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ESSAIS SUR LA FORMATION DE JURIDICTIONS ET LA SÉGRÉGATION

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Résumé

Cette thèse a pour objet l'étude de la formation endogène des juridictions et en particulier ses propriétés ségrégatives suite à l'introduction de différents facteurs susceptibles de les intensifier ou de les réduire.

Le premier chapitre est consacré à une revue de la littérature sur la formation endogène de juridictions basée sur les intuitions formulées par Tiebout : les ménages choisissent leur commune en fonction de la quantité de bien public disponible et du montant de taxe à y acquitter. Les différentes modélisations de ces hypothèses, les conditions sous lesquelles un équilibre existera, les possibles définitions de la ségrégation et les facteurs pro et anti-ségrégation développés par la littérature sont résumés et confrontés.

Le deuxième chapitre étudie l'impact de l'introduction d'un gouvernement central mettant en œuvre une politique de péréquation fiscale suivant un objectif bien-êtriste. Le gouvernement central peut ainsi taxer les ménages et/ou certaines communes afin de verser des subventions à d'autres communes. Bien que la péréquation fiscale soit susceptible de modifier l'ensemble des structures de juridictions stables, la condition nécessaire et suffisante pour que toute structure de juridictions stable soit ségrégée n'est pas affectée par l'introduction du gouvernement central si celui-ci cherche à maximiser une fonction de bien-être social utilitariste généralisée.

La présence d'un marché compétitif du logement et l'existence de plusieurs biens publics locaux sont introduites dans le chapitre 3. Si la condition nécessaire et suffisante à la ségrégation de toute structure de juridictions stable n'est pas affectée par l'introduction du marché du logement, et reste nécessaire s'il existe plusieurs biens publics locaux, une hypothèse sur les préférences doit être ajoutée pour que la condition reste suffisante.

Enfin, le quatrième chapitre relaxe l'hypothèse selon laquelle un bien public local ne souffre pas de problèmes de congestion et ne peut être consommé que par les membres de la juridiction qui le produit. Ainsi, s'il semble apparaître que la congestion favorise la ségrégation, alors que l'existence d'externalités positives générées par le bien public d'une juridiction dans les autres juridictions la réduit, la condition nécessaire et suffisante à la ségrégation de toute structure de juridictions stable est robuste à cette généralisation.

Mots clés : économie publique locale, formation endogène de juridiction, ségrégation, bien public local, vote, redistribution, marché du logement

Abstract

This thesis analyzes the endogenous jurisdictions formation process and its segregative properties after the introduction of some factors that may mitigate or increase them.

The first chapter is devoted to a survey of the literature on the endogenous formation of jurisdictions based on Tiebout's intuitions : households choose their place of residence according to a trade-off between the available amount of public good and the tax rate. The different models of these assumptions, the conditions under which an equilibrium exists, the possible definitions of segregation and the factors pro and anti-segregation developed in the literature are summarized and compared.

The second chapter examines the impact of the introduction of a welfarist central government implementing a equalization transfers policy. The central government can tax the household and/or certain jurisdictions in order to subsidize other jurisdictions. Though equalization transfers may modify stable jurisdictions structures, the necessary and sufficient condition to have any stable jurisdiction structure segregated is not affected by the introduction of the central government if it pursues a generalized utilitarian objective.

The presence of a competitive housing market and the existence of several local public goods are introduced in Chapter 3. If the necessary and sufficient condition for the segregation of any stable jurisdiction structure is not affected by the introduction of the housing market, and remains necessary if there are several local public goods, an additional assumption on the preference must be made for the condition to remain sufficient.

Finally, the fourth chapter relaxes the assumption that a local public good does not suffer from congestion and can be consumed only by the members of the jurisdiction that produces it. Though it seems that the congestion favors segregation, while the existence of positive externalities generated by a jurisdiction's public good in other jurisdictions mitigates them, the necessary and sufficient condition to ensure the segregation of any stable jurisdictions structure is robust to this generalization of the model.

Keywords : local public economics, endogenous jurisdictions formation, segregation, local public good, vote, redistribution, housing market

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Introduction générale

0.1 Introduction

Dans la plupart des pays, en plus de l'échelon central qu'est l'état, coexistent un ou plusieurs échelons de prise de décisions locaux. Par exemple, en France, depuis les lois de décentralisation de 1982, la Commune, le Département et la Région sont des collectivités locales dont les représentants sont élus par les citoyens. A ces collectivités locales s'ajoutent désormais l'échelon intercommunal, dont les compétences vont en s'amplifiant.

0.1.1 L'existence de juridictions multiples dans la réalité...

Une étude[68], menée en 2002 par l'organisme Europa[1], montre qu'en Europe, les dépenses des collectivités locales représentent 11 % du PIB européen, avec des écarts conséquents d'un pays à l'autre : 2,2 % du PIB en Grèce, contre plus de 30 % du PIB au Danemark. Cette part dépend des compétences que le gouvernement central délégue aux collectivités locales, et est en augmentation depuis la fin de la Seconde Guerre Mondiale.

La part de la dépense des collectivités locale dans la dépense publique en Europe est également conséquente : en Europe, 24 % des dépenses de fonctionnement et 67 % des dépenses d'investissement public sont le fait des collectivités locales.Cette importance grandissante des collectivités locales dans la dépense publique justifie l'existence de l'im-

[1] http ://unilim.fr/prospeur/fr/prospeur/ressources/finances/index.htm

portante littérature économique portant sur l'étude des structures à juridictions multiples.

Dans la plupart des pays démocratiques, les élus locaux, comme le gouvernement national, sont issus du vote des citoyens. Une fois élus, ils votent annuellement le budget de la juridiction qui détermine le taux de taxe s'appliquant dans la juridiction et quantité de biens publics locaux entrant dans le champs de compétences de l'échelon. Par exemple, en France, la mairie gère les écoles primaires, le département, les collèges et la Région, les lycées.

0.1.2 ... Et dans la littérature économique

Le gouvernement d'une juridiction a, parmi ses attributions, un rôle de production de bien public local, c'est-à-dire un bien à la fois non-rival et non-excluable pour les habitants de la juridiction. Un bien est non-rival si la consommation d'une unité d'un bien par un consommateur n'empêche pas la consommation de cette même unité par d'autres consommateurs. Un bien est non-excluable s'il est impossible (ou alors à un coût prohibitif) d'empécher individu de consommer ce bien lorsque celui-ci est disponible.

Le raccordement au câble ou au réseau téléphonique est un exemple de bien non-rival. La protection d'un quartier contre les risques de criminalité que fournit une patrouille policière constitue un exemple de bien non-excluable. La non-rivalité d'un bien rend non-désirable sur le plan de l'efficacité économique ex-post l'exclusion d'usage de ce bien alors que la non-excluabilité rend cette exclusion tout simplement impossible (ou très coûteuse).

Dans la mesure où l'exclusion d'usage est la caractéristique des biens qui permet de définir la propriété privée (comme droit d'exclusion d'usage), les biens publics ne peuvent

typiquement pas être fourni par des procédures marchandes basées sur leur appropriation privée. Ils sont donc souvent fourni par une autorité publique et financés par des prélèvements obligatoires que sont les impôts et les taxes.

Il est courant en économie publique de supposer que chaque ménage, lors de l'élection, vote pour le candidat (ou le programme) qui maximisera son utilité, toutes choses égales par ailleurs, et ce, en fonction de plusieurs paramètres, comme le revenu, les prix, la quantité de bien public qu'il recevra, etc.. L'économie publique locale fait également cette hypothèse et ne se distingue donc pas, sur ce plan, de l'économie publique traditionnelle.

Il existe toutefois 2 différences entre économie publique traditionnelle et économie publique locale :

1. En économie publique traditionnelle, il n'existe qu'une seule juridiction, que les individus ne peuvent pas quitter, m'me si le gouvernement conduit une politique qui ne leur convient pas. En revanche, en économie publique locale, plusieurs juridictions appartenant à un même échelon coexistent (coexistence horizontale) et les ménages ont la possibilité de choisir leur juridiction d'appartenance. Cette mobilité des ménages modifie substantiellement l'analyse des politiques que peuvent conduire les juridictions.

2. En plus de la coexistence horizontale de plusieurs juridictions de même échelons, l'économie publique locale intègre la coexistence verticale de plusieurs échelons de décision structurée de manière assez hiérarchique (gouvernement fédéral et provincial dans les pays fédéraux, Etat et collectivités locales en France, etc...). Cette pluralité verticale d'échelons pose notamment la question de la péréquation fiscale et de la redistribution entre juridictions d'un même échelon que peut décider le gouvernement

d'un échelon supérieur. Le problème de redistribution entre juridictions est évidemment absent de l'économie publique traditionnelle, du fait de l'unicité du niveau de prise de décisions.

Tiebout, dans son célèbre article, "A Pure Theory of Local Expenditures"[82], qui se voulait une réponse à l'article de Samuelson, "The Pure Theory of Public Expenditures"[78], fut l'un des premiers économistes à noter la spécificité de l'économie publique locale et les conséquences que cette spécificité pouvait avoir sur le problème plus traditionnel de l'économie d'allocations des ressources en présence de biens publics.

Au moment où Tiebout faisait paraître son article, la version la plus achevée de la théorie traditionnelle de l'allocation des ressources en présence de bien public était celle formulée par Samuelson. Ce dernier, après avoir développé une définition de la notion de bien public, posait le problème de la révélation par les individus de leurs préférences pour ces biens publics, et de la sous-optimalité qui pouvait en résulter.

En effet, en l'absence d'exclusion d'usage, la contribution d'un individu au bien public génère des externalités positives pour les autres membres de la société qui peuvent bénéficier sans coûts du bien public produit par cette contribution. Puisque chaque individu peut bénéficier sans coût du bien public, il tendra à s'en remettre aux autres pour financer le bien public ou, pour reprendre l'expression d'usage, à se comporter en "passager clandestin" ("free rider"). La généralisation de ce comportement engendrera typiquement une sous-production de bien public par rapport à ce qu'exigerait l'efficacité Parétienne. Cette inefficacité Parétienne sera d'autant plus sévère que le nombre de contributeurs potentiel est grand et que l'impact d'une contribution individuelle sur la production de bien public globale est faible. L'inefficacité vraisemblable à laquelle devait conduire, d'après Samuelson,

une production décentralisée de bien public du fait du phénomène du passager clandestin avait donc conduit cet auteur à justifier une production centralisée de bien public avec financement autoritaire par impôt.

L'article de Tiebout se voulait une contestation partielle de cette logique. D'après lui, sachant que, aux Etats-Unis, la moitié des dépenses publiques étaient d'origine locale, le choix par les individus du couple "taux de taxe-quantité de bien public" pouvait s'assimiler à un processus analogue à celui consistant à choisir un couple "prix des carottes-quantité de carottes" sur un marché. Ce processus conduit les agents, supposés rationnels, à révéler leurs préférences pour les couples "quantité de bien public-impôt" en comparant les différents couples, supposés très nombreux, entre eux et en choisissant celui qu'ils préfèrent.

Les hypothèses, non formalisées par Tiebout, sur lesquelles il fonde ses intuitions sont les suivantes :

- Il existe un grand nombre de juridictions (Tiebout parle de communautés), offrant différents couples taux de taxe-quantité de bien public où peuvent choisir de vivre les ménages ;
- Les ménages disposent d'une information parfaite sur les couples "taux de taxe-quantité de bien public" qui ont cours dans chacune des juridictions, et peuvent déménager sans coût d'une juridiction à l'autre ;
- Une juridiction ne profite pas du bien public généré par une autre juridiction. La production d'un bien public dans une juridiction n'a donc pas d'effets externes sur la fourniture de bien public dans une autre juridiction.

L'équilibre est atteint quand aucun ménage ne peut augmenter son utilité en quittant sa juridiction pour une autre.

Le résultat apporté par cet article est que les ménages, en "votant avec leur pied", c'est-à-dire, en quittant une juridiction dont le couple "taux de taxe-quantité de bien public" ne leur convient pas, révèlent leurs préférences de manière analogue à ce qu'ils feraient sur un marché. Les ménages désirant une grande quantité de bien public se rassembleront dans des juridictions à fort taux de taxe et importante provision de bien public, tandis que ceux moins disposés - ou moins capables - de payer pour le bien public préfèreront des juridictions moins pourvue en bien public où la pression fiscale est moindre. D'après Tiebout, ce mécanisme conduit à une allocation efficace de biens privés et de biens publics locaux sans qu'il n'y ait besoin d'une intervention publique centralisée.

La contribution de Tiebout s'applique essentiellement aux zones urbaines, dans lesquelles plusieurs communes situées autour d'une grande ville coexistent. Dans ce contexte, l'hypothèse d'un coût de mobilité faible est plausible. Elle le serait moins si on appliquait les intuitions de Tiebout aux juridictions formées par états-nations.

0.1.3 La formation de juridictions et la ségrégation

Depuis l'article de Tiebout, une abondante littérature s'est consacrée à clarifier ses intuitions et à préciser leur domaine de validité.

Dans son article, Tiebout évoque l'émergence, à l'équilibre, d'une structure de juridictions homogènes, c'est-à-dire dans laquelle les ménages se ressemblent par leur préférence ou leurs richesse.

Pourtant, Tiebout ne donne pas de définition précise et formalisée de la ségrégation.

Par conséquent, l'impact de la formation endogène de juridictions sur la ségrégation entre les juridictions reste flou. En effet, Tiebout ne précise pas dans quelle mesure et suivant quel critère (richesse, préférences...) les juridictions sont homogènes.

La ségrégation est un concept particulièrement étudié en sciences sociales. On entend par ségrégation la mise à l'écart d'un groupe d'individus en fonction de différents critères : origines, sexe, couleur de peau, religion, orientation sexuelle, classe sociale, richesse... . Nous ne traiterons dans cette thèse que de la ségrégation entre juridictions par la richesse.

La question de ce type de ségrégation intèresse au plus haut point les dirigeants politiques. En France par exemple, c'est l'aversion pour la ségrégation qui est à l'origine de la loi sur la Solidarité et le Renouvellement Urbain (loi SRU, votée en 2000) qui exige qu'un quota minimal de 20 % de logements sociaux soit imposé à toutes les communes de plus de 3500 habitants, situées dans une agglomération de plus de 50000 habitants et comprenant au moins une ville de plus de 15000 habitants, sous peine de voir la dotation qui leur est fournie par l'Etat diminuée.

En région parisienne, 7 ans après la promulgation de la loi SRU, près d'une commune sur 2 (83 sur 181) ne respecte pas le quota de 20 % de logements sociaux. De plus, la quasi-totalité de ces communes ne cherchent pas à se mettre en conformité avec la loi, et préfèrent payer une amende plutôt que de construire des logements sociaux.

Les différents articles que nous étudierons supposent que la formation des juridictions est endogène, c'est-à-dire que le processus de formation de juridictions se produit à l'intérieur du modèle considéré, et résultent donc de décisions prises par les agents individuels.

Les choix de résidence des individus dans telle ou telle juridiction ne sont pas exogènes,

mais résultent d'une décision rationnelle basée sur une comparaison des coûts des impôts qu'ils doivent payer et de la qualité/quantité disponible de bien public.

0.2 La modélisation de la formation endogène de juridictions et la ségrégation dans la littérature

Dans l'article de Tiebout, la répartition des ménages entre les différentes juridictions est endogène, c'est-à-dire que ce sont les ménages qui, en fonction de leurs préférences, choisissent la juridiction qui leur convient. La modélisation des intuitions de Tiebout soulevait plusieurs questions, notamment celle de l'existence d'un équilibre d'une structure de juridictions. Les hypothèses standards émises sur les préférences des ménages sont-elles suffisantes pour garantir l'existence d'un équilibre ?

Le fait que les ménages puissent choisir leur juridiction de résidence peut impliquer que les ménages ayant les mêmes préférences en matière de couple "taux de taxe-quantité de bien public" se regroupent au sein des mêmes juridictions. Selon les intuitions exprimées par Tiebout, la formation endogène de juridictions conduit à une homogénisation des populations au sein des différentes juridictions. Cette notion d'homogénisation n'étant pas formulée explicitement, il convient de définir le terme de ségrégation, en évitant les caricatures rencontrées lors de discussions ou dans certains articles de presse. Certaines villes souffrent en effet de préjugés concernant leur population : Nice est réputée peuplée de cheveux blancs dotés d'un niveau de vie conséquent, Neuilly-sur-Seine appara$_{itcommeunlieuuniquementremplidemnagesassujettisl'ImptdeSolidaritsurlaFortune,alorsqueSarcellesaunerputationdevil}$

0.2.1 La modélisation de Westhoff

L'une des premières tentatives de formalisation des intuitions de Tiebout se trouve dans l'article de Westhoff[84]. Cet article propose une modélisation des intuitions de Tiebout, en relaxant l'hypothèse selon laquelle le nombre de juridictions serait suffisamment grand pour que chaque ménage ait le choix entre un large spectre de couple "taux de taxe-quantité de bien public".

Cette hypothèse conduisait à ce que le budget de chaque juridiction soit approuvé unanimement, donc l'article de Tiebout ne s'intéressait pas aux mécanismes de prise de décision concernant le niveau de taxation. Westhoff intègre un mécanisme de prise de décision qui n'existait pas dans l'article de Tiebout, car le seul vote qui lui semblait intéressant était le vote "avec les pieds". Westhoff fait l'hypothèse que les habitants d'une juridiction choisissent le taux de taxe par un vote majoritaire. La taxe est assise sur la richesse des ménages.

Cette hypothèse revient à supposer que chaque juridiction adopte un taux de taxe tel qu'au moins la moitié des électeurs ne veut pas de taux de taxe inférieur à celui-ci tandis que l'autre moitié des électeurs ne veut pas de taux de taxe supérieur. Le choix de la juridiction concernant le taux de taxe sera donc celui de l'électeur médian. Le taux de taxe est ensuite appliqué à l'assiette fiscale de la juridiction, le montant perçu est entièrement consacré à la production du bien public.

Le ménage, en votant pour un taux de taxe, connaît la quantité de bien public correspondante. Les préférences des ménages pour les taux de taxe sont unimodales, c'est-à-dire que, à richesse privée et agrégée donnée, l'utilité du ménage par rapport au taxe de taxe est croissante si le taux de taxe est inférieur à son taux de taxe préféré puis décroissante

lorsque le taux de taxe est supérieur à son taux de taxe préféré.

Les 2 conditions nécessaires à l'application du théorème de Black[9], à savoir l'existence d'un ordre global faisant l'objet d'un consensus parmi les électeurs (ici, il s'agit tout simplement du domaine de définition du taux de taxe, compris entre 0 et 1), et des préférences unimodales, sont réunies. Le taux de taxe choisi par l'électeur médian (tel que 50 % des électeurs veulent un taux de taxe supérieur et 50 %, un taux inférieur) sera donc un vainqueur au sens de Condorcet : aucun taux de taxe ne peut lui être préféré par une majorité d'électeurs.

Westhoff suppose l'existence d'un continuum de ménages, repartis entre un nombre fixe de juridictions. Deux biens sont disponibles, un bien public et un bien privé composite. Le bien public est financé par une juridiction grâce à une taxe proportionnelle au revenu des ménages qui la composent. La recette fiscale de chaque juridiction est entièrement consacrée à la production du bien public de sorte que le budget public local soit équilibré. Une juridiction j est caractérisée par son taux de taxe et son niveau de bien public disponible.

Les ménages diffèrent par leurs préférences et par leur revenu. Les préférences des ménages sont représentées par une fonction d'utilité, différente selon les ménages, qui dépend des quantités de bien public et de bien privé.

Chaque ménage détermine le taux de taxe qui maximise son utilité, sachant que le taux de taxe retenu sera celui choisi par l'électeur médian. Une fois le taux de taxe établi au sein de chaque juridiction, un ménage peut quitter sa juridiction pour une autre qui lui permettra d'obtenir le niveau d'utilité le plus élevé. Le ménage prend sa décision de partir ou de rester en supposant que cette décision sera sans effet sur les taux de taxes et les assiettes fiscales existantes. Cette hypothèse est défendable dans la mesure où un ménage

individuel est arbitrairement petit par rapport à toute juridiction.

Une partition de l'ensemble des individus entre les m juridictions forme ce que Westhoff appelle à une structure de juridictions. Westhoff définit une structure de juridiction comme étant en équilibre si :

- chaque juridiction a un budget en équilibre,
- pour toute juridiction, sachant la contrainte budgétaire de la juridiction, au moins la moitié des membres de la juridiction ne veut pas plus de bien public et au moins la moitié des membres de la juridiction ne veut pas moins de bien public,
- aucun ménage ne peut accroître son utilité en changeant de juridiction.

Westhoff émet, dans cet article, 4 hypothèses standard décrivant le modèle, ainsi qu'une cinquième hypothèse, suffisante pour prouver l'existence d'un équilibre, quelque soit le nombre de juridictions :

1. dans chaque juridiction, les recettes proviennent d'une taxe proportionnelle au revenu de chaque ménage résidant dans cette juridiction,
2. la fonction de coût de production du bien public local est une fonction linéaire dépendant uniquement de la quantité de bien public local produit,
3. le revenu d'un ménage est une fonction positive, bornée et mesurable, de sorte que les ménages soient ordonnés en fonction de leur revenu,
4. toutes les fonctions d'utilité sont continues, croissantes au sens strict et quasi-concaves au sens strict par rapport à chacun de ses arguments, et respecte les conditions d'Inada[2] ;

[2] La condition d'Inada établit qu'un ménage préfèrera tout panier de bien composé de quantités strictement positives de chacun des 2 biens à tout autre panier où l'une des quantités est nulle

5. Le taux marginal de substitution du bien public au privé privé est une fonction continue et croissante par rapport au revenu pour tout niveau de bien public strictement positif et tout niveau de taxe inférieur à 1. En d'autres termes, la pente du taux marginal de substitution du niveau de bien public pour le bien privé est une fonction monotone par rapport à la richesse des ménages. Cela implique que, dans le plan "taux de taxe-quantité de bien public", les courbes d'indifférence de 2 ménages ne se croisent qu'une seule fois.

La condition 5 est suffisante non seulement pour garantir l'existence d'un équilibre de la structure de juridictions, mais également pour avoir ségrégation parfaite de la population entre les juridictions à l'équilibre, dans le sens où le ménage le plus riche d'une juridiction sera plus pauvre que le ménage les plus pauvre d'une juridiction à richesse moyenne plus élevée.

Notons que cette condition est suffisante, mais peut-être pas nécessaire.

0.2.2 L'article de Gravel et Thoron

L'article de Gravel et Thoron vise à identifier une condition explicite posée sur les préférences sous lesquelles la ségrégation des ménages par la richesse est systématiquement observée dans des structures de juridictions stables. Leur analyse est construite à partir du modèle de Westhoff : il existe un continuum de ménages et un nombre donné de lieux de résidence possible.

Les ménages disposent de ressources individuelles en bien privé différentes, mais, contrairement à Westhoff, leurs préférences pour le bien public et le biens privés sont identiques. Ces préférences sont représentées par une fonction d'utilité, identique pour chaque agent, qui dépend de la quantité de bien public et d'un bien privé composite. Cette fonction est

2 fois différentiable et est croissante et concave par rapport à chacun de ses arguments.

Comme dans l'article de Westhoff, le seul impôt local existant est une taxe proportionnelle assise sur la richesse privée des ménages. Il est entièrement consacré à la production du bien public. En revanche, le mécanisme de vote pour choisir le taux de taxe est plus général. La seule hypothèse qui est faite est que le taux de taxe choisi dans une juridiction doit être compris entre le plus bas et le plus élevé des taux de taxes préférés des habitants de cette juridiction.

Les effets externes et les problèmes de congestion sur le bien public étant ignorés, la quantité de bien public est égale à la richesse agrégée de la juridiction multipliée par le taux de taxe. De plus, en l'absence d'épargne, le montant disponible pour le bien privé composite est égal à la richesse privée du ménage après impôts.

Les ménages d'une juridiction déterminent leur taux de taxe préféré, c'est-à-dire celui qui maximisera leur utilité, étant donnés leur richesse privée et la richesse agrégée de leur juridiction. Le taux de taxe préféré d'un ménage est donc une fonction dépendant de la richesse privée et de la richesse agrégée.

Les ménages sont libres, une fois le taux de taxe de la juridiction fixe, de quitter leur juridiction pour une autre appliquant un couple "taux de taxe-quantité de bien public" augmentant leur utilité, et cela sans aucun coût.

Comme chez Westhoff, l'équilibre est atteint lorsque plus aucun ménage n'a intérêt à quitter unilatéralement sa juridiction. On parle alors de structure de juridictions stable.

Les auteurs définissent alors ce qu'ils entendent par ségrégation. La notion de stratifi-

cation par la richesse utilisée dans ce modèle est celle, très forte, sous-jacente à la notion de "consécutivité" utilisée par Westhoff, et proposé par Ellickson[32] : une structure de juridictions est ségrégée si, pour toute juridiction, exception faite des juridictions appliquant le même couple "taux de taxe-quantité de bien public", le ménage le plus pauvre de cette juridiction est plus riche que le ménage le plus riche de toute autre juridiction dont la richesse par habitant est inférieure. L'apport de l'article de Gravel et Thoron est d'identifier une condition sur les préférences des ménages qui est nécessaire et suffisante pour garantir la ségrégation de n'importe quelle structure de juridiction stable. On se souviendra que la condition 5) de Westhoff était suffisante pour cette ségrégation, mais rien n'était dit sur sa nécessité. D'autre part, cette condition était difficilement interprétable.

La condition identifiée par Gravel et Thoron s'exprime naturellement en terme de la relation de complémentarité/substituabilité brute entre le bien public et le bien privé.

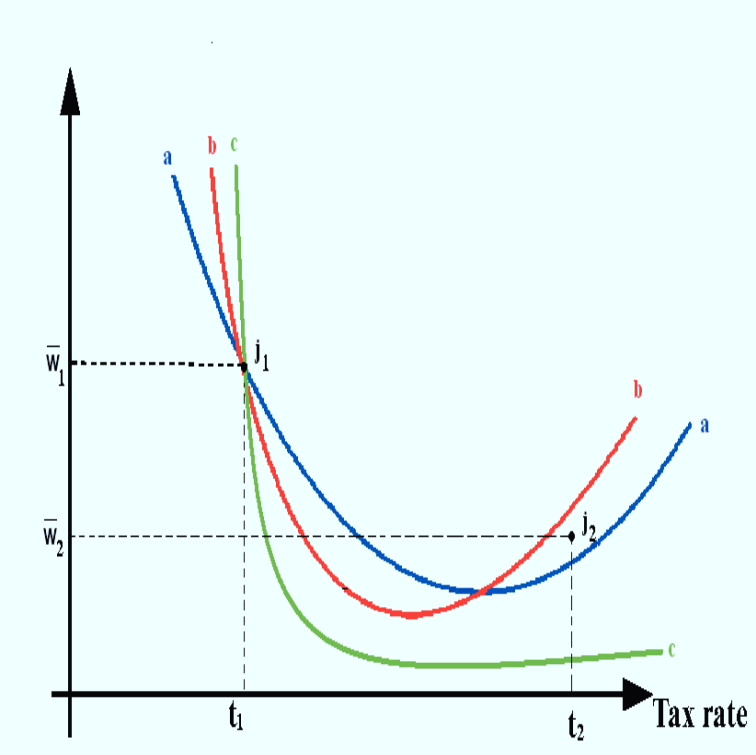
Par définition, le bien public est un complément (respectivement un substitut) brut au bien privé si la demande Marshallienne de bien public est décroissante (respectivement croissante) par rapport au prix du bien privé pour tout niveau de prix et de revenu.

La condition identifiée par Gravel et Thoron est la propriété selon laquelle la relation (complémentarité ou substituabilité) entre le bien public et le bien privé est indépendante du niveau des prix du bien public et du bien privé ainsi que du niveau de richesse du ménage.

Si et seulement si cette condition, dite de Complémentarité-Substituabilité Brut (GSC condition, pour Gross Substitutability-Complementarity condition) est respectée, alors le taux de taxe préféré d'un ménage est strictement monotone par rapport à sa richesse privée. Il s'en suit que, si on représente les préférences du ménage dans l'espace des différents taux de taxe et d'assiettes fiscales possibles, les courbes d'indifférences de deux ménages

dotées de richesses différentes seront ordonnée par rapport à la richesse en tout point et ne se croiseront, pour cette raison, qu'une seule fois.

Le graphique ci-dessous représente les courbes d'indifférence de 3 ménages a, b et c, tel que a est plus pauvre que b, lui-même plus pauvre que c.



Par exemple, le ménage a préférera, à la juridiction j_1, toute jurisdiction située au-dessus de la courbe bleue. Supposons que le bien public soit un substitut brut au bien privé (la preuve est la même si le bien public est un complément brut au bien privé). On voit que les ménages a et c préféreront aller dans la juridiction j_2, tandis que b préférera la juridiction j_1. La structure de juridictions ne sera donc pas stratifiée. Or les courbes d'indifférence de b et c se croisent à 2 reprises, ce qui contredit la propriété de "Single-Crossing" qu'implique la GSC condition.

L'objectif de cette thèse, après avoir présenté les principaux résultats dans la littérature traitant de la formation de juridictions dans le chapitre 1, sera de vérifier la robustesse de cette condition face à plusieurs importante généralisation du modèle.

0.3 La redistribution entre les juridictions par le gouvernement central

Dans la plupart des démocraties décentralisées, c'est-à-dire dans lesquelles certaines compétences sont transférées à un échelon inférieur, les juridictions n'ont bien évidemment pas toutes une richesse par habitant équivalente. Cet état de fait conduit donc les juridictions a fournir à leurs habitants un accès inégal aux services publics. Or il se heurte à un sentiment répandu d'aversion à l'inégalité que la fourniture de biens publics comme, par exemple, la qualité des écoles, ou encore les capacités d'actions des Centres communaux d'action sociale, soit fonction de la niveau de richesse des habitants des différentes juridictions. C'est une des raisons pour lesquelles, dans beaucoup de pays, le gouvernement central redistribue les richesses non seulement entre les individus, mais aussi entre les juridictions.

0.3.1 Assurer une certaine égalité entre les juridictions : la péréquation fiscale

Selon les pays, l'état central a mis en oeuvre différentes politiques afin d'assurer aux ménages une relative égalité d'accès au bien public, quelle que soit leur juridiction de résidence, par un mécanisme de redistribution entre juridictions : la péréquation.

La péréquation peut être horizontale, verticale, ou mixte. Dans un système de péréquation horizontale, le gouvernement central effectue une redistribution des richesses entre les juridictions appartenant à un même échelon, en taxant les juridictions "riches" pour subventionner les juridictions "pauvres". Dans le cadre d'une péréquation verticale, le gouvernement central, par les taxes et les impôts auxquels l'ensemble des citoyens sont soumis, verse des subventions aux juridictions en fonction de leur besoin. Le système de péréquation peut également être mixte, on qualifie alors ce système de "double péréquation".

En France, la péréquation est inscrite dans la Constitution depuis la modification de mars 2003 comme un objectif à valeur constitutionnelle. Le système de péréquation français est vertical, et s'articule principalement autour de la Dotation Globale de Fonctionnement (DGF). Elle est versée par l'État aux communes, aux départements et aux Régions. Cette DGF est calculé en fonction des ressources et des besoins des collectivités locales. Elle est prélevée sur les ressources de l'État, qui proviennent des impôts et taxes, et est redistribuée dans le budget des collectivités locales. D'autres pays, comme le Canada et le Japon, ont mis en place des politiques de péréquation verticale. On retrouve généralement le système vertical dans les pays peu décentralisés (exception faite du Canada).

Les pays scandinaves, comme la Suède, la Finlande et le Danemark, ont mis en place un système de péréquation horizontal. Le système danois, élaboré dans les années 1970, a pour objectif de permettre à chaque juridiction d'offrir à ses habitants un niveau minimal de bien public, quelles que soient ses ressources fiscales. Pour ce faire, la péréquation horizontale agit à la fois sur les recettes et sur les dépenses des gouvernements locaux. Les juridictions au sein desquelles les ménages ont une capacité fiscale supérieure à la moyenne nationale versent des subventions aux juridictions dont la capacité fiscale est inférieure à la moyenne nationale. De la même manière, les juridictions dont les besoins de dépenses sont

inférieurs à la moyenne nationale versent une subvention aux juridictions dont les besoins de dépenses sont supérieurs à la moyenne nationale.

L'état central joue un rôle important puisqu'il détermine à la fois le montant de la subvention et les critères de son obtention. Cette importance a fait l'objet de critiques, sur la base du fait qu'un gouvernement pourrait avoir tendance à modifier les règles pour favoriser les juridictions dirigées par ses alliés politiques. Par exemple, si le parti au pouvoir est principalement implanté dans les zones rurales, alors que l'opposition dirige les villes, le gouvernement pourrait établir comme critère d'obtention d'une subvention un nombre maximal d'habitants.

Le système allemand allie péréquation verticale, appelée "fédéralisme coopératif", et péréquation horizontale. L'État fédéral perçoit une partie des impôts et taxes, qu'il redistribue entre les différents Länder. D'autre part, la Constitution impose aux Länder les plus favorisés de soutenir financièrement les Länder les plus pauvres, afin d'éviter que ne se développent des inégalités de recettes par habitant trop importantes d'un Länder à un autre.

La plupart des pays ont donc mis en place un système de redistribution entre juridictions. La littérature économique se devait alors de réfléchir à un moyen de modéliser ces transferts. L'approche bien-êtriste peut être utilisée pour modéliser l'action du gouvernement central.

0.3.2 La modélisation de la péréquation

Comme en ce qui concerne la taxation, modéliser la redistribution par le gouvernement central entre les juridictions n'est pas chose facile, car il existe, en France notamment, plusieurs mécanismes de péréquation.

Les critères qui déterminent l'attribution et la grandeur de des transferts de péréquation fiscale - DGF régionale, Fonds de correction des déséquilibres régionaux, Contrat de Plan Etat-Région... - sont multiples : Produit Intérieur Brut (PIB) par habitant, revenu salarial net par habitant, taux de chômage, potentiel fiscal direct et indirect, densité de population... . Or, comme nous venons de le voir, la littérature économique actuelle ne prend pas en compte tous ces critères, généralement les travaux ne considèrent que la richesse agrégée de la juridiction, ou le niveau de bien public disponible.

Plus particulièrement, le problème de l'inégalité d'accès aux services publics entre deux collectivités locales de densités de population différentes est plus que jamais d'actualité : un département peu peuplé comme la Lozère ne peut disposer grâce à son financement propre de la même couverture hospitalière que la Seine-Saint-Denis. L'Etat intervient donc en faveur de la Lozère, car la richesse agrégée de ce département, et donc son assiette fiscale, sont inférieures à celle de la Seine-Saint-Denis.

Pourtant, la richesse par habitant en Lozère est supérieure à celle en Seine-Saint-Denis. On peut donc se demander si un tel transfert satisfait l'éthique. Les critères d'attribution d'une subvention du gouvernement central sont à déterminer. Faut-il s'intéresser à la seule provision en bien public ? Dans ce cas, des Départements ruraux, même avec une richesse par habitant élevée, devrait bénéficier d'aides, alors que des Départements plus peuplés, mais dont la population est moins riche, serait au contraire taxés. On peut, au contraire,

considérer que le fait de résider dans une zone rurale, et donc peu pourvue en bien public, relève d'un choix, et, par conséquent, ne retenir comme critère que la richesse par habitant. Mais ce serait prendre le risque de voir les zones rurales se désertifier, posant par la suite plusieurs problèmes, notamment d'ordre économique (difficulté pour l'agriculture, augmentation de la demande en logement dans les villes...).

Dans cette thèse, le gouvernement central sera "bien-êtriste", c'est-à-dire que sa politique n'aura pour objectif que la maximisation d'une fonction de bien-être social, dépendant du bien-être de chaque ménage.

La péréquation fiscale, présente dans la plupart des pays, a très certainement un impact sur la formation de structures de juridictions. Par conséquent, il était important de bien définir celle-ci, et de présenter les tentatives de modélisation dans la littérature, afin de pouvoir étudier son impact sur la ségrégation, dans le chapitre 2.

0.4 L'intégration de la terre : l'impact du foncier et de la taxation sur le logement

Aujourd'hui, les ménages consacrent en moyenne près du quart de leur budget au poste logement. On ne peut donc pas ignorer l'importance du foncier dans le choix d'un ménage pour telle ou telle juridiction. Le lieu de résidence d'un ménage influe évidemment sur son bien-être : un retraité ayant toujours vécu à Marseille sera prêt à payer plus pour pouvoir vivre à Marseille plutôt qu'en Picardie.

La taxation locale sur la propriété peut également jouer un rôle sur le bien-être des

ménages. Selon le niveau de taxation locale, un ménage pourra s'offrir un grand loft avec jardin, ou alors se contenter d'un appartement de deux pièces sans terrasse.

Or les modèles que nous avons étudiés jusqu'alors n'intègrent ni la terre, ni, a fortiori, la taxation locale sur le logement. Ces éléments sont susceptibles d'apporter des changements aux résultats obtenus précédemment. Les différents prix de l'immobilier entre les régions, et entre les communes d'une même région, ont-ils tendance à accentuer la ségrégation entre juridictions ? La taxation assise sur le logement influe-t-elle l'existence d'un équilibre ?

0.4.1 Le prix d'un bien immobilier

Entre les différentes régions, ou même entre les différentes communes d'une région, le loyer ou le prix d'achat par mètre carré demandé pour un logement varie énormément. Par exemple, d'après le site Internet d'annonces immobilières "seloger.com", le prix moyen de vente au mètre carré d'un appartement atteint environ 3800 euros à Aix-en-Provence, 2500 euros au Havre, 7200 euros à Neuilly-sur-Seine, ou encore 3000 euros à Strasbourg. Bien que ces moyennes ne tiennent pas compte de l'hétérogénéité des logements proposés à la vente, elles donnent une approximation du niveau des prix dans ces villes.

Le choix de résidence d'un ménage ne dépend donc pas uniquement du taux de taxe et de la quantité de bien public disponible dans une juridiction, mais également du prix que le ménage doit payer pour pouvoir dispose d'une unité de logement dans la juridiction en question. Une conséquence éventuelle de cette variation est l'accentuation possible de l'homogénisation de la population au sein de chaque juridiction qui pourrait en résulter. En effet, les grandes variations de prix d'une commune à une autre tendent à favoriser la ségrégation par le revenu. Par exemple, dans une ville comme Aix-en-Provence, le loyer

demandé pour un studio en centre-ville est rarement inférieur à 400 euros par mois, or il est demandé à un locataire de justifier de revenus 3 fois supérieurs au loyer toutes charges comprises. Un salarié célibataire, payé au SMIC (Salaire MInimum de Croissance), qui s'élève, au 1er juillet 2011, à 1072,07 euros net, ne peut donc pas se loger en centre-ville s'il ne dispose pas de revenus par ailleurs et que soit personne dans son entourage ne peut se porter caution solidaire, soit le propriétaire souhaite souscrire une assurance loyers impayés, incompatible avec la présence d'un garant.

Par ailleurs, le véritable prix d'un bien immobilier ne se limite pas à la valeur d'achat du logement. Il faut également compter avec la taxation locale sur les personnes, qui, en France comme aux Etats-Unis et dans la plupart des pays développés, n'est pas assise sur la richesse des ménages mais, plutôt, sur le logement (taxe d'habitation et foncier bâti en France).

0.4.2 L'intégration de la terre modélisée par Rose-Ackerman

Peu d'auteurs ont intégré le logement à leurs articles traitant des structures endogènes de gouvernance à juridictions multiples. L'une des premières analyses de cette question dans le cadre d'un modèle intégré de formation de juridictions est due à S. Rose-Ackerman[75], dans son article de 1979.

Cet article reprend la modélisation de Westhoff en y intégrant un marché du logement. Ce modèle comporte quatre parties interdépendantes, chacune d'entre elles devant être en équilibre : le marché de la terre, le choix des ménages parmi les différentes juridictions, le budget des juridictions et le vote des ménages sur le taux de taxe appliqué dans leur juridiction de résidence.

Les ménages ont des préférences différentes, représentées par une fonction d'utilité dépendant de la quantité de logement consommé par le ménage dans sa juridiction de résidence, des services publics proposées par cette même juridiction et d'un bien privé composite. Le revenu des ménages ainsi que leurs préférences sont exogènes. Les ménages votent sur le couple "taux de taxe-quantité de bien public" et peuvent, à la suite d'un vote qui ne leur convient pas, quitter (gratuitement) leur juridiction pour une juridiction appliquant un triplet "taux de taxe-quantité de bien public-prix de la terre" augmentant leur utilité.

Le marché de l'immobilier est relativement simple : il existe une quantité totale de terre fixe, divisée entre les différentes juridictions, chaque juridiction dispose donc d'une quantité fixe de terre. La terre est considérée comme étant un bien parfaitement divisible et homogène entre toutes les juridictions. Cependant, le prix d'une unité de terre dépend de la juridiction dans laquelle elle est située, car le marché de la terre suit les règles d'un marché compétitif (offre égale demande). Le bien privé composite -hors logement- est un bien numéraire dont le prix est par conséquent fixé à 1.

La quantité par habitant de bien public produit par une juridiction dépend du niveau de la taxe locale, qui est une taxe linéaire basée sur la valeur foncière. Tout comme dans les modèles de Westhoff, les effets externes d'un bien public d'une juridiction sur les ménages d'autres juridictions sont exclus.

