Three Essays on Labor Market Volatility
Monetary Policy and Real Wage Stickiness

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Introduction

The starting point of this dissertation is related to the “unemployment volatility puzzle” raised by the seminal contribution of Shimer (2005). Shimer argues that the search and matching class of models is unable to replicate the high volatility of unemployment rates. For the US, the canonical model would reproduce less than 10% of the unemployment standard deviation. The subsequent literature brought a large body of potential solutions. Notably, Sveen and Weinke (2008) and Hornstein, Krusell and Violante (2007) take into account alternative driving forces (like demand, preference or investment-specific shocks) and not solely labor productivity shocks. Alternatively, Andres, Domenech and Ferri (2006) consider various amplification mechanisms, such as intertemporal substitution or price rigidities while Hagedorn and Manovskii (2008) emphasize the critical impact of the calibration for some parameters.

Introducing real wage rigidities is the solution that has received most attention. In search and matching models, the real wage is traditionally determined by the standard Nash bargaining. It is further assumed that information is perfect and the threat points of the parties are their outside options, which are highly volatile. The resulting flexibility of the real wage implies that firms have weak incentive to create jobs after a positive shock, explaining the low volatility of the labor market. Instead, Hall (2005) and Shimer (2005) point that introducing some real wage stickiness considerably enhances the incentive to post vacancies and then sharply magnifies unemployment fluctuations.

In this dissertation, we argue that the design of real wage rigidities is crucial to raise unemployment volatility as well as to make central banks confronted with a meaningful policy trade-off. In the search and matching literature, firms usually adjust their labor demand through employment (the extensive margin). Once firms can adjust on hours per worker (the intensive margin), Sveen and Weinke (2008) stress that the ability of real wage rigidities to amplify unemployment movements would critically depend on the way hours are determined. Those rigidities would be effective in raising labor market volatility only under the strong assumption that hours are determined jointly by the employer and the worker. When this assumption is relaxed and hours supposed to be firms’s choices, real wage stickiness would loose their ability to increase unemployment variations. This result challenges the role of that wage stickiness in solving the unemployment volatility puzzle.

In the first chapter, we show that what is critical is instead the way real wage rigidities are introduced. Sveen and Weinke (2008), as many papers, integrate those rigidities through the lens of an ad-hoc wage norm (Hall (2005)). The resulting real wage is sticky with respect to

\footnote{Initially developed by Mortensen (1982) and Pissarides (1985).}
both labor market conditions and hours per worker, which creates an incentive for firms to adjust on hours. We replace the wage norm by the micro-founded credible bargaining (Hall and Milgrom (2008)) that implements the alternating offers model of Rubinstein (1982) to the wage bargaining. Hall and Milgrom (2008) point out that on a frictional labor market, outside options are not the credible threat points. The credible threat points are rather the a-cyclical payments obtained by the players during the bargaining. The equilibrium real wage is therefore sticky with respect to labor market conditions but flexible with respect to hours per worker, which does not create an incentive for firms to adjust on hours. We find that the real wage rigidities stemming from the credible bargaining sharply amplify the unemployment dynamics, whatever the way hours per worker are selected.

On the normative side, Blanchard and Galí (2007) stress that real wage rigidities are required to make the New Keynesian model consistent with a stabilization trade-off between inflation and unemployment. In the first chapter, we also argue that the design of real wage rigidities is critical to get such a trade-off when firms can adjust on hours per worker. We show that real wage stickiness implied by the credible bargaining generates a significant monetary policy trade-off, for both ways of determining hours, while the wage rigidities entailed by the wage norm produce a stabilization trade-off only for a joint determination of hours.

In the second chapter, we object that the credible bargaining displays a moderate degree of wage stickiness and thus requires questionable values for some parameters to completely replicate the labor market volatility. At the same time, we emphasize that the asymmetric information game, which was the second way investigated by the literature to provide micro-founded real wage rigidities, also delivers a moderate amount of wage stickiness and then needs an implausible calibration to fully solve the puzzle raised by Shimer. Alternatively, we claim that the alternating offers model with one-sided asymmetric information, which merges the two frameworks, brings a more satisfactory answer to the puzzle.

This model initially dealt with a seller of an item and a potential buyer who bargain over the item’s price. Both parties alternate in making proposals and the buyer’s valuation is private information. Grossman and Perry (1986), Gul and Sonnenschein (1988) notably determine the conditions under which there is a single equilibrium. In spite of its various applications to the wage bargaining, the alternating offers model with one-sided asymmetric information was not considered by the literature on the unemployment volatility puzzle. By delivering a higher degree of wage rigidity, this model produces much more unemployment volatility than its two components taken separately. The results are improved along two dimensions. First, the model completely replicates the labor market volatility for a realistic calibration. Secondly, it restitutes the empirical real wage elasticity and therefore gives a micro-founded explanation of the real wage stickiness characterizing labor markets.

In the third chapter, written with Camille Abeille-Becker, we take an alternative road to solve the puzzle. In the first two chapters, we use a canonical search and matching frame-
work in which the separation rate is exogenous and constant. Nevertheless, the empirical standard deviation of the unemployment rate is explained for an half by the standard deviation of the job finding rate, and for the other by that of the separation rate. Therefore, the appropriate framework is rather the search and matching model with endogenous job separations depicted by Mortensen and Pissarides (1994). Mortensen and Nagypál (2007), Pissarides (2009) and Fujita and Ramey (2012) introduce such an additional margin on which firms adjust the number of their jobs. In all these papers, however, the volatility of the separation rate is restituted but the job finding variability (and thus the unemployment one) is always far below its empirical counterpart. Moreover, the model fails in reproducing the Beveridge curve, i.e. the highly negative correlation between the unemployment and vacancy rates.

We determine a calibration that makes the search and matching model with endogenous separations capable to replicate simultaneously the volatility of the unemployment, job finding and separation rates, as well as the Beveridge curve. The strategy followed is close to the one used by Hagedorn and Manovskii (2008) for the constant separation rate model: the opportunity cost of employment is calibrated to match the job finding standard deviation. We also highlight a central mechanism of the model with endogenous separations: introducing cyclical separations amplifies the volatility of the job finding rate. Intuitively, since firms have the ability to adjust employment through job separations, we could expect that they would adjust less on job creations. The job finding rate should therefore decline. On the contrary, we stress the existence of an amplification mechanism, operating through the profit of the firm, that makes the job finding response increase with the introduction of cyclical separations. This mechanism implies that the value of the opportunity cost of employment required to replicate the standard deviation of the job finding rate is lower, and more realistic, than for the model with constant separations.
Chapter 1
A New Keynesian Framework with Unemployment and Credible Bargaining: Positive and Normative Implications

1 Introduction

Real wage rigidities were advocated to provide a solution to both positive and normative issues. On the positive side, Shimer (2005) and Hall (2005) argue that those rigidities are the required feature to solve the unemployment volatility puzzle, i.e. the weak labor market volatility produced by the canonical search and matching model. On the normative side, Blanchard and Gali (2007) demonstrate that introducing some real wage rigidity into the New Keynesian (NK) framework\(^2\) is necessary to establish the sub-optimality of zero inflation policies and makes the NK model consistent with a meaningful trade-off between stabilizing inflation and unemployment.

Most of the literature related to those issues retained frameworks for which firms only adjust on employment (the extensive margin). Once firms can adjust on hours per worker (the intensive margin), Sveen and Weinke (2008) point that the ability of real wage rigidity to amplify unemployment fluctuations would critically depend on the way hours are determined. Precisely, they show that real wage rigidities would raise unemployment volatility under the strong assumption that hours per worker are determined jointly by the firm and the worker. Conversely, for the case in which hours are firms’s decisions, real wage stickiness would loose the capacity to magnify labor market dynamics. Since the latter way of determining hours seems more relevant, real wage rigidities would neither be the solution to the unemployment volatility puzzle nor the required ingredient to provide a significant stabilization trade-off.

In this chapter, I argue that what is crucial is not the manner hours are determined but instead the way real wage rigidities are integrated. Sveen and Weinke (2008), as the literature traditionally does, introduce those rigidities through the lens of an ad-hoc wage norm (Hall (2005)). Here, I replace the wage norm by the micro-founded credible bargaining (Hall and Milgrom (2008)). I find that the resulting real wage rigidities sharply amplify the unemployment dynamics and produce a substantial inflation/unemployment stabilization trade-off, whatever the way hours per worker are selected.

\(^2\)See Gali (2008) for a presentation of the canonical NK model.
The credible bargaining applies the alternating offers model of Rubinstein (1982) to the wage bargaining. Hall and Milgrom (2008) emphasize that on a frictional labor market, the joint surplus of a match is such that leaving the wage bargaining, to get the highly cyclical outside option payoffs, is not a credible threat. The only credible threat consists in delaying the moment the parties reach an agreement. The credible threat points in this scheme are the a-cyclical payments obtained by the players during the bargaining, which generates some stickiness of the real wage.

Hall and Milgrom (2008) integrate the credible bargaining in an otherwise real search and matching model, in which firms only adjust on the extensive margin and productivity shocks are the only driving force. As emphasized by Sveen and Weinke (2008), productivity variations explain a small amount of unemployment fluctuations. The bulk of labor market volatility results from demand shocks that the standard search and matching model does not allow. It is required to add a monetary dimension that enables demand shocks. Our framework is therefore a NK model with search and matching frictions. We take account of both productivity and demand shocks. Firms can adjust on the intensive margin and two ways of determining hours per worker are investigated.

Until recently, hours per worker were supposed to be determined jointly by the worker and the employer in a privately efficient manner, i.e. to maximize the joint surplus of the match. Trigari (2006) notices that hours per worker are rarely the object of a negotiation. Furthermore, Sveen and Weinke (2008) stress that although simple and convenient, this assumption has the implausible implication that the cost and number of hours are independent from the wage. A growing part of the literature now considers that hours per worker are the result of the firm’s profit maximization. Hence, a satisfactory theory of labor market volatility should be consistent with hours per worker determined by the firms.

For the credible bargaining, the replacement of the outside options by the a-cyclical disagreement payoffs entails a rigidity of the real wage with respect to labor market conditions. However, the real wage does not display such a rigidity with respect to the marginal disutility of labor and then with respect to hours. This implies that the cost of a marginal hour is flexible while the cost of an additional worker is sticky. Firms will therefore adjust less on the intensive margin and more on employment. For the wage norm specification, the real wage of the current period is assumed to be a weighted average of the flexible Nash real wage and a norm, which is usually last period’s real wage or a constant wage. The real wage is thus not only sticky with respect to labor market conditions but also with respect to the disutility of work and hours per worker. The resulting stickiness in the cost of a marginal hour creates an incentive for firms to adjust more on hours per worker and less on employment.

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3On top of the contributions of Trigari and Sveen and Weinke, see notably Christoffel and Kuester (2008) and Christoffel and Linzer (2010) for considerations related to inflation dynamics.
I find that the credible bargaining considerably enhances unemployment fluctuations and reproduces a large amount of labor market standard deviations. This result holds for both ways of determining hours per worker. Instead, the wage norm specification fails in raising labor market variations when hours are firms’ choices. In line with Sveen and Weinke (2008), the unemployment standard deviations is even lower than for the flexible Nash bargaining wage. Interestingly, the unemployment volatility is decreasing with the degree of wage stickiness.

On the normative ground, we observe that for the credible bargaining, the unemployment rate under strict inflation targeting is much more volatile than the efficient rate. Zero inflation policies therefore induce highly inefficient unemployment fluctuations which leave central banks confronted with a substantial stabilization trade-off. That result applies whatever the way hours per worker are chosen. On the contrary, for the wage norm specification and hours determined by firms, the unemployment rate under strict inflation targeting is hardly more volatile than the efficient rate.

The unability of the wage norm to raise unemployment volatility and produce an inflation/unemployment stabilization trade-off, for hours per worker selected by the firms, could question the role of real wage rigidities in solving those issues. Since the credible bargaining is capable to amplify labor market fluctuations and generate a policy trade-off whatever the determination of hours, we conclude that real wage rigidities are the required mechanism. Recall that one of the main aims of the NK literature consists in giving strong micro-foundations to macro-economic relations. An interesting implication of this chapter is that using micro-founded real wage rigidities matters not only for theoretical elegance but above all improves quantitative properties.

The rest of the chapter is organized as follows. In the next section, we present the model. In Section 3, we calibrate it and assess its quantitative results relative to labor market volatility. In Section 4, we turn to normative issues. Section 5 concludes.

2 The Model

Our framework is a dynamic, stochastic, general equilibrium model of an economy characterized by search and matching frictions in the labor market. There are three types of agents: households, firms and the monetary authority. We follow the NK literature by assuming two subsets of firms: “producers” who produce an intermediate good sold to “retailers’. Vacancy posting, hours per worker and wage bargaining are determined at the producers level while the pricing of the consumption goods is set by the retailers.
2.1 Labor market frictions

Searching for a worker to fill a vacancy involves a fixed cost $\chi$. The number of new matches each period is given by a matching function $m(u_t, v_t)$, where $u_t$ and $v_t$ represent the number of unemployed workers and the number of open job vacancies, respectively. Since the labor force is normalized to one, $u_t$ and $v_t$ also represent the unemployment and vacancy rates.

The matching rate for unemployed workers, the job-finding rate, is given by:

$$\frac{m(u_t, v_t)}{u_t} = m(1, \theta_t) \equiv f(\theta_t)$$

which is increasing in market tightness $\theta_t$, the ratio of vacancies to unemployment. The rate at which vacancies are filled is given by:

$$\frac{m(u_t, v_t)}{v_t} = \frac{f(\theta_t)}{\theta_t} \equiv q(\theta_t)$$

and is decreasing in $\theta_t$.

Matches are destroyed at the exogenous rate $s$ at the end of each period. Following Krause et al. (2008), Faia (2009) and Blanchard and Gali (2010), we assume that hiring is instantaneous. Hence, the number of employed people at period $t$ is given by the number of employed people at period $t-1$ plus the flow of new matches concluded in period $t$:

$$n_t = (1-s)n_{t-1} + q(\theta_t)v_t$$  \hspace{1cm} (1)

2.2 Households

Following Merz (1995), we assume a large representative household in which a fraction $n_t$ of members are employed in a measure-one continuum of firms. The remaining fraction $u_t = 1 - n_t$ is unemployed and searching for a job. Equal consumption across members is ensured through the pooling of incomes. The welfare of the household is given by:

$$H_t = u(c_t) - \int_0^1 n_{it} h_{it}^{1+\eta} \frac{1}{1+\eta} di + \beta EtH_{t+1}$$

where $n_{it}$ represent the number of workers and $h_{it}$ the hours per worker in firm $i \in [0, 1]$ and

$$c_t = \int_0^1 [(c_{jt})^{1-1} \frac{1}{\tau} dj]^{\frac{1}{1-1}}$$
is a Dixit-Stiglitz aggregator of different varieties of goods, with $\epsilon$ measuring the elasticity of substitution across differentiated goods. The associated price index is defined as follows:

$$P_t = \int_0^1 [(P_{jt})^{1-\epsilon} dj]^{\frac{1}{1-\epsilon}}$$

where $P_{jt}$ is the price of good $j$. The household faces the sequence of real budget constraints:

$$\int_0^1 n_{it} w_{it}(h_{it}) di + (1 - n_{it})b + \frac{\Theta_t}{P_t} + (1 + \mu_t) \frac{B_{t-1}}{P_t} \geq c_t + \frac{B_t}{P_t}$$

where $w_{it}$ is the real hourly wage earned by workers in firm $i$, $b$ is the unemployment benefit received by unemployed members, $B_{t-1}$ is the holdings of one-period nominal bonds which pay a gross nominal interest rate $(1 + \mu_t)$ one period later and $\Theta_t$ is a lump-sum component of income that may notably include dividends from the firm sector or lump-sum taxes.

