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Electrical Control of the Electron spin dynamics in $\langle 111 \rangle$ -oriented GaAs/AlGaAs quantum wells

by

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To my family

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Abstract

We have studied the electron spin dynamics in $\langle 111 \rangle$ -oriented GaAs/AlGaAs quantum wells grown on $\langle 111 \rangle$ -substrate by time-resolved photoluminescence spectroscopy. By applying an external electric field about 50 kV/cm along growth direction, we observed the spectacular increase of electron spin which can attain values greater than 30 ns. This phenomenon comes from the electrical control of spin-orbit interaction in conduction band that make the Rashba term compensate exactly with the Dresselhaus term. The cancellation effect of these two terms results in the suppression of electron spin relaxation induced by D'yakonov-Perel mechanism which is dominant in undoped quantum wells and at the temperatures greater than 50K. The measurement under an external transverse magnetic field (Voigt configuration) demonstrates that the spin relaxation times in three spatial directions are also controlled simultaneously by electric field. The "total" control of electron spin relaxation can only be observed in $\langle 111 \rangle$ -oriented quantum wells. Finally, we also develop the model to interpret the experimental measurement of spin relaxation anisotropy depending on electric field in $\langle 111 \rangle$ -oriented quantum wells.

Résumé

Nous avons étudié la dynamique de relaxation de spin des électrons dans des puits quantiques GaAs/AlGaAs élaborés sur substrat d'orientation $\langle 111 \rangle$ par spectroscopie de photoluminescence résolue en temps. En appliquant un champ électrique externe d'environ 50 kV/cm le long de l'axe de croissance, nous avons observé une augmentation spectaculaire du temps de relaxation de spin de l'électron qui peut atteindre des valeurs plus grandes que 30 ns. Ceci est le résultat du contrôle électrique du décalage en énergie "spin-orbite" de la bande de conduction qui peut s'annuler lorsque le terme de Rashba compense exactement celui de Dresselhaus. Ceci entraîne une suppression du mécanisme de relaxation de spin de type D'Yakonov-Perel, mécanisme dominant dans les puits quantiques non-dopés à des températures supérieures à 50 K. Les mesures effectuées sous champ magnétique externe transverse (configuration Voigt) montrent que les temps de relaxation de spin pour les trois directions de l'espace peuvent être contrôlés simultanément par le champ électrique. Ce contrôle "total" de la relaxation de spin ne peut se produire que pour des puits quantiques élaborés sur une orientation $\langle 111 \rangle$. Nous avons finalement développé un modèle permettant d'interpréter les mesures expérimentales de la dépendance en champ électrique de l'anisotropie de la relaxation de spin dans ces puits quantiques $\langle 111 \rangle$.

Introduction

In recent years, spintronics has become one of the emerging disciplines that have been progressively investigated all over the world. Indeed, the spin of electron was ignored in classical electronics and only electron charge was used in information technologies up to now. The main aim of spintronics is to manipulate the spin of electrons in order to carry and process information; this could open the way for a next generation of semiconductor devices which could have some advantages such as increased data storage, low electrical power consumption, high data processing rate, ...etc compared with conventional semiconductor devices. Until now, spintronics can be divided into two distinct fields : metal spintronics [72] and semiconductor spintronics. The first one gave rise to several important applications such as Magnetic Hard Disk with reading heads based on tunnel magnetoresistance (TMR) effect. The latter one has been explored in fundamental science but is not exploited in the world of industrial applications until now. One of the first novel idea of semiconductor spintronic devices that has opened a new research field in condensed matter physics was the field effect spin transistor proposed by Datta and Das [12]. Within the framework of this thesis, we will focus on the study of spin physics in III-V zinc-blende semiconductors.

Thanks to the evolution of nanotechnologies, the dimensions of nanostructures have shrunk to sizes smaller than De-Broglie wavelength validating quantum effects in semiconductor devices and constituting new generations of quantum devices that upgrade information technology to quantum information. There are three quantum structures that have been studying up to now: quantum wells (2D), quantum wires (1D) and quantum dots (0D) with different physical characteristics. In this manuscript, we will study the electron spin properties in quantum wells grown on non-classical crystal orientation.

This thesis is divided into four chapters :

- (i) **Chapter 1** : The first chapter recalls some general bases in electronic and optical properties of zinc-blende semiconductor such as Bloch theorem, $\mathbf{k}\cdot\mathbf{p}$ method, optical selection rules and spin relaxation mechanisms which are necessary to understand the physical phenomenon presented in this thesis.
- (ii) **Chapter 2** : The second chapter introduces the samples structure and the time-resolved photoluminescence (TRPL) spectroscopy setup used for measuring the electron spin relaxation in $\langle 111 \rangle$ -oriented quantum wells under an adjustable external gate voltage.
- (iii) **Chapter 3** : The third chapter presents all the experimental results of electron spin relaxation time measured by TRPL and spin quantum beat techniques. These results demonstrate that the electron spin relaxation time increases under the influence of a transverse electric field created by the gate voltage and show the ability of electrically controlling spin relaxation anisotropy in quantum wells.
- (iv) **Chapter 4** : The fourth chapter presents two theoretical models : a simple model and a spin density matrix model in order to study the electrically controlled spin relaxation anisotropy. The simple model explains the role of linear and cubic terms of $\mathbf{k}$ -vector in the total Hamiltonian of electron which give contributions to the cancellation effect. The spin density matrix model is based on a more sophisticated description of the electron spin relaxation mechanism. A good agreement between the calculated and experimental results is obtained.

Chapter 1

Introduction to electronic and optical properties of zinc-blende III-V semiconductors

1.1 Bulk materials

To understand optical properties of semiconductors, it is necessary to know the electronic properties of semiconductors and the corresponding wave functions. Most of the semiconductors used for optical devices are direct bandgaps with many interesting optical phenomenon near the band edge. This section recalls some basic concepts of semiconductor band structures in bulk materials.

1.1.1 Zinc-blende structure

In the area of semiconductor optical devices, the most common crystal structure is the diamond lattices in which each atom has a covalent band with four adjacent atoms forming a tetrahedron. This lattice is constituted from two inter-penetrating face-centered-cubic lattice (FCC), which are displaced along the body diagonal of the larger cube by one quarter of that body diagonal. Two important elemental semiconductors that have a conventional diamond lattice are silicon (Si) and germanium (Ge) as depicted in figure 1.1(a).

Semiconductors are also formed by combining elements to make covalent bonds.

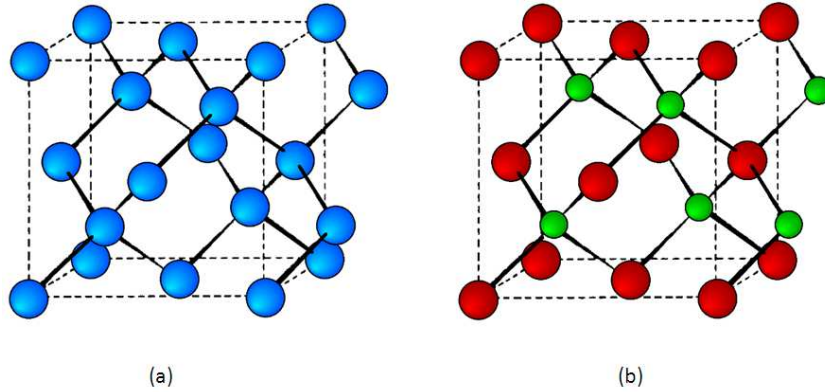


Figure 1.1: Tetrahedral crystal compound structures : (a) diamond lattice (b) zinc-blende lattice

They are compound semiconductors such as binary semiconductor (GaAs), ternary semiconductor (AlGaAs), quaternary semiconductor (InGaAsP),... which offer high performance (optical characteristics, bandgap engineering,...) due to the mixing of materials and greater device design flexibility. Zinc-blende semiconductors are the important compounds semiconductors used in optical devices. They have the same lattice structure with diamond except that the FCC is formed by two different materials. Actually, III-V zinc-blende semiconductor is the combi-

nation of one element of group III and one element of group V in periodic table. This is possible since group III elements have 3 electrons in their outermost shell and group V elements have 5 electrons in their outermost shell. The tetrahedron of III-V zinc-blende semiconductors is shown in figure 1.1(b).

1.1.2 Bloch theorem

In a crystal lattice, the electron moves in a periodic potential $V(\vec{r})$ which is generated by the periodic structure of solid atomic lattice and satisfied this equation :

$$V(\vec{r} + \vec{R}) = V(\vec{r})$$

where $\vec{R}$ is the Bravais lattice vector that are mapped by translational symmetry.

The stationary state for the electron obeys the Schrödinger equation :

$$H\psi = E\psi$$

In the periodic potential of solid, the Hamiltonian of an electron can be written as :

$$H = -\frac{\hbar^2}{m}\nabla_r^2 + V(\vec{r}) \quad (1.1)$$

Bloch's theorem states that the eigenstate ψ of the Hamiltonian H in equation 1.1 can be chosen to have the form of a plane wave multiplied by a function with the period of the Bravais lattice :

$$\psi_{nk}(\vec{r}) = e^{i\vec{k} \cdot \vec{r}} u_{nk}(\vec{r}) \quad (1.2)$$

where $u_{nk}(\vec{r} + \vec{R}) = u_{nk}(\vec{r})$ is the function having the same periodicity of crystal potential, the quantum number $n = 1, 2, 3, \dots$ is called the *band index*, $\mathbf{k}$ is a wavevector associated with an electron. It should be noted that the Bloch function $u_{nk}(\vec{r})$ has two subscripts $\mathbf{n}$ and $\mathbf{k}$ indicating the dependence of this function on $\mathbf{n}$ and $\mathbf{k}$.

1.1.3 k.p method

For calculating band structure at $k \neq 0$, **k.p** method is one of most important theories. The **k.p** method was originally an application of the perturbation theory used to study energy bands and wave functions in the vicinity of several important points in $\mathbf{k}$ space. Dresselhaus et al. [1] presented a rigorous **k.p** method for the empirical determination of band structure. Symmetry arguments

used by $\mathbf{k.p}$ show that band structure in a small region of $\mathbf{k}$ space depends on a few parameters. Extensive derivations and calculations of Kane using $\mathbf{k.p}$ method [68, 86] have transformed a perturbative approach to one of the main method to calculate band structures in semiconductor.

Principle

Taking into account the spin-orbit coupling term, the electron Hamiltonian is written as:

$$H = \frac{\mathbf{p}^2}{2m_0} + \mathbf{V}(\mathbf{r}) + \frac{\hbar}{4m_0^2c^2}(\boldsymbol{\sigma} \times \nabla \mathbf{V}) \cdot \mathbf{p}$$

where m_0 is the mass of free electron, $\boldsymbol{\sigma}$ is Pauli spin matrix, $\mathbf{p}$ is momentum operator.

Using the Bloch wavefunction $u_{nk}(\vec{r})$ and substituting momentum operator $\mathbf{p}$ by $-i\hbar\nabla$, we have a full equation on u_{nk}

$$\left(\frac{\mathbf{p}^2}{2m_0} + \mathbf{V}(\mathbf{r}) + \frac{\hbar^2 \mathbf{k}^2}{2m_0} + \frac{\hbar \mathbf{k} \cdot \mathbf{p}}{m_0} + \frac{\hbar}{4m_0^2c^2}(\boldsymbol{\sigma} \times \nabla \mathbf{V}) \cdot (\hbar \mathbf{k} + \mathbf{p})\right) u_{nk}(\mathbf{r}) = E_{nk} u_{nk}(\mathbf{r}) \quad (1.3)$$

With the notation

$$\boldsymbol{\pi} = \mathbf{p} + \frac{\hbar}{4m_0c^2}(\boldsymbol{\sigma} \times \nabla \mathbf{V})$$

equation 1.3 becomes :

$$\left(\frac{\mathbf{p}^2}{2m_0} + \mathbf{V}(\mathbf{r}) + \frac{\hbar^2 \mathbf{k}^2}{2m_0} + \frac{\hbar}{m_0} \mathbf{k} \cdot \boldsymbol{\pi} + \frac{\hbar}{4m_0^2c^2}(\boldsymbol{\sigma} \times \nabla \mathbf{V}) \cdot \mathbf{p}\right) u_{nk}(\mathbf{r}) = E_{nk} u_{nk}(\mathbf{r}) \quad (1.4)$$

For $\mathbf{k} = 0$, equation 1.4 becomes :

$$\left(\frac{\mathbf{p}^2}{2m_0} + \mathbf{V}(\mathbf{r}) + \frac{\hbar}{4m_0c^2}(\boldsymbol{\sigma} \times \nabla \mathbf{V}) \cdot \mathbf{p}\right) u_{n0}(\mathbf{r}) = E_{n0} u_{n0}(\mathbf{r})$$

Let us suppose that this equation is solved and the wavefunction u_{n0} is known. Now equation 1.4 is rewritten as :

$$(H_0 + \frac{\hbar}{m_0} \mathbf{k} \cdot \boldsymbol{\pi}) u_{nk}(\mathbf{r}) = (E_{nk} - \frac{\hbar^2 \mathbf{k}^2}{2m_0}) u_{nk}(\mathbf{r}) \quad (1.5)$$

with $H_0 = \frac{\mathbf{p}^2}{2m_0} + \mathbf{V}(\mathbf{r}) + \frac{\hbar}{4m_0c^2}(\boldsymbol{\sigma} \times \nabla \mathbf{V}) \cdot \mathbf{p}$. For small $\mathbf{k}$, the term $\frac{\hbar}{m_0} \mathbf{k} \cdot \boldsymbol{\pi}$ can be treated as a perturbative element compared with H_0 . By applying the nondegenerate perturbation theory under the condition that the linear term in $\mathbf{k}$ is zero since the energy band has a minimum at $\mathbf{k} = 0$, the solution up to second order part for the energy is :

$$E_{nk} = E_{n0} + \frac{\hbar^2 \mathbf{k}^2}{2m_0} + \frac{\hbar^2}{m_0^2} \sum_{m \neq n} \frac{|\mathbf{k} \cdot \langle n0 | \boldsymbol{\pi} | m0 \rangle|^2}{E_{n0} - E_{m0}} \quad (1.6)$$

and the wavefunction is given as :

$$u_{nk} = u_{n0} + \frac{\hbar}{m_0} \sum_{m \neq n} \frac{\mathbf{k} \cdot \langle n\mathbf{0} | \boldsymbol{\pi} | m\mathbf{0} \rangle}{E_{n0} - E_{m0}} u_{m0} \quad (1.7)$$

Kane model

In bulk materials, the carriers move freely along three directions of the space and the eigenfunction solutions of Schrödinger equation of electron describe different bands which are divided into conduction band and valence band. The eigenenergy corresponding to conduction band E_s is denoted by eigenfunction $|S\rangle$ due to its spherical symmetry; whereas in valence band, three fold degenerate solutions of eigenfunctions corresponding to eigenvalue E_p are denoted by $|X\rangle, |Y\rangle, |Z\rangle$ in analogy to p-state wavefunctions of Hydrogen atom.

Before taking into account the spin-orbit coupling term in H_0 , the band-edge functions are [11] :

1. Conduction band : $|S \uparrow\rangle, |S \downarrow\rangle$ with corresponding eigenvalue E_s .
2. Valence band : $|X \uparrow\rangle, |Y \uparrow\rangle, |Z \uparrow\rangle, |X \downarrow\rangle, |Y \downarrow\rangle, |Z \downarrow\rangle$ with corresponding eigenvalue E_p .

Now the spin-orbit coupling term is taken into account in H_0 , the diagonalization of H_0 at Γ center ($\mathbf{k} = 0$) yields eigenenergies listed in table 1.1 [7].

In table 1.1, u_1, u_2 represent the conduction band eigenfunctions; u_3, u_4 and

<i>Eigenfunction</i> u_n	<i>Atomic notation</i> $ J, m_j\rangle$	<i>Eigenenergy</i> $E_i(k=0)$
$u_1 = S \uparrow\rangle$	$ \frac{1}{2}, \frac{1}{2}\rangle$	E_s
$u_2 = S \downarrow\rangle$	$ \frac{1}{2}, -\frac{1}{2}\rangle$	E_s
$u_3 = \frac{1}{\sqrt{6}} (X + iY), \downarrow\rangle - \sqrt{\frac{2}{3}} Z, \uparrow\rangle$	$ \frac{3}{2}, \frac{1}{2}\rangle$	$E_{lh} = E_p + \frac{\Delta_0}{3}$
$u_4 = -\frac{1}{\sqrt{6}} (X - iY), \uparrow\rangle - \sqrt{\frac{2}{3}} Z, \downarrow\rangle$	$ \frac{3}{2}, -\frac{1}{2}\rangle$	$E_{lh} = E_p + \frac{\Delta_0}{3}$
$u_5 = \frac{1}{\sqrt{2}} (X + iY), \uparrow\rangle$	$ \frac{3}{2}, \frac{3}{2}\rangle$	$E_{hh} = E_p + \frac{\Delta_0}{3}$
$u_6 = \frac{1}{\sqrt{2}} (X - iY), \downarrow\rangle$	$ \frac{3}{2}, -\frac{3}{2}\rangle$	$E_{hh} = E_p + \frac{\Delta_0}{3}$
$u_7 = \frac{1}{\sqrt{3}} (X + iY), \downarrow\rangle + \frac{1}{\sqrt{3}} Z, \uparrow\rangle$	$ \frac{1}{2}, \frac{1}{2}\rangle$	$E_{SO} = E_p - \frac{2\Delta_0}{3}$
$u_8 = -\frac{1}{\sqrt{3}} (X - iY), \uparrow\rangle + \frac{1}{\sqrt{3}} Z, \downarrow\rangle$	$ \frac{1}{2}, -\frac{1}{2}\rangle$	$E_{SO} = E_p - \frac{2\Delta_0}{3}$

Table 1.1: Bloch eigenfunctions at Γ center of zinc-blende semiconductors [11]

u_5, u_6 represent the light holes and heavy holes eigenstates, respectively. Finally,

u_7, u_8 are the eigenfunctions of "split-off" band separated with light and heavy hole band by an energy equal to Δ_0 given by :

$$\Delta_0 = i \frac{3\hbar^2}{4m_0^2 c^2} \langle X | \mathbf{k} | Y \rangle$$

In GaAs, $\Delta_0 = 350$ meV [6].

In the first approximation, the eigenfunctions u_1, u_2 in conduction band which are calculated by the **k.p** method without taking into account the dependence of spin-orbit coupling term $H_{SO} = (\hbar^2/4m_0^2 c^2) \cdot (\boldsymbol{\sigma} \times \nabla V)$ lead to two degenerate parabola (see figure 1.2).

In order to describe more precisely the degenerate valence band, Luttinger-Kohn model [37, 36] can be used; in which, the coupling between two degenerate conduction bands is neglected and the Löwdin partitioning method [48] is employed to treat the six degenerate valence bands.

The general Hamiltonian for both cases of light and heavy holes are written as :

$$H_L = \frac{\hbar^2}{2m_0} \left[(\gamma_1 + \frac{5}{2}\gamma_2) \nabla^2 - 2\gamma_3 (\nabla \cdot \mathbf{J})^2 + 2(\gamma_3 - \gamma_2) (\nabla_x^2 \mathbf{J}_x^2 + c.p) \right] \quad (1.8)$$

where $\gamma_1, \gamma_2, \gamma_3$ are Luttinger parameters determined by experiments [3], $\mathbf{J} = (\mathbf{J}_x, \mathbf{J}_y, \mathbf{J}_z)$ is total angular momentum operator, c.p stands for cyclic permutation [85].

Diagonalizing the above Hamiltonian in equation 1.8, the eigenenergies of both light and heavy hole states are found in terms of Luttinger parameters :

$$E(k) = E_v(0) + \frac{\hbar^2}{m_0} [\gamma_1 k^2 \pm 2\sqrt{\gamma_2 \mathbf{k}^4 + 3(\gamma_3^2 - \gamma_2^2)(k_x^2 k_y^2 + k_y^2 k_z^2 + k_z^2 k_x^2)}] \quad (1.9)$$

Intuitively, equation 1.9 shows that heavy hole (negative sign) and light hole (positive sign) are degenerate at $\mathbf{k} = 0$ and split for $\mathbf{k} \neq 0$ into two subbands with different characteristics presented by different effective masses. In fact, the relationship between the effective masses of heavy and light holes of [100] equivalent direction can be expressed in the forms related to Luttinger parameters :

$$\begin{aligned} \frac{m_{hh}^*}{m_0} &= \frac{1}{\gamma_1 - 2\gamma_2} \\ \frac{m_{lh}^*}{m_0} &= \frac{1}{\gamma_1 + 2\gamma_2} \end{aligned}$$

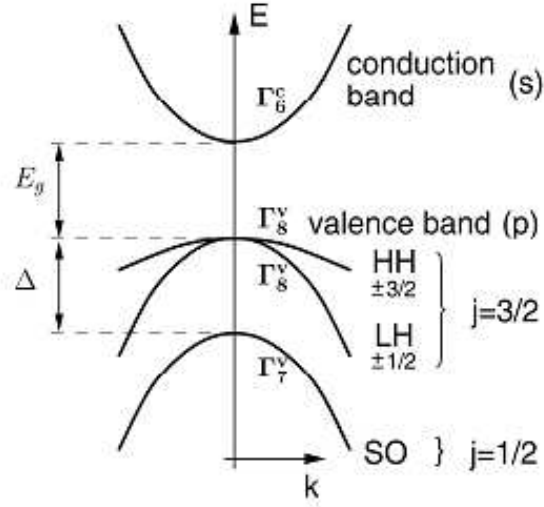


Figure 1.2: Schematic of band structure of zinc-blende bulk semiconductor near Γ center

Fourteen bands model (Extended Kane model)

The $\mathbf{k}\cdot\mathbf{p}$ method has been extended to full Brillouin zone for both bulk materials and heterostructures. The procedure is to extend first-order Kane model with the larger basis sets together with all the parameters related to energy gaps and momentum matrix elements at $\mathbf{k} = 0$. The 14×14 extended Kane model takes into account all $\mathbf{k}\cdot\mathbf{p}$ and spin-orbit interactions of fourteen band states including six bonding p-like valence band states, two anti-bonding s-like conduction band states and six anti-bonding p-like valence band states. The schematic band structure at the Γ -point is depicted in figure 1.3. In the model, the quantization axis of angular momentum is along to the crystallographic [001]-direction and the fourteen basis functions are given as [63]:

$$\begin{aligned}
 u_1 &= \left| \frac{3}{2}, \frac{3}{2} \right\rangle_{c'} = -\frac{1}{\sqrt{2}} |X' + iY', \uparrow\rangle \\
 u_2 &= \left| \frac{3}{2}, \frac{1}{2} \right\rangle_{c'} = -\frac{1}{\sqrt{6}} (|X' + iY', \downarrow\rangle - 2|Z', \uparrow\rangle) \\
 u_3 &= \left| \frac{3}{2}, -\frac{1}{2} \right\rangle_{c'} = \frac{1}{\sqrt{6}} (|X' - iY', \uparrow\rangle + 2|Z', \downarrow\rangle) \\
 u_4 &= \left| \frac{3}{2}, -\frac{3}{2} \right\rangle_{c'} = \frac{1}{\sqrt{2}} |X' - iY', \downarrow\rangle
 \end{aligned}$$

$$\begin{aligned}
u_5 &= \left| \frac{3}{2}, \frac{1}{2} \right\rangle_{c'} = -\frac{1}{\sqrt{3}}(|X' + iY', \downarrow\rangle + |Z', \uparrow\rangle) \\
u_6 &= \left| \frac{3}{2}, -\frac{1}{2} \right\rangle_{c'} = -\frac{1}{\sqrt{3}}(|X' - iY', \uparrow\rangle - |Z', \downarrow\rangle) \\
u_7 &= \left| \frac{1}{2}, \frac{1}{2} \right\rangle_c = |S, \uparrow\rangle \\
u_8 &= \left| \frac{1}{2}, -\frac{1}{2} \right\rangle_c = |S, \downarrow\rangle \\
u_9 &= \left| \frac{3}{2}, \frac{3}{2} \right\rangle_v = -\frac{1}{\sqrt{2}}|X + iY, \uparrow\rangle \\
u_{10} &= \left| \frac{3}{2}, \frac{1}{2} \right\rangle_v = -\frac{1}{\sqrt{6}}(|X + iY, \downarrow\rangle - 2|Z, \uparrow\rangle) \\
u_{11} &= \left| \frac{3}{2}, -\frac{1}{2} \right\rangle_v = \frac{1}{\sqrt{6}}(|X - iY, \uparrow\rangle + 2|Z, \downarrow\rangle) \\
u_{12} &= \left| \frac{3}{2}, -\frac{3}{2} \right\rangle_v = \frac{1}{\sqrt{2}}|X - iY, \downarrow\rangle \\
u_{13} &= \left| \frac{3}{2}, \frac{1}{2} \right\rangle_v = -\frac{1}{\sqrt{3}}(|X + iY, \downarrow\rangle + |Z, \uparrow\rangle) \\
u_{14} &= \left| \frac{3}{2}, -\frac{1}{2} \right\rangle_v = -\frac{1}{\sqrt{3}}(|X - iY, \uparrow\rangle - |Z, \downarrow\rangle)
\end{aligned}$$

The structure of the 14-band Hamiltonian is [28, 70] (see appendix A for more

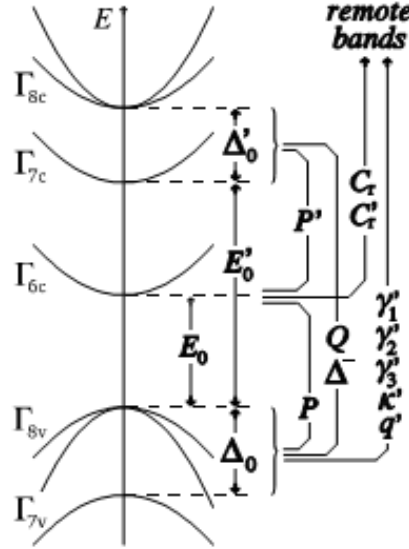


Figure 1.3: Schematic band structure for 14x14 extended Kane model of zinc-blende semiconductors at Γ point [70]

detail) :

$$H(\mathbf{k}) = \begin{pmatrix} H^{8c8c} & H^{8c7c} & H^{8c6c} & H^{8c8v} & H^{8c7v} \\ H^{7c8c} & H^{7c7c} & H^{7c6c} & H^{7c8v} & H^{7c7v} \\ H^{6c8c} & H^{6c7c} & H^{6c6c} & H^{6c8v} & H^{6c7v} \\ H^{8v8c} & H^{8v7c} & H^{8v6c} & H^{8v8v} & H^{8v7v} \\ H^{7v8c} & H^{7v7c} & H^{7v6c} & H^{7v8v} & H^{7v7v} \end{pmatrix} \quad (1.10)$$

with the irreducible momentum matrix elements and spin-orbit coupling terms in 14×14 extended Kane model defined as follow [63, 64]:

$$\begin{aligned} \mathbf{P} &= \frac{\hbar}{m_0} \langle S | \mathbf{p}_x | X \rangle = \frac{\hbar}{m_0} \langle S | \mathbf{p}_y | Y \rangle = \frac{\hbar}{m_0} \langle S | \mathbf{p}_z | Z \rangle \\ \mathbf{P}' &= \frac{\hbar}{m_0} \langle S | \mathbf{p}_x | X' \rangle = \frac{\hbar}{m_0} \langle S | \mathbf{p}_y | Y' \rangle = \frac{\hbar}{m_0} \langle S | \mathbf{p}_z | Z' \rangle \\ \mathbf{Q} &= \frac{\hbar}{m_0} \langle X | \mathbf{p}_y | Z' \rangle = \frac{\hbar}{m_0} \langle Y | \mathbf{p}_z | X' \rangle = \frac{\hbar}{m_0} \langle Z | \mathbf{p}_x | Y' \rangle \\ \Delta_0 &= -\frac{3i\hbar}{4m_0^2c^2} \langle X | [(\nabla V) \times \mathbf{p}]_y | Z \rangle = -\frac{3i\hbar}{4m_0^2c^2} \langle Y | [(\nabla V) \times \mathbf{p}]_z | X \rangle \\ &= -\frac{3i\hbar}{4m_0^2c^2} \langle Z | [(\nabla V) \times \mathbf{p}]_x | Y \rangle \\ \Delta'_0 &= -\frac{3i\hbar}{4m_0^2c^2} \langle X' | [(\nabla V) \times \mathbf{p}]_y | Z' \rangle = -\frac{3i\hbar}{4m_0^2c^2} \langle Y' | [(\nabla V) \times \mathbf{p}]_z | X' \rangle \\ &= -\frac{3i\hbar}{4m_0^2c^2} \langle Z' | [(\nabla V) \times \mathbf{p}]_x | Y' \rangle \\ \Delta^- &= -\frac{3i\hbar}{4m_0^2c^2} \langle X | [(\nabla V) \times \mathbf{p}]_y | Z' \rangle = -\frac{3i\hbar}{4m_0^2c^2} \langle Y | [(\nabla V) \times \mathbf{p}]_z | X' \rangle \\ &= -\frac{3i\hbar}{4m_0^2c^2} \langle Z | [(\nabla V) \times \mathbf{p}]_x | Y' \rangle \end{aligned}$$

1.2 Semiconductor heterostructures

A heterostructure is composed of different layers of dissimilar crystalline semiconductors with unequal bandgap energies. In practice, advanced epitaxial techniques such as Molecular Beam Epitaxy (MBE) or Metal-Organic Chemical Vapor Deposition (MOCVD) allow us to grow layers as thin as a few atomic layers, in which the motion of carriers is quantized. The heterostructural interfaces play a critical role in semiconductor heterostructure devices such as field-effect transistor (FET), bipolar transistor, light-emitting diode (LED), laser,... etc because they exhibit many interesting and useful electronic properties associated with their discontinuities of band structure at their interfaces [38]. This section recalls some basis on semiconductor heterostructures.

1.2.1 Envelope function approximation

As discussed before, the Bloch function only describes electron wavefunction under crystal periodic potential. However, in semiconductor heterostructure, the periodic condition is broken because of the interface between two different materials. In this situation, envelope function approximation (EFA) is implemented to find the wavefunction of electron in semiconductor heterostructures.

The Schrödinger equation for semiconductor heterostructures is written as [7]:

$$\left[\frac{-\hbar^2}{2m_0} \Delta_r + V_c(\vec{r}) + V_h(\vec{r}) \right] \psi_{nk}(\vec{r}) = E_{nk} \psi_{nk}(\vec{r}) \quad (1.11)$$

where n is band index, m_0 is the free electron effective mass, $V_c(\vec{r})$ is crystal potential, $V_h(\vec{r})$ is heterojunction interfacial potential. In EFA method, the interfacial potential $V_h(\vec{r})$ is considered as a perturbative component. In fact, the EFA method is valid under two assumed conditions : the envelope wavefunction and the density of current probability are continuous through the interface. Now the wavefunction of an electron in semiconductor heterostructure is found in this form :

$$\psi_n(\vec{r}) = F_{nk}(\vec{r}) u_{n0}(\vec{r})$$

where $F_{nk}(\vec{r})$ is the envelope function and $u_{n0}(\vec{r})$ is the Bloch wavefunction at $\mathbf{k} = 0$. The equation 1.11 becomes :

$$\left[\frac{-\hbar^2}{2m_c(\vec{r})} \Delta_r + V_h(\vec{r}) \right] F_{nj}(\vec{r}) = (E_{ni} - E_{n0}) F_{nj}(\vec{r}) \quad (1.12)$$

where $m_c(\vec{r})$ is the effective mass depending on semiconductor materials and $E_{nj} - E_{n0}$ is the confinement energy. In this approximation, the heterostructure is considered as a crystal with small perturbative potential $V_h(\vec{r})$.

1.2.2 Electronic properties of semiconductor quantum wells

A Quantum Well (QW) is a thin layer semiconductor structure sandwiched between two layers of another semiconductor. Semiconductor quantum wells can be divided into two categories : type I and type II [8]. We only concentrate on studying type I quantum well (GaAs/AlGaAs); in which, the material of barrier layers (AlGaAs) has wider bandgap than the quantum well layer (GaAs) as depicted in figure 1.4.

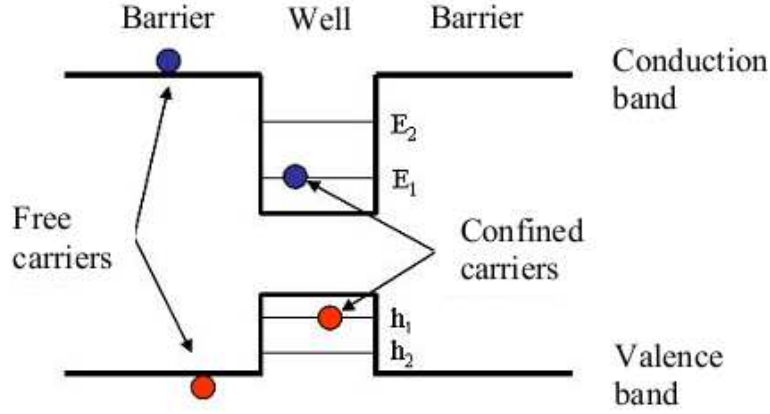


Figure 1.4: Schematic of band energy structure of type I quantum well [24]

Quantum well

For the band calculation, the potential for the finite quantum well is defined as :

$$V(z) = \begin{cases} 0 & \text{for } |z| \leq L/2 \\ V_0 & \text{for } |z| > L/2 \end{cases}$$

We denote material A for quantum well layer and material B for the barrier layers. Concerning $\Phi(z)$, the Schrodinger equation can be written as :

$$[E_s + V_s(z) - \frac{\hbar^2}{2\mu(z)}(\frac{\partial^2}{\partial z^2} + k_{\perp}^2)]\Phi(z) = E\Phi(z) \quad (1.13)$$

where E_s is the conduction band edge in quantum well which can be set as zero for the sake of simplicity, $V_s(z)$ is zero in quantum well and equal to V_0 in the barrier, $\mu(z)$ is the effective mass which is equal to m_A^* in quantum well and m_B^* in the barrier, $k_{\perp}$ is the in-plane wavevector ($k_{\perp}^2 = k_x^2 + k_y^2$), $\Phi(z)$ is the z-dependent part of the envelope function.

The boundary conditions are proposed in Ben Daniel-Duke model [7, 50] as follow :

$$\Phi_A(\pm \frac{L}{2}) = \Phi_B(\pm \frac{L}{2}) \quad (1.14)$$

and :

$$\frac{1}{m_A^*} \frac{d\Phi(z)}{dz} \Big|_{z=\pm L/2} = \frac{1}{m_B^*} \frac{d\Phi(z)}{dz} \Big|_{z=\pm L/2} \quad (1.15)$$

Due to the symmetry of potential, the wavefunction of a particle in this kind of quantum well can be expressed by even and odd functions. Solving the above

Schrodinger equation 1.13, the envelope wavefunction is obtained :

$$\Phi(z) = \begin{cases} Ae^{\beta z} & \text{for } z < -L/2 \\ B \cos(\alpha z) & \text{for } |z| \leq L/2 \text{ (even states)} \\ B \sin(\alpha z) & \text{for } |z| \leq L/2 \text{ (odd states)} \\ Ae^{-\beta z} & \text{for } z > L/2 \end{cases}$$

where :

$$\alpha = \sqrt{\frac{2m_A^*}{\hbar^2} E_n - k_{\perp}^2} \quad (1.16)$$

$$\beta = \sqrt{\frac{2m_B^*}{\hbar^2} (V_0 - E_n) + k_{\perp}^2} \quad (1.17)$$

and A, B are normalization constants. In the case of an infinite quantum well, the associated eigenenergies are of this form :

$$E_n(k_{\perp}) = \frac{\hbar^2}{m_A^*} \left(\left(\frac{n\pi}{L} \right)^2 + k_{\perp}^2 \right) \quad (1.18)$$

As for the bulk materials, the valence band is of more complicated treatment than the conduction band. The approach is to add a confinement potential in Luttinger-Kohn Hamiltonian. We can rebuild the hole Hamiltonian in the QW by referring to equation 1.8 : Firstly, the confinement potential term $V(z)$ should be added into Hamiltonian; then, the cubic term in equation 1.9 which is proportional to $(\gamma_3^2 - \gamma_2^2)$ is rather small in most zinc-blende semiconductors and we can omit it in a first approach. Finally, we assum that the Luttinger parameters are identical in QW and barriers. With those simplifications, the Hamiltonian of holes in QW becomes :

$$H_{QW} = \frac{\hbar^2}{2m_0} \left[\left(\gamma_1 + \frac{5}{2}\gamma_2 \right) \nabla^2 - 2\gamma_3 (\nabla \cdot \mathbf{J})^2 \right] + V(z)$$

The Hamiltonian for the motion in z-direction can be simplified in this form :

$$H_z = \frac{\hbar^2}{2m_0} \left[\left(\gamma_1 + \frac{5}{2}\gamma_2 \right) - 2\gamma_3 \nabla J_z^2 \right] \left(\frac{\partial}{\partial z} \right)^2 + V(z)$$

Figure 1.5 illustrates the schematic of hole dispersion relation in a typical 10nm GaAs/AlGaAs quantum well. As shown in this figure, the heavy hole (HH) subband is above the light hole (LH) subband at zone center because the confinement energy is inversely proportional to the effective mass (equation 1.18). As a result, the exciton E1-HH1 is dominant in photoluminescence spectroscopy of type I quantum well.

