,N\theta}^2 + 2 (1 - \theta) CS_{L,N\theta}^2$, as the surplus will have the same value at both dates. Similarly, let $CS_{L,F}$ denote the expected consumer surplus for the equilibrium $(L, F)$. We know already from Lemma 2 that the non-leading firm will follow at date 1 only when order flows reveal good news. $CS_{L,F}$ thus consists of two parts; the expected consumer surplus at date 2, which is equivalent to $\frac{1}{2} CS_{L,N}$, and the surplus $CS_{L,F}^3$ (at date 3). $CS_{L,F}^3$ includes $\frac{3}{4} \theta CS_{L,L\theta}$ when good news being revealed, $\frac{3}{4} (1 - \theta) CS_{L,N\theta}^2$ when bad news being revealed, and $\frac{1}{4} CS_{L,N}$ when order flows reveal no private information. $CS_{L,N\theta}^2 = CS_{L,L\theta}$ since the production cost of both firms remains unchanged if the innovation fails. The expression for $CS_{L,F}$ can then be simplified to $\frac{5}{8} CS_{L,N} + \frac{3}{8} CS_{L,L}$. The difference between $CS_{L,L}$ and $CS_{L,F}$ is thus $\frac{5}{8} (CS_{L,L} - CS_{L,N})$, which is positive because of the following.

The sum of consumer surplus over two stages conditioning on the innovation success
is the sum of $CS_{L,L|\theta}^2$ and $CS_{L,L|\theta}^3$. If innovation succeeds, the total consumer surplus equals to $\frac{2(a-c+\delta)^2}{(2-\gamma)^2(1+\gamma)}$, which can be expressed by $\frac{2}{1-\gamma}\pi_{c-e,c-e}$ by using equation (2.8). Similarly, if the innovation fails, the total consumer surplus over two stages is $\frac{2(\alpha-c)^2}{(2-\gamma)^2(1+\gamma)}$, expressed by $\frac{2}{1-\gamma}\pi_{e,e}$ by equation (2.9). $CS_{L,L}$ then equals $\frac{2}{1-\gamma}(\theta\pi_{c-e,c-e} + (1-\theta)\pi_{e,e})$ if both firms innovate at date 0.

$$CS_{L,L} - CS_{L,N}$$

$$= \frac{2}{1-\gamma}(\theta\pi_{c-e,c-e} + (1-\theta)\pi_{e,e}) - \left[2\theta CS_{L,N|\theta}^2 + \frac{2(1-\theta)}{1-\gamma}\pi_{e,e}\right]$$

$$= 2\theta \left(\frac{\pi_{c-e,c-e}}{1-\gamma} - CS_{L,N|\theta}^2\right)$$

By using formula (2.1), we can obtain $CS_{L,N|\theta}^2$:

$$CS_{L,N|\theta}^2 = \alpha^2 - \frac{2a-2c+\delta}{(2-\gamma)(1+\gamma)} - \frac{\delta^2}{2(1-\gamma)^2(2+\gamma)^2} - \frac{(1-\gamma)(a^2+\pi p_i)^2 - (a^2+\pi p_j)^2 + 2\pi p_i p_j}{(1-\gamma)^2},$$

where $p_i = c - \delta$, and $p_j = c$.

$$\frac{\pi_{c-e,c-e}}{1-\gamma} - CS_{L,N|\theta}^2 = \frac{\delta(2(a-c)(1-\gamma)(2+\gamma)^2 + \delta(4-3\gamma^2-2\gamma^3))}{2(2-\gamma)^2(1+\gamma)}$$

which is negative only when $\gamma$ is sufficiently close to 1. Note that when products become very close substitutes, firms will choose $(L, N)$ & $(N, L)$ in equilibrium and the consumer surplus for $(L, L)$ no longer concerns us. Therefore, $CS_{L,L}$ is greater than $CS_{L,N}$. $CS_{L,F}$ is then also greater than $CS_{L,N}$ since the difference between them is $\frac{3}{8}(CS_{L,L} - CS_{L,N})$.

At last the difference between $CS_{L,N}$ and $CS_{N,N}$ is $2\theta \left(\frac{CS_{L,N|\theta}^2 - \frac{\pi_{e,e}}{1-\gamma}}{1-\gamma}\right)$. It can be simplified to $\frac{\delta}{(2-\gamma)^2(1+\gamma)} (a-c + \frac{\delta(4-3\gamma^2)}{2(1-\gamma)(2+\gamma)^2})$, which is positive. We thus know that $CS_{L,N} > CS_{N,N}$. □

**Proof of Lemma 7.** To restrict the analysis to pure strategy equilibrium, I assume first that whether speculators acquire information is publicly observable. Next, if the parameters take values as such all noise traders quit trading and so do the speculators.
Expecting the exit of speculators, noise traders may however want to return to the market. To simplify the analysis, I assume the market maker’s pricing rule to be that he would consider the orders as being submitted by the speculators and set the prices disadvantageous to noise traders. I also let the information cost $\epsilon$ be trivial here to simplify the analysis, which however makes speculators strictly prefer not to participate when expecting to earn zero profit.

In the case where only firm $i$ innovates at date 0, it is easy to see $\Psi_i > \Psi_j > \Psi_j$ based on the computation of speculators’ expected profit in Section 2.3 and 2.4. Formula (2.14) then enables us to conclude that $\tilde{k}_i^* < k_i^* < \tilde{k}_j^* < k_j^*$.

When both firms find it optimal to innovate at date 0 with informed trading in stock market, their strategies stay the same with or without feedback effect. Due to the symmetry in speculators’ trading profit, either both speculators submit orders of equal size, that is, $k_i^* = k_j^* = \min\left(1 - \frac{\Psi_i (L, L)}{\tau}, 1\right)$. Or it occurs that $\tau$ is so low that both $k_i^*$ and $k_j^*$ fall to zero. Consequently no noise trader finds it profitable to trade and stock market breaks down.

We go back to the economy in the benchmark case. Firms’ optimal strategy in innovation remains unchanged, however.

Next, consider the case in which firms’ equilibrium strategies are affected by the feedback effect. For the case where firm $i$ leads in innovating and firm $j$ follows at a later date, it is easy to obtain $k_i^* = 1 - \frac{\theta}{\tau} (1 - \theta) (\pi_{c-\delta,c} - \pi_{e,c})$ and $k_j^* = 1 - \frac{\theta}{\tau} (1 - \theta) (\pi_{e,c} - \pi_{e,c-\delta})$, $k_i^* < k_j^*$. If $k_i^* = 0$ but $k_j^* > 0$, that is, noise traders quit trading firm $i$ and leave speculator $j$ the monopolist. The expected loss to the noise trader of firm $j$ is thus $2\theta (1 - \theta) (\pi_{e,c} - \pi_{e,c-\delta})$ that determines the new threshold for the noise traders of firm $j$, denoted by $\tilde{k}_j^*$, $\tilde{k}_j^* < k_j^*$. If
\( \tau \) is even lower than \( 2\theta (1 - \theta) (\pi_{c,c} - \pi_{c,c-\delta}) \), i.e., noise traders of firm \( j \) would incur a loss higher than their private benefit when speculator \( j \) is the monopolist. As a consequence, all noise traders quit and market breaks down completely. ■
CHAPTER 3.
FEEDBACK EFFECTS OF CERTIFICATIONS IN FINANCIAL MARKETS\textsuperscript{1}

3.1 Introduction

The role of certification intermediaries comes in a market with asymmetric information between buyers and sellers whereas either side cannot credibly disseminate their private information. These intermediaries are designed to acquire the signals about the privately informed parties and then to reveal to uninformed parties. Their credibility can be endorsed by laws and regulations, and/or determined by various mechanisms in different markets. Examples of certification intermediaries include auditors, industrial certification systems, credit rating agencies, and investment banks that evaluate the quality of firms that want to raise capital. The literature related to certification intermediaries focuses either on their strategies of information disclosure due to conflicts of interests between the users of the information and the intermediaries\textsuperscript{2}, or on the functionality of certifiers as a device for inspection or signalling\textsuperscript{3}.

In this paper, we look at the informational role of certification agencies in the financial markets. As an example, similar to other certifiers, a credit rating agency supposedly takes the role of providing an independent opinion on the credit quality of firms. Moreover, if ratings contain information, they may alter the expectation of market participants about the overall quality of a firm. As suggested by empirical studies, credit ratings have either

\textsuperscript{1}THIS IS A JOINT WORK WITH ALEXANDER GUEMBEL.  
\textsuperscript{2}See for example Viscusi (1978), Lizzeri (1999), Peyrache and Quesada (2004).  
\textsuperscript{3}See for example Fasten and Hofmann (2010), Stahl and Strausz (2011).
direct or indirect impact on the cost of capital of a borrowing firm⁴, and they may thus influence a firm’s investment and financing decisions⁵. This consequently raises the question how a certifier, such as rating agencies, affects the information production by speculators in the financial markets, whose payoffs are directly related to firms’ investment decisions. Being outsiders of a firm, speculators can actively acquire information on firm value and profit from trading. When the private information possessed by speculators is revealed via share prices, it may then improve the decision taken by the firm. As a result, by changing the firm’s cost of capital and subsequent investment actions, the announcement made by certification intermediaries influences speculators’ incentive of information acquisition and ultimately the total amount of useful information that can help to guide resource allocation. Despite its importance to market efficiency, the interaction between information production by certification agencies and private speculators has not been analyzed.

To address the impact of certifications on information production in the market, we build a model that incorporates the feedback effect from the announcement made by a certifier onto investment decisions. A firm has to decide whether to ascertain "in house" the prospect of a potential investment or to delegate this task to a certifier who can credibly reveal their evaluations to the outsiders. In our model, the difference between delegating and in-house production only lies in whether this piece of information is publicized by the certifier or remains private to the firm itself. The firm needs to decide, after updating its belief by using all available information in the market, whether to make the investment. Under delegation, lenders have access to the certifier’s evaluation, and hence the information

asymmetry between the firm and lenders is alleviated. So is the adverse selection problem at
the financing stage. On the other hand, while delegation increases the transparency of a
firm’s prospects, it may reduce the expected trading gain to the speculators, who now have
less information advantage. As a consequence, speculators may make less effort to acquire
information, which leads to a potential information crowding-out. Less information feeds
back to the firm’s investment decision. The firm has to trade off between this and a lower
cost of capital at the financing stage under delegation.

We show in this paper, for some parameter regions, if the firm chooses in-house
information production there is a separating equilibrium at the financing stage such that
the borrowing cost is higher than under delegation. This is because in-house production
entails more asymmetric information which generates a higher adverse selection discount.
The lenders thus need to set the interest rate at such that the firm can be screened as a
low-type borrower if it indeed receives a bad signal and asks for credit, i.e., the incentive
compatibility (IC) constraint of the low type has to be binding. We show that when a priori
it is more likely for the investment to realize a high payoff in the future, it is preferable for a
firm to choose delegation except when the prior belief about the investment is very high. The
causes are twofold. Firstly, under the regime of in-house production, a higher prior tightens
the IC constraint of the low type borrower who would be more tempted to mimic, which
consequently pushes up the lending interest rate. In contrast, under the regime of delegation
with the absence of adverse selection problem, a higher prior reduces the interest rate since
it gives a brighter prospect of the investment. This enlarges the difference in the financing
cost between two regimes. Moreover, with in-house information production, a higher prior
reduces the variance of payoff realizations of the investment and hence speculators’ incentive of information acquisition, which is again in the opposite direction compared to the regime of delegation. The crowding-out effect thus becomes less severe, and the firm more likely chooses to delegate.

We also show that if the quality of the private information to be acquired by the firm or the certifier is sufficiently high, the firm more likely chooses not to delegate. If the signal obtained and kept private by the firm predicts better the state of the world, the rent from mimicking is lower and thus the IC constraint of the low type borrower is less tight. The interest demanded by the lenders falls consequently, which increases the payoff variance and thus the information acquired by speculators under in-house production. As a comparison, when the information prevailing in the market is more precise under delegation, it becomes less valuable for speculators to acquire additional information, which thus reduces price informativeness in the stock market. As a consequence, when the firm expects to get a private signal with high precision, the advantage of delegation in having a lower financing cost is reduced while information crowding-out becomes more severe and dominant. In that case, the firm more likely chooses not to reveal its private signal through delegation. Using a similar reasoning, we show in addition that the firm prefers not to delegate when there is an increase in the expected payoff of the firm’s current assets in place without new investment.

Our paper is closely related to the literature on the link between stock market efficiency and its impact on the real economy. More specifically, we use the feedback mechanism based on the extended theory on efficient market hypothesis. The crucial assumption underlying our model is that stock prices are efficient not only in reflecting the available infor-
mation regarding the future cash flows, but also in aggregating private information explored by outside investors and being used to improve corporate decisions and resource allocation (Dow and Gorton, 1997). (See also Bond, Edmans and Goldstein (2012) for a survey on this active informational role of prices.) Empirical studies find strong evidence that firms use the information contained in stock prices when making decisions on corporate disclosure, cash savings, investment and takeovers.

Our paper also contributes to a better understanding of certification intermediaries in financial markets, which include investment bank, brokers, monitoring institutions, as well as credit rating agencies. Our model relies on an assumption commonly used in this research area that certifications in the financial market are informative to outsiders of a firm including capital providers and traders in the financial markets. Unlike the existing literature, however, we do not focus on the mechanisms used by these agencies in supplying information under various constraints and incentive schemes. It is also worth mentioning that the firm in our model makes the choice of delegation before knowing its type and thus does not use the certification as a signaling device for the outsiders. This aspect also differentiates our paper from the previous work on certifications.

When relating these two strands of literature, it is natural for one to wonder about the link between certifications and information efficiency in the stock market. The potential feedback effect of certifications is overlooked in previous work on certification intermediaries.

---


7There is a large body of literature on this subject. See for example Ramakrishnan and Thakor (1984), Chemmanur and Fulghieri (1994), Holmstrom and Tirole (1997), Bolton, Freixas and Shapiro (2012) among many others.
in financial markets. There are two exceptions. Manso (2013) incorporates in his model the feedback from credit ratings to firms’ optimal default decisions and focuses on welfare analysis under different rating policies. Boot, Milbourn, and Schmeits (2006) consider a situation in which some investors base their decisions on the announcements of credit rating agencies and consequently ratings have a real impact on the firm’s choice between a risky and a safe project. While under a similar feedback mechanism, the signal acquired by the certification agency in our paper is not more informative than the signal acquired by the firm itself. We thus do not study the actual level of information content in the certification or the disclosure policies of the certifier. Instead, we investigate the impact of such a certification on the strategies of other informed outsiders in the stock market, and further study how that impact feeds back to firms’ ex ante decision of committing itself to the certifier through delegation.