The intertemporal optimality condition is given by the standard Euler condition:

$$u'(c_t) = \beta (1 + \mu_t) E_t [\frac{P_t}{P_{t+1}} u'(c_{t+1})]$$

(2)

As usual, optimality also requires that a No-Ponzi condition is satisfied.

### 2.3 Producers

We first describe the producer’s job creation condition (section 2.3.1). We next present the wage bargaining (Section 2.3.2) which will be useful to understand how hours per worker are determined (Section 2.3.3).

#### 2.3.1 Job creation

There is a measure-one continuum of identical producers who produce a homogenous intermediate good which is sold to retailers at a perfectly competitive real price $\varphi_t$. Each firm $i$ employs $n_{it}$ workers. Each worker provides $h_{it}$ hours and receives the real hourly wage $w_{it}$.

Labor is transformed into output $x_{it}$ by means of the following production function:

$$x_{it} = A_t n_{it} h_{it}$$

where $A_t$ is a common labor productivity shock. The log of this shock, $a_t = \ln A_t$ follows an AR(1) process, with autoregressive coefficient $\rho_a$ and variance $\sigma_a^2$. The firm posts $v_{it}$
vacancies each period at a cost $\chi$. The firm is assumed to be large so $s$ and $q(\theta)$ are the fraction of the workers that separate from the firm and the fraction of vacancies that are filled by the firm, respectively. Employment at the firm level is given by the following law of motion:

$$n_{it} = (1 - s)n_{it-1} + q(\theta_t)v_{it}$$  \hspace{1cm} (3)

We denote by $\Pi_{it}$ the value of the firm $i$ at period $t$:

$$\Pi_{it} = \varphi_tA_th_{it} - w_{it}h_{it}n_{it} - \chi v_{it} + E_t\beta_t,t+1\Pi_{it+1}$$

where $\beta_{t,t+k}\equiv \beta^k u'(c_{t+k})/u'(c_t)$ is the stochastic discount factor between periods $t$ and $t+k$. We denote by $\vartheta$ the Lagrange multiplier with respect to constraint (3). The firm determines the state-contingent path $\{v_{it}, n_{it}\}$ that maximizes:

$$E_0 \sum_{t=0}^{\infty} \beta_{0,t} \{ \varphi_tA_th_{it} - w_{it}h_{it}n_{it} - \chi v_{it} + \vartheta [(1 - s)n_{it-1} + q(\theta_t)v_{it} - n_{it}] \}$$  \hspace{1cm} (4)

First-order conditions for the above problem read as follows:

$$\partial v_{it} : \frac{\chi}{q(\theta_t)} = \vartheta_{it}$$  \hspace{1cm} (5)

$$\partial n_{it} : \vartheta_{it} = \varphi_tA_th_{it} - w_{it}h_{it} + (1 - s)E_t\beta_{t,t+1}\vartheta_{it+1}$$  \hspace{1cm} (6)

Equation (5) is the job creation condition: the firm posts vacancies until the value of an additional worker equates the marginal cost of posting a vacancy. The value of an additional worker is given by (6).

Merging (5) and (6), the job creation condition also reads:

$$\frac{\chi}{q(\theta_t)} = \varphi_tA_th_{it} - w_{it}h_{it} + (1 - s)E_t\beta_{t,t+1}\frac{\chi}{q(\theta_{t+1})}$$  \hspace{1cm} (7)

Before explaining how the hours per worker are determined, we have to describe the wage bargaining.
2.3.2 The wage bargaining

We denote by $W_{it}$ the worker’s value of a match in firm $i$ at period $t$:

$$W_{it} = w_{it}h_{it} - \frac{h_{it}^{1+\eta}}{(1+\eta)u'(c_t)} + E_t\beta_{t,t+1}[(1-s)W_{it+1} + sU_{t+1}]$$

where the marginal disutility of labor is expressed in consumption units and $U_t$ is the unemployment value given by:

$$U_t = b + E_t\beta_{t,t+1}[f(\theta_{t+1})W_{t+1} + (1-f(\theta_{t+1}))U_{t+1}]$$

with $b$ the value of home production or unemployment benefits. We denote by $J_{it}$ the firm’s value of a filled match at period $t$:

$$J_{it} = \varphi_tA_{it}h_{it} - w_{it}h_{it} + E_t\beta_{t,t+1}[(1-s)J_{it+1} + sV_{it+1}]$$

where $V_{it}$ is the firm’s value of a vacancy. Given the job creation condition (7), $V_{it} = 0$.

In this chapter, we follow Hall and Milgrom (2008) by assuming that the worker and the employer alternate in making wage proposals, in a Rubinstein (1982) fashion. After a proposer makes an offer, the responding party has three options:

(i) accept the current proposal;
(ii) reject it, perceive a payment - the disagreement payoff - during this period and make a counter-offer next period;
(iii) abandon the negotiation and take her outside option.

The point of Hall and Milgrom (2008) is to show that on a frictional labor market, the surplus of a match is such that both the worker and the employer get higher payoffs by going to the end of the bargaining than leaving the negotiation to get their outside options. Consequently, outside options are not credible threat points and the solution of this strategic bargaining is the same as in the alternating offers game without outside options.

Consider an offer from the firm at period $t$ that would yield the worker the value $W_{it}$. In equilibrium, this proposal is such that the worker is indifferent between accepting this offer and rejecting it, taking the disagreement payoff $b$ at period $t$ and making a just acceptable counter-offer at period $t+1$ resulting in the value $W'_{it+1}$. That is:

$$W_{it} = b + E_t\beta_{t,t+1}W'_{it+1}$$

4In the credible bargaining considered by Hall and Milgrom (2008), there is a probability that the bargain will break before reaching an agreement. Here, like Mortensen and Nagypál (2007), we omit this case for two reasons. First, this probability does not exist in the Rubinstein (1982) model. Secondly, this case is purely exogenous and has no empirical value to be compared to.

5See Osborne and Rubinstein (1990) for a demonstration.
Symmetrically, let the worker makes a proposal at period \( t \) that would yield the firm the value \( J'_it \). In equilibrium, this proposal is such that the firm is indifferent between accepting this offer and rejecting it, incurring the cost \( \gamma \) at period \( t \) and making a just acceptable counter-offer at period \( t+1 \) resulting in \( J_{it+1} \).

\[
J'_it = -\gamma + E_t \beta_{t,t+1} J_{it+1}
\]

It is rather complicated to determine the real wage directly from those equations. Instead, we implement the main result of Binmore, Rubinstein and Wolinsky (1986)\(^6\): whenever the time interval between successive offers is sufficiently small, the solution of the alternating offers model converges to the solution of the corresponding static game. The solution to this game is found by the Nash solution (1953) with the appropriate threat points. The credible threat for a player consists in delaying the time at which an agreement is reached. The proper threat points are therefore the payments obtained during the negotiation - the disagreement payoffs - i.e. \( b \) for the worker and \( -\gamma \) for the employer.

The Nash solution is such that the value get by each party is the sum of her threat point and the share of the joint surplus of the match corresponding to her bargaining power. In flow rates, this surplus-sharing rule implies for the worker:

\[
w_{it} h_{it} = \frac{h_{it}^{1+\eta}}{(1+\eta)u'(c_t)} = b + \zeta S_{it}
\]

where \( \zeta \) denotes the worker’s bargaining power and \( S_{it} \) is the joint surplus in the current period, given by:

\[
S_{it} = \varphi_t A_t h_{it} - \frac{h_{it}^{1+\eta}}{(1+\eta)u'(c_t)} - b + \gamma
\]

The real wage income resulting from the credible bargaining is therefore:

\[
w_{it}^h h_{it} = \zeta[\varphi_t A_t h_{it} + \gamma] + (1 - \zeta)[b + \frac{h_{it}^{1+\eta}}{(1+\eta)u'(c_t)}]
\]

To understand how the wage rigidity works, we compare this real wage to those resulting from the Nash bargaining and the wage norm specification. Before Shimer (2005), the Nash bargaining was traditionally applied by the search and matching literature to get the real wage. In this case, the real wage is determined by the Nash solution with the outside options as threat points. The outside options are \( U_t \) for the worker and \( V_{it} = 0 \) for the employer. The real wage income for the Nash bargaining is thus:

\[
w_{it}^n h_{it} = \zeta[\varphi_t A_t h_{it} + (1 - s)E_t \beta_{t,t+1} \chi_{t+1}] + (1 - \zeta)[b + \frac{h_{it}^{1+\eta}}{(1+\eta)u'(c_t)}]
\]

\(^6\)Mortensen and Nagypál (2007) take the same route to determine the real wage.
Comparing equations (9) and (10), the wage stemming from the credible bargaining is more rigid than the wage resulting from the Nash bargaining since $\theta_{t+1}$ does not enter equation (9). This stickiness with respect to labor market conditions reflects that the threat points are the a-cyclical disagreement payoffs rather than the pro-cyclical outside options. However, the alternative threat points for the credible bargaining do not imply any rigidity with respect to the disutility of labor. Hence, both wages display the same flexibility with respect to hours per worker.

In order to introduce real wage rigidities, most of the literature that integrates NK and search and matching models assumes that the real wage is set as a weighted average of the Nash bargaining real wage and a real “wage norm”\(^7\). This norm can take many forms but last period’s real wage or a constant real wage are usually considered. Sveen and Weinke (2008) select a constant wage as a norm. Here, we alternatively retain a backward looking norm and show that their results carry over. The real hourly wage for the wage norm specification is therefore:

$$\begin{align*}
    w_{it}^{wn} = \alpha w_{it-1}^{wn} + (1 - \alpha) w_{it}^{nb}
\end{align*}$$

where $\alpha \in [0, 1]$ measures the degree of real wage rigidity. With such a wage rule, the real wage is sticky not only with respect to labor market conditions but also with respect to hours per worker.

2.3.3 Hours per worker

The literature has retained two alternative ways of determining hours per worker.

**Joint determination of hours per worker** A first way assumes that firm and worker determine $h_{it}$ together in a privately efficient manner, i.e. so as to maximize the joint surplus of their employment relationship. Maximizing (8) with respect to $h_{it}$ gives the following first-order condition:

$$\begin{align*}
    \varphi_t A_t = \frac{h_{it}^\eta}{u'(c_{it})}
\end{align*}$$

The firm and worker select $h_{it}$ such that the marginal revenue product of labor equals the worker’s marginal rate of substitution between consumption and leisure. This standard condition also applies to the Nash bargaining and the wage norm. The consumption/leisure

\(^7\)The wage norm was initiated by Hall (2005) in the search and matching literature. Krause and Lubik (2007), Faia (2008), Sveen and Weinke (2008), Blanchard and Galí (2007, 2010), Christoffel and Linzert (2010), Krause and Lubik (2007), among others, take some form of this approach when they integrate real wage rigidities into the NK model.
marginal rate of substitution is the cost of a marginal hour for the firm, whatever the specification of the wage. Hours are therefore independent of the wage, precisely because they are chosen to maximize the joint surplus.

Firm’s choice of hours per worker The joint determination of hours has the advantage of being simple. However, as noticed by Sveen and Weinke (2008), assuming that hours per worker are determined jointly by the worker and the employer, independently of the wage, is strong. Trigari (2006) notably argues that hours of work are rarely the object of bargaining agreements. Therefore a growing part of the recent literature considers that hours per worker are firm’s choice. Formally, we keep the assumption of Sveen and Weinke (2008) that hours are chosen by the firm to maximize per-period profit:

$$\max_{h_{it}} [\varphi_t A_t h_{it} - w_{it} h_{it}]$$

which gives the following first order condition:

$$\varphi_t A_t = w_{it}'(h_{it}) h_{it} + w_{it}$$

(13)

where \( w_{it}'(h_{it}) \) is the real marginal hourly wage. Hence, contrary to what happens under the joint determination of \( h_{it} \), the cost of an additional hour depends on the wage. Using (9), (10) and (11), the cost of a marginal hour for each wage specification is:

$$w_{it}^{nb}(h_{it}) h_{it} + w_{it}^{nb} = \zeta \varphi_t A_t + (1 - \zeta) \frac{h_{it}^n}{w'_{it}(c_l)}$$

(14)

$$w_{it}^{wn}(h_{it}) h_{it} + w_{it}^{wn} = (1 - \alpha)[\zeta \varphi_t A_t + (1 - \zeta) \frac{h_{it}^n}{w'_{it}(c_l)}] + \alpha w_{it}^{wn}$$

(15)

$$w_{it}^{cb}(h_{it}) h_{it} + w_{it}^{cb} = \zeta \varphi_t A_t + (1 - \zeta) \frac{h_{it}^n}{w'_{it}(c_l)}$$

(16)

The cost of a marginal hour is the same under Nash and credible bargainings. At the same time, since \( \alpha \in [0, 1] \), the cost of a marginal hour for the wage norm is stickier than for Nash and credible bargainings.

Comparing both assumptions Finally, let us compare the differences between the two ways of determining hours. Combining equations (13) with (14) and (13) with (16) gives:

$$\varphi_t A_t = \frac{h_{it}^n}{w'_{it}(c_l)}$$
which correspond to equation (12). For the Nash and credible bargainings, the cost of a marginal hour when hours are determined by the firm corresponds to the cost when hours are determined jointly. This cost is given by the marginal rate of substitution between consumption and leisure.

Combining equations (13) and (15) gives:

\[
\varphi_t A_t = \frac{\alpha w_t w_{t-1} + (1 - \alpha)(1 - \zeta) \frac{h_t}{w(c_t)}}{1 - (1 - \alpha)\zeta}
\]  

(17)

Since \( \alpha \in [0, 1] \), the right-hand side of (17) is less volatile than the marginal rate of substitution: for the wage norm specification, the cost of a marginal hour when hours are determined by the firm is stickier than the cost of an additional hour under private efficiency.

2.4 Retailers

There is a measure-one continuum of monopolistic retailers, each of them producing one differentiated consumption good. Cost minimization by households implies that demand for each retailer \( j \), \( y_{dj} \), can be written as:

\[
y_{dj} = \left( \frac{P_{jt}}{P_t} \right)^{-\epsilon} y_t^d
\]  

(18)

where \( y_t^d \) denotes aggregate demand. I assume that vacancy posting costs take the form of the same CES function as the one defining the consumption index. Aggregate demand is therefore given by:

\[
y_t^d = c_t + \chi v_t
\]

The production of \( y_{dj}^d \) units of good \( j \) requires the same amount of the intermediate output, purchased from producers at the real price \( \varphi_t \). Thus, \( \varphi_t \) represents the real marginal cost of production for retailers.