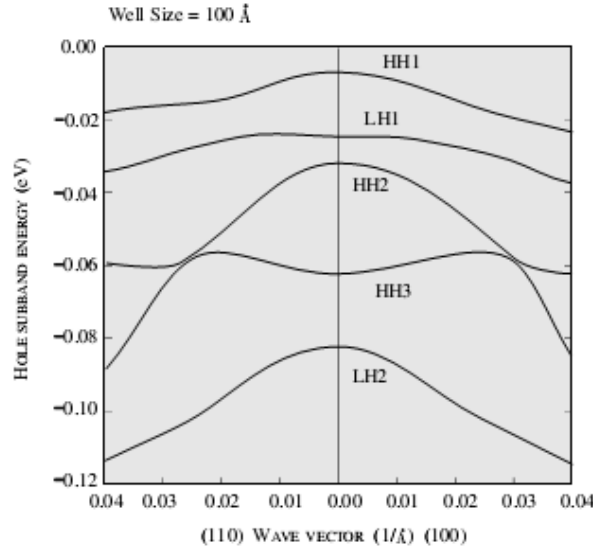


Figure 1.5: Hole relation dispersion in a 100 Å $GaAs/Al_{0.3}Ga_{0.7}As$ quantum well [76]

It should be noted that effective mass is anisotropic in the momentum space. In particular, heavy hole demonstrates heavy mass in longitudinal direction (z-axis) and light mass in x-y plane; vice versa for the light hole.

1.3 Optical orientation in zinc-blend semiconductor

Optical properties provide an important tool for studying many inner physical phenomenon in the solid. In fact, optical spectra reveal many information about electronic properties in semiconductor such as energy band, impurity level, localized states, exciton, ... etc. Optics is also very useful to study the spin properties of carriers in semiconductors. This section mentions some background of optical orientation in zinc-blende semiconductor.

1.3.1 Optical transitions

When electromagnetic waves interact with a direct bandgap semiconductor that has a bandgap smaller than photon energy, electrons in the lower energy states are excited to higher energy states followed by energy and momentum conservation law making an optical transition. There are different kinds of optical transitions in semiconductor. *Interband transitions* take place between conduction band and valence band, in which electrons in the occupied states of valence

band are excited in the empty states of conduction bands. *Intraband transitions* happen inside one band (conduction band or valence band) and involve only one type of carrier (electron or hole).

Electromagnetic radiation can be described by its electric field $E(\vec{r}, t)$ and magnetic field $H(\vec{r}, t)$ equivalent to a vector potential $\mathbf{A}(\vec{r}, t)$ which is defined as :

$$\begin{aligned} E(\vec{r}, t) &= -\frac{1}{c} \nabla \cdot \mathbf{A}(\vec{r}, t) \\ H(\vec{r}, t) &= \nabla \times \mathbf{A}(\vec{r}, t) \end{aligned}$$

The Hamiltonian of an electron in the presence of electromagnetic field is written as :

$$H = \frac{1}{2m_0} \left(\mathbf{p} - \frac{e\mathbf{A}}{c} \right)^2 + V(\mathbf{r}) \quad (1.19)$$

This Hamiltonian can be developped as :

$$H = H_0 - \frac{e}{m_0 c} \vec{\mathbf{A}} \cdot \vec{\mathbf{p}} + \frac{e^2}{2m_0 c} A^2$$

where $H_0 = \frac{p^2}{2m_0} + V(r)$. If the term of A^2 is neglected in the first approximation, the interaction Hamiltonian between an electron and electromagnetic field can be written as :

$$H_{int} = -\frac{e}{m_0 c} \vec{\mathbf{A}} \cdot \vec{\mathbf{p}} \quad (1.20)$$

where e is electron charge, m_0 is electron mass, c is the light velocity.

Using the first-order time-dependent perturbation theory, Fermi Golden Rule states that the transition rate of an electron from an initial state $|i\rangle$ with an energy E_i to a final state $|f\rangle$ with an energy E_f by absorbing an photon with the energy $\hbar\omega$ is defined as :

$$W_{if} = \frac{2\pi}{\hbar} |\langle i | H_{int} | f \rangle|^2 \delta(E_f - E_i - \hbar\omega) \quad (1.21)$$

where $M_{if} = |\langle i | H_{int} | f \rangle|^2$ is denoted as transition matrix element [44] whose values determine the allowed transitions and their relative intensities; this defines the optical selection rule. The Dirac function δ is responsible for energy conservation law in these transitions.

If the quantum states $|i\rangle$ and $|f\rangle$ are partially occupied, the transition probability is weighted by the occupancy factor given by Fermi-Dirac distribution as follow :

$$R_{i \rightarrow f} = W_{if} f(E_i) [1 - f(E_f)] \quad (1.22)$$

where $f(E) = (1 + e^{\frac{1}{k_B T}(E - \mu)})^{-1}$ is Fermi-Dirac distribution, in which k_B is Boltzmann constant, T is the temperature and μ is chemical potential of the electron.

1.3.2 Optical selection rules

For direct bandgap semiconductor such as GaAs and InAs,..., the lowest energy point in conduction band and the highest energy point in valence band have the same momentum at the Γ -point ($k=0$). Hence, interband transitions in these materials can occur without any contribution of phonons. Optical selection rules in this situation are presented as follow.

Let us consider an monochromatic electromagnetic wave :

$$E(\vec{r}, t) = F\vec{\epsilon}\cos(\omega t - \vec{k}\cdot\vec{r}) \quad (1.23)$$

where $\mathbf{F}$ is electric field strength, $\vec{\epsilon}$ is a polarization vector. For circular polarized light, the right-hand and left-hand polarization vector can be defined as $\vec{\epsilon} = \frac{1}{\sqrt{2}}(1, i, 0)$ and $\vec{\epsilon} = \frac{1}{\sqrt{2}}(1, -i, 0)$, respectively. For linear-polarized light, $\vec{\epsilon} = (1, 0, 0)$.

The potential vector is rewritten in this form :

$$A(\vec{r}, t) = -\frac{cF\vec{\epsilon}}{2i\omega}[e^{i(\omega t - \vec{k}\cdot\vec{r})} - e^{-i(\omega t - \vec{k}\cdot\vec{r})}] \quad (1.24)$$

and the interaction Hamiltonian is :

$$H_{int} = \frac{ieF}{2m_0\omega}(e^{i\omega t} - e^{-i\omega t})\vec{\epsilon}\cdot\vec{p} \quad (1.25)$$

under the dipole approximation $e^{i\vec{k}\cdot\vec{r}} \approx 1$ which is valid for the studied wavelength. As a result, the transition rate is given by :

$$W_{if} = \frac{2\pi}{\hbar} \left(\frac{e}{m_0\omega}\right)^2 \left|\frac{F(\omega)}{2}\right|^2 |\vec{\epsilon}\cdot\vec{p}_{if}|^2 \delta(E_f - E_i - \hbar\omega) \quad (1.26)$$

in which $p_{if} = \langle i|\vec{p}|f\rangle$ contains the selection rules information.

For quantum wells, the envelope functions for electrons in conduction band and valence band are taken into account in the matrix element $\langle i|\vec{p}|f\rangle$. We introduce the non-zero irreducible momentum matrix elements at Γ -point for zinc-blende structure calculated by group theory [57]:

$$\Pi = \frac{-i}{m_0} \langle S|p_x|X\rangle = \frac{-i}{m_0} \langle S|p_y|Y\rangle = \frac{-i}{m_0} \langle S|p_z|Z\rangle \quad (1.27)$$

<i>Polarization</i>	$\vec{\epsilon} \parallel \vec{x}$	$\vec{\epsilon} \parallel \vec{y}$	$\vec{\epsilon} \parallel \vec{z}$	<i>Type of transitions</i>
$\vec{k} \parallel \vec{z}$	$\Pi/\sqrt{2}$	$\Pi/\sqrt{2}$	<i>Impossible</i>	$HH \rightarrow E$
$\vec{k} \parallel \vec{x}$	<i>Impossible</i>	$\Pi/\sqrt{2}$	0	$HH \rightarrow E$
$\vec{k} \parallel \vec{y}$	$\Pi/\sqrt{2}$	<i>Impossible</i>	0	$HH \rightarrow E$
$\vec{k} \parallel \vec{z}$	$\Pi/\sqrt{6}$	$\Pi/\sqrt{6}$	<i>Impossible</i>	$LH \rightarrow E$
$\vec{k} \parallel \vec{x}$	<i>Impossible</i>	$\Pi/\sqrt{6}$	$2\Pi/\sqrt{6}$	$LH \rightarrow E$
$\vec{k} \parallel \vec{y}$	$\Pi/\sqrt{6}$	<i>Impossible</i>	$2\Pi/\sqrt{6}$	$LH \rightarrow E$

Table 1.2: Selection rules for interband transitions [7]

Table 1.2 shows the selection rules for different polarizations of electromagnetic waves for interband transitions in a quantum well :

Owing to optical selection rules, it is well-known that spin-polarized carriers can be generated by the absorption of circular-polarized light and vice versa, the recombination of spin-polarized charges results in the emission of circular-polarized light [25]. Let $\sigma^\pm$ represent the helicity of exciting light related to right and left-hand circular polarized light propagation along z-direction, respectively as shown in figure 1.6. In bulk zinc-blende semiconductors, the spin polarization of generated carriers are determined by the photon energy $\hbar\omega$. Under the condition $E_g < \hbar\omega < E_g + \Delta_{SO}$, the heavy and light hole subbands are excited simultaneously because they are degenerated at $\mathbf{k} = 0$ in zinc-blende structure. Denoting n_+ and n_- as the density of spin-polarized electron with $S_z = +1/2$ and $-1/2$ respectively, the spin polarization is defined as [31]:

$$P_n = \frac{n_+ - n_-}{n_+ + n_-} \quad (1.28)$$

At the moment of photoexcitation, the spin carriers are oriented against the direction of exciting light since the probability of heavy-hole transitions (E-HH) is three times larger than that of light-hole transitions (E-LH) (see in table 1.2) in zinc-blende structure. Therefore, the spin polarization is indicated by :

$$P_n = \frac{1 - 3}{1 + 3} = -\frac{1}{2} \quad (1.29)$$

The circular polarization of photoluminescence is :

$$P_{circ} = \frac{I_+ - I_-}{I_+ + I_-} = \frac{(n_+ + 3n_-) - (3n_+ + n_-)}{(n_+ + 3n_-) + (3n_+ + n_-)} = -\frac{P_n}{2} = \frac{1}{4} \quad (1.30)$$

where $I_\pm$ are defined as the radition intensities of corresponding helicity $\sigma_\pm$.

When the excited photon energy is over the split-off band ($\hbar\omega \geq E_g + \Delta_{SO}$),

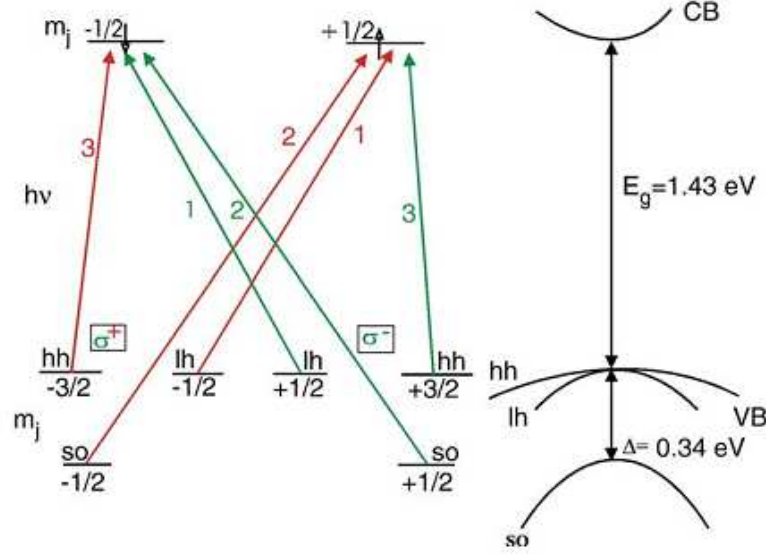


Figure 1.6: Optical selection rules from heavy-hole (HH), light-hole (LH) and split-off (SO) valence band (VB) to conduction band (CB) [53]

total carriers will not be spin-polarized because the numbers of spin-up and spin-down carriers are the same; which underlines the role of spin-orbit coupling in optical orientation.

For semiconductor quantum well, the heavy-hole subband is above the light-hole subband at zone center allowing the selective excitation of only one conduction subband ($|+1/2\rangle$ or $|-1/2\rangle$). This selective excitation can improve the spin polarization P_n up to 100%; which offers a nice feature to observe the spin relaxation phenomena.

1.3.3 Exciton

In direct bandgap semiconductor, a photon of energy higher than or equal to the bandgap of semiconductor is absorbed exciting an electron from valence band to conduction band. This phenomenon refers to photogenerated free electron-hole pair. When Coulomb interaction between electron and hole is taken into account, a bound electron-hole pair can be created by absorbing a photon of energy slightly smaller than semiconductor bandgap; the concept of exciton is developed.

Exciton is a quasi-particle used to describe electron-hole pairs coupled by Coulomb interaction in a manner similar to the hydrogen atom. The first theory of exciton was formulated by Frenkel [35] and Wannier [26] in 1930s. Exciton

can be classified into two types depending on the separation of electron-hole pair when compared with crystal unit cell : Frenkel exciton (smaller than unit cell) and Wannier-Mott exciton (larger than unit cell). In bulk material, all of the following concepts refer to Wannier-Mott exciton as it is correspondent to optical properties at low temperatures in GaAs-based structure.

The Schrödinger equations of exciton can be written as :

$$\left[-\frac{\hbar^2}{2m_e}\nabla_{r_e}^2 - \frac{\hbar^2}{2m_h}\nabla_{r_h}^2 - \frac{e^2}{4\pi\epsilon\epsilon_0|r_e - r_h|}\right]\psi_{ex} = E\psi_{ex} \quad (1.31)$$

where ϵ is the static dielectric constant. The wavefunction of the exciton can be expressed as [9]:

$$\psi_{ex} = \frac{1}{C} e^{i\mathbf{K}\cdot\mathbf{R}} \phi_e(\mathbf{r}_e) \phi_h(\mathbf{r}_h) \phi_{env}(\mathbf{r}_e - \mathbf{r}_h) \quad (1.32)$$

where C is a normalization constant; $\mathbf{K} = \mathbf{k}_e + \mathbf{k}_h$ is the total wavevector; $\mathbf{R} = (m_e\mathbf{r}_e + m_h\mathbf{r}_h)/(m_e + m_h)$ is the center of mass; ϕ_{env} is the hydrogen-like-atom envelope wavefunction describing the relative motion of electron and hole; ϕ_e, ϕ_h are the Bloch wavefunctions of electron and hole, respectively.

The excitonic energy level becomes :

$$E_n = E_g - R_y^* \frac{1}{n_B^2} + \frac{\hbar^2 \mathbf{K}^2}{2M} \quad (1.33)$$

where $M = m_e + m_h$ is the total mass of exciton, $n_B = 1, 2, 3, \dots$ is the principal quantum number, $R_y^* = \mu^* e^4 / 2m_0 (4\pi\epsilon\epsilon_0)^2 \hbar^2$ is the exciton binding energy in which μ^* is the reduced mass defined as $1/\mu^* = 1/m_e + 1/m_h$.

Exciton can be characterized by the pseudo Bohr radius which is the average distance between electron and hole :

$$a_B^{ex} = a_B^H \epsilon \frac{m_0}{\mu} \quad (1.34)$$

where a_B^H, a_B^{ex} are the Bohr radius of hydrogen atom and exciton, respectively.

Due to the influence of confinement in quantum nanostructures (2D, 1D), the exciton binding energy and exciton Bohr radius in these above equations are modified (see [9]). For a strictly 2D exciton, the exciton binding energy is four times larger than 3D exciton (bulk). The exciton radius in a real semiconductor quantum well comes from an intermediate solution between a 2D and 3D excitons.

1.4 Spin relaxation mechanism

Spin relaxation and spin dephasing are processes that lead an ensemble of carriers from spin-polarized state to equilibrium state. In brief, the spin relaxation

time T_{s1} refers to the nonequilibrium population decay time of spin-polarized energy eigenstates, whereas spin dephasing time T_{s2} is related to the destruction of phase coherence of these eigenstates. There are four major physical mechanisms mentioned in this section that are responsible for spin relaxation in semiconductor : Elliott-Yafet (EY), Dyakonov-Perel (DP), Bir-Aronov-Pikus (BAP) and hyperfine interaction processes.

1.4.1 Elliott-Yafet mechanism

Elliott-Yafet mechanism was first proposed by Elliott [67], which realized that electron spin in conduction band can relax via ordinary momentum scattering such as phonons or impurities due to the spin-orbit coupling as shown in figure 1.7.

The calculation of EY mechanism for III-V semiconductors can be performed

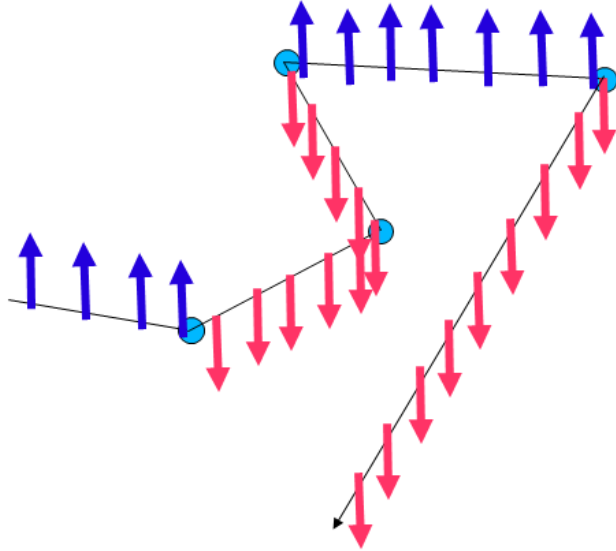


Figure 1.7: Schematics of Elliott-Yafet mechanism : spin flip process

analytically in a simple approach. It can be written as [31]:

$$\frac{1}{T_{s1}(E_k)} = A \left(\frac{\Delta_{SO}}{E_g + \Delta_{SO}} \right)^2 \left(\frac{E_k}{E_g} \right)^2 \frac{1}{\tau_p(E_k)} \quad (1.35)$$

where $\tau_p(E_k)$, $T_{s1}(E_k)$ are momentum scattering time and spin relaxation time at energy E_k , respectively. E_g is the energy bandgap, Δ_{SO} is the spin-orbit splitting of valence band. The numerical factor A depends on the dominant scattering mechanism (impurities at low temperatures and phonons at high temperatures).

Equation 1.35 shows that EY mechanism is important for narrow-gap semiconductor with large spin-orbit splitting.

1.4.2 Dyakonov-Perel mechanism

The Dyakonov-Perel (DP) mechanism is an efficient spin relaxation process for III-V and II-VI semiconductor proposed by Dyakonov and Perel in 1971. Due to the lack of inversion symmetry in III-V and II-VI type semiconductor (the elementary cell contains 2 different atoms in these compounds), the conduction band degeneracy for spin $|+1/2\rangle$ and $|-1/2\rangle$ electronic states is lifted. As a consequence, the spin splitting induced by this inversion asymmetry can be described by an intrinsic k -dependent effective magnetic field $\mathbf{B}(\mathbf{k})$ induced by the spin-orbit coupling (see figure 1.8).

The corresponding Hamiltonian of electron in the conduction band is :

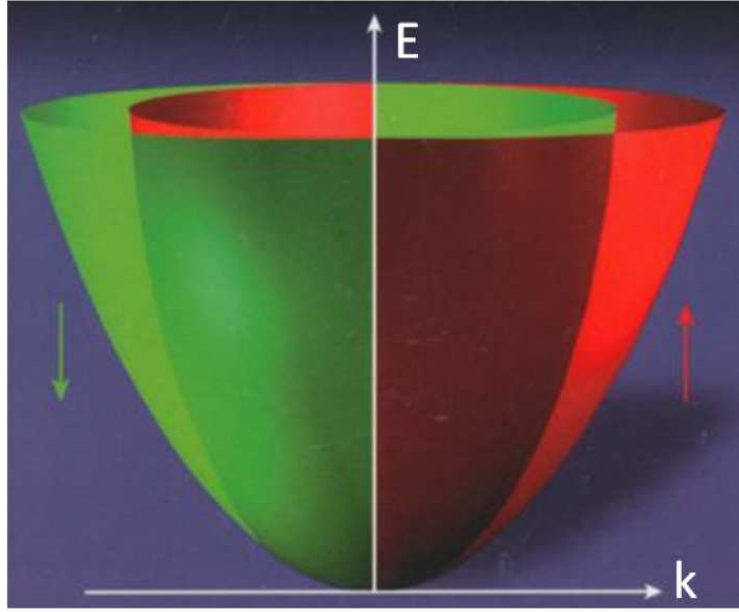


Figure 1.8: Schematics of conduction band spin splitting [20]

$$H(k) = \frac{\hbar}{2} \boldsymbol{\sigma} \cdot \boldsymbol{\Omega}(k) \quad (1.36)$$

where σ is Pauli matrices and $\Omega(k)$ is the Larmor frequency of electron spin precession around the effective magnetic field which is defined as $\boldsymbol{\Omega}(k) = (e/m)\mathbf{B}(k)$. Figure 1.9 illustrates the precession of electron spin around the effective spin-orbit field $\Omega(k)$. A scattering event modifying electron wavevector to $\mathbf{k}'$ leads the

electron spin to precess around a different effective field $\Omega(\mathbf{k}')$.

In fact, both the scattering events and the effective spin-orbit field give con-

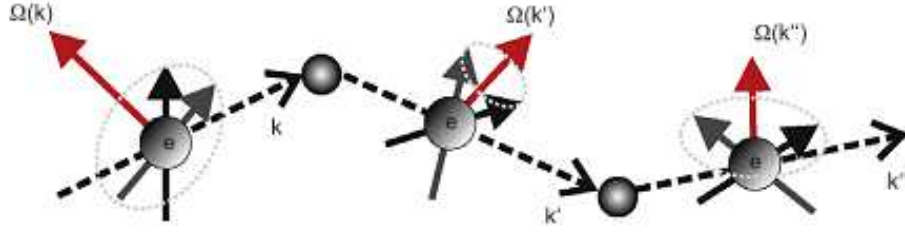


Figure 1.9: Schematics of Dyakonov-Perel mechanism : a random walk of electron spin [80]

tributions in different trends to spin relaxation process in DP mechanism. The scattering events make electron spins keep their spin orientation but the effective spin-orbit field make them precess and lose their spin orientation as a consequence. Therefore, there exists two regimes distinguished by the interplay between the average precession frequency $\bar{\Omega}$ and the momentum scattering time τ_p :

1. Weak scattering regime ($\bar{\Omega} \cdot \tau_p > 1$) : electron spins precess more than one full cycle about the effective field before the scattering event occurs. In this case, the spin relaxation time is proportional to momentum scattering time :

$$T_{s1} \propto \tau_p \quad (1.37)$$

2. Strong scattering regime ($\bar{\Omega} \cdot \tau_p < 1$) : electron spins scatter much more faster than the actions of spin-orbit field do. In this case, the spin-orbit field is characterized by rapid fluctuation. Contrary to the previous regime, the spin relaxation time is inversely proportional to the momentum scattering time :

$$\frac{1}{T_{s1}} \propto \bar{\Omega}^2 \cdot \tau_p \quad (1.38)$$

This regime is also referred as "motional-narrowing" regime. This regime is the most common situation to interpret the experiments performed in III-V or II-VI bulk or nanostructures.

We will focus on the study of Dyakonov-Perel mechanism in GaAs/AlGaAs quantum wells in chapter 3 and 4 in this manuscript.

1.4.3 Bir-Aronov-Pikus mechanism

In addition to the previous spin relaxation mechanisms, the Bir-Aronov-Pikus (BAP) [2] mechanism is an electron spin relaxation mechanism due to the presence of holes through direct exchange interaction between electron and hole. The Hamiltonian featuring this exchange interaction is given by :

$$H_{BAP} = A\mathbf{S} \cdot \mathbf{J}\delta(\mathbf{r}) \quad (1.39)$$

where A is the proportional factor related to exchange integral between electrons and holes, $\mathbf{J}$ is total angular momentum operator for holes, $\mathbf{S}$ is the electron spin operator and $\mathbf{r}$ is the relative position of electron and hole. BAP-type mechanism is important for excitons [60] as well as p-doped semiconductor with large hole concentration at low temperatures. In p-doped semiconductor heterostructures, the resident holes are unpolarized in thermal equilibrium and the exchange interaction simultaneously flip electron and hole spin states. Because the hole spins typically relax much more faster than electron spins do, nonequilibrium electron spins transfer their spin to the hole states and rapidly decay. The spin relaxation time can be calculated as follow [31]:

$$\frac{1}{T_{s1}} = \frac{2}{\tau_0} N_a a_B^3 \frac{v_k}{v_B} \left[\frac{p}{N_a} |\psi(0)|^4 + \frac{5}{3} \frac{N_a - p}{N_a} \right] \quad (1.40)$$

for nondegenerate holes. In the above equation, N_a is the concentration of "acceptor", a_B is the exciton Bohr radius; p is the density of free holes; τ_0 is an exchange splitting parameter : $\hbar/\tau_0 = (3\pi/64)\Delta_{ex}^2/E_B$ in which Δ_{ex} is the exchange splitting of excitonic ground state, E_B is Borh exciton energy; $v_B = \hbar/m_c a_B$; v_k is the electron velocity; $|\psi(0)|^2$ is the Sommeifield's factor.

If holes are degenerate, the spin relaxation time is given by :

$$\frac{1}{T_{s1}} = \frac{3}{\tau_0} p a_B^3 \frac{v_k}{v_B} \frac{k_B T}{E_{Fh}} \quad (1.41)$$

where E_{Fh} is the hole Fermi energy, v_k is the electron's velocity.

1.4.4 Hyperfine interaction mechanism

The hyperfine interaction is a mutual interaction between magnetic moments of electron and nuclei. It is a mechanism characterized by the fact that both electron and nuclear spins relax through their mutual magnetic field. The spins of lattice nuclei which are normally in disordered states create a random effective magnetic field that impacts on electron spins and makes them relax. Usually this

interaction is quite weak for free electron in bulk material and quantum wells but it plays an important role for localized electron (for example in quantum dots).

For s-type electrons, the Hamiltonian interaction between electron spin $\vec{S}$ and nuclear spins $\vec{I}$ is written in this form [31]:

$$H_{hf} = \frac{\mu_0}{4\pi} \frac{8}{3} \pi g_0 \mu_B \sum_i \gamma_{n,i} \hbar \vec{I}_i \cdot \vec{S} \delta(\mathbf{r} - \mathbf{R}_i)$$

where μ_0 is vacuum permeability, $g_0 = 2.0023$ is the g-factor of free electron, μ_B is Bohr magneton, $\gamma_{n,i}$ is the nuclear gyromagnetic ratio, $\mathbf{r}$ is the relative coordinate related to the nucleus, i is the label for nuclei at positions $\mathbf{R}_i$, $\mathbf{S}$ and $\mathbf{I}_i$. The spin relaxation time caused by this interaction is calculated through the rate of mutual spin flips between two spin systems : electron state $|k \uparrow\rangle$ and nuclear state $|I, \mu\rangle$ to other states $|k \downarrow\rangle$ and $|I, \mu + 1\rangle$, respectively.

1.4.5 Exciton spin relaxation mechanism

The exciton spin relaxation is another important process that gives an contribution to spin relaxation in semiconductor heterostructure at low temperature. The theory of exciton spin relaxation has been elaborated by Maialle et al. [60] and observed in GaAs quantum wells [32, 74, 71, 49], in which exciton spin relaxation mechanism is similar to DP mechanism and the electron-hole Coulomb exchange interaction can be considered as an effective magnetic field. This process involves two components : spin flip through an exchange interaction and the "motional-narrowing" process through the scattering of the center of mass of the exciton.

Figure 1.10 illustrates the exciton spin relaxation mechanism which corresponds to the direct exchange interaction between two optical active states : $|+1\rangle$ and $|-1\rangle$ (the similar process between two optical inactive states $|+2\rangle$ and $|-2\rangle$ is not drawn on this figure for clarity); the spin flip processes resulting from the transitions of exciton-bound single (electrons or holes) are also displayed. On the scale in figure 1.10, Δ is the short-range exchange splitting energy between two optical active and inactive states.

The electron-hole exchange interaction can be classified into two distinct parts : the short-range exchange interaction Δ and the long-range exchange interaction Δ_{LT} . In semiconductor quantum well, the long range part is found to be dominant in the spin relaxation process [60].

The exchange splitting for the long range part of lowest heavy-hole exciton in

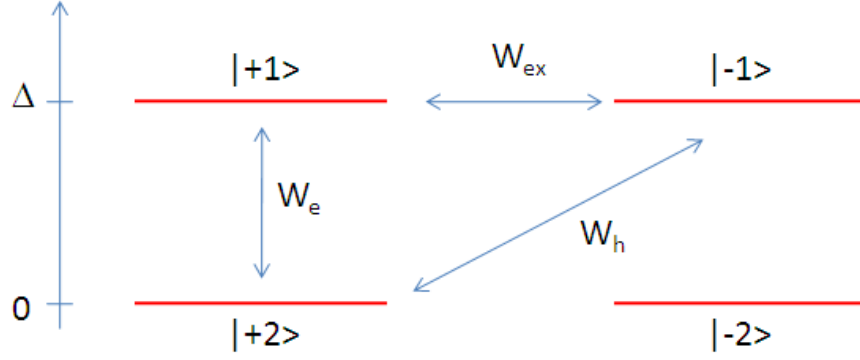


Figure 1.10: Schematics of exciton spin relaxation mechanism : exchange interaction rate (W_{ex}) and single particle spin flip rates (W_e, W_h)

quantum well is [29]:

$$\Delta_{LT}(K_{\perp}) \approx \frac{3}{16} \Delta_{LT}^{3D} |\langle \chi_{e1} | \chi_{h1} \rangle|^2 \left(\frac{E_B}{E_B^{3D}} \right)^2 a_B^{3D} K_{\perp}$$

where $K_{\perp}$ is the k-vector in the interface plane, Δ_{LT}^{3D} is the longitudinal exchange splitting of 3D semiconductor material ($\approx 0.1\text{meV}$ for GaAs and not depend on exciton wave vector), E_B, E_B^{3D} are the exciton binding energies of 2D and 3D semiconductor, respectively. χ_{e1}, χ_{h1} are the envelope wavefunctions of electron and hole, a_B^{3D} is the exciton Bohr radius in 3D semiconductor.

Following the DP mechanism, the exciton spin relaxation time in quantum well is given by :

$$\frac{1}{T} \propto \langle \overline{\Omega_{LT}^2} \rangle \tau_p$$

Conclusion

In conclusion, all the spin relaxation mechanisms discussed above can exist coincidentally. In the experiments, they are only distinguished by system variables such as carrier density, temperature dependence, doping, external field ... etc in order to determine which mechanism dominates. In the GaAs/AlGaAs quantum wells investigated in this thesis, the most efficient electron spin relaxation mechanism is the D'yakonov-Perel one [29, 25].

Chapter 2

Sample description and experimental setup

2.1 Samples description

In this thesis, three different samples have been studied. They were grown on n-doped (111)B GaAs substrates ($n=2 \times 10^{18} \text{cm}^{-3}$) by molecular beam epitaxy. These substrates are misoriented 3° towards (111)A.

In sample LAAS2311 and sample LAAS2328, these layers form a PIN device with the following structure sequence (figure 2.1): n-substrate / 500 nm GaAs layer ($n=2 \cdot 10^{18} \text{cm}^{-3}$) / 500 nm AlGaAs ($n=2 \cdot 10^{18} \text{cm}^{-3}$) / 100 nm non intentionally doped (nid) AlGaAs / 20 QWs / 100 nm nid AlGaAs / 500 nm p-AlGaAs ($p=2 \cdot 10^{18} \text{cm}^{-3}$) / 5 nm p+ GaAs ($p=1 \cdot 10^{19} \text{cm}^{-3}$). The bias is applied between the surface p-contact and the substrate back n-contact.

In sample LAAS2335, the n and p-doped layers are reversed in the sequence and a 500 nm nid GaAs layer followed by a 100 nm nid AlGaAs layer are grown in addition just above the substrate. In this NIP structure, the bias is applied between the un-etched top n-contact and the chemically etched p-contact.

Samples LAAS2328 (PIN) and LAAS2335 (NIP) have a $L_W = 15$ nm quantum well width and $L_B = 12$ nm $\text{Al}_{0.3}\text{Ga}_{0.7}\text{As}$ barrier width while sample LAAS2311 (PIN) has a 7.5 nm quantum well width and the barrier width $L_B = 12$ nm. These samples were grown by Dr. Chantal Fontaine at Laboratory for Analysis and Architecture of Systems (LAAS), Toulouse, France.

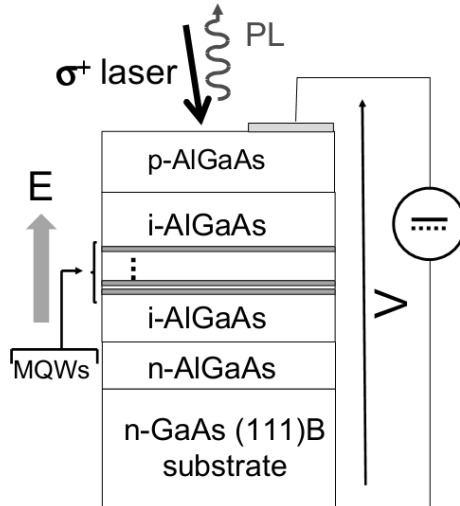


Figure 2.1: PIN/NIP AlGaAs/GaAs MQW sample structure [14]

2.2 Time and polarization resolved photoluminescence spectroscopy

The invention of laser in 1960 made photoluminescence spectroscopy (PL) become a powerful technique for studying material characterization and dynamic processes that occur in semiconductors. The use of pulsed excitation laser opens up the possibility of monitoring photoluminescence intensity as a function of time. Thanks to ultrafast laser sources, time-resolved photoluminescence became the most popular method to probe phenomena occurring on the time scale of hundreds of femtoseconds because this method allow us to measure the dynamics of PL spectra as function of time [73]. In this chapter, the setup of time-resolved photoluminescence spectroscopy used for measuring the PL intensity dynamics and the spin relaxation time as a function of external applied electric field is introduced.

2.2.1 Experimental setup

Figure 2.2 below represents the scheme of time-resolved photoluminescence setup used to measure the electron spin relaxation time in the different samples. The polarisation of the excitation pulse coming from a mode-locked Ti-Sapphire laser is controlled by a system of optical elements : a linear polarizer and a $\lambda/4$ waveplate. The first linear polarizer adjusts the excitation beam into absolute linear polarization; after that, the $\lambda/4$ plate changes the polarization of excitation beam from linear to left-hand or right-hand circular polarization. The luminescence signal emitted by the sample in the cryostat is collimated and passed through a $\lambda/4$ plate and a polarizer. It is then focused onto the entrance slit of a monochromator with its chosen polarization. Subsequently, this luminescence signal is selected automatically by a proper grating in the monochromator and sent to a streak camera (an ultrafast detector) synchronised with the laser pulse by a photodiode having ultrafast response. Finally, this signal is recorded by a high-sensitive CCD camera.

The different elements composing the experimental setup will be detailed below.

