Three Essays on the Economic Analysis of Marketing Practices on the Internet

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1. INTRODUCTION

A common observation is that price dispersion exists for homogeneous products. In Stigler (1961)'s example, the average asking price of an identical model of Chevrolet (in 1959) is 2436 dollars, ranging from 2350 to 2515, with a standard deviation of 42 dollars. The same model of Asus laptop is asked 349 euros in an on-line store, 399 euros in an electronic store and 499 euros in Auchan. These price variations are not coincidences. Other examples are food, clothing and airline tickets. A well-known fact is that informational structure (different consumers know different things about the market) is one cause of price dispersion. The 349 euros is targeted at internet shoppers, the 399 euros is targeted at consumers knowing electronic products, and the 499 euros is targeted mainly at consumers who do not know much about the market.

Consumers can learn the products through two channels, by searching the products and by directly receiving product information.

Consumers can search products in a sequential way or in a parallel way. If he searches in a sequential way, each time of search reveals information about one piece of product to the consumer. If he searches in a parallel way, the consumer commits a priori to search a number of firms and picks up the one maximizing his utility. On the other hand, consumers can search either in a random way or in an ordered way. By random search, each consumer has his own searching path, which is independent from others. By ordered search, all consumers search in the same path.
Products must be heterogeneous in certain dimensions; otherwise the outcome is predicted by [Diamond (1971)](known as the diamond paradox). In some markets (train tickets, stocks), products are homogeneous. Firms must employ mixed strategies in prices in order to motivate consumers to search. When products are differentiated, consumers search for lower prices, higher quality and better match.

Meanwhile, consumers are heterogeneous in their knowledge about the market. Some consumers have zero search cost and are labelled as shoppers. Other consumers are labeled as global searchers or local searchers. Recent attempts to estimate consumer search cost ([Hong and Shum (2006)], [Los Santos (2008)] and [Moraga-Gonzalez and Wildenbeest (2008)]) use structural methodologies to show that search cost is actually quite high (for instance, around 29 dollars for a textbook of around 65 dollars) and varies among the searched products and different education levels. [Wilson (2012)] finds a low search cost but a high switching cost: search cost is estimated to be around 0.01-0.3% of the maximum gain from search, while switching cost is estimated to be around 48-73%.

Some models allow the search cost to be chosen by firms, named obfuscation. Most search models assume free recall, meaning that once the product is searched, the consumer can pick it up at any time at no cost.

Consumers can also learn the products through the mass media or other consumers, either immediately or through a learning process. Prices can serve as incentives for better dissemination of product information. Group-buying, a recent shopping strategy originated in Asia is such an example. Consumers on the internet form groups in order to bargain for lower prices. Since individual consumer typically does not need multiples of one item or do not have the resources to buy in bulk, group buys allow (encourage) people to invite others to purchase in bulk jointly.
The rest of this chapter is organized as the following: section two elaborates certain theoretical models related to the thesis. Section three discusses contribution of the thesis to the search literatures.

1.1 Review on Theoretical Models

Search models date back to [Diamond (1971)] which predicts the market price to be the monopoly price if both consumers and firms are homogeneous. No consumers search under his context, which is far from real-world market. [Salop and Stiglitz (1977) and Varian (1980)] show that the diamond paradox can be avoided by assuming two types of consumers: the informed and the uninformed. As a result, there is price dispersion; some firms sell to informed consumers, others sell to uninformed consumers. [Burdett and Judd (1983)] constructs a parallel search model, where search cost is the same for every consumer. [Stahl (1989)] bridges the gap between the Diamond and Bertrand. In his model, consumers either have zero search cost, or positive search cost, and they conduct sequential search. In [Chen and Zhang (2011)], there are informed consumers, global searchers and local searchers, where global searchers conduct sequential search and local searchers search only once. [Janssen et al. (2009)] looks into the case where consumers can not observe firms’ marginal cost. [Janssen and Parakhonyak (2011)] analyses the sequential search model with costly recall.

Consumers search products in the hope of getting lower prices. When consumers search sequentially, they observe the price dispersion, and decide at which point to stop. Behind these decisions, they compare the expected incremental surplus from searching once more with search cost. If the expected incremental surplus is smaller, the optimal choice is to stop and make a
purchase. If the expected incremental surplus is larger than search cost, the optimal choice is to search the next firm. Consumers with low search cost on average search more firms than consumers with high search cost, that is to say that, their demand is more elastic. When firms sell products to consumers with varying search cost, it is as if they sell products to consumers with varying price elasticity of demand.

The optimal searching process takes into account the expected price dispersion. The price dispersion in equilibrium takes into account of consumers’ optimal searching process. One main result of these papers is that, prices increase in search cost. The more difficult to obtain product information, consumers tend to search less and each firm has larger local monopoly power.

The equilibrium price distribution in Stahl (1989) (and models based on that) has a larger probability mass in the high prices if the number of firms in the market increases. The reason is that as the number of firms increases, the probability of being the lowest-price firm observed by consumers decreases exponentially, which weakens the firm’s incentive to lower its price.

When the products are horizontally differentiated, consumers search not only for lower prices, but also for better match. Perloff and Salop (1985)’s spatial model shows that the equilibrium price is bounded away from marginal cost even if search cost is zero. Wolinsky (1986) obtains a sequential search model where consumers search for horizontally differentiated products. Anderson and Renault (1999) studies the effects of product diversity on prices. In Anderson and Renault (1999), the optimal number of firms for the society decreases in search cost, while in a free-entry equilibrium, the number of firms increases in search cost. Therefore, there is over-entry of firms. Zhou (2012) looks into the case where consumers search for several products. Armstrong and Zhou (2011) analyses the “buy-now discount”. Moraga-Gonzalez and Petrikaite (2013) discusses the incentive and welfare implications of horizontal mergers.
In Armstrong et al. (2009), one firm is prominent in the sense that each consumer will visit the prominent firm first. In Zhou (2011), each consumer searches a line of firms in the same ordering (as in Arbatskaya (2007)). Contrary to Arbatskaya (2007), Armstrong et al. (2009) and Zhou (2011) find that the firms searched by consumers first usually charge lower prices than rest of the firms. The conflicting results lie in the fact that consumers search only for lower prices in Arbatskaya (2007), but both prices and match in the context of Armstrong et al. (2009) and Zhou (2011). In Arbatskaya (2007), subsequent firms can use only lower prices to attract consumers. However, behind the price difference in Armstrong et al. (2009) and Zhou (2011), there are the prominent firms who lower the prices in order to prevent consumers from further search, and the subsequent firms, who infer from consumers’ optimal search rule that their visitors are those who do not obtain high surplus from the products of the prominent firms.

Three factors can be endogenized. First, even though firms are horizontally differentiated, firm can choose their extent of differentiation before choosing prices. Johnson and Myatt (2006) find that firms choose extreme designs. Larson (2011) demonstrates that search cost is one factor affecting firms’ decisions about differentiation. When search cost is low enough, firms maximize their product differentiation. When search cost is quite large, firms minimize their product differentiation. Bar-Isaac et al. (2012) finds that a larger fraction of niche firms arises as search cost drops.

Second, firms can choose the cost for consumers to obtain product information. Wilson (2010) shows that there is no equilibrium of full transparency. If one firm chooses obfuscation and the other one chooses no obfuscation, consumers are directed to the one without obfuscation, who raises the price. Ellison and Wolitzky (2012) develops a similar model where the level of obfuscation is not observable, and search cost is a non-linear function of obfuscation. Petrikaite
finds that obfuscation is profitable for a multi-product firm in order to internalize the externalities that one product imposes on others.

Third, the search order can be endogenized. The intermediary can choose an order to present the products to the consumers. In \cite{Hagiu2011}, the intermediary observes the preferences of consumers, and deliberately divert each consumer to the one less preferred. The reason to divert consumers is to reduce the profit difference between the two firms while maintaining the participation of both.

1.2 Summary of This Thesis

If consumers search in the same ordering, \cite{Arbatskaya2007} finds that prices decrease along consumers’ searching path. In the other model, \cite{Armstrong2009} shows that the first firm charges a lower price than the rest of the firms. Most aforementioned models study homogeneous firms or heterogeneous firms where the valuation of each firm’s product is under the same distribution. However, product ordering matters only when the products are different. Otherwise, their rankings have no implications on welfare. Chapter 2 looks for the optimal product ordering from an industry perspective and a social perspective. First, the chapter introduces two types of goods: niche goods and generic goods.

In practice, the distributions of valuations of products can differ: some products attract the mass population, whereas some products attract only a market niche. For instance, road bikes are generic goods designed for travelers on paved roads (although one can also ride it on rocks.). Mountain bikes are niche goods designed for off-road cycling (although one can also ride it on paved roads.)
Niche products do not have on average better quality than generic products. For instance, people riding bikes mostly on roads often find mountain bikes heavy and bulky. I start with a duopoly model (from [Wolinsky (1986)]) with two competing firms: a niche firm and a generic firm. When consumer search is frictionless, the unique price equilibrium has that both firms charge the same price.

A non-infinitesimal search cost somehow prevents consumers from searching the subsequent firm. I show that this pricing pattern is solid. The underlying mechanism is similar to [Armstrong et al. (2009) and Zhou (2011)]: the first firm lowers down the price in order to further prevent consumers from searching the second firm; the second firm infers from the optimal search rule that its visitors are those who have obtained poor match from the first firm.

By charging a lower price, the first firm actually always gains higher profit than the second firm, which does not automatically imply that each firm of each type gains higher profit when it is firstly searched by consumers than when it is searched by consumers after. Imagine that consumers see the generic firm first. Consumers immediately infer that the next firm is the niche firm, without knowing which niche the firm is actually in. As a result, more consumers wonder what is behind and search the second firm, resulting in intensified competition, and therefore lower price levels. By the same logic, the “niche goods first, generic goods second” ordering is optimal for the industry. The same ordering is preserved by consumers if consumers are believed to search the niche firm first, according to which firms set prices. The same ordering is also social optimal.

Firms agglomerates in different ways on the Internet and in real markets. In real markets, the clusters are formed more naturally and often without a planer: restaurants, hotels choose to be near the train station. On the Internet, wherever there are clusters, there are platforms
behind controlling the fees. More precisely, the platforms choose the number of firms on the Internet.

Chapter 3 provides an explanation to a subtle question: why do firms agglomerate on the Internet? Firms agglomerate in order to obtain larger profit than those who do not. They obtain higher profit by facing larger demand from those consumers visiting the cluster first. And the consumers visit the clusters first for lower prices. If this is the case, two firms are enough to attract all consumers. Then arises the following question: why do platforms on the Internet have thousands of firms?

In this chapter, I develop a search model à la Wolinsky (1986). In the model, there is a monopoly platform, infinitely many firms and one unit mass of consumers. Consumers can obtain any product information in the platform without cost, and outside the platform only through sequential search. I find an equilibrium where all consumers enter the platform first, no matter the platform imposes them a subscription fee or not. In the equilibrium, prices are lower in the platform due to competition intensifying effect inside the platform and the incentive for inside firms to prevent consumers from searching outside firms.

The answer to the last question lies in search cost. When search cost is large enough, consumers do not search and the optimal size of the platform is 2. When search cost is low, not only lower prices, but also more varieties are needed to hold these consumers. When several platforms (stores) compete, in a symmetric equilibrium, each platform (store) has the same number of firms, and the number of firms decreases in search cost.

Observations are also in real markets. A usual store in a centralized market for electronic products (called computer city in developing countries) often offers almost every brand. It is rare to see such a store selling only one type of product, since consumers will easily flow away.
The last chapter is aimed at discussing the profitability of a pricing strategy, referred as the group-buying. I use a location model with a monopolist and several consumers. Consumers, depending on their types, choose among informing another consumer, waiting or rejecting the offer. Different from the earlier chapters, no consumers search for product information, but some of them receive it from other consumers.

Two models are proposed. The first model assumes that consumers can not invite others. I find that the monopolist resorts to inter-temporal price discriminations (an early price and a late price), which suggests that the group-buying strategy is not optimal. The reason is that the monopolist is facing the same number of consumers, for whom group-buying is equivalent to a pure strategy in pricing in the first period, and a mixed strategy in pricing in the second period.

The second model allows each consumer to invite at most one more consumer. In equilibrium, the monopolist resorts to a “bait and switch” strategy. The monopolist gains less from successful deals than from unsuccessful ones. Low discounted prices are offered in order to motivate consumers to make invitations, and the monopolist places hope on that consumers arrive in insufficient number and some of them choose to purchase at the normal price. If the inviting cost is large enough, it is optimal to set the discounted price to zero.

Compared with monopoly pricing, the model with invitations decreases surplus of the first-cohort consumers and increases surplus of the second-cohort consumers. Overall it increases consumer surplus as long as inviting cost is not too large, and thereby, it can increase social welfare.
2. ORDERED SEARCH WITH ASYMMETRIC PRODUCT DESIGN

2.1 Introduction

It is common that consumers see through products in the same ordering. While visiting a supermarket or a web-store, most observe the products at the entrance or in the homepage before looking into a specific category. Even when products are listed in the same category, shoppers see through them in a top-down manner. Some experiments show that the click-through rate (CTR) of the first item can be 17 times higher than the eighth item. This huge difference implies that choosing the ordering is important, as different orderings can give rise to very different outcomes.

The consumers’ anticipation about their match with different types of products may be very different. The present paper focuses on the difference between generic products and niche products, in the terminology of Larson (2011). A generic product serves the mass population, while a niche product serves some small population better than the generic product, and others worse than the generic product. Different product types do not imply any superior (or inferior) quality. Take standard jeans as examples of generic products. Then niche products are either colourful jeans or ripped jeans. There is no evidence that standard jeans have on average higher quality than ripped jeans, besides the fact that ripped jeans usually target the young population.

The paper uses a duopoly model to study how the ordering of product presentation affects

1. Granka et al. (2004) employed eye-tracking analysis to explore both the click and attention distribution from Google. Evidence has shown that levels of attention depends on their locations.
the market. One firm (the niche firm) sells a niche product and the other firm (the generic firm) sells a generic product. The products can be arranged in two manners. In the niche-generic ordering, each consumer sees the niche product at the beginning. In the generic-niche ordering, each consumer sees the generic product at the beginning. In either ordering, consumers can always visit the other product after incurring a search cost.

In equilibrium, the firm that consumers visit first (the first firm) always sets a lower price than the second firm from the fact that the first firm has more elastic demand. Reasons are as follows. All consumers always observe the price of the first firm, but only those having received poor match from the first firm observe the price of the second firm. Therefore price deviations of the first firm affect the decisions of all consumers, while price deviations of the second firm affect only the decisions of a fraction of them. Behind this price pattern, the first firm prevents consumers from searching the second firm, who in turn charges a higher price to those failing to obtain good enough match from the first firm.

While charging a lower price, the first firm actually earns larger profits than the second firm. However, this does not imply that both types of firms earn higher profits being first than being second. This property, which would hold if both firms had identical types, remains valid in our setting for the firm selling the niche product. However, the generic firm may earn lower profits in the generic-niche ordering than in the niche-generic ordering, for some parameter configurations. After comparing the price equilibrium in both orderings, we find that the generic-niche ordering induces a larger proportion of consumers to search the second firm than the niche-generic ordering. More search intensifies competition, which brings down prices and industry profit. Therefore, the niche-generic ordering is optimal for the industry.

The framework is then extended. In section 5, consumers are free to choose the order in which
they search. According to Weitzman (1979), consumers optimally visit firms in the descending order of reservation prices. In equilibrium, this order of search is correctly anticipated by firms and consumers anticipate that firms price accordingly. There exists an equilibrium, where consumers search in the niche-generic manner thus yielding the largest industry profit. An equilibrium with the reverse order of search does not necessarily exist.

We then introduce an outside option by assuming that each consumer buys the product only if the product offers a positive surplus. Again, industry profits are always higher in the niche-generic ordering, whether we consider the duopoly setting or the case of a multiproduct firm selling both products.

Last, we analyze welfare. If the market is covered, simulations show that the niche-generic ordering decreases consumer surplus and increases social welfare when the difference between the two types of firms is large enough. The same ordering increases consumer surplus and decreases welfare if the difference is very small.

If the market is not covered, the niche-generic ordering increases consumer surplus when search cost is large because prices increase faster in search cost in the generic-niche ordering. The niche-generic ordering is socially efficient, where less price distortion is at presence.

The literature on search provides a theoretical basis for the paper. Wolinsky (1986) introduces sequential search in Perloff and Salop (1985) and obtained a model of true monopolistic competition. Anderson and Renault (1999) provide some general comparative statics for the oligopoly model and show that search costs exacerbate over entry in the market.

The paper is most closely related to the ordered search literature. Arbatskaya (2007) finds that prices for homogeneous products decline from the first to the last. However, Armstrong et al. (2009) obtains the opposite result by considering differentiated product: one prominent firm
charges a lower price than the other firms. Zhou (2011) further confirms that prices are indeed increasing along the searching path by considering ordered search with product differentiation and an arbitrary number of firms. Armstrong et al. (2009) further discusses products with varying quality and finds that the highest-quality firm has the greatest incentive to become prominent. Moraga-Gonzalez and Petrikaite (2013) looks into the incentives for horizontally differentiated firms to merger and find that in the short run the merged entity is searched last because it is expected to charge higher prices.

Finally, the paper draws on the literatures on product design. Johnson and Myatt (2006) study families of "variance-ordered" distributions and show that a monopolist's profit is maximized with either minimum or maximum preference diversity. Kuksov (2004) uses a duopoly model to study the impact of search cost on the endogenous level of product differentiation. Lower search cost leads firms to differentiate themselves more, which leads to higher prices and lower social welfare. Larson (2011) and Bar-Isaac et al. (2012) use sequential search models, in which firms are identical ex-ante and choose different levels of product differentiation according to other parameters. Hagiu and Jullien (2011) discuss intermediaries' decisions to divert some consumers to the unfavourable firm even after consumers have learned their types. Wang (2012) uses a monopoly model and looks at the impact of search cost on information disclosure.

Section 2 develops the basic model. Section 3 uses the basic model to find out the equilibrium in three cases: perfect information, generic-niche ordering and niche-generic ordering. After that, some comparative statics are done with respect to various parameters. Section 4 answers the main question of the paper: which ordering is optimal for the industry. Profit is first compared at the firm level and then at the industry level. Section 5 analyses consumers' incentives to deviate to another searching order. Section 6 introduces the outside option to the original model.
and studies two related cases. Section 7 provides numerical examples of consumer surplus and social welfare from each specific ordering. The last sections (8-9) concludes our paper and discuss possible future works.

2.2 The Model

Consumers. There is a large number of consumers with measure normalized to one. Each consumer looks for one unit of good in the market. We assume that if a consumer does not purchase any good, his surplus is low enough so that the market is covered. If consumer \( i \) purchases the product from firm \( j \), the utility derived from consuming the product consists of the price of the product \( j \in \{N, G\} \) and a random value \( \varepsilon_{ij} = \mu_j \epsilon_{ij} \). There are two components in \( \varepsilon_{ij} \): \( \mu_j \) is a firm-specific term that we will interpret in the next paragraph; \( \epsilon_{ij} \) is the match between consumer \( i \) and firm \( j \). We assume that \( \epsilon_{ij} \) is randomly distributed according to c.d.f. \( F(x) \). The match is independent across consumers and firms. Overall, consumer \( i \)'s net utility when purchasing product \( j \) at price \( p_j \) is given by:

\[
    u_{ij} = \mu_j \epsilon_{ij} - p_j
\]

We make the following assumption on c.d.f. \( F(.) \):

Assumption 1 (Symmetry). The density function \( f(\epsilon) \) exists and is continuous, symmetric, with support \([−a, a]\) with \( a > 0 \) and has zero mean.

Firms. The market has a niche firm (Firm N) and a generic firm (Firm G). Firms are producing products that are weak substitutes to each other. Firms incur neither fixed nor marginal cost.
The firm-specific term $\mu_j$ is a measure of the importance of match in a consumer’s overall utility function. We assume that

$$\mu_N > \mu_G$$

Given assumption 1, having a larger $\mu$ does not imply a superior or inferior quality but rather the importance of quirkiness on buyers’ purchase decisions. With this interpretation in mind (and following Johnson and Myatt (2006), Larson (2011) and Bar-Isaac et al. (2012)), we call firm N’s product the *niche good* and firm G’s product the *generic good*. The niche good (with a larger consumer heterogeneity) is more likely to provide extreme valuations which satisfy the needs of some small population, while missing the needs of others. On the other hand, the generic good has a smaller consumer heterogeneity and is aimed at the mass population.

Consumers know the type of each firm, but not its price and match. In order to learn the price and match information, consumers conduct product search in a sequential ordering. Each search costs $c$, which is identical for all consumers. Searching firm $j$ informs the consumer of the true price and the value of $\epsilon_{ij}$. After being informed about one firm, the consumer always has the free recall option for that firm. Firms cannot distinguish between the consumers coming for the first time from those choosing the recall option.

Firms are not only different in types, but also regarding the order in which they are searched. Use $\prec$ to denote the spatial order. For example, Firm N $\prec$ Firm G indicates that firms are in the niche-generic ordering. Denote firm $j$’s demand function by $D_j^{N\prec G}(p_N, p_G)$ when firms are in the niche-generic ordering, and its profit function by $\Pi_j^{N\prec G}(p_N, p_G)$. In the niche-generic ordering, each consumer visits the niche firm first. In the generic-niche ordering, each consumer visits the

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2. In previous literature, this parameter is often termed *preference intensity* or *consumer heterogeneity*.

3. Free recall is a standard assumption in search literatures because costly recall can lead to unstationary prices.
generic firm first. We call the firm that consumers visit first (second), the first firm (the second firm).

**Game Structure**: Each firm observes the type of the rival firm, and the product ordering, but not any match information. Consumers can acquire price and match information of the second firm only through searching that firm and paying search cost $c$. The game has the following timeline. In stage 1, firms choose prices simultaneously. In stage 2, consumers observe the price and match information about the first firm for free. Each individual consumer then either purchases the first product immediately or searches the second firm, correctly anticipating its type. In the latter case, he purchases the one offering higher surplus. In equilibrium, consumers hold correct beliefs about the prices set by the second firm, and form an optimal search rule. We skip the uninteresting equilibrium where no consumers search when they expect high prices and firms set prices to infinity.

A consumer’s pay-off is the surplus from the product minus the sum of search cost. The firm $j$’s pay-off is the expected profit, either $E[\Pi_j^{G<N}(p_N, p_G, c)]$ or $E[\Pi_j^{N<G}(p_N, p_G, c)]$.

### 2.3 Equilibrium Analysis

Start from the last stage and by backward inductions, we solve for the profit-maximizing prices and the most profitable ordering for the industry with two active firms. The benchmark case assumes zero search cost and later parts compare the benchmark case with the cases with positive search cost.
2. Ordered Search with Asymmetric Product Design

2.3.1 Perfect Information

If price and match information is costless to obtain, each consumer weakly prefers sampling two firms to sampling only one. For a consumer holding exact information about two products, he purchases the generic product if the following is satisfied:

\[ \mu_N \epsilon_N - p_N < \mu_G \epsilon_G - p_G \]

When the generic firm deviates to a new price \( p \), the new price pair becomes \((p_N, p)\), and the probability of the above inequity becomes \( F((\mu_G \epsilon_G + p_N - p)/\mu_N) \) for each realization of \( \epsilon_G \).

The demand function of the generic firm is the probability of offering larger surplus than the niche firm to a particular consumer:

\[
D_G(p, p_N; c = 0) = \int_{\max(-a, -\mu_N a - p_N + p)}^{a} F\left(\frac{\mu_G \epsilon + p_N - p}{\mu_N}\right) f(\epsilon) \, d\epsilon
\]  

(2)

Suppose that the generic firm sticks to \( p_G \), and the niche firm deviates to a new price \( p \). Since the market is covered, the demand share of the niche firm is the entire market minus the share of the generic firm. The below expression is the demand for the niche firm.

\[
D_N(p_G, p; c = 0) = 1 - \int_{\max(-a, -\mu_N a - p + p_G)}^{a} F\left(\frac{\mu_G \epsilon + p - p_G}{\mu_N}\right) f(\epsilon) \, d\epsilon
\]  

(3)

Clearly, both \( D_G \) and \( D_N \) can be rewritten into functions of the price difference. Stemming

4. Some consumers may skip the firm behind in case they expect zero gain from the search. For example, if goods are arranged in the niche-generic ordering, consumers who obtain values of match higher than \((\mu_G a - p_G + p_N)/\mu_N\) will skip the generic firm. To get around this, we assume that every consumer searches every firm.
from $D_G = 1 - D_N$, the derivatives of $D_G$ and $D_N$ with respect to $p$ are equal.

$$\frac{\partial}{\partial p} D_G(p, p_N; c = 0) = \frac{\partial}{\partial p} D_N(p_G, p; c = 0) = -\frac{1}{\mu_N} \int_{\max(-a, -\frac{\mu_N a - p_N}{\mu_G} - p)}^{a} f\left(\frac{\mu_G \epsilon + p_N - p}{\mu_N}\right) f(\epsilon) d\epsilon$$

In equilibrium, firms maximize their expected profits and do not deviate to other prices. By the first order conditions, we obtain the equilibrium $p_N$ and $p_G$:

$$p_N = p_G = \frac{\mu_N}{2} \int_{-a}^{a} f(\mu_G \epsilon / \mu_N) f(\epsilon) d\epsilon$$

(4)

The unique existing equilibrium satisfies $p_N - p_G + a(\mu_N - \mu_G) > 0$, therefore the consumer receiving $(-a, -a)$ purchases the generic good.

**Proposition 1.** Under perfect information, firms set identical prices and earn equal amounts of profit.

$$p_N = p_G = \frac{\mu_N}{2} \int_{-a}^{a} f(\mu_G \epsilon / \mu_N) f(\epsilon) d\epsilon$$

In Figure 1, the horizontal axis represents the match with the generic product, and the vertical axis represents the match with the niche product. The grey curve represents those consumers indifferent between two products. Then the consumers above it choose the niche product, and consumers below it choose the generic product. If prices are equal, this grey curve must cross the original point. If a consumer at $(\epsilon_G, \epsilon_N)$ purchases at Firm N, then the consumer at $(-\epsilon_G, -\epsilon_N)$ will purchase at Firm G, and vice versa.

Since $f(\epsilon)$ is symmetric, the integral above the grey curve is always as large as the integral below it. It is clear that the market is equally split. From equal prices and sizes of demand, firms always earn equal amounts of profit. On the other hand if one firm earns higher profit than the
other firm, then the other firm can always set the same price as the first firm and obtain equal demand shares.

Particularly if $F$ is uniform $[-1, 1]$, the common equilibrium price is $\mu_N$. In the polar case of $\mu_N \to 0$ (then $\mu_G \to 0$), the equilibrium price approaches the marginal cost, and the outcome becomes Bertrand:

$$\lim_{\mu_N \to 0} p_N, p_G = 0$$

When two products have an identical level of consumer heterogeneity, (4) becomes the Perloff and Salop (1985) price, where the equilibrium prices rise in consumer heterogeneity. While "consumer heterogeneity" levels differ, the equilibrium prices differ according to the shape of the distribution.
2.3.2 Imperfect Information

To simplify the analysis, we assume the followings in the remainder of the paper:

**Assumption 2.** $\epsilon$ is uniformly distributed on $[-1, 1]$.

Consider very large search cost ($c \geq \mu_G$). If consumer $i$ searches the generic firm, the surplus that he gains is $\mu_G\epsilon_i - p_N - c \leq 0$. To get around this situation, we make the third assumption in the following.

**Assumption 3.** Search cost is not too large, namely $c < \mu_G$.

In the subsection where we discuss the niche-generic ordering, the above is to ensure that the price elasticity of demand for the generic firm is negative.

**The Optimal Stopping Rule**

Suppose that firm $j$ is the first firm. After his match to the first firm (e.g. $(p_j, \epsilon_j)$), a consumer can either make a purchase from the current firm or sample the next firm $k$ at a cost $c > 0$. Suppose that the consumer expects the next firm $k$ to charge the price $p_k$. The expected gain in surplus from searching firm $k$ is

$$
\int_{x}^{1} \left( \mu_k \epsilon - \mu_j \epsilon_j + p_j - p_k \right) \frac{\partial \epsilon}{2} - c, \quad x = \frac{\mu_j \epsilon_j - p_j + p_k}{\mu_k}
$$

The integral represents the expected incremental benefit from searching another time given his current offer $u_{ij}$, and the second term is the search cost. The consumer will sample firm $k$ iff the benefit exceeds the cost, that is, the value of the above is larger than zero, and make a purchase
of the current good otherwise. Let \( \hat{\epsilon}_j \) solve the following equation:

\[
\int_1^{\hat{x}} \left( \mu_k \epsilon - \mu_j \hat{\epsilon}_j + p_j - p_k \right) \frac{\partial \epsilon}{2} = c, \quad \hat{x} = \frac{\mu_j \hat{\epsilon}_j - p_j + p_k}{\mu_k}
\]

The integral of (5) decreases in \( \hat{\epsilon}_j \), so the solution for \( \hat{\epsilon}_j \) exists uniquely and is a threshold determining the searching rule. If \( \epsilon_j > \hat{\epsilon}_j \), the expected incremental surplus is less than the search cost so that the consumer purchases the good and leaves the market. If \( \epsilon_j < \hat{\epsilon}_j \), the expected incremental surplus exceeds the search cost so that the consumer searches the second firm. Solve (5) for the threshold:

\[
(5) \Rightarrow \left( \frac{\mu_k \epsilon - \mu_j \hat{\epsilon}_j + p_j - p_k}{4\mu_k} \right)^4 \bigg|_{1}^{\hat{x}} = c
\]

\[
\Rightarrow \hat{\epsilon}_j(p_j, p_k) = \frac{\mu_k + p_j - p_k - 2\sqrt{\mu_k c}}{\mu_j}
\]

First, every time consumers have seen a higher price posted by the first firm, they are more willing to search for a lower price from the second firm. For this reason, (6) increases in current price and decreases in the expected price of the second firm. Second, (6) decreases in consumer heterogeneity with respect to the current product and increases in the others’ under a moderate search cost, because more consumers are likely to receive very good match and less are attracted by the other if the current offer is more consumer heterogeneous. In case the second product is more consumer heterogeneous, it leaves consumers a larger room (possibility) to gain from searching that firm and more consumers will move on. Last, (6) decreases in search cost.

Suppose there is a level of search cost that even the consumer receiving the poorest match from the first firm is indifferent between searching and not searching. Define this search cost by
2. Ordered Search with Asymmetric Product Design

$c_{\text{max}}$, and solve for $c_{\text{max}}$.

$$c_{\text{max}} = \min_j \left( \frac{\mu_j + \mu_j + p_j - p_k}{2} \right)^2 / \mu_k, \ j \in \{N, G\} \text{ and } j \neq k$$

Suppose there is the other level of search cost that even the consumer receiving the best match from the first firm is indifferent between searching and not searching. Define this search cost by $c_{\text{min}}$, and solve for $c_{\text{min}}$.

$$c_{\text{min}} = \max_j \left( \frac{\mu_k - \mu_j + p_j - p_k}{2} \right)^2 / \mu_k, \ j \in \{N, G\} \text{ and } j \neq k$$

We are not interested in any equilibrium satisfying $c > c_{\text{max}}$ or $c < c_{\text{min}}$. The reasons are as follows. When $c > c_{\text{max}}$, no consumers search the second firm. When $c < c_{\text{min}}$, each consumer searches each firm, then product orderings do not play any role. In the two following sub-cases, we look for equilibria (if they exist) when some positive proportion (less than proportion one) of consumers search.

**Generic-Niche Ordering**

According to the optimal stopping rule, those having received better match than $\hat{\epsilon}_G$ purchase at the first firm immediately. These consumers compose the *fresh demand* for the first firm. Other consumers have received worse match than $\hat{\epsilon}_G$, so they check out the second firm (niche firm). After these consumers have sampled the second firm, some return to the first firm because they find out that goods from the generic firm actually offer them larger surplus, and the others

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5. $c_{\text{max}}$ is very large. In the polar case, the first firm can charge price zero in order to prevent consumers from searching the second firm, who in turn also charges the price zero in order to attract consumers. Then $c > c_{\text{max}}$ is equivalent to $c > \min_j ((\mu_k + \mu_j)/2)^2 / \mu_k, j \neq k$. 
purchase at the niche firm. As in many ordered search models, the demand of the first firm (generic firm) is composed of two parts: fresh demand and returning demand. And the demand of the last firm (niche firm) consists of only the fresh demand.

To illustrate the two sources of demand, we depict the demand shares in the following figure: the vertical line splits the whole demand into two parts. The fraction on the right-hand side represents the fraction of consumers purchasing at Firm G immediately (fresh demand of the first firm). On the left-hand side are those consumers who decide to search, which are further divided into two parts by the curve $u_{iN} = u_{iG}$. Above the curve, consumers choose Firm N. Below the curve, consumers choose Firm G. 6 If Firm G deviates to a price $p$ while Firm N sets

---

6. The intercept of $u_{iN} = u_{iG}$ on the $\epsilon_N$ axis can be either greater or smaller than -1. Therefore the demand functions have different forms according to $\frac{p_G - p_N - \mu_N}{\mu_G}$. In case $\max(\frac{p_G - p_N - \mu_N}{\mu_G}, -1) = \frac{p_G - p_N - \mu_N}{\mu_G}$, the consumer receiving $(\epsilon_N, \epsilon_G) = (-1, -1)$ purchases the niche product. In case $\frac{p_G - p_N - \mu_N}{\mu_G} < -1$, consumers who receive poor match (between -1 and $\frac{p_N - p_G - \mu_G}{\mu_G}$) from the niche product switch to the generic product. We look for equilibria satisfying respectively $\frac{p_G - p_N - \mu_N}{\mu_G} > -1$, and $\frac{p_G - p_N - \mu_N}{\mu_G} < -1$. Since the unique existing equilibrium satisfies $\frac{p_G - p_N - \mu_N}{\mu_G} < -1$, we only draw this case in Figure 2 for simplicity.
the price right to the expectation, its demand function is given by:

\[
D_G^N(p_N, p) = \frac{1}{2} \left( 1 - \min \left( \epsilon_G(p, p_N), 1 \right) \right) + \frac{1}{4} \int_{\max \left( \frac{p - p_N - \mu_N}{\mu_G} \right)}^{\min \left( \epsilon_G(p, p_N), 1 \right)} \left( 1 + \frac{p_N - p + \mu_G \epsilon}{\mu_N} \right) \partial \epsilon \tag{7}
\]

From the above demand function, the first term corresponds to fresh demand, and the second term corresponds to returning demand. And if the niche firm deviates to another price \( p \) while the generic firm sets the price to the expectation, its demand is given by,

\[
D_G^N (p, p_G) = \frac{1}{2} \int_{\max \left( \frac{p - p_G - \mu_N}{\mu_G} \right)}^{\min \left( \epsilon_G(p, p_G), 1 \right)} \left( 1 - \frac{1}{2} \left( 1 + \frac{p - p_G + \mu_G \epsilon}{\mu_N} \right) \right) \partial \epsilon + \max \left( 1 + \frac{p_G - p - \mu_G}{\mu_N}, 0 \right) \tag{8}
\]

Both (7) and (8) depend on \( \max(1, \frac{p_G - p_N - \mu_N}{\mu_G}) \). In the appendix, we show that there is no equilibrium satisfying \( \frac{p_G - p_N - \mu_N}{\mu_G} > -1 \). Then we try to find out whether there is any equilibrium satisfying \( \frac{p_G - p_N - \mu_N}{\mu_G} < -1 \). The derivative of \( D_G^N(p_N, p) \) in \( p \) is,

\[
\frac{\partial}{\partial p} D_G^N(p_N, p) = -\frac{1}{2 \mu_G} + \frac{1}{4 \mu_G} \left( 1 + \frac{p_N - p - \mu_G}{\mu_N} \right)
\]

And the derivative of \( D_N^G(p_G, p) \) in \( p \) is,

\[
\frac{\partial}{\partial p} D_N^G(p_G, p) = -\frac{\mu_N + \mu_G - p_N + p_G - 2 \sqrt{\mu_N \mu_G}}{4 \mu_G}
\]

In any price equilibrium, each firm should have no incentive to deviate from the price expectations. From the first-order conditions, we find equilibrium prices. Let \( \Delta = p_N - p_G \). For the
Consider the generic firm’s profit function. When prices are set according to (9) and (10), we have,

**Lemma 1.** In the generic-niche ordering, the profit of the generic firm increases in search cost.

**Proof.** To see this, rewrite the niche firm’s best response (10) into the following form.

\[
\frac{3}{2}p_N = \frac{\mu_N + \mu_G + p_G + 2\sqrt{\mu_N c}}{2}
\]

Suppose that search cost increases. If the generic firm does not change its price \(p_G\), the above suggests that the niche firm’s best response is to increase its price \(p_N\). If \(p_N\) increases and \(p_G\) remains at the same level, the generic firm faces larger demand according to the optimal search rule, and thereafter higher profit. Therefore, the generic firm can at least keeps its price at the same level and still earns larger profit. Any change of the price is intended only to further increase its profit. \(\Box\)

When search cost is small enough, the generic firm’s profit can fall below its profit in the perfect information case. We may find a unique level of search cost, where the generic firm gets the same profit as if in the perfect information case. The curves in the following figure draw the profit when prices are set according to (9) and (10). It is clear that such a level of search cost
exists.