Comme nous le verrons dans le chapitre 1, l'introduction d'un marché de la terre rend plus difficile l'émergence d'un équilibre. D'autres auteurs ont toutefois identifiés des conditions suffisantes pour l'existence d'un équilibre. L'introduction d'un marché de la terre ayant un impact sur la formation d'un équilibre, il est naturel de penser qu'elle n'est pas

sans conséquence sur la ségrégation. Reste à savoir si elle la favorise, ou si au contraire elle atténue les propriétés ségrégatives de la formation endogène de juridictions.

0.4.3 Les systèmes de taxation locale et nationale dans la littérature économique

En France, d'après l'enquête de l'avocat P. Imbert pour l'atelier des taxes locale, les taxes locales sont devenues le second impôt en terme de montant récolté par le Trésor Public. En 2007, le Trésor Public perçut 131 milliards d'euros au titre de la TVA, 62 milliards d'euros au titre de la fiscalité locale directe, 51 milliards d'euros au titre de l'IS), et 50 milliards d'euros au titre de l'IRPP, alors qu'en 2003, la Taxe sur la Valeur Ajoutée (TVA) rapportait à l'Etat environ 110 milliards d'euros (soit 19.1 % d'augmentation), l'Impôt sur le Revenu des Personnes Physiques (IRPP), 53 milliards d'euros (5.7 % de diminution), et l'Impôt sur les Sociétés (IS), 35 milliards d'euros (45.7 % d'augmentation), le montant généré par la fiscalité locale directe s'élevait à 51 milliards d'euros (21.6 % d'augmentation). Depuis, la part de la fiscalité locale reste en expansion.

0.4.4 La fiscalité nationale

La fiscalité nationale est essentiellement basée sur la consommation, le revenu des ménages, et les profits des entreprises.

La TVA est un impôt indirect sur la consommation crée en 1954. Son principe est simple, sur le prix payé par le consommateur pour un bien donné, une partie (dont le taux dépend de la nature du bien) revient à l'Etat. Cet impôt présente l'avantage de ne taxer que le consommateur finale, et non chaque entreprise intermédiaire du circuit de produc-

tion. La plupart des Etats de l'Union Européenne ont d'ailleurs adopté cette taxe.

Concernant l'imposition directe des ménages, en France, l'IRPP, crée lors de la Première Guerre Mondiale, est un impôt progressif, c'est-à-dire que le taux moyen d'imposition augmente en même temps que le revenu annuel du ménage. Il est à noter que l'IRPP ne représente que 17 % des recettes de l'Etat, contre 42 % aux Etats-Unis et 53 % au Danemark.

Enfin, l'IS, payé directement par les entreprises, est un impôt assis sur les bénéfices réalisées par une entreprise sur le territoire franąis. C'est un impôt proportionnel (taux de 33,33 %), sauf si le bénéfice annuel est inférieur à 7,63 millions d'euros, auquel cas le taux est réduit.

Il existe d'autres impôts et taxes plus spécifiques contribuant au budget de l'Etat, comme les droit sur les successions, l'Impôt de Solidarité sur la Fortune... . Toutefois, leur apport au budget de l'Etat reste marginal.

0.4.5 La fiscalité locale

Contrairement aux principaux impôts nationaux, la fiscalité locale directe ne repose ni sur un impôt proportionnel ou progressif sur le revenu, ni sur une taxe sur la consommation. Cette fiscalité locale résulte, pour l'essentiel, de 4 taxes, dont le niveau est fixé par chacun des échelons administratifs - le Conseil Régional, le Conseil Général, le Conseil municipal et éventuellement le Conseil communautaire s'il existe une structure intercommunale - lors du vote du budget. Ces taxes sont :

1. la Taxe d'Habitation,
2. la Taxe Foncière sur les Propriétés Bâties

3. la Taxe Foncière sur les Propriétés Non-Bâties

4. la Taxe Professionnelle

Les trois premières taxes sont assises sur la valeur locative du bien immobilier, telle que celle-ci est calculée par les autorités centrales (dans de très nombreuses communes, il n'y a pas beaucoup de rapport entre les valeurs administratives et les valeurs marchandes des biens fonciers. La Taxe Professionnelle ne concerne quant à elle que les entreprises. Son montant est fonction du capital investi dans l'entreprise, c'est-à-dire le ou les biens immobiliers où l'activité est exercée (la valeur retenue pour le calcul de l'impôt sera la valeur locative cadastrale), ainsi que les immobilisations corporelles (machines, ordinateursâ), dont la valeur retenue pour le calcul de la taxe est le loyer s'il s'agit de location, ou 16 % de la valeur d'achat si ces actifs ont été acheté par l'entreprise.

La Taxe d'Habitation, payée par la personne qui occupe légalement le logement au 1er janvier de l'année (locataire ou propriétaire), est calculée sur la valeur locative administrative du logement. Bien que certains ménages puissent être exonérés selon des conditions sociales et économiques, le montant de la Taxe d'Habitation ne peut en aucun cas être considéré comme dépendant du revenu : un milliardaire qui souhaite vivre dans un studio paiera une Taxe d'Habitation infime tandis qu'un smicard ayant hérité d'un château devra s'acquitter d'une Taxe d'Habitation supérieure à plusieurs mois de salaire.

La Taxe Foncière sur les Propriétés Bâties est versée par la personne physique ou morale (par exemple une Société Civile Immobilière) propriétaire au 1er janvier de l'année considérée du bien immobilier concerné. Là encore, le montant de l'impôt est fonction uniquement de la valeur administrative du bien immobilier, le revenu ou le patrimoine autre qu'immobilier du propriétaire ne rentrent pas en compte dans le calcul de la taxe.

La Taxe Foncière sur les Propriétés Non Bâties fonctionne sur le même principe. Elle concerne essentiellement les terrains agricoles. Une fois encore, même si des exonérations peuvent être mises en place par les collectivités locales, le montant de la taxe ne dépend

pas du revenu du contribuable.

La Taxe Professionnelle, instaurée en 1975 par le Premier Ministre de l'époque, J. Chirac, n'a cessé de faire l'objet de débat quant à sa pertinence et à son efficacité. A sa création, le montant de cette taxe était calculé d'une part sur le capital de l'entreprise, comme c'est toujours le cas aujourd'hui, et d'autre part sur la masse salariale de l'entreprise.

Or la prise en compte des salaires dans le calcul de l'impôt était considérée par beaucoup d'acteurs politiques comme néfaste pour l'emploi et le pouvoir d'achat. En effet, une entreprise qui déciderait d'embaucher une personne supplémentaire, ou d'augmenter un salarié, voyait le montant de sa Taxe Professionnelle augmenter. C'est pourquoi, en 1998, le ministre de l'économie et des finances d'alors, D. STRAUSS-KAHN, mit en oeuvre la suppression progressive de la partie salariale du calcul de l'impôt.

L'annonce, par le Président CHIRAC, en 2004, de la suppression définitive de la Taxe Professionnelle est restée sans suite, notamment à cause de la crainte des élus locaux de ne pas recevoir de la part de l'Etat des transferts couvrant entièrement la disparition de l'impôt local qui rapporte le plus de recettes, et ce en période de transfert de compétences de l'Etat vers les collectivités locales.

Depuis le 1er janvier 2007, le montant de la Taxe Professionnelle dont une entreprise doit s'acquitter ne peut être supérieur à 3.5 % de sa valeur ajoutée, ce qui laisse supposer que la taxe professionnelle sera bientôt réformée, pour que son calcul soit basé sur la valeur ajoutée de l'entreprise, comme c'est le cas dans la plupart des pays européens.

Afin d'éviter une concurrence fiscale entre Communes membres d'une structure intercommunale, la Taxe Professionnelle Unique existe depuis 1999. Elle est obligatoire pour les Communautés Urbaines et les Communautés d'Agglomération, mais facultative pour les Communautés de Communes et les Syndicat d'Agglomération Nouvelle.

Dans la grande majorité des pays, la fiscalité locale est assise sur la valeur des biens fonciers, et pas sur le revenu ou sur le patrimoine global d'un ménage. Or, comme nous le verrons dans le chapitre 1, certains auteurs ayant intégré le logement dans un modèle de

formation de juridictions ont supposé une fiscalité locale assise sur le revenu, plutôt que sur la terre.

Bien qu'il existe évidemment un lien entre la richesse d'un ménage et la valeur de son patrimoine foncier, un modèle intégrant simultanément une taxe locale sur la propriété foncière et un impôt national sur la richesse pourrait obtenir des résultats sensiblement différents quant aux conditions suffisantes à l'existence d'un équilibre et la ségrégation.

0.5 Les propriétés des biens publics locaux

Dans la plupart des articles cités plus haut, les biens publics locaux sont considérés comme des biens publics locaux "purs", c'est-à-dire présentant les mêmes caractéristiques qu'un bien-club. Un bien-club est un bien non-rival, c'est-à-dire que la consommation de ce bien par un agent n'altère pas la consommation de ce même bien par un autre agent, et exclusif, ce qui signifie qu'il est aisé (et peu voire pas coûteux) d'empêcher un agent de consommer ce bien. Un exemple de bien-club est le raccordement au câble.

0.5.1 La non-rivalité et l'exclusivité des biens publics locaux

Toutefois, il est difficilement concevable qu'un bien public local ne génère pas d'externalités dans d'autres juridictions. En effet, une municipalité qui déciderait de construire une voie rapide améliorerait non seulement le quotidien de ses habitants, mais également de ceux des communes environnantes.

De la même manière, la plupart des biens publics locaux sont sensibles aux phénomènes de congestion, qu'il s'agisse des écoles, des routes ou encore des piscines publiques. Il est évident qu'à budget constant, une école publique n'offrira pas la même qualité d'enseigne-

ment selon qu'il y ait cent ou mille élèves inscrits.

Ces deux propriétés peuvent modifier substanciellement les résultats précédents en termes d'existence d'un équilibre. En effet, la congestion peut créer un phénomène dit de "pauvres chassant les riches" : une commune à haut niveau de richesse par tête disposerait d'une grande quantité de services publics locaux disponibles, qui attirerait des ménages plus pauvres, augmentant ainsi la congestion et donc diminuant la quantité disponible. En conséquence, les ménages riches pourraient choisir de quitter leur commune pour une autre moins peuplée, mais seraient ensuite rejoint par les ménages pauvres, et ainsi de suite.

De même, supposons que les biens publics locaux soient parfaitement non-exclusifs, c'est-à-dire qu'un ménage peut consomme le bien public d'une juridiction autre que la sienne comme s'il y résidait. Dans ce cas, le seul équilibre possible est celui oÃ¹ $toutes les juridictions appliquent$

Ayant un impact sur l'existence d'un équilibre, on peut légitimement pensait que la congestion et les externalités influencent également les propriétés ségrégatives de la formation endogène de juridiction.

0.5.2 La modélisation de la congestion et des externalités et leurs impacts sur la ségrégation

Concernant la congestion, la façon la plus générale de la modéliser est de considérer que le coût de production du bien public local est une fonction croissante non seulement de la quantité de bien public produit, mais aussi du nombre d'habitants dans la juridiction. Deux cas particuliers existent :

- si le bien public locaux est un bien public local pur, c'est-à-dire ne souffrant pas de

congestion, la fonction de coût de production sera constante par rapport au nombre d'habitants,

- si le bien public locaux est parfaitement rival, la fonction de coût de production sera linéaire (de coefficient 1) par rapport au nombre d'habitant.

Evidemment, selon la nature du bien considéré, la fonction de coût de production augmentera de manière plus ou moins forte par rapport au nombre d'habitants. Cependant, le dernier cas particuliers évoqués ci-dessus ne peut pas être exclus pour certains bien publics, si l'on en croit les études empiriques de Oates[71].

Les retombées économiques générées par un bien public dans une autre juridiction peuvent être modélisées de différentes manières. L'une d'elle, proposée par Bloch et Zenginobuz[13], consiste à supposer que la quantité de services publics disponibles dans une juridiction est égale à la quantité de services publics produite dans cette juridiction plus celles produites dans les autres juridictions pondérées par un coefficient compris entre 0 et 1. Au plus les coefficients sont proches de 0, au plus les biens publics locaux sont des biens clubs, ne pouvant être consommés que par les membres de la juridiction qui les produit. A contrario, au plus les coefficients sont proches de 1, au plus les biens publics locaux sont des biens publics au sens large, c'est-à-dire non-exclusifs.

Une autre possibilité, plus générale, consiste à définir la quantité de services publics disponibles comme une fonction dépendant de la quantité de services publics produite par la juridiction, mais aussi des quantités produites par les autres juridictions, pondérées par des coefficients.

La congestion et l'existence de retombées économiques entre juridictions sont susceptible de modifier les structures de juridictions à l'équilibre. En effet, si le bien public est

parfaitement non-rival, l'arrivée d'un ménage pauvre dans une juridiction à revenu moyen élevé ne suscitera pas l'opposition des habitants, car sa seule conséquence sera l'augmentation (même légère) de la base fiscale. Par contre, si le bien public est un bien rival, alors l'arrivée de ménages pauvres au sein de la juridiction pourrait avoir comme conséquence de diminuer la quantité disponible de bien public, si l'augmentation de la congestion qu'elle génère est supérieure à la hausse de la base fiscale. Par conséquent, les ménages seront plus tentés de vivre dans des juridictions composées de ménages au moins aussi riches qu'eux, ce qui encourage la ségrégation.

Dans le même temps, l'existence de retombées économiques inter-juridictionnelles pourrait mitiger les propriétés ségrégatives de la formation endogène de juridictions, en rendant les juridictions "plus égales" en termes de provision de bien public, décourageant ainsi les ménages de quitter leur juridiction pour une autre.

Toutefois, il reste à déterminer si et dans quelle mesure ces assertions sont valables. La congestion et l'existence de retombées entre les juridictions peuvent avoir d'autres impacts sur l'équilibre d'une structure de juridictions.

Par exemple, la congestion et l'existence de retombées positives peuvent conduire à ce que la provision de biens publics locaux soit inférieure au niveau optimal. Conscient de ce problème, le législateur français donc institué un nouvel échelon coopératif : l'intercommunalité.

0.5.3 Les structures intercommunales

L'inter-communalité se définit comme un groupement de municipalités au sein d'une structure reconnue légalement avec pour objectif de mener des projets en commun et de mu-

tualiser certains services publics, tels que le ramassage des déchets ménagers ou l'approvisionnement en eau.

Plusieurs pays développés ont créé un échelon intercommunal, comme les Etats-Unis (Consolidated city-county), la Belgique (Intercommunale), l'Espagne (Mancomunidad), le Luxembourg (Syndicats intercommunaux luxembourgeois) ou encore la France. Concernant la France, il existe 3 principaux types d'intercommunalité :

- les communautés de communes, regroupant n'importe quelles communes appartenant à un même tenant géographique, dont les compétences obligatoires sont les actions de développement économique intéressant l'ensemble de la communauté et l'aménagement de l'espace,

- les communautés d'agglomération, regroupant les communes d'un même tenant géographique à condition que l'ensemble de la communauté rassemble plus de 450 000 habitants, dont les compétences sont les mêmes que celle de la communauté de communes, ainsi que l'équilibre social de l'habitat, la politique de la vile et le transport urbain.

- les communautés urbaines, regroupant les communes d'un même tenant géographique à condition qu'une des communes compte plus de 10 000 habitants et que l'ensemble de la communauté rassemble plus de 50 000 habitants, dont les compétences sont les mêmes que celle de la communauté d'agglomération, ainsi que les services d'intérêt collectif (eau cimetière...) et l'environnement et le cadre de vie.

A noter qu'au sein de l'Union Européenne, les communes frontalières de pays différents peuvent se régrouper au sein d'un Euro-disctrict.

La création de l'inter-communalité a clairement pour objectif de mettre en commun

des services publics entre les juridictions membres, et ainsi d'éviter les phénomènes de passager clandestin, où une commune profiterait du bien public de la commune voisine sans en supporter aucun coût.

La loi prévoit qu'à l'horizon 2013, toute commune devra faire partie d'une structure intercommunale. Il sera alors temps d'observer les conséquences en terme d'efficacité économique, d'équité et de ségrégation.

0.6 Structure de la thèse

Le caractère endogène de la formation de la structure de juridiction pose la question de l'existence d'un équilibre, ainsi que la ségrégation de la population entre les différentes juridictions. Le premier chapitre de cette thèse sera consacrée à une revue de la littérature théorique et empirique sur la formation endogène de juridictions, les définitions possibles de la ségrégation et les conditions relatives à l'existence d'une ségrégation par la richesse entre les juridictions.

L'existence fréquente d'une politique de redistribution, effectuée par un gouvernement central, entre des juridictions, ainsi que les conséquences de cette politique font également l'objet de nombreuses recherches par certains économistes. Le but est de savoir si ces politiques jouent un rôle sur la formation d'un équilibre de la structure de juridictions, et sont efficaces pour réduire l'éventuelle ségrégation engendrée par le regroupement entre populations homogènes. Dans le deuxième chapitre de cette thèse, on étudiera l'effet des politiques de péréquation fiscale et de redistribution entre les juridictions mises en oeuvre par le gouvernement central sur la ségrégation.

Dans le troisième chapitre, nous nous intéresserons particulièrement à l'impact des prix

du foncier et de l'existence de plusieurs types de biens publics locaux sur la formation de juridictions. Plusieurs auteurs ont cherché à intégrer la terre aux modèles existants. L'intégration du foncier dans un modèle de formation de juridiction est également important pour modéliser la fiscalité locale. En effet, dans la plupart des pays développés, les impôts locaux que paient les ménages sont assis sur la valeur de leur logement de résidence ainsi que sur celle des biens immobiliers dont ils sont propriétaires. Or, la plupart des articles qui modélisent la formation des juridictions supposent que les biens publics locaux sont financés par un impôt sur la richesse individuelle. L'intégration de la terre permet d'identifier des conditions suffisantes à l'existence d'un équilibre et de comparer les effets de 2 systèmes de taxation locale possible, la taxation sur la richesse et la taxation sur la terre. C'est ce qui sera étudié dans le chapitre 3.

Une fois la terre incorporée au modèle, nous considérerons l'existence d'un phénomène de congestion du bien public, ainsi que la possibilité pour les ménages d'une juridiction de profiter du bien public d'autres juridictions. En effet, il est raisonnable de penser que 2 écoles disposant du même budget ne pourront pas fournir la même qualité de service si l'une est fréquentée par 1000 élèves et l'autre, par seulement 100. De même, 2 ménages vivant dans 2 communes distinctes mais similaires en termes de population et de richesse moyenne par habitant n'auront pas nécessairement accès à la même quantité de services publics si l'une est totalement isolée tandis que l'autre est proche d'une métropole. Ces éléments pourraient *a priori* être de nature à modifier de manière significative les conditions nécessaires et suffisantes à l'existence d'un équilibre, et donc la ségrégation de toute structure de juridictions stable. Cette question sera traitée dans le chapitre 4.

Chapter 1

Endogenous jurisdictions formation and its segregative properties: a survey of the literature

This paper provides a survey of the literature on human regrouping based on voluntary participation, with a focus on the theoretical and empirical articles dealing with the endogenous processes of jurisdictions formation and its segregative properties. We start from Tiebout's intuitions: households reveal their preferences for the public good by choosing the community according to a trade-off between the tax rate implemented by the different jurisdictions and the amount of public good they provide. Then, the paper analyzes how such intuitions have been modelled in the literature, and reviews the different questions that economists have examined: the existence of an equilibrium, its properties in terms of efficiency and the definition of segregation and its causes.

1.1 Introduction

According to historians and social scientists, human regrouping is defined as a "merging of similar individuals who want to live together in order to satisfy some needs such as security and poll their talents so as to make life easier"[1]. The first human regrouping are assumed to have taken place during the Palaeolithic.

The first communities were composed of nomadic hunters, that had to move frequently in order to find foods and to survive: in turn, each individual take care of the group while

[1] http://pages.usherbrooke.ca/manuel-histoire/definitions/paleo2.html

the others rest. Such communities can be considered as Clubs, since the guard is a non-rival and excludable service.

Although some human regrouping could be involuntary (such as esclavagism), or voluntary, but for other motivations than producing a good (for instance, people living close to a river to enjoy the proximity of the water), this paper will exclusively deal with voluntary regrouping which aim is to produce a collective good, that is to say a good that can be equally consumed by anyone that belonged to the community.

As it is well-known in economics, there exist 4 kinds of goods, according to the rivalry and the possibility of exclusion from the consumption of the good:

	Rival	**Non-rival**
Excludable	Private Good	Club good
Non-excludable	Common Resource	Public good

Gathering is obviously the most efficient way to produce a collective good at a lower cost, since the cost is shared between several individuals. However, the formation of a human group raises several questions: what leads individuals or households to join a group? Will people who chose to belong to the same group be alike in terms of wealth? in terms of preferences?...

To answer these questions, human beings started to develop societies so as to provides rules to answer those questions. A society, from the Latin word "socius", which means ally, partner, is defined by Joseph Fichter as "a model of organization, institutions and relations between individuals and groups which aim is to concertedly satisfy the collectivity's needs". Though there exists a wide varieties of societies, this paper will only consider formal local communities having a legal existence: local jurisdictions.

A jurisdiction is a public entity delimited by geographical boundaries, which aim is to provide local public services. Every individual living inside the jurisdiction's boundaries is automatically one of its members. The range of the jurisdictions' competencies is most of the time determined by the central government. For instance, the central government decides whether or not a local jurisdiction is in charge of primary school, and if it transfer

this competency to the jurisdiction, then the jurisdiction is free to choose what share of its budget it will devote to primary school. Nowadays, the share of the local public spendings out of the total public spendings can reach 50% in some countries, like the USA. This probably explains why a large body of the literature on local public economics has been developed in this country.

Local public economics is, to some extent, similar to public economics in general, in the sense that it deals with the intervention of a public entity in order to improve an inefficient situation that free market can not handle. Furthermore, most of the time, the voting rule to determine the optimal amount of public good that has to be provided are the same at the national and the local level.

However, local public economics differs from traditional public economics on two issues:

1. It is easier from a household to move from one jurisdiction to another than leaving its country,

2. The national level can have an impact on local decisions, for instance by implementing a fiscal equalization policy.

The co-existence of different levels of government that produce public goods is called "fiscal federalism". A lot of questions on fiscal federalism have raised: what is the optimal number of levels of political decision-maker? what is their optimal scope? what range of competencies should be decentralized?...

Some of these questions find answers in the literature, in particular in the survey provided by Oates[72]. But his survey focused more on normative questions, while this survey presents the positive results on the endogenous jurisdictions formation and its segregative properties.

The first article dealing with the endogenous formation of jurisdictions was actually a response to Samuelson, provided by Tiebout. In his article [78], Samuelson claimed that voluntary contributions to produce a public good can not be implemented, because households have no incentive to reveal their preferences for the public good, since, by definition, a public good is not excludable (or at a very high cost), hence the benefit a household can enjoy from the public good does not depend on how much it has spend to produce it, so households would have no incentive to reveal their preferences for the public good. Con-

sequently, taxing households on the basis of their preferences, as proposed by Lindhal[66], is not feasible. As a consequence, a public good may not be produced even if the benefits it provided is greater than its cost. Hence, public goods can not be provided through mercantile process based on exclusive ownership, but by an public authority, that finances the production cost with compulsory levies. Consequently, if households' preference can not be observed, the only possibility to have a public good produced is to tax households independently on their preferences over the public good.

As a response, Tiebout [82] asserted that, within any urban area, households can choose their place of residence among several jurisdictions, implementing different tax rate and so offering different quantity of local public good. Contrary to a pure public good, that is non-excludable, a local public good can be consumed only by households living in the jurisdiction that produces it.

In several articles we will explore, local public goods were supposed to be pure local public goods: a local public good does not suffer from congestion and can be consumed only by households living in the jurisdiction that produces it. Consequently, local public goods can be seen as Club goods, as defined by Buchanan[15]: they are non-rival and can be consumed only by members of the jurisdiction that produces them.

If an empirical article by Heikkila[54] confirmed the assumption that local public goods are club goods, i.e. non-rival and excludable, several articles relaxed it, by considering either that local public good can be partially consumed by households living in other jurisdictions, or that the public good may suffer from congestion effects.

The debate on the congestion indicates that local public goods are not pure club goods, but are in between club goods and private goods. Other economists, such as Nechyba[70] or Bloch and Zenginobuz[13] and [12], as we will see below, consider that local public goods are not totally excludable, which means that households living in a jurisdiction can partially or totally consume other jurisdictions' local public goods, and then are in between club goods and pure public goods.

Except for the case of private government, whose impacts on welfare and on the existence of an equilibrium has been studied by Helsley & Strange[55], a local public good can partially be consumed by households living in other jurisdictions.

In Tiebout's opinion, households, by choosing their place of residence according to a trade-off between the amounts of local public good provided by the jurisdictions and the tax rate they implement, reveal their preferences over the public good, since the choice among the different "tax rate-amount of public good" packages are similar to a choice between several "price paid-amount of good received" combinations.

Tiebout's intuitions must be understood in a context of an urban area with a large number of distinct jurisdictions, implementing different tax rates and offering different amounts of public good. Households are assumed to be able to move from one jurisdiction to another freely, and to have perfect knowledge about the policy implemented by every jurisdiction in the urban area. Under those assumptions, jurisdictions are not formed exogenously, but endogenously by the households that choose their place of residence according to a usual trade-off between a quantity of good and the price to paid to be able to enjoy such quantity. If the number of jurisdiction is large enough, then, for every household, there exists a jurisdiction applying exactly the policy it would pick if it were the dictator.

In his conclusion, Tiebout evoked the homogeneity of the jurisdictions structure: jurisdictions, at equilibrium, will be composed of households having exactly the same preferences over the public good, so the policy will be unanimously determined. Consequently, the endogeneity of the jurisdiction formation seems to have segregative properties, since it leads to assortive matching. However, Tiebout was not very specific about the definition of segregation, since the main objective of his article was to demonstrate that preferences over the public good can be observed.

Though very influential, Tiebout's article was not formal at all. It took two decades before a coherent model based on his intuitions could be developed.

Then, a wide literature explores the validity and the consequence of Tiebout's intuitions. First, the preferences over the public good can be observed only at the equilibrium, which raises several question: does an equilibrium always exist? if not, under which conditions is the existence of an equilibrium ensured? Second, the impact of the housing market and of the possible taxation scheme (the public good can be financed through, for instance, a tax on wealth, or through a tax based on the housing value) on the existence of an equilibrium and the segregation has been investigated by many economists. Third,

some authors asserts that local public good are not "pure" public good, they may either be partially consumed by households that do not live in the jurisdiction that produce them, or they may suffer from a congestion effect. Finally, a large number of articles investigate on the definition of the segregation, and on the segregative properties of endogenous jurisdictions structure formation.

The next section will present the different attempts at modelling Tiebout's intuitions, using microeconomic theory concepts, and sometimes the cooperative Game Theory tools. The third section will examine the main results on the existence of an equilibrium and its efficiency. The fourth section will focus on the definition of the segregation and its causes. Finally, the fifth section will conclude.

1.2 Modelling Tiebout's intuitions

1.2.1 The general approaches

Westhoff [84] was among the first economists to provide a formal model of multi-jurisdictional economies. The model is simple, there exist 2 goods: a local public good (Z) and a composite private good (x). Every jurisdiction produces a local public good through a tax based on households wealth, that is exogenous. There exist a continuum of households belonging to the interval $[0;1]$, and a set of jurisdictions $J \subset \mathbb{N}, \sharp(J) = M$. A jurisdiction j is composed of a measurable subset $I_j \subset [0;1]$. The number, possibly null, $\int\limits_{I_j} d\lambda$, where λ is the Lebesgue measure, can represent for instance the mass of households that live in jurisdiction j. Every households must choose an unique place of residence, so, $\forall (j,j') \in J, j \neq j', I_j \cap I_{j'} = \emptyset$ and $\bigcup\limits_{j \in J} I_j = [0;1]$. A jurisdiction j is characterized by the tax rate t_j it applies and by the amount of public good it produces (Z_j). Households differ in wealth and in preferences. For simplicity, Westhoff assumes that households are ordered by their wealth: the households' wealth distribution is modelled as a Lebesgue measurable function $\omega : [0;1] \to \mathbb{R}_+^*$ - household i is endowed with a wealth $\omega_i \in \mathbb{R}_+^*$ - with ω being increasing and bounded from above. Preferences are represented by a function that is continuous, increasing, twice differentiable and quasi-concave with respect to every argument:

$$U_i : \left| \begin{array}{ccc} \mathbb{R}_{++}^2 & \longrightarrow & \mathbb{R}_+ \\ (Z,x) & \longmapsto & U(Z,x) \end{array} \right.$$

Moreover, Westhoff assumed that $\forall i \in [0;1], \forall (Z, Z', x, x') \in \mathbb{R}^{4*}_{++}, U_i(Z, x) > U_i(Z', 0)$ and $U_i(Z, x) > U_i(0, x')$. In words, all households would prefer a consumption bundle with strictly positive amounts of the public good and the private good to any bundle with either no public good or no private good.

All the tax revenues are devoted to the production of the public good, that is produced through a linear cost function, so $Z_j = t_j \varpi_j$, where $\varpi_j = \int_{I_j} \omega_i d\lambda$ is the aggregated wealth in jurisdiction j. The budget constraint households must respect is given by $x \leq (1-t)\omega_i$. Since the utility is always increasing with respect to x, and since there is no savings, all the net-of-tax is consumed, so $x = (1-t)\omega_i$. Hence, the utility function can be re-written as follows:

$$U_i(Z, x) = U_i(t\varpi, (1-t)\omega_i)$$

Contrary to Tiebout, Westhoff does not assume that the number of jurisdiction is large enough to have every household living in a jurisdiction that applies exactly the policy that would maximizes its utility. As a consequence, Westhoff integrate a collective decision rule: the majority voting rule.

Using the median voter theorem, this assumption is equivalent to assuming that, in every jurisdiction, the tax rate is such that half of the households living in the jurisdiction would weakly prefer a higher tax rate while half of the households would not be worse-off with a lower tax-rate. Since the utility function is assumed to be concave with respect to each argument, one can prove that preferences are single-peaked over t, so, using Black's median voter theorem[9], we know that there exists $\bar{t}$ such that $\bar{t}$ is preferred by half of the voters to any tax rate lower than $\bar{t}$ and by half of the voters to any tax rate higher than $\bar{t}$. Once each jurisdiction has determined its tax rate, households are free to leave their jurisdiction for the one that would maximizes their utility.

A jurisdiction structure is the specific distribution of households among the different jurisdictions, and a $2M$-vector $(\{t_j, Z_j\}_{j \in J})$ representing the tax rates and the amounts of public goods produced by every jurisdiction. A jurisdiction structure is stable if and only if:

- Every jurisdiction has its budget balanced: $\forall j, Z_j = t_j \varpi_j$,
- In every jurisdiction, the tax rate is the one chosen by the median voter,

- No household has incentive to leave unilaterally its jurisdiction: $\forall j \in J, \forall i \in I_j, U_i(t_j\varpi_j, (1-t_j)\omega_i) \geq U_i(t_k\varpi_k, (1-t_k)\omega_i) \forall k \in J$

This notion of equilibrium is widely used in the literature of local public goods. It is known as the free mobility equilibrium. This notion considers only individual mobility, which is consistent with the assumption of a continuum of households: since every household has a null measure, its decision of leaving unilaterally its jurisdiction could not modify the tax base, nor the voting outcome. While mostly used, this notion is weaker than another definition of stability that consider group deviations (for instance [50] that we will mention later). Once the equilibrium is reached, the jurisdictions structure is stable.

Other economists have provided alternative models for Tiebout intuition. One of them is Ellickson. In his article [33], the set of households is discrete, and there are different private goods and public goods. The main difference is the non-divisibility of the local public goods. Contrary to Westhoff, Ellickson assumed that households could consume either one or zero unit of each public good, and at most one public good.

The fact that households consume a vector of private goods instead of a composite private good is not crucial, since different private goods prices vectors among jurisdictions would have the same effect than different tax rates: the private consumption will vary from one jurisdiction to another.

However, the non-divisibility of the public goods introduces a non-convexity of the consumption set, which, *a priori*, could have caused troubles to prove the existence of an competitive equilibrium. To avoid this difficulty, the author provided a convexified version of his model, by replacing the consumption set by its convex hull and using the demand correspondence as in [56].

Some economists provided models *à la* Tiebout using a discreet number of household instead of a continuum. For instance, Wooders ([85] and [86]) generalized Westhoff's model, by relaxing some assumptions such as the non-rivalry of the local public goods. She assumed that the public good suffers from congestion (called here "crowding effect"), which means that households' utility decreases with the number of people that consume the public good, for any given amount of public good produced. Those papers are relatively closer to Tiebout's intuitions in the sense that crowding effect are allowed in Tiebout's paper to justify his assumption that jurisdictions would not be too crowded.

Another way to model Tiebout's intuitions is to use hedonistic coalitions. Contrary to classical cooperative game theory, in which households' utility only depends on the payment they receive, in hedonistic coalition framework, households also value the coalitions themselves (see for instance [3]). Consequently, one household may prefer a coalition in which he would receives a lower payment than in another one, if it prefers this coalition for its intrinsic characteristics to the other one. Dreze & Greenberg [28] proposed a model in which households are assigned to a coalition, but are allowed to quit their coalition to another one. They consider in turns the case where no transfer are allowed between coalitions, and the case where those transfers are allowed.

The models explored in the previous subsection do not integrate the land market, so households do not consume housing, and do not pay any cost for living in a crowded jurisdiction. As a consequence, since the public good does not suffer from congestion, one may assume that, in many cases, households would be better-off if they merged into an unique jurisdiction to produce a large amount of public good at a lower cost (since the aggregated wealth would be higher) instead of staying separated into different smaller jurisdictions, as long as they do not differ "too much" on the tax rate that must be applied.

This assumption is quite unrealistic: one can observe the co-existence of big cities and small villages inside an urban area. This co-existence can be explained by the presence of a competitive land market: living in a big cities allows households to consume a large amount of public good, but housing is costly. On the contrary, the housing price is lower in a small jurisdiction, but the jurisdiction will provide a lower level of public good.

The introduction of a land market in local public goods economy models will be discussed in the next subsection.

1.2.2 The land market

Integrating a competitive land market into a model *'a la* Westhoff raises the question of the taxation scheme. In most countries, national taxation is based on households' wealth, while local taxation is based on the housing value. Those two taxation schemes may have different implications in terms of the existence of an equilibrium within the same meaning as Westhoff, in terms of social welfare, in terms of segregative properties...

Rose-Ackerman was among the first economists to introduce a competitive land market into a local public goods economy model [75]. Her article integrated a land market in a model *a la* Westhoff. Households differ by their wealth and by their preferences. Their utility functions depend on the amount of public good in their jurisdiction Z, on the amount of the composite private good x they enjoy, and also on the amount of land h they consume, whose before tax unit price, different from one jurisdiction to another one, is denoted p_j. As previously, households vote to determine their jurisdiction's tax rate, the difference is that the tax is linearly based on the housing value. The price paid by a household for one unit of housing in jurisdiction j is then $(1+d_j)p_j$, where d_j is the dwelling tax rate applied by jurisdiction j. Hence, the amount of public spending is $d_j p_j H_j$, where H_j is the total amount of land in jurisdiction j, that belongs to absentee landlords. In all jurisdictions, land is homogeneous and perfectly divisible.

To incorporate the land market and the dwelling tax to the definition of the stability, we define $B_{ij} \in \mathbb{R}^2_+, B_{ij} = \{(x,h) \in \mathbb{R}^2_+ : x + (1+d_j)p_j \leq \omega_i$ as the set consumption of private good and housing that respects the budget constraint faced by a household i living in jurisdiction j. The introduction of a land market requires to provide a definition of a voting equilibrium, whose existence is a necessary condition to have a jurisdictions structure stable.

A voting equilibrium is a tax rate, a before tax housing price and an amount of public good such that:

- Consumption of private good and housing is optimal for every household: $\forall x, h \in B_{ij}, U(Z_j, \bar{x}, \bar{h}) \geq U(Z_j, x, h)$, where $(\bar{x}, \bar{h}) \in B_{ij}$ is the current consumption,
- Every jurisdiction's public good is fully financed by the dwelling tax revenues: $\forall j \in J, Z_j = d_j p_j H_j$,
- The voting process must have a solution in every jurisdiction: for all $j \in J$, t_j must be such that 50% of the households living in j prefer t_j to any higher tax rate and 50% that prefer t_j to any lower tax rate„
- Housing market clear: $H_j = \int\limits_{i \in I_j} h_i d\lambda$, h_i being the amount of housing consumed by household i

With a land market, jurisdiction structure is stable if and only if:

1. A voting equilibrium must arise.

2. No household should have incentive to leave unilaterally its jurisdiction: $\forall j, j' \in J, \forall i \in I_j, \forall (x,h) \in B_{ij}, U_i(Z_j, \bar{x}, \bar{h}) \geq U_i(Z_{j'}, x, h)$.

This notion of stability incorporates the one defined in the previous subsection, but adds two elements: the land price must equal supply and demand in every jurisdiction, and be such that households would have no incentive to modify their current consumption of private good and housing. Those extra conditions make the definition of stability stronger than the one without land market.

As in Westhoff's article, local public goods do not generate spillovers in other jurisdictions and do not suffer from congestion effect. However, contrary to Westhoff, local public goods are assumed to be produced through a technology that may not be linear. The cost for producing an amount Z of public good is given by $\begin{array}{rccc} C: & \mathbb{R}_+ & \longrightarrow & \mathbb{R}_+ \\ & Z & \longmapsto & C(Z) \end{array}$ which is continuous and increasing. Hence, there exists a function $\begin{array}{rccc} C^{-1}: & \mathbb{R}_+ & \longrightarrow & \mathbb{R}_+ \\ & dpH & \longmapsto & C(dpH) = Z \end{array}$ that gives the amount of public good that will be produced with a budget of dpH.

In every jurisdiction, the tax rate is chosen by majority voting. Households determine their favorite tax rate considering that p_j is given and that households have perfect information over the public good cost function. The new budget constraint is given by $x + (1+d)ph \leq \omega_i$. As previously, there is no savings and the utility function is assumed to be always increasing with respect to x and h, so the budget constraint will always be saturated, hence one has $x = \omega_i - (1+d)ph$. Household i's utility function can then be re-written as follows:

$$U_i(Z, x, h) = U_i(C^{-1}(dpH), \omega_i - (1+d)ph, h)$$

Contrary to articles in which taxation was based on households' wealth, a voting equilibrium may not arise: suppose that households' favorite tax rate increases as the before tax housing price decreases, that the supply is perfectly inelastic (so the supply is constant with respect to the price, which would be the case if there are a sufficiently high number of suppliers and no cost for renting land), and that no inter-jurisdictional migration are allowed. As a consequence, after an increase of the tax rate, the suppliers will decrease the

before tax price to keep the demand constant. Then households will vote for a higher tax rate, and so on and so far, so the tax rate will tend to $+\infty$ and the before tax price, to 0, so no voting equilibrium will arise.

Let denote the indirect utility conditional upon the amount of public expenditure $E = dpH$, which is a function of the amount of public good, the net-of-tax $P = (1+d)p$ and the private wealth, as

$$V_i(E, P, \omega_i) = \max_{h \in [0; \frac{\omega_i}{P}} U_i(C^{-1}(E), \omega_i - Ph, h)$$

The MRS of the public good by the net-of-tax housing price is given by

$$a_i = \frac{\frac{\partial V_i(E,P,\omega_i)}{\partial E}}{\frac{\partial V_i(E,P,\omega_i)}{\partial P}}$$

Households are ranked according to a_i from the lowest to the highest: $i < i' \Rightarrow a_i < a_{i'}$. Several types of equilibria may exist. For example, there can exist a stable structure in which households are divided between the jurisdictions such that, in all jurisdictions, median voters have the same MRS of the public good by the net-of-tax housing price, so that the amount of public good and the net-of-tax housing price are identical in all jurisdictions.

Another possible stable structure is the one in which households are grouped according to their preferences for the public good. Thus, each jurisdiction would be only composed of households with the same a_i. Such an equilibrium can exist only if the number of jurisdictions is at least equal to the number of household types.

Finally, there can be mixed equilibria in which all jurisdictions have identical amounts of public good and net housing price, certain being composed of several types of households, but such that their median vote is of the same type, and others composed of only one type of household.

Equilibria in which several jurisdictions have the quantity of public good and the same unit gross price of the of land are not particularly interesting, because, in such cases, the fusion of these jurisdictions would not changed its households' utility, since the per capita amount of public good would be the same. So let only explore equilibria where two jurisdictions have different amounts of public good and net housing prices.

Suppose that every jurisdiction is composed of a connected subset of the continuum of households: jurisdiction 1 is composed of the interval $[0; j_1]$, jurisdiction 2, of the interval $[j_1; j_2]$,..., and jurisdiction M, of the interval $[j_{M-1}; 1]$. Obviously, at the equilibrium, one can not have 2 jurisdictions j and j', with $j < j'$ such that $Z_j > Z_{j'}$ and $P_j < P_{j'}$, because every households would prefer to live in j. Symmetrically, one can not have $Z_j < Z_{j'}$ and $P_j > P_{j'}$. Furthermore, jurisdiction j''s median voter would not choose a tax rate that would lead to an amount of public good less than Z_j, because households living in j', by definition, have their MRS greater than households living in j, so clearly $Z_j < Z_{j'}$ and $P_j < P_{j'}$.