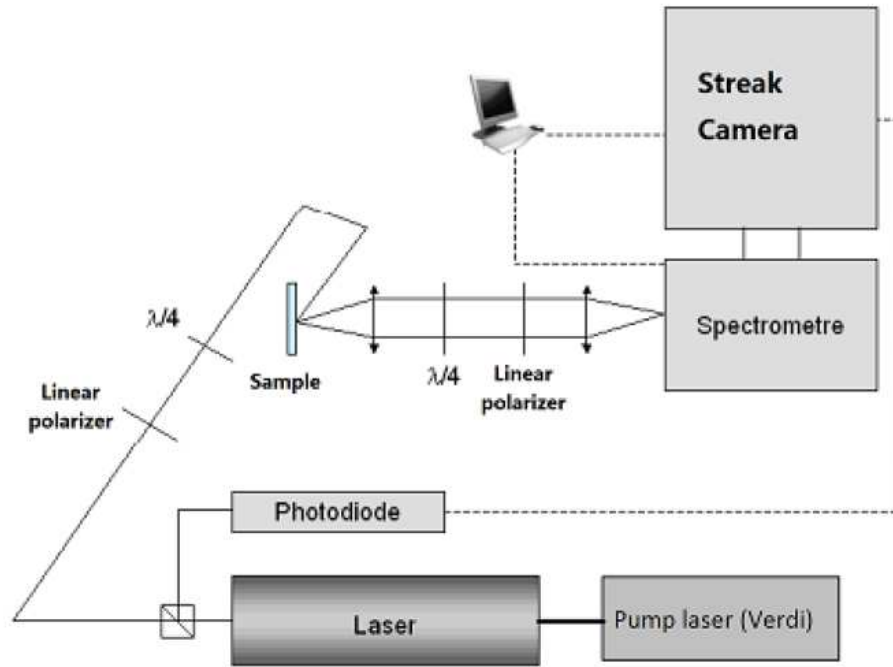


Figure 2.2: Time-resolved photoluminescence setup

2.2.2 Excitation source

For studying time-resolved effects and visualizing the dynamic processes in the samples, it is necessary to utilise a laser running in pulsed regime. In fact, the main excitation source in our experiment is a mode-locked Titanium-Sapphire (Ti:Sa) laser pumped by a continuous wave (CW) laser Coherent VERDI-V10. The Coherent Verdi V-10 laser is a solid state diode pumped frequency doubled laser system that can generate up to 10W CW optical power at single frequency 532 nm. This pump laser uses a $Nd : YVO_4$ (Neodymium Doped Yttrium Orthovanadate) crystal as amplifying medium that has an absorption wavelength of 809 nm and emission wavelength of 1064 nm. This crystal is pumped by 2 laser diode arrays placed in a monomode cavity and the emission wavelength of 532 nm is achieved by doubling the fundamental frequency (1064 nm) of a LBO crystal placed in the cavity. The stability of the VERDI laser output power is controlled by feedback circuitry on the measured crystal temperature and output power.

The key component of the mode-locked laser is the Ti:sapphire crystal produced

by introducing Ti_2O_3 into a melt of Al_2O_3 ; which causes Ti^{3+} ion substitute a small percentage of Al^{3+} ions. Consequently, the electronic ground state of Ti^{3+} ions is split into a pair of vibrationally broadened level making a wide tuning range for this laser [62] as shown in figure 2.3(a). The Ti:Sa laser can operate in

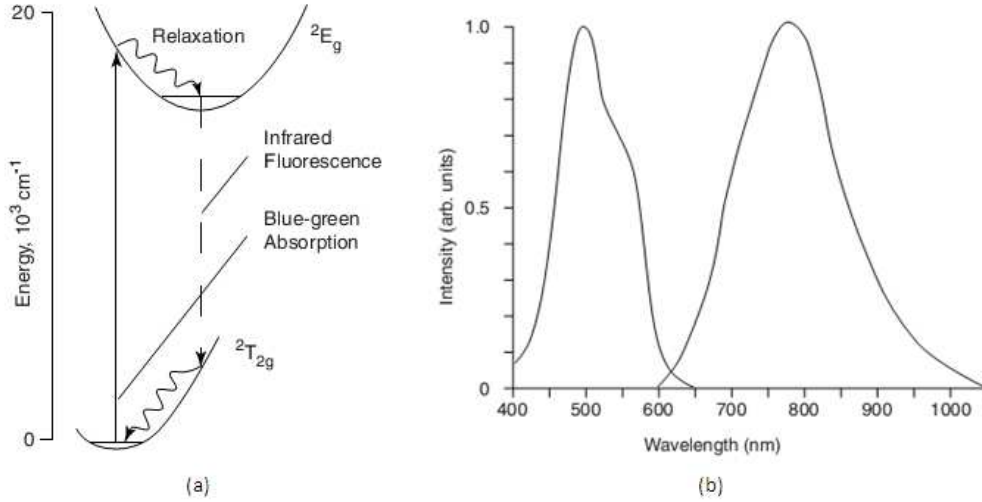


Figure 2.3: Ti:Sa characteristics :(a)Energy level structure of Ti^{3+} [62]

(b)Absorption and emission spectra of Ti:sapphire

pulsed regime with the repetition rate of 80 Mhz in two modes : (i) femtosecond mode generating 100 fs pulsed width; (ii) picosecond mode generating a pulse duration of about 1.5 ps. In our experiment, we use the picosecond mode due to the good compromise between two required parameters : spectral width and time resolution.

To achieve pulsed operation by mode-locking technique, Ti:Sa laser utilises self-phase modulation and Kerr effects in the crystal. The longitudinal optical modes in the laser cavity are synchronised by optical Kerr effect in the crystal. Due to non-uniform power density distribution of the laser beam characterized by a Gaussian distribution, the refractive index of the crystal changes across the beam profile; this makes the crystal act as a converging lens. This refractive index variation is responsible for nonlinear optical effects of self-focusing and self-phase modulation. As a result, the pulse length further shrinks to attain higher field strength at the center of optical pulse and the modes of the cavity are locked. In order to start the pulsed operation, an acoustic-optical modulator synchronised with laser pulse is placed in the cavity. While the beam travels along the cavity, the dispersion of group velocity needs to be compensated in

order to preserve the desired pulsed operation. This compensation is obtained by a Gires-Tournois Interferometer (GTI) installed into the mirror plate embedded in the cavity providing the necessary wavelength dispersion variation to make the different modes composing the output laser pulse in-phase. The GTI allows to adjust pulse width from 1 to 1.7 ps. The lasing wavelength is selected by a narrow tunable bandpass birefringence optical filter (Lyot filter). The characteristics of Ti:Sa laser are summarised in table 2.1.

Repetition Frequency	Spectral Range	Average Power	Pulse Energy	Pulse Duration	Spectral Width
80 MHz	680-990 nm	0.4-2 W	5-25 nJ	1.2-1.8 ps	≈ 1.2 meV

Table 2.1: Ti:Sa laser main characteristics

2.2.3 Streak camera detection

To detect the photoluminescence signal of the samples, streak camera is one of the highest sensitive, fastest detectors that can capture the signal in both spectral and temporal range. The principle of a streak camera is shown in figure 2.4(a). The measured light signal is guided onto the slit of streak tube and focused by the optical lens system on the photocathode. This high sensitive photocathode plays a key role to convert the light signal into photocurrent. Then, these photo-electrons are accelerated and passed between a pair of sweep electrodes controlled by a sweeping circuit that can apply a high voltage on two electrodes resulting in a high-speed sweep depending on time. The sweeping mechanism synchronised with the laser pulse by an ultrafast photodiode is shown in figure 2.4(b). Figure 2.5 depicts the spectral response of the streak tube (N5716-02 S1 photocathode) which is used in our experiments.

As a consequence, the electrons experience a vertical deflection proportional to the arrival time of photons on photocathode before reaching the MCP (micro-channel plate). When passing through this MCP, the electrons are multiplied by several thousands of times and then hit on a phosphoscreen where they are converted back to light and recorded by a CCD (Charged Coupled Device) camera. As a result, an obtained fluorescence image is arranged as follow : the earlier signal is spatially located at the top of the image and the photons which arrive latter yield a signal located at the bottom of the image. At the same time, the different wavelengths of the emission are spread horizontally on the image. In fact, the image retrieved from streak camera contains both spectral and tempo-

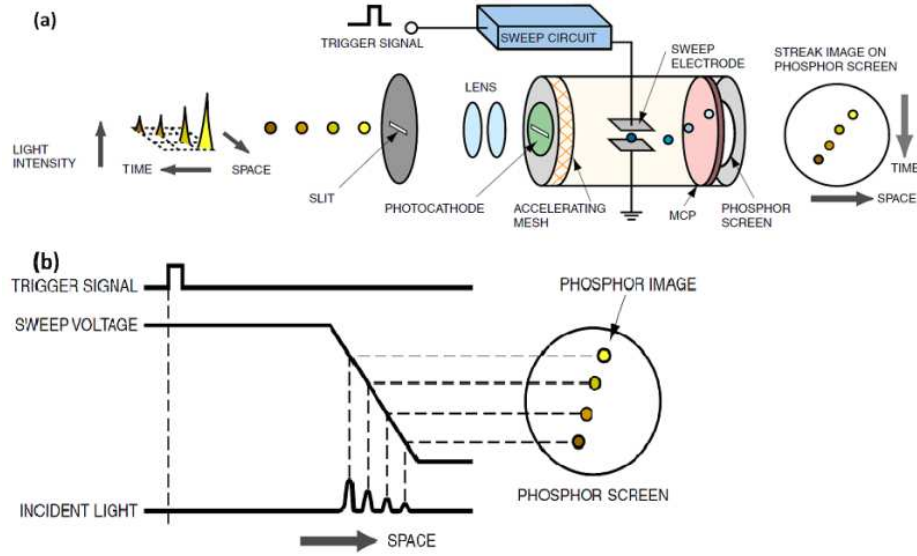


Figure 2.4: Streak camera operational principle: (a) Operating Principle of Streak Camera (b) Sweeping mechanism responsible for the time resolution

ral informations of incident light whose intensity is recorded in a colour scale as shown in figure 2.6.

2.2.4 Spectral and temporal resolution

The spectral resolution is obtained by a Jobin-Yvon Triax 320 monochromator, placed before the streak camera and equipped with 3 gratings presented in table 2.2.

The intrinsic time resolution of the streak camera is about 2ps. However, the

Grating (grooves/nm)	Blazing wavelength (nm)	Spectral width (nm)	Dispersion (nm/nm)	Resolution (nm)
80	870	188.0	39.6	0.6
300	1000	46.2	10.56	0.3
600	400-1300	22.2	5.28	0.1

Table 2.2: The characteristics of the Jobin-Yvon Triax 320 monochromator. The spectral resolution values are given for a 200 μm and 80 μm width of the entrance and exit slit respectively.

effective time resolution depends on the wavelength dispersion of the monochromator. A larger dispersion produces a larger optical path between the portion of light travelling on the side and on the central optical axis of the grating. Therefore, this difference generates a temporal dispersion which can be minimised by

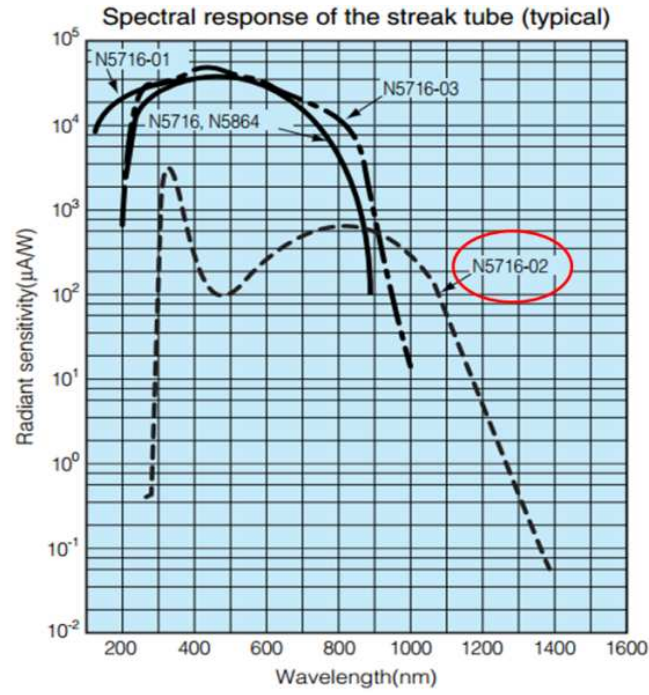


Figure 2.5: Spectral responsibility of the different streak tube S1

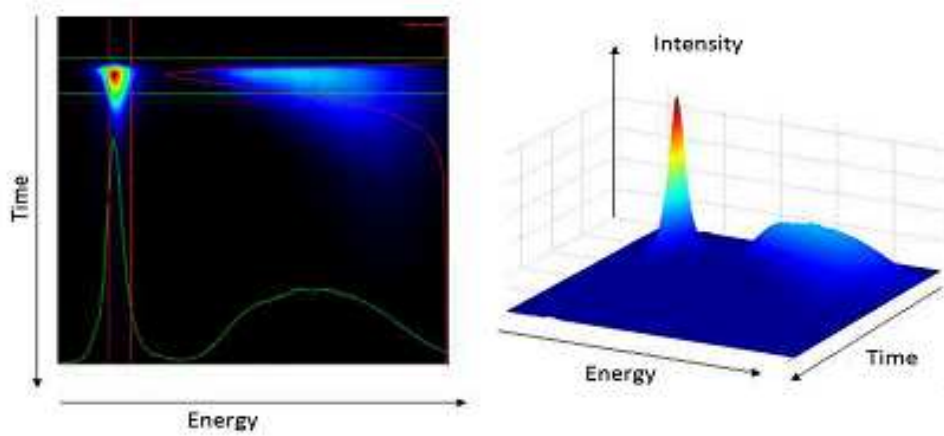


Figure 2.6: Example of a streak camera image

placing a vertical slit inside the monochromator to reduce the lateral dimension of the beam.

The streak camera can be operated in 4 different time ranges presented in table 2.3.

Time range index	Temporal span (ps)	Time resolution (ps) using the 80 grooves/mm grating
1	157	5
2	809	8
3	1542	12
4	2263	17

Table 2.3: Temporal mode of streak camera

2.2.5 Polarization resolution

For studying the temporal evolution of spin orientation of electrons in quantum heterostructures, time and polarization resolved photoluminescence experiment is used to detect spin polarization of photogenerated carrier by measuring the degree of circular polarization of emitted photoluminescence coming from the samples after excitation. In fact, the excitation on the sample generated by a circular-polarized laser pulse initializes spin orientation of the ensemble of photoexcited electrons and holes. The exciting laser polarization can be adjusted from linear (Σ^X) to elliptical or circular polarization (Σ^+ or Σ^-) by a Glan-Taylor polarizing cube together with a quarter-wave plate whose neutral axis can be rotated (see figure 2.2). The incident laser beam then is focused onto the samples with a $50\mu\text{m}$ diameter of spot size by the lens. Because of optical selection rules (see section 1.3.2), the photoluminescence emitted from the samples contains the counter-polarized (I^-) and co-polarized (I^+) light with respect to the laser excitation. The PL polarization analysis is performed by a quarter-wave plate followed by the Glan-Taylor cube on the collimated beam before going to the optical collection system of the detector. The relevant images of the two PL components are stored and analysed separately.

The circular polarization degree is defined as :

$$P_{\text{circ}} = \frac{I^+ - I^-}{I^+ + I^-} \quad (2.1)$$

2.2.6 Cooling system and magnetic field

Cooling system

To perform measurements at low temperatures, samples were placed into a Helium closed cycle cryostat. The sample is mounted at the end of the cold finger fixed in the inner region of the cryostat thermally isolated with outer environment by a vacuum chamber with a typical pressure of 10^{-6}mbar . The temperature

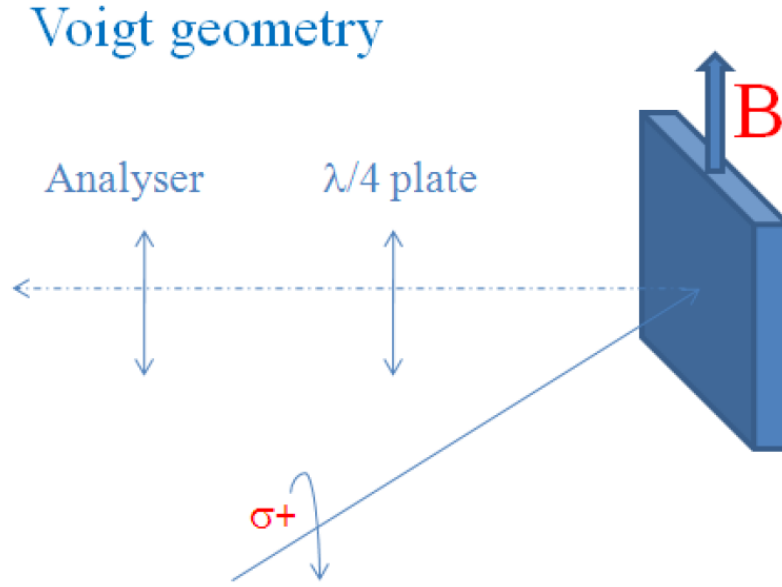


Figure 2.7: Spin quantum beat spectroscopy setup in Voigt geometry

in the cryostat can be varied from 10K to 300K. This temperature is stabilized by a heating element placed in the cryostat. This element is controlled by a PID controller outside the cryostat which was used to monitor the temperature through two sensors placed in the cryostat and adjust the heater accordingly. In addition, for the purpose of applying an electric field, the sample is mounted on an Alumina plate in order to make an isolation with the metallic cold finger. The DC voltage applied on the sample is generated by a precise Source Meter Keithley 2400. The cryostat has the glass windows that allow an optical access to the sample.

Magnetic field

To measure the tranverse electron spin relaxation time (see chapter 3), spin quantum beat spectroscopy is used as shown in figure 2.7. In that experiment, an external magnetic field is applied in Voigt geometry (magnetic field parallel to the sample plane and perpendicular to the excitation light wave vector). A quasi-homogeneous magnetic field in the range of 200 mT to 800 mT is produced by two permanent magnets $\approx 650\text{mT}$ each in Hemholtz configuration. In order to change magnetic field strength, the mutual distance between two magnets can be varied symmetrically from the center of the sample.

Chapter 3

Electrical control of the electron spin relaxation time

3.1 State of the art

The change in dimensionality of semiconductor nanostructures modifies the electronic band structure and as a consequence the characteristics of electron spin relaxation. For III-V and II-VI semiconductor 2D structure (quantum well), DP and BAP spin relaxation mechanisms dominate as discussed in section 1.4. Theoretical calculation by means of the fully microscopic kinetic spin Bloch equation (KSBE) [41] shows that DP mechanism plays the most important role in undoped GaAs quantum well and is comparable to BAP mechanism in p-doped narrow quantum well. Time-resolved measurements of the electron spin relaxation in GaAs quantum wells performed on $\langle 001 \rangle$ substrate demonstrated that the DP mechanism is the dominant mechanism in the temperature range from 50K to 300K [22, 30]. As all of the work in this manuscript focuses on undoped GaAs quantum well, we consider only DP mechanism. The key ingredient of the DP mechanism is an effective magnetic field Ω resulting from the interplay between the spin-orbit interaction and the lack of inversion symmetry of III-V zinc-blende semiconductor (see section 1.4.2). There are three components of this inversion symmetry in semiconductor quantum well: interfacial inversion asymmetry (IIA), bulk inversion asymmetry (BIA) and structural inversion asymmetry (SIA) [29]. To detail the contributions of these different components in GaAs/AlGaAs quantum well, the role of each component is analysed below :

1. Interfacial inversion asymmetry : IIA originates from the difference of chemical bonding structures at the interface of quantum well [43] because the quantum well and barrier layers do not share a common atom breaking the natural quantum well symmetry and making the precession vector anisotropic. For symmetric GaAs/AlGaAs quantum well, this term can be neglected.
2. Bulk inversion asymmetry : In III-V zinc-blende semiconductor with the presence of two different kinds of atoms in the lattice structure of the bulk material, the symmetry is broken and the precession vector Ω_{BIA} resulting from BIA (usually called Dresselhaus term [15]) depends on the crystallographic growth direction and the confinement energy in quantum well. Under the approximation that the cubic term is weak in the Hamiltonian i.e $\langle k_z^2 \rangle \gg k_x^2, k_y^2$, the precession vector corresponding to the effective internal magnetic field for the lowest electron subband depends on the growth

direction $\mathbf{z}$ [14] :

$$\Omega_{BIA}(\mathbf{k}_\perp) = \frac{\gamma}{\hbar} \langle k_z^2 \rangle (-k_x, k_y, 0) \quad \text{for } \mathbf{z} \parallel [001] \quad (3.1a)$$

$$\Omega_{BIA}(\mathbf{k}_\perp) = \frac{\gamma}{2\hbar} \langle k_z^2 \rangle (0, 0, k_y) \quad \text{for } \mathbf{z} \parallel [100] \quad (3.1b)$$

$$\Omega_{BIA}(\mathbf{k}_\perp) = \frac{2\gamma}{\hbar\sqrt{3}} \langle k_z^2 \rangle (k_y, -k_x, 0) \quad \text{for } \mathbf{z} \parallel [111] \quad (3.1c)$$

where $\langle k_z^2 \rangle$ is the averaged squared wavevector along the growth direction and $\mathbf{k}_\perp$ the in-plane wavevector.

3. Structural inversion asymmetry : SIA is an inversion asymmetry resulting from the potential profile of quantum well and is commonly called Rashba term [84]. This potential can be a built-in or an external potential. Recent experiments showed that the SIA spin splitting can be tuned by applying an external electric field [51] or using an asymmetric doping profile [52]. For an electric field applied along the growth axis, this SIA term is written as:

$$\Omega_{SIA}(k_\perp) = \frac{\alpha}{\hbar} (k_y, -k_x, 0) \quad (3.2)$$

where $\alpha = a_6^4 \cdot E$ is the Rashba coefficient (E is the electric field amplitude and a_6^4 is a constant). It is very important to note that the SIA effective magnetic field spin precession vector is always in the plane of quantum well for transverse electric field ($\vec{E} \parallel \mathbf{Oz}$) regardless of the crystallographic growth direction of the quantum well.

As shown in equation 3.1 and 3.2, the strength of Dresselhaus term and Rashba term can be tuned by varying the confinement energy through quantum well width ($\langle k_z^2 \rangle \propto 1/L_W^2$) and applied electric field through the gate voltage, respectively. Figure 3.1 shows the schematic of Dresselhaus and Rashba effective magnetic fields for different growth directions of quantum well [18].

For standard $\langle 001 \rangle$ -oriented quantum well, the Dresselhaus and Rashba precession vectors are oriented in the quantum well plane. As seen in figure 3.1(a), the orientation of the two effective field are different for a given wavevector. As a consequence, the Dresselhaus component can not be compensated by the Rashba one and the electron spin relaxation time can not be controlled by the application of the external electric field. However, there is an exception. If the electron spin is oriented along the $[\bar{1}10]$ -direction (or $[110]$), it turns out that Ω_{BIA} and Ω_{SIA} are parallel and this compensation can occur. If the electron spin is oriented in another direction, this compensation effect is impossible. As a result, the spin

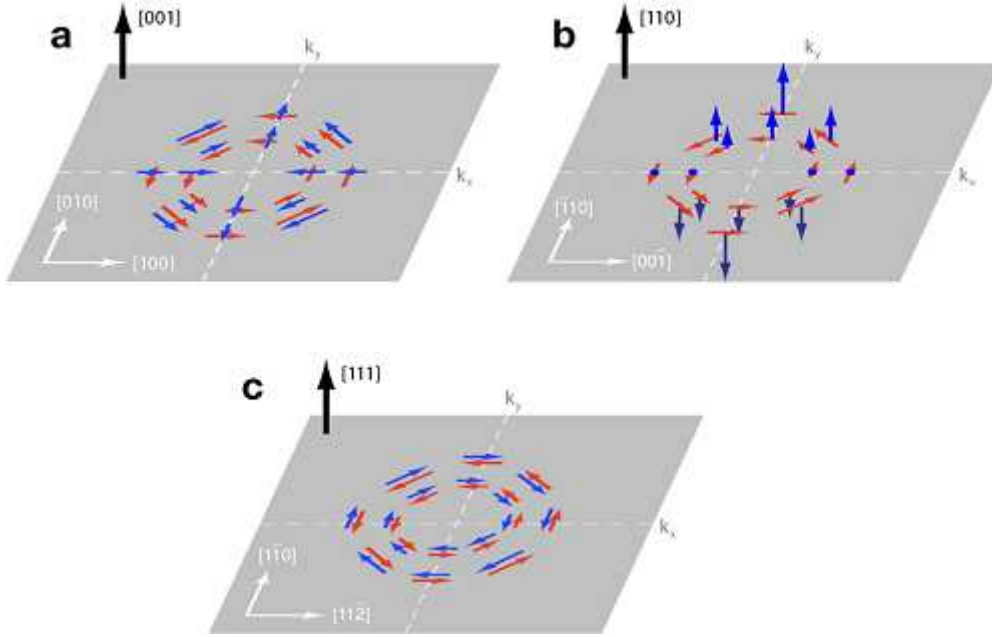


Figure 3.1: Spin-orbit effective fields in a zinc-blend quantum well : (a)<001> (b)<110> (c)<111>. The Dresselhaus term is indicated in blue whereas the Rashba term is indicated in red. For <111>-quantum well, two terms are in-plane with the same momentum-dependent structure [18]

relaxation time for electron spin oriented along $[\bar{1}10]$ -axis (or $[110]$ -axis) is very long but much more shorter for other spin orientation; this introduces a large spin relaxation anisotropy [61](see figure 3.2(a)).

For <110>-oriented quantum well, the BIA precession vector is entirely out of plane. Without an external field, electron spins created along the $[110]$ -axis do not relax as predicted initially by Dyakonov and Kachorovskii [55] and the longitudinal spin relaxation time is as long as 20 ns at room temperature [51]. However, if the electron spin lies in quantum well plane, it feels the out of plane Dresselhaus field. Consequently, the corresponding spin relaxation time is very short. When electric field is applied along $\mathbf{Oz}$, the Rashba term is in-plane, i.e it is perpendicular to the Dresselhaus one. By embedding <110>-oriented GaAs QW in PIN structure, it was experimentally demonstrated indeed that the relaxation time of electron spins oriented in $\mathbf{z}$ -axis decreases when the reverse bias increases [51, 42]. In brief, for <001> and <110> oriented quantum well, DP spin relaxation mechanism can only be suppressed in one direction and demonstrates a large spin relaxation anisotropy as shown in the calculation presented in figure 3.2.

Contrary to two previous growth directions, the situation is different for $\langle 111 \rangle$ -oriented quantum well which offers the possibility of complete suppression of DP mechanism (see figure 3.2(c)). Figure 3.1(c) shows that the Dresselhaus and Rashba term have the same direction whatever the wavevector $\mathbf{k}$ is. In the first order approximation of the Hamiltonian (neglecting the cubic Dresselhaus terms), the total effective field vanishes at a proper applied electric field $\mathbf{E}$ which fullfills the equation :

$$\frac{2}{\sqrt{3}} \langle k_z^2 \rangle = -a_6^4 E \quad (3.3)$$

In this chapter, the measurement of the electron spin dynamics in $\langle 111 \rangle$ -oriented GaAs-AlGaAs structure is presented, focusing on the effect of the electrical field control of the spin relaxation time in quantum well grown on $\langle 111 \rangle$ B substrate embedded in PIN or NIP structure. The suppression of DP spin relaxation mechanism is observed through time-resolved photoluminescence spectroscopy. The cancellation effect of Dresselhaus and Rashba terms only occurs in NIP structure when a reverse bias is applied and the obtained results are in good agreement with theoretical predictions [17, 33, 5].

3.2 Sample LAAS2311

In order to observe the influence of electric field on the electron spin relaxation time in $\langle 111 \rangle$ -oriented GaAs QW, a first sample LAAS2311 was grown. It consists of a PIN structure with twenty 7.5 nm GaAs QWs separated by 12 nm $Al_{0.3}Ga_{0.7}As$ barrier layers (see section 2.1 for the detailed characteristics of this sample).

The electron spin relaxation time in this sample is probed directly by time-resolved PL measurement under nonresonant condition. In this experiment, the sample is excited by circular-polarized laser pulse having photon energy which is above the transition of first-subband electron and first-subband light hole (E1-LH1) in quantum well. Through the measurement of the circular polarization dynamics of the luminescence, we observed directly the electron spin relaxation time because of two reasons : (i) hole spin relaxation time is very fast (in the order of $\tau_h \approx 1ps$ [74]) under nonresonant excitation conditions because of the strong spin-orbit coupling in the valence band [46]; (ii) the exciton spin relaxation time due to the electron-hole exchange interaction is inefficient because of the short τ_h .

For sample LAAS2311 ($L_W = 7.5$ nm), the excitation wavelength of the Ti:Sa

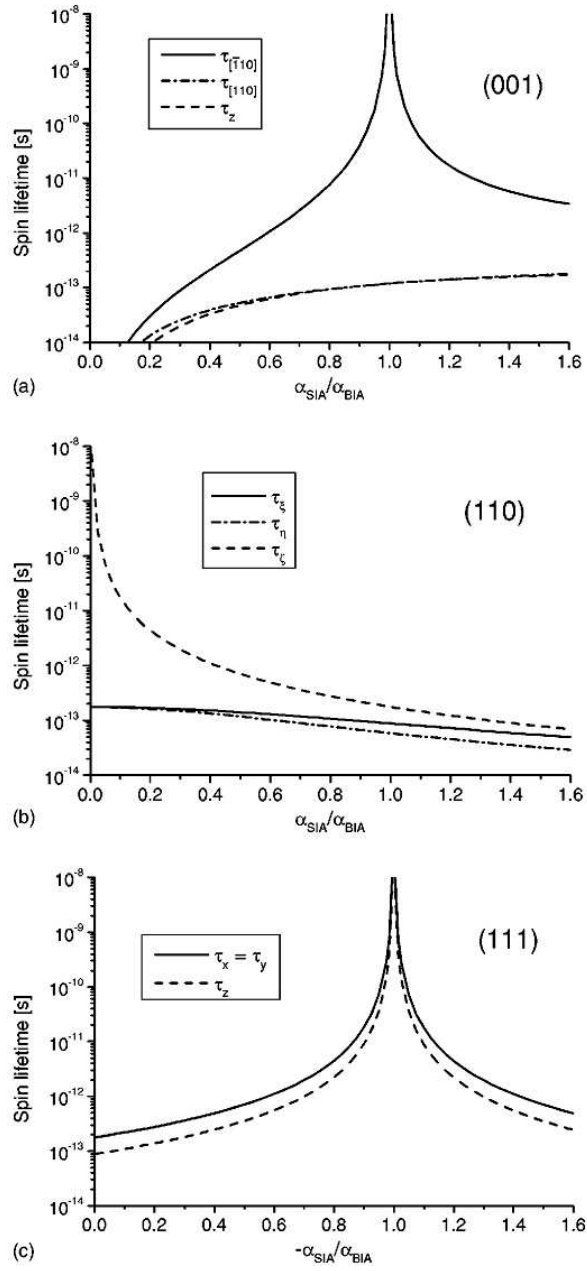


Figure 3.2: The comparison of spin relaxation times for the three spin components in $\langle 001 \rangle$, $\langle 110 \rangle$ and $\langle 111 \rangle$ -grown QW (theoretical calculation by X.Cartoxà et al. in [17]). In the figure, $\alpha_{SIA} = a_6^4 \cdot E$ and $\alpha_{BIA} = (2/\sqrt{3}) \langle k_z^2 \rangle$

laser operating in pulsed regime was set at 785 nm with 1mW optical power. The PL peak wavelength corresponding to the E1-HH1 transition at 8K is 792 nm.

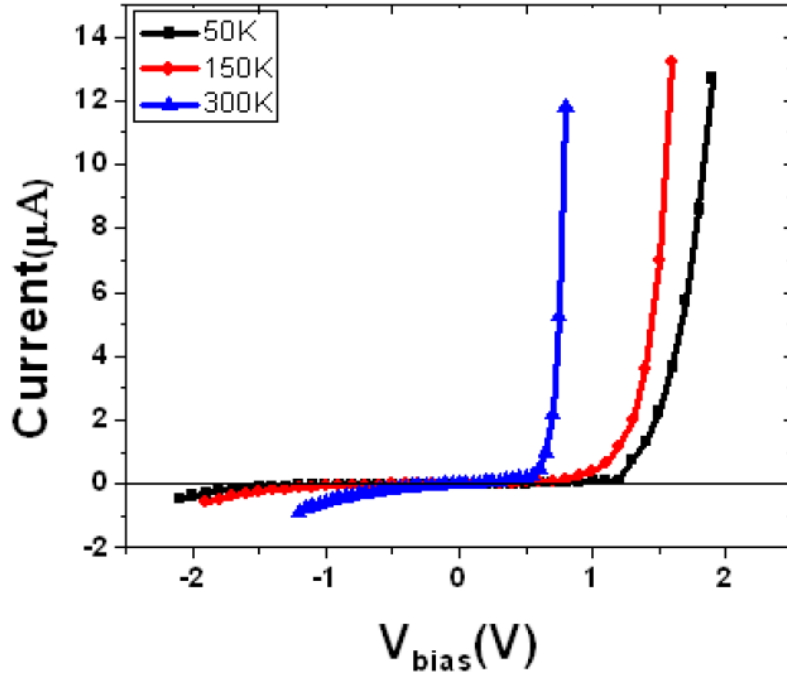


Figure 3.3: The current voltage measurement (in the dark) of sample LAAS2311 for different temperatures.

3.2.1 I-V measurement

Figure 3.3 depicts the current-voltage measurement of sample LAAS2311 (PIN structure) for different temperatures in order to check the quality of the PIN junction in different experimental conditions. We observed the well expected diode characteristics. Even at room temperature, a large reverse bias can be applied with a rather small reverse current. This will allow us to apply large external electric fields on the quantum well located in the intrinsic region of the diode.

3.2.2 Spin relaxation time depending on electric field

Figure 3.4 shows the decay of the PL circular polarization for different bias voltages at 8K. Within the uncertainty of this experiment, we do not observe any significant variation of electron spin relaxation time τ_s when the bias voltage changes in the range from -3V to +2V. We measured that $\tau_s \approx 60$ ps. This value is very close to the results measured in GaAs QW with the same quantum well width grown on <001> substrate.

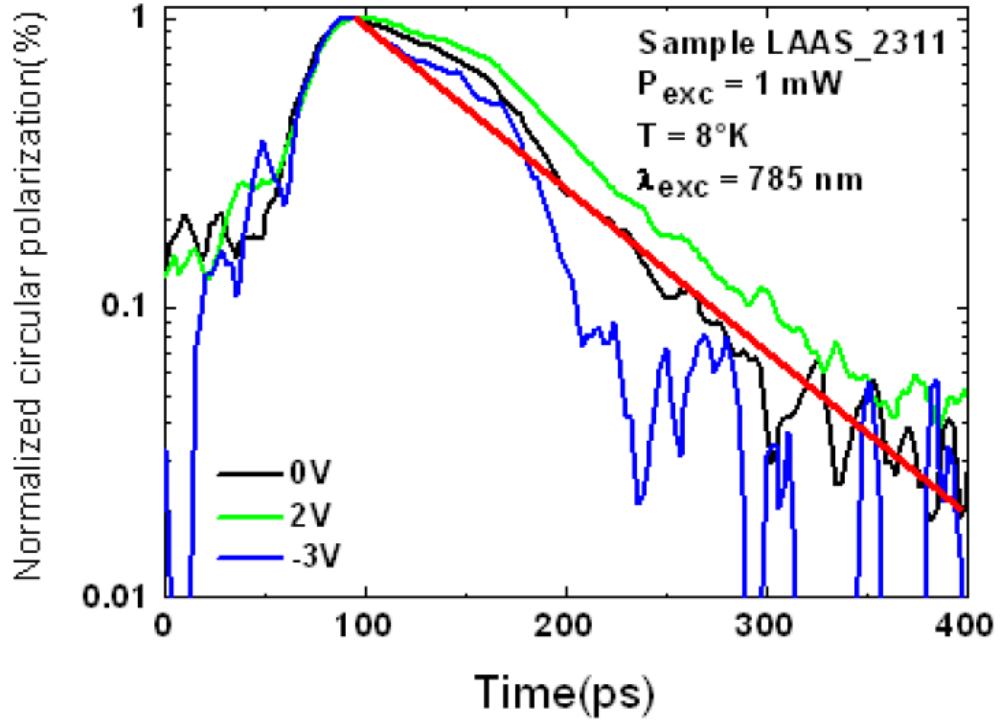


Figure 3.4: PL circular polarization dynamics for different bias voltages in sample LAAS2311 ($L_W = 7.5nm$)

3.2.3 Electric field effect on the photoluminescence spectrum

In the aforementioned results, the spin relaxation times were measured under different applied bias voltages. To verify the actual electric field presented in the sample, the quantum confined Stark effect (QCSE) can be used [47, 78, 79, 9]. Figure 3.5 depicts the Stark shift of the peak wavelength of quantum well photoluminescence when the bias voltage changes from forward bias to reverse bias. This figure shows an increase of the peak PL wavelength of about 3.5 nm (a shift of about 6.8 meV of heavy-hole exciton in QW) when the bias voltage changes from 3V to -3V. This proves that the electric field acting on the electrons in the quantum wells increases when the reverse bias increases.

