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**Modélisation des écoulements de gravité et des
ondes longues. Application à l'évaluation des
risques de catastrophes naturelles dans les
Antilles Françaises**

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Résumé

La thèse est consacrée à la recherche des catastrophes marines (tsunamis, ondes de tempête) dans les Antilles Françaises, en utilisant des modèles analytiques et numériques de mécanique des fluides. L'accent est mis sur le développement de modèles de mouvement de glissements de terrain et des vagues causées par ces glissements. Le modèle le plus simple du glissement de terrain «solide block» est utilisé pour évaluer les caractéristiques des flux pyroclastiques du volcan Soufrière-Hills (Montserrat). Un modèle plus complexe de modélisation de glissement de terrain (modèle de Savage-Hutter) a été étudié analytiquement, donnant ainsi une nouvelle famille de solutions exactes décrivant le mouvement de l'écoulement par gravité non linéaire dans une vallée ou dans un canyon sous-marin. Le modèle comprend des ondes simples («Riemann waves»), le cas d'un barrage qui cède («dam-break problem»), des solutions auto-similaires dans la M-forme et «parabolic cap». Grâce à la théorie linéaire de l'eau peu profonde nous avons étudié le processus de génération de tsunamis par des glissements de terrain de volume variable, se déplaçant à une vitesse variable dans un bassin de profondeur variable. Dans le cas d'un fond marin particulier (cas sans «réflexion»), les phénomènes de résonance ont été étudiés dans un bassin à profondeur variable. Nous avons utilisé des méthodes numériques pour la résolution non-linéaire des équations des eaux peu profondes afin d'analyser des catastrophes marines réelles (les ondes de tempête causées par le cyclone Lili en 2002, le tsunami volcanique de 2003 à Montserrat) et probables (un tsunami près des côtes de la Martinique). Des données sur les catastrophes naturelles marines près de la Guadeloupe ont été obtenues lors des enquêtes de terrain (tsunami volcanique en 2003, les ondes de tempête causée par l'ouragan Dean en 2007). Des catalogues des tsunamis et des ondes de tempête sont construits et des analyses statistiques effectuées, basées sur des données historiques, donnant des résultats de modélisation numérique ainsi que des études de terrain.

Abstract

The dissertation is devoted to research in the field of marine natural hazards (tsunamis, storm surges) in the French West Indies, using analytical and numerical models of fluid mechanics. Emphasis is placed on the development of models of landslide motion and generated tsunami waves. The simple «solid block» model is used to evaluate the characteristics of pyroclastic flow Soufriere Hills volcano (Montserrat). The «fluid model» of a landslide (so called Savage-Hutter model) is studied analytically; within this model a new family of exact solutions that describe the motion of nonlinear gravity flow in a valley or underwater canyon is found: nonlinear Riemann wave, dam break problem, self-similar solutions (M - wave and parabolic cap). In the framework of the linear shallow water theory the process of generation of tsunami waves by landslides of variable volume moving with variable velocity above the basin of variable depth is studied. For the specific bottom profile («reflectionless» one) the resonant phenomena is investigated in the basin of variable depth. Numerical methods are used to analyze marine hazards: historical (storm surges, caused by Cyclone Lili in 2002; volcanic tsunami 2003 on Montserrat) and possible events (possible tsunami of the coast of Martinique). Various data on marine natural disasters are obtained during field surveys (volcanic tsunami in 2003, storm surges caused by Hurricane Dean in 2007). Designed catalogs of tsunamis and storm surges are created based on results of numerical modeling and field studies; some statistical analysis is performed.

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Introduction

L'étude des catastrophes naturelles a une orientation pratique évidente. Au cours des seules 10 dernières années, plusieurs événements importants se sont produits. Parmi eux le tsunami en Indonésie de 2004 qui a tué 300.000 personnes; l'ouragan Katrina en 2005 dans l'océan Atlantique, qui a «noyé» la Nouvelle-Orléans et provoqué la mort de 1836 personnes; l'éruption volcanique en Islande en 2010, paralysant le trafic aérien dans toute l'Europe; un glissement de terrain en Chine en 2010, causant la disparition de deux mille personnes ; la sécheresse en France continentale en 2003. Les catastrophes naturelles sont fréquentes dans les Antilles Françaises (dont certaines sont examinées ci-dessous.) Dans le cadre de la thèse, nous considérerons surtout les catastrophes naturelles marines (tsunamis et houles cycloniques) qui occupent la première place en terme de dommages (environ 40%). L'amélioration des méthodes de prévision des catastrophes exige des efforts dans nombreuses branches de la recherche: géophysique, ingénierie, informatique servant à la simulation numérique.

Dans cette étude, nous mettons en évidence les aspects de modèles hydrodynamiques qui sont utilisés pour décrire les vagues des tsunamis et les houles cycloniques. Ces méthodes sont efficaces dans la théorie de mécanique des fluides. Tsunamis et houles cycloniques sont connus comme étant des vagues longues (en comparaison avec la profondeur de l'océan), et l'approximation hydrostatique est très souvent utilisée. Cette approximation permet d'écrire les équations dans deux dimensions pour le déplacement du niveau de l'eau et la vitesse d'écoulement. Ces équations sont hyperboliques, et la théorie mathématique est très bien conçue. Il y a par exemple un grand nombre de logiciels informatiques pour les résoudre. Plusieurs d'entre eux sont spécialement conçus pour résoudre les problèmes géophysiques : calculer les caractéristiques des tsunamis et des ondes de tempête. Néanmoins, la prévision des catastrophes naturelles marines est encore loin d'être parfaite. Par exemple, un récent tsunami au Chili (le 27 Février 2010), a traversé l'océan Pacifique et est venu atteindre l'île de Paramushir (îles Kouriles, Russie) avec une hauteur de 2 mètres une heure après le retrait de l'alerte de tsunami.

Dans la physique des vagues de tsunami, le mécanisme de génération de tsunamis par des glissements de terrain est un problème très intéressant. On estime que les glissements de terrain causés par les tremblements de terre peuvent générer des tsunamis importants, voir, par exemple le livre du Yalciner et al (2001). Actuellement, il y a un grand nombre de programme numériques développés pour le calcul des vagues de tsunami en fonction de différentes caractéristiques des glissements de terrain. Ces programmes sont largement utilisés pour la modélisation de phénomènes qui n'ont pas besoin d'une grande précision. Dans un même temps, les propriétés mathématiques de ces modèles, la possibilité d'obtenir des solutions analytiques et les critères de leur existence restent dans les «coulisses» des recherches géophysiques. Évidemment, l'application des méthodes de mécanique des fluides et la physique des vagues non-linéaires va aider à comprendre plus clairement l'applicabilité des modèles existants à ces problèmes. La solution analytique simplifiée peut trouver une application dans la reconstruction des événements historiques qui sont mal connus. Ainsi, le développement de la théorie de la génération de vagues par glissement de terrain, ainsi que l'étude de la dynamique des glissements de terrain reste un problème réel d'une grande importance pratique.

Cependant, il existe le désir d'application «immédiate» de nouvelles méthodes pour résoudre des problèmes pratiques concernant la prévision des catastrophes naturelles marines. Dans ce cas, il est intéressant d'avoir un «terrain» où les catastrophes naturelles d'origine marines sont relativement fréquentes. Ce sont, en particulier, les Petites Antilles dans les Caraïbes, où cyclones, éruptions volcaniques et tsunamis sont présents. Récemment, l'opinion selon laquelle il peut survenir des tsunamis de forte intensité générés par de forts tremblements de terre ou par glissements de terrain a fait surface (Grindlay et al., 2005). L'analyse des catastrophes marines dans des zones différentes et leur comparaison permettra de mieux comprendre les caractéristiques régionales des événements «extrêmes», permettant d'améliorer leur prévision. Ainsi, l'étude conjointe des différents types de catastrophes marines (tsunamis, houles cycloniques, glissements de terrain) dans une zone donnée est un véritable problème, qui a d'importantes applications pratiques.

Ce sont les problèmes abordés dans la thèse. Dans le premier chapitre une approche analytique pour étudier la dynamique des flux de gravité est développée. Dans le paragraphe 1.1, le mouvement des glissements de terrain est étudié. Dans le cadre de ce problème on cherche le régime critique de la propagation du glissement. Nous montrons que pour une description correcte du mouvement de glissement de terrain, il est nécessaire d'avoir des cartes topographiques de bonne résolution et donc les données les plus précises possibles du coefficient de frottement. Dans le paragraphe 1.2, les solutions analytiques qui décrivent la dynamique non-linéaire des glissements «liquides» dans le cadre du modèle de Savage-Hutter lorsque le glissement se déplace dans un canal incliné (canyons sous-marins et dans les montagnes). Il est montré que le profil transversal du canal influence fortement sur les caractéristiques du processus. Dans le paragraphe 1.3 nous étudions le mouvement de la «langue» du glissement de terrain en utilisant la transformation d'hodographe. Il nous permet de déterminer dans quels cas la pente peut s'effondrer.

Le deuxième chapitre examine le tsunami dans les Antilles Françaises. Le paragraphe 2.1 présente des données historiques sur les tsunamis et leur analyse. Il démontre que la région a connu des événements de toutes origines, y compris les teletsunamis, les tsunamis tectoniques et volcaniques. Dans ce dernier cas, les flux pyroclastiques sont entrés dans l'eau et ont continué à se déplacer comme les glissements de terrain sous-marin. Au paragraphe 2.2, nous étudions le tsunami provoqué par une éruption volcanique à Montserrat en 2003. On présente les données d'étude terrain et les résultats de la simulation numérique de la propagation du tsunami dans la théorie des eaux peu profondes. On étudie la question de l'impact de la profondeur du tremblement de terre sur les caractéristiques des vagues de tsunami dans le paragraphe 2.3 (l'application pour un tsunami possible près de Martinique). Dans le paragraphe 2.4 on a abordé le problème linéaire de génération de vagues par des glissements de terrain sous-marins de masse et vitesse variable dans un bassin de profondeur variable. Ces solutions analytiques peuvent clarifier le rôle de la résonance dans le processus de génération d'ondes.

Le troisième chapitre analyse les cyclones et tempêtes dans les Antilles françaises. Dans le paragraphe 3.1 les données historiques sur le passage du cyclone sur Guadeloupe sont analysées ainsi que leurs caractéristiques en fonction de la force du vent et de la chute de pression atmosphérique. Dans le paragraphe 3.2, les résultats d'étude terrain sur les houles cycloniques (cyclone Dean en 2007) en Guadeloupe sont résumés. Les simulations numériques des vagues causées par le cyclone Lili en 2002, sont discutées dans le paragraphe 3.3.

Les résultats principaux sont résumés dans la conclusion. Ils sont publiés dans 12 articles et ont été présentés lors des sessions annuelles de l'Union européenne de géophysique et plusieurs autres conférences.

1. Modélisation des écoulements de gravité provenant de la pente et les canyons sous-marins

Les glissements de terrain se déplaçant dans les canyons des volcans émergés et sous-marins et génèrent souvent des grosses vagues. L'exemple le plus célèbre est le glissement de terrain de 600 mètres de hauteur dans La baie Lituya (en Alaska) en 1958, associé à des éclaboussures d'eau de 524 m sur le côté opposé de la montagne, et des vagues à la sortie de la baie d'une hauteur de 23 mètres (Pelinovsky, 1996; Fritz et al., 2001). Autre exemple dans ma ville natale: le glissement de terrain s'est déclenché en haut du versant de la vallée de la rivière Volga près de Nijni Novgorod en 1597 (Didenkulova et Pelinovsky, 2006). Il a conduit à la formation d'une «angoisse terrible» dans la rivière, et les navires se sont échoués sur le rivage, à 43 m au dessus du niveau normal de l'eau. Depuis 1997, les écoulements pyroclastiques de Soufrière Hills (Montserrat) ont provoqué trois tsunامي enregistrés en Guadeloupe. Le séisme sous-marin déclenche des glissements de terrain qui peuvent conduire à la génération des vagues de tsunami (Yalciner et al., 2003). Ce phénomène nous montre l'importance d'étudier le rôle du mouvement de glissement de terrain au cours de la génération des vagues.

Les flux de glissements de terrain peuvent être «solide» (ils ne changent pas pendant le mouvement) et «liquide» (par exemple, de boue ou de écoulement pyroclastiques). Ils peuvent avoir une rhéologie complexe, ces flux peuvent être composites, contenant les fractions différentes. Pour les deux types, on utilise en mécanique des fluides et solides l'approximation de flux de «gravité».

Au début, dans les travaux théoriques qui ont décrit la genèse des vagues par les glissements de terrain (généralement sous-marins), le glissement a été considéré comme un corps rigide (bloc), se déplaçant à une vitesse constante. Aujourd'hui il existe des modèles plus réalistes qui prennent en compte la dynamique du glissement de terrain: la variabilité de la vitesse, la déformation de son profil, les processus d'érosion et d'accumulation (Aranson et al, 2008; Pirulli et Mangeney, 2008; Bouchut et Westdickenberg, 2004; Bouchut et al, 2003; Savage et Hutter, 1989, 1991; Heinrich et al., 1998, 2001). Dans le domaine de l'informatique, la plupart des modèles développés existe sous la forme de programmes informatiques utilisés pour résoudre des problèmes pratiques, qui ne nécessitent pas une grande précision. Dans ce cas, on ne fait pas attention à la physique du processus et aux propriétés (mathématiques et physiques) du modèle même. De nombreuses solutions pour évaluer la précision des solutions informatiques et pour juger l'applicabilité de ces modèles sont extrêmement fiables. Voici quelques travaux analytiques sur le mouvement des «solides» (Harbitz, 1992; Pelinovsky et Poplavsky, 1996; Watts, 2000) et «liquides» (Mangeney et al, 2000; Fernandez-Feria, 2006; Pudasaini et Hutter, 2007) de glissements de terrain et la génération de vagues par glissements de terrain (Tinti et al, 2001; Liu et al, 2003; Pelinovsky, 2003; Sammarco et Renzi, 2008).

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DIDENKULOVA I. I., NIKOLKINA I. F., PELINOVSKY E. N. Resonant amplification of tsunami waves generated by an underwater landslide. *Dokl. Earth Sci.*, 2011, vol. 436, n°1, p. 66-69.

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ZAHIBO N., PELINOVSKY E., TALIPOVA T., NIKOLKINA I. Self-similar solutions in the theory of the underwater landslide dynamics in inclined canyons. *Open Oceanogr. J.*, 2010b, vol. 4, p. 92-98.

1.1 Le mouvement du glissement «solide» sur la pente d'inclinaison variable

La dynamique du glissement de terrain «solide» sur la pente d'inclinaison variable (Fig. 1.1) est appliquée aux écoulements pyroclastiques qui descendent du volcan dans la mer.

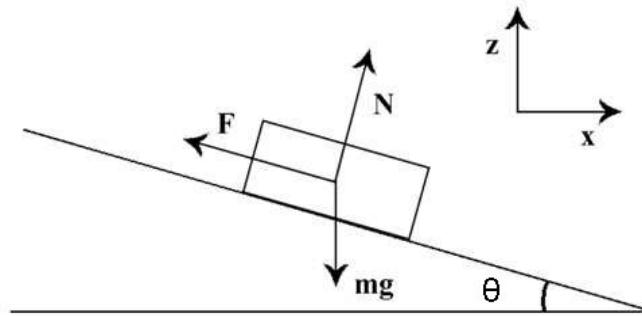


Fig. 1.1. Géométrie du problème: un bloc solide de masse m descend sur un plan incliné sous la force de la gravité mg , du frottement de Coulomb F et de la réaction du support N

Dans ce cas, l'écoulement est décrit par la 2ème loi de Newton

$$\frac{d^2x}{dt^2} = g \cos \theta(x) \sin \theta(x) - \mu g \cos^2 \theta(x), \quad (1.1)$$

où x est la coordonnée horizontale, g est l'accélération de la pesanteur, $\theta(x)$ est un angle variable d'inclinaison et μ est le coefficient de frottement de Coulomb. Selon ce modèle, on calcule le temps de descente depuis sommet et la vitesse d'entrée dans l'eau pour évaluer les paramètres initiaux du raz de marée. La principale difficulté est associée à l'incertitude du coefficient de frottement et de à faible résolution de topographie des cartes du volcan.

Les calculs sont effectués pour l'écoulement pyroclastique de volcan Soufrière Hills, Montserrat (le mer Caraïbes) où l'éruption continue des 15 dernières années a détruit la capitale Plymouth et provoqué trois cas de tsunamis documentés. Les calculs ont permis d'obtenir une régression entre les caractéristiques d'écoulement.

1.2 La dynamique non linéaire d'un glissement terrain «liquide»: des solutions exactes

La dynamique non linéaire de glissement de terrain «liquide» dans le canyon sous-marin ou dans une gorge de la montagne est étudiée dans le cadre du modèle de Savage-Hutter. Les équations basiques du modèle pour l'écoulement moyenné sur la section sont de la forme

$$\frac{\partial h}{\partial t} + u \frac{\partial h}{\partial x} + \frac{m}{m+1} h \frac{\partial u}{\partial x} = 0, \quad \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + g \cos \theta \frac{\partial h}{\partial x} = g \alpha, \quad (1.2)$$

où h est la hauteur de l'écoulement le long de l'axe du canal, u est la vitesse moyenne d'écoulement, θ - comme avant, l'angle d'inclinaison de la surface et $\alpha = \sin \theta - \mu \cos \theta$. Ici, nous utilisons l'approximation des invariants de Riemann de la forme

$$I_{\pm} = u \pm 2 \sqrt{\frac{m+1}{m}} \sqrt{gh \cos \theta - \alpha g t}, \quad (1.3)$$

Cela nous permet d'appliquer des approches connues dans la dynamique des gaz et de l'acoustique non linéaire. En particulier, l'onde de Riemann

$$\Gamma(x, t) = \Gamma_0 \left(x - \frac{\alpha g t^2}{2} - \Gamma t \right), \quad \Gamma = \frac{3m+2}{\sqrt{m(m+1)}} \sqrt{gh \cos \theta}, \quad u = 2 \sqrt{\frac{m+1}{m}} \sqrt{gh \cos \theta - \alpha g t}, \quad (1.4)$$

montre une tendance d'effondrement de la pente qui se trouve devant l'avalanche. Les effets non linéaires dans un des canaux plus étroits se développent plus rapidement. La déformation du glissement de terrain dans l'espace (dans un système de coordonnées en mouvement uniformément accéléré) est représentée sur la Fig. 1.2 pour les canaux des sections différentes.

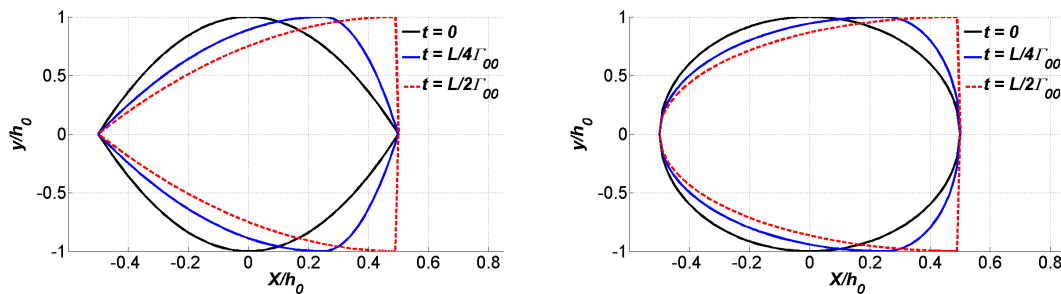


Fig. 1.2. Contours du glissement de terrain (à gauche) dans le canal triangulaire ($m = 1$) et (à droite) parabolique ($m = 2$)

Le problème de barrage de stockage (dam-break) pour un canal parabolique est résolu; la forme du glissement terrain est représentée sur la Fig. 1.3.

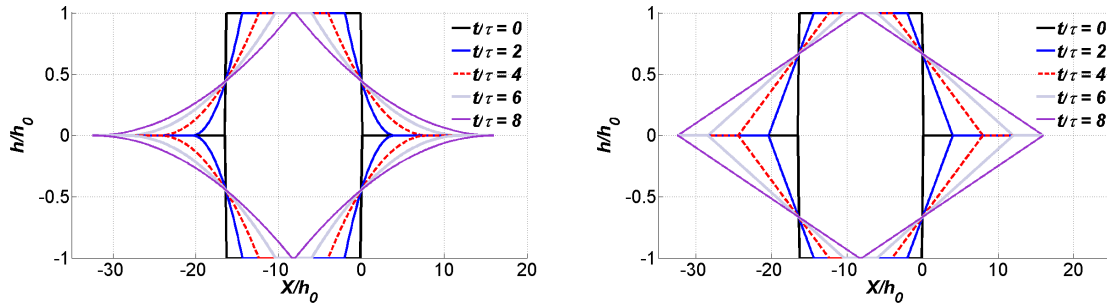


Fig. 1.3. Evolution d'un glissement de terrain initialement rectangulaire dans le cas (à gauche) du canal triangulaire ($m = 1$) et dans le cas (à droite) parabolique ($m = 2$)

Les divers types de solutions auto-similaires sont trouvés pour l'écoulement dans les canaux inclinés. Pour les coordonnées uniformément accélérées une de ces solutions (cap parabolique) a la forme

$$h = h_0(t) \left\{ 1 - [X / L(t)]^2 \right\} \quad h_0(t) = \frac{L d^2 L / dt^2}{2g \cos \theta}, \quad u = \frac{dL}{dt} \frac{X}{L(t)}, \quad L^{\frac{2m+1}{m+1}} \frac{d^2 L}{dt^2} = q, \quad (1.5)$$

et dans un canal plus étroit, la propagation du glissement de terrain est plus lent (Fig. 1.4).

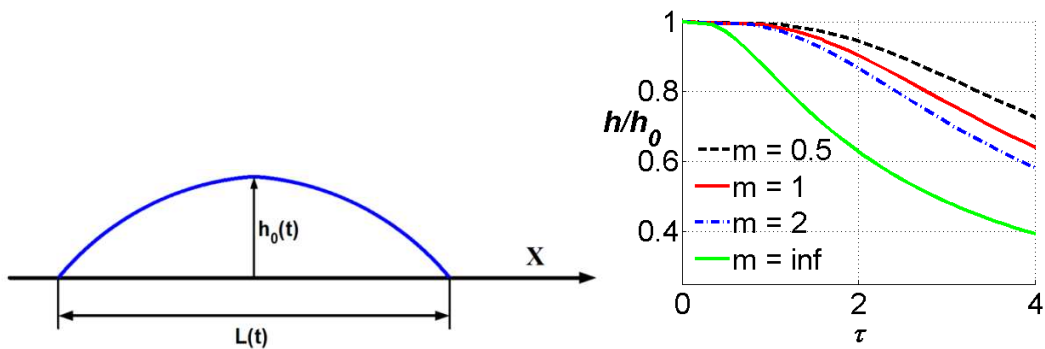


Fig. 1.4. Le cap parabolique dans la section longitudinale (à gauche); la hauteur du glissement de terrain en fonction du temps sans dimension (à droite)

1.3 La dynamique du front de l'avalanche

La dynamique non linéaire du front de l'avalanche dans un canal de section parabolique ($m = 2$) est étudiée. Les équations (1.2) ont réduit une équation d'onde linéaire (1.6) en utilisant la transformation hodographe où la hauteur des glissements de terrain H , la vitesse V , la coordonnée (dans un système uniformément accéléré) X et T , temps sont déterminées par la fonction d'onde Φ grâce à l'équation (1.7).

$$\frac{\partial^2 \Phi}{\partial \lambda^2} - \frac{\partial^2 \Phi}{\partial \sigma^2} - \frac{2}{\sigma} \frac{\partial \Phi}{\partial \sigma} = 0, \quad (1.6)$$

$$H = \frac{\sigma^2}{6}, \quad X = \frac{V^2}{2} + \frac{\sigma^2}{6} - \frac{1}{3} \frac{\partial \Phi}{\partial \lambda}, \quad T = V - \lambda, \quad V = \frac{1}{\sigma} \frac{\partial \Phi}{\partial \sigma}. \quad (1.7)$$

Pour le glissement terrain qui est initialement au repos ($u = 0$), les conditions initiales ont la forme suivante

$$\Phi(\lambda = 0, \sigma) = 0, \quad (1.8)$$

$$\left. \frac{\partial \Phi}{\partial \lambda} \right|_{\lambda=0} = \frac{\sigma^2}{2} - 3\alpha g x(\sigma), \quad (1.9)$$

où $x(\sigma)$ est déterminée à partir de la forme initiale du glissement terrain dans la section longitudinale,

$$h(x) = \frac{1}{4g \cos \theta} \frac{2}{3} \sigma^2. \quad (1.10)$$

L'un des conditions limites sur le bord du glissement terrain ($\sigma = 0$) est déterminé par la limitation de la fonction d'onde $\Phi(0, \lambda)$. En général, l'autre limite de glissements de terrain ($h = h_{\max}$, ce qui correspond à $\sigma = \sigma_{\max}$) n'est pas définie et doit être retrouvée à partir des mêmes équations. Il y a deux exceptions: a) la hauteur initiale du glissement de terrain tend vers une constante (problème de la destruction d'une avalanche), et b) la hauteur d'un glissement terrain augmente de façon monotone avec la distance de front d'avalanche. Ici, nous considérons le deuxième cas,

et l'équation (1.6) est résolue pour $0 < \sigma < \infty$. Ensuite, la condition limite à $\sigma \rightarrow \infty$ est la condition de radiation de Sommerfeld (pas de sources à l'infini). La solution de l'équation (1.6) a la forme

$$\Phi(\lambda, \sigma) = \frac{\Psi(\lambda, \sigma)}{\sigma}, \quad \Psi(\lambda, \sigma) = \begin{cases} \Omega(\sigma + \lambda) - \Omega(\lambda - \sigma) & 0 < \sigma < \lambda, \\ \Omega(\sigma + \lambda) - \Omega(\sigma - \lambda) & \lambda < \sigma, \end{cases} \quad (1.11)$$

L'évolution du glissement de terrain ($H \sim X^{0.5}$) dépend de sa courbure initiale: dans un cas, le flux du front est «étendu» (Fig. 1.5), tandis que dans l'autre, il devient plus raide (Fig. 1.6).

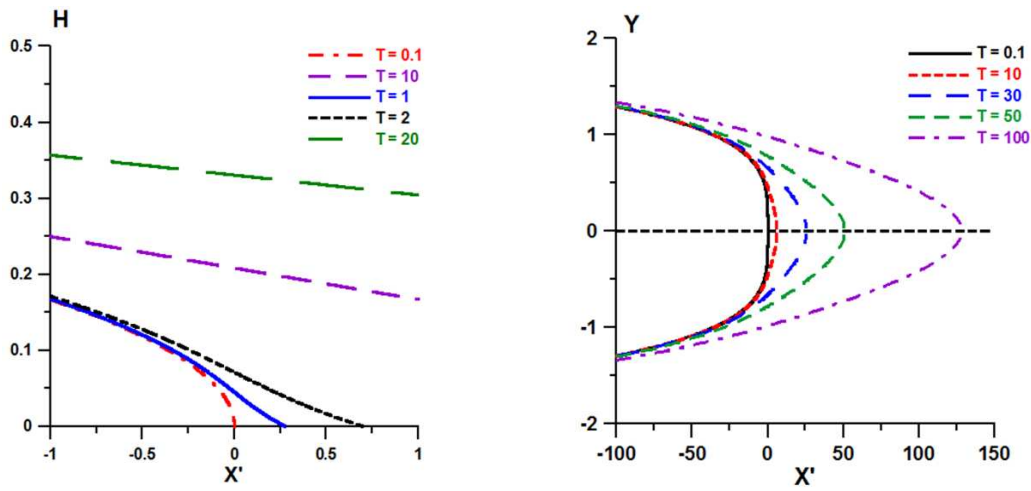


Fig. 1.5 Evolution de la partie frontale du glissement de terrain en fonction du temps (à gauche) et de l'espace (à droite) (la forme initiale $H \sim X^{0.5}$)

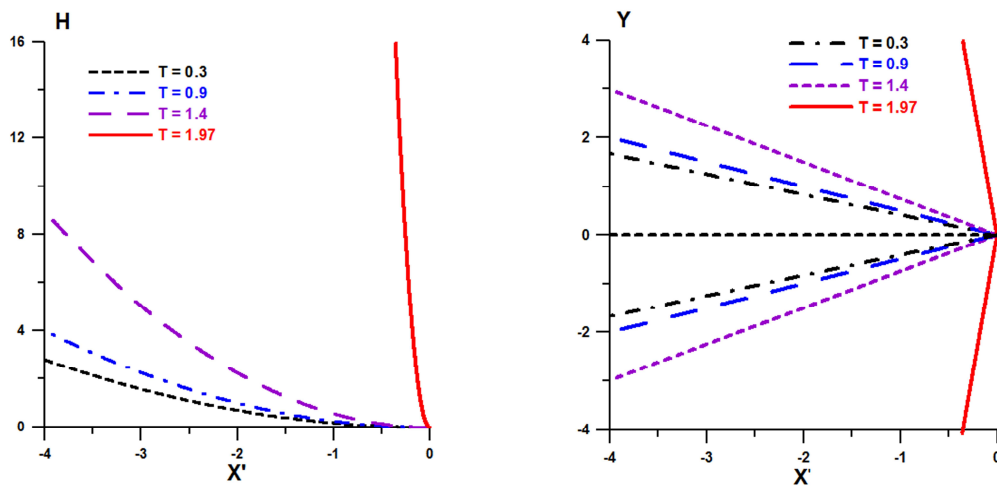


Fig. 1.6. (a) Evolution d'une forme de glissement de terrain parabolique avant l'effondrement ($q = 1$, $H \sim X^2$); (b) structure spatiale d'un glissement terrain parabolique pour des moments différents

1.4 Génération de tsunami par les glissements de terrain de masse et vitesse variable

Dans ce paragraphe, nous présentons de nouvelles solutions analytiques pour décrire la génération des vagues longues générées par les glissements de terrain de masse et vitesse variable se déplaçant dans un bassin de profondeur variable. Elles ont été obtenues dans le cadre d'un problème linéaire lorsque le système d'équations en eau peu profonde peut être réduit à l'équation des ondes à coefficients variables

$$\frac{\partial^2 \eta}{\partial t^2} - \frac{\partial}{\partial x} \left[c^2(x) \frac{\partial \eta}{\partial x} \right] = \frac{\partial^2 z_b}{\partial t^2}, \quad c = \sqrt{gh(x)}, \quad (1.12)$$

Ici z_b est la surface du glissement de terrain en mouvement. Le profil du fond est sélectionné dans la forme $h \sim x^{4/3}$. Un tel profil n'ayant pas de réflexion, les vagues qui se propage dans des directions opposées n'interagissent pas les unes avec les autres (Didenkulova et al, 2009; Didenkulova et Pelinovsky, 2010). Par conséquence, après le changement de variables, l'équation (1.12) peut être réduite à une équation d'onde homogène à coefficients constants; et sa solution générale est présentée sous la forme de l'intégrale de Duhamel. En particulier, si le glissement terrain de masse variable se déplace à vitesse variable :

$$z_b(\tau, t) = A(\tau) Z(\tau - Fr \cdot t), \quad \tau = \int_{x_0}^x dx' / c(x'), \quad (1.13)$$

ou Fr est le nombre de Froude; ainsi le champ des vagues est décrit dans la forme explicite :

$$\eta(x, t) = \frac{Fr^2}{Fr^2 - 1} A(x) Z \left[\int \frac{dx}{c(x)} - Fr \cdot t \right] - \frac{A(x)}{2(Fr - 1)} Z \left[\int \frac{dx}{c(x)} - t \right] + \frac{A(x)}{2(Fr + 1)} Z \left[\int \frac{dx}{c(x)} + t \right], \quad (1.14)$$

Cela nous permet d'étudier les modes possibles de propagation, tels que les régimes supercritiques ou sous-critiques. La première vague est forcée, et se déplace au-dessus du glissement de terrain. Sa longueur augmente avec la distance, et son amplitude baisse. L'onde forcée est positive («peigne») dans le régime supercritique ($Fr > 1$) et négative («socket») dans le régime sous-critique ($Fr < 1$). La deuxième vague est libre, et se déplace à vitesse variable $c(x)$ derrière le glissement de terrain dans le régime supercritique ($Fr > 1$) et devant lui dans le régime sous-critique ($Fr < 1$). Au fil du temps, son amplitude diminue lorsque sa longueur augmente. La troisième vague est libre, et au fur à mesure qu'elle s'approche de la côte, son amplitude augmente et sa longueur diminue. La Fig. 1.7 montre les amplitudes des trois vagues: la ligne bleue en pointillée dessine le premier terme

de l'équation (1.14), la ligne rouge en tirets et en points le deuxième, et la ligne noire en continue le troisième.

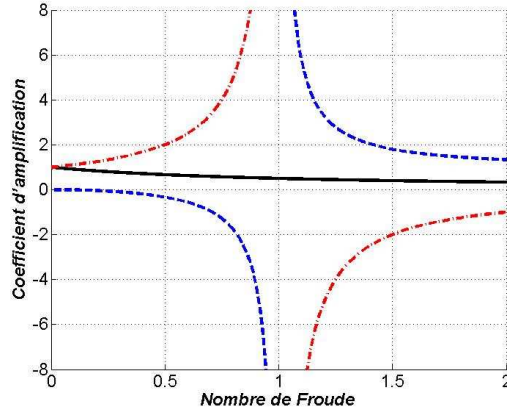


Fig. 1.7. Coefficient d'amplification des vagues au-dessus d'un fond concave ($h \sim x^{4/3}$) en fonction du nombre de Froude: la vague forcée (ligne bleue en pointillée), la vague libre de la côte (ligne rouge en tirets et en points) et vers la côte (ligne noire solide)

Dans le cas de la résonance ($Fr=1$) le champ des vagues est limité; tandis que dans le cas de la profondeur constante, la résonance conduit à une augmentation illimitée de l'amplitude des vagues:

$$\eta(x,t) = -\frac{A(x)t}{2} \frac{\partial Z}{\partial \tau} [\tau(x)-t] + \frac{3A(x)}{4} Z[\tau(x)-t] + \frac{A(x)}{4} Z[\tau(x)+t]. \quad (1.15)$$

Les calculs sont effectués pour les conditions typiques des glissements de terrain près de Marie-Galante (mer Caraïbe), et les résultats sont présentés sur la Fig. 1.8. Nous avons également calculé les caractéristiques des vagues générées par le changement du mode de glissement de terrain de mouvement (le passage par la résonance) selon différentes hypothèses concernant le volume (constant, en augmentation ou diminution).