I assume that firms reset their price in a Calvo (1983) fashion. Each period, only a randomly selected fraction \( (1 - \delta) \) of firms are able to change their price. When a firm has the chance of resetting its price, it maximizes:

\[
E_t \sum_{k=0}^{\infty} \delta^k \beta_{t,t+k} \left( \frac{P_{jt}}{P_{t+k}} - \varphi_{t+k} \right) y_{jt+k}^d
\]
with respect to $P_{jt}$, subject to (18). The optimal price setting rule for a firm resetting its price in period $t$ is:

$$E_t \sum_{k=0}^{\infty} \delta^k \beta_{t+k} P^*_{t+k} y^d_{t+k} \left( \frac{P^*_t}{P^*_{t+k}} - \frac{\epsilon}{\epsilon-1} \phi_{t+k} \right) = 0$$

(19)

where $P^*_t$ is the common price chosen by all price-setters. Hence, price-setters target a constant mark-up $M = \frac{\epsilon}{\epsilon-1} > 1$ over real marginal costs for the expected duration of the price set in period $t$.

We immediately write the optimal price setting condition in a log-linearized form around a zero-inflation steady-state. Let “hats” denote log-deviations of a variable around its steady-state value. Equation (19) could be rewritten as:

$$\log P^*_t = (1 - \beta \delta) E_t \sum_{k=0}^{\infty} (\beta \delta)^k \{ \hat{\phi}_{t+k} + \log P_{t+k} \}$$

(20)

The law of motion for the price level is given by:

$$P_t^{1-\epsilon} = \delta P_{t-1}^{1-\epsilon} + (1 - \delta)(P^*_t)^{1-\epsilon}$$

which has the following log-linear approximation:

$$\log P^*_t - \log P_t = (1 - \delta) \pi_t$$

(21)

where $\pi_t \equiv \frac{P_t}{P_{t-1}}$ is the inflation rate. Combining (20) and (21), we obtain the New-Keynesian Phillips Curve (NKPC):

$$\pi_t = \beta E_t \pi_{t+1} + \kappa \hat{\phi}_t$$

(22)

where $\kappa$ reads as follows:

$$\kappa = \frac{(1 - \delta \beta)(1 - \delta)}{\delta}$$

(23)

### 2.5 Aggregate output and market clearing

Aggregate output $y_t$ is obtained by aggregating the final goods of each retailer:

$$y_t \equiv \int_0^1 \left[ (y_{jt})^{\frac{1-\epsilon}{\epsilon}} \right]^{\frac{\epsilon}{1-\epsilon}} dt$$
The final goods market clearing condition is:

\[ y_t = y^d_t \]

which implies:

\[ y_t = c_t + \chi v_t \]  \hspace{1cm} (24)

We also derive the aggregate relation between final and intermediate goods:

\[ x_t \equiv \int_0^1 [x_{it} di] = y_t \int_0^1 \left[ \left( \frac{P_{jt}}{P_t} \right)^{-\epsilon} dj \right] \]  \hspace{1cm} (25)

We denote by \( D_t \equiv \int_0^1 \left[ \left( \frac{P_{jt}}{P_t} \right)^{-\epsilon} dj \right] \) the term capturing the inefficiency resulting from dispersion in the quantities consumed of the different final goods, which is itself a consequence of the price dispersion entailed by staggered price setting. In the neighborhood of the zero inflation steady state, we have \( D_t \approx 1 \) up to a first order approximation \(^8\). From (25), we also obtain the approximate aggregate production function:

\[ y_t = A_t n_t h_t \]  \hspace{1cm} (26)

### 2.6 Monetary policy

We follow much of the NK literature by assuming that monetary policy is described by a Taylor interest rate rule:

\[ i_t = \rho_i i_{t-1} + (1 - \rho_i)(\phi_x \pi_t + \phi_y \hat{y}_t) + m_t \]  \hspace{1cm} (27)

where \( \rho_i \) captures the degree of interest rate smoothing, \( \phi_x \) and \( \phi_y \) the responses to inflation and output variations, and \( m_t \) is an exogenous policy shock that follows an AR(1) process with autoregressive coefficient \( \rho_m \) and variance \( \sigma_m^2 \).

\(^8\)See Galí (2008, chapters 3 and 4) for details.
3 Positive implication: labor market volatility

3.1 Calibration

From now onwards, we assume the following functional forms for the preferences over consumption and the matching technology:

\[
    u(c) = \frac{c^{1-\sigma}}{1-\sigma}
\]

\[
    m(u_t, v_t) = m_0 u_t^\varsigma v_t^{1-\varsigma}
\]

Our model is composed by 9 equations: the employment law of motion (equation (1)), the Euler equation which describes the evolution of consumption (2), the job creation condition (7), the real wage (9), the real marginal cost (12), the NKPC (22), the final goods market clearing condition (24), the aggregate production function (26) and the Taylor rule (27). The log-linear equations are listed in the Appendix.

Preferences and price rigidities Time is measured in quarters. We set standard values for the discount factor \( \beta = 0.99 \) (corresponding to an annual interest rate equal to 4%) as well as for the intertemporal elasticity of substitution \( \sigma = 1 \). From the estimates of Domeij and Flodén (2006), we set \( \eta = 2 \), corresponding to a standard labor supply elasticity \((1/\eta)\) of 0.5.

Following micro evidence, the average duration of a price contract is approximately a year, which implies \( \delta = 0.75 \). The elasticity of substitution between differentiated goods \( \epsilon \) equals 7.67 (Woodford (2005)).

Labor market flows We set the separation rate \( s \) to 0.08 (Hall (1995)) and target the steady state value of the job finding rate to 0.7, which corresponds to monthly rates at less than 0.03 and 0.3, respectively. We also target the steady state value for the vacancy-to-unemployment ratio \( \theta \) to 0.72\(^9\). The resulting unemployment rate at the steady state is 10%.

This rate is somewhat higher than the observed rate but allows for potential participants in the matching market such as discouraged workers and workers loosely attached to the labor force. For the elasticity of the matching function with respect to unemployment, we select \( \varsigma = 0.5 \), in the range of Petrongolo and Pissarides (2001).

\(^9\)The sample mean for \( \theta \) in 1960-2006 (Pissarides (2009)).
**Shocks and monetary policy** The aggregate productivity shock process follows an AR(1) and based on the RBC literature is calibrated with a standard deviation $\sigma_a$ and an autocorrelation coefficient $\rho_a$ set to 0.80% and 0.95, respectively. The monetary policy is described by a Taylor rule with interest rate smoothing and standard values for the parameters $\phi_\pi = 1.5$, $\phi_y = 0.5/4$ and $\rho = 0.9$. The standard deviation $\sigma_m$ of the exogenous policy shock process is calibrated at 0.132%, to replicate the standard deviation of real output. While this is not a robust procedure, this is not essential here since the model is not evaluated along this dimension. The autocorrelation coefficient $\rho_m$ is set to 0.5.

**Wage bargaining parameters and vacancy posting costs** The worker’s bargaining power $\zeta$ is chosen at 0.5, a common practice that implies a symmetric bargaining. The flow value of unemployment $b$ is selected at 0.4, from Shimer (2005). The partial adjustment coefficient $\alpha$ of the wage norm is set to 0.5, the value commonly used in the literature.

Two parameters remain to calibrate: the cost borne by the employer during the wage bargaining $\gamma$ and the vacancy posting cost $\chi$. Since $\gamma$ is specific to the credible bargaining, $\chi$ is the last parameter to calibrate for the Nash and wage norm specifications. There are no empirical counterparts for these costs. In order to assign values to those parameters, we proceed in two steps. We begin by determining the value of $\chi$ that solves the job-creation condition at the steady state for the Nash bargaining. We next replace $\chi$ by this value and find the value of $\gamma$ that closes the job creation condition at the steady state for the credible bargaining.

This strategy has two advantages. First, there is a single value for the parameters that are common to the three wage specifications. Secondly, the real wages at the steady state under the three specifications are identical. This last point is critical for the labor market volatility: as Hagedorn and Manovskii (2008) stress, the labor market is all the more volatile as the steady state profit is low (and therefore as the steady state real wage is high). By implying identical real wages at the steady state, our calibration does not favour any particular specification.

The value of $\chi$ that closes the job creation condition at the steady state for the Nash solution is 0.23. Given this value for $\chi$, the job creation condition at the steady state under the credible bargaining is solved for $\gamma = 0.15$. All the parameters are summarized in the following table.

---

11This value is also retained by Galí (2008, 2010).
12The resulting value for $\chi$ is also the value that solves the job creation condition under the wage norm, since at the steady state the real wages for the Nash bargaining and the wage norm are identical.
13Since the resulting value for $\gamma$ equals $(1 - s)3\chi\theta$. 

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<table>
<thead>
<tr>
<th>Parameter</th>
<th>Definition</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>Discount factor</td>
<td>0.99</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Intertemporal elasticity of substitution</td>
<td>1</td>
</tr>
<tr>
<td>$\eta$</td>
<td>Convexity of labor disutility</td>
<td>2</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Fraction of unchanged prices</td>
<td>0.75</td>
</tr>
<tr>
<td>$\epsilon$</td>
<td>Elasticity of demand curves</td>
<td>7.67</td>
</tr>
<tr>
<td>$s$</td>
<td>Separation rate</td>
<td>0.08</td>
</tr>
<tr>
<td>$\varsigma$</td>
<td>Elasticity matching fct wrt vacancies</td>
<td>0.5</td>
</tr>
<tr>
<td>$\sigma_a$</td>
<td>SD of productivity shock</td>
<td>0.80%</td>
</tr>
<tr>
<td>$\rho_a$</td>
<td>AC of productivity shock</td>
<td>0.95</td>
</tr>
<tr>
<td>$\sigma_m$</td>
<td>SD of policy shock</td>
<td>0.132%</td>
</tr>
<tr>
<td>$\rho_m$</td>
<td>AC of policy shock</td>
<td>0.5</td>
</tr>
<tr>
<td>$\phi_\pi$</td>
<td>Response to inflation in the Taylor rule</td>
<td>1.5</td>
</tr>
<tr>
<td>$\phi_y$</td>
<td>Response to output gap in the Taylor rule</td>
<td>0.5/4</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Interest rate smoothing</td>
<td>0.9</td>
</tr>
<tr>
<td>$\varsigma$</td>
<td>Worker’s bargaining power</td>
<td>0.5</td>
</tr>
<tr>
<td>$b$</td>
<td>Flow value of unemployment</td>
<td>0.4</td>
</tr>
<tr>
<td>$\chi$</td>
<td>Vacancy posting cost</td>
<td>0.23</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Employer’s cost of delay</td>
<td>0.15</td>
</tr>
</tbody>
</table>

### 3.2 Labor market volatility

Tables 1 and 2 display the standard deviations of the main labor market variables for the three wage specifications. Those tables provide the results when the source of the fluctuations are productivity shocks (“Prod.”), monetary policy shocks (“Mon.”) and both types of shocks together.

Two results emerge from these tables. First, the credible bargaining replicates a great part of the labor market volatility for both ways of determining hours. Secondly, the wage norm also raises unemployment fluctuations when hours are determined jointly; when hours are firms’ choice, the wage norm produces less unemployment volatility than the Nash bargaining.
Table 1: Labor market volatility/ Credible and Nash bargainings

<table>
<thead>
<tr>
<th>Std Deviations</th>
<th></th>
<th>Credible Bargaining</th>
<th></th>
<th>Nash Bargaining</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Data&lt;sup&gt;a&lt;/sup&gt;</td>
<td>Prod.</td>
<td>Mon.</td>
<td>Both</td>
</tr>
<tr>
<td>Output</td>
<td>0.016</td>
<td>0.009</td>
<td>0.013</td>
<td>0.016</td>
</tr>
<tr>
<td>Unemployment</td>
<td>6.90</td>
<td>4.30</td>
<td>5.61</td>
<td>5.23</td>
</tr>
<tr>
<td>Vacancies</td>
<td>8.25</td>
<td>5.56</td>
<td>8.28</td>
<td>7.51</td>
</tr>
<tr>
<td>Tightness</td>
<td>14.95</td>
<td>9.80</td>
<td>13.70</td>
<td>12.58</td>
</tr>
<tr>
<td>Hours per worker</td>
<td>0.35</td>
<td>0.62</td>
<td>0.40</td>
<td>0.48</td>
</tr>
<tr>
<td>Real wage</td>
<td>0.70</td>
<td>0.77</td>
<td>1.04</td>
<td>0.96</td>
</tr>
</tbody>
</table>

<sup>a</sup>Statistics for the US economy are computed using quarterly (with a smoothing parameter of 1600) HP-filtered data from 1964:1 to 2002:3. The standard deviations of all variables are relative to output.
Table 2: Labor market volatility/ Wage norm

<table>
<thead>
<tr>
<th>Std Deviations</th>
<th>Data</th>
<th>Prod.</th>
<th>Mon.</th>
<th>Both</th>
<th>Prod.</th>
<th>Mon.</th>
<th>Both</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output</td>
<td>0.016</td>
<td>0.008</td>
<td>0.014</td>
<td>0.016</td>
<td>0.008</td>
<td>0.013</td>
<td>0.015</td>
</tr>
<tr>
<td>Unemployment</td>
<td>6.90</td>
<td>1.93</td>
<td>2.83</td>
<td>2.63</td>
<td>3.12</td>
<td>4.92</td>
<td>4.51</td>
</tr>
<tr>
<td>Vacancies</td>
<td>8.25</td>
<td>2.80</td>
<td>4.14</td>
<td>3.84</td>
<td>4.93</td>
<td>7.80</td>
<td>7.19</td>
</tr>
<tr>
<td>Tightness</td>
<td>14.95</td>
<td>4.67</td>
<td>6.86</td>
<td>6.38</td>
<td>7.86</td>
<td>12.47</td>
<td>11.48</td>
</tr>
<tr>
<td>Hours per worker</td>
<td>0.35</td>
<td>0.72</td>
<td>0.69</td>
<td>0.70</td>
<td>0.63</td>
<td>0.45</td>
<td>0.51</td>
</tr>
<tr>
<td>Real wage</td>
<td>0.70</td>
<td>0.69</td>
<td>0.83</td>
<td>0.79</td>
<td>0.80</td>
<td>1.05</td>
<td>1.01</td>
</tr>
</tbody>
</table>
Consider first what happens for the credible bargaining. Let us assume that the economy is hit by a positive (productivity or demand) shock. A producer increases production by raising hours per worker (intensive margin) and employment, through vacancy creations (extensive margin). The relative adjustment between the two margins depends on their relative costs. For both Nash and credible bargainings, equation (12) teaches that the cost of a marginal hour is given by the worker’s marginal rate of substitution between consumption and leisure. This marginal rate is increasing and convex in the number of hours. At the same time, the cost of a marginal worker is given by the real wage and the cost to post a vacancy. In the credible bargaining, the real wage is sticky, so the cost of a marginal worker increases less than for the Nash bargaining. This implies that the adjustment rests more on vacancy creations in the credible bargaining than under Nash bargaining and explains the higher standard deviations of vacancies and unemployment and the lower standard deviation of hours in Table 1.

It is worth recalling here that the cost of marginal hour for the credible and Nash bargainings is the same for both ways of determining hours per worker. The standard deviations of all variables in Table 1 then hold for both ways: the credible bargaining is capable to provide a high amount of labor market volatility, whatever the way hours are assumed to be determined.