Fig. 2.3: Profit patterns in the generic-niche ordering ($\mu_G/\mu_N = 1/2$).

Define two levels of search cost: $c', c''$, where $c'$ is the maximum level of search cost at which
the generic firm earns weaker lower profit than the perfect information case, and the use of $c''$
will be explained in the following text. The figure draws both search cost levels.

$$c' = \max\{c | \Pi_G^{\geq N}(\Delta^*(c'), c') \leq \mu_N/2, c \leq \mu_G\}, c'' = \frac{1}{\mu_N} \left(\frac{\mu_N - \mu_G}{2}\right)^2$$  \hspace{1cm} (11)

Fig. 2.4: Levels of $c'$ (blue) and $c''$ (red).

Lemma 2. If $c' > 0$ exists, the unique price equilibrium in the generic-niche ordering is $p_N = p_G = \mu_N$ for $c < \min\{c', c''\}$.

Proof. When search cost is below $c'$, the generic firm prefers the "perfect information" profit.

The niche firm prefers the "perfect information" profit as well, because otherwise its profit
cannot exceed the profit of the generic firm. The remaining question are: once the generic firm charges $p_G = \mu_N$ as in the "perfect information" case, 1) whether it is also optimal for the niche firm to charge $p_N = \mu_N$; 2) whether each consumer searches each firm under this price setting.

The answers lie in whether $c < \min\{c', c''\}$. Suppose that both firms set $p_N = p_G = \mu_N$. According to the optimal search rule, each consumer visits the niche firm if

$$\hat{\epsilon}_G(\mu_N, \mu_N) > 1 \Rightarrow c < c''$$  \hspace{1cm} (12)

Then consider that the generic firm sets $p_G = \mu_N$ and the niche firm sets the price according to (10) ($p_N' = \frac{2\mu_N + \mu_G + 2\sqrt{T_m}}{3}$). The optimal search rule shows that,

$$\hat{\epsilon}_G(\mu_N, p_N') > 1 \Rightarrow c < c''$$  \hspace{1cm} (13)

Therefore, when search cost is lower than $\min\{c', c''\}$ and the generic firm sets price to $\mu_N$, each consumer visits the niche firm no matter whether the niche firm sets the price at $\mu_N$ or according to (10). Then it is logically to think that the niche firm will set the price to $\mu_N$. This price pattern is illustrated in the following figure.

---

**Fig. 2.5:** Discontinuous prices in the generic-niche ordering ($\mu_G/\mu_N = 1/2$).

---

**Proposition 2.** In the generic-niche ordering, two types of equilibria exist and depend on the
When search cost is higher than \( \min\{c', c''\} \), prices are set from (9) and (10). The first firm sets a lower price than the second firm.

When search cost is lower than \( \min\{c', c''\} \), equilibrium prices are given by \( p_N = p_G = \mu_N \).

Prices \( p_j(\mu_N, \mu_G, c) \) and price differences \( \Delta(\mu_N, \mu_G, c) \) are homogeneous of degree one.

\[ \begin{tabular}{cccc}
\hline
& \( p_{\text{niche}} \) & \( p_{\text{generic}} \) & \( \Pi_{\text{niche}} \) & \( \Pi_{\text{generic}} \) \\
\hline
\( \mu_N \) & + & + & + & + \\
\( \mu_G \) & + & + & + & + \\
search cost & + & + & ambiguous & + \\
\hline
\end{tabular} \]

To see what the comparative statics of prices and profit with respect to each parameter are, we summarize them in the table. When the products are more consumer heterogeneous, prices and profit generally increase. When search cost increases, prices increase at different speeds; the price difference \( \Delta = p_N - p_G \) first increases then decreases; the generic firm’s profit is always larger, but the niche firm’s profit can first increase then decrease.
Niche-Generic Ordering

When firms are arranged in the *niche-generic* manner, consumers must visit the niche firm at the beginning. The search pattern is similar to the previous section. From the optimal stopping rule, the second firm (Firm G) knows that those who get to them must have received poorer match than $\hat{\epsilon}_N$ from the niche firm. As in the previous case, each demand function can be written in different forms according to the sign of $p_N - p_G + \mu_N - \mu_G$. We employ the same trick by assuming the sign of $p_N - p_G + \mu_N - \mu_G$ and verifying the sign in the equilibrium. An increase of $p_N$ can be observed by each consumer immediately: more consumers search the generic firm and more consumers choose the generic product after having seen the two products. However, consumers cannot observe Firm G’s price directly. Hence a deviation by the generic firm changes only the decisions of those who observe it. If the generic firm deviates to a new price $p$ while the other firm keeps its price to the expectation, its demand function is the conditional probability

\[
\begin{align*}
  &\text{Firm}_N \\
  \hat{\epsilon}_N &\quad (\text{fresh demand}) \\
  (u_N = u_G) &\quad \text{(searchers)} \\
  (-1,-1) &\quad \text{Firm}_G
\end{align*}
\]
of offering a consumer higher surplus than Firm N:

\[ D_G^{N \triangleleft G}(p_N, p) = \]
\[ \frac{1}{2} \int_{\max(-1, \frac{p_N - p - \mu_G}{\mu_N})}^{\epsilon_N(p_N, p_G)} \left( 1 - \frac{1}{2} \left( 1 + \frac{p - p_N + \mu_N \epsilon}{\mu_G} \right) \right) \partial\epsilon + \max \left( \frac{1}{2} \left( 1 + \frac{p_N - p - \mu_G}{\mu_N} \right), 0 \right) \]

The derivative of \( D_G^{N \triangleleft G}(p_N, p) \) with respect to \( p \) is, for \( \frac{p_N - p - \mu_G}{\mu_N} < -1 \),

\[ \frac{\partial}{\partial p} D_G^{N \triangleleft G}(p_N, p) = -\frac{\mu_G + \mu_N + p_N - p_G - 2\sqrt{\mu_G c}}{4\mu_N} \]

For \( \frac{p_N - p - \mu_G}{\mu_N} > -1 \),

\[ \frac{\partial}{\partial p} D_G^{N \triangleleft G}(p_N, p) = -\frac{1}{2\mu_N} + \frac{p_G - p + 2\sqrt{\mu_G c}}{4\mu_N \mu_G} \]

At \( p = p_G \), the above is \( -\frac{\sqrt{\mu_G(c(\sqrt{\mu_G} - \sqrt{c}))}}{2\mu_N \mu_G} \). To ensure that the price elasticity of demand is negative, we need \( c < \mu_G \). Accordingly, the demand function of Firm N is 1 minus the demand of Firm G:

\[ D_N^{\triangleleft G}(p, p_G) = \frac{1}{2} \left( 1 - \epsilon_N(p_G, p) \right) + \frac{1}{4} \int_{\max(-1, \frac{p_G - p - \mu_G}{\mu_N})}^{\epsilon_N(p_G, p)} \left( 1 + \frac{p_G - p + \mu_N \epsilon}{\mu_G} \right) \partial\epsilon \]

The derivative of \( D_N^{\triangleleft G}(p, p_G) \) with respect to \( p \) is, for \( \frac{p_G - p - \mu_N}{\mu_N} < -1 \),

\[ \frac{\partial}{\partial p} D_N^{\triangleleft G}(p, p_G) = -\frac{1}{2\mu_N} + \frac{1}{4\mu_N} \left( 1 + \frac{p_G - p - \mu_N}{\mu_G} \right) \]
For $\frac{p_{-G} - \mu_{G}}{\mu_N} > -1$, $D_{N,G}^N(p, p_G) = -\frac{1}{2\mu_N}$. By the first order conditions we obtain the following proposition.

**Proposition 3.** In the niche-generic ordering, two types of price equilibria exist and depend on the parametrizations. Let $\Delta = p_N - p_G$.

- **Equilibrium 1** satisfies $p_N = \mu_N + c - \Delta$, $p_G = \mu_G(\mu_N + \Delta - c)/(\mu_G - \sqrt{\mu_Gc})$ and $(p_N - p_G) + (\mu_N - \mu_G) > 0$.

- **Equilibrium 2** satisfies $p_N = (4\mu_G(\mu_N + c - \Delta) - (\Delta + (\mu_N - \mu_G))^2)/2(\mu_G + \mu_N + \Delta)$, $p_G = (\mu_N + \mu_G + \Delta + 2\sqrt{\mu_Gc})/2$ and $(p_N - p_G) + (\mu_N - \mu_G) < 0$.

The first firm sets a lower price than the second firm. Prices $p_j(\mu_N, \mu_G, c)$ and price differences $\Delta(\mu_N, \mu_G, c)$ are homogeneous of degree one.

**Proof.** See the appendix.

We illustrate the equilibria and their parameter ranges in the following figure.

![Fig. 2.7: Two existing equilibria in the niche-generic ordering](image-url)
Depending on values of $\frac{\mu_G}{\mu_N}$ and $c$, there exist two different sets of equilibrium prices. The second firm always charges a higher price than the first firm.\footnote{There exist no equilibrium satisfying $p_N - p_G + \mu_N - \mu_G < 0$ and $\mu_G/\mu_N > 12/13$.} By increasing (reducing) parameters in the same proportion, the equilibrium prices (price difference) and profit change again in the same proportion. Hence, we say that these functions are homogeneous of degree one.

Henceforth in this section, we focus the discussion on the properties of equilibrium 1, which is relevant when taste intensities for the two products are different enough. Larger search cost discourages consumers from searching and grants each firm more market power. By lowering its price the first firm actually further discourages consumers from searching the subsequent firm. The second firm learns from the optimal stopping rule, that the consumers searching the second firm have obtained match lower than $\hat{\epsilon}_N(p_N, p_G)$, which is decreasing in the search cost. Hence the upward pressure in the generic product price, which is searched second, is very strong and, as we now see it is stronger than the upward pressure on the niche product’s price. Solving for $\Delta = p_N - p_G$ we have

$$\Delta = \frac{-\mu_N \sqrt{c} - c \sqrt{c} + 2c \sqrt{\mu_G}}{3 \sqrt{\mu_G} - 2 \sqrt{c}}$$

Its derivative with respect to $c$ is,

$$\frac{-(3\mu_N + 13c + 12\sqrt{\mu_G c}) \sqrt{\mu_G} + 4c \sqrt{c}}{2(3 \sqrt{\mu_G} - 2 \sqrt{c})^2 \sqrt{c}},$$

Which is negative for $c$ small enough. Hence, the price difference is larger with larger search costs, although both prices increase as the next result shows. The following corollary summarizes comparative statics results with respect to search costs as well as taste intensities for the two products.
Corollary 1. In the niche-generic ordering, the impact of consumer heterogeneity and search cost on prices and profit can be summarized from the below table.

<table>
<thead>
<tr>
<th></th>
<th>$p_{\text{niche}}$</th>
<th>$p_{\text{generic}}$</th>
<th>$\Pi_{\text{niche}}$</th>
<th>$\Pi_{\text{generic}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu_N$</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>$\mu_G$</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>search cost</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>ambiguous</td>
</tr>
</tbody>
</table>

The above results show that an increase in taste heterogeneity for the niche product is consistent with the standard market power effect, whereby more heterogeneity leads to higher prices for both firms. However, more taste heterogeneity for the generic product has the opposite impact on prices. If the second product is more consumer heterogeneous, a consumer is more likely to visit the second firm because by doing that he expects a larger probability of obtaining a better match than with the first firm. As a result, more consumers compare the two prices. Demands for the two firms become more elastic. In short, more consumer heterogeneity regarding the second product may lead to more intense competition. This is analogous to the result in Anderson and Renault (1999), who find that, in a random search setting, more taste heterogeneity may lead to lower prices. Although the additional incentive to search for the second product is also present in the generic/niche ordering, it is outweighed by the increased market power afforded by more taste heterogeneity pertaining to the niche product.

A related argument that also relies on the consumers’ incentive to search the second firm can be made by comparing the generic-niche ordering with the niche-generic ordering in terms of the percentages of searchers. The prediction is that consumers have higher reservation match if firms are in a generic-niche ordering, that is when the second product exhibits more consumer
taste heterogeneity: \((\hat{\epsilon}_G > \hat{\epsilon}_N)\).

**Lemma 3.** The generic-niche ordering induces more searching than the reverse order if search cost is not too large \((\mu_G \gg c)\) and \(\mu_G/\mu_N < 12/13\).  

**Proof.** See the appendix. \(\square\)

Once more consumers sample both products, they make comparisons and buy the better offer. The direct outcome is more intense competition, which should be seen from the prices. To see this, we draw \((p_G^{G\prec N}, p_N^{N\prec G})\) and \((p_N^{G\prec N}, p_G^{N\prec G})\) in the following figure.

**Fig. 2.8:** Price levels for each ordering (location) at \(c = 1/20\).

The price \(p_G^{G\prec N}\) is lower than \(p_N^{N\prec G}\) and \(p_N^{G\prec N}\) is lower than \(p_G^{N\prec G}\). Therefore the generic-niche ordering leads to a higher level of price competition. Note that the usual prices in the niche-generic ordering are higher than the perfect information case \((p_N = p_G = \mu_N)\). To see this, compare \(p_N^{N\prec G}\) with \(\mu_N\).

\[
p_N^{N\prec G} - \mu_N = (\mu_N + c - \Delta_N^{N\prec G}) - \mu_N = c - \Delta_N^{N\prec G} > 0
\]

---

8. By simulations, we find that more consumers search in the generic-niche ordering for \(\mu_G/\mu_N > 12/13\). We also find that the reverse can hold when \(\mu_G\) is large and search cost is very large. For example, \(\mu_G = 0.8, c = 0.75\).
Since $p_N^{N<G} < p_G^{N<G}$, we also have $p_G^{N<G} > \mu_N$. The market being covered, consumers pay at least $p_N^{N<G}$ in the niche-generic ordering. We next show that the higher price level leads to higher joint profits.

### 2.4 Analysis of Profit

From the analysis above, it should be expected that having the niche firm in front of the other will give rise to higher industry profit than the other way around. The following results confirms that this is the case.

**Proposition 4.** There exists a cut-off value $\tilde{\mu}$ such that the industry profit is higher in niche-generic ordering if $\frac{\mu_G}{\mu_N} < \tilde{\mu}$.  

**Proof.** The main proof is in the appendix. The idea is as follows. Previous sections have found that $\Delta^{G<N} > 0$ and $\Delta^{N<G} < 0$. Provided that the market is covered, consumers pay at least $p_N^{N<G}$ in the niche-generic setting and $p_G^{G<N}$ in the reverse setting. In case $p_N^{N<G} > p_G^{G<N}$, the joint profit will be larger in the niche-generic ordering. Therefore which ordering induces higher joint profit can be seen on which ordering induces higher price of the niche firm. \(\square\)

In the previous literatures on ordered search, the firm obtains larger profit if it is visited by consumers first. In [Armstrong et al. (2009)](#), the prominent firm earns more than the non-prominent firms. In [Zhou (2011)](#), firm $k$ receives a higher profit than firm $k + 1$. If all firms are identical, they predict that the firm is willing to pay more to be either prominent or placed in a higher rank. This is also our result for the niche firm.

---

9. We focus on two most common equilibria with $\mu_G/\mu_N < 12/13$, which requires that firms always choose extreme designs as in [Johnson and Myatt (2006)](#) and [Larson (2011)](#).
Proposition 5. The niche firm receives a higher profit in the niche-generic ordering than the reverse ordering if $\mu_G/\mu_N < 12/13$.

Proof. See the appendix.

The result is intuitive. In the niche-generic ordering, less consumers search the second firm according to the optimal stopping rule. Therefore, prices are higher. Meanwhile, the niche firm captures a larger market share as opposed to its demand share in the generic-niche ordering. This result remains valid for $\mu_G/\mu_N \geq 12/13$.

When search cost is large enough, we find that the generic firm receives larger profit in the generic-niche ordering. However the locational preference of the generic firm is not so straightforward if search cost is small. To illustrate this, we show the generic firm’s profit in the figure.

When $\mu_G \simeq \mu_N$, the generic firm prefers the first position, as in most ordered search literatures because the difference in price level from two orderings is dominated by the fact that firm G takes larger market share in the generic-niche ordering. When $\mu_G$ is small, less consumers purchase the generic product in the niche-generic ordering as opposed to the reverse ordering, but prices are high enough so that Firm G actually gains more from the subordinate position.

Fig. 2.9: Profit of the generic firm at each ordering at $c = 1/20$. 
2.5 Rational Consumers

Suppose that consumers can actually choose the order of search. In the new game, firms have some prior beliefs about the manners in which consumers search firms. In stage 1, firms set prices. In stage 2, consumers choose which firm to visit first, and then make their search and purchase decisions as in previous sections.

We focus on two situations: either firms believe that each consumer visits the niche firm first, or firms believe that each consumer visits the generic firm first. We assume throughout, \( \mu_G/\mu_N < 12/13 \).

If firms believe that consumers search in the niche-generic manner, prices are set according to
\[
p_N = \mu_N + c - \Delta
\]
and
\[
p_G = \frac{\mu_G(\mu_N + \Delta - c)}{\mu_G - \sqrt{\mu_G c}}
\]

The optimal stopping rule is determined by the threshold \( \hat{\epsilon}_N(p_N, p_G) \) from (6). Such an equilibrium exists as we now show.

**Proposition 6.** There exists an equilibrium where consumers search the niche firm first.

**Proof.** We prove the above result by applying [Weitzman (1979)](#). In [Weitzman (1979)](#), the optimal strategy is to open the box with the highest reservation price \( r_j, j \in \{N, G\} \), where \( r_j \) satisfies,
\[
\int_{r_j}^{\infty} (U_j - r_j) dF_j(U_j) = c,
\]
where \( c \) is search cost as before. \( U_j = \mu_j \epsilon_j - p_j \) is the utility of the consumer, a random variable under distribution function \( F_j(U_j) \). Since \( \epsilon_j \) is uniformly distributed on \([-1, 1]\), \( U_j \) is uniformly
distributed on \([-\mu_j - p_j, \mu_j - p_j]\) with density \(1/2\mu_j\). Solve the above for \(r_j\).

\[
r_j = \mu_j - 2\sqrt{\mu_j c} - p_j
\]

\(r_j\) increases in \(\mu_j\) because

\[
\frac{dr_j}{d\mu_j} = 1 - \frac{\sqrt{c}}{\mu_j} > 0
\]

Knowing that \(p_N < p_G\) from proposition 3, it is clear that \(r_N > r_G\). We apply the results in Weitzman (1979), and conclude that any consumer will search the niche firm first.

When firms believe that consumers search in the generic-niche manner, prices are set according to the following

\[
p_G = \frac{(4\mu_N(\mu_G + c + \Delta) - ((\mu_N - \mu_G) + \Delta)^2)}{2(\mu_N + \mu_G - \Delta)}
\]

and \(p_N = \frac{(\mu_N + \mu_G - \Delta + 2\sqrt{\mu_N c})/2}{\mu_N + \mu_G - \Delta}\), for \(c > \min\{c', c''\}\) (\(c', c''\) are defined in (11)). For \(c \leq \min\{c', c''\}\), \(p_N = p_G = \mu_N\).

According to Weitzman (1979), the optimal strategy for consumers is that, they visit the generic firm first if \(r_G > r_N\), and visit the niche firm first if otherwise. So far we know \(\mu_N - 2\sqrt{\mu_N c} > \mu_G - 2\sqrt{\mu_G c}\) and \(p_G < p_N\). It is ambiguous which firm consumers will visit first. For convenience, we draw consumers’ decisions in the following graph. If the parameters are in the blue area, each consumer visits the niche firm first. If parameters are in the grey area, consumers visit the generic firm first.

Some numerical examples can show us exactly how large is the loss (gain) surplus from consumer
deviations. We assume that consumers receive two units of fixed surplus from each good and $c = 1/5$. We assume that all consumers are expected to search in the niche-generic way. Then we have noticed a 3.3%-27% drop in consumer surplus if one chooses the other way round (generic-niche). We now assume that all consumers are expected to search in a generic-niche way. Expected consumer surplus will change between 0% and 3.7% after choosing the other way round (for $\frac{\mu_N}{\mu_N} \leq 0.85$).

Take one step further. Since every consumer have no prior information about the products besides the expected prices, it is obvious that they will visit the same firm first. In case the parameters are in the blue region in the above figure, firms will not expect consumers to search in the generic-niche way, because firms should have expected consumers to deviate. Finally if consumers search in the niche-generic way, then it is clearly not optimal for each firm to set the price as if in the generic-niche ordering.
2.6 The Outside Option

Suppose that firms in the duopoly case charge the price pair \((p_N, p_G)\). For firm \(j\), the probability of receiving negative surplus from its good is \(\Pr(\epsilon_j < p_j/\mu_j) = F(p_j/\mu_j)\). In the following model, we assume that zero surplus is the outside option for any consumer, so he does not buy the good from firm \(j\) if \(\epsilon_j < p_j/\mu_j\). Further, the proportion of consumers receiving negative surplus from both goods is the multiplication of two probabilities: \(F(p_N/\mu_N)F(p_G/\mu_G)\). These consumers quit the market without buying anything and the surplus is zero minus the search cost. Hence, the lowest possible surplus for any consumer is \(-c\). The below reiterates the outside option in detail.

**Assumption 4.** any consumer purchases a good only if the good offers him positive surplus : \(u_{ij} \geq 0\) for \(j \in \{N, G\}\). (outside option)

The question remains when for consumers to drop out. There are two possibilities: first, some consumers sample the first firm and decides to drop out immediately. Second, the same consumers decide to drop out only after sampling both firms. We can show that the existence of the first possibility depends essentially on the level of search cost (in 6.11-6.12). If the search cost is too large, even those having received bad match decide to quit the market since otherwise they have to pay the large sampling cost only to find out if the second good fits them. To simplify our analysis, we assume away the first possibility. So the present model has three new features: first, market is not always covered. Second, prices are lower to avoid negative surplus on the consumers’ part. Therefore, consumer surplus is more often a positive number unless he drops out without any purchase after searching. Third, those consumers receiving negative surplus from the first firm will search the second firm so that we avoid the existence of consumers who
do not purchase any good and do not search the next firm. To summarize the new restriction on the model, we impose the following condition.

**Assumption 5.** $c \leq \min\{\mu_j/16\}, j \in \{N, G\}$. Therefore anyone receiving negative surplus from the first firm searches the second firm. (search cost)

Compared with our previous assumption that $c < \mu_G$, assumption 5 is very restrictive but intuitive. If the threshold $\hat{\epsilon}$ is lower than the point that the consumer obtains zero surplus, then any consumer receiving positive surplus from the first firm will purchase immediately and a fraction of those receiving negative surplus do not buy and do not search. The second firm, can decrease the price to attract this relatively inelastic demand since these consumers do not buy from the first firm because of negative surplus, until the the matching threshold rises above to the point the consumer obtains zero surplus. Then follows the assumption 5.

**2.6.1 Competing Firms**

For the most of this paper, we have made an assumption that sellers are competing: shops leasing spaces from commercial centres, firms selling goods on EBay. A third party determines the ordering of firms in the first step, then competing firms simultaneously choose prices. The market is divided into five parts from each ordering (see details in appendix c).

**Generic-Niche Ordering**

In the generic-niche order, if the niche firm keeps its price to the price expectation $p_N$ and the generic firm deviates to a new price $p$, the demand function of the generic firm is given by,

$$
\tilde{D}_G^{<N}(p_N, p) = (1 - \min(\hat{\epsilon}_G, 1))/2 + \int_{p/\mu_G}^{\min(\hat{\epsilon}_G, 1)} P_r(u_iG > u_iN) f(\epsilon_G) \partial\epsilon_G/4
$$

(15)
The demand function $\tilde{D}_G^{G\prec N}(p_N, p)$ consists of two parts: the first part $(1/2)(1 - \min(\hat{\epsilon}_G(p, p_N), 1))$ corresponds to the fraction of consumers who stop and purchase immediately at Firm G, and the second part corresponds to the fraction of consumers who have sampled both firms and decide to return to Firm G. Deviations by Firm G have impacts on both parts because it is observed by each consumer.

$$\tilde{D}_G^{G\prec N}(p, p_G) = \int_{p/\mu_N}^{\hat{\epsilon}_N} Pr(u_{iN} > u_{iG}) f(\epsilon_N) d\epsilon_N/4 + (1 - \hat{\epsilon}_N)(1 + \min(\hat{\epsilon}_G, 1))/4 \tag{16}$$

The demand $\tilde{D}_N^{G\prec N}(p, p_G)$ consists of merely visitors for the first time: the first part of $\tilde{D}_N^{G\prec N}(p, p_G)$ corresponds to demand for $\epsilon < \hat{\epsilon}_N$; the second part of $\tilde{D}_N^{G\prec N}(p, p_G)$ is $F(\hat{\epsilon}_G)(1 - F(\hat{\epsilon}_N))$ representing those more into the niche good, where

$$\hat{\epsilon}_i(p, p_i, p_j) = \frac{\mu_j \min(\hat{\epsilon}_j(p_i, p_j), 1) + p - p_j}{\mu_i}$$

and its value should be smaller than or equal to the upper bound of the distribution. Any deviation by Firm N can only be observed by those who search it. In an equilibrium, no firms have incentives to deviate. The FOC deliver the equilibrium prices:

$$p_G = (\mu_G + p_N - p_G + c) - \frac{(p_N + \mu_N)^2}{4\mu_N}$$

and

$$p_N = \frac{\mu_N + \mu_G - p_N + p_G + 2\sqrt{\mu_N c}}{2} - \frac{(p_G + \mu_G)^2}{2(\mu_N + \mu_G + p_G - p_N - 2\sqrt{\mu_N c})}$$

Even after defining $\Delta = p_N - p_G$, there is no closed-formed expression for $\Delta$, which is the
main reason that our analysis stops at the numerical stage. If \( c = \frac{\mu_N}{16} \), the above yields the following solution: each firm behaves like a monopolist.

\[
p_N = \frac{\mu_N}{2} \text{ and } p_G = \frac{\mu_G}{2}
\]

If \( p_N = \frac{\mu_N}{2} \), the optimal stopping rule becomes

\[
\hat{\epsilon}_G(p_N, p_G) = \frac{\mu_N + p_G - p_N - 2\sqrt{\mu_N c}}{\mu_G} = \frac{p_G}{\mu_G}
\]

Niche-Generic Ordering

In the niche-generic order, if the generic firm keeps its price at the price expectation \( p_G \) and the niche firm deviates to a new price \( p \), the demand function of the niche firm is given by,

\[
\tilde{D}_N^{\prec G}(p, p_G) = \left(1 - \min(\hat{\epsilon}_N, 1)/2 + \int_{\min(\hat{\epsilon}_N, 1)}^1 Pr(u_{iN} > u_{iG}) f(\epsilon_N) d\epsilon_N / 4 \right)
\]

The demand function \( \tilde{D}_N^{\prec G}(p, p_G) \) consists of: the first part \((1/2)(1 - \min(\hat{\epsilon}_N(p, p_G), 1))\) corresponds to fresh demand of Firm N, and the second part corresponds to returning demand.

\[
\tilde{D}_G^{\prec G}(p_N, p) = \int_{p/\mu_G}^{\hat{\epsilon}_G} Pr(u_{iG} > u_{iN}) f(\epsilon_G) d\epsilon_G / 4 + (1 - \hat{\epsilon}_G)(1 + \min(\hat{\epsilon}_N, 1))/4
\]

The demand \( \tilde{D}_G^{\prec G}(p_N, p) \) consists of fresh demand. Deviations by the niche firm can be observed directly by each consumer, but any deviation by the generic firm can be observed only by those who search that generic firm. In an equilibrium, no firms deviate. The FOC deliver the
equilibrium prices:

\[ p_N = (\mu_N - p_N + p_G + c) - \frac{(p_G + \mu_G)^2}{4\mu_G} \]

and

\[ p_G = \frac{(\mu_N + \mu_G + p_N - p_G + 2\sqrt{\mu_Gc})}{2} + \frac{(p_N + \mu_N)^2}{2(\mu_N + \mu_G + p_N - p_G - 2\sqrt{\mu_Gc})} \]

If \( c = \frac{\mu_G}{16} \), \( p_N = \frac{\mu_N}{2} \) and \( p_G = \frac{\mu_G}{2} \) are the solutions of the above. The optimal stopping rule becomes,

\[ \hat{\epsilon}_N(p_N, p_G) = \frac{\mu_G + p_N - p_G - 2\sqrt{\mu_Gc}}{\mu_N} = \frac{p_N}{\mu_N} \]

When search cost is larger than \( \frac{\mu_G}{16} \), \( \hat{\epsilon}_N(p_N, p_G) < \frac{p_N}{\mu_N} \). There are some consumers who do not purchase at the first firm, and who do not search the second firm.

2.6.2 The Merged Firm

Occasionally firms do not make price decisions independently. The first possibility is that, the firm is a two-product firm. In practice, lots of firms develop more than one product. For instance, a restaurant typically has plenty of dishes to offer. The second possibility is that, firms can cooperate on their price decisions and share the joint profit. There are two stages in the game. In the first stage, the firms (or the multi-product firm) choose the ordering. In the second stage, the firms (or the multi-product firm) choose prices. From either ordering, consumers observe the price of the first firm directly and make their search decisions after. In the generic-niche ordering, the expected joint profit is a function of the price pair \((p_G, p)\) given
consumers’ belief about $p_N$:

$$\Pi_{col}^{G\prec N}(p_G, p) = p_G \tilde{D}_G^{G\prec N}(p_G, p) + p \tilde{D}_N^{G\prec N}(p_G, p)$$ (19)

In the niche-generic ordering, the expected joint profit is a function of the price pair $(p, p_N)$ given consumers’ belief about $p_G$:

$$\Pi_{col}^{N\prec G}(p, p_N) = p \tilde{D}_G^{N\prec G}(p, p_N) + p_N \tilde{D}_N^{N\prec G}(p, p_N)$$ (20)

Provided a price pair $(p_N, p_G)$, the market is divided in the same way as Figure 16. However, the two forms, along with the previous cases, are found to have quite different pricing patterns.

2.6.3 Prices

Firstly, prices are lower (more than 50%) if we apply assumption 4 to the model. This result is not surprising because with high prices only few consumers buy; one firm can deviate to a lower price and increases its profit.

Secondly, firms should have charged the same price since the expectation of the match is identical for each firm and each consumer. However from some ordered search models (Armstrong et al. (2009) and Zhou (2011)), the firm that is searched at the beginning charges a lower price than the rest. We call this pricing pattern distortionary pricing. We observe similar patterns from our duopoly model without the outside option. The distortion is primarily caused by search cost because it enforces the ordered search and prevents a consumer from getting better match. If the search cost is larger, the price difference becomes larger, which further discourages
consumers from searching. The distortion can also be caused by competition because the first firm intentionally lowers down the price to capture fresh demand. Accordingly the generic-niche ordering which induces more intense competition results in more distortion than the reverse ordering.

If the model considers the non-covered market, the price distortion either softens or completely disappears. Under duopoly competition, we still observe that $p_N - p_G$ is smaller under the niche-generic ordering at small search cost. It completely disappears for the merged firm, which is from the fact that the multi-product firm internalizes the competition. From our numerical results, the price of the second good is not necessarily larger than the price of the first good. For example, in the niche-generic ordering where $\mu_G/\mu_N = 0.8$, $p_N \approx 0.54$ and $p_G \approx 0.42$. In the perfect information case, this price pattern (firms charge the same price) completely disappears.

2.6.4 Percentage of Searchers

Price distortion results in the distortion of consumer search. In the perfect information case, there is clearly no distortion because price distortion does not play any role except lowering down its own profit. In many random search models (Wolinsky (1986) and Anderson and Renault (1999)), firms are identical and there is no distortion. When firms differ in order, they charge different prices, consumers no longer search only for what match them better, but they consider also the price difference.

When $\mu_G/\mu_N$ is too small and $c < c_{\text{min}}$, all consumers sample every firm as in the perfect information case. In general, percentage of searchers decreases in $\mu_{\text{first}}/\mu_{\text{second}}$ as (6) predicts, taking into account of the price difference. As a summary, the niche-generic ordering induces less searching activities.
Here is a reference with huge search cost and no outside options. When $\mu_G/\mu_N = 3/5$ and $c = 0.55$, 5.9% of consumers search the niche firm, compared with 4.8% of them to search the generic firm. This pattern is consistent in each case with the outside option. Here we merely present some numbers instead of a lengthy investigation. When two firms are competing (with the outside option), $\mu_G/\mu_N = 3/5$ and $c = 1/30$, there are 85.4% of searchers in the generic-niche ordering, compared with 75.8% in the reverse ordering. Even in the merged case (with the outside option), at the same parameters, there are 89% of searchers in the generic-niche ordering, compared with 77.5% in the reverse ordering.

\[ \text{Fig. 2.11: Percentage of searchers from the competition outcome} \]

\[ \begin{array}{c}
\text{Percentage of Searchers } c=0.025 \\
\text{Percentage of Searchers } c=0.01 \\
\end{array} \]

2.6.5 Industry Profit

If consumers consistently search more intensively in the generic-niche ordering, then products must be also more compared. Firstly, more comparison results in more intense competition and lowers down prices and industry profit. Secondly, more search infers better match quality, then more surplus can be extracted by firms through prices. Thirdly, lower prices actually contribute
to more purchases (less drop-offs). As for industry profit, the first effect is a direct reduction of profit, and the rest two are indirect and increase profit. Choosing product ordering is a trade-off between price and quantity.

Figure 12 compares industry profit for models with the outside option. The horizontal axis stands for the consumer heterogeneity ratio. The vertical axis stands for the joint profit.

Even though more consumers purchase goods and the market for each firm can be sometimes

**Fig. 2.12:** Industrial profit comparison under models with the outside option.

better targeted in the generic-niche ordering, Figure 12 shows that the niche-generic ordering does lead to a higher joint profit. Therefore the direct effect from softened competition dominates the rest two effects.

First, this result holds for the duopoly model, and the model for a multi-product firm. The numerical results are solid wherever $c_{min} < c < \mu_j/16$ for $j \in \{N, G\}$. Second, curves are upward sloping, which suggests that the joint profit increases in the general heterogeneity of consumer taste. Last but not least, joint profit is slightly higher under collusion.
2. Ordered Search with Asymmetric Product Design

2.7 Welfare Consequences

Even though the previous model suggests to us that the niche-generic ordering increases industry profit, the primary issue is that the market is covered therefore firms tend to set very high prices and from equilibrium prices consumers typically obtain negative surplus. Moreover, the previous parts are silent about consumer surplus and welfare. We try other models to test the solidarity of our previous results. The following subsections concentrate on two types of models: in the first type, two firms are competing and the market is covered. We focus on three sub-cases: perfect information, generic-niche and niche-generic. The second family discusses the same set of sub-cases under the uncovered market: perfect information, generic-niche and niche-generic. And for each sub-case, we briefly discuss two situations. In the first situation, two firms are competing. In the second situation, there is a multi-product firm. The total nine cases shed light on how surplus is allocated between firms and consumers, and how product orderings have impacts on social welfare.