3.2.4 Average electric field estimation

Figure 3.6 presents the band diagram of a quantum well under the influence of an external electric field. With an increasing electric field oriented perpendicular

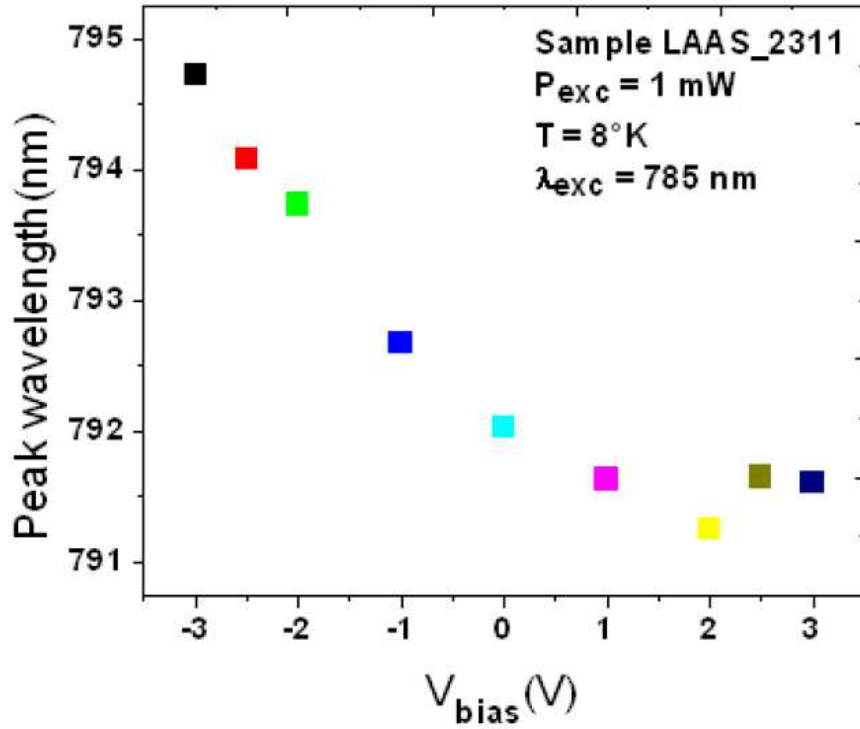


Figure 3.5: Stark shift of the peak PL wavelength depending on applied bias

to the quantum well plane, the energy band tends to tilt. Neglecting the internal charge distribution of the PIN junction of the sample, the average electric field is calculated as followed :

$$|E| = \frac{V_b + V}{d} \quad (3.4)$$

where V_b is the built-in threshold of the PIN junction, V is the applied bias voltage and d is the intrinsic region thickness (see figure 3.6).

In this structure, 20 periods of 7.5 nm QWs separated by 12 nm barriers are grown between two 0.1 μm undoped buffer layer of AlGaAs. We can calculate the width of the intrinsic region by :

$$d = 20 \times 0.0075 + 21 \times 0.012 + 2 \times 0.1 = 0.602(\mu\text{m}) \quad (3.5)$$

At 8K, the measured threshold of PIN diode is about 1.5 V (corresponding to a built-in electric field in the sample structure of 20kV/cm); so, at -3V reverse bias, the absolute value of applied electric field is:

$$|E| = \frac{V_{built-in} + V_{bias}}{d} = \frac{1.5 + 3}{0.602} = 74.7(\text{kV/cm}) \quad (3.6)$$

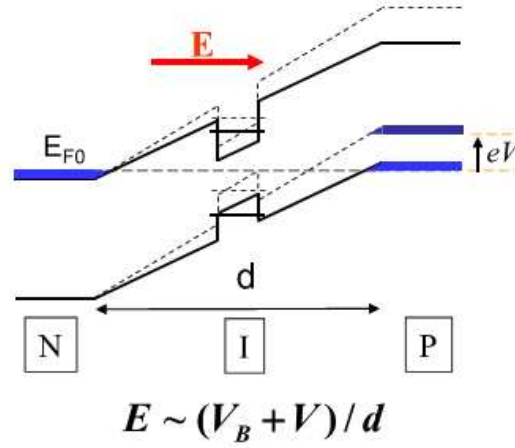


Figure 3.6: Schematic of the PIN structure and average electric field calculation in quantum wells [65]

G. Lengyel et al. [23] used a semi-empirical model to evaluate the QCSE shift

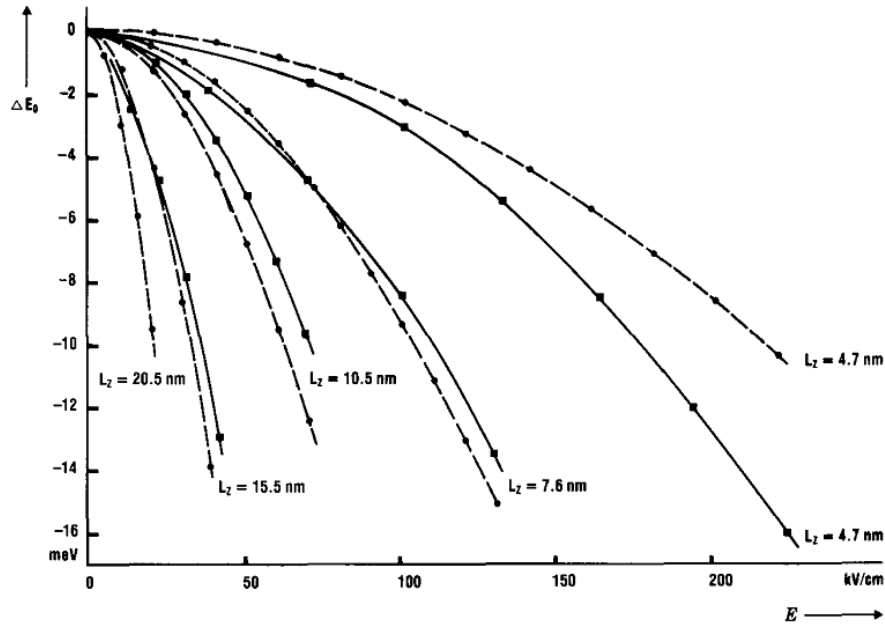


Figure 3.7: The shift of E1-HH1 exciton with applied electric field (Experiment : solid line, model calculation : dashed line) in GaAs/AlGaAs quantum wells [23]

of heavy-hole exciton (E1-HH1) with applied field. This model shows a remarkable agreement with experiments for GaAs/AlGaAs quantum wells ranging from 5 to 20 nm. Figure 3.7 presents this empirical model to estimate the electric field

amplitude in the sample. The measured shift of the heavy-hole exciton in sample LAAS2311 is about :

$$\Delta E_{XH} = 1.2395 \times \left(\frac{1}{0.79473} - \frac{1}{0.79125} \right) \approx -6.9(\text{meV}) \quad (3.7)$$

This measurement is in quite good agreement with the model of G.Lengyel presented in figure 3.7. Deduced from the model in figure 3.7, the 6.9 meV shift of heavy-hole exciton corresponds to 80 kV/cm (compared with the estimated value 74.7 kV/cm from the thickness of sample in equation 3.6). Clearly, we can use equation 3.6 to evaluate the average electric field amplitude in quantum wells.

3.2.5 Analysis of the carriers lifetime

In figure 3.8, we observe that the photoluminescence decay time reduces when a negative voltage is applied. With an increasing electric field in QWs due to the negative bias, the band structure of QWs inclines; so, confined electrons and holes in QWs have a trend to tunnel through the barriers out of the quantum well. Therefore, the PL decay time decreases with the larger reverse bias. Let us recall that the barrier width of this sample is only 12 nm. If the barrier layers become larger, this effect is less pronounced. Actually, in the samples with larger quantum well width and larger tunnel barrier width, we even observe the expected increase of the PL decay time when the reverse voltage increases as a consequence of the weaker overlap between electron and hole wavefunctions (QCSE).

3.3 Sample LAAS2335 and LAAS2328

As described in previous section, we did not observe any clear variation of the spin relaxation time depending on the applied electric field in the sample LAAS2311 ($L_W = 7.5$ nm). Indeed, the interpretation is that the Dresselhaus field is too large to be compensated in this sample because the Dresselhaus field is inversely proportional to the well width (see section 3.1) :

$$\Omega_{BIA} \approx \frac{2}{\sqrt{3}} \langle k_z^2 \rangle \approx \frac{2}{\sqrt{3}} \left(\frac{\pi}{L_W} \right)^2$$

in which $\langle k_z^2 \rangle \approx (\pi/L_W)^2$ (see appendix B for more detail). As a consequence, narrow QW such as sample LAAS2311 requires large electric field to make Rashba field compensate the contribution of Dresselhaus field. In order to evidence the electric field control of spin relaxation time, samples with larger QWs were grown. Two samples with twenty quantum wells ($L_W = 15$ nm) were embedded in a PIN

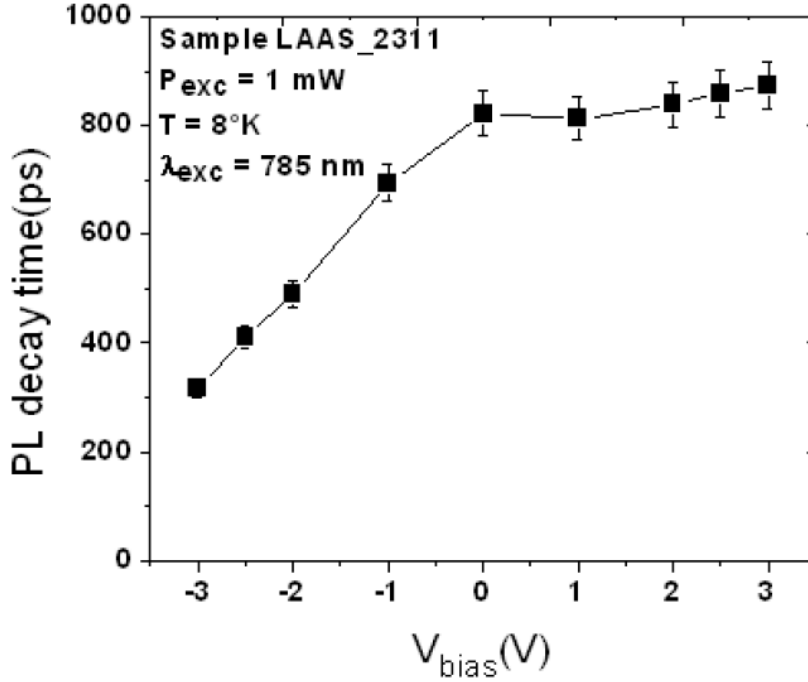


Figure 3.8: Dependence of the PL decay time as a function of the bias voltage

structure (sample LAAS2328) and NIP structure (sample LAAS2335). The electron spin relaxation times in the two structures were measured by time-resolved PL measurement under nonresonant condition. The laser wavelength was set at 792 nm corresponding to excitation energy about 1.56 eV (larger than E1-LH1 exciton of quantum well). The average excitation power (P_{exc}) was 1mW with $50\mu\text{m}$ spot size focused on the samples in the cryostat at 50K.

3.3.1 I-V measurement

Figure 3.9 depicts the current-voltage measurement of sample LAAS2335 (NIP structure) both under the illumination of Ti:Sa laser with wavelength of 792 nm and in the dark. In the dark, the device shows a typical behaviour of normal diode with an exponential increase in the forward bias and no current in reverse bias. Under the illumination of laser with different excitation powers, this device shows an increase of the current in reverse bias. As shown in the figure 3.9, the reverse current increases up to 0.12 mA at -2V under 50mW laser light.

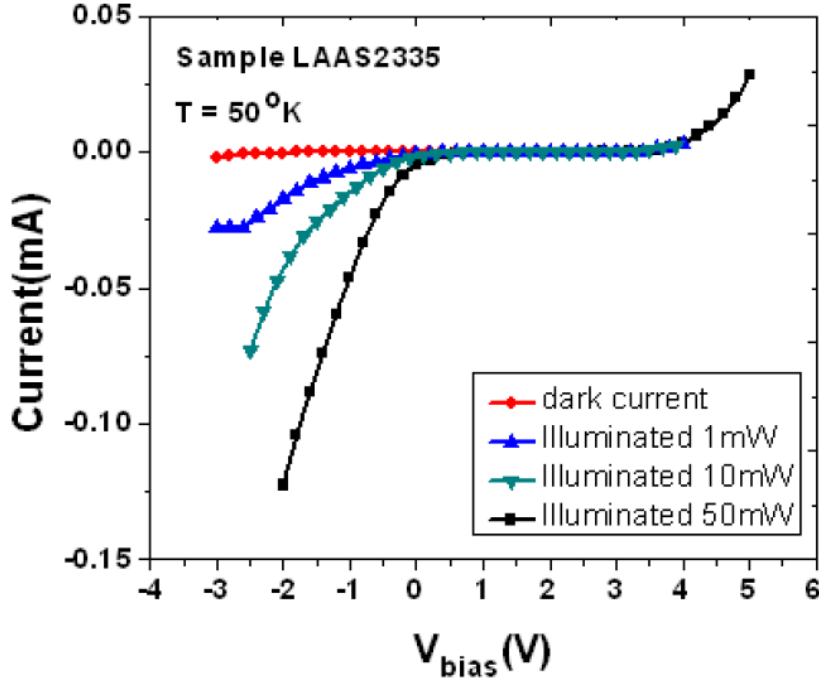


Figure 3.9: The current voltage measurement of sample LAAS2335 at 50K

3.3.2 Electric field control of spin relaxation time

Figure 3.10 displays the PL circular polarization dynamics P_c for different bias voltages V in sample LAAS2328 at $T=50$ K. We observe a clear variation of the electron spin relaxation time with V : the relaxation time *decreases* when the reverse bias increases, *i.e.* when the electrons in the quantum wells feel a larger electric field $|E|$. This variation rules out a possible role played by the exciton spin relaxation mechanism (induced by the exchange interaction). Indeed it was shown that the application of an external electric field when excitons are excited strictly resonantly yields an *increase* of the exciton spin relaxation time τ_X due to the reduction of the exchange interaction (weaker overlap between the electron and hole wavefunction) [59] (see figure 3.11). Thus, the results in figure 3.10 demonstrate that the electron spin relaxation time in $\langle 111 \rangle$ quantum wells can be tuned by the Rashba effect. In the narrow MQW ($L_W=7.5$ nm, sample LAAS2311), we see in the inset of figure 3.10 that the electric field has almost no effect on the spin relaxation compared to the one in sample LAAS2328 as a consequence of the larger BIA splitting due to the stronger $\langle k_z^2 \rangle \approx (\pi/L_W)^2$ term. However, in these PIN devices, it turns out that the Dresselhaus and Rashba

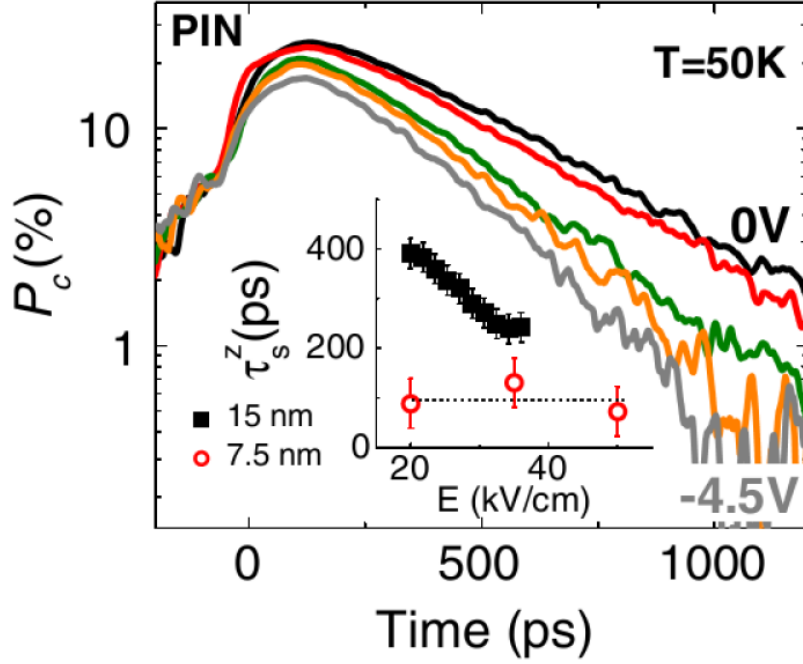


Figure 3.10: Circular polarization dynamics for different reverse bias voltage V for sample LAAS2328 (PIN structure). Inset : Dependence of the electron spin relaxation time with the electric field for sample LAAS2328 ($L_W = 15$ nm) and LAAS2311 ($L_W = 7.5$ nm)

vectors Ω_{BIA} and Ω_{SIA} have the same direction and same sign. In other words, these two contributions to the conduction band spin splitting add up: the total effective field Ω increases with E and the DP spin relaxation time decreases with the reverse bias.

In order to check this assumption, we have measured the electron spin dynamics in sample LAAS2335 (NIP structure) where the applied electric field direction is reversed with respect to the one in sample LAAS2328. In this case we expect to reduce the CB spin splitting (the BIA and SIA terms will subtract to each other) and as a consequence an *increase* of the electron spin relaxation time with the applied reverse bias should be observed. Figure 3.12 presents the variation of the electron spin dynamics with the applied reverse bias in sample LAAS2335. A spectacular increase of the electron spin relaxation time is observed when the electric field modulus $|E|$ increases. The dependence of the electron spin relaxation time $\tau_{s,z}$ as a function of E is displayed in figure 3.12 : the electron spin relaxation time increases from 500 ps to $\sim 30\,000$ ps when $|E|$ varies from 20 to 55 kV/cm. For $|E| > 50$ kV/cm, the electron spin relaxation time ($\tau_{s,z} > 10$

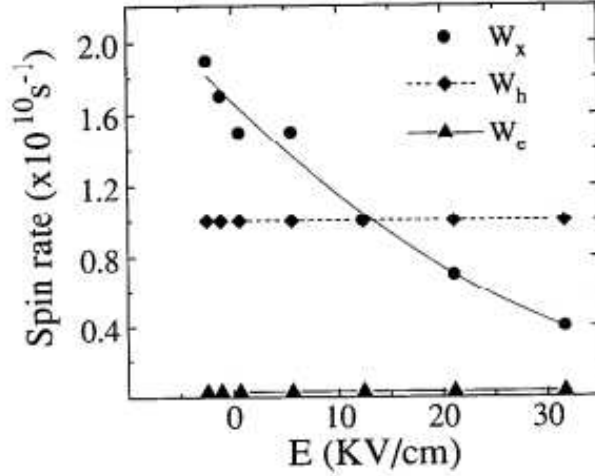


Figure 3.11: Dependence of spin relaxation rate $W_X(1/\tau_X)$, $W_h(1/\tau_h)$, $W_e(1/\tau_e)$ on applied electric field for 15nm GaAs/ $Al_{0.7}Ga_{0.3}As$ quantum well [59]

ns) becomes much longer than the carrier lifetime so that it is difficult to measure it in time-resolved PL. For larger $|E|$ we expect that an efficient DP spin relaxation should be switched on again because the spin splitting would not be anymore zero [17]. In this experiment, -5.5V is the highest reverse bias which could be used to avoid going to breakdown regime of this device. This voltage is limited by avalanche photocurrent in reverse bias. This limitation of this NIP device characteristics did not allow us to test this prediction (55 kV/cm being the maximum achievable electric field).

Assuming that $E = -55$ kV/cm is close to the cancellation of the spin splitting on the basis of the very long measured $\tau_{s,z}$, we can extract from equation 3.3 the ratio between the Dresselhaus γ and the Rashba α coefficients. We find $\alpha/\gamma = 5 \cdot 10^{16} \text{ m}^{-2}$. No measurements were made before in $\langle 111 \rangle$ QW for comparison. Nevertheless, using the measured γ value in a $\langle 001 \rangle$ QW with similar confinement energy ($|\gamma| = 11 \text{ eV} \cdot \text{\AA}^3$) [30], gives a Rashba coefficient $\alpha = 550 \cdot 10^{-15} \text{ eV} \cdot \text{\AA}$ [69], in good agreement with the coefficient measured in a $\langle 110 \rangle$ MQW sample with an applied field of 60 kV/cm ($\alpha = 500 \cdot 10^{-15} \text{ eV} \cdot \text{\AA}$). These results demonstrate that the electron spin relaxation time in $\langle 111 \rangle$ quantum wells can be tuned with an applied electric field thanks to the Rashba-induced variation of the CB spin splitting.

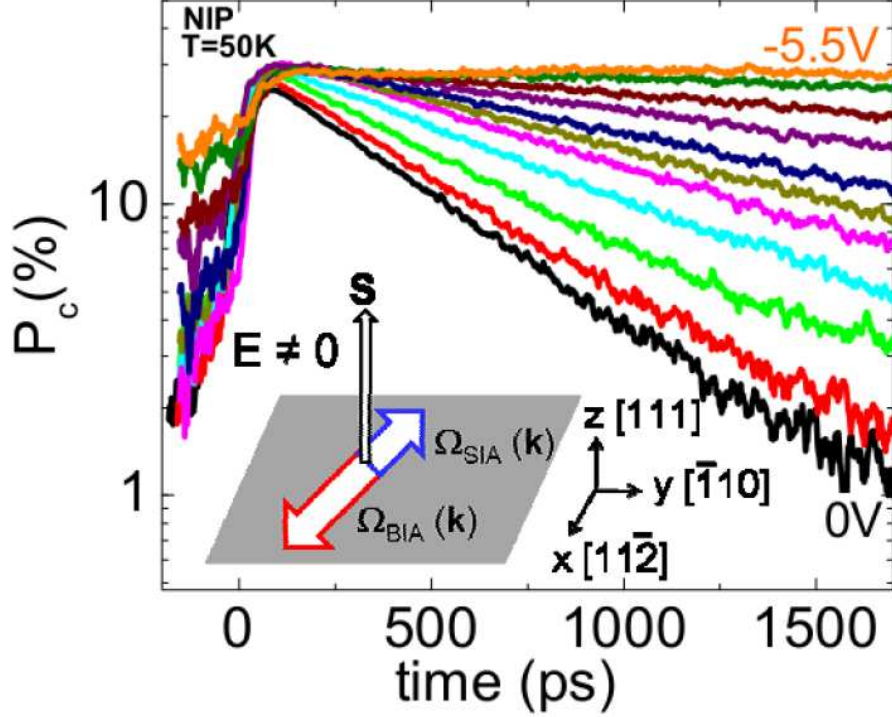


Figure 3.12: Electron spin dynamics of sample LAAS2335 for different reverse voltages at 50K. Inset : Schematic of the BIA and SIA precession vector in $\langle 111 \rangle$ GaAs QW. $\mathbf{S}$ is the photogenerated electron spin at $t=0$.

3.3.3 Anisotropic spin relaxation time measurement

We have shown so far that very long electron relaxation times for spins which are oriented along the $\langle 111 \rangle$ growth direction can be obtained with the reverse bias in the NIP structure. We recall that in the optical spin orientation experiments presented here, the photogenerated electron spin is parallel to the growth direction which is also the propagation direction of the excitation laser beam. In order to test that the relaxation time can also be controlled electrically for spins along the $\mathbf{x}$ and $\mathbf{y}$ in-plane QW direction, which is the key feature of $\langle 111 \rangle$ quantum wells, we have measured the electron spin dynamics in transverse magnetic field (Voigt configuration). Following the circularly polarized laser excitation, the electron spins will be first oriented along the $\mathbf{z}$ axis but will then precess around the in-plane external magnetic field. As a consequence, the electron spins should experience both in-plane and out-of plane spin relaxation processes. Figure 3.14 depicts the circular polarization dynamics under a transverse magnetic field along $\mathbf{y}$ axis ($B_y = 730$ mT) for different bias voltages. We observe clearly the oscil-

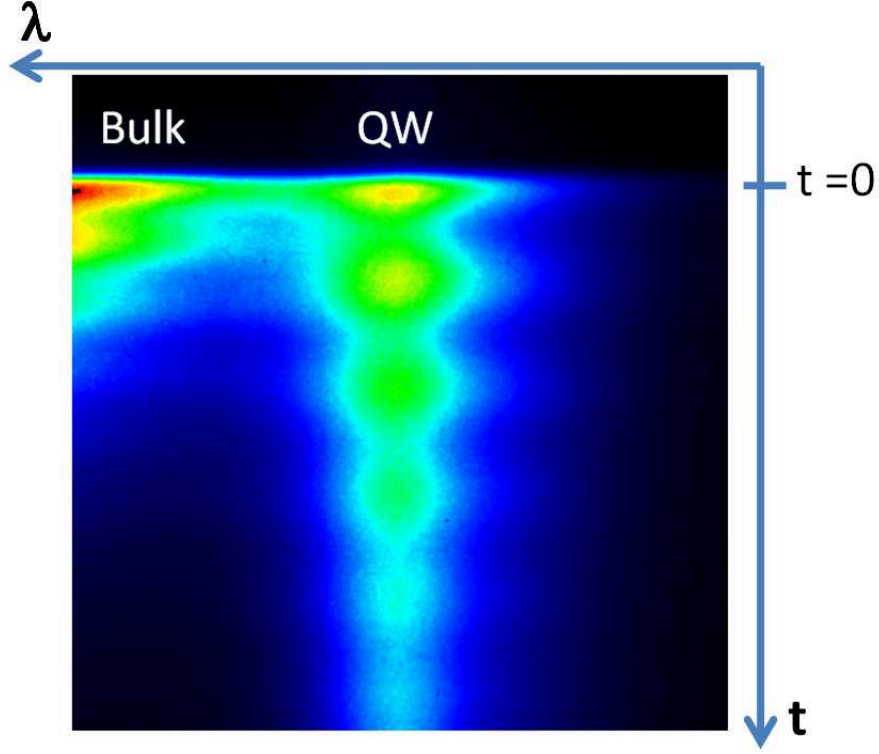


Figure 3.13: PL intensity dynamics of σ^+ polarized luminescence of sample LAAS2335 (NIP structure, $L_W = 15$ nm, $V_{bias} = -3$ V, $T = 50$ K) detected directly by the streak camera

lations due to the electron spin precession (see figure 3.13 for the oscillations of PL intensity with a given circular polarization). The striking feature in figure 3.14 is that the decay time of the envelope of the oscillations increases when the reverse bias increases. This is an indication that both S_x and S_z components of spin relaxation time increases when the electric field in the QW increases.

A classical approach to determine the ensemble spin relaxation and spin dephasing in quantum well is to use spin Bloch equation [25, 58]. Figure 3.15 shows the crystallographic axes and the direction of the in-plane applied magnetic field $\mathbf{B}$.

The time dependent spin polarization S_x, S_z for spins that precess around the y-axis is given by the reduced differential equation [16]:

$$\frac{\partial}{\partial t} \begin{pmatrix} S_x \\ S_z \end{pmatrix} = - \begin{pmatrix} \Gamma_x & -\omega \\ \omega & \Gamma_z \end{pmatrix} \begin{pmatrix} S_x \\ S_z \end{pmatrix} \quad (3.8)$$

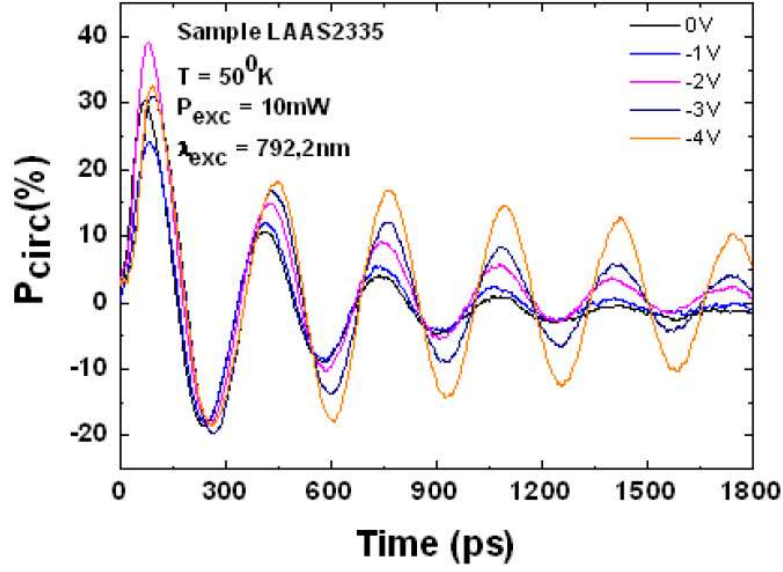


Figure 3.14: Circular polarization dynamics of sample LAAS2335 (NIP structure, $L_W = 15$ nm, $T = 50$ K) for different bias voltages under a transverse magnetic field $B_y = 730$ mT

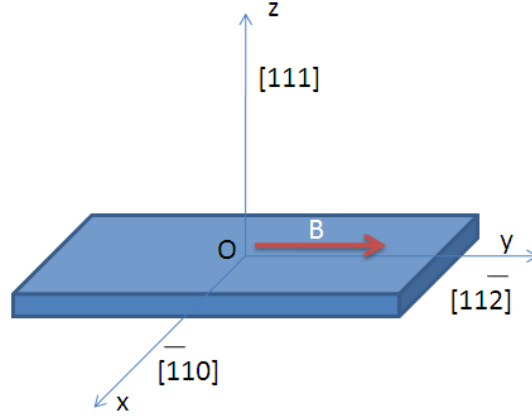


Figure 3.15: Schematic of sample crystallographic and applied magnetic field axis ($\vec{B} \parallel Oy$)

where Γ_x, Γ_z are the two components of spin dephasing tensor on x-axis and z-axis, respectively. $\omega = (g_e \mu_B B) / \hbar$ is the measured spin oscillation pulsation; where g_e is the Landé factor of electron in quantum well and μ_B is Bohr's magneton.

Solving equation 3.8, the solution for the measured $S_z(t)$ is given by :

$$S_z(t) = S_z(0) \frac{e^{-\frac{t}{2}(\Gamma_x + \Gamma_z)}}{\cos(\phi)} \cos(\omega' t) \quad (3.9)$$

under the condition that $|\Gamma_x - \Gamma_z| \ll 2\omega$ (in our experimental condition, this condition is satisfied); and $S_z(0)$ is the initial degree of the ensemble electron spins polarization, $\tan(\phi) = (\Gamma_x - \Gamma_z)/(2\omega')$, $\omega' = \sqrt{\omega^2 - (\Gamma_x - \Gamma_z)^2/4}$ is the effective spin oscillation frequency slowed down by the possible dephasing anisotropy $\Gamma_x - \Gamma_z$. The spin relaxation times τ_x, τ_z are inversely proportional to the relevant components of spin dephasing tensor Γ_x, Γ_z , respectively.

Figure 3.16 presents the measured pulsation as a function of the applied

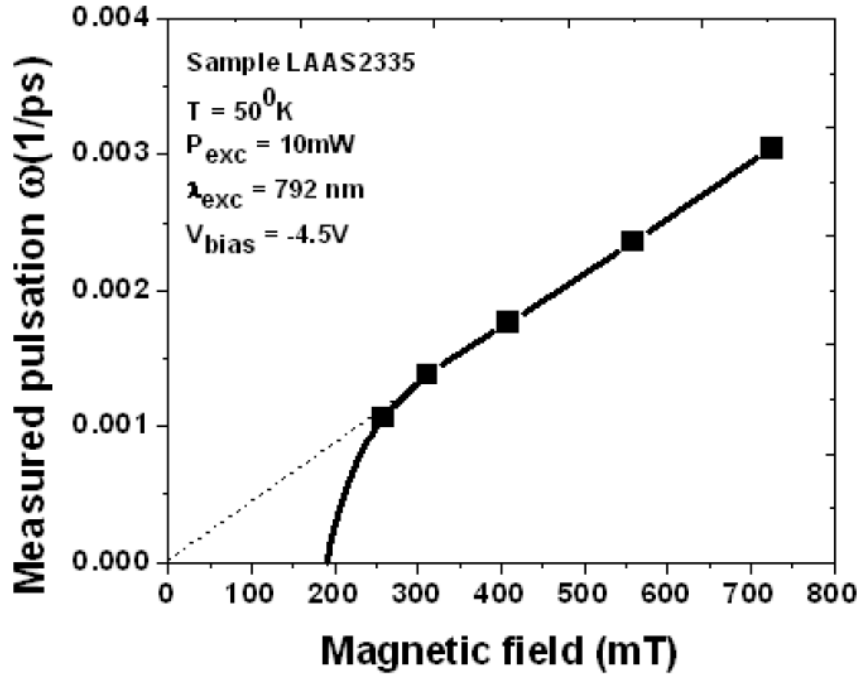


Figure 3.16: Measured spin oscillation pulsation of quantum beats as a function of the magnetic field. The strength of magnetic field is deduced from quantum beats of bulk GaAs of sample LAAS2335 using the well-known electron Landé g-factor of bulk GaAs $|g_e| \approx 0.445$ at 50K [56]

magnetic field. We observe a rather linear dependence, demonstrating the weak influence of the spin relaxation anisotropy on the measured spin quantum beat frequency. Thus, the time evolution of the average electron spin polarization along

z -axis in a transverse B_y magnetic field is described by the simple equation :

$$S_z(t) = S_z(0)e^{-\frac{t}{2}(\Gamma_x + \Gamma_z)} \cos(\omega t) \quad (3.10)$$

Figure 3.17 depicts how to determine the in-plane electron spin relaxation

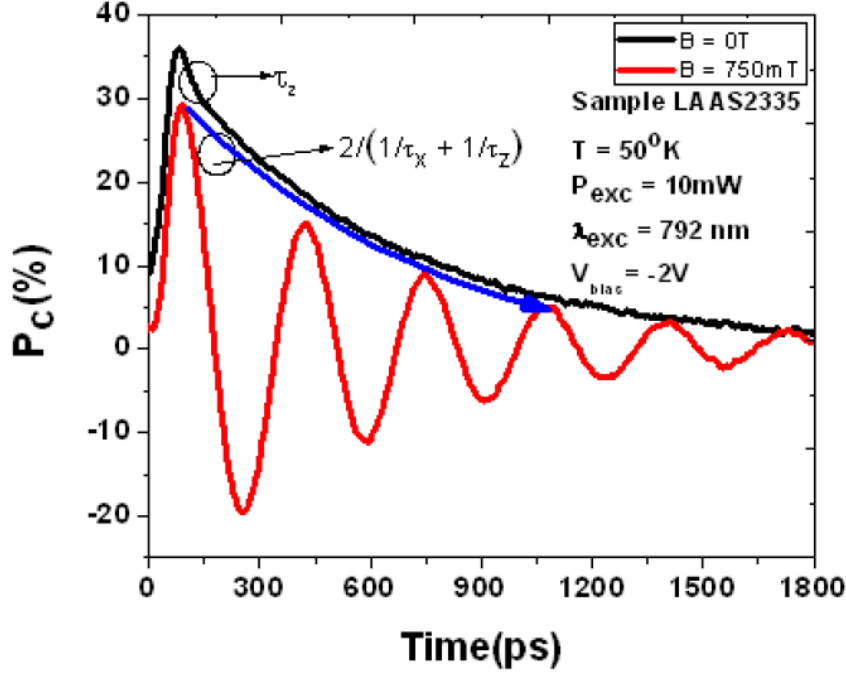


Figure 3.17: Longitudinal and transverse spin relaxation time calculation issued from the measurement of the spin dynamics

times using equation 3.10. First, without external magnetic field, the average electron spin is oriented along z -axis; so, circular polarization evolution reveals the longitudinal spin relaxation time τ_z (along the growth direction). Second, the external magnetic field is applied while keeping the same voltage. As a result, the Larmor precession of electron spin is observed with its envelope decaying with a time $2\tau_x\tau_z/(\tau_x + \tau_z)$. After fitting the quantum beats with the equation 3.10, the in-plane spin relaxation time is deduced. For example in figure 3.17, at $B=0$ and $-2V$ bias voltage, the longitudinal spin relaxation time τ_z is 578 ps. Fitting the envelope of the four beats by first-order exponential decay function with decay time of 636 ps, we find the in-plane spin relaxation time τ_x about 707 ps. This procedure is repeated with the different reverse bias and two in-plane magnetic field orientation B_x and B_y .