Consider now the consequences of the positive shock for the wage norm specification. When hours per worker are determined jointly, the cost of a marginal hour is given by the worker’s marginal rate of substitution. Since the real wage is sticky, firms adjust more on the extensive margin and the unemployment fluctuations are raised. When hours are firms’s choices, equation (17) makes clear that the cost of a marginal hour is less volatile than the marginal rate of substitution: the cost of an additional hour increases by less as compared to the Nash and credible bargainings. This creates an incentive for firms to adjust more on hours per worker. This incentive is such that firms less adjust on employment than in the Nash bargaining. This lower unemployment standard deviation is in line with Sveen and Weinke (2008). In the Appendix, Table 3 further points that the unemployment volatility is collapsing in the degree of real wage stickiness $\alpha$: the higher the wage stickiness, the more rigid the cost of a marginal hour and the higher the adjustment on hours.

Tables 1 and 2 raise a critical question: why is the real wage rigidity induced by the credible bargaining capable to magnify labor market volatility while the real wage stickiness entailed by the wage norm is not, when hours per worker are firms’s decisions? The explanation is as follows. Recall that the solution of the credible bargaining corresponds to the Nash solution with the credible threat points. On a frictional labor market, those threat points are no longer the outside options, which depend on labor market conditions, but rather the a-cyclical disagreement payoffs. The real wage resulting from the credible bargaining is thus sticky with respect to labor market fluctuations. Nevertheless, the a-cyclical threat points of the credible bargaining do not deliver any wage stickiness with respect to the disutility of work and then with respect to hours per worker: as for the Nash bargaining, the cost
of marginal hour is equal to the worker’s marginal rate of substitution, which is increasing
and convex in the number of hours. With a real wage rigid with respect to labor market
conditions but flexible with respect to hours per worker, firms adjust less on hours and
more on vacancies. Instead, the wage norm specification sets the real wage as a weighted
average of the Nash bargaining wage and the last’s period wage. Consequently, the real
wage is sticky with respect to both labor market fluctuations and hours per worker. The
rigidity of the wage with respect to hours encourages firms to adjust more on hours per
worker.

Sveen and Weinke (2008) introduce real wage rigidities through the wage norm specification
and conclude that the capacity of those rigidities to solve the puzzle raised by Shimer (2005)
critically depends on the way hours per worker are chosen. Their inability to magnify
unemployment fluctuations when hours are firms’s choices would imply that real wage
rigidities are not the answer to this puzzle. From Table 1, we instead argue that what is
critical is the way that rigidities are introduced. Once real wage rigidities result from the
credible bargaining, they considerably enhance labor market volatility, whatever the way
hours are determined. Real wage rigidities are therefore the required ingredient to solve
the puzzle.

Finally, another interesting feature of the credible bargaining is related to the degree of
wage stickiness. From Table 1, the relative standard deviation of the real wage to the real
output is slightly higher than what is observed empirically. The credible bargaining then
avoids the usual criticism which states that models with real wage rigidities generate a real
wage sharply more sticky than in the data. This criticism is relevant for standard search and
matching models, which are purely real and allow no adjustment on the intensive margin.
In that models, the only variable component of the real wage is the labor productivity,
itself weakly volatile. In our monetary model with adjustment on the intensive margin, the
real marginal cost and hours per worker are other variables that enter the wage equation.
The resulting real wage is stickier than Nash bargaining wage but displays slightly more
volatility than in the data.

4 Normative implication: an inflation/unemployment stabi-
lization trade-off

Blanchard and Galí (2007, 2010) argue that for the canonical NK model, in which the real
wage is flexible, the strict inflation targeting policy is efficient. Indeed, they demonstrate
that a full stabilization of the inflation rate implies that the fluctuations of the unemploy-
ment rate mimic those of the constrained-efficient allocation, which would be the allocation
chosen by a benevolent planner. This absence of a stabilization trade-off between inflation and unemployment, at odds with conventional wisdom, is what Blanchard and Galí call the divine coincidence. Nevertheless, when real wage rigidities are introduced, the unemployment fluctuations resulting from the zero inflation policy are much more volatile than that of the constrained-efficient rate: with real wage stickiness, strict inflation targeting entails inefficient fluctuations of the unemployment rate and the central bank faces a stabilization trade-off.

In this section, I point that when firms can adjust on the intensive margin, the capacity of real wage rigidities for providing a policy trade-off critically depends on the way those rigidities are introduced. I find that for the credible bargaining, strict inflation targeting produces inefficient unemployment variations whatever the determination of hours per worker while for the wage norm specification, inefficient unemployment movements appear only for hours determined jointly.

4.1 The constrained-efficient allocation

The social planner chooses the state-contingent path of $c_t$, $h_t$, $v_t$ and $n_t$ that maximizes the joint welfare of households and managers:

$$\sum_{t=0}^{\infty} \beta^t \left\{ u(c_t) - n_t \frac{h_t^{1+\eta}}{1+\eta} \right\}$$

subject to the aggregate resource constraint:

$$A_t n_t h_t + b(1-n_t) = c_t + \chi v_t$$  \hspace{1cm} (28)

and the law of motion of employment:

$$n_t = (1-s)n_{t-1} + q(\theta_t)v_t$$  \hspace{1cm} (29)

Since the benevolent planner avoids any inefficient dispersion in relative prices, the price dispersion term $D_t$ equals 1. Using (28) to substitute for $c_t$ in the objective function, the social planner is left with the choice of $h_t$, $v_t$ and $n_t$. The first-order condition with respect to $h_t$ is given by:

$$A_t = \frac{h_t^{\eta}}{u'(c_t)}$$  \hspace{1cm} (30)

From (30), the social planner equalizes the marginal product of labor and the marginal rate of substitution between consumption and leisure. Merging first order conditions with
respect to $n_t$ and $v_t$ delivers the job creation condition for the constrained-efficient allocation:

$$
\frac{X}{q(\theta_t)} = (1 - \zeta)[A_t h_t - b - \frac{h_t^{1+\eta}}{(1 + \eta)u'(c_t)}] + (1 - s)E_t \beta_{t,t+1}[-\zeta \chi_{t+1} + \frac{X}{q(\theta_{t+1})}] \tag{31}
$$

In the equilibrium allocation of the decentralized economy, when the real wage is determined by the Nash bargaining, the first-order condition with respect to $h_{it}$ was instead given by equation (12) (whatever the way hours per worker are chosen):

$$
A_t \varphi_t = \frac{h_t^n}{u'(c_t)}
$$

The job creation condition for this decentralized allocation is obtained by substituting (10) into (7). This gives:

$$
\frac{X}{q(\theta_t)} = (1 - \zeta)[\varphi_t A_t h_t - b - \frac{h_t^{1+\eta}}{(1 + \eta)u'(c_t)}] + (1 - s)E_t \beta_{t,t+1}[-\zeta \chi_{t+1} + \frac{X}{q(\theta_{t+1})}] \tag{32}
$$

The correspondence between equations (30) and (12) on one side, and equations (31) and (32) on the other, is ensured under the three following conditions:

- $\varsigma = \zeta \quad (i)$
- $\varphi = 1 \quad (ii)$
- $\varphi_t = 1 \ \forall t \quad (iii)$

Condition (i) is the well-known Hosios (1990) condition stating that the efficient number of vacancy creations is such that the bargaining power of workers $\zeta$ equals their share in the matching technology $\varsigma$. Vacancy posting decisions generate a negative externality in the form of vacancy posting costs that reduce the resources available for consumption. The costs to post vacancies are increasing in $\varsigma$. The higher these costs, the lower the efficient number of vacancy creations. An higher $\zeta$ is therefore required since the implied higher wage induces fewer vacancy openings.

Condition (ii) deals with the goods market and requires the absence of a market power for final goods firms. Recall that $\varphi$ is the real marginal cost for retailers, which is inversely related to their mark-up. The usual way to eliminate this mark-up is to assume that retailers sales are subsidized (through lump-sum taxes) at the rate $\frac{1}{1+\epsilon}$.

Conditions (i) and (ii) imply an efficient steady state.

Condition (iii) states that the real marginal cost should be stabilized at one in every period. From the NKPC, stabilizing $\varphi_t$ corresponds to a full stabilization of the price level: a strict inflation targeting policy implements the constrained-efficient allocation.
Hence, for an efficient steady state, the zero inflation policy is optimal when the real wage is Nash-bargained. This is the point raised by Blanchard and Galí (2007): when real wages are flexible, there is divine coincidence between stabilizing inflation and stabilizing unemployment around its rate at the first best.

4.2 Strict inflation targeting and inefficient unemployment fluctuations

In what follows, we assume that the steady state is efficient. We thus set $\zeta = \varsigma$ and $\varphi = 1$. Nevertheless, for the credible bargaining, there is an additional condition required to ensure the efficiency of the steady state. The job creation condition under this wage bargaining is:

$$\frac{X}{q(\theta_t)} = (1 - \zeta)(\varphi_t A_t h_t - b - (1 - \zeta)\frac{h_t^{1+\eta}}{(1 + \eta)u'(c_t)}) - \zeta\gamma + (1 - s)\beta_{t+1}E_t \frac{X}{q(\theta_{t+1})}$$

Evaluated at the steady state and setting $\zeta = \varsigma$ and $\varphi = 1$, this equation becomes:

$$\frac{X}{q(\theta)}(1 - \beta(1 - s)) = (1 - \varsigma)[Ah - b - (1 - \varsigma)\frac{h^{1+\eta}}{(1 + \eta)u'(c)}] - \varsigma\gamma$$

Equation (33) corresponds to (34) for $\gamma = (1 - s)\beta\chi\theta$. This condition provides a positive relation between the employer’s cost borne during the wage bargaining $\gamma$ and the vacancy posting costs $\chi\theta$. The explanation is the same as the one given for the Hosios condition: the higher the costs to post vacancies, the higher is the $\gamma$ consistent with the efficient allocation since the resulting higher wage (see equation (9)) entails fewer vacancy creations.

The plain lines in Figure 1 display the efficient responses of the unemployment rate and hours per worker to a 1% negative productivity shock. The dashed lines represent the responses of that variables under the strict inflation targeting policy, i.e. the policy that keeps the price level constant in every period. This policy is implemented by fully stabilizing the real marginal cost each period. All the responses are shown in percentage points.
Figure 1: impulse responses to a 1% negative productivity shock

**Credible Bargaining**

Unemployment

Hours per worker

**Wage Norm / joint determination of hours**

Unemployment

Hours per worker

**Wage Norm / hours determined by firms**

Unemployment

Hours per worker
For the credible bargaining, the unemployment rate under strict inflation targeting is much more volatile than the efficient rate: the zero inflation policy entails large and inefficient unemployment fluctuations. At the same time, hours per worker are hardly more responsive under strict inflation targeting than the first best. The real wage stickiness resulting from the credible bargaining therefore creates a meaningful stabilization trade-off between inflation and unemployment, whatever the determination of hours per worker.

The results for the wage norm specification are still dependent on the way hours are chosen. When hours are determined jointly, the dynamics of unemployment and hours resemble to the credible bargaining. In this case, the real wage rigidities induced by the wage norm break the divine coincidence. Conversely, when hours are firms’s choices, the movements in unemployment are weak and close to the efficient rate while hours per worker display much larger variations. In this case, the real wage rigidities implied by the wage norm generate a stabilization trade-off between inflation and hours per worker, but no longer between inflation and unemployment.

Why does the real wage rigidity provided by the credible bargaining deliver an inflation/unemployment trade-off while the wage stickiness resulting from the wage norm does not, when hours are firms’ decisions? Recall that for the credible bargaining, the real wage is sticky with respect to labor market conditions but flexible with respect to disutility of work. As a result, the cost of a marginal hour is very flexible and makes firms less adjust on the intensive margin and more on employment. The unemployment rate thus displays larger fluctuations than the rate resulting from the Nash bargaining, which corresponds to the efficient rate under zero inflation policy.

For the wage norm specification, the real wage is sticky with respect to both labor market conditions and disutility of work. As we have pointed in the positive analysis, the resulting stickiness in the cost of an additional hour creates an incentive for firms to adjust more on hours per worker. This explains the higher response of hours while unemployment does not exhibit larger variations than the efficient rate.

This inability of the wage norm to provide a stabilization trade-off contracts with the result of Blanchard and Galí (2007, 2010) who introduce real wage rigidities through the lens of a wage norm. However, their analysis is led in a model in which labor supply is inelastic. In such a framework, firms can only adjust on employment and the wage stickiness resulting from the wage norm generates inefficient variations of the unemployment rate.

The failure of the wage norm to bring an inflation/unemployment stabilization trade-off when hours are determined through firms’ profit maximization could challenge the role of real wage rigidities in generating such a trade-off, as Sveen and Weinke (2008) did for the labor market volatility. We again argue that the way of introducing wage rigidities is critical. Since for the credible bargaining zero inflation policies produce large and inefficient unemployment fluctuations, whatever the way hours are determined, real wage rigidities are the required ingredient to produce a meaningful policy trade-off.
5 Conclusion

Sveen and Weinke (2008) have argued that the ability of real wage rigidities to amplify labor market volatility crucially depends on the way hours per workers are determined. In this chapter, we have pointed out that what is critical is instead the way those rigidities are introduced. We have replaced the traditional ad-hoc wage norm by the credible bargaining into a NK model with search frictions. We have shown that the resulting micro-founded real wage rigidities raise unemployment fluctuations and provide a significant inflation/unemployment stabilization trade-off, whatever the way hours are determined. Since the real wage is sticky with respect to labor market conditions but flexible with respect to disutility of work, firms adjust more and employment and less on the intensive margin. Conversely, for the wage norm specification, the real wage is sticky with respect to both labor market conditions and disutility of work. This creates an incentive for firms to adjust more on hours per worker and less on employment.
Appendix: log-linearized equilibrium conditions

We restrict attention to a log-linear approximation to the equilibrium dynamics around a zero inflation steady state. A variable without a time subscript denotes the steady state value of that variable while hats are meant to indicate log-deviations of a variable around its steady state value.