2.7.1 Consumer Surplus

Recall that a consumer's surplus is its utility minus the search cost he has incurred.

\[ u_{ij} - kc \]  

(21)

\( k \) is the number of times the consumer has searched. After visiting the first firm, he decides whether to invest some search cost to visit the next firm. If so and the match of the new product is higher, he takes the new one. If not, he sticks to the old one and pays the search cost. The search cost is sunk. If both products deliver negative utilities to the consumer, he chooses not
to buy anything.

Changing the ordering causes at least three effects from prices, product match and search cost. Under the \textit{niche-generic} ordering, on average less consumers visit the next firm, suggesting less incurred search cost. Fewer searches dampen competition and both firms set higher prices. Consumers pay more and more consumers drop off. On the contrary under the \textit{generic-niche} ordering, more consumers search. Prices are lower and consumers pay more search cost, which is as if some parts of search cost are converted into prices and two orderings produce trade-offs between prices, product match and search cost: either one searches more and pays less, or one searches less and pays more.

\textbf{Fig. 2.13:} Surplus comparison under covered market

If the market is covered, the consumer surplus shows one pattern. Consumer surplus is higher in the \textit{generic-niche} ordering. The reasons are that prices are much more higher in the \textit{niche-generic} ordering, and on average, more consumers search and obtain better match in the \textit{generic-niche} ordering.

The duopoly model with the outside option shows two patterns. First, consumer surplus increases in the overall consumer heterogeneity because the probability of obtaining a high match is larger when products are more consumer heterogeneous.\footnote{When we do not consider the prices, the probability of receiving lower matches than $\eta$ from both firms is \((1 + \eta/\mu_N)(1 + \eta/\mu_G)/4\), which decreases in both $\mu_N$ and $\mu_G$.} Second, the niche-generic ordering can induce higher consumer surplus than the other ordering when the search cost is...
large. Being exposed to wider possibilities of values of match can lead to higher surplus when the search cost is large and the search cost is not too much converted into prices. When the search cost is small, the effect on prices dominates the other two effects (on product match and search cost), so the niche-generic ordering makes consumers worse off.

But how many of them are actually better (worse) off from each ordering? At $c = 1/20$ and $\mu_G = 1/2$, the niche-generic ordering actually increases the surplus of around $25\frac{3}{8}\%$ of all consumers and decreases the surplus of around $20\frac{1}{8}\%$ of them, while the rest (around $54\frac{1}{2}\%$) are indifferent. At $c = 1/50$ and $\mu_G = 1/2$, the niche-generic ordering increases surplus of around $21\frac{2}{5}\%$ of consumers and decreases $22\frac{11}{10}\%$ of them, while the rest are indifferent.

Particularly with the merged firm, as we pointed out, the price distortion disappears. Therefore the effects on better match and search cost dominate the effect on prices. Then the niche-generic ordering delivers higher consumer surplus under collusion. Under the original model without the outside option, the niche-generic way makes consumers always worse off, probably because prices are too high in the niche-generic ordering.
2.7.2 Social Welfare

The figures below clearly support our view that the niche-generic ordering delivers more social surplus in general, except some rare cases with a huge search cost. Second, the social welfare increases in product heterogeneity of Firm G (when the other normalized to one) so it infers that more product heterogeneity increases social welfare. Last, the figures show that collusive outcome delivers less social welfare than the competitive outcome.

Fig. 2.15: Welfare comparison assuming the covered market.

![Figure 2.15](image1)

Fig. 2.16: Welfare comparison with assumption 4.
(1. Duopoly competition. 2. The merged firm)

![Figure 2.16](image2)

In any case when welfare comes mainly from consumers’ satisfactions from their match, prices are transfers and the only playing factor is how consumers are matched under different
circumstances. And the two main factors affecting product match are the search cost and the 
*price distortion*. In our original model without the outside option, both the search cost and the 
*price distortion* affect consumers’ search decision in a detrimental way for the social welfare. For 
instance under \( \text{Firm}_j \prec \text{Firm}_m \), consumers should have chosen \( \text{Firm}_j \) in case 
\[ \epsilon_j > \left( \frac{\mu_m}{\mu_j} \right) \epsilon_m \] 
but not \[ \epsilon_j > \hat{\epsilon}_j(p_j,p_m) \]. The search cost and the *price distortion* enter the searching threshold in a 
way that distorts the optimal search rule and prevents consumers to obtain their best match. 
From this reasoning, we are not surprised to see that the product ordering (the generic-niche 
ordering) which induces more distortion lowers down the social welfare. And this pattern is 
consistent in both the competition and collusion models, with and without the outside option.

2.8 Discussion

2.8.1 Implementation

In practice, product ordering is usually set by a third party. The third party can be an on-line 
store or a multi-product firm. In these cases, the third party chooses the product ordering which 
maximizes its profit, which is usually a share of the industry profit. From proposition 4 and 
some numerical results, we know that products are more often arranged in the *niche-generic* 
manner. If the third party is a social planner, it chooses the *niche-generic* ordering. Last, the 
product ordering can be determined by an auction.

2.8.2 Niche Market

According to the definition, a niche market is a small subset of market. Hence the niche 
product normally satisfies merely the needs of a certain minority group. However in our model,
the niche firm usually accounts for the majority of demand. Then the niche firm becomes actually the mass firm and vice-versa.

One solution is allowing varying qualities. Then the niche product is the one with lower quality but higher preference intensity, and a generic product is the one with higher quality but lower preference intensity. Then which ordering implies larger industry profit depends on the trade-off between anticipated quality difference and the willingness to search for better match. But the new model becomes less tractable.

Since the niche firm gains more than the generic firm in the original model, perhaps an easier alternative is to let more niche firms enter the market and keep the number of generic firms small. In practice, it is difficult for firms to target a broad population and increasing competition drives many firms into niche markets, partitions the market into many segments, makes the long-tail.

2.9 Conclusion

This paper tries to address the problem of optimal product ordering. There are two types of firms in the market, the niche firms and the generic firms. If the search cost is zero, firms are setting the same price and obtain the same amount of profit. If the search cost is positive, the second firm always sets a price larger than the first firm. The niche-generic ordering outcome augments industrial profit. The same ordering is preserved by the strategic consumers if firms believe them to visit in the niche-generic manner. Numerical examples show that the niche-generic ordering delivers higher consumer surplus only under a large search cost in a duopoly model with the outside option. Finally, numerical examples show that social welfare is almost always higher under the niche-generic way, because the same way has the least price distortion.
The major limitation is the uniform distribution used to simplify our analysis. Search models without assumptions about the distribution function are often-times not tractable. A usual assumption is the log-concavity of the density function. So far, there have been no papers applying this condition in search models with product heterogeneity.

In section 6, the model assumes that search cost is low enough that the situation never occurs when some consumers do not search the second product and do not buy the first product. If we relax this assumption for the two firms \( c > \max\{\mu_j/16\} \), the model degenerates and the two firms set prices to monopoly prices. If only one assumption is relaxed, then there are monopoly prices only in the \textit{niche-generic} ordering. It remains to see which ordering implies larger industry profit.

The paper restricts each consumer to unit demand. The model does not capture the fact that consumers are tempted to purchase more than one product upon different search paths. The \textit{impulsive purchase}, arising from a mixture of socio-economic and psychological effects, not only triggers new incentives for rearrangements in the usual sense, but also opens up the possibility of \textit{bundling}.

\section*{A Appendix A}

\subsection*{A.1 Proof of Proposition 1}

\textit{Proof.} We first look for equilibrium satisfying \( p_N - p_G + a(\mu_N - \mu_G) > 0 \). The derivative of \( D_G(p_N, p; c = 0) \) with respect to \( p \) is:

\[
D_G'(p_N, p; c = 0) = -\frac{1}{\mu_N} \int_{-a}^{a} f\left(\frac{\mu_G \epsilon_G + p_N - p}{\mu_N}\right)f(\epsilon_G)d\epsilon_G
\]  

\hspace{1cm} (A.1)
From the FOC conditions \( p_G = -\frac{D_G(p_N, p_G; c = 0)}{D_G'(p_N, p_G; c = 0)} \) and \( p_N = -\frac{D_N(p_N, p_G; c = 0)}{D_N'(p_N, p_G; c = 0)} \), we obtain the equilibrium prices as the following,

\[
p_G = \mu_N \frac{\int_{-a}^{a} F\left(\frac{\mu_G e_G + \Delta}{\mu_N}\right) f(e_G) \, \text{d}e_G}{\int_{-a}^{a} f\left(\frac{\mu_G e_G + \Delta}{\mu_N}\right) f(e_G) \, \text{d}e_G}, \quad p_N = \mu_N \frac{1 - \int_{-a}^{a} F\left(\frac{\mu_G e_G + \Delta}{\mu_N}\right) f(e_G) \, \text{d}e_G}{\int_{-a}^{a} f\left(\frac{\mu_G e_G + \Delta}{\mu_N}\right) f(e_G) \, \text{d}e_G}
\]

Since \( f \) is symmetric, we have \( F(x) = 1 - F(-x) \). Change the integrating variable from \( e_G \) to \(-e_G\).

\( p_N \) can be rewritten as,

\[
p_N = \mu_N \frac{\int_{-a}^{a} F\left(-\frac{\mu_G e_G + \Delta}{\mu_N}\right) f(e_G) \, \text{d}e_G}{\int_{-a}^{a} f\left(\frac{\mu_G e_G + \Delta}{\mu_N}\right) f(e_G) \, \text{d}e_G} = \mu_N \frac{\int_{-a}^{a} F\left(\frac{\mu_G e_G - \Delta}{\mu_N}\right) f(e_G) \, \text{d}e_G}{\int_{-a}^{a} f\left(\frac{\mu_G e_G + \Delta}{\mu_N}\right) f(e_G) \, \text{d}e_G}
\]

Subtract the above two equations. Then \( \Delta = 0 \) is an evident solution of the below equation.

\[
\Delta = \mu_N \int_{-a}^{a} \{ F\left(\frac{\mu_G e_G - \Delta}{\mu_N}\right) - F\left(\frac{\mu_G e_G + \Delta}{\mu_N}\right) \} f(e_G) \, \text{d}e_G
\]

Second, we look equilibrium satisfying \( p_N - p_G + a(\mu_N - \mu_G) < 0 \). The demand faced by the generic firm is,

\[
D_G(p, p_N; c = 0) = \int_{p - p_N - \mu_N a}^{p} \frac{p_N - p + \mu_G e_G}{\mu_N} \, \text{d}f(\epsilon_G)
\]

The derivative of the above with respect to \( p \) is,

\[
\frac{\partial}{\partial p} D_G(p, p_N; c = 0) = -\frac{1}{\mu_N} \int_{p - p_N - \mu_N a}^{p} f\left(\frac{p_N - p + \mu_G e_G}{\mu_N}\right) f(\epsilon_G) \, \text{d}e_G
\]

The first-order condition states that,

\[
p_G = \mu_N \frac{\int_{p - p_N - \mu_N a}^{p} \frac{p_N - p + \mu_G e_G}{\mu_N} f(e_G) \, \text{d}e_G}{\int_{p - p_N - \mu_N a}^{p} f\left(\frac{p_N - p + \mu_G e_G}{\mu_N}\right) f(e_G) \, \text{d}e_G}
\]
On the other hand, we have,

\[
 p_N = \mu_N \left( 1 - \int_{p_G=p_N-\mu_N}^{p_N+\mu_N} \frac{F\left(\frac{p_N-p_G+\mu_G\epsilon_G}{\mu_N}\right)}{\int_{p_G=p_N-\mu_N}^{p_N+\mu_N} F\left(\frac{p_N-p_G+\mu_G\epsilon_G}{\mu_N}\right) f(\epsilon_G) d\epsilon_G} f(\epsilon_G) d\epsilon_G \right)
\]

For \( p_N - p_G + a(\mu_N - \mu_G) < 0 \), \( p_N - p_G < -a(\mu_G - \mu_N) < 0 \). From the above equations, this is to say that,

\[
 p_N - p_G = \mu_N \left( 1 - 2 \int_{p_G=p_N-\mu_N}^{p_N+\mu_N} \frac{F\left(\frac{p_N-p_G+\mu_G\epsilon_G}{\mu_N}\right)}{\int_{p_G=p_N-\mu_N}^{p_N+\mu_N} F\left(\frac{p_N-p_G+\mu_G\epsilon_G}{\mu_N}\right) f(\epsilon_G) d\epsilon_G} f(\epsilon_G) d\epsilon_G \right) < 0
\]

which is equivalent to,

\[
 \int_{p_G=p_N-\mu_N}^{p_N+\mu_N} F\left(\frac{p_N-p_G + \mu_G\epsilon_G}{\mu_N}\right) f(\epsilon_G) d\epsilon_G > \frac{1}{2}
\]

The above is exactly the expression of \( D_G \). Since \( p_G > p_N \), it is not possible that firm G faces larger demand than firm N, which poses a contradiction. \( \square \)

### A.2 Proof of Proposition 2

**Proof.** On one hand, we look for price equilibrium satisfying \( p_N - p_G + \mu_N - \mu_G > 0 \). Substitute (9) and (10) into each other and we obtain the following

\[
 \Lambda(\Delta) = 2\Delta^2 - (3\mu_N + 3\mu_G + \sqrt{\mu_Nc})\Delta + (\mu_N - \mu_G)^2 - 2\mu_Nc + (\mu_N + \mu_G)\sqrt{\mu_Nc}
\]

The price equilibrium satisfies \( \Lambda(\Delta) = 0 \). Note first that the vertical axis of symmetry of this parabola is

\[
 \Delta = \frac{3\mu_N + 3\mu_G + \sqrt{\mu_Nc}}{4} > 0
\]
Second, \( \Lambda(0) = (\mu_N - \mu_G)^2 + (\mu_N + \mu_G)\sqrt{\mu_N c} - 2\mu_N c > 0 \). Therefore, the two roots of \( \Lambda(\Delta) = 0 \) are positive.

On the other hand, we look for price equilibrium satisfying \( p_N - p_G + \mu_N - \mu_G < 0 \), the solutions are the following equations:

\[
p_N = \frac{\mu_N(\mu_G - \Delta - c)}{(\mu_N - \sqrt{\mu_N c})}
\]

and \( p_G = c + \mu_G + \Delta \). Substitute one into the other, we obtain

\[
\Delta = \frac{(c + \sqrt{\mu_G})\sqrt{c} - 2c\sqrt{\mu_N}}{3\sqrt{\mu_N} - 2\sqrt{c}}
\]

Then

\[
p_N - p_G + \mu_N - \mu_G = \frac{(c + \sqrt{\mu_G})\sqrt{c} - 2c\sqrt{\mu_N} + (3\sqrt{\mu_N} - 2\sqrt{c})(\mu_N - \mu_G)}{3\sqrt{\mu_N} - 2\sqrt{c}}
\]

Normalize \( \mu_N \to 1 \). Then,

\[
p_N - p_G + \mu_N - \mu_G = \frac{3 + c - 2\mu_G + 2\mu_G\sqrt{c} - 4\sqrt{c}}{3 - 2\sqrt{c}}
\]

Let \( h(\mu_G, c) = 3 + c - 2\mu_G + 2\mu_G\sqrt{c} - 4\sqrt{c} \). It is clear that \( h(\mu_G, c) \) decreases in \( \mu_G \). Therefore \( h(1, c) > 0 \) is a sufficient condition for \( h(\mu_G, c) > 0 \). \( h(1, c) = (1 - \sqrt{c})^2 \geq 0 \). Therefore we have \( p_N - p_G + \mu_N - \mu_G > 0 \), which is a contradiction. \( \Box \)
A.3 Proof of Proposition 3


Equilibrium 1: We have the root satisfying

$$\Delta_1 = \frac{2c\sqrt{\mu_G} - \mu_N \sqrt{c} - c\sqrt{c}}{3\sqrt{\mu_G} - 2\sqrt{c}}$$

For $p_N - p_G + \mu_N - \mu_G > 0$, we have

$$(2c\sqrt{\mu_G} - \mu_N \sqrt{c} - c\sqrt{c}) + (\mu_N - \mu_G)(3\sqrt{\mu_G} - 2\sqrt{c}) > 0$$

which is equivalent to,

$$(\sqrt{\mu_G} - \sqrt{c})(c - \sqrt{\mu_G}c + 3\mu_N - 3\mu_G) > 0$$

For $c - \sqrt{\mu_G}c + 3\mu_N - 3\mu_G > 0$, we need either $13\mu_G < 12\mu_N$ or

$$13\mu_G \geq 12\mu_N \text{ and } c \in [0, c_1] \cup [c_2, c_G], c_1 = \left(\frac{\sqrt{\mu_G} - \sqrt{13\mu_G - 12\mu_N}}{2}\right)^2, c_2 = \left(\frac{\sqrt{\mu_G} + \sqrt{13\mu_G - 12\mu_N}}{2}\right)^2$$

Equilibrium 2: We find out that the price difference of the first equilibrium satisfies

$$\Delta = \frac{(4\mu_G(\mu_N - \Delta + c) - (\mu_G - \Delta - \mu_N)^2)}{2(\Delta + \mu_N + \mu_G)} - \frac{(\mu_N + \mu_G + \Delta + 2\sqrt{\mu_Gc})}{2}$$

Arranging it gives $\lambda(\Delta) = 0$, where

$$\lambda(\Delta) = 4\mu_G(\mu_N - \Delta + c) - (\mu_G - \mu_N - \Delta)^2 - (\mu_N + \mu_G + \Delta + 2\sqrt{\mu_Gc})(\Delta + \mu_N + \mu_G) - 2\Delta(\Delta + \mu_N + \mu_G)$$
which is a quadratic function in $\Delta$, with axis $\Delta = -(6\mu_N + 6\mu_G + 2\sqrt{\mu_Gc})/8 < 0$. Therefore, to prove both roots of $\lambda(\Delta) = 0$ to be negative, it suffices to prove that $\lambda(0) < 0$.

\[
\lambda(0) = -2(\mu_N^2 - 2\mu_N\mu_G + \mu_G^2) - 2\sqrt{\mu_Gc}(\mu_N - 2\mu_Gc + \mu_G) < 0
\]

Hence the equation $\lambda(\Delta) = 0$ has only negative roots and $\Delta = p_N - p_G < 0$. Denote the roots of $\lambda(\Delta) = 0$ as $\Delta_{11}$ and $\Delta_{12}$ (let $\Delta_{11} < \Delta_{12}$). From

\[
\lambda(-(\mu_N + \mu_G)) = 4\mu_G(2\mu_N + c) > 0
\]

we know that $\Delta_{11} < -(\mu_N + \mu_G)$, in which case

\[
p_N = \frac{4\mu_G(\mu_N + c - \Delta_{11}) - (\Delta_{11} + \mu_N - \mu_G)^2}{2(\Delta_{11} + \mu_N + \mu_G)} < 0
\]

Therefore, we exclude $\Delta_{11}$. For $\Delta_{12}$ to satisfy $p_N - p_G + \mu_N - \mu_G < 0$, we must have $\lambda(\mu_G - \mu_N) = 4\mu_G(c - \sqrt{\mu_Gc} + 3\mu_N - 3\mu_G) \leq 0$, which is equivalent to

\[
13\mu_G \geq 12\mu_N \text{ and } c \in [c_1, c_2]
\]

\[\square\]

### A.4 Proof of Corollary 1

**Proof.** Substitute the equations from proposition 3 into each other and we obtain that: $p_G = \sqrt{\mu_G}(3\mu_N - c)/(3\sqrt{\mu_G} - 2\sqrt{c})$. Take its partial derivative in $\mu_N$, $\mu_G$ and $c$, we obtain the following,

\[
\partial p_G/\partial \mu_N = 3\mu_G/(3\mu_G - 2\sqrt{c}) > 0, \partial p_G/\partial \mu_G = -\frac{\sqrt{c}(3\mu_N - c)}{\mu_G(3\sqrt{\mu_G} - 2\sqrt{c})^2} < 0
\]
\[
\frac{\partial p_G}{\partial c} + 2\sqrt{\mu_G(3\sqrt{\mu_G} - c)} > 0
\]

Firm N’s price,
\[
\frac{\partial p_N}{\partial \mu_N} = 1 + \sqrt{c}/(3\sqrt{\mu_G} - 2\sqrt{c}) > 0, \quad \frac{\partial p_N}{\partial \mu_G} = -\frac{\sqrt{c}(3\mu_N - c)}{2\sqrt{\mu_G}(3\sqrt{\mu_G} - 2\sqrt{c})^2} < 0
\]
\[
\frac{\partial p_N}{\partial c} = (4c\sqrt{c} + 3\mu_N\sqrt{\mu_G} + 6\mu_G\sqrt{c} - 11c\sqrt{\mu_G})/2\sqrt{c}(3\sqrt{\mu_G} - 2\sqrt{c})^2 > 0
\]

Firm G’s profit, \(\Pi_N^{<G} = \frac{\sqrt{\mu_G(\sqrt{\mu_G} - \sqrt{c})}(3\mu_N - c)^2}{4\mu_N\sqrt{\mu_G}(3\sqrt{\mu_G} - 2\sqrt{c})^3} \). Its derivatives in \(\mu_G\) and \(\mu_N\) are given by the below.
\[
\frac{\partial \Pi_N^{<G}}{\partial \mu_G} = -\frac{\sqrt{c}(3\mu_N - c)^2(\sqrt{\mu_G} - 2\sqrt{c})}{4\mu_N\sqrt{\mu_G}(3\sqrt{\mu_G} - 2\sqrt{c})^3} < 0
\]
\[
\frac{\partial \Pi_N^{<G}}{\partial \mu_N} = \frac{\mu_G(9\mu_N^2 - c^2)(\sqrt{\mu_G} - \sqrt{c})}{2\mu_N(3\sqrt{\mu_G} - 2\sqrt{c})^2} > 0
\]

Firm N’s profit, \(\Pi_N^{<G} = \frac{1}{2\mu_N}(c + \mu_N - \frac{(2c\sqrt{\mu_G} - \mu_N\sqrt{\mu_G} - c\sqrt{c})^2}{(3\sqrt{\mu_G} - 2\sqrt{c})} \). Its derivatives in \(\mu_G\) and \(\mu_N\) are given by the below.
\[
\frac{\partial \Pi_N^{<G}}{\partial \mu_N} = \frac{9\mu_G\mu_N^2 + \mu_N^2c + 2c^2\sqrt{\mu_Gc} - 6\mu_N^2\sqrt{\mu_Gc} - \mu_Gc^2 - c^3}{2\mu_N(2\sqrt{c} - 3\sqrt{\mu_G})^2} > 0
\]

The above is positive and the below is negative for \(c < \mu_G\). The niche firm’s profit increases in the consumer heterogeneity of the niche firm, and decreases in the consumer heterogeneity of the generic firm.
\[
\frac{\partial \Pi_N^{<G}}{\partial \mu_G} = \frac{\sqrt{c}(c - 3\mu_N)(c\sqrt{c} + \mu_N\sqrt{c} - c\sqrt{\mu_G} - 3\mu_N\sqrt{\mu_G})}{2\mu_N\sqrt{\mu_G}(2\sqrt{c} - 3\sqrt{\mu_G})^3} < 0
\]
\[
\frac{\partial \Pi_N^{<G}}{\partial c} = -\frac{(c\sqrt{c} + \mu_N\sqrt{c} - c\sqrt{\mu_G} - 3\mu_N\sqrt{\mu_G})(4c\sqrt{c} - 11c\sqrt{\mu_G} + 3\mu_N\sqrt{\mu_G} + 6\mu_G\sqrt{c})}{2\sqrt{c}\mu_N(3\sqrt{\mu_G} - 2\sqrt{c})^3} > 0
\]
In the last case, if the search cost is larger, more consumers are stuck by the first firm, which further raises the market power of both firms and both prices. Therefore the profit of the first firm increases, confirmed by the below.

\[ \text{A.5 Proof of Proposition 4} \]

\begin{proof}
there exists a unique equilibrium for the generic-niche ordering, but two sets of equilibria for the niche-generic ordering. We focus exclusively on the case that firms always choose extreme designs: \( \mu_G / \mu_N < 12/13 \). Prices, price differences and profits are homogeneous of degree one.\[11\]

From \( \Delta^{N<G} < 0 \) and \( \Delta^{G<N} > 0 \), consumers pay at least \( p_N^{N<G} \) in the niche-generic ordering, and at most \( p_N^{G<N} \) in the reverse ordering. Therefore it suffices to show \( p_N^{N<G} > p_N^{G<N} \). Let

\[ \varphi(\mu_N, \mu_G, c) = 2(3\sqrt{\mu_G} - 2\sqrt{c})(p_N^{N<G} - p_N^{G<N}) \]

The expression of \( p_N^{G<N} \) decreases in \( \Delta^{G<N} \). Use 0 to replace \( \Delta^{G<N} \) in the expression of \( p_N^{G<N} \) in \( \varphi(\mu_N, \mu_G, c) \) and obtain

\[ \psi(\mu_N, \mu_G, c) = (3\sqrt{\mu_G} - 2\sqrt{c})(\mu_N - \mu_G + 2c - 2\sqrt{\mu_N c}) - 2\sqrt{c}(2\sqrt{\mu_G c} - \mu_N - c) \quad (A.2) \]

Then \( \varphi(\mu_N, \mu_G, c) > \psi(\mu_N, \mu_G, c) \). Take the second derivative of \( \psi(\mu_N, \mu_G, c) \) in \( \mu_G \)

\[ \psi''_{\mu_G}(\mu_N, \mu_G, c) = \frac{-2c - 3\mu_N - 9\mu_G + 6\sqrt{\mu_N c}}{4\mu_G \sqrt{\mu_N}} < 0 \]

11. By increasing the values of \( \mu_N, \mu_G \) and \( c \) by a certain percentage, the equilibrium price and profit will increase by the same percentage. Therefore, the equilibrium prices and profits are homogeneous of degree one in search cost and consumer heterogeneity.
Because,

\[-3\mu_N - 9\mu_G + 6\sqrt{\mu_N c} = -3(\mu_N + 3\mu_G - 2\sqrt{\mu_N c}) < -3(\mu_N + 3\mu_G - 2\sqrt{\mu_N \mu_G}) < 0\]

Therefore \(\psi(\mu_N, \mu_G, c)\) is concave in \(\mu_G\). For each value of \(c\) and each \(\mu_G \in (c, \tilde{\mu}]\), there exists a \(\gamma\) that \(\mu_G = \gamma c + (1 - \gamma)\tilde{\mu}\). From the concavity of \(\psi(\mu_N, \mu_G, c)\), there is automatically

\[\psi(\mu_N, \mu_G, c) > \gamma \psi(\mu_N, c, c) + (1 - \gamma)\psi(\mu_N, \tilde{\mu}, c)\]

Normalize \(\mu_N \rightarrow 1\). Therefore, to prove \(\psi(\mu_N, \mu_G, c) > 0\) as \(\{\mu_G, c\} \in (c, \tilde{\mu}] \times [c, \tilde{c}]\), it is sufficient to verify the signs of two polynomials: \(\psi(\mu_N, c, c) = -\sqrt{c}(\sqrt{c} + 1)(\sqrt{c} - 3) > 0\) and \(\psi(\mu_N, \tilde{\mu} = 0.7, c) = 0.75 - 3.62\sqrt{\tilde{c}} + 5.67c - 2c\sqrt{\tilde{c}} > 0\). Then we reach our conclusion that the niche-generic ordering induces higher industry profit than the reverse for \(0 < \mu_G \leq 0.7\).

\[\square\]

\section*{A.6 Proof of Lemma 1}

\textit{Proof.} This corollary is equivalent to saying that \(\hat{\epsilon}_G > \hat{\epsilon}_N\). From (6), we define a function,

\[\psi(\mu_N, \mu_G, c) = (\mu_N + p^G_{\leq N} - p^G_N - 2\sqrt{\mu_N c})/\mu_G - (\mu_G + p_N^{N<\hat{N}} - p_N^G - 2\sqrt{\mu_G \tilde{c}})/\mu_N\]

The above decreases in \(\Delta^{G<\hat{N}}\). Define another function

\[\tilde{\psi}(\mu_N, \mu_G, c) = \mu_G(3\sqrt{\mu_G} - 2\sqrt{\tilde{c}})\psi(\mu_N, \mu_G, c)\]

We replace \(\Delta^{G<\hat{N}}\) with \(\tilde{\eta}(\mu_N, \mu_G, c)\), which is the upper bound function for \(\Delta^{G<\hat{N}}\), and obtain

\[\varphi(\mu_N, \mu_G, c) = (3\sqrt{\mu_G} - 2\sqrt{\tilde{c}})(\mu_N(\mu_N - \tilde{\eta} - 2\sqrt{\mu_N c}) - \mu_G\mu_G + 2\mu_G\sqrt{\mu_G \tilde{c}}) - \mu_G\sqrt{\tilde{c}}(2\sqrt{\mu_G \tilde{c}} - \mu_N - c)\]
Then $\varphi(\mu_N, \mu_G, c) > 0$ is the sufficient condition for $\psi(\mu_N, \mu_G, c) > 0$. First, $\varphi(\mu_N, \mu_G, c)$ is a concave function in $\mu_G$ because

$$\varphi''(\mu_N, \mu_G, c) = 16\sqrt{c} - \frac{9(5\mu_G + 2c)}{4\sqrt{\mu_G}} - \frac{3(\mu_N(4\mu_N - 8\sqrt{\mu_N c} - 3) + (9\mu_G - 4.94\sqrt{\mu_N c}))}{16\mu_G \sqrt{\mu_G}} < 0$$

in case $\mu_G \gg c$. Normalize $\mu_N \to 1$. Let $\bar{\mu} = 0.7$. Then for each $c$, and each $\mu'_G \in (c, \bar{\mu}]$. There must exist a $\gamma$ that $\mu'_G = \gamma c + (1-\gamma)\bar{\mu}$. From the strict concavity of $\varphi$, we have that

$$\varphi(1, \mu'_G, c) > \gamma \varphi(1, c, c) + (1-\gamma)\varphi(1, \bar{\mu}, c)$$

Therefore it suffices to prove that $\varphi(1, c, c) > 0$ and $\varphi(1, \bar{\mu}, c) > 0$. Second, $\varphi(1, c, c) = \sqrt{c}(1 + c + \sqrt{35c - 9\sqrt{c}})/4 > 0$ while $c \in [0, 0.135]$. Third, $\varphi(1, \bar{\mu}, c) \approx 1.18 - \sqrt{c}(2.95 - 0.98\sqrt{c} - 0.7c) > 0$ while $c \in [0, 0.28]$. We reach our conclusion that the niche-generic ordering induces less consumers to check out both firms.

\[\square\]

### A.7 Proof of Proposition 5

**Proof.** There are three parts in the proof.

(i) From $p_{N \prec G}^N < p_{G \prec G}^N$, the niche firm’s demand share is more than half. Hence the niche firm’s profit is larger than $p_{N \prec G}^N$ in the niche-generic ordering. In the generic-niche ordering, the niche firm’s demand share is less than half, because $p_{G \prec N}^G > p_{G \prec N}^G$. Therefore the niche firm’s profit is lower than $p_{N \prec G}^N$ in the generic-niche ordering. The section A.5 shows that $p_{N \prec G}^N > p_{G \prec N}^G$ for $\frac{\mu_G}{\mu_N} < \frac{7}{10}$. Then it is clear the niche firm receives larger profit in the niche-generic ordering.
(ii) Next, we will look for conditions for the below to hold when $\mu_G \in \left[\frac{7}{10}, \frac{12}{13}\right]$: 

\[
\Pi_N^{N\prec G} - \Pi_N^{G\prec N} = \frac{(\mu_N + c - \Delta_N^{N\prec G})^2}{2\mu_N} - \frac{(\mu_N + \mu_G - \Delta_N^{G\prec N} + 2\sqrt{\mu_Nc})^2(\mu_N + \mu_G - \Delta_N^{G\prec N} - 2\sqrt{\mu_Nc})}{16\mu_G\mu_N} > 0 \tag{A.3}
\]

Since $\Pi_N^{G\prec N}$ decreases in $\Delta_N^{G\prec N}$, we use zero to replace $\Delta_N^{G\prec N}$. Define a function

\[
\Gamma(\mu_G, c) = 8\mu_G(\mu_N + c - \Delta_N^{N\prec G})^2 - (\mu_N + \mu_G + 2\sqrt{\mu_Nc})^2(\mu_N + \mu_G - 2\sqrt{\mu_Nc})
\]

Hence $\Gamma > 0$ is a sufficient condition for $\Pi_N^{N\prec G} - \Pi_N^{G\prec N} > 0$.

First $\Gamma$ is a concave function in $\mu_G$ for $c < \frac{4}{25}$. Take the second derivative in $\mu_G$ of the first term and we obtain,

\[
\frac{\partial^2}{\partial \mu_G^2} 8\mu_G(\mu_N + c - \Delta_N^{N\prec G})^2 = \frac{4\sqrt{c}(e - 3)(6c^2 - 24\sqrt{\mu_Gc} - 8c\sqrt{\mu_Gc} + 9\mu_G + 6c + 3\mu_Gc)}{\sqrt{\mu_G}(3\sqrt{\mu_G} - 2\sqrt{c})^4} < 0
\]

Further

\[
- \frac{\partial^2}{\partial \mu_G^2} (\mu_N + \mu_G + 2\sqrt{\mu_Nc})^2(\mu_N + \mu_G - 2\sqrt{\mu_Nc}) = -(6 + 6\mu_G + 4\sqrt{c}) < 0
\]

Therefore it suffices to show that $\Gamma(\frac{7}{10}, c) > 0$ and $\Gamma(\frac{12}{13}, c) > 0$ for $c \in [0, \frac{4}{25}]$. For the limit of space, we will show it in the on-line appendix.