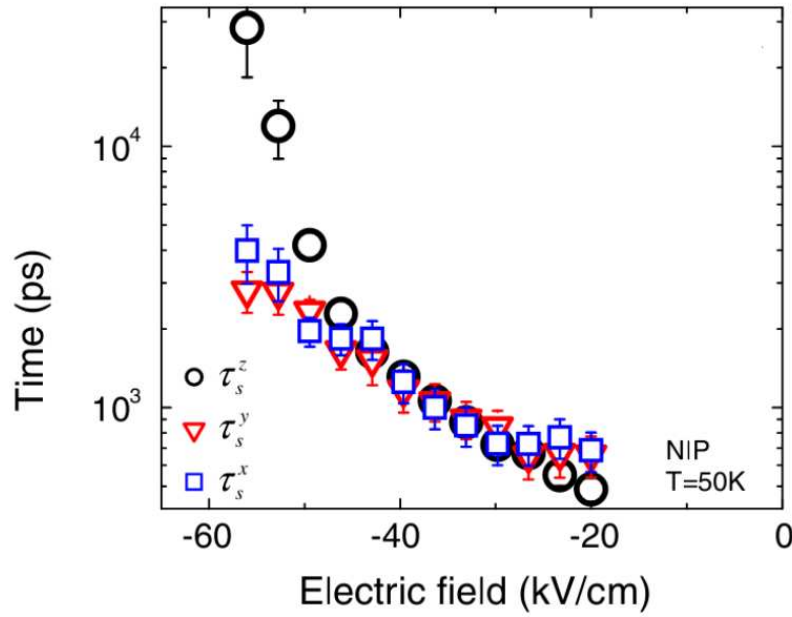


Figure 3.18: Spin relaxation time anisotropy: Dependence of three components of electron spin relaxation times τ_z (circles), τ_y (triangles), τ_x (squares) as a function of the applied electric field

Figure 3.18 presents the three measured components of electron spin relaxation times depending on applied electric field. The large increase of the in-plane spin relaxation time τ_x and τ_y confirmed that not only longitudinal spin relaxation time but also transverse spin relaxation times can be controlled by external electric field. These results show the superior characteristics of $\langle 111 \rangle$ quantum wells when compared with $\langle 001 \rangle$ and $\langle 110 \rangle$ quantum wells [83, 82, 75].

3.3.4 Temperature dependence of electrical control of spin relaxation

An important requirement for future semiconductor spintronic devices is the ability to work at room temperature. The results presented in the previous section were obtained at $T = 50\text{K}$. This section presents all the experimental results of sample LAAS2335 at higher temperatures. The results show that the electric field control of the spin relaxation can be observed up to 150K but above this temperature, it is very difficult to discriminate the emission of quantum wells from that of bulk GaAs.

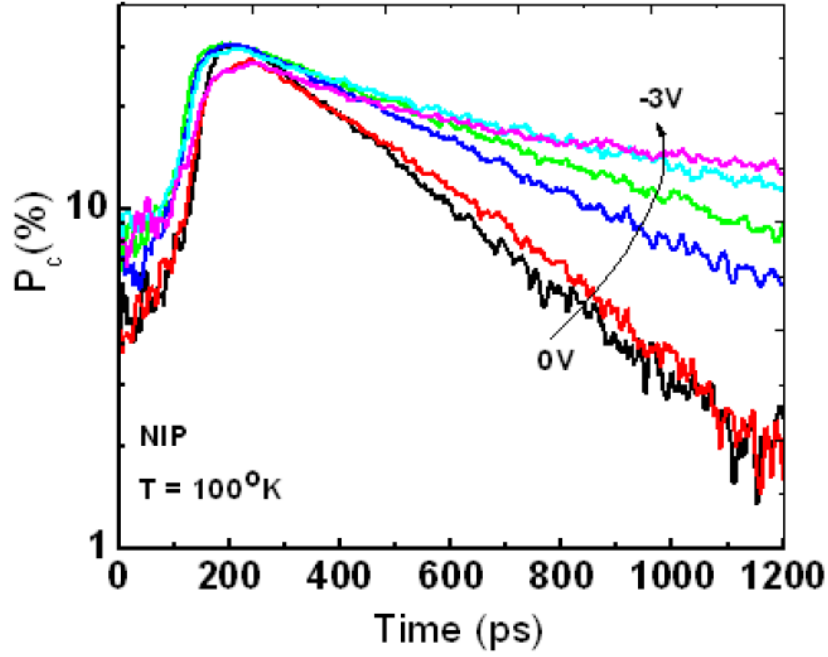


Figure 3.19: Circular polarization dynamics of sample LAAS2335 (NIP structure, $L_W = 15$ nm, $T = 100$ K) for different bias voltages

PL measurement at 100K

Figure 3.19 presents the measured electron spin dynamics of sample LAAS2335 at 100K for different applied reverse bias. It is shown that the compensation effect of Dresselhaus and Rashba fields works well at 100K but is weaker when compared with the effect at 50K. This emphasizes the role of cubic terms in Dresselhaus field which becomes much more important at high temperature. This problem will be discussed in detail in the next chapter.

Chapter 4

Electric field dependence of spin relaxation anisotropy

4.1 Experimental evidence of electric field dependence of spin relaxation anisotropy

In the previous chapter, we showed that the electron spin relaxation time can be electrically tuned for any electron spin direction in $\langle 111 \rangle$ -oriented GaAs/AlGaAs quantum wells. Nevertheless, for a given applied electric field, the longitudinal $\tau_{s,z}$ and transverse $\tau_{s,\perp}$ spin relaxation time are usually different. Figure 4.1 presents the measured circular polarization dynamics P_c under the influence of an external transverse magnetic field. The black curve presents the time evolution of P_c without the magnetic field (we have $\tau_{s,z}$ deduced from the decay time of P_c at $B = 0$) and the red one presents the case with the applied magnetic field (from which we deduce $\tau_{s,\perp}$). Note that in Voigt configuration the decay time of the polarization beats envelope is equal to $1/\tau_{s,env} = 1/2(1/\tau_{s,z} + 1/\tau_{s,\perp})$. If the black curve stays above the envelope of the red one (figure 4.1(a)), this shows that $\tau_{s,z} > \tau_{s,\perp}$. In the figure 4.1(c), we see clearly that for small $\vec{E}$ (due to the built-in field at $V = 0$), the longitudinal spin relaxation time is shorter than the transverse one ($\tau_{s,z} < \tau_{s,\perp}$). This corresponds to the "classical" anisotropy in QWs initially predicted by D'yakonov and Perel [55]. For larger electric field (figure 4.1(a)), we observe that the opposite situation occurs : $\tau_{s,z} > \tau_{s,\perp}$. Moreover, we see in figure 4.1(b) that both spin relaxation times are equal for a specific value of $\vec{E}$ (corresponding here to a bias $V = -2.3$ volt).

In this chapter we present the models which explain this very interesting effect. All the results presented in this chapter have been obtained at $T = 10K$.

4.2 Simple model approach

As discussed in section 1.4.2, DP mechanism is the most important spin relaxation mechanism in III-V zinc-blende semiconductor quantum wells. There are two terms which give contributions to the effective magnetic field around which the electron spins precess : Dresselhaus term and Rashba term. In $\langle 111 \rangle$ -oriented quantum well, the two terms can compensate each other by proper gate voltage as proved experimentally in the previous chapter. This section is devoted to the presentation of a simple way to evaluate theoretically the spin relaxation components in $\langle 111 \rangle$ -oriented quantum well. The simulated results are in good agreement with experimental ones.

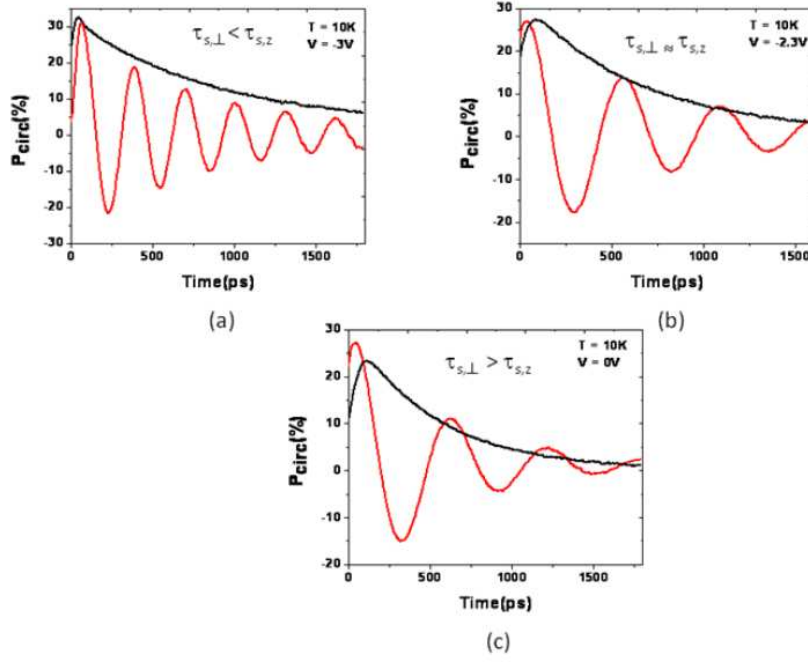


Figure 4.1: Comparison between longitudinal and transverse spin relaxation time $\tau_{s,z}, \tau_{s,\perp}$ measured through PL circular polarization dynamics P_c with and without magnetic field at $T = 10K$: (a) $\tau_{s,z} > \tau_{s,\perp}$; (b) $\tau_{s,z} \approx \tau_{s,\perp}$; (c) $\tau_{s,z} < \tau_{s,\perp}$

4.2.1 Dresselhaus effective magnetic field in $\langle 111 \rangle$ -oriented quantum well

The Dresselhaus term was calculated by perturbation theory for bulk zinc-blende semiconductor [15] and written in this form [25] :

$$\hat{H} = \frac{\hbar^2 k^2}{2m_e} + \frac{\hbar}{2} \boldsymbol{\sigma} \cdot \boldsymbol{\Omega}_D(\mathbf{k}) \quad (4.1)$$

where

$$H_D = \frac{\hbar}{2} \boldsymbol{\sigma} \cdot \boldsymbol{\Omega}_D(\mathbf{k}) = \gamma [k_x(k_y^2 - k_z^2) \boldsymbol{\sigma}_x + k_y(k_z^2 - k_x^2) \boldsymbol{\sigma}_y + k_z(k_x^2 - k_y^2) \boldsymbol{\sigma}_z] \quad (4.2)$$

is the Dresselhaus Hamiltonian term (BIA spin splitting), $\boldsymbol{\sigma}$ are the Pauli matrices and the term $\boldsymbol{\Omega}_D(\mathbf{k})$ presents Dresselhaus effective magnetic field vector following this equation :

$$\boldsymbol{\Omega}_D(\mathbf{k}) = \frac{2\gamma}{\hbar} [k_x(k_y^2 - k_z^2) \vec{e}_x + k_y(k_z^2 - k_x^2) \vec{e}_y + k_z(k_x^2 - k_y^2) \vec{e}_z] \quad (4.3)$$

in which γ is the Dresselhaus coefficient, $\vec{e}_x = (1,0,0)$, $\vec{e}_y = (0,1,0)$, $\vec{e}_z = (0,0,1)$ are three unit vectors along crystallographic axes of the unit cell.

To calculate the Dresselhaus effective magnetic field for <111>-oriented quantum well, it is convenient to choose a new basis set of axis in order to facilitate the evaluation of spin relaxation time. We use three new unit vectors written as :

$$\vec{e}_x = (\frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{-2}{\sqrt{6}}), \vec{e}_y = (\frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0), \vec{e}_z = (\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}})$$

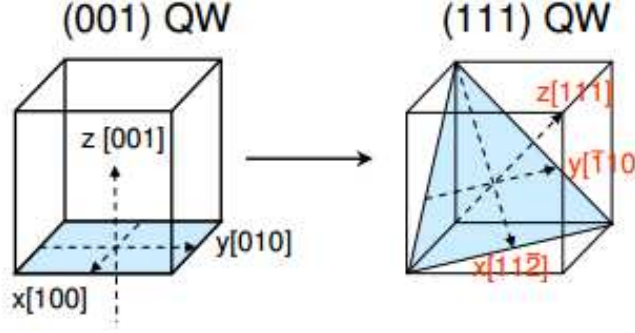


Figure 4.2: The axis used for (111) quantum well [19]

The corresponding rotation matrix is :

$$R = \begin{bmatrix} \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & \frac{-2}{\sqrt{6}} \\ \frac{-1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \end{bmatrix} \quad (4.4)$$

Using this rotation matrix, the effective magnetic field Ω_D can be written in the new basis as (see the detail in appendix C.1):

$$\Omega_{D'_x} = \frac{2\gamma}{\hbar} [\frac{1}{\sqrt{3}}k'_y(2k_z'^2 - \frac{1}{2}k_x'^2 - \frac{1}{2}k_y'^2) - \sqrt{\frac{2}{3}}k'_xk'_yk'_z] \quad (4.5)$$

$$\Omega_{D'_y} = \frac{2\gamma}{\hbar} [\frac{1}{2\sqrt{3}}k'_x(k_x'^2 + k_y'^2) - \frac{1}{\sqrt{6}}k'_z(k_x'^2 + 2\sqrt{2}k'_xk'_z - k_y'^2)] \quad (4.6)$$

$$\Omega_{D'_z} = \frac{2\gamma}{\hbar} [\frac{1}{\sqrt{6}}k'_y(3k_x'^2 - k_y'^2)] \quad (4.7)$$

In conclusion, the Dresselhaus Hamiltonian in the coordinate (e'_x, e'_y, e'_z) is :

$$\begin{aligned} H_D &= \frac{\hbar}{2} \boldsymbol{\sigma}' \cdot \boldsymbol{\Omega}_D(\mathbf{k}') \\ &= \gamma [(\frac{1}{\sqrt{3}}k'_y(2k_z'^2 - \frac{1}{2}k_x'^2 - \frac{1}{2}k_y'^2) - \sqrt{\frac{2}{3}}k'_xk'_yk'_z)\sigma'_x \\ &\quad + (\frac{1}{2\sqrt{3}}k'_x(k_x'^2 + k_y'^2) - \frac{1}{\sqrt{6}}k'_z(k_x'^2 + 2\sqrt{2}k'_xk'_z - k_y'^2))\sigma'_y \\ &\quad + (\frac{1}{\sqrt{6}}k'_y(3k_x'^2 - k_y'^2))\sigma'_z] \end{aligned} \quad (4.8)$$

In a quantum well, the conduction subband splitting due to confinement is much greater than the electron spin coupling term induced by spin-orbit interaction. We can thus replace H_D by its average on the electron state for each confined state $\Sigma_n \langle \mathbf{H}_D \rangle_n$ (first-order perturbation theory). Due to the localisation, $\langle \mathbf{k}'_z \rangle_{nn} = \langle \psi_n | \mathbf{k}'_z | \psi_n \rangle = 0$ in which ψ_n is the wavefunction of electron in the subband index n (see appendix B). For the lowest conduction band ($n = 1$) of a symmetric infinite quantum well, we have $\langle k_z'^2 \rangle = (\pi/L_w)^2$ in which L_w is the quantum well width (appendix B). We can approximate the Dresselhaus Hamiltonian in $\langle 111 \rangle$ -oriented quantum well:

$$\begin{aligned} H_{D(n=1)} = & \gamma \left[\frac{1}{\sqrt{3}} k'_y (2 \langle k_z'^2 \rangle - \frac{1}{2} k_x'^2 - \frac{1}{2} k_y'^2) \sigma'_x \right. \\ & + \left(\frac{1}{2\sqrt{3}} k'_x (k_x'^2 + k_y'^2) - \frac{2}{\sqrt{3}} k'_x \langle k_z'^2 \rangle \right) \sigma'_y \\ & \left. + \frac{1}{\sqrt{6}} k'_y (3k_x'^2 - k_y'^2) \sigma'_z \right] \end{aligned} \quad (4.9)$$

When eliminating the cubic term of $\mathbf{k}$ -vector, linear-term Dresselhaus Hamiltonian is :

$$H_{D(n=1)}^{(1)} = \frac{2\gamma}{\sqrt{3}} \langle k_z'^2 \rangle (k'_y \sigma'_x - k'_x \sigma'_y) \quad (4.10)$$

4.2.2 Rashba effective magnetic field in $\langle 111 \rangle$ -oriented quantum well

In semiconductor quantum structures, the spin splitting degeneracy can be lifted away not only by bulk inversion asymmetry of the underlying crystal structures but also by a structural inversion asymmetry of confining potential $V(\mathbf{r})$ that stems from a built-in or external potential. In our experiments, structural inversion asymmetry (SIA) is controlled by external electric field that can be tuned by an applied gate voltage on the sample. The SIA spin splitting in Γ_6^c conduction band is given by Rashba term [70] :

$$H_R = \frac{\hbar}{2} \boldsymbol{\sigma} \cdot \boldsymbol{\Omega}_R \quad (4.11)$$

where $\boldsymbol{\Omega}_R = a_6^4 (\vec{k} \times \vec{E})$ is the Rashba effective magnetic field in quantum well in which a_6^4 is given as [70, 20] :

$$a_6^4 = \frac{eP^2}{3} \left[\frac{1}{E_0^2} - \frac{1}{(E_0 + \Delta_0)^2} \right] + \frac{eP'^2}{3} \left[\frac{1}{(E_0 - E'_0)^2} - \frac{1}{(E_0 - E'_0 - \Delta'_0)^2} \right] \quad (4.12)$$

where e is electron charge; $\vec{k}, \vec{E}$ are wavevector and external electric field. $P, P', E_0, E'_0, \Delta_0, \Delta'_0$ are momentum matrix elements, energy band edges and spin-orbit interaction, respectively in 14×14 extended Kane model for band structure

(see section 1.1.3).

For <111>-oriented quantum well, the Rashba effective magnetic field can be written as :

$$\mathbf{\Omega}_R(\vec{k}') = \begin{bmatrix} k'_x \\ k'_y \\ k'_z \end{bmatrix} \times \begin{bmatrix} E'_x \\ E'_y \\ E'_z \end{bmatrix}$$

$$\Rightarrow \mathbf{\Omega}_R(\vec{k}') = (k'_y E'_z - k'_z E'_y) \vec{e}_x + (k'_z E'_x - k'_x E'_z) \vec{e}_y + (k'_x E'_y - k'_y E'_x) \vec{e}_z \quad (4.13)$$

With an external electric field applied along growth axis, we have $\vec{E}' = (0, 0, E)$; so, equation 4.13 can be rewritten as :

$$\mathbf{\Omega}_R(\vec{k}') = E(k'_y \vec{e}_x - k'_x \vec{e}_y) \Rightarrow H_R = a_6^4 E (k'_y \sigma'_x - k'_x \sigma'_y) \quad (4.14)$$

4.2.3 The cancellation effect in <111>-oriented quantum well

Without taking the cubic Dresselhaus term into account, the total Hamiltonian of an electron in the lowest conduction band in quantum well from equations 4.10 and 4.14 is :

$$H_{(n=1)} = H_D + H_R = \left(\frac{2\gamma}{\sqrt{3}} \langle k_z'^2 \rangle + a_6^4 E \right) (k'_y \sigma'_x - k'_x \sigma'_y) \quad (4.15)$$

Obviously, the cancellation effect occurs when the external electric field E is applied; so that :

$$E = \frac{-2\gamma \langle k_z'^2 \rangle}{\sqrt{3} a_6^4} \quad (4.16)$$

In addition, when taking the cubic Dresselhaus term into account, the total Hamiltonian can be written as :

$$\begin{aligned} H_{(n=1)} &= \underbrace{\left(\frac{2\gamma}{\sqrt{3}} \langle k_z'^2 \rangle + a_6^4 E \right) (k'_y \sigma'_x - k'_x \sigma'_y)}_{\text{linear term}} \\ &\quad + \underbrace{\gamma \left[\frac{-1}{2\sqrt{3}} k'_y (k_x'^2 + k_y'^2) \sigma'_x + \frac{1}{2\sqrt{3}} k'_x (k_x'^2 + k_y'^2) \sigma'_y + \frac{1}{\sqrt{6}} k'_y (3k_x'^2 - k_y'^2) \sigma'_z \right]}_{\text{cubic term}} \\ &= \left(\frac{2\gamma}{\sqrt{3}} \langle k_z'^2 \rangle + a_6^4 E - \frac{\gamma}{2\sqrt{3}} (k_x'^2 + k_y'^2) \right) (k'_y \sigma'_x - k'_x \sigma'_y) \\ &\quad + \frac{\gamma}{\sqrt{6}} k'_y (3k_x'^2 - k_y'^2) \sigma'_z \end{aligned} \quad (4.17)$$

As shown in equation 4.17, the spin relaxation time τ_s can't be fully suppressed by external electric field because of the field-independent cubic term along z-axis.

4.2.4 Calculation of longitudinal and transverse spin relaxation times

In order to evaluate the longitudinal and transverse spin relaxation times, the components of effective magnetic field Ω along x' , y' , z' axis (the growth direction of quantum well oriented along z' -axis) are deduced from the total Hamiltonian in equation 4.17 :

$$\Omega_x = \frac{2}{\hbar}(\alpha_T - \frac{\gamma}{2\sqrt{3}}k_{\perp}'^2)k_y' \quad (4.18a)$$

$$\Omega_y = -\frac{2}{\hbar}(\alpha_T - \frac{\gamma}{2\sqrt{3}}k_{\perp}'^2)k_x' \quad (4.18b)$$

$$\Omega_z = \frac{2\gamma}{\hbar\sqrt{6}}k_y'(3k_x'^2 - k_y'^2) \quad (4.18c)$$

where $\vec{k}_{\perp}' = (k_x', k_y', 0) = |k_{\perp}'|(\sin(\theta), \cos(\theta), 0)$ is the in-plane k-vector perpendicular to growth direction (θ is the angular measure of k' -vector in the quantum well plane and $k_{\perp}'$ ($|k_{\perp}'|^2 = k_x'^2 + k_y'^2$) is the magnitude of in-plane k-vector), α_T is given by :

$$\alpha_T = \frac{2\gamma}{\sqrt{3}}\langle k_z'^2 \rangle + a_6^4.E \quad (4.19)$$

In zinc-blende semiconductors, the Dyakonov-Perel mechanism can be simply expressed in the form [29] :

$$\frac{1}{\tau_{s,i}} = \langle \Omega_{\perp}^2 \rangle \tau_p^*$$

where $\tau_{s,i}$ is the spin relaxation time of electron along a particular axis i , $\langle \Omega_{\perp}^2 \rangle$ is the mean square precession vector in the plane perpendicular to i , τ_p^* is the momentum relaxation time of an individual electron (see chapter 1).

Under the *nondegenerate regime*, let us recall some important parameters for an infinite quantum well as follow :

$$\rho_c^{2D} = \frac{m_c^*}{\pi\hbar^2} \quad (4.20)$$

$$|k_{\perp}'| = \frac{(2m_c^*E_{\perp})^{\frac{1}{2}}}{\hbar} \quad (4.21)$$

$$E_c = E_{n,\parallel} + E_{\perp} = \frac{\hbar^2\pi^2}{2m_c^*L_W^2}n_z^2 + E_{\perp}, \quad n_z = 1, 2, 3, \dots \quad (4.22)$$

where ρ_c^{2D} is the two-dimensional density of states per unit electron energy per unit area of electrons in conduction band, m_c^* is the effective mass of electron in conduction band, $E_{\perp}$ is the kinetic energy of an electron in x' - y' plane, $E_{n,\parallel}$ is the quantized kinetic energy of an electron in z' -axis, E_c is the total energy of an electron in conduction band, L_W is the quantum well width.

In undoped nondegenerate semiconductor, under thermal equilibrium condition, the energy distribution in x'-y' plane follows the Maxwell-Boltzmann distribution. Combining equation 4.18, 4.20 with 4.21, we can find the average value of the mean square of the components of an effective magnetic field which drives the spin relaxation time in DP mechanism (see the detailed calculation in appendix C.2) :

$$\langle \overline{\Omega_z^2} \rangle = \frac{2\gamma^2}{\hbar^2} \left(\frac{2m_c^* k_B T}{\hbar^2} \right)^3 \quad (4.23)$$

$$\langle \overline{\Omega_x^2} \rangle = \langle \overline{\Omega_y^2} \rangle = \frac{2}{\hbar^2} \left[\frac{2m_c^* k_B T \alpha_T^2}{\hbar^2} + \frac{\gamma^2}{2} \left(\frac{2m_c^* k_B T}{\hbar^2} \right)^3 - \frac{2}{\sqrt{3}} \alpha_T \gamma \left(\frac{2m_c^* k_B T}{\hbar^2} \right)^2 \right] \quad (4.24)$$

where k_B is the Boltzmann constant, T is the temperature. Equations 4.23 and 4.24 can be rewritten by using the thermal wavevector $k_\theta = \sqrt{\frac{2m_c^* k_B T}{\hbar^2}}$ as follow:

$$\langle \overline{\Omega_z^2} \rangle = \frac{2\gamma^2}{\hbar^2} k_\theta^6 \quad (4.25)$$

$$\langle \overline{\Omega_x^2} \rangle = \langle \overline{\Omega_y^2} \rangle = \frac{2}{\hbar^2} k_\theta^2 \left[\alpha_T^2 + \frac{\gamma^2}{2} k_\theta^4 - \frac{2}{\sqrt{3}} \alpha_T \gamma k_\theta^2 \right] \quad (4.26)$$

Following the DP mechanism, we have a system of equations for the components of spin relaxation time :

$$\begin{aligned} \frac{1}{\tau_{s,x}} &= \langle \overline{\Omega_y^2 + \Omega_z^2} \rangle \tau_p^* = \frac{2}{\hbar^2} k_\theta^2 \left[\left(\alpha_T - \frac{\gamma k_\theta^2}{\sqrt{3}} \right)^2 + \frac{7}{6} \gamma^2 k_\theta^4 \right] \tau_p^* = \frac{1}{\tau_{s,y}} \\ \frac{1}{\tau_{s,z}} &= \langle \overline{\Omega_x^2 + \Omega_y^2} \rangle \tau_p^* = \frac{4}{\hbar^2} k_\theta^2 \left[\left(\alpha_T - \frac{\gamma k_\theta^2}{\sqrt{3}} \right)^2 + \frac{1}{6} \gamma^2 k_\theta^4 \right] \tau_p^* \end{aligned} \quad (4.27)$$

where $\tau_{s,x}$ ($\tau_{s,y}$) and $\tau_{s,z}$ are transverse and longitudinal spin relaxation times, respectively.

From equation 4.27, the ratio between longitudinal and transverse spin relaxation times is :

$$\frac{\tau_{s,z}}{\tau_{s,x}} = \frac{1}{2} \left[1 + \frac{\gamma^2 k_\theta^4}{\left(\alpha_T - \frac{\gamma k_\theta^2}{\sqrt{3}} \right)^2 + \frac{1}{6} \gamma^2 k_\theta^4} \right] \quad (4.28)$$

4.2.5 Comparison between simulated and experimental results

Using the approximation $\langle k_z^2 \rangle \approx (\pi/L_w)^2$ for infinite quantum well, equation 4.28 is written as :

$$\begin{aligned} \frac{\tau_{s,z}}{\tau_{s,x}} &= \frac{1}{2} \left[1 + \frac{\gamma^2 \left(\frac{2m_c^* k_B T}{\hbar^2} \right)^2}{\left(\frac{2\gamma}{\sqrt{3}} \left(\frac{\pi}{L_w} \right)^2 + a_6^4 \cdot E - \frac{\gamma}{\sqrt{3}} \cdot \frac{2m_c^* k_B T}{\hbar^2} \right)^2 + \frac{1}{6} \gamma^2 \left(\frac{2m_c^* k_B T}{\hbar^2} \right)^2} \right] \\ &= \frac{1}{2} \left[1 + \frac{\left(\frac{2m_c^* k_B T}{\hbar^2} \right)^2}{\left(\frac{2}{\sqrt{3}} \left(\frac{\pi}{L_w} \right)^2 + \frac{a_6^4}{\gamma} \cdot E - \frac{1}{\sqrt{3}} \cdot \frac{2m_c^* k_B T}{\hbar^2} \right)^2 + \frac{1}{6} \left(\frac{2m_c^* k_B T}{\hbar^2} \right)^2} \right] \end{aligned}$$

Applying these parameters for sample LAAS2335 :

$$\begin{aligned} m_c^* &= 0.067m_0 \\ T &= 10K \\ \gamma &= 23.6eV(A^\circ)^3 [20] \\ L_w &= 15nm \end{aligned}$$

and using the Rashba coefficient as a fit parameter, the curve in figure 4.3 is obtained with the value $a_6^4 \approx 28.8e(A^\circ)^2$. As shown in this figure, the *black* line presents the experimental data whereas the *red* line presents the simulated data taken from the above equation. We can see that the simulated result is in general good agreement with experimental data in the high electric field region. In particular, it predicts an isotropic point where the longitudinal and transverse spin relaxation times are equal. But this model does not work well at low field region as shown in figure 4.3. The reasonable interpretation is that in this first simple model, we eliminate the nature of the different scattering mechanisms in the momentum scattering time τ_p which is very important to evaluate spin relaxation time. The next section describes a more detailed model where the electron scattering processes are explicitly taken into account. The second model follows the approach initially developed by Averkiev and Golub for <001>-oriented quantum wells [45].

4.3 Spin density matrix model

4.3.1 Dyakonov-Perel mechanism based on density matrix formalism

In the kinetic theory of Boltzmann, a single-particle probability distribution in the phase space of two noncommutative variables $\mathbf{r}$ and $\mathbf{p}$ was introduced to

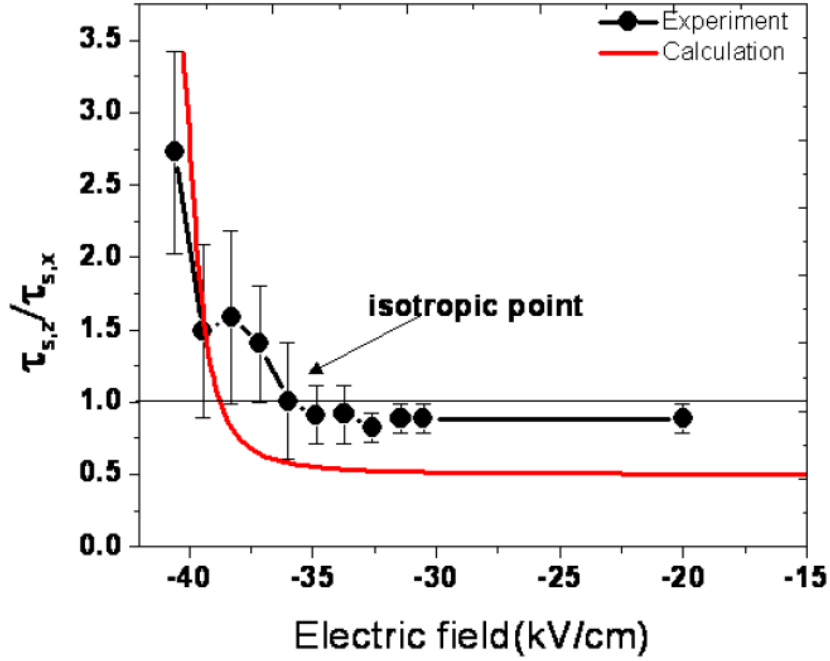


Figure 4.3: The measured and calculated ratio between longitudinal and transverse spin relaxation times as a function of the applied electric field at $T = 10\text{K}$

describe the evolution of a single-particle distribution. This Boltzmann distribution function is denoted as $F(\mathbf{r}, \mathbf{p}, t)$. In the limit of classical Hamiltonian theory, the time-evolution of Boltzmann kinetic distribution function of a single-particle is given by [39]:

$$\frac{\partial F(\mathbf{r}, \mathbf{p}, t)}{\partial t} = \frac{\partial F}{\partial t} + \frac{d\mathbf{r}}{dt} \cdot \nabla_{\mathbf{r}} F + \frac{d\mathbf{p}}{dt} \cdot \nabla_{\mathbf{p}} F = \left. \frac{\partial F}{\partial t} \right|_{\text{collision}} \quad (4.29)$$

where the distribution function $(\partial F / \partial t)|_{\text{collision}}$ describes the effect of the collisions under the influence of other particles in the gas. From equation 4.29, the spin Boltzmann kinetic equation which describes the temporal evolution of electrons in time is given by [27] :

$$\frac{\partial \rho(k)}{\partial t} + \frac{i}{\hbar} [H'(k), \rho(k)] + \frac{\partial \rho(k)}{\partial (\hbar k)} F_k + \frac{\partial \rho(k)}{\partial r} v_k = I_k(t) \quad (4.30)$$

where $\rho(k)$ is the 2×2 spin density matrix operator, $I_k(t)$ is the collision integral term, $H'(k)$ is the Hamiltonian term of spin splitting caused by Dresselhaus and Rashba field. Assuming a homogeneous distribution of electrons ($\partial \rho(k) / \partial r = 0$) with the absence of external fields ($F_k = 0$) under the condition $\tau \gg \tau_1$ in which

τ is the particle life time and τ_1 is the momentum scattering time (considering $H'(k)$ is a small perturbation), equation 4.30 is reduced in this form :

$$\frac{\partial \rho(k)}{\partial t} + \frac{i}{\hbar} [H'(k), \rho(k)] = I_k(t)$$

To simplify this equation, the other following assumptions are used : all particles are in the elastic scattering regime and have isotropic energy; the above equation is rewritten as :

$$\frac{\partial \rho(k)}{\partial t} + \frac{i}{\hbar} [H'(k), \rho(k)] = - \sum_{k'} W_{k,k'} (\rho(k) - \rho(k')) \quad (4.31)$$

where $I_k(t) = - \sum_{k'} W_{k,k'} (\rho(k) - \rho(k'))$ with $W_{k,k'}$ is the probability of scattering from wavevector state $\mathbf{k}$ to $\mathbf{k}'$.

For convenience, the spin density matrix can be decomposed in the form [61]:

$$\rho(k) = \overline{\rho(k)} + \rho'(k)$$

where $\overline{\rho(k)}$ is the quasiequilibrium component presenting uniform spin polarization (isotropic part of spin density matrix) and $\rho'(k)$ is the nonequilibrium component (anisotropic part of spin density matrix which satisfies $\overline{\rho'(k)} = 0$). The bar means the averaging over the direction of $\mathbf{k}$. Taking k-average equation 4.31, we obtain :

$$\frac{\partial \overline{\rho(k)}}{\partial t} + \frac{i}{\hbar} [\overline{H'(k)}, \overline{\rho(k)}] = 0 \quad (4.32)$$

Because $H'(k)$ is an odd function of k , $[\overline{H'(k)}, \overline{\rho(k)}] = 0$ and the right hand side vanishes upon averaging over k . We also decomposed equation 4.31 as :

$$\frac{\partial \overline{\rho(k)}}{\partial t} + \frac{\partial \rho'(k)}{\partial t} + \frac{i}{\hbar} [H'(k), \overline{\rho(k)} + \rho'(k)] = - \sum_{k'} W_{k,k'} (\rho(k) - \rho(k')) \quad (4.33)$$

Using equation 4.32 in equation 4.33, we obtain :

$$\begin{aligned} -\frac{i}{\hbar} [\overline{H'(k)}, \overline{\rho'(k)}] + \frac{\partial \rho'(k)}{\partial t} + \frac{i}{\hbar} [H'(k), \overline{\rho(k)}] + \frac{i}{\hbar} [H'(k), \rho'(k)] &= \\ &= - \sum_{k'} W_{k,k'} (\overline{\rho(k)} + \rho'(k) - \overline{\rho(k')} - \rho'(k')) \\ &= - \sum_{k'} W_{k,k'} (\rho'(k) - \rho'(k')) \end{aligned} \quad (4.34)$$

Based on the order of spin-orbit coupling term, we assume equation 4.34 that [66] :

1. $\overline{\rho(k)}$ is the zeroth order term

2. $\rho'(k)$ is the first order term
3. $\partial\overline{\rho(k)}/\partial t$ is the second order term
4. $\partial\rho'(k)/\partial t$ is the third order term

Under the quasi-static limit $(\partial\rho'(k)/\partial t) = 0$, the anisotropic part of the density matrix should return to isotropy on a typical time corresponding to the momentum scattering time [66]. Keeping the first order of spin-orbit coupling term, equation 4.34 can be simplified as :

$$-\frac{i}{\hbar}[H'(k), \overline{\rho(k)}] = \sum_{k'} W_{k,k'}(\rho(k) - \rho(k')) \quad (4.35)$$

Three dimensions solution

Using spherical harmonic approximation to decompose the spin-orbit Hamiltonian, we write :

$$H'(k) = \sum_l \sum_{n=-l}^l H_{l,n} Y_l^n(\vartheta_k, \phi_k)$$

in which ϑ_k is taken as the polar coordinate and ϕ_k as the azimuthal (longitudinal) coordinate of $\mathbf{k}$ -vector.