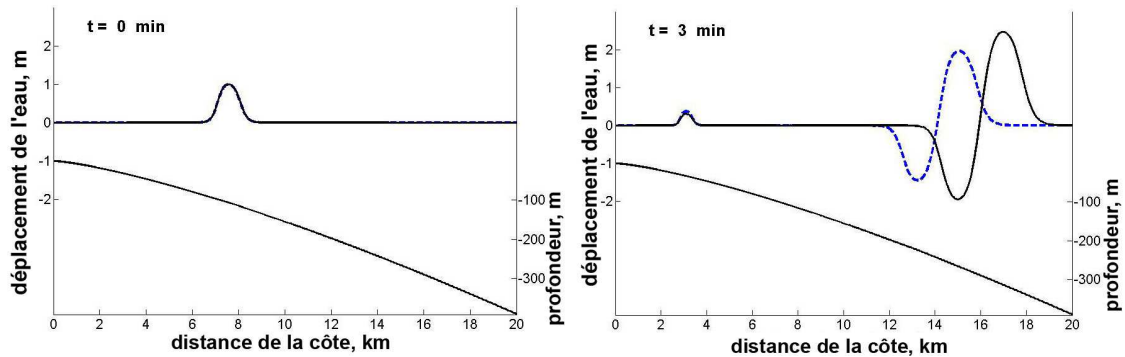


Fig. 1.8. Le déplacement de la surface de l'eau pour le profil de fond concave $h \sim x^{4/3}$ dans le régime sous-critique (ligne en pointillée) et supercritique (trait plein)

Tsunami waves generated by submarine landslides of variable volume: analytical solutions for a basin of variable depth

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Abstract. Tsunami wave generation by submarine landslides of a variable volume in a basin of variable depth is studied within the shallow-water theory. The problem of landslide induced tsunami wave generation and propagation is studied analytically for two specific convex bottom profiles ($h \sim x^{4/3}$ and $h \sim x^4$). In these cases the basic equations can be reduced to the constant-coefficient wave equation with the forcing determined by the landslide motion. For certain conditions on the landslide characteristics (speed and volume per unit cross-section) the wave field can be described explicitly. It is represented by one forced wave propagating with the speed of the landslide and following its offshore direction, and two free waves propagating in opposite directions with the wave celerity. For the case of a near-resonant motion of the landslide along the power bottom profile $h \sim x^\gamma$ the dynamics of the waves propagating offshore is studied using the asymptotic approach. If the landslide is moving in the fully resonant regime the explicit formula for the amplitude of the wave can be derived. It is demonstrated that generally tsunami wave amplitude varies non-monotonically with distance.

1 Introduction

The landslide motion is now recognized as an important source of tsunami wave generation, since it is the second most frequent tsunami source after earthquakes responsible for about 10% of all tsunami waves (Gusiakov, 2009).

At least, two special workshops specialized on this problem should be mentioned (Keating et al., 2000; Yalciner et al., 2003).

For example, an underwater landslide on 17 July 1998 caused the destructive Papua New Guinea tsunami, when the 15-m wave at the coast devastated three villages and took more than 2200 lives (McSaveney et al., 2000; Synolakis et al., 2002). On 17 August 1999 a shore slump in the Izmit Bay (Turkey) generated a 2.5 m high tsunami (Altinok et al., 2001). The problem of tsunami generation by landslides has also been actively discussed in the Atlantic and Caribbean. In particular, three tsunamis in the Lesser Antilles were generated by avalanche flows from the Montserrat volcano in 1997, 2003 and 2006 (O'Loughlin and Lander, 2003; Pelinovsky et al., 2004; Zahibo, 2006). Deplus et al. (2001) reported large-scale debris avalanche deposits around the Lesser Antilles. Particularly, a large area covered by mega blocks up to 2.8 km long and 260 m high is located off Dominica. Large coastal landslides and associated tsunami hazard in the Caribbean are described in (Teeuw et al., 2009). According to their study possible tsunami waves in the vicinity of Dominica can reach 3–10 m due to probable collapse of coastal blocks. Thus, the problem of tsunami generation by landslides is of current scientific interest.

For description of tsunami wave generation by submarine and aerial landslides various numerical models, which take into account the rheological properties of landslide body and real bathymetry are used (see, for instance, Imamura and Gica, 1996; Heinrich et al., 1999; Assier-Rzadkiewicz et al., 2000; Mangeney et al., 2000; Fine et al., 2003, 2005; Kuo et al., 2008; among others). In all these numerical models the volume of the landslide is conserved. However, Pudasaini and Hutter (2007) in their book pointed out the importance



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of the erosion processes in avalanches, so that the deposited mass in the runout zone can be much larger than the initial mass in the starting zone. This effect of variable volume of the landslide has never been analyzed in the problem of tsunami wave generation and the given study is focused on this problem.

The number of analytical methods for analysis of tsunami wave generation by submarine landslides is limited even for the cases when the landslide is moving with a constant speed and conserves its volume. 1-D wave field around landslide moving with a constant speed in a basin of constant depth is computed in (Pelinovsky, 1996, 2003; Tinti and Bortolucci, 2000; Tinti et al., 2001; Okal and Synolakis, 2003). The 2-D and 3-D motion of a constant-speed landslide is studied analytically for shallow-water approximation in the framework of the potential theory in (Novikova and Ostrovsky, 1978; Pelinovsky and Poplavsky, 1997; Ward, 2001). The landslide tsunami generation along a sloping beach in 1-D and 2-D cases is discussed in (Liu et al., 2003; Sammarco and Renzi, 2008), in the last case the landslide also induces edge waves propagating along a plane beach.

An interesting analytical example of tsunami wave generation in the shallow water basin of variable depth is given in (Tinti et al., 2001). It was shown that in the case of bottom profile $h \sim x^{4/3}$ (h is the water depth and x is coordinate directed offshore) the wave equation with an external force can be simplified and the same methods as for a constant depth basin can be applied. It was found later in (Didenkulova et al., 2009) that this bottom profile can be interpreted as a “non-reflecting” one. Waves propagating along this profile in the opposite directions do not interact between each other and are able to transfer the wave energy over large distances. At the same time there is another quartic bottom profile, $h \sim x^4$, which has the same “non-reflecting” nature (Didenkulova and Pelinovsky, 2010) and the analytical solution of the tsunami wave generation by landslides along this profile is studied below in detail. In fact, the number of “non-reflecting” profiles in 1-D geometry exceeds two, but other bottom configurations lead to the wave dispersion even in the shallow-water approximation (Grimshaw et al., 2010).

At the same time real bathymetries in the coastal zone can be well approximated by these two “non-reflecting” profiles. For example, Fig. 1 demonstrates the applicability of “non-reflecting” approximation to bottom profiles near Guadeloupe, Caribbean Sea (Gusiakov, 2002).

Here we present an analytical study of tsunami wave generation by landslides of a variable volume moving with a variable speed in 1-D case. The mathematical model that is linear shallow-water theory is briefly described in Sect. 2. The process of tsunami wave generation along a special beach profile $h \sim x^{4/3}$ is studied in Sect. 3. Here, in contrast to Tinti et al. (2001) the landslide is assumed to have a volume variable per unit cross-section and velocity variable during its motion. The runup of generated tsunami waves

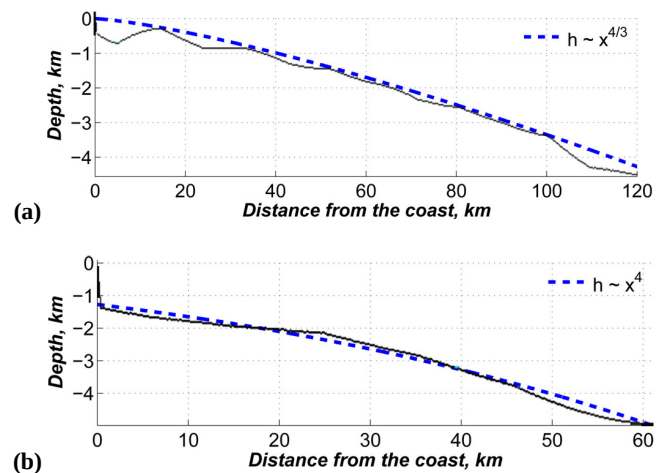


Fig. 1. Bottom profiles near Guadeloupe, Caribbean Sea (solid line) and their “non-reflecting” approximations (dashed line); (a) Marie-Galante and (b) La Desirade.

on this beach is also analyzed. The generation of tsunami waves along a quartic bottom profile $h \sim x^4$ is discussed in Sect. 4 and the new analytical solution for landslide induced tsunamis is studied in detail. The case of the resonant generation of tsunami waves by landslides above slowly varying “power” bottom profile and its propagation offshore is considered in Sect. 5. Using the asymptotic approach, the wave field is expressed analytically in the integral form and discussed for various scenarios, when the volume of the landslide increases, decreases or stays constant with time. It is shown that generally the wave amplitude varies with offshore distance non-monotonically due to competing effects of the resonant growth and the wave attenuation with a depth. The main results are summarized in Sect. 6.

2 Basic model

The governing equations of 1-D linear shallow-water theory for tsunami wave generation by underwater landslides are (Pelinovsky, 1996; Tinti et al., 2001; Liu et al., 2003):

$$\frac{\partial \eta}{\partial t} + \frac{\partial}{\partial x} [h(x)u] = W(x, t) = \frac{\partial z_b}{\partial t}, \quad (1)$$

$$\frac{\partial u}{\partial t} + g \frac{\partial \eta}{\partial x} = 0, \quad (2)$$

where $\eta(x, t)$ is the water displacement, $u(x, t)$ is the depth-averaged flow velocity, g is the gravity acceleration, $W(x, t)$ is the vertical velocity of the moving bottom boundary $z_b(x, t)$; see Fig. 2.

Initial conditions for the shallow-water system (1)–(2) are applied to both the water displacement $\eta(x, t)$ and depth-averaged flow velocity $u(x, t)$. If at the initial moment the ocean rests, they are

$$\eta(x, 0) = 0, \quad u(x, 0) = 0. \quad (3)$$

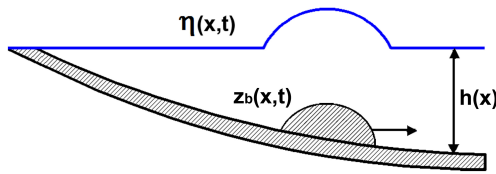


Fig. 2. The geometry of the problem: the water displacement $\eta(x, t)$ is generated by the landslide with the shape $z_b(x, t)$ above the variable bottom profile $h(x)$.

These initial conditions (3) are used in (Tinti et al., 2001; Liu et al., 2003). Meanwhile, if the vertical velocity of the landslide, W , is nonzero at the initial moment, that is a typical approximation in this kind of a problem. It follows from Eq. (1) that the vertical velocity of the fluid ($\partial\eta/\partial t$) is also nonzero. In fact, flow distribution around the body with nonzero velocity is more complicated and should be found by solving the full system of hydrodynamic equations. So the ratio between vertical and horizontal components of velocity is not evident in the shallow-water approximation. For example, Pelinovsky (1996, 2003) assumed that the vertical velocity ($\partial\eta/\partial t$) is zero at the initial moment.

Here we consider the situation of the landslide movement triggered by the earthquake that in the initial moment the landslide body lifts up and then slides down. Of course, the shallow water theory does not give a correct representation of the early fluid motion from rest. In the literature this paradox has been specially studied for the piston-model of tsunami wave motion. As it has been shown by (Kajiura, 1963) and then reproduced by (Kervella et al., 2007), if the size of the landslide is four times larger than the water depth, in shallow water theory and in full hydrodynamic model the shape of the free water surface (for the most part of the initial disturbance) coincide. The study was based on the rectangular initial disturbance and the only discrepancy between these two models was found at the sharp ends of the rectangular pulse, and this discrepancy can be smoothed by filtering. This explains why the shallow water theory is widely used for the description of the initial stage of the landslide motion. In all our calculations below we consider only smooth initial disturbances that justify the applicability of the shallow water model.

In this case the appropriate initial conditions should be derived from the assumption that the landslide velocity is zero at the initial moment as it is rigorously applied to the problem of wave generation by a moving bottom (“piston” model). Using the analogy with the piston model of tsunami generation let us consider the following approximation of the moving bottom:

$$z_b(x, t) = Y(t)Z_b(x, t), \quad (4)$$

where $Z_b(x, t)$ is a “smooth” function, that describes the landslide motion (for example, constant-speed motion), and

$Y(t)$ is almost the step function, which reflects the transition process when the landslide suddenly starts its motion. After integrating Eqs. (1) and (2) over an infinitesimal time interval the new initial conditions for the system (1)–(2) with an external forcing $W = \partial Z_b/\partial t$ can be obtained

$$\eta(x, 0) = Z_b(x, 0), \quad u(x, 0) = 0. \quad (5)$$

These effective initial conditions (5) are well known for the piston shift of the bottom during the earthquake. Mathematically, nonzero initial conditions appear in the hyperbolic equations with an external forcing if the Cauchy problem is posed in the class of generalized functions (in our case it is a step function, which reflects sudden motion of the landslide) (Courant and Hilbert, 1989).

Equations (1) and (2) of the shallow-water system can be reduced to the wave equation for the water displacement (Z_b is replaced by z_b)

$$\frac{\partial^2 \eta}{\partial t^2} - \frac{\partial}{\partial x} \left[c^2(x) \frac{\partial \eta}{\partial x} \right] = \frac{\partial^2 z_b}{\partial t^2}, \quad c = \sqrt{gh(x)}, \quad (6)$$

which satisfies the following initial conditions

$$\eta(x, 0) = z_b(x, 0), \quad \frac{\partial \eta}{\partial t}(x, 0) = \frac{\partial z_b}{\partial t}(x, 0). \quad (7)$$

Alternatively, the wave equation for the flow velocity

$$\frac{\partial^2 u}{\partial t^2} - \frac{\partial^2}{\partial x^2} \left[c^2(x)u \right] = -g \frac{\partial^2 z_b}{\partial t \partial x}, \quad (8)$$

satisfies the initial conditions

$$u(x, 0) = 0, \quad \frac{\partial u}{\partial t}(x, 0) = -g \frac{\partial z_b}{\partial x}(x, 0). \quad (9)$$

We would like to stress that both initial conditions (7) and (9) are derived from a more general assumption (5) and are equivalent.

Each form of the wave equation: Eq. (6) or Eq. (8) has its advantages in different problems and for different bottom configurations. Analytical solutions of both Eqs. (6) and (8) are discussed in Sects. 3 and 4, respectively. It is also important to mention here that the landslide described by the function $z_b(x, t)$ can have a variable volume and move with a variable speed. Different scenarios of the landslide body change, when its volume increases, decreases or stays constant are studied in Sect. 5.

3 Tsunami generation by the landslide motion along the bottom profile $h \sim x^{4/3}$

Let us consider here the bottom profile described by

$$h(x) = px^{4/3}, \quad (10)$$

where p is an arbitrary coefficient with a dimension $m^{-1/3}$. Here as before axis x is directed offshore and the point

$x = 0$ corresponds to the shoreline. As it has been shown in Didenkulova et al. (2009), long waves propagating along a bottom profile (10) do not interact between each other and conserve their shape, while their amplitude satisfies the Green's law ($h^{-1/4}$). In comparison with the water displacement, the velocity field has a more complicated form: the shape of the wave of velocity changes in time and its amplitude does not satisfy the Green's law. Formally, the water displacement disturbance can be considered as a traveling wave transferring the potential wave energy over large distances. The detailed analysis of the long wave dynamics along the bottom profile (10) including the process of the wave runup on the coast is given in (Didenkulova et al., 2009) for various initial disturbances.

Mathematically, this “non-reflecting” behavior is related to the possibility of reduction of the variable-coefficient wave equation to the constant-coefficient wave equation. In the case of the bottom profile (10) this reduction was first applied to the problem of tsunami generation by a moving landslide by (Tinti et al., 2001). The transformation (x_0 and h_0 represent the coordinate and the depth of the initial landslide location, respectively)

$$\eta(x, t) = A(x) H(\tau(x), t), \quad \tau = \int_{x_0}^x \frac{dx'}{c(x')},$$

$$A(x) = A_0 \left[\frac{h_0}{h(x)} \right]^{1/4}, \quad (11)$$

reduces Eq. (6) to the constant-coefficient wave equation with an external forcing

$$\frac{\partial^2 H}{\partial t^2} - \frac{\partial^2 H}{\partial \tau^2} = \frac{\partial^2}{\partial t^2} \left[\frac{z_b(\tau, t)}{A(\tau)} \right]. \quad (12)$$

The solution of Eq. (12) satisfying the initial conditions (7) can be expressed in the Duhamel integral form (Courant and Hilbert, 1989):

$$H(\tau, t) = \frac{1}{2} \left[\frac{z_b(\tau - t)}{A(\tau - t)} + \frac{z_b(\tau + t)}{A(\tau + t)} \right] + \frac{1}{2} \int_{\tau-t}^{\tau+t} \frac{1}{A(\sigma)} \frac{\partial z_b}{\partial t}(\sigma, 0) d\sigma + \frac{1}{2} \int_0^t d\rho \int_{\tau-(t-\rho)}^{\tau+(t-\rho)} \frac{1}{A(\zeta)} \frac{\partial^2 z_b}{\partial \rho^2}(\zeta, \rho) d\zeta. \quad (13)$$

Similar solution with some modifications due to other initial conditions was used in (Tinti et al., 2001) for tsunami waves generated by the landslide moving with a constant speed. Obtained results were compared with those for a flat bottom and the difference was relatively small.

Equation (13) can be presented in the explicit form for the following landslide motion:

$$z_b(\tau, t) = A(\tau) Z(\tau - Fr \cdot t), \quad (14)$$

where Fr is the constant Froude number, which determines the variable speed of the landslide

$$V(x) = \frac{dx}{dt} = c(x) \cdot Fr. \quad (15)$$

The landslide body according to Eq. (14) has the variable volume (mass) per unit cross-section

$$M(t) = \int z_b(x, t) dx = \int c(\tau) A(\tau) Z(\tau - Fr \cdot t) d\tau. \quad (16)$$

If $Z(\tau)$ is defined as a stepwise function, where τ_0 and T are arbitrary constants, which define the location and the length of the landslide,

$$Z(\tau) = \begin{cases} 0, & \tau < \tau_0, \\ 1, & \tau_0 < \tau < \tau_0 + T, \\ 0, & \tau > \tau_0 + T, \end{cases} \quad (17)$$

Eq. (16) simplifies to

$$M(t) = M_0 \left[1 + \frac{Fr}{\tau_0 + T/2} t \right]. \quad (18)$$

Thus, the landslide moves with acceleration proportional to t ($\sim x^{1/3}$ or $\sim h^{1/4}$), its height decreases with time as t^{-1} ($\sim x^{-1/3}$), but the volume grows with time and with distance ($\sim x^{1/3}$). The length of the landslide also increases ($\sim x^{2/3}$). Therefore not all the points of the landslide move at the same speed. The growth of the landslide volume can be connected to the bottom sediment friction; so that the landslide “absorbs” the surrounding materials and makes bottom particles “cling” to it. This physical process is well known for avalanches and described in (Pudasaini and Hutter, 2007). It is important to emphasize that variations of the landslide parameters (its acceleration, height and volume) with the depth are relatively slow ($\sim h^{1/4}$) and therefore, the process of the wave generation by the landslides along the bottom profile (10) can be considered as qualitatively the same as in the basin of constant depth.

Equation (13) for the special landslide shape (14) transforms into

$$H(\tau, t) = \frac{Fr^2}{Fr^2 - 1} Z(\tau - Fr \cdot t) - \frac{1}{2(Fr - 1)} Z(\tau - t) + \frac{1}{2(Fr + 1)} Z(\tau + t). \quad (19)$$

Solutions, similar to Eq. (19) are known for tsunami waves induced by landslides moving with a constant speed in a basin of constant depth (Pelinovsky, 1996, 2003; Tinti et al., 2001). In physical variables the solution (19) has the following form

$$\eta(x, t) = \frac{Fr^2}{Fr^2 - 1} A(x) Z \left[\int \frac{dx}{c(x)} - Fr \cdot t \right] - \frac{A(x)}{2(Fr - 1)} Z \left[\int \frac{dx}{c(x)} - t \right] + \frac{A(x)}{2(Fr + 1)} Z \left[\int \frac{dx}{c(x)} + t \right], \quad (20)$$

and represents three waves. The first one is the forced wave, which is located above the landslide and moves with the variable velocity (15); its length increases with a distance and its amplitude is proportional to the landslide height and decreases with a distance. The forced wave is positive (wave of elevation) in super-critical regime ($Fr > 1$) and negative (wave of depression) in sub-critical regime ($Fr < 1$). The second wave is a free wave, which moves with a variable velocity $c(x)$ behind the landslide in the super-critical regime ($Fr > 1$) and in front of it in sub-critical regime ($Fr < 1$). Its amplitude decreases and its length increases in time. The third (free) wave moves shoreward with an increase in its amplitude and decrease in length. The relation between amplitudes of these waves for different values of the Froude number at the moment of wave generation is illustrated in Fig. 3.

Here the amplification factor is the ratio H/Z marked in dashed blue for the first term, dash-dotted red – for the second and solid black – for the third. The resonance between waves and the landslide ($Fr \approx 1$) dramatically increases amplitudes of waves moving offshore (in the direction of the landslide). At the same time these waves do not split in space in the vicinity of the resonance and, therefore, the resulting amplitude also stays bounded for a long time. For exact resonance ($Fr = 1$) Eq. (20) transforms into

$$\eta(x, t) = -\frac{A(x)t}{2} \frac{\partial Z}{\partial \tau} [\tau(x) - t] + \frac{3A(x)}{4} Z[\tau(x) - t] + \frac{A(x)}{4} Z[\tau(x) + t]. \quad (21)$$

The forced wave propagating offshore has a sign-variable shape. Taking into account that in the vicinity of the resonance $\tau(x) \approx t$, the resonant wave can be written as

$$\eta_{\text{res}}(x, t) = -\frac{A(x)\tau(x)}{2} \frac{\partial Z}{\partial \tau} [\tau(x) - t]. \quad (22)$$

As a result, wave amplitude grows with the distance and tends to the asymptotic value

$$A_{\infty} = \frac{3x_0}{c_0} A_0 \max(\partial Z / \partial \tau). \quad (23)$$

In contrast to the basin of a constant depth, where the resonance leads to the infinite growth of the wave amplitude, the resonant effect in the basin of a variable depth can be bounded. Detailed study of these effects is given in Sect. 5.

The wave dynamics along the beach profile (10) with the same parameter p as for the coast of Marie-Galante (Fig. 1a) is calculated below for the initial landslide shape (Fig. 4), described by

$$z_b(x, t=0) = \frac{A(x)}{2} \left[\tanh\left(\frac{\tau(x) - \tau_0 + T/2}{a}\right) - \tanh\left(\frac{\tau(x) - \tau_0 - T/2}{a}\right) \right], \quad (24)$$

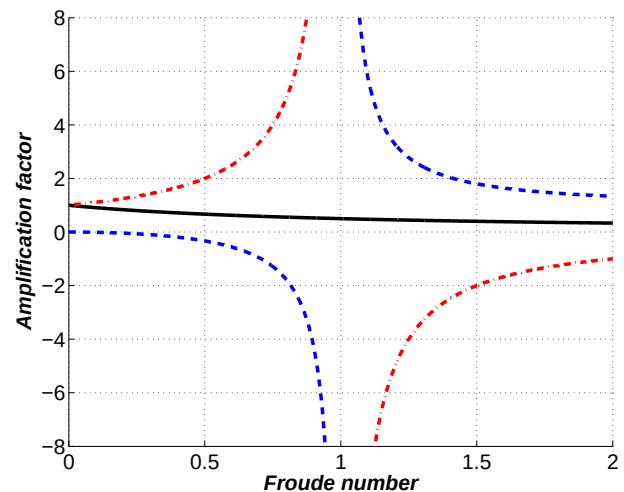


Fig. 3. Amplification factor for generated tsunami waves along the bottom profile $h \sim x^{4/3}$ plotted against the Froude number: the forced wave (dashed blue line) and free waves propagating offshore (dash-dotted red line) and onshore (solid black line).

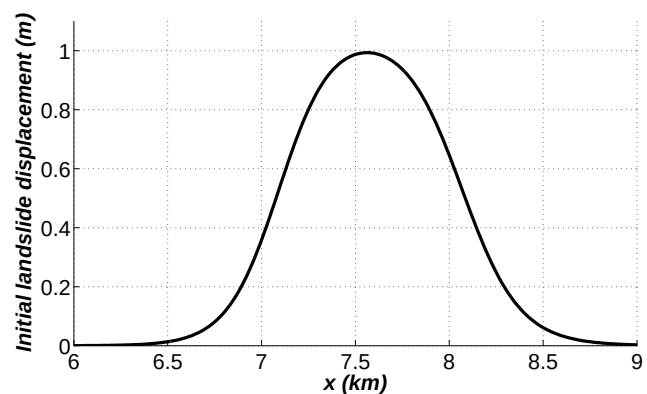


Fig. 4. Initial landslide shape (24) along the bottom profile $h \sim x^{4/3}$.

where $T = 30$ s and $\tau_0 = 11$ min, providing the location of 2 km long and 1 m high landslide at the distance 7.5 km offshore at the water depth 100 m. Parameter $a = 0.3T$ is chosen to provide smoothing of the landslide shape. Similar parameters of the landslide were used in the paper by Tinti et al. (2001), and they are also in agreement with estimations of possible and real submarine landslides in the Lesser Antilles (Ten Brink et al., 2006; Teeuw et al., 2009).

The results of calculation are presented in Fig. 5 for two different regimes of the wave motion: subcritical ($Fr < 1$) and supercritical ($Fr > 1$). In the case of the supercritical regime ($Fr = 1.2$) the forced wave, described by the first term in Eq. (19), propagates with the same velocity as the landslide body and represent the leading wave of elevation. In the case of the subcritical regime ($Fr = 0.8$) it is the second wave of depression moving offshore. Waves propagating offshore become longer with time (Fig. 5). The variations

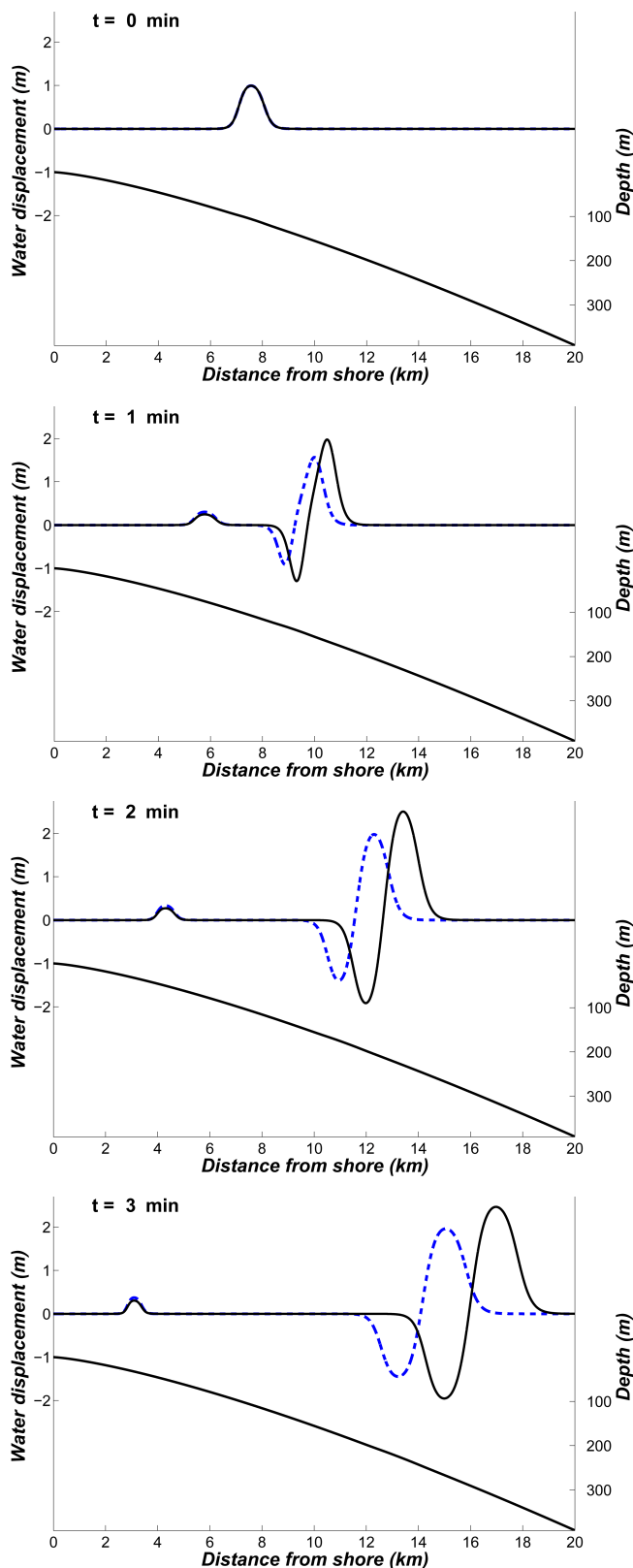


Fig. 5. Wave dynamics along the bottom profile $h \sim x^{4/3}$ in subcritical (blue dashed line) and supercritical (black line) regimes.

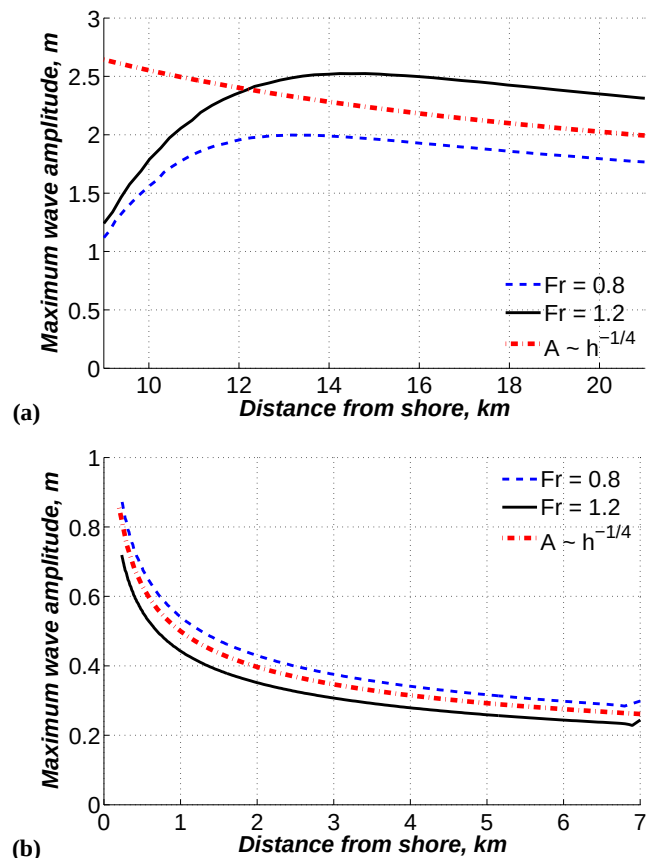


Fig. 6. Variations of the maximum amplitude of tsunami waves during its propagation along the bottom profile $h \sim x^{4/3}$: for positive waves propagating offshore (a) and onshore (b).

of their amplitudes are presented in Fig. 6a. In the beginning for a short time amplitudes of individual waves propagating offshore are increased by the resonant effect up to the factor of 2.5 in the case of the supercritical regime ($Fr = 1.2$) but at the distance of about 2–3 wavelengths they start to be determined by the function $A(x) \sim h(x)^{-1/4} \sim x^{-1/3}$, which reflects the Green's law (Fig. 6a). The main difference between two studied regimes is that waves propagating offshore are higher and faster in the case of the supercritical regime even if the deviation from the critical value of the Froude number is the same.

The free wave of the third term in Eq. (19) moves onshore with a weak initial amplitude that grows when the wave approaches the coast (Fig. 7). The wave approaching the beach increases in its amplitude according to the Green's law (Fig. 6b) and reduces in length. The wave amplitude is higher in the subcritical regime rather than in supercritical one. The shoaling effect becomes significant near the shore and gives wave amplification in several times (2–3 times).