(iii) Finally we consider the case for $c > \frac{4}{25}$ and $\mu_G \in \left[\frac{7}{10}, \frac{12}{13}\right]$. Define,

\[
\bar{\Gamma}(\mu_G, c) = 8\mu_G(\mu_N + c)^2 - (\mu_N + \mu_G + 2\sqrt{\mu_Nc})^2(\mu_N + \mu_G - 2\sqrt{\mu_Nc})
\]

12. Then for each $\mu_G' \in \left(\frac{7}{10}, \frac{12}{13}\right]$. There must exist a $\gamma$ that $\mu_G' = \frac{\gamma}{10} + \frac{12(1-\gamma)}{13}$. From the concavity of $\Gamma$, we have that $\bar{\Gamma}(\mu_G', c) > \gamma\bar{\Gamma}(\frac{7}{10}, c) + (1-\gamma)\bar{\Gamma}(\frac{12}{13}, c)$.
Normalize $\mu_N \to 1$. Since $\Delta^{N < G} < 0$, it is clear that $\tilde{\Gamma} < \Gamma$. Hence $\tilde{\Gamma} > 0$ is a sufficient condition.

\[
\frac{\partial \tilde{\Gamma}}{\partial c} = \frac{4\sqrt{c}(1 + 3\sqrt{c} + 4\mu_G c + 5\mu_G) - (1 + \mu_G^2)}{\sqrt{c}}
\]

Since $\sqrt{c} > \frac{2}{5}$, it is easy to show that $\frac{\partial \tilde{\Gamma}}{\partial c} > 0$. It can be shown that, for $\mu_G \in \left[\frac{7}{10}, \frac{13}{13}\right]$,

\[
\tilde{\Gamma}(\mu_G, c; c = 0) = 32\mu_G - (1 + \mu_G)^3 > 0
\]

\[\Box\]

B Bound Functions of Price Difference

From the generic-niche order, we suppose that $\Delta^{G < N}$ satisfies the below:

\[
\Delta^{G < N} < \eta(\mu_N, \mu_G, c)
\]

where the bound function is defined as below:

\[
\eta(\mu_N, \mu_G, c) = \left(3\mu_N + 3\mu_G + \sqrt{\mu_N c} - \sqrt{35\mu_N \mu_G}\right)/4 \tag{B.1}
\]

Proof. Assume $c < \mu_G$. From transformations, it suffices to prove that

\[
\sqrt{35\mu_N \mu_G} < \sqrt{\mu_G^2 + \mu_N(17c + \mu_N - 2\sqrt{\mu_N c}) + 2\mu_G(17\mu_N - \sqrt{c})}
\]

Square both sides, the above can be reduced to,

\[
\mu_G^2 + 17\mu_N c + (\mu_N - \mu_G)(\mu_N + 2\sqrt{\mu_N c}) > 0
\]
The above is clearly positive, since \((\mu_N - \mu_G) > 0\). When the search cost is small, the difference between \(\Delta\) and \(\hat{\eta}(\mu_N, \mu_G, c)\) is smaller. We use the bound function to simplify the price difference function in the proof of Lemma 1.

\[\square\]

### C Incomplete Information w. Outside Option

#### C.1 Market Share

In the below figure, demand shares in the generic-niche order are shown on the left-hand side, and demand shares in the niche-generic order are shown on the right-hand side. \(D_{1,2}\) and \(T_{3,4}\) represent the demand of Firm G. \(T_{1,2}\) and \(D_{3,4}\) represent the demand of Firm N. \(D_5\) and \(T_5\) represent those drop-offs. Independent of the product order, \(D_5\) and \(T_5\) always have the same size. Assumption 4 ensures that \(D_5\) and \(T_5\) always have positive sizes. And, assumption 5 ensures that \(D_{2,4}\) and \(T_{2,4}\) always have positive sizes.

\[\text{Fig. 2.17: Demand share comparison from both orderings.}\]
C.2 The Merged Firm

The generic-niche ordering: $p_N$ is the expected price of the niche product. When the merged firm charges $(p, p_G)$, the profit function of the industry is,

$$\Pi^{G \ prec N}(p, p_G) = p_G \left( \frac{1 - \hat{\epsilon}_G}{2} + \frac{1}{4} \int_{p_G}^{p} \left( 1 + \frac{p - p_G + \mu_G \epsilon_G}{\mu_N} \right) \partial \epsilon_G \right)$$

$$+ \frac{p}{4} \left( \int_{p_N}^{p - p_N + \mu_G \epsilon_G} \frac{\mu_N \epsilon - p + p_G}{\mu_G} \partial \epsilon_N + (1 + \hat{\epsilon}_G) \left( \frac{2 \sqrt{\mu_N c} - p + p_N}{\mu_N} \right) \right)$$

The first-order conditions are,

$$\frac{\partial}{\partial p_G} \Pi^{G \ prec N}(p_N, p_G) = -\frac{3p_G^2 - 4p_G \mu_G - \mu_N (4c + 4\mu_G - \mu_N - 8p_G)}{8\mu_N \mu_G} = 0$$

and $\lim_{p \to p_G} \frac{\partial}{\partial p} \Pi^{G \ prec N}(p, p_G) = 3p_G^2 - 4\mu_G c + 4(p_N - p_G) \sqrt{\mu_N c} + \mu_N (4p_G + \mu_N + 2\mu_G) - 2p_N (3p_G + 2(\mu_N + \mu_G)) = 0$.

The niche-generic ordering: $p_G$ is the expected price of the generic product. When the merged firm charges $(p_N, p)$, the profit function of the industry is,

$$\Pi^{N \ prec G}(p_N, p_G) = p_N \left( \frac{1 - \hat{\epsilon}_N}{2} + \frac{1}{4} \int_{p_N}^{p} \left( 1 + \frac{p - p_N + \mu_N \epsilon_N}{\mu_G} \right) \partial \epsilon_N \right)$$

$$+ \frac{p}{4} \left( \int_{p_G}^{p - p_G + \mu_N \epsilon_G} \frac{\mu_G \epsilon - p + p_N}{\mu_N} \partial \epsilon_G + (1 + \hat{\epsilon}_N) \left( \frac{2 \sqrt{\mu_G c} - p + p_G}{\mu_G} \right) \right)$$

The first-order conditions are,

$$\frac{\partial}{\partial p_N} \Pi^{N \ prec G}(p_N, p_G) = -\frac{3p_N^2 - 4p_G \mu_G - \mu_G (4c - 8p_N + 4\mu_N - \mu_G)}{8\mu_N \mu_G} = 0$$

and $\lim_{p \to p_N} \frac{\partial}{\partial p} \Pi^{N \ prec G}(p_N, p_G) = 3p_G^2 - 4\mu_G c + 4(p_G - p_N) \sqrt{\mu_G c} + \mu_G (4p_N + \mu_G + 2\mu_N) - 2p_N (3p_G + 2(\mu_N + \mu_G)) = 0$. 
\[ 2(\mu_G + \mu_N) = 0 \]

## D Perfect Information w. Outside Option

### D.1 The Duopoly Model

Under perfect information, it is a weakly dominant strategy for each consumer to visit each firm. Hence each price and match information is perfectly observable by each consumer. Consider the case that \( \Delta < \mu_N - \mu_G \). From the price pair \((p_N, p_G)\), Firm N receives the demand given by,

\[
D_N(p_N, p_G; c = 0) = \frac{1 - (\mu_G + p_N - p_G)/\mu_N}{2} + \int_{p_N/\mu_N}^{(\mu_G + p_N - p_G)/\mu_N} Pr(u_{iN} > u_{iG}) f(\epsilon_N) d\epsilon_N
\]

And provided the same price pair, Firm G receives the demand given by,

\[
D_G(p_N, p_G; c = 0) = \int_{p_G/\mu_G}^{1} Pr(u_{iG} > u_{iN}) f(\epsilon_G) d\epsilon_G
\]

Under the price equilibrium, none of them has incentives to change their prices. Therefore FOC deliver price equilibrium characterized by

\[
p_N = \frac{\mu_G(\mu_N - \Delta) - (\mu_G + p_G)^2}{4\mu_G}
\]

and

\[
p_G = \frac{(\mu_N + \mu_G + p_N - p_G)^2 - (\mu_N + p_N)^2}{2(\mu_N + \mu_G + p_N - p_G)}
\]

Consider the case that \( \Delta > \mu_N - \mu_G \), we get the price equilibrium,

\[
p_N = \frac{(\mu_N + \mu_G + p_G - p_N)^2 - (\mu_G + p_G)^2}{2(\mu_N + \mu_G + p_G - p_N)}
\]
2. Ordered Search with Asymmetric Product Design

and

\[ p_G = \frac{(\mu_N + p_N)^2 - 4\mu_N(\mu_G + p_N - p_G)}{4\mu_N} \]

From simulations we find that the equilibrium prices from above never satisfy \( \Delta > \mu_N - \mu_G \), which is a contradiction.

The equivalent prices and demand shares do not hold any more under this setting, because each of them faces a different outside option. Even by charging the same price, firms would not necessarily receive the same demand. Apart from the duopoly case provided above, the multi-product firm case is similar in that it borrows the same form of demand functions except having the objective function maximizing industry profit.

**D.2 The Merged Firm**

We consider first the case \( \Delta < \mu_N - \mu_G \). The firm receives profit from two products. From the generic product, the firm receives the first term of the following. From the niche product, the firm receives the second term of the following. The merged firm’s profit is,

\[
\Pi(p_G, p_N) = p_G \left( \frac{1}{4} \int_{\frac{p_G}{p_G}}^{1} \left( 1 + \frac{\mu_G \epsilon_G + p_N - p_G}{\mu_N} \right) d\epsilon_G \right) + \\
p_N \left( \frac{1}{2} \left( 1 - \frac{\mu_G + p_N - p_G}{\mu_N} \right) + \frac{1}{4} \int_{\frac{p_N}{p_N}}^{\frac{\mu_G + p_N - p_G}{\mu_G}} \left( 1 + \frac{\mu_N \epsilon_N + p_G - p_N}{\mu_G} \right) d\epsilon_N \right)
\]

The first-order conditions are \( 4\mu_G(\mu_N + p_G - 2p_N) + 2p_G(\mu_G - p_G) - (\mu_G + p_G)^2 = 0 \) and \( \mu_N(\mu_G - p_G)(2\mu_N + 2p_N + \mu_G - p_G) - 2p_G(\mu_N + \mu_G + p_N - p_G) - 2p_N(p_G - \mu_G) = 0 \).
Then consider the case $\Delta > \mu_N - \mu_G$. The merged firm’s profit is,

$$\Pi(p_G, p_N) = p_G \left( \frac{1}{2}(1 - \frac{p_G - p_N + \mu_N}{\mu_G}) \right) + \frac{1}{4} \int_{p_G}^{p_G - p_N + \mu_N} \left( 1 + \frac{p_N - p_G + \mu_G \epsilon_G}{\mu_N} \right) \partial \epsilon_G \right) +$$

$$p_N \left( \frac{1}{4} \int_{\epsilon_N}^{1} \left( 1 + \frac{p_G - p_N + \mu_N \epsilon_N}{\mu_G} \right) \partial \epsilon_N \right)$$

from the generic product

from the niche product

The first-order conditions are $4\mu_N(\mu_G + p_N - 2p_G) + 2p_N(\mu_N - p_N) - (\mu_N + p_N)^2 = 0$ and $\mu_G(\mu_N - p_N)(2\mu_G + 2p_G + \mu_G - p_N) - 2p_N(\mu_G + \mu_N + p_G - p_N) - 2p_G(p_N - \mu_N) = 0$.

From simulations, we could not find equilibrium prices satisfying $\Delta > \mu_N - \mu_G$. The only equilibrium prices (if they exist), satisfy $\Delta < \mu_N - \mu_G$. 
3. SEARCH COST AND FIRM AGGLOMERATION: THE CASE OF ON-LINE PLATFORMS

3.1 Introduction

Firm agglomeration is ubiquitous. Hotels, restaurants and malls form clusters near central business districts and train stations (and airports). For on-line shopping, these clusters are platforms (EBay, Amazon and Taobao.com). Many consumers prefer visiting the clusters to stand-alone firms. They visit these on-line stores because going through these goods takes less time. In case there is not any good match, they will consider search engines or firms around the corner and look for surprises. One observation is that, the market in the cluster is far more competitive than local markets. Take mountain bikes as the example, there are 58,828 listings in EBay and 69,070 listings in Amazon, but a couple of them in a sports shop. The other observation is that, each store in the platform typically lists many brands. For example, Asus, Lenovo and Sony are usually found in the same laptop shop, but not in separate shops.

What causes firm agglomeration? The usual explanation is that competition implies lower prices which attract consumers to the cluster first. However intuitively, more than two firms in the cluster are enough to form lower prices. Each firm in the cluster gains larger profit than a firm outside the cluster. If the cost to search a firm outside the cluster is larger, more firms enter the cluster, leading to greater agglomeration.
However, this can not explain the large-scale agglomeration on the Internet where search cost is small. Firm agglomeration on the Internet is different in that the cost for firms to join the cluster is always endogenous and easy to change. Firms are either advertised in the platform, or reached only through a search engine by consumers. This paper tries to fill the gap by delving into the relationship between search cost and firm agglomeration in Internet markets with and without platforms.

We use a sequential search model and borrows product differentiation from Perloff and Salop (1985). Consumers with unit demand can either look for price and match information from the platform, or search it outside the platform. Each search entails an identical cost for each consumer. The platform sets a membership fee for firms who make simultaneous decisions to join the platform and provides consumers with product information in the platform for free.

The model exhibits some first results with an exogenous number of inside firms: prices inside the platform are lower than prices outside the platform and drop in the number inside firms. Prices inside and outside the platform increase in search cost because firms have more monopoly power if it is harder to obtain price and product information. When there are finitely many firms in the market, all prices are higher than the previous case with infinitely many firms because consumers have less choices so that each firm has more market power. Particularly in this case, prices of outside firms increase in the number of inside firms.

Lower prices are charged inside the platform for two reasons: 1) inside firms face more intense competition and their demand is more elastic. 2) Outside firms hold more match information of consumers than inside firms. Accordingly, outside firms set even higher prices after having observed more inside firms because more match information can be inferred from the visitors. Having expected this price pattern, consumers will indeed visit the platform before going elsew-
here. And expecting consumers’ searching path, firms do have incentives to pay some amount to join the platform.

The platform can choose the number of firms it hosts by adjusting the membership fee. The monopoly platform under-supplies product information to avoid too much inside competition which reduces the market performance. We show that this under-provision can be worsened because of larger search cost: the platform reduces its capacity if consumers find it harder to sample products. On the contrary, the platform expands its capacity if search cost is small, which increases the size of agglomeration. This relationship is solid when we consider a finite total number of firms in the market. In the presence of multiple platforms (or multiple shops in a single platform), each platform (shop) lists the same number of brands, and the number decreases in search cost. This relationship can explain the large-scale firm agglomeration on the Internet.

This paper is first related to search literatures: Wolinsky (1986) presents a sequential search model in which consumers search for price and match information. Anderson and Renault (1999) proves that prices increase in search cost and decrease in number of firms. Since firms do not always receive the same amount of attention, this paper is more closely related to Armstrong et al. (2009) in which the prominent firm charges the same price as the rest if the market has infinitely many firms, and a lower price if the market has finitely many firms. Armstrong et al. (2009) also finds out that the prominent firm earns more than a non-prominent firm, and earns more than it would with random search. For that reason, firms have incentives to be visited first. Zhou (2009) provides a more generalized framework, where finitely many firms are into two groups, and consumers incur different cost to search the two groups.

Two papers Arbatskaya (2007) and Zhou (2011) investigate ordered search. In case many
firms are located one after each other and consumers follow the same searching path, both find declining profit along the search path. Arbatskaya (2007) shows decreasing prices while Zhou (2011) finds the opposite price pattern. A simple explanation is that goods in Arbatskaya (2007) are homogeneous but are horizontally differentiated in Zhou (2011). In Zhou (2011), some consumers search even when they expect higher and higher prices, which would never happen in Arbatskaya (2007) where consumers look only for lower prices.

Fischer and Harrington (1996) is the closest paper by far, which extends Wolinsky (1983) and Wolinsky (1986) to study the incentives for firms to agglomerate. An agglomeration also takes place in our paper, yet only when chosen by a platform. The two papers are different in that the cost to join the cluster is exogenous in Fischer and Harrington (1996), but is set by the platform in our work. This difference in setting provides some different properties. From some parameter ranges, Fischer and Harrington (1996) predicts a larger cluster while our work predicts the opposite. Further, Konishi (2005) analyses a similar case of geographical concentration of stores. Moraga-Gonzalez and Petrikaite (2013) studies conditions under which firms selling horizontally differentiated products have incentives to merge.

On two-sided markets, Baye and Morgan (2001) studies an internet gatekeeper gathering all price information and charging fees on both sellers and buyers. Firms and buyers choose whether to subscribe to the platform. In Galeotti and Moraga-Gonzalez (2009), firms and buyers play mixed strategies on entering the platform or staying outside.

The rest of the paper is structured as the following: Section 2 shows the model with a monopoly platform. In Section 3, the prices in equilibrium are found along with the profit maximizing number of firms in the platform. In section 4, we briefly discuss the competition among stores in the platform. In section 5, the platform can also charge the buyers who become
subscribers. As a result, the social optimum is restored. In section 6, we illustrate some simple results with finitely many firms. Section 7 concludes.

3.2 The Model

There exists one unit mass of consumers and an infinite number of firms with zero marginal cost in the market. Consumers are restricted to purchase at most one product, as in other discrete choice models. As in Perloff and Salop (1985), consumer $i$’s utility if she buys product $j$ is:

$$u_{ij} = \epsilon_{ij} - p_j$$

$\epsilon_{ij}$ is the match between firm $j$’s product and consumer $i$, which is an i.i.d. random variable under a distribution function $F$ with support $[v, \bar{v}]$ satisfying $(\bar{v} > 0 \text{ and } v \leq 0)$. $p_j$ is the price set by firm $j$. There is a platform in the market. Some firms are in the platform, which we later call "inside firms". The rest of the firms are called "outside firms". Denote the number of inside firms as $m$. Without loss of generality, these firms are indexed from 1 to $m$. The platform informs consumers on products and prices of inside firms. Consumers can either enter the platform and check out all inside goods for free or see through outside firms, each at the search cost $c > 0$. The cost is identical for all consumers and for each search. For simplicity, we assume that all firms expect consumers to visit inside firms first.\[1\]

Having visiting all firms in the platform, a consumer faces three choices: 1) stay in the

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1. We assume these so that: first, demand is always positive. Second, the market is not always covered.
2. A tourist finds that the restaurant is not interesting only after he travelled and visited it. A computer user may find a software package unattractive only after 15 days’ free trial.
3. Otherwise, firms can expect consumers to search goods in many possible paths: 1) consumers may run into the platform while searching goods randomly. 2) Consumers may refrain from the platform while expecting high prices there. We make this assumption on the consumers’ part and show after that other strategies are weakly dominated.
platform and buy a good. 2) Leave the platform and search outside firms. 3) Leave without buying anything. A consumer searches not only for better prices, but also for the goods fitting his own tastes. Before searching outside firms, he has formed expectations about their prices. Each round of search offers him price and match information of one firm. Having searched one firm, the consumer can freely recall that firm at any time. The consumer purchases at one firm if the three following conditions are satisfied: 1) the product offers him a positive utility (i.e. $u_{ij} > 0$). 2) The same product offers him the highest utility hitherto. 3) Searching another firm yields lower expected surplus. If he has visited all firms and all of them offer him a negative utility, he chooses not to make any purchase.

We denote firm $j$’s strategy as $\sigma_j = \{I_j, p_j\}$. A firm makes two decisions: the first decision is whether to join the platform. $I_j = \text{in}$ indicates that firm $j$ joins the platform. The opposite ($I_j = \text{out}$) indicates that it chooses not to join the platform. The second decision is the price. Let $p_{I_j}$ be the expected symmetric equilibrium price set by firm $j$. Therefore, inside firms and outside firms are expected to charge $p_{\text{in}}$ and $p_{\text{out}}$. Let $\Delta$ be the expected price difference $p_{\text{in}} - p_{\text{out}}$. The demand function $D_{I_j}(p_j, p_{-j}; m)$ indicates firm $j$’s expected demand when it charges $p_j$ and other firms charge $p_{-j}$ provided $m$ inside firms. Denote $\Pi_{I_j}(p_j, p_{-j}, w; m)$ as firm $j$’s expected profit. All inside firms must pay membership fees $w$ to the platform for entrance, which is the only source of revenue for the platform (i.e. no per transaction fees).

**Timing**

in this extensive-form game with imperfect information, three types of players are: the platform, firms and consumers. Match realization is decided by nature. The platform does not observe any match information (neither can firms) while setting \( w \), although it knows the distribution of the match value. A firm can observe other firms’ decisions of whether to enter the platform before setting its price. Firms set prices simultaneously. Consumers can observe the platform’s size and prices set by inside firms. They learn price and match information of outside firms only through searching them. However, they have beliefs about the distribution of prices outside the platform.

Suppose there are already \( m' \) entries. Having observed the \( m' \) entries, the marginal firm compares its expected future profit after his entrance with the entrance fee \( w \). If its expected future profit exceeds the fee, it decides to join the platform. Otherwise, it stays outside.

The pay-off of a consumer \( i \) is simply his surplus denoted by \( u_{ij} - kc \) from purchasing a good, where \( k \) is the number of times he has searched, and zero by walking away. The pay-off of firm \( j \) is its expected profit, denoted by

\[
\Pi_i(p_j, p_{-j}, w; m) = p_jD_{I_j}(p_j, p_{-j}; m) - w \times I_j
\]
3. Search Cost and Firm Agglomeration: The Case of Online Platforms

The pay-off of a platform is its collection of entrance fees:

$$\Pi^p(w) = m(w)w$$

### 3.3 Equilibrium Analysis

#### 3.3.1 Optimal Search Rule

Since firms inside the platform are freely accessible, every consumer then samples all inside firms to find out the best offer. Suppose that a consumer has his current best offer from firm $j$, which is a pair $\{\bar{\epsilon}, p\}$. Then his next step is to choose between taking the current best offer immediately or searching for a better offer. If he samples an additional firm and the newly sampled product offers $\{\epsilon, p'\}$, he will prefer the new product if $\epsilon - p' > \bar{\epsilon} - p$. Based on his expectation about prices outside the platform $p_{out}$ and the search cost $c$, the expected incremental surplus from sampling another firm is,

$$\int_{p + p_{out} - p}^{\bar{\epsilon}} (\epsilon - p_{out} - (\bar{\epsilon} - p))dF(\epsilon) - c$$

The first (second) term above is the expected incremental benefit (cost) from another search. From the cost-benefit analysis, he will sample another outside firm if the above is positive, and take the current best offer if otherwise. The optimal search rule is for the consumer to decide at which points he should stop and purchase the current best offer. Since the expected incremental benefit from another search decreases in $\bar{\epsilon} - p + p_{out}$, there exists at most one point where the
consumer is indifferent between both options (i.e. (2) holds).

\[
\int_{\bar{\epsilon}+p_{out}-p}^{\bar{\epsilon}} (\epsilon - p_{out} - (\bar{\epsilon} - p))dF(\epsilon) = c
\]  

(2)

In a symmetric equilibrium \((p = p_{out})\), denote the unique solution for \(\bar{\epsilon}\) by \(a\), so that \(a\) satisfies 
\[
\int_{a}^{\bar{\epsilon}} (\epsilon - a)dF(\epsilon) = c.
\]

When the current best match is larger than \(a\), the consumer purchases the best match. Otherwise, he moves on to another firm. From \[Anderson and Renault (1999),\] this stopping rule is stationary and does not depend on the number of firms left. Therefore consumers face the same threshold while deciding whether to search the 1st outside firm, and the \(N^{th}\) \((N > 1)\) outside firm.

If firms charge different prices \((p \neq p_{out})\), the corresponding threshold becomes \(a - p_{out} + p\). The threshold increases in current price and decreases in expected price of outside firms. When a solution for (2) exists, the optimal search rule can be summarized as the following:

\[
|a - p_{out}|
\begin{cases}
> \bar{\epsilon} - p \text{ search again} \\
< \bar{\epsilon} - p \text{ buy the best match}
\end{cases}
\]

The threshold further decreases in search cost if the search cost is not too large. Therefore consumers are more likely to take the current offer if it is costly to sample another product.

In case the search cost becomes too large, consumers do not necessarily participate in searching firms. While \(a < p_{out}\), \(a - p_{out}\) is even below the outside option. Each consumer is then content with the outside option and will not perform any search.
3.3.2 Profit Maximization by Firms

Suppose other inside (outside) firms are charging the same price $p_{in}(p_{out})$ and firm $j$ is freely choosing price $p$. After checking out all inside firms at the beginning, a consumer faces three choices: choose firm $j$ to buy a good, search randomly outside the platform or drop out without any purchase. The first choice is better if the following two conditions are satisfied:

- Firm $j$’s good is the best match for him in the platform: $\epsilon_j - p \geq \max\{\epsilon_k - p_{in}, 0\}$, $\forall k \leq m$.

- A random search will not give him a higher incremental utility than the search cost: $\epsilon_j > a - p_{out} + p$.

For a given realization of $\epsilon_j$, the probability for firm $j$ to be a better match than any other inside firm is $F^{m-1}(\epsilon_j + p_{in} - p)$. For the second condition to be satisfied, $\epsilon_j$ must be larger than $a - p_{out} + p$, which partly gives the lower bound for the integral of (3). So the joint probability is $\Pr(\epsilon_j - p > \max\{\epsilon_k - p_{in}, a - p_{out}\})$ for $a \geq p_{out}$, and $\Pr(\epsilon_j - p > \max_{k \neq j}\{\epsilon_k - p_{in}, 0\})$ for $a < p_{out}$. In case $a + p - p_{out} < \bar{\epsilon}$ or $a - p_{out} < 0$, no consumers leave the platform and outside firms receive no demand. We have assumed that $\bar{\epsilon} < 0$, which has two implications: 1) $\bar{\epsilon} - p < 0$. 2) $a + p - p_{out} < \bar{\epsilon}$ then implies $a - p_{out} < 0$. Here is the demand function of an inside firm:

$$D_{in}(p, p_{out}, p_{in}; m) = \int_{\max\{p, a + p - p_{out}, \bar{\epsilon}\}}^{\bar{\epsilon}} F^{m-1}(\epsilon + p_{in} - p) dF(\epsilon) \tag{3}$$

If $a \geq p_{out}$, fraction $F^m(a - p_{out} + p_{in})$ of them are not satisfied by any good in the platform after checking out all inside firms. And they leave to search outside the platform. An outside

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4. In case where $a < p_{out}$, the firms set prices to $p_{out}$ if some consumers search. We learn from the previous example that no consumers conduct search after considering their expected surplus. Hence $p > a - p_{out} + p$. If $\tilde{\epsilon} < p$, the consumer chooses not to buy anything. The lower bound of the integration needs to be adjusted to $p$. 
firm could be the consumer’s 1st visit (2nd, 3rd, 4th...∞th). The probability that it is his Sth visit is \( \frac{S}{N} \), where N is the total number of firms. Assume that the ratio of the number of consumers to the number of firms is 1. By adding up the probabilities, we obtain the term \( \sum_{k=0}^{\infty} a^k \). An outside firm’s demand is:

\[
D_{Out}(p, p_{out}, p_{in}; m) = \begin{cases} 
F^m(a - p_{out} + p_{in})(\sum_{k=0}^{\infty} a^k)(1 - F(a - p_{out} + p)), & \text{for } a \geq p_{out} \\
0, & \text{otherwise}
\end{cases}
\]

(4)

From \( p_{out} = \frac{D_{Out}(p_{out}, p_{out}, p_{in}; m)}{\partial D_{Out}(p_{out}, p_{out}, p_{in}; m) / \partial p} \), it is easy to obtain the equilibrium price for outside firms:

\[
p_{out} = \frac{1 - F(a)}{f(a)}, \text{ for } a \geq p_{out}
\]

(5)

Outside firms are targeting only their fresh demand because everyone visits inside firms first and once a consumer passes an outside firm, he never returns. The equilibrium price increases in search cost, which is in line with Anderson and Renault (1999).

For \( a < p_{out} \), outside firms are not active because they receive no visitors. Fraction \( F^m(p_{in}) \) of consumers drop out without any purchase.

Consider \( a \geq p_{out} \) for inside firms, \( \max(p, a - p_{out} + p, \underline{\epsilon}) = a - p_{out} + p \). The derivative of firm \( j \)'s expected demand in \( p \) is,

\[
\frac{\partial}{\partial p} D_{In}(p, p_{out}, p_{in}; m) =
\]

\[
- F^{m-1}(a + p_{in} - p_{out})f(a + p - p_{out}) - (m - 1) \int_{a+p-p_{out}}^{\underline{\epsilon}} F^{m-2}(\epsilon + p_{in} - p)f(\epsilon + p_{in} - p)f(\epsilon) d\epsilon
\]

5. While \( a \geq p_{out} \), the only required condition for the consumer to purchase at an inside firm is \( \epsilon > a - p_{out} + p \). Because once he stops there, the surplus he obtain is \( \epsilon - p > a - p_{out} > 0 \). While \( a < p_{out} \), consumers choose not to search since from searching they expect a negative surplus.
Similarly for $a < p_{out}$ and $p_{in} = -\frac{D_{In}(p_{out},p_{out},p_{in},m)}{\partial D_{In}(p_{out},p_{out},p_{in},m)}$ we obtain the following:

$$p_{in} = \frac{1 - F_{m}(\max\{p_{in},a + \Delta\})}{m(f(\bar{v}) - \int_{\max\{p_{in},a + \Delta\}}^{\max\{p_{in},a + \Delta\}} F_{m-1}(\epsilon) f'(\epsilon) d\epsilon)} \quad (7)$$

**Proposition 1.** If $m$ firms in the market are inside the platform, there exists an equilibrium where all inside firms charge identical prices and all outside firms charge identical prices under the log-concavity of $f(\epsilon)$. Equilibrium prices are: (let $\Delta = p_{in} - p_{out}$)

$$a \geq p_{out} : \begin{cases} p_{out} = \frac{1-F(a)}{f(a)} \\ p_{in} = \frac{1-F_{m}(a + \Delta)}{m(f(\bar{v}) - \int_{a + \Delta}^{\max\{p_{in},a + \Delta\}} F_{m-1}(\epsilon) f'(\epsilon) d\epsilon)} \end{cases} \quad \text{and} \quad a < p_{out} : \begin{cases} p_{out} = \text{undefined} \\ p_{in} = \frac{1-F_{m}(p_{in})}{m(f(\bar{v}) - \int_{p_{in}}^{\max\{p_{in},a + \Delta\}} F_{m-1}(\epsilon) f'(\epsilon) d\epsilon)} \end{cases} \quad (8)$$

The outside firms set a higher (or equal) price than inside firms, whose both prices decrease in number of inside firms if $\epsilon$ is under uniform distribution or $f(\bar{v}) - F_{m}(a + \Delta)f(a + \Delta) < 0$.\[6\]

Both the inside price and the outside price increase in search cost.

**Proof.** See the appendix. \[7\]

The existence of price equilibrium inside the platform needs the log-concavity of $f(\epsilon)$\[7\] and implies the existence of price equilibrium outside the platform. The former suggests that each inside firm’s profit function is quasi-concave in its own price according to Caplin and Nalebuff (1991) and the latter is suggested by Anderson and Renault (1999).\[8\]

Another entrance can affect the inside prices in two directions: 1) competition becomes more intense so that each demand is more elastic. 2) Total inside demand becomes larger so

---

6. Besides the uniform case which we have proved separately. This list has included exponential, gamma, Laplace, normal and Weibull distribution.

7. The log-concavity of $f(\epsilon)$ means that the log of that function is concave.

8. This price equilibrium is also a natural extension of Armstrong et al. (2009) except that competition inside the platform is similar to Perloff and Salop (1985).
that there is a small market-expansion effect. In our model, the first effect outweighs the other so that inside prices drop in number of inside firms. The outside prices depend only on the level of search cost because changes can not have effects if not being observed. In short, inside firms compete with each other and the search option, and outside firms only compete with the search option. Last, consumers will indeed visit inside firms first because lower prices are charged in the platform.

An inside firm’s expected profit is:

$$\Pi_{In}(p_{in}, p_{out}, w; m) = D_{in}(p_{in}, p_{in}, p_{out}; m) p_{in} - w$$  \hspace{1cm} (9)$$

$w$ is the set-up cost of joining the platform. To simplify the analysis, we substitute $F$ with the uniform distribution from $[0, 1]$ from now on. In the uniform case, the condition for consumers

to search rationally \( a \geq p_{\text{out}} \) becomes \( a \geq \frac{1}{2} \) or \( c \leq \frac{1}{8} \). The equilibrium prices are:

\[
\begin{align*}
\text{\( a \geq \frac{1}{2} \) (active search):} & \quad p_{\text{out}} = 1 - a, \quad p_{\text{in}} = \frac{1 - (a + \Delta)}{m}, \quad a < \frac{1}{2} \\
\text{\( \text{(no search):} \) } & \quad p_{\text{out}} = \text{undefined,} \quad p_{\text{in}} = \frac{1 - p_{\text{out}}}{m}
\end{align*}
\]

(10)

We are about to show that the profit \( \Pi_{\text{in}}(p_{\text{in}}, p_{\text{out}}, w; m) \) also decreases in \( m \) so that the demand for entrance drops in number of existing inside firms. The platform thus faces a downward sloping demand for entrance.

Take the derivative of \( \Pi_{\text{in}} \) in \( m \). Since \( D_{\text{in}}(p_{\text{in}}, p_{\text{out}}, w; m) = p_{\text{in}} \) in the uniform case,

\[
\frac{\partial \Pi_{\text{in}}(p_{\text{in}}, p_{\text{out}}, w; m)}{\partial m} = 2p_{\text{in}} \frac{\partial p_{\text{in}}}{\partial m} < 0, \text{ since } \frac{\partial p_{\text{in}}}{\partial m} < 0
\]

On the effects from a higher search cost, the sign of the partial derivative of \( \Pi_{\text{in}} \) in \( a \) can be seen from the sign of the partial derivative of \( p_{\text{in}} \) in \( a \):

\[
\frac{\partial \Pi_{\text{in}}(p_{\text{in}}, p_{\text{out}}, w; m)}{\partial a} = 2p_{\text{in}} \frac{\partial p_{\text{in}}}{\partial a}, \quad \frac{\partial p_{\text{in}}}{\partial a} = \frac{(a + \Delta)^{m-1}}{1 + (a + \Delta)^{m-1}} \left( \frac{\partial p_{\text{out}}}{\partial a} - 1 \right), \text{ where } \frac{\partial p_{\text{out}}}{\partial a} = -1 \quad (11)
\]

Because \( p_{\text{in}} \) increases in \( c \), both inside (outside) prices increase in search cost.

**Proposition 2.** Under a Uniform(0,1) distribution, an inside firm receives higher demand and profit under higher search cost. Given a fixed set-up cost \( w \), an inside firm receives higher demand and profit under a lower number of inside firms.

Lower search cost means an easy access to outside goods. Consumers are therefore more tempted to visit outside firms, who charge also a lower price according to (11). To keep consumers in the platform, the inside firms have to lower prices which immediately lowers the threshold
\[ \hat{\epsilon} = a + \Delta. \]

With an infinitesimal search cost \((a \to \hat{\epsilon})\), \(\lim_{a \to \hat{\epsilon}} \frac{\partial p_{in}}{\partial a} = -1\), that is, the inside price falls almost one for one in \(a\). By comparison with \(p_{in}\), we have as well that \(\lim_{a \to \hat{\epsilon}} \frac{\partial p_{out}}{\partial a} = -1\). Under negligible search cost, the boundary between both types of firms is blurred. Price dispersion vanishes and the equilibrium profit for a firm drops to almost zero. On the contrary, when the search cost approaches infinity \((a \leq p_{out})\), no outside firms are then active. The inside firms’ profits are then not connected to the search cost since no consumers search.\(^9\)

Turning to the threshold, \(d\hat{\epsilon}/da = d(a + \Delta)/da = 1 + d(p_{in} - p_{out})/da\). Again from (11), it is clear that \(\hat{\epsilon}\) decreases in search cost in the uniform case. Lower search cost makes consumers more choosy about goods even when price decisions are endogenized. Last but not least, the whole industrial profit increases in \(c\), provided number of inside firms constant, because of less intense competition induced by fewer searching behaviours. However the platform might responds to a different search cost by different set-up costs, which is discussed in the next section.

### 3.3.3 Platform’s Pricing

Having observed \(w\), firms decide simultaneously whether to be "in" or "out". In this entry game, if one firm decides to be "in", it pays \(w\) to the platform and enters. Since an outside firm’s profit is negligible compared to an inside firm’s profit, the expected difference in profit between an inside firm and an outside firm is actually approximate to \(\Pi_{in}(p_{in}, p_{out}; m)\). \(\Pi_{in}(p_{in}, p_{out}; m)\) decreases in number of entrances and equals \(-w\) when there exist infinite entrances. The only (pure strategy) SPNE is a number \(m^*\) satisfying:

\[ D_{in}(p_{in}, p_{in}, p_{out}; m^*) p_{in} \geq w \]

and

\[ D_{in}(p_{in}, p_{in}, p_{out}; m^* + 1) p_{in} < w. \]

After the entrances, the outside firms are indifferent be-

---

9. For instance, if a certain number of brands sell goods in the city center and the rest of them are far from both the city center and each other. Then, whether they are 20 miles or 30 miles away will affect none of the prices since nobody visit them.
ween entering the platform and not entering it because the entrance fee is too high; the inside firms prefer to stay in the platform because they have already paid the fees.

On top of entry decisions and price decisions on firms’ part, the platform sets the set-up cost \( w \), which is the only way for the platform to determine its size \( m \), under a relation indicated by (3), (7) and \( w = D_{\text{in}}(p_{\text{in}}, p_{\text{in}}, p_{\text{out}}; m^*) p_{\text{in}} \). Hence choosing an entrance fee is indeed equivalent to choosing a size of the platform. Since an inside firm’s profit decreases in number of inside firms, the demand for entrance is downward sloping in \( w \). Therefore the platform’s profit could increase if each individual firm’s profit does not fall too fast.