The model in our paper is related to Khanna, Slezak and Bradley (1994) who illustrate in a different context a crowding-out effect on information production. These authors consider the competition in trading between informed outsiders in the stock market and the managers (insiders) when being allowed to trade, the result of which affects the information production by the outsiders. Similarly to ours, managers in their model allocate resources based on both their own information and outside information that is revealed through share prices. These authors show that when managers are allowed to trade, it becomes more likely that order flows reveal the state of the world, which reduces outsiders’ trading profit. Consequently, outsiders have lower incentive to improve the quality of their information acquisition. In the meanwhile the firm may benefit from a higher initial offer price paid by
liquidity traders, who expect to bear a smaller trading loss against informed traders. Different from Khanna et al (1994), our paper do not model the competition in the trading game between informed traders inside and outside of the firm. If a firm chooses to delegate, it commits itself to supply the "insider" information to the outsiders in a credible way.

The paper proceeds as follows. Section 3.2 presents the setup of the model. Firms’ investment strategies are computed in Section 3.3. Section 3.4 discusses the delegation choice of the firm. Section 3.5 includes an extension and Section 3.6 concludes. Proofs are relegated to the Appendix.

3.2 The Model

The timeline There are four dates. At date 0, a firm faces an uncertainty about the payoff of a project that requires a certain amount of investment. At the same date, the firm decides whether to delegate to an outside agency the task of acquiring additional information about the project. At date 1, a private signal can be obtained by either the outside agency or the firm itself, depending on the delegation choice. If the firm chooses delegation, the outside agency is going to publicize this signal at the same date. At date 2, speculators in the stock market can acquire a costly private signal regarding the firm’s project. The stock market opens. Both speculators and noise traders submit orders to a market maker who sets the trading price. After observing the order flow in the stock market, the firm makes the investment decision and the credit market decides the interest rate if the firm asks for credit. At the final date (date 3), the payoff of the project is realized.
The firm's choices and payoffs A firm is currently operating a project with an uncertain future payoff. The final payoff of this project depends on both the firm's action $A$ at this point and a state of the world $\omega$. If the firm decides to continue investing in the project, it needs to finance $I$, and this action is denoted by $C$. The firm can also choose an action $S$, which is to stop investing and maintain the status quo. Depending on the state of the world $\omega$ which takes a value from \{0, 1\}, each action in \{C, S\} will lead to a different set of payoff realizations. $\omega$ is assumed to be distributed between the state 1 and 0 with probabilities $\eta$ and $1 - \eta$, respectively. The distribution of $\omega$ is independent from the action $A$.

We use the notation $V_A^\omega$ for the payoff of the project. If the firm chooses $C$, the payoff is $V_c^1$ if $\omega = 1$ and $V_c^0$ otherwise. The payoff realizations of the assets in place, when the firm chooses $S$, are denoted by $V_s^1$ and $V_s^0$ for $\omega = 1$ or 0, respectively. We assume that $V_c^1 > V_c^0$, $V_s^1 > V_s^0$, and in addition the variance of the payoff is higher under the action to continue investing, i.e., $V_c^1 - V_c^0 > V_s^1 - V_s^0$. We also assume that the optimal action for the firm in a world with perfect information is to invest if $\omega = 1$ and to stop if otherwise, i.e.,
$V^1_c - I > V^1_s$ and $V^0_c < I + V^0_s$.

**Financing problem** Assume that the firm can only choose debt financing from a competitive credit market, and lenders demand an interest rate $R$ based on their belief of $\omega$. Creditors do not possess any private information about $\omega$ and therefore have the same prior as the firm at date 0. We assume that creditors do not actively search for information on their own, but they can update their belief if additional information is produced and revealed by other agents. Let us now use $\theta$ to denote the posterior belief of the creditors as well as other agents who only have access to public information.

Consider that the firm chooses $C$ and obtains $I$ from lenders at a gross interest rate $R$. The firm will pay back $IR$ to lenders if the state of the world is high ($\omega = 1$). If the state of the world is low ($\omega = 0$), the firm will have to default and pay only $V^0_c$ and be left with zero payoff as it is protected by the limited liability. This assumption then implies $V^0_c < I$.

Let the risk free interest rate be normalized to zero. The minimum value of $R$ is hence 1. The participation constraint of creditors then follows,

$$\theta IR + (1 - \theta)V^0_c \geq I. \quad (PC1)$$

When this constraint is binding, lenders are assured to break even from lending $I$ at the interest rate $\tilde{R}$,

$$\tilde{R} = \frac{1}{\theta I} \left( I - (1 - \theta)V^0_c \right). \quad (3.1)$$

On the other hand, whether the firm chooses to continue the project depends on whether its own participation constraint is satisfied for a given interest rate $R$, that is,

$$\theta \left( V^1_c - IR \right) \geq \theta V^1_s + (1 - \theta) V^0_s. \quad (PC2)$$
Private information There may be two sources of additional information about the realization of \( \omega \). The first one may come from an in-house information production conducted by the firm itself, which will generate a private signal \( z, z \in \{0, 1\} \). The precision of this signal, denoted by \( \gamma \), is defined as \( \gamma = \Pr (z = \omega) \). We assume that the signal \( z \) is informative but never perfect, and thus \( \gamma \in (\frac{1}{2}, 1) \). The precision \( \gamma \) is exogenously given and publicly known. The firm can update its belief based on \( z \) and take the decision on whether to continue the project.

The same source of information (signal \( z \)) may also be obtained if the firm delegates information acquisition to a certification agency. The agency assesses the prospect of the project and reveals their signal to the public. To simply the analysis and to focus on the difference that is subject only to the delegation choice itself, we assume that the quality of signal \( z \) is unaffected by delegation. In other words, the signal received by the agency has the same precision \( \gamma \) as the one received by the firm under in-house information production. This assumption also makes the point that the certification agency cannot acquire better or more information regarding the investment project than the firm itself. The difference only lies in whether \( z \) is publicized by the outside agency or remains private to the firm itself. We also assume that the cost incurred by the firm to obtain \( z \) is the same whether or not to delegate, which is thus neglected in the model for simplification. In addition, the firm is not obliged to delegate if it prefers otherwise.

The second source of information regarding \( \omega \) may come from informed trading in the stock market. Let us first put down the assumptions. There are three types of agents in the stock market. First of all, there exist a continuum of speculators of measure one, who are
risk neutral. Each of them can acquire a private signal $s$ about $\omega$ after exerting an effort $\lambda$. Similarly as $z$, the precision of $s$ is defined as $\Pr(s = \omega)$. We match the precision of $s$ with the effort level $\lambda$ chosen by each speculator, i.e., $\lambda = \Pr(s = \omega)$. We assume that each speculator can buy or sell only one unit of share due to some exogenous wealth constraints. Given that all the speculators are identical apart from each receiving an independent signal $s$, they share the same cost function of exerting effort and therefore choose the same $\lambda$ in equilibrium. The information cost, $c(\lambda)$, is defined by the following,

$$c(\lambda) = \beta \left( \lambda - \frac{1}{2} \right)^2$$  \hspace{1cm} (3.2)

where $\beta > 0$. We restrict our attention to the scenario where $\beta$ is sufficiently large such that the signal $s$ is informative but never perfect. In other words, each speculator chooses $\lambda$ from the open interval $\left( \frac{1}{2}, 1 \right)$. In the meanwhile, there also exist noise traders in the stock market who trade an aggregate quantity $y$ that follows a uniform distribution, $y \sim U(-1, 1)$. Finally there is a competitive market maker who provides liquidity by trading against speculators and noise traders. The market maker sets the share price at his rational expectation of the firm’s payoff based on the total order flow, and he earns zero profit in expectation.

Based on these assumptions, we can now look at the information production in the stock market. Let $x$ denote the aggregate size of the orders submitted by the speculators. Suppose the true state of the world is $1$, each speculator has a probability $\lambda$ to obtain a signal $s = 1$. By the law of large numbers, there will be a fraction $\lambda$ of speculators who obtain a correct signal and submit a buying order, and also a fraction $(1 - \lambda)$ who obtain a wrong signal and submit a selling order. The aggregate order size is therefore $2\lambda - 1$. Similarly, the speculators submit orders with an aggregate size $x = 1 - 2\lambda$ conditional on that the state $\omega$
Let $X$ denote the total order flow, $X = x + y$. $X$ thus follows the distribution below,

$$
X \sim \begin{cases} 
U(2\lambda - 2, 2\lambda), & \text{if } \omega = 1 \\
U(-2\lambda, 2 - 2\lambda), & \text{if } \omega = 0
\end{cases}
$$

of which the density is $\frac{1}{2}$, $\forall \omega$. Given that $\lambda \in \left(\frac{1}{2}, 1\right)$, we know that $-2\lambda < 2\lambda - 2 < 2 - 2\lambda < 2\lambda$. When being between $2\lambda - 2$ and $2 - 2\lambda$, the total order flow $X$ is not informative, which occurs with the following probability,

$$
\Pr(X \in (2\lambda - 2, 2 - 2\lambda)) = \int_{2\lambda - 2}^{2 - 2\lambda} \frac{1}{2} dX = 2 - 2\lambda.
$$

(3.3)

(3.4)

This probability stays the same for $\omega$ being either 1 or 0, since the aggregate size of noise traders’ orders is symmetrically distributed around zero. When $X$ is higher than $2 - 2\lambda$ (or lower than $2\lambda - 2$), $X$ reveals perfectly that $\omega$ is high (or low), which occurs with the complementary probability $2\lambda - 1$. The speculators can therefore profit from trading with a probability $2 - 2\lambda$. As a consequence, the probability that the order flow is informative is endogenized by speculators’ choice of $\lambda$ in equilibrium. The more effort the speculators exert in information acquisition, the more precise their signals are, and the more likely the share price perfectly reveals the value of $\omega$ to the firm and other agents.

To summarize, the firm can learn about $\omega$ from the signal $z$ which is acquired either privately on its own or by the certification agency under delegation. Besides, the firm may also benefit from the information production in the stock market.
3.3 Investment Decisions Under Two Regimes

Regime 1: To delegate, \( z \) is publicized. When the signal \( z \) is going to be revealed by the agency under delegation, there is no asymmetric information between the firm and the lenders. The posterior belief \( \theta \) is therefore a function of \( z \) and also the order flow \( X \). Remember that when \( X \) is informative, agents learn perfectly about \( \omega \) and thus \( \theta = \omega \). The investment problem is solved naturally. That is, if \( X > 2 - 2\lambda \) and thus \( \theta = 1 \), the firm chooses \( C \) (to continue investing), borrows at a zero interest rate, and realizes a payoff \( (V^1_c - I) \). Or if \( X \) is below \( 2 - 2\lambda \) and reveals \( \omega = 0 \), the firm chooses \( S \) (to stop) and obtains \( V^0_s \).

We therefore only need to look at the situation when \( X \) is not informative, i.e., when \( X \in (2\lambda - 2, 2 - 2\lambda) \). In this case, the interest rate \( R \) required by the lenders is determined by the binding constraint (PC1), and is hence equal to \( \tilde{R} \) as defined in (3.1). We thereafter denote the interest rates that are conditional on the publicized signal \( z \) by \( R_{z=1} \) and \( R_{z=0} \), respectively. Similarly, the posterior belief conditioning on \( z = 1 \) and \( z = 0 \) are denoted respectively by \( \theta^1 \) and \( \theta^0 \). We can obtain \( \theta^1 \) and \( \theta^0 \) by Bayesian updating,

\[
\begin{align*}
\theta^1 &= \frac{\eta \gamma}{\eta \gamma + (1 - \eta) (1 - \gamma)} \quad (3.5) \\
\theta^0 &= \frac{\eta (1 - \gamma)}{\eta (1 - \gamma) + (1 - \eta) \gamma} \quad (3.6)
\end{align*}
\]

It is easy to show that \( \theta^1 > \theta^0 \) and hence \( R_{z=1} < R_{z=0} \).

Substituting \( R_{z=1} \) and \( R_{z=0} \) to the firm’s participation constraint (PC2), we can obtain the conditions for the investment decision, that is, the firm chooses \( C \) when

\[
\begin{align*}
\gamma > \gamma^1 &\equiv \frac{(1 - V^0_c + V^0_s)(1 - \eta)}{(V^1_c - V^1_s - I) \eta} \quad \text{if } z = 1, \quad (3.7) \\
\gamma < \gamma^0 &\equiv \frac{(V^1_c - V^1_s - I) \eta}{(1 - V^0_c + V^0_s)(1 - \eta) + (V^1_c - V^1_s - I) \eta} \quad \text{if } z = 0. \quad (3.8)
\end{align*}
\]
(3.7) and (3.8) give the expressions for the thresholds of $\gamma$ conditioning on $z$, namely, $\gamma^1$ and $\gamma^0$. To simplify the notation, let $\mu_C$ and $\mu_S$ denote the firm’s expected payoff from choosing $C$ and $S$ based on the prior belief $\eta$.

\begin{align*}
\mu_C &= \eta V^1_c + (1 - \eta)V^0_c \\
\mu_S &= \eta V^1_s + (1 - \eta)V^0_s
\end{align*}

(3..9)

(3..10)

Notice that $\gamma^1$ and $\gamma^0$ are the same when the required investment $I$ equals $\mu_C - \mu_S$. Using these notations, we can write down the investment decisions under the regime of delegation, when the order flow does not reveal $\omega$.

Lemma 8 When the order flow is not informative, the firm takes the investment decision under regime 1 as follows.

(i). Given $I \leq \mu_C - \mu_S$ (i.e., $\gamma^1 < \frac{1}{2} < \gamma^0$), the firm chooses $C \forall z$, if $\gamma \in \left(\frac{1}{2}, \gamma^0\right)$; and the firm chooses $C$ only when $z = 1$, if $\gamma \in (\gamma^0, 1)$.

(ii). Given $I > \mu_C - \mu_S$ (i.e., $\gamma^0 < \frac{1}{2} < \gamma^1$), the firm chooses $S \forall z$, if $\gamma \in \left(\frac{1}{2}, \gamma^1\right)$, and the firm chooses $C$ only when $z = 1$, if $\gamma \in (\gamma^1, 1)$.

The lenders ask for an interest $R_{z=1}$ or $R_{z=0}$, depending on the publicized signal $z$.