The proof of the existence of an equilibrium is highly challenged when a land market is introduced. For instance, Rose-Ackerman failed to identify sufficient conditions to ensure the existence of an equilibrium using Kakutani's fixed point theorem, because of the non-convexity of the consumption set. Let us consider a household endowed with a private wealth of 100 and living in a jurisdiction with $H = 100$ and $p = 2$. The cost function is given by $C(Z) = \sqrt{Z}$. The consumption bundles B_1, composed of amounts of public of 10000, of private good 40 and of housing 20, and 2 composed of amounts of public of 40000, of private good 60 and of housing 10, both respect the budget constraint $x+(1+d)p = 100$, the first one, by applying a tax rate of $\frac{1}{2}$, so the net-of-tax housing price is 3, which respect the budget constraint, the second one, by applying a tax rate equal to 1, so the housing price is 4. However, a linear combination of budget 1 and 2, given by $Z = 25000$, $x = 50$ and $h = 15$ is not feasible, which proves that, in this case, the consumption set is not convex.

Another possible modelling of land is the one proposed by Dunz [29]: housing is indivisible, but there exist several type of housing within jurisdiction. Using this modelling, sufficient conditions to ensure the existence of an equilibrium can be found.

Another question raised by the introduction of land is its hedonic value, as developed by Rosen [76] that may depends on its location: people may be willing to spend more money for a same amount of housing in a jurisdiction 1 than in a jurisdiction 2, even if both jurisdictions offered the same amount of public good with the same tax rate, because of their intrinsic value, for instance if jurisdiction 1 is nearby the sea, while jurisdiction 2 is not.

Some authors tried to take into account the jurisdiction's intrinsic value into their analysis, as Greenberg [48] and Nechyba [70]. Nechyba proposed a model with Dunz's land market modelling ([29] and [30]) and taxation on land in which households' utility depends on their place of residence for itself, besides the public good policy applied, and in which public good in one jurisdiction can generate spillovers in others. However, among the sufficient conditions he identified to ensure the existence of a equilibrium, and also segregation, one is the indifference among 2 jurisdictions offering the same among of public good and in which the net-of-tax housing price is the same. Since this condition is sufficient to ensure the existence of an equilibrium, but has not been proved to be necessary, assuming that jurisdictions may have intrinsic value does not improve the results.

Other articles, by Epple, Filimon and Romer [34] and [35], find necessary and sufficient conditions to ensure the existence of an equilibrium, if restrictive assumption are made on the preferences and on the public good production function: the MRS of the public good by the net-of-tax housing price must be always increasing with the private wealth, the public good cost function must be an strictly affine function (and not a linear function), and the Marshallian demand for housing must not depend on the available amount of public good.

If taxation, as in Westhoff's article, is still based on wealth, then Konishi [64] proved the existence of an equilibrium in a model with housing market, using Kakutani's fixed point theorem. In his model, as in Rose-Ackerman's, housing is perfectly divisible.

The different possible modelling of the housing market in multi-jurisdictional economy models has unquestionably an impact on the existence of an equilibrium. However, it seems that the equilibrium, if it exists, will be stratified, either in terms of preferences for the public good, or in terms of wealth.

Now that the existence of an equilibrium can be ensured, let explore the question of the its properties in terms of Pareto-efficiency.

1.2.3 Are equilibria always Pareto-efficient?

In this section, we will examine the properties of the equilibrium in models à la Tiebout in terms of Pareto-efficiency. Let us first remind the mostly used notion for stability.

A jurisdictions structure is stable if and only if:

- No household i has incentive to modify its current consumption bundle X_i or to leave unilaterally its jurisdiction for another one: $\forall j, j' \in J, \forall i \in I_j, U_i(Z_j, X_i) \geq V_i(Z_{j'}, P_{j'}, R_{ij'})$ where $V_i(Z_{j'}, P_{j'}, R_{ij'})$ represents the maximal utility that household i can obtain when the amount of public good is $Z_{j'}$, the net-of-tax prices of every private goods $P_{j'}$, and the available wealth is $R_{ij'}$.

- A voting equilibrium is reached in every jurisdiction.

- Every jurisdiction presents a balanced budget: the local public goods are fully financed by the fiscal revenues, that are devoted only to the production of the local public goods.

The definition of Pareto-efficiency (or Pareto-optimality) is widely known: an allocation of public and private goods is Pareto-efficient if no other conceivable allocation of public and private goods can increase strictly the utility of one household, while all the others are not worse-off. A jurisdictions structure is Pareto-efficiency if it leads to a Pareto-efficient allocation of public and private goods.

Bewley [7] presented several counter-examples in which either no equilibrium exists, or, if an equilibrium exists, it is not Pareto-optimal. He distinguished pure public goods, that do not suffer from congestion and pure public services, which cost is proportional to the number of users. He also consider in turns the possible objective followed by local governments: a democratic one, whose goals is to maximize a certain social welfare function, and an entrepreneurial one, which is independent from households' welfare (for example, maximizing the probability of being re-elected, or the number of inhabitants).

In order to prove the existence of an equilibrium, most authors used either Brouwer's or Kakutani's fixed point theorem. Some conditions are required to allow the use of a fixed point theorem. For instance, Westhoff's main result is the identification of a sufficient condition to have all stable jurisdictions structure segregated, in addition to the standard properties assumed on the preferences: the Marginal Rate of Substitution (MRS), given by $\frac{\frac{\partial U_i}{\partial Z}}{\omega_i \frac{\partial U_i}{\partial x}}$, must be a continuous and increasing function with respect to i.

If this condition is satisfied, then an equilibrium will be reached. But the equilibrium is not necessarily Pareto-optimal. Consider any case where there is only one type

of households, with half living in one jurisdiction, and the other half, in a second one. Clearly, those jurisdictions will be identical, since the tax bases will be the same, so will be the median voters' preferred tax rate (actually, every household will be the median voter). Consequently, no household can be better-off by moving from its jurisdiction to the another one, so such a jurisdiction structure is stable. However, the jurisdictions structure where all households are merged into a grand jurisdiction would make all households better-off, since the tax base would be multiplied by 2 and every household would be the median voter, so the tax rate implemented would be the tax rate that maximizes its utility.

Furthermore, Hamilton [53] claimed that the conditions under which the provision of local public goods is Pareto-optimal according to Tiebout, such as the number of households in every jurisdiction that is assumed to be "optimal", which means that the desired amount of public good can be produced at the lowest average cost, do not hold in reality. His main contribution is to show that a land-restriction policy must be implemented to ensure the existence of a Pareto-optimal equilibrium: to be a member of the jurisdiction, a household must consume more than a defined threshold of housing.

The existence of spillovers and congestion, along with the differences in preferences, can be part of the reason why it may be efficient to have households separated between different jurisdiction, instead of having all households merged into a grand jurisdiction. Let us first present a definition of spillovers.

A local public good produced by jurisdiction j generates positive spillovers in another jurisdiction j' if and only if the available amount of public good in j' is strictly increasing with respect to the amount of public good produced in j.

Among the several possible way to model the spillovers, Greenberg's modelling [48] is the more general: households' utility function depends on the amount of public good produced by the jurisdiction and on the number of households that live in it. The impact of the mass of the population on the public good is not defined, which is a very general, and an interesting way to model the congestion effects. Greenberg [48] also generalized Westhoff's model by introducing the presence of profit-maximizing firms that produce several kinds of private goods, by relaxing the assumption that local taxes are proportional. The only assumption made on the tax scheme is that no households would be asked to paid a tax greater than its wealth. However, under standard assumptions, Greenberg proved that the existence an equilibrium can be ensured.

Papers such as [13] and [12] provide a modelling for spillovers generated by local public goods: the available amount of public good households living in jurisdiction j can enjoy is given by

$$Z_j = \sum_{k \in J} \alpha_{jk} \zeta_k$$

where α_{jk} is the spillovers coefficient from jurisdiction k to jurisdiction j. The spillovers coefficient matrix $A = [\alpha_{jk}]$ does not need to be symmetric, the only assumption is that, $\forall (j,k) \in J^2, \alpha_{jj} = 1$ and $0 \leq \alpha_{jk} \leq 1$.

Using a discrete number of households could have caused problems to prove the existence of an equilibrium. The problem is known as "the integer problem" [85]. Since Brouwer's theorem can be applied only to continuous function, it mazy not be applied if the set of households is not continuous, as if it the case here. However, Wooders, by replicating the economy, found a solution to prove the existence of an equilibrium.

The introduction of congestion and spillovers gives the intuition why having households separated between different jurisdiction may be unanimously preferred to the grand jurisdiction, while models *a la* Westhoff conclude that the grand jurisdiction is always a Pareto-efficient equilibrium.

Local public goods suffer from congestion effects if and only if the available amount of local public goods households can enjoy, Z, depends positively on the public expenditure E and negatively on the mass of households that can consume them, μ_j.

There exist several approaches to model congestion. One possible but extreme way of modelling congestion is to assume that the public good cost function depends on the per capita amount of public good. It is equivalent to assume that households share the local public good equally, so, if the taxation is based on wealth, the available amount of public good each household can consume is given by $Z_j = \frac{t_j \varpi_j}{\mu_j}$ where $\mu_j = \int -i \in I_j d\lambda$ is the mass of households in jurisdiction j. The local public good can then be considered as a perfectly rival common resource. Some articles modelled congestion in such a way (see for instance [16]).

Such a modelling is quite restrictive and unrealistic: a million households endowed with the same wealth would enjoy the same amount of public good if all lived in a grand

jurisdiction as if they all loved separately into a million jurisdictions. This modelling can be applied only to a very few kind of public good, for example a road.

A more realistic modelling is the one provided by Oates [71]. Oates modelled the available amount of public good $Z_j = \frac{\zeta_j}{\mu_j^\beta}$, where ζ_j is the amount of public good produced by jurisdiction j, and empirically estimated the parameter β. Though, for most public goods, the parameter β will be included between 0 and 1, there exist some kinds of public good for which the hypothesis $\beta = 1$ can not be excluded.

If local public goods do no suffer from congestion effects, the grand jurisdiction is always Pareto-optimal, for a simple reason: the median voter(s) in the grand jurisdiction will always be worse-off if there are at least 2 distinct jurisdictions, because the tax base of its jurisdiction will be lower, so even if it is still the median voter, its utility will be lower. The same reasoning can be used to demonstrate that an equilibrium in Wooders' model may not be Pareto-optimal if the crowding effect is not too strong.

However, if congestion effects are strong enough, then the grand jurisdiction might not be Pareto-efficient: Suppose that the local public good is perfectly rival, so the available amount of public good is the amount of public expenditure divided by the mass of households, and that there exist two groups of households, having the same wealth, but different preferences, one group wants a high tax rate and a high amount of public good, while the other group would choose a lower tax rate. The jurisdiction structure composed of the grand jurisdiction is clearly not Pareto-efficient, because the group having the greatest mass will choose the tax rate. If we consider the jurisdictions structure where the two groups were separated into two distinct jurisdictions, the tax base would be the same in every jurisdiction, and both groups could have its favorite tax rate. Consequently, households belonging to the group with the greatest mass would be indifferent between living in the grand jurisdiction and living in a jurisdiction composed only of other households of its group, while households belonging to the other group would strictly prefer the second jurisdictions structure to the first one.

However, the grand jurisdiction is not necessarily socially efficient, as shown by Guesnerie & Oddou ([51] and [52]). They used the notion of super-additivity developed in the cooperative game theory. A game is super-additive if and only if the pay-off of the union of two coalitions is greater than the sum of the two separated coalitions' pay-offs. Applied to

the jurisdiction formation theory, the super-additivity means that two jurisdictions could produce a greater amount of public if they merged than the sum of the amount of public good they could produce in they remain separated apart. They proved that, in a model *à la* Westhoff, if the game is super-additive, then the grand jurisdiction is always efficient according to any individualistic social welfare function. If the game is not necessarily super-additive (which would be the case is there exist some congestion effects), clearly it might not be the case.

Their model is quite imperfect though, because the existence of an equilibrium can not be proved. Konishi, Le Breton & Weber [65] even show that such a model may not admit a pure strategy Nash equilibrium if the local tax based on wealth is proportional and preferences are quasi-linear with respect to the private consumption. A pure strategy Nash equilibrium is a jurisdictions structure where no household have incentive to move unilaterally to another jurisdiction nor to create its own jurisdiction.

If there exist several local public goods, then there may not exist a voting equilibrium within a jurisdiction, so, even if mobility is not allowed, or if there exists only one jurisdiction, the stability of a jurisdictions structure could be compromised. To ensure the existence of a voting equilibrium if there exist q local public goods, a voting rule has been identified by Greenberg [47] and used later by Greenberg & Shitovitz [49]: the q-majority rule.

This rule states that an existing allocation $(Z_1, ..., Z_q)$ of local public goods will be replaced by another feasible allocation $(Z'_1, ..., Z'_q)$ if and only if the fraction of households that prefer the second allocation to the existing one is greater than $\frac{q}{q+1}$. Obviously, if $q = 1$, the q-majority rule degenerates to the simple majority rule.

Most articles considering the existence of several public goods use this rule in order to ensure the existence of a voting equilibrium, such as Konishi [64]. Konishi's proof of the existence of an equilibrium has been adapted in other articles to prove the existence of an equilibrium, for example by Bloch & Zenginobuz in [13].

The authors, using Konishi's proof of the existence of an equilibrium, ensured the existence of an equilibrium under 2 assumptions: the public good and the composite private good are both normal (which is a pretty standard assumption), and the MRS of the private

good for the public good tends to 0 when the available amount of public good tends to 0, and, for an available amount of public good $\bar{Z} \in [0;1]$, tends to something greater or equal to $\bar{Z}$.

Moreover, if the spillovers coefficient matrix is symmetric, the authors proved that all stable jurisdictions structure are symmetric, while if it is asymmetric, there exist an unique equilibrium if spillovers coefficients are low enough, and several equilibria if they are high enough.

1.3 The definition of segregation

Among economists, sociologists, political scientists, and more generally social scientists, the discussion on segregation by income between neighbourhoods or communities became more and more important as more and more people moved to a metropolitan area. Debates exists about whether or not segregation is ineluctable, efficient or even desirable from a welfarist point of view.

Although there exist lots of arguments against or in favor of segregation by wealth (or by income), one may just argue that households who are similar in terms of wealth would have the same preferences over the public good, and so decisions would be easier to take and more respected. On the contrary, one may object that poverty concentration in a neighbourhood seems to be correlated with high criminal activity, not only in this very neighbourhood, but in the whole metropolitan area.

More generally , the existence of an impact of spatial inequalities on metropolitan areas' growth and development is widely admitted by economists and social scientists. However, this section will not provide arguments for or against segregation, but present the possible definitions of segregation and its causes. One possible way to define segregation is to measure it through an index.

Although there exists several indexes measuring segregation within groups if there are only two different types (blacks and whites, or males and females), such as Duncan's (1955), Theil and Finizza's (1971), Hutchens' (2000) or Frankel and Volij's (2005), only few indexes that can measures segregation between more than two types, or with a continuous

variable. Let's just mention the Neighbourhood Gini Coefficient [57], the Neighbourhood Sorting Index (NSI) [58], the Gini Coefficient of Segregation (NCS) [59] and the Centile Gap Index (CGI) [83].

We denote μ as the mass of households in the area, and μ_j as the mass of households living in jurisdiction j, ω^{mean} (resp. ω^{med}) as the mean (resp. median) income in the whole area, and ω_j^{mean} (resp. ω_j^{med})as the mean income (resp. median) in jurisdiction j. Finally, we denote σ_j as the standard income deviation in jurisdiction j, and σ, the standard income deviation in the whole area. A jurisdictions structure is defined as the triplet $S = (J, \{I_j\}_{j\in J}, \{\omega_i\}_{i\in I})$

The segregation index of a jurisdictions structure S is a function $I(S) \in [0; 1]$, the closer $I(S)$ is to 1, the more the jurisdictions structure S is segregated. Of course, to compute a segregation index, we will require that the considered area is composed of at least 2 jurisdictions and at least 2 different levels of wealth.

Let's formally write the previously mentioned income segregation indexes :

- The Neighborhood Gini's Coefficient (NGC) : $NGC = \frac{M+1}{M-1} - \frac{2}{M(M-1)\omega^{med}} \sum_{j=1}^{M} R_j \omega_j^{med}$, where R_j is the rank of jurisdiction j, such that the rank of jurisdiction with the highest median wealth is 1, and the rank of jurisdiction with the lowest median wealth is M.

- The Neighborhood Sorting Index (NSI) : $NSI = \frac{\sum_{j=1}^{M} \mu_j (\omega_j^{mean} - \omega^{mean})^2}{\sigma^2}$.

  The NSI is the ratio Variance inter-jurisdiction over total variance.

- The Gini Coefficient of Segregation (GCS) : $NCS = \frac{\int_{i=0}^{1} (\int_{i'=0}^{1} |\omega_{ni} - \omega_{ni'}| di') di}{\sum_{i=0}^{1} (\sum_{i'=0}^{1} |\omega_i - \omega_{i'}| di') di}$
  where ω_{ni} is the mean wealth in i's jurisdiction.

- The Centile Gap Index (CGI) : $CGI = 1 - \frac{4 \int_{i=0}^{1} |P_i - P_i^{med}|}{\mu}$
  where P_i is the estimated percentile in the whole area distribution of household i , and P_i^{med} is the estimated percentile in the whole area distribution of the median

household of the jurisdiction in which household i lives.

So far, no segregation index respecting several desirable properties has been identified. Among the desirable properties a segregation index should respect, there are:

- Scale Invariance: $\forall k \in \mathbb{N}^*, \forall S, S^k$, where $S = (J, \{I_j\}_{j\in J}, \{\omega_i\}_{i\in I})$ and $M^k = (J^k, \{I_j^k\}_{j\in J}, \{\omega_{i^k}\}_{i^k\in N^k})$ such that $i \in I_j \Rightarrow \forall i^k \in [k(i-1)+1; ik], i^k \in N_j^k$ and $\forall i, \forall i^k \in [k(i-1)+1; ik], \omega_{i^k} = \omega_i$ then $I(S) = I(S^k)$. In words, a segregation index that respects this property will remains constant if each household is "cloned" k times. This axiom is necessary to compare 2 metropolitan areas of different population sizes.

- No Monetary Illusion: $\forall c \in \mathbb{R}_*^+, \forall M, M'$ where $M = (N, J, (N_j)_{j\in J}, (\omega_i)_{i\in N})$ and $M' = (N, J, (N_j)_{j\in J}, (c\omega_i)_{i\in N})$, then $SI(M) = SI(M')$. This axiom is useful to compare 2 metropolitan area from different monetary zones or to observe the evolution of the segregation within a metropolitan area between two periods, to avoid the inflation issue.

- Sensitivity to per capital wealth differences: $I(S) = 0 \Rightarrow \omega_j^{mean} = \omega_{j'}^{mean}, \forall (j, j') \in J^2$. Consider a metropolitan area M composed of 2 jurisdictions. The first jurisdiction is composed of 25 homeless households, with no wealth, 1 household endowed with 10 K\$ and 25 households endowed with 15 K\$, while the second jurisdiction is composed of 25 households endowed with 5 K\$, 1 household endowed with 10 K\$ and 25 households endowed with 20 K\$. Here, $\varpi_1 \approx 7.55$\$ and $\varpi_2 \approx 12.45$\$, but both the NGC and the CGI will be equal to 0.

- Sensitivity to skewness differences: $I(S) = 0 \Rightarrow \gamma_j \gamma_{j'} > 0, \forall (j, j') \in J^2$, with $\gamma_j = \frac{\omega_j^{mean} - \omega_j^{med}}{\sigma_j}$ being Pearson's second skewness coefficient for jurisdiction j. This property is desirable for many reasons, the most important one being the difference in public good provision between 2 jurisdictions having the same average wealth, but different distribution skewness. This property is not respected by the NGC and the CGI. In the last example, $\gamma_1 \approx -0.33$ and $\gamma_2 =\approx 0.33$, so $\gamma_1 \neq \gamma_2$ but NGC=CGI=0. The NSI and the GCS don't respect this property neither. Let's consider a metropolitan area M composed of 2 jurisdictions. The first jurisdiction is composed of 20 homeless households, with no wealth, and 80 households endowed with 10 K\$, while the second jurisdiction is composed of 99 households endowed

with 5 K$ and 1 household endowed with 305 K$. In this example, $\gamma_1 = -\frac{1}{2}$ and $\gamma_2 = \frac{1}{3\sqrt{11}}$, so $\gamma_1 \neq \gamma_2$. Those 2 jurisdictions have the same average and aggregated wealth, so NSI=GCS=0, but they would certainly not have the same type of public good, schools will not be frequented by the same population of students...

- Monotonicity: $\forall S^A = (J, (I_j^A)_{j\in J}, (\omega_i)_{i\in I}), S^B = (J, (I_j^B)_{j\in J}, (\omega_i)_{i\in I})$, where :
  1. $i \in I_j^A, i' \in I_{j'}^A, \omega_i > \omega_{i'}$,
  2. $\omega_{j^A}^{mean} < \omega_{j'^A}^{mean}$,
  3. $\omega_{j^A}^{med} \leq \omega^{med}$,
  4. $\omega_{j'^A}^{med} \geq \omega^{med}$,
  5. $I_j^B = I_j^A \cup \{i'\} \setminus \{i\}, I_{j'}^B = I_{j'}^A \cup \{i\} \setminus \{i'\}$,
  6. $\omega_{j^B}^{med} \leq \omega^{med}$,
  7. $\omega_{j'^B}^{med} \geq \omega^{med}$,

  $I(S^A) < I(S^B)$.

This last property implies that any population movement, which increases the difference in average wealth between jurisdictions, without changing the relative position of their respective with respect to the median wealth of the whole area, should increase the segregation index too. The NGC violates this properties. Let's consider a metropolitan area M composed of 2 jurisdictions. The first jurisdiction is composed of 20 homeless households, with no wealth, 10 households endowed with 10 K$, and 20 households endowed with 11 K$ while the second jurisdiction is composed of 20 households endowed with 5 K$, 10 household endowed with 11 K$ and 20 households endowed with 15 K$. Replacing one household from the first jurisdiction endowed with 11 K$ with one household from the second jurisdiction endowed with 5 K$, neither the median nor the rank of each jurisdiction will change, so neither would the NGC.

The CGI does not respect the third property, while the NSI and the GSC do not respect the fourth one and the NGC, the third and the last one. Moreover, to be able to compute the GSC and the CGI, one must know the exact distribution of wealth in every jurisdiction, which may be very costly.

For those reasons, from the best of my knowledge, economists who studied the causes of segregation do not use either one of those measures in theoretical models.

Another possible definition of segregation can be expressed in terms of consecutiveness of the continuum of households, as it is known in coalition theory, see for example [50]: a jurisdictions structure is segregated if and only if every jurisdiction is composed of a connected subset of the continuum of households.

In plain English, this definition means that a jurisdictions structure is segregated by wealth if and only if, for every pair of jurisdictions, the richest households of the jurisdiction with the lowest per capita wealth is poorer than the poorest households living in the other jurisdiction. Clearly, this is an extreme notion of segregation, but that also makes it interesting, because it allows economists to identify the the segregative forces that would lead to such a pure segregation in a stylized world.

In most articles dealing with the wealth-stratification of stable jurisdictions structure, authors consider this definition of the the segregation. Although it presents some default, one of them being the fact the grand jurisdiction, i.e. a jurisdiction structure composed of only one jurisdiction gathering all households, would be considered as segregated.

Despite our awareness of this definition's imperfections, from now, that is the one we will use in the rest of the survey.

1.4 The segregative properties of endogenous jurisdictions structure formation in the theoretical literature

The first article that provided a condition to ensure the wealth-stratification, expressed in terms of the consecutiveness of the set of households when ordered by their wealth, of any stable jurisdictions structure is from Ellickson[32]. This condition is the single crossing of households indifference curves in the "tax-rate-amount of public good" space. One can notice that, as we previously mention it, this condition is also sufficient to ensure the existence of an Equilibrium.

This definition is widely used in the theoretical literature on segregation. However, this extreme notion of segregation does not allow to compare two different areas, or to examine the evolution of an urban area at two different periods, unless one has all its jurisdiction compose of connected subsets and not the other. Moreover, let's consider an urban area composed of millions of inhabitants that is segregated in a certain period according to Ellickson's definition. Suppose that in next period, two households from different towns exchange their respective housing. Then the area is not considered as segregated anymore, although the change is relatively minor.

One strong assumption of models *à la* Tiebout is the fact that households choose their place of residence according to a trade-off between the amounts of public good provided by the jurisdictions and the tax they would pay there. According to Percy and ali[73], this assumption can not be rejected. By using data on inter-jurisdictional migrations among 55 communities, they show that the decision of voting with one's feet and the choice of the new jurisdiction depend greatly on the levels of tax rates and of the amount of public services available in the different jurisdictions.

Empirical evidences brought by Banzhaf and Walsh[4] also show that households actually choose their place of residence according to local amenities that could be improved by the presence of a local public good. In particular, rich households are willing to pay higher taxes in order to enjoy high air quality.

Another article [45] identified a necessary and sufficient condition on the preferences to have all possible stable jurisdictions structures segregated in a model very close to Westhoff's: there is a continuum of households who differ by their wealth, they vote over the tax rate applied by their jurisdiction, and they can leave their jurisdiction for another that would provide a higher utility level. However, contrary to Westhoff's model, no specific collective decision rule is imposed, the authors only assume that the tax rate applied by the jurisdiction is democratically chosen, that is to say it is between the lowest tax rate preferred by one households and the highest tax rate preferred by another one.

The main difference between the articles is that households have identical preferences over the public good and the composite private good. This assumption, while restrictive, enables the authors to identify a necessary and sufficient condition to ensure the segrega-

tion of every stable jurisdictions structure: the Gross Substitutability/Complementarity (GSC) condition.

The public good is a gross substitute (resp. a gross complement) if and only if, $\forall p_Z, p_x, \omega_i \in \mathbb{R}_+, \frac{Z^M(p_Z, p_x, \omega_i)}{p_x} > 0$ (resp. < 0). The GSC condition is necessary in the sense that, for any violation of the condition, one can always construct a stable and yet no segregated jurisdictions structure. However, it does not mean that a stable jurisdictions structure could not be segregated if the GSC condition was violated. It is also sufficient because it is equivalent to the sufficient condition identified by Westhoff.

While stringent, this condition is not outlandish. It is equivalent to have the preferred tax rate being a monotonic function of the private wealth, for any given amount of aggregated wealth. The validity of this condition can be challenged by several generalization of the model. For instance, so far, households are supposed to be freely mobile. Obviously, if the cost for moving from one jurisdiction to another is extremely high, no household would leave its jurisdiction, even if another jurisdiction implemented a more desirable package "tax rate-amount of public good".

When households share the same preferences, the GSC condition implies the condition identified by Westhoff to ensure the existence of an equilibrium. More precisely, under the assumption that households have the same utility function, the condition identified by Westhoff is equivalent to have the public good to be a gross complement to the composite private good (see [45] lemma 5).

Consequently, Gravel and Thoron's contribution is to identify a condition that is not only sufficient, but also necessary to have every stable jurisdictions structure segregated by wealth, while Westhoff and other authors only identify sufficient conditions. However, Westhoff's model is more general, since households may have different preferences.

1.5 Do people self-sort themselves into homogeneous jurisdictions? : Empirical results

1.5.1 The segregative factors

A reasonable definition of segregation is still to be invented, in order to provide empirical tests of the existing theoretical models. However, there exist some attempts to prove empirically that jurisdictions structures are segregated, such as Bergstrom and Goodman's article [6].

In this article, empirical evidences suggested that the richer a household is, the more it is willing to pay taxes so as to increase its consumption of public good, for any other given parameters. It means that a household's preferred tax rate is a monotonic function of its private wealth, which is equivalent to the condition identified by Gravel & Thoron to ensure the wealth stratification of any stable jurisdictions structure.

Those results are confirmed by another empirical article by Deacon and Schapiro[24] dealing with households' votes. The authors demonstrated that households' predisposition to vote in favor of the improvement of a collective good is positively correlated with the income.

Those articles estimated the demand for the public good through aggregate data using a median voter model. The principle is quite simple: examining an eventual correlation between the provision of public good in one jurisdiction and the characteristics of the "median household" of the jurisdiction (the median income, the median political preferences...).

However, those articles suffer from the assumption that households are not mobile among jurisdictions. The imperfection is corrected by another article provided by Goldstein and Pauly[42]. In this article, they emphasize the impossibility to use a median voter econometric model to estimate the demand for the public good when households are free to leave their jurisdiction for another one, because of what they called the "Tiebout's bias": ignoring that households may vote with their will lead to a estimated demand for the public good that will be biased in a direction that can be determined.

Even when the assumption that voters are myopic, the results remains robust, accord-

ing to Epple and Sieg[40]. They tested the validity of Ellickson's condition to ensure the wealth-stratification of any stable jurisdictions structure: the single crossing of households' indifference curves in the "tax rate-amount of public good" space. They found out that this hypothesis can not be significantly rejected.

The same results exist when local taxation is progressive, according to a empirical study realized by Schmidheiny[79]. However, in his article, he found that rich households prefer to move to low-tax jurisdictions, which contradicts the previous articles.

Peer-effects may also increase the segregative properties of endogenous jurisdictions formation, if we compare the results obtained by Epple and al. in [39] and [16] showing that segregation by wealth, among households sharing the same preference, is present inn both case, both results are more significant when peer-effects are allowed.

Those results are confirmed by Alesina and ali[1]. In their article, they presented evidences that households may choose a smaller jurisdiction -that produces a lower amount of public good- in order to avoid income heterogeneity, which prove the impact of peer-effect in favor of the segregation.

Another article, by de Bartolome & Ross[20] (confirmed in [22]), provided an model based on a "monocentric city" model, *à la* Alonso[2], with 2 types of households (rich or poor) showing that, if peer effects are neither too strong nor too weak, 2 equilibria may arise, one with poor households living in the center of the city while rich people live in the suburbs, and the other where, on the contrary, rich people live in the center while poor households live in the suburbs. This model is empirically tested and consistent with the data.

To conclude this section, the main factors that favors segregation, according to the theoretical and the empirical literature, are the monotonicity of the willingness to pay for the public good with respect to the wealth, the congestion and the peer group effects.

1.5.2 The anti-segregation forces and the empirical evidence

In most articles, under the sufficient conditions identified to ensure the existence of an equilibrium, all stable jurisdictions structures will be segregated. However, Nechyba [70]

identified extra conditions to have stable jurisdictions structure segregated, according to different possible definitions of the segregation. The extra condition to ensure the wealth-stratification of any jurisdictions structure is to have all households sharing the same preferences over the public good, the private good and the housing.

Though the model is more realistic than Gravel & Thoron's one, the condition is only sufficient, and may not be necessary, in the sense that a violation of the condition may not always allow to construct a jurisdictions structure that is both stable and non-segregated.

Consequently, not only all stable jurisdictions structure are not segregated, but even under conditions that are sufficient to ensure the existence of an equilibrium, a stable jurisdictions structure may not be segregated.

Although many models conclude that the endogenous jurisdictions structure formation leads to have any stable jurisdictions structure segregated, there does not exist a single metropolitan area that is segregated in the sense of Greenberg and Westhoff. This could mean that jurisdictions structure are not stable yet, but converge to a stratified equilibrium, so endogenous jurisdictions formation does lead households to self sort themselves into homogeneous jurisdictions.

However, one must keep in mind that Tiebout's intuitions are based on stringent assumptions, such as the absence of mobility costs, no spatial environment (and consequently no job location), the large number of jurisdictions inside an urban area, no housing market...

The relaxation of one (or several) of these assumptions may contradict Tiebout's predictions, for instance, if mobility costs were too high, no household would leave its jurisdiction, even if another one apply a better policy.

One of the first scepticisms of Tiebout's predictions come from Dowding and ali[27]. They considered that Tiebout's assumptions of free mobility is not consistent with the empirical studies, and, consequently, the outcome can not be as predicted by Tiebout. Furthermore, they questioned the validity of another Tiebout's assumption: households' choice of location could be determined not only by the tax rates and the amounts of public good produced by the jurisdictions of their metropolitan area, but also by other factors, such as the job location or the type of public good provided.

A empirical survey provided by Rhode and Strumpf[74] suggests that fiscal federalism does not lead to segregation. Even though, on one hand, their article claims that segregation increases as mobility costs decrease, on the other hand, they also asserts that their data do not present significant evidences that jurisdictions structure converge to a segregated equilibrium. This implies that there must exist factors that mitigate the segregative properties of endogenous jurisdictions formation, such as differences in preferences, employment opportunities, or maybe other reasons. This article proved not only that endogenous jurisdictions formation does not lead households to self-sort themselves according to their private wealth, contrary to Tiebout's intuitions, but also that it may decrease income heterogeneity across jurisdictions.

Those results have been partially confirmed by a difference-in-difference study provided by Farnham and Sevak[41], although stronger evidences show that migrations *à la* Tiebout are present when differences between the states' taxation schemes are controlled for.

A strong force that mitigates the segregative properties of endogenous jurisdictions structure formation is the difference in terms of preferences for the public good. If, *ceteris paribus*, empirical evidences suggest that local public goods are a substitute to the private consumption, one must take into account that households can not be characterized only by their private wealth.

For instance, the willingness to pay for the public good seems to be increasing with respect the age, because elderies could become unable to drive anymore, and prefer to take the public transportation, or with respect to the number of children. Consequently, a young and poor household with no child could desire the same amount as an old and richer one, so there would be no segregation.

Others reasons that could lead to a non-stratified equilibrium are the fiscal competition, housing prices and the existence of commuting costs, as another monocentric city model provided by de Bartholome and Ross[21] suggests. In their article, 2 kinds of equilibria may arise: one segregated and one non-segregated in which rich and poor households are indifferent between living in the city with a majority of poor households (and, consequently, their preferred policy) and living in the suburbs with a majority of rich households.

Such an hypothesis is confirmed by Nechyba's article[70], in which 2 other types of segregation may arise: segregation by housing and/or by preferences, if they do not share the same preferences over the local public goods and if there exist different types of housing. Different preferences seems to mitigate the segregative properties of jurisdictions formation *à la* Tiebout.

The proposition that differences in preferences over the housing is a factor that decreases the segregative forces is partially confirmed by Schmidheiny[80] in a two jurisdictions model: there will be an imperfect segregation, e. g. at the equilibrium, some rich households will live in the poor jurisdiction and vice-versa.

Several other articles provided a theoretical model and empirical evidences that stable jurisdictions structures are not segregated because of the differences in taste for the public good, such as [39] and [16].

In these articles, the authors developed a model in which households differ in wealth and in preferences. Households' preferences are represented by an utility function depending on the amount of public[2] that is financed through a dwelling tax, on the amounts of housing and of a composite private good they own, and finally on a taste parameter $\alpha \in [0;1]$ representing their preferences over the public good[3]

The differences in preferences could also decrease wealth segregation if local political institutions have other competencies than just the provision of a local public good, as claimed in Kollman and ali[63]: for instance, if a jurisdiction has the competencies to organize a referendum on alcohol prohibition, then clearly, wealth segregation may not occur, because even if the single crossing of the indifference curves in the tax rate-provision of public good holds, there can be two distinct jurisdictions both composed of rich, poor and middle-class households, one in which alcohol is legal and the other one in which it is prohibited.

To ensure the existence of an equilibrium, the authors assumed that the slopes of

[2] In [16], the amount of public good is replaced by the quality of the public good, that depends on the per capita jurisdiction's spendings for the public good, and also on the mean income in the jurisdiction (which measures the peer effect).

[3] for instance, if preferences are represented by a CES function, the utility function could be of the shape $U(Z,x,h) = (Z^{\alpha\rho} + xh^{\rho})^{\frac{1}{\rho}}$.

households' indifference curves in the "tax rate-quality of the local public good" space are increasing with respect to their private wealth and to their taste for the public good. Consequently, for households having the same preferences for the public good, two indifference curves will cross only once, so, if households have the same preferences, any stable jurisdictions structure will be segregated. However, when households do not share the same preferences, this is not the case anymore.

Furthermore, the parameter representing the taste for the public good is not significantly correlated with the private wealth, which implies that there exist poor households who would vote for the same tax rate as rich people, so those households would co-exist in the same jurisdiction.

Rubinfeld and ali's article[77] provided a discussion on Tiebout bias introduced in the previous section and an attempt to correct it. Using micro data, they found out that the correlation between the demand for public schooling and the income is lower than when aggregate data are used and is not significant anymore.

We can conclude this section that the main forces that mitigate the segregative properties of endogenous jurisdictions formation are the differences in tastes, over both the public good, the housing and the location, the mobility costs and the existence of spillovers generated by local public goods in other jurisdictions.

1.6 Conclusion

The question of the modelling of jurisdictions structure formation, inspired by Tiebout's intuitions, have been explored by several economists, in order to provide a realistic model so as to investigate on the consequence on segregation, on social welfare, and on the provision of public goods that fiscal federalism can have.

It is hard to provide a perfectly realistic model that could be applied to any country, since fiscal federalism structure is not the same from one country to another one. Those differences could be explored in order to identify what are the causes of segregation.

Another interesting question would be the definition of the segregation. Tiebout, fol-

lowed by other economists, assume that households living the same jurisdictions are homogeneous in terms of preferences, others define the segregation in terms of consecutiveness of the continuum of households.

Several indexes measuring segregation by wealth have been proposed, but none of them respects simultaneously several desirable properties. Developing an index satisfying all those properties, or, alternatively, an impossibility theorem, would be a major improvement of the literature.

If most of the theoretical models based on Tiebout intuitions concluded that, under some stringent assumptions, households will self-sort themselves into homogeneous jurisdictions, some of these assumptions are not consistent with empirical data, that is the reason why perfect stratification does never occur in the real life.

Finally, according to the existing literature, the main factors that favors segregation across jurisdictions are the monotonicity of the willingness to pay for the public good with respect to the private wealth, the congestion and the peer group effects.

The factors that mitigate the segregative properties of endogenous jurisdictions formation are the differences in preferences over the public good, the presence of a competitive housing market, the mobility costs, the intrinsic value of the possible locations and the existence of spillovers generated by local public goods in other jurisdictions.

An interesting extension of this survey would consist in examining whether further generalizations of models *à la* Tiebout favor or mitigate the segregative properties of endogenous jurisdictions formation, and in testing those results empirically.

Chapter 2

The segregative properties of endogenous jurisdictions formation with a welfarist central government[1]

This paper examines the segregative properties of Tiebout processes of jurisdiction formation in the presence of a central government which makes equalization transfers to jurisdictions in such a way as to maximize a welfarist objective. All agents - the households, the local governments and the central government - are assumed to make their choices simultaneously, taking as given the choices of others. The central government is assumed to pursue a generalized utilitarian objective. The main result of the analysis is that, if the utility function used by generalized utilitarian central government is additively separable, the class of preferences that guarantees the wealth segregation of any stable jurisdiction structure is unaffected by the presence of a central government.

2.1 Introduction

There is a wide presumption that decentralized processes of jurisdiction formation *à la* [82] lead individuals to self-sort into homogenous communities. [45] investigate the validity of this intuition within the classical model of jurisdictions formation developed by [84] (see also [50], [25] and [62] among many others). In this model, unequally wealthy households with the same preference for a local public good and a private good choose simultaneously their place of residence from a finite set of locations. Households who choose the same

[1] this chapter reviews a joint article with Rongili Biswas and Nicolas Gravel, submitted to Social Choice and Welfare, with a "revise and re-submit" notification.

location form a *jurisdiction* and produce a local public good by applying a democratically chosen tax rate to all residents' wealth. Any such simultaneous choice of residence by the households is referred to as a *jurisdiction structure*.

The analysis of this literature concerns *stable* jurisdiction structures, which satisfy the additional property of being *robust* to *individual deviations*. The question raised by [45] is whether stable jurisdiction structures lead households to self sort, or segregate, themselves by their wealth. The notion of segregation used is that known under the heading of *consecutiveness* in the coalition formation literature (see e.g. [50]). A jurisdiction structure is segregated in this sense if, for any two jurisdictions with different *per capita* wealth, the richest individual in the poorer jurisdiction is (weakly) poorer than the poorest individual in the richer jurisdiction. [45] identify a condition on households preferences that is necessary and sufficient for the segregation of any stable jurisdiction structure. The condition requires the public good to *always* be a gross complement to, or *always* be a gross substitute for, the private good. While stringent, and violated by several well-known preferences, including additively separable ones, this Gross Substitutability-Complementarity (GSC) condition is not implausible. For this reason, [45] seems to provide theoretical ground for the belief that decentralized processes of jurisdiction formation are inherently segregative.

In this paper, we investigate the extent to which this conclusion is affected by the introduction of a *central government*. Introducing a central government in models of endogenous jurisdiction formation strikes us as an important step toward improving the realism of these models. In many countries, one finds indeed a juxtaposition of several levels of government: central and local. It is also commonly observed that the central government puts into place *equalization payment schemes* that redistribute funds across jurisdictions so as to achieve specific normative objectives. It seems, therefore, of some interest to examine the consequence of central government's intervention on the segregative properties of the endogenous formation of local jurisdictions by freely mobile households.[2]

Doing this requires one to specify:

1. the instruments available to the central government,

[2]To the best of our knowledge, the only theoretical paper that has introduced a central government in a setting with Tiebout-like processes of local jurisdiction formation is [70]. Yet, the setting considered by Nechyba, which involves the purchase of location-specific indivisible house, and where the central government provides central public goods is quite different from the classical Whestoff one examined here in.