Now the solution for equation 4.35 is [25, 54]:

$$\rho(k) = -\frac{i}{\hbar} \sum_l \sum_{n=-l}^l \tau_l Y_l^n(\vartheta_k, \phi_k) [H_{l,n}, \overline{\rho(k)}] \quad (4.36)$$

and :

$$\begin{aligned} \sum_{k'} W_{k,k'}(\rho(k) - \rho(k')) &= -\frac{i}{\hbar} \sum_l \sum_{n=-l}^l \tau_l Y_l^n(\vartheta_k, \phi_k) [H_{l,n}, \overline{\rho(k)}] \times \\ &\times \int_0^\pi \sin(\theta) W(\theta) (1 - P_l(\cos \theta)) d\theta \end{aligned} \quad (4.37)$$

where $P_l(\cos \theta)$ is the Legendre polynomials, $W_{k,k'} \rightarrow W(\theta)$ under the assumption that the scattering mechanism is elastic and the scattering probability depends only on the angle θ between the wavevectors $\mathbf{k}$ and $\mathbf{k}'$. The scattering time τ_l can be expressed as :

$$\frac{1}{\tau_l} = \int_0^\pi \sin(\theta) W(\theta) (1 - P_l(\cos \theta)) d\theta \quad (4.38)$$

Substituting equation 4.36 and 4.37 into equation 4.32, we have :

$$\frac{\partial \overline{\rho(k)}}{\partial t} = -\frac{1}{4\pi\hbar^2} \sum_l \sum_{n=-l}^l (-1)^n \tau_l [H_{l,-n}, [H_{l,n}, \overline{\rho(k)}]]$$

To determine the time evolution of spin components, the Pauli matrices σ is taken into account :

$$\mathbf{s} = \overline{Tr(\rho(k) \cdot \sigma)} = Tr(\overline{\rho(k)} \cdot \sigma) \quad (4.39)$$

Using the standard Bloch-like equation :

$$\frac{ds_i}{dt} = -\frac{1}{\tau_{ij}} s_j$$

the *relaxation tensor* is given as :

$$\frac{1}{\tau_{ij}} = \frac{1}{8\pi\hbar^2} Tr\left(\sum_l \sum_{n=-l}^l (-1)^n \tau_l [H_{l,-n}, [H_{l,n}, \sigma_j]] \sigma_i\right) \quad (4.40)$$

in which Tr denotes the trace of the matrix.

Two dimensions solution

The two dimensions solution is found in a similar way with the three dimensions solution but the spin-orbit Hamiltonian is expanded in terms of Fourier transform ($e^{in\varphi}$ function) instead of spherical harmonic :

$$H'(k) = \sum_{n=-\infty}^{\infty} H_n(k) e^{in\varphi}$$

where $H_n(k)$ is the coefficient defined as :

$$H_n(k) = \int_0^{2\pi} \frac{d\varphi}{2\pi} H'(k) e^{-in\varphi} \quad (4.41)$$

and $\mathbf{k}$ is the in-plane wavevector considering the motion in x-y plane, φ is the angular measure of wavevector $\mathbf{k}$. The scattering time τ_n is given as :

$$\frac{1}{\tau_n} = \int_0^{2\pi} \sin(\theta) W(\theta) (1 - \cos(n\theta)) d\theta \quad (4.42)$$

in which $\theta = \varphi - \varphi'$ is the angle between $\mathbf{k}$ and $\mathbf{k}'$. Now the angular average is obtained as :

$$\frac{\partial \overline{\rho(k)}}{\partial t} = -\frac{1}{\hbar^2} \sum_{n=-\infty}^{\infty} \tau_n [H_{-n}, [H_n, \overline{\rho(k)}]] \quad (4.43)$$

Then the components of the *relaxation tensor* for two dimensions are found :

$$\frac{1}{\tau_{ij}} = \frac{1}{2\hbar^2} Tr\left(\sum_{n=-\infty}^{\infty} \tau_n [H_{-n}, [H_n, \sigma_j]] \sigma_i\right) \quad (4.44)$$

4.3.2 Electron spin relaxation times in <111>-oriented QW

Let us recall the Dresselhaus and Rashba contributions to the Hamiltonian of an electron in conduction band of <111>-QW, we have the first order and third order of Hamiltonian as follow:

$$\begin{aligned} H^{(1)} &= \left(\frac{2\gamma}{\sqrt{3}} \langle k'_z \rangle^2 + a_6^4 E \right) (k'_y \sigma_x - k'_x \sigma_y) = \alpha_T (k'_y \sigma_x - k'_x \sigma_y) \\ H^{(3)} &= \gamma \left[\frac{-1}{2\sqrt{3}} k'_y (k_x'^2 + k_y'^2) \sigma_x + \frac{1}{2\sqrt{3}} k'_x (k_x'^2 + k_y'^2) \sigma_y \right. \\ &\quad \left. + \frac{1}{\sqrt{6}} k'_y (3k_x'^2 - k_y'^2) \sigma_z \right] \end{aligned}$$

We denote $\vec{k}_\perp = (k'_x, k'_y, 0) = |k'_\perp| (\cos(\varphi_k), \sin(\varphi_k), 0)$. In order to calculate the spin relaxation tensor from equation 4.44, the Fourier transform of the two Hamiltonian components should be calculated. For convenience, we use the raising and lowering spin operators $\sigma_\pm = \sigma_x \pm i\sigma_y$ to define the Hamiltonian (see appendix D.1):

$$H^{(1)} = -\frac{i}{2} \alpha_T |k'_\perp| [\sigma_- e^{i\varphi_k} - \sigma_+ e^{-i\varphi_k}] \quad (4.46a)$$

$$H^{(3)} = \frac{i\gamma k_\perp'^2}{4\sqrt{3}} [\sigma_- e^{i\varphi_k} - \sigma_+ e^{-i\varphi_k}] + \frac{\gamma |k'_\perp|^3}{2i\sqrt{6}} \sigma_z (e^{3i\varphi_k} - e^{-3i\varphi_k}) \quad (4.46b)$$

The Fourier transforms of the first and third order of Hamiltonian are given as (see appendix D.1):

$$H_{(1)}^{(1)} = -\frac{i}{2} \alpha_T |k'_\perp| \sigma_- \quad (4.47a)$$

$$H_{(-1)}^{(1)} = \frac{i}{2} \alpha_T |k'_\perp| \sigma_+ \quad (4.47b)$$

$$H_{(1)}^{(3)} = \frac{i |k'_\perp|^3}{4\sqrt{3}} \sigma_- \quad (4.47c)$$

$$H_{(-1)}^{(3)} = -\frac{i |k'_\perp|^3}{4\sqrt{3}} \sigma_+ \quad (4.47d)$$

$$H_{(3)}^{(3)} = \frac{|k'_\perp|^3}{2i\sqrt{6}} \sigma_z \quad (4.47e)$$

$$H_{(-3)}^{(3)} = -\frac{|k'_\perp|^3}{2i\sqrt{6}} \sigma_z \quad (4.47f)$$

in which the lower subscripts denote the order of Fourier transform, the upper subscripts denote the order of Hamiltonian depending on wavevector $\mathbf{k}$.

From equation 4.47, we summarize the Fourier transform of total Hamiltonian

as follow :

$$H'_{(1)} = -\frac{i}{2}(\alpha_T |k'_\perp| - \frac{\gamma |k'_\perp|^3}{2\sqrt{3}})\sigma_- \quad (4.48a)$$

$$H'_{(-1)} = \frac{i}{2}(\alpha_T |k'_\perp| - \frac{\gamma |k'_\perp|^3}{2\sqrt{3}})\sigma_+ \quad (4.48b)$$

$$H'_{(3)} = \frac{|k'_\perp|^3}{2i\sqrt{6}}\sigma_z \quad (4.48c)$$

$$H'_{(-3)} = -\frac{|k'_\perp|^3}{2i\sqrt{6}}\sigma_z \quad (4.48d)$$

From equation 4.48 we realize that $H'_{(1)} = H'^*_{(-1)}$, $H'_{(3)} = H'^*_{(-3)}$ in which the subscript * denotes the conjugate operator. Due to this reason, the spin relaxation tensor in equation 4.44 can be rewritten for <111>-oriented QW as :

$$\frac{1}{\tau_{ij}} = \frac{1}{\hbar^2} Tr(\sum_{n=0}^{\infty} \tau_n [H_{-n}, [H_n, \sigma_j]] \sigma_i) \quad (4.49)$$

From equation 4.48, we obtain the spin relaxation tensor as (see appendix D.2 for more detailed calculation):

$$\Gamma = \begin{bmatrix} \Gamma_{xx} & 0 & 0 \\ 0 & \Gamma_{yy} & 0 \\ 0 & 0 & \Gamma_{zz} \end{bmatrix} \quad (4.50)$$

in which the spin relaxation rates $\Gamma_{xx}, \Gamma_{yy}, \Gamma_{zz}$ are given by :

$$\Gamma_{xx} = \Gamma_{yy} = \frac{2\tau_1}{\hbar^2} [(\alpha_T |k'_\perp| - \frac{\gamma |k'_\perp|^3}{2\sqrt{3}})^2 + \frac{\tau_3}{6\tau_1} \gamma^2 |k'_\perp|^6] \quad (4.51)$$

$$\Gamma_{zz} = \frac{4\tau_1}{\hbar^2} (\alpha_T |k'_\perp| - \frac{\gamma |k'_\perp|^3}{2\sqrt{3}})^2 \quad (4.52)$$

Finally, the spin lifetimes for electrons in the *nondegenerate regime* are determined by doing thermal average of the corresponding scattering rates [45] :

$$\begin{aligned} \frac{1}{\tau_{xx}} &= \langle \Gamma_{xx} \rangle_\theta = \frac{2}{\hbar^2} [\alpha_T^2 \langle \tau_1 k'^2_\perp \rangle_\theta - \frac{\gamma \alpha_T}{\sqrt{3}} \langle \tau_1 k'^4_\perp \rangle_\theta + \\ &\quad + \frac{1}{12} (1 + \frac{2\tau_3}{\tau_1}) \gamma^2 \langle \tau_1 k'^6_\perp \rangle_\theta] \end{aligned} \quad (4.53a)$$

$$\frac{1}{\tau_{yy}} = \langle \Gamma_{yy} \rangle_\theta = \frac{1}{\tau_{xx}} \quad (4.53b)$$

$$\begin{aligned} \frac{1}{\tau_{zz}} &= \langle \Gamma_{zz} \rangle_\theta = \frac{4}{\hbar^2} [\alpha_T^2 \langle \tau_1 k'^2_\perp \rangle_\theta - \frac{\gamma \alpha_T}{\sqrt{3}} \langle \tau_1 k'^4_\perp \rangle_\theta + \\ &\quad + \frac{\gamma^2}{12} \langle \tau_1 k'^6_\perp \rangle_\theta] \end{aligned} \quad (4.53c)$$

in which the symbol $\langle \dots \rangle_\theta$ means the thermal average.

Moreover, we have [17]:

$$\langle \tau_1 k^{2n} \rangle_\theta = \frac{(n + \nu)!}{(\nu + 1)!} \tau_p k_\theta^{2n} \quad (4.54)$$

for *non-degenerate* electron gas where τ_p is the transport mobility scattering time, ν is the energy dependent coefficient which depends on the scattering process when we assume that the scattering time depends on the energy $\tau \propto E^\nu$; $k_\theta = \sqrt{(2m_c^* k_B T)/(\hbar^2)}$ is the thermal wavevector in which m_c^* is the electron effective mass, k_B is the Boltzmann constant and T is the temperature.

Applying equation 4.54 to equation 4.53, we obtain :

$$\frac{1}{\tau_{zz}} = \frac{4\tau_p}{\hbar^2} k_\theta^2 \left[(\alpha_T - \frac{2+\nu}{2\sqrt{3}} \gamma^2 k_\theta^2)^2 + \frac{2+\nu}{12} \gamma^2 k_\theta^4 \right] \quad (4.55a)$$

$$\frac{1}{\tau_{xx}} = \frac{1}{\tau_{yy}} = \frac{2\tau_p}{\hbar^2} k_\theta^2 \left[(\alpha_T - \frac{2+\nu}{2\sqrt{3}} \gamma^2 k_\theta^2)^2 + \frac{2+\nu}{12} (1 + 2(3+\nu) \frac{\tau_3}{\tau_1}) \gamma^2 k_\theta^4 \right] \quad (4.55b)$$

where $\alpha_T = \frac{2\gamma}{\sqrt{3}} \langle k_z'^2 \rangle + a_6^4 \cdot E$.

It should be noted that τ_{zz}^{-1} and τ_{xx}^{-1} are symmetrical functions of electric field around the peak condition E_p where E_p is the electric field defined as :

$$\alpha_T(E_p) - \frac{2+\nu}{2\sqrt{3}} \gamma^2 k_\theta^4 = 0 \quad (4.56)$$

4.3.3 Comparison with experiments

From equation 4.55, we can see that there are two important parameters that must be defined such as the momentum scattering time τ_p and the energy dependent coefficient ν depending on the scattering mechanisms. The first parameter can be measured by Hall mobility measurement and the latter one is determined by the ratio of scattering times τ_3/τ_1 . In addition, we can deduce from equation 4.55 that at the compensation condition E_p , the ratio between $\tau_{zz,p}$ and $\tau_{xx,p}$ is given by :

$$\frac{\tau_{zz,p}}{\tau_{xx,p}} = \frac{1}{2} [1 + 2(3 + \nu) \frac{\tau_3}{\tau_1}] \quad (4.57)$$

in which the subscript p denotes the peak value. According to Kainz et al. [81], the values of ν for different scattering mechanisms are listed in the table 4.1.

Figure 4.4(a) compares the simulated and the experimental results for the in-plane spin relaxation time τ_{xx} (τ_{yy}). In this figure, the full line presents the simulated results whereas the dots present the experimental results. The ratio

Scattering mechanism	ν	$\frac{\tau_3}{\tau_1}$	$\frac{\tau_{zz,p}}{\tau_{xx,p}}$
<i>Type I : acoustic phonons (deformation potential)</i> <i>optical phonons (deformation potential)</i> <i>ionized impurities (screened)</i> <i>neutral impurities</i> <i>alloy scattering</i> <i>inter face roughness</i>	0	1	$\frac{7}{2}$
<i>Type II : acoustic phonon (polar , piezoelectric)</i> <i>optical phonon (polar)</i>	1	$\frac{1}{3}$	$\frac{11}{6}$
<i>Type III : ionized impurities</i> <i>(weakly screened)</i>	2	$\frac{1}{9}$	$\frac{19}{18}$

Table 4.1: Characteristic exponent ν in the scattering time for different scattering mechanisms in a 2D electron gas [81]

$\tau_{\parallel}/\tau_{\perp} \equiv \tau_{zz,p}/\tau_{xx,p}$ displayed in figure 4.4(b) increases up to a maximum value determined by the remaining cubic Dresselhaus terms. Since $\tau_{\parallel}/\tau_{\perp} > 11/6$, this means that the dominant scattering mechanism is type I in the table 4.1. From the peak ratio $\tau_{zz,p}/\tau_{xx,p}$ resulting from experiment results, the energy dependent coefficient $\nu = 0$ corresponding to $\tau_3/\tau_1 = 1$ is thus determined. In the calculated curve, the parameters are :

$$\begin{aligned}
 T &= 10K \\
 \gamma &= 22eV(A^\circ)^3 \\
 a_6^4 &= 24e(A^\circ)^2 \\
 \tau_p &= 6ps \\
 L_W &= 15nm
 \end{aligned}$$

This model shows a resonable agreement between theoretical and experimental work. We clearly see that this model predicts well the reversal of the anisotropic spin relaxation when the applied electric field varies. We observe that for $|E_i| \approx 38$ kV/cm, $\tau_{xx} \approx \tau_{zz}$ whereas for $|E| < |E_i|$, $\tau_{zz} < \tau_{xx}$ and for $|E| > |E_i|$, $\tau_{zz} > \tau_{xx}$.

Compared with the simple model in the previous section, the spin density matrix model describes the electron spin relaxation in a more accurate way because the nature of collision term is taken into account. In fact, equation 4.55 shows that the ratio between τ_{zz} and τ_{xx} contains the collision term but in the simple model, this term is eliminated. Following the spin density matrix model, the microscopic

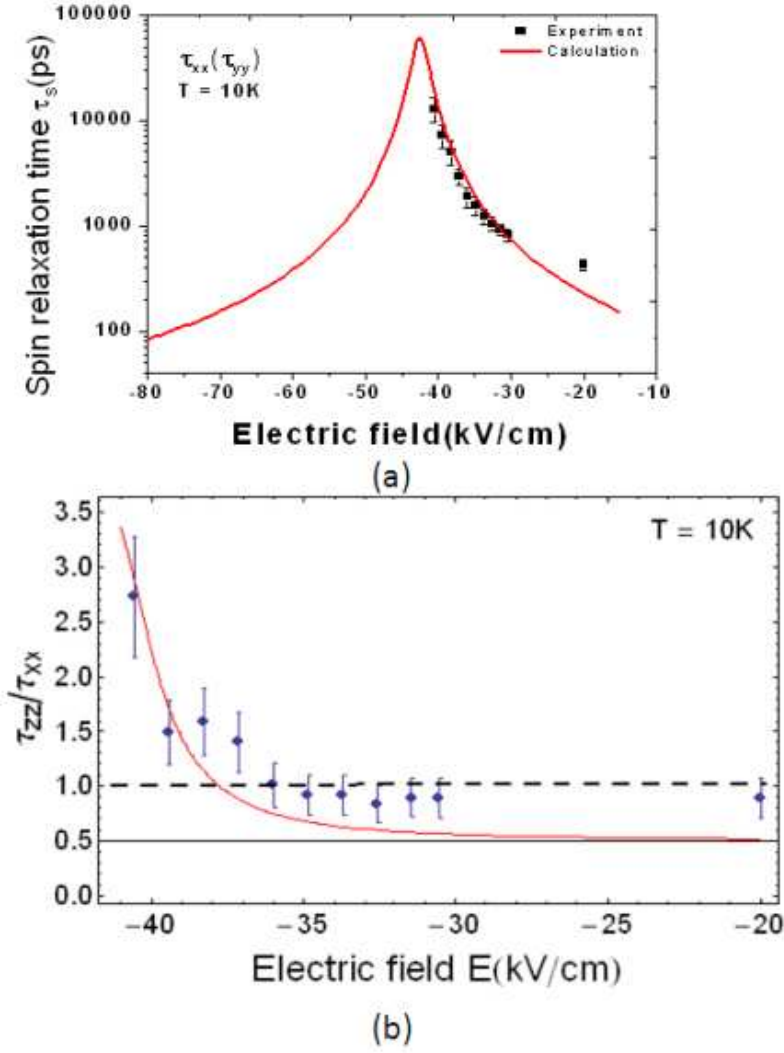


Figure 4.4: Comparison between simulated and experimental results (see the parameters in the text) : (a) in-plane spin relaxation time; (b) ratio between longitudinal and transverse spin relaxation times

picture of collision mechanisms depending on the nature of the samples is revealed. Finally, among the different type I processes in table 4.1, we think that the reasonable dominant scattering process is the scattering on the QW interface roughness due to unavoidable characteristics of MBE grown QW.

From equation 4.55, the compensation electric field at resonant conditions is given by :

$$a_6^4 \cdot E_p(T) + \frac{2\gamma}{\sqrt{3}} \left(\frac{\pi}{L_W} \right)^2 - \frac{1 + \nu/2}{2} k_\theta^2 = 0 \quad (4.58)$$

in which $E_p(T)$ is the electric field value at resonant condition which depends on the temperature. Hence, using the value of electric field E_p obtained from experiments, we can deduce the ratio between Rashba and Dresselhaus coefficients α/γ (where $\alpha = a_6^4 \cdot E$). In this model, we find $\alpha/\gamma \approx 5 \cdot 10^{16} \text{ m}^{-2}$ which is very close to the estimated value in chapter 3 (when the cubic terms were neglected).

Moreover, the striking feature of this model is that we can deduce directly from experimental results three parameters such as ν , τ_p and α which are difficult to measure in experiments. This model points out that the momentum scattering time τ_p can be evaluated from the electric field dependence of the spin relaxation tensor in $\langle 111 \rangle$ -oriented quantum wells. In fact, we do not need some complementary mobility measurements in order to determine τ_p as in $\langle 001 \rangle$ and $\langle 110 \rangle$ -oriented quantum wells. Through the peak value of the longitudinal spin relaxation time $\tau_{zz,p}$ obtained from the experimental results, the momentum scattering time τ_p is deduced in this equation :

$$\frac{1}{\tau_{zz,p}(E_p)} = \frac{\tau_p}{\hbar^2} \frac{2 + \nu}{3} \gamma^2 k_\theta^6 \quad (4.59)$$

4.3.4 Variation of parameters and corresponding trends

Motional narrowing effect

Figure 4.5 shows the motional-narrowing effect in $\langle 111 \rangle$ -oriented QWs at $T = 10\text{K}$ as deduced from the calculated spin relaxation time. The momentum scattering time τ_p is varied from 13.5 ps to 0.135 ps. The three pairs of curves correspond to the change of momentum scattering time τ_p from 0.135 ps, 1.35 ps and 13.5 ps, respectively. The two lowest curves correspond to the case $\tau_p = 13.5$ ps and the highest ones correspond to the case $\tau_p = 0.135$ ps. We can observe that the isotropic point is not strongly dependent on the momentum scattering time τ_p .

Quantum well width dependence

Figure 4.6 shows the calculated dependence of the electric field required for compensation (the value at peak E_p) in $\langle 111 \rangle$ -oriented QWs at $T = 10\text{K}$. The larger the quantum well, the smaller the required electric field that make Rashba term compensate the Dresselhaus term as shown in chapter 3.

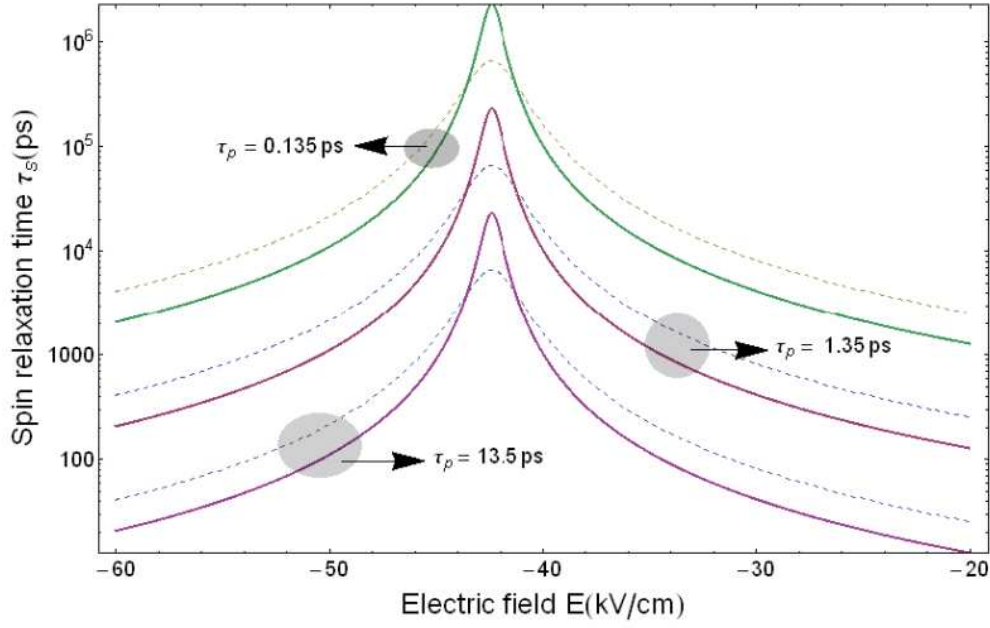


Figure 4.5: Calculated spin relaxation times τ_{zz} (full lines), $\tau_{xx}(\tau_{yy})$ (dashed lines) for different momentum scattering times τ_p at $T = 10K$

Temperature dependence of spin relaxation

At the compensation condition, we have :

$$\begin{aligned} a_6^4 \cdot E + \frac{2\gamma}{\sqrt{3}} \gamma \langle k_z'^2 \rangle - \frac{2+\nu}{2\sqrt{3}} \gamma k_\theta^2 &= 0 \\ \Rightarrow a_6^4 \cdot (E - E_p) &= 0 \end{aligned}$$

where :

$$a_6^4 \cdot E_p + \frac{2\gamma}{\sqrt{3}} \gamma \langle k_z'^2 \rangle - \frac{2+\nu}{2\sqrt{3}} \gamma k_\theta^2 = 0$$

Combined with equation 4.55, the ratio between τ_{zz} and τ_{xx} is given as :

$$\frac{\tau_{zz}}{\tau_{xx}} = \frac{1}{2} \left[1 + \frac{2(2+\nu)(3+\nu)}{12} \frac{\tau_3}{\tau_1} \frac{\gamma^2 k_\theta^4}{[a_6^4 (E - E_p)]^2 + \frac{2+\nu}{12} \gamma^2 k_\theta^4} \right] \quad (4.60)$$

At the isotropic point, $\tau_{zz}/\tau_{xx} = 1$; so, the electric field at these points are :

$$E_i(\pm) = E_p(T) \pm \left[\frac{2+\nu}{12} (2(3+\nu)) \frac{\tau_3}{\tau_1} - 1 \right]^{\frac{1}{2}} \gamma k_\theta^2 \quad (4.61)$$

Equation 4.61 shows that two isotropic points are temperature-dependent and move away from point $E_p(T)$ when the temperature increases. Figure 4.7 depicts the calculated variation of spin relaxation times depending on temperature; in which the temperature changes from 10K to 100K and we assume in the first

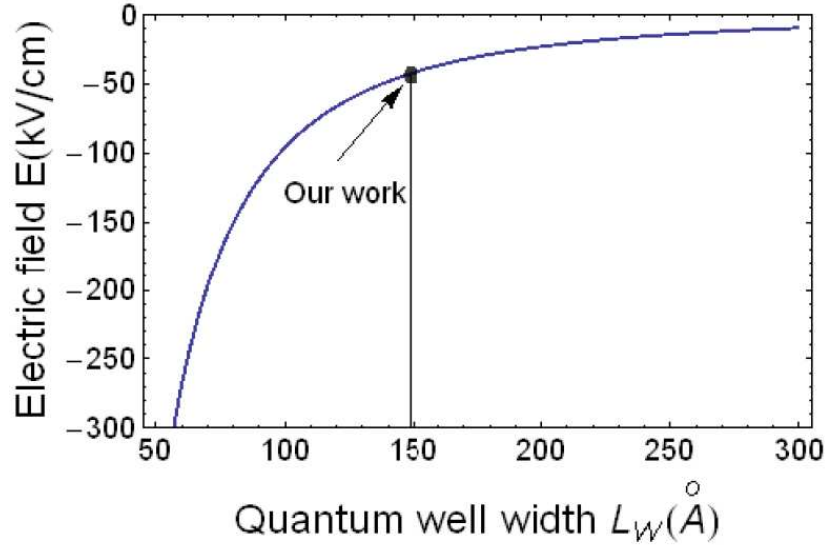


Figure 4.6: The calculated dependence of the electric field E_p required for compensation as a function of quantum well width at $T = 10\text{K}$

approximation that the momentum scattering time τ_p is unchanged when the temperature increases. It can be seen that both the value of electric fields at the peak E_p of τ_{zz} (τ_{xx}) and isotropic points $E_i(\pm)$ are strongly dependent on the temperature as expected in equation 4.61.

4.4 Conclusion

This chapter introduced the theoretical models of electron spin relaxation in $\langle 111 \rangle$ -oriented QWs. We have presented two models :

1. The simple model: This model describes the first interpretation of experimental results in which the role of cubic term in electron Hamiltonian is emphasized. This model shows that we only compensate the total field (Dresselhaus field and Rashba field) in terms of first-order Hamiltonian by external electric field. The cubic term can not be compensated and become more important at high temperatures as shown by experimental results in chapter 3.
2. Spin density matrix model : This model presents the Dyakonov-Perel mechanism by spin density matrix formalism in *nondegenerate* regime, in which the nature of collision term is taken into account. The electron spin relaxation is described in a more accurate way. The model shows the im-

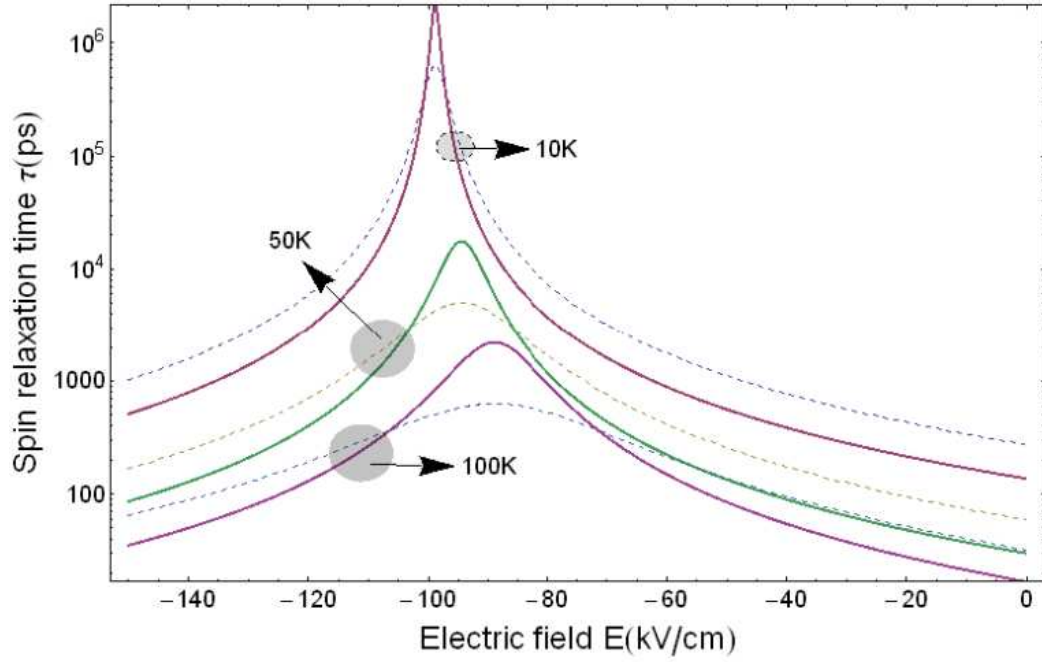


Figure 4.7: Calculated temperature dependence of spin relaxation times (full lines present the longitudinal spin relaxation times τ_{zz} , dashed lines present the transverse spin relaxation times $\tau_{xx}(\tau_{yy})$)

portant role of scattering mechanisms which give a strong contribution on electron spin relaxation. The simulated results are in reasonably good agreement with the experimental result and improve the weakness of the simple model. Moreover, the model also presents the way to extract the important parameter from experimental results such as momentum scattering time τ_p which depends on scattering mechanisms and usually need to be measured by independent Hall mobility experiment. The model also emphasizes an interesting phenomenon i.e. there exists an isotropic point where the spin relaxation tensor is scalar. This effect opens up the possibility of electrically controlling spin relaxation anisotropy in zinc-blende semiconductor.

Conclusions

In this manuscript, we have studied the electron spin dynamics in $\langle 111 \rangle$ -oriented GaAs/AlGaAs quantum well. Observed by time-resolved photoluminescence (TRPL) spectroscopy, this type of quantum wells reveals remarkable properties of electrically-controlled electron spin relaxation. In fact, for this special quantum well growth direction, the Dresselhaus (originated from bulk inversion asymmetry) and Rashba (originated from the structural inversion asymmetry) spin-orbit coupling terms which give the most important contributions to electron spin relaxation by Dyakonov-Perel mechanism (DP) are in-plane and have the same direction whatever the $\mathbf{k}$ -vector is. Using an external gate voltage applied on the sample to create an electric field across the quantum well allowed us to tune the Rashba term to compensate the Dresselhaus one; the electron spin relaxation time can be increased by about two orders of magnitude.

We have also demonstrated that the electron spin relaxation anisotropy can be electrically controlled. By tuning the gate voltage, there exists an isotropic point where all three components of electron spin relaxation time are equal. In order to study these interesting effects, two models are developed. A simple model introduces a good understanding of the role of linear term and cubic term in electron Hamiltonian. The simple model shows that an external gate voltage can only cancel the total field (Dresselhaus and Rashba fields) in the linear-term but the cubic term always exists. The experimental evidence is that the cancellation effect is much weaker at high temperature because the cubic term gives more contribution in electron spin relaxation. However, the weakness of this model is that the scattering mechanisms which are very important in momentum scattering time are eliminated. Therefore, a spin density matrix model has been developed to improve the description. When the role of collision term is considered, the model demonstrates a reasonably good agreement between simulated and exper-

imental results. Moreover, this model also introduces a simple way to evaluate an important parameter which is momentum scattering time (τ_p). It is difficult to measure experimentally this characteristic time.

The work in this thesis opens interesting perspectives for the future. The key feature of prolonging electron spin relaxation in $\langle 111 \rangle$ -oriented quantum well by tuning the Rashba term to compensate the Dresselhaus one is only based on symmetry consideration. Thus it could be demonstrated in semiconductor III-V and II-VI structures. Instead of an adjustable external gate voltage, an electric field across the quantum wells could also be created by asymmetric doping profile in quantum well structures. We could also use the piezoelectric field in strained quantum wells.

The electrical control of electron spin relaxation could also be performed in the future in *wurtzite* semiconductor quantum wells such as GaN, AlN and ZnO. Indeed, the theoretical study by **k.p** method in $\langle 001 \rangle$ wurtzite structure shows that the Dresselhaus term in $\langle 001 \rangle$ -grown wurtzite Hamiltonian is very similar with $\langle 111 \rangle$ -grown zinc-blende Hamiltonian [10, 40]. It means that the Dresselhaus and the Rashba terms in $\langle 001 \rangle$ -grown wurtzite QWs have the same direction as $\langle 111 \rangle$ -grown zinc-blende QWs have. Thus, $\langle 001 \rangle$ -grown wurtzite QWs are potential candidates to demonstrate the cancellation effect of D'yakonov-Perel electron spin relaxation.

Résumé en Français

Conformément aux dispositions votées au Conseil Scientifique de l'INSA, une thèse rédigée en anglais doit être accompagnée d'un résumé détaillé en français présenté ci-dessous.

Introduction

Le contrôle du spin des électrons dans les semiconducteurs pour des applications potentielles dans des dispositifs électroniques ou pour des systèmes de traitement de l'information quantique a fait l'objet de nombreux travaux dans les dernières années [25, 29, 55].

Dans les nanostructures bi-dimensionnelles réalisées à partir de semiconducteurs III-V ou II-VI, le mécanisme dominant de perte de mémoire de spin est appelé mécanisme D'Yakonov-Perel (DP) [55]. Dans ces matériaux, l'absence de symétrie d'inversion et le couplage Spin-Orbite (SO) entraîne une levée de dégénérescence des états électroniques de spin $|1/2\rangle$ et $-|1/2\rangle$ de la bande de conduction. Ce clivage en énergie joue un rôle crucial sur les phénomènes de transport et de manipulation de spin dans ces semiconducteurs. Comme ce clivage dépend fortement de la symétrie du cristal et de la nanostructure, il peut être façonné de manière efficace. Ce travail de thèse en est une illustration.

Le clivage Spin-Orbite de la bande de conduction peut être vu comme étant la conséquence de l'action d'un champ magnétique effectif sur le spin de l'électron, champ magnétique dont la direction et l'amplitude dépendent du vecteur d'onde de l'électron. Le spin de l'électron va précesser autour de ce champ avec un vecteur de précession Ω , dont l'amplitude correspond au clivage SO de la bande de conduction. Ce champ magnétique effectif varie dans le temps du fait des chan-

gements du vecteur d'onde de l'électron.