The platform does not take every firm since then the joint profit is zero. Nevertheless, taking too few firms makes more consumers leave the platform and search outside firms. There is a trade-off between in-house competition and attractiveness of the platform, essentially the trade-off between price and quantity. The platform chooses the optimal number of brands exposed to consumers. For instance, if it contains only one brand, then the total profit will be \( (1 - F(a))^2 \) from (3) and (5).

The platform’s problem is to maximize its own profit, denoted as \( \Pi_p \):

\[
\max \, \Pi_p(w) = m(w) w = mD_{\text{in}}(p_{\text{in}}, p_{\text{in}}, p_{\text{out}}; m)p_{\text{in}} \ , \text{s.t.} \ p_{\text{in}} = \frac{1 - (\max(p_{\text{in}}, a + \Delta))m}{m} \quad (12)
\]

Define the elasticity \( \varepsilon(m) = -\frac{m\partial p_{\text{in}}}{p_{\text{in}}\partial m} \) as the percentage of drop of inside price with regard to the increase of the platform’s size. We assume first that the platform’s profit is quasi-concave in its size. Thus a globally optimal size exists. Take the first derivative of (12) with regard to \( m \) and
let it be zero.

\[ \frac{\partial}{\partial m} \Pi^p(m) = p_m^2 + 2mp_m \frac{\partial p_m}{\partial m} = p_m^2 \left( 1 - 2 \varepsilon(m) \right) = 0 \Rightarrow \varepsilon(m) = 1/2 \] (13)

The monopolist continues to take new firms until the elasticity reaches 1/2. While maintaining this elasticity at 1/2, we have the following proposition regarding different levels of search cost.

**Proposition 3.** in case the inside firms are charged for entrance fees and the buyers enter the platform for free: there exist a number of brands \( m \) maximizing the platform’s profit satisfying:

\[
(1 - (a + \Delta)^m(1 - \log(a + \Delta)^m))/(1 + (a + \Delta)^m - 1)(1 - (a + \Delta)^m) = 1/2
\]

In an active search equilibrium, the number of brands chosen by a monopoly platform decreases in search cost. In a no search equilibrium, search cost has no effects on its choice of brands.

**Proof.** See the appendix. ■

The below figure illustrates two cases: \( a = 1/2, a = 3/4 \). The search cost is higher when \( a = 1/2 \). The solid curves represent the decreasing relations between the equilibrium price and the number of inside firms. The dashed curves are iso-profit curves. The platform chooses the points where the isoprofit curve (12) and the price curve (10) are tangent to each other. The figure makes two points: 1) the platform receives a higher profit when \( a = 1/2 \). 2) The platform finds a smaller number of inside firms more profitable under a higher search cost. While \( a = 1/2 \), its most profitable size is roughly around 2; while \( a = 3/4 \), the platform prefers a size at around 4 or 5. To put differently, the platform offers two (4-5) brands, if by searching costly outside the platform, one naturally stops at an offer better than average (75th percentile). If the search
cost is extremely high \((c \text{ larger than } 1/8)\), the platform accepts two firms (see claim 1-2 in the appendix), and each outside firm sets its price to the monopoly price (in our case the monopoly price is exactly \(\frac{1}{2}\)). With an extremely low search cost \((c \approx 0)\), the platform sets an extremely low fee and provides consumers infinitely many choices. The large size of a platform on the Internet is due to low fees which can be further explained by low search cost. For example, Amazon and Taobao.com charge $0 for each listing and EBay charges $0.50 for almost every listing.

From (11), the platform faces a larger demand for prominence under a larger \(c\), and it is natural to offer more "slots" to meet the rising demand. However, more firms cause more competition, which might be counter-productive for the market performance. The usual "business stealing"
effect from a new entrant is internalized because the monopoly platform will finally extract all profits from the inside firms. The platform can only pull out consumer surplus through the inside firms. Due to the "non-appropriability of social surplus", it contains less than the efficient number of firms. From the standpoint of consumers, shops from one industry look more clustered than others if to look for product information in the former industry is easier (for example, electronic shops). This result is clearly different from Fischer and Harrington (1996) where the joining cost does not change corresponding to higher demand to join the cluster (platform). From their framework, the number of inside firms will be larger at higher search cost where the agglomeration should more likely take place, whereas our work predicts the opposite pattern.

To see how large is the loss of consumer surplus, we illustrate the impact of search cost (horizontal axis) on the expected consumer surplus (vertical axis) in the figure below. The platform takes 4 firms when search cost is around \( \frac{1}{25} \) and the expected consumer surplus is denoted by the up-left corner. Along the horizontal axis, the search cost gets larger and larger. If the platform keeps its number of inside firms unchanged, expected consumer surplus slowly decreases from the solid line because searches are harder and firms then charge higher prices. However at certain points \( c=0.053 \) and \( c=0.097 \), the platform reduces its size to 3 and 2, which de facto pushes more consumers towards outside firms. The dash line with two jumps is the actual curve of the expected consumer surplus after considering the choice by the platform. The gray region denotes the amount of surplus which the platform fails to appropriate from some higher search costs, which can be as large as total consumer surplus when the search cost is large.

The equilibrium fee charged by the platform is also unique. Suppose first that \( \frac{\partial \varepsilon(m)}{\partial m} \) is positive. From (13), there must exist a \( m^* \) where the value of \( p_{in}^2 (1 - 2\varepsilon(m)) \) is positive for \( m < m^* \).
suggesting that taking more inside firms leads to higher profit while \( \varepsilon(m) \) is small because the fall of \( w \) can be offset by the increase of \( m \). For \( m > m^* \), the platform’s profit starts to decrease in number of inside firms. The above suggests that the profit function for the platform \( \Pi_p(m) \) is quasi-concave. We take the FOC of (13) in \( a \) and obtain the following

\[
\frac{\partial \varepsilon(m)}{\partial a} + \frac{\partial \varepsilon(m)}{\partial m} \frac{\partial m}{\partial a} = 0 \Rightarrow sgn \frac{\partial \varepsilon(m)}{\partial m} = -sgn \left[ \frac{\partial \varepsilon(m)}{\partial a} \left( \frac{1}{\partial m/\partial a} \right) \right]
\]

The sign of \( \frac{\partial \varepsilon(m)}{\partial m} \) can be unveiled by the other two terms. The uniqueness of a maximum of \( \Pi_p(m) \) requires that \( \varepsilon(m) \) falls in \( a \), for which we have the following:

**Corollary 1.** the elasticity between the inside price and number of slots increases in search cost and number of firms. Therefore, the equilibrium membership fee is unique.

**Proof.** See the appendix.

Having more firms in the platform increases the elasticity. In order to keep \( \varepsilon(m) \) at \( \frac{1}{2} \), the platform prefers choosing fewer firms under a higher search cost, where \( \varepsilon(m) \) is generally larger. Particularly for \( a < \frac{1}{2} \), we show in appendix a.1 that the platform only selects two firms.

From simulations, the result is solid while taking a more general form of consumer utility function, or in other words, when the platform is able to provide consumers some surplus ei-
ther from some "consumer support" service or non-commercial contents exclusively provided
to buyers.\footnote{For example, some websites offer better delivery service, coupons, free product trials, free CDs and the
access to some other on-line content to whoever made a purchase on the website.} The other reason we introduce $\alpha$ is to compare the number of firms taken by the
platforms in the asymmetric duopoly case from section 4. To start, we use the alternative form
of the utility function,
\[ u'_{ij} = \alpha_1 + \epsilon_{ij} - p_i \]
where $\alpha_1$ is the surplus offered by platform 1. Then the inside price is determined as the
below.
\[ p_{in} = \frac{1 - \left( \max(p_{in} - \alpha_1, a - \alpha_1 + \Delta) \right)^m}{m} \]
When we take $\alpha_1 = 0.25$, the platform takes 5 firms when $a = 0.75$, and 11 firms when $a = 0.9$.
When we take the small surplus $\alpha_1 = 0.05$, the platform takes 5 firms when $a = 0.75$, and 12
firms when $a = 0.9$.

3.4 The Oligopoly Case

Here we consider a new situation: firms agglomerate in stores, and then stores compete
with each other. Each store is a platform in the previous section. Provided free entrance on
consumers’ part, each consumer typically checks out each store before making any decision.
Searching products across stores is seen as costless.\footnote{Also since each store offers free entrance for consumers, there exists no multi-homing on the firms’ side
because no firms are willing to pay twice the entrance fee.} Since firms produce weak substitutes to
each other according to (1), stores are homogeneous and differ in the number of inside firms.

Consider the following game: In the first stage, each store chooses its respective fee for the
firms. In the second stage, firms observe the fees and decide which store (platform) to join. In the third stage, firms set prices. In the last stage, consumers visit the stores.

We look for a symmetric equilibrium of store sizes. First consider the duopoly model. Store 1 takes the size of store 2 as given, and looks for its optimal size. Denote the expected sizes of the two stores respectively by \( \bar{m} \) and \( \bar{k} \). Its maximization program is:

\[
\max \quad \Pi_p(w) = m(w)w = mp_{in}^2, \text{ s.t. }\quad p_{in} = \frac{1 - (\max(p_{in}, a + \Delta))^{m+k}}{m+k}
\]  

(14)

**Proposition 4.** There exists a symmetric equilibrium, where two stores have an equal size and the total size is larger than one store. The industry profit is lower with two stores than with the monopoly platform.

**Proof:** In section 3, the monopoly platform (store) is maximizing \( mp_{in}^2(m) \). The monopoly platform (store) takes new entrants until \( \epsilon(m) = \frac{1}{2} \). However, in the duopoly case, two stores are maximizing respectively \( mp_{in}^2(m + \bar{k}) \) and \( kp_{in}^2(\bar{m} + k) \), and each takes new entrants until \( \epsilon(m + k) = \frac{1}{2} \), as the following:

\[
\frac{m \partial p_{in}(m + \bar{k})}{p_{in}(m + k) \partial m} = \frac{m \partial p_{in}(m + \bar{k})}{p_{in}(m + k) \partial (m + k)} = -\frac{1}{2}, \quad \text{and} \quad \frac{k \partial p_{in}(k + \bar{m})}{p_{in}(k + \bar{m}) \partial (k + \bar{m})} = -\frac{1}{2}
\]

If there exists an equilibrium, \( m = \bar{m} \) and \( k = \bar{k} \). Summing up the above yields:

\[
\frac{(k + \bar{m}) \partial p_{in}(k + \bar{m})}{p_{in}(k + \bar{m}) \partial (k + \bar{m})} = \epsilon(\bar{m} + \bar{k}) = 1
\]

---

12. Two remarks here: first, the competition between two stores is quantity competition rather than price competition due to the limit of space in the store (yellow pages are usually a few pages long). The conditions are similar to Kreps and Scheinkman [1983], where both choose their capacities in the first stage then the price in the second stage. Their results coincide with the Cournot model. Second, this competition happens in the 1st stage of the game.
We already know that \( \varepsilon(m) \) increases in \( m \) from corollary 1. Therefore the above suggests that two stores take more firms than one, consistent with most previous duopoly literatures. Besides, a higher resulting elasticity leads to a lower joint profit than the monopoly case, because the joint profit of two stores containing respectively \( m \) and \( k \) is equivalent to the profit of the single store with \( m + k \) entries. ■

For the oligopoly model, we have found similar results.

**Proposition 5.** there exists a symmetric equilibrium, where the stores' total size increases in number of existing stores, each existing store has the equal size and the size decreases in search cost.

**Proof**: suppose that there are \( H \) stores, with indices from 1 to \( H \). \( i \) is here the index of a store. Each store \( i \) chooses its number of firms inside \( q_i \) to maximize its profit function \( \Pi_i^p \) given that a certain store \( k \) is choosing \( \bar{q}_k \). This is equivalent to adding more firms until

\[
\varepsilon_i(q_i + \sum_{k=1,k\neq i}^H \bar{q}_k) = \varepsilon_i(q_i + \sum_{k=1,k\neq i}^H \bar{q}_k) = \varepsilon_i\left(\frac{q_i \partial p_{in}(q_i + \sum_{k=1,k\neq i}^H \bar{q}_k)}{p_{in}(q_i + \sum_{k=1,k\neq i}^H \bar{q}_k)}\partial q_i = -\frac{q_i \partial p_{in}(q_i + \sum_{k=1,k\neq i}^H \bar{q}_k)}{p_{in}(\sum_{k=1,k\neq i}^H \bar{q}_k)}\partial q_i + \sum_{k=1,k\neq i}^H \bar{q}_k = \frac{1}{2} \right)
\]

If there exists an equilibrium, \( q_i = \bar{q}_i \). Aggregating the above with all \( H \) stores, we have

\[
\varepsilon(\sum_{k=1}^H \bar{q}_k) = \sum_{i=1}^H \varepsilon_i(\sum_{k=1}^H \bar{q}_k) = -\frac{\sum_{k=1}^H \bar{q}_k \partial p_{in}(\sum_{k=1}^H \bar{q}_k)}{p_{in}(\sum_{k=1}^H \bar{q}_k)}\partial \sum_{k=1}^H \bar{q}_k = \frac{H}{2}
\]

From corollary 1, we know that \( \varepsilon(m) \) increases in \( m \). From the above, \( \varepsilon(\sum_{k=1}^H \bar{q}_k) \) increases in \( H \). Hence a larger \( H \) infers a larger \( \sum_{k=1}^H \bar{q}_k \), which is the total size of the stores. Again from corollary 1 we know that \( \varepsilon(m) \) increases in search cost. For the search cost \( c' > c \), the existence
of $\varepsilon(\sum_{k=1}^{H} q_k; c) = H/2$ infers that $\varepsilon(\sum_{k=1}^{H} q'_k; c') \geq H/2$. Since $\varepsilon(m)$ increases in $m$, the existence of $\varepsilon(\sum_{k=1}^{H} q'_k; c') = H/2$ further infers that $\sum_{k=1}^{H} q'_k \leq \sum_{k=1}^{H} q_k$. Since each store takes the same number of brands in a symmetric equilibrium, the number of brands taken by each store decreases in search cost.

Numerical results show that if $m \leq 2$ is the size for store 1 with $a < \frac{1}{2}$, then it is most profitable for store 2 to adjust its size to 2 ($k = 2$). However if store 1’s size is $m = \bar{n} > 2$, then it is optimal for store 2 to adjust its size to $k = \bar{n}$. In equilibrium, two stores in the platform usually have the same size. The similar case can be shown under $a > \frac{1}{2}$, but generally with more inside firms than when $a < \frac{1}{2}$.

Observations can be seen from electronic stores in a few developing countries (China, India, etc.), where most are found in a centralized market (e.g. the computer city). Each store in the market corresponds to a platform in our model. Travelling between stores takes few seconds and is considered costless. In this centralized market, one typically find each store sell almost every brand of cellphones and laptops. Few stores focus on one brand, because consumers will easily flow away.

More inside firms and lower prices suggest that consumers generally obtain better match, pay lower prices and search less, which further suggests that they pay less search cost in general. This answers how multiple platforms imply larger consumer surplus and lower industry profit.
3.5 Charging Consumers

From the model in section 2-4, consumers are certainly willing to pay some amount for the entrance to the platform, which may in turn impose a fee on the consumers. In practice, consumers purchase membership from on-line music stores (e.g. Deezer) and on-line TV (e.g. iTVmediaPlayer and Sky Go). While some platform contents are exclusively available through a hardware device (e.g. Amazon Kindle), some consumers purchase the devices. Because of this subscription fee, some consumers stay out of the platform.

Firms can charge consumers different prices, if they find out whether the consumers are subscribers or not. A price discrimination strategy might include three prices.\footnote{The first price is the price charged by the inside firms on members. The inside firms charge the non-members for the second price. And the outside firms charge all consumers for the third price.} Further, if the number of firms in the market is finite, this always leads to an ordered search and an unstationary search rule, because rational non-members can always identify inside (outside) firms by prices, and recall the number of them he has visited.\footnote{For a non-member with perfect memory, the current search is never the same as the next search. If the firm he just visited is an inside (outside) firm, he expects an higher (lower) price for the next search. Thus the demand function of a firm must include all possible orders a consumer may have.} If the number of firms is infinite, the chance to come across a particular inside firm is negligible, so that consumers would not take any past search into his optimal search rule. In order to have a stationary search rule, we only consider the case with infinitely many firms. Henceforth price discrimination is no longer an issue.

Denote the fraction of subscribers by $\rho$, which is chosen by the platform by the subscription fee (for consumers) $s$ and exogenous afterwards. $\tilde{p}_{in}$ and $\tilde{p}_{out}$ are the corresponding price expectations of an inside firm and an outside firm. For simplicity, we only consider the case where $a \geq 1/2$. For a subscriber, his searching threshold is given by $a + \tilde{p}_{in} - \tilde{p}_{out}$ in a similar way to (2). For a non-subscriber, his searching threshold is always $a$.
If an inside rm deviates to a new price \( p \) while other rms are keeping their prices to the expectation, the inside rm receives the demand given by:

\[
\hat{D}_{In}(p, \tilde{p}_{out}, \tilde{p}_{in}; m) = \rho \int_{a+p-\tilde{p}_{out}}^{\min\{1,1+p-\tilde{p}_{in}\}} (\epsilon - p + \tilde{p}_{in})^{m-1} d\epsilon + \rho \max\left(0, \tilde{p}_{in} - p\right) \tag{15}
\]

The above expression shows that the inside rm faces demand approximately equal to a proportion \( \rho \) of the demand from (3). Let \( \lambda = n_c/(n-m) \) be the ratio of number of consumers \( n_c \) to number of outside rms \( (n-m) \). If an outside rm deviates to a new price \( p \) while other rms are keeping the same price, its demand is given as follows:

\[
\lim_{n \to \infty} \hat{D}_{Out}(p, \tilde{p}_{out}, \tilde{p}_{in}; m) = \lambda \left\{ \rho(a + \tilde{p}_{in} - \tilde{p}_{out})^m + (1 - \rho) \frac{n-m}{n} \left( \sum_{k=0}^{n-m-1} a^k \right) \left( 1 - a + \tilde{p}_{out} - p \right) \right\} \tag{16}
\]

Provided that \( \rho \) is a positive constant and there exist infinitely many rms in the market, the demand from the non-subscribers is infinitesimal compared with demand from subscribers so that inside rms consider only subscribers while making price decisions. The outside rms face a blend of subscribers and non-subscribers. In case there exists an equilibrium price pair, both types of firms have no incentive to deviate to other prices. From the FOC, the equilibrium prices are again given by (10).

Next we turn to the consumer i’s surplus: let \( \hat{\epsilon} = a + \tilde{p}_{In} - \tilde{p}_{Out} \). We use \( \hat{V}_{In} \) to denote the expected surplus of a subscriber and \( \hat{V}_{Out} \) for that of a non-subscriber.

\[
E(\hat{V}_{In}) = \Pr\left( \max_{i,j,m} \epsilon_{ij} \leq m \geq a + \tilde{p}_{In} - \tilde{p}_{Out} \right) E\left[ \max_{\epsilon_{ij},m} \epsilon_{ij} \leq m, \max_{\epsilon_{ij},m} \epsilon_{ij} \leq m \geq a + \tilde{p}_{In} - \tilde{p}_{Out} \right] \\
\quad + \Pr\left( \max_{i,j,m} \epsilon_{ij} \leq m < a + \tilde{p}_{In} - \tilde{p}_{Out} \right) E\left[ \hat{V}_{Out} \mid \max_{\epsilon_{ij},m} \epsilon_{ij} \leq m < a + \tilde{p}_{In} - \tilde{p}_{Out} \right] - s \tag{17}
\]

$$E(\tilde{V}_{Out}) = E[\epsilon_i | \epsilon_i > a] - \sum_{k=0}^{\infty} a^{k-1} \left(1 - a\right)kc - \tilde{\rho}_{Out}$$ (18)

(17) and (18) can be understood by the fact that if the consumer is satisfied with one product in the platform, he obtains the conditional expected surplus given that that particular match is above the threshold $\hat{\epsilon}$, which corresponds to the $1^{st}$ term in (17). In case all inside firms provide match below $\hat{\epsilon}$, he obtains another expected quality conditioned on the fact that one particular match is above the threshold $a$ after some product search, which corresponds to the $2^{nd}$ term in (17). If the consumer does not attend the platform and searches directly outside firms, then his expected surplus is given by (18). (17) is equivalent to the following:

$$E(\tilde{V}_{In}) = \int_{a+\hat{\rho}_{In}-\tilde{\rho}_{Out}}^{1} m(\epsilon - \hat{\rho}_{In})\epsilon^{m-1}d\epsilon + \Pr\left(\max \epsilon_{ij,j} < \hat{\epsilon}\right) E[\tilde{V}_{Out} | \max \epsilon_{ij,j} < \hat{\epsilon}] - s$$

Particularly,

$$E(\tilde{V}_{Out}) = E[\epsilon | \epsilon > a] - c(1-a) \sum_{k=0}^{\infty} ka^{k-1} - \tilde{\rho}_{Out} = a - \tilde{\rho}_{Out}$$

15 On the consumers’ part, the subscription fee is the amount equating $E(\tilde{V}_{In})$ with $E(\tilde{V}_{Out})$,

$$s = \int_{a+\hat{\rho}_{In}-\tilde{\rho}_{Out}}^{1} m(\epsilon - \hat{\rho}_{In})\epsilon^{m-1}d\epsilon + (a - \tilde{\rho}_{Out}) \left((a + \hat{\rho}_{In} - \tilde{\rho}_{Out})^{m} - 1\right)$$ (19)

The above subscription fee is composed of two parts: the first part is the gain from information in the platform. The second part comes from the fact that subscribers have smaller chances to visit outside firms. The difference in profit between an inside firm and an outside firm is $\rho mp^2$.

15 The last equality comes from the relation between the threshold and the search cost. $c = (1-a)^2/2$. 
Since the platform is a monopoly and can extracts the whole surplus from both inside firms and subscribers. The platform’s profit is given by:

\[ \bar{\Pi}^p(w, s) = \rho m \tilde{D}_{In}(\tilde{p}_{in}, \tilde{p}_{out}, \tilde{p}_{in}; m) \tilde{p}_{In} + \rho s \]  

(20)

The above is monotonic in \( \rho \). Therefore the platform will continuously increase \( \rho \) until it reaches 1. In the end, the platform’s profit function is:

\[ \bar{\Pi}^p(w, s) = \frac{m}{(m + 1)} \left( 1 - (a + \tilde{p}_{In} - \tilde{p}_{Out})^{m+1} \right) - (a - \tilde{p}_{Out}) \left( 1 - (a + \tilde{p}_{In} - \tilde{p}_{Out})^m \right) \]  

(21)

**Proposition 6.** with an infinite number of firms in the market, if the platform is able to charge the consumers for subscription fees, then regardless of the value of search cost, the platform will enrol all consumers \( \rho = 1 \) and its profit increases in \( m \).

**Proof.** See the appendix. ■

From section 2-4, the platform charges only the firms, and an insufficient number of firms enter. The subscription fee s allows the platform to internalize consumer surplus, partially from the product match, and partially from search cost. The platform decreases gradually the firms' fee and increases the consumers' fee until it captures the whole difference in surplus between subscribers and non-subscribers. For this reason, consumers are indifferent between joining the platform and staying outside. It thus makes no difference to say "some consumers join the platform" and "the platform picks these consumers in". Social optimality is almost restored because the platform will include infinitely many firms, consumers have infinitely many choices and pay nearly zero for the product. The main difference from section 2-4 is that here the firms
will enter the platform for free and the platform charges mainly the consumers. Some comparable results are shown in Galeotti and Moraga-Gonzalez (2009). In their paper with finitely many firms, all consumers know only in advance the expected gain from joining the platform and the platform charges this expected gain. In Rochet and Tirole (2003), both buyers and sellers know in advance their exact benefits, thus there exist always some sellers (buyers) outside the platform.

Table 1 shows that, with an infinite number of firms (sellers), a positive search cost, it is more profitable for the platform to charge only the consumers rather than only the firms (when \( c > 0.02 \)). \( \max \Pi^p(w; c) \) is calculated according to section 3. \( \min \lim \tilde{\Pi}^p(w, s; c) \) can be found from (21) by taking \( c \) to the lower bound and \( m \) to infinity. Since we assume infinitely many firms, the example may also apply to cases with a large number of firms.

<table>
<thead>
<tr>
<th>( \bar{c} )</th>
<th>( \bar{s} )</th>
<th>( \max \Pi^p(c) )</th>
<th>( \min \lim \tilde{\Pi}^p(c) )</th>
<th>( \bar{c} )</th>
<th>( \bar{s} )</th>
<th>( \max \Pi^p(c) )</th>
<th>( \min \lim \tilde{\Pi}^p(c) )</th>
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<td>0.0688</td>
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</tr>
<tr>
<td>0.022</td>
<td>0.016</td>
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<td>0.36</td>
<td>0.0061</td>
<td>0.0052</td>
<td>0.0612</td>
<td>0.204</td>
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</table>

The remaining issue is that, we assume always the number of inside firms to be negligible compared with the total number of firms in the proof. Yet the assumption is undermined if the platform accredits as many firms as possible. The problem can be eased if the entry of firms is costly to the platform.
3.6 Finitely Many Firms

In case there exists a fixed cost of production, each firm, by entering the market, pays the fixed cost. Still the firm pays the membership fee to be in the platform. Denote this fixed cost by \( \kappa \), and denote as before the membership fee by \( w \). There would not be as many firms as under no fixed cost. Therefore, this setting is more general to test the solidarity of our previous results.

Suppose under a free-entry equilibrium, \( n \) out of an infinite number of firms are active, and \( m \) out of \( n \) firms become inside firms. If consumers expect the price charged by outside firms to be \( p_{\text{out}} \), then section 3.1 suggests that the optimal search rule is indicated by (2).

From the optimal search rule, both types of firms receive two sources of demand: fresh demand and returning demand. Denote the returning buyers towards an inside firm by \( r_{d_{\text{in}}} \) and the returning buyers towards an outside firm by \( r_{d_{\text{out}}} \). For simplicity, we only consider the case \( a + \Delta \geq 0 \). If one of inside firms deviates to a new price \( p \) and other firms keep the price at consumers’ expectation, its demand becomes

\[
D_{\text{in}}(p, p_{\text{out}}, p_{\text{in}}; n, m) = \int_{a+p-p_{\text{out}}}^{\min\{1,1-p_{\text{in}},p\}} (\epsilon - p + p_{\text{in}})^{m-1}\epsilon + \max\left(0, p_{\text{in}} - p\right) + r_{d_{\text{in}}}(p, p_{\text{out}}, p_{\text{in}}; n, m)
\]

(22)

The first term represents the fresh demand, the second term is a technical term and the third term represents the returning demand, which can be elaborated as the following,

\[
rd_{\text{in}}(p, p_{\text{out}}, p_{\text{in}}; n, m) = \int_{p}^{a+p-p_{\text{out}}} (\epsilon + p_{\text{in}} - p)^{m-1}(\epsilon + p_{0} - p)^{n-m}\epsilon d\epsilon
\]

\[
= \int_{0}^{a-p_{\text{out}}} (\epsilon + p_{\text{in}})^{m-1}(\epsilon + p_{\text{out}})^{n-m}\epsilon d\epsilon
\]

(23)
If an outside firm deviates to a new price $p$, it faces the demand as:

$$D_{Out}(p,p_{out},p_{in};n,m) = \frac{(a + \Delta)^m}{n - m} \left( \sum_{k=0}^{n-m-1} a^k \right) (1 - a + p_{out} - p) + rd_{out}(p,p_{out},p_{in};n,m)$$  \hspace{1cm} (24)$$

where the returning demand of an outside firm can be elaborated as,

$$rd_{out}(p,p_{out},p_{in};n,m) = \int_{p_{out}}^{a+p_{out}} (\epsilon + p_{in} - p)(\epsilon + p_{out} - p)^{n-m-1} d\epsilon$$

$$= \int_{0}^{a-p_{out}} (\epsilon + p_{in})^m(\epsilon + p_{out})^{n-m-1} d\epsilon$$  \hspace{1cm} (25)$$

Notably, the determination of $rd_{out}$ and $rd_{in}$ aren’t relevant to $p$. This is because all returning consumers have viewed all prices (including $p$) so that they take $p$ into both search and purchase decisions and outside options in a linear form. In case the firm has no intention to deviate, we derive the inside price from $p_{in} = -\frac{D_{In}(p_{in},p_{out},p_{in};n,m)}{D'_{In}(p_{in},p_{out},p_{in};n,m)}$:

$$p_{in} = 1 - \frac{(a + \Delta)^m}{m} + rd_{in}(p_{in},p_{out},p_{in};n,m)$$  \hspace{1cm} (26)$$

It is notable that the first term is the same as (8) and the second term comes directly from the returning consumers. Further the returning demand enters the equilibrium price in a linear form. The same pattern appears also in the price of an outside firm. If an outside firm does not deviate, we then derive the price equilibrium $p_{out}$ from the first order condition:

$$p_{out} = (1 - a) + \frac{n - m}{(a + \Delta)^m} \frac{1}{\sum_{k=0}^{n-m-1} a^k} rd_{out}(p_{out},p_{out},p_{in};n,m)$$  \hspace{1cm} (27)$$

**Proposition 7.** with a finite number of firms in the market, there exists a symmetric equilibrium
given by (26) and (27), where each inside firm sets a lower price than each outside firm.

Proof. See the appendix. ■

In a symmetric equilibrium, inside firms charge one price and outside firms charge the other price. The prevailing price of an outside firm is the term in (5) plus a weighted returning demand. If their second terms $r_{out}(p_{out}, p_{out}, p_{in}, n, m)$ and $r_{in}(p_{out}, p_{out}, p_{in}, n, m)$ are positive, the equilibrium $p_{out}, p_{in}$ are larger than their counterparts in (5) and (7), because the returning demand provides the firms with more market power even when consumers may choose the outside option if prices are too high. The larger the returning demand, the higher both prices are. Further, the inside price is always lower than the outside price, which is the corresponding version of proposition 1 under finitely many firms. Finally when the total number of firms tends to infinity, returning demand tends to zero and we return to the infinite case. The below figure draws each price equilibrium.

If only one firm is prominent, the inside price and the outside price coincide, as in Armstrong et al. (2009). Provided more inside firms, the inside price is decreasing as in section 3. Interestingly, the outside firms raise gradually their prices if more inside firms are present. The probable
reason is that with more inside firms and less outside firms, more information is revealed to the latter if the consumers turn to the firms outside the platform. The outside firms can not observe any match between consumers and inside firms, but can update their beliefs about the match as soon as they see a consumer come to visit an outside firm. The updated probability of having received poor matches among inside firms increases. And the larger the number of inside firms, the poorer match it must be. Therefore, the outside firms will increase their prices. From the decreasing inside price, the price gap between an inside firm and an outside firm enlarges more and more. Keeping the number of inside firms unchanged, this price gap is larger than the case with infinitely many firms. To see this, we compare the inside price with that in section 3:

The inside price with limited number of firms is larger than that with infinite number of firms. This result might be due to the presence of returning buyers, which corresponds to \( r_{d_{in}}(p_{out}, p_{out}, p_{in}, n, m) \) in (23). In the figure, with a smaller number of inside firms, the difference is more noticeable, because then their prominence is more notable. Besides, all inside firms have higher market power and higher profits.

In the usual case, an inside firm’s profit has the form of \( \Pi_{in}(p_{in}, p_{out}; n, m) = p_{in}^2 - w - \kappa \) and an outside firm’s profit has the form of \( \Pi_{out}(p_{in}, p_{out}; n, m) = \frac{(a+\Delta)^m}{n-m} (\sum_{k=0}^{n-m-1} a^k) p_{out}^2 - \kappa. \)
From simulations, both $\Pi_{In}$ and $\Pi_{Out}$ decrease in $n$, number of active firms. Further both $\Pi_{In}$ and $\Pi_{Out}$ decrease in $m$, number of inside firms. Since it is a free entry equilibrium, firms will keep entering the market until $\Pi_{Out} = 0$. Last, the profit difference $\Pi_{In} - \Pi_{Out}$ decreases in $m$.

Having observed the platform’s fee, firms pay that membership fee and become inside firms until this profit difference vanishes. The platform reaps the rest of profit difference until $\Pi_{In} = \Pi_{Out}$. Therefore in such an equilibrium, $\kappa = \frac{(a+\Delta)^m}{n-m} \left( \sum_{k=0}^{n-m-1} a^k \right) p_{out}^2$. And the difference in profit, turns out to be the platform’s membership fee $w$.

$$w(m, n, a) = p_{in}^2 - \frac{(a + \Delta)^m}{n - m} \left( \sum_{k=0}^{n-m-1} a^k \right) p_{out}^2$$  \hspace{1cm} (28)

The platform has another option to take all active firms, so that there are no standalone firms in the market. In this case, consumers do not search any firm because firms outside the platform are not active. The profit of an inside firm still has the form of $\Pi_{In} = p_{in}^2 - w - \kappa$ and the rest earns zero. Finally, the membership fee becomes $w = p_{in}^2 - \kappa$.

An increase in $\kappa$ first drives some firms out of the market. Among the fewer firms in the market, the platform chooses a new number of them. The new size of the platform will also have an impact on rest of firms. In case the new platform size is smaller, the profit of an outside firm increases. Secondly several firms who dropped out of the market will be active again. And after observing the returning of firms, the platform will consider again a new size. This process goes on and on until both the platform and firms have no incentives to move. A similar process applies to a decrease in the fixed cost. The table below summarizes the stable points after each impact of fixed cost.

By leaving several firms alone, the platform can only charge the profit difference between
two types of firms. And by taking all active firms, the platform can obtain industry profit. If the search cost is small, the profit difference is usually less than the profit of a standalone firm. Therefore sometimes, the platform opts to taking all standalone firms provided a large fixed cost even though this engenders more intense in-house competition. If the fixed cost drops and too many firms enter the market, then taking another firm eventually becomes more costly than letting it alone. From our numerical sample, the choice of number of firms can even drop in the market size. On the other hand, a large search cost can maintain a certain level of profit difference, thereby the platform is less tempted to adding in all active firms. The above table predicts that the agglomeration of firms will more likely take place when the fixed cost is large and search cost is small (e.g. auto-mobiles, computers).

To compare this case with section 2-3, the profit of the platform is generally smaller with finitely many firms, from the reason that the outside firm receives a non-negligible profit and the platform charges the profit difference. Therefore, the platform can not extract as large the difference in profit as before. The optimal size of the platform is smaller with infinite firms, which infers that the most profitable size of the platform decreases in the total number of firms in the market. The ratio of \( \frac{\text{Number of Inside firms}}{\text{Total Market Size}} \) is decreasing in market size.

### Tab. 3.2: Joint impact of search cost and fixed cost on number of firms.

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<td>0.95</td>
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Note: Number of active firms in parentheses.
3.7 Conclusion

The paper constructs an equilibrium where each consumer visits the Internet platform first and standalone firms in case he can not find any good match inside the platform. The equilibrium has price dispersion. Prices inside the platform decrease in the size of the platform, and are generally lower than prices outside the platform.

Consumer search and firm agglomeration have been discussed since Wolinsky (1983). The idea is that firms in the cluster use lower prices to attract consumers to visit the cluster first. The larger the search cost, the larger the cluster can be sustained. None of the papers endogenize cost of entry to the cluster, which is the case of most Internet platforms. The paper shows that the opposite relationship between size of the platform and search cost can be found after endogenizing cost of entry, which is due to the "non-appropriability of social surplus". To ease the information under-provision, we can either introduce more platforms or charge both consumers and firms.

The model has several major limitations. First, there is no heterogeneity among consumers in search cost. If there are high-cost and low-cost searchers, low-cost searchers are more likely to obtain better match. The price equilibrium normally depends on the percentages of each type of consumers. Second, there is no quality difference among firms. If some firms provide products with higher quality, these firms may want to pay more to enter the platform. This improves industry profit and social welfare. Third, a different fee structure does not change the main result when the platform is a monopoly. When the platform instead charges per transaction fees, it still grabs all the profit.

Last, searching in the platform may be costly as well and the platform can control the search
cost and number of inside firms. One sure thing is that the platform will not set search cost larger than the cost to search an outside firm. If search cost inside the platform is lower, it is logical to think that consumers still search inside firms first. This requires the generalized framework of Zhou (2009).