2. the objective of the central government,

3. the nature of the interaction between the central government, the households and the local governments.

As for the first point, we assume that the central government taxes households at a fixed (possibly negative) rate and redistributes tax revenues between jurisdictions in such a way as to maximize some objective function. Although stylized, this modelling of the redistribution performed by the central government does not provide a bad approximation of many *existing* systems of equalization payments.

It is consistent both with the so-called *horizontal* equalization payments scheme of the sort existing in Scandinavian countries and Germany - and the *vertical* schemes observed in several other countries (like Belgium, France, Canada, Australia, Switzerland and India, to mention just a few).[3]

As for the objective of the central government, we assume it to be *welfarist*. We consider more specifically the somewhat large family of *generalized utilitarian* social objectives that compare alternative packages of equalization grants and tax rates on the basis of the sum of some (increasing and concave) transformation of the households' utilities. This family, characterized in classical choice theory by plausible axioms (see e.g. [10], ch. 4), contains the standard utilitarian criterion as well as several others like, for instance, the symmetric mean of order r.

As for the interaction between the households and the governments (central and local), we assume that agents make their decisions *simultaneously*, taking the behavior of others as given. In this setting, we define a *stable jurisdiction structure with a central government* to be an assignment, to every location, of a local tax rate, a central government net transfer, and a set of households that is immune to unilateral deviation from the part of every agent.

The question addressed is whether the GSC condition on households' preferences remains necessary and sufficient for ensuring the segregation of any stable jurisdiction structure with a generalized utilitarian central government We first discuss, by means of examples, how the presence of a generalized-utilitarian central government may significantly

[3]See e.g. [43] for a theoretical normative analysis of equalization payment in federations and [81] for an empirical investigation in the Canadian context. A broader discussion of equalization in structures with multiple levels of governance is provided by [14].

affect the set of stable jurisdiction structures. Yet we show that, if the households' preferences are additively separable, the GSC condition remains necessary and sufficient for the segregation of any stable jurisdiction structure with a generalized utilitarian central government. Hence, it appears that, at least for households with additively separable preference, the presence of a generalized utilitarian central government does not affect the segregative properties of decentralized processes of jurisdiction formation, even though it affects a great deal the set of stable jurisdiction structures.

The rest of the paper is organized as follows. The next section introduces the model and illustrates by some examples how the presence of a central government affects the set of stable jurisdiction structures. Section 3 states and proves the main results and section 4 concludes.

2.2 The model

As in [45] and [84], we consider economies with a continuum of households indexed by the interval $[0,1]$ interval. Any such economy consists of three elements.

First, there is a *wealth distribution* modeled as a Lebesgue measurable, increasing and bounded from above function $\omega : [0,a] \to \mathbb{R}_{++}$ that associates to each household $i \in [0,1]$ its strictly positive private wealth ω_i.[4] Assuming the function ω to be increasing is a convention according to which households are ordered by their wealth ($i \leq i' \Longrightarrow \omega_i \leq \omega_{i'}$).

The *second* ingredient is a specification of the household's preferences, taken to be the same for all households. We assume that household's preference for the local public good (Z) and the private good (x) is represented by a twice differentiable, strictly increasing and strictly concave utility function $U : \mathbb{R}^2_+ \to \mathbb{R}$ bounded from below.[5] For the result on the necessity of the GSC condition for segregation, we make the additional assumption that the utility function that represents the household's preference and that is used by the

[4] To alleviate notation, for any $i \in [0,1]$, any set A and any function $f : [0,1] \to A$, we write f_i rather than $f(i)$.

[5] The assumption of strict concavity is an inessential simplification that garantees the uniqueness of the solution to the standard consumer program. The assumption that the utility is bounded from below guarantees that the maxmin criterion that is approached in the limit by some members of the generalized utilitarian family of social objectives used by the central government is well-defined.

central government is *additively separable* so that it can be written, for every $(\overline{Z},\overline{x}) \in \mathbb{R}^2_+$, as:

$$U(\overline{Z},\overline{x}) = f(\overline{Z}) + h(\overline{x}) \tag{2.1}$$

for some twice differentiable increasing and concave real valued functions f and h having both $\mathbb{R}_+$ as domain. Given any bundle of public and private good $(\overline{Z},\overline{x}) \in \mathbb{R}^2_+$, we denote by $MRS(\overline{Z},\overline{x})$ the *marginal rate of substitution of public good to private good* evaluated at $(\overline{Z},\overline{x})$ defined by:

$$MRS(\overline{Z},\overline{x}) = \frac{\partial U(\overline{Z},\overline{x})/\partial Z}{\partial U(\overline{Z},\overline{x})/\partial x} \tag{2.2}$$

We also denote by $Z^M(p_Z,p_x,R)$ and $x^M(p_Z,p_x,R)$ the households' Marshallian demands for the public and private good (respectively) when the prices of these goods are p_Z and p_x and the household's income is R. Marshallian demand functions are the (unique under our assumptions) solution of the program:

$$\max_{Z,x} U(Z,x) \text{ subject to } p_Z Z + p_x x \leq R$$

Given again our assumptions, Marshallian demands are differentiable functions of their arguments (except, possibly, at the boundary of $\mathbb{R}^3_+$). We emphasize that we view Marshallian demands as dual representations of preferences rather than descriptions of actual behavior (after all households rarely if ever purchase local public goods on competitive markets). The indirect utility function corresponding to U is denoted by V and is defined as usual by:

$$V(p_Z,p_x,R) = U(Z^M(p_Z,p_x,R), x^M(p_Z,p_x,R))$$

We further assume that the Marshallian demand for the local public good satisfies the following additional *regularity* condition (introduced and discussed in [45]).

Condition 1: If there exists a public good price $\overline{p}_Z$, an income level R and a non-degenerate interval I of strictly positive real numbers such that, $Z^M(\overline{p}_Z,p_x,R) = Z^M(\overline{p}_Z,p'_x,R)$ for all prices p'_x and p_x in I, then, for all $(p_Z,p_x,R) \in \mathbb{R}^3_+$, we must have $Z^M(p_Z,p_x,R) = h(p_Z,R)$ for some function $h : \mathbb{R}^2_{++} \to \mathbb{R}_+$.

We let $\mathcal{U}$ denote the set of all bounded from below utility functions that are twice differentiable, strictly increasing, strictly concave and whose Marshallian demand functions satisfy condition 1 and we let $\mathcal{U}^{\mathcal{A}}$ denote the subset of those functions that are additively

separable.

Finally, the *third* element in the description of an economy is a finite set $\mathbb{L} = \{1, ..., L\}$ of possible locations.

The problem considered is that of identifying the properties of the *jurisdiction structures with a central government* that can emerge when households freely choose their location and share, when they locate at the same place, the benefit of a local public good produced by local tax revenues and a central government grant. This intervention of the central government is the main distinctive ingredient of our model as compared to what is done in the literature (e.g. [25], [45], [48], [62],[84], [50]).

Specifically, we define a *jurisdiction structure with a central government* for the economy $(\omega, U, \mathbb{L})$ to be a Lebesgue measurable *location* function $L : [0,1] \to \mathbb{L}$, a (local) tax vector $t \in \mathbb{R}^L$, a central government grant vector $g \in \mathbb{R}^L$ and a central government wealth tax rate $c \in \mathbb{R}$ that satisfy:

$$_{l\in\mathbb{L}} g_l \leq c_{[0,1]} \omega_i d\lambda \tag{2.3}$$

and, for every $l \in \mathbb{L}$:

$$t_l \varpi_l + g_l \geq 0 \tag{2.4}$$

and:

$$1 - t_l - c \geq 0 \tag{2.5}$$

where λ denotes the Lebesgue measure over (Lebesgue measurable) subsets of $[0,1]$ and where, for every location $l \in \mathbb{L}$, $\varpi_l =_{L_l^{-1}} \omega_i d\lambda$. The interpretation given to the (Lebesgue measurable) set $L_l^{-1} = \{i \in [0,1] : L_i = l\}$ is that it is the community of households assigned to l by the location function L. This community will form a jurisdiction. We let $\mu_l^L = \lambda(L_l^{-1})$ denote the "measure of households living at l" under the location function L and we denote by $\omega_{L_l^{-1}}$ the restriction of the measurable function ω to the measurable set L_l^{-1}. The possibility that $\mu_l^L = 0$ for some l is, of course, not ruled out. We interpret $\omega_{L_l^{-1}}$ as the distribution of wealth in jurisdiction L_l^{-1}. Condition (2.3) requires the central government to balance its budget so that that the sum of the (possibly negative) grants given to the jurisdictions does not exceed the revenues obtained from taxing the households at rate c. Conditions 2) and 3) limit the fiscal power of the central and the local governments to raise taxes and to give grants to the extent compatible with non-negative consumption

of public (2.4) and private (2.5) spending. We emphasize that negative local tax rates are possible in a world with a central government. A household living in a jurisdiction receiving a large positive grant may prefer local tax rates to be negative and, therefore, use part of the central grant in private spending. Similarly, a central government may want to subsidize private consumption and to choose, for this purpose, a negative income tax rate t.

This modeling of the central government covers both the possibility that it transfers money between jurisdictions without taxing households ("horizontal" equalization) and the combination of horizontal and vertical equalization. But our model, by restricting the taxation power of the central government to linear schemes, rules out the possibility of using wealth tax for redistributive purposes (by making it progressive for instance). Of course providing the central government with the *full* power of redistributing the exogenous individual wealth (by choosing the tax paid by each household for instance based on its characteristic) would devoid the problem examined in this paper of much of its interest. For any central government that is averse to wealth inequality would obviously choose, if given such a power, to equalize wealth perfectly within a given jurisdiction. But between the full power given to a central government of taxing individually each household, and the extremely small one considered here of taxing all of them at the same rate, there is a large spectrum of possibilities that deserve, perhaps, a closer analysis.

Denote by $\Phi(\tau, \varpi, \omega_i, \gamma, c) = U(\tau\varpi+\gamma, (1-\tau-c)\omega_i)$ the utility received by a household with wealth ω_i living in a jurisdiction with local tax rate τ, aggregate wealth ϖ and central government grant γ when the central government household wealth tax rate is c (provided of course that these parameters satisfy conditions (2.4) and (2.5)). The function Φ so defined has several properties that we record in the following lemma (whose straightforward proof is omitted).

Lemma 2.2.1. *Let U be a utility function in $\mathcal{U}$ and let $(\tau, \varpi, \omega_i, \gamma, c) \in \mathbb{R}^5$ be such that inequalities (2.4) and (2.5) hold strictly. Then Φ is a twice differentiable function of its five arguments, is strictly increasing and concave with respect to ω_i, ϖ and γ (taking τ and c as given) and is strictly concave and single peaked[6] with respect to τ (taking ω_i, ϖ, γ and c as given).*

One important property of Φ is its strict single peakedness. It implies that a house-

[6] A function $f : A \to \mathbb{R}$ $(A \subset \mathbb{R})$ is strictly single peaked if, for all a, b and $c \in A$ such that $a < b < c$, $f(c) > f(b) \Rightarrow f(b) > f(a)$ and $f(a) > f(b) \Rightarrow f(b) > f(c)$.

hold with wealth ω_i facing a central tax rate of c has a unique favorite local tax rate of $\tau^*(\varpi, \omega_i, \gamma, c)$ in any jurisdiction with tax base ϖ and central government grant γ to which it may belong. This unique favorite local tax rate is the solution of the program:

$$\max_{\tau} \Phi(\tau, \varpi, \omega_i, \gamma, c) \text{ subject to (2.4) and (2.5)} \tag{2.6}$$

and is, for this reason, a continuous function of its four arguments.

As in [45], the behavior of this favorite local tax rate function is an important ingredient of the analysis. This behavior is very closely related to that of the Marshallian demand functions, as established in the following lemma whose proof, similar to that of lemma 2 in [45], is omitted.

Lemma 2.2.2. *Let* $(\varpi, \omega_i, \gamma, c) \in \mathbb{R}^2_{++} \times \mathbb{R} \times [0,1]$. *Then for all* U *in* $\mathcal{U}$ *,* $\frac{1}{\varpi}[Z^M(\frac{1}{\varpi}, \frac{1}{\omega_i}, 1 - c + \frac{\gamma}{\varpi}) - \gamma]$ *is the solution of (2.6).*

Lemma 2.2.2 states that, in a jurisdiction with aggregate wealth ϖ and central government transfer γ, the favorite tax rate of a household with a (net of central government tax) wealth $\omega_i(1-c)$ can be viewed as the *expenditure* that the household would like to devote to local public good in excess of the central government grant if the prices of public and the private goods were $\frac{1}{\varpi}$ and $\frac{1}{\omega_i}$, and if this household had an income of $1-c+\frac{\gamma}{\varpi}$.

We are interested in the possible segregative properties of the likely outcome of a free choice of location by households in the presence of a central government. This likely outcome must be *stable* in the usual sense of being immune to individual deviations from the part of all agents (households, local governments and central government). Providing a precise definition of stability requires one to specify first the objective pursued by each category of agents.

Households' objectives are clear. Each household seeks for the jurisdiction that offers the (utility) best package of tax burden and (local) public good provision.

In so far as the *local governments* are concerned, we adopt the view that each jurisdiction's choice of local tax rate is minimally democratic in the sense that it is the favorite tax rate of some household whose preference are not too distant from those of the jurisdiction's members. The particular rule used for choosing this "dictator" is inconsequential for the

results of this paper. In many models of endogenous jurisdiction formation with public good provision where voting is assumed, such as [84], the jurisdiction tax rate would be the one that occupies the *median position* in the jurisdiction's distribution of favorite taxes. While the analysis of this paper applies to this particular positional rule of selection of the dictator, they are valid for other rules as well. Formally, given an economy $(\omega, U, \mathbb{L})$, and a jurisdiction structure $J = (L, t, g, c)$ with a central government for this economy, we let m_J^l be the "dictator" in l for that jurisdiction structure. The only assumption made on this dictator is that his (her) favorite tax rate be contained between the *infimum* and the *supremum* of the favorite tax rates of the jurisdiction's members. Formally, for the jurisdiction structure with a central government (L, t, g, c), we define, for every location $l \in \mathbb{L}$ the infimum and supremum favorite tax rates t_{l*} and t_l^* respectively by:

$$\begin{aligned} t_{l*} &= \inf_{i \in L_l^{-1}} \arg\max_{\tau} \Phi(\tau, \varpi_l, \omega_i, g_l, c) \text{ subject to (2.4) and (2.5) and} \\ t_l^* &= \sup_{i \in L_l^{-1}} \arg\max_{\tau} \Phi(\tau, \varpi_l, \omega_i, g_l, c) \text{ subject to(2.4) and (2.5)} \end{aligned}$$

As for the *central government*, we adopt the *welfarist* view point that it compares alternative packages of equalization grants and household wealth tax rate on the basis of the (Lebesgue measurable) distributions of utility levels that they generate. We view any such distribution of utility levels as the graph of a (Lebesgue measurable) bounded function $\mathfrak{u} : [0,1] \to \mathbb{R}$ that maps household $i \in [0,1]$ into utility level $\mathfrak{u}_i$. When convenient, we alternatively denote by $< \mathfrak{u}_i >_{i \in [0,1]}$ such a function. The graph of these functions are compared by a social ordering [7] R with the usual interpretation that $\mathfrak{u}\ R\ \mathfrak{u}'$ means "distribution of utilities $\mathfrak{u}$ is socially weakly better than distribution $\mathfrak{u}'$". We shall, somewhat more specifically, assume that the central government uses a Generalized Utilitarian (GU) ordering. There are many justifications that could be given to this choice (see e.g. [10] ch. 4, theorem 4.7) in a classical social choice setting with a finite number of households. But social choice theory is not very developed for populations with a continuum of individuals (see however [17], [18], [61] and [60]) and we are not aware of axiomatic results that would justify a generalized utilitarian objective in such a setting. We simply observe that the GU family encompasses many welfarist criteria used in the literature. The Generalized Utilitarian ordering R^{GU} is defined by:

$$\mathfrak{u}\ R^U\ \mathfrak{u}' \Longleftrightarrow \int_{[0,1]} \Psi(\mathfrak{u}_i) d\lambda \geq \int_{[0,1]} \Psi(\mathfrak{u}_i') d\lambda$$

[7] An ordering is a reflexive, complete and transitive binary relation.

where $\Psi : \mathbb{R} \to \mathbb{R}$ is a function that evaluates individual utility functions from a social viewpoint. The Pareto principle requires Ψ to be increasing while utility-inequality aversion considerations suggest that Ψ be concave. We assume throughout that Ψ is differentiable. A well-known example of such a generalized utilitarian criterion is the global symmetric mean of order r (for some real number $r \leq 1$) in which Ψ is defined by:

$$\begin{aligned} \Psi(u) &= u^r \text{ if } r \in]0,1] \\ &= \ln u \text{ if } r = 0 \\ &= -u^r \text{ if } r < 0 \end{aligned}$$

This family contains the standard utilitarian criterion (for $r = 1$) and approaches the *maxmin* criterion (as r approaches $-\infty$).

We are now equipped to define formally what is meant for jurisdiction structure with a central government to be stable.

Given an economy $(\omega, U, \mathbb{L})$, we say that the jurisdiction structure $J = (L, t, g, c)$ with a central government endowed with a social ordering R is stable if
1) For every l, $l' \in \mathbb{L}$ and all $i \in L_l^{-1}$, $\Phi(t_l, \varpi_l, \omega_i, g_l, c) \geq \Phi(t_{l'}, \varpi_{l'}, \omega_i, g_{l'}, c)$.
2) For all $l \in \mathbb{L}$, $t_l = \tau^*(\varpi_l, \omega_{m_J^l}, g_l, c)$
3) $< U(t_{L_i}\varpi_{L_i} + g_{L_i}, (1 - t_{L_i} - c)\omega_i) >_{i\in[0,1]} R \ < U(t_{L_i}\varpi_{L_i} + g'_{L_i}, (1 - t_{L_i} - c')\omega_i) >_{i\in[0,1]}$
for all $g' \in \mathbb{R}^l$ and $c' \in \mathbb{R}$ satisfying (2.3)-(2.5).

This definition of stability rides of course on the assumption that households as well as local and central governments take their decision simultaneously, considering as given the behavior of others. This assumption generalizes naturally the simultaneous Westhoff framework. Yet it may be viewed as restricting unduly the central government's power of shaping the process of jurisdiction formation. An alternative could have been to assume a *two-stage* setting in which the central government would play *before* households and local governments and would choose its redistributive grants and households tax by anticipating the impact of its choice on the stable jurisdiction structure that would emerge in the second stage. While this alternative would certainly be worth exploring in detail, we have chosen to leave it aside in this paper that represents, to the best of our knowledge, the first attempt to introduce a welfarist central government in Tiebout-like models of jurisdiction formation. [8] We simply notice that the two-stage setting for integrating a central govern-

[8] As mentionned in introduction, [70] also introduces a central government in a Tiebout-like

ment in a Tiebout-like economy involves delicate modeling issues. Let us mention three of them that come to our mind.

First, choosing a set of equalization grants and wealth tax rate before knowing the jurisdiction structure that prevail raises the problem of the financial viability of the equalizations grants. What if the central government chooses, in the first stage, an equalization scheme which imposes a tax burden to a jurisdiction which, in the second stage, will be empty ? Second, and more importantly, as illustrated in the examples below, there may be many stable jurisdiction structures that correspond to a given central government equalization and taxation scheme. If this is the case, how is the central government going to predict which of the stable jurisdiction structures will emerge ? Third, everything else being the same, the central government, at least if it uses a Pareto inclusive social objective, *may* have a tendency to favour the "trivial" grand jurisdiction structure in which all households are put in the same jurisdiction and where, therefore, there is no role for a central government. The reason for this tendency is that, the local public good being non-rival, producing any quantity of it in a larger jurisdiction tend to be preferable from a social welfare view point because its cost can be shared by a larger number of tax payers.[9]

In order to identify the condition on households preferences that is necessary and sufficient for the wealth-segregation of any stable jurisdiction structure, one needs first to define what is meant by a wealth-segregation. We take the definition to be that used in [45] (see also [84] and [50]).

A jurisdiction structure with a central government $J = (L, t, g, c)$ for the economy $(\omega, U, \mathbb{L})$ is wealth-segregated if, for every locations l, $l' \in \mathbb{L}$ for which $\lambda(L_l^{-1}) \neq 0$ and $\lambda(L_l^{-1}) \neq 0$ and every households h, i and $k \in [0,1]$ such that $\omega_h < \omega_i < \omega_k$, h, $k \in L_l^{-1}$ and $i \in L_{l'}^{-1}$ imply that $t_l = t_{l'} = \frac{g_{l'} - g_l}{\varpi_l - \varpi_{l'}}$.

In words, a jurisdiction structure is wealth-stratified if, whenever a jurisdiction j contains two households h and k with different levels of wealth, it also contains all households whose wealth levels are strictly between that of h and k (if there exist such households

model of local jurisdiction formation. Yet Nechyba's model, that has an (indivisible) land market, is quite different from ours. The central government in [70] does not merely redistribute wealth and local public goods accross jurisdiction but also provides central public good. Moreover it makes its choice through (some form of) majority voting.

[9] This is true at least if the central government has full information onhouseholds' wealths. See [44] for an analysis of the (difficult) problem of the optimal choice of a jurisdiction structure when the central government is imperfectly informed about households'wealth.

of course) or, if it does not contain those households, it is because they belong to some (non-null) jurisdiction that offers the same tax rate and the same amount of public good as j.

Before establishing, in the next section, that the introduction of a GU central government does not affect the segregative properties of stable jurisdiction structures, it is probably worth discussing a bit how the introduction of a GU central government nonetheless modifies significantly the set of stable jurisdiction structures.

In many economies, the introduction of a central GU government will tend to sharply reduce this set. This can be probably best seen by considering a central government that pursues a *maxmin objective*. While such an objective does not - strictly speaking - belong to the class of GU criteria, it can be approximated with any degree of precision by some member of this class (for instance a symmetric mean of order r that uses a sufficiently negative value of r). The formal definition of the maxmin ordering, denoted $R^{\min}$, defined over any two bounded Lebesgue measurable functions $\mathfrak{u}$ and $\mathfrak{u}'$, is

$$\mathfrak{u}\ R^{\min}\ \mathfrak{u}' \Longleftrightarrow \inf_{i\in[0,1]} \mathfrak{u}_i \geq \inf_{i\in[0,1]} \mathfrak{u}'_i$$

If the central government uses such a maxmin objective, then the set of stable jurisdiction structures that can emerge with such a government is rather limited. For, as established in the following proposition, the only stable jurisdiction structure that can be observed with such a central government are those where the jurisdictions' poorest households have the same wealth in all jurisdictions. The trivial "grand jurisdiction" structure in which all households are in the same jurisdiction is, of course, a particular example of such jurisdiction structures.

Proposition 1. *A jurisdiction structure (L, g, c, t) with a maxmin central government is stable as per definition 2.2 only if for all l and l', $\inf_{i\in L_l^{-1}} \omega_i = \inf_{h\in L_{l'}^{-1}} \omega_h$.*

Proof.

By contraposition, assume that (L, g, c, t) is a stable jurisdiction structure with a welfarist central government in which there are locations l and $l' \in \mathbb{L}$ for which $L_l^{-1} \neq \varnothing$ and $L_{l'}^{-1} \neq \varnothing$ such that $\inf_{i\in L_l^{-1}} \omega_i \neq \inf_{h\in L_{l'}^{-1}} \omega_h$. We wish to show that the welfarist central government can not be maxmin. Without loss of generality, assume that the set L_l^{-1} contains a household

who is the worst off in the whole population. Because this household must belong to the set of poorest households in L_l^{-1}*, one has:*

$$\Phi(t_l, \varpi_l, \inf_{i \in L_l^{-1}} \omega_i, g_l, c) \leq \Phi(t_k, \varpi_k, \omega_h, g_k, c) \tag{2.7}$$

for all $k \in \mathbb{L}$ *and* $h \in L_k^{-1}$*. By clause (1) of definition of stability, one has also:*

$$\Phi(t_l, \varpi_l, \inf_{i \in L_l^{-1}} \omega_i, g_l, c) \geq \Phi(t_{l'}, \varpi_{l'}, \inf_{i \in L_l^{-1}} \omega_i, g_{l'}, c) \tag{2.8}$$

and:

$$\Phi(t_{l'}, \varpi_{l'}, \inf_{h \in L_{l'}^{-1}} \omega_h, g_{l'}, c) \geq \Phi(t_l, \varpi_l, \inf_{h \in L_{l'}^{-1}} \omega_h, g_l, c) \tag{2.9}$$

Either (i) $\inf_{h \in L_{l'}^{-1}} \omega_h < \inf_{i \in L_l^{-1}} \omega_i$ *or:*
(ii) $\inf_{h \in L_{l'}^{-1}} \omega_h > \inf_{i \in N_l^{--1}} \omega_i$*. Yet assuming (i) would imply, using (2.8) and the fact that* Φ *is increasing with respect to private wealth, that:*

$$\Phi(t_l, \varpi_l, \inf_{i \in L_l^{-1}} \omega_i, g_l, c) > \Phi(t_{l'}, \varpi_{l'}, \inf_{h \in N_{l'}^{-1}} \omega_h, g_{l'}, c)$$

in contradiction with (2.7). Hence (i) can not hold. If (ii) holds, then one has, because of (2.9) and the monotonicity of Φ *with respect to private wealth, that:*

$$\Phi(t_{l'}, \varpi_{l'}, \inf_{h \in L_{l'}^{-1}} \omega_h, g_{l'}, c) > \Phi(t_l, \varpi_l, \inf_{i \in L_l^{-1}} \omega_i, g_l, c)$$

so that the household whose income is (arbitrarily close to) $\inf_{i \in L_l^{-1}} \omega_i$ *is strictly worse off than the poorest household in jurisdiction* l'*. In that case, let* W *be defined by:*
$W = \{l'' \in \mathbb{L} : \Phi(t_{l''}, \varpi_{l''}, \inf_{i \in L_{l''}^{-1}} \omega_i, g_{l''}, c) = \Phi(t_l, \varpi_l, \inf_{i \in L_l^Z} \omega_i, g_l, c)\}$*. The set* W *is clearly non-empty since* $l \in W$*. Consider then taking away from jurisdiction* l' *some amount of grant* Δ *and dividing it up equally among all jurisdictions in* W *so as to keep constant the central government budget constraint. For a suitably small* Δ*, this change in the central government transfer policy increases the well-being of all households in the jurisdictions contained in* W *(including the worst off) while keeping the well-being of the worst off household in jurisdiction* l' *above. This shows that the original equalization grant vector* g *was not maximizing a maxmin ordering whatever the value of* c *might have been.*

The intuition behind this result is simple. A maxmin government wants to transfer

money to the jurisdiction that contains one of the worst off households (which must clearly be the poorest in its jurisdiction). By stability, any such worst off household prefers staying in its jurisdiction than moving elsewhere while the households in other jurisdictions - including the poorest in them - also prefer staying where they are than moving to the jurisdiction containing the worst off households. Except in the case when the wealth of these poorest households in all jurisdictions happens to be the same, these two conditions for stability imply that worst-off households are strictly worse off than at least one household which is the poorest in its jurisdiction. But if this is the case, then the transfers given by the central government to jurisdictions are not optimal from a maxmin point of view. Notice that the reasoning holds irrespective of the income tax rate chosen by the central government.

Jurisdiction structures in which all non-empty jurisdictions contain some of the poorest households in the population are, admittedly, rather special. For example, if all stable jurisdiction structures are stratified - in the sense of definition 2.2 above- then the *only* stable jurisdiction structure with a maxmin central government is the trivial "grand" one in which everybody but a set of measure zero lives in one place. By contrast, as illustrated in the following quasi-linear example, the absence of a central government is commonly compatible with *several* stable jurisdiction structures.

Consider the quasi-linear economy $(\omega, U, \mathbb{L})$ defined by:

$$\begin{aligned} \omega_i &= 1 \text{ if } i \in [0, 4/7[\\ &= 2 \text{ if } i \in [4/7, 6/7] \\ &= 10 \text{ if } i \in]6/7, 1] \end{aligned}$$

$$U(Z, x) = \ln 7Z + x, \tag{2.10}$$

and $\mathbb{L} = \{1, 2, 3\}$. Suppose there is no central government. Because the preferences represented by U satisfy what will be called below the GSC condition (see also [45]), we know that all stable jurisdiction structures will be segregated as per definition 2.2. Up to irrelevant permutations of locations, there are therefore only four such segregated jurisdiction structure:
1) The perfectly segregated structure in which each of the three intervals $[0, 4/7[$, $[4/7, 6/7]$ and $]6/7, 1]$ form a jurisdiction,
2) the "poor-mixed" structure in which the two poor households categories belonging to

the interval $[0, 6/7[$ form a jurisdiction and the rich households in the interval $]6/7, 1]$ form another,
3) the somewhat opposite "rich-mixed" structure where the two "rich" categories of households in $[4/7, 1]$ form a jurisdiction and where the poor households in $[0, 4/7[$ form another and
4) the "grand jurisdiction" structure in which the all households form a unique jurisdiction. Assume for the sake of this example that majority voting in each jurisdiction so that the dictator in each jurisdiction is the household whose favorite tax rate is the median of the favorite tax rates of the jurisdiction's members. Solving program (2.6) for $\gamma = c = 0$ (no central government) and for U defined by (2.10), one obtains easily that this favorite tax rate is $1/\omega_i$. This enables one to conclude that the perfectly segregated jurisdiction structure 1) is stable in this example. Indeed if one denotes by 1, 2, and 3 the jurisdictions in which households in the intervals $[0, 4/7[$, $[4/7, 6/7]$ and $]6/7, 1]$ respectively live, one obtains from clause (2) of the definition of stability that $t_1 = 1$, $t_2 = 1/2$ and $t_3 = 1/10$. This leads to a provision of public good of $Z_1 = t_{1[0,4/7]}\omega_i d\lambda = 4/7$, $Z_2 = t_{2[4/7,6/7]}\omega_i d\lambda = 2/7$ and $Z_3 = t_{3[0,4/7]}\omega_i d\lambda = 1/7$. We remark that this structure satisfies clause (1) of the definition of stability. Indeed for any household $i \in [0, 4/7[$, one has $U(Z_1, 0) = \ln 4 > U(Z_2, (1-t_2)) = \ln 2 + 1/2$ and $U(Z_1, 0) = \ln 4 > U(Z_3, (1-t_3)) = 9/10$. Similarly a household belonging to $[4/7, 6/7]$ and living in jurisdiction 2 has a utility of $U(Z_2, 2(1 - t_2)) = \ln 2 + 1$ at its place of residence while it would obtain a utility of $U(Z_1, 0) = \ln 4$ if it were to move to jurisdiction 1 and a utility of $U(Z_3, 2(1-t_3)) = 9/5$ if it were to move to the rich jurisdiction 3. Finally, the utility of $U(Z_3, (1-t_3)10) = 9$ achieved by any rich household in jurisdiction 3 is larger than the utility of $U(Z_2, (1-t_2)10) = \ln 2 + 5$ it would get if it were to move to jurisdiction 2, and the utility of $U(Z_1, 0) = \ln 4$ it would get if it were to move to jurisdiction 1. The "poor-mixed" structure is also stable in this economy. Indeed, in this structure, the (median) tax rate at jurisdiction 1 (inhabited by households in $[0, 6/7]$) is $t_1 = 1$ while the tax rate that will prevail at the (rich) jurisdiction 2 inhabited by households in $[6/7, 1]$ is $t_2 = 1/10$. We have here $Z_1 = t_1[_{[0,4/7]}\omega_i +_{[4/7,6/7]} \omega_i]d\lambda = 8/7$. Clearly:

$$U(Z_1, 0) = \ln 8 > U(Z_2, 2(1 - t_2)) = 9/5$$

so that no household in $[4/7, 6/7]$ wants to move to jurisdiction 2. Similarly:

$$U(Z_1, 0) = \ln 8 > U(Z_2, 1 - t_2) = 9/10$$

so that no household in $[0, 4/7[$ wants to move to jurisdiction. Lastly:

$$U(Z_2, (1-t_2)10) = 9 > U(Z_1, 0) = \ln 8$$

so that the rich prefer staying in jurisdiction 2. It can be checked however that the "rich-mixed" jurisdiction structure is not stable no matter how the local tax rate in the "rich-mixed" jurisdiction is set (provided of course that this tax rate lies in the interval $[1/10, 1/2]$ as it should if the structure satisfy clause 2) of the definition of stability. Of course the grand jurisdiction structure is stable (because no household with preferences as in this example would choose to move to a desert jurisdiction with zero public good provision. Hence, there are three stable jurisdiction structures in this example without a central government. Thanks to proposition 1, we know that only the grand jurisdiction structure can be stable with a maxmin central government.

While this example and proposition 1 both suggest that the introduction of a GU central government in a Westhoff model tends to reduce the number of stable jurisdiction structures, it is not difficult to think of economies where the opposite is true. Indeed, in the following example, we have an economy where the introduction of a utilitarian central government increases the number of stable jurisdictions.

Consider the Cobb-Douglas economy $(\omega, U, \mathbb{L})$ defined by:

$$\begin{aligned} \omega_i &= 1 \text{ if } i \in [0, 1/4[\\ &= 3 \text{ if } i \in [1/4, 1] \end{aligned}$$

$$U(Z, x) = \ln Z + \ln x, \tag{2.11}$$

and $\mathbb{L} = \{1, 2\}$. Hence we consider here a very simple economy with only two possible place of residence and two types of households: poor ones (in proportion $1/4$) and rich ones (in proportion $3/4$, admittedly not a very realistic distribution). Solving program (2.6) for U defined by (2.11), one obtains:

$$\frac{1}{\varpi}[Z^M(\frac{1}{\varpi}, \frac{1}{\omega_i}, 1 - c + \frac{\gamma}{\varpi}) - \gamma] = \frac{1-c}{2} - \frac{\gamma}{2\varpi} \tag{2.12}$$

Suppose first there is no central government so that $c = \gamma = 0$. Then, any household has a favorite tax rate of $1/2$ no matter where it lives. In this (very simple) world without government, the grand jurisdiction is stable. But the (segregated) jurisdiction structure where poor households live in one jurisdiction and rich ones live in another is not stable.

Indeed, since the two jurisdiction structure must offer the same tax rate of $1/2$ for being stable as per clause (2) of the definition of stability, any poor household will have incentive to leave its jurisdiction and to move to the rich one where, for a same tax payment, it will get a larger public good provision. However, the segregated structure may become stable if one introduces a GU central government. Indeed, consider specifically a utilitarian government and the jurisdiction structure (L, t, g, c) defined by:

$$\begin{aligned} L(i) &= 1 \text{ if } i \in [0, 1/4[\text{ and} \\ &= 2 \text{ if } i \in [1/4, 1] \end{aligned}$$

$$\begin{aligned} t_1 &= 5/4 \\ t_2 &= 25/12 \end{aligned}$$

$$\begin{aligned} \gamma_1 &= 0 \\ \gamma_2 &= -15/4 \\ c &= -3/2 \end{aligned}$$

This jurisdiction structure, in which the central government subsides everybody's income at a rate of 50% and finances this subsidy by collecting a grant of $15/4$ from the rich jurisdiction, satisfies clearly conditions (2.3)-(2.5). Notice also that that each t_i satisfies condition (2.12) for its g_i (for $i = 1, 2$) and that:

$$\begin{aligned} \Phi(t_1, \varpi_1, \omega_i, g_1, c) &= \Phi(\frac{5}{4}, \frac{1}{4}, 1, 0, -3/2) \\ &= \ln \frac{5}{16} + \ln \frac{5}{4} = 2 \ln \frac{5}{4} - \ln 4 \\ &= \Phi(\frac{25}{12}, \frac{3}{4}, 1, \frac{-15}{4}, -3/2) \\ &= \Phi(t_2, \varpi_2, \omega_i, g_2, c) \end{aligned}$$

and,

$$\begin{aligned}\Phi(t_2, \varpi_2, \omega_i, g_2, c) &= \Phi(\frac{25}{12}, \frac{3}{4}, 3, \frac{-15}{4}, -\frac{3}{2}) \\ &= \ln\frac{15}{16} + \ln\frac{5}{4} = \ln\frac{3}{4} + 2\ln\frac{5}{4} \\ &= \Phi(\frac{5}{4}, \frac{1}{4}, 3, 0, -3/2) \\ &= \Phi(t_1, \varpi_1, \omega_i, g_1, c).\end{aligned}$$

Hence every household prefers (weakly) to live in its jurisdiction then to move. We finally invite the reader to verify that g_1, g_2 and c satisfy the 1st order conditions for the following program solved by the central government (after substituting the budget constraint $c_{[0,1]}\omega_i = \frac{5c}{2} = g_1 + g_2$ into the objective):.

$$\max_{g_1,g_2} \frac{1}{4}[\ln(\frac{5}{16} + g_1) + \ln(1 - \frac{2(g_1+g_2)}{5} - \frac{5}{4})] + \frac{3}{4}[\ln(\frac{75}{48} + g_2) + \ln((1 - \frac{2(g_1+g_2)}{5} - \frac{25}{12})3)]$$

Given the strict concavity of the objective function, this is sufficient to conclude that this jurisdiction structure with a central government is stable.

These two examples make clear that the introduction of a GU central government may affect substantially the set of stable jurisdiction structures. This set may be shrunk, as in the first example, or enlarged, as in the second.

We now show that, if households preferences are additively separable, the effect of that the central government can have on the endogenous formation of jurisdiction process, as important as it may be, does not affect the segregative properties of the process.

2.3 Results

As in [45], the monotonicity of τ^* with respect to household's wealth (given jurisdiction's wealth and central government grant) is a key element for guaranteeing the wealth segregation of stable jurisdiction structures. By lemma 2, this monotonicity of the household's favorite tax rate with respect to private wealth is equivalent to requiring the public good to be, at any price of public good, either always a gross *complement* to (if Z^M is monotonically *decreasing* with respect to p_x) or always a gross *substitute* for (if Z^M is monotonically *increasing* with respect to p_x) the private good. We state formally this state of affairs,

proved in [45], using the regularity condition 1 as follows.

Lemma 2.3.1. *For every $U \in \mathcal{U}$, the function τ^* that solves (2.6) is monotonic with respect to ω_i for any given jurisdiction wealth level ϖ and per capita wealth if and only if the public good is always either a gross complement to, or a gross substitute for, the private good.*

We refer to this property according to which the substitutability/ complementarity relationship between the public and private good is independent from all possible prices as to the **Gross Substitutability/ Complementarity (GSC) condition**. Although not unreasonable, the GSC condition is nonetheless a significant restriction that, as discussed in [45], can be violated even by additively separable preferences.

An information used in [45] to show that the GSC condition is necessary and sufficient for guaranteeing the segregation of any stable jurisdiction structure is the structure of households' indifference curves in the tax-jurisdiction's wealth space. While this information is also useful in the present context, we need to account for the fact that the relevant space of location characteristics is now *three*, rather than *two*, dimensional and must include central government grant as well as local tax rate and jurisdiction's aggregate wealth. Specifically, the indifference surface of a household with (net of central government) wealth $(1-c)\omega_i$ passing through some point $(\overline{\tau}, \overline{\omega}, \overline{\gamma}) \in \mathbb{R}^3$ such that $\Phi(\overline{\tau}, \overline{\omega}, \omega_i, \overline{\gamma}, c) = \overline{\Phi}$ is the graph of the implicit function $f^{\overline{\Phi}} : [\frac{-\overline{\gamma}}{\overline{\omega}}, 1] \times \mathbb{R} \times \mathbb{R}_+ \longrightarrow \mathbb{R}_+$ defined by $\Phi(\tau, f^{\overline{\Phi}}(\tau, \gamma, c; \omega_i), \omega_i, \gamma, c) \equiv \overline{\Phi}$. The assumption imposed on U guarantees that the function $f^{\overline{\Phi}}$ exists and is derivable everywhere. Its partial derivative $f^{\overline{\Phi}}_{\tau}(\overline{\tau}, \overline{\gamma}, c; \omega_i)$ with respect to τ evaluated at $(\overline{\tau}, \overline{\gamma}, c, \omega_i)$ is given by:

$$f^{\overline{\Phi}}_{\tau}(\overline{\tau}, \overline{\gamma}, c; \omega_i) = \frac{1}{\overline{\tau}}[\frac{\omega_i}{MRS(\overline{\tau}\varpi + \overline{\gamma}, (1-c-\overline{\tau})\omega_i)} - \varpi] \tag{2.13}$$

where $\varpi = f^{\Phi}(\overline{\tau}, \overline{\gamma}, c; \omega_i)$. Figure 1 below illustrates the shape of these indifference curves in the (τ, ϖ) plane for given values of $\overline{\gamma}$ and c. Specifically, indifference curves of a household with private wealth $(1-c)\omega_i$ are U-shaped and reach a minimum at this household's most preferred tax rate for the corresponding jurisdiction wealth level. It can be seen indeed that, at the minimum of an indifference curve, the term within the bracket of (2.13) is zero thanks to the first order conditions of (2.6)). As in [45], and despite what figure 1 suggests, indifference curves need *not* be globally convex. The only property that indifference curves possess is that of being "single-through" (monotonically decreasing at

the left of the minimum and monotonically increasing at the right).