Tightness:
\[ \hat{\theta}_t = \hat{v}_t - \hat{u}_t \]

Employment law of motion:
\[ \hat{n}_t = (1 - s)\hat{n}_{t-1} + m_0 u^\gamma v^{1-\gamma}(\varsigma \hat{u}_t + (1 - \varsigma)\hat{v}_t) \]

Unemployment:
\[ \hat{u}_t = -\frac{n}{u} \hat{\kappa}_t \]

Euler equation:
\[ \hat{c}_t = E_t \hat{c}_{t+1} - (\hat{v}_t - E_t \pi_{t+1}) \]

Aggregate production function:
\[ \hat{y}_t = \hat{A}_t + \hat{h}_t + \hat{n}_t \]

Final goods market clearing condition:
\[ \hat{y}_t = \frac{c}{y} \hat{\kappa}_t + \frac{\chi}{y} \hat{v}_t \]

NKPC:
\[ \pi_t = \beta E_t \pi_{t+1} + \kappa \hat{\phi}_t \]

Real marginal cost:
\[ \hat{\phi}_t = \eta \hat{h}_t + \hat{c}_t - \hat{\theta}_t \]
Job-creation condition:

\[ \frac{\chi}{q(\theta)} q_t = \varphi A h [\hat{\varphi}_t + \hat{A}_t + \hat{h}_t] - w h [\hat{w}_t + \hat{h}_t] + (1 - s) \beta E_t \left[ \frac{\chi}{q(\theta)} q_{t+1} + \tilde{c}_t - \tilde{c}_{t+1} \right] \]

Real hourly wage:

\[ \hat{w}_t = \left\{ \zeta \varphi A [\hat{\varphi}_t + \hat{A}_t] - \zeta \gamma \hat{h}_t - (1 - \zeta) b \hat{h}_t + \frac{1 - \zeta}{1 + \eta} c [\eta \hat{h}_t + \tilde{c}_t] \right\} \frac{1}{w} \]
Table 3: Labor market volatility/ Wage norm - determination of hours by firms

<table>
<thead>
<tr>
<th>Std Deviations</th>
<th>Data</th>
<th>WN (α = 0.2)</th>
<th>WN (α = 0.4)</th>
<th>WN (α = 0.6)</th>
<th>WN (α = 0.8)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output</td>
<td>0.016</td>
<td>0.015</td>
<td>0.015</td>
<td>0.017</td>
<td>0.019</td>
</tr>
<tr>
<td>Unemployment</td>
<td>6.90</td>
<td>3.14</td>
<td>2.81</td>
<td>2.40</td>
<td>1.82</td>
</tr>
<tr>
<td>Vacancies</td>
<td>8.25</td>
<td>4.68</td>
<td>4.16</td>
<td>3.50</td>
<td>2.53</td>
</tr>
<tr>
<td>Tightness</td>
<td>14.95</td>
<td>7.72</td>
<td>6.87</td>
<td>5.83</td>
<td>4.29</td>
</tr>
<tr>
<td>Hours per worker</td>
<td>0.35</td>
<td>0.65</td>
<td>0.69</td>
<td>0.72</td>
<td>0.77</td>
</tr>
<tr>
<td>Real wage</td>
<td>0.70</td>
<td>1.28</td>
<td>0.95</td>
<td>0.64</td>
<td>0.32</td>
</tr>
</tbody>
</table>
Chapter 2
Alternating Offers with Asymmetric Information and the Unemployment Volatility Puzzle

1 Introduction

The alternating offers model with one-sided asymmetric information (henceforth “AOMOSAI”) initially considers a seller of an item and a potential buyer who bargain over the item’s price. Both parties alternate in making proposals in a Rubinstein (1982) fashion. Moreover, information is asymmetric since the seller’s valuation is common knowledge whereas the buyer’s valuation is known only to herself. In such a framework, there is a multiplicity of equilibria which explains that a literature was addressed to narrow down the range of predicted bargaining outcomes. Notably, Grossman and Perry (1986) and Gul, Sonnenschein and Wilson (1986) develop respectively the concepts of stationary equilibrium and perfect sequential equilibrium. Gul and Sonnenschein (1988) refine the conditions over strategies and time interval between successive offers that ensure a single equilibrium.

The wage bargaining is a natural implementation of that framework. In this case, the worker and the employer alternate in making wage proposals while the productivity of the match is privately observed by the employer. Within this set-up, Menzio (2007) determines the conditions under which vague noncontractual statements (found in help wanted ads) by the firms are correlated to actual wages and partially direct the search strategy of the workers. However, the AOMOSAI was not investigated by the large literature that follows the influential paper by Shimer (2005) on the unemployment volatility puzzle. Instead, this literature focuses on each component separately, i.e. the alternating offers bargaining on one hand and the asymmetric information game on the other. The point of the present chapter is to show that considering the whole model provides a more satisfactory answer to the puzzle raised by Shimer.

In order to bring real wage rigidities with strong micro-foundations, we saw in Chapter 1 that Hall and Milgrom (2008) replaced the Nash bargaining by the alternating offers bargaining. They pointed out that on a frictional labor market, the joint surplus of a match is such that the threat to leave the negotiation before reaching an agreement is not credible: the pro-cyclical outside options are not credible threat points. The only credible
threat is to delay the moment they agree. The credible threat points are therefore the a-cyclical payments obtained during the bargaining, called the disagreement payoffs.

The asymmetric information game was investigated by Kennan (2010). Firms would be subject to both aggregate and specific productivity shocks and the latters are supposed to be pro-cyclical. It is also assumed that only the employer is able to observe the specific productivity component. Kennan shows, in a generalization of the Nash bargaining to cases with private information, that the worker is prudent by considering that the specific productivity is the lowest. This strategy avoids losing the match if the realization of the shock was low whereas the worker bargains considering that it was high. The bargained real wage is therefore insensitive to the larger number of matches realizing a higher specific productivity in cyclical booms, and then delivers some rigidity.

In this chapter, I argue that the alternating offers bargaining and the asymmetric information game, separately, display only a limited real wage stickiness and thus require implausible calibration values to amplify labor market volatility. As Hagedorn and Manovskii (2008) demonstrate, what drives job creations is the variation of the firms’ profit in percentage terms. The real wage has therefore to be high and sticky. With a limited real wage rigidity in both models, this wage should be very high. Since the level of the real wage crucially depends on the disagreement payoffs, the required values for these payoffs are high and questionable. I show that for lower disagreement payoffs, the labor market volatility collapses in both models. Another calibration feature open to criticism is specific to the asymmetric information game. Indeed, to provide a sufficient amount of wage rigidity, this game needs that all the labor productivity variations result from privately observed idiosyncratic shocks. We stress that for a realistic contribution of those shocks, the unemployment volatilityplummets.

By combining the two frameworks, the AOMOSAI brings a higher level of wage stickiness that considerably increases the labor market response to aggregate shocks. The results are improved along two dimensions. First and foremost, the model solves the unemployment volatility puzzle for realistic values of the disagreement payments and a plausible contribution of specific shocks to productivity fluctuations. Secondly, the model produces the right wage elasticity with this calibration. The alternating offers model with one-sided asymmetric information then provides a completely micro-founded explanation of the real wage rigidity characterizing labor markets.

The rest of the chapter is organized as follows. In the next section, I derive the equations of the model. In Section 3, I calibrate and assess its quantitative properties. We conclude in Section 4.
2 The alternating offers model with one-sided asymmetric information

2.1 The basic structure

I consider an economy populated by a continuum of workers and a continuum of firms with measures 1. Every agent is risk-neutral and has a life of indefinite length. The current state is denoted by $i$. A job match of type $j$ produces an output at flow rate $p_i + y_j$, where $p_i$ is an aggregate component common to all matches, and $y_j$ is a random idiosyncratic variable drawn from a commonly known state varying CDF $F_i(y)$ that has strictly positive density $f_i(y)$ over the fixed interval $[y_L, y_H]$, with $y_L > 0$ and $\int_{y_L}^{y_H} f_i(y) dy = 1$.

We assume that there is a positive covariance between $p_i$ and the average (or expected) idiosyncratic productivity $\int_{y_L}^{y_H} y f_i(y) dy$, which is an important feature of Kennan (2010). This positive covariance means that the average idiosyncratic productivity is procyclical: during an economic expansion, there is an improvement in the distribution of the idiosyncratic productivity and the amount of matches with higher types increases. Kennan (2010) gives some evidence\textsuperscript{14} that supports this assumption.

The average value of total productivity (henceforth the “average productivity”) in this economy at state $i$ is given by:

$$\rho_i = p_i + \int_{y_L}^{y_H} y f_i(y) dy$$ (35)

Following a positive shock on aggregate productivity, $\rho_i$ rises both because $p_i$ and the proportion of matches with higher types increase. Note that $\rho_i$ is the productivity that we observe in the empirical data.

The rest of the framework is analogous to the standard search and matching model. The opportunity cost of employment to the worker and the cost of posting a vacancy to a firm are denoted by $z$ and $c$, respectively. The number of new matches each period is given by a matching function $m(u_i, v_i)$, where $u_i$ and $v_i$ represent the number of unemployed workers and the number of open job vacancies, respectively. Since the number of workers is normalized to 1, $u_i$ and $v_i$ also represent the unemployment and vacancy rates. The job-finding rate $f(\theta_i) = \frac{m(u_i, v_i)}{u_i} = m(1, \theta_i)$ is increasing in market tightness $\theta_i$, the ratio of vacancies to unemployment. The rate at which vacancies are filled is denoted by $q(\theta_i) = \frac{m(u_i, v_i)}{v_i} = \frac{f(\theta_i)}{\theta_i}$, and is decreasing in $\theta_i$. The form of the matching function is assumed to be Cobb-Douglas, with $m(u_i, v_i) = m_0 u_i^\eta v_i^{1-\eta}$. This implies $f(\theta_i) = m_0 \theta_i^{1-\eta}$ and $q(\theta_i) = m_0 \theta_i^{-\eta}$.

\textsuperscript{14}From Dunne, Foster, Haltiwanger and Troske (2004).
Finally, matches are destroyed at the exogenous rate $s$ and all agents have the same discount rate $r$.

I denote by $U_i$ the value of unemployment, $W_{ij}$ the worker’s value of a match of type $j$, $J_{ij}$ and $V_{ij}$ the employer’s values of a filled job and a vacancy of type $j$. All these values are determined by the Bellman equations:

\begin{align}
  rU_i &= z + f(\theta_i)(W_{ij} - U_i) + \lambda(E_iU_{i'} - U_i) \\
  rW_{ij} &= w_i(y_j) - s(W_{ij} - U_i) + \lambda(E_iW_{i'j} - W_{ij}) \\
  rJ_{ij} &= p_i + y_j - w_i(y_j) - sJ_{ij} + \lambda(E_iJ_{i'j} - J_{ij}) \\
  rV_{ij} &= -c + q(\theta_i)(J_{ij} - V_{ij}) + \lambda(E_iV_{i'j} - V_{ij})
\end{align}

where $\lambda$ represents the arrival rate of aggregate productivity shocks and $E_i$ the expectation operator conditional on the current state $i$.

Free entry is assumed on the goods market, such that the expected profit of opening a vacancy is zero ($V_{ij} = 0$). For a type $j$ match, the zero-profit condition is:

\[
\frac{c}{q(\theta_i)} = \frac{p_i + y_j - w_i(y_j) + \lambda E_i J_{i'j}}{r + s + \lambda}
\]

For the whole economy, this condition is:

\[
\frac{c}{q(\theta_i)} = \frac{\rho_i - \omega_i + \lambda E_i J_{i'}}{r + s + \lambda}
\]

with $\omega_i$ the average wage (the wage observed in the data) given by:

\[
\omega_i = \int_{y_L}^{y_H} w_i(y)f_i(y)dy
\]

Wages are assumed to be renegotiated after every aggregate shock, so the real wages determined in the next subsection only depend on the current state $i$.

### 2.2 The wage bargaining

Before Shimer (2005), the MP class of models traditionally retained the Nash bargaining to derive the equilibrium wage. This wage bargaining applies the Nash solution (1953) and identifies outside options - $U_i$ for the worker and $V_{ij} = 0$ for the employer - with threat points. The resulting joint surplus in flow rates for a type $j$ match is $p_i + y_j - rU_i$. The Nash solution is such that each party obtains the amount of her threat point and a share of the
joint surplus proportional to her bargaining power. For a type $j$ match, this surplus-sharing rule implies:

\[ w_{ij} = rU_i + \beta(p_i + y_j - rU_i) \]  

(42)

where $\beta$ denotes the worker’s bargaining power. Under the Nash surplus-sharing rule, and making use of the job creation condition,

\[ rU_i = z + \frac{\beta}{1-\beta}c\theta_i \]

Inserting this equation into (42) gives the following outcome for the real wage:

\[ w_{ij} = (1 - \beta)z + \beta(c\theta_i) + \beta(p_i + y_j) \]

The average wage given by the Nash bargaining is thus:

\[ \omega_i = (1 - \beta)z + \beta(c\theta_i) + \beta(p_i) \]  

(43)

In this chapter, I replace the Nash bargaining by the alternating offers bargaining with one-sided asymmetric information. Before turning to this game, it is useful to consider the alternating offers bargaining with perfect information. As in Rubinstein (1982), the worker and the employer are assumed to make offers alternately until they reach an agreement. After a proposer makes an offer, the responding party has three options:

(i) accept the current proposal;

(ii) reject it, perceive her disagreement payoff during this period and make a counteroffer next period;

(iii) abandon the negotiation and take her outside option.

The disagreement payoffs are the flow values received by the players during the negotiation. Without search on-the-job, the disagreement payoff of the worker is her opportunity cost of employment $z$. Since the job is idle during the wage bargaining, the disagreement payoff of the employer is assumed to be 0.

Hall and Milgrom (2008) argue that on a frictional labor market (with $f(\theta_i) < 1$), both the worker and the employer gain more by going to the end of the bargaining process rather than leaving it to take the outside options. The threat to leave the wage bargaining is then not credible. The only credible threat is to delay it.

\[15\] In the alternating offers bargaining considered by Hall and Milgrom (2008), there is an exogenous probability $\delta$ that the bargain will break before reaching an agreement. Here, like Mortensen and Nagypal (2007), we omit this case for three reasons. First, this probability does not exist in the Rubinstein (1982) model. Secondly, that probability increases the volatility of the wage (since the worker perceives the procyclical unemployment value in this case). Since the alternating offers bargaining already generates too little wage stickiness without this probability, it is pointless to add it. Thirdly, this case is purely exogenous and has no empirical value to be compared to.
To determine the equilibrium wage, we apply the main result of Binmore, Rubinstein and Wolinsky (1986), demonstrating that the strategic model of Rubinstein corresponds to the axiomatic Nash solution with the appropriate threat points. The credible threat points are not the outside options but the payments the players obtain when the bargain is delayed, i.e. the disagreement payoffs. The joint surplus of a type \( j \) match in flow rates is therefore given by \( p_i + y_j - z \), which implies the following real wage:

\[
w_i(y_j) = z + \beta(p_i + y_j - z) = (1 - \beta)z + \beta(p_i + y_j)
\]

(44)

We now introduce asymmetric information into the alternating offers bargaining to form the AOMOSAI. We assume that only the employer is able to observe the type of his match. The worker knows the distribution of \( y \) over \([y_L, y_H]\) but he is unable to observe the type of the match.

Consider the wage bargaining game between a worker and an employer that privately observes the type \( j \) of his match, with \( y_j \in [y_L, y_H] \). The worker’s beliefs at the opening of the wage bargaining are given by the CDF \( F_i(y) \) over the interval \([y_L, y_H]\). Those beliefs are consistent, that is to say updated from Bayes’ law whenever possible. Menzio (2007) argues that any sequential equilibrium of such a game has the following recursive structure. The worker proposes a wage \( w_t \). If \( y_j \) is sufficiently high, the employer accepts the proposal and the bargaining ends. If this \( y_j \) is sufficiently low, the employer rejects this proposal and makes an unacceptable counteroffer. For intermediate values of \( y_j \), the employer rejects \( w_t \) and makes a counteroffer that the worker is just willing to accept.

With such a structure, one can prove that the equilibrium wage resulting from this game cannot be higher than \( w_i(y_j) \) (given by equation (44)) which is the wage outcome of the perfect information game between the two players, and cannot be lower than \( w_i(y_L) \) which is the wage outcome of the perfect information game between a worker and an employer with the lowest match’s type. \( w_i(y_L) \) is given by:

\[
w_i(y_L) = (1 - \beta)z + \beta(p_i + y_L)
\]

(45)

The intuition behind this result is the following. By delaying the agreement, the employer can always signal that the match has relatively low idiosyncratic productivity. Notably, by refusing to trade at the wage \( w_i(y_j) \), the employer can convince the worker that the productivity of the match lies somewhere between \( y_L \) and \( y_j \).