A Appendix A

A.1 Proof of Claim 1: Under the no-search equilibrium \( a < 1/2 \), the most profitable size of the monopoly platform is two.

Denote the highest match value the consumer could obtain from inside goods as \( \bar{\epsilon} \). Then either \( \bar{\epsilon} < a - p_{out} + p_{in} \) or \( \bar{\epsilon} > a - p_{out} + p_{in} \). For \( \bar{\epsilon} < a - p_{out} + p_{in} \), these consumers leave the platform if \( a - p_{out} + p_{in} > 0 \). For \( \bar{\epsilon} > a - p_{out} + p_{in} \), they stay in the platform with or without buying. Under the no-search equilibrium, the outside price is constantly 0.5 and the inside price is solely determined by (10). First, we compare three cases: \( m = 1, 2, 3 \). Then we prove that \( \Pi^p \) decreases in \( m \) when \( m > 3 \).

For \( m = 1 \), \( p_{in} = 0.5 \) and \( \Pi^p = 0.25 \). For \( m = 2 \), \( p_{in} \approx 0.4142 \) and \( \Pi^p \approx 0.3431 \). For \( m = 3 \), \( p_{in} \approx 0.322 \) and \( \Pi^p \approx 0.311 \). Therefore 2 is the most profitable size for the platform if \( m \leq 3 \).

Take the derivative of \( \Pi^p \) w.r.t. \( m \) and obtain:

\[
\frac{\partial \Pi^p}{\partial m} = p_{in}^2 + 2m p_{in} \frac{\partial p_{in}}{\partial m} \tag{A.1}
\]

We substitute \( \frac{\partial p_{in}}{\partial m} \) from (A.3), substitute \( p_{in} \) with \( (1 - p_{in}^m)/m \) and expand the expression, we obtain:

\[
\frac{\partial \Pi^p}{\partial m} = p_{in} \left( p_{in}^{m-1} (1 + p_{in} - p_{in}^m - p_{in} \log(p_{in}^m)) - 1 \right) / m \left( 1 + p_{in}^{m-1} \right)
\]

\[
\eta(p_{in}, m)
\]

It suffices to prove that: \( \eta(p_{in}, m) < 1 \). \( \eta(p_{in}, m) \) increases in \( p_{in} \). Since \( p_{in} < \frac{1}{m} \), we then substitute
3. Search Cost and Firm Agglomeration: The Case of Online Platforms

$p_m$ with $\frac{1}{m}$ to achieve its upper bound. We want to show that this upper bound is smaller than 1 when $m \geq 3$.

$$f(m) = \frac{(1/m)^m + (1/m)^{m-1} - (1/m)^{2m-1} - 2(1/m)^m \log((1/m)^m)}{g(m)}$$

$f(m)$ decreases in $m$. The last term decreases in $m$ when $m > 3$. Let $\gamma = (1/m)^m$, $g'(\gamma) = 2 + 2 \log \gamma > 0$, for $\gamma < 1/27$. In the end, it suffices to examine the case with $m=3$, whose value is approximately $0.3881 < 1$. Therefore when search cost is large, the platform never obtains higher profit than the case when $m=2$. Further, search cost does not play any role since consumers do not search as long as $a < 0.5$.

Hence if a result holds for any $a' < 0.5$, it must hold for all $a' < 0.5$. We arrive at the result.

A.2 Proof of Claim 2: The monopoly platform always admit at least two firms under the active search equilibrium $a \geq 1/2$.

When $a \geq 1/2$, all consumers who stay in the platform make a purchase. We compare $\Pi^p$ in two cases $m = 1, 2$.

$$\Pi^p = \begin{cases} (1 - a)^2, & m = 1 \\ 2(2\sqrt{a} - 2a)^2, & m = 2 \end{cases}$$

(A.2)

Let $\Pi^p_m$ denote the platform’s profit by taking $m$ inside firms:

$$\Pi^p(m = 1) < \Pi^p(m = 2) \iff 1 - a < 2\sqrt{2}(\sqrt{a} - a) \iff 1 + \sqrt{a} < 2\sqrt{2}a \iff \left(\frac{1}{2\sqrt{2} - 1}\right)^2 \approx 0.3 < a$$

A.3 Proof of Claim 3: $\hat{\epsilon} < 1 - 1/m$

Since $\hat{\epsilon}$ decreases in $m$, the above means that at a given search cost, the monopoly platform adds firms until the threshold $\hat{\epsilon}$ is below $\hat{\epsilon} = 1 - 1/m$. From (13), it suffices to show that $1 - 2\varepsilon(m) > 0$ at

Let $\zeta(m) = \varepsilon(m)$. That is to prove the below

$$\zeta < \frac{1}{2} \Leftrightarrow 2(1 - 1/m)^m(1 - \log(1 - 1/m)^m) + (1 - (1 - 1/m)^m)(1 + (1 - 1/m)^{m-1}) > 2$$

Let $\gamma = (1 - 1/m)^m$, $f(m)$ increases in $m$ ($m \geq 2$) because $f(m)$ increases in $\gamma$ for $\gamma \in [0, 1]$ and $\gamma$ increases in $m$. Therefore $\min f(m) = f(2) \approx 1.193$. But $g(m)$ decreases in $m$. Let

$$\tilde{g}(m) = (1 - 1/m)^{1-m} g'(m)$$

$$= (1-(1-1/m)^m) (1/m + \log(1 - 1/m)) - (1-(1+1/m)^{m-1})(1-1/m) (1/(m - 1) + \log(1 - 1/m)) < 0$$

From $\lim f(m) + g(m) = 1.193 + (1 - 1/e)(1 + 1/e) > 2$, we arrive at the result.

A.4 Extension of A.6

(i) The proof of $\psi(\tilde{\epsilon}, m) > 0$ can be reduced to $h(m) = (1-1/m)^{m-1} - 2m(1-1/m)^m \log(1-1/m) > 1$. We take its first-order derivative in $m$:

$$h'(m) = -\frac{m-2}{m} (1 - 1/m)^{m-1} (\log(1 - 1/m) + 1/(m - 2)) - (\log(1 - 1/m) + 1/(m - 1))$$

Further, $h(2) = 1.193$, $h'(m) < 0$. Finally its limit is, $\lim_{m \to \infty} h(m) = \frac{3}{e} > 1$.

(ii) The proof of $\varphi(\epsilon, m) > 0$ when $m \geq 2$ can be reduced to the following:

$$(2m - 1)(1 - 1/m)^{m-1} + 2m^2 \log(1 - 1/m) < 0, \text{while } m \geq 2$$  \hspace{1cm} (A.3)

Its derivative in $m$ is the following: $2(1 - 1/m)^{m-1} + 4m \log(1 - 1/m) + 2m/(m - 1) + (2m - 1)(1 - 1/m)^{m-1} \zeta(m)$, where $\zeta(m) = \log(1 - 1/m) + 1/m$. From $\zeta(m) < 0$, we know that, $2(1 - 1/m)^{m-1} +
(2m - 1)(1 - 1/m)^{m-1} \zeta(m) < 2(1 - 1/m)^{m-1} < 2/e \approx 0.74. The rest two terms are defined by \\
\phi(m) = 4m \log(1-1/m) + 2m/(m-1). These last two terms are decreasing in m because, \( \partial \phi(m)/\partial m = 4(\log((m-1)/m) + 1/(m-1)) - 2/(m-1)^2 < 0 \)

The above can be proven by,

\[
\lim_{m \to 2} \frac{\partial \phi(m)}{\partial m} \approx -0.7726, \quad \lim_{m \to \infty} \frac{\partial \phi(m)}{\partial m} \approx 0
\]

and, \( \partial^2 \phi(m)/\partial m^2 = 4/m(m-1) - 4/(m-1)^2 + 4/(m-1)^3 > 0 \). Since,

\[
\lim_{m \to 2} \phi(m) \approx -1.545
\]

The first two terms have an upper bound of 2/e. The last two terms are decreasing and have an upper of -1.545. Therefore, the value of \((2m - 1)(1 - 1/m)^{m-1} + 2m^2 \log(1 - 1/m)\) is always decreasing and

\[
\lim_{m \to 2} (2m - 1)(1 - 1/m)^{m-1} + 2m^2 \log(1 - 1/m) \approx -4.04
\]

We arrive at the conclusion.

(iii) To prove that \( \Gamma(\hat{\epsilon}, m) < 0 \), we take some linear transformations and it could be simplified to the proof that \( \varphi(m) = 1/m - 2(1 - 1/m)^m - 2(m-1) \log(1 - 1/m) > 0 \). Take the derivative of \( \varphi(m) \) in m,

\[
\frac{\partial \varphi(m)}{\partial m} = -1/m^2 - 2(1 - 1/m)^m (\log(1 - 1/m) + 1/(m-1)) - (\log(1 - 1/m) + 2/m) < 0
\]

Therefore, \( \varphi(m) \) decreases in m. \( \varphi(2) \approx 1.386 \) and \( \lim_{m \to \infty} \varphi(m) \approx 2 - \frac{2}{e} > 0 \). Therefore, the expression for \( \varphi(m) \) is always positive.
A.5 Proof of Proposition 1

Uniform Case

We focus on the uniform case for \( a - p_{\text{out}} \geq 0 \). For other cases, the way to prove this proposition is quite similar. First we look at the uniform case. From (12), we can get:

\[
\left( 1 + \frac{(a - p_{\text{out}} + p_{\text{in}})^{m-1}}{f(\bar{v})} \right) f(\bar{v}) \frac{\partial p_{\text{in}}}{\partial m} = \frac{(a - p_{\text{out}} + p_{\text{in}})^{m}[1 - \log(a - p_{\text{out}} + p_{\text{in}})^{m}] - 1}{m^2} \tag{A.4}
\]

Here we only consider the case where \( m \geq 1 \). Since \( 0 < x(1 - \log[x]) < 1 \) as long as \( x \in [0, 1] \), the right hand-side is positive. Thus one obtains that \( \frac{\partial p_{\text{in}}}{\partial m} < 0 \). Similarly we can prove the case where \( a < p_{\text{out}} \).

Non-Uniform Cases

\[
p_{\text{in}} = \frac{1 - F^m(\Delta + a)}{m \left( f(\bar{v}) - \int_{\Delta+a}^{\bar{v}} F^{m-1}(\epsilon)f'(\epsilon)d\epsilon \right)} , \quad a - p_{\text{out}} > 0 \tag{A.5}
\]

To prove the above decreases in \( m \), which is equivalent to,

\[
\frac{1 - F^m(\Delta + a)}{m \left( f(\bar{v}) - \int_{\Delta+a}^{\bar{v}} F^{m-1}(\epsilon)f'(\epsilon)d\epsilon \right)} > \frac{1 - F^{m+1}(\Delta + a)}{(m+1) \left( f(\bar{v}) - \int_{\Delta+a}^{\bar{v}} F^m(\epsilon)f'(\epsilon)d\epsilon \right)}
\]

Expand the above,

\[
(m + 1) f(\bar{v}) - (m + 1) \int_{\bar{v}}^{\bar{v}} F^m(\epsilon)f'(\epsilon)d\epsilon - (m + 1) F^m(\bar{v}) - (m + 1) F^m(\bar{v}) + (m + 1) F^{m+1}(\bar{v}) \int_{\bar{v}}^{\bar{v}} F^{m+1}(\epsilon)f'(\epsilon)d\epsilon
\]

\[
> m f(\bar{v}) - m \int_{\bar{v}}^{\bar{v}} F^{m-1}(\epsilon)f'(\epsilon)d\epsilon - m F^{m+1}(\bar{v}) + m F^{m+1}(\bar{v}) \int_{\bar{v}}^{\bar{v}} F^{m-1}(\epsilon)f'(\epsilon)d\epsilon
\]

16. Normally, one should take the derivative on both sides and use implicit function theorem. Since we will provide the proof that \( \frac{\partial \text{RHS}}{\partial p_{\text{in}}} < 0 \) in A.4.3. We omit that proof in this part.
Since, \( m + 1 ) f(\overline{v}) - (m + 1) F^m(\hat{\epsilon}) f(\overline{v}) > m f(\overline{v}) - m F^{m+1}(\hat{\epsilon}) f(\overline{v}) \)

it only suffices to show that,

\[
(m + 1)(1 - F^m(\hat{\epsilon})) \int_{\hat{\epsilon}}^{\overline{v}} F^m(\epsilon) f'(\epsilon) d\epsilon < m (1 - F^{m+1}(\hat{\epsilon})) \int_{\hat{\epsilon}}^{\overline{v}} F^{m-1}(\epsilon) f'(\epsilon) d\epsilon
\]

which is further simplified to the below because \( F(\hat{\epsilon}) < 1 \),

\[
(m + 1) \int_{\hat{\epsilon}}^{\overline{v}} F^m(\epsilon) f'(\epsilon) d\epsilon < m \int_{\hat{\epsilon}}^{\overline{v}} F^{m-1}(\epsilon) f'(\epsilon) d\epsilon
\]

The left is expanded from integration by parts,

\[
f(\overline{v}) - F^m(\hat{\epsilon}) f(\hat{\epsilon}) < m \int_{\hat{\epsilon}}^{\overline{v}} F^{m-1}(\epsilon) f'(\epsilon) d\epsilon - m \int_{\hat{\epsilon}}^{\overline{v}} F^m(\epsilon) f'(\epsilon) d\epsilon + m \int_{\hat{\epsilon}}^{\overline{v}} F^{m-1}(\epsilon) f(\epsilon) d\epsilon
\]

On the right side it becomes

\[
m \int_{\hat{\epsilon}}^{\overline{v}} F^{m-1}(\epsilon) (f(\epsilon)(1 - F(\hat{\epsilon})) + f^2(\epsilon)) d\epsilon > 0
\]

The above is met since \( F(\epsilon) > 0 \) and \( 1 - F(\epsilon) \) is log-concave.

\textit{Search Cost}

Assume the log-concavity of \( 1 - F \),

- \( p_{in} \) increases in search cost \( c \).

\[ 17. \ 1 + m F^{m+1}(\hat{\epsilon}) - (m + 1) F^m(\hat{\epsilon}) > 0 \] for any \( 0 \leq \hat{\epsilon} \leq 1 \). First, \( m F^{m+1}(\hat{\epsilon}) - (m + 1) F^m(\hat{\epsilon}) \) falls in \( \hat{\epsilon} \). To see this, the derivative in \( \hat{\epsilon} \) is \( -m(m + 1)(1 - F(\hat{\epsilon})) F^{m-1}(\hat{\epsilon}) f(\hat{\epsilon}) < 0 \). Then \( 1 + m F^{m+1}(\hat{\epsilon}) - (m + 1) F^m(\hat{\epsilon}) > 1 + m F^{m+1}(\overline{v}) - (m + 1) F^m(\overline{v}) = 0 \).
Let

\[ p_{in} = \frac{1 - F^m(\Delta + a)}{m(f(\bar{v}) - \int_{\Delta+a}^0 F^{m-1}(\epsilon)f'(\epsilon)d\epsilon)} = \lambda \]

Using the chain rule, we obtain from the above :

\[ \frac{\partial p_{in}}{\partial a} - \frac{\partial \lambda}{\partial p_{in}} = \frac{\partial \lambda}{\partial a} \]

Denote \( \hat{\epsilon} = \Delta + a \), indeed \( \frac{\partial \lambda}{\partial p_{in}} = \frac{\partial \lambda}{\partial a} = \frac{\partial \lambda}{\partial \hat{\epsilon}} \) assuming that \( p_{in} \) is not linked with a while taking the derivative. Hence we focus on showing \( \frac{\partial \lambda}{\partial \hat{\epsilon}} < 0 \), which is to show that :

\[ m F^{m-1}(\hat{\epsilon})f'(\hat{\epsilon})D'_{In}(p_{in}) - F^{m-1}(\hat{\epsilon})f'(\hat{\epsilon})(1 - F^m(\hat{\epsilon})) < 0 \]

which reduces to the following

\[ m f(\hat{\epsilon})D'_{In}(p_{in}) - f'(\hat{\epsilon})(1 - F^m(\hat{\epsilon})) < 0 \]

The above is met because \( D'_{In}(p_{in}) = -f(\bar{v}) + \int_{\Delta+a}^0 F^{m-1}(\epsilon)f'(\epsilon)d\epsilon \). From the log-concavity of \( 1 - F \), we know that the hazard rate is increasing which suggests that \( f'(\hat{\epsilon}) > -\frac{f^2(\hat{\epsilon})}{1 - F(\hat{\epsilon})} \). Substitute it to the above, it suffices to show :

\[ m D'_{In}(p_{in})(1 - F(\hat{\epsilon})) + f(\hat{\epsilon})(1 - F^m(\hat{\epsilon})) < 0 \]

which reduces to the following because \( D_{in} = \frac{1 - F^m(\hat{\epsilon})}{m} \)

\[ D'_{In}(p_{in})\frac{1 - F(\hat{\epsilon})}{f(\hat{\epsilon})} + D_{In}(p_{in}) < 0 \]
\[ \hat{\epsilon} = a - p_{\text{out}} + p_{\text{in}}. \] From proposition 1, \( p_{\text{in}} < p_{\text{out}} \), hence \( \hat{\epsilon} < a \). From increasing hazard rate, we know that

\[
\frac{1 - F(\hat{\epsilon})}{f(\hat{\epsilon})} > \frac{1 - F(a)}{f(a)} = p_{\text{out}} > p_{\text{in}}
\]

From FOC conditions on (3), we know that \( D'_{in}(p_{in})p_{in} + D_{in}(p_{in}) = 0 \) and \( D'_{in}(p_{in}) < 0 \). Hence we reach the conclusion.

### A.6 Proof of Proposition 3

The problem of the platform is to maximize the total sum of profit by inside firms,

\[
\max \Pi^p(w) = m p_{in} D_{in}(p_{in}, p_{in}, p_{out}; m(w))
\]

FOC indicate that,

\[
p_{in}^2 + 2mp_{in}(\partial p_{in}/\partial m) = 0 \Rightarrow p_{in} + 2m(\partial p_{in}/\partial m) = 0
\]

Replace \( p_{in} \) alone with \( \frac{1 - (a - p_{out} + p_{in})}{m} \), and \( \frac{\partial p_{in}}{\partial m} \) according to (A.1). Let \( \hat{\epsilon} = a - p_{out} + p_{in} \). We obtain,

\[
\left(1 - \frac{\hat{\epsilon}^m}{m}\right) + 2m\left(1 + \hat{\epsilon}^{m-1}\right)^{-1}\left(\frac{\hat{\epsilon}^m(1 - \log \hat{\epsilon}^m) - 1}{m^2}\right) = 0
\]

Multiply on two sides \( m(1 + \hat{\epsilon}^{m-1}) \). Define \( \psi(\hat{\epsilon}, m) = m(1 + \hat{\epsilon}^{m-1}) (p_{in} + 2m\frac{\partial p_{sa}}{\partial m}) \). Then \( \psi(\hat{\epsilon}, m) = (1 - \hat{\epsilon}^m)(1 + \hat{\epsilon}^{m-1}) + 2\hat{\epsilon}^m(1 - \log(\hat{\epsilon}^m)) - 2 = 0 \). From the chain rule \( (m'_{a} = m'_{a}) \) and \( 0 < \hat{\epsilon}'_{a} < 2 \), it suffices to determine \( \text{sgn}(m'_{a}) \). From the implicit function theorem:

\[
\frac{\partial m}{\partial \hat{\epsilon}} \bigg|_{\psi(\hat{\epsilon}, m) = 0} = -\frac{\psi'}{\psi} \bigg|_{\psi(\hat{\epsilon}, m) = 0} \tag{A.8}
\]
1. First let \( \varphi = \psi' \) and let \( \tilde{\varphi} = \tilde{\psi}' = (m - 1) - m \tilde{e} - (2m - 1)\tilde{e}m - 2m^2 \tilde{e} \log(\tilde{e}) \). \( \tilde{\varphi}(0, m) > 0 \) and \( \tilde{\varphi}(1, m) = -2m < 0 \). Since \( \tilde{\varphi}(\tilde{e}, m) \) is a continuous function in \([0, 1] \times [1, \infty)\), there must exist at least one point where \( \tilde{\varphi}(\tilde{e}, m) = 0 \). Its derivative \( \tilde{\varphi}' = -(m + 2m^2) + (m - 2m^2)\tilde{e}m - 2m^2 \log(\tilde{e}) \). For \( m \geq 2 \), \( \tilde{\varphi}'(\tilde{e}, m) \) declines from \( \tilde{\varphi}'(0, m) > 0 \) to \( \tilde{\varphi}'(1, m) < 0 \). Therefore we can say that \( \tilde{\varphi} \) increases in \( \tilde{e} \) until some \( \tilde{e}' > 0 \) then becomes negative till \( \tilde{\varphi}(1, m) = -2m < 0 \). It becomes clear that \( \tilde{\varphi}'(\tilde{e}, m) \) and \( \tilde{\varphi}(\tilde{e}, m) \) are quasi-concave and start from positive values. Hence \( \varphi(\tilde{e}, m) = 0 \) must have at most one root between \((0, 1]\) for any \( m \).

2. Second let

\[
\Gamma(\tilde{e}, m) = \psi'_{m}(\tilde{e}, m) = \left( \tilde{e}^{m-1} - \tilde{e}^m - 2\tilde{e}^2m - 2m\tilde{e}^m \log(\tilde{e}) \right) \log(\tilde{e}) = \tilde{\Gamma}(\tilde{e}, m) \tilde{e}^{m-1} \log(\tilde{e})
\]

Let \( \tilde{\Gamma}'(\tilde{e}, m) = -1 - 2m - 2m\tilde{e}^m - 2m \log(\tilde{e}) \). \( \tilde{\Gamma}'(\tilde{e}, m) \) decreases from positive to negative and changes the sign only once. Particularly \( \tilde{\Gamma}(0, m) = 1 \) and \( \tilde{\Gamma}(1, m) = -2 \). Therefore both \( \tilde{\Gamma}(\tilde{e}, m) \) and \( \tilde{\Gamma}'(\tilde{e}, m) \) are quasi-concave. \( \tilde{\Gamma}(\tilde{e}, m) = 0 \) must have the unique root between \([0, 1]\).

3. \( \tilde{\varphi}'(0, m) > 0, \tilde{\varphi}'(1, m) < 0, \tilde{\varphi}(0, m) > 0, \tilde{\varphi}(1, m) = -2m < 0, \psi(0, m) = -1 \) and \( \psi(1, m) = 0 \) infer that \( \psi(\tilde{e}, m) \) increases from negative to zero and \( \psi(\tilde{e}, m) = 0 \) has the unique root between \((0, \tilde{e})\). Since \( \varphi(\tilde{e}, m) = 0 \) has at most one root between \((0, 1]\), \( \psi(\tilde{e}, m) \) can not increase between \((\tilde{e}, 1)\), which implies that \( \psi(\tilde{e}, m) \) is quasi-concave and \( \psi(\tilde{e}, m) = 0 \) has the unique root between \((0, 1]\).

4. Finally **construct** \( \tilde{e}(m) = 1 - \frac{1}{m} \). The platform chooses \( \psi(\tilde{e}^*, m) = 0 \). A.4, (i) proves that \( \psi(\tilde{e}, m) > 0 \), for which we have \( \tilde{e}^* < \tilde{e} \). Hence the platform never chooses thresholds above \( \frac{m-1}{m} \). (ii) proves that \( \varphi(\tilde{e}, m) > 0 \) for \( \tilde{e} \in (0, \tilde{e}] \). (iii) proves that \( \Gamma(\tilde{e}, m) < 0 \). Then \( \Gamma(\tilde{e}, m) < 0 \) for each \( \tilde{e} \in (0, \tilde{e}] \). Therefore \( \varphi \) and \( \Gamma \) do not change their signs until \( \tilde{e} \). Henceforth \( \Gamma(\tilde{e}^*, m) \varphi(\tilde{e}^*, m) < 0 \).

Then (A.5) and the chain rule imply that equilibrium number of inside firms decreases in search.
cost. From numerical examples, the number of firms increases to twelve when \( a = 0.9 \), and decreases to two when \( a = 0.5 \). Finally the unique root of \( \psi(\hat{\epsilon}, m) = 0 \) between \((0,1)\) implies that for each level of search cost, there will be only one size of the platform and a unique equilibrium membership fee optimizing its profit. We arrive at the result.

### A.7 Proof of Corollary 1

Suppose that \( \varepsilon(m) \) decreases in \( m \), then the term \( 1 - 2\varepsilon(m) \) would increase in \( m \). Then there would not exist any \( m \) where the above expression has the value zero unless at \( m = 0 \).

\[
\frac{\partial}{\partial m} \Pi^p(m) = p^2_m + 2mp_m \frac{\partial p_m}{\partial m} = p^2_m \left( 1 - 2\varepsilon(m) \right)
\]

From proposition 3 we know that, the unique equilibrium for the membership fee exists and is larger than zero, which contradicts the above. Therefore it can only be that \( \varepsilon(m) \) starts from a low value, and increases in \( m \). The following expression gives us the negative sign of \( \partial \varepsilon(m) / \partial a \).

\[
\frac{\partial \varepsilon(m)}{\partial a} + \frac{\partial \varepsilon(m)}{\partial m} \frac{\partial m}{\partial a} = 0 \Rightarrow \text{sgn} \frac{\partial \varepsilon(m)}{\partial a} = -\text{sgn} \left[ \frac{\partial \varepsilon(m)}{\partial m} \times \frac{\partial m}{\partial a} \right]
\]

Therefore when the search cost is larger, by increasing a certain number of slots, the platform is obliged to give away a larger percentage of profit per slot.

### A.8 Proof of Proposition 6

Instead of a rigorous proof which is available on-line, we try to prove this proposition by reasoning.

From the monotonicity of (20) in \( \rho \), we know that on the top the platform will capture all consumers by setting the subscription fee exactly to the expected surplus gain from a subscription to the platform.

While prices are transfers, what matters to the platform is how much it can internalized the sum of
consumer surplus (essentially from the match) rather than leave it to outside firms, which is essentially how much the optimal search rule prevents consumers from searching. Ideally, consumers should have searched a firm as long as a positive probability exists to obtain a better match. However, the search cost and the price distortion caused by the search cost, enter the searching threshold (as shown below) and prevent consumers from a better match.

\[ \hat{\epsilon} = a \frac{\text{search cost}}{\text{price distortion}} + p_{in} - p_{out} \]

If more firms are in the platform, consumers clearly gain more from more variety in the platform and lower inside prices. Therefore less consumers will visit outside firms, which infers that less surplus will be wasted in transportation and less surplus will go to outside firms. Therefore the monopoly platform clearly prefers taking as many firms as possible. After taking all firms, prevailing prices inside the platform drop to zero. Consumers do not incur search cost and they visit a new firm as long as a positive probability exists of obtaining a better match. Distortionary pricing disappears, and the platform actually maximizes social surplus. Since there is no cost of taking another inside firm, it is straightforward to say that the platform will take as many firms as possible.

A.9 Proof of Proposition 7

We assume that \( p_{in} > p_{out} \), and build a contradiction. Then from \( a + p_{in} - p_{out} > a \), we have when \( m > 1 \),

\[ (1 - (a - p_{out} + p_{in})^m)/m < (1 - a^m)/m < 1 - a \]
Therefore, it must be that the second term of (26) is larger than the second term of (27). Further we
know that $\frac{n-m}{\sum_{k=0}^{n-m-1} a_k} > 1$ and $\frac{1}{(a+\Delta)^m} > 1$. Then it must be that, for each value of $a,$

$$\int_{a-p_{\text{out}}}^{0} (\epsilon + p_{\text{in}})^{m-1}(\epsilon + p_{\text{out}})^{m-1}d\epsilon > \int_{0}^{a-p_{\text{out}}} (\epsilon + p_{\text{in}})^{m}(\epsilon + p_{\text{out}})^{n-1}d\epsilon$$

From $p_{\text{in}} > p_{\text{out}},$ for each $\epsilon \in [0, a - p_{\text{out}}],$ there is,

$$(\epsilon + p_{\text{in}})^{m-1}(\epsilon + p_{\text{out}})^{n-m} < (\epsilon + p_{\text{in}})^{m}(\epsilon + p_{\text{out}})^{n-m-1}$$

Taking all above and (26)-(27) into consideration, we can only have

$$p_{\text{in}} < p_{\text{out}}$$

Finally, this is a contradiction to our assumption.
4. WHEN DOES GROUP-BUYING INCREASE PROFIT?

4.1 Introduction

Group-buying or collectively buying is a phenomenon mostly seen on the Web. The mechanism works as follows. A firm posts a normal price, a discounted price, the minimum required number of participants and a date limit on the group-buying website. Upon the deadline, if the number of consumers having promised to make a purchase exceeds the minimum requirement (sometimes buyers are required to pledge some amount.), the firm sends a voucher to whoever promised to buy the product, and the consumer can use the voucher to buy the product at the discounted price. Otherwise, transactions still take place but only at the normal price. A consumer can either buy the product immediately at the normal price, or wait to see whether the price will be lower on the deadline. Furthermore, during the waiting period he can inform other consumers. This mechanism is similar to a referral reward program except that subsequent visitors are offered the same discounted price. Therefore, any invitation not only benefits the inviter but also the invitees. From Groupon to "Google Offers" and LivingSocial, thousands of new deals are offered every day. A leader such as Groupon achieved a market capitalization of 17.4 billion dollars.

Even though such deals mostly take place on the Web, group-buying is not a new phenomenon and can be dated back to the pre-Internet era. For example, some airlines and railway companies
offer group travel packages. KFC introduced the "family barrel" package. The other form used by Groupon is similar to an on-line promotion. Consumers not knowing preference information about other consumers choose whether to accept the deal simultaneously. Meanwhile the group-buying websites offer tools (Email, Facebook, Twitter, etc.) to make invitations. Firms often use some heavily discounted prices ($\geq 70\%$) to attract consumers. For example, a concert ticket costs 30 Euros for an individual, but merely 12 Euros if a minimum of 30 people commit to attend.

This practice has several features that set it apart from other popular pricing schemes. First, different from quantity discount models, consumers usually have unit demand. Second, the firm is no longer the only messenger for product information. The group-buying strategy allows buyers themselves to be messengers and invite potential buyers. Consumers might have more information about consumers around than the firm. Group-buying aligns the incentives for consumers to look for lower prices, with the incentives for firms to look for consumers. It is for this reason, that I find group-buying as an interesting and important mechanism, from which one may draw some market implications.

I first analyse the model with no invitations, in which $N$ consumers decide simultaneously whether to participate in the group buying option or to buy immediately at the undiscounted price. The results are rather unfavourable to the use of group buying. Given that the monopolist is can commit to a discounted price (that is, it is committed to not lowering it) group buying is dominated by a simple inter-temporal price discrimination scheme. It is optimal to price discriminate because consumers discount the future more than the firm. Intuitively, group buying would amount to committing to a mixed strategy on prices in the second period, which is shown to always be dominated by offering a discounted price with certainty.
In the second analysis, each consumer can not only wait, but also inform one more who are similar to them in terms of willingness to pay. The model is restricted to have two consumers in the first cohort, therefore at most two more consumers in the second cohort. The monopolist sets prices and the minimum requirement and each consumer decides whether to inform one more consumer.

I find that, when the inviting cost is low, the monopolist always finds it optimal to set a high minimum and heavily discounted prices. Heavily discounted prices are used to motivate the high-type consumers to invite other high type consumers, and a high minimum helps sorting out the high-type consumers from the rest. This is profitable because if the number of consumers turns out to be insufficient for the discount, it is likely that these consumers will purchase the product at a large normal price. While setting the low minimum implies no sales at the normal price, setting the high minimum implies more than half of profit coming from selling the product at the normal price. When the inviting cost is large, the group-buying strategy is dominated by either a simple one-price setting, or a time-dependent price setting.

The group-buying phenomenon attracts recent attention from a number of researchers. The paper that is most closely related to my work is that of Chen and Zhang (2013) who consider a similar setting but assume a discrete demand with some demand uncertainty and rule out inter-temporal price discrimination. They find that group-buying may be a profitable strategy, even without invitations. In Jing and Xie (2011), consumers are initially heterogeneous in their information about the product. The group-buying strategy is a contract that induces the informed consumer to disseminate information to the uninformed consumers. When the information gap between experts and novice consumers is neither too high or too low, this word-of-mouth

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1. According to recent surveys, the average rate of discount is over 50% for deals in Groupon.
model dominates traditional individual-selling strategies. Chen and Li (2013) discusses a particular case when buyers form a group and commit to purchase exclusively from one of two firms who differ in their incentives to invest in quality improvement. In Hu et al. (2013), two cohorts of consumers observe the group-buying deal one after the other. Provided exogenous prices and sizes of two cohorts, their paper compares the simultaneous mechanism and the sequential mechanism and finds out that the deal success rate is higher under the sequential mechanism. Gerstner and Libai (2001) studies the referral rewards program.

The paper is also related to the price discrimination literatures. Stokey (1979), Landsberger and Meilijson (1985) show that the inter-temporal price discrimination is profitable when consumers are heterogeneous in time preference. Sobel (1984) studies the cyclical pricing patterns, where firms face continuous inflows of consumers. Doyle (1986) presents a similar model where firms advertise introductory offers to hold new consumers. Chen and Wang (1999) consider a seller facing a sequence of potential buyers whose valuations are drawn from the distribution unknown to the seller. The seller learns gradually the distribution of buyers’ willingness to pay and lowers down the price over time.

On joint-buying and price discrimination, Alger (1999), using a two-type setting (high-valuation and low-valuation consumers), studies the optimal menu of price-quantity pairs when a continuum of consumers are able to purchase multiple times or jointly. Using a similar two-type setting, Jeon (2005) characterizes a complete (collusion-proof) contract for the monopolist to achieve the same profit regardless of whether or not buyers can form a coalition. Prices and quality can sometimes vary from time to time, for the purpose of exploiting differences in consumers’ reservation prices.

I considers in turn two monopoly models. In section 2, I propose the N-consumer model where
consumers in the first cohort determine simultaneously whether to accept the deal. In section 3, I propose a two-consumer model where each consumer is able to invite another consumer to accept the deal. Some proofs are relegated to the appendix.

4.2 Without Invitations

The market consists of a durable-goods monopolist (M) and \( N \geq 2 \) risk-neutral consumers indexed by \( i = 1, 2, ..., N \), each having unit demand. There is a linear city from zero to \( \bar{u} \). M is located at the point zero. Denote the locations of consumers by the vector \( \Theta = (\theta_1, \theta_2, ..., \theta_N) \). Each element of the vector is independently distributed under distribution function \( F(\theta) \) on \([0, \bar{u}]\). The reservation price for each consumer is \( \bar{u} \), and the transportation cost per unit is 1.

The utility function for the consumer purchasing at price \( p \) is given by

\[
U(\theta, p) = \bar{u} - \theta - p
\]

Only the distribution of consumer locations is common knowledge. The firm has no additional information whereas each consumer knows her location, her type, but not that of other consumers.

The game has four stages.

- In the first stage, nature chooses the consumers’ locations.

- In the second stage, M offers a package \((p_R, p_G, \kappa)\) to the \( N \) consumers, characterized by the normal price \( p_R \), the group buying price \( p_G \) and a minimum requirement on the number of participants for group buying to take place, \( \kappa \).

- In the third stage, each consumer observes her location and the firm’s offer, and choose
one of three options: 1) purchase the product immediately at the normal price. 2) register to participate in the group buying option; participation costs \( w > 0 \) and the utility is discounted by a factor \( \beta \in (0, 1) \). 3) reject the offer.

- In the last stage, if more than \( \kappa \) consumers have accepted to wait for the group buying opportunity, \( M \) offers \( p_G \) to each of them. If not, consumers have two options: 1) buying at \( p_R \). 2) Reject \( p_R \).

The firm is committed throughout to the prices announced in stage 2. The pay-off of \( M \) is the expected profit. The pay-off of a consumer is the (discounted) surplus over the two periods. I look for Perfect Bayesian Equilibria where the consumers’ strategies are symmetric in the sense that, at any information set, all consumers with identical type make the same choice. Let \( m_i \) be the number of consumers (excluding consumer \( i \)) accepting to wait in equilibrium. It is a random variable which, thanks to the symmetry restriction, is the same for all consumers, \( i \). It is henceforth denoted \( m \). Let \( \Gamma = \Pr(m \geq \kappa - 1) \) be the probability of a deal success if consumer \( i \) also accepts to wait. In equilibrium, this probability can be anticipated without bias by both \( M \) and consumers after having observed the offer.