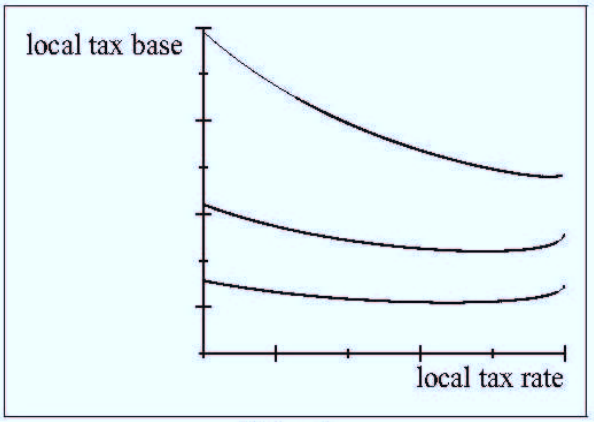


Figure 1

Analogously, one can fix local tax rate at $\overline{\tau}$ and examine the property of the derivative of $f^{\overline{\Phi}}$ with respect to the central government's grant. This partial derivative $f^{\overline{\Phi}}_{\gamma}(\overline{\tau}, \overline{\gamma}, c; \omega_i,)$ with respect to γ evaluated at $(\overline{\tau}, \overline{\gamma}, c, \omega_i)$ is given by:

$$f^{\overline{\Phi}}_{\gamma}(\overline{\tau}, \overline{\gamma}, c; \omega_i) = \frac{-1}{\overline{\tau}}$$

Hence, when looked in the $(\gamma, \overline{\omega})$ space, indifference surfaces are straight line with negative slope (if at least $\overline{\tau}$ is positive). There is therefore a constant marginal trade off between tax base and central government grant as envisaged by a mobile household. This is of course not surprising since both central government grant and local tax base are perfectly substitutable ways of getting public expenditure in a given jurisdiction. The rate at which the household is willing to sacrifice local tax base in order to get more central government transfer depends obviously upon the local tax rate that converts tax base into public spending. Figure 2 shows a typical indifference surface in the tax rate, tax base and central government space in which location choice by households is made.

We first establish, in the following lemma, that the ordering of the *slopes* of these indifference curves at every point in the tax- jurisdictions wealth space (for a given level of central government grant) coincides with the ordering of the households' wealth when preferences for the public and the private good satisfy the GSC condition. The proof of this lemma, which mimics very closely that of lemma 5 in [45], is omitted.

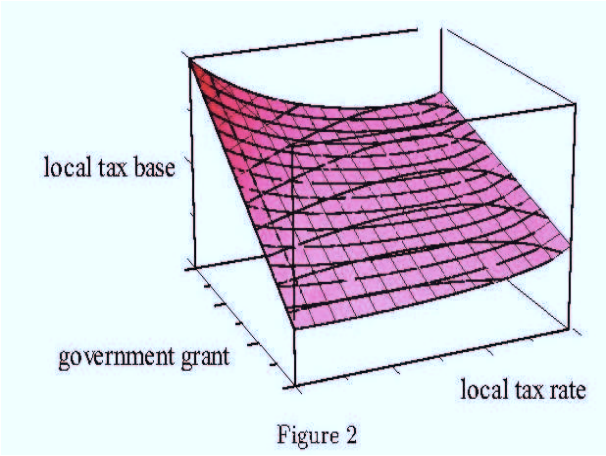


Figure 2

Lemma 2.3.2. *Assume that households preferences are represented by a utility function in* $\mathcal{U}$*. Then,* Z^M *is everywhere a gross substitute (resp. complement) to the private good if and only if one has, at any* $(\overline{\tau}, \varpi, \overline{\gamma}, c) \in \mathbb{R}^4$ *satisfying* $\overline{\gamma} + \overline{\tau}\varpi \geq 0$ *and* $1 - \overline{\tau} - c \geq 0$*,* $f_\tau^{\overline{\Phi}_j}(\overline{\tau}, \overline{\gamma}; \omega_h) \leq$ *(resp.* $\geq$*)* $f_\tau^{\overline{\Phi}_k}(\overline{\tau}, \overline{\gamma}, \omega_k)$ *for every* h*,* k *such that* $\omega_h < \omega_k$ *where, for every* i*,* $\overline{\Phi}_i = \Phi(\overline{\tau}, \varpi, \overline{\gamma}, c; \omega_i)$*.*

In plain English, this lemma says that the GSC condition is equivalent to the requirement that, for any central government grant, the slope of the indifference surfaces in the local tax rate and jurisdiction wealth space be *monotonic* with respect to private wealth at any point of that space. In proposition 3 below, we shall show that this monotonicity of the slopes in the two-dimensional space of tax rate and aggregate wealth for a given government grant holds true as well in the three dimensional space of government grants, local tax and jurisdiction's wealth.

We now establish that, if preferences are additively separable, and if the utility function aggregated by the generalized-utilitarian central government is the additively separable numerical representation of those preferences, then the GSC condition is *necessary* for the wealth segregation of any stable jurisdiction structure for the two definition of stability.

Proposition 2. *Assume that households preferences are represented by a utility function belonging to* $\mathcal{U}^{\mathcal{A}}$*. Then, a stable jurisdiction structure with a generalized-utilitarian central government that uses an additively separable numerical representation of these preferences as utility function is segregated only if the preferences satisfies the GSC condition.*

Proof.

Assume that the GSC condition is violated. Then there are private good prices p_x^0, p_x^1 *and* p_x^2 *satisfying* $0 < p_x^0 < p_x^1 < p_x^2$ *and public good price* $p_Z > 0$ *such that:*

$$\begin{aligned} Z^M(p_Z, p_x^0, 1) &= Z^M(p_Z, p_x^2, 1) \\ &> Z^M(p_Z, p_x^1, 1) \text{ or} \\ &< Z^M(p_Z, p_x^1, 1) \end{aligned}$$

Our objective is to show the existence of an economy where a non-segregated stable jurisdiction structure can be constructed. We provide the construction by assuming the first of these two inequalities, the argument being symmetric for the second. Consider an economy where a mass μ_0 *of households have wealth* $1/p_x^0$, *a mass* μ_1 *of households have wealth* $1/p_x^1$ *and a mass* μ_2 *of households have wealth* $1/p_x^2$ *and let* $\mathbb{L} = \{1, 2\}$. *In order to construct a stable jurisdiction structure that is not segregated, we are going to locate households with wealth* $1/p_x^0$ *and* $1/p_x^2$ *in location 1 and households with wealth* $1/p_x^1$ *in location 2. For this purpose we are going to choose positive numbers of households* μ_0, μ_1 *and* μ_2 *so that:*

$$\frac{\mu_0}{p_x^0} + \frac{\mu_2}{p_x^2} = \frac{\mu_1}{p_x^1} = \frac{1}{p_Z} \tag{2.14}$$

There is clearly no difficulty, given any p_x^0, p_x^1, p_x^2 *and* p_Z *satisfying the properties above, of finding positive numbers* μ_i *(for* $i = 0, 1, 2$*) satisfying (2.14). Without a central government, such a jurisdiction structure, that is clearly non-segregated, would be stable because the two jurisdictions created would have the same tax base* $\frac{1}{p_Z}$ *and each household would get its favorite local tax given this tax base in its jurisdiction of residence. With a central government however, this jurisdiction structure can be stable only if the central government finds optimal, given the two jurisdictions and local tax rates, to perform no equalization grants and to raise no taxes. Hence, we must prove that positive numbers* μ_0, μ_1 *and* μ_2 *can be found in such a way that a generalized-utilitarian central government that uses the additively separable numerical representation of households' preferences provided by (2.1) would find it optimal to give no equalization grants to jurisdictions and to collect no taxes*

as a result. We must prove that positive numbers μ_0, μ_1 and μ_2 such that:

$$\begin{aligned}(0,0) \quad \in \quad & \arg \max_{(\gamma,c)\in\mathbb{R}\times[0,1]} \mu_0\Psi(f(Z^M(p_Z,p_x^0,1)+\gamma)+h(x^M(p_Z,p_x^0,1)-\frac{c}{p_x^0})) \\ & +\mu_1\Psi(f(Z^M(p_Z,p_x^1,1)+\frac{2c}{p_Z}-\gamma)+h(x^M(p_Z,p_x^1,1)-\frac{c}{p_x^1})) \\ & +\mu_2\Psi(f(Z^M(p_Z,p_x^2,1)+\gamma)+h(x^M(p_Z,p_x^2,1)-\frac{c}{p_x^2})) \end{aligned} \tag{2.15}$$

can be found. Since the objective function of the program (2.15) is concave, any combination of values of jurisdiction 1's grant γ^* and wealth tax c^* that satisfy the first order conditions of this program is a solution to it. Using the (assumed) fact that $Z^M(p_Z,p_x^2,1) = Z^M(p_Z,p_x^0,1)$, the 1st order conditions of (2.15) for $\gamma^*=0$ and $c^*=0$ write:

$$(\mu_0\Psi^{0\prime}+\mu_2\Psi^{2\prime})\frac{\partial f(Z^M(p_Z,p_x^2,1))}{\partial Z} = \mu_1\Psi^{1\prime}\frac{\partial f(Z^M(p_Z,p_x^1,1))}{\partial Z} \tag{2.16}$$

and:

$$\begin{aligned}&\frac{\mu_0}{p_x^0}\Psi^{0\prime}\frac{\partial h(x^M(p_Z,p_x^0,1))}{\partial x}+\frac{\mu_1}{p_x^1}\Psi^{1\prime}\frac{\partial h(x^M(p_Z,p_x^1,1))}{\partial x}+\frac{\mu_2}{p_x^2}\Psi^{2\prime}\frac{\partial h(x^M(p_Z,p_x^2,1))}{\partial x} \\ &= \frac{2\mu_1}{p_Z}\Psi^{1\prime}\frac{\partial f(Z^M(p_Z,p_x^1,1))}{\partial Z}\end{aligned} \tag{2.17}$$

where, for $k=0,\,1,\,2$, $\Psi^{k\prime}>0$ denotes the derivative of Ψ evaluated at $f(Z^M(p_Z,p_x^k,1))+h(x^M(p_Z,p_x^k,1))$. We notice that, thanks to the decreasing monotonicity of the indirect utility function with respect to prices $f(Z^M(p_Z,p_x^2,1)+h(x^M(p_Z,p_x^2,1))<f(Z^M(p_Z,p_x^1,1)+h(x^M(p_Z,p_x^1,1))<f(Z^M(p_Z,p_x^0,1)+h(x^M(p_Z,p_x^0,1))$. Since Ψ is concave, one has therefore:

$$\Psi^{2\prime}>\Psi^{1\prime}>\Psi^{0\prime}>0 \tag{2.18}$$

By definition of the Marshallian demands, condition (2.17) writes (thanks to additive separability):

$$\begin{aligned}&\frac{\mu_0}{p_Z}\Psi^{0\prime}\frac{\partial f(Z^M(p_Z,p_x^0,1))}{\partial Z}+\frac{\mu_1}{p_Z}\Psi^{1\prime}\frac{\partial f(Z^M(p_Z,p_x^1,1))}{\partial Z}+\frac{\mu_2}{p_Z}\Psi^{2\prime}\frac{\partial f(Z^M(p_Z,p_x^2,1))}{\partial Z} \\ &= \frac{2\mu_1}{p_Z}\Psi^{1\prime}\frac{\partial f(Z^M(p_Z,p_x^1,1))}{\partial Z}\end{aligned}$$

or (since $Z^M(p_Z, p_x^2, 1) = Z^M(p_Z, p_x^0, 1)$*):*

$$(\mu_0\Psi^{0\prime} + \mu_2\Psi^{2\prime})\frac{\partial f(Z^M(p_Z, p_x^2, 1))}{\partial Z} = \mu_1\Psi^{1\prime}\frac{\partial f(Z^M(p_Z, p_x^1, 1))}{\partial Z}$$

which is nothing else than condition (2.16). Hence, the only other condition that the numbers μ_0*,* μ_1 *and* μ_2 *must satisfy beside (2.14) is condition (2.16). Since* p_x^0*,* p_x^1*,* p_x^2 *and* p_Z *are given positive numbers, we have that* $\mu_1 = p_x^1/p_Z > 0$*. Hence, in order for the proposed jurisdiction structure to be stable with a generalized utilitarian government who chooses (optimally) not to intervene, we only need to find positive numbers* μ_0 *and* μ_2 *such that equalities:*

$$\mu_2 = \frac{p_x^2}{p_Z} - \frac{p_x^2}{p_x^0}\mu_0 \tag{2.19}$$

and:

$$\mu_2 = \frac{p_x^1\Psi^{1\prime}\partial f(Z^M(p_Z, p_x^1, 1))/\partial Z}{p_Z\Psi^{2\prime}\partial f(Z^M(p_Z, p_x^2, 1))/\partial Z} - \frac{\Psi^{0\prime}}{\Psi^{2\prime}}\mu_0$$

hold. From the first order condition that defines Marshallian demands, we can write the later equality (using additive separability) as: :

$$\mu_2 = \frac{p_x^2\Psi^{1\prime}\partial h(x^M(p_Z, p_x^1, 1))/\partial x}{p_Z\Psi^{2\prime}\partial h(x^M(p_Z, p_x^2, 1))/\partial x} - \frac{\Psi^{0\prime}}{\Psi^{2\prime}}\mu_0 \tag{2.20}$$

Since the preferences represented by the utility function are additively separable, no good can be inferior and, as a result, the private good is not a Giffen good. Hence x^M *is decreasing with respect to its own price so that, since* $p_x^1 < p_x^2$*,* $x^M(p_Z, p_x^1, 1) > x^M(p_Z, p_x^2, 1)$ *and, since* h *is concave,* $\partial h(x^M(p_Z, p_x^1, 1))/\partial x < \partial h(x^M(p_Z, p_x^2, 1))/\partial x$*. Since by (2.18),* $\Psi^{1\prime} < \Psi^{2\prime}$*, we conclude that the intercept of the linear equation (2.20) is smaller than that of equation (2.19). Moreover, the abscissa at the origin of the linear equation (2.19) is* p_x^0/p_Z *while the abscissa at the origin of equation (2.20), denoted* $\mu^0(2)$ *is:*

$$\begin{aligned}
\mu^0(2) &= \frac{\Psi^{1\prime}p_x^2\partial h(x^M(p_Z, p_x^1, 1))/\partial x}{\Psi^{0\prime}p_Z\partial h(x^M(p_Z, p_x^2, 1))/\partial x} \\
&= \frac{\Psi^{1\prime}p_x^1\partial f(Z^M(p_Z, p_x^1, 1))/\partial Z}{\Psi^{0\prime}p_Z\partial f(Z^M(p_Z, p_x^2, 1))/\partial Z} \text{ (by separability and definition of Marshallian demands)} \\
&= \frac{\Psi^{1\prime}p_x^1\partial f(Z^M(p_Z, p_x^1, 1))/\partial Z}{\Psi^{0\prime}p_Z\partial f(Z^M(p_Z, p_x^0, 1))/\partial Z} \text{ (since } Z^M(p_Z, p_x^0, 1) = Z^M(p_Z, p_x^2, 1)\text{)} \\
&= \frac{\Psi^{1\prime}p_x^0\partial h(x^M(p_Z, p_x^1, 1))/\partial x}{\Psi^{0\prime}p_Z\partial h(x^M(p_Z, p_x^0, 1))/\partial x} \text{ (by separability and definition of Marshallian demands)}
\end{aligned}$$

Now, again, since the private good is not inferior (and therefore not Giffen) we obtain,

using concavity of h and inequality (2.18), that $\mu^0(2) > p_x^0/p_Z > 0$. *Hence the abscissa at the origin of the linear equation (2.20) is larger than that of equation (2.19) so that the two straight lines represented by those two equations are just as in figure 3) and cross in the strictly positive orthant. This shows the existence of positive numbers* μ_0 *and* μ_2 *satisfying equations (2.19) and (2.20) and this completes the proof.*

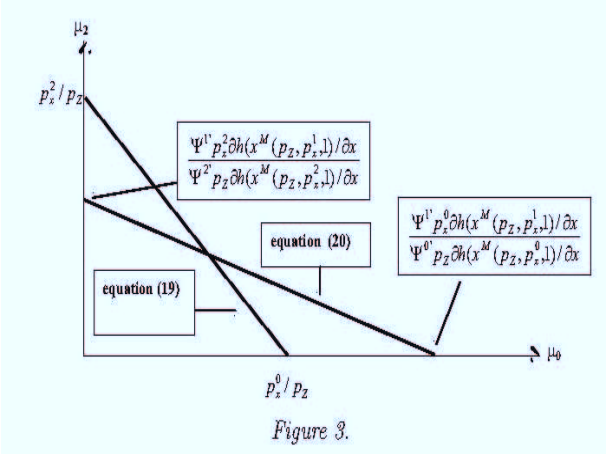


Figure 3.

The difference between the proof of this proposition and the corresponding one in [45] is worth mentioning. The reason for the difference is the additional constraint imposed by the optimality of the central government (non) intervention in the construction of the stable but non-segregated jurisdiction structure for any violation of the GSC condition. This constraint obviously increases the difficulty of the construction of the economy giving rise to such a stable non-segregated jurisdiction structure. The additive separability condition plays a key role in this construction and we do not know whether we could obtain the construction without such an assumption. We emphasize, however, that proposition 2 proves in fact something slightly stronger than what is required. Indeed, what is established in proposition 2 is that, for any violation of the GSC condition obtained with additively separable preferences, one can find a non-segregated stable jurisdiction structure in which a generalized utilitarian government finds it optimal to perform zero equalization (and accordingly to levy no wealth taxes). The possibility of proving, less demandingly, that any violation of the GSC condition can give rise to a non-segregated stable jurisdiction structure in which the utilitarian central government performs non-zero equalization without assuming additive separability of households' utility function remains an open, if not difficult, question.

We notice also that the assumption that there is a continuum of households plays a role

in this proof. Specifically, the proof establishes, for any violation of the GSC condition, the existence of positive *real* numbers of households that can be put in a non-segregated but yet stable jurisdiction structure. Yet, we are not capable of proving, for any violation of the GSC condition, the existence of *integer* numbers of households that can be put in a stable but non-segregated jurisdiction structure.

We now establish, without any further condition on household's preferences, the converse proposition that the GSC condition is sufficient for the wealth segregation of any stable jurisdiction structure. No additive separability is needed for this proposition.

Proposition 3. *Assume that households' preferences satisfy the GSC condition and are represented by a utility function in $\mathcal{U}$. Then, any stable jurisdiction structure with a GU central government as per either definition 2.2 is wealth segregated.*

Proof.
We sketch the argument for the case where the local public good is everywhere a gross complement to the private good. Assume therefore that Z^M is decreasing with respect to p_x and, by contradiction, let (L, g, c, t) be a jurisdiction structure that is not wealth-stratified. Hence, there are locations l and $l' \in \mathbb{L}$ (with $l \neq l'$), and households h, i and $k \in [0,1]$ with $h < i < k$ for which one has h and $k \in L_l^{-1}$, $i \in L_{l''}^{-1}$ with $\lambda(L_l^{-1}) > 0$ and, $\lambda(L_{l''}^{-1}) > 0$ and either $t_l \neq t_{l'}$ or $t_l\varpi_l + g_l \neq t_{l'}\varpi_{l'} + g_{l'}$. It is clear that if only one of the two inequalities $t_l \neq t_{l'}$ and $t_l\varpi_l + g_l \neq t_{l'}\varpi_l + g_{l'}$ holds, then the jurisdiction structure can not be stable because there would be unanimity of the inhabitants of one of the jurisdictions l and l' to go to the jurisdiction with the low tax rate (if $t_l \neq t_{l'}$ and $t_l\varpi_l + g_l = t_{l'}\omega_{l'} + g_{l'}$) or to the jurisdiction with the largest public good provision (if $t_l = t_{l'}$ and $t_l\varpi_l + g_l \neq t_{l'}\omega_l + g_{l'}$). Hence one can assume that both $t_l \neq t_{l'}$ and $t_l\varpi_l + g_l \neq t_{l'}\varpi_l + g_{l'}$ hold. Define now ϖ by:

$$\begin{aligned} t_l\varpi + g_{l'} &= t_l\varpi_l + g_l \\ &\Leftrightarrow \\ \varpi &= \varpi_l + \frac{g_l - g_{l'}}{t_l} \end{aligned}$$

Clearly one has:

$$\begin{aligned} U(t_l\varpi + g_{l'}, (1-c-t_l)\omega_m) &= \Phi(t_l, \varpi, \omega_m, g_{l'}) \\ &= U(t_l\varpi_1 + g_l, (1-c-t_l)\omega_m) \\ &= \Phi(t_l, \varpi_l, \omega_m, g_l, c) \end{aligned} \tag{2.21}$$

for every household m. For this non-stratified jurisdiction structure to be stable, one must have:

$$\begin{aligned} \Phi(t_l, \varpi_l, \omega_h, g_l, c) &\geq \Phi(t_{l'}, \varpi_{l'}, \omega_h, g_{l'}, c) \\ \Phi(t_l, \varpi_l, \omega_i, g_l, c) &\leq \Phi(t_{l'}, \varpi_{l'}, \omega_i, g_{l'}, c) \end{aligned}$$

and

$$\Phi(t_l, \varpi_l, \omega_k, g_l, c) \geq \Phi(t_{l'}, \overline{\omega}_{l'}, \omega_k, g_{l'}, c)$$

or, using (2.21):

$$\Phi(t_l, \varpi, \omega_h, g_{l'}, c) \geq \Phi(t_{l'}, \varpi_{l'}, \omega_h, g_{l'}, c) \tag{2.22}$$

$$\Phi(t_l, \varpi, \omega_i, g_{l'}, c) \leq \Phi(t_{l'}, \overline{\omega}_{l'}, \omega_i, g_{l'}, c) \tag{2.23}$$

and

$$\Phi(t_l, \varpi, \omega_k, g_{l'}, c) \geq \Phi(t_{l'}, \overline{\omega}_{l'}, \omega_k, g_{l'}, c) \tag{2.24}$$

By lemma 2.3.2, the slopes of indifference curves in the space of all combinations of local tax rate and jurisdiction aggregate wealth are ordered by individual wealth for any level of central government grant and, therefore, for the level $g_{l'}$. Hence indifference curves of households h, i and k at (t_l, ϖ) must be as they are depicted in figure 4. Clearly from this figure, unless indifference curves of households k and i or h and i cross in the "wrong" order at a point such as (τ, ϖ''), the set of combinations of local tax rates and jurisdiction aggregate wealth that i considers as weakly worse (given central government grant $g_{l'}$) than (t_l, ϖ) is contained in the set of such combinations that either household h or household k considers as strictly worse than (t_l, ϖ). Hence, unless indifference curves of two households cross in the wrong order at (τ, ϖ''), inequalities (2.22)-(2.24) can not simultaneously hold for distinct combinations (t_l, ϖ) and $(t_{l'}, \varpi_{l'})$ of local tax rates and jurisdiction tax rates.

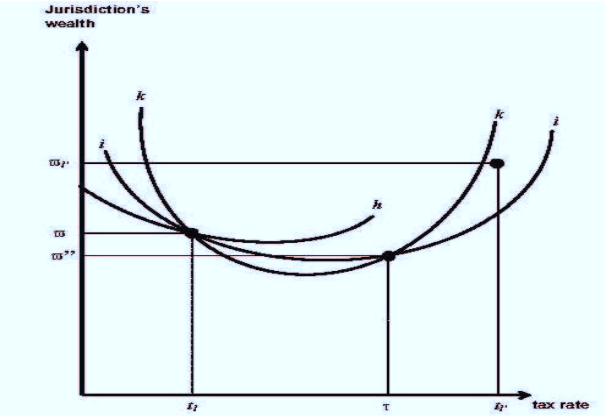


Figure 4.

This prove that unless the GSC condition is violated, the jurisdiction structure can not be stable.

2.4 Conclusion

The main conclusion of this paper is that the welfarist - in fact generalized-utilitarian - intervention of a central government does not alter the segregative properties of endogenous jurisdiction formation, at least when this jurisdiction formation is modelled within a framework *à la* Westhoff. Specifically, the GSC condition that it is necessary and sufficient to impose on household preferences for guaranteeing the wealth segregation of any stable jurisdiction is not affected by the presence of a central government as modelled in this paper. This of course does not mean that central government intervention does not affect jurisdiction formation. As examples 1 and 2 discussed in section 2 reveal, the redistributive behavior of the GU central government affects quite sharply the set of stable jurisdiction structures. Yet the jurisdictions structures that remain stable under a generalized utilitarian central government are segregated under exactly the same conditions on household's preferences than would be the case without a central government.

While we believe that the message according to which central government intervention does not modify the segregative forces underlying Tiebout-like processes of jurisdiction formation is of some interest, it is worth recalling the limitations of the analysis on which it stands. For one thing, the result is obtained, at least for its necessity part, under the assumption that preferences are additively separable. It would be nice if this assumption could be relaxed. Another limitation of the analysis lies, perhaps, in the simultaneous

setting in which the decisions by households, central and local government are considered. A third limitation of the analysis is the rather limited power given to the central government in our model to redistribute private wealth. Extending the analysis of this paper over these limitations, as well as many others, seems to us a worthy objective for future research.

Chapter 3

The segregative properties of endogenous jurisdictions formation with a land market[1]

This paper examines the segregative properties of Tiebout-like endogenous processes of jurisdiction formation in presence of a competitive land market. In the model considered, a continuum of households with different wealth levels and the same preferences for local public goods, private spending and housing choose a location from a finite set. Each location has an initial endowment of housing that is priced competively and that belongs to absentee landlords. Each jurisdiction is also endowed with a specific technology for producing public goods. Households' preferences are assumed to be homothetically separable between local public goods on the one hand and private spending and housing on the other. Public goods provision is financed by a given, but unspecified, mixture of (linear) wealth and housing taxes. We show that stable jurisdiction structures are always segregated by wealth only if households view any public good conditionally on the quantities of the other public goods as either always a gross substitute, or either always a gross complement, to private spending. We also show that, if there are more than one public good, this condition is not sufficient for segregation unless households preferences are additively separable. Since this condition is necessary and sufficient for the segregation of stable jurisdiction structures without land market and with only one public good, our results suggests that introducing a land market does not affect the segregative properties of endogenous jurisdiction formation but that increasing the number of public goods mitigates segregation.

[1] this chapter reviews a joint article with Nicolas Gravel

3.1 Introduction

It is widely believed that endogenous processes of jurisdiction formation, at least when driven by perfectly mobile households who make a [82] trade-off between local taxes and local public provision, are *self-sorting* and *segregative*. That is to say, such processes lead to the formation of homogenous jurisdictions inhabited by households with "similar" characteristics. [45] investigates the validity of this belief in the context of a classical model of endogenous jurisdiction formation due to [84]. In this model, households with different wealth and the same preference for local public spending and private spending choose to locate in a finite set of possible places of residence. Households who choose the same place form a jurisdiction that democratically decides of local taxes (paid by households on their private wealth) and local public spending. The analysis focuses on *stable* jurisdiction structures. These are partitions of the set of households that is immune to individual deviations, under the assumption that an individual move has no effect on jurisdictions' wealth, tax rates and public spending. [45] identify a condition on households' preference - the GSC condition - that is necessary and sufficient to ensure the wealth-segregation of any such stable jurisdiction structure. As in [84], the definition of "segregation" used by [45] is that underlying the notion of *consecutiveness* (see also [50]). It defines as "wealth-segregated" any jurisdiction structure in which the richest household of a jurisdiction is, weakly, poorer than the poorest household of any other jurisdiction with a strictly larger *per capita* wealth. The GSC condition requires households to consider local public spending to be either always a gross complement, or always a gross substitute, to the private good. In [8], the necessity and sufficiency of the GSC condition for segregation is also established for a generalized version of the model that allows for the presence of a generalized-utilitarian redistributive central government, under the additional assumption that households preferences are additively separable. While the GSC condition is stringent, and may be violated even by additively separable preferences, it is certainly not an outlandish condition. For this reason, [45] and [8] results may be seen as providing support to the widespread intuition that endogenous processes of jurisdiction formation *à la* Tiebout are inherently segregative.

Yet these results are obtained in a model with only one public good and without a dwelling market and where, as a result, households can reside for free in the jurisdiction that offers their favorite public good and tax package. This contrasts somewhat with *actual* processes of jurisdiction formation in which households must consume housing in order to have access to the public good and tax package available at a particular jurisdiction. Does the requirement for households to consume housing in order to benefit from local tax and

public good package affect the segregative properties of endogenous processes of jurisdiction formation? More specifically, what are the conditions - if any - that households preferences must satisfy in order for any stable jurisdiction structure to be segregative when housing consumption is required for living in a jurisdiction ? This is the main question examined in this paper. Realism is only one reason for addressing that question. Another motivation for identifying the conditions on households preferences that are necessary and sufficient for the segregation of stable jurisdiction structure in presence of housing markets is that those markets provide an easy way of testing the conditions through hedonic methods (see e.g. [76] or [5]).

Addressing this question requires of course a model of jurisdiction formation driven by household's optimizing decisions with respect to local public good and taxes under perfect mobility that incorporates housing consumption. At least three approaches to this modeling have been proposed in the literature.

The first, explored notably by [75], [34] , [35], assumes that housing, owned by absentee landlords, is a perfectly divisible good available in (possibly) different quantities in a finite number of locations. Households have preference for housing, local public spending and non-housing private spending and must consume a positive amount of housing at most one place. Purchase of housing is made on competitive markets that equalize local supply of housing with local demand. Households who choose to consume housing at the same location form a jurisdiction and choose by majority voting a *property tax* rate which, when applied to the (before tax) market value of local land, finances local public spending. An important issue discussed in this literature is the difficulty of establishing *existence* of stable jurisdiction structures. [34] and [35] have provided conditions on households' preferences that are sufficient for the existence of stable jurisdiction structures. As shown by these authors, the conditions also guarantee that the stable jurisdiction structures will be "segregated" in the same (consecutive) sense than above. Models that satisfy these assumptions have been the object of intensive empirical research in the last ten years or so (see e.g. [37], [40], [39] and [36]).

Another approach, explored by [26], [49] and [64] among others, considers a similar setting than the previous stream but with the important difference that local public good provision is assumed to be financed by wealth taxation. To that extent, this setting is closer in spirit to that of Westhoff to which it is easy to compare. Another advantage

of this setting is that it eases considerably the problem of existence of stable jurisdiction structures (see for instance [64] for a quite general existence theorem). On the other hand, assuming the financing of local public spending by wealth taxation is clearly at odd with what is observed in most institutional settings that we are aware, as property tax is by far the most widely use financing device for local public spending.

The third approach has been proposed by [70], and builds on the (largely unpublished) work of [29], [30] and [31]. It differs from the two previous ones in that it assumes that housing is an indivisible good that is available in various types in the various jurisdictions. In such a setting, [70] proves the existence of stable jurisdiction structure under both wealth and dwelling taxation. He also provides conditions that are sufficient for the segregation of stable jurisdiction structures.

In this paper, we stick to the perfectly divisible land (or housing) framework but we consider a financing scheme of local public good provision that combines wealth and dwelling taxation. Moreover, we allow for the possibility that jurisdictions produce several public goods rather than a single one. Yet, we adopt a more general view than the one typically taken in the literature with respect to the mechanism used by local jurisdictions to select public good provisions and taxes. Indeed, except for the linearity of the taxation (on both housing and wealth) and the balancing of the local government budget, we do not make any assumption on the process by which taxes and local public good are chosen. By contrast, much of the literature assume that local taxes are chosen by some voting mechanism (e.g. tax rates are the favorite ones of the median individual). It is actually this voting assumption which, together with competitive pricing of lands, create problems of existence of stable jurisdiction structures. By abstracting from the particular mechanism used by jurisdictions to decide upon local public good provision and taxes, our analysis thus escapes from the difficulties raised by possible inexistence of stable jurisdiction structures.

While the abstraction from the particular intra-jurisdiction collective choice goes toward a generalization of the approach favoured in the literature, we conduct the analysis by making the (significantly) simplifying assumption that households preferences are *homotheticaly separable* in the sense of [11] (3.4.2) between local public goods on the one hand and private spending (on housing and private consumption) on the other. This assumption is admittedly restrictive. Yet, it is not grossly inconsistent with the available empirical evidence (see for instance [19]) that indicates that the budget share devoted to housing

is remarkably stable both across locations (who differ in local public good provision) and across households (who differ in their wealth).

In this framework, we show that the propensity of stable jurisdiction structures to lead to segregation is not affected by the introduction of the housing market. For we show that a suitable generalization of the GSC condition remains necessary and sufficient for any stable jurisdiction to be segregated if there is only one public good. While we interpret this result as indicative of a somewhat robust connection between the GSC condition and the segregative properties of endogenous jurisdiction formation, we emphasize that the assumption of separable homotheticity is crucial for the result. Moreover, our analysis also suggests that the GSC condition may not be sufficient for segregation if preferences are not additively separable and there are more than one public goods. This suggests, somewhat intuitively, that increasing the number of public goods by which jurisdictions can be distinguished may mitigate the segregative feature of endogenous jurisdiction formation.

The plan of the rest of this paper is as follows. In the next section, we introduce the main notation and concepts. Section 3 provide the results for housing taxation while section 4 show how the results extend to the case with a welfarist central government. Section 5 provides some conclusion.

3.2 The model

As in the literature, we consider economies with a continuum of households represented by the $[0,1]$ interval, and we denote by λ the Lebesgue measure defined over all (Lebesgue measurable) subsets of $[0,1]$. For any Lebesgue measurable subset S of $[0,1]$, we interpret $\lambda(S)$ as "the fraction of households" in the set S. An economy is made of the three following ingredients.

First, there is a *wealth distribution* modeled as a Lebesgue measurable, increasing and bounded from above function $\omega : [0,1] \to \mathbb{R}_{++}$ that associates to every household $i \in [0,1]$ its strictly positive private wealth ω_i. Limiting attention to increasing functions is a convention according to which households are ordered by their wealth ($i \leq i' \Longrightarrow \omega_i \leq \omega_{i'}$).

The *second* ingredient in the description of an economy is a specification of the house-

holds' preferences, taken to be the same for all households. These preferences are defined over k local public goods ($\mathbf{Z} = (Z_1, ..., Z_k)$), private spending ($x$) and housing ($h$) and are represented by a twice differentiable, strictly increasing and strictly quasi-concave[2] utility function $U : \mathbb{R}_+^{k+2} \to \mathbb{R}$. We sometimes focus attention on some particular public good j. On such occasions, we may find convenient to write a particular bundle $\mathbf{Z}$ of public goods as $\mathbf{Z} = (Z_j; \mathbf{Z}_{-j})$ where the bundle $\mathbf{Z}_{-j}$ of the $k-1$ other public goods is defined by $Z_{-jh} = Z_h$ if $h < j$ and $Z_{-jh} = Z_{h+1}$ if $h > 1$. All preferences that are considered in this paper are also assumed to satisfy the following *regularity condition* with respect to the public goods.

(Regularity) If $k \geq 2$, then, for any bundle $(x, h) \in \mathbb{R}_+^2$ of private goods and any two bundles $\mathbf{Z}$ and $\mathbf{Z}' \in \mathbb{R}_+^k$ of public goods, there exists a public good $j \in \{1, ..., k\}$ for which a quantity quantity $\overline{Z}_j \in \mathbb{R}_+$ can be found for which $U(\overline{Z}_j; \mathbf{Z}_{-j}, x, h) = U(\mathbf{Z}', x, h)$.

This weak regularity condition, that applies only if there are more than one public good, rules out the possibility for indifference surfaces in the public goods space (conditional on any bundle of private goods) to have "vertical or horizontal" asymptotes in the interior of $\mathbb{R}_+^k$. If indifference surfaces in the public goods space have vertical or horizontal asymptotes, then these asymptotes must be the axes of the plane. For instance, preference that would generate indifference curves in $\mathbb{R}_+^2$ as in figure 1 below, are ruled out by this condition.

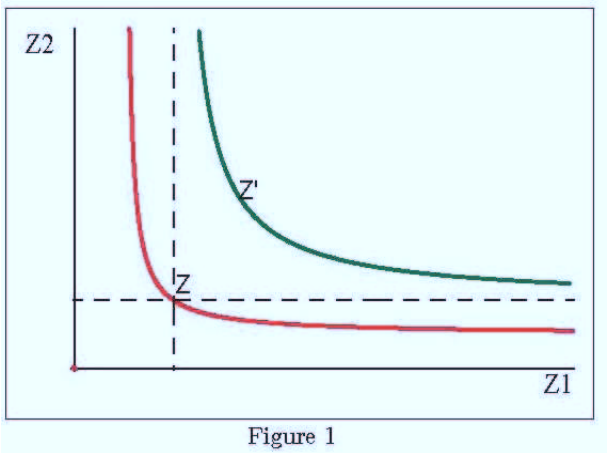


Figure 1

[2] A function $f : A \to \mathbb{R}$ (where A is some convex subset of $\mathbb{R}^l$) is strictly quasi-concave if, for every a, b, $x \in A$, with $a \neq b$ and every $\alpha \in]0, 1[$, $f(a) \geq f(x)$ and $f(b) \geq f(x)$ imply $f(\alpha a + (1-\alpha)b) > f(x)$.

As mentioned above, we also assume that the households preferences are *homothetically separable*, in the sense of [11] (3.4.2) between the k public goods on the one hand and the two private goods on the other. This assumption amounts to say that, for all $\mathbf{Z} \in \mathbb{R}^k_+$ and $(x,h) \in \mathbb{R}^2_+$, U can be written as:

$$U(\mathbf{Z}, x, h) = G(\mathbf{Z}, \Phi(x,h)) \tag{3.1}$$

for some twice continuously differentiable increasing and strictly quasi-concave function $G : \mathbb{R}^{k+1}_+ \to \mathbb{R}$ and some twice continuously differentiable, increasing and homogenous of degree 1 function $\Phi : \mathbb{R}^2_+ \to \mathbb{R}_+$. For proving the sufficiency of the GSC condition when the number of public goods is larger than 1, we shall assume that households preference are not only homothetically separable but are also *additively separable* so that the function G of expression (3.1) can be written, for any $Z \in \mathbb{R}^k_+$ and $\phi \in \mathbb{R}_+$, as $G(Z,\phi) = g(Z) + \Gamma(\phi)$ for some increasing and strictly quasi-concave function $g : \mathbb{R}^k_+ \to \mathbb{R}_+$. and some continuous and increasing function $\mathbb{R}_+ \to \mathbb{R}$.

We denote by $Z^M_j(\mathbf{p}^Z; p_x, p_h, R)$, $x^M(\mathbf{p}^Z; p_x, p_h, R)$ and $h^M(\mathbf{p}^Z, p_x, p_h, R)$ the household's Marshallian demands for public good j (for $j = 1, ..., k$), private consumption and housing (respectively) when the prices of local public goods are $\mathbf{p}^Z = (p^Z_1, ..., p^Z_k)$, the prices of private spending and housing are p_x and p_h and when its income is R. These Marshallian demand functions are the - unique under our assumptions - solution of the program:

$$\max_{Z,x,h} U(\mathbf{Z}, x, h) \text{ subject to } \mathbf{p}^Z.\mathbf{Z} + p_x x + p_h h \leq R \tag{3.2}$$

and are differentiable functions of their arguments (except, possibly, at the boundary of $\mathbb{R}^{k+2}_+$).

We emphasize that we view Marshallian demand functions as a dual way of representing households preference for the $k+2$ goods rather than as a behavioral description of households behavior. After all real households rarely purchase local public goods on competitive markets. For any given vector $\mathbf{Z} \in \mathbb{R}^k_+$ of public goods, we denote by $V^{\mathbf{Z}}$ the conditional (upon $\mathbf{Z}$) indirect utility function defined, for any $(p_x, p_h, y) \in \mathbb{R}^3_+$, by:

$$V^{\mathbf{Z}}(p_x, p_h, y) = \max_{x,h} U(\mathbf{Z}, x, h) \text{ subject to } p_x x + p_h h \leq y \tag{3.3}$$

This conditional indirect utility function plays an important role in the analysis. It de-

scribes indeed the maximal utility achieved by a household endowed with a (net of tax) wealth y/p_x (using private good as *numéraire*) when living in a jurisdiction offering quantities Z of the local public goods and (net of tax) dwelling price p_h/p_x. We denote by $h^{\mathbb{Z}M}$ and $x^{\mathbb{Z}M}$ the conditional (upon $\mathbf{Z}$) Marshallian demands of the two private goods that solve program 3.3. Under our assumptions, these conditional Marshallian demands are continuous functions of their three arguments that satisfy all the usual properties of Marshallian demands.

We also note that, thanks to homothetic separability, it is possible to describe program (3.2) by means of a *two-step budgeting* procedure (see e.g. [11], ch. 5).