In order to restrict the range of potential outcomes, Menzio (2007) confines attention to stationary equilibria (Gul, Sonnenschein and Wilson (1986)). Those equilibria correspond to

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16 Mortensen and Nagypál (2007) follow the same strategy to determine the real wage.
18 Together with monotonic out-of-equilibrium conjectures.
the sequential equilibria in which the employer’s strategy is stationary and monotonic. Stationarity means that the employer’s strategy depends on some history exclusively through the effect this history has on the worker’s beliefs. Monotonicity implies that if after some history the worker has more optimistic beliefs about the match’s type than after some other history, then the employer’s acceptance wage is at least as high in the former case than in the latter. When those conditions are met, then the assumptions of Theorem 1 in Gul and Sonnenschein (1988) are satisfied and they prove that, as the delay between two proposals converges to zero, all types in the interval \([y_L, y_H]\) trade instantaneously at the wage \(w_i(y_L)\) given by equation (45). Therefore, whatever the realization of the idiosyncratic productivity, the equilibrium wage corresponds to the wage outcome of the perfect information game between the worker and the employer with the lowest match’s type \(w_i(y_L)\).

This result is a generalization of the well-known “Coase Conjecture”\(^20\) to the case of alternating offers game. Intuitively, the worker is not able to threaten the firm with long delay if he is allowed to make a new proposal soon after the previous one has been rejected.

If the equilibrium wage is \(w_i(y_L)\) whatever the realization of \(y\), the average wage is also \(w_i(y_L)\). Then, \(w_i\) in the alternating offers model with one-sided asymmetric information is given by:

\[
\omega_i = (1 - \beta)z + \beta(p_i + y_L)
\]  

In order to understand how the real wage stickiness works in this model, we compare equation (46) to the average wage given by the standard Nash bargaining (equation (43)). The first source of wage rigidity is related to the alternating offers bargaining: the threat points are no longer the pro-cyclical outside options but instead the a-cyclical disagreement payoffs. This is reflected in equation (46) by the absence of the market tightness \(\theta_i\). The second source is related to the asymmetric information: the real wage is not affected by the larger amount of matches realizing higher idiosyncratic values during cyclical booms. This is reflected in equation (46) by the productivity of the lowest type, \(p_i + y_L\), that replaces the average productivity \(\rho_i\). Recall that there is a positive covariance between \(p_i\) and the average idiosyncratic productivity \(\int_{y_L}^{y_H} yf_i(y)dy\). When there is a positive shock on \(p_i\), the average productivity rises both because the aggregate productivity and the proportion of matches with higher types increase. Rather, the productivity of the lowest type rises only because \(p_i\) increases.

\(^19\)In Menzio (2007), after having observed their matches’ types, firms send messages at the time they advertise their vacancies. A worker in touch with a firm having sent a message \(s_k\) has initial beliefs about the firm’s type given by the CDF \(G(y)\) over the interval \([y_k, \bar{y}_k]\), with \(y_L \leq y_k \leq \bar{y}_k \leq y_H\). In this case, the equilibrium wage corresponds to the perfect information game outcome between the worker and the firm with the lowest type on the support of the worker’s beliefs, \(w_i(y_k)\). In our framework, firms do not send messages. Hence, the worker’s initial beliefs are given by the function \(F(y)\) over \([y_L, y_H]\) and the lowest type on the support of the worker’s beliefs corresponds to the lowest effective type \(y_k\).

\(^20\)In its original form, the Coase Conjecture states that a durable goods monopolist selling those goods to atomistic buyers would lose its monopoly power if it could make frequent price offers.
The elasticity of $\omega_i$ with respect to $\rho_i$ is obtained by dividing the log change in $\omega_i$ by the log change in $\rho_i$. For the AOMOSAI, this elasticity is given by:

$$\epsilon_\omega = \frac{\beta \rho_i \frac{d \rho_i}{d \rho}}{(1 - \beta)z + \beta(p_i + y_L)}$$

(47)

with $\frac{d \rho_i}{d \rho_i}$ the relative contribution of aggregate shocks to the average productivity fluctuations. Reciprocally, $1 - \frac{d \rho_i}{d \rho}$ is the relative contribution of privately observed specific shocks. $\frac{d \rho_i}{d \rho}$ is related to the asymmetric information side of the game and the lower $\frac{d \rho_i}{d \rho}$, the lower the wage elasticity $\epsilon_\omega$. In words, a high relative contribution of specific shocks to the average productivity variations (i.e. a low $\frac{d \rho_i}{d \rho}$) means that a high amount of matches realize higher idiosyncratic productivity levels in good states. Since the equilibrium real wage is insensitive to this idiosyncratic distribution improvement, the average real wage $\omega_i$ is all the more rigid with respect to the average productivity $\rho_i$ as $\frac{d \rho_i}{d \rho}$ is low.

It is useful for the next section to determine the equilibrium real wages implied, separately, by the alternating offers bargaining and the asymmetric information game. In the alternating offers bargaining, a type $j$ match is paid $w_i(y_j)$ (since information is perfect), given by equation (44). Hence, the average wage is:

$$\omega_i = (1 - \beta)z + \beta \rho_i$$

(48)

The wage elasticity for the alternating offers bargaining is:

$$\epsilon_\omega = \frac{\beta \rho_i}{(1 - \beta)z + \beta \rho_i}$$

(49)

The wage bargaining in the asymmetric information game considered by Kennan (2010) lasts one round. In the case of asymmetric information, the standard Nash bargaining is precluded. Instead, Kennan applies the Myerson’s (1984) neutral bargaining solution, which is a generalization of the Nash solution to the case of imperfect information. Kennan argues that the worker always bargains under the assumption that the match’s type is the lowest. With such a strategy, the worker avoids to loose the match, which would be an outcome if the match’s type was lower than the type assumed by the worker. Whatever the type of the match, the equilibrium wage is then given by the Nash solution over the lowest surplus. The average wage in the asymmetric information game is therefore given by:

$$\omega_i = (1 - \beta)z + \beta(ch_i) + \beta(p_i + y_L)$$

(50)

And the wage elasticity is:

$$\epsilon_\omega = \frac{\beta \rho_i (\frac{dp_i}{dp} + \frac{d\theta_i}{dp})}{(1 - \beta)z + \beta(ch_i) + \beta(p_i + y_L)}$$

(51)
3 Quantitative Analysis

In this section, we compare the quantative implications of four wage bargaining specifications: the Nash bargaining, the asymmetric information game, the alternating offers bargaining and the AOMOSAI. We follow Hall and Milgrom (2008) and Kennan (2010) by evaluating the labor market volatility implied by those specifications from a comparative static exercise that compares steady states at different realizations of $p_i$. We assume two states, 1 and 2, with $p_2 = 1.01p_1$ and compute the elasticity of the market tightness with respect to the average productivity$^{21}$. This elasticity is obtained by setting $\lambda = 0$ in equation (40) and by dividing the log change in $\theta$ by the log change in $\rho$. We get:

$$\epsilon_\theta = \frac{d\ln(\theta)}{d\ln(\rho)} = \frac{1}{\eta} \frac{p - \epsilon_\omega}{\rho - \omega}$$

As argued by Shimer (2005) and Pissarides (2009), this approach is a proper approximation of stochastic simulations if the productivity process is sufficiently persistent, which is the case in Shimer’s data.

3.1 Calibration

Preferences and labor market flows Time is measured in months. The discount rate $r$ is set to the standard value $0.05 \frac{12}{12}$. Following Shimer (2005), the exogenous destruction rate is chosen at 0.036. The labor market tightness at state 1, $\theta_1$, is set to 0.72 (Pissarides (2009)). The elasticity of the matching function with respect to unemployment, $\eta$, is selected at 0.6, the middle of the range of values determined by Petrongolo and Pissarides (2001).

Productivity To compute the tightness elasticity, we have to assign values to $\rho_1$, $p_1$, $y_L$ and $\frac{d\rho}{d\rho}$. We follow Kennan (2010) by normalizing $\rho_1$ at one and by assuming that in state 1, all matches realize $y_L$. This implies that $p_1 + y_L = \rho_1$. For the sake of simplicity, we set $y_L$ to be 10% of the average productivity in state 1, which results in $p_1 = 0.9$ and $y_L = 0.1$. Note that the assumption that all matches realize $y_L$ and the values retained for $p_1$ and $y_L$ have no impact on the quantitative results.

In the data, we observe $\rho_i$ and its fluctuations. However, we are not able to distinguish$^{21}$ The indicator that received most attention in the literature.
what proportions of those fluctuations are attributable to aggregate and idiosyncratic shocks
since, as Kennan (2010) argues, the latters are privately observed. Hence, there is no em-
pirical counterpart for $\frac{dp}{dp}$ to be compared to.

From equation (47), this parameter has a direct impact on the wage elasticity of the AOMOSAI. We therefore calibrate $\frac{dp}{dp}$ such that this bargaining replicates the empirical wage elasticity. Hagedorn and Manovskii (2008) find a wage elasticity of 0.45 over 1951-2004 from BLS data. The value of $\frac{dp}{dp}$ that allows to reproduce this wage elasticity is 0.63. We discuss the realism of that value in the next sub-section. It is worth recalling that $\frac{dp}{dp}$ has no impact on the wage elasticity of both the Nash and alternating offers bargainings, since these latters are led under perfect information.

**Wage bargaining parameters and vacancy posting costs** From Shimer (2005), the
flow value of unemployment $z$ is selected at 0.4, at the upper hand of the government
replacement rate in the US. Following a common practice, the worker’s bargaining power
$\beta$ is chosen at 0.5. The equilibrium real wage resulting from the AOMOSAI is independent
from the cost to post a vacancy $c$. Therefore, this parameter has no impact on the tightness
elasticity of this bargaining. Instead, this cost matters for the determination of the wage for
the asymmetric information game. The value of $c$ is chosen so as to close the job-creation
condition (equation (40)) under this wage specification at state 1. All the parameters are
summarized in the following table.

<table>
<thead>
<tr>
<th>Table 1: Parameter values</th>
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</thead>
<tbody>
<tr>
<td>Parameters</td>
</tr>
<tr>
<td>$\rho_1$</td>
</tr>
<tr>
<td>$p_1$</td>
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<tr>
<td>$y_L$</td>
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<tr>
<td>$\delta p$</td>
</tr>
<tr>
<td>$z$</td>
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<tr>
<td>$\eta$</td>
</tr>
<tr>
<td>$\beta$</td>
</tr>
<tr>
<td>$\theta_1$</td>
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<tr>
<td>$s$</td>
</tr>
<tr>
<td>$c$</td>
</tr>
<tr>
<td>$r$</td>
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</tbody>
</table>
3.2 Results and Discussion

Table 2: Results for the baseline calibration

<table>
<thead>
<tr>
<th></th>
<th>Tightness elasticity $\epsilon_\theta$</th>
<th>$\epsilon_\theta$ differential with AOMOSAI</th>
<th>Wage elasticity $\epsilon_\omega$</th>
</tr>
</thead>
<tbody>
<tr>
<td>AOMOSAI</td>
<td>3.81</td>
<td>-</td>
<td>0.45</td>
</tr>
<tr>
<td>Alternating Offers</td>
<td>2.79</td>
<td>37%</td>
<td>0.71</td>
</tr>
<tr>
<td>Asymmetric Information</td>
<td>2.61</td>
<td>46%</td>
<td>0.96</td>
</tr>
<tr>
<td>Nash</td>
<td>1.79</td>
<td>113%</td>
<td>1.00</td>
</tr>
</tbody>
</table>

The empirical value for $\epsilon_\theta$ is 7.56\textsuperscript{22}.

The AOMOSAI improves the results on two grounds. First, it considerably amplifies the labor market dynamics. Under the baseline calibration, the volatility generated by this framework is 37% and 46% greater than in the alternating offers bargaining and the asymmetric information game, respectively. This is related to the higher amount of real wage stickiness produced by the AOMOSAI. The wage rigidity in this model is higher than in the alternating offers bargaining since the real wage is not affected by the larger number of matches realizing higher idiosyncratic productivity levels during cyclical booms. At the same time, the AOMOSAI implies more wage stickiness than in the asymmetric information game since the threat points are no longer the pro-cyclical outside options but instead the a-cyclical disagreement payoffs.

Secondly, the AOMOSAI is able to replicate the observed rigidity of the real wage and thus provides an explanation of that rigidity. The model restitutes the wage elasticity for $\frac{dp}{dp}$ equals 0.63. This value means that almost two thirds of the average productivity variations would be related to aggregate shocks and one third to privately observed idiosyncratic shocks. Recall that there is no empirical counterpart for $\frac{dp}{dp}$ to be compared to. However, this proportion seems quite plausible. Indeed, in order to provide some support to the assumption of pro-cyclical specific shocks\textsuperscript{23}, Kennan (2010) finds a negative correlation between the productivity dispersion series given by Dunne et al. (2004) and the unemployment rate. Kennan concludes that this finding supports his assumption. Nevertheless, the

\textsuperscript{22}See Mortensen and Nagypál (2007) and Pissarides (2009) for details.

\textsuperscript{23}Or equivalently to the assumption of a positive covariance between $p_i$ and the average idiosyncratic productivity.
correlation identified by Kennan is moderate, which implies that privately observed specific shocks are moderately pro-cyclical. The contribution of one third of that shocks to the average productivity variations seems therefore rather realistic.

There is still a debate concerning the real wage cyclical behavior. On one side, Pissarides (2009) reviews a body of studies based on individual data, showing that there is no real wage stickiness, with a wage elasticity of 1 for new matches and 0.5 for old matches. On the other side, Hagedorn and Manovskii (2008) find an elasticity of 0.45 from both aggregate and individual data, for new matches as well as for ongoing matches. Gertler, Huckfeldt and Trigari (2008) show econometrically that the finding of Pissarides (2009) and others is due to a “cyclical composition effect” that biases the results towards more wage flexibility. The wage elasticity estimated by Gertler, Huckfeldt, Trigari (2008) and Gertler and Trigari (2009) for new matches is close to the Hagedorn and Manovskii value. The debate is not closed but if one believes that real wage stickiness is a feature characterizing labor markets, the half response of the real wage to productivity movements is a good empirical reference.

Even though the AOMOSAI delivers more labor market volatility than its two components taken separately, this model produces a tightness elasticity far under its empirical value. This is the result of the low value of the opportunity cost of employment $z$ retained under our calibration. From (46), this opportunity cost accounts for a large part of the wage. As Hagedorn and Manovskii (2008) demonstrate, what matters for the labor market variability is the response of the profit in percentage terms. To get a responsive profit in proportion, the real wage has to be high (such that the profit be small) and sticky (such that the profit be pro-cyclical). The low value of $z$ retained under our baseline calibration results in a low real wage. The alternating offers bargaining and the asymmetric information game display a limited real wage stickiness. The combination of a low real wage and a limited wage stickiness explains the weak volatility generated by these two specifications. By providing more wage rigidity, the AOMOSAI partly offsets the negative impact of the low real wage on the proportional profit’s response but only replicates half of the empirical tightness elasticity.