Regime 2: In-house information production Under the second regime, the firm chooses not to delegate and therefore acquires the signal $z$ by itself. Since the firm cannot creditably reveal its signal, $z$ remains private in this case. This leads to information asymmetry between the lenders and the firm. To facilitate the illustration, we thereafter call the firm as a type 1 borrower if $z = 1$ and a type 0 borrower otherwise. After receiving the signal $z$, the firm becomes aware of its type, and it may learn more about $\omega$ from the order flow at the next
date when the stock market opens and trading orders are submitted. Remember that the uncertainty on $\omega$ is resolved completely if the order flow reveals the state of the world. If the order flow is not informative and the firm decides to invest, it then goes to the lenders for raising the required capital. We assume that there are many lenders in a competitive credit market. The firm is going to borrow from whoever offers the lowest interest rate.

Under Regime 2, the lenders cannot distinguish a type 1 borrower from a type 0, and consequently they chooses an interest rate to first satisfy the following participation constraint,

$$\Pr(z = 1) \left( \theta^1 IR + (1 - \theta^1) V^0_c \right) + \Pr(z = 0) \left( \theta^0 IR + (1 - \theta^0) V^0_c \right) \geq I. \quad (PC3)$$

Let $\bar{R}$ denote the interest rate for $PC3$ to be binding,

$$\bar{R} = \frac{I - (1 - \eta) V^0_c}{\eta I} . \quad (3.11)$$

When a pooling equilibrium at the rate $\bar{R}$ is not feasible, the interest rate is then chosen to screen the type 0 borrower such that the following incentive compatibility constraint is satisfied,

$$\theta^0 (V^1_c - IR) \leq \theta^0 V^1_s + (1 - \theta^0) V^0_s. \quad (IC)$$

By binding the $IC$ constraint, we obtain the interest rate, $R_{IC}$, that can prevent the type 0 borrower from mimicking the type 1 and borrowing,

$$R_{IC} = \frac{1}{I} \left( V^1_c - V^1_s - \frac{1 - \theta^0}{\theta^0} V^0_s \right) . \quad (3.12)$$

**Lemma 9** A type 1 borrower is always willing to borrow at the interest rate $R_{IC}$.
The following lemma states the interest rate demanded by the lenders when the order flow is not informative under the second regime.

**Lemma 10** If the order flow is not informative under the regime of in-house information production, and if \( R_{z=1} < \bar{R} < R_{z=0} \leq R_{IC} \), the lenders demand an interest rate at \( \bar{R} \) to attract both type 1 and type 0 borrower;

- If \( R_{z=1} < \bar{R} < R_{IC} \leq R_{z=0} \), the lenders demand \( \bar{R} \);
- If \( R_{z=1} < R_{IC} \leq \bar{R} < R_{z=0} \), the lenders demand \( R_{IC} \) to screen the type 0 borrower;
- And if \( R_{IC} \leq R_{z=1} < \bar{R} < R_{z=0} \), the lenders demand \( R_{z=1} \) to attract only the type 1 borrower.

Once we know how the interest rate is decided in the credit market, we can now check the investment decision under Regime 2 when the order flow is not informative. Based on Lemma 10, Lemma 11 follows immediately.

**Lemma 11** When the order flow is not informative, the firm takes the following investment decisions under Regime 2:

(i). Given \( I \leq \mu_C - \mu_S \) (i.e., \( \gamma^1 < \frac{1}{2} < \gamma^0 \)), if \( \gamma \in \left(\frac{1}{2}, \gamma'\right) \) the firm always chooses \( C \), and lenders ask for the interest rate \( \bar{R} \); if \( \gamma \in (\gamma', 1) \), the firm chooses \( C \) only when \( z = 1 \) and it borrows either at \( R_{IC} \forall \gamma \in (\gamma', \gamma'') \) or at \( R_{z=1} \forall \gamma \in (\gamma'', 1) \);

(ii). Given \( I > \mu_C - \mu_S \) (i.e., \( \gamma^0 < \frac{1}{2} < \gamma^1 \)), if \( \gamma \in \left(\frac{1}{2}, \gamma^1\right) \) the firm always chooses \( S \), and lenders set the interest rate to \( R_{z=0} \) to exclude the borrower of both types; if \( \gamma \in (\gamma^1, 1) \), the firm chooses \( C \) only when \( z = 1 \) and it borrows either at \( R_{IC} \forall \gamma \in (\gamma^1, \gamma'') \) or at \( R_{z=1} \forall \gamma \in (\gamma'', 1) \).
where $\gamma^1$ and $\gamma^0$ are defined in (3.7) and (3.8), $\gamma'$ and $\gamma''$ are the threshold values for $\gamma$ at which $R_{IC} = \bar{R}$ and $R_{IC} = R_{z=1}$, respectively, $\gamma'' > \gamma' > 0$.

Comparing Lemma 11 to Lemma 8, we can see the differences between two regimes in both the firm’s action and the interest rate set by the lenders, which shows the first effect due to delegation. Note that when $\gamma$ is either sufficiently high, i.e., $\gamma \in (\gamma'', 1)$, the firm has exactly the same strategy and financing cost under two regimes. Similarly the firm always chooses $S$ when the required investment is very high ($I > \mu_C - \mu_S$) and $\gamma \in \left(\frac{1}{2}, \gamma^1\right)$.

Remember that choosing delegation or not does not affect the acquisition of the private signal $z$, but changes whether $z$ is publicly known. When the firm commits itself to a credible disclosure via delegation, it eliminates the information asymmetry between the firm and the lenders. The financing cost falls and the participation constraint of the firm is relaxed. This is particularly true in the case in which the firm has the same strategy under two regimes but bears a higher financing cost ($R_{IC}$) under in-house information production than under delegation (the interest rate being $R_{z=1}$)\(^8\).

The reduced information asymmetry under delegation then leads to changes in the optimal effort level (the precision of the signal $s$) chosen by speculators. The intuition is that speculators’ incentive to acquire information is not only affected by the variance of the payoff realizations related to the equilibrium investment decision, but also driven by their information advantage relative to the other outside agents. The more uncertainty the outsiders have about the state of the world $\omega$, the higher effort is chosen by speculators.

\(^8\)This occurs, as shown by Lemma 11, when $\gamma \in (\gamma', \gamma'')$ if $I \leq \mu_C - \mu_S$ and when $\gamma \in (\gamma^1, \gamma'')$ if $I > \mu_C - \mu_S$. 87
Delegation may therefore induce a crowding-out effect of information production in the financial market, which reduces the probability of the stock price being informative. Being less likely perfectly informed about the prospect of the investment, the firm may generate a lower payoff.

To see more clearly the trade-off between these two effects, we thereafter focus our analysis of the firm’s delegation choice in the parameter region where a separating equilibrium arises with $R_{IC}$ at the financing stage, as defined by Lemma 11. Compared to the same parameter region under in-house information acquisition, there is no distortion of investment decision but a higher financing cost for the firm when $z = 1$ and the order flow is not informative. A quick discussion for the case with both distortion of investment decision and borrowing cost (i.e., $I \leq \mu_C - \mu_S$ and $\gamma \in (\gamma^0, \gamma')$) is presented in the extension.

3.4 Delegation choice

After computing the firm’s investment strategies under two regimes, we are interested in knowing when it is preferable for the firm to choose delegation. Under regime 1, $z$ is public information and therefore the effort choice of speculators is conditional on $z$. Remember that the cost function of speculators, defined in (3.2), is convex in the effort choice $\lambda$, and speculators may profit from trading with a probability $(2 - 2\lambda)$. The expected trading profit, netted out the effort cost, is thus a concave function of $\lambda$. By deriving the first order condition, we can obtain the optimal effort choice for $z = 1$ and 0, which are denoted respectively by $\lambda_{c,z=1}^*$ and $\lambda_{s,z=0}^*$. All the speculators, being identical ex ante, choose the same effort level in equilibrium. The subscripts of $\lambda^*$ indicate the action taken by the firm.
when the order flow is not informative, for a given signal $z$.

**Lemma 12** Under the regime of delegation, $\forall \gamma \in \{\gamma^1, \gamma', \gamma''\}$, speculators choose the following effort level based on the publicized signal $z$,

$$
\lambda^*_{c,z=1} = \frac{1}{2} + \frac{2\theta^1 (1 - \theta^1) (V^1_c - IR_{z=1})}{\beta + 4\theta^1 (1 - \theta^1) (V^1_c - IR_{z=1})} 
$$

$$
\lambda^*_{s,z=0} = \frac{1}{2} + \frac{2\theta^0 (1 - \theta^0) (V^1_s - V^0_s)}{\beta + 4\theta^0 (1 - \theta^0) (V^1_s - V^0_s)},
$$

(3.13) (3.14)

Using $\lambda^*_{c,z=1}$ and $\lambda^*_{s,z=0}$, we can then obtain the firm’s payoff. Let $\pi_d$ denote the expected payoff the firm by choosing delegation. $\pi_d$ is composed of the conditional payoffs $\pi_{d,z=1}$ and $\pi_{d,z=0}$, respectively, for $z = 1$ and $z = 0$. Remember that the order flow is not informative with probability $2 - 2\lambda^*_{c,z=1}$, and it is perfectly informative with probability $2\lambda^*_{c,z=1} - 1$. The same logic applies when $z = 0$. We thus have $\pi_{d,z=1}$ and $\pi_{d,z=0}$ as follows,

$$
\pi_{d,z=1} = (2 - 2\lambda^*_{c,z=1}) \theta^1 (V^1_c - IR_{z=1}) + (2\lambda^*_{c,z=1} - 1) \left[ \theta^1 (V^1_c - I) + (1 - \theta^1) V^0_s \right]
$$

$$
\pi_{d,z=0} = (2 - 2\lambda^*_{s,z=0}) \left[ \theta^0 V^1_s + (1 - \theta^0) V^0_s \right] + (2\lambda^*_{s,z=0} - 1) \left[ \theta^0 (V^1_c - I) + (1 - \theta^0) V^0_s \right]
$$

(3.15) (3.16)

The ex ante expected payoff under delegation, $\pi_d$, is thus,

$$
\pi_d = Pr (z = 1) \pi_{d,z=1} + Pr (z = 0) \pi_{d,z=0}
$$

(3.17)

Under the regime of in-house information production, a speculator’s expected trading profit and thus his effort level in equilibrium is no longer conditional on $z$. For the case in which the firm chooses $C$ only when $z = 1$ and pays $R_{IC}$ when the order flow is uninformative, the equilibrium effort choice $\lambda^*_{R_{IC}}$ is defined in Lemma 13.
Lemma 13  Under the regime of in-house information production, \( \forall \gamma \in (\max \{ \gamma^1, \gamma' \}, \gamma'' ) \), speculators exert effort \( \lambda^*_{RIc} \) in equilibrium,

\[
\lambda^*_{RIc} = \frac{1}{2} + \frac{2\eta(1-\eta)[\gamma(V^1_c - IR_{IC} - V^0_s) + (1 - \gamma)V^1_s]}{\beta + 4\eta(1-\eta)[\gamma(V^1_c - IR_{IC} - V^0_s) + (1 - \gamma)V^1_s]} \tag{3.18}
\]

Using Lemma 13, we can write down the expected payoff of the firm, denoted by \( \pi \),

\[
\pi = (2 - 2\lambda^*_{RIc}) \left[ \gamma \eta \left( V^1_c - IR_{IC} \right) + (1 - \gamma) \eta V^1_s + \gamma (1 - \eta) V^0_s \right] \\
+ \left( 2\lambda^*_{RIc} - 1 \right) \left[ \eta \left( V^1_c - I \right) + (1 - \eta) V^0_s \right]. \tag{3.19}
\]

By comparing (3.17) and (3.19), we observe that delegation changes not only the payoff of investment conditioning on \( \omega \), but also the amount of information production in the stock market (\( \lambda^* \)) that is determined by the variance of payoff realizations for a given investment strategy, as shown by (3.13), (3.14) and (3.18). Therefore, \( \lambda^* \) is affected by speculators’ belief after observing \( z \) (i.e., \( \theta^1 \) or \( \theta^0 \)) under delegation, or \( \eta \) when the firm chooses not to delegate. The posterior belief may further affect \( \lambda^* \) through changing the payoff of the investment conditioning on \( \omega = 1 \) (i.e., \( V^1_c - IR^* \) with \( R^* \) being the lending rate in equilibrium). To better understand when and why a firm wants to delegate, we are going to examine how the firm’s choice is affected by the changes in parameter values. We focus on the impact of the prior \( \eta \), the precision \( \gamma \) of the signal \( z \), the payoff realization \( V^1_s \). \( \eta \) affects the prospect of the investment, \( \gamma \) may be interpreted as the difficulty in assessing a project value, and \( V^1_s \) may be interpreted as the prospect of the current assets in place. To simplify the derivation and to disentangle the effect of each force, we impose separately some simplifications on \( V^1_s, V^0_s \) and \( \eta \).
The impact of the prior belief \( \eta \). The size of \( \eta \) determines the ex-ante likelihood of the project being at a good state of the world (i.e., a better project). In order to focus on the impact of \( \eta \), we now assume \( V_s^1 = V_s^0 = V_s \) while all other assumptions remain unchanged. Consequently, speculators’ optimization problem becomes trivial if they anticipate the firm to choose \( S \) when the order flow is not informative, as speculators do not exert effort given there is no uncertainty in the final payoff related to \( S \), i.e., \( \lambda^*_{s,z=0} = \frac{1}{2} \).

Under the regime of delegation, an increase in \( \eta \) first increases the posterior \( \theta^1 \) and thus pushes down the interest rate \( R_{z=1} \). This leads to a higher conditional payoff \( (V_c^1 - IR_{z=1}) \) when \( z = 1 \) and the order flow uninformative, which consequently increases the incentive of information acquisition. If however \( \theta^1 \) becomes closer to 1, the reduced uncertainty decreases the payoff variance and thus has an opposite effect on \( \lambda^*_{c,z=1} \). The negative effect dominates when \( \eta \) becomes sufficiently large.

Similarly, an increase in \( \eta \) has two effects on \( \lambda^*_{RI_{IC}} \). It first leads to a higher posterior of the type 0 borrower who then has more incentive to mimic the type 1. As a result, the IC constraint is tightened, and lenders demand a higher lending rate \( R_{IC} \). A lower payoff \( (V_c^1 - IR_{IC}) \) reduces the incentives of speculators to acquire information, and thus leads to a smaller \( \lambda \) in equilibrium. On the other hand, \( \eta \) also affects the ex ante uncertainty about \( \omega \). The closer \( \eta \) is getting towards \( \frac{1}{2} \), a higher incentive speculators have for information acquisition. We can show that the negative effect always dominates. Lemma 14 thus follows

**Lemma 14** Under the regime of in-house information production, \( \lambda^*_{RI_{IC}} \) decreases in the prior belief \( \eta \), \( \forall \eta \in (0, 1) \);

Under the regime of delegation, speculators’ effort level \( \lambda^*_{c,z=1} \) increases in the prior be-
lief $\eta$, $\forall \eta \in \left(0, \frac{(I-2V_0^0+V_1^1)(1-\gamma)}{V_e^0-V_e^0-(2\gamma-1)(I-V_0^0)}\right)$, and $\lambda^*_{x=1}$ decreases in $\eta$, $\forall \eta \in \left(\frac{(I-2V_0^0+V_1^1)(1-\gamma)}{V_e^0-V_e^0-(2\gamma-1)(I-V_0^0)}, 1\right)$.