The *first* step of the procedure is described by the program:

$$\max_{(Z;\phi)\in\mathbb{R}_+^{k+1}} G(\mathbf{Z},\phi) \text{ s. t. } \mathbf{p}^Z.\mathbf{Z} + \overline{E}(p_x,p_h)\phi \leq R. \tag{3.4}$$

where the function $\overline{E}:\mathbb{R}_+^2$ is defined by the dual program:

$$\overline{E}(p_x,p_h)\phi = E(p_x,p_h,\phi) = \min_{x,h} p_x x + p_h h \text{ s. t. } \Phi(x,h) \geq \phi \tag{3.5}$$

As is well-known indeed the expenditure function associated to a homogeneous utility function is linear in utility. As is also well-known from standard microeconomic theory that $\overline{E}$ is continuous, homogeneous of degree 1, increasing and concave. Hence, in this first step, the household is depicted as allocating its wealth R between the local public goods $\mathbf{Z}$ and the "utility of private spending" (measured by $\phi = \Phi(x,h)$), taking as given the public price vector p^Z and the "aggregate" price $p^X = \overline{E}(p_x,p_h) > 0$ of (the utility of) private spending. We notice that the function $\overline{E}$ is also involved in the definition of the conditional indirect utility function $V^{\mathbf{Z}}$ defined in program (3.3). Indeed, it is immediate to verify that $V^{\mathbf{Z}}$ writes:

$$V^{\mathbf{Z}}(p_x,p_h,R) = G(\mathbf{Z},\frac{R}{\overline{E}(p_x,p_h)}) \tag{3.6}$$

Denote now by $\mathbf{Z}^*(\mathbf{p}^Z,\overline{E}(p_x,p_h),R)$ and $\phi^*(\mathbf{p}^Z,\overline{E}(p_x,p_h),R)$ the (unique under our assumptions) solution to program (3.4). Denote also by $e(\mathbf{p}^Z,p_x,p_{h,},R)$ the optimal expenditure on the "utility of private spending" that results from the solution of program (3.4)

and that is defined by:

$$e(\mathbf{p}^Z, p_x, p_{h,}, R) = \overline{E}(p_x, p_h)\phi^*(\mathbf{p}^Z, \overline{E}(p_x, p_h), R) \tag{3.7}$$

The *second* step of the procedure consists in solving the program :

$$\max_{x,h} \Phi(x,h) \text{ s. t. } p_x x + p_h h \leq e(\mathbf{p}^Z, p_x, p_{h,}, R)) \tag{3.8}$$

Denote by $x^*(p_x, p_h, e(\mathbf{p}^Z, p_x, p_{h,}, R))$ and $h^*(p_x, p_h, e(\mathbf{p}^Z, p_x, p_{h,}, R))$ the solution of program (3.8). Thanks to the homotheticity of the (separable from the public goods) preferences for private goods represented by the function Φ, we know from standard microeconomic theory that the functions x^* and h^* so defined can be written as:

$$x^*(p_x, p_h, y) = \digamma^x(p_x, p_h)y$$

and

$$h^*(p_x, p_h, y) = \digamma^h(p_x, p_h)y$$

where, thanks to Roy's identity, $\digamma^x$ and $\digamma^h$ can be written as

$$\digamma^h(p_x, p_h) = \frac{\partial \overline{E}(p_x, p_h)/\partial p_h}{\overline{E}(p_x, p_h)} \tag{3.9}$$

and :

$$\digamma^x(p_x, p_h) = \frac{\partial \overline{E}(p_x, p_h)/\partial p_x}{\overline{E}(p_x, p_h)} \tag{3.10}$$

Moreover, one has

$$h^{\mathbf{Z}M}(p_x, p_h, y) = h^*(p_x, p_h, y)$$

and

$$x^{\mathbf{Z}M}(p_x, p_h, y) = h^*(p_x, p_h, y)$$

for any bundle $\mathbf{Z}$ of local public goods (Marshallian demands of housing and private spending are independent from public good provision). As a result of theorem 5.8 in [11], it follows that:

$$Z^M(\mathbf{p}^Z, p_x, p_h, R) = Z^*(\mathbf{p}^Z; \overline{E}(p_x, p_h), R) \tag{3.11}$$

$$x^M(\mathbf{p}^Z, p_x, p_h, R) = \digamma^x(p_x, p_h)e(\mathbf{p}^Z, p_x, p_{h,}, R) \tag{3.12}$$

$$h^M(\mathbf{p}^Z, p_x, p_h, R) = \digamma^h(p_x, p_h)e(\mathbf{p}^Z, p_x, p_{h,}, R) \tag{3.13}$$

These results will be used later on. We finally notice that this description of the household's preference is valid for any number of public goods whatsoever. As it turns out, a large part of the analysis deals with *conditional Marshallian demand* functions which are denoted, for any public good $j = 1, ..., k$, and any given specification $\overline{\mathbf{Z}}_{-j} = (\overline{Z}_1, ..., \overline{Z}_{j-1}, \overline{Z}_{j+1}, ..., \overline{Z}_k) \in \mathbb{R}_+^{k-1}$ of the quantities of the $k-1$ other public goods, by $Z_j^M(\overline{\mathbf{Z}}_{-j}; p_j^Z; p_x, p_h, R)$, $x^M(p_j^Z; p_x, p_h, R)$ and $h^M(\overline{\mathbf{Z}}_{-j}; p_j^Z; p_x, p_h, R)$. These conditional demands are defined to be the (unique) solution of program (3.2) for the conditional utility function $U^{\overline{\mathbf{Z}}_{-j}} : \mathbb{R}_+^3 \to \mathbb{R}$ defined, for any $(Z_j, h, x) \in \mathbb{R}_+^3$, by:

$$U^{\overline{\mathbf{Z}}_{-j}}(Z_j, x, h) = U(\overline{Z}_1, .., \overline{Z}_{j-1}, Z_j, .\overline{Z}_{j-1}, ..., \overline{Z}_k, x, h) \tag{3.14}$$

It can be checked that the aforementioned properties of U, including homothetic separability, are also possessed by $U^{\overline{\mathbf{Z}}_{-j}}$ for any $\overline{\mathbf{Z}}_{-j} \in \mathbb{R}_+^{k-1}$. We accordingly denote by $G^{\overline{\mathbf{Z}}_{-\mathbf{j}}}$ the (conditional) specification of the function G of expression (3.1) above when U is replaced by $U^{\overline{\mathbf{Z}}_{-j}}$.

The *last two* elements of our description of an economy are a *common finite* set $\mathbb{L}$ of $\bar{l}$ locations available to households together with a specification, for each location $l \in \mathbb{L}$, of the amount of land $L^l \in \mathbb{R}_{++}$ exogenously assigned to l, and, for each public good $j = 1, ..., k$, of a cost function $C_j^l : \mathbb{R}_+ \to \mathbb{R}_+$ of producing public good j at l . We assume that these cost functions satisfy $C_j^l(0) = 0$ and are increasing at every location l and for every public good j. Allowing the cost of producing a given public good to differ across locations seems natural to us (it is more costly to provide a given access to a sand beach in Paris than in Miami). We assume throughout that the endowments of land belong to absentee landlords that play no role in the economy. We denote by $\mathbb{D}$ the domain of all economies $(\omega, U, \mathbb{L}, \{L^l, C_1^l, ..., C_k^l\}_{l \in \mathbb{L}})$ that satisfy these assumptions and by $\mathbb{D}^A$ the subset of $\mathbb{D}$ that made of economies where households have additively separable preferences.

A **jurisdiction structure** for the economy $(\omega, U, \mathbb{L}, \{L^l, C_1^l, ..., C_k^l\}_{l \in \mathbb{L}})$ is a list $(j, \{p^l, t_h^l, t_w^l, \mathbf{Z}^l\}_{l \in \mathbb{L}})$ where :

- $j : [0, 1] \to_{l \in \mathbb{L}} \{l\} \times \mathbb{R}_{++}$ is a Lebesgue measurable function that assigns to each household i in $[0, 1]$ a unique combination $j(i) = (l_i, h_i^l)$ of a place of residence and a housing consumption at that place of residence and, for every $l \in \mathbb{L}$:

- $p^l \in \mathbb{R}_{++}$ is the (before tax) housing price at location l.

- $t_h^l \in \mathbb{R}_{++}$ is the housing tax rate prevailing at location l
- $t_w^l \in [0,1]$ is the wealth tax rate prevailing at location l
- $\mathbf{Z}^l \in \mathbb{R}_+^k$ is the bundle of local public goods available at location l.

For any such jurisdiction structure, and for any $l \in \mathbb{L}$, we denote by $j^l = \{i \in [0,1] : j(i) = (l,a) \text{ for some } a > 0\}$. Hence j^l is the (Lebesgue measurable) set of all households who have chosen to locate at l in the considered jurisdiction structure. The possibility that $\lambda(j^l) = 0$ (l is a "desert" jurisdiction) is of course not ruled out. We restrict attention throughout to **feasible** jurisdiction structures that satisfy the additional conditions that:

$$\int_{j^l} h_i^{l_i} d\lambda \leq L^l \text{ and} \tag{3.15}$$

$$t_h^l p^l \int_{j^l} h_i^{l_i} d\lambda + t_w^l \int_{j^l} \omega_i d\lambda \geq {}_{h=1}^{k} C_h^l(Z_h^l) \tag{3.16}$$

at every location l. That is to say, feasible jurisdiction structures are such that, at any location, aggregate housing consumption does not exceed the total amount of land that is available there (3.15), and that tax revenues raised are sufficient to cover the cost of providing the available bundle of public goods (3.16). Notice that the later inequality, together with the assumption that the cost functions are increasing and satisfy $C_j^l(0) = 0$, implies that the only public good package Z^l that can be observed in a "desert" jurisdiction is $Z^l = 0^l$.

Given an economy $(\omega, U, \mathbb{L}, \{L^l, C_1^l, ..., C_k^l\}_{l\in\mathbb{L}})$ and a jurisdiction structure $(j, \{p^l, t_h^l, t_w^l, \mathbf{Z}^l\}_{l\in\mathbb{L}})$ for this economy, we denote by $H^l = \int_{j^l} h_i^l d\lambda$ and $\varpi^l = \int_{j^l} \omega_i d\lambda$ the aggregate consumption of land and wealth (respectively) at location l.

We remark that our definition of jurisdiction structures is quite general and covers several models of endogenous jurisdiction formation with a housing market examined in the literature. It covers in particular models like [64] where local public good provision is assumed to be financed by (linear) wealth taxation as well as models such as [75], [34], [35], [38], [37], [40], [39], [36]) in which local public goods are financed by dwelling (or property) tax only. We believe that allowing for both types of taxation is consistent with what is observed in several real world institutional settings. For instance several states in the US

have "property tax relief" features that reduce the tax burden of specific categories of tax payers on the basis of their income. Similarly in France, many households are exempt from the so-called "taxe d'habitation" (dwelling tax) on the basis of their income.

We now turn to our definition of a *stable jurisdiction structure*, which we formally state as follows.

A feasible jurisdiction structure $(j, \{p^l, t_h^l, t_w^l, \mathbf{Z}^l\}_{l\in\mathbb{L}})$ for the economy $(\omega, U, \mathbb{L}, \{L^l, C_1^l, ..., C_k^l\}_{l\in\mathbb{L}})$ is stable if, for every $l \in \mathbb{L}$ such that $\lambda(j^l) > 0$, one has, for every $i \in j^l$, $H^l = L^l$ and $U(\mathbf{Z}^l, \omega_i(1 - t_w^l) - p^l(1 + t_h^w)h_i^l, h_i^l) \geq V^{\mathbf{Z}^{l'}}(1, p^{l'}(1 + t_h^{l'}), \omega_i(1 - t_w^{l'}))$ for every $l' \in \mathbb{L}$ not necessarily distinct from l.

In words, a feasible jurisdiction structure is stable if, in every jurisdiction, land consumption is equal to land supply and the bundle of land and private spending obtained by every household is considered, by this household, better than any bundle that it could afford (given its wealth, dwelling tax and land prices) either in its jurisdiction of residence or elsewhere.

We notice that our definition of stability does not assume any specific mechanism for choosing local public good provision and tax rate. By contrast, much of the literature dealing with endogenous formation of jurisdictions assumes that local jurisdiction choose their tax rate and/or public good provision by some voting mechanism. It is well-known that combining a voting mechanism with a competitive pricing of the land may raise serious existence problem, that are exacerbated if voting concerns the housing tax rate (see e.g. [75], [34], [35], [38] and [37]). These existence problems are alleviated here by avoiding the requirement for the tax rate to be result of a voting procedure. For instance the (grand) jurisdiction structure in which all households are put into one jurisdiction will be stable if a Inada's condition on one of the public good is assumed (as no household would want to unilaterally move to a desert jurisdiction with zero public goods in that case, even if land is free).

We now investigate under which condition any stable jurisdiction structure is wealth-segregated. This requires a definition of wealth-segregation that we provide as follows.

A feasible jurisdiction structure $(j, \{p^l, t_h^l, t_w^l, \mathbf{Z}^l\}_{l\in\mathbb{L}})$ for the economy $(\omega, U, \mathbb{L}, \{L_l, C_1^l, ..., C_k^l\}_{l\in\mathbb{L}})$ is wealth-segregated if, for every households h, i and $k \in [0, 1]$ such that $\omega_h < \omega_i < \omega_k$,

$h \in j^l$, $k \in j^l$ and $i \in j^{l'}$ for some $l' \neq l$ imply that $V^{\mathbf{Z}^l}(1, p^l(1+t_h^l), \omega_h(1-t_w^l)) = V^{\mathbf{Z}^{l'}}(1, p^{l'}(1+t_h^{l'}), \omega_h(1-t_w^{l'}))$ for every $h \in j^l \cup j^{l'}$.

In words, a jurisdiction structure is wealth-segregated if any jurisdiction j^l containing two households with strictly different levels of wealth also contains any household with wealth in between of those two or, if it does not contain this household, it is because it resides in another jurisdiction $j^{l'}$ that is "identical" to jurisdiction j^l in the sense that everybody living in either of these jurisdiction is indifferent between the two.

In [45], in a model without housing and with one public good, it was established that the Gross Substitutability/Complementarity (GSC) condition according to which the (non-symmetric) relation of gross substitutability or complementarity (as it may be) of the unique public good *vis-à-vis* private consumption is independent from all prices is necessary and sufficient for the wealth segregation of any stable jurisdiction structure. In the current context where land is present and where there are, possibly, many public goods, it turns out that the GSC condition applied to the conditional Marshallian demand of every local public is necessary and sufficient for securing the wealth-segregation - as per definition 3.2 - of any stable jurisdiction structure as per definition 3.2. This statement of this generalized GSC condition is the following.

(Generalized GSC) The household's preference satisfies the generalized GSC condition if, for every local public good j, the function $Z_j^M(\overline{\mathbf{Z}}_{-j}; p_j^Z, p_x, p_h, R)$ is monotonic with respect to p_x for any $(p_j^Z, p_h, R) \in \mathbb{R}_+^3$.

We notice that, thanks to (3.11), one can write

$$Z_j^M(\overline{\mathbf{Z}}_{-j}; p_j^Z, p_x, p_h, R) = Z^{\overline{\mathbf{Z}}_{-j}*}(p_j^Z; \overline{E}^{\overline{\mathbf{Z}}_{-j}*}(p_x, p_h), R) \tag{3.17}$$

where the functions $Z^{\overline{\mathbf{Z}}_{-j}*}$ and $\overline{E}^{\overline{\mathbf{Z}}_{-j}*}$ are nothing else than the functions Z^* and $\overline{E}$ defined for the utility function $U^{\overline{\mathbf{Z}}_{-j}}$. Since the function $\overline{E}^{\overline{\mathbf{Z}}_{-j}*}$ is increasing in its two arguments, requiring the function $Z_j^M(\overline{\mathbf{Z}}_{-j}; p_j^Z, p_x, p_h, R)$ to be monotonic with respect to p_x is equivalent to requiring the function $Z^{\overline{\mathbf{Z}}_{-j}*}$ to be monotonic with respect to the *aggregate* private goods price index $p_X^{\overline{\mathbf{Z}}_{-j}*} = \overline{E}^{\overline{\mathbf{Z}}_{-j}*}(p_x, p_h)$.

Using this fact, we now establish that this generalization of the GSC condition is necessary and sufficient for the wealth segregation of any stable jurisdiction structure. As it turns out, this condition will not be sufficient in the most general version of the model

presented here. It will be sufficient either if we make the additional assumption that there is only one local public good, or that the household's preferences are additively separable between local public goods on the one hand and the private goods on the other.

Yet, as established in the following proposition, the condition will be necessary for segregation.

Proposition 4. *The GSC condition is necessary for the wealth segregation of any stable jurisdiction structure for an economy* $(\omega, U, \mathbb{L}, L^l, C_1^l, ..., C_k^l\}_{l\in\mathbb{L}})$ *in* $\mathbb{D}$.

Proof.
Suppose that the GSC condition is violated. This means that there exists a public good $j \in \{1, ..., k\}$, some private spending prices p_x^0, p_x^1 and p_x^2 satisfying $0 < p_x^0 < p_x^1 < p_x^2$ for which one has:

$$Z_j^M(\overline{\mathbf{Z}}_{-j}; p_j^Z, p_x^0, p_h, R) = Z_j^M(\overline{\mathbf{Z}}_{-j}; p_j^Z, p_x^2, p_h, R) > Z_j^M(\overline{\mathbf{Z}}_{-j}; p_j^Z, p_x^1, p_h, R) \tag{3.18}$$

(the argument is similar if we assume instead $Z_j^{MC}(\overline{\mathbf{Z}}_{-j}; p_j^Z, p_x^0, p_h, R) = Z_j^M(\overline{\mathbf{Z}}_{-j}; p_j^Z, p_x^2, p_h, R) < Z_j^M(\overline{\mathbf{Z}}_{-j}; p_j^Z, p_x^1, p_h, R)$) for some public good j price $p_j^Z \in \mathbb{R}_{++}$, housing price $p_h \in \mathbb{R}_{++}$, some vector $\overline{\mathbf{Z}}_{-j} \in \mathbb{R}_+^{k-1}$of quantities of the other public goods and some income $R > 0$). Denote by $\mathbf{Z}^1$ and $\mathbf{Z}^2$ the vector of public goods defined by:

$$\mathbf{Z}^1 = (Z_j^M(\overline{\mathbf{Z}}_{-j}; p_j^Z, p_x^1, p_h, R); \overline{\mathbf{Z}}_{-j})$$

and:

$$\begin{aligned} \mathbf{Z}^2 &= (Z_j^M(\overline{\mathbf{Z}}_{-j}; p_j^Z, p_x^2, p_h, R); \overline{\mathbf{Z}}_{-j}) \\ &= (Z_j^M(\overline{\mathbf{Z}}_{-j}; p_j^Z, p_x^0, p_h, R); \overline{\mathbf{Z}}_{-j}). \end{aligned}$$

Using (3.17)) and homogeneity of degree 0 of Marshallian demands, expression (3.18) can be written as:

$$\begin{aligned} Z_j^{\overline{Z}_{-j}*}(q_j^Z; \widetilde{E}(p_x^0, p_h), 1) &= Z_j^{\overline{Z}_{-j}*}(q_j^Z; \widetilde{E}(p_x^2, p_h), 1) \\ &> Z_j^{\overline{Z}_{-j}*}(q_j^Z; \widetilde{E}(p_x^1, p_h), 1) \end{aligned} \tag{3.19}$$

where:

$$\begin{aligned} q_j^Z &= p_j^Z/R \text{ and} \\ \widetilde{E}(p_x^j, p_h) &= \overline{E}^{\overline{\mathbf{Z}}_{-j^*}}(p_x^j, p_h)/R \text{ for } j = 0, 1, 2 \end{aligned}$$

Of course, since $0 < p_x^0 < p_x^1 < p_x^2$ and the function $\overline{E}^{\overline{\mathbf{Z}}_{-j^*}}$ is increasing with respect to its arguments, one has $0 < \widetilde{E}(p_x^0, p_h) < \widetilde{E}(p_x^1, p_h) < \widetilde{E}(p_x^1, p_h)$. Let us show that we can find an economy in $\mathbb{D}$ for which a stable jurisdiction structure can be non-segregated. For this sake, consider the economy where $\mathbb{L} = \{1, 2\}$ and where the wealth distribution function ω is such that there are α and $\beta \in]0, 1[$ satisfying $\alpha < \beta$ for which one has:
$\omega_i = 1/\widetilde{E}(p_x^0, p_h)$ for all $i \in [0, \alpha[$,
$\omega_i = 1/\widetilde{E}(p_x^1, p_h)/R$ for all $i \in [\alpha, \beta[$ and,
$\omega_i = 1/\widetilde{E}(p_x^2, p_h)/R$ for all $i \in [\beta, 1]$,
and let there be a mass $\mu_t > 0$ of household of type t (for $t = 0, 1, 2$) with the masses chosen in such a way as to satisfy:

$$\mu_0\omega_0 + \mu_2\omega_2 = \frac{1}{q_j^Z} = \mu_1\omega_1 \tag{3.20}$$

It is clearly possible to find such positive real numbers μ_t (for $t = 0, 1, 2$). One simply set

$$\mu_1 = \frac{1}{\omega_1 q_j^Z} > 0$$

and observe that there are several strictly positive values of μ_0 and μ_2 that satisfy:

$$\mu_2 = \frac{1}{\omega_2}\left(\frac{1}{q_j^Z} - \omega_0\mu_0\right)$$

Consider any increasing and convex cost functions C_g^l (for $g = 1, ..., k$) and $l = 1, 2$) such that ${}_{g\neq j}C_g^1(Z_g) = {}_{g\neq j}C_g^1(Z_g) = c$ for some non-negative real number c that we leave, for the moment, unspecified. Let also $C_j^1 = C_j^2 = C$ for some **strictly increasing** and convex cost function C satisfying:

$$\begin{aligned} C(Z_j^{\overline{Z}_{-j^*}}(q_j^Z; \widetilde{E}(p_x^1, p_h), 1)) &< \\ C(Z_j^{\overline{Z}_{-j^*}}(q_j^Z; \widetilde{E}(p_x^0, p_h), 1)) &= C(Z_j^{\overline{Z}_{-j^*}}(q_j^Z; \widetilde{E}(p_x^2, p_h), 1)) \\ &< \mu_0\omega_0 + \mu_2\omega_2 \\ &= \mu_1\omega_1 \end{aligned} \tag{3.21}$$

Consider the jurisdiction structure $(j, \{p^l, t_h^l, t_w^l, \mathbf{Z}^l\}_{l=1,2})$ defined by:

$$j(i) = (2, F^h(p_x^0, p_h)(e^{\overline{Z}-j}(q_j^Z, p_x^0, p_{h,}, 1)) \text{ for } i \in [0, \alpha[, \tag{3.22}$$

$$j(i) = (1, F^h(p_x^1, p_h)e^{\overline{Z}-j}(q_j^Z, p_x^1, p_{h,}, 1)) \text{ for } i \in [\alpha, \beta[, \tag{3.23}$$

$$j(i) = (2, F^h(p_x^2, p_h)e^{\overline{Z}-j}(q_j^Z, p_x^2, p_{h,}, 1)) \text{ for } i \in [\beta, 1], \tag{3.24}$$

$$p^1(1 + t_h^1) = p^2(1 + t_h^2) = p_h \tag{3.25}$$

$$t_h^1 = t_h^2 = 0,$$

$$t_w^2 = q_j^Z C(Z_j^{\overline{Z}-j*}(q_j^Z; \widetilde{E}(p_x^2, p_h), 1)) \text{ and} \tag{3.26}$$

and

$$t_w^1 = q_j^Z C(Z_j^{\overline{Z}-j*}(q_j^Z; \widetilde{E}(p_x^1, p_h), 1)) \tag{3.27}$$

where the function $e^{\overline{Z}-j}$ is the analogue, for program 3.14, to the function e defined in expression (3.7) for program (3.4). Observe that, thanks to (3.20) and (3.21), one has $t_w^l \in [0, 1]$ for $l = 1, 2$. Observe also that equation (3.25) leaves complete freedom for choosing before-tax housing price and dwelling tax rates p^l and t_h^l satisfying $p^l(1+t_h^l) = p_h$ for $l = 1, 2$. Set now the quantities of land $L^1$and L^2 so that:.

$$\mu_1 F^h(p_x^1, p_h)e^{\overline{Z}-j}(q_j^Z, p_x^1, p_{h,}, 1) = L^1 \tag{3.28}$$

and:

$$\mu_0 F^h(p_x^2, p_h)e^{\overline{Z}-j}(q_j^Z, p_x^2, p_{h,}, 1) + \mu_2 F^h(p_x^2, p_h)e^{\overline{Z}-j}(q_j^Z, p_x^2, p_{h,}, 1) = L^2 \tag{3.29}$$

There are clearly no difficulties in finding such L^1 and L^2. Given L^l, set the yet undetermined positive real numbers t_h^l and p^l so that :

$$t_h^l p_h^l L^l = c$$

(for $l = 1, 2$). It is clear that this equality, which says that the cost of producing the public goods other than j in the two jurisdiction is financed by housing taxation, requires to set $t_h^l = \frac{c}{p_h^l L^l}$ for $l = 1, 2$. Substituting this back into equation (3.25) yields:

$$p^l = p_h - \frac{c}{L^l}$$

for $l = 1, 2$. It is clear that the before tax housing price p^l so defined will be positive if the cost functions C_g^l (for $g = 1, ..., k$) and $l = 1, 2$) are chosen in such a way that c is sufficiently small. Let us show that this non-segregated jurisdiction structure is stable. Our choice of L^1 and L^2 already guarantees (equations (3.28)-3.29) that land markets clear at the two locations. Since, as was just established, the cost of producing public goods other than j in the two jurisdiction is exactly covered by housing tax revenues, equations (3.26)-(3.27) imply that the total tax raised in each of the two jurisdictions covers exactly the cost of public good provision. We only need to show that no household has incentive to modify its consumption of public good and private goods either at its location or at the other. We provide the argument for a household of type 1 living at jurisdiction 1, leaving to the reader the task of verifying that the same argument holds for a type 0 and type 2 household living at jurisdiction 2 Consider therefore household 1 who has private $\omega_1 = 1/\widetilde{E}(p_x^t, p_h)$ and who lives at location 1 where it consumes the bundle $\mathbf{Z}^1 = (Z_j^{\overline{Z}_{-j}*}(q_j^Z; \widetilde{E}(p_x^1, p_h), 1)), \overline{Z}_{-j})$ of public goods and has $\omega_1(1 - t_w^1)$ to spend on housing and private spending. Yet:

$$\begin{aligned}
\omega_1(1 - t_w^1) &= \omega_1[1 - q_j^Z C(Z_j^{\overline{Z}_{-j}*}(q_j^Z; \widetilde{E}(p_x^1, p_h), 1))] \\
&= \frac{[1 - q_j^Z C(Z_j^{\overline{Z}_{-j}*}(q_j^Z; \widetilde{E}(p_x^1, p_h), 1))]}{\widetilde{E}(p_x^1, p_h)} \\
&= \phi^{\overline{Z}_{-j}*}(q_j^Z, \widetilde{E}(p_x^1, p_h), 1) \qquad (3.30)
\end{aligned}$$

thanks to the budget constraint associated to the program 3.4 applied to the conditional consumer's program (3.14). Now, since a type 1 household consumes $F^h(p_x^1, p_h)(e^{\overline{Z}-j}(q_j^Z, p_x^1, p_{h,}, 1)$ units of housing (equation (3.22) purchased at price p_h, such a household has

$$\omega_1(1 - t_w^1) - p_h F^h(p_x^1, p_h)[e^{\overline{Z}-j}(q_j^Z, p_x^1, p_{h,}, 1)]$$

available for private spending. Yet we know that

$$\omega_t(1 - t_w^1) - p_h F^h(p_x^t, p_h)(e^{\overline{Z}-j}(q_j^Z, p_x^1, p_{h,}, 1)$$

$$\begin{aligned}
&= \phi^{\overline{Z}_{-j}*}(q_j^Z, \widetilde{E}(p_x^1, p_h), 1) - p_h F^h(p_x^1, p_h)(e^{\overline{Z}-j}(q_j^Z, p_x^1, p_{h,}, 1) \\
&= p_x^t F^x(p_x^1, p_h)(e^{\overline{Z}-j}(q_j^Z, p_x^1, p_{h,}, 1)
\end{aligned}$$

Since $(F^h(p_x^1, p_h)(e^{\overline{Z}-j}(q_j^Z, p_x^1, p_{h,}, 1), F^x(p_x^1, p_h)(e^{\overline{Z}-j}(q_j^Z, p_x^1, p_{h,}, 1))$ solves program (3.8), a type 1 - household can not find a bundle of private spending x and housing h satisfying

$p_h h + x \leq \phi^{\overline{Z}_{-j}*}(q_j^Z, \overline{E}(p_x^1, p_h), 1)) = \omega_t(1 - t_w^1)$ that is strictly preferred to:

$$(F^h(p_x^1, p_h)(e^{\overline{Z}_{-j}}(q_j^Z, p_x^1, p_{h,}, 1), p_x^1 F^x(p_x^1, p_h)[e^{\overline{Z}_{-j}}(q_j^Z, p_x^1, p_{h,}, 1)]$$

Hence a type 1 household has no incentive to change its private consumption pattern within its jurisdiction. It has also no incentive to move to location 2. Indeed, if it were to move there, it would obtain the bundle:

$$\mathbf{Z}^2 = (Z_j^{\overline{Z}_{-j}*}(q_j^Z; \widetilde{E}(p_x^0, p_h), 1)), \overline{Z}_{-j}) = (Z_j^{\overline{Z}_{-j}*}(q_j^Z; \widetilde{E}(p_x^2, p_h), 1)), \overline{Z}_{-j})$$

for which it would pay $t_w^2 \omega_1$ amount of tax and would have $\omega_1(1 - t_w^2)$ units of numéraire to spend on private matters. Observe now that, thanks to (3.26) and (3.30):

$$\begin{aligned}
[t_w^1 + (1 - t_w^1)]\omega_1 &= C(Z_j^{\overline{Z}_{-j}*}(q_j^Z; \widetilde{E}(p_x^1, p_h), 1))/\mu_1 + \phi^{\overline{Z}_{-j}*}(q_j^Z, \widetilde{E}(p_x^1, p_h), 1) \\
&= [t_w^2 + (1 - t_w^2)]\omega_1 \\
&= C(Z_j^{\overline{Z}_{-j}*}(q_j^Z; \widetilde{E}(p_x^2, p_h), 1))/\mu_1 + (1 - t_w^2)]\omega_1
\end{aligned}$$

Define now the function $\widetilde{G}^{\overline{Z}_{-j}}$:$\mathbb{R}_+^2 \to \mathbb{R}_+$. by:

$$\widetilde{G}^{\overline{Z}_{-j}}(c, \phi) = G^{\overline{Z}_{-j}}(C^{-1}(c), \phi)$$

where C^{-1} is the inverse cost function. This function is well-defined if C is strictly increasing. Since $(C(Z_j^{\overline{Z}_{-j}*}(q_j^Z; \widetilde{E}(p_x^1, p_h), 1)), \phi^{\overline{Z}_{-j}*}(q_j^Z, \widetilde{E}(p_x^1, p_h), 1))$ solves the program:

$$\max_{(Z;\phi) \in \mathbb{R}_+^{k+1}} G^{\overline{Z}_{-j}}(Z_j, \phi) \; s.t. \; Z_j/\mu_1 + \phi \leq \omega_1.$$

it follows from a standard revealed preference argument that

$$\widetilde{G}^{\overline{Z}_{-j}}(C(Z_j^{\overline{Z}_{-j}*}(q_j^Z; \widetilde{E}(p_x^1, p_h), 1)), \phi^{\overline{Z}_{-j}*}(q_j^Z, \widetilde{E}(p_x^1, p_h), 1))$$

$$\geq \widetilde{G}^{\overline{Z}_{-j}}(Z_j^{\overline{Z}_{-j}*}(q_j^Z; \widetilde{E}(p_x^2, p_h), 1))/\mu_1 + (1 - t_w^2)]\omega_1.$$

Hence type 1 household prefer staying in 1 than moving to 2.

We now turn to the question of the *sufficiency* of the GSC condition for segregation. As in [45] or [84], doing this analysis requires some knowledge of the households preferences defined in the space of all parameters that affect its choice of place of residence, preferences that are described, as mentioned earlier, by the conditional indirect utility function

defined in (3.3). Under homothetic separability, we know from (3.6) that we can write this conditional indirect utility function $V^{\mathbf{Z}}$ as:

$$V^{\mathbf{Z}}(1, q_h, \omega_i(1-t_\omega)) = G(\mathbf{Z}, \frac{\omega_i(1-t_\omega)}{\widehat{E}(q_h)})$$

where $\widehat{E}(q_h) = \overline{E}(1, q_h)$. Suppose now that we focus on some public good j and that we fix the quantities $\overline{\mathbf{Z}}^{-j} \in \mathbb{R}_+^{k-1}$ of the other $k-1$ public goods. We can then represent a typical indifference curve of a household of wealth ω_i in the space $[0,1] \times \mathbb{R}_+$ of all combinations of wealth tax rate and public good j by means of the implicit function $z^I : [0,1] \to \mathbb{R}$ defined by

$$G^{\overline{\mathbf{Z}}_{-\mathbf{j}}}(z^I(t, \omega_i), \frac{\omega_i(1-t)}{\widehat{E}(q_h)}) \equiv a \tag{3.31}$$

for some a. Since $G^{\overline{\mathbf{Z}}_{-\mathbf{j}}}$ is a twice differentiable concave and strictly increasing function of its two arguments, it is clear that the implicit function z^I is well-defined and twice differentiable. Differentiating (3.31) with respect to t yields:

$$\partial z^I(\overline{t}, \overline{\omega}_i)/\partial t \equiv \frac{\omega_i}{\widehat{E}(q_h)\frac{G_{Z_j}^{\overline{\mathbf{Z}}_{-\mathbf{j}}}}{G_{\phi}^{\overline{\mathbf{Z}}_{-\mathbf{j}}}}} \tag{3.32}$$

If we now differentiate (3.32) with respect to ω_i, we obtain:

$$\frac{\partial^2 z^I(\overline{t}, \overline{\omega}_i)}{\partial t \partial \omega_i} \equiv \frac{\frac{G_{Z_j}^{\overline{\mathbf{Z}}_{-\mathbf{j}}}}{G_{\phi}^{\overline{\mathbf{Z}}_{-\mathbf{j}}}} - \frac{(1-\overline{t})\omega_i}{\widehat{E}(q_h)[G_{\phi}^{\overline{\mathbf{Z}}_{-\mathbf{j}}}]^2}(G_{\phi}^{\overline{\mathbf{Z}}_{-\mathbf{j}}} G_{Z_j\phi}^{\overline{\mathbf{Z}}_{-\mathbf{j}}} - G_{Z_j}^{\overline{\mathbf{Z}}_{-\mathbf{j}}} G_{\phi\phi}^{\overline{\mathbf{Z}}_{-\mathbf{j}}})}{\widehat{E}(q_h)[\frac{G_{Z_j}^{\overline{\mathbf{Z}}_{-\mathbf{j}}}}{G_{\phi}^{\overline{\mathbf{Z}}_{-\mathbf{j}}}}]^2} \tag{3.33}$$

Notice also that the Marhallian demand for public good j conditional upon the quantities $\overline{\mathbf{Z}}^{-j}$of the other $k-1$ public goods can be defined to be the solution of the following program:

$$\max_{Z_j} G^{\overline{\mathbf{Z}}_{-\mathbf{j}}}(Z_j, \frac{\omega_i - p_Z Z}{\widehat{E}(q_h)}) \tag{3.34}$$

that is characterized (under our conditions) by the first order condition:

$$\frac{G_{Z_j}^{\overline{\mathbf{Z}}_{-\mathbf{j}}}}{G_{\phi}^{\overline{\mathbf{Z}}_{-\mathbf{j}}}} \equiv \frac{p_j^Z}{\widehat{E}(q_h)} \tag{3.35}$$

If we differentiate identity (3.35) with respect to $\widehat{E}(q_h) = p_X$, we obtain (upon manipulations):

$$\frac{\partial Z^{\overline{\mathbf{Z}}_{-j}*}(p_j^Z; p_X, \omega_i)}{\partial p_X} \equiv \frac{-\frac{G_\phi^{\overline{\mathbf{Z}}_{-\mathbf{j}}}}{\widehat{E}(q_h)}[\frac{G_{Z_j}^{\overline{\mathbf{Z}}_{-\mathbf{j}}}}{G_\phi^{\overline{\mathbf{Z}}_{-\mathbf{j}}}} - \frac{\omega_i - p_Z Z^{\overline{\mathbf{Z}}_{-j}*}(.)}{\widehat{E}(q_h)[G_\phi^{\overline{\mathbf{Z}}_{-\mathbf{j}}}]^2}(G_\phi^{\overline{\mathbf{Z}}_{-\mathbf{j}}} G_{Z_j\phi}^{\overline{\mathbf{Z}}_{-\mathbf{j}}} - G_{Z_j}^{\overline{\mathbf{Z}}_{-\mathbf{j}}} G_{\phi\phi}^{\overline{\mathbf{Z}}_{-\mathbf{j}}})]}{G_{Z_jZ_j}^{\overline{\mathbf{Z}}_{-\mathbf{j}}} - \frac{2p_j^Z}{\widehat{E}(q_h)} G_{Z_j\phi}^{\overline{\mathbf{Z}}_{-\mathbf{j}}} + \frac{(p_j^Z)^2}{[\widehat{E}(q_h)]^2} G_{\phi\phi}^{\overline{\mathbf{Z}}_{-\mathbf{j}}}} \tag{3.36}$$

If the generalized GSC condition holds, the sign of the left hand side of identity (3.36) is the same for all values of $(p_j^Z; p_X, \omega_i)$ and all quantities $\overline{\mathbf{Z}}^{-j}$ of the other public goods. As the denominator of the right hand side of (3.36) is negative thanks to the second order condition of program (3.34), the sign of the right hand sign is completely determined by the sign of $\frac{G_{Z_j}^{\overline{\mathbf{Z}}_{-\mathbf{j}}}}{G_\phi^{\overline{\mathbf{Z}}_{-\mathbf{j}}}} - \frac{\omega_i - p_Z Z^{\overline{\mathbf{Z}}_{-j}*}(.)}{\widehat{E}(q_h)[G_\phi^{\overline{\mathbf{Z}}_{-\mathbf{j}}}]^2}(G_\phi^{\overline{\mathbf{Z}}_{-\mathbf{j}}} G_{Z_j\phi}^{\overline{\mathbf{Z}}_{-\mathbf{j}}} - G_{Z_j}^{\overline{\mathbf{Z}}_{-\mathbf{j}}} G_{\phi\phi}^{\overline{\mathbf{Z}}_{-\mathbf{j}}})]$ which must be constant under the GSC condition. Because the sign of $\frac{G_{Z_j}^{\overline{\mathbf{Z}}_{-\mathbf{j}}}}{G_\phi^{\overline{\mathbf{Z}}_{-\mathbf{j}}}} - \frac{(1-\bar{t})\omega_i}{\widehat{E}(q_h)[G_\phi^{\overline{\mathbf{Z}}_{-\mathbf{j}}}]^2}(G_\phi^{\overline{\mathbf{Z}}_{-\mathbf{j}}} G_{Z_j\phi}^{\overline{\mathbf{Z}}_{-\mathbf{j}}} - G_{Z_j}^{\overline{\mathbf{Z}}_{-\mathbf{j}}} G_{\phi\phi}^{\overline{\mathbf{Z}}_{-\mathbf{j}}})$ is also what determines the change in the slope of indifference curves as given by (3.32) brought about by a change in the household wealth, we therefore have that the slope of this implicit function evaluated at any given (t, z) is monotonic with respect to ω_i. This monotonicity property, which implies that any two indifference curves belonging to households with different wealth can cross at most once, will play a key role in the proof of proposition 2 below. It is illustrated in figure 2 below.

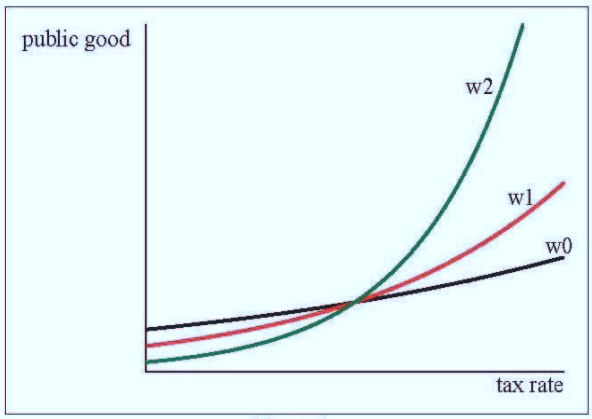


Figure 2

Using this important single-crossing property, we now establish that, if we restrict attention to economies in $\mathbb{D}^A$ in which households have additively separable preferences, or

if we assume that there is only one local public good, then the GSC condition is sufficient for the wealth segregation of any stable jurisdiction structure.

Proposition 5. *If households preferences satisfy the generalized GSC condition, then any stable jurisdiction structure associated to an economy in $\mathbb{D}^A$, or to an economy in $\mathbb{D}$ if $k = 1$, is wealth-segregated.*

Proof.