Par conséquent, la précession de spin autour de ce champ dans les intervalles entre deux collisions conduit à une relaxation de spin. Dans le cas classique des collisions fréquentes, le temps de relaxation d'un spin d'électron orienté le long de la direction i s'écrit :

$$(\tau_s^i)^{-1} = \langle \Omega_{\perp}^2 \rangle \tau_p^*$$

Où $\Omega_{\perp}^2$ est la moyenne du vecteur de précession au carré dans le plan perpendiculaire à i ($i = x, y$ ou z) et τ_p^* est le temps de relaxation du moment de l'électron. Ceci conduit à des temps de relaxation de spin typique de quelques dizaines à quelques centaines de picosecondes dans des puits quantiques III-V ou II-VI. Une réduction du clivage SO doit entraîner une augmentation du temps de relaxation de spin si celui-ci est dominé par le mécanisme D'Yakonov-Perel.

Dans les semiconducteurs qui cristallisent dans la structure Blende de Zinc, le terme SO qui provient de l'asymétrie d'inversion du matériau massif, appelé Bulk Inversion Asymmetry (BIA) ou terme de Dresselhaus est déterminé par [15] :

$$\Omega_{BIA}^{3D} = \frac{\gamma}{\hbar} [k_x(k_y^2 - k_z^2), k_y(k_z^2 - k_x^2), k_z(k_x^2 - k_y^2)]$$

où γ est le coefficient de Dresselhaus et $\vec{k} = (k_x, k_y, k_z)$ est le vecteur d'onde de l'électron. Dans un puits quantique où la composante du moment le long de l'axe de croissance (Oz) est quantifiée, le terme de Dresselhaus pour la première sous-bande de conduction s'écrit :

$$\begin{aligned} \Omega_{BIA}(\mathbf{k}_{\perp}) &= \frac{\gamma}{\hbar} \langle k_z^2 \rangle (-k_x, k_y, 0) & \text{for } \mathbf{z} \parallel [001] \\ \Omega_{BIA}(\mathbf{k}_{\perp}) &= \frac{\gamma}{2\hbar} \langle k_z^2 \rangle (0, 0, k_y) & \text{for } \mathbf{z} \parallel [110] \\ \Omega_{BIA}(\mathbf{k}_{\perp}) &= \frac{2\gamma}{\hbar\sqrt{3}} \langle k_z^2 \rangle (k_y, -k_x, 0) & \text{for } \mathbf{z} \parallel [111] \end{aligned}$$

où $\langle k_z^2 \rangle$ est le vecteur d'onde au carré moyen le long de la direction de croissance et $k_{\perp}(k_x, k_y)$ est le vecteur d'onde de l'électron dans le plan du puits quantique. Les termes cubiques de l'Hamiltonien d'amplitude plus faible ont ici été négligés car $\langle k_z^2 \rangle \gg k_x^2, k_y^2$; leur influence sera discutée plus tard dans ce résumé.

Si un champ électrique externe est appliqué ou si le puits quantique comporte un dopage asymétrique (qui génère un champ électrique interne), une contribution supplémentaire apparaît dans le clivage Spin-Orbite de la bande de conduction, contribution appelée Structural Inversion Asymmetry (SIA) ou terme de Rashba [84].

Pour un champ électrique appliqué le long de l'axe de croissance (Oz), le terme SIA s'écrit :

$$\Omega_{SIA}(k_{\perp}) = \frac{\alpha}{\hbar}(k_y, -k_x, 0)$$

où $\alpha = a_6^4 E$ est le coefficient de Rashba (E est l'amplitude du champ électrique et a est une constante positive). Le vecteur SIA est toujours perpendiculaire au vecteur d'onde de l'électron et au champ électrique ($\Omega \propto E \times k_{\perp}$).

Les équations précédentes montrent qu'à la fois le terme BIA et le terme SIA peuvent être contrôlés. En effet, la modification du confinement électronique dans le puits permet de contrôler l'amplitude du clivage Spin-Orbite du type BIA (via la dépendance de $\langle k_z^2 \rangle$ avec la largeur du puits). Le terme SIA peut quant à lui être facilement modifié en faisant varier l'amplitude du champ électrique externe appliqué.

Pour les puits quantiques élaborés sur l'orientation cristalline standard $\langle 001 \rangle$, la présence simultanée des termes BIA et SIA conduit à un clivage Spin-Orbite dans le plan anisotrope [61]. Grâce au contrôle du terme de Rashba dans des puits quantiques asymétriques, une augmentation du temps de relaxation pour des spins alignés le long de la direction $\langle 110 \rangle$ a pu être observée expérimentalement [83, 4, 13, 77]. Cependant cette augmentation du temps de relaxation de spin ne peut être observée que pour cette direction cristallographique.

Pour des puits quantiques épitaxiés sur un substrat d'orientation $\langle 110 \rangle$, le champ magnétique effectif lié au terme BIA est orienté le long de l'axe de croissance $\langle 110 \rangle$. En conséquence, le mécanisme de relaxation de spin D'Yakonov-Perel est supprimé pour des spins d'électron orientés selon cette direction, comme prévu théoriquement par D'Yakonov et Kacharovski en 1986 [55]. Des temps de relaxation de spin pouvant atteindre 20 ns ont ainsi pu être mesurés par différentes équipes [21, 16]. Cependant ces temps longs ne sont observables que pour des spins alignés le long de l'axe de croissance. Pour des spins d'électrons orientés dans le plan du puits quantique, le mécanisme DP est à nouveau opérationnel conduisant à des temps de relaxation de l'ordre d'une centaine de picosecondes [16, 51].

Contrairement aux puits quantiques $\langle 110 \rangle$ et $\langle \bar{1}\bar{1}0 \rangle$, pour lesquels la relaxation de spin de type D'Yakonov-Perel peut être supprimée pour une seule direction de spin, cette relaxation peut en principe être supprimée pour des spins orientés selon les trois directions de l'espace pour des puits quantiques $\langle 111 \rangle$. En effet, pour ce type de puits, les vecteurs BIA et SIA ont exactement la même direction dans le plan, quel que soit le vecteur d'onde (voir les équations ci-dessus).

La bande de conduction peut ainsi devenir dégénérée en spin (au premier ordre) [17, 33, 5].

Si le champ électrique appliqué le long de l'axe de croissance du puits quantique $\langle 111 \rangle$ vérifie la relation :

$$\frac{2\gamma}{\sqrt{3}} = -a_6^4 \cdot E$$

la relaxation de spin de type D'yakonov-Perel devrait être supprimée pour les spins S_x, S_y et S_z dans la mesure où toutes les composantes de Ω sont nulles. Le principal objectif de cette thèse a été la mise en évidence expérimentale de cette manipulation électrique de la relaxation de spin dans des puits quantiques $\langle 111 \rangle$.

Echantillons étudiés

Nous avons principalement étudiés trois échantillons élaborés par épitaxie par jets moléculaires par Chantal Fontaine et Alexandre Arnoult au Laboratoire d'Analyses et d'Architectures des Systèmes (LAAS) Toulouse. Ces échantillons ont été épitaxiés sur des substrats dopés n ($2 \times 10^{18} \text{cm}^{-3}$) avec une désorientation de 3° selon la direction $\langle 111 \rangle_A$.

Pour les trois échantillons, la région centrale est constituée de multi-puits quantiques $GaAs/Al_{0.3}Ga_{0.7}As$ avec une épaisseur de barrière de $L_B = 12$ nm. Les échantillons LAAS2328, LAAS2335 and LAAS2311 ont des largeurs de puits de $L_W = 15$ nm et 7.5 nm respectivement. Dans les échantillons LAAS2311 et LAAS2328, ces puits quantiques sont insérés dans une structure de type PIN (Figure 4.8(a)). Pour appliquer un champ électrique le long de l'axe de croissance, une tension est appliquée entre le contact de surface de type p et le contact arrière sur le substrat de type n. Dans l'échantillon LAAS2335, les couches n et p de la structure ont été permutées conduisant à une structure NIP et une attaque chimique a permis de prendre le contact arrière de type p. Le champ électrique interne de la jonction pour ces structures PIN ou NIP est de l'ordre de 20 KV/cm. L'application d'une tension inverse dans ces structures permet de moduler le terme de Rashba (SIA) des puits quantiques $\langle 111 \rangle$. L'amplitude de ce champ est déterminée à partir des dimensions de la structure et de la tension appliquée ; la mesure du décalage du spectre de photoluminescence lié au "Quantum Confined Stark Effect" permet de vérifier expérimentalement cette estimation du champ électrique.

Technique expérimentale de photoluminescence résolue en temps et en polarisation

La mesure des temps de relaxation de spin dans les échantillons à puits quantiques a été réalisée par spectroscopie de photoluminescence résolue en temps. Les puits quantiques sont excités par des impulsions d'une durée d'1.5 ps polarisés circulairement (σ^+) générés par un laser Ti :Sa à modes bloqués ayant une fréquence de répétition de 80 MHz. La puissance typique d'excitation est de 10 mW et le diamètre de spot de 50 microns. L'énergie des photons du laser d'excitation est de 1.56 eV pour les échantillons LAAS2328 and LAAS2335, c'est-à-dire une énergie supérieure à la transition E1-LH1 du puits quantique. La cinétique de photoluminescence de la transition E1-HH1 est enregistrée par une caméra à balayage de fentes synchroscan Hamamatsu avec une résolution temporelle typique de 8 ps. Les cinétiques co-polarisées (I^+) et contra-polarisées (I^-) avec la polarisation du laser d'excitation sont enregistrées et le taux de polarisation circulaire de la photoluminescence PC est défini par :

$$P_c = \frac{I^+ - I^-}{I^+ + I^-}$$

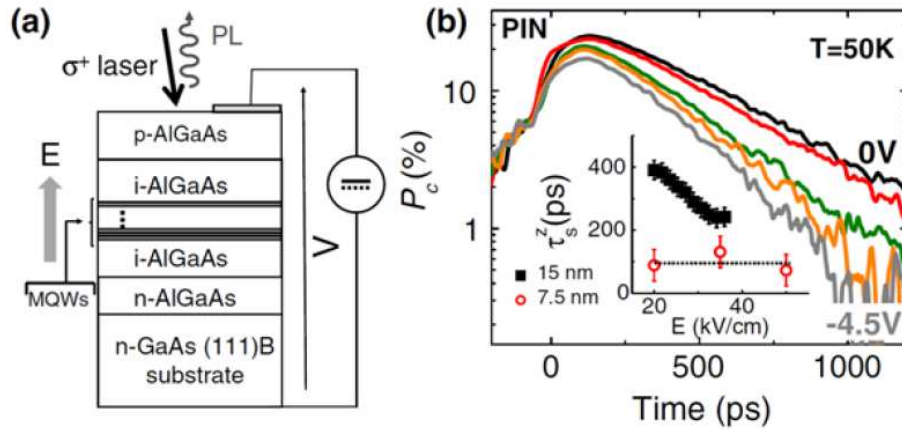


Figure 4.8: (a) Structure de type PIN à multi-puits quantique GaAs/AlGaAs; (b) Dynamique de polarisation circulaire de la photoluminescence pour différentes tensions en polarisation inverse sur l'échantillon LAAS2328 (15nm). Insert : Dépendance du temps de relaxation de spin avec le champ électrique appliqué sur les échantillons LAAS2328 and LAAS2311(7.5nm)

Dynamique de relaxation de spin dans les structures PIN

La figure 4.8(b) présente les dynamiques de polarisation circulaire de photoluminescence P_c pour différentes tensions appliquées en inverse sur l'échantillon I à 50 K. Rappelons que P_c sonde directement la dynamique de relaxation de spin des électrons du puits quantique dans ces conditions d'excitation non résonnante car (i) le temps de relaxation de spin du trou est extrêmement court (≈ 1 ps) [25, 34] et (ii) le temps de relaxation de spin excitonique (lié à l'interaction d'échange électron-trou) n'est pas efficace du fait du temps de relaxation de spin très court du trou.

Figure 4.8(b) montre que le temps de relaxation de spin des électrons décroît quand la tension en inverse augmente, i.e. quand le champ électrique appliqué aux puits quantiques augmente. Cette observation expérimentale permet d'écarter un effet possible lié à la relaxation de spin excitonique dont la variation avec le champ devrait être opposée [59]. Par conséquent, les résultats présentés sur la figure 4.8(b) suggèrent que le temps de relaxation de spin électronique peut être modulé par l'effet Rashba. Pour le puits quantique plus fin ($L_W = 7.5$ nm), l'application du champ électrique n'entraîne aucune variation du temps de relaxation de spin (insert de la figure 4.8(b)). Ceci s'explique facilement par le fait qu'un puits fin est caractérisé par un terme BIA de plus forte amplitude du fait du terme $\langle k_z^2 \rangle = (\pi/L_W)^2$. Il faudrait appliquer un champ électrique beaucoup plus important pour observer une variation du temps de relaxation de spin.

Cependant, il apparaît que dans ces structures de type PIN, les vecteurs BIA et SIA ont la même direction et le même sens. Autrement dit, ces deux contributions s'ajoutent conduisant à une augmentation du clivage Spin-Orbite de la bande de conduction (et donc une réduction du temps de relaxation de spin) quand le champ électrique appliqué augmente. Pour vérifier cette hypothèse, nous avons mesuré la dynamique de spin dans l'échantillon LAAS2335 de type NIP. Dans ce cas, on s'attend à réduire le clivage Spin-Orbite en augmentant le champ électrique (les deux composantes BIA et SIA se soustrayant) et nous devrions donc observer une augmentation sensible du temps de relaxation de spin électronique.

Dynamique de relaxation de spin dans les structures NIP

Figure 4.9(a) présente la variation de la dynamique de spin avec la tension inverse appliquée dans l'échantillon LAAS2335 (structure NIP). Une augmenta-

tion spectaculaire du temps de relaxation de spin électronique est observée quand l'amplitude du champ électrique (en valeur absolue) augmente. La dépendance du temps de relaxation de spin τ_s^z en fonction du champ électrique appliqué est présentée sur la figure 4.9(b). Le temps de relaxation de spin varie de 500 ps à une valeur supérieure à 30 000 ps quand le champ électrique varie de 20 à 55 kV/cm. Pour des champs électriques supérieurs à 50 KV/cm (en valeur absolue), le temps de relaxation de spin devient difficile à évaluer dans la mesure où celui-ci est très supérieur au temps de vie des électrons. En supposant que la valeur de -55 KV/cm est proche de la valeur où le clivage Spin-Orbite est nul, on peut déterminer le rapport entre les coefficients de Dresselhaus et de Rashba : nous trouvons $\alpha/\gamma \approx 5 \times 10^{16} m^{-2}$. Cette valeur est proche de mesures réalisées sur des puits quantiques $\langle 110 \rangle$ ou $\langle 110 \rangle$ [30, 69], en précisant toutefois que la mesure simultanée des deux coefficients dans le même échantillon est difficile. Ces résultats démontrent que le temps de relaxation de spin des électrons dans les puits quantiques $\langle 111 \rangle$ peut être contrôlé avec un champ électrique externe grâce à la variation du clivage Spin-Orbite induite par le terme de Rashba.

Cependant nous avons mesuré jusqu'ici les temps de relaxation longs pour des

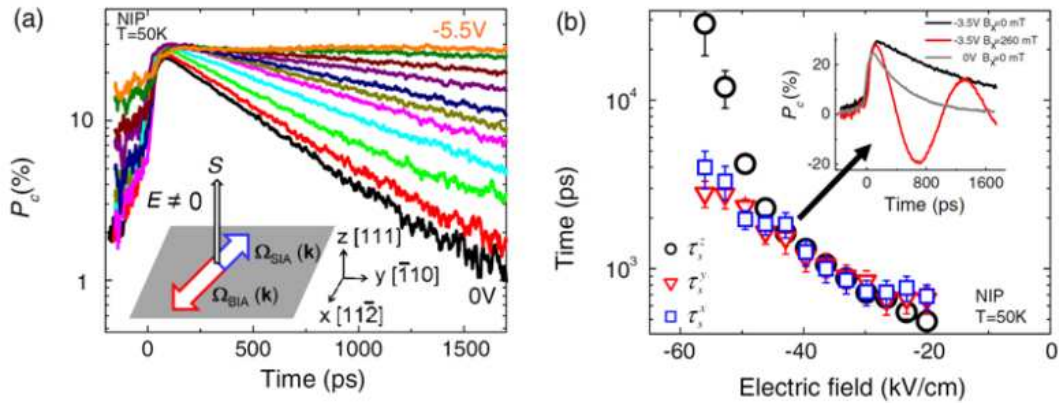


Figure 4.9: Echantillon NIP. (a) Dynamique de spin électronique à $T = 50K$ pour différentes tensions inverses appliquées. Insert : diagramme schématique des vecteurs BIA et SIA dans les puits quantiques $\langle 111 \rangle$; S est le spin d'électron photogénéré à $t=0$; (b) Dépendances des temps de relaxation de spin τ_s^z (cercles), τ_s^y (triangles), τ_s^x (carrés) en fonction du champ électrique appliqué. Insert : dynamique de polarisation circulaire pour $B_x = 0$ et $B_x = 0.26$ T ($V = -3.5$ volts, voir la flèche). La dynamique de polarisation circulaire pour $V=0$ et $B_x = 0$ est également présentée.

spins d'électrons orientés le long de l'axe de croissance $\langle 111 \rangle$. En effet, dans les expériences de pompage optique orienté où la propagation de la lumière excita-

trice se fait le long de l'axe Oz, les spins d'électron photogénérés sont parallèles à cette direction.

Afin de démontrer que le temps de relaxation peut aussi être contrôlé pour des spins orientés dans le plan du puits quantique (Ox ou Oy), nous avons mesuré la dynamique de polarisation circulaire de la photoluminescence en champ magnétique transverse. Après la photogénération, les spins initialement orientés selon l'axe z vont précesser autour du champ magnétique externe. En conséquence, les spins d'électron vont être sensibles à la fois à la relaxation de spin suivant z et à la relaxation de spin dans le plan du puits. L'insert de la figure 4.9(b) présente la dynamique de polarisation circulaire pour $B_x = 0$ et $B_x = 0.26$ T pour une tension inverse appliquée de -3.5 V (le déclin pour $V=0$ est également tracé).

Pour $B_x = 0.26$ T, on observe les oscillations attendues de la polarisation circulaire de la photoluminescence consécutive à la précession de Larmor du spin d'électron autour du champ magnétique B_x . La mesure de la période de précession permet de déterminer le facteur de Landé effectif de l'électron ; nous trouvons $g=0.22$.

Le point clé sur l'insert de la figure 4.9(b) est que le temps de déclin de la polarisation circulaire de la photoluminescence en présence du champ magnétique est quasi identique à celui mesuré sans champ. Des résultats comparables ont été obtenus pour différentes valeurs de champ magnétique dans la gamme 0,26 - 0,7 T. Nous mesurons un temps de relaxation de spin $(1/\tau_s^z + 1/\tau_s^y)/2$ qui conduit à $\tau_s^y = 1530 \pm 300$ ps. En effectuant le même type d'expérience pour un champ magnétique orienté selon l'axe Oy, nous mesurons $\tau_s^x = 1830 \pm 300$ ps.

L'augmentation très claire des temps de relaxation de spin τ_s^x et τ_s^y en fonction du champ électrique observé sur la figure 4.9(b) démontre que la relaxation de spin dans les puits quantiques $\langle 111 \rangle$ peut non seulement être contrôlée pour des spins S_z mais également pour des spins S_x ou S_y . Cette propriété très intéressante est spécifique à cette orientation cristalline et ne peut s'observer sur des puits quantiques élaborés sur des orientations plus standards, comme $\langle 001 \rangle$ ou $\langle 110 \rangle$ [83, 4, 13, 51].

Dépendance de l'anisotropie de la relaxation de spin en fonction du champ électrique

Sur la figure 4.9(b), nous observons que les temps de relaxation de spin dans le plan du puits quantique sont pour les fortes valeurs de champ (en valeur abso-

lue) plus courts que le temps de relaxation de spin le long de l'axe de croissance. Ceci est dû aux termes cubiques de l'Hamiltonien Dresselhaus qui ont jusqu'ici été négligés. En effet, en prenant en compte rigoureusement ces termes, le clivage Spin-Orbite total s'écrit [17, 33, 5] :

$$\Omega = \Omega_{BIA} + \Omega_{SIA} \equiv \begin{cases} \Omega_x(k_{\parallel}) = \frac{1}{\hbar} \left(\frac{\gamma}{\sqrt{3}} \left(2\langle k_z^2 \rangle - \frac{k_{\parallel}^2}{2} \right) + a_6^4 \cdot E \right) k_y \\ \Omega_y(k_{\parallel}) = -\frac{1}{\hbar} \left(\frac{\gamma}{\sqrt{3}} \left(2\langle k_z^2 \rangle - \frac{k_{\parallel}^2}{2} \right) + a_6^4 \cdot E \right) k_x \\ \Omega_z(k_{\parallel}) = \frac{1}{\hbar} \left(\frac{3k_x^2 k_y - k_y^3}{\sqrt{6}} \right) \end{cases}$$

Cette équation montre que pour un vecteur d'onde donné $\mathbf{k}$, la compensation

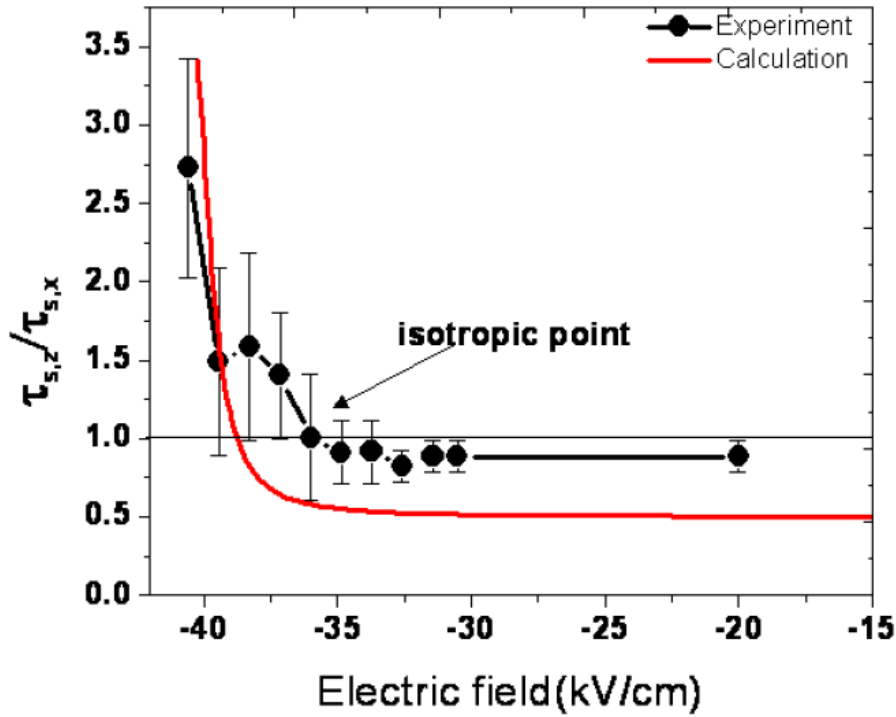


Figure 4.10: Dépendance du rapport : des temps de relaxation de spin longitudinaux et transverse en fonction du champ électrique appliqué

stricte des termes de Rashba et Dresselhaus ne peut se produire que pour les composantes dans le plan Ω_x et Ω_y . Par conséquent, les spins d'électrons orientés le long de l'axe Oz vont avoir des temps de relaxation de spin plus longs que ceux orientés dans le plan car ces derniers vont précesser autour d'un champ effectif non nul perpendiculaire au puits quantique. De plus cette équation montre que la compensation stricte ne peut pas de produite simultanément pour différents

vecteurs d'onde. Quand la température augmente, les électrons vont occuper des états d'énergie de plus grand $\mathbf{k}$ et les termes cubiques de Dresselhaus vont donc jouer un rôle plus important. Nous avons cependant réussi à observer la dépendance des temps de relaxation de spin en fonction du champ électrique jusqu'à une température typique de 150 K.

Finalement, nous avons développé un modèle complet permettant de calculer les temps de relaxation de spin en fonction du champ électrique en prenant en compte à la fois les termes linéaires et les termes cubiques de l'Hamiltonien de Dresselhaus. Ces calculs ont permis de décrire de manière très satisfaisante la dépendance mesurée en fonction du champ électrique de l'anisotropie des temps de relaxation de spin, comme le montre la figure 4.10.

En conclusion, nous soulignons que le contrôle voire la suppression de la relaxation de spin du type D'Yakonov-Perel démontré dans cette thèse pour des puits quantiques GaAs/AlGaAs est également possible pour tout puits quantique III-V ou II-VI de type Blende de Zinc dans la mesure où le phénomène n'est basé que sur des arguments de symétrie. On pourrait imaginer par la suite réduire de manière significative la relaxation de spin dans des puits quantiques contraints (InGaAs/GaAs $\langle 111 \rangle$ par exemple) grâce au contrôle du champ électrique interne piézoélectrique. Ceci ouvrirait sûrement de nouvelles perspectives dans le domaine de la spintronique.

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Appendices

Appendix A

Extended Kane model

The structure of the 14-band Hamiltonian is [70]:

$$H(\mathbf{k}) = \begin{pmatrix} H^{8c8c} & H^{8c7c} & H^{8c6c} & H^{8c8v} & H^{8c7v} \\ H^{7c8c} & H^{7c7c} & H^{7c6c} & H^{7c8v} & H^{7c7v} \\ H^{6c8c} & H^{6c7c} & H^{6c6c} & H^{6c8v} & H^{6c7v} \\ H^{8v8c} & H^{8v7c} & H^{8v6c} & H^{8v8v} & H^{8v7v} \\ H^{7v8c} & H^{7v7c} & H^{7v6c} & H^{7v8v} & H^{7v7v} \end{pmatrix} \quad (\text{A.1})$$

where :

$$H_{8c8c} = \begin{bmatrix} E'_g + \Delta'_0 & 0 & 0 & 0 \\ 0 & E'_g + \Delta'_0 & 0 & 0 \\ 0 & 0 & E'_g + \Delta'_0 & 0 \\ 0 & 0 & 0 & E'_g + \Delta'_0 \end{bmatrix}; H_{8c7v} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$H_{8c6c} = \begin{bmatrix} \frac{1}{\sqrt{2}}iP'k_- & 0 \\ -\sqrt{\frac{2}{3}}iP'k_z & \frac{1}{\sqrt{6}}iP'k_- \\ -\frac{1}{\sqrt{6}}iP'k_+ & -\sqrt{\frac{2}{3}}iP'k_z \\ 0 & \frac{1}{\sqrt{2}}iP'k_+ \end{bmatrix}; H_{8c8v} = \begin{bmatrix} \frac{\Delta^-}{3} & \frac{i}{\sqrt{3}}Qk_+ & \frac{i}{\sqrt{3}}Qk_z & 0 \\ -\frac{i}{\sqrt{3}}Qk_- & \frac{\Delta^-}{3} & 0 & \frac{i}{\sqrt{3}}Qk_z \\ -\frac{i}{\sqrt{3}}Qk_z & 0 & \frac{\Delta^-}{3} & -\frac{i}{\sqrt{3}}Qk_+ \\ 0 & -\frac{i}{\sqrt{3}}Qk_z & \frac{i}{\sqrt{3}}Qk_- & \frac{\Delta^-}{3} \end{bmatrix}$$

$$\begin{aligned}
H_{8c7v} &= \begin{bmatrix} -\frac{1}{\sqrt{6}}iQk_+ & -\sqrt{\frac{2}{3}}iQk_z \\ 0 & \frac{1}{\sqrt{2}}iQk_+ \\ -\frac{1}{\sqrt{2}}iQk_- & 0 \\ -\sqrt{\frac{2}{3}}iQk_z & \frac{1}{\sqrt{6}}iQk_- \end{bmatrix}; H_{7c7v} = \begin{bmatrix} E'_g & 0 \\ 0 & E'_g \end{bmatrix} \\
H_{7c6c} &= \begin{bmatrix} \frac{1}{\sqrt{3}}iP'k_z & \frac{1}{\sqrt{3}}iP'k_- \\ \frac{1}{\sqrt{3}}iP'k_+ & -\frac{1}{\sqrt{3}}iP'k_z \end{bmatrix}; H_{7c8v} = \begin{bmatrix} \frac{1}{\sqrt{6}}iQk_- & 0 & \frac{1}{\sqrt{2}}iQk_+ & \sqrt{\frac{2}{3}}iQk_z \\ \sqrt{\frac{2}{3}}iQk_z & -\frac{1}{\sqrt{2}}iQk_- & 0 & -\frac{1}{\sqrt{6}}iQk_+ \end{bmatrix} \\
H_{7c7v} &= \begin{bmatrix} -\frac{2i\Delta_-}{3} & 0 \\ 0 & -\frac{2i\Delta_-}{3} \end{bmatrix}; H_{6c6c} = \begin{bmatrix} E_g + \frac{\hbar^2 k^2}{2m_0}\gamma'_c & 0 \\ 0 & E_g + \frac{\hbar^2 k^2}{2m_0}\gamma'_c \end{bmatrix} \\
H_{6c8v} &= \begin{bmatrix} -\frac{1}{\sqrt{2}}Pk_+ & \sqrt{\frac{2}{3}}Pk_z & \frac{1}{\sqrt{6}}Pk_- & 0 \\ 0 & -\frac{1}{\sqrt{6}}Pk_+ & \sqrt{\frac{2}{3}}Pk_z & \frac{1}{\sqrt{2}}Pk_- \end{bmatrix}; H_{6c7v} = \begin{bmatrix} -\frac{1}{\sqrt{3}}Pk_z & -\frac{1}{\sqrt{3}}Pk_- \\ -\frac{1}{\sqrt{3}}Pk_+ & \frac{1}{\sqrt{3}}Pk_z \end{bmatrix} \\
H_{8v8v} &= \begin{bmatrix} -\frac{\hbar^2}{2m_0}(\gamma'_1 + \gamma'_2)k_{\parallel}^2 & 2\sqrt{3}\frac{\hbar^2}{2m_0}\gamma'_3k_-k_z & \sqrt{3}\frac{\hbar^2}{2m_0}\gamma'_2\mathbf{K} + Ck_z & -\frac{\sqrt{3}}{2}Ck_- \\ -\frac{\hbar^2}{2m_0}(\gamma'_1 - 2\gamma'_2)k_z^2 & -\frac{1}{2}Ck_+ & -2i\sqrt{3}\frac{\hbar^2}{2m_0}\gamma'_3k_xk_y & -\frac{\sqrt{3}}{2}Ck_- \\ 2\sqrt{3}\frac{\hbar^2}{2m_0}\gamma'_3k_+k_z & -\frac{\hbar^2}{2m_0}(\gamma'_1 - \gamma'_2)k_{\parallel}^2 & \frac{\sqrt{3}}{2}Ck_+ & \sqrt{3}\frac{\hbar^2}{2m_0}\gamma'_2\mathbf{K} - Ck_z \\ -\frac{1}{2}Ck_- & -\frac{\hbar^2}{2m_0}(\gamma'_1 + 2\gamma'_2)k_z^2 & \frac{\sqrt{3}}{2}Ck_+ & -2i\sqrt{3}\frac{\hbar^2}{2m_0}\gamma'_3k_xk_y \\ \sqrt{3}\frac{\hbar^2}{2m_0}\gamma'_2\mathbf{K} + Ck_z & \frac{\sqrt{3}}{2}Ck_- & -\frac{\hbar^2}{2m_0}(\gamma'_1 - \gamma'_2)k_{\parallel}^2 & -2\sqrt{3}\frac{\hbar^2}{2m_0}\gamma'_3k_-k_z \\ 2i\sqrt{3}\frac{\hbar^2}{2m_0}\gamma'_3k_xk_y & \frac{\sqrt{3}}{2}Ck_- & -\frac{\hbar^2}{2m_0}(\gamma'_1 + 2\gamma'_2)k_z^2 & -\frac{1}{2}Ck_+ \\ -\frac{\sqrt{3}}{2}Ck_+ & \sqrt{3}\frac{\hbar^2}{2m_0}\gamma'_2\mathbf{K} - Ck_z & -2\sqrt{3}\frac{\hbar^2}{2m_0}\gamma'_3k_+k_z & -\frac{\hbar^2}{2m_0}(\gamma'_1 + \gamma'_2)k_{\parallel}^2 \\ -\frac{\sqrt{3}}{2}Ck_+ & 2i\sqrt{3}\frac{\hbar^2}{2m_0}\gamma'_3k_xk_y & -\frac{1}{2}Ck_- & -\frac{\hbar^2}{2m_0}(\gamma'_1 - 2\gamma'_2)k_z^2 \end{bmatrix} \\
H_{8v7v} &= \begin{bmatrix} -\sqrt{6}\frac{\hbar^2}{2m_0}\gamma'_3k_-k_z + \frac{1}{2\sqrt{2}}Ck_- & -\sqrt{6}\frac{\hbar^2}{2m_0}\gamma'_2\mathbf{K} + 2i\sqrt{6}\frac{\hbar^2}{2m_0}\gamma'_3k_xk_y + \frac{1}{\sqrt{2}}Ck_z \\ -\sqrt{2}\frac{\hbar^2}{2m_0}\gamma'_2(k_{\parallel}^2 - 2k_z^2) & 3\sqrt{2}\frac{\hbar^2}{2m_0}\gamma'_3k_-k_z - \frac{\sqrt{3}}{2\sqrt{2}}Ck_+ \\ 3\sqrt{2}\frac{\hbar^2}{2m_0}\gamma'_3k_+k_z + \frac{\sqrt{3}}{2\sqrt{2}}Ck_- & \sqrt{2}\frac{\hbar^2}{2m_0}\gamma'_2(k_{\parallel}^2 - 2k_z^2) \\ \sqrt{6}\frac{\hbar^2}{2m_0}\gamma'_2\mathbf{K} + 2i\sqrt{6}\frac{\hbar^2}{2m_0}\gamma'_3k_xk_y + \frac{1}{\sqrt{2}}Ck_z & -\sqrt{6}\frac{\hbar^2}{2m_0}\gamma'_3k_+k_z - \frac{1}{2\sqrt{2}}Ck_- \end{bmatrix} \\
H_{7v7v} &= \begin{bmatrix} -\frac{\hbar^2 k^2}{2m_0}\gamma'_1 - \Delta_0 & 0 \\ 0 & -\frac{\hbar^2 k^2}{2m_0}\gamma'_1 \end{bmatrix}
\end{aligned}$$

in which $E'_g = E_v + E'_0$, $k^2 = k_x^2 + k_y^2 + k_z^2$, $k_{\parallel} = k_x^2 + k_y^2$, $k_{\pm} = k_x \pm ik_y$ and $\mathbf{K} = k_x^2 - k_y^2$. The three modified Luttinger parameters $\gamma'_1, \gamma'_2, \gamma'_3$ account for remote band effects on electron mass in conduction and valence bands. The parameter C is a small coefficient for k-linear term originated from the second order on momentum matrix elements.

Appendix B

Infinite quantum well

Let us consider the one-dimensional motion of a particle in a quantum well grown on z-axis with quantum well width L. For simplicity, the effective mass of particle is assumed to be constant regardless of the different material layers. The one-dimensional Schrödinger equation for this quantum well is :

$$-\frac{\hbar^2}{2m^*} \frac{\partial^2}{\partial z^2} \psi(z) + V(z)\psi(z) = E\psi(z) \quad (\text{B.1})$$

where $V(z)$ is confinement energy of quantum well, m^* is the constant effective mass. For infinite quantum well, $V(z) = 0$ for quantum well layer and $V(z) = \infty$ outside the quantum well. In this case, the boundary conditions are $\psi(0) = \psi(L) = 0$. Thus, the eigenvalues are [47]:

$$E_n = \frac{\hbar^2 k^2}{2m^*} = \frac{\hbar^2 \pi^2 n^2}{2m^* L^2} \quad n = 0, 1, 2, \dots \quad (\text{B.2})$$

with $kL = n\pi$. The wavefunctions are given as :

$$\psi(z) = \begin{cases} \frac{2}{L} \sin(\frac{n\pi}{L} z) & n = 2, 4, 6, \dots \\ \frac{2}{L} \cos(\frac{n\pi}{L} z) & n = 1, 3, 5, \dots \end{cases} \quad (\text{B.3})$$

For the lowest conduction band ($n = 1$) of a symmetric infinite quantum well, we have the system of equation :

$$\begin{aligned}\langle k_z \rangle &= \frac{\langle p_z \rangle}{\hbar} = \frac{\langle \psi_1 | -i\hbar \frac{\partial}{\partial z} | \psi_1 \rangle}{\hbar} \\ &= \frac{-i\pi}{L} \int_{-L/2}^{L/2} \left(\frac{\cos \pi z}{L} \right) \left(\frac{\sin \pi z}{L} \right) dz = 0\end{aligned}\quad (\text{B.4})$$

$$\begin{aligned}\langle k_z^2 \rangle &= \frac{\langle \psi_1 | (-i\hbar \frac{\partial}{\partial z})^2 | \psi_1 \rangle}{\hbar^2} = \left(\frac{\pi}{L} \right)^2 \langle \psi_1 | \psi_1 \rangle \\ &= \left(\frac{\pi}{L} \right)^2\end{aligned}\quad (\text{B.5})$$

where L is quantum well width, ψ_1 is the wavefunction of an electron at the lowest energy level ($n=1$) in quantum well. Equation B.5 is a key ingredient to understand the quantum well width dependence of the Dresselhaus spin splitting discussed in chapter 3 and 4 of this manuscript.