In other words, except when $\eta$ is sufficiently high, an increase in $\eta$ reduces the amount of information production under regime 2, compared to regime 1. The crowding-out effect becomes less significant in this case.

On the other hand, $\eta$ affects also the firm’s conditional payoff when the investment takes place. An increase of $\eta$ decreases $R_{x=1}$ as it increases the posterior $\theta^1$ under delegation. $R_{IC}$ however increases in $\eta$ since the incentive compatibility constraint is tightened for the type 0 borrower. This means that the cost advantage at the financing stage becomes more significant under delegation when $\eta$ increases. Using the results from Lemma 14, we can obtain Proposition 6 as below.

**Proposition 6** An increasing prior belief $\eta$ makes it more likely that the advantage of having a low financing cost dominates the crowding-out effect of information production, except when $\eta$ gets close to 1, and thus it is more likely that the firm wants to delegate the information production.

The impact of $\gamma$ Next, let us look at the impact of $\gamma$, that is, the quality of the signal obtained by either the firm or the certification agency. This is of interest because the precision of signal $\gamma$ affects both information production and the conditional payoffs of the firm through changing the belief updating of the speculators as well as the lenders. In addition, the level of $\gamma$ is relevant in practice, since it can be related to the complexity of the investment which affects the difficulty and the result of information acquisition.  

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It may be also related to the level of internal corporate governance. When the internal governance is more effective, managers work more efficiently in evaluating a future project and it is also easier for an outside agency to obtain information and to fully understand a firm’s profitability concerning an investment.

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In order to restrict attention and simplify the analysis, we keep the assumption $V_s^1 = V_s^0 = V_s$ and thus $\lambda_{c,z=0}^* = \frac{1}{2}$. In addition, we assume $\eta = \frac{1}{2}$, that is, a priori the state of the world $\omega$ takes the value 1 or 0 with equal probabilities. This assumption simplifies $\theta^1$ to $\gamma$ and $\theta^0$ to $(1 - \gamma)$. The expressions of the optimal $\lambda$ can then be simplified,

$$\lambda_{c,z=1}^* = \frac{1}{2} + \frac{2\gamma (1 - \gamma)}{\beta + 4\gamma (1 - \gamma)} \frac{V^1_c - IR_{z=1}}{V^1_c - IR_{z=1}}$$

$$\lambda_{R_{IC}}^* = \frac{1}{2} + \frac{\gamma (V^1_c - IR_{IC} - V_s) + (1 - \gamma) V_s}{2[\beta + \gamma (V^1_c - IR_{IC} - V_s) + (1 - \gamma) V_s]}$$

where $R_{z=1} = \frac{1}{\beta} (I - (1 - \gamma) V_s^0)$, $R_{IC} = \frac{\gamma}{2} \left( V^1_c - \frac{1}{1-\gamma} V_s \right)$. The comparative statics with respect to $\gamma$ is stated in Lemma 15.

**Lemma 15** Under the regime of delegation, speculators’ effort level $\lambda_{c,z=1}^*$ increases in the precision level $\gamma$ of the signal $z$, $\forall \gamma \in \left( \frac{1}{2}, \frac{V^1_c + I - 2V_s^0}{2(V^1_c - V_s^0)} \right)$, and $\lambda_{c,z=1}^*$ decreases in $\gamma$, $\forall \gamma \in \left( \frac{V^1_c + I - 2V_s^0}{2(V^1_c - V_s^0)}, 1 \right)$;

Under the regime of in-house information production, $\lambda_{R_{IC}}^*$ increases $\gamma$, $\forall \gamma \in \left( \frac{1}{2}, 1 \right)$.

The intuition is as follows. When the signal $z$ becomes more precise, the posterior $\theta^1$ increases for $z = 1$, and thus the interest rate $R_{z=1}$ is lowered. Consequently, the conditional payoff $V^1_c - IR_{z=1}$ increases in $\gamma$, which increases speculators’ optimal effort level $\lambda_{c,z=1}^*$. Meanwhile, a higher $\gamma$ reduces the uncertainty about $\omega$ when $z$ is revealed under delegation, which then reduces the information advantage of speculators and thus their effort level $\lambda_{c,z=1}^*$. If $\gamma$ is sufficiently large, the negative effect dominates and $\lambda_{c,z=1}^*$ decreases in $\gamma$. Similarly, an increase of $\gamma$ has two effects on $\lambda_{R_{IC}}^*$. On the one hand, it loosens the incentive compatibility constraint of the type 0 borrower and thus lowers the interested rate $R_{IC}$. This has a positive impact on $\lambda_{R_{IC}}^*$, due to a higher variance of investment payoff. On the other hand, an increase
of \( \gamma \) pushes up the expected firm value and thus the market price when the order flow is not informative, which affects \( \lambda^*_R \) negatively. This effect is however dominated by the positive one (when \( \eta = \frac{1}{2} \)), and therefore \( \lambda^*_R \) increases in \( \gamma \). As a result, with \( \gamma \) being sufficiently high (i.e., \( \gamma < \left( \frac{V_1^1 + I - 2V_0^0}{2(V_2^2 - V_0^0)}, 1 \right) \)), the more precise the signal \( z \) is expected to be, the more significant the crowding-out effect would be on information production under delegation.

The disadvantage from having a higher financing cost becomes less significant due to the inverse relationship between \( \gamma \) and \( R IC \). We can therefore conclude in Proposition 7.

**Proposition 7** When the quality of the signal \( z \) is sufficiently high, the crowding-out of information production becomes more severe, compared to the loss due to the higher financing cost. As a result, the firm more likely chooses the in-house information acquisition.

The impact of \( V_s^1 \) At last, we want to understand how the prospect of the firm’s current assets in place affects the delegation choice. Let us denote \( V_s^0 \) as \( V_s^1 - \delta \) and focus on the expected payoff of the strategy \( S \) by varying the value of \( V_s^1 \). Note that we keep the assumption that ex ante the state of the world \( \omega \) can be high or low with equal probabilities (i.e., \( \eta = \frac{1}{2} \)).

Under the regime of delegation, an increase in \( V_s^1 \) does not change the variance of the payoff realizations either when \( z \) is revealed to be 0 and the firm chooses \( S \) or when \( z = 1 \) and the firm chooses \( C \). As a result, \( \lambda^*_{s, z=0} \) and \( \lambda^*_{c, z=1} \) are not affected. A higher \( V_s^1 \) however relaxes the incentive compatibility constraint of the type 0 borrower, and thus decreases the interest rate \( R IC \) demanded in equilibrium. This would then increases the variance of payoffs associated with the action \( C \) under the regime of in-house information production. Speculators’ incentive of information acquisition is thus higher and so is \( \lambda^*_R \).

The crowding-out effect on information production becomes more severe, and in the mean-
while the disadvantage at the financing stage is less significant. We can thus conclude in Proposition 8.

**Proposition 8** When the assets in place have a higher expected payoff, it is preferable for the firm to choose the regime of in-house information production.

3.5 Extension

In the previous section, we examine in isolation the case in which the firm’s investment decision remains unchanged (its action $A$ depends on the signal $z$) whether or not $z$ is public while the financing cost differs between two regimes. It is worth mentioning that the changes of parameter values may shift the equilibrium from one to another. The equilibrium with an adjacent parameter region is that, under delegation, the firm invests only when $z = 1$ at the interest rate $R_{z=1}$, versus the firm invests regardless of $z$ and borrows at $\bar{R}$ under in-house information production. We know from Lemma 11 that, under the regime of in-house production, the equilibrium at the financing stage is shifted from a separating one to a pooling equilibrium when $I \leq \mu_C - \mu_S$ and $\gamma$ falls below $\gamma'$ such that $R_{IC} > \bar{R}$.

To understand whether the results in the previous section may be altered when changes in the parameter values changes the equilibrium at the financing stage, we first compute the effort choice of speculators, denoted by $\lambda_R^*$, in the parameter region with $R_{IC} > \bar{R}$.

**Lemma 16** When $z$ remains private in the regime of in-house information production, if $I \leq \mu_C - \mu_S$ and $\gamma^0 < \gamma < \gamma'$, speculators choose the following effort level,

$$
\lambda_R^* = \frac{1}{2} + \frac{2\eta (1 - \eta) \left(V_c^1 - IR\right)}{\beta + 4\eta (1 - \eta) \left(V_c^1 - I\bar{R}\right)}.
$$
Speculators choose, under the regime of delegation, $\lambda^*_{c,z=1}$ and $\lambda^*_{s,z=0}$ that are defined by (3.13) and (3.14) in Lemma 12.

First of all, $\lambda^*_{R}$ is not affected by any change in $\gamma$ since the signal $z$ is hidden and speculators’ effort choice is independent from $\gamma$. Neither is the firm’s expected payoff as the firm always chooses to invest as long as the order flow is not informative. Based on Lemma 15, we know that the $\lambda^*_{c,z=1}$ decreases in $\gamma$ when $\gamma > \frac{\eta}{2(V^1_c - V^0_c)}$ (for $\eta = \frac{1}{2}$), and therefore in this region the information production by speculators is reduced when the firm chooses delegation compared to otherwise. As a consequence, when $\gamma$ is below the threshold $\frac{\eta}{2(V^1_c - V^0_c)}$, an increase in $\gamma$ makes it preferable for the firm to choose delegation. Combining with Proposition 7, we observe that, for $\gamma$ at a relatively low level, the firm more likely prefers to delegate and to borrows at $R_{z=1}$, when $\gamma$ takes a higher value. When $\gamma$ continues to increase and reaches the threshold $\frac{\eta}{2(V^1_c - V^0_c)}$, the firm may switch its preference to the regime of in-house information production, under which the equilibrium at the financing stage changes from pooling to separating, that is, the firm invests when $z = 1$ and borrows at $R_{IC}$ given the order flow is uninformative.

Secondly, an increase of $V^1_s$ may shift the equilibrium at the financing stage, under in-house information production, from pooling at $\bar{R}$ to separating with $R_{IC}$, as an increase of $V^1_s$ reduces $R_{IC}$ but does not affect $\bar{R}$. Notice that both $\lambda^*_{R}$ and $\lambda^*_{z=1}$ are not affected by $V^1_s$, an increase of $V^1_s$ has a bigger positive impact on the conditional payoff of the firm under delegation. This is because that under delegation, the firm chooses $S$ not only when the order flow reveals the state of the world to be low but also when the certification agency reveals $z = 0$. The firm’s payoff is thus more often associated with strategy $S$. Combining
Proposition 8, we see that when $V^1_s$ is relatively low, an increase in $V^1_s$ makes it more likely that the firm should choose delegation. When $V^1_s$ is sufficiently high such that the firm would borrow at $R_{IC}$ (i.e., $R_{IC} < \bar{R}$) if the signal $z$ is hidden, the firm may switch from the regime of delegation to the regime of in-house production.

Finally, it is easy to show that $R_{IC} - \bar{R}$ moves in the same direction as the prior belief $\eta$, and hence an increasing $\eta$ may shift the equilibrium from separating to pooling in the regime of in-house production. For the pooling equilibrium with the interest rate $\bar{R}$, $\eta$ affects $\lambda_{\bar{R}}^*$ positively via reducing the interest rate $\bar{R}$ and thus pushing up the payoff variance of the investment. On the other hand, depending on whether $\eta$ is above $\frac{1}{2}$, a change in $\eta$ affects the level of uncertainty about $\omega$, which consequently influences speculators’ incentive of information acquisition. When two effects combined, $\lambda_{\bar{R}}^*$ increases in $\eta$ for $\eta < \frac{V^1_c + I^2 - 2V^0_c}{2(V^1_c - V^0_c)}$, and vice versa. For $\eta$ relatively small, the firm would borrow at $R_{IC}$ at the separating equilibrium under the regime of in-house production. Combining Lemma 14, we know an increasing $\eta$ alleviates the crowding out of information production, and hence the firm may prefer to delegate, which is consistent with Proposition 6. When $\eta$ reaches a certain threshold such that $R_{IC} > \bar{R}$ and the firm would be in the pooling equilibrium at $\bar{R}$ if $z$ is kept private, the crowding-out effect increases with a higher prior $\eta$. In this case, the firm’s preference is ambiguous, however.

3.6 Conclusion

We study in this paper the impact of certifications on information production in financial market and how it feeds back to investment decisions. We build a model in which
a firm needs to take decisions on a potential investment that requires new capital from the credit market. The firm can choose to acquire additional information in order to ascertain the investment prospect and keep that private. If however the firm decides to commit itself to credible disclosures in order to alleviate the adverse selection problem at the financing stage, it can delegate the information production to a certification agency that disseminates their evaluations to the public. While reducing the financing cost, delegation may nevertheless crowd out private information that could be acquired by speculators and reduce information efficiency in the stock market, which may worsen the investment outcome. The firm thus needs to trade it off with a lower cost of capital under delegation. We show that it is preferable for the firm to choose delegation when the prior belief about the investment prospect is relatively high. The firm may however choose not to delegate, when its own signal is expected to be more precise and/or its current assets can generate a higher expected payoff.

The model contributes to a better understanding of the informational effect of certifications in the financial market. This is of interest to practitioners and regulators as our results provide new insights on both corporate strategies and market policies on information disclosure. To provide clearer empirical and regulatory implications, we yet need to work on the full characterization of the equilibrium. This is left for our future research.
REFERENCES CITED


3.7 Appendix

**Proof of Lemma 8.** The necessary condition for both (3.9) and (3.10) being satisfied is \( \gamma^1 < \gamma^0 \) (i.e., \( I < \mu_C - \mu_S \)). If, however, \( \gamma^1 > \gamma^0 \) and \( \gamma \) falls in the interval \((\gamma^0, \gamma^1)\), it can never be the case that firm chooses \( C \) regardless of \( z \), given that the order flow is uninformative. Finally, if \( \gamma > \max \{\gamma^0, \gamma^1\} \), the inequality in (3.7) is satisfied but not the inequality in (3.8), and thus the firms chooses \( C \) only when a good signal is revealed, \( z = 1 \). □

**Proof of Lemma 9.** By intuition, provided that the type 0 borrower is indifferent from borrowing or not at the rate \( R_{IC} \), the type 1 borrower with a higher posterior about
\omega should be attracted by the same interest rate. It can be proved easily that the following inequality holds

\[ \theta^1 (V^1_c - IR_{IC}) > \theta^1 V^1_s + (1 - \theta^1) V^0_s. \]  

(3.22)

Proof of Lemma 10. When the interest rate \( R_{IC} \) is higher than \( \bar{R} \), intuitively PC2 is satisfied at \( \bar{R} \) with \( \theta \) being replaced by \( \theta^1 \), and thus the type 1 borrower can be attracted by \( R_{IC} \) if lenders ask for \( R_{IC} \). The competition in the credit market will however drive down the lending interest to \( \bar{R} \) that leaves PC3 binding.