There are two possible scenarios of the runoff of this wave on a beach: with or without breaking. Formally, there is no rigorous criterion of the wave breaking on the beach,

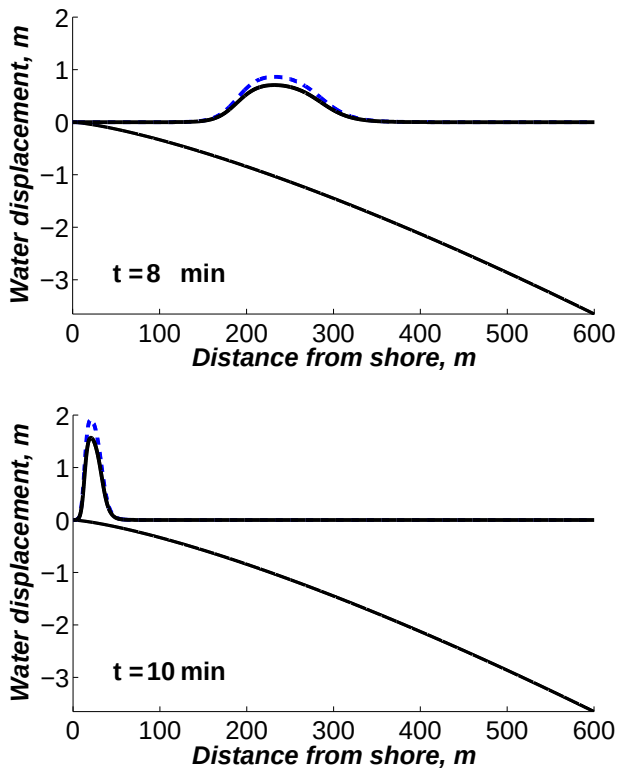


Fig. 7. Tsunami wave runup on the beach $h \sim x^{4/3}$ in subcritical (blue dashed line) and supercritical (black line) regimes.

described by Eq. (10) in comparison with a plane beach (Synolakis, 1987; Didenkulova et al., 2007, 2008), since the linear wave equation is solved. Estimates of the wave runup on the beach, described by Eq. (10), give unfeasibly high runup height (Didenkulova et al., 2009). That is why we match the non-reflecting beach (Eq. 10) with the beach of constant slope, for which the nonlinear problem of runup is solved, in the point, where the wave amplitude is small in comparison with the water depth. In our case it is the distance $X_1 \sim 500 - 600$ m from the shore. The process of the wave runup on a plane beach is well-studied (Synolakis, 1987; Didenkulova et al., 2007, 2008) and the runup height can be determined (Didenkulova et al., 2008)

$$R_{\text{plane}} \approx 3.5A\sqrt{X_1/\lambda}, \quad (25)$$

where A and λ are amplitude and wavelength of the approaching wave at the distance X_1 from the shore respectively. Taking into account wave parameters at the distance $X_1 \sim 500$ m from the shore: amplitude $A \sim 0.7$ m and wavelength is $\lambda \sim 200$ m, the runup height can be estimated from Eq. (25) as 3.9 m. It follows from Eq. (20) that amplitude of incoming wave decreases with an increase in the Froude number. Hence, the runup height also decreases with an increase in the Froude number as $\sim 1/(Fr+1)$.

At the same time, since the water depth in the near shore is too small (a few meters) it is reasonable that the dissipation of wave energy in the near-bottom turbulent flow and the wave breaking lead to the decrease in the tsunami runup height. As it is shown in (Didenkulova et al., 2010) usually landslide generated tsunami waves break at the coast. Thus, the results presented here can be considered as the upper estimate for the possible tsunami runup heights.

4 Tsunami generation by the landslide motion along the quartic bottom profile $h \sim x^4$

Another bottom profile, which allows analytical solution for the problem of tsunami wave generation by landslides, is

$$h(x) = qx^4, \quad (26)$$

where q is an arbitrary coefficient with the dimension m^{-3} .

As it has been recently shown in (Didenkulova and Pelinovsky, 2010), similarly to the wave dynamics along the bottom profile (10) discussed in Sect. 3, long waves of the flow velocity do not interact between each other along the quartic bottom profile, described by Eq. (1). These propagating waves of flow velocity do not change their shape in time and their amplitudes satisfy the Green's law for velocity ($h^{-3/4}$). In contrast to the case of the bottom profile, described by Eq. (10), the water displacement along the quartic bottom profile does not have a simple representation: the wave shape is not conserved and its amplitude does not satisfy the Green's law. The detailed analysis of the wave dynamics in this case for various initial disturbances is given in (Didenkulova and Pelinovsky, 2010).

Similarly to the case discussed in Sect. 3, the simplified behavior along the quartic bottom profile is related to the possibility of reduction of the variable-coefficient wave equation to the constant-coefficient wave equation. The following transformation

$$u(x, t) = B(x)U(\tau(x), t), \quad \tau = -\int \frac{dx}{c(x)}, \quad B(x) = bh^{-3/4}(x), \quad (27)$$

where b is an arbitrary constant with dimension $\text{m}^{7/4} \text{s}^{-1}$, reduces the wave equation for the flow velocity with an external forcing (8) to the inhomogeneous constant-coefficient wave equation

$$\frac{\partial^2 U}{\partial \tau^2} - \frac{\partial^2 U}{\partial \tau^2} = -\frac{g}{B(\tau)c(\tau)} \frac{\partial^2 z_b(\tau, t)}{\partial \tau \partial t}. \quad (28)$$

Initial conditions for Eq. (3) follow from Eqs. (9) for velocity flow that is identical to Eqs. (7) for water displacement

$$U(\tau, 0) = 0, \quad \frac{\partial U}{\partial \tau}(\tau, 0) = -\frac{g}{B(\tau)c(\tau)} \frac{\partial z_b}{\partial \tau}(\tau, 0). \quad (29)$$

Here the spatial domain ($0 < x < \infty$) transforms to the new domain ($\infty < \tau < 0$) with an opposite direction that differs

from the previous case of the beach (10), described in Sect. 3. It means that in the new variables the landslide moves to the left.

Similarly to the case described in Sect. 3, the solution of Eq. (3) satisfying the initial conditions (4) can be represented in the form of the Duhamel integral (Courant and Hilbert, 1989):

$$U(\tau, t) = -\frac{1}{2} \int_{\tau-t}^{\tau+t} \frac{g}{B(\sigma)c(\sigma)} \frac{\partial z_b}{\partial \sigma}(\sigma, 0) d\sigma - \frac{1}{2} \int_0^t d\rho \int_{\tau-(t-\rho)}^{\tau+(t-\rho)} \frac{g}{B(\zeta)c(\zeta)} \frac{\partial^2 z_b}{\partial \rho \partial \zeta}(\zeta, \rho) d\zeta. \quad (30)$$

The new forcing function

$$P(\tau, t) = \int \frac{g}{B(\tau)c(\tau)} \frac{\partial z_b(\tau, t)}{\partial \tau} d\tau, \quad (31)$$

simplifies Eqs. (28) and (29) and transforms them to

$$\frac{\partial^2 U}{\partial t^2} - \frac{\partial^2 U}{\partial \tau^2} = -\frac{\partial^2 P(\tau, t)}{\partial t \partial \tau}, \quad (32)$$

$$U(\tau, 0) = 0, \quad \frac{\partial U}{\partial t}(\tau, 0) = -\frac{\partial P}{\partial \tau}(\tau, 0). \quad (33)$$

If the new forcing function (31) of the landslide motion can be presented in the following way

$$P(\tau, t) = P(\tau + Fr \cdot t), \quad (34)$$

Eq. (30) transforms into a simple explicit solution for the velocity field

$$U(\tau, t) = -\frac{Fr}{Fr^2 - 1} P(\tau + Fr \cdot t) + \frac{1}{2(Fr - 1)} P(\tau + t) + \frac{1}{2(Fr + 1)} P(\tau - t), \quad (35)$$

which represents three waves of velocity, the first of which is the forced one propagating offshore (the first term in Eq. 35), and other two are free waves propagating offshore and onshore respectively. The amplitude of the forced wave propagating offshore in dimensionless variables varies linearly with time in the vicinity of the resonance ($Fr = 1$)

$$U(\tau, t) = -\frac{t}{2} \frac{\partial P(\tau + t)}{\partial \tau} - \frac{1}{4} P(\tau + t) + \frac{1}{4} P(\tau - t). \quad (36)$$

The water displacement can be found from Eq. (2)

$$\frac{\partial \eta}{\partial \tau} = -bq^{1/4} \tau \frac{\partial U}{\partial t}, \quad (37)$$

using the condition of zero displacement at infinity $x \rightarrow \infty$ that corresponds to $\tau = 0$.

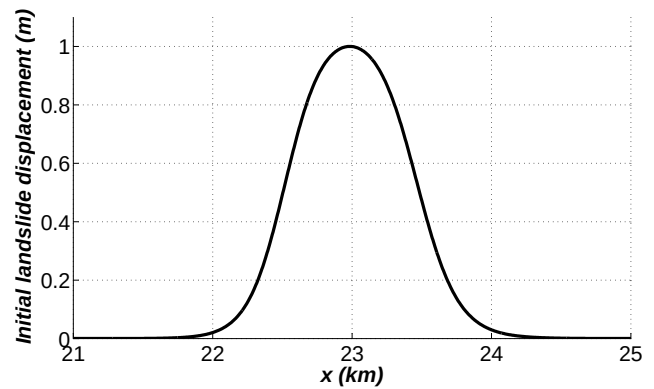


Fig. 8. Initial landslide shape (38) along the quartic bottom profile $h \sim x^4$.

The long wave dynamics along the quartic bottom profile is discussed for the landslide shape (Fig. 8) given by

$$z_b(x, t = 0) = \frac{A_0}{2} \left[\tanh\left(\frac{\tau(x) - \tau_0 + T/2}{a}\right) - \tanh\left(\frac{\tau(x) - \tau_0 - T/2}{a}\right) \right], \quad (38)$$

where τ_0 represents the travel time to the deep water. The landslide shape in Eq. (38) differs from the one discussed in Sect. 3 (Eq. 24) by the amplitude A . It is constant in Eq. (38), while in Eq. (24) it is a function of coordinate $A(x)$, representing the Green's law. Though the solution (Eq. 35) describing the long wave dynamics along the quartic profile is very similar to the one, discussed in Sect. 3, there are some differences, which should be mentioned. The depth of a quartic bottom profile is a rapid function of the distance far from the coast that leads to the fact that long waves cross the deep water in a fixed time. It is evident that the shallow-water approximation breaks down on large depths and the solution given by Eq. (35) does not work there.

At the same time the water depth becomes very shallow in the near shore and very soon the wave amplitude starts to be comparable with the water depth and the wave should break due to the nonlinearity. The reflection from such a beach is also very small and almost negligible. At the same time it can be significant if we place the vertical wall at some distance from the shore and study the wave dynamics in this new bottom geometry. This situation of a sudden change of the depth just at the shore occurs quite often for rocky and cliffy coasts. One of such examples is shown on Fig. 1b for Desirade. In this case the wave can be reflected from the vertical wall without breaking. Such a situation is considered in (Didenkulova and Pelinovsky, 2010). Anyhow we are not going to do it here and consider our solution given by Eqs. (35) and (36) only in the main part of the basin, excluding deep water part and the vicinity of the shoreline.

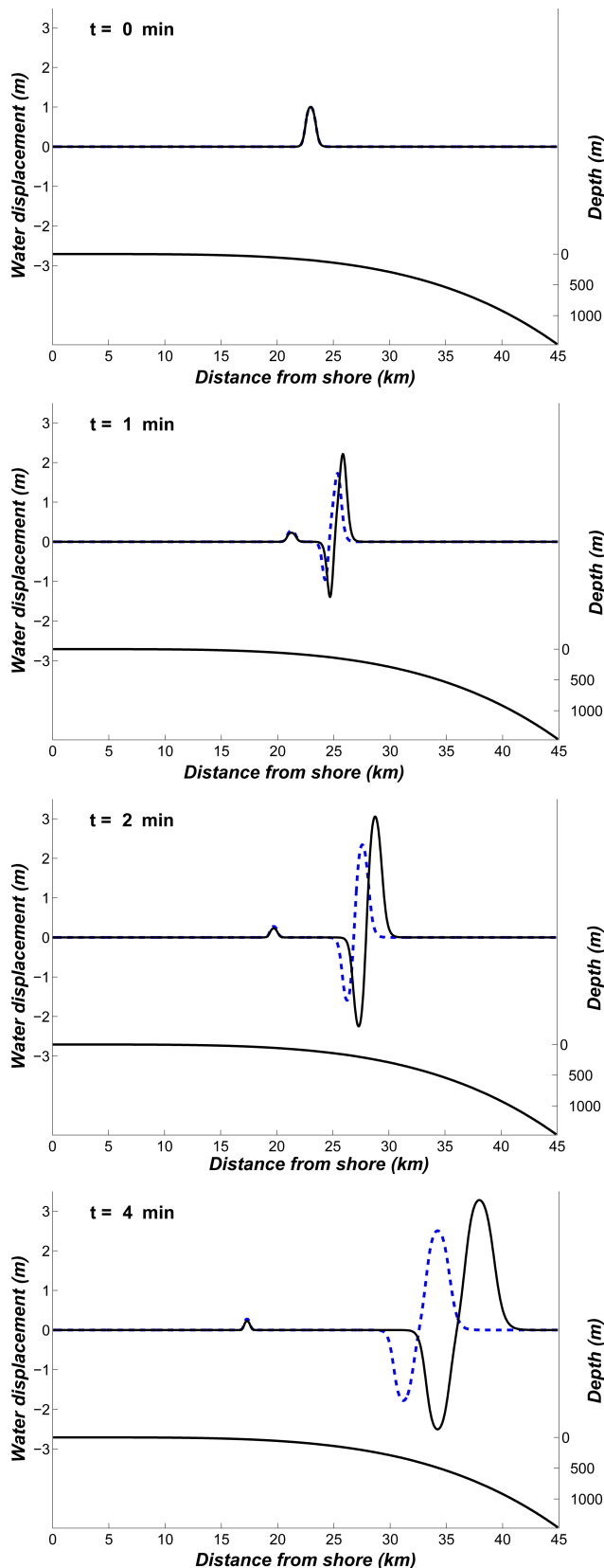


Fig. 9. Wave dynamics along the quartic bottom profile $h \sim x^4$ in subcritical (blue dashed line) and supercritical (black line) regimes.

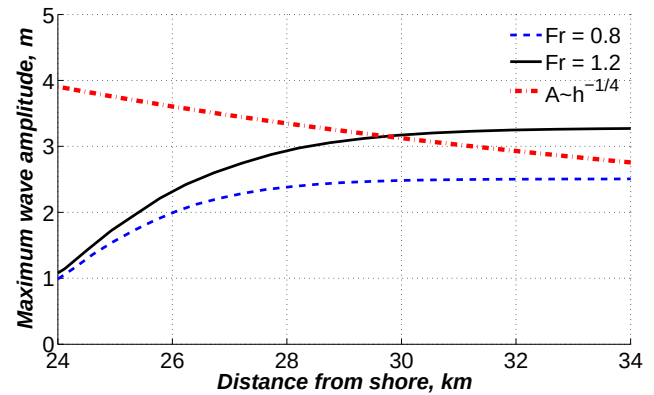


Fig. 10. Variations of the maximum amplitude of tsunami waves during its propagation along the quartic bottom profile $h \sim x^4$ for positive waves propagating offshore.

The calculated long wave dynamics along the quartic bottom profile with the parameter q taken from the bathymetry near Desirade (Fig. 1b) for the landslide shape (38) is illustrated in Fig. 9 for sub- ($Fr = 0.8$) and supercritical ($Fr = 1.2$) regimes. The landslide of 1 m high and 2 km long is located at the distance of 23 km from the shore at the depth of 100 m, the travel time to the deep water τ_0 is 12 min, T is 30 s and $a = 0.3T$.

In the initial moment the water displacement has the same shape as the landslide and initial velocity is zero. As it has been discussed for the beach profile (10) in Sect. 3, the waves that travel upward the quartic bottom profile are larger and faster in case of supercritical regime.

Variations of wave amplitudes with the distance are presented in Fig. 10. In the beginning amplitudes of waves propagating offshore grow due to the resonant effect and then stabilize with some value at deep waters. It can be seen that the wave amplitude does not follow the Green's law (Fig. 10). As it has been mentioned above, it is related to very fast changes of the water depth offshore that is contradictory to the shallow water theory.

Thus, generally the wave dynamics along the quartic bottom profile is very similar to the one, discussed in Sect. 3 for the beach profile, described by Eq. (10). The landslide motion generates three waves, two of which propagate offshore in the direction of the landslide and one propagates onshore. Amplitudes of offshore propagating waves grow due to the resonance effect and reach 3.3 m in subcritical and 2.5 m in supercritical regimes. For comparison, in the case of the bottom profile (10), discussed in Sect. 3, for the similar shape of the landslide, located at the same depth, the maximum values of the offshore propagating waves are 2.5 m in subcritical and 2 m in supercritical regimes. Hence two analytical solutions presented here give similar estimates for amplitudes of landslide generated waves.

We should mention that rigorous analytical solutions discussed here are found within shallow water theory and do not take into account wave dispersion. However in the case of a weak dispersion the processes of tsunami wave generation and propagation can be considered independently. The process of tsunami wave generation can be studied in the framework of the shallow water theory, while the wave propagation over long distances, when dispersive effects start to be significant, can be studied within Bousinesq-type models. Here we focus on the process of tsunami wave generation and obtained solutions of the shallow water theory are adequate.

5 Tsunami generation by the resonantly moving landslide along an arbitrary bottom profile

Two analytical examples of tsunami wave generation by the landslides, described in Sects. 3 and 4, demonstrate the important role of the resonance, when the landslide and the sea waves are moving with the same speeds ($Fr = 1$). As it has been shown in Sect. 3 the resonant wave amplitude can be bounded in the basin of variable depth, whereas it tends to infinity if the depth of the basin is constant. It makes clear that the resonance phenomenon should be considered not only for specific bottom configurations but for a more general case that should give us understanding about the sensitivity of these results to variations of the bottom profile and landslide characteristics.

Here this effect is studied for a basin of a slowly varying depth. Anyhow we do not analyze the full wave field, since the dynamics of the “non-resonant” wave approaching the coast is not related to the landslide motion and we do not expect large variations of its amplitude for all other regimes of tsunami generation by the landslide. Here we concentrate on the “resonant” waves propagating offshore. In the vicinity of the resonance these waves propagate with speeds close to the local wave celerity $c(x)$, which slowly changes with a distance, since the depth is a slow function of the distance. For this case let us introduce new adequate coordinates in Eq. (6)

$$s = \tau(x) - t, \quad x' = x, \quad (39)$$

where $\tau(x)$ is a travel time as before. After substitution of Eq. (39) the wave equation (6) has the following form

$$2c(x') \frac{\partial^2 \eta}{\partial s \partial x'} + \frac{dc}{dx'} \frac{\partial \eta}{\partial s} + \frac{\partial}{\partial x'} \left[c^2(x') \frac{\partial \eta}{\partial x'} \right] = - \frac{\partial^2 z_b}{\partial s^2}. \quad (40)$$

The last term in LHS of Eq. (40) is small according to the assumptions given above and can be neglected. After integration of Eq. (40) and some simple manipulations the solution for the water displacement can be found (prime is omitted):

$$\eta(x, s) = - \frac{1}{2\sqrt{c(x)}} \int_{x_0}^x \frac{1}{\sqrt{c(y)}} \frac{\partial z_b(y, s)}{\partial s} dy + \eta(x_0, s), \quad (41)$$

where the location ($x = x_0$) is chosen at the right border of the initial location of the landslide and the function $\eta(x_0, s)$ is the tsunami time series in this location. Eq. (41) corresponds to the first two terms in Eq. (21), describing the waves propagating offshore along the bottom profile $h \sim x^{4/3}$. Here we focus on the dynamics of the resonant wave, described by

$$\eta_{\text{res}}(x, s) = - \frac{1}{2\sqrt{c(x)}} \int_{x_0}^x \frac{1}{\sqrt{c(y)}} \frac{\partial z_b(y, s)}{\partial s} dy. \quad (42)$$

Equation (42) can be used for study of tsunami wave generation by the landslide of a variable volume (mass) moving with an arbitrary speed. It is restricted by the speed of the landslide, which should be close to the wave speed.

Let us consider the resonantly moving landslide of an arbitrary shape and variable volume

$$z_b(x, t) = Q(x)Z(s) = Q(x)Z[\tau(x) - t], \quad (43)$$

which according to Eq. (41) generates tsunami waves of variable amplitude $D(x)$

$$\eta_{\text{res}}(x, t) = -D(x) \frac{\partial Z(\tau - t)}{\partial \tau}, \quad D(x) = \frac{1}{2\sqrt{c(x)}} \int_{x_0}^x \frac{Q(y)dy}{\sqrt{c(y)}}, \quad (44)$$

along a general “power” bottom profile

$$h(x) = h_0 \left(\frac{x}{x_0} \right)^\gamma, \quad \gamma > 0. \quad (45)$$

As it has been mentioned above the resonant wave has a sign-variable shape for any kind of the bottom profile. For understanding of all possible regimes of the tsunami wave generation it is important to consider different scenarios of the landslide volume variations.

The first case is when the landslide height changes with depth according to the Green’s law (this assumption is used in Sect. 3)

$$Q_{Gr}(x) = Q_0 \left(\frac{h}{h_0} \right)^{-1/4} = Q_0 \left(\frac{x}{x_0} \right)^{-\gamma/4}. \quad (46)$$

In this case the wave amplitude $D(x)$ is

$$D_{Gr}(x) = \frac{Q_0 x_0}{2\sqrt{gh_0}} N_{Gr}(x),$$

$$N_{Gr}(x) = \begin{cases} \frac{1}{1-\gamma/2} \left[(x/x_0)^{1-3\gamma/4} - (x/x_0)^{-\gamma/4} \right], & \gamma \neq 2 \\ \ln(x/x_0) / (x/x_0)^{1/2}, & \gamma = 2 \end{cases}, \quad (47)$$

where the function $N_{Gr}(x)$ is displayed in Fig. 11. It follows

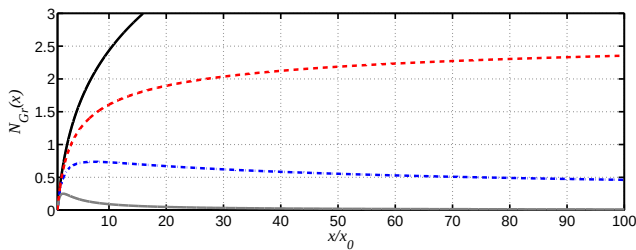


Fig. 11. Wave amplitude variations versus distance (Eq. 47) for $\gamma = 1$ (black solid line), $\gamma = 4/3$ (red dashed line), $\gamma = 2$ (blue dash-dotted line), $\gamma = 4$ (green dotted line).

from Eq. (47) that the wave amplitude grows up to infinity for $\gamma < 4/3$, but slower than in the basin of constant depth. In the case of the “non-reflecting” beach ($\gamma = 4/3$) discussed in Sect. 3, the wave amplitude asymptotically tends to its maximum. For all values $\gamma > 4/3$, the competition of two effects: resonance and increasing water depth leads to a non-monotonic behavior of the wave amplitude. If the depth increases with distance more rapidly the amplification becomes smaller and the amplification zone becomes narrower.

For analysis to be completed we need to calculate the volume of the landslide. In the case of $\gamma < 2$ the volume of the landslide can be computed explicitly

$$M_{Gr}(t) = \int z_b(x, t) dx = \int c(\tau) Q(\tau) Z(\tau - t) d\tau \sim \int \tau^{\frac{\gamma}{2(2-\gamma)}} Z(\tau - t) d\tau \sim t^{\frac{\gamma}{2(2-\gamma)}}. \quad (48)$$

It increases as $t^{1/2}$ along the plane beach and as t along the “non-reflecting” beach. So, the landslide volume grows with the distance while the wave amplitude decreases for $\gamma > 4/3$.

The second case is the landslide of a constant volume, which height and length vary with a distance. It follows from Eq. (48) that this can be realized if $Q \sim c^{-1}$:

$$Q_C(x) = Q_0 \left(\frac{h}{h_0} \right)^{-1/2} = Q_0 \left(\frac{x}{x_0} \right)^{-\gamma/2}, \quad (49)$$

for which

$$N_C(x) = \begin{cases} \frac{1}{1-3\gamma/4} \left[(x/x_0)^{1-\gamma} - (x/x_0)^{-\gamma/4} \right], & \gamma \neq 4/3 \\ \frac{\ln(x/x_0)}{(x/x_0)^{1/3}}, & \gamma = 4/3 \end{cases}. \quad (50)$$

The function $N_C(x)$ is bounded for $\gamma \geq 1$, and for all $\gamma > 1$ the wave amplitude decreases with the distance (Fig. 12). The maximum amplification along the plane beach ($\gamma = 1$) is $N_C(x \rightarrow \infty) = 4$.

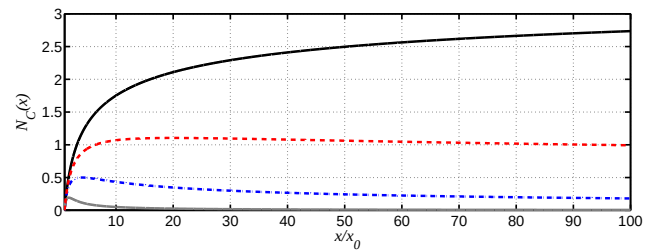


Fig. 12. Wave amplitude variations versus distance (Eq. 50) for $\gamma = 1$ (black solid line), $\gamma = 4/3$ (red dashed line), $\gamma = 2$ (blue dash-dotted line), $\gamma = 4$ (green dotted line).

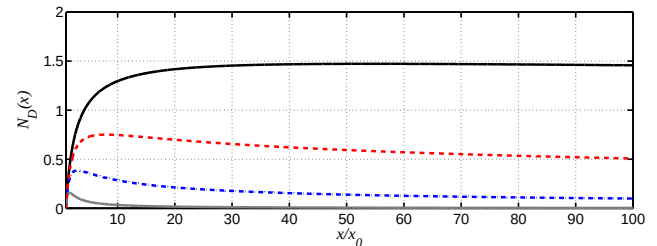


Fig. 13. Wave amplitude variations versus distance (Eq. 53) for $\gamma = 1$ (black solid line), $\gamma = 4/3$ (red dashed line), $\gamma = 2$ (blue dash-dotted line), $\gamma = 4$ (green dotted line).

And the third case is the landslide motion with the decrease in its volume, which we study on the following example:

$$Q_D(x) = Q_0 \left(\frac{h}{h_0} \right)^{-3/4} = Q_0 \left(\frac{x}{x_0} \right)^{-3\gamma/4}. \quad (51)$$

For the “non-reflecting” bottom profile (10) and $Z(\tau)$ defined by Eq. (17) the volume of the landslide can be calculated explicitly.

$$M_D(t) = Q_0 x_0 \ln \left[1 + \frac{T}{\tau_0 + t} \right]. \quad (52)$$

As before, the wave amplitude is described by the function $N_D(x)$

$$N_D(x) = \begin{cases} \frac{1}{1-\gamma} \left[(x/x_0)^{1-5\gamma/4} - (x/x_0)^{-\gamma/4} \right], & \gamma \neq 1 \\ \frac{\ln(x/x_0)}{(x/x_0)^{1/4}}, & \gamma = 1 \end{cases}. \quad (53)$$

Now the wave amplitude decreases for all $\gamma > 4/5$ including the plane beach (Fig. 13).

Thus, we demonstrate that the joint action of the resonance and increasing depth generally leads to non-monotonic variations of the wave amplitude with a distance. The result is sensitive to the landslide volume variations in time and the amplification zone reduces with a decrease in the landslide volume.

6 Conclusions

The tsunami wave generation by submarine landslides of a variable volume moving with a variable speed in a basin of variable depth is studied analytically.

The theory of tsunami wave generation by landslides is developed for basins of “non-reflecting” bottom configurations $h \sim x^{4/3}$ and $h \sim x^4$. For such bottom configurations the shallow-water system can be reduced to the constant-coefficient wave equation with an external forcing, and its analytical solution can be expressed in the form of the Duhamel integral. Analytical solutions for tsunami wave generation by landslides of variable volume and speed are obtained and discussed. They consist of three waves: the forced wave propagating in the same direction as the landslide and two free waves propagating in the opposite directions. The forced wave is located above the landslide and propagates with a variable velocity; its length increases with a distance and its amplitude is proportional to the landslide height and decreases with a distance. The forced wave is presented by the wave of elevation in supercritical regime ($Fr > 1$) and by the wave of depression in subcritical regime ($Fr < 1$). One of the free waves propagates offshore with a variable velocity $c(x)$. It moves behind the landslide in the supercritical regime ($Fr > 1$) and in front of it in subcritical regime ($Fr < 1$). In the beginning its amplitude increases due to the resonant effect and then decreases according to the Green’s law for the beach $h \sim x^{4/3}$ or stabilizes at deep waters for the beach $h \sim x^4$. The wavelengths of the waves propagating offshore increase in time. Another free wave propagates onshore with an increase in its amplitude and decrease in its length.

The runup of the landslide generated wave approaching the coast $h \sim x^{4/3}$ is studied analytically assuming that the wave does not break at the shoreline. The runup characteristics depend on the Froude number.

The process of the resonant generation of tsunami waves by landslides is studied in a basin of slowly varying depth. Using the asymptotic approach, the wave field can be expressed analytically in the integral form. The main result here is that even for the landslide moving with the resonant speed, generally the wave amplitudes remain bounded and decrease at large distances for most of “power-like” beaches.

Since the resonance leads to the maximum amplification, the obtained expressions can be used for evaluation of prognostic tsunami waves generated by the submarine landslides of variable volume and speed. These analytical results can also be used for testing numerical schemes.

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References

- Altinok, Y., Tinti, S., Alpar, B., Yalciner, A. C., Ersoy, S., Bortolucci, E., and Armigliato, A.: The tsunami of August 17, 1999 in Izmit Bay, Turkey, *Nat. Hazards*, 24, 133–146, 2001.
- Assier-Rzadkiewicz, S., Heinrich, P., Sabatier, P. C., Savoyer, B., and Bourillet, J. F.: Numerical modeling of a landslide-generated tsunami: the 1979 Nice event, *Pure Appl. Geophys.*, 157, 1707–1727, 2000.
- Courant, R. and Hilbert, D.: *Methods of Mathematical Physics*, John Wiley & Sons Inc, 598 pp., 1989.
- Deplus, C., Le Friant, A., Boudon, G., Komorowski, J.-C., Villemant, B., Harford, C., Ségoufin, J., and Cheminée, J.-L.: Submarine evidence for large-scale debris avalanches in the Lesser Antilles Arc, *Earth Planet. Sc. Lett.*, 192(2), 145–157, 2001.
- Didenkulova, I., Pelinovsky, E., Soomere, T., and Zahibo, N.: Runup of nonlinear asymmetric waves on a plane beach, in: *Tsunami and Nonlinear Waves* edited by: Anjan Kundu, Springer, Germany, 173–188, 2007.
- Didenkulova I., Pelinovsky, E., and Soomere, T.: Run-up characteristics of tsunami waves of “unknown” shapes, *Pure Appl. Geophys.*, 165(11/12), 2249–2264, 2008.
- Didenkulova, I., Pelinovsky, E., and Soomere, T.: Long surface wave dynamics along a convex bottom, *J. Geophys. Res.-Oceans*, 114, C07006, doi:10.1029/2008JC005027, 2009.
- Didenkulova, I. and Pelinovsky, E.: Traveling water waves along quartic bottom profile, *P. Est. Acad. Sci.*, 59(2), 166–171, doi:10.3176/proc.2010.2.16, 2010.
- Didenkulova, I., Pelinovsky, E., and Soomere, T.: Can the waves generated by fast ferries be a physical model of tsunami?, *Pure Appl. Geophys.*, in press, 2010.
- Fine, I. V., Rabinovich, A. B., Thomson, R. E., and Kulikov, E. A.: Numerical modeling of tsunami generation by submarine and subaerial landslides, in: *Submarine landslides and tsunamis*, in NATO Science Series: IV. Earth Env. Sci., 21, edited by: Yalçiner, A., Pelinovsky, E., Okal, E., and Synolakis, E., Kluwer, 69–88, 2003.
- Fine, I. V., Rabinovich, A. B., Bornhold, B. D., Thomson, R. E., and Kulikov, E. A.: The Grand Banks landslide-generated tsunami of November 18, 1929: preliminary analysis and numerical modeling, *Mar. Geol.*, 215, 45–57, 2005.
- Gusiakov, V. K.: Historical Tsunami Database, Natl. Geophys. Data Cent., Novosibirsk, Russia, available at: http://www.ngdc.noaa.gov/hazard/tsu_db.shtml (last access: October 2010), 2002.
- Gusiakov, V. K.: Tsunami history: recorded in The Sea, in: *Tsunamis*, edited by: Robinson, A. and Bernard, E., Harvard University Press, Cambridge, 15, 23–54, 2009.
- Grimshaw, R., Pelinovsky, D., and Pelinovsky, E.: Homogenization of the variable – speed wave equation, *Wave Motion*, 47(8), 496–507, doi:10.1016/j.wavemoti.2010.03.001, 2010.
- Heinrich, P., Guibourg, S., Mangeney, A., and Roche, R.: Numerical modeling of a landslide-generated tsunami following a potential explosion of the Montserrat volcano, *Phys. Chem. Earth Pt. A*, 24(2), 163–168, 1999.