Consider first any economy $(\omega, U, \mathbb{L}, \{L^l, C_1^l, ..., C_k^l\}_{l\in\mathbb{L}})$ in $\mathbb{D}^A$ and a jurisdiction structure $(j, \{p^l, t_h^l, t_w^l, \mathbf{Z}^l\}_{l\in\mathbb{L}})$ for this economy and denote by q^l the after-tax dwelling price in jurisdiction l defined by $q^l = p^l(1 + t_h^l)$. Proceed by contraposition and assume that the jurisdiction structure is not wealth-segregated. This means that there are households a, b and c in $[0, 1]$ endowed with private wealth $\omega_a < \omega_b < \omega_c$, and 2 jurisdictions l and l', with $a, c \in j^l$ and $b \in j^{l'}$. Either this jurisdiction structure is not stable, and the proof is over, or it is stable. If it is stable than one must have (exploiting the additive separability of the preferences):

$$g(\mathbf{Z}^l) + \Gamma(\widehat{E}(q^l)(1 - t_\omega^l)\omega_a) \geq g(\mathbf{Z}^{l'}) + \Gamma(\widehat{E}(q^{l'})(1 - t_\omega^{l'})\omega_a) \tag{3.37}$$

$$g(\mathbf{Z}^l) + \Gamma(\widehat{E}(q^l)(1 - t_\omega^l)\omega_b) \leq g(\mathbf{Z}^{l'}) + \Gamma(\widehat{E}(q^{l'})(1 - t_\omega^{l'})\omega_b) \tag{3.38}$$

$$g(\mathbf{Z}^l) + \Gamma(\widehat{E}(q^l)(1 - t_\omega^l)\omega_c) \geq g(\mathbf{Z}^{l'}) + \Gamma(\widehat{E}(q^{l'})(1 - t_\omega^{l'})\omega_c) \tag{3.39}$$

where for some continuous and increasing function $\Gamma : \mathbb{R}_+ \to \mathbb{R}$ with at least one inequality being strict (to avoid universal indifference). Suppose that $q^l \geq q^{l'}$ (the proof being symmetric if $q^l < q^{l'}$. Since both the functions Γ and $\widehat{E}$ are continuous and increasing, we know from the intermediate value theorem that there exists some $\bar{t}_\omega \in [0; 1]$ such that $\Gamma(\widehat{E}(q^l)(1 - t_\omega^l)) = \Gamma(\widehat{E}(q^{l'})(1 - \bar{t}_\omega))$ It follows that:

$$g(\mathbf{Z}^l) + \Gamma(\widehat{E}(q^l)(1 - t_\omega^l)\omega_a) = g(\mathbf{Z}^l) + \Gamma(\widehat{E}(q^{l'})(1 - \bar{t}_\omega)\omega_a) \tag{3.40}$$

$$g(\mathbf{Z}^l) + \Gamma(\widehat{E}(q^l)(1 - t_\omega^l)\omega_b) = g(\mathbf{Z}^l) + \Gamma(\widehat{E}(q^{l'})(1 - \bar{t}_\omega)\omega_b) \tag{3.41}$$

$$g(\mathbf{Z}^l) + \Gamma(\widehat{E}(q^l)(1 - t_\omega^l)\omega_c) = g(\mathbf{Z}^l) + \Gamma(\widehat{E}(q^{l'})(1 - \bar{t}_\omega)\omega_c) \tag{3.42}$$

Using the regularity condition on preferences, let $\bar{Z}_j$ be the amount of some public

good j such that $g(\mathbf{Z}^{l'}_{-j}, \bar{Z}_j) = g(\mathbf{Z}^l)$. Hence one has:

$$\begin{aligned} g(\mathbf{Z}^l) + \Gamma(\widehat{E}(q^l)(1-t^l_\omega)\omega_a) &= g(\mathbf{Z}^{l'}_{-j}, \bar{Z}_j) + \Gamma(\widehat{E}(q^{l'})(1-\bar{t}_\omega)\omega_a) \\ g(\mathbf{Z}^l) + \Gamma(\widehat{E}(q^l)(1-t^l_\omega)\omega_b) &= g(\mathbf{Z}^{l'}_{-j}, \bar{Z}_j) + \Gamma(\widehat{E}(q^{l'})(1-\bar{t}_\omega)\omega_b) \\ g(\mathbf{Z}^l) + \Gamma(\widehat{E}(q^l)(1-t^l_\omega)\omega_c) &= g(\mathbf{Z}^{l'}_{-j}, \bar{Z}_j) + \Gamma(\widehat{E}(q^{l'})(1-\bar{t}_\omega)\omega_c) \end{aligned}$$

and, as a result, inequalities (3.37)-(3.39) write:

$$g(\mathbf{Z}^{l'}_{-j}, \bar{Z}_j) + \Gamma(\widehat{E}(q^{l'})(1-\bar{t}_\omega)\omega_a) \geq g(\mathbf{Z}^{l'}) + \Gamma(\widehat{E}(q^{l'})(1-t^{l'}_\omega)\omega_a) \quad (3.43)$$

$$g(\mathbf{Z}^{l'}_{-j}, \bar{Z}_j) + \Gamma(\widehat{E}(q^{l'})(1-\bar{t}_\omega)\omega_b) \leq g(\mathbf{Z}^{l'}) + \Gamma(\widehat{E}(q^{l'})(1-t^{l'}_\omega)\omega_b) \quad (3.44)$$

$$g(\mathbf{Z}^{l'}_{-j}, \bar{Z}_j) + \Gamma(\widehat{E}(q^{l'})(1-\bar{t}_\omega)\omega_c) \geq g(\mathbf{Z}^{l'}) + \Gamma(\widehat{E}(q^{l'})(1-t^{l'}_\omega)\omega_c) \quad (3.45)$$

which violates the single-crossing implication of the generalized GCS condition in the plane of all housing tax rates and quantity of public good j. We leave to the reader the task of verifying that the same argument can be established under separability only if there is only one public good.

Additive separability (along with regularity) plays a key role in the proof if the number of public good is larger than one. We further emphasize this by providing an example of an economy in $\mathbb{D}$ (but not in $\mathbb{D}^A$) where a stable jurisdiction structure can be non-segregated even when the GSC condition holds. Hence, the GSC condition is not sufficient for segregation if there are several public goods and if preferences are homothetically separable - but not additively so:

Consider and economy $(\omega, U, \mathbb{L}, \{L^l\}_{l\in\mathbb{L}})$ in $\mathbb{D}$ where the households preferences are represented by the utility function:

$$U(Z_1, Z_2, x, h) = Z_1 + Z_2 \ln(1 + 2(xh)^{\frac{1}{2}})$$

Such an utility function is continuous, increasing and strictly-quasi concave with respect to all its arguments. Furthermore, the function is homothetically separable - but not additively separable - between the 2 public goods on one hand and the two private goods on the other hand. Consequently,using the two-step budgeting procedure, maximizing the utility function subject to the budget constraint is equivalent to solving the program:

$$\max_{(Z_1;Z_2;\phi)\in\mathbb{R}^3_+} Z_1 + Z_2 \ln(1+\phi) \text{ s. t. } p^Z_1 Z_1 + p^Z_2 Z_2 + p_\phi \phi \leq R \quad (3.46)$$

where $p_\phi = \overline{E}(p_x, p_h)$. For any amount $\bar{Z}_2$ of public good 2, the marshallian demand for Z_1 is given by:

$$\begin{aligned} Z_1^{MC}(p_1^Z, p_\phi, R) &= \frac{R+p_\phi}{p_1^Z} - \bar{Z}_2 \text{ if } \frac{R+p_\phi}{p_1^Z} > \bar{Z}_2 \\ Z_1^{MC}(p_1^Z, p_\phi, R) &= 0 \text{ otherwise} \end{aligned}$$

which is always (weakly) increasing with respect to p_ϕ. Even though it is difficult to provide an explicit definition of the marshallian demand for public good 2 (equal to the conditional demand for that public good thanks to additive separability between the two public goods), we can prove that it is always decreasing with respect to p_ϕ. Indeed, the Marshallian demand for public good 2 conditional upon public good 1 is the solution of the following program

$$\max_{Z^2} Z_2 \ln(\frac{p_\Phi + R - p_2^Z Z_2}{p_\Phi}) \tag{3.47}$$

and is characterized therefore by the 1st order condition:

$$\ln(\frac{p_\Phi + R - p_2^Z Z_2^M(Z_2;.)}{p_\Phi}) - \frac{p_2^Z Z_2^M(Z_2;.)}{p_\Phi + R - p_2^Z Z_2^M(Z_2;.)} \equiv 0$$

Differentiating this identity with respect to p_Φ and rearranging yields:

$$\frac{\partial Z_2^M(Z_2;.)}{\partial p_\phi} = \frac{\frac{p_\Phi}{p_\Phi+R-p_2^Z Z_2^M(Z_2;.)}[\frac{R-p_2^Z Z_2^M(Z_2;.)}{p_\Phi^2}] + \frac{p_2^Z Z_2^M(Z_2;.)}{(p_\Phi+R-p_2^Z Z_2^M(Z_2;.))^2}]}{\frac{-p_2^Z}{p_\Phi+R-p_2^Z Z_2^M(Z_2;.)}[2 + \frac{1}{(p_\Phi+R-p_2^Z Z_2^M(Z_2;.)}]} \leq 0 \tag{3.48}$$

Hence, the Marshallian demand for public good 2 conditional on public good 1 is decreasing with respect to the price of the private good so that the generalized GSC condition holds Let us construct a stable but yet non-segregated jurisdiction. For this sake, consider a jurisdiction structure with jurisdictions j^1 and j^2 where $Z_1^1 = 1/100$, $Z_2^1 = 2$, $p^1 = 1$, $t_h^1 = 0$ and $t_\omega^1 = 7/10$, and j^2, by $Z_1^2 = 0$, $Z_2^2 = 1$, $p^2 = 1$, $t_h^2 = 0$, and $t_\omega^2 = 1/1000$, and 3 types of households a, b, c with $\omega_a = 0, 1$, $\omega_b = 1$ and $\omega_c = 10$. Assume also that $C_1^1 = C_2^1 = C_1^2 = C_2^2 = C$ with $C(x) = x$. Households of types a and c will prefer to live in jurisdiction 1 while households of type b will be better-off in jurisdiction 2. Indeed, their utility would be (approximately):
0.011 in j^1 and 0.001 in j^2 for households of type a,
0,535 in j^1 and 0.693 in j^2 for households of type b,
2,783 inj^1 and 2,397 in j^2 for households of type c.

There is obviously no difficulty in finding land endowments and mass μ_a, μ_b and μ_c of these households such that $7\mu_c = 1/100 + 2$ and $\frac{\mu_b}{1000} = 1$ and such that that the demand of land by each of these households equals the available amount of land at these land prices.

3.3 Conclusion

The main lesson of this paper holds in one sentence. If a continuum of households with differing wealth but with the same regular and homothetically additively separable preferences for local public goods, private spending and housing are free to choose their favorite combination of dwelling tax rates, wealth tax rates and local public good provision, then any stable jurisdiction structure that results from such a free choice will involve perfect wealth stratification of those households if and only if the households preferences satisfies a generalization of the GSC condition of [45]. Yet, as illustrated by the example, the generalization of the GSC condition is not sufficient to ensure the segregation of any stable jurisdiction structure if there is more than one public good if preferences are homothetically separable, but not additively so. While we believe this main lesson to be of some interest, it is clear that more work needs to be done to understand the extent to which this GSC condition is necessary or sufficient for segregation in the case where households preferences are not homothetically separable. It would also be important to test whether the GSC condition is actually verified by households who populate the jurisdictions of the real world. The fact that we have provided such condition in a model with competitive land market opens up the way for empirical testing using the housing market. We plan to provide these empirical tests in our future work.

Chapter 4

The effect of spillovers and congestion on the segregative properties of endogenous jurisdiction structure formation[1]

This paper analyzes the effect of spillovers and congestion of local public services on the segregative properties of endogenous formation of jurisdictions. Households choosing to live at the same place form a jurisdiction whose aim is to produce congested local public services, that can create positive spillovers to other jurisdictions. In every jurisdiction, the production of the local public services is financed through a local tax based on households' wealth. Local wealth tax rates are democratically determined in all jurisdictions. Households also consume housing in their jurisdiction. Any household is free to leave its jurisdiction for another one that would increase its utility. A necessary and sufficient condition to have every stable jurisdictions structure segregated by wealth, for a large class of congestion measures and any spillovers coefficients structure, is identified: the public services must be either a gross substitute or a gross complement to the private good and the housing.

4.1 Introduction

In most countries, local public spending accounts for a large share of the public spending (almost 50% in the USA), and this share has been increasing since the end of the Second World War. As a consequence of the growing role played by local jurisdictions, another phenomenon appeared: jurisdictions belonging to the same urban area seem to be more

[1]This chapter reviews a CTN-FEEM working paper number 2011.46

differentiated in terms of their inhabitants' wealth (see for instance [67]). A possible explanation is provided by Tiebout's 1956 article [82]. According to his intuitions, individuals choose their place of residence according to a trade-off between local tax rates and amounts of public services provided, which leads every jurisdiction to be homogeneous. The formation of jurisdictions structure is endogenous, due to the free mobility of households, that can "vote with their feet", that is to say leave their jurisdiction to another one, if they are unsatisfied with their jurisdiction's tax rate and amount of public services. An important literature dealing with the endogenous jurisdictions formation *à la* Tiebout exists. A widely spread belief is the self-sorting mechanism of the endogenous formation process: agents will live in homogeneous jurisdictions. This homogeneity can be expressed in terms of wealth, preferences on public services, on housing, on economic activity...

Westhoff [84] was among the first economists to provide a formal model based on Tiebout's intuitions. In this model, households can enjoy 2 goods, a local public good, financed through a local tax on wealth, which is a pure club good (only households living in the jurisdiction that produced it can enjoy the local public good, that does not suffer from congestion effect), and a composite private good, whose amount is equal to the after-tax wealth. He found a condition that ensures the existence of an equilibrium. This condition is for the slopes of individuals' indifference curves in the tax rate-amount of public good space to be ordered by their private wealth. If this condition is respected, not only an equilibrium will exist, but, at equilibrium, the jurisdictions structure will be segregated.

Gravel and Thoron [45] identified a necessary and sufficient condition that ensures the segregation, within Westhoff's meaning, of every stable jurisdictions structure: the public good must be, for all level of prices and wealth, either always a complement or always a substitute to the private good. This condition is called the Gross Substitutability/Complementarity (GSC) condition. This condition is equivalent to have the preferred tax rate being a monotonous function of the private wealth, for any level of prices and wealth. Biswas, Gravel and Oddou [8] integrated a welfarist central government to the model, whose purpose is to maximize a generalized utilitarian social welfare function by implementing an equalization payment policy. Equalization payment can be either vertical (the government taxes households and redistributes the revenues to jurisdictions), horizontal (the government redistributes local tax revenues between the jurisdictions), or mixed. They found that the GSC condition remains necessary and sufficient.

Greenberg [46] proposed a model of local public goods with spillovers among jurisdictions, but did not consider the possibility for households to leave their jurisdiction for another that offer a better "tax rate - amount of public good" package. He proved the existence of an equilibrium under the d-majority voting rule.

Nechyba [70] developed a model with spillovers and housing, but contrary to Rose-Ackerman [75], housing is modelled as a discreet good, which differ on type, and households own their house, so wealth is not exogenous anymore, since housing price may vary. In his model, spillovers between jurisdictions were allowed, because households' utility depends not only on the amounts of local public good provided by its jurisdiction and of the national public good, but also on the amounts of public good provided by all other jurisdictions. After having ensured the existence of an equilibrium under certain conditions, he identified sufficient conditions for a stable jurisdictions structure to be segregated. Unfortunately, one of these sufficient conditions was the absence of spillovers between jurisdictions, which is a pretty strong assumption, that might not be necessary.

The effect of spillovers on the provision of public goods and on the equilibrium have been analyzed by Bloch and Zenginobuz [13] and [12]. However, the authors do not examine the consequences in terms of segregation the existence of spillovers may generate.

This paper generalizes Gravel & Thoron's model by assuming that local public goods may suffer from congestion and create spillovers. Households choose a location, each set of households living in the same place forms a jurisdiction. In each jurisdiction, absentee landlords use the land available in the jurisdiction to produce housing. Housing price is competitive, so, at the equilibrium, the housing supply is equal to the housing demand for that price. Then every jurisdiction democratically determines its tax rate (which is applied to households' wealth), and the revenues generated by this tax are fully used to financed local public services, that may suffer of congestion.

Furthermore, households may benefit from other jurisdictions' local public services. This assumption differs from the main part of the literature on local public goods. However, considering that small towns belonging to a metropolitan area benefit from the public services provided by the main city is not outlandish.

Households are assumed to be freely mobile, so, once all jurisdictions have determined

their tax rate, households can leave their jurisdiction for another one that would increase their utility. Equilibrium is reached when no household has incentive to leave unilaterally its jurisdiction or to modify its consumption bundle, the housing price clears the market in every jurisdiction and the local tax rates are democratic. We assume that preferences are homothetically separable between local public services on one hand and private spending and housing on the other hand within the meaning of Blackorby and alii[11]. This assumption, though restrictive, seems to be relevant with respect to recent data [19].

This paper aims at examining the segregative properties of the endogenous jurisdiction structure formation in such a framework. The article is organized as follows. The next section introduces the formal model. Section 3 provides an example of how congestion and spillovers can modify a jurisdictions structure. Section 4 states and proves the results. Finally, section 5 concludes.

4.2 The formal model

We consider a model *à la* Gravel & Thoron, improved by the presence of a competitive land market, and by the existence of spillovers and congestion effects. There is a continuum of households on the interval $[0; A]$ with Lebesgue measure λ, where, for any subset $I \subset [0; A]$, the mass of household in I is given by $\lambda(I)$. Households' wealth distribution is modelled as a Lebesgue measurable function $\omega : [0; A] \to \mathbb{R}_+^*$ - household i is endowed with a wealth $\omega_i \in \mathbb{R}_+^*$ - with ω being an increasing and bounded from above function.

Households have identical preferences, represented by a twice differentiable, increasing and concave utility function

$$U : \left| \begin{array}{ccc} \mathbb{R}^3_{++} & \longrightarrow & \mathbb{R}_+ \\ (Z, h, x) & \longmapsto & U(Z, h, x) \end{array} \right.$$

where

1. Z is the available amount of public services households can enjoy,
2. h is their amount of housing,
3. x is the amount of the households' expenditures for other things than housing.

We assume that public services are a non-Giffen good.

The utility function is assumed to be homothetically separable in the sense of [11] between the available public services on one hand, and the housing and other expenditures on the other hand. This property implies that the share of their after tax wealth households will spend on housing and on other expenditure does not depend on their after tax wealth, nor on the amount of the available amount of public services. Consequently, the indirect utility function, conditional to the amount of public services, is given by

$$V^C : \left| \begin{array}{rcl} \mathbb{R}_+ & \longrightarrow & \mathbb{R}_+ \\ \psi(p_h, p_x)(\omega_i - p_Z \bar{Z}) & \longmapsto & V^C(\bar{Z}; \psi(p_h, p_x)(\omega_i - p_Z \bar{Z})) \end{array} \right.$$

where

$$V^C(\bar{Z}; \psi(p_h, p_x)(\omega_i - p_Z \bar{Z}) = \max_{h,x} U(\bar{Z}, h, x)$$

$$\text{s.t. } p_h h + p_x x \leq \omega_i - p_Z \bar{Z}$$

and $\psi : \left| \begin{array}{rcl} \mathbb{R}_+^2 & \longrightarrow & \mathbb{R}_+ \\ (p_h, p_x) & \longmapsto & \psi(p_h, p_x) \end{array} \right.$ being a differentiable, increasing and quasi-convex[2] function.

We denote $\mathbb{U}$ as the set of all functions satisfying the properties defined above.

Each household has to choose a place of residence among all the conceivable location. $\mathbb{L} \subset \mathbb{N}$ is the finite set of locations.

Every location l has an amount $H_l \in \mathbb{R}_+^*$ of housing, belonging to absentee landlords that rent it at the unit price p_l. Since housing is costly, and that households only enjoy the housing that is located in their own location, then, obviously, no household will consume housing in more than one location.

We do not rule out the possibility for some locations to be empty. If a location is empty, then we assume that $p_l = 0$ and that no tax will be collected. Households living at the same location form a jurisdiction. We denote $J \subseteq \mathbb{L}$ the set of jurisdictions, with $card(J) = M$.

We denote μ_i as the measure of households with private wealth ω_i.

Since local public services create spillovers in other jurisdictions, the total amount

[2] A function f is quasi-convex if $\forall x, y \in \mathbb{R}_+$ with $f(x) \geq f(y)$ and $\forall \lambda \in [0; 1], f(\lambda x + (1-\lambda)y) \leq f(x)$

of public services a household in jurisdiction j can enjoy (Z_j) depends not only on the available amount of public services produced by jurisdiction j, but also on the amount of public services produced by the other jurisdictions. This amount is given by

$$Z_j = \pi(\zeta_j, S_j)$$

with :

- $\pi : \mathbb{R}_+^2 \to \mathbb{R}_+$ being a non-decreasing (strictly increasing with respect to ζ_j), twice differentiable, concave function such that, $\forall S \in \mathbb{R}_+, \pi(0, S) = 0$,
- ζ_j is the available amount of public services produced by jurisdiction j,
- S_j is the amount of spillovers from other jurisdictions in jurisdiction j, given by

$$S_j = \sum_{k \in J - \{j\}} \beta_{jk} \zeta_k$$

  where $\beta_{jk} \in \mathbb{R}_+$ represents the spillovers coefficient of jurisdiction k's local public services in jurisdiction j.

Let us denote B as the square matrix of order M, that represents the spillovers coefficients, with $\forall (j, j') \in J^2, \beta_{jj'} \in [0; 1]$ and $\beta_{jj} = 1$. No assumption needs to be made on B. It can be exogenously determined[3], or depends positively or negatively on the amount of public services provided by the jurisdictions that generate and/or receive them. The matrix B is not necessarily symmetric. We denote $\mathbb{B}$ as the set of all square matrix of order M such that, $\forall (j, j') \in J^2, \beta_{jj'} \in [0; 1]$ and $\beta_{jj} = 1$

The amount of available local public services *produced* by jurisdiction j is given by

$$\zeta_j = \frac{t_j \varpi_j}{C_j}$$

with :

- t_j being the local tax rate,

[3]For instance, spillovers coefficients can represent the distance between two jurisdictions (the closer jurisdiction j is to jurisdiction j', the closer to 1 will be $\beta_{jj'}$), or be affected by political agreements concluded between jurisdictions...

- ϖ_j being the aggregated wealth in j,
- $C_j = C(\{\mu_k\}_{k\in J}, \{\beta_{kj}\}_{k\in J})$ is the congestion function, with $C : \mathbb{R}_+^{2M} \to [1; +\infty[$, continuous and non-decreasing (with respect to every argument).

Assuming that the intensity of the congestion faced by jurisdiction j's public services caused by the mass of households in jurisdiction j' depends on the spillovers coefficient j's public services creates in j' is quite reasonable, since public services will suffer more from an important mass of households in a jurisdiction that receives a lot of spillovers from these public services than from a important mass of households that receives little spillovers. Moreover, it is assumed that if jurisdiction j's public services create no spillovers in jurisdiction j' (i.e. $\beta_{j'j} = 0$), then the congestion function is constant with respect to $\mu_{j'}$. Formally,

$$\beta_{j'j} = 0 \Rightarrow \frac{\partial C(\{\mu_k\}_{k\in J}, \{\beta_{kj}\}_{k\in J})}{\partial \mu_{j'}} = 0$$

However, one could specify the congestion measure in such a way that there would be no relation between congestion and the spillovers coefficients. The present definition of the congestion measure does not exclude such an assumption. The only properties assumed on the congestion function are that:

1. $\forall (j,k) \in J^2, \forall \{\mu_l\}_{l\in J} \in \mathbb{R}^M, \forall \{\beta_{lj}\}_{l\in J} \in [0;1]^M$:

$$\lim_{\mu_k \to +\infty} \frac{\partial C_j(\{\mu_l\}_{l\in J}, \{\beta_{lj}\}_{l\in J})}{\partial \mu_k} = 0$$

2. $\forall (j,k,k') \in J^3, \forall \{\mu_l\}_{l\in J} \in \mathbb{R}^M, \forall \{\beta_{lj}\}_{l\in J} \in [0;1]^M$:

$$\frac{\partial^2 C_j(\{\mu_l\}_{l\in J}, \{\beta_{lj}\}_{l\in J})}{\partial \mu_k \partial \mu_{k'}} \leq 0$$

The first property is certainly restrictive, but not unreasonable though, it requires that the marginal congestion in one jurisdiction generated by a infinitesimal increase of the mass of household, in this very jurisdiction or another one, tends to be null when the mass of households is infinite. Homogeneous functions of degree less than 1, for instance, respect this property.

The second property is more natural since, to a certain extent, the masses of households in different jurisdictions are substitutable concerning the congestion they provide in

one jurisdiction.

We denote Γ as the set of all congestion function satisfying the properties defined above.
A economy is composed of 5 elements:

- A wealth distribution ω
- Preferences represented by the utility function $U \in \mathbb{U}$
- A congestion measure $C \in \Gamma$
- A spillovers coefficient matrix $B \in \mathbb{B}$
- A set of location $\mathbb{L} \in \mathbb{N}$

We denote Δ as the set of all conceivable economies.
For simplicity, we denote $F_j = \frac{\varpi_j}{C_j}$ as jurisdiction j's fiscal potential, that is to say the maximal available amount of public services jurisdiction j can produce (if $t_j = 1$).

The demands for housing and private consumption of a household i depend on his after-tax wealth $(1 - t_j)\omega_i$ and on the housing price in jurisdiction j, p_j. In every jurisdiction, p_x is normalized to 1. We defined $h^M(p_j, (1-t_j)\omega_i)$ and $x^M(p_j, (1-t_j)\omega_i)$ as respectively the Marshallian demand for housing and for private consumption[4] of a household i in a jurisdiction j, e.g.

$$(h^M(p_j, (1-t_j)\omega_i), x^M(p_j, (1-t_j)\omega_i) \in \arg\max_{h,x} U(Z, h, x)$$

subject to

$$p_j h + x = (1 - t_j)\omega_i$$

The local tax rate is determined according to a democratic rule. Hence, every household has to determine its favorite tax rate, denoted $t^* : \mathbb{R}^4_+ \to [0; 1]$, which is a function of:

- the fiscal potential F,
- the spillovers from other jurisdictions' public services S (taken as given),

[4]Under the homothetic separability assumption, those Marshallian demands do not depend on the amount of public services, that is the reason why Z is withdrawn from the arguments of the Marshallian demands.

- the housing price p,
- the private wealth ω_i.

Formally,

$$t^*(F,S,p,\omega_i) \in \arg\max_t U(\pi(tF,S), h^M(p,(1-t)\omega_i), (1-t)\omega_i - ph^M(p,(1-t)\omega_i))$$

Lemma 4.2.1. *For all utility functions belonging to* $\mathbb{U}$, $\forall(F,S,p,\omega_i) \in \mathbb{R}^4_+$ *and for all functions* $\pi : \mathbb{R}^2_+ \to \mathbb{R}_+$ *satisfying the above properties, preferences are single-peaked with respect to* t, *so* $t^*(F,S,p,\omega_i)$ *exists* .

Proof.

The proof of this lemma can be easily obtained by showing that the utility function is concave with respect to t, so there exists an unique

$$t^* \in \arg\max_{t\in[0;1]} U(\pi(tF,S), h^M(p,(1-t)\omega_i), (1-t)\omega_i - ph^M(p,(1-t)\omega_i))$$

Let us denote

g_{ij} :

$$The first derivative of this function with respect to t is : \frac{\partial g_{ij}(t)}{\partial t} = U_Z F\pi'(tF,S) - \omega_i[\frac{\partial h^M(p,R)}{\partial R}U_h - \frac{\partial x^M(p,R)}{\partial R}U_x]$$

Let us define

$$\underline{t}^*_j = \min_{i\in I_j} t^*(F_j, S_j, p_j, (1-t_j)\omega_i)$$

and

$$\bar{t}^*_j = \max_{i\in I_j} t^*(F_j, S_j, p_j, (1-t_j)\omega_i)$$

as respectively the lowest and the highest tax rate preferred by a household living in jurisdiction j.

A jurisdictions structure is a vector $\Omega = (J, \{I_j\}_{j\in J}; (\{t_j\}_{j\in J}); (\{p_j\}_{j\in J}), (\{S_j\}_{j\in J})$.

A jurisdictions structure $\Omega = (J, (\{I_j\}_{j\in J}); (\{t_j\}_{j\in J}); (\{p_j\}_{j\in J}), (\{S_j\}_{j\in J})$ is **stable** in the economy $(\omega, U, C, B, \mathbb{L})$ if and only if

1. $\forall j, j' \in J, \forall i \in I_j, U(Z_j, h_{ij}, (1-t_j)\omega_i - p_j h_{ij}) \geq V(Z_{j'}, \psi(p_{j'})(1-t_{j'})\omega_i)$, with h_{ij} being the amount of housing in j consumed by household i,

2. $\forall j \in J, \int\limits_{I_j} h^M(p_j, (1-t_j)\omega_i)d\lambda = H_j$
3. $\forall j \in J, t_j \in [\underline{t}_j^*; \bar{t}_j^*]$

In words, a jurisdictions structure is stable if and only if :

1. No household can increase its utility by modifying its consumption bundle or by leaving its jurisdiction,
2. The housing prices are competitive in every jurisdiction (supply equals demand),
3. The tax rate is democratically chosen in every jurisdiction.

Let us now express formally the definition of the segregation, which is the same definition as in [84].

A jurisdictions structure $\Omega = (J, (\{I_j\}_{j\in J}); (\{t_j\}_{j\in J}); (\{p_j\}_{j\in J}), (\{S_j\}_{j\in J})$ in the economy $(\omega, U, C, B, \mathbb{L})$ is **segregated** if and only if $\forall \omega_h, \omega_i, \omega_k \in \mathbb{R}_+$ such that $\omega_h < \omega_i < \omega_k$, $(h,k) \in I_j$ and $i \in I_{j'} \Rightarrow Z_j = Z_{j'}$ and $\forall \omega \in \mathbb{R}_+, V^C(Z_j, p_j, (1-t_j)\omega) = V^C(Z_{j'}, p_{j'}, (1-t_{j'})\omega)$

In words, a jurisdictions structure is wealth-segregated if, except for groups of jurisdictions offering the same amount of public services and in which every household would have the same utility, the poorest household of a jurisdiction with a high per capita wealth is (weakly) richer than the richest household in a jurisdiction with a lower per capita wealth.

Let us define $J_j = \{k \in J : Z_k = Z_j \text{ and } \psi(p_k)(1-t_k) = \psi(p_j)(1-t_j)\}$. In words, J_j is the set of all jurisdictions offering the same amount of available public services as j, and such that households have the same purchasing power within the meaning of Hicks[5]. Obviously, for all $j \in J$, households are indifferent between all jurisdictions belonging to J_j.

Formally, a jurisdictions structure is segregated if and only if, when, for all $j \in J$, the interval $\bigcup\limits_{k\in J_j} I_k$ is a connected subset of I.

4.3 Examples

This section presents 2 examples of economies, where congestion and spillovers are introduced in turn, so as to examine their impact on the jurisdictions structure at the equilib-

[5] 2 vectors of prices and wealth $(p_1, ...p_K, R)$ and $(p'_1, .., p'_K, R')$ provide the same purchasing power within the meaning of Hicks if and only if $V(p_1, ...p_K, R) = V(p'_1, .., p'_K, R')$

rium. In the first example, households' preferences are homothetically separable between the local public services on one hand, and the private spendings on the other hand, but violate the GCS condition (and so the monotonicity of the preferred tax rate function with respect to the private wealth).

In the second one, on the contrary, the GCS condition and the monotonicity of the preferred tax rate function hold, but not the homothetical separability. However, we construct a stable and yet non-segregated jurisdictions structure, which show that neither the GSC condition nor the monotonicity of the preferred tax rate function are sufficient to ensure the segregation of any stable jurisdictions structure if preferences are not homothetically separable. We also show that the GSC condition, that implies the condition identified by Westhoff to ensure the existence of an equilibrium, is not anymore sufficient when congestion effects are allowed.

4.3.1 First example: the effect of congestion and spillovers on a stable jurisdictions structure

In this example, we start from a situation where the jurisdictions structure is stable and non segregated. We first assume that public services do not generate spillovers nor suffer from congestion. Then, we introduce congestion effects, which will lead to instability. The new jurisdictions structure will then be segregated. Finally, we consider that local public services generate spillovers, and again, the jurisdictions structure will not be stable anymore, and the first jurisdictions structure will arise. This example suggests that congestion effects increase the segregative properties of endogenous jurisdictions formation, while the presence of spillovers mitigates them.

Let us consider the example provided by Gravel & Thoron, improved by the presence of housing. Households' preferences are represented by

$$U(Z,h,x) = \begin{cases} \ln(Z) + 8\sqrt{hx} - 4hx & \text{if } \sqrt{hx} \leq \frac{7}{4} \\ \ln(Z) + (1 - \frac{14}{4(ln(1.75)-1)})\sqrt{hx} + \frac{49}{16(ln(1,75)-1)} \ln(2\sqrt{hx}) & \text{otherwise} \end{cases}$$

Such an utility function is continuous, twice differentiable, increasing and concave with respect to every argument. The indirect utility function conditional to the public services is given by

$$V^C(Z; p, (1-t)\omega_i)$$

$$=$$

$$\begin{cases} \ln(Z) + \frac{4(1-t)\omega_i}{\sqrt{p}} - \frac{(1-t)^2\omega_i^2)}{p} & \text{if } \frac{(1-t)\omega_i}{p} \leq \frac{7}{4} \\ \ln(Z) + (0,5 - \frac{7}{4(ln(1.75)-1)})\frac{(1-t)\omega_i}{\sqrt{p}} + \frac{49}{16(ln(1,75)-1)}\ln(\frac{(1-t)\omega_i)}{p}) & \text{otherwise} \end{cases}$$

Consider an economy with 2 jurisdictions j_1 and j_2 and 3 types of households a, b, c with private wealth $\omega_a = 2 - \sqrt{2}$, $\omega_b = 1.5$ and $\omega_c = 3$ and whose masses are $\mu_a = \frac{11.9}{2-\sqrt{2}}$, $\mu_b = 8$ and $\mu_c = \frac{1}{30}$. For simplicity, let us assume that the tax rate is determined through the majority voting rule, and that the housing supply is perfectly elastic with respect to its price, that will be considered as fixed to 1 in both jurisdictions.

For all $\omega_i \leq \frac{3+\sqrt{7}}{2}$, the preferred tax rate function is given by

$$t^*(F, S, \omega_i) = \frac{\omega_i - 2 + \sqrt{(\omega_i - 2)^2 + 2}}{2\omega_i}$$

Determining the preferred tax rate function of an households endowed with a private wealth greater than $\frac{3+\sqrt{7}}{2}$ will not be required, since households of type c will never be majoritary in their jurisdiction, so their preferred tax rate will never be applied. One can observe that the preferred tax rate does not depend on the fiscal potential, which will greatly facilitate the example.

Let us assume first that there is no congestion and no spillovers. Then, the available amount of public services in a jurisdiction j is simply the tax revenue: $Z_j = t_j\varpi_j$. Suppose that households of type a and c live in j_1, while households of type b live in j_2. Then, in both jurisdictions, the aggregate wealth will be equal to 12, the tax rate in j_1, denoted t_1, will be equal to $\frac{1}{2}$, while $t_2 = \frac{1}{3}$. Since, at first, it is assumed that $C_1 = C_2 = 1$ and $S_1 = S_2 = 0$, one has $Z_1 = 6$ and $Z_2 = 4$. Such a jurisdictions structure is stable, since the fiscal potential is the same in both jurisdictions, households of type a and b have their favorite tax rate in their respective jurisdiction, and households of type c are better-off in j_1, in which they enjoy an utility level equal to $ln(6) + \frac{15}{4} \approx 5.54$, against $ln(4) + 4 \approx 5.51$ if they would move to j_2.

Now, let us reconsider the example when the local public services suffer from congestion, with

$$C_k = \frac{30 + \sqrt{\mu_k}}{30}$$

Consequently,

$$Z_j = \frac{30(t_j \varpi_j)}{30 + \sqrt{\mu_k}}$$

Then, the amount of public services in j_1 will be equal to

$$Z_1 = \frac{180}{30 + \sqrt{\frac{11.9}{2-\sqrt{2}} + \frac{1}{30}}} \approx 5.216$$

and, in j_2,

$$Z_2 = \frac{120}{30 + \sqrt{8}} \approx 3.655$$

which will leads households of type c to move to j_2, in which they will enjoy an utility level of $ln(\frac{120}{30+\sqrt{8}}) + 2(0.5 - \frac{7}{4(ln(1.75)-1)}) + \frac{49ln2)}{16(ln(1,75)-1)} \approx 5.424$ while their utility level would have been $ln(\frac{180}{30+\sqrt{\frac{11.9}{2-\sqrt{2}}+\frac{1}{30}}}) + 15\frac{}{4 \approx 5.402}$ if they had stayed in j_1. Households of type a and b would not have incentive to move, since households of type a would enjoy an utility level of approximatively 2.74 in j_1 against 2.71 in j_2, and households of type b, an utility level of 4.30 in j_2 and 4.09 in j_1.

Is the new jurisdictions structure stable after that households of type c had moved from j_1 to j_2? Since the preferred tax rate function is constant with respect to the fiscal potential, the tax rate will be the same in every jurisdiction. Once households of type c moved from j_1 to j_2, the new amount of public services in j_1 will be

$$Z_1 = \frac{178.5}{30 + \sqrt{\frac{11.9}{2-\sqrt{2}}}} \approx 5.173$$

and in j_2,

$$Z_2 = \frac{120 + \frac{1}{3}}{30 + \sqrt{8 + \frac{1}{30}}} \approx 3.665$$

so households of type a will get a higher utility level in j_1 than in j_2 (approximatively 2.73 against 2.71, while households of type b and of type c can enjoy a higher utility level by staying in j_2 than if they moved to j_1, respectively with 4.30 against 4.08 for households of type b and 5.43 against 5.39 for households of type c, so this new structure is stable and segregated, while the previous one was stable and non-segregated as long as no congestion effects were assumed. In this very specific example, the congestion seems to increase the segregative properties of the endogenous jurisdictions structure formation.

Let us now introduce spillovers in the example to observe what impact they can have. Suppose that jurisdiction j_1's local public services generates spillovers in jurisdiction j_2, and vice-versa. For instance, suppose that the available amount in a jurisdiction j is given by

$$Z_j = \frac{t_j \varpi_j (1 + S_j)}{C_j}$$

with $S_j = \sum_{k \in J \setminus \{j\}} \beta_{jk} \frac{t_k \varpi_k}{C_k}$. Since local public services generate spillovers in other jurisdictions, it is not unreasonable to assume that the congestion function also depends on the mass of households in other jurisdictions and on the spillovers coefficients. Let us then redefine the congestion function as follows:

$$C_j = 1 + \frac{\sqrt{\mu_j + \sum_{k \in J \setminus \{j\}} \beta_{kj} \mu_k}}{30}$$

Although j_1 produces more public services than j_2, suppose that jurisdiction j_1 receives more spillovers from j_2's public services than vice-versa [6]:

$$S_1 = \frac{40 + \frac{1}{9}}{30 + \sqrt{8 + \frac{1}{30}}} \approx 1.222$$

and

$$S_2 = \frac{36.3}{30 + \sqrt{\frac{11.9}{2 - \sqrt{2}}}} \approx 1.034$$

hence $\beta_{12} = \frac{1}{3}$ and $\beta_{21} = \frac{1}{5}$.

With such spillovers coefficients, the available amounts of public services households can enjoy are now

$$Z_1 = \zeta_1 (1 + \frac{1}{3} \zeta_2) \approx 11.29$$

and

$$Z_2 = \zeta_2 (1 + \frac{1}{5} \zeta_1) \approx 7.18$$

As a consequence, households of type c will move back to j_1, in which they will be able to enjoy an utility level of approximatively 6.17, against 6.10 if they stayed in j_2. Households

[6] for instance, j_1 may have implemented a restrictive policy in order to prevent households living in j_2 from congesting its public services.

of type a and b can not increase their utility by voting with their feet, since households of type a's utility is about 3.51 in j_1 while it would be about 3.38 in j_2, and households of type b have an utility level of approximatively 4.97 in j_2 against 4.86 if they moved to j_1.

Finally, the jurisdictions structure in which jurisdiction j_1 is composed of households of type a and c, and j_2 is composed of households of type b is stable: with households of type c in j_1 instead of j_2,

$$Z_1 \approx 11.513$$

and

$$Z_2 \approx 7.18$$

One can observe that, due to the existence of spillovers between jurisdictions, households of type c's change of jurisdiction has almost no impact on the available amount of public services in each jurisdiction, then the utility levels will remains almost the same for all type of households.

As a conclusion, this example suggests that congestion favors the segregative properties of endogenous jurisdictions formation, whereas the existence of spillovers tends to decrease the number of stable segregated jurisdictions structures.

However, in the next section, the validity of the GCS condition, that was necessary and sufficient to ensure the segregation of every stable jurisdictions structure, is established within the existence of congestion effect and spillovers.