If \( R_{IC} < \bar{R} \), \( \bar{R} \) no longer satisfies the IC for the type 0 borrower. A pooling equilibrium is no longer possible and the competition in the credit market will push down the interest rate. And if \( R_{IC} > R_{z=1} \), lenders ask for \( R_{IC} \) which satisfies their participation constraint and also screens out the type 0.

If however \( R_{IC} \leq R_{z=1} < \bar{R} \), lenders will set the interest at \( R_{z=1} \), since \( R_{z=1} \) suffices to prevent the type 0 from mimicking and also satisfies PC1. 

Proof of Lemma 11. First, we can show that \( I > \mu_C - \mu_S \) is the sufficient condition for \( R_{IC} < \bar{R} \), but not the necessary condition. Therefore, when \( I > \mu_C - \mu_S \), there cannot exist the pooling equilibrium with the interest rate \( \bar{R} \) at the financing stage. Based on Lemma 8 and Lemma 10, we know that for \( I \leq \mu_C - \mu_S \) the participation constraint of both type 0 and 1 borrower can be satisfied at the interest rate \( R_{z=0} \) and \( R_{z=1} \). Now if \( R_{IC} > R_{z=0} > \bar{R} \), the type 1 borrower’s PC can be satisfied when borrowing at \( \bar{R} \). By solving \( R_{IC} - \bar{R} = 0 \), we can obtain the threshold condition in \( \gamma \). When \( \gamma^0 < \gamma < \gamma' \), with

\[ \gamma' = \frac{I - V^0_c + (V^0_c - V^1_s + V^1_s)}{I - V^0_c - V^0_c + (V^0_c - V^1_s + V^1_s + V^2_s)}, \]

there is a pooling equilibrium at the interest rate \( \bar{R} \).
The second threshold condition in $\gamma$ is obtained by solving $R_{IC} - R_{z=1} = 0$. If $R_{IC} < R_{z=1} < \tilde{R}$, the type 0 will prefer not to borrow and thus it is sufficient for the lenders to ask for $R_{z=1}$ to attract only the type 1. When $\gamma'' < \gamma < 1$, with $\gamma'' = \frac{2(I-V_{c}^{0})-(3I-2V_{b}^{0}-V_{c}^{1}+V_{s})\eta}{2(I-V_{b}^{0}+V_{b}^{0}-(2I-V_{b}^{0}-V_{c}^{1}+V_{b}^{0}+V_{s}))\eta} + \frac{4V_{b}^{0}(2\eta-1)(I-V_{b}^{0})+\eta^2(I^2+4V_{b}^{0}V_{c}^{0}+(V_{c}^{1}+V_{s}^{0})^2-2I(V_{c}^{1}+2V_{b}^{0}-V_{s}^{0}))}{(I+V_{b}^{0}+V_{b}^{0}(1-\eta)-2\eta I+\eta V_{c}^{1}+\eta(V_{c}^{1}+V_{s}^{0}))}$, the firm borrows at $R_{z=1}$ upon receiving the signal $z = 1$. ■

**Proof of Lemma 12.** Let us denote the expected trading profit, netted out the effort cost, of a speculator by $\Psi$, and the market maker price by $P$. Conditional on receiving $z = 1$, the expected payoff of a speculator anticipating the firm to continue, is then denoted by $\Psi_{c,z=1}$,

$$
\Psi_{c,z=1} = \left(2 - 2\bar{\lambda}_{c,z=1}\right) \left[\left(\theta^1 \lambda_{c,z=1} - \theta^1 \left(1 - \lambda_{c,z=1}\right)\right) (V_{c}^{1} - IR_{z=1} - P) \right. \\
+ \left. \left(1 - \theta^1 \right) \lambda_{c,z=1} - \left(1 - \theta^1 \right) \left(1 - \lambda_{c,z=1}\right) \right] P \\
- \beta \left(\lambda_{c,z=1} - \frac{1}{2}\right)^2
$$

where $\lambda_{c,z=1}$ is the precision choice of this speculator, while $\bar{\lambda}_{z=1}$ depends on the precision level chosen by all the other speculators. Note that $\left(2 - 2\bar{\lambda}_{z=1}\right)$ is the probability that all the speculators can hide the identity of their trading orders. The market maker price $P$ equals $\theta^1 (V_{c}^{1} - IR_{z=1})$ for $z = 1$ and $X$ uninformative. The above can then be simplified to

$$
\Psi_{c,z=1} = 2\theta^1 (1 - \theta^1) \left(2 - 2\bar{\lambda}_{c,z=1}\right) (2\lambda_{c,z=1} - 1) (V_{c}^{1} - IR_{z=1}) - \beta \left(\lambda_{c,z=1} - \frac{1}{2}\right)^2
$$

Using the first order condition of $\Psi_{c,z=1}$, we can obtain the optimal $\lambda_{c,z=1}^*$,

$$
\lambda_{c,z=1}^* = \frac{1}{2} + \frac{1}{\beta} 2\theta^1 (1 - \theta^1) \left(2 - 2\bar{\lambda}_{c,z=1}\right) (V_{c}^{1} - IR_{z=1}).
$$

Note that in the equilibrium all the speculators, being identical ex ante, choose the same
level of effort, which means \( \tilde{\lambda}_{z=1} = \lambda^*_z = 1 \). Solving the equation gives the expression of \( \lambda^*_{c,z=1} \)

\[
\lambda^*_{c,z=1} = \frac{1}{2} + \frac{2\theta^1 (1 - \theta^1) (V^1_c - IR_{z=1})}{\beta + 4\theta^1 (1 - \theta^1) (V^1_c - IR_{z=1})} \tag{3.26}
\]

Similarly, we can obtain the expected trading profit \( \Psi_{s,z=0} \) of a speculator for \( z = 0 \)

\[
\Psi_{s,z=0} = 2\theta^0 (1 - \theta^0) \left( 2 - 2\tilde{\lambda}_{s,z=0} \right) (2\lambda_{s,z=0} - 1) (V^1_s - V^0_s) - \beta \left( \lambda_{s,z=0} - \frac{1}{2} \right)^2 \tag{3.27}
\]

\[
\lambda^*_{s,z=0} = \frac{1}{2} + \frac{2\theta^0 (1 - \theta^0) (V^1_s - V^0_s)}{\beta + 4\theta^0 (1 - \theta^0) (V^1_s - V^0_s)} \tag{3.28}
\]

**Proof of Lemma 13.** Speculators have rational expectations about both the continuation decision of the firm and also the interest rate asked by the lenders. Let us use \( \Psi_{RIC} \) to denote the expected payoff of a speculator,

\[
\Psi_{RIC} = \left( 2 - 2\tilde{\lambda}_{RIC} \right) (2\lambda_{RIC} - 1) \left[ \eta \gamma (V^1_c - IR_{RIC} - P) + \eta (1 - \gamma) (V^1_s - P) \
- (1 - \eta) \gamma (V^0_s - P) + (1 - \eta) (1 - \gamma) P \right] \\
- \beta \left( \lambda_{RIC} - \frac{1}{2} \right)^2, \tag{3.29}
\]

where the market maker price \( P = \eta \gamma (V^1_c - IR_{RIC}) + \eta (1 - \gamma) V^1_s + (1 - \eta) \gamma V^0_s \). We can simplify (3.29) to

\[
\Psi_{RIC} = 2 (1 - \eta) \left( 2 - 2\tilde{\lambda}_{RIC} \right) (2\lambda_{RIC} - 1) \left[ \eta \gamma (V^1_c - IR_{RIC} - V^0_s) + \eta (1 - \gamma) V^1_s \right] \\
- \beta \left( \lambda_{RIC} - \frac{1}{2} \right)^2, \tag{3.30}
\]

and then obtain the solution to the optimal effort level \( \lambda^*_{RIC} \) chosen by speculators in equilibrium,

\[
\lambda^*_{RIC} = \frac{1}{2} + \frac{2\eta (1 - \eta) [\gamma (V^1_c - IR_{RIC} - V^0_s) + (1 - \gamma) V^1_s]}{\beta + 4\eta (1 - \eta) [\gamma (V^1_c - IR_{RIC} - V^0_s) + (1 - \gamma) V^1_s]} \tag{3.31}
\]
Proof of Lemma 14. Observing the expression of $\lambda^*$ in (3.13), (3.14) and (3.18), we know that $\lambda^*$ increases monotonically in the nominator. By computing the total derivatives, we can decide the sign of the first order derivative of $\lambda^*$ with respect to a certain parameter by observing the sign of the derivative of the nominator with respect to that parameter.

For $\lambda^*_{c,z=1}$, we thus take the first order derivative of $2\theta^1 (1 - \theta^1) (V^1_c - IR_{z=1})$ with respect to $\eta$, which is positive for $\eta \in \left(0, \frac{(I-2V^0_c+V^1_c)(1-\gamma)}{V^1_c-V^0_c-(2\gamma-1)(I-V^0_c)}\right)$. Similarly for $\lambda^*_R$, we take the first order derivative of its nominator $2\eta (1 - \eta) \left[\gamma (V^1_c - IR_R - V^0_s) + (1 - \gamma) V^1_s\right]$ with respect to $\eta$, which is negative $\forall \eta \in (0,1)$.

Proof of Proposition 6. The proof is under the assumption $V^1_s = V^0_s = V_s$. We decompose the first order derivative of $(\pi_d - \pi)$ into part (i) and (ii) as below, in order to understand how a change in $\eta$ affects separately the information production and the firms’ payoff conditioning on the posterior and the corresponding action ($C$ or $S$).

(i) The variation in information production:

$$2 \frac{\partial (\lambda^*_{c,z=1} - \lambda^*_R)}{\partial \eta} \left[ \eta \gamma (V^1_c - I) + (1 - \eta) (1 - \gamma) V_s \right] + 2 \frac{\partial (\frac{1}{2} - \lambda^*_R)}{\partial \eta} \left[ \eta (V^1_c - I) + (1 - \eta) V_s \right]$$

$$+ \eta \gamma \left[ \frac{\partial (2 - 2\lambda^*_{c,z=1})}{\partial \eta} (V^1_c - IR_{z=1}) - \frac{\partial (2 - 2\lambda^*_R)}{\partial \eta} (V^1_c - IR_R) \right]$$

(ii) The variation in conditional payoffs:

$$2 (\lambda^*_{c,z=1} - \lambda^*_R) \frac{\partial [\eta \gamma (V^1_c - I) + (1 - \eta) (1 - \gamma) V_s]}{\partial \eta} + 2 \left( \frac{1}{2} - \lambda^*_R \right) \frac{\partial [\eta (V^1_c - I) + (1 - \eta) V_s]}{\partial \eta}$$

$$+ (2 - 2\lambda^*_{c,z=1}) \frac{\partial [\eta \gamma (V^1_c - IR_{z=1})]}{\partial \eta} - (2 - 2\lambda^*_R) \frac{\partial [\eta \gamma (V^1_c - IR_R)]}{\partial \eta}$$
We can then further simplify part (i) to be

\[ 2 \frac{\partial \lambda^{*}_{c,z=1}}{\partial \eta} \left[ \eta \gamma (V^1_c - I) + (1 - \eta)(1 - \gamma) V_s - \eta \gamma (V^1_c - IR_{z=1}) \right] \]

\[ + 2 \frac{\partial \lambda^{*}_{R_{IC}}}{\partial \eta} \left[ \eta \gamma (V^1_c - IR_{IC}) - \eta (1 + \gamma)(V^1_c - I) - (1 - \eta)(2 - \gamma) V_s \right] \]

It can be shown easily that term A is positive while term B is negative. Using the results from Lemma 14, we can prove that, except when \( \eta \) is very large, an increase in \( \eta \) decreases speculators’ incentives to acquire their own information when \( z \) is kept private to the firm. The crowding-out effect becomes less significant in this case.

Now let us look at part (ii) which can be further simplified to

\[ (2 - 2\lambda^{*}_{c,z=1}) \frac{\partial [\eta \gamma (I - IR_{z=1}) - (1 - \eta)(1 - \gamma) V_s]}{\partial \eta} \]

\[ + (2 - 2\lambda^{*}_{R_{IC}}) \frac{\partial [\eta \gamma (IR_{IC} - I) + (1 - \eta)(2 - \gamma) V_s + \eta (V^1_c - I)]}{\partial \eta} \]

\[ + \left( - \frac{\partial [\eta (V^1_c - I) + (1 - \eta) V_s]}{\partial \eta} \right) \]

It is obvious that term C is positive since \( R_{z=1} \) decreases in \( \eta \) thanks to an increased posterior \( \theta^1 \). Term D is also positive as \( R_{IC} \) increases in \( \eta \) when the IC constraint is tightened. The last term E is however negative. In this case, we can compute again the derivative of part (ii) with respect to \( \eta \) in order to understand the speed of changing with respect to \( \eta \). We already know that \( \frac{\partial \lambda^{*}_{R_{IC}}}{\partial \eta} < 0 \) and \( \frac{\partial \lambda^{*}_{c,z=1}}{\partial \eta} \) is negative for \( \eta \in \left( \frac{(I - 2V^0_c + V^1_c)(1 - \gamma)}{V^1_c - V^0_c - (2\gamma - 1)(1 - V^0_c)^2}, 1 \right) \), while term C, D, and E are all constants. This means that for \( \eta \) sufficiently large, an increase of \( \eta \) makes it more preferable for a firm to choose delegation in order to have the cost advantage at
the financing stage. Combining the analysis for both part (i) and (ii), Proposition 6 follows immediately.