- Imamura, F. and Gica, E. C.: Numerical model for tsunami generation due to subaqueous landslide along a coast, *Sci. Tsunami Hazards*, 14(1), 13–28, 1996.
- Keating, B. H., Waythomas, C. F., and Dawson, A. G.: Landslides and Tsunamis, *Pageoph, Topical Volumes*, 157, 871–1313, 2000.
- Liu, P. L.-F., Lynnett, P., and Synolakis, C. E.: Analytical solutions for forced long waves on a sloping beach, *J. Fluid Mech.*, 478, 101–109, 2003.
- Mangeney, A., Heinrich, P. H., Rachel, R., Boudon, G., and Cheminee, J.-L.: Modeling of Debris Avalanche and Generated Water Waves: Application to Real and Potential Events in Montserrat, *Phys. Chem. Earth Pt. A*, 25(9–11), 741–745, 2000.
- McSaveney, M. J., Goff, J. R., Darby, D. J., Goldsmith, P., Barnett, A., Elliott, S., and Nongkas, M.: The 17 July 1998 tsunami, Papua New Guinea: Evidence and initial interpretation, *Mar. Geol.*, 170, 81–92, 2000.
- Kajiura, K.: The leading wave of a tsunami, *B. Earthq. Res. Inst. Tokyo*, 41, 535–571, 1963.
- Kervella, Y., Dutykh, D., and Dias, F.: Comparison between three-dimensional linear and nonlinear tsunami generation models, *Theor. Comp. Fluid Dyn.*, 21, 245–269, 2007.
- Kuo, C. Y., Tai, Y. C., Bouchut, F., Mangeney, A., Pelanti, M., Chen, R. F., and Chang, K. J.: Simulation of Tsaoiling landslide, Taiwan, based on Saint Venant equations over general topography, *Eng. Geol.*, 104(3–4), 181–189, doi:10.1016/j.enggeo.2008.10.003, 2008.
- Novikova, L. E. and Ostrovsky, L. A.: Excitation of tsunami waves by a travelling displacement of the ocean bottom, *Mar. Geod.*, 2, 365–380, 1978.
- Okal, E. O. and Synolakis, C. E.: Theoretical comparisons of tsunamis from dislocations and slides, *Pure Appl. Geophys.*, 160, 2177–2188, 2003.
- O’Loughlin, K. F. and Lander, J. F.: *Caribbean Tsunamis, a 500-Year History from 1498–1998*, Kluwer Academic Publishers, Netherlands, 263 pp., 2003.
- Pelinovsky, E. N.: *Hydrodynamics of tsunami waves*, Institute of Applied Physics, Nizhny Novgorod, 276 pp., 1996 (in Russian).
- Pelinovsky, E. N.: Analytical models of tsunami generation by submarine landslides, in: *Submarine landslides and tsunamis*, NATO Science Series: IV. Earth Env. Sci., 21, edited by: Yalçiner, A., Pelinovsky, E., Okal, E., and Synolakis, E., Kluwer, 111–128, 2003.
- Pelinovsky, E. and Poplavsky, A.: Simplified model of tsunami generation by submarine landslides, *Phys. Chem. Earth*, 21, 13–17, 1997.
- Pelinovsky, E., Zahibo, N., Dunkley, P., Edmonds, M., Herd, R., Talipova, T., Kozelkov, A., and Nikolkina, I.: Tsunami Generated by the Volcano Eruption on July 12–13, 2003 at Montserrat, Lesser Antilles, *Sci. Tsunami Hazards*, 22(1), 44–57, 2004.
- Pudasaini, S. P. and Hutter, K.: *Avalanche Dynamics: Dynamics of Rapid Flows of Dense Granular Avalanches*, Springer, 602 pp., 2007.
- Sammarco, P. and Renzi, E.: Landslide tsunamis propagating along a plane beach, *J. Fluid Mech.*, 598, 107–119, 2008.
- Synolakis, C. E.: The runup of solitary waves, *J. Fluid Mech.*, 185, 523–545, 1987.
- Synolakis, C. E., Bardet, J., Borrero, J. C., Davies, H. L., Okal, E. A., Silver, E. A., Sweet, S., and Tappin, D. R.: The slump origin of the 1998 Papua New Guinea Tsunami, *P. Roy. Soc. Lon. Ser.-A*, 458, 763–789, 2002.
- Teeuw, R., Rust, D., Solana, C., Dewdney, C., and Robertson, R.: Large Coastal Landslides and Tsunami Hazard in the Caribbean, *EOS T. Am. Geophys. Un.*, 90(10), 81–88, doi:10.1029/2009EO100001, 2009.
- Ten Brink, U. S., Geist, E. L., and Andrews, B. D.: Size distribution of submarine landslides and its implication to tsunami hazard in Puerto Rico, *Geophys. Res. Lett.*, 33, L11307, doi:10.1029/2006GL026125, 2006.
- Tinti, S. and Bortolucci, E.: Analytical investigation of tsunamis generated by submarine slides, *Ann. Geofis.*, 43, 519–536, 2000.
- Tinti, S., Bortolucci, E., and Chiavettieri, C.: Tsunami excitation by submarine slides in shallow-water approximation, *Pure Appl. Geophys.*, 158, 759–797, 2001.
- Ward, S. N.: Landslide tsunami, *J. Geophys. Res.*, 106, 11201–11215, 2001.
- Yalçiner, A. C., Pelinovsky, E. N., Okal, E., and Synolakis, C. E.: Submarine landslides and tsunamis, in: *NATO Science Series: IV. Earth Env. Sci.*, Kluwer, 2003.
- Zahibo, N.: Deshaies, Guadeloupe tsunami, 20 May 2006, Montserrat, Lesser Antilles, Soufriere volcano tsunami, *Tsunami Newsletter*, 38(2): available at: http://www.palais-decouverte.fr/expos/vst_2k7/vrst_2k7/m7_2.html (last access: October 2010), 2006.

GEOFYSICS

Resonant Amplification of Tsunami Waves Generated by an Underwater Landslide

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Generation and resonant amplification of tsunami waves by underwater landslides is studied within the framework of shallow water theory in a varied depth basin. Two cases of landslide motion are considered: a variable mass landslide moving resonantly with tsunami waves and a constant mass one moving with constant velocity. In both cases, the amplitude of the tsunami wave varies nonmonotonically in time. Characteristics of the resonant zone are examined in detail.

At present, landslides are considered to be one of the main sources of tsunami waves; e.g., in 1998, an underwater landslide caused a 15-meter tsunami wave in Papua New Guinea [13]. Later, in 2006, 9-meter tsunami wave occurred as the result of a landslide seriously damaging territory in the Columbia River valley [9].

Various numerical models are used for description of tsunami wave generation by landslides; they take into consideration rheologic properties of landslide and real bathymetry change for a constant volume of the landslide [3, 13]. Due to the fact that landslides really move with variable velocities [4, 7, 10], the appearance of resonant effects is possible because of the variable depth of the basin and the variable velocity of the landslide motion. The peculiarities of such effects have been examined in the present work.

The phenomenon of wave resonance is observed from the coincidence of the motion velocity of the source, which generates the wave, and the wave's own velocity in the medium [1]. In accurate resonance, the wave grows in time (or in space), and this growth can be limited by the appearance of velocity detuning, nonlinear effects, or other causes. An example is unlimited increase in resistance with the motion of a flying vehicle with sonic speed in the atmosphere. Taking nonlinearity into consideration leads here to the appearance of a finite maximum in the depen-

dence of resistance on flight velocity, and this maximum is shifted to the supersonic area. In the problem on generation of tsunami waves by landslides, this phenomena becomes more complicated, because the velocities of both the source and the wave are not constant and depend on coordinates.

We will use the simplest model of the linear shallow wave theory, based on the wave equation, for analysis [5, 8, 12]:

$$\frac{\partial^2 \eta}{\partial t^2} - \frac{\partial}{\partial x} \left[c^2(x) \frac{\partial \eta}{\partial x} \right] = \frac{\partial^2 z_b}{\partial t^2}, \quad c = \sqrt{gh(x)}, \quad (1)$$

where $\eta(x, t)$ is the displacement of the water surface, g is gravity acceleration, $z_b(x, t)$ is movement of the bottom caused by a landslide (in the general case, a landslide with variable volume). It is solved at the following initial conditions:

$$\eta(x, 0) = z_b(x, 0), \quad \frac{\partial \eta}{\partial t}(x, 0) = \frac{\partial z_b}{\partial t}(x, 0). \quad (2)$$

In the case of resonance, when the landslide and water surface move synchronously (the Froude number is $Fr = 1$), in basin of constant depth, a resonant wave with monotonically increasing amplitude is formed [5], while it can appear to be finite due to the variable depth. In the vicinity of resonance, waves propagating offshore have a velocity close to $c(x) = \sqrt{gh(x)}$, which is varied with distance. In this case, it is favorable to introduce new variables

$$s = \tau(x) - t, \quad x' = x, \quad (3)$$

where $\tau(x) = \int \frac{dx}{c(x)}$ is the propagation time. Then initial equation (1) is transformed into the following form:

$$2c(x') \frac{\partial^2 \eta}{\partial s \partial x'} + \frac{dc}{dx'} \frac{\partial \eta}{\partial s} + \frac{\partial}{\partial x'} \left[c^2(x') \frac{\partial \eta}{\partial x'} \right] = -\frac{\partial^2 z_b}{\partial s^2}. \quad (4)$$

In the general form, Eq. (4) is not solved; hence, the approximation of slowly varying depth (WKB approximation) is used here. In this case, the third

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term of Eq. (4) is negligibly small and can be omitted. Then, solution of Eq. (4) for zero initial conditions is written as

$$\eta(x, s) = -\frac{1}{2\sqrt{c(x)}} \int_{x_0}^x \frac{1}{\sqrt{c(y)}} \frac{\partial z_b(y, s)}{\partial s} dy, \quad (5)$$

where point x_0 corresponds to the initial position of the landslide. Taking into consideration the initial conditions from Eq. (2) leads to addition of nonresonant terms, which are small by their amplitude. In the present work, only resonant waves propagating to the deep-water side will be examined. The tsunami wave would also go to the shore side, but with a wave that is nonresonant and relatively small by amplitude; however, it can cause a significant runup at the coast [2].

Equation (5) can be used for studying the process of tsunami generation by a landslide, which is moved with an arbitrary velocity close to the velocity of wave motion. Let us consider the case when the landslide slips down the slope with constant angle

$$h(x) = h_0 \frac{x}{x_0} \quad (6)$$

(h_0 is depth at point x_0), though many results obtained here would be true for more complicated bottom profiles as well. Let the landslide with variable volume have arbitrary form and move resonantly:

$$z_b(x, t) = Q(x)Z[\tau(x) - t]. \quad (7)$$

It generates a tsunami wave with variable amplitude,

$$\eta(x, t) = -D(x) \frac{\partial Z(\tau - t)}{\partial \tau}, \quad (8)$$

$$D(x) = \frac{1}{2\sqrt{c(x)}} \int_{x_0}^x \frac{Q(y)dy}{\sqrt{c(y)}}.$$

Let us consider the most popular case first, when the landslide has constant volume

$$M = \int z_b(x, t) dx = \int c(\tau) Q(\tau) Z(\tau - \text{Fr} \cdot t) d\tau = \text{const}, \quad (9)$$

and its height and length vary with distance. It follows from Eq. (9) that it is possible at $Q \sim c^{-1}$

$$Q_c(x) = Q_0 \left(\frac{h}{h_0} \right)^{-1/2} = Q_0 \left(\frac{x}{x_0} \right)^{-1/2}. \quad (10)$$

Then the amplitude of the wave is determined by function $N_c(x)$

$$D_c(x) = \frac{Q_0 x_0}{2\sqrt{gh_0}} N_c(x), \quad N_c(x) = 4 \left[1 - \left(\frac{x}{x_0} \right)^{-1/4} \right]. \quad (11)$$

The behavior of function $N_c(x)$ is shown in Fig. 1 (solid line). It is seen that in the case of the occurrence of a landslide with constant volume, the amplitude of tsunami waves increases with distance but stays lim-

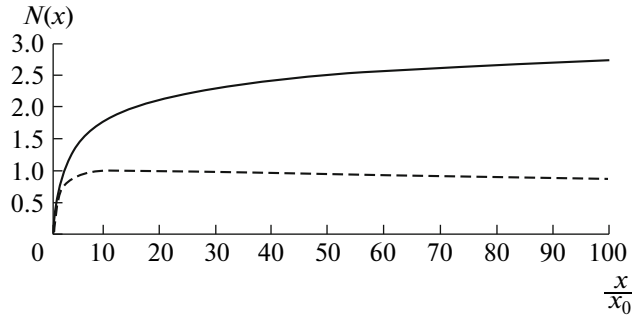


Fig. 1. Variation of function $N(x)$ with distance for a landslide with constant (solid line) and decreasing (dashed line) volume.

ited: the maximal increase in the amplitude of the wave above the flat slope is $N_c(x \rightarrow \infty) = 4$.

However, the mass of the landslide can vary during its motion, in particular, caused by erosion processes [1]; hence, we will consider the case when the volume of the landslide decreases, as well; e.g., in accordance with the law

$$Q_D(x) = Q_0 \left(\frac{h}{h_0} \right)^{-1} = Q_0 \left(\frac{x}{x_0} \right)^{-1}. \quad (12)$$

Then the amplitude of the wave is described by the function $N_D(x)$ analogously to (11) (Fig. 1, dashed line):

$$N_D(x) = 4 \left(\frac{x}{x_0} \right)^{-1/4} \left[1 - \left(\frac{x}{x_0} \right)^{-1/4} \right]. \quad (13)$$

In this case, the amplitude of the wave first increases up to its maximum $N_D(x = 16x_0) = 1$, and then reduces. The mass of the landslide reduces inversely to the motion time t^{-1} and inversely to the square root of distance $x^{-1/2}$, which explains the slow amplitude decrease for tsunami waves caused by a landslide in Fig. 1. At a stronger law of landslide mass reduction, the decrease would be more significant, as well.

Thus, the combined contribution of resonance and depth increase lead to nonmonotonic variation of the wave amplitude with distance. This result depends on the character of landslide volume variation, and the region of resonant amplification becomes less when the landslide volume decreases.

Let us consider separately the transition of the landslide through resonance with its motion along a flat slope. Let the landslide of constant volume have a Gaussian form and move with a constant velocity V :

$$z_b(t, x) = Q_0 \exp \left(-\frac{4 \left(\frac{x}{V} - t \right)^2}{T^2} \right). \quad (14)$$

Let us introduce dimensionless variables

$$X = \frac{x}{x_0}, \quad S = \frac{c_0 s}{x_0}, \quad \text{Fr} = \frac{V}{c_0}, \quad \theta = \frac{c_0 T}{x_0}, \quad (15)$$

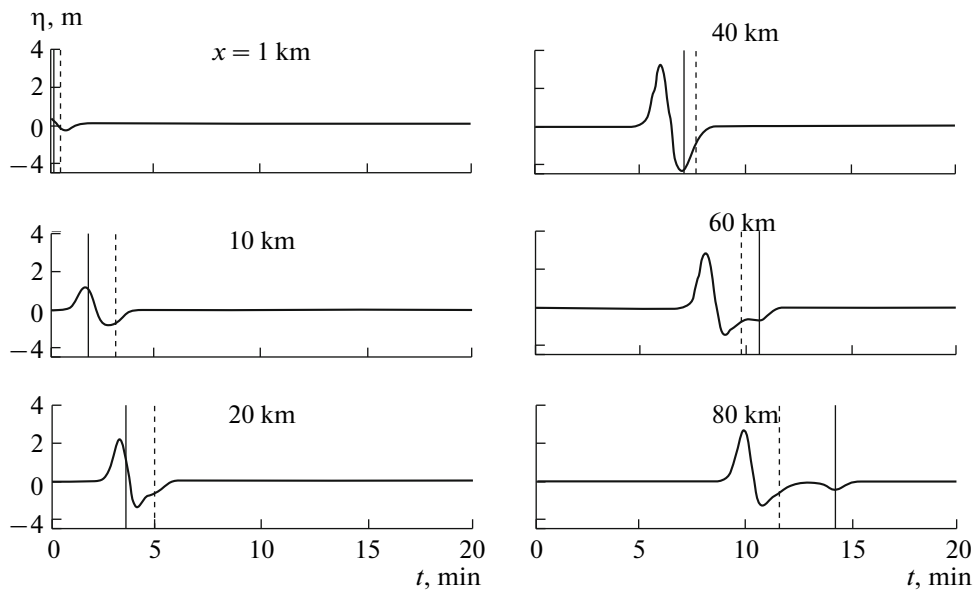


Fig. 2. Tsunami wave records at different distances for $Fr = 3$ and $\theta = 1$. The solid red line shows the location of the landslide; the dashed line marks the moment of free wave arrival $t = \tau$.

where the last term is the ratio between the length of the landslide ($L = c_0 T$) and the characteristic distance of depth variation x_0 . Then, the argument in (14) has the following form:

$$\xi = \frac{S - \left[2(\sqrt{1 + X} - 1) - \frac{X}{Fr} \right]}{\theta}. \quad (16)$$

Equation (5) is also transformed with the use of dimensionless variables:

$$\eta(S, X) = -\frac{Q_0}{2\theta(1 + X)^{1/4}} \int_0^X \frac{1}{(1 + y)^{1/4}} \frac{dZ(\xi)}{d\xi} dy, \quad (17)$$

$$Z(\xi) = \exp(-4\xi^2).$$

The calculated records of tsunami waves at different distances from the source for the initial Froude number $Fr = 3$ and relative length of landslide $\theta = 1$ are given in Fig. 2. At the initial moment of time, a landslide of 2 km length is located at 100 m depth.

In this case, with the propagation of a wave up to the distance $x = 16$ km, the supercritical regime acts ($Fr > 1$), and then the subcritical regime ($Fr < 1$) occurs and the landslide starts moving more slowly than the tsunami wave (see Fig. 2). In the supercritical regime, the induced wave with positive polarity propagates first and moves with the same velocity as the landslide does, and the second (free) wave follows it, moving with the velocity of surface waves $c(x)$ and having negative polarity. After passing through resonance, the resonant wave transforms into a free one, staying positive, and continues propagation the first.

Thus, the polarity of first wave differs for the sub- and supercritical regimes, though tsunami wave records for various Froude numbers appear the same.

In the case when the landslide is shorter (θ is less), the mareogram is not changed qualitatively, but the amplitude of wave increases, because the tsunami is generated mostly by leading and trailing fronts of the landslide.

Spatial variation of positive A_+ and negative A_- amplitudes of tsunami waves for different Froude numbers is shown in Fig. 3. As the Froude number increases, the resonance region becomes more shifted relative to the initial position; the increase of the wave in the resonance region is more significant, than beyond it. This example demonstrates the difference between resonant and nonresonant processes of tsunami wave generating.

Taking the effects of nonlinearity and dispersion in the vicinity of resonance into consideration can be implemented within the framework of the forced Korteweg–de Vries equation [6]. Then small-scale soliton-shape waves appear on the body of a long wave, but they do not have any influence on the characteristics of long waves. Hence, the obtained approximate solutions can be used for determination of the parameters of tsunami waves generated by underwater landslide of variable volume, which moves with variable velocity.

As has been shown above, at the initial Froude number $Fr = 3$, a landslide of one meter height and 2 km length can generate a tsunami wave of more than 5 m height that is in good agreement with the real events mentioned above. At the initial Froude number $Fr = 1$, when the resonant effects are weaker and there

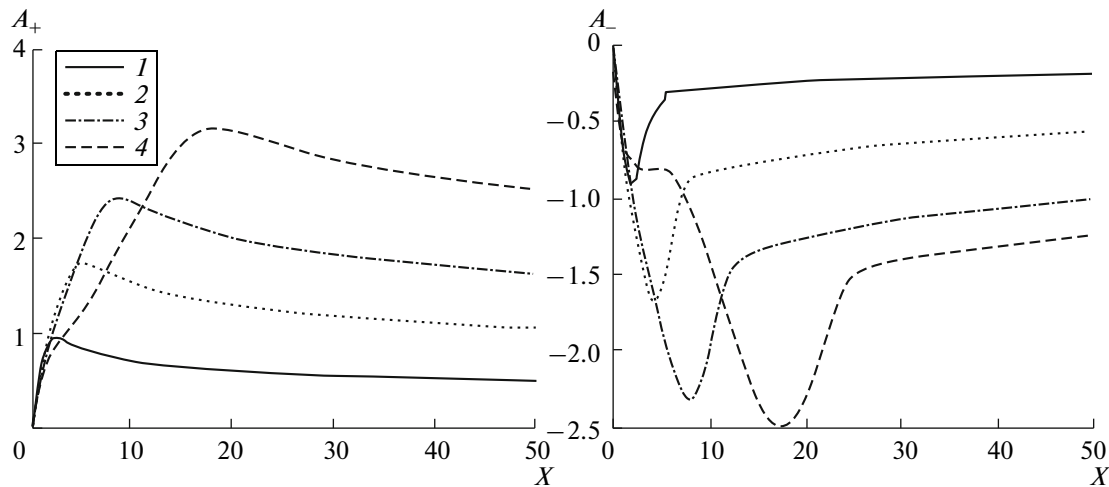


Fig. 3. Spatial variation of positive A_+ and negative A_- amplitudes of tsunami wave for $\theta = 1$; $Fr = 1$ (1), 1.5 (2), 2 (3), 3 (4).

is no passage through the resonance is absent, the height of the tsunami wave does not exceed two meters.

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REFERENCES

1. M. B. Vinogradova, O. V. Rudenko, and A. P. Sukhorukov, *Theory of Waves* (Nauka, Moscow, 1990) [in Russian].
2. I. I. Didenkulova, N. Zahibo, A. A. Kurkin, and B. V. Levin, *Doklady. Akad. Nauk*, **410**, 676 (2006) [*Doklady Earth Sci.* **411**, 1241 (2006)].
3. B. V. Levin and M. A. Nosov, *Physics of Tsunami* (Nauka, Moscow, 2005) [in Russian].
4. I. F. Nikolkina, E. N. Pelinovskii, and T. G. Talipova, *Doklady Akad. Nauk* **432** (5), 489–492 (2010).
5. E. N. Pelinovskii, *Hydrodynamics of Tsunami Waves* (Nizhni Novgorod, IPF RAN, 1996) [in Russian].
6. E. N. Pelinovskii, *Non-Linear Waves* (N. Novgorod, IPF RAN, 2003), pp. 199–210 [in Russian].
7. O. V. Rudenko, A. L. Sobisevich, and L. E. Sobisevich, *Doklady Akad. Nauk* **416**, 384 (2007) [*Doklady Earth Sci.* **416**, 1109 (2007)].
8. P. L.-F. Liu, P. Lynnett, and C. E. Synolakis, *J. Fluid. Mech.* **478**, 101 (2003).
9. National Geophysical Data Center, <http://www.ngdc.noaa.gov>.
10. E. Pelinovsky and A. Poplavsky, *Phys. Chem. Earth*, **21** (12), 13 (1996).
11. S. P. Pudasaini and S. Hutter, *Avalanche Dynamics: Dynamics of Rapid Flows of Dense Granular Avalanches* (Springer, 2007).
12. S. Tinti, E. Bortolucci, and C. Chiavettieri, *Pure and Appl. Geophys.* **158**, 759 (2001).
13. A. C. Yalciner, E. N. Pelinovsky, E. Okal, and C. E. Synolakis, *Submarine Landslides and Tsunamis. NATO Science Ser. IV. Earth and Environmental Sciences* (Kluwer, 2003).

GEOPHYSICS

Nonlinear Dynamics of Gravity Flows in Sloping Channels

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New analytical solutions, which allow us to estimate the landslide parameters in narrow mountainous valleys or sea canyons, were found within a nonlinear model of the Savage-Hutter slush landslide.

Gravity flows (avalanches and landslides) flowing down from mountains and volcanoes in the coastal zones of seas frequently cause tsunami waves. The most striking example is avalanching in Lituya Bay (Alaska, United States) in 1958, which caused a tsunami with a height of about 500 m [1]. Different analytical and numerical models are applied to describe tsunami generation by landslides [4]. In some publications, a landslide is modeled as a solid body moving under the force of gravity and friction over the underlying surface [2, 5, 7]. More realistic models take into account the deformation of the landslide body in the course of its motion [3, 4, 6, 8]. A popular model is the model of a slush landslide by Savage-Hutter moving under the force of gravity and Coulomb friction [7]. A number of exact solutions were obtained, which allowed us to test the numerical codes applied to calculate avalanches and landslides [3, 6, 8]. In all of these cases, a flat landslide moves along a sloping plane.

In realistic conditions, landslides and avalanches move in channels (mountainous valleys, sea canyons). In this work, we obtained analytical solutions for avalanches in sloping channels of a parabolic section. It will be shown that the spatial structure of flows appears variable, and the channel form influences significantly the rates of nonlinear deformation of the landslide body.

The main equations for the landslide flow with a homogeneous vertical structure moving under the force of gravity and Coulomb friction (Savage-Hutter model) in narrow channels in the coordinate system related to topography are written as

$$\frac{\partial S}{\partial t} + \frac{\partial}{\partial x}(Su) = 0, \quad \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + g \cos \theta \frac{\partial h}{\partial x} = g \alpha, \quad (1)$$

where S is the square of the cross section occupied by the flow; h is the height of the flow along the channel axis, u is the mean velocity of the flow, g is the acceleration due to gravity, θ is the slope angle of the plane, $\mu = \tan \varphi$ is the Coulomb friction coefficient, φ is the dynamic friction angle, and $\alpha = \sin \theta - \mu \cos \theta$. The value of φ depends on the material of the landslide, in particular, for the avalanches flowing from volcanoes its value is estimated as 17° – 35° . Equations (1) differ from the known shallow water equations only by the dissipative term, which is usually accepted as quadratic friction for water.

We assume for the closure of this system that the cross section of the channel has a parabolic form $z(y) \sim |y|^m$ ($S \sim h^{(m+1)/m}$) and $m > 0$. As a result, the first equation in (1) is replaced by

$$\frac{\partial h}{\partial t} + u \frac{\partial h}{\partial x} + \frac{m}{m+1} h \frac{\partial u}{\partial x} = 0 \quad (2)$$

and system (1), (2) becomes closed. The cross section of the channel is manifested here by means of coefficient m , and in the case when $m \rightarrow \infty$ ($S \sim h$), system (1), (2) describes a landslide flow over a plane. In the remaining part of this study, we assume that the slope of the channel is constant and sufficient for initiating the flow motion ($\theta > \varphi$).

Equation system (1), (2) is a hyperbolic one with constant coefficients. Therefore, many solutions of this system can be obtained using standard approaches. In particular, Riemann invariants are found explicitly:

$$I_{\pm} = u \pm 2 \sqrt{\frac{m+1}{m}} g h \cos \theta - \alpha g t, \quad (3)$$

which allows us to rewrite system (1), (2) as

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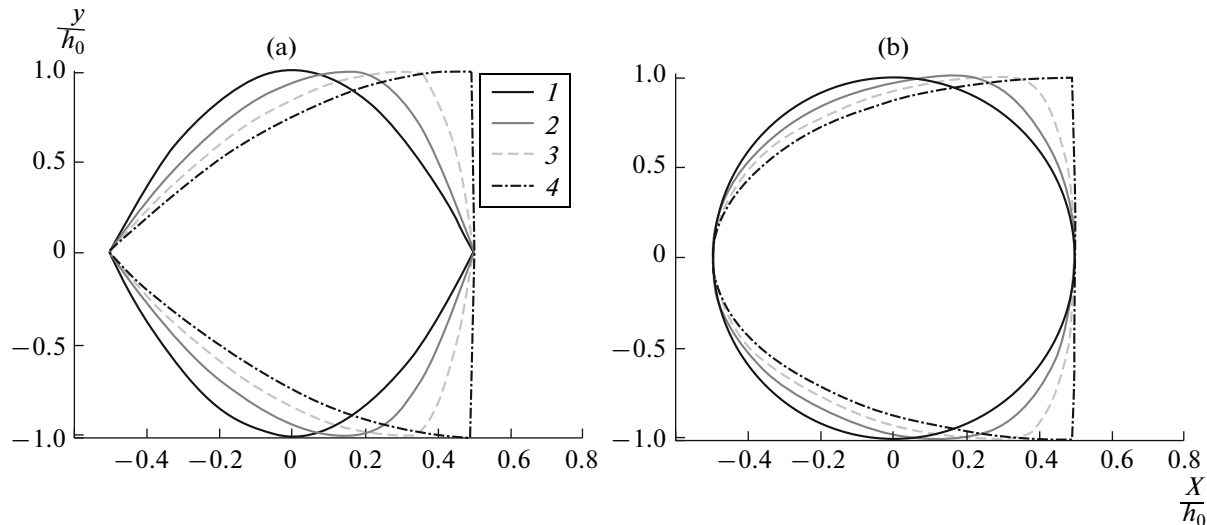


Fig. 1. Contours of the landslide at different time moments in triangular (a) and parabolic (b) cross sections: T_{br} is collapse time; $t = 0$ (1), $\frac{1}{3} T_{br}$ (2), $\frac{2}{3} T_{br}$ (3), T_{br} (4).

$$\begin{aligned} \frac{\partial I_{\pm}}{\partial t} + c_{\pm} \frac{\partial I_{\pm}}{\partial x} &= 0, \\ c_{\pm} &= \frac{3m+2}{4(m+1)} I_{\pm} + \frac{m+2}{4(m+1)} I_{\mp} + \alpha g t. \end{aligned} \quad (4)$$

Equation system (4) was studied in the theory of waves on the water surface [1]. However, Coulomb friction was not considered and the bottom slope was considered small. The formulation of the problem was also different: in the study of waves in water, the initial conditions correspond to weakly nonlinear waves, while in the gravity waves, nonlinearity is always strong. Nevertheless, the mathematical similarity between water waves and flow equations allows us to skip some calculations and present part of the results without deduction of relations.

RIEMANN WAVES

Solutions of this type were already found for landslide flows over a slope plane without dry friction [3]. They can easily be rewritten for the flows in sloping channels. The Riemann wave is described as

$$\begin{aligned} \Gamma(x, t) &= \Gamma_0 \left(x - \frac{\alpha g t^2}{2} - \Gamma t \right), \quad \Gamma = \frac{3m+2}{\sqrt{m(m+1)}} \sqrt{gh \cos \theta}, \\ u &= 2 \sqrt{\frac{m+1}{m}} gh \cos \theta + \alpha g t. \end{aligned} \quad (5)$$

The physical sense of this solution is clear: landslide flows down under the influence of reduced gravity and rotates at the front. In channels of different forms, the landslide is deformed in the longitudinal and transversal directions, which is demonstrated in

Fig. 1 (in the coordinate systems moving with uniform acceleration). In the course of time, the landslide front becomes plane; at the same time, its rear slope is sensitive to the form of the transversal section: it has a triangle form in the channel of a triangular cross section ($m = 1$) and a parabolic form in the channel of parabolic cross section ($m = 2$).

During a finite time period, the landslide front becomes vertical and begins to collapse. The time of the collapse can easily be found (it is not given here). The time depends on the height of the landslide and the inclination angle of the channel to the horizontal plane. It also depends on the form of the transversal section of the channel and decreases in narrow channels ($m \rightarrow 0$).

After collapsing, this solution is formally not valid, and the broken domain should be replaced by a jump. The method of calculation of the jump location is well known in hydrodynamics and nonlinear acoustics. It was applied for flat landslides in [3]. However, we will not calculate the amplitude of the jump because our main goal is finding the analytical solutions, which are not found explicitly for the landslides with a broken front.

THE DAM BREAK PROBLEM

An analytical solution of this problem is well known for water flow over a flat bottom. This analysis was used in [6] for a landslide flow over a sloping plane with Coulomb friction. Actually, the new solution was obtained from the old one after a transition to the coordinate system, which moves with acceleration. The corresponding transformations can also be found for Eq. (4). Zero velocity and a sharp jump in the water depth from 0 to h_0 are the initial conditions in the dam

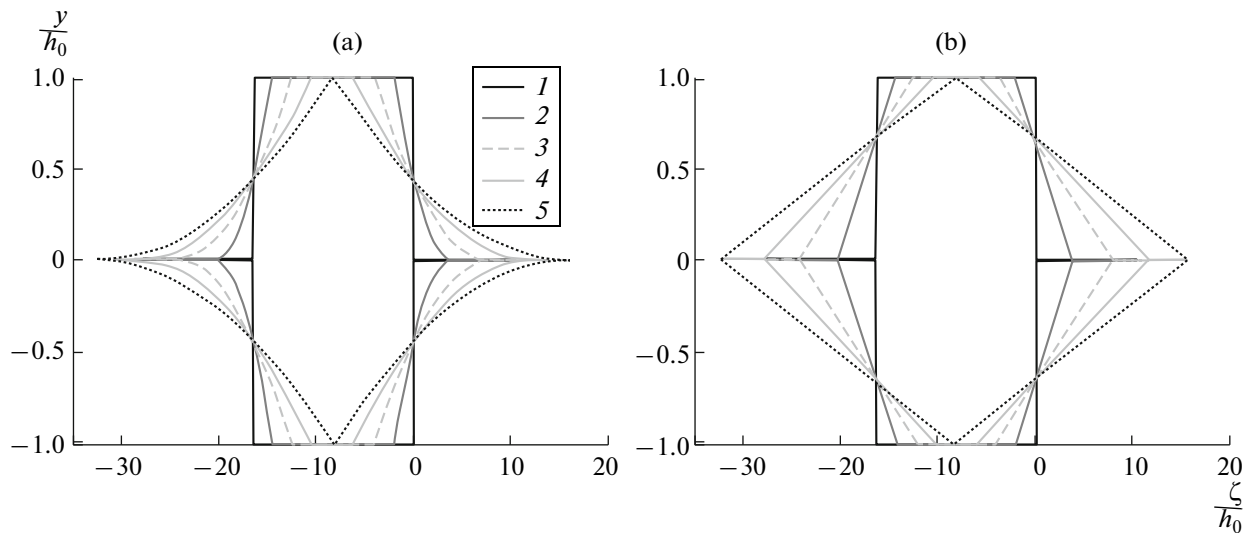


Fig. 2. Evolution of a rectilinear landslide in a triangle (a) and parabolic sections of the channel: $\tau = \frac{h_0}{c_0}$, $\frac{t}{\tau} = 0$ (1), 2 (2), 4 (3), 6 (4), 8 (5).

break problem. This condition does not change in the transformed coordinate system. Therefore, we present the solution without deduction:

$$h(\zeta, t) = \frac{h_0}{9} \left(2 - \frac{\zeta}{c_0 t} \right)^2 \quad (6)$$

in the interval $-c_0 t < \zeta < 2c_0 t$ and 0 beyond this interval. Here, c_0 is found through the height of the landslide using the relation $c_0 = \left[(m+1) \frac{gh_0 \cos \theta}{m} \right]^{1/2}$. The

landslide front moves with a maximum velocity $2c_0$ (in a uniformly accelerated coordinate system), and the domain of the immobile landslide moves with velocity $-c_0$. The velocity of the current within the transition zone changes linearly and increases toward the landslide front.