Consider the four options of consumer \( i \). Note that group buying is attractive only if it allows a consumer to purchase at a lower price than if she buys on her own. Hence we necessarily have \( p_G < p_R \). The consumer’s surplus when rejecting the offer is zero. If she purchases the product immediately, her utility is given by \( U_A(\theta_i) = \bar{u} - \theta_i - p_R \). If she waits and purchases the product at the lowest available price no matter whether the discounted price is realized or not, her expected utility is

\[
U_B(\theta_i) = \beta \Gamma (\bar{u} - \theta_i - p_G) + \beta (1 - \Gamma)(\bar{u} - \theta_i - p_R) - w
\]
4. When Does Group-buying Increase Profit?

Last, if she purchases only at the discounted price, her utility is $U_C(\theta_i) = \beta \Gamma(\bar{u} - \theta_i - p_R) - w$.

4.2.1 Consumer Behaviour

Consider a consumer in the third stage. Let $\hat{\theta}_G = \bar{u} - p_G$ and $\hat{\theta}_R = \bar{u} - p_R$. Hence, $\hat{\theta}_R < \hat{\theta}_G$.

A consumer $i$ with type $\theta_i > \hat{\theta}_G$ never buys and receives a pay-off of 0. If $\theta_i \leq \hat{\theta}_R$, then the consumer necessarily buys and chooses between buying immediately at price $p_R$, thus receiving utility $U_A$ or waiting and buying the product at either $p_G$ or $p_R$, which yields pay-off $U_B$. The marginal consumer type is at the cut-off $\hat{\theta}_1$ solution to

$$U_A(\theta_1) = U_B(\theta_1) \iff (\beta - 1)(\bar{u} - \hat{\theta}_1 - p_R) - w + \beta \Gamma(p_R - p_G) = 0$$

$$\iff \hat{\theta}_1 = \frac{(\beta - 1)\hat{\theta}_R - w + \beta \Gamma(\hat{\theta}_G - \hat{\theta}_R)}{\beta - 1} \quad (2)$$

The first term in (2) refers to the full surplus loss from waiting until the last stage if it is not associated with a drop in price. The second term refers to the expected gain from the price difference. High-type consumers (low $\theta_i$) are more tempted to give up the group discount.

For $\hat{\theta}_R < \theta_i \leq \hat{\theta}_G$, the consumer buys only if she benefits from the group discount. Because there is a positive cost, $w$, of waiting around for the offer, she chooses between dropping out immediately thus obtaining zero surplus or waiting to see whether the group deal takes place with corresponding expected surplus $U_C(\theta_i)$. The indifferent type is at the cut-off location $\hat{\theta}_3$ that satisfies

$$U_C(\hat{\theta}_3) = \beta \Gamma(\bar{u} - \hat{\theta}_3 - p_G) - w = 0 \quad (3)$$
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Fig. 4.1: Consumer $i$'s Decision on the Linear Segment.

| $0$ | $\hat{\theta}_1$ | $\hat{\theta}_2$ | $\hat{\theta}_3$ | $\bar{u}$ |

Depending on prices, on the required minimum for group buying and on parameter values, a consumer’s behaviour may be characterized by up to three cut-offs, $\hat{\theta}_1$ solution to (2), $\hat{\theta}_2 = \theta_R$ and $\hat{\theta}_3$ solution to (3). If all three cut-offs are relevant, they are ordered as follows.

$$\hat{\theta}_1 < \hat{\theta}_2 = \hat{\theta}_R < \hat{\theta}_3 < \bar{u}$$

If all regions exist: high valuation consumers, $\theta_i \leq \hat{\theta}_1$, buy immediately at price $p_R$; consumers with intermediate/high valuations, $\hat{\theta}_1 < \theta_i \leq \hat{\theta}_2$ choose to wait and buy at whichever lowest price is available in the last stage; those with intermediate/low valuations, $\hat{\theta}_2 < \theta_i \leq \hat{\theta}_3$, choose to participate and buy in the last stage only if group buying is an option; low valuation consumers, $\theta_i > \hat{\theta}_3$ do not buy. When $M$ sets a very large $\kappa$, the probability of a deal success is close to 0, $\hat{\theta}_3$ approaches $\hat{\theta}_2$ and the mechanism becomes similar to individual sales at $p_R$. On the other hand, setting a low $\kappa$ implies that $\Gamma$ is close to 1, meaning that $\hat{\theta}_3$ approaches $\hat{\theta}_G - \frac{w}{\beta}$ and the mechanism becomes similar to individual sales at $p_G + \frac{w}{\beta}$ and $p_R$ (some form of inter-temporal price discrimination).
4.2.2 Equilibrium Prices and Profits

The firm’s profit function can be written as, (Note that consumers whose types \( \theta_i \in [\hat{\theta}_1, \hat{\theta}_3] \) accept to wait.)

\[
\Pi(p_R^*, p_G^*, \kappa^*) = \max_{p_R, p_G, \kappa} \Pi = p_G \left( \sum_{i=k}^{N} i \binom{N}{i} (F(\hat{\theta}_3) - F(\hat{\theta}_1))^i (1 - (F(\hat{\theta}_3) - F(\hat{\theta}_1)))^{N-i} \right) + \\
+ \mathbb{E} \left( \sum_{i=0}^{k-1} \binom{N}{i} (F(\hat{\theta}_3) - F(\hat{\theta}_1))^i (1 - (F(\hat{\theta}_3) - F(\hat{\theta}_1)))^{N-i} \right) w \left( \frac{\bar{u} - \hat{\theta}_3 - p_G}{\beta} \right)^{-1}
\]

where \( p_G = \bar{u} - \hat{\theta}_G, p_R = \bar{u} - \hat{\theta}_2 \)

Indeed, the expected number of consumers, \( j \), whose types are \( \theta_j \leq \hat{\theta}_1 \), where the expectation
is taken over all realization of \( i \), the number of consumers accepting the group deal.

\[
\frac{F(\hat{\theta}_1)}{1 - (F(\hat{\theta}_3) - F(\hat{\theta}_1))} \sum_{i=0}^{N} (N - i) \binom{N}{i} (F(\hat{\theta}_3) - F(\hat{\theta}_1))^i (1 - (F(\hat{\theta}_3) - F(\hat{\theta}_1)))^{N-i}
\]  \hspace{1cm} (6)

For any binomial distribution,

\[
N \sum_{i=0}^{N} \binom{N}{i} (F(\hat{\theta}_3) - F(\hat{\theta}_1))^i (1 - (F(\hat{\theta}_3) - F(\hat{\theta}_1)))^{N-i} = N
\]

And for the other term,

\[
\sum_{i=0}^{N} i \binom{N}{i} (F(\hat{\theta}_3) - F(\hat{\theta}_1))^i (1 - (F(\hat{\theta}_3) - F(\hat{\theta}_1)))^{N-i} = N (F(\hat{\theta}_3) - F(\hat{\theta}_1))
\]

Substitute these two expressions into (6), the right-hand side becomes exactly \( NF(\hat{\theta}_1) \). Then \( N p_R F(\hat{\theta}_1) \) is the expected profit from those purchasing the goods immediately. The last two terms in (5) refer to the expected profit from consumers postponing the decisions until the last stage. To see how they are obtained, note that the binomial combination in (4) can be rewritten
as,

\[
\sum_{i=k}^{N} i \binom{N}{i} (F(\hat{\theta}_3) - F(\hat{\theta}_1))^i (1 - (F(\hat{\theta}_3) - F(\hat{\theta}_1)))^{N-i} \\
= N \sum_{i=k}^{N} \binom{N-1}{i-1} (F(\hat{\theta}_3) - F(\hat{\theta}_1))^i (1 - (F(\hat{\theta}_3) - F(\hat{\theta}_1)))^{N-i} \\
= N \sum_{i=k-1}^{N-1} \binom{N-1}{i} (F(\hat{\theta}_3) - F(\hat{\theta}_1))^{i+1} (1 - (F(\hat{\theta}_3) - F(\hat{\theta}_1)))^{N-i-1} \\
= N (F(\hat{\theta}_3) - F(\hat{\theta}_1)) \sum_{i=k-1}^{N-1} \binom{N-1}{i} (F(\hat{\theta}_3) - F(\hat{\theta}_1))^{i+1} (1 - (F(\hat{\theta}_3) - F(\hat{\theta}_1)))^{N-i-1} \\
= N (F(\hat{\theta}_3) - F(\hat{\theta}_1)) \Gamma = N (F(\hat{\theta}_3) - F(\hat{\theta}_1)) \frac{w}{\beta (\bar{u} - \hat{\theta}_3 - p_G)}
\]

The last equation comes from (3). Seen from (5), the profit is proportional to the total number of consumers. M’s problem is to maximize the expected profit per consumer. Its problem can be fully reduced to choosing the cutting points. To solve this non-linear programming problem, I write the Lagrangian as the following,

\[
\mathcal{L}(\hat{\theta}_2, \hat{\theta}_3, \hat{\theta}_G) = (\bar{u} - \hat{\theta}_2) F(\hat{\theta}_2) + \frac{w}{\beta (\hat{\theta}_G - \hat{\theta}_3)} \left( (\bar{u} - \hat{\theta}_G) F(\hat{\theta}_3) - (\bar{u} - \hat{\theta}_2) F(\hat{\theta}_2) + (\hat{\theta}_G - \hat{\theta}_2) F(\hat{\theta}_1) \right) \\
- \lambda_1 (\hat{\theta}_3 - \hat{\theta}_G + \frac{w}{\beta}) - \lambda_2 (\hat{\theta}_2 - \hat{\theta}_3)
\]

(7)

Appendix A.1 solves the Lagrangian problem in detail. The solution has either of the constraints binding. If \( \hat{\theta}_2 \leq \hat{\theta}_3 \) turns out to be binding, the group-buying strategy has \( \kappa > N \). Otherwise if \( \hat{\theta}_3 \leq \hat{\theta}_G - \frac{w}{\beta} \) is binding, the group-buying strategy has \( \kappa = 1 \). After comparing the two configurations \( \kappa > N \) and \( \kappa = 1 \), I find the following.

**Proposition 1.** If the discount factor \( 0 < \beta < 1 \), M sets \( \kappa = 1 \). The optimal group-buying
4. When Does Group-buying Increase Profit?

strategy degenerates to an early price and a late price.

Proof. See the appendix.

The idea behind the result is as follows. The group buying strategy is a way for the firm to commit to a pricing scheme where the consumer may either pay $p_R$ immediately or wait in order to have access to a discounted price $p_G$ that he may obtain with probability $\sigma$. The proof shows that if the firm had to select such a random pricing scheme, it would select $\sigma = 1$.

In view of this negative result as to the profitability of group buying for the firm, I now turn to an analysis of how it might be profitable if consumers have the opportunity to invite other consumers who, otherwise, would not have been aware of the availability of the product.

4.3 With Invitations

There are two ways in practice in which consumers can make invitations. First, consumers can inform directly other consumers. Second, group-buying websites often offer tools (Facebook, Twitter, etc.) facilitating communication with potential buyers. Sharing deals takes time and effort. What prompts consumers to invite others is that, when more consumers are informed about the deal, more are likely to participate in the deal, and the minimum is more likely to be met.

Consider the following monopoly model with two consumers ($A$ and $B$), whose locations are indicated by the vector $\Theta = (\theta_A, \theta_B)$. Each consumer has unit demand and reservation price $\bar{u}$. $\theta_i$ is distributed under the distribution function $F(\theta)$ for $\theta_i \in [0, \bar{u}]$. The monopolist ($M$) is located at the point zero, with zero marginal cost of production. Transportation cost per unit is 1.
Each of the two consumers observes the deal and can at most inform one more consumer about the deal. Inviting cost is $c \leq \bar{u}$. Assume that the consumer invited by consumer $A$ is different from the consumer invited by consumer $B$. Assume that the newly informed consumer has the same type as the consumer informing her.

For simplicity, I assume that future utility is not discounted ($\beta = 1$) and waiting is costless ($w = 0$). Taking $I$ as the decision to inform the other consumer, the utility function of the consumer of type $\theta$ is,

$$U(\theta, p, I) = \bar{u} - \theta - p - I \times c$$

(8)

The distribution $F(\theta)$ can be observed by consumers and the monopolist, while the type of each consumer can be observed only by herself.

The game has three players: $M$ and consumer $A, B$ and the following timing.

- In the first stage, nature chooses locations of consumers.
- In the second stage, $M$ proposes the offer $(p_R, p_G, \kappa)$ to consumer $A$ and $B$.
- In the third stage, consumers observe the offer and choose simultaneously one of the three options: 1) accepting the offer and informing another consumer. 2) Accepting the offer without informing anyone. 3) Rejecting the offer.
- In the fourth stage, the demand is realized. If the minimum is met, the discounted price is offered to the consumers who accept the offer. Otherwise, only the normal price is available. Again, the firm is committed throughout to the prices announced in stage 2. As in the model without invitations, I look for Perfect Bayesian Equilibria, assuming consumers’ strategies are such that consumers with identical types make the same choice.

There are two consumers in the first cohort, so that $\kappa$ can be ranged from 2 to 4. Based on $\kappa$, ...
optimal prices and consumers’ best responses differ. The next few sections discuss the optimal pricing strategy for each value of $\kappa$.

4.3.1 Low Minimum ($\kappa = 2$)

When $\kappa = 2$, there are two manners for the minimum to be met. The first manner is that one of the consumers informs the other consumer. By this manner, the discounted price is offered with probability 1. The second manner is that both consumers accept the offer and wait. Without losing generality, consider consumer $A$’s choices. If she informs the other consumer, her utility is,

$$U_1(\theta_A, p_G, p_R) = \bar{u} - \theta_A - p_G - c$$

If she simply accepts the offer without informing anyone, her expected utility is

$$U_2(\theta_A, p_G, p_R) = \begin{cases} \left( \frac{\bar{u} - p_G}{\bar{u}} \right) \left( \bar{u} - \theta_A - p_G \right) + \frac{p_G}{\bar{u}} (\bar{u} - \theta_A - p_R), & 0 \leq \theta_A \leq \hat{\theta}_R \\
\left( \frac{\bar{u} - p_G}{\bar{u}} \right) (\bar{u} - \theta_A - p_G), & \hat{\theta}_R \leq \theta_A \leq \hat{\theta}_G \end{cases}$$

$(\bar{u} - p_G) / \bar{u}$ is the probability that the other consumer accepts the offer. $p_G / \bar{u}$ is the probability that the other consumer rejects the offer. Suppose that there is a threshold of $\theta$, say $\hat{\theta}_1$. If consumer $A$’s type $\theta_A = \hat{\theta}_1$, she is indifferent between informing the other and not informing. For $0 \leq \theta_A \leq \hat{\theta}_R$, the difference

$$U_1(\theta_A, p_G, p_R) - U_2(\theta_A, p_G, p_R) = (p_R - p_G) \frac{p_G}{\bar{u}} - c$$
The difference does not depend on $\theta_A$, implying that consumer $A$'s optimal decision is the same for all $0 \leq \theta_A \leq \hat{\theta}_R$. If prices are set such that $(p_R - p_G)\frac{p_G}{u} - c < 0$, no consumers make invitations, which is certainly not an interesting case. For this reason, I focus on the optimal price setting where $(p_R - p_G)\frac{p_G}{u} - c \geq 0$. For $\hat{\theta}_R \leq \theta_A \leq \hat{\theta}_G$, let

$$U_1(\hat{\theta}_1, p_G, p_R) = U_2(\hat{\theta}_1, p_G, p_R) \Leftrightarrow \hat{\theta}_1(\kappa = 2) = \bar{u} - p_G - \frac{c\bar{u}}{p_G}$$

When $\theta_A$ is smaller than the threshold, accepting the offer and informing the other consumer dominates the other two options.

The normal price is set merely to ensure that consumers do not purchase at the normal price. To see this, suppose that $p_R < p_G + c$. Consumers receiving positive surplus from accepting the offer and informing another consumer would rather purchase at the normal price. We are then back to the model without invitations.

If consumer $A$ and $B$ inform another consumer, the total demand is 4. When only one consumer makes the invitation and the other consumer participates without making any invitation, the total demand is 3. When only one consumer makes the invitation and the other consumer does not participate, the total demand is 2. When both consumers participate without inviting any other consumer, the total demand is 2. The following figure illustrates the composition of demand.

By changing the left panel into the right panel does not change the demand, but simplifies the demand function. The demand function is,

$$D(p_R, p_G; \kappa = 2) = \frac{4}{u} \left( \bar{u} - p_G - \frac{c\bar{u}}{p_G} \right) + \frac{2c}{\bar{u}p_G} (\bar{u} - p_G)$$ (9)
The derivative of $D(p_R, p_G; \kappa = 2)$ with respect to $p_G$ is,

$$\frac{\partial}{\partial p_G} D(p_R, p_G; \kappa = 2) = \frac{2c}{p_G} - \frac{4}{u}$$

From the first-order condition,

$$p_G^*(\kappa = 2) = \frac{2\bar{u} - c}{4}$$  \hspace{1cm} (10)

The indirect profit function is given by,

$$\Pi(p_R^*, p_G^*; \kappa = 2) = \bar{u} + \frac{c^2}{4\bar{u}} - 3c$$  \hspace{1cm} (11)

As $p_G$ is set according to (10), the equilibrium $\hat{\theta}_1$ can be zero if the inviting cost is large enough. Solve $\hat{\theta}_1 = \bar{u} - p_G - \frac{\bar{u}}{p_G} = 0$ for $p_G$. I have $p_G = (\bar{u} - \sqrt{\bar{u}^2 - 4c\bar{u}})/2$. Solve the following for the inviting cost.

$$p_G = \frac{\bar{u} - \sqrt{\bar{u}^2 - 4c\bar{u}}}{2} = \frac{2\bar{u} - c}{4}$$

I obtain the solution $c = 2(\sqrt{17} - 4)\bar{u}$. For $c > 2(\sqrt{17} - 4)\bar{u}$, setting price according to (10) leads
no consumers to informing other consumers and consumers at most are willing to accept the offer without informing others. If this happens, we are back to the model without invitations.  

4.3.2 Medium Minimum \((\kappa = 3)\)

Similarly, there are two manners for the minimum to be met. By the first manner, both consumer \(A\) and \(B\) inform the other consumer, and the total demand will be 4. By the second manner, one of them informs another consumer and the other consumer only accepts to wait. In case consumer \(A\) informs the other consumer, her expected utility is given by

\[
U_1(\theta_A, p_G, p_R) = \begin{cases} 
\frac{(\bar{u} - p_G)(\bar{u} - \theta_A - p_G - c) + \frac{p_G}{\bar{u}}(\bar{u} - \theta_A - p_R - c)}{0 \leq \theta_A \leq \hat{\theta}_R} \\
\frac{(\bar{u} - p_G)(\bar{u} - \theta_A - p_G - c) + \frac{p_G}{\bar{u}}(-c)}{\hat{\theta}_R \leq \theta_A \leq \hat{\theta}_G}
\end{cases}
\] (12)

\((\bar{u} - p_G)\) refers to the probability that consumer \(B\) participates in the group-offer. \(\frac{p_G}{\bar{u}}\) refers to the probability that consumer \(B\) rejects the offer. If consumer \(A\) accepts the offer without informing anybody, her expected utility is given by,

\[
U_2(\theta_A, p_G, p_R) = \begin{cases} 
\frac{\hat{\theta}_1}{\bar{u}}(\bar{u} - \theta_A - p_G) + \frac{\bar{u} - \theta_A - p_R}{\bar{u}}, 0 \leq \theta_A \leq \hat{\theta}_R \\
\frac{\hat{\theta}_1}{\bar{u}}(\bar{u} - \theta_A - p_G), \hat{\theta}_R \leq \theta_A \leq \hat{\theta}_G
\end{cases}
\] (13)

where \(\frac{\hat{\theta}_1}{\bar{u}}\) stands for the probability that consumer \(B\) accepts the offer and informs another consumer. Since \(U_1(\theta_A, p_G, p_R)\) decreases continuously in \(\theta_A\) and \(\lim_{\theta_A \to \bar{u}} U_1(\theta_A, p_G, p_R) < 0,\)

2. Substitute (10) into the expression of \(\hat{\theta}_1\). I obtain,

\[
\hat{\theta}_1 = \frac{1}{4} \left( 2\bar{u} + c - \frac{16\bar{u}c}{2\bar{u} - c} \right), \quad \frac{d\hat{\theta}_1}{dc} = \frac{c^2 - 4\bar{uc} - 28\bar{u}^2}{4(2\bar{u} - c)^2} < 0
\]
there must be a cut-off point where consumer \( A \) is indifferent between informing another consumer, and not.

Suppose that this cut-off point is \( \hat{\theta}_1 \). There is \( \hat{\theta}_1 \geq \hat{\theta}_R \). To see this, suppose \( \hat{\theta}_1 < \hat{\theta}_R \). Calculate the difference in utility for \( 0 \leq \theta_A \leq \hat{\theta}_R \),

\[
U_1(\theta_A, p_G, p_R) - U_2(\theta_A, p_G, p_R) = (\frac{\hat{\theta}_G - \hat{\theta}_1}{\bar{u}})(p_R - p_G) - c
\]

If consumer \( A \) informs the other consumer, the chance of getting the discounted price is \( (\frac{\hat{\theta}_G}{\bar{u}}) \). If he simply participates in the group-offer without informing anyone, the chance is \( (\frac{\hat{\theta}_1}{\bar{u}}) \). The difference in chance is \( (\frac{\theta_G - \theta_1}{\bar{u}}) \). Hence the price difference must compensate for the cost, that is \( p_R - p_G \geq c\frac{\bar{u}}{\theta_G - \theta_1} \) in order for consumer \( A \) to make the invitation. The utility difference does not depend on \( \theta_A \). If at some \( \theta_A < \hat{\theta}_R \), consumer \( A \) is indifferent between informing somebody and not informing, then consumer \( A \) must be indifferent at any \( \theta_A < \hat{\theta}_R \). Suppose that she is indifferent at any \( \theta_A < \hat{\theta}_R \), it follows that no consumers invite anyone, and we come back the model without invitations.

For this reason, the unique existing \( \hat{\theta}_1 \) satisfies \( \hat{\theta}_1 \geq \hat{\theta}_R \). Solving \( U_1(\hat{\theta}_1, p_G, p_R) = U_2(\hat{\theta}_1, p_G, p_R) \) for \( \hat{\theta}_1 \) yields.

\[
\hat{\theta}_1(\kappa = 3) = \bar{u} - \sqrt{\bar{u}}c - p_G
\]

The demand is illustrated in the following figure.

The right panel demonstrates the same demand pattern as the left panel. The profit function
is given by,

\[
\Pi(p_R, p_G; \kappa = 3) = \frac{4p_G}{\bar{u}^2} \left( \bar{u} - p_G - \sqrt{uc} \right)^2 + \frac{6p_G}{\bar{u}^2} \left( \bar{u} - p_G - \sqrt{uc} \right) \sqrt{uc} + 4p_R \left( \frac{\bar{u} - p_R}{\bar{u}} \right) \left( \frac{p_G}{\bar{u}} \right).
\]

subject to \( \hat{\theta}_R \leq \hat{\theta}_1 \) and \( U(\theta, p_R) \leq U_1(\theta, p_G, p_R) \) for \( \theta \leq \hat{\theta}_1 \)

(14)

Consider the case where both constraints are inactive. The first-order condition for \( p_R \) is

\[
\partial \Pi(p_R, p_G; \kappa = 3) / \partial p_R = 0,
\]

and apparently the optimal normal price is \( p^*_R = \bar{u}/2 \). To see this, suppose that consumer A informs another consumer and consumer B rejects the offer so that the minimum is not met in the last stage. Then clearly consumer A and the informed consumer have types \( \theta \leq \hat{\theta}_1 \). Setting the normal price to \( p_R \), M’s profit is then \( \frac{2p_R(\bar{u} - p_R)}{\bar{u}} \). Solving \( \partial^2 \Pi(p_R, p_G; \kappa = 3) / \partial p_R = 0 \) yields \( p^*_R = \bar{u}/2 \). Solving \( \partial \Pi(p_R, p_G; \kappa = 3) / \partial p_G = 0 \) yields the following.

\[
p^*_G(\kappa = 3) = \frac{1}{6} \left( 4\bar{u} - \sqrt{u}c - \sqrt{\bar{u}^2 + 7uc} - 2\bar{u}\sqrt{uc} \right)
\]

The above contradicts \( \hat{\theta}_1 > \hat{\theta}_R \). Substitute the above into \( \hat{\theta}_1(\kappa = 3) \), I have \( \hat{\theta}_1(\kappa = 3) < \bar{u}/2 \).
(shown in the appendix). Consider the binding case ($\hat{\theta}_1 = \hat{\theta}_R$) where $p_R$ is set so that each consumer who did not reject the offer purchases at $p_R$, that is $p_R = \bar{u} - \hat{\theta}_1 (\kappa = 3)$. The profit function is then

$$P(p_R, p_G; \kappa = 3) = \frac{4p_G}{\bar{u}^2} (\bar{u} - p_G - \sqrt{\bar{u}c})^2 + \frac{6p_G}{\bar{u}^2} (\bar{u} - p_G - \sqrt{\bar{u}c}) \sqrt{\bar{u}c} + 4(p_G + \sqrt{\bar{u}c}) \left( \frac{\hat{\theta}_1}{\bar{u}} \right) \left( \frac{p_G}{\bar{u}} \right)$$

First-order condition in $p_G$ yields that

$$p_G^* = \frac{\bar{u} - \sqrt{\bar{u}c}}{2}, \quad p_R^* = \frac{\bar{u} + \sqrt{\bar{u}c}}{2}$$

Consider any $\theta < \hat{\theta}_1 = \hat{\theta}_R$. Examine the constraint $U(\theta, p_R) \leq U_1(\theta, p_G, p_R)$ for the optimal prices from above.

$$U_1(\theta, p_G, p_R) - U(\theta, p_R) = \left( \frac{\bar{u} - p_G}{\bar{u}} \right) (\bar{u} - \theta - p_G - c) + \left( \frac{p_G}{\bar{u}} \right) (\bar{u} - \theta - p_R - c) - (\bar{u} - \theta - p_R)$$

$$= (\sqrt{\bar{u}c} - c)/2 > 0$$

Therefore no consumers purchase the product immediately and leave the market. The indirect profit function is,

$$P(p_R^*, p_G^*; \kappa = 3) = \frac{(\sqrt{\bar{u}} - \sqrt{\bar{e}})^2 (2\sqrt{\bar{u}} + 3\sqrt{\bar{e}})}{2\sqrt{\bar{u}}}$$

**4.3.3 High Minimum ($\kappa = 4$)**

As opposed to the low (medium) minimum, there is only one manner by which the minimum can be met. Either both consumer $A$ and $B$ inform other consumers, or the minimum is not
met. If consumer $i$ does not inform anybody, either she buys at the normal price and receives $\bar{u} - \theta - p_R$ or she rejects the offer and receives 0. Suppose that if a consumer’s type is $\hat{\theta}_1$, she is indifferent between informing somebody and not informing. Intuitively, for any consumer-type $\theta_i < \hat{\theta}_1$, she chooses to inform. Write consumer $A$’s expected utility when he informs somebody as,

$$U_1(\theta_A, p_G, p_R) = \begin{cases} 
\left(\frac{\hat{\theta}_1}{\bar{u}}\right)(\bar{u} - \theta_A - p_G - c) + \left(\frac{\bar{u} - \hat{\theta}_1}{\bar{u}}\right)(\bar{u} - \theta_A - p_R - c), & 0 \leq \theta_A \leq \hat{\theta}_R \\
\left(\frac{\hat{\theta}_1}{\bar{u}}\right)(\bar{u} - \theta_A - p_G - c) + \left(\frac{\bar{u} - \hat{\theta}_1}{\bar{u}}\right)(-c), & \hat{\theta}_R \leq \theta_A \leq \hat{\theta}_G 
\end{cases} \quad \text{(16)}$$

If consumer $A$ does not inform anyone, his utility is,

$$U_2(\theta_A, p_G, p_R) = \begin{cases} 
\bar{u} - \theta_A - p_R, & 0 \leq \theta_A \leq \hat{\theta}_R \\
0, & \hat{\theta}_R \leq \theta_A \leq \bar{u}
\end{cases}$$

First, I show that there exists not a $\hat{\theta}_1$ satisfying $\hat{\theta}_1 < \hat{\theta}_R$. Suppose that such a $\hat{\theta}_1$ exists. Calculate the utility difference for $0 \leq \theta_A \leq \hat{\theta}_R$,

$$U_1(\theta_A, p_G, p_R) - U_2(\theta_A, p_G, p_R) = \frac{\hat{\theta}_1}{\bar{u}}(p_R - p_G) - c$$

The above is clearly positive. By definition, the chance of getting a discounted price is $(\frac{\hat{\theta}_1}{\bar{u}})^2$. Therefore, $M$ has to offer a price difference $p_R - p_G \geq (\frac{\bar{u}}{\hat{\theta}_1})^2 c$ in order to get consumers to invite others. Hence, $U_1(\theta_A, p_G, p_R) - U_2(\theta_A, p_G, p_R) = 0$ has no solutions for $0 \leq \theta_A \leq \hat{\theta}_R$. 
For $\hat{\theta}_1 \geq \hat{\theta}_R$, solve $U_1(\hat{\theta}_1, p_G, p_R) = 0$ for $\hat{\theta}_1$.

$$\hat{\theta}_1(\kappa = 4) = \frac{1}{2} \left( \bar{u} - p_G + \sqrt{(\bar{u} - p_G)^2 - 4\bar{u}c} \right)$$ \hspace{1cm} (17)

The demand is illustrated in the following figure. Again, the right panel illustrates the same demand function as the left panel. The monopolist’s problem is given by

$$\max \Pi(p_R, p_G; \kappa = 4) = \begin{cases} 4p_G \left( \frac{\hat{\theta}_1}{\bar{u}} \right)^2 & \text{the minimum is met} \\ 4p_R \left( \frac{\bar{u} - p_R}{\bar{u}} \right) \left( \frac{\bar{u} - \hat{\theta}_1}{\bar{u}} \right) & \text{the minimum is not met} \end{cases}$$ \hspace{1cm} (18)

subject to $\hat{\theta}_R \leq \hat{\theta}_1$ and $U(\theta, p_R) \leq U_1(\theta, p_G, p_R)$ for $\theta \leq \hat{\theta}_1$

First, consider that neither of the constraints is active. First-order condition in $p_R$ yields that $p_R = \bar{u}/2$. Solving $\hat{\theta}_R = \hat{\theta}_1$ yields that $p_G = \left( \frac{\bar{u} - 4c}{2} \right)$. To show that the optimal $p_G$ from first-order conditions does not satisfy $\hat{\theta}_R < \hat{\theta}_1$, it suffices to demonstrate that $\Pi(p_R, p_G; \kappa = 4)$ keeps increasing in $p_G$ even at $p_G = \left( \frac{\bar{u} - 4c}{2} \right)$, as long as $c \leq \bar{u}$. Therefore $M$ keeps increasing $p_G$...
until the constraint $\hat{\theta}_R \leq \hat{\theta}_1$ is binding.

$$\frac{\partial \Pi(\bar{u}/2, (\bar{u} - 4c)/2; \kappa = 4)}{\partial p_G} = \frac{2c}{\bar{u}^2} \left( \frac{2c}{\bar{u}^2} \left( \bar{u}^2 + 24c^2 + 6c(|\bar{u} - 4c| - \bar{u}) \right) > 0 \right)$$

Then the constraint $\hat{\theta}_R \leq \hat{\theta}_1$ is binding, equivalent to that $p_R = \bar{u} - \hat{\theta}_1$. $p_R$ is low enough that when the minimum is not met, then those informing the other consumer always make a purchase. The monopolist’s problem is then,

$$\max \Pi(p_R, p_G; \kappa = 4) = 4p_G \left( \frac{\hat{\theta}_1}{\bar{u}} \right)^2 + 4(\bar{u} - \hat{\theta}_1) \left( \frac{\hat{\theta}_1}{\bar{u}} \left( \frac{\bar{u} - \hat{\theta}_1}{\bar{u}} \right) \right)$$

subject to $\hat{\theta}_R = \hat{\theta}_1$ and $U(\theta, p_R) \leq U_1(\theta, p_G, p_R)$ for $\theta \leq \hat{\theta}_1$

$p_G$ can be written into an expression of $\hat{\theta}_1$. Solve (17) for $p_G$, I have

$$p_G(\kappa = 4) = \frac{\bar{u} \hat{\theta}_1 - \bar{u} c - \hat{\theta}_1^2}{\hat{\theta}_1}$$

Plug (20) into (19). Solving the first-order conditions of (19) with respect to $\hat{\theta}_1$ yields $\hat{\theta}_1 = \frac{u - c}{2}$ and the optimal discounted price

$$p_G^*(\kappa = 4) = \frac{\bar{u}^2 - 4\bar{u} c - c^2}{2(\bar{u} - c)}$$

Next, I examine the constraint $U(\theta, p_R) \leq U_1(\theta, p_G, p_R)$.

$$U(\theta, p_R^*) - U_1(\theta, p_G^*, p_R^*) = (\bar{u} - \theta - p_R^*) - \left( (\bar{u} - \theta - p_G^* - c) \frac{\hat{\theta}_1}{\bar{u}} + \left( \frac{\bar{u} - \hat{\theta}_1}{\bar{u}} \right) (\bar{u} - \theta - p_R^* - c) \right) = 0$$
The above means that prices are set such that consumers who make purchase in the last stage are always indifferent between informing other consumers and not informing. The profit function is given by, for \( c \leq (\sqrt{5} - 2)\bar{u} \),

\[
\Pi(p_R^*, p_G^*; \kappa = 4) = \frac{(\bar{u} - c)^2}{\bar{u}} \quad \text{for} \quad c < (\sqrt{5} - 2)\bar{u}
\]  

(22)

According to (21), the optimal discounted price \( p_G = 0 \) for \( (\sqrt{5} - 2)\bar{u} < c \leq \bar{u}/4 \). Suppose that the optimal discounted price \( p_G \neq 0 \), then we are back to (18) and (19). Fixing \( p_G = 0 \), (17) is equivalent to

\[
\hat{\theta}_1 = \frac{\bar{u} + \sqrt{u^2 - 4uc}}{2} > \frac{\bar{u}}{2}
\]

The monopolist’s problem is given by,

\[
\max \Pi(p_R, p_G; \kappa = 4) = 4p_R\left(\frac{\bar{u} - p_R^R}{\bar{u}}\right)\left(\frac{\bar{u} - \hat{\theta}_1}{\bar{u}}\right) \quad \text{subject to} \quad \hat{\theta}_R \leq \hat{\theta}_1 \quad \text{and} \quad U(\theta, p_R) \leq U_1(\theta, p_G, p_R) \quad \text{for} \quad \theta \leq \hat{\theta}_1
\]

If neither of the constraints are binding, first-order conditions yield \( p_R = \bar{u}/2 \), which violates \( U(\theta, p_R) \leq U_1(\theta, p_G, p_R) \) for \( \theta \leq \hat{\theta}_1 \) (proof omitted). Therefore the constraint \( U(\theta, p_R) \leq U_1(\theta, p_G, p_R) \) is binding. Solving the constraint \( U(\theta, p_R) = U_1(\theta, p_G, p_R) \) for \( p_R \) yields \( p_R = \frac{uc}{\hat{\theta}_1} \).

Replace \( p_R \) in the above profit function with \( \frac{uc}{\hat{\theta}_1} \). Rearranging the terms gives the profit function below. The optimal price also satisfies \( \hat{\theta}_R = \hat{\theta}_1 \).

\[
\Pi(p_R^*, p_G^*; \kappa = 4) = 2c(1 + \sqrt{1 - \frac{4c}{\bar{u}}}) \quad \text{for} \quad (\sqrt{5} - 2)\bar{u} < c \leq \bar{u}/4
\]
For \( c > \bar{u}/4 \), there are no solutions for \( U_1(\hat{\theta}_1, p_G, p_R) = 0 \). No invitations take place and we are back to the model without invitations.