**Proof of Lemma 15.** By taking the first order derivative of the nominator of $\lambda^*_{c,z=1}$ with respect to $\gamma$, we can show that $\frac{\partial \lambda^*_{c,z=1}}{\partial \gamma} > 0$ for $\gamma \in \left(\frac{1}{2}, \frac{V^1_c + I - 2V^0_s}{2(V^1_c - V^0_s)}\right)$, and $\frac{\partial \lambda^*_{c,z=1}}{\partial \gamma} < 0$ for $\gamma \in \left(\frac{V^1_c + I - 2V^0_s}{2(V^1_c - V^0_s)}, 1\right)$. Similarly for $\lambda^*_{R_{IC}}$, we take the first order derivative of its nominator $2\eta (1 - \eta) \left[\gamma (V^1_c - IR_{IC} - V^0_s) + (1 - \gamma) V^1_s\right]$ with respect to $\gamma$, which is positive $\forall \gamma \in \left(\frac{1}{2}, 1\right)$.

**Proof of Proposition 7.** The proof is under the additional simplifications with $\eta = \frac{1}{2}$ and $V^1_s = V^0_s = V_s$. Using the same algorithm as in the proof of Proposition 6, we can compute the first order derivative of the payoff difference between two regimes with respect to $\gamma$ and decompose it to two parts (i) and (ii),

(i) The variation in information production:

$$\frac{\partial \lambda^*_{c,z=1}}{\partial \gamma} \left[\gamma (V^1_c - I) + (1 - \gamma) V_s - \gamma (V^1_c - IR_{z=1})\right]$$

A (+)

$$\frac{\partial \lambda^*_{R_{IC}}}{\partial \gamma} \left[\gamma (V^1_c - IR_{IC}) - (1 + \gamma) (V^1_c - I) - (2 - \gamma) V_s\right]$$

B (-)

(ii) The variation in conditional payoffs:

$$\left(1 - \lambda^*_{c,z=1}\right) \frac{\partial \left[\gamma (I - IR_{z=1}) - (1 - \gamma) V_s\right]}{\partial \gamma}$$

F (+)

$$\left(1 - \lambda^*_{R_{IC}}\right) \frac{\partial \left[\gamma (IR_{IC} - I) + (1 - \gamma) V_s\right]}{\partial \gamma}$$

G

In part (i), we know from the previous proof that term A is positive and B is negative. Using the result from Lemma, we observe that with $\gamma$ being sufficiently high (i.e., $\gamma \in \left(\frac{1}{2}, 1\right)$, the terms F and G dominate.
\[
\left( \frac{V_1^3 + I - 2V_0^3}{2(V_c^3 - I V_c^3)}, 1 \right), \text{ both the first term and the second term in part (i) are negative. That is, when a firm expects the signal } z \text{ to be more precise, there would be more crowing out on information production in the stock market.}
\]

Next, let us look at part (ii). It is obvious that term F is positive since \( R_{z=1} \) decreases in \( \gamma \). Term G is positive for \( \gamma \in \left\{ \frac{1}{2}, 1 - \sqrt{\frac{V_s}{V_c^3 - I V_c^3}} \right\} \) and \( V_c^1 - I - 5V_s > 0 \), that is, when \( \gamma \) is relatively small and \( V_c^1 \) is very large. Therefore, if \( \gamma \) is sufficiently large, term G is negative and the advantage from a lower cost of capital is less significant. Combining the analysis for part (i), Proposition 8 follows.

**Proof of Proposition 8.** Let \( V_s^0 \) be denoted as \( V_s^1 - \delta \). With the simplification \( \eta = \frac{1}{2} \), we can write down the expression of \( \lambda \) in equilibrium,

\[
\lambda_{s,z=0}^* = \frac{1}{2} + \frac{2\gamma (1 - \gamma) \delta}{\beta + 4\gamma (1 - \gamma) \delta} \tag{3.32}
\]

\[
\lambda_{c,z=1}^* = \frac{1}{2} + \frac{2\gamma (1 - \gamma) (V_c^1 - I R_{z=1})}{\beta + 4\gamma (1 - \gamma) (V_c^1 - I R_{z=1})} \tag{3.33}
\]

\[
\lambda_{R_{IC}}^* = \frac{1}{2} + \frac{[\gamma (V_c^1 - I R_{IC} - V_s^1 + \delta) + (1 - \gamma) V_s^1]}{2[\beta + \gamma (V_c^1 - I R_{IC} - V_s^1 + \delta) + (1 - \gamma) V_s^1]} \tag{3.34}
\]

where \( R_{z=1} = \frac{1}{I \gamma} (I - (1 - \gamma) V_c^0) \), \( R_{IC} = \frac{1}{I} \left( V_c^1 - V_s^1 - \frac{\gamma}{1 - \gamma} (V_s^1 - \delta) \right) \). Note that \( V_s^1 \) enters only the expression of \( \lambda_{R_{IC}}^* \). We can thus write the first order derivative of the payoff difference between two regimes with respect to \( V_s^1 \) into two parts,

(i) The variation in information production:

\[
\frac{\partial \lambda_{R_{IC}}^*}{\partial V_s^1} \left[ \frac{\gamma (V_c^1 - I R_{IC}) + (1 - \gamma) V_s^1 + \gamma (V_s^1 - \delta)}{(V_c^1 - I + V_s^1 - \delta)} \right] \tag{3.35}
\]

(ii) The variation in the conditional payoff:

\[
\left( \lambda_{s,z=0}^* - \lambda_{R_{IC}}^* \right) \frac{\partial \left[ (1 - \gamma) V_s^1 + \gamma (V_s^1 - \delta) \right]}{\partial V_s^1} - (1 - \lambda_{R_{IC}}^*) \frac{\partial \left[ \gamma (V_c^1 - I R_{IC}) \right]}{\partial V_s^1} \tag{3.36}
\]
In part (i), term H is negative since $IR_{IC} > I$ and $V_s^1 < V_c^1 - I$ (by assumption). We can obtain \( \frac{\partial \lambda_{R_{IC}}^*}{\partial V_s} = \frac{\beta (1-\gamma)^2}{2(1-\gamma)\beta + (1-\gamma) V_c^2 + \gamma (2\gamma - 1) V_s^2} \), which is positive. Part (ii) can be simplified to \( (\lambda_{s,z=0}^* - \lambda_{R_{IC}}^*) - \frac{\gamma}{1-\gamma} (1 - \lambda_{R_{IC}}^*) \), which is negative. An increase of $V_s^1$ does not affect the firm’s payoff when $z = 1$ in the regime of delegation and thus has a bigger impact on the payoff with in-house information acquisition. The proof is thus completed.

**Proof of Lemma 16.** Since $z$ is now private to the firm itself, the expected trading profit of a speculator and thus his effort level in equilibrium is no longer conditional on $z$,

\[
\Psi_R = 2\eta (1 - \eta) \left( 2 - 2\tilde{\lambda}_{c,R} \right) (2\lambda_{c,R} - 1) (V_c^1 - I \tilde{R}) - \beta \left( \lambda_{c,R} - \frac{1}{2} \right)^2. 
\]  

By taking the first order derivative of $\Psi_R$ with respect to $\lambda$, we can then obtain the optimal effort level \( (\lambda_{c,R}^*) \) chosen by speculators in this case,

\[
\lambda_R^* = \frac{1}{2} + \frac{2\eta (1 - \eta) (V_c^1 - I \tilde{R})}{\beta + 4\eta (1 - \eta) (V_c^1 - I \tilde{R})}. 
\]  

\[ (3.38) \]
CHAPTER 4.
MANAGERIAL INCENTIVE IN A SPATIAL COMPETITION WITH
UNCERTAIN PRODUCT QUALITY

4.1 Introduction

There is a common view in the literature that product market competition should have an important influence on managerial decisions and hence the firm value.\(^1\) There are however more debates on how exactly competition should affect managerial incentives and the underlying driving forces. Greater competition may lead to stronger incentives for agents because principals can have an additional means to be better informed about their agents’ actions (Hart, 1983), or because greater effort is required to decrease the disutility cost to be born by an agent (Schmidt, 1997). Some other studies suggest that whether competition may substitute for incentives rather depends on the characterization of agents’ preference or the specific form of competition. For example, Scharfstein (1988) shows that competition may lead to managerial slack when a manager’s marginal utility from income is strictly positive. Graziano and Parigi (1998) show that a lower degree of product market differentiation reduces the manager’s optimal effort choice while an increase in the number of competing firms has an ambiguous effect.

To further explore the complexity of the relationship between market competition and managerial incentive, I build a model that considers both horizontal and vertical differentiations between firms. Two firms are assigned to exogenous locations on a circular city. Consumers are uniformly distributed on the circle, and they need to pay a transportation

\(^1\)See Nickell (1996) for a discussion.
cost for making a purchase. The firms therefore enjoy a local market power. The transportation cost is thus used as a proxy for the level of competition between two firms. At the beginning of the game, both firms anticipate a future uncertainty in their product qualities. They simultaneously offer incentive contracts to the managers in order to induce an effort level such that the expected firm profit is maximized.

I show that competition has two opposite effects on the equilibrium effort level. A lower transportation cost and thus more competition impairs a firm’s local market power. This reduces the marginal benefit that a firm may enjoy from producing a high quality product, particularly when its competitor also produces a high quality product. Competition may thus affect adversely incentives. On the other hand, greater competition reduces a firm’s profit if it fails to produce high quality products. The effect increases the optimal effort level and becomes dominant if the magnitude of quality improvement is relatively large compared to the cost of exerting effort. Both effects are less significant when firms are located further away from each other and thus more differentiated horizontally.

Moreover, I show that a large decrease in the transportation cost may change the market structure, such that the firm with better quality goods attracts all the demand from consumers. This makes more significant the positive effect of competition on the effort level. The results seem to suggest that the relationship between competition and incentive depends on the absolute level of competition on top of the size of vertical differentiation as well as the cost of effort. For example, Beiner, Schmid and Wanzenried (2011) find that more competition reduces incentives when competition levels are low. For higher levels of competition, increased competition intensity leads to stronger incentives.
It may be more appropriate to relate the model in this paper to the applications of an oligopoly industry in service sector than in manufacturing sector, as it links directly the effort choice of managers to the product quality. In addition, it is usually more difficult to evaluate and verify the quality of services, which justifies the assumption that firms cannot write a contract directly on the quality of the output. The locations of firms may also have relevant interpretations for the service sector. For example, it is of interest to understand how the managerial effort in the financial sector is affected by the product differentiation, which includes not only the traditional interpretation as banks’ geographical location choice but also the designs of credit products, for instance.

The paper is related to the managerial incentive literature in the context of product market competition. There are mainly three strands of theoretical research on this subject. Early works mostly focus on the role competition in providing additional information that can be used by the principal to infer the effort level chosen by their agents. For example, Hart (1983) formalizes the idea that competition may reduce the managerial slack when there is a common component in production cost between managerial firms and entrepreneurial firms. In his model, managers target at a fixed profit level. Competition between the two types of firms forces managers to exert more effort for cost minimization, typically when the common component of cost falls in the industry. Relying crucially on the characterization of a manager’s preference, e.g., a discontinuity in their utility function, Hart’s result can be reversed if the manager’s marginal utility strictly increases with income, as shown by Scharfstein (1988). The ambiguity in the informational effect on managerial incentive is also confirmed in Hermalin (1992).
While there is a common component in firms’ costs in the model of Hart (1983), contracts are only based on an individual firm’s performance. Informational role of market competition may arise also when the performance of competing managers can help a firm infer the effort choice of its own manager. A large body of literature is devoted to analyze the use of relative performance evaluation (RPE).\textsuperscript{2} My model in this paper, while employing one manager for each firm, does not consider this particular feature at the contracting stage. The realization of one manager’s effort is independent from the competing manager, although it spills over across firms through competition in the product market. A manager is thus rewarded more for a superior performance than its competitor.

Another strand of literature focuses on the impact of market competition on the disutility of managerial effort. Schmidt (1997) considers a risk-neutral manager’s incentive in cost reduction. The author models an increase in competition as a higher probability of bankruptcy which provides an incentive for the manager to exert effort to reduce the disutility of liquidation. Competition can also affect managerial incentive inversely since it may lower the marginal benefit from reducing production cost, such that the principal no longer finds it optimal to induce higher effort. The aspect of liquidation threat on managers’ action is also considered by Aghion, Dewatripont and Rey (1999). The impact of liquidation on managerial incentive is not modelled in my paper. I demonstrate nevertheless when the transportation cost is sufficiently small and hence the competition is very intense, the firm with better quality goods can sell to the entire market and its competitor earns zero profit. In this case, the effort level increases with competition except when the quality difference is

\textsuperscript{2}The early research on relative performance evaluation can be found in Holmstrom (1982), Nalebuff and Stiglitz (1983), Aggarwal and Samwick (1999) among others.
significant compared to the effort cost. The positive effect is more compelling than in the scenario where firms always share the market. Such an effect, although driven by a different force, prevails in my model even when a firm is able to observe the manager’s effort choice.

Some recent works endogenize the market structure under changes in competition in an industry. Raith (2003) use an oligopoly framework in which firms can enter and exit a market on a circle, depending on their profitability, while agents are given incentives to reduce production cost. Raith shows that competition always leads to greater incentives, when the number of firms in the market becomes endogenous and depends on market fundamentals such as product substitutability, market size or cost of entry. A related paper by Golan, Parlour and Rajan (2010) examines how competition affects the optimal effort that a firm or its shareholders wish to induce in order to increase the likelihood of producing high-quality product. Different from Raith (2003), these authors endogenize both cost and benefit from generating a particular quality level with the market structure, and show that competition in a market of network goods may reduce effort level and hence the average product quality in the industry.

The paper proceeds as follows. Section 4.2 presents the setup of the model. Section 4.3 provides the solution to the model and Section 4.4 discusses the link between competition and incentives. Section 4.5 concludes. Proofs and additional discussions are relegated to the Appendix.
4.2 The Model

The timeline There are two firms in this model, indexed by $i$ and $i \in \{1, 2\}$, each producing a single product. There are four periods, date 0, 1, 2 and 3. At date 0, firms’ locations, and hence the location of their future products, are exogenously fixed in a circular city. Meanwhile, both firms anticipate a future uncertainty in their product quality and each of them offer an incentive compatible contract to its manager. Contracts are written simultaneously at date 0. At date 1, managers of firm 1 and 2 choose effort level simultaneously, and they work to increase the likelihood of producing high-quality goods. During the next period (date 2), product qualities are revealed and become common knowledge. Production is completed in both firms and firms choose their product prices at the same date. Finally, at date 3, consumers observe firms’ locations, product qualities and prices. Each of them choose goods and make the purchase. All agents receive their payoffs at the end of date 3.

The managers I assume that each firm hires one risk neutral manager for maximizing firms’ profit. Managers have zero initial wealth but they are protected by the limited liability constraints. It is common knowledge that both managers have the same ability in organizing production, but the effort level chosen by managers can affect the probability of producing high-quality goods.