In the course of time, the frontal part of the landslide becomes smoother and no collapse occurs at any moment of time. It seems that the solutions of the two problems mentioned above contradict each other: in the first case, collapse of the front is mandatory, in the other case, the front remains smooth. The principal difference between these solutions is related to the distribution of velocity inside the landslide: in the first case, the velocity of the current increases from the edge of the landslide; in the second, it reaches the maximum at the edge point. Therefore, in the first case the rear particles press the leading ones; in the second case they are retarded.

The dam break problem is usually considered in a semi-infinite medium. In the case of a landslide of a finite length, the solution described above is valid at both ends of the landslide until its slopes are separated from each other. The spatial evolution of such a landslide is shown in Fig. 2 for the channels with a triangle

and parabolic sections. The form of the channel also influences the rates of landslide spreading (at the same height). Coulomb friction influences only the acceleration of the landslide but not the rates of its spreading.

SELF-SIMILAR SOLUTIONS

Such solutions for a plane landslide were found in [8]. In order to find them, it was necessary to make a transition to self-similar solutions of the following type $\eta = x - \frac{\alpha g t^2}{2}$, $t = t$; then the initial equations are transformed to

$$\begin{aligned} \frac{\partial}{\partial t} h^{(m+1)/m} + \frac{u - \eta \frac{dL}{dt}}{L} \frac{\partial}{\partial \eta} h^{(m+1)/m} + \frac{h^{(m+1)/m}}{L} \frac{\partial u}{\partial \eta} &= 0, \\ \frac{\partial u}{\partial t} + \frac{u - \eta \frac{dL}{dt}}{L} \frac{\partial u}{\partial \eta} + \frac{g \cos \theta}{L} \frac{\partial h}{\partial \eta} &= 0. \end{aligned} \quad (7)$$

If we assume the following distribution of velocity in the landslide (in the coordinate system associated with the landslide) $u = \eta \frac{dL}{dt}$, then Eqs. (7) are transformed to the usual differential equations (with respect to different variables η and t), and they are integrated:

$$\begin{aligned} h &= h_0(t)(1 - \eta^2), \quad h_0(t) = \frac{L \frac{d^2 L}{dt^2}}{2g \cos \theta}, \\ L^{(2m+1)/(m+1)} \frac{d^2 L}{dt^2} &= q. \end{aligned} \quad (8)$$

Here, parameter q determines the initial tension of the landslide. We see that a landslide of arbitrary form of the channel has a parabolic form called a “parabolic cap” in [8], where this solution was obtained only for plane geometry. The spatial form of the landslide in the channels of different sections is completely equivalent to the one shown in Fig. 1 for the initial moment of time.

The last equation in (8) is integrated in quadratures in the general form. It is important to note that, at large times, the longitudinal size of the landslide increases almost linearly with time (in the channels of arbitrary form). The landslide height decreases with time as $h \sim t^{-m/(m+1)}$ and depends on the form of the channel. In narrower channels, the landslide spreads more slowly than in wide ones.

The analytical solutions obtained here give better possibilities to take into account the actual specific properties of landslide flows spreading in narrow mountainous valleys and sea canyons. In particular, calculations show that an avalanche with an initial height of 40 m and length of 200 m collapses while avalanching from the Klyuchevskaya Sopka volcano (Kamchatka, Russia) so that its front becomes very steep. Characteristic parameters were assumed in the calculations for Klyuchevskaya Sopka: $m = 1$, inclination angle 30° , friction angle 11° , distance 2 km.

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REFERENCES

1. E. N. Pelinovsky, *Hydrodynamics of Tsunami Waves* (IPF RAN, Nizh. Novgorod, 1996) [in Russian].
2. E. N. Pelinovsky, “Nonlinear Models of Tsunami Generation by Moved Origins,” in *Nonlinear Waves-2002* (IPF, Nizh. Novgorod, 2003), pp. 199–210 [in Russian].
3. O. V. Rudenko, A. L. Sobisevich, and L. E. Sobisevich, *Dokl. Akad. Nauk* **416**, 384–387 (2007) [*Dokl. Earth Sci.* **416**, 1109 (2007)].
4. F. Bouchut, A. Mangeney-Castelnau, B. Perthame, and J.-P. Vilotte, *C.R. Acad. Sci. Paris, Ser. I* **1336**, 531–536 (2003).
5. P. L.-F. Liu, P. Lynett, and C. E. Synolakis, *J. Fluid Mech.* **478**, 101–109 (2003).
6. A. Mangeney, P. Heinrich, and R. Roche, *PAGEOPH* **157**, 1081–1096 (2000).
7. E. Pelinovsky and A. Poplavsky, *Phys. Chem. Earth* **21** (12), 13–17 (1996).
8. S. P. Pudasaini and K. Hutter, *Avalanche Dynamics: Dynamics of Rapid Flows of Dense Granular Avalanches* (Springer, New York, Berlin, 2006).
9. A. C. Yalciner, E. N. Pelinovsky, E. Okal, and C. E. Synolakis, *Submarine Landslides and Tsunamis* (Kluwer, London, 2003), Vol. 21.

Savage-Hutter model for avalanche dynamics in inclined channels: Analytical solutions

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[1] The Savage-Hutter model is applied to describe gravity driven shallow water flows in inclined channels of parabolic-like shapes modeling avalanches moving in mountain valleys or landslide motions in underwater canyons. The Coulomb (sliding) friction term is included in the model. Several analytical solutions describing the nonlinear dynamics of avalanches are obtained: the nonlinear deformed (Riemann) wave, the dam break problem, self-similar solutions and others. Some of them extend the known solution for an inclined plate (one-dimensional geometry). The cross-section shape of the inclined channels significantly influences the speed of avalanche propagation and characteristic time of dynamical processes. Obtained analytical solutions can be used to test numerical models and to give insights into the structure of avalanche flow and to highlight basic mechanisms of avalanche dynamics.

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1. Introduction

[2] Gravitational flows like submarine and aerial landslides, debris avalanches and pyroclastic flows from volcanoes are frequently reported as the sources of tsunami waves. In 1958, such an avalanche from Fairweather Mountain induced a huge swash of 524 m on the opposite side of the Lituya bay, Alaska, and tsunami waves reached 21 m near the bay entrance [Miller, 1960]. The pyroclastic flows were generated several times on Montserrat Island (the Lesser Antilles, Caribbean) due to continual volcano activity and caused three tsunami events recorded in Guadeloupe and Montserrat in the last decade [Heinrich et al., 1998; Mangeney et al., 2000a; Pelinovsky et al., 2004; Zahibo et al., 2008b]. The various analytical and numerical models are developed to describe tsunamis generated by landslides. In many studies [Pelinovsky and Poplavsky, 1996; Watts, 2000; Tinti et al., 2001; Liu et al., 2003; Pelinovsky, 2003; Sammarco and Renzi, 2008], the landslide is assumed as a solid block moving under the joint action of gravity, the Coulomb friction and hydraulic resistance in turbulent flow. More realistic models of tsunami generation include the “two-layer” approach which jointly describes tsunami wave and a landslide propagation influencing one another [Imamura and Gica, 1996; Assier-Rzadkiewicz et al., 2000; Mangeney et al., 2000a; Heinrich et al., 1999, 2001; Bouchut et al., 2003; Fine et al., 2003, 2005;

Mangeney-Castelnau et al., 2003, 2005; Le Friant et al., 2006; Fernandez-Nieto et al., 2008]. Some attempts have been made to simulate submarine landslides using the full three-dimensional (3-D) Navier-Stokes equations [Heinrich et al., 1998, 1999; Mangeney et al., 2000b]. Similar models developed to describe dry subaerial avalanches, pyroclastic and debris flows including possible sources of tsunami, have aroused much interest in the last decade [see also Gray et al., 1999; Mangeney et al., 2000b; Heinrich et al., 2001; Bouchut et al., 2003; Mangeney-Castelnau et al., 2003, 2005; Le Friant et al., 2006; Fernandez-Feria, 2006; Rudenko et al., 2007; Mangeney et al., 2007a; Bouchut et al., 2008; Pirulli and Mangeney, 2008; Pelanti et al., 2008; Yu et al., 2009; Luca et al., 2009]. Most of such models of landslides and avalanches are based on homogeneous shallow water flows that are deformed during propagation. These models use various approximations of the vertical structure of the velocity field (uniform as in the ideal fluid, or parabolic as for a viscous fluid), as well as different approximations of the friction law (the Coulomb friction, Bagnold behavior, Pouliquen law); the derivation of the mentioned models can be found in the recently published book by Pudasaini and Hutter [2007]. Although most of these models are realized in numerical codes and are applied for modeling real and prognostic events, analytical solutions can be used to test numerical models and to gain insights into the structure of avalanche flow, and to highlight basic mechanisms of avalanche dynamics.

[3] In this paper special attention is paid to the analytical description of dry subaerial avalanche dynamics basing on the Savage-Hutter model. The number of the analytical solutions of the gravity driven shallow water flow is limited. Savage and Hutter [1989, 1991] constructed some self-similar solutions called parabolic cap and M waves due to their shapes; see also the book by Pudasaini and Hutter

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Figure 1. The 2 December 2008 pyroclastic surge effects on St. George's Hill, Montserrat (see <http://www.mvo.ms>).

[2007]. *Mangeney et al.* [2000b] found the analytical solution to a one-dimensional dam break problem over inclined plane also taking into account the Coulomb friction. It was widely used to test the debris avalanche numerical models applied to evaluate the hazard of the volcano eruption in the Lesser Antilles [*Heinrich et al.*, 1999, 2001; *Mangeney et al.*, 2000a; *Le Friant et al.*, 2003, 2006]. *Fernandez-Feria* [2006] extended this analysis to arbitrary bottom slopes; the axisymmetric dam break problem is solved by *Kerswell* [2005]. *Rudenko et al.* [2007] found the exact solution that represented the nonlinear deformed wave in an ideal flow (neglected friction) along a plate of a constant slope.

[4] The model we used here includes sliding friction (the Coulomb law). Although the simple friction law suggests simplified behavior of the flow which is expected to be different from the natural one, some basic mechanisms can be deduced. For instance, natural avalanches are submitted to erosion processes that can significantly change their dynamics and generate surge waves [*Mangeney et al.*, 2007b]. Furthermore the fluid phase generally plays a significant role as described, for example, by *Pelanti et al.* [2008]. It must be said that the simple model based on Coulomb friction is very convenient to predict potential events and to analyze historical events because it contains only one empirical parameter (friction angle) as pointed out by *Pirulli and Mangeney* [2008]. That is why this kind of friction is used in our study.

[5] Analytical solutions of the gravity driven shallow water flow with Coulomb friction are obtained in the literature for 1-D geometry only. However, gravity driven masses move in basins of more complicated geometry. As a rule, they move in inclined channels (valleys on mountains and submarine canyons) which are generally diverging, converging or twisted in various parts. Some of them have straight parabolic-like shapes at least in a part of flank (Figure 1). We provide here several new analytical solutions for gravity driven shallow water flow in inclined channels of constant slope with the specific parabolic-like cross section. New effects in the avalanche dynamics comparable to those known for the 1-D avalanche motion along an inclined plate are related with the parabolic-like cross-

section shape of the channel which influence the speed of avalanche propagation and characteristic time of dynamical processes.

[6] The paper is organized as follows. The basic model to describe the avalanche motion that represents the gravity driven shallow water flow (simplified Savage-Hutter model) in a narrow inclined channel of parabolic-like cross section is given in section 2. Nonlinear deformation of the avalanche shape for the nonzero sign constant initial distribution of the particle velocity in the avalanche body is studied in section 3. The dam break problem for gravity driven flow in an inclined channel with initial zero velocity distribution is investigated in section 4; it is the extension of the well-known 1-D solution described by *Stoker* [1957] and *Mangeney et al.* [2000b]. Other self-similar solutions for the avalanche of finite length and symmetrical shape in the inclined channels are also found in section 4; they are extensions of the M wave and parabolic cap solutions for the 1-D avalanche given by *Pudasaini and Hutter* [2007]. The analytical solution for an avalanche initially at rest in a parabolic channel is obtained in section 5; from a mathematical point of view it is the extension of the Carrier-Greenspan transformation [*Carrier and Greenspan*, 1958] used early in the theory of water waves in particular tsunami waves [*Zahibo et al.*, 2006; *Choi et al.*, 2008]. The discussion of obtained results is given in the conclusion.

2. Basic Model of the Gravity Driven Shallow Water Flow

[7] To describe the gravity driven flow (debris avalanche or submarine landslide) we follow *Pudasaini and Hutter* [2007] using the two-dimensional depth-integrated shallow water equations for the incompressible fluid with the Coulomb-type friction term (Savage-Hutter model); the earth pressure coefficients in the Savage-Hutter model have been set to 1. This model as pointed above [*Pirulli and Mangeney*, 2008] is very convenient to predict potential events and also to analyze historical events because it contains only one empirical parameter (friction angle). In the system of coordinates linked to the topography the equations of mass and momentum conservation have the following form [see *Mangeney et al.*, 2000b]:

$$\frac{\partial h}{\partial t} + \frac{\partial}{\partial x}(hu) + \frac{\partial}{\partial y}(hw) = 0, \quad (1)$$

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + w \frac{\partial u}{\partial y} + g \cos \theta \frac{\partial h}{\partial x} = g \sin \theta_x - \mu g \cos \theta \frac{u}{\sqrt{u^2 + w^2}}, \quad (2)$$

$$\frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} + w \frac{\partial w}{\partial y} + g \cos \theta \frac{\partial h}{\partial y} = g \sin \theta_y - \mu g \cos \theta \frac{w}{\sqrt{u^2 + w^2}}, \quad (3)$$

where

x, y local slope and parallel coordinates, m;
 $h(x, y, t)$ flow height perpendicular to the local slope, m;

u, w depth-averaged velocities along x and y axes, respectively, m/s;
 g acceleration due to gravity, m/s²;
 θ steepest slope angle, deg;
 θ_x, θ_y slope angles along x and y axes, deg;
 $\mu = \tan(\varphi)$ sliding friction coefficient, nondimension;
 φ dynamic friction angle, deg.

[8] The value of φ depends on the landslide material, and it has not been studied well. Usually it is considered to be relatively small for sand-textured debris (17–30° [Heinrich et al., 2001]) and relatively large for a granular debris avalanche (20–40° [Le Friant et al., 2003]). Very often this angle is estimated using comparison between the observed data and the results of numerical modeling. In particular, for the debris avalanche on the Montserrat volcano the dynamic friction angle is tested in a wide range as 13–35° and for the simple friction law the value 13–14.5° has been chosen. [Voight et al., 2002; Heinrich et al., 2001]. Le Friant et al. [2003] state that this angle for the flank collapse event on the Montagne Pelée, Martinique, has been estimated as 6.6° from observations and has been taken about 7° for numerical simulations.

[9] The given equations are simplified compared to the advanced version of the Savage-Hutter models of complex topography recently proposed by Bouchut and Westdickenberg [2004] and Mangeney et al. [2007a]. We would like to point out that the given equations differ from the well-known “fluid” shallow water system in the friction term, which is usually proportional to $u|u|$ (Chezy friction); this Saint Venant model is also used in avalanche dynamics [Bouchut et al., 2003; Mangeney-Castelnau et al., 2003, 2005]. In fact, it is rather complicated to describe and to parameterize the dissipative processes in landslides and avalanches, and various formulas are applied in practice [see Pudasaini and Hutter, 2007]. Meanwhile, the Coulomb-type friction has only one empirical parameter (friction angle, φ) and as mentioned above, this model is very convenient to predict potential events and also to analyze historical events [see Pirulli and Mangeney, 2008]. Here we use Coulomb-type friction in order to obtain analytical solutions of equations (1)–(3).

[10] Avalanches and landslides usually move in narrow channels (valleys and canyons), see for instance Figure 1. Assuming that the channel axis is straight and the flow is almost uniform in the cross section, the shallow water equations may be integrated on the cross section and effective 1-D system may be derived (typical approach for river hydraulics). This procedure is demonstrated quite extensively by Pudasaini and Hutter [2007] and yields

$$\frac{\partial S}{\partial t} + \frac{\partial}{\partial x}(Su) = 0, \quad (4)$$

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + g \cos \theta \frac{\partial h}{\partial x} = g \sin \theta - \mu g \cos \theta \operatorname{sgn} u, \quad (5)$$

where $S(x, t)$ area of the cross section of the channel, m²; $h(x, t)$ avalanche height along x axis of channel, m; and $u(x, t)$ mean velocity flow, m/s. To close this system, the function $S(h)$ which is given as an integral expression over

the width of a channel should be determined. If the cross section has a parabolic-like shape,

$$z(y) = k|y|^m, \quad (6)$$

with positive arbitrary constants, k and m (Figure 2), the function S is

$$S = \frac{2m}{(m+1)k^{1/m}} h^{(m+1)/m}. \quad (7)$$

As a result, equation (4) transforms to

$$\frac{\partial h}{\partial t} + u \frac{\partial h}{\partial x} + \frac{m}{m+1} h \frac{\partial u}{\partial x} = 0, \quad (8)$$

and the system of equations (5) and (8) becomes closed. It differs from the “classical” one-dimensional equations of the simplified Savage-Hutter model in the constant coefficient $m/(m+1)$, and may be transformed to them as $m \rightarrow \infty$ ($S \sim h$).

[11] Below we assume the constant slope of the inclined channel, θ , which should be relatively high ($\theta > \varphi$) to provide the gravity driven motion of the landslide. In this case $u > 0$ throughout and equation (5) is simplified to

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + g \cos \theta \frac{\partial h}{\partial x} = g \sin \theta - \mu g \cos \theta. \quad (9)$$

[12] The basic system of the gravity driven flow, (8) and (9), is analyzed below. Mathematically, it is a hyperbolic system with constant coefficients and as for water waves above inclined bottom [Stoker, 1957; Carrier and Greenspan, 1958; Zahibo et al., 2006; Choi et al., 2008]; the Riemann invariants can be found explicitly

$$I_{\pm} = u \pm 2 \sqrt{\frac{m+1}{m}} gh \cos \theta - g \alpha t, \quad (10)$$

where

$$\alpha = \sin \theta - \mu \cos \theta > 0, \quad (11)$$

and the system (8)–(9) can be rewritten in the form

$$\frac{\partial I_{\pm}}{\partial t} + c_{\pm} \frac{\partial I_{\pm}}{\partial x} = 0, \quad (12)$$

where the characteristic speeds are

$$c_{\pm} = \frac{3m+2}{4(m+1)} I_{\pm} + \frac{m+2}{4(m+1)} I_{\mp} + g \alpha t. \quad (13)$$

It should be mentioned that equations (12) and (13) appear in the problems of long water wave run-up on the coast [Stoker, 1957; Carrier and Greenspan, 1958; Zahibo et al., 2006; Choi et al., 2008] but the main difference here is that the bottom slope is not small (as in “run-up problems”) and the Coulomb-type friction is not neglected. Mathematically, the initial and boundary conditions for water waves and avalanche dynamics also differ. That is why the “avalanche

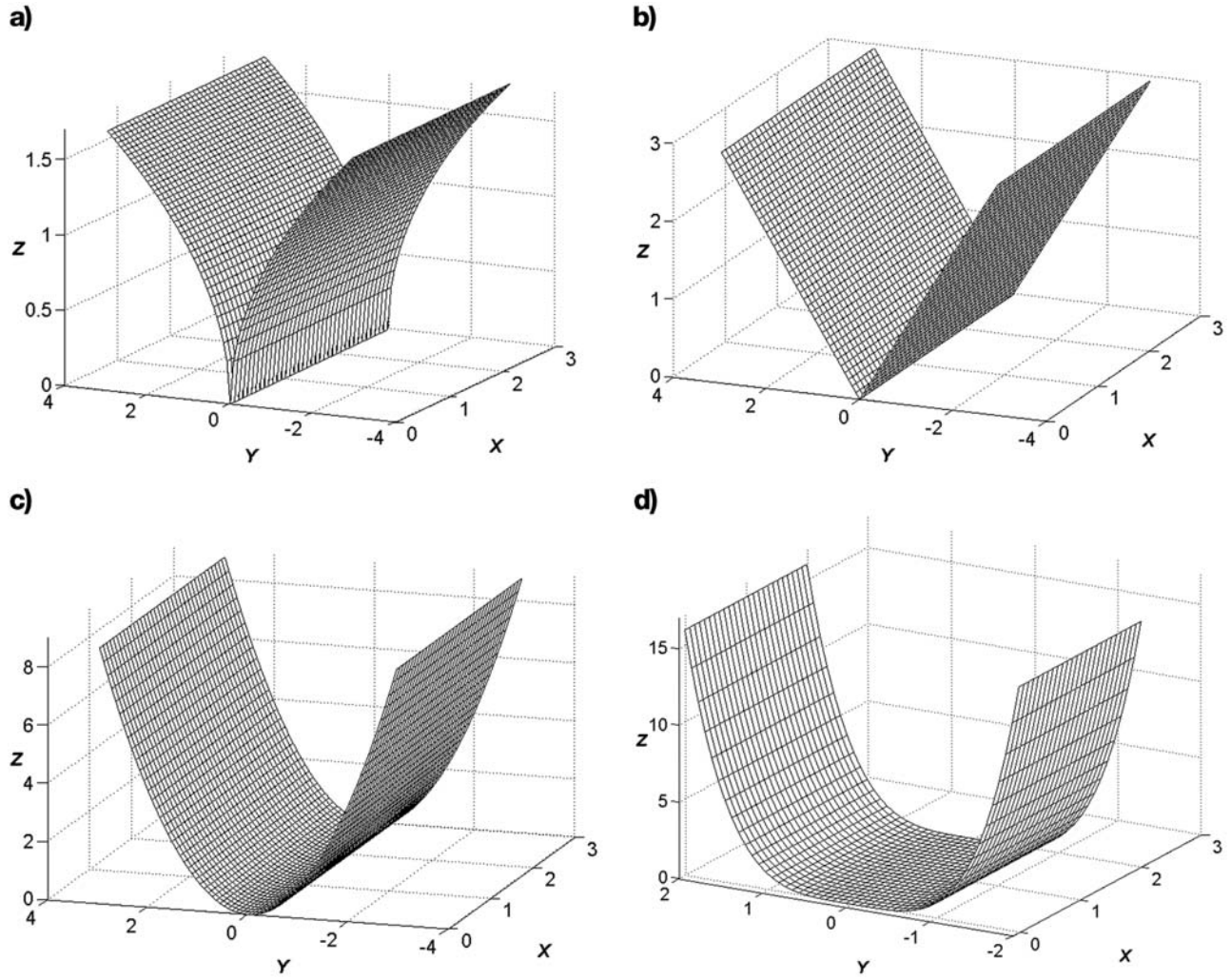


Figure 2. Different channel shapes of the cross channel: (a) $m = 0.5$, (b) $m = 1$, (c) $m = 2$, (d) $m = 4$.

solutions” cannot be directly obtained from the “water solutions.” Physical variables, $h(x, t)$ and $u(x, t)$ are found from the algebraic equations (10) and the downslope component of the velocity field ($g\alpha t$) propagating under the action of the gravity force can be eliminated by use of

$$u(x, t) = g\alpha t + v(x, t), \quad (14)$$

$$v(x, t) = \frac{I_+ + I_-}{2}, \quad (15)$$

$$h = \frac{m}{16g \cos \theta (m+1)} (I_+ - I_-)^2, \quad (16)$$

where $\alpha > 0$ depends on the slope angle and friction coefficient, see (11). Another useful development is the change of coordinates, the accelerated reference system can be written as

$$X = x - \frac{g\alpha t^2}{2}, t = t. \quad (17)$$

As a result, system (12) is reduced to the nonlinear system with constant coefficients

$$\frac{\partial J_{\pm}}{\partial t} + C_{\pm} \frac{\partial J_{\pm}}{\partial X} = 0, \quad (18)$$

where the modified Riemann invariants $J_{\pm}$ are

$$J_{\pm} = v \pm 2\sqrt{\frac{m+1}{m}} gh \cos \theta, \quad (19)$$

and the modified characteristic speeds are

$$C_{\pm} = \frac{3m+2}{4(m+1)} J_{\pm} + \frac{m+2}{4(m+1)} J_{\mp}. \quad (20)$$

The system (18)–(20) now does not include the downslope component of the velocity field ($g\alpha t$). The new parameter m characterizing the cross section of the shape of the channel is added compared to the plane case. Equations (18)–(20)

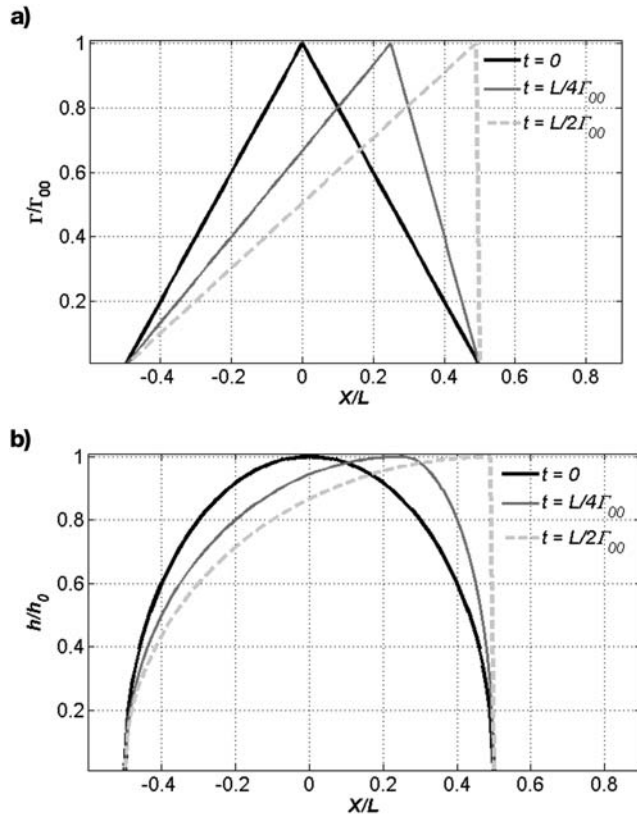


Figure 3. (a) Velocity and (b) height distributions in longitudinal direction for times 0, $L/4\Gamma_{00}$, and $L/2\Gamma_{00}$.

present the basic model of the avalanche motion in the inclined channel of parabolic-like cross section.

3. Nonlinear (Riemann) Wave Presentation of the Gravity Driven Flow

[13] A particular solution of the system (18) is $J_- = 0$, that leads to the following relation between the velocity and the flow height

$$v(x, t) = 2\sqrt{\frac{m+1}{m}}gh \cos \theta. \quad (21)$$

Using this solution equation (18) for J_+ can be reduced to the nonlinear PDE of the first order

$$\frac{\partial \Gamma}{\partial t} + \Gamma \frac{\partial \Gamma}{\partial X} = 0, \quad (22)$$

where

$$\Gamma = \frac{3m+2}{2(m+1)}v(X, t) \quad (23)$$

is the normalized velocity. The solution of (22) is expressed through the so-called Riemann wave [Stoker, 1957]

$$\Gamma(x, t) = \Gamma_0(X - \Gamma t), \quad (24)$$

where $\Gamma_0(X)$ describes the initial longitudinal landslide shape (exactly, $\sqrt{h_0(X)}$).

[14] The solution (24) in initial coordinates has the following evident physical meaning: the landslide moves with acceleration, $g\alpha$ (we consider the case when the gravity force is not balanced by Coulomb friction) and the shape of landslide deforms during the motion whereas the front steepness increases.

[15] The solution (24) has been obtained by Rudenko *et al.* [2007] for the particular case of free gravity flow (no friction) above an inclined plate ($m \rightarrow \infty$). Similar solutions have been studied for water waves above a horizontal bottom and in channels with parabolic-like cross section [Pelinovsky and Troshina, 1994; Zahibo *et al.*, 2008a]. Our solution includes both, the inclined channel of parabolic cross section and Coulomb friction.

[16] The nonlinear evolution of the avalanche body can be analyzed for any initial distribution of the height in the longitudinal direction and is illustrated here for a parabolic initial distribution of avalanche height. In this case the velocity distribution has triangular shape (as $\Gamma_0(X) \sim \sqrt{h_0(X)}$), with the initially equal front and back slopes, see Figure 3a. Analytically, the longitudinal evolution of the flow within the avalanche in the accelerated reference system (17) is described by

$$\frac{\Gamma(X, t)}{\Gamma_{00}} = \begin{cases} \frac{X + L/2}{\Gamma_{00}t + L/2} & -L/2 < X < \Gamma_{00}t \\ \frac{X - L/2}{\Gamma_{00}t - L/2} & \Gamma_{00}t < X < L/2, \end{cases} \quad (25)$$

where L is the avalanche length, and Γ_{00} is the maximum velocity in the center of the avalanche. In time, the steepness of the front slope increases, and the steepness of the back slope decreases

$$s(t) = \frac{s_0}{1 \mp s_0 t}, \quad (26)$$

where $s_0 = d\Gamma_0/dx$ is the initial steepness, and $\pm$ corresponds to the back/front slope. The longitudinal avalanche shape evolution is described by the following formula deducible from (25)

$$h(X, t) = h_0 \begin{cases} \left[1 - \frac{(X - \Gamma_{00}t)^2}{(-L/2 - \Gamma_{00}t)^2} \right] & -L/2 < X < \Gamma_{00}t \\ \left[1 - \frac{(X - \Gamma_{00}t)^2}{(L/2 - \Gamma_{00}t)^2} \right] & \Gamma_{00}t < X < L/2, \end{cases} \quad (27)$$

and is presented in Figure 3b.

[17] The given solutions are valid only for bounded times less than the breaking time

$$T_{br} = \frac{L}{2\Gamma_{00}} = \frac{L\sqrt{m(m+1)}}{2(3m+2)\sqrt{gh_0 \cos \theta}}, \quad (28)$$

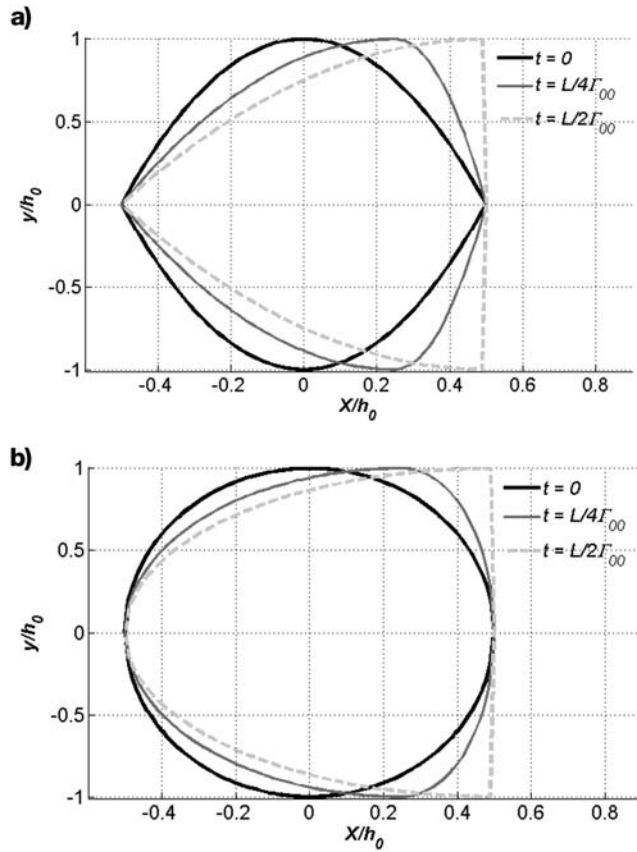


Figure 4. Avalanche contours for different times in the accelerated reference system for times 0, $\frac{L}{4\Gamma_{00}}$ and $\frac{L}{2\Gamma_{00}}$, (a) triangular channel ($m = 1$), (b) parabolic channel ($m = 2$).

when the avalanche front steepness is infinite. The breaking time depends on the slope angle and increases with the steepness of the slopes. It also depends on the cross section of the channel (parameter m) and decreases for narrow channels ($m \rightarrow 0$). We would like to mention that the breaking time does not depend on the friction coefficient, and the nonlinear deformation of the avalanche body in time is the same both for the friction and frictionless cases. However, the distance accomplished by an avalanche before breaking depends on friction.

[18] Let us investigate the influence of the cross section of an inclined channel on the evolution of the flow. It follows from (21) that the particle velocity increases when m decreases. For instance, for a channel of parabolic shape ($m = 2$) this speed is $(1.5)^{1/2} = 1.225$ times that on a plane slope, for the channel of triangle shape ($m = 1$) this factor is $2^{1/2} = 1.414$. As a result, an avalanche with the same height deforms quickly in narrow channels ($m \rightarrow 0$) and so, the flow is more energetic. The spatial structure of the deformed avalanche computed from (6) depends on the cross section of the channel (Figure 4). The avalanche rear tail in the channel of the triangular cross section is beak shaped, and in the parabolic channel it is parabolic like. In time the avalanche front becomes flattened in both cases. All the conclusions made above are valid for any initial distribution of the velocity (not only for triangles). In particular,

equation (26) describes the time evolution of the maximum steepness of the avalanche longitudinal shape (in terms $d\Gamma/dx$), the breaking time is also determined definitely through the maximum value of $d\Gamma_0/dx$.