To prove strongly the condition $\langle k_z \rangle = 0$, we use the conjugate operator $\mathbf{p}_z = \hbar \mathbf{k}_z$ which satisfies this condition $\mathbf{p}_z = \mathbf{p}_z^*$ where the subscript $*$ denotes the conjugate operator. We have :

$$\begin{aligned}\langle \psi_n | \mathbf{p}_z | \psi_{n'} \rangle &= \langle \psi_{n'} | \mathbf{p}_z | \psi_n \rangle^* \\ \left\langle \psi_n \left| -i\hbar \frac{\partial}{\partial z} \right| \psi_{n'} \right\rangle &= \left\langle \psi_{n'} \left| -i\hbar \frac{\partial}{\partial z} \right| \psi_n \right\rangle^* \\ -i\hbar \left\langle \psi_n \left| \left(\frac{\partial}{\partial z} \right) \right| \psi_{n'} \right\rangle &= i\hbar \left\langle \psi_{n'} \left| \left(\frac{\partial}{\partial z} \right) \right| \psi_n \right\rangle^*\end{aligned}$$

in which $\psi_n, \psi_{n'}$ is the wavefunctions of electron in the subbands index n and n' , respectively.

When $n = n'$, we can obtain :

$$\left\langle \psi_n \left| \left(\frac{\partial}{\partial z} \right) \right| \psi_n \right\rangle = - \left\langle \psi_n \left| \left(\frac{\partial}{\partial z} \right) \right| \psi_n \right\rangle$$

Therefore:

$$\text{Re} \left(\left\langle \psi_n \left| \left(\frac{\partial}{\partial z} \right) \right| \psi_n \right\rangle \right) = 0. \quad (\text{B.6})$$

Since the Hamiltonian whose characteristic terms are without spin-orbit interaction or external magnetic field in real, we can choose a real basis for the conduction states $\psi_n z$. The relation B.6 becomes $\langle \frac{\partial}{\partial z} \rangle_n = 0$; so that $\langle k_z \rangle = 0$ regardless of the subband index n . This relation is due to the localization along z -axis regardless of the influence of external electric field.

Appendix C

Simple model

C.1 Dresselhaus term in the new basis

The rotation matrix translates the orthogonal coordinate (e_x, e_y, e_z) to the other (e'_x, e'_y, e'_z) is :

$$R = \begin{bmatrix} \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & \frac{-2}{\sqrt{6}} \\ \frac{-1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \end{bmatrix} \quad (\text{C.1})$$

Accompanying with the operation R, all vectors in the new coordinates write (we consider an effective magnetic field as a pseudo-vector):

$$\begin{bmatrix} \Omega_{D'_x} \\ \Omega_{D'_y} \\ \Omega_{D'_z} \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & \frac{-2}{\sqrt{6}} \\ \frac{-1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \end{bmatrix} \begin{bmatrix} \Omega_{D_x} \\ \Omega_{D_y} \\ \Omega_{D_z} \end{bmatrix} \quad (\text{C.2})$$

$$\begin{bmatrix} k'_x \\ k'_y \\ k'_z \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & \frac{-2}{\sqrt{6}} \\ \frac{-1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \end{bmatrix} \begin{bmatrix} k_x \\ k_y \\ k_z \end{bmatrix} \quad (\text{C.3})$$

$$\Rightarrow k = R^{-1}k' \Rightarrow \begin{bmatrix} k_x \\ k_y \\ k_z \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{6}} & \frac{-1}{\sqrt{2}} & \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{3}} \\ \frac{-2}{\sqrt{6}} & 0 & \frac{1}{\sqrt{3}} \end{bmatrix} \begin{bmatrix} k'_x \\ k'_y \\ k'_z \end{bmatrix} \quad (\text{C.4})$$

where $\Omega_{D'_x}, \Omega_{D'_y}, \Omega_{D'_z}$, and k'_x, k'_y, k'_z are Dresselhaus effective magnetic fields and the wavevectors in the new coordinate (e'_x, e'_y, e'_z) , respectively.

From equation C.2 and C.4, we have the system of equations :

$$\Omega_{D'_x} = \frac{1}{\sqrt{6}}\Omega_{D_x} + \frac{1}{\sqrt{6}}\Omega_{D_y} + \frac{-2}{\sqrt{6}}\Omega_{D_z} \quad (\text{C.5})$$

$$\Omega_{D'_y} = \frac{-1}{\sqrt{2}}\Omega_{D_x} + \frac{1}{\sqrt{2}}\Omega_{D_y} \quad (\text{C.6})$$

$$\Omega_{D'_z} = \frac{1}{\sqrt{3}}\Omega_{D_x} + \frac{1}{\sqrt{3}}\Omega_{D_y} + \frac{1}{\sqrt{3}}\Omega_{D_z} \quad (\text{C.7})$$

$$k_x = \frac{1}{\sqrt{6}}k'_x + \frac{-1}{\sqrt{2}}k'_y + \frac{1}{\sqrt{3}}k'_z \quad (\text{C.8})$$

$$k_y = \frac{1}{\sqrt{6}}k'_x + \frac{1}{\sqrt{2}}k'_y + \frac{1}{\sqrt{3}}k'_z \quad (\text{C.9})$$

$$k_z = \frac{-2}{\sqrt{6}}k'_x + \frac{1}{\sqrt{3}}k'_z \quad (\text{C.10})$$

Using equations 4.3 and C.5, C.6, C.7, the equations of Dresselhaus effective magnetic field for [111]-oriented quantum well can be written as :

$$\Omega_{D'_x} = \frac{2\gamma}{\hbar} \left[\frac{1}{\sqrt{6}}k_x(k_y^2 - k_z^2) + \frac{1}{\sqrt{6}}k_y(k_z^2 - k_x^2) + \frac{-2}{\sqrt{6}}k_z(k_x^2 - k_y^2) \right] \quad (\text{C.11})$$

$$\Omega_{D'_y} = \frac{2\gamma}{\hbar} \left[\frac{-1}{\sqrt{2}}k_x(k_y^2 - k_z^2) + \frac{1}{\sqrt{2}}k_y(k_z^2 - k_x^2) \right] \quad (\text{C.12})$$

$$\Omega_{D'_z} = \frac{2\gamma}{\hbar} \left[\frac{1}{\sqrt{3}}k_x(k_y^2 - k_z^2) + \frac{1}{\sqrt{3}}k_y(k_z^2 - k_x^2) + \frac{1}{\sqrt{3}}k_z(k_x^2 - k_y^2) \right] \quad (\text{C.13})$$

Using equations C.8, C.9, C.10, the effective magnetic field on each axis of new coordinate depend on new k-vector (k'_x, k'_y, k'_z) can be calculated :

$$\begin{aligned} \Omega_{D'_x} &= \frac{2\gamma}{\hbar} \left(\frac{1}{\sqrt{6}}[k_x(k_y^2 - k_z^2) + k_y(k_z^2 - k_x^2) - 2k_z(k_x^2 - k_y^2)] \right) \\ &= \frac{2\gamma}{\hbar} \left(\frac{1}{\sqrt{6}}[(k_x k_y^2 - k_y k_x^2) + (k_y k_z^2 - k_x k_z^2) - k_z(k_x - k_y)(2k_x + 2k_y)] \right) \\ &= \frac{2\gamma}{\hbar} \left(\frac{1}{\sqrt{6}}(k_y - k_x)(k_x k_y + k_z^2 + 2k_z k_x + 2k_z k_y) \right) \\ &= \frac{2\gamma}{\hbar} \left(\frac{1}{\sqrt{6}}(k_y - k_x)[k_x k_y + k_z(k_z + 2k_x + 2k_y)] \right) \\ &= \frac{2\gamma}{\hbar} \left(\frac{1}{\sqrt{6}}\sqrt{2}k'_y \left[\frac{-1}{2}k_y'^2 + \left(\frac{1}{\sqrt{6}}k'_x + \frac{1}{\sqrt{3}}k'_z \right)^2 + \left(\frac{-2}{\sqrt{6}}k'_x + \frac{1}{\sqrt{3}}k'_z \right) \left(\frac{2}{\sqrt{6}}k'_x + \frac{5}{\sqrt{3}}k'_z \right) \right] \right) \\ &= \frac{2\gamma}{\hbar} \left(\frac{1}{\sqrt{3}}k'_y \left(2k_z'^2 - \frac{1}{2}k_x'^2 - \frac{1}{2}k_y'^2 - \sqrt{2}k'_x k_y \right) \right) \\ &= \frac{2\gamma}{\hbar} \left[\frac{1}{\sqrt{3}}k'_y \left(2k_z'^2 - \frac{1}{2}k_x'^2 - \frac{1}{2}k_y'^2 \right) - \sqrt{\frac{2}{3}}k'_x k'_y k'_z \right] \end{aligned} \quad (\text{C.14})$$

$$\begin{aligned}
\Omega_{D'_y} &= \frac{2\gamma}{\hbar} \left[\frac{1}{\sqrt{2}} k_y (k_z^2 - k_x^2) - \frac{1}{\sqrt{2}} k_z (k_x^2 - k_y^2) \right] \\
&= \frac{2\gamma}{\hbar} \left[\frac{1}{\sqrt{2}} (k_y k_z^2 - k_y k_x^2 - k_x k_y^2 + k_x k_z^2) \right] \\
&= \frac{2\gamma}{\hbar} \left[\frac{1}{\sqrt{2}} (k_y + k_x) (k_z^2 - k_x k_y) \right] \\
&= \frac{2\gamma}{\hbar} \left[\frac{1}{\sqrt{2}} \left(\frac{2}{\sqrt{6}} k'_x + \frac{2}{\sqrt{3}} k'_z \right) \left[\left(\frac{1}{\sqrt{3}} k'_z - \frac{2}{\sqrt{6}} k'_x \right)^2 - \left(\frac{1}{\sqrt{6}} k'_x + \frac{1}{\sqrt{3}} k'_z \right)^2 + \frac{1}{2} k_y'^2 \right] \right] \\
&= \frac{2\gamma}{\hbar} \left[\frac{1}{2\sqrt{3}} k'_x (k_x'^2 + k_y'^2) - \frac{1}{\sqrt{6}} k'_z (k_x'^2 + 2\sqrt{2} k'_x k'_z - k_y'^2) \right] \tag{C.15}
\end{aligned}$$

$$\begin{aligned}
\Omega_{D'_z} &= \frac{2\gamma_c}{\hbar} \left[\frac{1}{\sqrt{3}} k_x (k_y^2 - k_z^2) + \frac{1}{\sqrt{3}} k_y (k_z^2 - k_x^2) + \frac{1}{\sqrt{3}} k_z (k_x^2 - k_y^2) \right] \\
&= \frac{2\gamma}{\hbar} \left[\frac{1}{\sqrt{3}} [k_x k_y^2 - k_x k_z^2 + k_y k_z^2 - k_y k_x^2 + k_z (k_x - k_y) (k_x + k_y)] \right] \\
&= \frac{2\gamma}{\hbar} \left[\frac{1}{\sqrt{3}} [k_x k_y (k_y - k_x) + k_z^2 (k_y - k_x) - (k_y - k_x) (k_z k_x + k_z k_y)] \right] \\
&= \frac{2\gamma}{\hbar} \left[\frac{1}{\sqrt{3}} (k_y - k_x) (k_x k_y + k_z^2 - k_z k_x - k_z k_y) \right] \\
&= \frac{2\gamma}{\hbar} \left[\frac{1}{\sqrt{3}} (k_y - k_x) [k_x (k_y - k_z) + k_z (k_z - k_y)] \right] \\
&= \frac{2\gamma}{\hbar} \left[\frac{1}{\sqrt{3}} (k_y - k_x) (k_y - k_z) (k_x - k_z) \right] \\
&= \frac{2\gamma}{\hbar} \left[\frac{1}{\sqrt{3}} \sqrt{2} k'_y \left(\frac{3}{\sqrt{6}} k'_x - \frac{1}{\sqrt{2}} k'_y \right) \left(\frac{3}{\sqrt{6}} k'_x + \frac{1}{\sqrt{2}} k'_y \right) \right] \\
&= \frac{2\gamma}{\hbar} \left[\sqrt{\frac{2}{3}} k'_y \left(\frac{3}{2} k_x'^2 - \frac{1}{2} k_y'^2 \right) \right] \\
&= \frac{2\gamma}{\hbar} \left[\frac{1}{\sqrt{6}} k'_y (3k_x'^2 - k_y'^2) \right] \tag{C.16}
\end{aligned}$$

C.2 Effective magnetic field calculation

The x-component of the effective magnetic field taken from equation 4.18 is :

$$\Omega_x = \frac{2}{\hbar} (\alpha_T - \frac{\gamma}{2\sqrt{3}} k_\perp'^2) k'_y = \frac{2}{\hbar} (\alpha_T - \frac{\gamma}{2\sqrt{3}} k_\perp'^2) k'_\perp \cos(\theta) \tag{C.17}$$

Because the surface of constant energy of electron in quantum well is a circle around the point (0,0) in the plane (e'_x, e'_y) [?], the energy distribution in x' - y' plane is isotropic with θ . Therefore, we can neglect an energy distribution

function in x'-y' plane. The angular average of Ω_x^2 over θ is obtained as :

$$\begin{aligned}
\langle \overline{\Omega_x^2} \rangle_\theta &= \frac{\int_0^{2\pi} \langle \Omega_x^2 \rangle d\theta}{\int_0^{2\pi} d\theta} = \frac{4}{\hbar^2} \frac{\int_0^{2\pi} (\alpha_T - \frac{\gamma}{2\sqrt{3}} k'_\perp{}^2)^2 k'_\perp{}^2 \cos^2 \theta d\theta}{\int_0^{2\pi} d\theta} \\
&= \frac{4}{\hbar^2} (\alpha_T - \frac{\gamma}{2\sqrt{3}} k'_\perp{}^2)^2 k'_\perp{}^2 \frac{\int_0^{2\pi} (1 + \cos(2\theta)) d\theta}{2 \int_0^{2\pi} d\theta} = \frac{2}{\hbar^2} (\alpha_T - \frac{\gamma}{2\sqrt{3}} k'_\perp{}^2)^2 k'_\perp{}^2 \\
&= \frac{2}{\hbar^2} (\alpha_T - \frac{\gamma}{2\sqrt{3}} \cdot \frac{2m_c^* E_\perp}{\hbar^2})^2 \frac{2m_c^* E_\perp}{\hbar^2} \\
&= \frac{2}{\hbar^2} [\frac{2m_c^* E_\perp}{\hbar^2} \alpha_T^2 + \frac{\gamma^2}{12} (\frac{2m_c^* E_\perp}{\hbar^2})^3 - 2 \frac{\alpha_T \gamma}{2\sqrt{3}} (\frac{2m_c^* E_\perp}{\hbar^2})^2] \quad (C.18)
\end{aligned}$$

The mean square of Ω_x^2 in quantum wells is given as :

$$\langle \overline{\Omega_x^2} \rangle = \frac{\int_0^\infty \langle \overline{\Omega_x^2} \rangle_\theta \frac{m_c^*}{\pi \hbar^2} e^{-\frac{E_\perp}{k_B T}} dE_\perp}{\int_0^\infty \frac{m_c^*}{k_B T} e^{-\frac{E_\perp}{k_B T}} dE_\perp} \quad (C.19)$$

Replacing $x = \frac{E_\perp}{k_B T}$ in equation C.18 and equation C.19, we have :

$$\begin{aligned}
\langle \overline{\Omega_x^2} \rangle &= \frac{2}{\hbar^2} \frac{\frac{2m_c^* \alpha_T^2}{\hbar^2} (k_B T) \int_0^\infty x e^{-x} dx + \frac{\gamma^2}{12} (\frac{2m_c^*}{\hbar^2})^3 (k_B T)^3 \int_0^\infty x^3 e^{-x} dx}{\int_0^\infty e^{-x} dx} - \\
&\quad - \frac{2}{\hbar^2} \frac{2 \frac{\alpha_T \gamma}{2\sqrt{3}} (\frac{2m_c^*}{\hbar^2})^2 (k_B T)^2 \int_0^\infty x^2 e^{-x} dx}{\int_0^\infty e^{-x} dx} \\
&= \frac{2}{\hbar^2} [\frac{2m_c^* k_B T}{\hbar^2} \alpha_T^2 \times 1 + \frac{\gamma^2}{12} (\frac{2m_c^* k_B T}{\hbar^2})^3 \times 6 - 2 \frac{\alpha_T \gamma}{2\sqrt{3}} (\frac{2m_c^* k_B T}{\hbar^2})^2 \times 2] \\
&= \frac{2}{\hbar^2} [\frac{2m_c^* k_B T \alpha_T^2}{\hbar^2} + \frac{\gamma^2}{2} (\frac{2m_c^* k_B T}{\hbar^2})^3 - \frac{2\alpha_T \gamma}{\sqrt{3}} (\frac{2m_c^* k_B T}{\hbar^2})^2] \quad (C.20)
\end{aligned}$$

The y-component of the effective magnetic field is :

$$\Omega_x = -\frac{2}{\hbar} (\alpha_T - \frac{\gamma}{2\sqrt{3}} k'_\perp{}^2) k'_x = -\frac{2}{\hbar} (\alpha_T - \frac{\gamma}{2\sqrt{3}} k'_\perp{}^2) k'_\perp \sin(\theta) \quad (C.21)$$

The angular average of Ω_y over θ is :

$$\begin{aligned}
 \langle \overline{\Omega_y^2} \rangle_\theta &= \frac{\int_0^{2\pi} \langle \Omega_y^2 \rangle d\theta}{\int_0^{2\pi} d\theta} = \frac{4}{\hbar^2} \frac{\int_0^{2\pi} (\alpha_T - \frac{\gamma}{2\sqrt{3}} k'_\perp{}^2) k'_\perp{}^2 \sin^2 \theta d\theta}{\int_0^{2\pi} d\theta} \\
 &= \frac{4}{\hbar^2} (\alpha_T - \frac{\gamma}{2\sqrt{3}} k'_\perp{}^2) k'_\perp{}^2 \frac{\int_0^{2\pi} (1 - \cos(2\theta)) d\theta}{2 \int_0^{2\pi} d\theta} = \frac{2}{\hbar^2} (\alpha_T - \frac{\gamma}{2\sqrt{3}} k'_\perp{}^2) k'_\perp{}^2 = \langle \overline{\Omega_x^2} \rangle_\theta
 \end{aligned} \tag{C.22}$$

$$\Rightarrow \langle \overline{\Omega_y^2} \rangle = \langle \overline{\Omega_x^2} \rangle.$$

The z-component of the effective magnetic field is :

$$\Omega_z = \frac{2\gamma}{\sqrt{6}\hbar} k'_y (3k'_x{}^2 - k'_y{}^2) = \frac{2\gamma}{\sqrt{6}\hbar} |k'_\perp|^3 \cos \theta (3 \sin^2 \theta - \cos^2 \theta) = -\frac{2\gamma}{\sqrt{6}\hbar} |k'_\perp|^3 \cos(3\theta) \tag{C.23}$$

The angular average of Ω_z^2 is :

$$\begin{aligned}
 \langle \overline{\Omega_z^2} \rangle_\theta &= \frac{\int_0^{2\pi} |\Omega_z^2|_\theta d\theta}{\int_0^{2\pi} d\theta} = \frac{\gamma^2}{3\pi\hbar^2} |k'_\perp|^6 \int_0^{2\pi} \cos^2(3\theta) d\theta \\
 &= \frac{\gamma^2}{3\pi\hbar^2} |k'_\perp|^6 \int_0^{2\pi} \left(\frac{1 + \cos 6\theta}{2} \right) d\theta \\
 &= \frac{\gamma_c^2}{3\pi\hbar^2} |k'_\perp|^6 \times \pi = \frac{\gamma^2 |k'_\perp|^6}{3\hbar^2} \\
 \Rightarrow \langle \overline{\Omega_z^2} \rangle &= \frac{\int_0^\infty |\Omega_z^2|_{(E_\perp)} \frac{m_c^*}{\pi\hbar^2} e^{\frac{-E_\perp}{k_B T}} dE_\perp}{\int_0^\infty \frac{m_c^*}{\pi\hbar^2} e^{\frac{-E_\perp}{k_B T}} dE_\perp} = \frac{\gamma^2}{3\hbar^2} \left(\frac{2m_c^*}{\hbar^2} \right)^3 \frac{\int_0^\infty E_\perp^3 e^{\frac{-E_\perp}{k_B T}} dE_\perp}{\int_0^\infty e^{\frac{-E_\perp}{k_B T}} dE_\perp} \\
 &= \frac{\gamma^2}{3\hbar^2} \left(\frac{2m_c^*}{\hbar^2} \right)^3 \times 6(k_B T)^3 = \frac{2\gamma^2}{\hbar^2} \left(\frac{2m_c^* k_B T}{\hbar^2} \right)^3 \tag{C.24}
 \end{aligned}$$

Appendix D

Spin density matrix model

D.1 Fourier transform of Hamiltonian

Using the raising and lowering spin operator $\sigma_{\pm}$ for equation 4.45, we simplify the first and third order of Hamiltonian as follow :

$$\begin{aligned} H^{(1)} &= \alpha_T(k'_y\sigma_x - k'_x\sigma_y) = \alpha_T |k'_{\perp}| (\sin(\varphi_k)\sigma_x - \cos(\varphi_k)\sigma_y) \\ &= \alpha_T |k'_{\perp}| \left(\frac{e^{i\varphi_k} - e^{-i\varphi_k}}{2i} \sigma_x - \frac{e^{i\varphi_k} + e^{-i\varphi_k}}{2} \sigma_y \right) \\ &= -\frac{i}{2} \alpha_T |k'_{\perp}| [(e^{i\varphi_k} - e^{-i\varphi_k})\sigma_x - i\sigma_y(e^{i\varphi_k} + e^{-i\varphi_k})] \\ &= -\frac{i}{2} \alpha_T |k'_{\perp}| [e^{i\varphi_k}(\sigma_x - i\sigma_y) - e^{-i\varphi_k}(\sigma_x + i\sigma_y)] \\ &= -\frac{i}{2} \alpha_T |k'_{\perp}| (\sigma_- e^{i\varphi_k} - \sigma_+ e^{-i\varphi_k}) \end{aligned} \tag{D.1}$$

$$\begin{aligned} H^{(3)} &= \gamma_c \left[\frac{-1}{2\sqrt{3}} k'_y (k_x'^2 + k_y'^2) \sigma_x + \frac{1}{2\sqrt{3}} k'_x (k_x'^2 + k_y'^2) \sigma_y + \frac{1}{\sqrt{6}} k'_y (3k_x'^2 - k_y'^2) \sigma_z \right] \\ &= \frac{\gamma_c |k'_{\perp}|^3}{2\sqrt{3}} [\cos(\varphi_k)\sigma_y - \sin(\varphi_k)\sigma_x] + \frac{\gamma_c |k'_{\perp}|^3}{\sqrt{6}} \sin(\varphi_k) (3\cos^2(\varphi_k) - \sin^2(\varphi_k)) \sigma_z \end{aligned} \tag{D.2}$$

We have :

$$\begin{aligned} \sin(\varphi_k)(3\cos^2(\varphi_k) - \sin^2(\varphi_k)) &= \sin(\varphi_k)(2\cos^2(\varphi_k) + \cos(2\varphi_k)) \\ &= 2\sin(\varphi_k)\cos(\varphi_k)\cos(\varphi_k) + \sin(\varphi_k)\cos(2\varphi_k) \\ &= \sin(2\varphi_k)\cos(\varphi_k) + \sin(\varphi_k)\cos(2\varphi_k) \\ &= \sin(3\varphi_k) \end{aligned}$$

Equation D.2 becomes :

$$\begin{aligned}
H^{(3)} &= \frac{\gamma_c |k'_\perp|^3}{2\sqrt{3}} \left[\frac{e^{i\varphi_k} + e^{-i\varphi_k}}{2} \sigma_y - \frac{e^{i\varphi_k} - e^{-i\varphi_k}}{2i} \sigma_x \right] + \frac{\gamma_c |k'_\perp|^3}{\sqrt{6}} \frac{e^{3i\varphi_k} - e^{-3i\varphi_k}}{2i} \sigma_z \\
&= \frac{i\gamma_c |k'_\perp|^3}{4\sqrt{3}} [e^{i\varphi_k} \sigma_- - e^{-i\varphi_k} \sigma_+] + \frac{\gamma_c |k'_\perp|^3}{2i\sqrt{6}} (e^{3i\varphi_k} - e^{-3i\varphi_k}) \sigma_z \quad (D.3)
\end{aligned}$$

From equation D.1 we can see obviously that the first order of Hamiltonian $H^{(1)}$ only has the first order Fourier transform ($n=1$ or -1). Actually, following equation 4.41 we have :

$$\begin{aligned}
H_{(-1)}^{(1)} &= \int_0^{2\pi} H^{(1)} e^{i\varphi_k} \frac{d\varphi_k}{2\pi} = - \int_0^{2\pi} \frac{i}{2} \alpha_T |k'_\perp| (\sigma_- e^{i\varphi_k} - \sigma_+ e^{-i\varphi_k}) \frac{d\varphi_k}{2\pi} \\
&= -\frac{i}{4\pi} \alpha_T |k'_\perp| \sigma_- \int_0^{2\pi} e^{2i\varphi_k} d\varphi_k + \frac{i}{4\pi} \alpha_T |k'_\perp| \sigma_+ \int_0^{2\pi} d\varphi_k \\
&= -\frac{i}{4\pi} \alpha_T |k'_\perp| \sigma_- \frac{e^{2i\varphi_k}}{2i} \Big|_0^{2\pi} + \frac{i}{4\pi} \alpha_T |k'_\perp| \sigma_+ \varphi_k \Big|_0^{2\pi} \\
&= \frac{i}{2} \alpha_T |k'_\perp| \sigma_+ \quad (D.4a)
\end{aligned}$$

$$\begin{aligned}
H_{(1)}^{(1)} &= \int_0^{2\pi} H^{(1)} e^{-i\varphi_k} \frac{d\varphi_k}{2\pi} = - \int_0^{2\pi} \frac{i}{2} \alpha_T |k'_\perp| (\sigma_- - \sigma_+ e^{-2i\varphi_k}) \frac{d\varphi_k}{2\pi} \\
&= -\frac{i}{4\pi} \alpha_T |k'_\perp| \sigma_- \int_0^{2\pi} d\varphi_k + \frac{i}{4\pi} \alpha_T |k'_\perp| \sigma_+ \int_0^{2\pi} e^{-2i\varphi_k} d\varphi_k \\
&= -\frac{i}{2} \alpha_T |k'_\perp| \sigma_- \quad (D.4b)
\end{aligned}$$

In the similar way, the third order of Hamiltonian $H^{(3)}$ in equation D.3 has the first and third order of Fourier transform ($n = \pm 1, \pm 3$). We have :

$$H_{(1)}^{(3)} = \int_0^{2\pi} H^{(3)} e^{-i\varphi_k} \frac{d\varphi_k}{2\pi} = \frac{i |k'_\perp|^3}{4\sqrt{3}} \sigma_- \quad (D.5a)$$

$$H_{(-1)}^{(3)} = \int_0^{2\pi} H^{(3)} e^{i\varphi_k} \frac{d\varphi_k}{2\pi} = -\frac{i |k'_\perp|^3}{4\sqrt{3}} \sigma_+ \quad (D.5b)$$

$$H_{(3)}^{(3)} = \int_0^{2\pi} H^{(3)} e^{-3i\varphi_k} \frac{d\varphi_k}{2\pi} = \frac{|k'_\perp|^3}{2i\sqrt{6}} \sigma_z \quad (D.5c)$$

$$H_{(-3)}^{(3)} = \int_0^{2\pi} H^{(3)} e^{3i\varphi_k} \frac{d\varphi_k}{2\pi} = -\frac{|k'_\perp|^3}{2i\sqrt{6}} \sigma_z \quad (D.5d)$$

D.2 Spin relaxation tensor

We recall to the Pauli spin matrices :

$$\begin{aligned}
\sigma_x &= \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}; \sigma_y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}; \sigma_z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \\
\sigma_+ &= \begin{bmatrix} 0 & 2 \\ 0 & 0 \end{bmatrix}; \sigma_- = \begin{bmatrix} 0 & 0 \\ 2 & 0 \end{bmatrix}
\end{aligned}$$

and several useful equations :

$$\sigma_x^2 = \sigma_y^2 = \sigma_z^2 = I; [\sigma_a, \sigma_b] = 2i \sum_c \epsilon_{abc} \sigma_c$$

where the square bracket denotes comutation operator; $\mathbf{I}$ is the 2x2 identity matrix; a,b,c denote for x,y,z axes; ϵ_{abc} is *Levi-Civita* symbol.

Following equation 4.49, we calculate the spin relaxation tensor as follow :

$$\begin{aligned} \Gamma_{xx} &= \frac{1}{\hbar^2} Tr \left(\sum_{n=0}^{\infty} \tau_n [H'_{-n}, [H'_n, \sigma_x]] \sigma_x \right) \\ &= \frac{1}{\hbar^2} Tr (\tau_1 [H'_{(-1)}, [H'_{(1)}, \sigma_x]] \sigma_x + \tau_3 [H'_{(3)}, [H'_{(3)}, \sigma_x]] \sigma_x) \\ &= \frac{1}{\hbar^2} \left(\frac{1}{4} (\alpha_T |k'_\perp| - \frac{\gamma_c |k'_\perp|^3}{2\sqrt{3}})^2 \tau_1 Tr([\sigma_+, [\sigma_-, \sigma_x]] \sigma_x) + \frac{|k'_\perp|^6}{24} \tau_3 Tr([\sigma_z, [\sigma_z, \sigma_x]] \sigma_x) \right) \end{aligned}$$

Using matrix commutator, we have :

$$\begin{aligned} [\sigma_-, \sigma_x] &= \begin{bmatrix} 0 & 0 \\ 2 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} -2 & 0 \\ 0 & 2 \end{bmatrix} \\ [\sigma_+, [\sigma_-, \sigma_x]] &= \begin{bmatrix} 0 & 2 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} -2 & 0 \\ 0 & 2 \end{bmatrix} - \begin{bmatrix} -2 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 0 & 2 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 8 \\ 0 & 0 \end{bmatrix} \\ [\sigma_+, [\sigma_-, \sigma_x]] \sigma_x &= \begin{bmatrix} 0 & 8 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 8 & 0 \\ 0 & 0 \end{bmatrix} \\ Tr([\sigma_+, [\sigma_-, \sigma_x]] \sigma_x) &= 8 \end{aligned}$$

$$\begin{aligned} [\sigma_z, \sigma_x] &= \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix} \\ [\sigma_z, [\sigma_z, \sigma_x]] &= \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix} - \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 4 \\ 4 & 0 \end{bmatrix} \\ [\sigma_z, [\sigma_z, \sigma_x]] \sigma_x &= \begin{bmatrix} 0 & 4 \\ 4 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix} \\ Tr([\sigma_z, [\sigma_z, \sigma_x]] \sigma_x) &= 8 \end{aligned}$$

The spin relaxation rate Γ_{xx} is obtained as :

$$\begin{aligned} \Gamma_{xx} &= \frac{2\tau_1}{\hbar^2} (\alpha_T |k'_\perp| - \frac{\gamma_c |k'_\perp|^3}{2\sqrt{3}})^2 + \frac{\tau_3}{3\hbar^2} |k'_\perp|^6 \\ &= \frac{2\tau_1}{\hbar^2} [(\alpha_T |k'_\perp| - \frac{\gamma_c |k'_\perp|^3}{2\sqrt{3}})^2 + \frac{\tau_3}{6\tau_1} \gamma_c^2 |k'_\perp|^6] \end{aligned} \quad (D.6)$$

Using the same method for spin relaxation rate Γ_{xy} , we have :

$$\begin{aligned}
\Gamma_{xy} &= \frac{1}{\hbar^2} Tr \left(\sum_{n=0}^{\infty} \tau_n [H'_{-n}, [H'_n, \sigma_y]] \sigma_x \right) \\
&= \frac{1}{\hbar^2} Tr (\tau_1 [H'_{(-1)}, [H'_{(1)}, \sigma_y]] \sigma_x + \tau_3 [H'_{(3)}, [H'_{(3)}, \sigma_y]] \sigma_x) \\
&= \frac{1}{\hbar^2} \left(\frac{1}{4} (\alpha_T |k'_\perp| - \frac{\gamma_c |k'_\perp|^3}{2\sqrt{3}})^2 \tau_1 Tr([\sigma_+, [\sigma_-, \sigma_y]] \sigma_x) + \frac{|k'_\perp|^6}{24} \tau_3 Tr([\sigma_z, [\sigma_z, \sigma_y]] \sigma_x) \right)
\end{aligned}$$

where :

$$\begin{aligned}
[\sigma_-, \sigma_y] &= \begin{bmatrix} 0 & 0 \\ 2 & 0 \end{bmatrix} \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} - \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} 2i & 0 \\ 0 & -2i \end{bmatrix} \\
[\sigma_+, [\sigma_-, \sigma_y]] &= \begin{bmatrix} 0 & 2 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 2i & 0 \\ 0 & -2i \end{bmatrix} - \begin{bmatrix} 2i & 0 \\ 0 & -2i \end{bmatrix} \begin{bmatrix} 0 & 2 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & -8i \\ 0 & 0 \end{bmatrix} \\
[\sigma_+, [\sigma_-, \sigma_y]] \sigma_x &= \begin{bmatrix} 0 & -8i \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} -8i & 0 \\ 0 & 0 \end{bmatrix} \\
&\rightarrow Re[Tr([\sigma_+, [\sigma_-, \sigma_y]] \sigma_x)] = 0
\end{aligned}$$

$$\begin{aligned}
[\sigma_z, \sigma_y] &= \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} - \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 0 & -2i \\ -2i & 0 \end{bmatrix} \\
[\sigma_z, [\sigma_z, \sigma_y]] &= \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 0 & -2i \\ -2i & 0 \end{bmatrix} - \begin{bmatrix} 0 & -2i \\ -2i & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 0 & -4i \\ 4i & 0 \end{bmatrix} \\
[\sigma_z, [\sigma_z, \sigma_y]] \sigma_x &= \begin{bmatrix} 0 & -4i \\ 4i & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} -4i & 0 \\ 0 & 4i \end{bmatrix} \\
&\rightarrow Tr([\sigma_z, [\sigma_z, \sigma_y]] \sigma_x) = 0
\end{aligned}$$

Therefore, the spin relaxation rate $\Gamma_{xy} = 0$. From equation 4.44, we can realize that this spin relaxation tensor is symmetric. Thus, we also obtain $\Gamma_{yx} = 0$. Calculating in the same way for other spin relaxation rates, we'll find that :

$$\Gamma_{xz} = \Gamma_{zx} = 0 \quad (D.7)$$

$$\Gamma_{yy} = \Gamma_{xx} = \frac{2\tau_1}{\hbar^2} \left[(\alpha_T |k'_\perp| - \frac{\gamma_c |k'_\perp|^3}{2\sqrt{3}})^2 + \frac{\tau_3}{6\tau_1} \gamma_c^2 |k'_\perp|^6 \right] \quad (D.8)$$

$$\Gamma_{zz} = \frac{4\tau_1}{\hbar^2} \left(\alpha_T |k'_\perp| - \frac{\gamma_c |k'_\perp|^3}{2\sqrt{3}} \right)^2 \quad (D.9)$$

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