Since there are only two possible realizations for the product quality, I assume that the probability of producing a high-quality product, denoted by $\mu$, directly matches the effort level. Using $e$ to denote the effort level, $\mu (e) = e$, $e \in [0, 1]$. As effort is costly, the manager incurs a disutility $\psi(e)$, $\psi(e) = \frac{1}{2}e^2$. $\psi(e)$ is increasing and strictly convex in $e$,
\[ \psi'(e) = \gamma e > 0 \text{ and } \gamma > 0. \] I assume that the parameter \( \gamma \) is sufficiently large such that the effort level is always below 1. That is, it is never optimal to induce an effort that ensures a high product quality with certainty.

I assume that firms’ profit is contractible and contracts offered to managers are publicly observable. In addition, ex-post renegotiations are not allowed in this model. By writing the contract, the firm decides a transfer \( w \) to remunerate its manager. Under the assumption that both firms and their managers are identical ex ante, managers’ utility functions are thus of the same form,

\[
w = \frac{\gamma e^2}{2}. \tag{4.1}
\]

The circular city and the product market The model setup of the circular city follows Salop (1979). Two firms are located on the circumference of a circular city, of which the perimeter equals 1. (See Figure 4.1.) Assuming that there is no border in this city, I assign firm 1 to a reference point \( x_0 \). The relative location of firm 2 is defined by the central angle between the radius connecting firm 1 to the center of circle and the radius connecting firm 2. This central angle is assumed to be positive when measured anti-clockwise and negative otherwise. Everyone can travel only along the sphere. The location of firm 2 is denoted by \( x_\beta \) when it is located with an angle \( \beta \) to firm 1, and the distance between two firms is \( \frac{\beta}{2\pi} \).

Since the semi sphere at both sides of firm 1 are symmetric, I only discuss the case when \( \beta \) is positive and \( \beta \in [0, \pi] \), that is, firm 2 is assumed to be located on the left to firm 1. For simplicity, I assume that firms produce at zero production cost. The products of firm 1 and 2 are geographically differentiated, and they may also be vertically differentiated by the quality of their products, as the product quality \( q \) has two possible realizations, \( q^h \) and \( q^l \).
Apart from two firms, there also exists a continuum of consumers of measure 1, who are located uniformly on the circumference. Each consumer buys at most one unit of goods that are not divisible. Consumers are identical except their locations at the circular city, which are also defined by the central angle between the radius of firm 1 and the ones of consumers. Let $t$ denote the travel distance of a consumer. Assuming that the transportation cost of a consumer is a quadratic and strictly convex function of $t$, a consumer’s utility is then defined by,

$$u = q_i - \tau t^2 - p_i,$$  \hfill (4.2)$$

where $p_i$ and $q_i$ denotes respectively the price and the quality of firm $i$'s product, if this consumer buys from firm $i$, $i = 1, 2$, and $\tau > 0$. 

$q'$ with $q^h > q'$. 

Figure 4.1: The Circular City
Note that firm 1 is located at the reference point, and thus $x_1 = x_0$. The reservation utility of consumers is assumed to be zero for simplicity. Based on the utility function, consumers naturally choose the shortest arc to travel to the location of their desired product.

For the purpose of illustration, I restrict the angle between any consumer to the firms between $[-\pi, \pi]$. For a consumer located with an angle $\alpha$ to $x_0$, his travel distance to firm 1 is thus $\left|\frac{\alpha}{2\pi}\right|$, $\alpha \in [-\pi, \pi]$. His travel distance to firm 2 located at $x_\beta$ is $\min\left\{\frac{2\pi - \beta - |\alpha|}{2\pi}, \frac{\beta - |\alpha|}{2\pi}\right\}$ for $\alpha \in [-\pi, \pi]$.

### 4.3 Solution to the Model

In this paper, I mainly consider the situation that the entire market is covered, i.e., every consumer buys one unit of product from either firm 1 or firm 2. An example for the market being partially covered is presented in the appendix. There is a special case that firms are both located at the reference point $x_0$, i.e., $\beta = 0$. Either firms’ products have the same quality, $q_1 = q_2$, and two firms share the market but earn zero profit due to perfect competition between them. Or $q_1 \neq q_2$, the firm producing better-quality goods attracts the entire market. I neglect the discussion of this case since the analysis is trivial in the absence of horizontal differentiation and it does not add new insights to the link between market competition and managerial incentive problem.

Given firms are separately located, if $q_1 = q_2$, there are two marginal consumers who are indifferent from purchasing from firm 1 and firm 2, one on each of the two circular arcs between firms. If $q_1 \neq q_2$, either two firms share the market, and there are two marginal consumers, or the firm with better-quality goods attracts the entire market when the quality
difference between firms is sufficiently large. I first discuss the case where both firms stay active for any \(\{q_1, q_2\}\), and then look at the case where the firm with lower-quality good is driven out of the market.

**Product pricing** We can find the location(s) of the marginal consumer(s) by using the utility function defined in (4.1). For the case with \(q_1 \neq q_2\), let us use \(q_1 = q^h\) and \(q_2 = q^l\) as an example for illustration. The result in demand and pricing holds for the reverse case, since two firms are always symmetrically located on the circle, regardless of the distance between them. I denote the locations of the marginal consumers by \(\alpha^*\). The first one can be found at \(\alpha^*_1\) on the arc between \([0, \pi]\), and the following equation must be satisfied,

\[
q_1 - \tau \left( \frac{\alpha^*_1}{2\pi} \right)^2 - p_1 = q_2 - \tau \left( \frac{\beta - \alpha^*_1}{2\pi} \right)^2 - p_2. \tag{4.3}
\]

When firms share the market, there must be another marginal consumer located at \(\alpha^*_2\) on the arc between \([-\pi, 0]\), for which the equation below has to be satisfied,

\[
q_1 - \tau \left( \frac{|\alpha^*_2|}{2\pi} \right)^2 - p_1 = q_2 - \tau \left( \frac{2\pi - \beta - |\alpha^*_2|}{2\pi} \right)^2 - p_2. \tag{4.4}
\]

Solving (4.3) and (4.4), we can find the locations of the marginal consumers, which then give the demand for each firm. Let us denote the demand for firm 1 and firm 2 by \(d_1\) and \(d_2\), respectively. \(d_1\) and \(d_2\) are given in the following lemma.

**Lemma 17** When the market is fully covered and shared by two firms, the demands are

\[
d_1 = \frac{1}{2} + \frac{2\pi^2}{\beta \tau (2\pi - \beta)} (q_1 - q_2 - p_1 + p_2) \tag{4.5}
\]

\[
d_2 = \frac{1}{2} - \frac{2\pi^2}{\beta \tau (2\pi - \beta)} (q_1 - q_2 - p_1 + p_2) \tag{4.6}
\]
Remember that firms incur zero production cost. The profit of firm $i$, denoted by $S_i$, is therefore $p_i d_i$, $i = 1, 2$. Using the demand functions in Lemma 17, each firm can then maximize its profit $p_i d_i$. It is easy to find the reaction in product price,

$$R_1(p_2) = p_1 = \frac{\beta \tau (2\pi - \beta)}{8\pi^2} + \frac{1}{2} (q_1 - q_2 + p_2)$$

(4..7)

with $q_1 > 0$ and $q_2 > 0$. The reaction function of firm 2, $R_2(p_1)$, is symmetric to $R_1(p_2)$ in (4..7). Solving the system of price reactions, we obtain the equilibrium price,

$$p_1 = \frac{1}{3} (q_1 - q_2) + \frac{\tau \beta (2\pi - \beta)}{4\pi^2},$$

(4..8)

and $p_2$ is symmetric to (4..8). Substituting both $p_1$ and $p_2$ into (4..5) and (4..6), we obtain the demand for each firm’s product,

$$d_1 = \frac{1}{2} + \frac{2\pi^2}{3\tau \beta (2\pi - \beta)} (q_1 - q_2),$$

(4..9)

and similarly $d_2$ is symmetric to (4..9). Note that the participation constraint of the consumer at both $\alpha_1^*$ and $\alpha_2^*$ needs to be satisfied, for which the parameter region lies in $q^h > q^l > \frac{\beta (8\pi - 3\beta)}{16\pi^2} \tau$, and $q^h - q^l < \frac{3\beta (2\pi - \beta)}{4\pi^2} \tau$, such that the market is fully covered and firms share the market even when their products are of different qualities.

If one firm gets the entire market and its competitor quits, when this firm’s product has a higher quality. This occurs when the size of quality improvement is sufficiently large, i.e., $q^h - q^l > \frac{3\beta (2\pi - \beta)}{4\pi^2} \tau$. Since the production cost is normalized to zero, the firm producing lower-quality goods reduces its product price to zero under competition, and its competitor sets the price $p_m$ at

$$p_m = \frac{\Delta q - \tau \beta (2\pi - \beta)}{4\pi^2}$$

(4..10)
where \( \Delta q = q^h - q^l \). Let \( S \) denote the profit function of the firms, and \( S_i = p_i d_i, \ i = 1,2 \).

We can obtain some of the properties of \( S_i \) based on (4.8) and (4.9), which are consistent with the standard literature in industry organization.

**Lemma 18** If the market is fully covered and two firms always share the market, for any realization of product quality, their profit increases in the horizontal differentiation (\( \beta \)) and also consumers’ transportation cost (\( \tau \)).

The results in Lemma 18 are straightforward. The further away two firms are located from each other and the more consumers need to spend on travelling, the lower competition there is between two firms. The increased local market power pushes up the profit of the firms.

**Incentive contract** Let us now look at the incentive contracts written by the firms. We check the contracts when managers’ effort is observable and discuss the case with unobservable effort in the extension\(^3\). Note that the transfers and thus the optimal contract are conditional only on the realizations of product quality. Consequently, firm \( i \)’s objective function can be reduced to its expected payoff conditional on its own product quality given all the possible realizations of the other firm’s product quality. Let us use the notations \( \bar{S}_i \) and \( \underline{S}_i \) for firm \( i \)’s expected payoff when \( q_i \) equals \( q^h \) and \( q^l \), respectively, \( i = 1,2 \). \( \bar{S}_i \) and \( \underline{S}_i \) are functions of the effort level \( e_i \) chosen by firm \( i \)’s managers,

\[
\bar{S}_i = e_i S_i(q^h, q^h) + (1 - e_i) S_i(q^h, q^l) \tag{4.11}
\]

\(^3\) The qualitative results still hold when the effort is unobservable. They are however amplified when it is more costly to induce effort.
where the subscription $i$ refers to the competitor of firm $i$, $i = 1, 2$.

Firm $i$ chooses the transfer between $\{\bar{w}, w\}$,

$$\max_{\{\bar{w}, w\}} e_i (\bar{S}_i - \bar{w}) + (1 - e_i) (S_i - w)$$

s.t. $\bar{w} \geq 0, w \geq 0$, and $e_i \bar{w} + (1 - e_i) w - \frac{\gamma e_i^2}{2} \geq 0$.

With a verifiable effort, a manager receive $w^*, w^* = \frac{\gamma}{2} e_i^2$, which is just sufficient to compensate the effort cost. The expected payoff of firm $i$, $E[S_i]$, is thus concave in the effort level chosen by its manager. Let us use $S_1(q_1, q_2)$ to denote firm 1’s payoff for the pair of product quality $(q_1, q_2)$. Since managers are identical ex ante and they both receive a contract at date 0, the problem is symmetric between firm 1 and firm 2.

**Lemma 19** When effort is observable, the first best effort choice of firm $i$’s manager is, $i = 1, 2$,

$$e_i^{FB} = \frac{S_i(q_i, q_i') - S_i(q_i', q_i'}{\gamma + S_i(q_i, q_i') - S_i(q_i', q_i') - [S_i(q_i, q_i) - S_i(q_i', q_i')]}.$$  \hspace{1cm} (4.15)

The effort level $e_i^{FB}$ increases in both $S_i(q_i, q_i') - S_i(q_i', q_i')$ and $S_i(q_i, q_i) - S_i(q_i', q_i')$, but decreases in the difference between two terms. In the meanwhile, the scale of disutility cost $\gamma$ affects negatively the optimal effort choice. The first impression is that firms want to induce managerial incentive when managers’ effort is not too costly but brings large profit improvement, particularly when the marginal benefit from improving product quality is less sensitive to the quality realization of the competing firm. In the next section, we look more closely how the effort choice is affected by competition.
4.4 Competition and Effort Choice

After computing the pricing functions of the firms and managers’ optimal effort choice, I discuss next how the market competition changes the incentive of managers. I discuss separately the scenario where both firms stay active regardless of their product quality and the scenario where competition forces the firm with low-quality product to quit the market.

Scenario (I) Let us first look at the scenario where two firms stay active for all realizations of \( \{q_1, q_2\} \), for \( \Delta q < \frac{3\beta(2\pi - \beta)}{4\pi^2} \tau \). As stated in Lemma 18, a decrease in transportation cost \( \tau \) lowers the local monopoly power of a firm and intensifies the competition between two firms. It brings two opposing effects. On the one hand, a lower \( \tau \) reduces a firm’s profit less significantly when it succeeds to produce a high quality product while its competitor fails, i.e., \( S_i(q^h, q^l) - S_i(q^l, q^l) \) increases for a lower \( \tau \). Competition has a positive impact on incentive. On the other hand, there is also a negative effect of competition. A decrease in \( \tau \) impairs the marginal benefit from improving quality realization when its competitor also manages to produce high-quality goods, that is, \( S_i(q^h, q^h) - S_i(q^l, q^l) \) decreases for a lower \( \tau \). Furthermore, when travelling becomes less costly to consumers, a firm’s profit becomes more sensitive to its competitor’s effectiveness in quality improvement. That is, when \( \tau \) falls, the difference between \( S_i(q^h, q^l) - S_i(q^l, q^l) \) and \( S_i(q^h, q^h) - S_i(q^l, q^l) \) goes up. Two effects combined, the net result is dependent on the relative magnitude of \( \Delta q \) to \( \gamma \).

In addition, when firms’ products are more differentiated horizontally, i.e., \( \beta \) increases, both the positive effect and negative of \( \tau \) on incentives becomes less significant. This can be shown by the second order mixed derivatives of \( S_i(q^h, q^l) - S_i(q^l, q^l) \) and \( S_i(q^h, q^h) -
$S_i(q^l,q^h)$ with respect to $\tau$ and $\beta$.\textsuperscript{4} It is intuitive as when firms’ products are located further away (that is, a lower degree of production substitution), consumers’s preference for the local market becomes more important. An increase in $\beta$ thus eliminates partially the impact of increased competition caused by a lower transportation cost.