4.3.4 Comparative Statics

The above pricing strategies can be summarized into the following proposition.

**Proposition 2.** While setting \( \kappa = 2 \), the pricing functions and the profit function are,

\[
p^*_G(\kappa = 2) = \frac{2\bar{u} - c}{4} \quad \text{and} \quad \Pi(p^*_R, p^*_G; \kappa = 2) = \bar{u} + \frac{c^2}{4\bar{u}} - 3c
\]

While setting \( \kappa = 3 \), the pricing functions are,

\[
p^*_G = \frac{\bar{u} - \sqrt{\bar{u}c}}{2}, \quad p^*_R = \frac{\bar{u} + \sqrt{\bar{u}c}}{2}, \quad \Pi(p^*_R, p^*_G; \kappa = 3) = \frac{(\sqrt{\bar{u}} - \sqrt{c})^2(2\sqrt{\bar{u}} + 3\sqrt{c})}{2\sqrt{\bar{u}}}
\]

While setting \( \kappa = 4 \), price equilibrium satisfies, for \( c \leq (\sqrt{5} - 2)\bar{u} \),

\[
p^*_G(\kappa = 4) = \frac{\bar{u}^2 - 4\bar{u}c - c^2}{2(\bar{u} - c)} \quad \text{and} \quad p^*_R(\kappa = 4) = \bar{u} - \frac{1}{2} \left( \bar{u} - p_G + \left| \frac{\bar{u}^2 - 6\bar{u}c + c^2}{2(\bar{u} - c)} \right| \right)
\]

\[
\Pi(p^*_R, p^*_G; \kappa = 4) = \frac{(\bar{u} - c)^2}{\bar{u}}
\]

for \( (\sqrt{5} - 2)\bar{u} < c \leq \bar{u}/4 \),

\[
p^*_G = 0, \quad p^*_R = \frac{\bar{u} - \sqrt{\bar{u}^2 - 4\bar{u}c}}{2}, \quad \Pi(p^*_R, p^*_G; \kappa = 4) = 2c(1 + \sqrt{1 - \frac{4c}{\bar{u}}})
\]

\( p_G \) decreases in inviting cost and \( p_R \) increases in inviting cost. Industry profit decreases in inviting cost. All price and profit functions are homogeneous of degree one in \( \bar{u} \) and \( c \).
4. When Does Group-buying Increase Profit? 

Proof. See the appendix.

Intuitively, larger inviting cost forces M to set a lower discounted price (or a higher normal price) to motivate consumers to inform others, particularly when a higher minimum is set so that consumers faces a lower probability that the minimum is met in the last stage. Besides, larger inviting cost helps sort out consumers having high valuations in the sense that only those with higher valuations invite others. Thus, a higher normal price can be offered to each of them if the minimum is not met. The following illustrates the optimal pricing strategy at different minimums.

Fig. 4.5: Optimal discounted prices under various $\kappa$ ($\bar{u} = 1$).

The following figure shows that the expected profit decreases in inviting cost. Both the $\kappa = 2$ configuration and the $\kappa = 3$ configuration are dominated by the $\kappa = 4$ configuration. In the polar case when inviting cost is very large, the group-buying strategy is suboptimal.

**Proposition 3.** Setting the high minimum always dominates setting the low (and medium) minimum.

There exists a threshold of inviting cost $c' = \bar{u}/4$. For $c < c'$, the monopolist uses the group-buying strategy. For $c \geq c'$, the monopolist uses a single-price setting.

Proof. See the appendix.
Setting different prices is about encouraging high-type consumers to invite other consumers, and the trade-off between motivating more consumers to inform others and setting lower discounted prices. The best situation is when both consumer $A$ and $B$ invite others, as they invite only high-type consumers willing to pay a higher discounted price. A worse situation is when only one consumer invites another and the other does not and free-rides on the first one. The discounted price must be low enough to let the free-rider accept the offer. The worst situation is when there are two free-riders.

The minimum should be set in the same vein, and if possible, to sort out the high-type consumers. The $\kappa = 2$ configuration offers the discounted price even to the situation with two free-riders, while the $\kappa = 3$ configuration offers the discounted price to the situations allowing at most one free-rider. Both configurations indeed encourage high-type consumers, while they fails to keep the discounted price away from the low-type. In the equilibrium under large inviting cost, selling two only to the high-type is sometimes more profitable than selling three to both the high-type and the low-type.

Here is a related example. Suppose that consumer $A$ is located at $x$ (therefore can pay at most $\bar{u} - x$), and consumer $B$ is located at $y \gg x$ (therefore can pay at most $\bar{u} - y$). The monopolist observes the location information (without knowing who is who). Setting $\kappa = 3$ and
selling 3 give the profit $3(\bar{u} - y - c)$, whereas setting $\kappa = 4$ and selling 2 to consumer $A$ and the informed give $2(\bar{u} - x - c)$. $\bar{u} - y$ being small enough, $2(\bar{u} - x - c) > 3(\bar{u} - y - c)$; selling 2 is more profitable than selling 3.

The $\kappa = 4$ configuration keeps all low-type away because each consumer has to make the invitation for the possibility of getting the discounted price. No matter whether the minimum is not realized, only the high-type consumers purchase. In equilibrium, consumers having made invitations purchase at the normal price no matter the minimum is met or not in the last stage. The share coming from failing to sell at the discounted price can be always above 50% for $\kappa = 4$ (above 40% for $\kappa = 3$), illustrated in Figure 7. To see this, consider the indirect profit function for $\kappa = 3$

$$\Pi(p^*_R, p^*_G; \kappa = 3) = \frac{\bar{u}\sqrt{\bar{u}} + 2c\sqrt{c} - 3c\bar{u}}{2\sqrt{\bar{u}}} + \frac{(\sqrt{\bar{u}} - \sqrt{c})^2(\sqrt{\bar{u}} + \sqrt{c})}{2\sqrt{\bar{u}}}$$

(23)

The percentage of the second term out of the total profit is

$$\lambda(p^*_R, p^*_G; \kappa = 3) = \frac{\sqrt{\bar{u}} + \sqrt{c}}{2\sqrt{\bar{u}} + 3\sqrt{c}} \geq 40\%$$

And for $\kappa = 4$,

$$\lambda(p^*_R, p^*_G; \kappa = 4) = \frac{(\bar{u} + c)^2}{2\bar{u}(\bar{u} - c)} \geq 50\%$$

This percentage can decrease or increase in inviting cost. In the polar case, $p_G$ is set to zero. Under the $\kappa = 3$ configuration, the chance that the "group offer" is carried out in equilibrium is between 0% and 33%. Under the $\kappa = 4$ configuration, the chance that the "group offer" is carried
out in equilibrium is between 0% and 25%. M provides a low $p_G$ as the bait for invitations, gains nothing from the successful "group offer" and expects that the "group offer" cannot be carried out and some consumers purchase at $p_R$.

**Corollary 1.** In the optimal configuration ($\kappa = 4$), the monopolist gains larger profit from the normal price than from the discounted price.

**Proof.** $\lambda(p_R^*, p_G^*; \kappa = 4) = \frac{(\bar{u}+c)^2}{2\bar{u}(u-c)} > \frac{1}{2} \iff -2c(3\bar{u} + c) < 0.$

---

### 4.3.5 Consumer Surplus

There are two measures of consumer surplus, respectively from the first cohort and the second cohort. Take (8) as the form of utility function. It is difficult to measure consumer surplus of the second cohort because low-type consumers are never informed.
First Cohort

For the first cohort of consumers, her expected consumer surplus when $\kappa$ is set to 2 is given by, (here $\hat{\theta}_1 = \hat{\theta}_1(\kappa = 2)$)

$$CS^{(1)}(\kappa = 2) = \frac{1}{\bar{u}} \int_0^{\hat{\theta}_1} (\bar{u} - \theta - p_G - c)d\theta + \frac{1}{\bar{u}} \int_{\hat{\theta}_1}^{\bar{u} - p_G} \left( \frac{\bar{u} - p_G}{\bar{u}} \right) (\bar{u} - \theta - p_G)d\theta$$ (24)

For $\kappa = 3$, let $\hat{\theta}_1 = \hat{\theta}_1(\kappa = 3)$,

$$CS^{(1)}(\kappa = 3) = \frac{1}{\bar{u}} \int_0^{\hat{\theta}_1} \left( \frac{\bar{u} - p_G}{\bar{u}} \right) (\bar{u} - \theta - p_G - c)d\theta + \left( \frac{p_G}{\bar{u}} \right) (\bar{u} - \theta - p_R - c)d\theta + \frac{1}{\bar{u}} \int_{\hat{\theta}_1}^{\hat{\theta}_R} \left( \frac{\bar{u} - p_G}{\bar{u}} \right) (\bar{u} - \theta - p_G)d\theta$$

$$= (\bar{u} - c)/8 < \bar{u}/8$$ (25)

For $\kappa = 4$, let $\hat{\theta}_1 = \hat{\theta}_1(\kappa = 4)$,

$$CS^{(1)}(\kappa = 4) = \frac{1}{\bar{u}} \int_0^{\hat{\theta}_1} \left( \frac{\bar{u} - p_G}{\bar{u}} \right) (\bar{u} - \theta - p_G - c)d\theta + \frac{1}{\bar{u}} \int_{\hat{\theta}_1}^{\hat{\theta}_1} \left( \frac{\bar{u} - \hat{\theta}_1}{\bar{u}} \right) (\bar{u} - \theta - p_R - c)d\theta$$

$$= (\bar{u} - c)^2/8\bar{u} < \bar{u}/8$$ (26)

Here I draw the comparison of consumer surplus when different minimums are set. The "$\kappa = 2$" case always delivers lower consumer surplus than other cases. The "$\kappa = 3$" case always delivers larger consumer surplus than other group-buying cases.

**Proposition 4.** in the optimal configuration, the group-buying strategy lowers down the surplus of the first-generation consumers.
4. When Does Group-buying Increase Profit?

**Fig. 4.8:** First-cohort consumer surplus under various $\kappa$ ($\bar{u} = 1$).

![Graph showing consumer surplus under various $\kappa$ values](image)

**Proof.** I plug the respective optimal price into (24), and rearrange the terms. $CS^{(1)}(\kappa = 2)$ can be simplified as below. $CS^{(1)}(\kappa = 2) > \bar{u}/8$ iff $c < (\sqrt{1129} - 31)\bar{u}/7$.

$$CS^{(1)}(\kappa = 2) = \frac{8\bar{u}^3 + 62\bar{uc}^2 - 28\bar{u}^2c + 7c^3}{64\bar{u}^2 - 32\bar{uc}}$$

Comparing with monopoly pricing, consumers of types $\theta \in [0, \bar{u}/2)$ are worse off under the $\kappa = 2$ configuration because they pay more ($p^*_G + c = (2\bar{u} + 3c)/4 > \bar{u}/2$). Consumers of types $\theta \in (\bar{u}/2, (2\bar{u} + c)/4)$ are better off because chances are that they pay a lower price. Summing up these effects yields lower total surplus.

**Fig. 4.9:** Consumer surplus under group-buying and monopoly pricing ($c = 0.15$, $\bar{u} = 1$).

![Graph showing consumer surplus under group-buying and monopoly pricing](image)
4. When Does Group-buying Increase Profit?

Under the $\kappa = 3$ (and $\kappa = 4$) configuration, the surplus function is illustrated as the blue curve in Figure 9. Again consumers of high types are worse off compared with monopoly pricing, while consumers of low-types are better off. Summing up these two effects yields lower expected surplus for a consumer of the first generation.

Second Cohort

For the second cohort of consumers, I have to assume that there is a one-to-one relationship between each consumer of the second cohort and his counterpart in the first cohort. For example, for each second-cohort consumer having $\theta < \hat{\theta}_1$, there must exist a first-cohort consumer having $\theta < \hat{\theta}_1$ who invites him. For the $\kappa = 2$ configuration, I have,

$$CS^{(1)}(\kappa = 2) = \frac{1}{\bar{u}} \int_0^{\hat{\theta}_1} (\bar{u} - \theta - p_G) d\theta$$  \hspace{1cm} (28)$$

For the $\kappa = 3$ configuration, let $\hat{\theta}_1 = \hat{\theta}_1(\kappa = 3)$,

$$CS^{(1)}(\kappa = 3) = \frac{1}{\bar{u}} \int_0^{\hat{\theta}_R} \left( \frac{\bar{u} - p_G}{\bar{u}} \right) (\bar{u} - \theta - p_G) + \left( \frac{p_G}{\bar{u}} \right) (\bar{u} - \theta - p_R) d\theta$$

$$= \frac{1}{8} (\sqrt{\bar{u}} - \sqrt{\bar{c}})^2 (1 - 2\sqrt{\bar{u}\bar{c}})$$  \hspace{1cm} (29)$$

For the $\kappa = 4$ configuration, let $\hat{\theta}_1 = \hat{\theta}_1(\kappa = 4)$,

$$CS^{(1)}(\kappa = 4) = \frac{1}{\bar{u}} \int_0^{\hat{\theta}_1} \left( \frac{\hat{\theta}_1}{\bar{u}} \right) (\bar{u} - \theta - p_G) + \left( \frac{\bar{u} - \hat{\theta}_1}{\bar{u}} \right) (\bar{u} - \theta - p_R) d\theta$$

$$= \frac{(\bar{u} - c)(\bar{u}^2 - \bar{u}\bar{c} + 4\sqrt{\bar{u}\bar{c}})}{8\bar{u}^2}$$  \hspace{1cm} (30)$$
When inviting cost increases, there are largely two effects. The larger the invitation cost, the lower the discounted price and the less chance consumers are invited. The first effect increases the surplus of second-cohort consumers, and the second effect decreases it. The first effect dominates the second in the $\kappa = 4$ configuration and the opposite holds in the $\kappa = 3$ configuration. Besides, the reason for the lowest surplus for the $\kappa = 3$ configuration is that, the chance for consumers to pay the normal price is too large.

**Fig. 4.10:** Second-cohort consumer surplus under various $\kappa (\bar{u} = 1)$.

### Two Cohorts

In the end, I compare the sum of consumer surplus of the first cohort and the second cohort with the monopoly pricing case. It is clear that consumer surplus is larger using the group-buying strategy as long as inviting cost is not too large, which along with our result in industry profit, demonstrates that social welfare is larger using group-buying.

### 4.4 Conclusion

This paper builds two tractable models, both offering clear-cut results. In the model without invitation, the group-buying strategy is always suboptimal with respect to either a single-price
setting or a time-dependent price setting. In the model with invitation, the monopolist uses the group-buying strategy with a high minimum when inviting cost is low, and comes back to the single-price setting when inviting cost is large. Consumers of high types are worse off, and consumers of lower types are better off.

The paper has three major limitations. From the most important to the least important, the first limitation is that consumers are merely allowed to inform one more consumer. If this restriction is relaxed, then consumers with highest types are likely to inform more than one consumers, which may exacerbate the "free rider" problem because low-types consumers are more ready to wait.

The second limitation is that consumers merely invite other consumers with the same consumer-types. I have considered a similar model where each consumer is either a local or non-local, with probability $\rho$ and $1 - \rho$. A local informs another consumer with the same type, and a non-local informs another consumer whose type is randomly drawn from $U[0, \bar{u}]$. Some preliminary simulation results have suggested that prices are lower when $\rho$ is smaller.

The third limitation is that this is a monopoly model. As soon as I introduce competition, the model becomes less tractable. For example, there are no longer closed-form solutions for
the minimums under $\kappa = 3$ and $\kappa = 4$. Further, asymmetries can arise such that firms choose different minimums.

A Appendix A

A.1 Proof of Proposition 1

Proof. 1) Solving the optimization problem by Lagrangian.

In what follows, I abstract from the fact that in a group buying arrangement, the probability $\Gamma$ should be given by the cumulative of a binomial distribution, so I solve a relaxed problem where $\Gamma$ can take any value in $[0,1]$. From (3), it can be rewritten as a function of thresholds,

$$\Gamma = \frac{w}{\beta (\hat{\theta}_G - \hat{\theta}_3)}$$

I check ex post that the solution to the relaxed problem can be implemented by merely offering two prices without resorting to group buying.

Plug the above into (2), and rewrite $\hat{\theta}_1$ into the expression of $\hat{\theta}_2$, $\hat{\theta}_3$ and $\hat{\theta}_G$.

$$(\beta - 1)(\bar{u} - \hat{\theta}_1 - p_R) - w + \frac{w(\hat{\theta}_G - \hat{\theta}_2)}{\hat{\theta}_G - \hat{\theta}_3} = 0 \iff \hat{\theta}_1 = \hat{\theta}_2 - \frac{w(\hat{\theta}_3 - \hat{\theta}_2)}{(1 - \beta)(\hat{\theta}_G - \hat{\theta}_3)} \quad \text{(A.1)}$$

The three derivatives of $\hat{\theta}_1$ in other variables are,

$$\frac{\partial \hat{\theta}_1}{\partial \hat{\theta}_2} = 1 + \frac{w}{(1 - \beta)(\hat{\theta}_G - \hat{\theta}_3)}, \quad \frac{\partial \hat{\theta}_1}{\partial \hat{\theta}_3} = \frac{-w(\hat{\theta}_G - \hat{\theta}_2)}{(1 - \beta)(\hat{\theta}_G - \hat{\theta}_3)^2}, \quad \frac{\partial \hat{\theta}_1}{\partial \hat{\theta}_G} = \frac{w(\hat{\theta}_3 - \hat{\theta}_2)}{(1 - \beta)(\hat{\theta}_G - \hat{\theta}_3)^2}$$
Replace $p_R$ by $\bar{\theta}_2$, and $p_G$ by $\bar{\theta}_G$. The Lagrangian is,

$$\mathcal{L}(\hat{\theta}_2, \hat{\theta}_3, \hat{\theta}_G) = (\bar{\theta}_2 - \hat{\theta}_2) F(\hat{\theta}_2) + \frac{w}{2} (\bar{\theta}_G - \hat{\theta}_3) \left( (\bar{\theta}_2 - \hat{\theta}_2) F(\hat{\theta}_3) - (\bar{\theta}_G - \hat{\theta}_3) F(\hat{\theta}_4) \right)$$

$$- \lambda_1 (\hat{\theta}_3 - \hat{\theta}_G) + \frac{w}{\beta} - \lambda_2 (\hat{\theta}_2 - \hat{\theta}_3) \tag{A.2}$$

where $\hat{\theta}_1$ satisfies (A.1). Then the platform’s choice satisfies $\mathcal{L}''_{\hat{\theta}_2} = 0$, $\mathcal{L}''_{\hat{\theta}_3} = 0$, $\mathcal{L}''_{\hat{\theta}_G} = 0$ and two Kuhn-Tucker conditions. They are, after some rearrangements,

$$\mathcal{L}''_{\hat{\theta}_2} = 0 \iff (\bar{\theta}_2 - \hat{\theta}_2) F(\hat{\theta}_2) + w ((\bar{\theta}_2 - \hat{\theta}_2) F(\hat{\theta}_3) - (\bar{\theta}_G - \hat{\theta}_3) F(\hat{\theta}_2)$$

$$+ f(\hat{\theta}_1) (\hat{\theta}_G - \hat{\theta}_3) \frac{\partial \hat{\theta}_1}{\partial \hat{\theta}_2} - \lambda_2 \beta (\hat{\theta}_G - \hat{\theta}_3) = 0 \tag{A.3}$$

$$\mathcal{L}''_{\hat{\theta}_3} = 0 \iff w ((\bar{\theta}_2 - \hat{\theta}_2) F(\hat{\theta}_3) - (\bar{\theta}_G - \hat{\theta}_2) F(\hat{\theta}_2) + (\hat{\theta}_G - \hat{\theta}_2) F(\hat{\theta}_1) + w (f(\hat{\theta}_3)(\hat{\theta}_G - \hat{\theta}_3)(\bar{\theta}_G - \hat{\theta}_G)$$

$$- \frac{w f(\hat{\theta}_1)(\hat{\theta}_G - \hat{\theta}_2)^2}{(1 - \beta)(\hat{\theta}_G - \hat{\theta}_3)} - \lambda_1 \beta (\hat{\theta}_G - \hat{\theta}_3)^2 + \lambda_2 \beta (\hat{\theta}_G - \hat{\theta}_3)^2 = 0 \tag{A.4}$$

$$\mathcal{L}''_{\hat{\theta}_G} = 0 \iff w ((\bar{\theta}_2 - \hat{\theta}_2) F(\hat{\theta}_2) + (\hat{\theta}_G - \hat{\theta}_2) F(\hat{\theta}_1)) + w ((\bar{\theta}_G - \hat{\theta}_3) F(\hat{\theta}_3)$$

$$(\bar{\theta}_2 - \hat{\theta}_2) F(\hat{\theta}_2) + (\hat{\theta}_G - \hat{\theta}_2) F(\hat{\theta}_1)) + \lambda_1 \beta (\hat{\theta}_G - \hat{\theta}_3)^2 = 0 \tag{A.5}$$

$$\lambda_1 \geq 0 \ (\lambda_1 = 0 \text{ if } \hat{\theta}_3 - \hat{\theta}_G + \frac{w}{\beta} < 0), \lambda_2 \geq 0 \ (\lambda_1 = 0 \text{ if } \hat{\theta}_2 - \hat{\theta}_3 < 0) \tag{A.6}$$

2) Look for solutions while both constraints are not binding.

If there are no solutions when both constraints are inactive, then I am done. If either one of constraints is binding, $M$ resorts to an early price plus a late price. Suppose that $\lambda_1 = 0$ and $\lambda_2 = 0$. I will show that $\hat{\theta}_1 = \hat{\theta}_2 = \hat{\theta}_3$ is the solution. Indeed when $\hat{\theta}_2 = \hat{\theta}_3$, (A.1) suggests that
\( \hat{\theta}_1 = \hat{\theta}_2 \). Adding (A.4) to (A.5) yields the following after some transformations,

\[
(1 - \beta)(\hat{\theta}_G - \hat{\theta}_3)(F(\hat{\theta}_1) - F(\hat{\theta}_3)) - w f(\hat{\theta}_1)(\hat{\theta}_G - \hat{\theta}_2) + (1 - \beta)(\hat{\theta}_G - \hat{\theta}_3)f(\hat{\theta}_3)(\bar{u} - \hat{\theta}_G) = 0
\]

Then if I apply \( \hat{\theta}_1 = \hat{\theta}_2 = \hat{\theta}_3 \neq \hat{\theta}_G \), the above is reduced to,

\[
\bar{u} - \hat{\theta}_G - \frac{w}{1 - \beta} = 0
\] (A.7)

Plug (A.7) into (A.3) and apply \( \hat{\theta}_1 = \hat{\theta}_2 = \hat{\theta}_3 \neq \hat{\theta}_G \). After cancelling out some terms, (A.3) can be rewritten into,

\[
((\bar{u} - \hat{\theta}_2)f(\hat{\theta}_2) - F(\hat{\theta}_2))\beta(\hat{\theta}_G - \hat{\theta}_3) + w\left(\frac{w f(\hat{\theta}_1)}{1 - \beta} - f(\hat{\theta}_1)(\bar{u} - \hat{\theta}_G)\right) = 0
\] (A.8)

The second term of the above is clearly zero from (A.7). Since \( \hat{\theta}_3 \neq \hat{\theta}_G \), the above implies the monopoly price: \( (\bar{u} - \hat{\theta}_2)f(\hat{\theta}_2) - F(\hat{\theta}_2) = 0 \), which along with \( \hat{\theta}_1 = \hat{\theta}_2 = \hat{\theta}_3 \) and (A.7), is a solution in the admissible set. Hence, I find a binding constraint by assuming that perhaps both constraints are not binding. In this solution, M sets a target which is not achievable. Then all the buyers purchases the products at the normal price determined by

\[
p_R = \frac{F(\bar{u} - p_R)}{f(\bar{u} - p_R)}
\]

3) Look for solutions while either of the constraints is binding.

On the other hand, the constraints are binding: either \( \hat{\theta}_3 - \hat{\theta}_G + \frac{w}{\beta} \leq 0 \) or \( \hat{\theta}_2 - \hat{\theta}_3 \leq 0 \). If the constraint \( \hat{\theta}_2 - \hat{\theta}_3 \leq 0 \) is binding, we come back to the above monopoly price. If the constraint
\( \hat{\theta}_3 - \hat{\theta}_G + \frac{w}{\beta} \leq 0 \) is binding, (3) implies that \( \Gamma = 1 \), meaning that each consumer will be offered the discounted price if he waits until the last stage. From (A.1), we know that \( \hat{\theta}_1 \neq \hat{\theta}_2 \neq \hat{\theta}_3 \). \( \hat{\theta}_1 \) and \( \hat{\theta}_3 \) are determined by

\[
\hat{\theta}_1 = \bar{u} + \frac{w - p_R + \beta p_G}{1 - \beta}, \quad \hat{\theta}_3 = \bar{u} - p_G - \frac{w}{\beta}
\]

Note that \( p_R \neq p_G \) implies that \( \hat{\theta}_1 \neq \hat{\theta}_3 \). \( \mathbf{M} \) maximizes the joint profit, which is given by

\[
\Pi(p_G, p_R) = p_G \left( F(\bar{u} - p_G - \frac{w}{\beta}) - F(\bar{u} + \frac{w - p_R + \beta p_G}{1 - \beta}) \right) + p_R F(\bar{u} + \frac{w - p_R + \beta p_G}{1 - \beta})
\]

The profit function can be rewritten as the following by replacing \( p_R, p_G \) with \( \hat{\theta}_1, \hat{\theta}_3 \).

\[
\Pi(\hat{\theta}_1, \hat{\theta}_3) = (\bar{u} - \hat{\theta}_3 - \frac{w}{\beta}) F(\hat{\theta}_3) + ((1 - \beta)(\hat{\theta}_3 - \hat{\theta}_1) + \frac{w}{\beta}) F(\hat{\theta}_1)
\]

First-order conditions are

\[
\frac{\partial \Pi(\hat{\theta}_1, \hat{\theta}_3)}{\partial \hat{\theta}_1} = -(1 - \beta) F(\hat{\theta}_1) + ((1 - \beta)(\hat{\theta}_3 - \hat{\theta}_1) + \frac{w}{\beta}) f(\hat{\theta}_1) \quad (A.9)
\]

\[
\frac{\partial \Pi(\hat{\theta}_1, \hat{\theta}_3)}{\partial \hat{\theta}_3} = -F(\hat{\theta}_3) + (\bar{u} - \hat{\theta}_3 - \frac{w}{\beta}) f(\hat{\theta}_3) + (1 - \beta) F(\hat{\theta}_1) \quad (A.10)
\]

Suppose that \( \hat{\theta}_1 = \hat{\theta}_3 \). Then the above two conditions yield,

\[
\frac{F(\hat{\theta}_1)}{f(\hat{\theta}_1)} = \frac{w}{\beta(1 - \beta)}, \quad \frac{F(\hat{\theta}_1)}{f(\hat{\theta}_1)} = \frac{\bar{u} - \hat{\theta}_1 - \frac{w}{\beta}}{\beta}
\]

Clearly, there is not a single \( \hat{\theta}_1 \) satisfying the above two. \( \square \)
A.2 Proof of Proposition 2

Proof. First I show that for the $\kappa = 3$ configuration, by charging

$$p^*_G(\kappa = 3) = \frac{1}{6} \left( 4\bar{u} - \sqrt{\bar{u}c} - \sqrt{\bar{u}^2 + 7\bar{u}c - 2\bar{u}\sqrt{\bar{u}c}} \right)$$

we have $\hat{\theta}_1(\kappa = 3) < \frac{\bar{u}}{2}$. Note that

$$\hat{\theta}_1(\kappa = 3) = \bar{u} - \sqrt{\bar{u}c} - \frac{1}{6} \left( 4\bar{u} - \sqrt{\bar{u}c} - \sqrt{\bar{u}^2 + 7\bar{u}c - 2\bar{u}\sqrt{\bar{u}c}} \right)$$

After some transformations, to show $\hat{\theta}_1(\kappa = 3) < \frac{\bar{u}}{2}$ is equivalent to,

$$6\bar{u}(3c + 2\sqrt{uc}) > 0$$

Second, I try to show that for the $\kappa = 4$ configuration, for inviting cost small enough, the normal price increases in the inviting cost. From (21), we know that,

$$p^*_R(\kappa = 4) = \bar{u} - \frac{1}{2} \left( \bar{u} - \frac{\bar{u}^2 - 4\bar{u}c - c^2}{2(\bar{u} - c)} + \frac{\bar{u}^2 - 6\bar{u}c + c^2}{2(\bar{u} - c)} \right)$$

If the inviting cost is small enough such that $\bar{u}^2 - 6\bar{u}c + c^2 > 0$, the above can be simplified as,

$$p^*_R(\kappa = 4) = \frac{\bar{u} + c}{2}$$

Apparently, the normal price increases in the inviting cost. \(\square\)
A.3 Proof of Proposition 3

Proof. First I focus on the comparison between the \( \kappa = 2 \) configuration and the \( \kappa = 3 \) configuration.

\[
\Pi(p_R^*, p_G^*; \kappa = 2) - \Pi(p_R^*, p_G^*; \kappa = 3) = (\bar{u} + \frac{c^2}{4\bar{u}} - 3c) - \frac{1}{2\sqrt{u}}(\sqrt{\bar{u}} - \sqrt{c})^2(2\sqrt{\bar{u}} + 3\sqrt{c})
\]

\[
= \frac{\sqrt{c}(c\sqrt{c} + 2\bar{u}\sqrt{\bar{u}} - 6c\sqrt{\bar{u}} - 4\bar{u}\sqrt{c})}{4\bar{u}} \quad (A.11)
\]

Let \( \varphi(c) = c\sqrt{c} + 2\bar{u}\sqrt{\bar{u}} - 6c\sqrt{\bar{u}} - 4\bar{u}\sqrt{c} \). \( \varphi(\bar{u}) = -7\bar{u}\sqrt{\bar{u}} \) and \( \varphi'(c) = 3\sqrt{c}/2 - 6\sqrt{\bar{u}} - 2\bar{u}/\sqrt{c} < 0 \).

Therefore, there exists a threshold \( c' \) such that \( c > c' \) if \( \Pi(p_R^*, p_G^*; \kappa = 2) < \Pi(p_R^*, p_G^*; \kappa = 3) \).

Consider the profit function when \( \kappa \) is set to 4.

\[
\Pi(p_R^*, p_G^*; \kappa = 4) = \frac{(\bar{u} - c)^2}{\bar{u}} \quad (A.12)
\]

Second I focus on the comparison between the \( \kappa = 2 \) configuration and the \( \kappa = 4 \) configuration.

\[
\Pi(p_R^*, p_G^*; \kappa = 2) - \Pi(p_R^*, p_G^*; \kappa = 4) = (\bar{u} + \frac{c^2}{4\bar{u}} - 3c) - \left(\frac{\bar{u} - c}{\bar{u}}\right)^2 = -\frac{c(4\bar{u} + 3c)}{4\bar{u}} < 0
\]

Third I focus on the comparison between the \( \kappa = 3 \) configuration and the \( \kappa = 4 \) configuration.

\[
\Pi(p_R^*, p_G^*; \kappa = 4) - \Pi(p_R^*, p_G^*; \kappa = 3) = \frac{(\bar{u} - c)^2}{\bar{u}} - \frac{(\sqrt{\bar{u}} - \sqrt{c})^2(2\sqrt{\bar{u}} + 3\sqrt{c})}{2\sqrt{\bar{u}}}
\]

\[
= \frac{\sqrt{c}(\bar{u}\sqrt{\bar{u}} + 2c\sqrt{c} - 3c\sqrt{\bar{u})}}{2\bar{u}} > 0 \quad (A.13)
\]
Last I focus on the comparison between the $\kappa = 4$ configuration and a single-price setting.

$$\Pi(p^*_R, p^*_G; \kappa = 4) - \Pi(p^*_R, p^*_G; \kappa = 1) = \frac{(\bar{u} - c)^2}{\bar{u}} - \frac{\bar{u}}{2} > 0 \iff c < (1 - \frac{\sqrt{2}}{2})\bar{u}$$
4. When Does Group-buying Increase Profit?
5. RÉSUMÉ

Dans le premier chapitre de cette thèse nous avons proposé un modèle permettant de maximiser le profit sur l’ordonnancement de différents produits. Nous avons utilisé, d’une part, un produit de niche, spécifique au goût de certains consommateurs pris de manière hétérogène ; d’autre part, nous avons utilisé un produit générique.

De manière générale, les consommateurs, dans leur recherche séquentielle d’un produit, se concentrent sur le produit et le prix. Nous postulons que cette attitude est généralisable à tous les consommateurs. Dans un premier temps, ils recherchent soit le produit de niche, soit le produit générique. Notre étude a montré qu’indépendamment de l’ordre recherche suivi par le consommateur, la seconde entreprise visitée fixait un prix plus élevé que la première. Les prix ne sont pas monotones en ce qui concerne l’hétérogénéité de la préférence des consommateurs pour le second produit.

Les performances du marché sont supérieures quand le produit de niche est visitée en premier par le consommateur. Ceci est dû au fait que cet ordonnancement restreint la recherche du consommateur, relâchant ainsi la concurrence. Ainsi, cet ordonnancement profite sans ambiguïté à l’entreprise proposant les produits de niche. Pour autant, l’entreprise spécialisée en produits génériques peut y trouver un intérêt. Le résultat « niche précédant générique » reste un équilibre dans la mesure où les consommateurs choisissent librement le type de produit qu’ils recherchent.

Enfin, nos simulations ont montrées que l’ordonnancement « niche précédant générique »
implique un surplus du consommateur plus grand sous certaines conditions. Ce même ordonnancement réduit généralement les distorsions de prix et sera de ce fait maximise le bien-être social.

Dans le second chapitre, nous avons utilisé une variation basée sur le modèle de recherche de Wolinsky (1986) afin de d’examiner de nouveau les relations entre les coûts de recherche et la concentration des entreprises. Les consommateurs recherchent un prix et recoupent les informations sur une plateforme de commerce électronique monopolistique ou bien par le biais de recherches séquentielles. Les entreprises proposant les produits peuvent ou non, adhérer à ce type de plateformes. Par ailleurs, la plateforme détermine le nombre d’entreprises qu’elle référence. Notre étude a montré que le degré de concentration voulu par la plateforme est inversement proportionnel à l’évolution du coût de recherche. Nous avons retrouvé le même schéma dans le cas d’un contexte concurrentiel entre plusieurs plateformes.

Dans le dernier chapitre, nous nous sommes concentrés sur les achats groupés. Une stratégie d’achats groupés se réfère à un système de tarification basé sur trois variables : le prix normal ; le prix réduit et le nombre minimum requis de participants pour constituer un accord d’achats groupés.

Alors que le prix normal est disponible à tout moment et pour tous, le prix réduit est réservé à un groupe de consommateurs atteignant le nombre minimum requis de participants. Cette stratégie s’est récemment fortement développée dans les ventes sur internet.

Notre étude a proposé d’étudier deux modèles monopolistiques. Dans un premier modèle, sans invitation pré-requise, nous avons démontré que la stratégie d’achats groupés est toujours sous-optimale comparée à un système fondé sur la simple dépendance du prix au facteur temps. Dans le second modèle, sur invitations, nous avons montré que la stratégie de prix, caractérisée
par un niveau moyen (ou bas) minimum requis, est toujours dominée par celle, caractérisée par un niveau élevé de minimum requis. En outre, la stratégie d’achats groupés est sous-optimale à un niveau élevé de coût d’invitation. De manière générale, les prix réduits sont d’autant plus bas et les prix normaux d’autant plus élevés que le coût de l’invitation augmente.

Comparée aux prix dans un système monopolistique, la stratégie d’achats groupés réduit le surplus de la première génération de consommateurs (ceux qui ont l’opportunité d’avoir accès aux accords d’achats groupés sans invitation), et augmente le surplus de la seconde génération de consommateurs.
BIBLIOGRAPHIE