[19] After breaking, the avalanche profile becomes multivalued in the framework of the solution (24). Usually, multivalued solutions of hyperbolic equations are replaced by single-valued solutions with a discontinuity in height on the front to satisfy the approximation of the uniform distribution of the velocity along the depth of the shallow water model. Taking into account that the mass of the avalanche is conserved, the real configuration of the avalanche body with the shock front can be found from the multivalued solution using the law of “square equality.” According to this approach which is popular in nonlinear waves [Stoker, 1957; Whitham, 1974] the front location is determined from the condition $S_1 = S_2$, as illustrated in Figure 5.

[20] The avalanche height h_L decreases in time, and its length increases so as to conserve the avalanche mass. For large times the height and the length of the avalanche satisfy the asymptotic expressions

$$X_{fr} \approx \sqrt{\frac{2L\Gamma_{00}t}{3}}, h_L \approx h_0 \sqrt{\frac{8}{3\Gamma_{00}t}}. \quad (29)$$

As a result, the avalanche shape in the longitudinal direction being initially parabolic tends to a triangle.

[21] The formation of the shock profiles of the avalanche body should be obtained in various shallow water models of the avalanche due to their hyperbolicity; this process is specially investigated numerically in the framework of the two-phase model of the shallow granular flow [Pelanti *et al.*, 2008].

4. Self-Similar Solutions

[22] The famous self-similar analytical solution of the shallow water system is the dam break solution on the

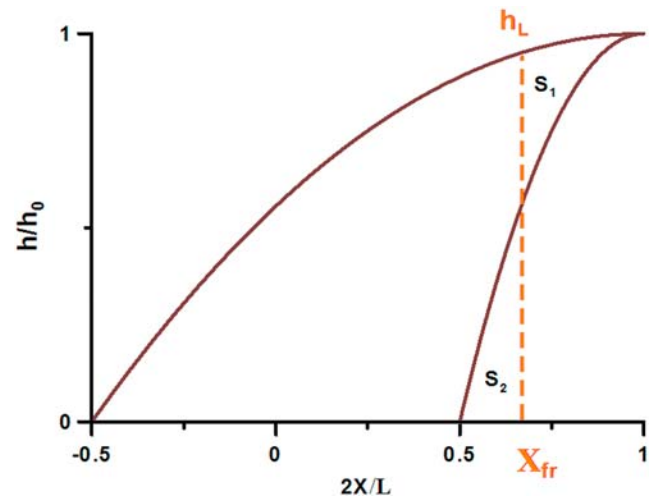


Figure 5. Avalanche shape after breaking in the longitudinal direction. The red line corresponds to the front location.

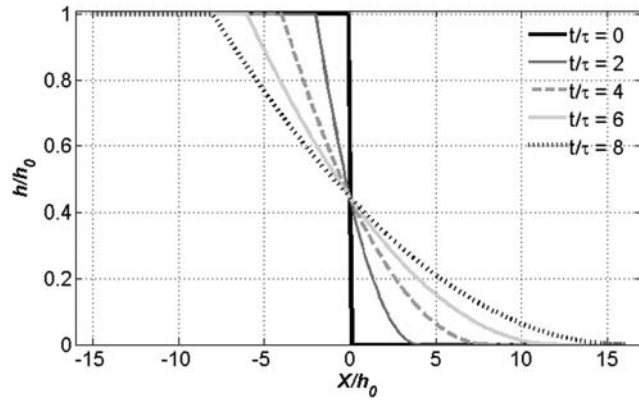


Figure 6. Avalanche shape evolution in the dam break problem for time slices with time increment $\tau = h_0/c_0$.

horizontal plane bottom described in the book by *Stoker* [1957]. This analysis was extended by *Mangeney et al.* [2000b] for the debris avalanche flow along an inclined plate. Our system of gravity-driven flow in an inclined channel of parabolic-like cross section (18) can be also reduced to equations studied by *Stoker* [1957].

[23] Let us introduce a new variable of the dimension of velocity rather than avalanche depth:

$$c = \sqrt{\frac{m+1}{m}} gh \cos \theta. \quad (30)$$

With it, equations (18) can be rewritten in the form

$$\frac{\partial}{\partial t}(v \pm 2c) + (v \pm c) \frac{\partial}{\partial X}(v \pm 2c) = 0, \quad (31)$$

which is exactly the same as the equations studied by *Stoker* [1957] and by *Mangeney et al.* [2000b].

[24] The initial condition ($t = 0$) for the dam break problem on the inclined channel is

$$u(x, 0) = 0, h(x, 0) = \begin{cases} h_0 & x < 0 \\ 0 & x > 0. \end{cases} \quad (32)$$

In the modified variables it is reduced to

$$v(X, 0) = 0, c(X, 0) = \begin{cases} c_0 & X < 0 \\ 0 & X > 0, \end{cases} \quad (33)$$

which is by *Stoker* [1957] as the initial condition for the dam break problem on the horizontal plane. This means that the analytical solution given in the cited book can be used to obtain the solution of the dam break problem for the avalanche in the inclined channel with Coulomb friction as it was first shown by *Mangeney et al.* [2000b] for 1-D geometry.

[25] The classical solution to the dam break problem on the horizontal plane obtained by *Stoker* [1957] has the following form

$$c(X, t) = \begin{cases} c_0 & X < -c_0 t \\ \frac{1}{3} \left(2c_0 - \frac{X}{t} \right) & -c_0 t < X < 2c_0 t \\ 0 & 2c_0 t < X, \end{cases} \quad (34)$$

$$v(X, t) = \begin{cases} 0 & X < -c_0 t \\ \frac{2}{3} \left(c_0 + \frac{X}{t} \right) & -c_0 t < X < 2c_0 t \\ 2c_0 & 2c_0 t < X. \end{cases} \quad (35)$$

The “front” edge point moves with the highest speed, $2c_0$, the “quiet region” has speed c_0 ; both variables v and c vary linearly in the transition zone.

[26] Returning to the dam break problem for the gravity-driven flow in the inclined channel, we may say that the cross section of the channel and its slope influence the speed of the dam break process through the relation between c_0 and h_0 as follows

$$c_0 = \sqrt{\frac{m+1}{m}} gh_0 \cos \theta. \quad (36)$$

[27] The process of “dam breaking” of the avalanche of the same height, h_0 , is intensified in narrow channels ($m \rightarrow 0$) and along gentle slopes. The bottom friction influences the acceleration of the avalanche (through the accelerated system of coordinates) but not the dam break process. The avalanche shape in the longitudinal direction is described by the formula

$$h(X, t) = \begin{cases} h_0 & X < -c_0 t \\ \frac{h_0}{9} \left(2 - \frac{X}{c_0 t} \right)^2 & -c_0 t < X < 2c_0 t \\ 0 & 2c_0 t < X, \end{cases} \quad (37)$$

which follows from (35). The evolution of the avalanche shape is demonstrated in Figure 6. It is clearly seen that the initial discontinuity is diluted in time and its front becomes smoother.

[28] Formally, the found solutions oppose each other: whereas the Riemann wave always breaks, the “dam break” front becomes gentler as time proceeds. The difference between the two solutions is connected with the value of the initial velocity that can be zero (dam break problem) and nonzero (the Riemann wave). In the dam break problem «back» particles drop behind, which leads to the front extension. In the other case the particles of high speed, located in the «back» of the landslide overrun slow moving particles in the front. Thus, the avalanche dynamics is sensitive to the initial conditions.

[29] Some extensions of the classical dam break problem for the avalanche on an inclined plate have been done. *Fernandez-Feria* [2006] extended the 1-D dam break solution to more complicated initial condition different from (32); the avalanche shape is not constant at $x < 0$. In fact, *Fernandez-Feria* [2006] found an analytical solution only in

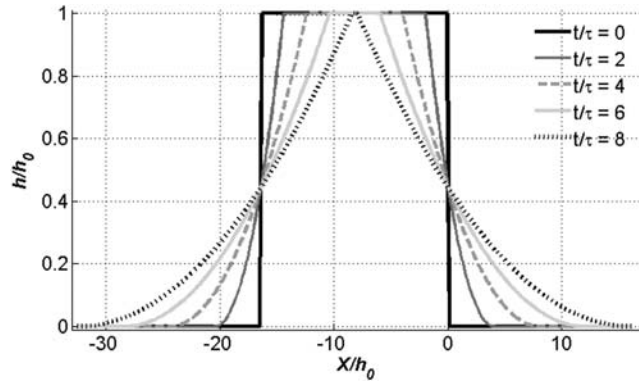


Figure 7. Avalanche shape evolution in the longitudinal direction for stepwise (rectangular) initial shape with increment $\tau = h_0/c_0$.

the vicinity of the avalanche edge, of which the motion does not depend on the avalanche shape behind the front, at least for small times. The detailed evolution of the avalanche shape has been obtained numerically in the cited paper. We would also like to point out a paper by *Mangeney-Castelnau et al.* [2005], where the detailed study of the spreading of the column of a frictional material has been investigated numerically. The scaling laws based on analytical solutions have been used successfully to reproduce experimental results on granular collapse. With the adequate scaling these solutions can be applied to analyze the avalanche evolution in the inclined channel.

[30] The solution of the dam break problem given above is one of the particular self-similar solutions of hyperbolic equations, and it is valid for a semi-infinity avalanche. If the avalanche has a table-top shape of total length L , both ends are evolving independently on the first stage according to the self-similar scenario. The dynamics of the front slope is described by (37), and the rear slope may be described by the following formula obtained from (37) by changing X to $-(X + L)$ and accordingly u to $-u$:

$$h(X, t) = \begin{cases} h_0 & X + L > c_0 t \\ \frac{h_0}{9} \left(2 + \frac{X + L}{c_0 t} \right)^2 & -2c_0 t < X + L < c_0 t \\ 0 & X + L < -2c_0 t. \end{cases} \quad (38)$$

[31] The evolution of the avalanche shape is shown in Figure 7 for $L/h_0 = 16.4$. The independent dynamics of the avalanche tails is valid for $t < L/c_0$, after that both tails influence each other and the landslide height decreases.

[32] The spatial structure of the avalanche computed with the use of (6) is shown in Figure 8. The avalanche edge is beak shaped in the channel of both, triangular and parabolic cross sections.

[33] If the initial avalanche length is relatively short, its shape evolution can be described by another kind of self-similar solutions. Such solutions (called parabolic cap and M wave) were found by *Savage and Hutter* [1989, 1991] and described by *Pudasaini and Hutter* [2007] for the avalanche along the inclined plane. The same solutions can be derived for avalanche dynamics in the inclined channel of parabolic-like cross section. It is again convenient

to use the “accelerated” coordinates (17) and “reduced” velocity (15). In this case the basic equations (8) and (9) transform to

$$\frac{\partial h}{\partial t} + v \frac{\partial h}{\partial X} + \frac{m}{m+1} h \frac{\partial v}{\partial X} = 0, \quad (39)$$

$$\frac{\partial v}{\partial t} + v \frac{\partial v}{\partial X} + g \cos \theta \frac{\partial h}{\partial X} = 0. \quad (40)$$

[34] The self-similar solutions of this system are sought in the form

$$h(\xi, t) = \frac{1}{t^a} F(\xi), v(\xi, t) = \frac{1}{t^b} G(\xi), \xi = X/t^d, \quad (41)$$

where the functions F , G and the constants a , b , d should be determined. The first relation between the constants can be obtained from the avalanche mass conservation law (39) or from (4) and (7). This yields

$$\int_{-\infty}^{+\infty} h^{(m+1)/m}(X, t) dX = \text{const}, \quad (42)$$

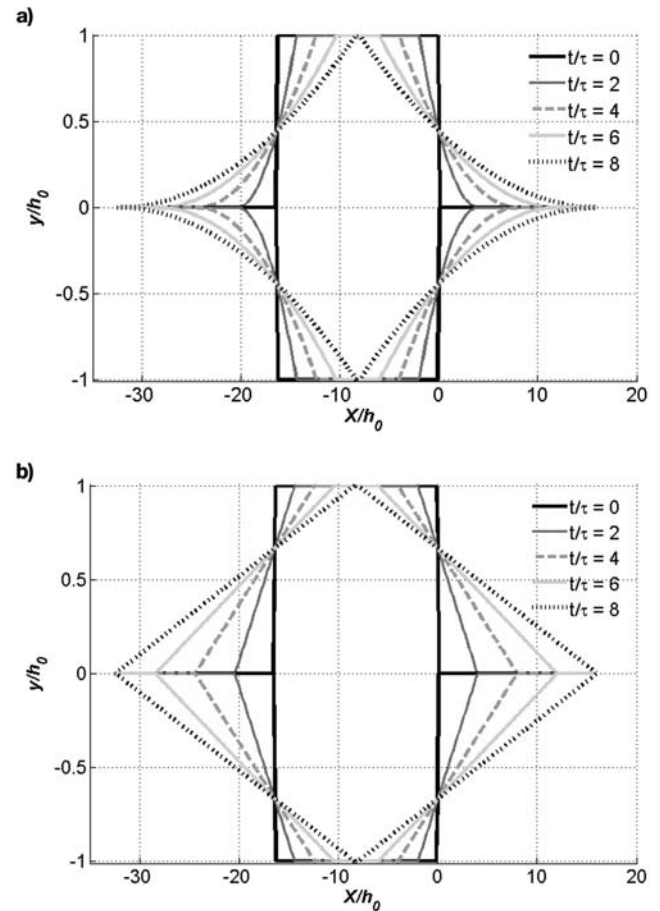


Figure 8. Avalanche contours in dam break problem: (a) channel of triangle cross section, and (b) channel of parabolic cross section, $\tau = h_0/c_0$.

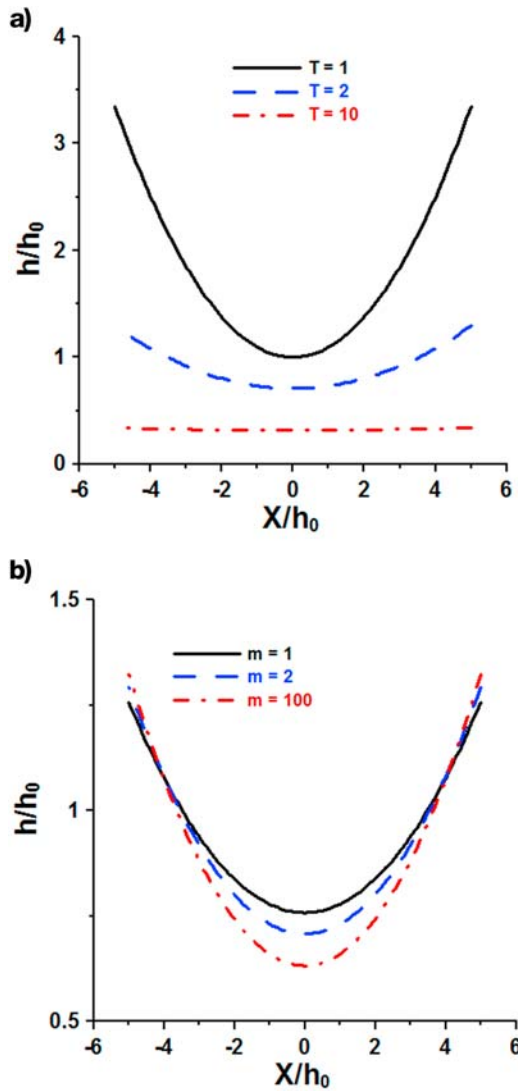


Figure 9. M wave avalanche shape in longitudinal direction (a) $m = 2$ (b) $T = 2$.

as well as

$$f = a \frac{m+1}{m}. \quad (43)$$

[35] After substitution of (41) in (39) and (40), these equations become

$$-aF - f\xi \frac{dF}{d\xi} + t^{-b-f+1} \left[G \frac{dF}{d\xi} + \frac{m}{m+1} F \frac{dG}{d\xi} \right] = 0 \quad (44)$$

$$-bG - f\xi \frac{dG}{d\xi} + t^{-b-f+1} G \frac{dG}{d\xi} + t^{-a-f+b+1} g \cos \theta \frac{dF}{d\xi} = 0. \quad (45)$$

Both equations would be ODEs when the time is not included. This argument leads to the algebraic equations for the coefficients, which are explicitly determined:

$$a = \frac{2m}{3m+2}, b = \frac{m}{3m+2}, f = \frac{2(m+1)}{3m+2}. \quad (46)$$

With these choices the resulting equation (44) can be integrated to yield

$$[-f\xi + G(\xi)]F^{\frac{m+1}{m}}(\xi) = \text{const.} \quad (47)$$

One of the solutions is chosen for the zero constant in (47), it determines the velocity

$$G(\xi) = \frac{2(m+1)}{3m+2} \xi. \quad (48)$$

With the use of (48) equation (45) is reduced to

$$g \cos \theta \frac{dF}{d\xi} = bf\xi, \quad (49)$$

and is easily integrated, with the result

$$F(\xi) = F_0 + \frac{m(m+1)}{g \cos \theta (3m+2)^2} \xi^2. \quad (50)$$

[36] The self-similar solution (41) expressed in the initial physical variables has the final form

$$h(\tilde{X}, T) = h_0 \left[T^{-\frac{2m}{3m+2}} + \frac{m(m+1)}{(3m+2)^2} \frac{\tilde{X}^2}{T^2} \right], \quad (51)$$

$$v(X, t) = 2\sqrt{gh_0 \cos \theta} \frac{m+1}{3m+2} \frac{\tilde{X}}{T},$$

where

$$h_0 = \frac{F_0^{1+m/2}}{(g \cos \theta)^{m/2}}, \tilde{X} = \frac{X}{h_0}, T = t \sqrt{\frac{g \cos \theta}{h_0}}.$$

[37] For $m \rightarrow \infty$ the solution (51) transforms to the M wave solution obtained by *Savage and Hutter* [1989] and is described by *Pudasaini and Hutter* [2007]. As it follows from (51), its shape in the longitudinal direction is universal for all values m and T . The time evolution of the avalanche shape in the parabolic channel is demonstrated in Figure 9a, it becomes gentle with the lapse of time. The avalanche shape as a function of the channel shape is shown in Figure 9b. This self-similar solution has an evident physical interpretation: landslide particles are “dragged apart” in two directions due to the opposite flows that are strong far from the center. It is interesting to mention that the avalanche height decreases slower in the center than the periphery. The cross-sectional shape of the avalanche influences the speed of the avalanche height, in particular decreasing in extremely narrow channels ($m \rightarrow 0$).

[38] Another self-similar solution of (39) and (40) can be obtained by using the self-similar transformation of coordinates

$$\eta = \frac{X}{L(t)}, t = t, \quad (52)$$

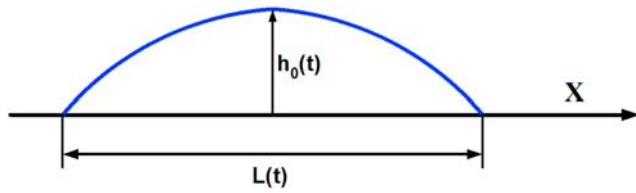


Figure 10. Parabolic cap shape of the avalanche in longitudinal direction.

which transforms equations (39) and (40) to

$$\frac{\partial}{\partial t} \left(h^{\frac{m+1}{m}} \right) + \frac{v - \eta dL/dt}{L} \frac{\partial}{\partial \eta} \left(h^{\frac{m+1}{m}} \right) + \frac{h^{\frac{m+1}{m}}}{L} \frac{\partial v}{\partial \eta} = 0 \quad (53)$$

$$\frac{\partial v}{\partial t} + \frac{v - \eta dL/dt}{L} \frac{\partial v}{\partial \eta} + \frac{g \cos \theta}{L} \frac{\partial h}{\partial \eta} = 0. \quad (54)$$

[39] Following *Pudasaini and Hutter* [2007] we determine the velocity distribution in the avalanche body as

$$v = \eta \frac{dL}{dt}. \quad (55)$$

[40] After the substitution of (55) in (53) and (54) we obtain two equations with various arguments

$$\frac{\partial}{\partial t} \left(h^{\frac{m+1}{m}} \right) + \frac{dL/dt}{L} h^{\frac{m+1}{m}} = 0, \quad (56)$$

$$\frac{\partial h}{\partial \eta} + \frac{\eta L d^2 L / dt^2}{g \cos \theta} = 0. \quad (57)$$

Both equations are integrated and the corresponding choice of constants can be written as

$$h = h_0(t) (1 - \eta^2), \quad h_0(t) = \frac{L}{2g \cos \theta} \frac{d^2 L}{dt^2}, \quad (58)$$

where $L(t)$ is a solution of

$$L^{\frac{2m+1}{m+1}} \frac{d^2 L}{dt^2} = q, \quad (59)$$

and q is determined by the initial spreading of the avalanche. So, the avalanche has a parabolic cap shape in the longitudinal direction for any m (Figure 10), but its height and length depend on the cross section of the inclined channel. For a plate, the parabolic cap solution was obtained by *Savage and Hutter* [1989] and is reproduced by *Pudasaini and Hutter* [2007] and its time evolution is quantitatively investigated.

[41] In general, equation (59) is integrated once

$$\left(\frac{dL}{dt} \right)^2 = \frac{2q}{L_0^{\frac{m}{m+1}}} \frac{m+1}{m} \left[1 - \left(\frac{L_0}{L} \right)^{\frac{m}{m+1}} \right], \quad (60)$$

here we used the initial condition $L(t=0) = L_0$. In the variables

$$l = \frac{L}{L_0}, \quad \tau = t \sqrt{\frac{2q(m+1)}{mL_0^{\frac{3m+2}{m+1}}}}. \quad (61)$$

Equation (60) is presented in the simplified form

$$\tau = \int_1^l \frac{dx}{\sqrt{1 - x^{-\frac{m}{m+1}}}}. \quad (62)$$

[42] For each value of m (i.e., specific cross-sectional shapes of the valley) the integral (62) can be evaluated. For $m = \infty$ (inclined plate) it is computed by *Savage and Hutter* [1989] and reproduced by *Pudasaini and Hutter* [2007]. For channels of triangular and parabolic cross section (62) is also calculated explicitly

$$\begin{aligned} \tau &= \left(l + \frac{3}{2} \sqrt{l} \right) \sqrt{1 - \frac{1}{\sqrt{l}}} + \frac{3}{4} \log \left[2 \left(\sqrt{1 - \frac{1}{\sqrt{l}}} + 1 \right) \sqrt{l} - 1 \right], \\ (m=1) \\ \tau &= \left(l - l^{1/3} \right) \sqrt{1 - l^{-2/3}}, \quad (m=2) \end{aligned} \quad (63)$$

The length of the parabolic cap varies in time depending on the shape of the cross section; see Figure 11 (in dimensionless variables).

[43] For large times the avalanche length varies almost linearly in time

$$l(\tau) \approx \tau - \frac{1+m}{2} \tau^{\frac{1}{1+m}}. \quad (64)$$

This leads to the following large-time asymptotic form for the avalanche height

$$h(t) \sim L^{-\frac{m}{m+1}} \sim t^{-\frac{m}{m+1}}. \quad (65)$$

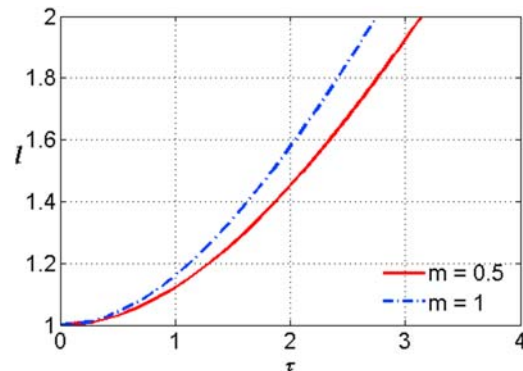


Figure 11. Time evolution of the length of the parabolic cap for various values of m .

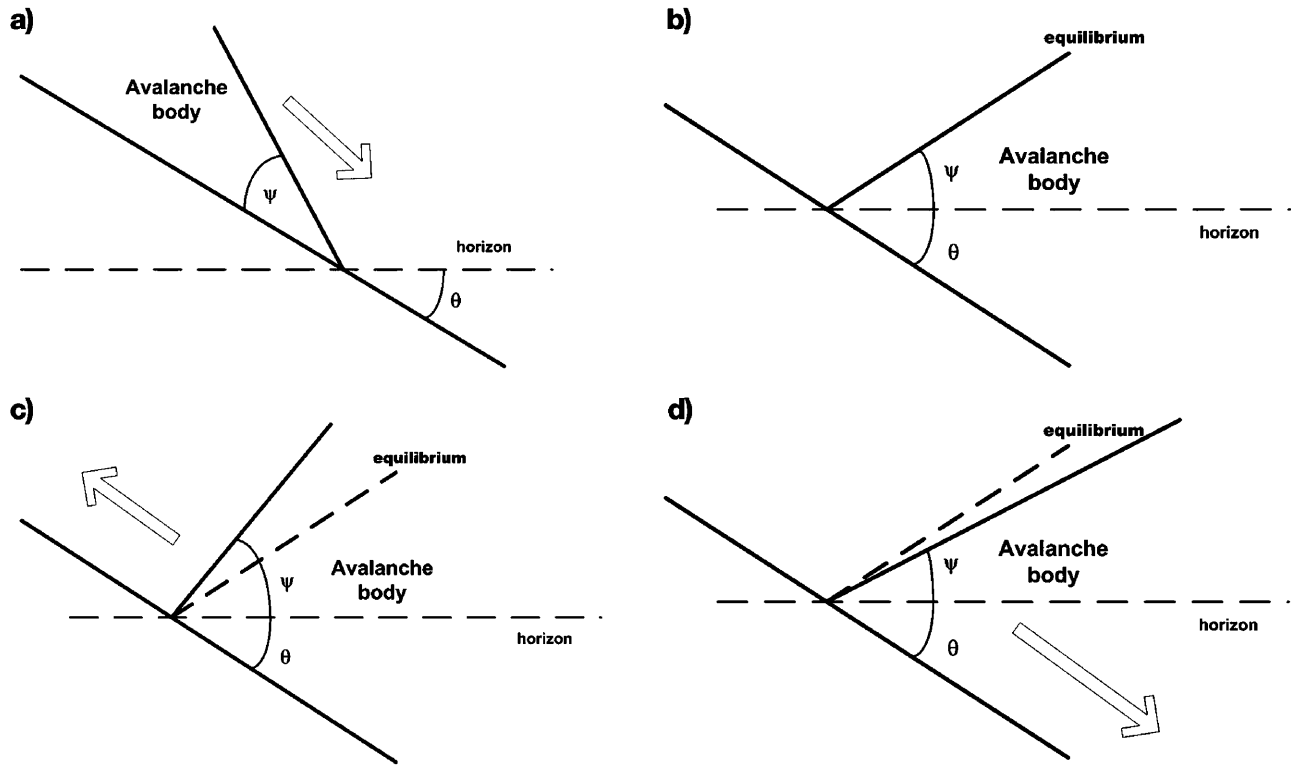


Figure 12. Dynamics of the avalanche with linearly growing height for various surface slopes.

For narrow channels the avalanche height decreases slowly, in particular for channels of parabolic cross section, $L \sim t^{-2/3}$.

[44] Another kind of self-similar solution of the system (5) and (8) can be obtained for the avalanche with linear growing height

$$\begin{aligned} h(x, t) &= \beta[x - x_0(t)], \\ u(t) &= \frac{dx_0}{dt} = g[\sin \theta - \mu \cos \theta \operatorname{sgn} u - \beta \cos \theta]t, \end{aligned} \quad (66)$$

in which β is the slope of the avalanche surface.

[45] The avalanche slides down when $\beta < 0$ ($\Psi = \arctan(\beta) > \pi/2$) with constant acceleration that exceeds the sliding component of gravity acceleration reduced by Coulomb friction. In this case the avalanche travels down as a whole (Figure 12a). In the case $\beta > 0$ ($\Psi < \pi/2$) formula (66) describes a rear end of the avalanche. In the frictionless case ($\mu = 0$) the avalanche is nonmoving (static equilibrium) if $\theta = \Psi$ (Figure 12b). If $\theta < \Psi$ the avalanche moves up (Figure 12c) and if $\theta > \Psi$ the avalanche moves down (Figure 12d). When $\mu \neq 0$ the Coulomb friction does not influence the slope of the equilibrium plane. In the vicinity of this equilibrium plane the solution of (66) is sensitive to the friction law (the “jump” function $\operatorname{sgn} u$ should replace the smooth function of u) and time evolution is more complicated.

[46] Of course, approximation of an avalanche surface of the constant slope is valid on relatively small distances from the edge, and the solution given here describing the initial dynamics of the real avalanche edge (front or rare) demon-

strates that its acceleration differs from the sliding component of the gravity force.

5. Analytical Solutions for Avalanche Dynamics in the Vicinity of its Ends

[47] More general solutions of system (12) can be obtained with the use of the hodograph (Legendre) transformation. Multiplying equation (12) by the Jacobian $\partial(t, x)/\partial(I_+, I_-)$, and assuming that it is not zero (this is discussed later), it can be transformed to

$$\frac{\partial x}{\partial I_+} - c_{\pm} \frac{\partial t}{\partial I_+} = 0. \quad (67)$$

System (67) is nonlinear due to the dependence of $c_{\pm}$ on t , see (13). However, it can be reduced to a linear equation by eliminating x

$$\frac{\partial^2 t}{\partial I_+ \partial I_-} + \frac{3m+2}{2m(I_+ - I_-)} \left(\frac{\partial t}{\partial I_-} - \frac{\partial t}{\partial I_+} \right) = 0. \quad (68)$$

Let us introduce new independent variables:

$$\lambda = \frac{I_+ + I_-}{2} = u - \alpha g t \quad (69)$$

$$\sigma = \frac{I_+ - I_-}{2} = 2\sqrt{\frac{m+1}{m}} g h \cos \theta. \quad (70)$$

Then, equation (68) takes the form

$$\frac{\partial^2 t}{\partial \lambda^2} - \frac{\partial^2 t}{\partial \sigma^2} - \frac{3m+2}{m\sigma} \frac{\partial t}{\partial \sigma} = 0. \quad (71)$$

We should also determine $x(\lambda, \sigma)$. From the system (67), using the new variables (69) and (70), it follows by eliminating t ,

$$\alpha g \frac{\partial x}{\partial \sigma} = u \frac{\partial u}{\partial \sigma} + \frac{m\sigma}{2(m+1)} - \frac{m\sigma}{2(m+1)} \frac{\partial u}{\partial \lambda}. \quad (72)$$

This equation can be integrated when the velocity, u , is expressed in terms of the wave function $\Phi(\sigma, \lambda)$

$$u = \frac{1}{\sigma} \frac{\partial \Phi}{\partial \sigma}. \quad (73)$$

An equation for this wave function can be deduced from (71). First, with the use of (69) it transforms to the wave equation for u and second, after the substitution of (73) it can be once integrated over σ

$$\frac{\partial^2 \Phi}{\partial \lambda^2} - \frac{\partial^2 \Phi}{\partial \sigma^2} - \frac{m+2}{m\sigma} \frac{\partial \Phi}{\partial \sigma} = 0. \quad (74)$$

[48] The integration of (72) together with (69), (70) and (73) allows determination of all physical variables, which are given below

$$h = \frac{1}{4g \cos \theta} \frac{m}{m+1} \sigma^2, u = \frac{1}{\sigma} \frac{\partial \Phi}{\partial \sigma} \quad (75)$$

$$\alpha g x = \frac{u^2}{2} + \frac{m\sigma^2}{4(m+1)} - \frac{m}{2(m+1)} \frac{\partial \Phi}{\partial \lambda}, t = \frac{u - \lambda}{g\alpha}. \quad (76)$$

[49] So, the initial set of the nonlinear shallow water equations for gravity driven flow in an inclined channel of parabolic-like cross section is reduced to the linear wave equation (74), and all physical variables can be found via the wave function Φ using simple operations. The main advantage of this form is that the moving (unknown) avalanche edge corresponds now to $\sigma = 0$ (since its height $h = 0$) and, therefore, equation (74) should be solved in the half-space $\sigma \geq 0$ with the fixed boundary. Such a transformation generalizes the original Carrier – Greenspan transformation for water waves above a beach of constant slope [Carrier and Greenspan, 1958], and is reduced to it for a plane beach ($m \rightarrow \infty$) and zero friction. In fact, it was also derived for inviscid water waves in an inclined channel [Zahibo et al., 2006; Choi et al., 2008], and our approach extends it to the avalanche flow with Coulomb friction. In the cited papers the wave shoaling, reflection and run-up were analyzed, however, for avalanche dynamics “non-wave” solutions are more important.

[50] This approach is used below to study the evolution of a landslide initially at rest ($u = 0$). According to the right equation (76) the initial conditions should be formulated at

$\lambda = 0$. They follow from (73) and the left equation (76); thus,

$$\Phi(\sigma, \lambda = 0) = 0 \quad (77)$$

$$\left. \frac{\partial \Phi}{\partial \lambda} \right|_{\lambda=0} = \frac{\sigma^2}{2} - \frac{2(m+1)\alpha g}{m} x(\sigma), \quad (78)$$

where $x(\sigma)$ is found from the initial landslide shape in the longitudinal direction, $h(x) = \frac{1}{4g \cos \theta} \frac{m}{m+1} \sigma^2$, see (75).

[51] The boundary condition on the avalanche edge ($\sigma = 0$) is the boundedness of the physical variables, and therefore, the wave function, $\Phi(0, \lambda)$. In general, another boundary ($h = h_{\max}$ or $\sigma = \sigma_{\max}$) is not fixed, and it should be determined from the solution. It is fixed only if: i) the initial avalanche height tends to a constant (as in the dam break problem); and ii) the avalanche height grows monotonically with the distance from the edge. For simplicity the last variant is analyzed here and we solve the wave equation (74) in the domain $0 \leq \sigma < \infty$. In this case the boundary condition for $\sigma \rightarrow \infty$ is the Sommerfeld radiation condition (no disturbances come from infinity).