4.3.2 Second example: what if preferences are not homothetically separable?

The example follows 2 aims: showing that the GSC condition is not sufficient anymore if preferences are not homothetically separable between public services on one hand and the composite private good and the housing on the other, and then emphasizing the fact that more restrictive conditions must be found to ensure the existence of an equilibrium when public services suffer from congestion.

In this example, preferences are represented by the following function:

$$U : \left| \begin{array}{ccc} \mathbb{R}^3_+ & \longrightarrow & \mathbb{R}_+ \\ (Z,x,h) & \longmapsto & ln(Z)+ln(1+x)+ln(h) \end{array} \right.$$

Then, the Marshallian demands are given by:

$$Z^m(p_Z,p_x,p_h,R) = \frac{R+p_x}{3p_Z}$$

$$x^m(p_Z,p_x,p_h,R) = \frac{R}{3p_x} - \frac{2}{3}$$

$$h^m(p_Z,p_x,p_h,R) = \frac{R+p_x}{3p_h}$$

if $R \geq 2p_x$, and by:

$$Z^m(p_Z,p_x,p_h,R) = \frac{R}{2p_Z}$$

$$x^m(p_Z,p_x,p_h,R) = 0$$

$$h^m(p_Z,p_x,p_h,R) = \frac{R}{2p_h}$$

otherwise.

Clearly, local public services are a gross substitute to both the private good, and preferences are not homothetic between the composite private good and the housing. One can notice that it is equivalent to the condition identified by Westhoff to ensure the existence of an equilibrium (see lemma 4 below).

For a price of public service equal to $\frac{\omega_i}{\varpi}$ and a price of the composite private good normalized to 1, the preferred tax rate function is given by :

$$t^*(\varpi,p_h,R) = \left\{ \begin{array}{ll} \frac{1}{2} & \text{if } R<2 \\ \frac{R+1}{3R} & \text{otherwise} \end{array} \right.$$

One can see that the preferred tax rate function is constant and then strictly decreasing with respect to the private wealth.

Let us suppose that there are 2 jurisdictions j_1 and j_2 and three types of households a,b,c respectively with private wealth $0,5$, 2 and 5 with $\mu_a = 2$, $\mu_b = 5$ and $\mu_c = 4.2$. Let assume first that there is no congestion effect nor spillovers, and that the technology

is linear, so $Z = t\varpi$.

Suppose that households of type a and c live in j_1, while households of type b live in j_2. Consequently, if the tax rate is determined through the majority voting rule, then, one has $\varpi_1 = 22$, $t_1 = 0,4$ so $Z_1 = 8,8$, and $\varpi_2 = 10$, $t_2 = 0,5$ so $Z_1 = 5$. If we assume that housing prices are respectively 2 and 1 in j_1 and j_2 and the available amount of land, 4.5 and 5, then the jurisdictions structure is stable: households of type a have an utility of 0.28 while it would be only 0.22 in j_2, households of type b, an utility of 0.92 (against 0.88 in j_1) and households of type c, an utility of 2.17 (against 1.88 in j_2).

If congestion are introduced, with $Z = \frac{t\varpi}{C}$, with $C = 1 + \sqrt{\mu}$, then the jurisdictions structure is not stable anymore, because households of type a would be better of by moving to j_2, since their utility would -0.86 against -0.97 if they stayed in j_1. The new jurisdictions structure will have households of type a moving in j_2, while other households will stay in their jurisdiction. But this jurisdictions structure will not be stable neither, because, due to the evolution of the total demand in each jurisdiction, housing prices will increase in j_2 to 1.1, and decrease in j_2 to $\frac{28}{15}$. Consequently, households of type a will have incentive to move back to j_1, because their utility would be -0.9 against -0.95 if they stayed in j_2, while other households could not increase their utility by moving to the other jurisdiction. This will lead to a cycle, so no equilibrium will arise.

Such a result proves that, contrary to Westhoff's model, the single crossing of the indifference curves in the (t, Z) space is not sufficient to ensure the existence of an equilibrium when there is a competitive housing market and when public services suffer from congestion effect.

4.4 Results

The main result of this paper is the robustness of the GSC condition to the existence of spillovers and congestion effect on the local public services to have all stable jurisdictions structures segregated. As in Gravel and Thoron (2007), this condition is equivalent to the monotonicity of the preferred tax rate function with respect to the private wealth, for any given amount of the other arguments. To prove this equivalence, let us first establish the following lemma.

Let us define $\pi^{-1}: \left| \begin{array}{ccc} \mathbb{R}_+ & \longrightarrow & \mathbb{R}_+ \\ Z & \longmapsto & \pi^{-1}(Z;\bar{S}) \end{array} \right.$ as the amount of available public services produced in a jurisdiction needed to have a total available amount of public services Z if the amount of spillovers is $\bar{S}$. Formally, $\pi(\pi^{-1}(Z;\bar{S}),\bar{S}) = Z$

Since $\forall \bar{S} \in \mathbb{R}_+, \pi(\zeta,\bar{S})$ is continuous and strictly increasing with respect to ζ, $\pi^{-1}(Z;S)$ always exists.

Lemma 4.4.1. *$\forall U \in \mathbb{U}$, $\forall (F,S,p,\omega_i) \in \mathbb{R}^4_+$, the preferred tax rate function is a monotonic function of the Marshallian demand for the public good:*

$$t^*(F,S,p,\omega_i) \equiv \frac{1}{F}\pi^{-1}[Z^M(\frac{1}{F\pi_\zeta(t^*(F,S,p,\omega_i)F,S)},\frac{p}{\omega_i},\frac{1}{\omega_i},1);S] \tag{4.1}$$

Proof.

At the optimum, the Marginal Rate of Substitution (MRS) is equal to the price ratio. Then:

$$\frac{U_Z(Z,h,x)}{U_x(Z,h,x)} = \frac{p_Z}{p_x} \tag{4.2}$$

The FOC of the utility maximization program with respect to t implies that:

$$\frac{U_Z(\pi(t^*F,S),h^M(p,(1-t^*)\omega_i),(1-t^*)\omega_i - ph^M(p,(1-t^*)\omega_i))}{U_x(\pi(t^*F,S),h^M(p,(1-t^*)\omega_i),(1-t^*)\omega_i - ph^M(p,(1-t^*)\omega_i))} = \frac{\omega_i}{F\pi_\zeta(t^*(F,S,p,\omega_i)F,S)} \tag{4.3}$$

because, using the envelop theorem, we know that $pU_x = U_h$. Then, using (4.2) and (4.3), we know that:

$$\pi(t^*(F,S,p,\omega_i)F,S) \equiv Z^M(\frac{\omega_i}{F\pi_\zeta(t^*(F,S,p,\omega_i)F,S)},p,1,\omega_i) \tag{4.4}$$

Since Marshallian demands are homogeneous of degree 0, we can divide all the arguments of the Marshallian demand for the public services by ω_i.

$$\pi(t^*(F,S,p,\omega_i)F,S) \equiv Z^M(\frac{1}{F\pi_\zeta(t^*(F,S,p,\omega_i)F,S)},\frac{p}{\omega_i},\frac{1}{\omega_i},1) \tag{4.5}$$

Then, using the definition of $\pi^{-1}(Z;S)$, we know that:

$$t^*(F,S,p,\omega_i)F,S) \equiv \pi^{-1}(Z^M(\frac{1}{F\pi_\zeta(t^*(F,S,p,\omega_i)F,S)},\frac{p}{\omega_i},\frac{1}{\omega_i},1);S) \tag{4.6}$$

This lemma states that the favorite tax rate function is equivalent to an increasing function of the Marshallian demand for public services. This lemma is used to prove that the favorite tax rate function is monotonic with respect to the private wealth if and only if the public services are either always a substitute or always a complement to the housing and the available wealth. This condition is called the Gross Substitutability Complementarity (GSC) condition.

Definitionsolution If the GCS condition holds, then, one has either $\frac{Z^M(p_Z,p_x,p_h,R)}{p_x} \leq 0 \forall (p_Z,p_x,p_h,R)$ (if Z is a gross complement to x) or $\frac{Z^M(p_Z,p_x,p_h,R)}{p_x} \geq 0 \forall (p_Z,p_x,p_h,R)$ (if Z is a gross substitute to x)

Remarksolution For all utility function that are homothetically separable between the public services on one hand and the housing and private consumption on the other, the public services is a complement (resp. a substitute) to the private consumption if and only if it is also a complement (resp. a substitute) to the housing.

Lemma 4.4.2. *If public services are a non-Giffen good, then, for all utility functions belonging to* $\mathbb{U}$ *and all production function* π *that are increasing and concave with respect to* ζ*, the favorite tax rate function is always monotonic with respect to private wealth if and only if the public services are a gross substitute or a gross complement to the 2 other goods.*

Proof.

To prove this lemma, we will show that the derivative of the preferred tax rate function with respect to the private wealth can be expressed as a negative function of the derivative of the Marshallian demand for the public services with respect to the housing price. Consequently, the preferred tax rate function will be monotonic with respect to the private wealth if and only if the Marshallian demand for public services are monotonic with respect to the housing price (or the composite private good price).

By deriving (4.1) with respect to the private wealth, one gets :

$$\frac{\partial t^*(F,S,p,\omega_i)}{\partial \omega_i}$$

$$=$$

$$\frac{-1}{\pi_\zeta(\zeta,S)}\left[\frac{\pi_{\zeta\zeta}(\zeta,S)}{F\pi_\zeta^2(\zeta,S)}\frac{\partial Z^M(\frac{1}{F\pi_\zeta(\zeta,S)},\frac{p}{\omega_i},\frac{1}{\omega_i},1)}{\partial p_Z}\frac{\partial t^*(F,S,p,\omega_i)}{\partial \omega_i}+\frac{p+k}{\omega_i^2}\frac{\partial Z^M(\frac{1}{F\pi_\zeta(\zeta,S)},\frac{p}{\omega_i},\frac{1}{\omega_i},1)}{\partial p_h}\right]$$

with

$$\frac{\partial Z^M(\frac{1}{F\pi_\zeta(\zeta,S)},\frac{p}{\omega_i},\frac{1}{\omega_i},1)}{\partial p_x} = k\frac{\partial Z^M(\frac{1}{F\pi_\zeta(\zeta,S)},\frac{p}{\omega_i},\frac{1}{\omega_i},1)}{\partial p_h}$$

with $k > 0$, then one has :

$$(1+\frac{\pi_{\zeta\zeta}(\zeta,S)}{F\pi_\zeta^2(\zeta,S)}\frac{\partial Z^M(\frac{1}{F\pi_\zeta(\zeta,S)},\frac{p}{\omega_i},\frac{1}{\omega_i},1)}{\partial p_Z})\frac{\partial t^*(F,S,p,\omega_i)}{\partial \omega_i} = \frac{-(p+k)}{\omega_i^2}\frac{\partial Z^M(\frac{1}{F\pi_\zeta(\zeta,S)},\frac{p}{\omega_i},\frac{1}{\omega_i},1)}{\partial p_h}$$

Since the public services is assumed to be a non-Giffen good, and since $\pi(\zeta,S)$ is concave with respect to ζ, then $(1+\frac{\pi_{\zeta\zeta}(\zeta,S)}{F\pi_\zeta^2(\zeta,S)}\frac{\partial Z^M(\frac{1}{F\pi_\zeta(\zeta,S)},\frac{p}{\omega_i},\frac{1}{\omega_i},1)}{\partial p_Z}) > 0$, so

$$sign(\frac{\partial t^*(F,S,p,\omega_i)}{\partial \omega_i}) = -sign(\frac{\partial Z^M(\frac{1}{F\pi_\zeta(\zeta,S)},\frac{p}{\omega_i},\frac{1}{\omega_i},1)}{\partial p_h})$$

The GSC condition is restrictive enough to be discussed, but nevertheless it is not outlandish. For instance, suppose that the only competence the central government has transferred to the studied jurisdictions level is social aid. Then, one may assume that the public services will be a substitute to the two other goods. On the contrary, if the jurisdiction is competent only in cultural activities, then the public services will probably be a complement. Suppose now that the jurisdiction is in charge of primary schools. In this case, the relation between the public services and the other goods is not trivial, and may vary with respect to the jurisdiction's parameters and the private wealth.

To prove the sufficiency of the GSC condition, the notion of indifference curve has to be introduced.

$\forall(t,\bar{u},p,S,\omega_i) \in [0;1]\times\mathbb{R}_+^4$, let us define $F^{\bar{u}}(\bar{u},t,p,S,\omega_i)$:

$$U(\pi(tF^{\bar{u}}(\bar{u},p,t,S,\omega_i),S),h^M(p,(1-t)\omega_i),x^M(p,(1-t)\omega_i)) \equiv \bar{u}$$

as the indifference curve of a household with private wealth ω_i, that is to say the amount of fiscal potential the household needs to reach utility $\bar{u}$ in a jurisdiction with housing price p, tax rate t and spillovers S.

The assumptions imposed on the utility function and on $\pi(.)$ ensure the existence and the derivability of F^u.The slope of the indifference curve in the plane (t,F) is given by

$$F_t^{\bar{u}}(\bar{u},t,p,S),\omega_i) = \frac{1}{t}[\frac{\omega_i}{\pi_\zeta(tF,S)MRS^{\bar{u}}(\pi(t\bar{F},S),(1-t)\omega_i)} - \bar{F}] \tag{4.7}$$

The next lemma, that will be used to prove the sufficiency of the GSC condition to have every stable jurisdictions structure segregated, states that the GSC condition implies the ordering of the indifference curves slopes with respect to wealth.

Lemma 4.4.3. *For any preferences belonging to* $\mathbb{U}$ *and any production function that is increasing and concave with respect to each argument, one has* $\frac{\partial F^{\bar{u}}(\bar{u},t,p,S,\omega_i)}{\partial t} \leq$ *(resp.* $\geq$ $)\frac{\partial F^{\bar{u}}(\bar{u},t,p,S,\omega_k)}{\partial t} \forall (t,p,S) \in [0;1] \times \mathbb{R}_+^2, \omega_i < \omega_k$ *and* $\forall U \in \mathbb{U}$ *if the public services is a gross substitute for (resp. a gross complement to) the two other goods*[7].

Proof.

This lemma states that if the GSC condition holds, then, for any given housing price, any given amount of spillovers and any tax rate, the slope of the indifference curves in the (t, F) space is monotonic with respect to the private wealth. We prove this lemma by using the definition of $F^{\bar{u}}(\bar{u}, t, p, S, \omega_i)$ introduced above. The proof is provided for the gross complementary case, the gross substitutability case being symmetric. Assume that the public services is a gross complement to the two other goods. Then, by definition, $\frac{\partial Z^M(p_Z,p_h,p_x,\omega_i}{\partial p_h} < 0$ and $\frac{\partial Z^M(p_Z,p_h,p_x,\omega_i}{\partial p_x} < 0$. Let $(t, F, S, p) \in [0;1] \times \mathbb{R}_+^4$ be a certain combination of tax rate, fiscal potential, spillovers and housing price and $(a, b) \in \mathbb{R}_+^2$ two amount of private wealth $(a < b)$. Let us define $\mathbb{F}(a)$ and $\omega(a)$ such that

$$Z^M(\frac{1}{\pi_\zeta(tF,S)F(a)}, \frac{p}{\omega(a)}, \frac{1}{\omega(a)}, 1) = \pi(tF, S)$$

and

$$ph^M(\frac{1}{\pi_\zeta(tF,S)F(a)}, \frac{p}{\omega(a)}, \frac{1}{\omega(a)}, 1) + m^M(\frac{1}{\pi_\zeta(tF,S)F(a)}, \frac{p}{\omega(a)}, \frac{1}{\omega(a)}, 1) = (1-t)a$$

Hence, the Marginal Rate of Substitution between the public services on one hand, and the housing and the other expenditure on the other hand, which is a function $MRS^u(Z, h+m)$ is equal, at the optimum, to the price ratio, and, by definition, the chosen bundle respects the budget constraint :

$$MRS^u(\pi(tF,S), (1-t)a) = \frac{\omega(a)}{\pi_\zeta(tF,S)F(a)} \tag{4.8}$$

$$\frac{\pi(tF,S)}{\pi_\zeta(tF,S)F(a)} + \frac{(1-t)a}{\omega(a)} = 1 \tag{4.9}$$

[7]Actually, the ordering of the indifference curve slopes with respect to the private wealth is equivalent to the GSC condition, but the implication is sufficient to prove our theorem. However, it shows that the GSC condition implies the condition identified by Westhoff to ensure the existence of an equilibrium, when households have identical preferences.

Combining (4.8) and (4.9) yields :

$$\frac{1-t}{\pi_\zeta(tF,S)F(a)+\pi(tF,S)} = \frac{MRS^u(\pi(tF,S),(1-t)a)}{a}$$

Let us now define $\omega(b)$ such that $\frac{p}{\omega(b)}$ and $\frac{1}{\omega(b)}$ are respectively the highest housing price and available money price that would allow a household with private wealth 1 to afford the bundle (not necessarily the optimal one) $(\pi(tF,S),h,x)$, with $\frac{ph+x}{\omega(b)} = (1-t)b$, if the public services price is still $\frac{1}{\pi_\zeta(tF,S)F(a)}$. Given the budget constraint, one has :

$$\omega(b) = \frac{\pi_\zeta(tF,S)F(a)(1-t)b}{\pi_\zeta(tF,S)F(a)-\pi(tF,S)} > \omega(a) \tag{4.10}$$

Since the public services is a complement, then one must have :

$$Z^M(\frac{1}{\pi_\zeta(tF,S)F(a)},\frac{p}{\omega(a)},\frac{1}{\omega(a)},1) \leq Z^M(\frac{1}{\pi_\zeta(tF,S)F(a)},\frac{p}{\omega(b)},\frac{1}{\omega(b)},1)$$

Moreover, the slope of the indifference curve must be, in absolute value, more than the price ratio $\frac{\omega(k)}{\pi_\zeta(tF,S)F(a)}$:

$$MRS^{\bar{u}}(\pi(tF,S),(1-t)b) \geq \frac{\omega(k)}{\pi_\zeta(tF,S)F(a)}$$

which is equivalent to

$$\frac{MRS^{\bar{u}}(\pi(tF,S),(1-t)b)}{b} \geq \frac{(1-t)}{\pi_\zeta(tF,S)F(a)-\pi(tF,S)}$$

Using (4.10), one obtains:

$$\frac{MRS^{\bar{u}}(\pi(tF,S),(1-t)b)}{b} \geq \frac{MRS^{\bar{u}}(\pi(tF,S),(1-t)a)}{a}$$

$$\Leftrightarrow$$

$$\frac{b}{MRS^{\bar{u}}(\pi(tF,S),(1-t)b)} \leq \frac{a}{MRS^{\bar{u}}(\pi(tF,S),(1-t)a)}$$

Using the definition of $F_t^{\bar{u}}$ given by (4.7), the implication is established.

This lemma is particularly important to prove the sufficiency of this article's main result, which is the following theorem.

Theorem 1. *For any possible economy* $(\omega, U, C, B, \mathbb{L}) \in \Delta$*, every stable jurisdictions structure will be segregated if and only if* U *is such that the GSC condition is satisfied.*

Let us begin the proof of this theorem by the sufficiency of the condition.

Proposition 6. *For all economies* $(\omega, U, C, B, \mathbb{L}) \in \Delta$*,* ***if*** *the GSC condition holds, then every stable jurisdictions structure is segregated.*

Proof.

To prove this proposition, we use the lemma 4 to demonstrate that if a non-segregated jurisdictions structure arise at the equilibrium, then the GSC condition is not respected. This proof needs no assumption on how the spillovers coefficients are determined. Suppose that there exist 2 jurisdictions j_1 and j_2 with respective parameters (F_1, S_1, t_1, p_1) and (F_2, S_2, t_2, p_2) and 3 households with private wealth a, b, c, $a < b < c$, such that :

$$V^C(\pi(t_1F_1), S_1), \psi(p_1)(1-t_1)a) > V^C(\pi(t_2F_2, S_2), \psi(p_2)(1-t_2)a)$$

$$V^C(\pi(t_1F_1), S_1), \psi(p_1)(1-t_1)b) < V^C(\pi(t_2F_2, S_2), \psi(p_2)(1-t_2)b)$$

$$V^C(\pi(t_1F_1), S_1), \psi(p_1)(1-t_1)c) > V^C(\pi(t_2F_2, S_2), \psi(p_2)(1-t_2)c)$$

Suppose, with no loss of generality, that $p_1 > p_2$.

Consider the hypothetical jurisdiction j_0 with parameters (F_0, S_2, t_0, p_2) with $t_0 = 1-(1-t_1)\frac{\phi(p_1)}{\phi(p_2)}$ and $F_0 = \frac{\pi^{-1}(\pi(t_1F_1, S_1); S_2)}{t_0}$ Hence, every household is indifferent between j_1 and j_0, because in both jurisdictions, the amount of available public services is the same and their purchasing power is the same. Then,

$$V^C(\pi(t_0F_0), S_2), \psi(p_2)(1-t_0)a) > V^C(Z_2, \psi(p_2)(1-t_2)a)$$

$$V^C(\pi(t_0F_0), S_2), \psi(p_2)(1-t_0)b) < V^C(Z_2, \psi(p_2)(1-t_2)b)$$

$$V^C(\pi(t_0F_0), S_2), \psi(p_2)(1-t_0)c) > V^C(Z_2, \psi(p_2)(1-t_2)c)$$

which, according to the lemma 4, is impossible if the GSC condition holds.

Now that the sufficiency of the GCS condition to have all stable jurisdictions structure segregated has been proved, the following proposition states that it is also necessary, by showing that any violation of the GCS condition allows to construct a non-segregated but

yet stable jurisdictions structure.

Proposition 7. *For all economies belonging to Δ, every stable jurisdictions structure will be segregated **only if** the GSC condition holds.*

Proof.

The proof of this proposition consists in constructing a stable and yet non-segregated jurisdictions structure. We are free to determine the number of jurisdictions, their available amount of housing, the mass of each type of households in every jurisdiction, and the spillovers coefficients matrix, in order to generate the housing price, the fiscal potential and the amount of spillovers for which the violation of the GSC condition arise.

Consider an utility function violating the GCS condition for some $(\bar{F}, \bar{S}, \bar{p}) \in \mathbb{R}_+^3$ and some non-degenerated interval $W \subset \mathbb{R}_+$. Using lemma 3, the monotonicity of the favorite tax rate function is known to be equivalent to the GSC condition, so we know for sure that there exist $(a, b, c) \in W^3$, with $a < b < c$, such that $t^*(\bar{F}, \bar{S}, \bar{p}, a) = t^*(\bar{F}, \bar{S}, \bar{p}, c) > t^*(\bar{F}, \bar{S}, \bar{p}, b)$ (the proof is the same if the favorite tax rate is increasing and then decreasing with respect to the private wealth). Then one can always construct a stable and non-segregated jurisdictions structure. Let us create 2 subsets of jurisdictions, both jurisdictions having a fiscal potential $\bar{F}$, a housing price p and receiving an amount of spillovers $\bar{S}$:

- jurisdictions belonging to J_1 are composed of certain measures μ_a and μ_c of households endowed with private wealth a and c, and apply a tax rate $t_1 = t^*(\bar{F}, \bar{S}, \bar{p}, a) = t^*(\bar{F}, \bar{S}, \bar{p}, c)$
- jurisdictions belonging to J_2 are composed of a certain measure μ_b of households endowed with private wealth b, and apply a tax rate $t_2 = t^*(\bar{F}, \bar{S}, \bar{p}, b)$.

Such a jurisdictions structure is clearly non-segregated, because the 2 different types of jurisdiction provide different amounts of public services. Moreover, no household has incentive to leave its jurisdiction, since its favorite tax rate is applied, and other parameters are the same in all the other jurisdictions. Let us now prove that there always exist positive measures μ_a, μ_b, μ_c, integers M_1 and M_2 of jurisdictions of respectively type 1 and type 2, a matrix of spillovers coefficients and available amount of housing H_1 and H_2 such that, in each jurisdiction :

- Spillovers are equal to $\bar{S}$,
- Fiscal potential is equal to $\bar{F}$,
- Housing price $\bar{p}$ is competitive.

We define respectively $\zeta_1 = t_1\bar{F}$ and $\zeta_2 = t_2\bar{F}$ as the amount of public services produced by jurisdictions of type 1 and of type 2. For simplicity, jurisdictions in J_1 will not create any spillovers for jurisdictions belonging to J_2, and vice-versa. Since the function $S(.)$ is non-bounded from above, we know that

$$\forall S \in \mathbb{R}_+, \forall(\tilde{\beta}, \tilde{\zeta}) \in \mathbb{R}_+^{*2}, \exists M \in \mathbb{N} : (M-1)\tilde{\beta}\tilde{\zeta} < S \leq M\tilde{\beta}\tilde{\zeta}$$

In words, for any strictly positive amount of produced public services and spillovers coefficient, by duplicating a jurisdiction a certain number of times, any amount of spillovers can be bounded from below and from above, even if spillovers coefficients depend on the amount of public services produced by the jurisdiction can generate or receive spillovers. Since $S = 0$ when the spillovers coefficient are null, one can deduce, using the Theorem of Intermediate Value, that $\exists \beta^* \in [0; \bar{\beta}] : M\beta^*\tilde{\zeta} = S$

As a consequence, we can always find (M_1, β_1) and (M_2, β_2) such that

$$M_1\beta_1\zeta_1 = M_2\beta_2\zeta_2 = \bar{S}$$

Now, let us prove that we can always find a positive measures μ_a, μ_b, μ_c such that, in each jurisdiction, the fiscal potential is $\bar{F}$. Let us consider a jurisdiction j belonging to J_2. This jurisdiction is only composed of households endowed with a private wealth b. Since all jurisdictions in J_2 have the same measure of households and the same spillovers coefficients, the congestion function can be re-written as a function of the measure of households, the spillovers coefficient and the number of jurisdictions in J_2. Let us define $C^{J_2}(\beta_2, \mu_b, M_2) = C(\{\mu_b\}_{j\in J_2}, \{\beta_2\}_{l\in J_2})$. Hence, the fiscal potential of this jurisdiction j is $F_j = \frac{\mu_b b}{C(\beta_2, \mu_b, M_2)}$.

Using the properties assumed on the congestion function, we can prove that

$$\lim_{\mu_b \to +\infty} \frac{\mu_b b}{C(\beta_2, \mu_b, M_2)} \to +\infty$$

Indeed, since

$$\forall(j, k, k') \in J^3, \forall\{\mu_l\}_{l\in J} \in \mathbb{R}^M, \forall\{\beta_{lj}\}_{l\in J} \in [0; 1]^M$$

one has

$$\frac{\partial^2 C_j(\{\mu_l\}_{l\in J}, \{\beta_{lj}\}_{k\in J})}{\partial\mu_k \partial\mu_{k'}} \leq 0$$

Then $\forall(j,k,k') \in J^3, \forall(\mu_k, \mu_{k'}) \in \mathbb{R}^2_{++}$, and $\forall \mu'_{k'} > \bar{\mu}_{k'}$, one has

$$0 < \frac{\partial C_j(\mu_1, ..., \mu_k, ..., \mu'_{k'}, ..., \mu_{M_2}, \{\beta_{lj}\}_{l\in J})}{\partial \mu_k} \leq \frac{\partial C_j(\mu_1, ..., \mu_k, ..., \bar{\mu}_{k'}, ..., \mu_{M_2}, \{\beta_{lj}\}_{l\in J})}{\partial \mu_k}$$

As a consequence,

$$0 < \lim_{\mu'_{k'}\to+\infty} \frac{\partial C_j(\mu_1, ..., \mu_k, ..., \mu'_{k'}, ..., \mu_{M_2}, \{\beta_{lj}\}_{l\in J})}{\partial \mu_k} \leq \frac{\partial C_j(\mu_1, ..., \mu_k, ..., \bar{\mu}_{k'}, ..., \mu_{M_2}), \{\beta_{lj}\}_{l\in J}}{\partial \mu_k}$$

By definition, one has $\lim_{\mu_k\to+\infty} \frac{\partial C_j(\mu_1,...,\mu_k,...,\bar{\mu}_{k'},...,\mu_{M_2},\{\beta_{lj}\}_{l\in J})}{\partial \mu_k} = 0$. Then, using the version of the Squeeze theorem[8] provided in Matousek[69][9], one can deduce that

$$\lim_{\mu_k\to+\infty} \lim_{\mu'_{k'}\to+\infty} \frac{\partial C_j(\mu_1, ..., \mu_k, ..., \mu'_{k'}, ..., \mu_{M_2}, \{\beta_{lj}\}_{l\in J})}{\partial \mu_k} = 0$$

This proof can be re-iterated to show that, $\forall(j,k) \in J^2, \forall\{\mu_l\}_{l\in J} \in \mathbb{R}^M_{++}, \forall\{\beta_{lj}\}_{l\in J} \in \mathbb{R}^M_{++}$,

$$\lim_{\mu_1\to+\infty} \lim_{\mu_2\to+\infty} ... \lim_{\mu_M\to+\infty} \frac{\partial C(\{\mu_l\}_{l\in J}, \{\beta_l j\}_{l\in J})}{\partial \mu_k} = 0$$

So, $\forall\beta_b \in [0;1]$ and $\forall M_2 \in \mathbb{N}$, one has

$$\lim_{\mu_b\to+\infty} \frac{\partial C^{J_2}(\mu_b, \beta_2, M_2)}{\partial \mu_b} = 0$$

Using L'hopital's rule[10] [23], one can show that:

$$\lim_{\mu_b\to+\infty} \frac{\mu_b b}{C^{J_2}(\mu_b, \beta_2, M_2)} = \lim_{\mu_b\to+\infty} \frac{b}{\frac{\partial C^{J_2}(\mu_b,\beta_2,M_2)}{\partial \mu_b}} = +\infty$$

Moreover, if the mass of households in a jurisdiction is null, then so is its fiscal potential. Hence, by the Intermediate Value Theorem, we know that there exists μ^*_b such that $\frac{\mu^*_b b}{C(\beta_2,\mu^*_b,M_2)} = F$. The same reasoning can be applied for jurisdictions in J_1, taking a constant mass of households with private wealth a over mass of households with private wealth c ratio. We now choose the available amount of housing H_1 and H_2 respectively in

[8] The Squeeze theorem (also called the Sandwich rule) states that if $\forall x \in E, u(x) \leq f(x) \leq v(x)$ then, $\forall \bar{x} \in E$ such that $\lim_{x\to\bar{x}} u(x) = l$ and $\lim_{x\to\bar{x}} v(x) = l$, one has $\lim_{x\to\bar{x}} f(x) = l$.

[9] 2003, Chap. 3, Section 1, the Ham Sandwich Theorem

[10] L'Hopital's rule states that $\lim_{x\to\bar{x}} \frac{u(x)}{v(x)} = \lim_{x\to\bar{x}} \frac{u'(x)}{v'(x)}$ if u and v are differentiable.

jurisdictions in J_1 and J_2 such that the housing price $\bar{p}$ is competitive, i.e.

$$H_1 = \mu_a h^M(\bar{p}, (1-t_1)a) + \mu_c h^M(\bar{p}, (1-t_1)c)$$

$$H_2 = \mu_b h^M(\bar{p}, (1-t_2)b)$$

Then, for any violation of the monotonicity of the preferred tax rate function with respect to the private wealth, one can always construct a stable and yet non-segregated jurisdictions structure.

4.5 Conclusion

The conclusion of this paper is that neither the congestion nor the existence of spillovers across jurisdictions modify the necessity or the sufficiency of the GSC condition to ensure the segregation of every stable jurisdictions structure in a model *a la* Westhoff.

However, this result does not imply that introducing congestion and spillovers into a model *a la* Westhoff would have no impact on the stability or on the segregative properties of endogenous jurisdictions structures formation, as the example provided in section 3 proved it.

The presence of congestion effects obviously mitigates the existence of an equilibrium. Consequently, stronger conditions must be found in order to ensure that a stable jurisdictions structure will arise.

This condition is robust to several generalizations of the model. Searching for a generalization that would make the condition either too weak or too strong to have all stable jurisdictions structures would be a interesting objective for further researches.

Conclusion générale

L'étude des déterminants de la ségrégation par la richesse entre les communes d'une même zone urbaine est un enjeu important en terme de politiques publiques. En effet, la ségrégation par la richesse présente des avantages, tels que la possibilité rendue plus aisée d'opérer une politique de redistribution entre les juridictions, et l'amélioration du lien social entre ménages similaires, et des inconvénients, comme l'accroissement de la criminalité aux alentours de juridictions durement touchées par la pauvreté extrême, ou l'absence de mixité sociale. S'il est vrai que les recherches portant sur la nature positive ou négative de l'impact de la ségrégation sur le bien-être social sont rares, il n'en reste pas moins nécessaire de s'intéresser aux causes de la ségrégation, afin de pouvoir soit la favoriser soit l'empêcher.

Ce constat conduit à s'intéresser à la formation de juridiction en premier lieu. Suivant les intuitions de Tiebout, les ménages choisissent leur juridiction en fonction d'un compromis entre le taux de taxe à acquitter et la quantité de bien public disponible. La formation de juridictions est donc endogène, puisque les ménages peuvent quitter sans coût leur juridiction pour une autre qui applique un taux de taxe et propose une quantité de bien public disponible plus avantageux. Toujours selon les intuitions de Tiebout, ce processus de formation endogène de juridictions conduit les ménages à se répartir dans des juridictions homogènes.

Les intuitions de Tiebout, non formalisées, se basent sur des hypothèses fortes : absence

de coût de mobilité, absence de marché du logement, unicité, non-rivalité et exclusivité parfaites du bien public local dans chaque juridiction, absence de gouvernement central...

L'étude des propriétés ségrégatives de la formation endogène de juridictions soulève quatre principales questions traitées dans les différents chapitre de cette thèse :

- Comment la littérature économique a-t-elle modélisée la formation endogène de juridictions, quelle(s) définition(s) de la ségrégation propose-t-elle, et quels les résultats théoriques et empiriques en matière d'existence d'un équilibre et de son caractère ségrégé nous apporte-t-elle ?

- L'existence d'une politique de péréquation fiscale mise en œuvre par un gouvernement central poursuivant un objectif bien-êtriste est-elle de nature à modifier les propriétés ségrégatives de la formation endogènes de juridictions ?

- Comment l'introduction d'un marché compétitif du logement, ainsi que l'existence de plusieurs biens publics locaux, modifient-elles les structures de juridictions stables ?

- Admettre que le bien public local d'une juridiction est susceptible de générer des retombées économiques dans les autres juridictions et de souffrir d'effets de congestion a-t-il un impact sur l'équilibre et la ségrégation des structures de juridictions ?

Le premier chapitre de cet ouvrage apporte un aperÃ§u de la littérature théorique et empirique sur la formation endogène de juridictions. Nous commençons à partir des intuitions de Tiebout : les ménages révèlent leurs préférences pour le bien public en choisissant leur communauté de résidence en fonction d'un compromis entre le taux d'imposition en œuvre par les juridictions différents et la quantité de bien public qu'elles produisent. Ensuite, le document analyse la faÃ§on dont de telles intuitions ont été modélisées dans la littérature, et les différentes questions et critiques que les économistes ont examinées : l'existence d'un équilibre, ses propriétés en termes d'efficacité, la définition de la ségréga-

tion, et enfin les facteurs pro et anti-ségrégation.

Dans le modèle à deux biens (un bien public et un bien privé) proposé par Westhoff, une condition suffisante à l'existence d'un équilibre est la monotonie par rapport à la richesse individuelle des pentes des courbes d'indifférence entre le taux de taxe (assise sur la richesse des ménages) et la quantité de bien public disponible. Cette condition est également suffisante pour que toute structure de juridictions stable soit également ségrégée.

Les articles théoriques et empiriques développés depuis indiquent que la ségrégation est favorisée par la monotonie par rapport à la richesse de l'acceptation à payer pour le bien public, toute chose égale par ailleurs, et par le fait que le bien public puisse être partiellement ou totalement rival.

Par contre, les différences de préférences pour le bien public, l'existence d'un marché compétitif du logement, l'existence de coûts de mobilité et la valeur intrinsèque des localités sont des éléments venant mitiger les propriétés ségrégatives du processus de formation endogène de juridictions.

Des éléments de réponse à la seconde question sont apportés par le second chapitre. Ce chapitre examine les propriétés ségrégative des processus endogènes de la formation de juridiction à la Tiebout, en présence d'un gouvernement central qui met en place une politique de péréquation fiscale de manière à maximiser un objectif bien-êtriste. Le choix du lieu par les ménages, de la quantité de bien public local produite par les administrations, des transferts et du taux d'imposition déterminés par le gouvernement central sont supposées être faits simultanément, en considérant les choix des autres comme donné.

Si le gouvernement central poursuit un objectif maxmin, il est facile de montrer que

les seules structures de juridictions stables qui peuvent émerger sont celles dans lesquelles les ménages les plus pauvres de chaque juridiction ont tous la même richesse.

Une classe de structures de juridictions stables plus riche est compatible avec un gouvernement central dont l'objectif est l'utilitarisme généralisé. Pourtant, il s'avère que, si l'analyse est limité aux ménages ayant des préférences additivement séparable, la classe de préférences qui garantissent la séparation des richesses de toute structure stable n'est pas affectée par la présence d'un gouvernement central.

Le troisième chapitre de cette thèse répond à la question 3. Ce chapitre examine les propriétés ségrégatives des processus endogènes de formation de juridictions à la Tiebout, en présence d'un marché concurrentiel du logement. Dans le modèle considéré, un continuum de ménages avec différents niveaux de richesse et les mêmes préférences pour les biens publics locaux, les dépenses privées et le logement choisissent un emplacement parmi un ensemble fini. Chaque emplacement dispose d'une dotation initiale de logements dont le prix est concurrentiel appartenant à des propriétaires absents du modèle et dispose d'une technologie pour produire les biens publics.

Les préférences des ménages sont supposés être homothétiquement séparables entre les biens publics d'une part et les dépenses privées et de logement, de l'autre. La production de biens publics est financée par un donné, mais indéterminée, mélange donné, mais indéterminée, de taxation (linéaire) assise sur la richesse et sur la valeur du logement. Ce chapitre montre que, si les préférences sont additivement séparables entre les biens publics d'un part et les dépenses privées (logement et bien privé) d'autre part, ou s'il n'existe qu'un seul type de bien public, les structures stables de juridictions seront ségrégées par la richesse si et seulement si tout bien public, pour toutes quantités fixées des autres biens publics, est soit toujours un substitut brut, ou soit toujours un complément de brut aux dépenses privées. Comme cette condition est aussi nécessaire et suffisante pour la sépara-

tion des structures de la juridiction stable sans marché foncier et avec un seul bien public, ce résultat montre que la terre l'introduction d'un marché du logement affecte moins les propriétés ségrégatives des structures stables de juridictions que l'existence de plusieurs biens publics.

Le quatrième et dernier chapitre de cette thèse analyse l'effet des externalités et la congestion des services publics locaux sur les propriétés ségrégatives de la formation endogène de juridictions. Les ménages choisissant de vivre au même endroit forment une juridiction, dont le but est pour produire des services publics locaux pouvant souffrir de congestion et créer des retombées positives dans d'autres juridictions. Dans chaque juridiction, la production de services publics locaux est financée par une taxe locale assise sur la richesse des ménages. Les taux d'imposition sur la richesse locaux sont déterminés démocratiquement dans toutes les juridictions. Les ménages consomment également du logement dans leur juridiction. Tout ménage est libre de quitter son juridiction pour une autre qui augmenterait son utilité.

Une condition nécessaire et suffisante pour que chaque structure de juridictions stable soit ségrégée par la richesse, pour une large classe de mesures de la congestion et toute structure de coefficients de retombées, est identifiée : les services publics locaux doivent être soit un substitut brut ou un complément brut du bien privé et du logement.

Les exemples fournis dans ce chapitre suggère que la condition suffisante à l'existence d'un équilibre ne suffit plus à le garantir en présence d'effets de congestion sur le bien public local, que la congestion favorise la ségrégation alors que les retombées économiques générées par le bien public d'un juridiction dans les autres juridictions la réduise, et enfin que la condition nécessaire et suffisante pour avoir toute structure stable de juridictions ségrégée par la richesse n'est plus suffisante, mais reste nécessaire. La présence d'un marche du logement affecte donc plus les propriétés ségrégatives de la formation endogène de juridictions

que l'existence d'effets de congestion et de retombées économiques inter-juridictionnelle si les préférences ne sont pas homothétiquement séparables entre le bien privé et le logement.

Proposer une définition plus réaliste de la ségrégation est nécessaire pour pouvoir juger de l'évolution des structures de juridictions et de la pertinence du qualificatif de "ségrégatif" associé au processus endogène de formation de juridictions. Il serait intéressant, lors de futures recherches, de chercher à identifier un indice mesurant la ségrégation qui satisferait plusieurs propriétés souhaitables.

Bien que n'étant plus suffisante suite à certaines généralisations du modèle, la condition de Complémentarité/ Substituabilité Brute a fait preuve d'une relative robustesse. Tester empiriquement cette condition est un objectif à poursuivre.

D'autres généralisation du modèle peuvent également avoir un impact sur le caractère ségrégé de la structure de juridictions à l'équilibre, comme la présence d'entreprises dans les différentes juridictions ayant plusieurs conséquences : le paiement de taxes, la création de pollution, la localisation de l'emploi...
Il serait intéressant, au cours de futures recherches, d'étudier théoriquement et empiriquement le rôle des entreprises dans la formation endogène de juridictions.

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