Substituting the product price and demand functions in (4.8) and (4.9) into each profit function $S$ in the optimal effort (4.15), we can obtain the exact effort level $e_i^{FB}$,

$$
e_i^{FB} = \frac{\Delta q (2\pi^2 \Delta q + 3\beta \tau (2\pi - \beta))}{4\pi^2 \Delta q^2 + 9\gamma \beta \tau (2\pi - \beta)}.
$$

(4.16)

Its first order derivative with respect to $\tau$ is thus,

$$
\frac{\partial e_i^{FB}}{\partial \tau} = \frac{6\pi^2 \Delta q^2 \beta (2\pi - \beta)}{(4\Delta q^2 + 9\gamma \beta \tau (2\pi - \beta) \tau \gamma)^2} (2\Delta q - 3\gamma)
$$

(4.17)

We can observe if $\Delta q$ is sufficiently large compared to the disutility cost $\gamma$ ($\Delta q > \frac{3}{2}\gamma$), the second effect dominates the first one, and thus the optimal effort level is inversely related to competition (i.e., positively related to $\tau$). If however $\Delta q < \frac{3}{2}\gamma$, the quality difference is not sufficiently high such that it is worth less effort for the firm to ensure a realization of $q^h$.

I summarize these results in the following proposition.

\textbf{Proposition 9} When a market is fully covered and firms stay in competition for every realization of $\{q_1,q_2\}$, the optimal effort level increases in the transportation cost if the quality improvement is significant relative to the cost of exerting effort ($\Delta q > \frac{3}{2}\gamma$), and product market competition reduces incentive. The effort level decreases in the transportation cost if the quality improvement is not sufficient compared to the cost of effort, in which case competition enhances incentives.

\textsuperscript{4}The second order mixed derivatives are,

$$
\frac{\partial^2 S_i(q^l,q^h) - S_i(q',q')}{\partial \tau \partial \beta} = \frac{4\pi^2 \Delta q^2 (\pi - \beta)}{9\beta^2 \tau^2 (2\pi - \beta)^2}, \text{ and } \frac{\partial^2 S_i(q^l,q^h) - S_i(q',q')}{\partial \tau \partial \beta} = \frac{4\pi^2 \Delta q^2 (\pi - \beta)}{9\beta^2 \tau^2 (2\pi - \beta)^2}.
$$
The results from Proposition 9 are still valid qualitatively in a world with unobservable effort and limited liability, which I discuss briefly in the Appendix A.

Scenario (II) Keeping the size of quality improvement \( q^h - q^l \) fixed, when the transportation cost is sufficiently low, the firm producing lower quality goods may lose the market completely and its competitor attracts all the consumers. This occurs when \( q^h - q^l > \frac{3\beta(2\pi - \beta)}{4\pi^2} \tau \), and the firm with better quality goods set the price as in (4..10). In this case, \( S_i(q^h, q^h) = 0 \), and \( S_i(q^h, q^l) = \Delta q - \frac{\beta(2\pi - \beta)}{4\pi^2} \tau \). \( S_i(q^h, q^l) \) decreases in the transportation cost \( \tau \) and the horizontal differentiation \( \beta \). This is because when \( \tau \) goes up, the firm with lower quality goods gains more local market power and thus there is more price pressure to the firm with high quality goods. This then leads to a lower incentive for the firm to induce effort. The positive effect of competition on incentive is now more significant, compared to the first scenario.

To show this mathematically, we can compute the optimal effort induced by the contracts and its first order derivative with respect to \( \tau \),

\[
E_i^{FB} = \frac{\pi^2 \Delta q - \frac{3 \beta \tau (2\pi - \beta)}{8}}{\pi^2 (\Delta q + \gamma) - \frac{1}{2} \beta \tau (2\pi - \beta)} \tag{4..18}
\]

\[
\frac{\partial E_i^{FB}}{\partial \tau} = \frac{\pi^2 (2\pi - \beta) \beta}{2 (2\pi^2 (\Delta q + \gamma) - \beta \tau (2\pi - \beta))^2} (\Delta q - 3\gamma) \tag{4..19}
\]

Obviously, the optimal effort level increases in \( \tau \) if \( \Delta q > 3\gamma \), and vice versa. Comparing to (4..17) in the first scenario, we observe that except when the quality improvement is sufficiently high compared to the disutility cost (i.e., \( \Delta q > 3\gamma \)), a lower transportation cost (and thus greater competition) leads to a higher incentive.

We thus observe that the relationship between competition and the optimal effort level
largely depends on the size of quality improvement relative to the cost of exerting effort. In addition, when the transportation cost falls significantly and thus the competition becomes even more intense, market structure may change subsequently. When the loss to the firm upon producing a low-quality product is heavier, competition triggers more incentives for a manager to differentiate vertically his product from his competitor. The positive effect becomes increasingly important.

At last, when the transportation cost becomes so high that \( q^l < q^h < \frac{\beta(8\pi - 3\beta)}{16\pi^2} \), a special case arises that firms never compete with each other but only attract their local market. In this case, the marginal consumer would find it indifferent between making a purchase from the closest firm and not buying. One firm’s product price no longer depends on the product quality of its competitor. I give an example in the Appendix B for this situation.

4.5 Concluding Remarks

The relationship between product market competition and incentives has long been a popular research topic, not only because of its ambiguity and complexity that provide a fruitful area for exploration, but because of the great relevance of the subject to either firms or regulators in the real world. This paper is another attempt to explore this research area. I consider a spatial competition model in which firms are differentiated both horizontally and vertically, and I employ an optimal contracting framework with a multi-agent setting to examine managerial incentives in a duopoly market. Managers are provided incentives to improve product quality. I show that the ambiguity in the relationship between competition
and incentives may also come from the relative degree of vertical differentiation compared to the cost of effort. More specifically, if the size of quality improvement is relatively large, competition reduces incentives due to the negative impact of competition on the marginal benefit from improving the product quality. In addition, I show that the change in transportation cost may alter the market structure under competition, which further influences the impact of competition on incentives.

There is still much to be done with this model. The future work may include two parts. The first part is to study the location choice before the quality realization within the optimal contracting framework. It is also interesting to solve the location problem from the perspective of a central planner and have a closer look at the welfare implications. This is then related to the second part of future research, which is to establish applications in the real economy, especially in the industry of financial services. For example, the competition in the credit rating market and the incentive of rating agencies, among other possible applications, may prove to be interesting to relate to this model.
REFERENCES CITED


4.6 Appendix

A. Effort unobservable When managers’ effort choice cannot be observed, they choose an optimal level of $e$ to maximize their payoff,

$$
e^* = \arg \max_{e \in [0,1]} e\bar{w} + (1 - e) \overline{w} - \frac{\gamma}{2} e^2.
$$

(4.20)

First order approach gives $\bar{w} - \overline{w} = le$. The profit maximization problem for the firms now becomes

$$
\max_{\{e, \bar{w}, \overline{w}\}} e (\bar{S} - \bar{w}) + (1 - e) (\overline{S} - \overline{w})
$$

(4.21)

s.t. $\bar{w} \geq 0, \overline{w} \geq 0, \text{ and } \bar{w} - \overline{w} = \gamma e.$

(4.22)
Similarly, we can obtain the second-best effort choice.

**Lemma 20**  When effort is not observable, the second best effort choice of firm i’s manager is, $i = 1, 2$,

$$e^i_{SB} = \frac{S_i \left(q^h, q^l\right) - S_i \left(q^l, q^l\right)}{2\gamma + S_i \left(q^h, q^l\right) - S_i \left(q^l, q^l\right) - \left[S_i \left(q^h, q^h\right) - S_i \left(q^l, q^h\right)\right]}.$$  \hspace{1cm} (4.23)

The second best effort choice is

$$e^i_{SB} = \frac{\Delta q \left(2\pi^2 \Delta q + 3\beta \tau \left(2\pi - \beta\right)\right)}{4\pi^2 \Delta q^2 + 18\gamma \beta \tau \left(2\pi - \beta\right)}.$$  \hspace{1cm} (4.24)

of which the first order derivative with respect to $\tau$ equals

$$\frac{\partial e^i_{SB}}{\partial \tau} = \frac{3\pi^2 \Delta q^2 \beta \left(2\pi - \beta\right)}{\left(2\Delta q^2 + 9\gamma \beta \tau \left(2\pi - \beta\right)\right)^2} \left(\Delta q - 3\gamma\right).$$  \hspace{1cm} (4.25)

In this case, the quality improvement needs to be larger than in $e^i_{FB}$ to have a positive relationship between optimal effort level and travelling cost. The intuition is as follows. It is more costly to induce managers’ incentive when their effort choice is unobservable. Consequently, it is more difficult to incentivize managers to exert effort due to an increase in local monopoly power caused by a higher travelling cost.

**B. A special case** To show the qualitative result in the special case in which both firms attract only the local market, I use an example with $\beta = \pi$ (i.e., firms are located furthest apart) for the purpose of illustration. In this case, even when a firm produces a product with quality $q^h$, the marginal consumer is indifferent from purchasing from this firm and not purchasing any product. We then have two marginal consumers for each firm, who are located symmetrically around the firm. The equation below holds for firm 1,

$$q_i - \tau \left(\frac{\alpha^*}{2\pi}\right)^2 - p_i = 0.$$  \hspace{1cm} (4.26)
The problem is symmetric for firm 2. Maximizing firms’ profit by the first order condition, we can obtain the product price

\[ p_i = \frac{2}{3} q_i \]  \hspace{1cm} (4.27)

and the profit of the firm for a given \( q_i \) is

\[ S_i = \frac{4}{3} \left( \frac{q_i}{3\tau} \right)^{\frac{1}{2}}. \]  \hspace{1cm} (4.28)

Using the same formula in (4.15), we can compute the optimal effort,

\[ e_i^{FB} = \frac{4}{3\gamma} \left[ q^h \left( \frac{q^h}{3\tau} \right)^{\frac{1}{2}} - q^l \left( \frac{q^l}{3\tau} \right)^{\frac{1}{2}} \right]. \]  \hspace{1cm} (4.29)

Given \( q^h > q^l \), the optimal effort always decreases in the transportation cost \( \tau \), which is not due to competition. Instead, when \( \tau \) increases, the decrease in a firm’s profit is more significant when the product quality is high than otherwise and thus the benefit from improving product quality shrinks. This then depresses managers’ incentives to exert effort.

C. Proofs

**Proof of Lemma 17.** The location of the first marginal consumer can be obtained directly from solving equation (4.2), and

\[ \alpha_1^* = \frac{\beta}{2} + \frac{2\pi^2}{\beta\tau} (q_1 - q_2 - p_1 + p_2). \]  \hspace{1cm} (4.30)

where \( \alpha_1^* \in (0, \pi) \).

Similarly, we can obtain the location of the second marginal consumer by solving equation (4.3),

\[ \alpha_2^* = -\frac{2\pi - \beta}{2} - \frac{2\pi^2}{(2\pi - \beta)\tau} (q_1 - q_2 - p_1 + p_2). \]  \hspace{1cm} (4.31)

where \( \alpha_2^* \in (-\pi, 0) \).
Since the circumference of the circle and thus the total market size is 1, the demand for firm 1’s product $d_1$ is thus $\frac{\alpha_1^* - \alpha_2^*}{2\pi}$, and for firm 2, $d_2 = \frac{2\pi - \alpha_1^* + \alpha_2^*}{2\pi}$. For the ease of illustration, for the case $q_1 \neq q_2$, we compute $\alpha_1^*$ and $\alpha_2^*$ for the example that firm 1 producing the high quality products. This should not affect the size of demands, since firms are symmetric in locations. Substituting (4.30) and (4.31) into $d_1$ and $d_2$, we obtain (4.5) and (4.6).

**Proof of Lemma 18.** If two firms always share the market, firm $i$’s profit $S_i$ is

$$S_i = \frac{(4\pi^2(q_i - q_{-i}) + 6\pi \beta \tau - 3\beta^2 \tau)^2}{72\pi^2 (2\pi - \beta) \beta \tau}$$

(4.32)

Taking the first order derivative with respect to $\beta$ and $\tau$, we can obtain

$$\frac{\partial S_i}{\partial \beta} = \frac{(\pi - \beta)}{36\pi^2 \tau} \left( 9\tau - \frac{16\pi^4 (q_i - q_{-i})^2}{(2\pi - \beta)^2 \beta^2} \right)$$

(4.33)

$$\frac{\partial S_i}{\partial \tau} = \frac{2\pi (q_i - q_{-i})^2}{9 (2\pi - \beta) \beta \tau^2}$$

(4.34)

The subscription ”$- i$” denotes the competitor of firm $i$. That is, if $i = 1$, $q_{-i}$ is the product quality of firm 2.

If $q_i = q_{-i}$, it is obvious that both $\frac{\partial S_i}{\partial \beta}$ and $\frac{\partial S_i}{\partial \tau}$ are positive. If $q_i \neq q_{-i}$, $(q_i - q_{-i})^2$ is equivalent to $\Delta q^2$, $\Delta q = q^h - q^l$. Given that $q^h - q^l > \frac{\tau \beta (2\pi - \beta)}{12\pi^2}$ for the case that firms share the market even when one firm’s product has a better quality than its competitor, we can show easily that both $\frac{\partial S_i}{\partial \beta}$ and $\frac{\partial S_i}{\partial \tau}$ are positive.

If the firm with better quality product can seize the entire market. It is straightforward that its price $p_m$ defined in (4.10) and thus its profit for the fixed quantity of sales decreases in both $\tau$ and $\beta$.

**Proof of Lemma 19.** For firm 1, given the realizations of the product quality $\{q_1, q_2\}$ and the effort level chosen by the managers $\{e_1, e_2\}$, the expected profit of firm 1 is
then \( e_1 \tilde{S}_1 + (1 - e_1) S_1 \), namely,

\[
E[S_1] = e_1 e_2 S_1(q^h, q^h) + e_1 (1 - e_2) S_1(q^h, q') + (1 - e_1) e_2 S_1(q', q^h) + (1 - e_1) (1 - e_2) S_1(q', q') - \frac{\gamma e_2^2}{2} \tag{4.35}
\]

With \( \gamma > 0 \), for a given \( e_2 \), \( E[S_1] \) is thus a concave function of \( e_1 \). Using the first order condition, we can obtain the best response in effort level chosen by firm \( i \)'s manager,

\[
e_1 = \frac{1}{\gamma} \left[ e_2 S_1(q^h, q^h) + (1 - e_2) S_1(q^h, q') - e_2 S_1(q', q^h) - (1 - e_2) S_1(q', q') \right]. \tag{4.36}
\]

The response function for firm 2’s manager is symmetric to (4.36). Solving the system, we can obtain the effort choice stated in Lemma 19. 

**Proof of Proposition 9.** The discussion prior to the proposition suffices to prove this proposition. 

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