We follow Hall and Milgrom (2008) by introducing the value of leisure forgone into the opportunity cost of employment $z$. From a CES utility function, a labor supply elasticity set to one and hours per worker normalized at one, they estimate the value of leisure at approximately 0.3. Combined with the unemployment benefits, this value implies an opportunity cost of employment at 0.7.

![Table 3: Results for $\gamma = 0.7$](image-url)
With $z = 0.7$, the equilibrium real wage is increased for all specifications, the profit’s proportional response to shocks is higher and every model generates more labor market volatility. The AOMOSAI provides a tightness elasticity which is one quarter and one half higher than what is produced by the alternating offers bargaining and asymmetric information game, respectively. Furthermore, the AOMOSAI restitutes 91% of the empirical elasticity. Under this calibration of $z$, the value of $\frac{dp}{d\rho}$ that allows the AOMOSAI to reach a wage elasticity at 0.45 is 0.77: the average productivity variations would be explained by less than one quarter by privately observed idiosyncratic shocks. This contribution is in line with the moderate pro-cyclicality of those shocks mentioned above. Therefore, under a realistic calibration for the disagreement payoffs and a plausible relative contribution of specific shocks, the AOMOSAI replicates both the tightness and wage elasticities.

Separately, the alternating offers bargaining and asymmetric information game exhibit a limited wage rigidity. To offset this weak wage stickiness, those models require high and questionable values for the disagreement payoffs as well as for the relative contribution of idiosyncratic shocks (in the case of the asymmetric information game). Hall and Milgrom (2008) notably assume that the employer bears a cost $\gamma$ during the wage bargaining. The disagreement payoff of the employer is no longer zero but $-\gamma$ and the equilibrium real wage is increased by $\beta\gamma$. Hall and Milgrom point out that the alternating offers bargaining would generate enough volatility for $z = 0.7$ and $\gamma = 0.27$. This calibration is questionable for three reasons. First, the sum of the disagreement payoffs, which is the opportunity cost of a match, equals 0.97, i.e. just below labor productivity. This value of the opportunity cost of a match is very large and almost equivalent to the required value found by Hagedorn and Manovskii (2008) for the Nash bargaining to restitute the tightness elasticity. Secondly, the labor share that would emerge from these payoffs would be close to 100%, far from the empirical observation. Thirdly, the value of $\gamma$ is implausible: it is rather difficult to imagine that the employer bears a cost representing approximately 30% of labor productivity each period of the wage negotiation. Hall and Milgrom argue that this cost could alternatively been seen as the cost for the employer to formulate a counteroffer. Again, assuming that

<table>
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<th>Wage elasticity $\epsilon_\omega$</th>
</tr>
</thead>
<tbody>
<tr>
<td>AOMOSAI</td>
<td>6.86</td>
<td>-</td>
<td>0.45</td>
</tr>
<tr>
<td>Alternating Offers</td>
<td>5.54</td>
<td>24%</td>
<td>0.59</td>
</tr>
<tr>
<td>Asymmetric Information</td>
<td>4.65</td>
<td>47%</td>
<td>0.94</td>
</tr>
<tr>
<td>Nash</td>
<td>3.47</td>
<td>98%</td>
<td>0.99</td>
</tr>
</tbody>
</table>
the employer looses almost 30% of labor productivity to formulate a counteroffer seems strong.

The wage stickiness displayed by the asymmetric information game highly depends on \( \frac{d\rho}{d\theta} \); the lower this term, the higher the real wage rigidity. With the high values we retain for \( \frac{d\rho}{d\theta} \), the asymmetric information game delivers a small wage rigidity. Decreasing \( \frac{d\rho}{d\theta} \) could be a way to raise the wage rigidity and the labor market dynamics. In practice, this solution has only a weak impact on the wage stickiness. Indeed, recall that the threat points of the asymmetric information game are the pro-cyclical outside options. This implies that the real wage (as for the Nash bargaining) depends on the labor market tightness \( \theta \) (see equation (50)). When \( \frac{dp}{d\rho} \) is decreased, the initial effect is to make the real wage stickier, which amplifies tightness fluctuations. However, these higher variations for \( \theta \) makes in turn the real wage more volatile. This latter effect partly offsets the initial impact of \( \frac{dp}{d\rho} \) on the wage rigidity and \( \frac{dp}{d\rho} \) has to be sharply decreased to raise the wage stickiness and the labor market volatility. Precisely, the asymmetric information model would replicate the tightness elasticity only for \( \frac{dp}{d\rho} = 0 \) (with \( z = 0.7 \)). \( \frac{dp}{d\rho} = 0 \) means that the aggregate productivity would be constant across states and that the average productivity variations would be exclusively related to privately observed specific shocks: the asymmetric information game solves the puzzle raised by Shimer (2005) for highly procyclical specific shocks, at odds with Kennan’s findings.

4 Conclusion

The alternating offers bargaining and asymmetric information game provide the main solutions to the unemployment volatility puzzle resting on real wage rigidities. In this chapter, I have pointed that the alternating offers model with one-sided asymmetric information (Grossman and Perry (1986), Gul and Sonnenschein (1988)), which merges the two frameworks, gives a more satisfactory answer to the puzzle. Separately, each bargaining brings a limited wage stickiness and thus requires questionably high disagreement payoffs and relative contribution of privately observed shocks to productivity fluctuations. The AOMOSAI produces more wage rigidity that amplifies the labor market response to aggregate shocks.

This model improves the results on two grounds. First and foremost, it better replicates the labor market volatility for plausible disagreement payoffs and specific shocks relative contribution. Notably, when the value of leisure is introduced into the worker’s disagreement payoff, the model almost completely solves the puzzle. Secondly, the AOMOSAI is capable to display the right wage elasticity for a realistic pro-cyclicality of specific shocks. This model therefore gives a completely micro-founded explanation for the real wage stickiness characterizing actual labor markets.
Chapter 3
The cyclical behavior of the unemployment, job finding and separation rates

Joint with Camille Abeille-Becker

1 Introduction

The unemployment volatility puzzle, following the well-known contribution of Shimer (2005), embodies the inability of the Mortensen-Pissarides (henceforth MP) class of models to replicate the volatility of the unemployment rate in the US. Empirically, the standard deviation of the unemployment rate is explained for an half by the standard deviation of the job finding rate, and for the other by that of the separation rate (Fujita and Ramey (2009), Elsby and al.(2010)), even though this proportion is still debated. However, the first attempts to solve this puzzle narrowed on the canonical version of the matching model, in which the separation rate is purely exogenous and constant. In his paper, Shimer (2005) targeted the volatility of the job finding rate but this solution reproduces only half of the unemployment standard deviation. Hagedorn and Manovskii (2008) targeted the unemployment variability in their calibration of the worker’s opportunity cost of employment. In this case, all the unemployment volatility stems from the job finding, which clearly overstates the job finding standard deviation.

Since the seminal work of Mortensen and Pissarides (1994), the cyclical behavior of the separation rate has received an increasing attention. Mortensen and Nagypál (2007), Pissarides (2009) and Fujita and Ramey (2012) notably introduce an additional margin on which the firms adjust the number of their jobs. In all these papers, the volatility of the separation rate is restituted but the job finding variability (and thus the unemployment volatility) is always far below its empirical counterpart. Moreover, the model fails in reproducing the Beveridge curve, i.e. the highly negative correlation between the unemployment and vacancy rates.

In this chapter, we determine a calibration that makes the search and matching model with endogenous separations capable to replicate simultaneously the volatility of the unemployment, job finding and separation rates, as well as the Beveridge curve. The strategy followed is close to the one applied by Hagedorn and Manovskii (2008) for the constant separation
rate model: the value of some key parameters is chosen to match the various standard deviations and the unemployment/vacancy correlation. Particularly, the opportunity cost of employment is calibrated to reproduce the job finding standard deviation.

We also highlight a central mechanism of the model with endogenous separations: introducing cyclical separations amplifies the volatility of the job finding rate. Intuitively, since firms have the ability to adjust employment through job separations, we may expect that they would adjust less on job creations. The job finding rate would therefore be less volatile and we should find some trade-off between the job finding and separation rates variabilities. On the contrary, we stress the existence of an amplification mechanism, operating through the profit of the firm, which makes the job finding response rise with the introduction of cyclical separations. This amplification is an increasing and convex function of the opportunity cost of employment.

The amplification mechanism has two implications. First, the value of the opportunity cost of employment (around 85% of labor productivity) required to replicate the standard deviation of the job finding rate is lower, and more plausible, than for the model with constant separations. Secondly, the search and matching model with endogenous separations delivers the right standard deviations for a real wage determined by the symmetric Nash bargaining. The amplification mechanism therefore makes the model able to solve the unemployment volatility puzzle without resting on any real wage stickiness.

The rest of the chapter is organized as follows. In the next section, we present the model with endogenous separations. We also describe the amplification mechanism and how this mechanism depends on the opportunity cost of employment. In the third section, we find a calibration that replicates the right standard deviations for the unemployment, job finding and separation rate, as well as the correlation between the unemployment and vacancy rates. We also illustrate the amplification. Section 4 concludes.

2 The model with endogenous separations and the amplification mechanism

The framework is derived from Mortensen and Pissarides (1994). Here we use the presentation of Fujita and Ramey (2012).
2.1 The framework

2.1.1 Basic structure

The model with endogenous separations sets the same assumptions than the canonical one. The only element that differs is the modelisation of the separation rate, which is no longer an exogenous probability but depends on the productivity of the match.

There are two types of agents in the economy, the workers and the firms. In every period $t$, a worker is either employed, and receives a wage $w_t$, or unemployed, and receives a flow benefit $z$ (which may include unemployment benefits, the value of leisure and home production). $z$ is also the opportunity cost of employment for the worker. A firm is either matched with a worker or posting a vacancy. In this latter case, the firm bears a cost $c$.

The number of vacant jobs is denoted by $v_t$ while the number of unemployed workers is given by $u_t$. Since the labor force is normalized to one, $v_t$ and $u_t$ are also the vacancy and unemployment rates. We denote by $\theta_t = \frac{v_t}{u_t}$ the labor market tightness. The number of matches created every period is given by the following matching function $m(u_t, v_t) = A u_t^\alpha v_t^{1-\alpha}$, which is a Cobb-Douglas function with constant return to scale. We define the probability $q(\theta_t)$ of filling a vacancy by $q(\theta_t) = \frac{m(u_t, v_t)}{v_t} = A \theta_t^{1-\alpha}$ and the job finding rate by $\theta_t q(\theta_t) = \frac{m(u_t, v_t)}{u_t} = A \theta_t^{1-\alpha}$.

There are two sources of productivity in the economy: $p_t$, the aggregate productivity (macro productivity) and $x_t$, a match specific productivity factor (micro productivity). $x_t$ is a random variable, which evolves according to the c.d.f $G(x)$. Following a common practice in the literature, every match begins at the maximum micro productivity $x_h$ and has a probability $\lambda$ of being hit by a shock each period.

There are also two sources of job separation: exogenous and endogenous. In every period, a match has an exogenous probability $s$ of being destroyed. We denote by $S_t(x_t)$ the value of the joint surplus of a match. In every period $t$, The worker and the firm agree to continue the match if $S_t(x_t) > 0$. Instead, they separate if separation is jointly optimal, i.e. if $S_t(x_t) = 0$. This case occurs whenever the micro productivity falls under a certain level $R_t$, called the reservation productivity.

In period $t$, the value of a firm with a vacant job satisfies the following Bellman equation:

$$ V_t = -c + \beta E_t[q(\theta_t) J_{t+1}(x_h) + (1 - q(\theta_t)) V_{t+1}] $$

with $\beta$ the discount factor. The firm’s value of a filled match, when the continuation of this match is chosen, is given by:

$$ J_t(x_t) = p_t x_t - w_t(x_t) + \beta E_t[(1 - s)(\lambda \int_{R_t+1}^{x_h} J_{t+1}(y)dG(y) + (1 - \lambda)J_{t+1}(x_t)) + sV_{t+1}] $$
In period $t$, the unemployment value is:

$$U_t = z + \beta E_t[\theta t q(\theta t) W_{t+1}(x_h) + (1 - \theta t q(\theta t))U_{t+1}]$$

The worker’s value of a match, after continuation of this match is chosen, is:

$$W_t(x_t) = w_t(x_t) + \beta E_t[(1 - s)(\lambda \int_{R_{t+1}}^{x_h} W_{t+1}(y)dG(y) + (1 - \lambda)W_{t+1}(x_t)) + sU_{t+1}]$$

The joint surplus of a match is shared following the standard Nash bargaining. The value of the joint surplus is then defined by:

$$S_t(x_t) = J_t(x_t) + W_t(x_t) - V_t - U_t$$

The Nash bargaining entails the following surplus-sharing rule:

$$W_t(x_t) - U_t = \zeta S_t(x_t)$$
$$J_t(x_t) - V_t = (1 - \zeta)S_t(x_t)$$

with $\zeta$ the bargaining power of the worker.

### 2.1.2 Creation and destruction

The free entry condition implies that vacancies are opened as long as the expected profit is higher than the expected cost. Hence, $V_t = 0$ in equilibrium, which gives the job creation condition:

$$\beta E_t J_{t+1}(x_h) = \frac{c}{q(\theta t)}$$

Given the surplus-sharing rule, the job creation condition also reads:

$$\beta (1 - \zeta) E_t S_{t+1}(x_h) = \frac{c}{q(\theta t)} \quad (53)$$

From the definitions of $U_t$, $W_t$, $J_t$ and the surplus-sharing rule, the value of the joint surplus, when continuation of the match is chosen, can be expressed as:

$$S_t(x_t) = p_t x_t - z + \beta E_t[(1 - s)(\lambda \int_{R_{t+1}}^{x_h} S_{t+1}(y)dG(y) + (1 - \lambda)S_{t+1}(x_t)) - \theta_t q(\theta_t)\zeta S_{t+1}(x_h)]$$

$$\quad (54)$$
This equation leads to the job destruction condition. The reservation productivity corresponds to the micro productivity which cancels out the joint surplus, i.e. $S(R_t) = 0$. The job destruction condition is thus written:

$$p_t R_t = z - \beta E_t[(1-s) \lambda \int_{R_{t+1}}^{x_h} S_{t+1}(y) dG(y) - \theta_t q(\theta_t) \zeta S_{t+1}(x_h)]$$  \hspace{1cm} (55)

From equations (54) and (55), $S_t(x_t)$ also reads:

$$S_t(x_t) = p_t(x_t - R_t) + \beta(1-s)(1-\lambda)E_t S_{t+1}(x_t)$$  \hspace{1cm} (56)

### 2.2 Trade-off or amplification?

In this sub-section, we examine how the volatility of the separation rate affects the job finding standard deviation.

When endogenous separations are integrated into the canonical MP class of models, one could intuitively expect that the volatility of the separation rate would come at the expense of the vacancies and job-finding volatilities, since firms have now an additional margin on which they could adjust. Following a positive aggregate shock, firms could reduce the number of destructions and/or increase the number of vacancy creations, whereas in the constant destructions model, they are only able to adjust the number of creations.