[52] The solution of the Cauchy problem for the wave equation (74) for the semiaxis is well known [Courant and Hilbert, 1953], and in general it can be expressed through Bessel functions. It is simplified for the parabolic cross section of the channel ($m = 2$). For case $m = 2$, the wave equation (74), after the substitution

$$\Phi(\sigma, \lambda) = \frac{\Psi(\sigma, \lambda)}{\sigma}, \quad (79)$$

transforms to the constant-coefficient wave equation

$$\frac{\partial^2 \Psi}{\partial \lambda^2} - \frac{\partial^2 \Psi}{\partial \sigma^2} = 0, \quad (80)$$

which should be solved for $\sigma \geq 0$. Its solution is

$$\Psi(\sigma, \lambda) = \begin{cases} \Omega(\sigma + \lambda) - \Omega(\lambda - \sigma) & 0 < \sigma < \lambda, \\ \Omega(\sigma + \lambda) - \Omega(\sigma - \lambda) & \lambda < \sigma, \end{cases} \quad (81)$$

and satisfies the initial condition (77), reduced to $\Psi(\sigma, 0) = 0$, and the boundary condition of the bounded quantity of $\Phi(\lambda, \sigma = 0)$ reduced to $\Psi(0, \lambda) = 0$.

[53] The function $\Omega(\xi \geq 0)$ is found from the initial avalanche shape (78) that, after substitution of (81), transforms into the ODE

$$\frac{2}{\sigma} \frac{d\Omega(\sigma)}{d\sigma} = \frac{\sigma^2}{2} - 3\alpha g x(\sigma), h(x) = \frac{1}{6g \cos \theta} \sigma^2. \quad (82)$$

Below the dimensionless form of these equations is applied. Dimensionless variables are

$$\begin{aligned} \sigma'|\lambda' &= \frac{\sigma|\lambda}{\sqrt{gh_0}}, \Phi' = \frac{\Phi}{(gh_0)^{3/2}}, H = \frac{h \cos \theta}{h_0}, \\ V &= \frac{u}{\sqrt{gh_0}}, X = \frac{\alpha}{h_0} x, T = \alpha t \sqrt{\frac{g}{h_0}}, \end{aligned} \quad (83)$$

where h_0 is the characteristic avalanche height. We keep X and T as nondimensional variables of the spatial coordinate and time and introduce $X' = X - T^2/2$ for the accelerated reference system. Parameters of the hodograph transformation are rewritten in the final form (primes are omitted)

$$\begin{aligned} H &= \frac{\sigma^2}{6}, X = \frac{V^2}{2} + \frac{\sigma^2}{6} - \frac{1}{3} \frac{\partial \Phi}{\partial \lambda}, \\ T &= V - \lambda, \frac{2}{\sigma} \frac{d\Omega(\sigma)}{d\sigma} = \frac{\sigma^2}{2} - 3X(\sigma, 0). \end{aligned} \quad (84)$$

[54] Let us consider the dynamics of the avalanche front when its initial shape in the longitudinal direction is described by

$$X(T=0) = -q\sigma^\gamma = -q(6H)^{\gamma/2}, q > 0. \quad (85)$$

Substitution of (85) into the last expression in (84) after integrating leads to the explicit definition of the function $\Omega(\sigma)$ as follows:

$$\Omega(\sigma) = \frac{\sigma^4}{16} + \frac{3q}{2(\gamma+2)} \sigma^{\gamma+2}, \quad (86)$$

and, therefore, using (79) and (81) the function $\Phi(\sigma, \lambda)$ is determined for all values of its arguments.

[55] As a first example, an avalanche with linearly growing height ($\gamma = 2$) is analyzed; this front is curvature free. In this case (the details of calculations are given in Appendix A)

$$\Phi(\lambda, \sigma) = \frac{1+6q}{2} (\sigma^2 \lambda + \lambda^3), \quad (87)$$

Which, with the use of (84), leads to an explicit formula for the avalanche height as a function of time,

$$H(X, T) = \frac{1}{6q} \left(\frac{VT}{2} - X \right), V(T) = \frac{(1+6q)}{6q} T. \quad (88)$$

This solution coincides with the self-similar solution (66) obtained directly from the basic equations. The avalanche shape in the longitudinal direction does not change in time, and it falls off with constant acceleration depending on the avalanche slope.

[56] Another example is an avalanche with the square root singularity on its front ($\gamma = 4$, $H \sim X^{1/2}$). As the curvature of the avalanche surface in the frontal zone is positive ($d^2H/dx^2 > 0$) the function is concave. In this case (we will not give here details of the calculations which are similar to those given in Appendix A)

$$\Omega(\sigma) = \frac{\sigma^4}{16} + \frac{q}{4} \sigma^6, \quad (89)$$

and the function $\Phi(\sigma, \lambda)$ is fully determined for $\sigma \geq 0$

$$\Phi = \frac{1}{2} (\sigma^2 \lambda + \lambda^3) + q(3\sigma^4 \lambda + 10\sigma^2 \lambda^3 + 3\lambda^5). \quad (90)$$

As a result, the parametric equations for the avalanche characteristics can be found

$$\begin{aligned} T &= q(12\sigma^2 \lambda + 20\lambda^3), \\ X &= q(2\sigma^2 \lambda^2 + 15\lambda^4 - \sigma^4) + \frac{T^2}{2}, V = \lambda + T. \end{aligned} \quad (91)$$

[57] At small times and far from the front ($\lambda \sim T/12q\sigma^2$) the avalanche shape is described by the explicit asymptotic expression followed from (91)

$$X(H, T) \approx -36qH^2 + \left(1 + \frac{1}{216qH}\right) \frac{T^2}{2}, \quad (92)$$

which is valid far from the front edge

$$H \gg \left[\frac{1}{q72\sqrt{18/5}} \right]^{2/3} T^{2/3}. \quad (93)$$

So, the main part of the avalanche begins to move with an acceleration equal in absolute value to 1 (the projection of the gravity acceleration reduced by Coulomb friction in dimensional system is $g\alpha$), in the zone of large heights. The part of the avalanche with moderate heights falls with larger acceleration, and its value grows with decreasing height. Near the front edge where the height satisfies equation (93) with the opposite inequality sign, the asymptotic expression followed from (91) is valid

$$X(H, T) \approx -36qH^2 + \frac{3}{4\sqrt[3]{20q}} T^{4/3} + \frac{T^2}{2}, \quad (94)$$

and this part moves with variable acceleration. Therefore, the front part of the avalanche is stretched in time. These conclusions, made on the basis of the asymptotic analysis, are confirmed by direct calculations of equation (91); see Figure 13 where the avalanche shape is drawn in dimensionless accelerated reference coordinates X' .

[58] The front moves quickly, and the avalanche shape tends to the equilibrium state (63) that corresponds to the avalanche with the linearly growing height. The distance covered by the avalanche front is described by the curve

$$X(T) = \frac{3}{4\sqrt[3]{20q}} T^{4/3} + \frac{T^2}{2}, \quad (95)$$

following from (91) at $\sigma = 0$. At large times the front also moves with acceleration 1 ($g\alpha$ in the dimensional system), whereas the main avalanche body confirms its tendency to the equilibrium state.

[59] The avalanche contour in time is displayed in Figure 14 for different times. It is evident that the avalanche contour on the X' - Y plane tends from almost straight line for small X' to the “tongue” shape with time.

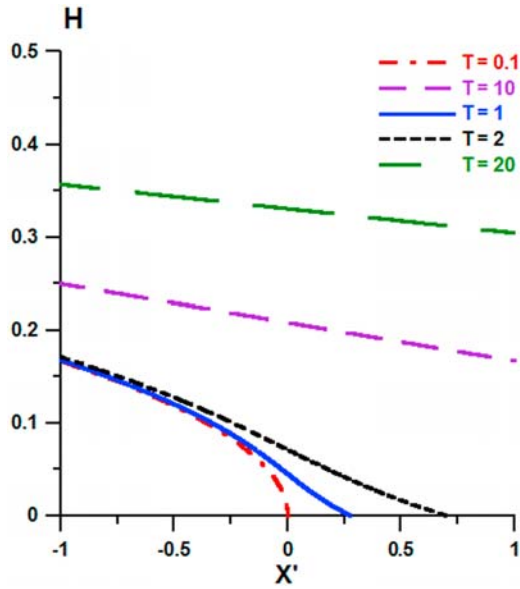


Figure 13. Time evolution of the frontal zone of avalanche for $q = 1$ (initial shape is $H \sim X^{0.5}$).

[60] The third example is an avalanche whose initial shape is described by equation (85) with $\gamma = 1$ when the avalanche height grows as X^2 with distance from the edge (a beak-shaped front). As the curvature of this front is negative, the wave function is found in the form (mathematical details are the same as given in Appendix A)

$$\Phi(\sigma, \lambda) = \begin{cases} \frac{\lambda}{2}(\sigma^2 + \lambda^2) + q(\sigma^2 + 3\lambda^2) & \sigma < \lambda \\ \frac{\lambda}{2}(\sigma^2 + \lambda^2) + q\frac{\lambda}{\sigma}(3\sigma^2 + \lambda^2) & \sigma > \lambda. \end{cases} \quad (96)$$

Consequently, the parametric expressions for the avalanche shape can be obtained: for $\sigma < \lambda$ we obtain

$$X(\sigma, \lambda) = 2q\lambda - 4q^2, T(\sigma, \lambda) = 2q, \quad (97)$$

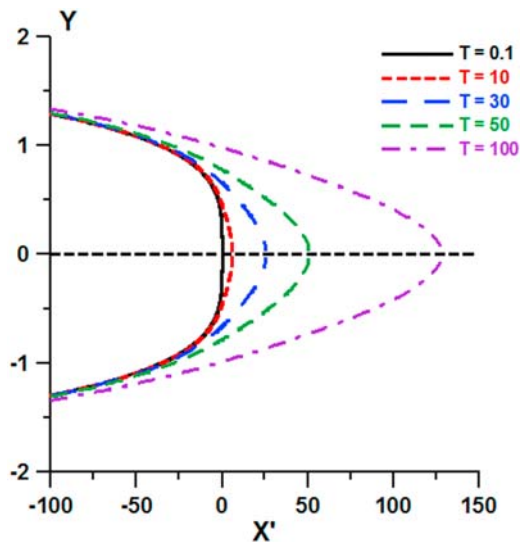


Figure 14. Spatial structure of the avalanche for times $T = 10, 30, 50, 100$ (initial shape is $H \sim X^{0.5}$).

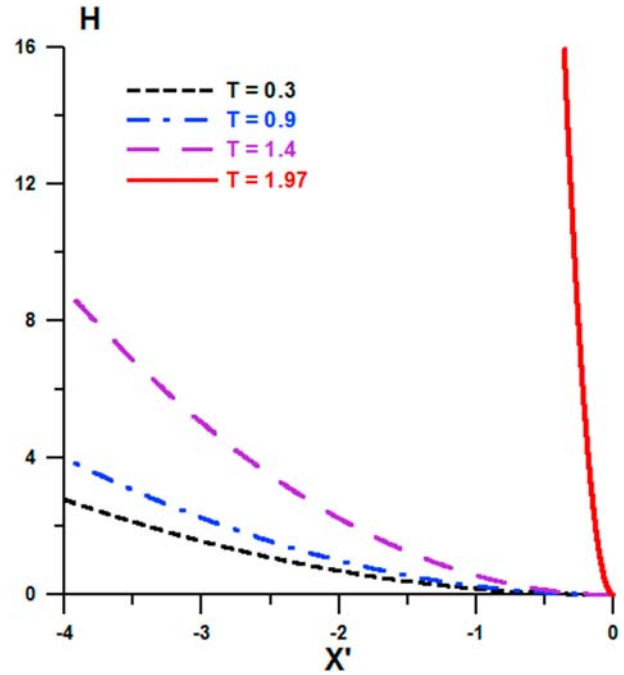


Figure 15. Initially parabolic avalanche profiles for various times before shock formation ($q = 1, H \sim X^2$).

while for $\sigma > \lambda$ we deduce

$$X(\sigma, \lambda) = q\frac{\lambda^2}{\sigma}\left(2 - \frac{\lambda^2}{\sigma^2}\right) - q\sigma + \frac{q^2\lambda^2}{2\sigma^2}\left(3 - \frac{\lambda^2}{\sigma^2}\right)^2, \quad (98)$$

$$T(\sigma, \lambda) = \frac{q\lambda}{\sigma}\left(3 - \frac{\lambda^2}{\sigma^2}\right).$$

On the avalanche front, expression (95) becomes inconsistent. The Jacobian of the hodograph transformation vanishes there which leads to the appearance of multivalued functions. Physically, this means that a shock is formed on the avalanche front. Far from the edge and at small times the avalanche shape in the longitudinal direction is described by the asymptotic expression

$$H(X, T) \approx \chi(T)[X - X_0(T)]^2, \quad (99)$$

$$X_0(T) = \frac{T^2}{2}, \chi(T) = \frac{1}{6q^2}\left(1 + \frac{2T^2}{9q^2}\right).$$

[61] The avalanche falls under the action of gravity acceleration (1 in the dimensionless system), and its slope increases with time. In finite time the steepness of the flow increases for each point on the avalanche which causes breaking. The breaking time, estimated from the exact formula (97), for $\lambda \sim \sigma$ corresponds to $T_{br} \sim 2q$. The avalanche shape for various instances is shown in Figure 15 for $q = 1$. The evident tendency to shock formation for the time close to $2q$ is observed. It is necessary to note that the avalanche shape is close to a parabolic curve, $H \sim X^2$ everywhere, not only in the asymptotic case predicted by (99).

[62] To consider the further evolution of the avalanche with the beak-shaped front it is necessary to take into account the real configuration of the flow shape at large

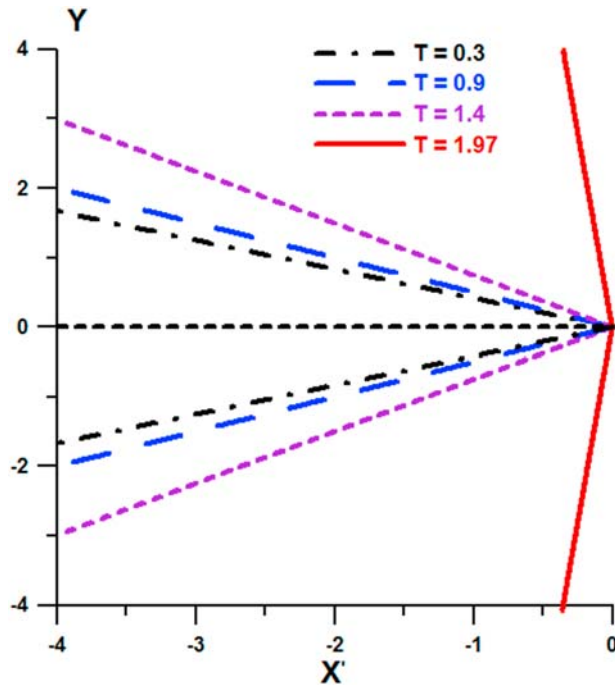


Figure 16. Spatial structure of the initially parabolic avalanche for various times.

heights which is expected far from the parabola. Anyhow the beak-shaped front should break. The avalanche contours in time for this case are presented in Figure 16. In this case the avalanche contour on X' - Y plane tends from a triangular shape for small T to almost a straight line for T tending to 2.

[63] Thus, tendencies of the evolution of the avalanche front depend on the curvature. Flows with positive curvature in the front spread in time, their fronts become gentle, and their curvatures tend to zero. The curvature increases and breaks for the avalanche with a negative curvature in the front. The equilibrium shape has no curvature on its front and moves as a whole.

[64] The same approach can be applied to study the behavior of the rear end of an avalanche (for simplicity we assume here $\mu = 0$). For an avalanche with linearly growing height ($\gamma = 2$) on the rear end, the solution can be obtained from (88) by changing the sign of q . There are three different kinds of solutions that correspond to various values of q . When $q = -1/6$, the avalanche surface represents a horizontal plane (inclined by the Coulomb friction), and there is no movement. When $q < -1/6$, the rear end of the avalanche is inclined down, and it falls down as a whole with constant acceleration. When $q > -1/6$ the rear end is inclined up and moves upward. As mentioned above, this solution coincides with the self-similar solution (63) obtained directly from the basic equations.

[65] For an avalanche with the square root singularity on the rear end ($\gamma = 4$, $H \sim X^{1/2}$) the design formula is (91) where $q < 0$. New features of avalanche dynamics follow from the approximate equation (92) for small times as the coefficient changes its sign (that represents the acceleration). The zone of big avalanche heights $H > H_{cr} = \frac{1}{216q}$ should move down, whereas a rear end of avalanche should move up. This difference can be understood if we compare the shapes of avalanches with linear and square root dependences on

height. On the rear end, the avalanche with square root singularity has high steepness like the avalanche with the linearly growing height inclined down to horizon. As shown, such avalanches move up. The periphery of the avalanche has small steepness and the avalanche surface is inclined down (i.e., below the horizon); such avalanches move down. In Figure 17 the curves $X(T)$ for various points on the rear slope computed from (91) for $q = -1$ are shown. The behavior of the curves corresponds to what was predicted in the framework of the perturbation theory based on equation (92). At small times the avalanche rear part moves up, though at large times the avalanche falls down as a whole. All curves diverge in time, and therefore, the avalanche shape remains smooth.

[66] For the trajectory of the rear point of the avalanche ($\sigma = 0$) a simple analytical formula can be given

$$X(T) = -\frac{3}{4\sqrt[3]{20|q|}}T^{4/3} + \frac{T^2}{2}, \quad (100)$$

to compare with (95). This curve is plotted in Figure 18 for $q = -1$. The rear end of the avalanche moves up until $T^* = (20|q|)^{-1/2}$, and then it falls. The maximum height of avalanche in the upward motion is

$$|X|^* = 1/80|q|. \quad (101)$$

[67] The evolution of the rear end with the initial square root singularity in the usual coordinate system is shown in Figure 19a for $q = -1$. It is demonstrated that these points move up and then fall, which confirms our asymptotic analysis. Only the main body of the avalanche falls. The shape of the varied rear end becomes more linear in time. This is clearly seen in Figure 19b where the avalanche shape is shown in the accelerated reference system, and in this case the avalanche moves to the left. So, the rear end of the avalanche has the tendency to become linear. The given examples show the applicability of the hodograph transformation to describe analytically the various scenarios of avalanche dynamics in inclined channels of parabolic-like

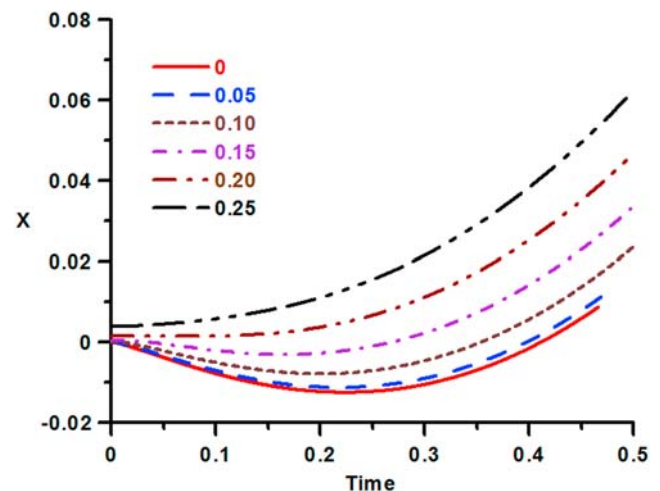


Figure 17. Curves $X(T)$ for various points of rear slope (numbers indicate values of σ).

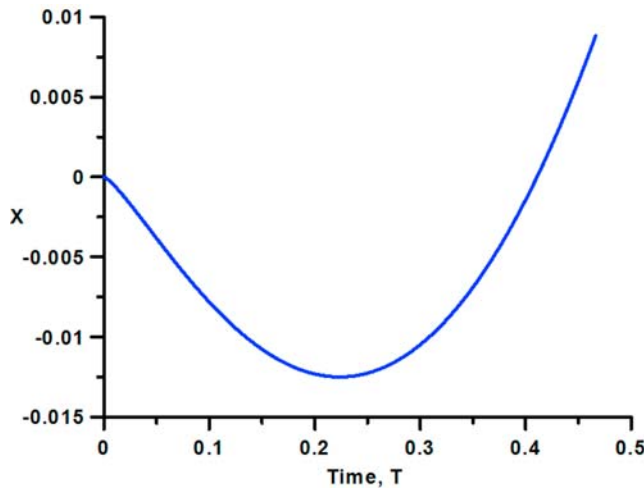


Figure 18. Trajectory of the rear end of avalanche for $q = 1$ (initial shape is $H \sim X^{0.5}$).

shape. In comparison with the self-similar solutions that describe the nonbreaking dynamics of the avalanche initially at rest (section 4), new solutions demonstrate that, depending on the initial shape, the avalanche can move with a smooth shape as well as with a shock front. The formation of shock fronts in landslides and two-phase granular flows has been pointed out in numerical experiments performed by *Fine et al.* [2003] and *Pelanti et al.* [2008].

6. Conclusion

[68] Computing avalanche dynamics is an extremely important task in natural hazard assessments, and various physical and numerical models are developed. The analytical solutions allow studying the physical features of such processes (the beginning of movement, the direction of the avalanche motion, spreading, forming of shocks and so on). Several analytical solutions for avalanche dynamics in mountain valleys approximated by narrow channels (sometimes, granular flows in wide channels can be propagated as self-channeling narrow flows, and this effect has been reproduced recently numerically by *Mangeney et al.* [2007a]) of parabolic-like cross section are obtained here in the framework of the simplified version of the Savage-Hutter model. Some of them generalize known solutions for avalanche propagation along the inclined plate (the dam break problem, the parabolic cap, the M wave, the Riemann wave). The quality of avalanche dynamics in the longitudinal direction is similar to that of avalanches along channels of arbitrary cross section. However, the spatial structure of the avalanche is different in channels of different cross sections. Quantitative characteristics of avalanche dynamics depend on the cross-sectional shape and, in narrow channels, for instance, nonlinear processes develop more intensively for the same avalanche heights.

[69] In the present paper a new approach to investigate avalanche dynamics is suggested. It is based on the hodograph (Legendre) transformation and can be applied for avalanches of different initial shapes. The derived solutions show that the linear avalanche shape is the equilibrium shape which moves along the mountain flank without

changing its shape (it also follows from the direct self-similar solution). Avalanche frontal shapes that may be approximated by the equation with power dependence on the coordinate $h \sim x^a$ on the front, have a tendency to break if the power a is more than one and a tendency to form the frontal “tongue” if the power a is less than one. Furthermore, the rear end of the avalanche is studied and the possibility of nonmonotonic motion of the rear end is found. All the solutions obtained above describe avalanche dynamics above solid basalt. In fact, an erodible bed can increase the mobility of the avalanche which can propagate in the form of traveling waves; this interesting geophysical phenomenon is in the process of being analyzed [*Mangeney et al.*, 2007b; *Lucas and Mangeney*, 2007].

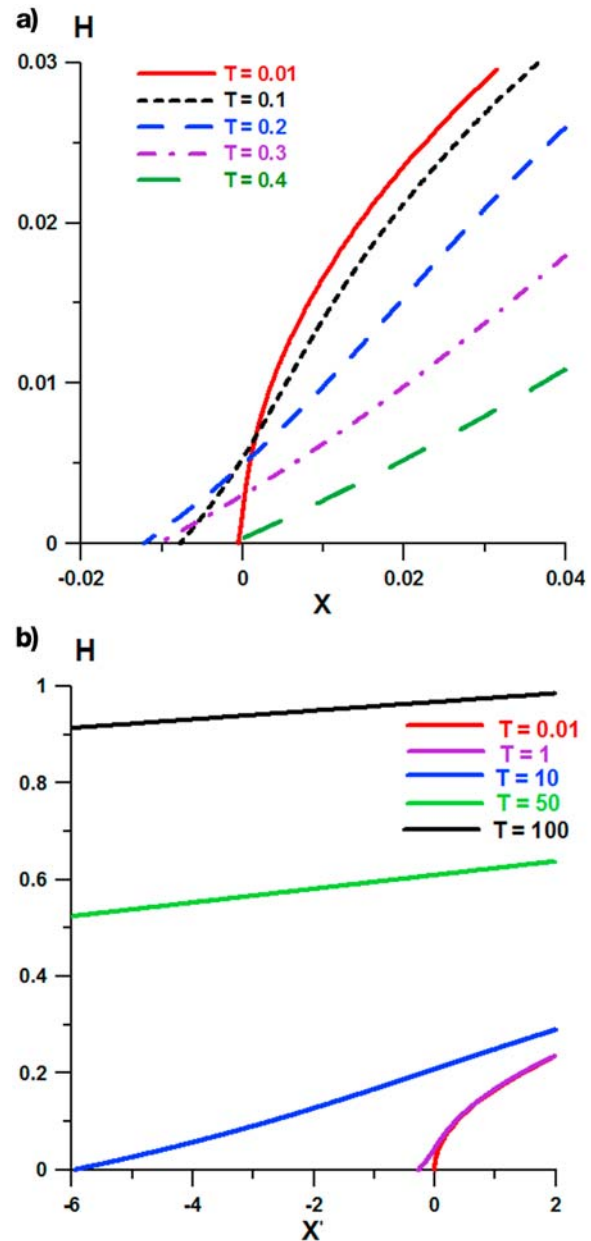


Figure 19. Evolution of the rear end of avalanche for $q = 1$ (initial shape is $H \sim X^{0.5}$) (a) in the usual coordinate system and (b) in the accelerated reference system.

[70] In fact, the simplified frictional rheology (constant friction angle) is used to obtain analytical solutions. As pointed by *Pirulli and Mangeney* [2008], various rheological models can be applied to model avalanche dynamics but only the frictional rheology has allowed the back analysis of the historical events of the rock avalanches: Frank (Canada, 1903) and Val Pola (Italy, 1987). The main advantage of this simplified model is that only one parameter (the friction angle) has to be calibrated. In the case of the inclined channel the second parameter (the shape factor) should be taken into account and it can be determined for real mountain valleys. This is very important for the prediction of potential events, and the analytical solutions described above allow estimating possible scenarios taking into account the lack of data of the potential events. The simple analytical solutions presented here may be very useful to analyze and recover scaling laws observed in laboratory experiments of natural landslides [e.g., *Mangeney-Castelnau et al.*, 2005; *Lucas and Mangeney*, 2007]. They can be also used to test numerical codes of avalanche dynamics in the valleys of complicated geometry.

Appendix A

[71] Let us consider avalanche dynamics with linear growing height in the front ($\gamma = 2$). In this case equation (86) yields

$$\Omega(\sigma) = \frac{1 + 6q}{16} \sigma^4. \quad (\text{A1})$$

[72] Substituting (A1) into (81) we may compute the function Ψ

$$\Psi(\sigma, \lambda) = \frac{1 + 6q}{2} \sigma \lambda (\sigma^2 + \lambda^2), \quad (\text{A2})$$

and then the wave function (79)

$$\Phi(\sigma, \lambda) = \frac{1 + 6q}{2} \lambda (\sigma^2 + \lambda^2). \quad (\text{A3})$$

Now we may calculate all physical variables. In particular, the velocity is given by

$$V = \frac{1}{\sigma} \frac{\partial \Phi}{\partial \sigma} = (1 + 6q) \lambda. \quad (\text{A4})$$

The third equation in (84) determines the time

$$T = V - \lambda = 6q \lambda. \quad (\text{A5})$$

As a result, arguments λ and σ can be expressed explicitly through physical variables T and H (as the first equation in (84) determines the relation between H and σ). Finally, the velocity is a function of time only

$$V = \frac{1 + 6q}{6q} T, \quad (\text{A6})$$

and X is computed from the second equation (84)

$$X = \frac{1 + 6q}{12q} T^2 - 6qH, \quad (\text{A7})$$

or

$$H = \frac{1}{6q} \left(\frac{VT}{2} - X \right), \quad (\text{A8})$$

see (88). In other cases of the initial distribution of the avalanche height in the longitudinal direction, formulas for physical variables are implicit and the computer is required to perform algebraic manipulations.

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References

- Assier-Rzadkiewicz, S., P. Heinrich, P. C. Sabatier, B. Savoyer, and J. F. Bourillet (2000), Numerical modeling of a landslide-generated tsunami: The 1979 Nice event, *Pure Appl. Geophys.*, *157*, 1707–1727, doi:10.1007/PL00001057.
- Bouchut, F., and M. Westdickenberg (2004), Gravity driven shallow water models for arbitrary topography, *Commun. Math. Sci.*, *2*, 359–389.
- Bouchut, F., A. Mangeney-Castelnau, B. Perthame, and J.-P. Vilotte (2003), A new model of Saint Venant and Savage-Hutter type for gravity driven shallow water flows, *C. R. Acad. Sci., Ser. I*, *336*, 531–536.
- Bouchut, F., E. D. Fernandez-Nieto, A. Mangeney, and P.-Y. Lagree (2008), On new erosion models of Savage-Hutter type for avalanches, *Acta Mech.*, *199*, 181–208, doi:10.1007/s00707-007-0534-9.
- Carrier, G. F., and H. P. Greenspan (1958), Water waves of finite amplitude on a sloping beach, *J. Fluid Mech.*, *4*, 97–109, doi:10.1017/S0022112058000331.
- Choi, B. H., E. Pelinovsky, D. C. Kim, I. Didenkulova, and S. B. Woo (2008), Two- and three-dimensional computation of solitary wave run-up on non-plane beach, *Nonlinear Process. Geophys.*, *15*(3), 489–502.
- Courant, R., and D. Hilbert (1953), *Methods of Mathematical Physics*, 561 pp., John Wiley, New York.
- Fernandez-Feria, R. (2006), Dam-break flow for arbitrary slopes of the bottom, *J. Eng. Math.*, *54*, 319–331, doi:10.1007/s10665-006-9034-5.
- Fernandez-Nieto, E. D., F. Bouchut, D. Bresch, M. J. Castro Diaz, and A. Mangeney (2008), A new Savage-Hutter type model for submarine avalanches and generated tsunami, *J. Comput. Phys.*, *227*, 7720–7754, doi:10.1016/j.jcp.2008.04.039.
- Fine, I. V., A. B. Rabinovich, R. E. Thomson, and E. A. Kulikov (2003), Numerical modeling of tsunami generation by submarine and subaerial landslides, in *Submarine Landslides and Tsunamis*, vol. 21, *Earth and Environmental Sciences, NATO Sci. Ser.*, vol. 4, edited by A. Yalçiner et al., pp. 69–88, Kluwer, Boston.
- Fine, I. V., A. B. Rabinovich, B. D. Bornhold, R. E. Thomson, and E. A. Kulikov (2005), The Grand Banks landslide-generated tsunami of November 18, 1929: Preliminary analysis and numerical modeling, *Mar. Geol.*, *215*, 45–57, doi:10.1016/j.margeo.2004.11.007.
- Gray, J. M. N. T., M. Wieland, and K. Hutter (1999), Gravity-driven free surface flow of granular avalanches over complex basal topography, *Proc. R. Soc. London, Ser. A*, *455*, 1841–1874, doi:10.1098/rspa.1999.0383.
- Heinrich, P., A. Mangeney, S. Guibourg, R. Roche, G. Boudon, and J. Cheminée (1998), Simulation of water waves generated by a potential debris avalanche in Montserrat, Lesser Antilles, *Geophys. Res. Lett.*, *25*(19), 3697–3700, doi:10.1029/98GL01407.
- Heinrich, P., S. Guibourg, A. Mangeney, and R. Roche (1999), Numerical modeling of a landslide-generated tsunami following a potential explosion of the Montserrat volcano, *Phys. Chem. Earth*, *24*(2), 163–168, doi:10.1016/S1464-1895(99)00013-7.
- Heinrich, P., G. Boudon, J. C. Komorowski, R. S. J. Sparks, R. Herd, and B. Voight (2001), Numerical simulation of the December 1997 debris avalanche in Montserrat, Lesser Antilles, *Geophys. Res. Lett.*, *28*(13), 2529–2532, doi:10.1029/2001GL012968.