More precisely, three effects, going in opposite directions, impact the job creation condition when endogenous separations are introduced. Recall that this condition is given by equation (53):

$$\beta(1-\zeta)E_t S_{t+1}(x_h) = \frac{c}{q(\theta_t)}$$

and the following expression for $S_t(x_h)$ (from equation (56)):

$$S_t(x_h) = p_t(x_h - R_t) + \beta(1-s)(1-\lambda)E_t S_{t+1}(x_h)$$

The first two effects are related to the discounted profit of a vacancy creation (the left-hand side of this condition) while the third effect impacts the discounted cost of such a creation (the right-hand side).

Let us assume that there is a positive aggregate shock. The first consequence is a direct increase in the flow profit associated to a vacancy and then in the discounted profit. This effect, which obviously raises the number of vacancy creations and the job-finding rate, is
present and quantitatively identical for separations being endogenous or exogenous. It is all the more powerful as $z$ is high. Indeed, Hagedorn and Manovskii (2008) stress that the dynamics of vacancy creations is driven by the variations of the firms’ profit in percentage terms. From equation (55), a higher value for $z$ raises the steady state value of $R$, which reduces the steady state joint surplus and then makes the profit more responsive in proportion.

The second effect is a reduction in the reservation productivity that increases the flow and discounted profit linked to a new job. This effect only stands in the endogenous separations model. The strength of this effect is also increasing in $z$: with a high value for $z$, the profit at the steady state is low and even a small decrease in the reservation productivity entails a sharp rise in the discounted profit in proportion.

At this stage, the job-finding volatility is increasing in $z$, with or without cyclical separations. This volatility is nonetheless higher in the endogenous separation framework because of the second effect. Consequently, adding some cyclical behavior in the separation rate amplifies job finding variations. This amplification is furthermore rising in $z$ since the second effect depends positively on this parameter.

The third effect is related to the discounted cost of vacancy posting. The unemployment rate falls following a positive aggregate shock, which entails an increase in the market tightness, a decrease in the probability for firms to fill a vacancy and then an increase in the expected duration of vacancy posting. The resulting rise in the discounted cost of vacancy posting reduces the incentive to create jobs and thus job finding fluctuations. Nevertheless, this effect is stronger in the endogenous separations model, since the fall in unemployment following the positive shock is higher than in the constant destructions model: this effect implies that introducing cyclical separations comes at the expense of job finding variations and illustrates the intuitive trade-off between the job finding and separation rates volatilities.

Whether endogenous separations amplify or reduce the job finding volatility depends on the relative strength of the second and third effects. The value of $z$ is prominent in this comparison since the quantitative impact of the second effect crucially rests on this parameter while the third effect is independent from it. For low values for $z$, the second effect is weak and should be dominated by the third effect. In this case, the job finding volatility is lower in the endogenous separations model than in the constant separations model, implying the intuitive trade-off. Alternatively, with higher values for $z$, the second effect is strong and should more than offset the third effect. The job-finding volatility is now amplified by endogenous separations.
3 Calibration strategy and the amplification in practice

3.1 Calibration

The empirical series come from the CPS $^{24}$. The unemployment, separation and job finding rates were calculated for the 1976 - 2010 period. These monthly data were seasonally adjusted and then HP filtered. The mean unemployment, job finding and separation rates are respectively 0.061, 0.017 and 0.305. The standard deviations of those rates are given in Table 2.

In order to calibrate the model with cyclical separations, we follow the strategy initiated by Hagedorn and Manovskii (2008) for the model with constant separations: the values of some key parameters are determined to replicate the standard deviations of the unemployment, job finding and separation rates, as well as the correlation between vacancy and unemployment rates. Three parameters are critical to get the right volatilities and the Beveridge curve: the opportunity cost of employment $z$, the standard deviation of specific shocks $\sigma_x$ and their arrival rate $\lambda$.

We have argued above that the opportunity cost of employment has a strong impact on the variations of the job finding rate. A high value for $z$ implies a high volatility for this rate, since the firm’s profit is more responsive in proportion. We therefore select the value of $z$ so as to reproduce the empirical standard deviation of the job finding rate. We obtain a value of 0.86 for this parameter: the search and matching model with endogenous separations generates the right volatility of the job finding rate for an opportunity cost of employment representing 86% of labor productivity. This value is between the evaluation of Mortensen and Nagypál (2007) (0.73) and the value found by Hagedorn and Manovskii (2008) for the model with exogenous separations (0.96). The evaluation of Mortensen and Nagypál includes unemployment benefits and the value of leisure forgone amounting to 30% and 43% of labor productivity, respectively. Recall that $z$ could also include the value of home production. The value we find for the model with cyclical separations means that home production would represent 13% of labor productivity.

The standard deviation of specific shocks has a strong impact on the variations of the separation rate. Indeed, $\sigma_x$ determines the size of specific shocks. A high value for this parameter means that specific shocks are large. The movements of the reservation productivity are therefore magnified, making the separation rate more volatile. We thus choose the value of $\sigma_x$ to replicate the empirical standard deviation of the separation rate. We obtain a value of 0.035 for $\sigma_x$, which is close to Fujita and Ramey (2012).

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$^{24}$This data was constructed by Robert Shimer. For additional details, see Shimer (2007) and his webpage http://sites.google.com/site/robertshimer/research/flows.
The arrival rate of specific shocks has an impact on both the vacancy and unemployment rates. A high value for $\lambda$ means that the probability for a new match to be hit by a negative specific shock is high, which reduces the expected duration of this match. Hence, the incentive to post vacancies is also reduced. At the same time, a high value for $\lambda$ implies that specific shocks are weakly persistent. This creates an incentive for firms to dampen job separations following a negative specific shock. The separation and unemployment rates are consequently less volatile. By affecting both vacancy and unemployment rates, $\lambda$ has an impact on the Beveridge curve. We select the value that allows the model to deliver the empirical correlation between those rates and find a value of 0.1 for $\lambda$. Again, this value is close to Fujita and Ramey (2012) and represents a mean waiting time of three months between two shocks.

The remaining parameters are chosen in a standard way. The aggregate productivity is simulated with an AR(1) process $p_t = \mu_p p_{t-1} + \sigma_p$, with $\mu_p = 0.9895$ and $\sigma_p = 0.0034$. This is the process retained by Hagedorn and Manovskii (2008) and followed by Fujita and Ramey (2012). The discount factor is equal to $\beta = \frac{1}{1+r}$, with $r = 0.04/12$. This corresponds to an annual interest rate of 4%, consistent with the data. The wage bargaining is assumed to be symmetric, an assumption often made in the literature. This implies a value of 0.5 for $\zeta$. The elasticity of the matching function $\alpha$ is set to 0.5, in the range determined by Petrongolo and Pissarides (2001). The efficiency parameter of the matching function $A$ is determined in order to replicate the empirical level of the job finding rate. The cost of posting a vacancy ensures that the job creation always holds at sample mean, for a value of the tightness equal to 0.72 at steady-state (Pissarides (2009)). $G(x)$ is assumed to be a log normal distribution function, of zero mean. The highest match-specific productivity $x_h$ is set to 1. Finally, The exogenous separation rate $s$ is selected to replicate the empirical value of the separation rate.

Table 1 summarizes all the parameters.

Table 1: Parameter values
### Table 2: Data and results

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Baseline calibration</th>
<th>Target</th>
</tr>
</thead>
<tbody>
<tr>
<td>$z$</td>
<td>0.86</td>
<td>job finding standard deviation</td>
</tr>
<tr>
<td>$\sigma_x$</td>
<td>0.035</td>
<td>separation rate standard deviation</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>0.1</td>
<td>Beveridge curve</td>
</tr>
<tr>
<td>$\mu_p$</td>
<td>0.98</td>
<td>HM (2008)</td>
</tr>
<tr>
<td>$\sigma_p$</td>
<td>0.0034</td>
<td>HM (2008)</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.99</td>
<td>annual interest rate of 4%</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.5</td>
<td>Petrongolo-Pissarides (2001)</td>
</tr>
<tr>
<td>$\zeta$</td>
<td>0.5</td>
<td>symmetric bargaining</td>
</tr>
<tr>
<td>$\mu_x$</td>
<td>0</td>
<td>Ramey (2008)</td>
</tr>
<tr>
<td>$x_h$</td>
<td>1</td>
<td>Mortensen Nagypál (2007)</td>
</tr>
<tr>
<td>$A$</td>
<td>0.36</td>
<td>job finding rate</td>
</tr>
<tr>
<td>$c$</td>
<td>0.13</td>
<td>mean $\theta$</td>
</tr>
<tr>
<td>$s$</td>
<td>0.015</td>
<td>separation rate</td>
</tr>
<tr>
<td>$\theta$</td>
<td>0.72</td>
<td>CPS</td>
</tr>
<tr>
<td>$A\theta_i^{1-\alpha}$</td>
<td>0.305</td>
<td>CPS</td>
</tr>
</tbody>
</table>

### Data, Mean values, Source

| $\mu_p$   | 0.98 | HM (2008) |
| $\sigma_p$| 0.0034 | HM (2008) |
| $\beta$   | 0.99 | annual interest rate of 4% |
| $\alpha$  | 0.5 | Petrongolo-Pissarides (2001) |
| $\zeta$   | 0.5 | symmetric bargaining |
| $\mu_x$   | 0 | Ramey (2008) |
| $x_h$      | 1 | Mortensen Nagypál (2007) |
| $A$        | 0.36 | job finding rate |
| $c$        | 0.13 | mean $\theta$ |
| $s$        | 0.015 | separation rate |
| $\theta$  | 0.72 | CPS |
| $A\theta_i^{1-\alpha}$ | 0.305 | CPS |

#### 3.2 The amplification in practice

To illustrate how the amplification mechanism works and depends on $z$, we compare the job finding standard deviations for, on the one hand, the MP framework with endogenous separations and, on the other, the framework with a constant separation rate. Table 3 displays the standard deviations of the job finding rate for each value of $z$.

In order to get the standard deviation of the job finding rate for the model with a constant separation rate, we set $s$ at 0.017 and $\lambda$ at 0. In this case, the match specific productivity is drawn at the first period and does not change thereafter, given that the probability to be hit by a match-specific productivity shock is now equal to zero.
Table 3: Job finding volatility

<table>
<thead>
<tr>
<th>$z$</th>
<th>0.3</th>
<th>0.4</th>
<th>0.5</th>
<th>0.6</th>
<th>0.7</th>
<th>0.8</th>
<th>0.86</th>
<th>0.93</th>
<th>0.96</th>
<th>0.99</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)</td>
<td>0.011</td>
<td>0.014</td>
<td>0.017</td>
<td>0.022</td>
<td>0.029</td>
<td>0.047</td>
<td>0.068</td>
<td>0.088</td>
<td>0.109</td>
<td>0.154</td>
</tr>
<tr>
<td>(2)</td>
<td>0.011</td>
<td>0.013</td>
<td>0.016</td>
<td>0.019</td>
<td>0.025</td>
<td>0.040</td>
<td>0.056</td>
<td>0.070</td>
<td>0.082</td>
<td>0.111</td>
</tr>
<tr>
<td>(1) - (2)</td>
<td>0</td>
<td>0.001</td>
<td>0.001</td>
<td>0.003</td>
<td>0.004</td>
<td>0.007</td>
<td>0.012</td>
<td>0.018</td>
<td>0.027</td>
<td>0.043</td>
</tr>
</tbody>
</table>

(1): job finding standard deviation in the endogenous separation framework
(2): job finding standard deviation in the exogenous separation framework

For a worker’s opportunity cost below 0.4, the job finding standard deviations for the two frameworks are quite similar, meaning that the second and third effects depicted in Section 2.2 cancel each other out. In this range of values for $z$, introducing a cyclical behavior for the separations neither amplifies nor reduces the job finding volatility.

From $z = 0.4$, the job finding standard deviation is higher for the endogenous separations model than for the exogenous one. The second effect dominates the third effect. Introducing cyclical separations now amplifies the job finding response to aggregate productivity shocks. As shown in Table 3, this amplification is increasing and convex in the value of $z$. For $z = 0.4$, the standard deviations difference is only equal to 0.001 while for $z = 0.86$, this difference amounts to 0.012: for this value of $z$, the model with endogenous separations produces a volatility of the job finding rate 22% higher than the model with a constant separation rate. As $z$ approaches 1, the amplification becomes considerable: at $z = 0.99$, the job finding standard deviation difference amounts to 0.043, which is two thirds the empirical value (0.0687).

This amplification has two consequences. First, the value of $z$ required to replicate the job finding standard deviation for the model with endogenous separations is lower than for the model with a constant separation rate. For this latter model, the required value for $z$ is around 0.93 (and 0.96 for the calibration of Hagedorn and Manovskii (2008)). Introducing cyclical separations thus enables the search and matching model to restitute the job finding volatility for a lower, and more realistic, value of the opportunity cost of employment.

Secondly, our calibration does not imply any real wage stickiness. The elasticity of the real wage with respect to productivity is equal to 0.96. Therefore, the amplification makes the model with endogenous separations able to reproduce the various standard deviations and then to solve the unemployment volatility puzzle without resting on real wage rigidities. This is another difference with the model with exogenous separations: even with their high value for $z$, Hagedorn and Manovskii (2008) need sharp wage rigidities to solve this puzzle.
4 Conclusion

In this chapter, we have followed a strategy close to Hagedorn and Manovskii (2008) by determining a calibration that makes the MP framework replicate the unemployment standard deviation with the right contributions of the job finding and separation rates, and the Beveridge curve. We have also stressed a central mechanism of the model with endogenous separations: introducing cyclical separations amplifies the volatility of the job finding rate. This mechanism dominates the intuitive trade-off between job finding and separation rates variabilities. The amplification mechanism has two implications. First, the required value for the opportunity cost of employment in the model with cyclical separations is lower, and more plausible, than in the model with exogenous separations. Secondly, the framework with endogenous separations solves the unemployment volatility puzzle with the symmetric Nash bargaining, i.e. without resting on real wage rigidities.
Conclusion

In this dissertation, we have pointed out the critical role of the design of real wage stickiness for both raising labor market volatility and producing a monetary policy trade-off.

In the first chapter, we have stressed that contrary to what Sveen and Weinke (2008) argued, what matters for unemployment fluctuations is not how firms determine hours per worker but instead the way real wage rigidities are introduced. We have replaced the traditional ad-hoc wage norm by the credible bargaining into a NK model with search frictions and shown that the resulting real wage stickiness magnifies unemployment variations and delivers a significant inflation/unemployment stabilization trade-off, whatever the determination of hours.

In the second chapter, we have objected that the credible bargaining displays a moderate amount of real wage stickiness and thus requires questionable values for some key parameters to completely reproduce the labor market volatility. We have found that the same result applies to the asymmetric information game, which was the other way retained by the literature to generate micro-founded wage rigidities. We have emphasized that merging the two frameworks brings a more satisfactory answer to the unemployment volatility puzzle. The alternating offers model with one-sided asymmetric information not only replicates the labor market volatility for a realistic calibration but also provides a micro-founded explanation of real wage rigidities.

In the third chapter, we have taken another road to solve the puzzle by considering the search and matching model with cyclical job separations. We have determined a calibration that allows this model to restitute the volatility of the unemployment rate with the right contributions for the job finding and separation rates, as well as the Beveridge curve. We have also pointed a central mechanism of that framework: introducing cyclical separations amplifies the volatility of the job finding rate. This amplification mechanism makes the model with endogenous separations able to produce the right volatilities for a lower, and more realistic, value of the opportunity cost of employment than for the model with constant separations.
References