- Imamura, F., and E. C. Gica (1996), Numerical model for tsunami generation due to subaqueous landslide along a coast, *Sci. Tsunami Hazards*, 14(1), 13–28.
- Kerswell, R. R. (2005), Dam break with the Coulomb friction: A model for granular slumping?, *Phys. Fluids*, 17, 057101, doi:10.1063/1.1870592.
- Le Friant, A., P. Heinrich, C. Depluch, and G. Boudon (2003), Numerical simulation of the last flank-collapse event of Montagne Pelée, Martinique, Lesser Antilles, *Geophys. Res. Lett.*, 30(2), 1034, doi:10.1029/2002GL015903.
- Le Friant, A., G. Boudon, J.-C. Komorowski, P. Heinrich, and M. P. Semet (2006), Potential flank-collapse of Soufriere Volcano, Guadeloupe, Lesser Antilles? Numerical simulation and hazards, *Nat. Hazards*, 39, 381–393, doi:10.1007/s11069-005-6128-8.
- Liu, P. L.-F., P. Lynett, and C. E. Synolakis (2003), Analytical solutions for forced long waves on a sloping beach, *J. Fluid Mech.*, 478, 101–109, doi:10.1017/S0022112002003385.
- Luca, L., K. Hutter, Y. C. Tai, and C. Y. Kuo (2009), A hierarchy of avalanche models on arbitrary topography, *Acta Mech.*, 205, 121–149, doi:10.1007/s00707-009-0165-4.
- Lucas, A., and A. Mangeney (2007), Mobility and topographic effects for large Valles Marineris landslides on Mars, *Geophys. Res. Lett.*, 34, L12021, doi:10.1029/2007GL029835.
- Mangeney, A., P. H. Heinrich, R. Roche, G. Boudon, and J. L. Cheminee (2000a), Modeling of debris avalanche and generated water waves: Application to real and potential events in Montserrat, *Phys. Chem. Earth*, 25(9–11), 741–745, doi:10.1016/S1464-1895(00)00115-0.
- Mangeney, A., P. Heinrich, and R. Roche (2000b), Analytical solution for testing debris avalanche numerical models, *Pure Appl. Geophys.*, 157, 1081–1096, doi:10.1007/s000240050018.
- Mangeney, A., F. Bouchut, N. Thomas, J. P. Vilotte, and M. O. Bristeau (2007a), Numerical modeling of self-channeling granular flows and of their levee-channel deposits, *J. Geophys. Res.*, 112, F02017, doi:10.1029/2006JF000469.
- Mangeney, A., L. S. Tsimring, D. Volfson, I. S. Aranson, and F. Bouchut (2007b), Avalanche mobility induced by the presence of an erodible bed and associated entrainment, *Geophys. Res. Lett.*, 34, L22401, doi:10.1029/2007GL031348.
- Mangeney-Castelnau, A., J.-P. Vilotte, M. O. Bristeau, B. Perthame, F. Bouchut, C. Simeoni, and S. Yerneni (2003), Numerical modeling of avalanches based on Saint Venant equations using a kinetic scheme, *J. Geophys. Res.*, 108(B11), 2527, doi:10.1029/2002JB002024.
- Mangeney-Castelnau, A., F. Bouchut, J. P. Vilotte, E. Lajeunesse, A. Aubertin, and M. Pirulli (2005), On the use of Saint Venant equations to simulate the spreading of a granular mass, *J. Geophys. Res.*, 110, B09103, doi:10.1029/2004JB003161.
- Miller, D. J. (1960), Alaska Earthquake on July 10, 1958: Giant wave in Lituya Bay, *Bull. Seismol. Soc. Am.*, 50(2), 253–266.
- Pelanti, M., F. Bouchut, and A. Mangeney (2008), A Roe-type scheme for two-phase shallow granular flows over variable topography, *Math. Modell. Numer. Anal.*, 42, 851–885, doi:10.1051/m2an:2008029.
- Pelinovsky, E. (2003), Analytical models of tsunami generation by submarine landslides, in *Submarine Landslides and Tsunamis*, vol. 21, *Earth and Environmental Sciences, NATO Sci. Ser.*, vol. 4, edited by A. Yalçiner et al., pp. 111–128, Kluwer, Boston.
- Pelinovsky, E., and A. Poplavsky (1996), Simplified model of tsunami generation by submarine landslides, *Phys. Chem. Earth*, 21(12), 13–17, doi:10.1016/S0079-1946(97)00003-7.
- Pelinovsky, E. N., and E. N. Troshina (1994), Propagation of long waves in straits, *Phys. Oceanogr.*, 5(1), 43–48, doi:10.1007/BF02197568.
- Pelinovsky, E., N. Zahibo, P. Dunkley, M. Edmonds, R. Herd, T. Talipova, A. Kozelkov, and I. Nikolkina (2004), Tsunami generated by the volcano eruption on July 12–13 2003 at Montserrat, Lesser Antilles, *Sci. Tsunami Hazards*, 22(1), 44–57.
- Pirulli, M., and A. Mangeney (2008), Results of back-analysis of the propagation of rock avalanches as a function of the assumed rheology, *Rock Mech. Rock Eng.*, 41(1), 59–84, doi:10.1007/s00603-007-0143-x.
- Pudasaini, S. P., and K. Hutter (2007), *Avalanche Dynamics: Dynamics of Rapid Flows of Dense Granular Avalanches*, 602 pp., Springer, New York.
- Rudenko, O. V., A. L. Sobisevich, and L. E. Sobisevich (2007), Nonlinear dynamics of slope flows: Simple models and exact solutions, *Dokl. Earth Sci.*, 416(1), 1109–1113, doi:10.1134/S1028334X07070288.
- Sammarco, P., and E. Renzi (2008), Landslide tsunamis propagating along a plane beach, *J. Fluid Mech.*, 598, 107–119, doi:10.1017/S0022112007009731.
- Savage, S. B., and K. Hutter (1989), The motion of a finite mass of granular material down a rough incline, *J. Fluid Mech.*, 199, 177–215, doi:10.1017/S0022112089000340.
- Savage, S. B., and K. Hutter (1991), Dynamics of avalanches of granular materials from initiation to run-out. Part 1: Analysis, *Acta Mech.*, 86, 201–223, doi:10.1007/BF01175958.
- Stoker, J. J. (1957), *Water Waves: The Mathematical Theory with Applications*, Interscience, New York.
- Tinti, S., E. Bortolucci, and C. Chlavettieri (2001), Tsunami excitation by submarine slides in shallow-water approximation, *Pure Appl. Geophys.*, 158, 759–797, doi:10.1007/PL00001203.
- Voight, B., J.-C. Komorowski, G. Norton, A. Belousov, M. Belousova, G. Boudon, P. W. Francis, R. S. J. Sparks, and S. R. Young (2002), The 26 December (Boxing Day) 1997 sector collapse and debris avalanche at Soufriere Hills Volcano, *Geol. Soc. London Mem.*, 21, 363–407, doi:10.1144/GSL.MEM.2002.021.01.17.
- Watts, P. (2000), Tsunami features of solid block underwater landslides, *J. Waterw. Port Coastal Ocean Eng.*, 126(3), 144–152, doi:10.1061/(ASCE)0733-950X(2000)126:3(144).
- Whitham, G. B. (1974), *Linear and Nonlinear Waves*, 660 pp., Wiley, New York.
- Yu, B., K. Dalbey, A. Webb, M. Bursik, A. Patra, E. B. Pitman, and C. Nichita (2009), Numerical issues in computing inundation areas over natural terrains using Savage-Hutter theory, *Nat. Hazards*, 50, 249–267, doi:10.1007/s11069-008-9336-1.
- Zahibo, N., E. Pelinovsky, V. Golinko, and N. Osipenko (2006), Tsunami wave run-up on coasts of narrow bays, *Int. J. Fluid Mech. Res.*, 33(1), 106–118, doi:10.1615/InterJFluidMechRes.v33.i1.70.
- Zahibo, N., I. Didenkulova, A. Kurkin, and E. Pelinovsky (2008a), Steepness and spectrum of nonlinear deformed shallow water wave, *Ocean Eng.*, 35(1), 47–52, doi:10.1016/j.oceaneng.2007.07.001.
- Zahibo, N., E. Pelinovsky, A. Kurkin, and I. Nikolkina (2008b), Tsunami hazard for the French West Indies, Lesser Antilles, in *Integrated Coastal Zone Management*, edited by R. Krishnamurthy, pp. 517–535, Res. Publ., Singapore.

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Self-Similar Solutions in the Theory of Underwater Landslide Dynamics in Inclined Canyons

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Abstract: The landslide motion in inclined underwater canyons is described in the framework of basic equations of the gravity driven shallow-water flows. A Coulomb (sliding) friction term is included in the model. The analytical self-similar solutions are obtained in the explicit form. The spatial structure of avalanche that depends on the cross-section of the inclined channel is studied. Temporal variation of landslide parameters for different cross-sections of the channel is analyzed. Obtained results can be used to test 2D numerical models of landslide motion and tsunami generation by underwater landslides.

Keywords: Landslide, Underwater

1. INTRODUCTION

The submarine and aerial landslides are frequently reported as the sources of tsunami waves. In fact, such situation is typical for Lesser Antilles in Caribbean. The pyroclastic flows were generated several times on the Montserrat Island due to continued volcano activity and caused three tsunami events recorded in Guadeloupe and Montserrat in the last decade [1-3] the repetition of such events is rather high. Various data of tsunami generation and propagation can be found in books [4-6] Diverse analytical and numerical models are developed to describe tsunamis generated by landslides. In series of papers [1, 7-14] the landslide is assumed as a solid block moving under the joint action of gravity and friction. As its motion is known the hydrodynamic equations for tsunami waves are solved with determined external source. More realistic models of tsunami generation include “two-layer” approach which jointly describes tsunami wave and a landslide propagation influencing one another. Such models of underwater landslides are based on homogeneous shallow-water flows that are deformed during propagation [15-22]. These models use various approximations of vertical structure of velocity field (uniform as in ideal fluid, or parabolic for viscous fluid), as well as different approximations of the friction law (Coulomb friction, Chezyquadratic friction and so on); the derivation of the mentioned models can be found for instance [23].

The number of analytical solutions of gravity driven shallow-water flow is limited. self-similar solutions called parabolic cap and M-wave similarity solutions with application to the avalanche dynamics were constructed in [24, 25]. Such solutions are obtained for a chute of constant inclination

and constant bed friction angle. [16] found the analytical solution of a one-dimensional dam-break problem over inclined plane taking into account Coulomb friction. It was actively used to test the numerical models applied to evaluate the hazard of the volcano eruption in the Lesser Antilles and tsunami waves induced by landslides [21].

However, real underwater landslides move in basins of more complicated bottom geometry. As a rule, they move in submarine canyons which are generally diverging, converging or twisted. Anyway some of them have straight parabolic-like shapes (Fig. 1). We provide here new analytical solutions for underwater landslides in inclined channels of constant slope with the specific parabolic-like cross-section taking into account Coulomb friction. New effects in the landslide dynamics comparably to those known for the 1D motion along an inclined plate are related with the parabolic-like cross-section shape of the canyon.

The paper is organized as follows. The basic model to describe the landslide motion that represents gravity driven shallow-water flow in a narrow inclined channel of parabolic-like cross-section is briefly given in section 2. For narrow channels the basic equations are also 1D but they contain additional coefficients that determine the parabolic-like cross-section. The self-similar solution for the landslide of finite length and symmetrical shape in the inclined canyons is found in the section 3. The discussion of obtained results is given in the conclusion.

2. GOVERNING EQUATIONS OF GRAVITY DRIVEN SHALLOW-WATER FLOW

The shallow-water system is the basic system to describe gravity driven flow (submarine landslide), various analytical and numerical models realized the shallow-water equations can be found in [23,26]. Here we will assume that the landslide can be presented by the flow of incompressible fluid

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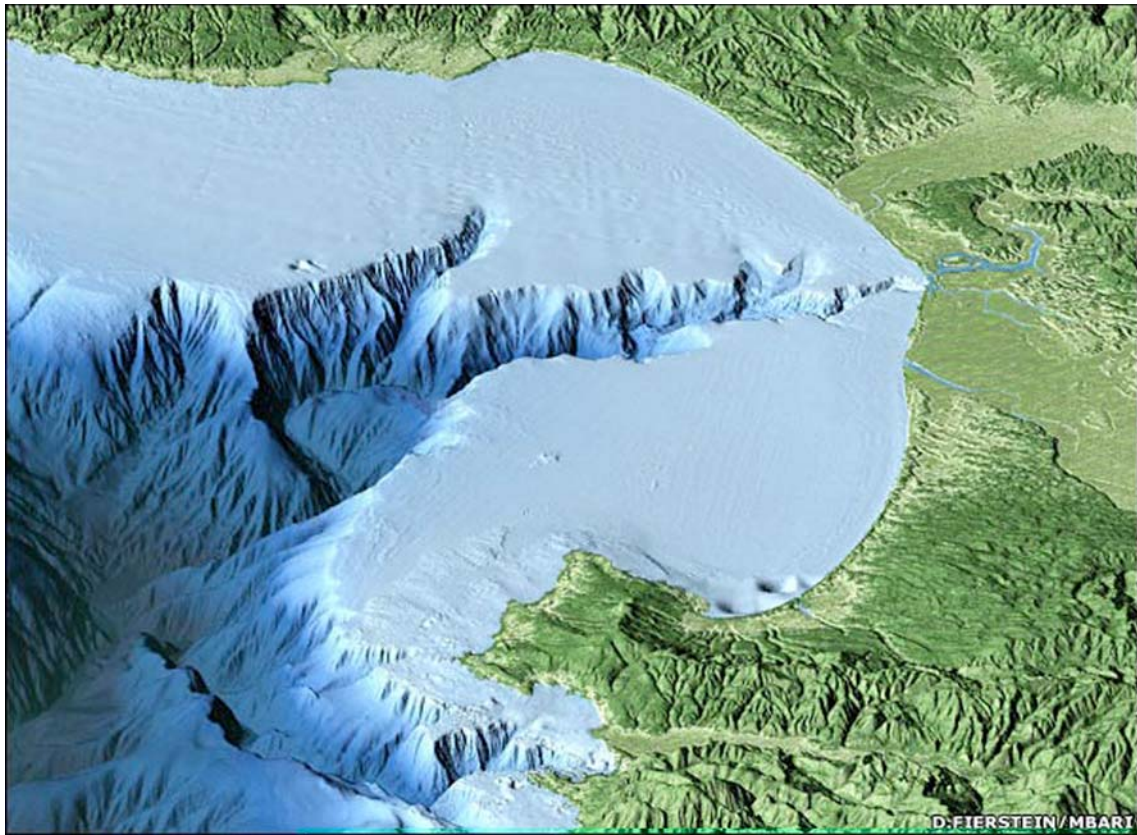


Fig. (1). View of Monterey, Soquel and Carmel Canyons (Monterey Bay Aquarium Research Institute data [28]).

with the Coulomb-type friction term. In the first approximation we will assume that the dynamics of the underwater landslide can be described independently from tsunami waves induced by landslides see [7,8]. Formally, it is possible if the landslide density exceeds significantly the water density. In the system of coordinates linked to the topography the equations of mass and momentum conservation have the following form

$$\frac{\partial S}{\partial t} + \frac{\partial}{\partial x}(Su) = 0, \quad (1)$$

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + g \cos \theta \frac{\partial h}{\partial x} = g \sin \theta - \mu g \cos \theta, \quad (2)$$

where $S(x, t)$ is the area of the cross-section of the channel, $h(x, t)$ is the height along x -axis of channel, and $u(x, t)$ is the mean velocity flow, g is the acceleration due to gravity, θ is the slope angle along x -axes, $\mu = \tan(\varphi)$ is the sliding friction coefficient, and φ is the dynamic friction angle. The value of φ depends on the landslide material, and is small for sand and large for granular debris avalanche. One-dimensional equations are valid if the inclined channel (submarine canyon) is narrow, and water flow is uniform in the transversal direction. This 1D system can be derived from two-dimensional equations of gravity-driven flow given for instance in books [26, 27].

To close this system, the function $S(h)$ should be determined. If the cross-section has a parabolic-like shape (see Fig. 2),

$$z(y) = k |y|^m, \quad (3)$$

with positive arbitrary constants, k and m , the function, S is calculated explicitly

$$S = \frac{2m}{(m+1)k^{1/m}} h^{(m+1)/m}. \quad (4)$$

In general cross-section of the channel is parabolic, triangular or concave for different values of m , (see Fig. (2)).

As a result, the equation (1) transforms to

$$\frac{\partial h}{\partial t} + u \frac{\partial h}{\partial x} + \frac{m}{m+1} h \frac{\partial u}{\partial x} = 0, \quad (5)$$

and the system of equations (2) and (5) becomes closed. It differs from the “classical” one-dimensional shallow-water equations [16, 26] in the constant coefficient $m/(m+1)$, and may be transformed to them as $m \rightarrow \infty$ ($S \sim h$).

Below we assume the constant slope of the inclined channel θ , which should be relatively high ($\theta > \varphi$) to provide the landslide motion.

The basic system of gravity driven flow, (2) and (5), is analyzed below. Mathematically, it is a hyperbolic system with constant coefficients. As for water waves above inclined bottom [27, 29-31], the Riemann invariants can be found explicitly

$$I_{\pm} = u \pm 2 \sqrt{\frac{m+1}{m}} gh \cos \theta - gat, \quad (6)$$

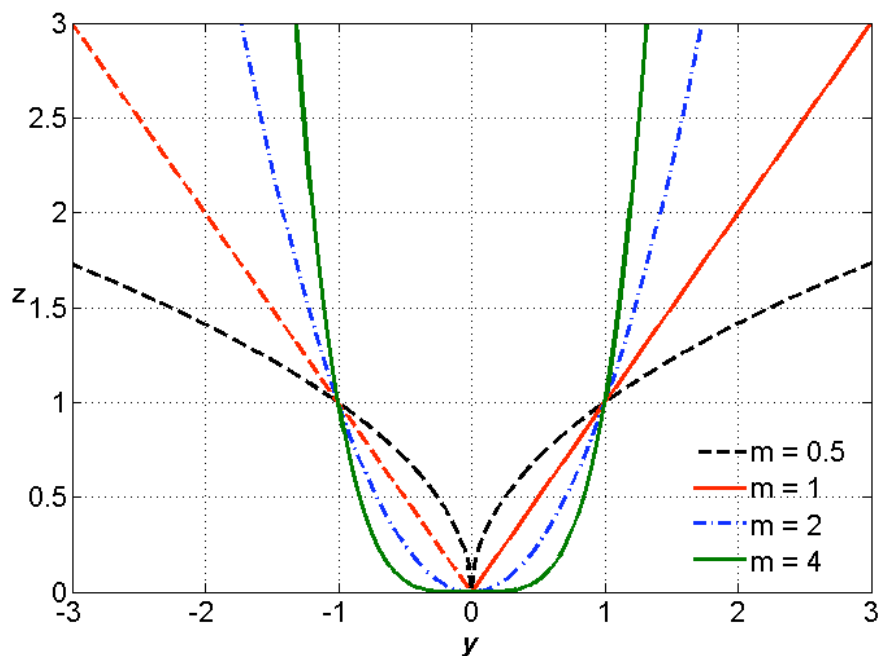


Fig. (2). Cross-section of the channel computed for $k = 1$.

where

$$\alpha = \sin \theta - \mu \cos \theta > 0, \quad (7)$$

and the system (5) – (2) can be re-written in the form

$$\frac{\partial I_{\pm}}{\partial t} + c_{\pm} \frac{\partial I_{\pm}}{\partial x} = 0, \quad (8)$$

where characteristic speeds are

$$c_{\pm} = \frac{3m+2}{4(m+1)} I_{\pm} + \frac{m+2}{4(m+1)} I_{\mp} + g\alpha t. \quad (9)$$

It should be mentioned that the equations (8) – (9) appear in the problems of long water wave runup on the coast including tsunami wave runup [27, 29-31] but the main difference here is that the bottom slope is not small (as in “runup problems”) and the Coulomb-type friction is not neglected. The “landslide solutions” can not be directly obtained from the “tsunami solutions” because mathematically, the initial and boundary conditions for tsunami waves and landslide dynamics differ. Physical variables, $h(x, t)$ and $u(x, t)$ are found from the algebraic equations (6) and the down slope component of the velocity field ($g\alpha t$) propagated under the action of gravity can be eliminated by use of

$$u(x, t) = g\alpha t + v(x, t), \quad (10)$$

$$v(x, t) = \frac{I_{+} + I_{-}}{2}, \quad (11)$$

$$h = \frac{m}{16g \cos \theta (m+1)} (I_{+} - I_{-})^2. \quad (12)$$

where $\alpha (> 0)$ depends on the slope angle and friction coefficient, see (7). Another useful development is the change of coordinates, the accelerated reference system can be written as

$$X = x - \frac{g\alpha t^2}{2}, \quad t = t. \quad (13)$$

In this case the basic equations (2) and (5) transform to

$$\frac{\partial h}{\partial t} + v \frac{\partial h}{\partial X} + \frac{m}{m+1} h \frac{\partial v}{\partial X} = 0, \quad (14)$$

$$\frac{\partial v}{\partial t} + v \frac{\partial v}{\partial X} + g \cos \theta \frac{\partial h}{\partial X} = 0. \quad (15)$$

The system (14) – (15) will be used below to study the unsteady dynamics of the landslide in the inclined canyon.

3. SELF-SIMILAR SOLUTION

Let us try to find self-similar solutions of the equations (14) and (15). They can be obtained using the self-similar transformation of coordinates

$$\eta = \frac{X}{L(t)}, \quad t = t, \quad (16)$$

which transforms equations (14) and (15) to

$$\frac{\partial}{\partial t} \left(h^{\frac{m+1}{m}} \right) + \frac{v - \eta dL/dt}{L} \frac{\partial}{\partial \eta} \left(h^{\frac{m+1}{m}} \right) + \frac{h^{\frac{m+1}{m}}}{L} \frac{\partial v}{\partial \eta} = 0, \quad (17)$$

$$\frac{\partial v}{\partial t} + \frac{v - \eta dL/dt}{L} \frac{\partial v}{\partial \eta} + \frac{g \cos \theta}{L} \frac{\partial h}{\partial \eta} = 0. \quad (18)$$

Similar system of the equations (at $m \rightarrow \infty$) with application to the avalanche dynamics is studied in the [26]. Second term in both equations can be eliminated if we determine velocity distribution in the landslide body as

$$v = \eta \frac{dL}{dt} = \frac{X}{L} \frac{dL}{dt}. \quad (19)$$

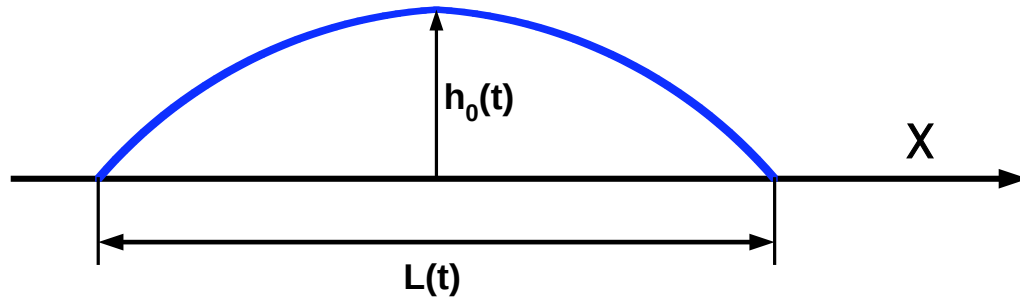


Fig. (3). Parabolic cap shape of the avalanche in longitudinal direction.

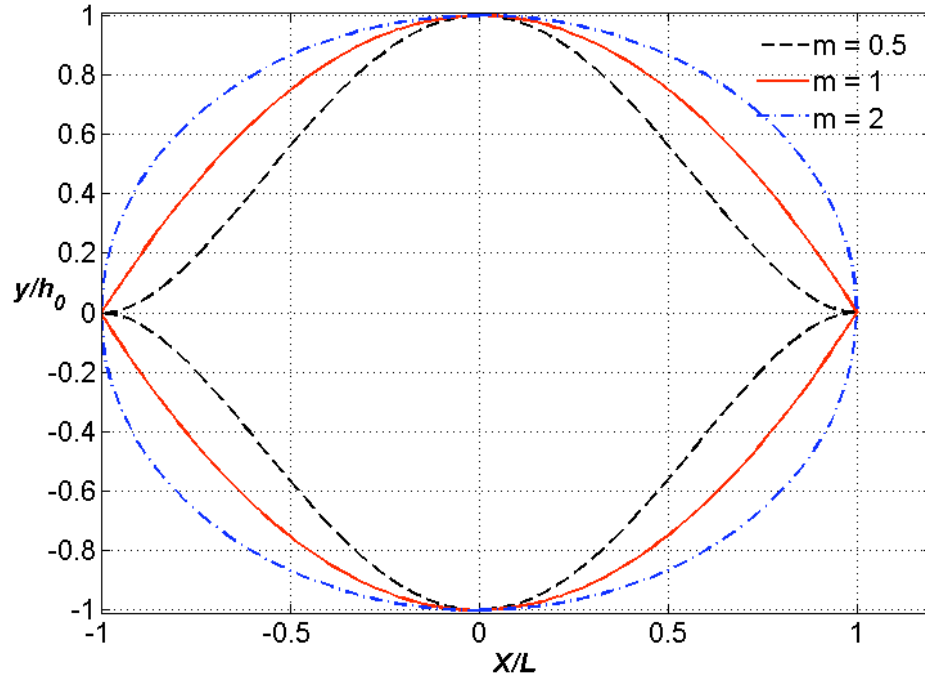


Fig. (4). Landslide contours in the accelerated reference system in different channels.

After substitution of (19) in (17) and (18) we obtain two differential equations for landslide height with respect to variables t and η

$$\frac{\partial}{\partial t} \left(h^{\frac{m+1}{m}} \right) + \frac{dL/dt}{L} h^{\frac{m+1}{m}} = 0, \quad (20)$$

$$\frac{\partial h}{\partial \eta} + \frac{\eta L d^2 L / dt^2}{g \cos \theta} = 0. \quad (21)$$

Both equations are integrated and the adequate choice of constants can be written as

$$h = h_0(t) (1 - \eta^2), \quad h_0(t) = \frac{L d^2 L / dt^2}{2g \cos \theta}, \quad (22)$$

where the landslide length $L(t)$ is a solution of

$$\frac{L^{\frac{2m+1}{m+1}} d^2 L}{dt^2} = q, \quad (23)$$

and q is determined by the initial spreading of the landslide. So, the underwater landslide has a parabolic cap shape in the longitudinal direction for any m (Fig. 3), but its height and

length depend on the cross-section of the inclined channel. For an inclined plate, the parabolic cap solution was obtained in [24] and its time evolution is quantitatively investigated. This solution is reproduced in the book [24].

The spatial structure of avalanche computed using the equation (3) depends on the cross-section of the channel (Fig. 4). The avalanche edge is quasi beak-shaped in the channel of the cross section ($m = 0.5$) and near lozenge-shaped in the triangular cross section ($m = 1$); further increasing of the incurvature of channel leads to formation of ellipsoidal-shaped gravity flows ($m > 1$).

In general, equation (23) is integrated once

$$\left(\frac{dL}{dt} \right)^2 = p^2 \left[1 - \left(\frac{L_0}{L} \right)^{\frac{m}{m+1}} \right], \quad (24)$$

where

$$p = \sqrt{\frac{2q(m+1)}{mL_0^{\frac{m}{m+1}}}}. \quad (25)$$

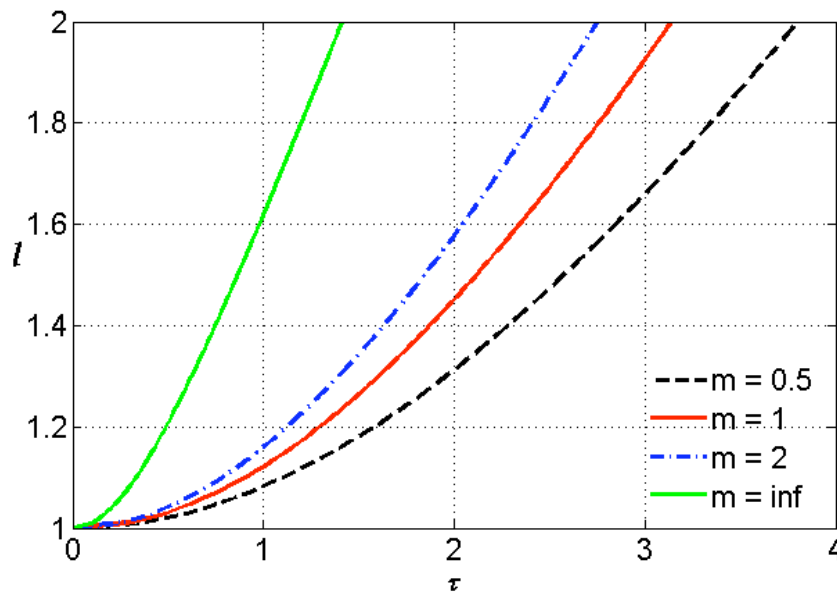


Fig. (5). Time evolution of the length of the parabolic cap landslide in different channels.

here we used the initial condition $L(t = 0) = L_0$. In the variables

$$l = \frac{L}{L_0}, \quad \tau = tp^2 L_0^2. \quad (26)$$

The equations (24) are solved by quadratures

$$\tau = \int_1^l \frac{dx}{\sqrt{1 - x^{-\frac{m}{m+1}}}}, \quad (27)$$

For some values of m (specific cross-section of shape of the canyon) the integral (27) can be evaluated:

$$\tau = \sqrt{1 - \frac{1}{l^{1/3}}} \left(l + \frac{5}{4} l^{2/3} + \frac{15}{8} l^{1/3} \right) + \frac{15}{16} \log \left[2l^{1/3} \left(\sqrt{1 - \frac{1}{l^{1/3}}} + 1 \right) - 1 \right], \quad (m = 0.5),$$

$$\tau = \sqrt{1 - \frac{1}{l^{1/2}}} \left(l + \frac{3}{2} l^{1/2} \right) + \frac{3}{4} \log \left[2 \left(\sqrt{1 - \frac{1}{l^{1/2}}} + 1 \right) \sqrt{l} - 1 \right], \quad (m = 1), \quad (28)$$

$$\tau = \sqrt{1 - \frac{1}{l^{2/3}}} (l + 2l^{1/3}), \quad (m = 2),$$

$$\tau = \sqrt{1 - \frac{1}{l^{3/4}}} \cdot l + 5(-1)^{7/6} \cdot \sqrt{1 - (-1)^{1/3}} \cdot \left[(3 + i\sqrt{3}) E(\eta_1 | k_1) + (-1 - i\sqrt{3}) E(\eta_1 | k_1) \right] \Gamma, \quad (m = 3),$$

$$\tau = l \sqrt{1 - \frac{1}{l^{4/5}}} - \frac{3 \cdot l^{2/5}}{\sqrt{1 - l^{4/5}}} \sqrt{1 - \frac{1}{l^{4/5}}} (E(\eta_2 | k_2) - F(\eta_2 | k_2)), \quad (m = 4),$$

$$\tau = \sqrt{l(l-1)}, \quad (m = \infty),$$

where

$$\Gamma = \frac{(l^{3/4} - 1)^{3/2} l^{-1/4} \sqrt{((-1)^{1/3} l^{1/4} + 1)((-1)^{2/3} l^{1/4} - 1)}}{2 \left(2 \left(1 + (-1)^{2/3} \right) + l^{3/4} + \sqrt{l} + l^{1/4} \right) \sqrt{l^{1/4} - 1}}, \quad (29)$$

and $F(x, k)$ and $E(x, k)$ are the elliptic integrals of first and second order correspondingly

$$F(\eta, k) = \int_0^\eta \frac{d\eta}{\sqrt{1 - \eta^2} \sqrt{1 - k^2 \eta^2}}, \quad E(\eta, k) = \int_0^\eta \frac{\sqrt{1 - k^2 \eta^2} d\eta}{\sqrt{1 - \eta^2}}, \quad (30)$$

and

$$\eta_1 = l^{1/5}, \quad k_1 = -1, \quad \eta_2 = \sqrt{\frac{2l^{1/4}(3 + i\sqrt{3})}{l^{1/4} - 1}}, \quad k_2 = \frac{-i + \sqrt{3}}{i + \sqrt{3}}.$$

Solution (28) for $m = \infty$ (inclined plate) is also given in Pudasaini and Hutter (2007).

The length of the parabolic cap varies in time depending on the cross-section of shape; see Fig. (5) (in dimensionless variables), it increases faster for concave channels (at the same values of spreading coefficient pL_0).

For large times the landslide length varies almost linearly in time

$$l(\tau) \approx \tau - \frac{1+m}{2} \tau^{\frac{1}{1+m}}. \quad (31)$$

Using (22) and (28) the maximal landslide height can be calculated, Fig. (6).

On the large times the landslide height is described by power asymptotic

$$h(t) \sim L^{\frac{m}{m+1}} \sim t^{\frac{m}{m+1}}. \quad (32)$$

For narrow channels the landslide height decreases slowly, in particular for channels of parabolic cross section, $L \sim t^{2/3}$.

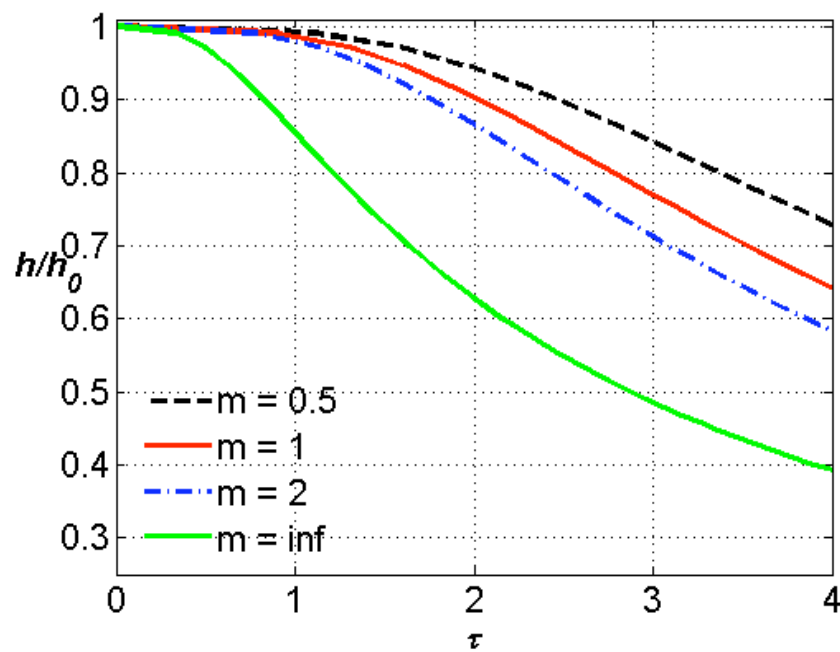


Fig. (6). Time evolution of height of parabolic cap landslide in different channels ($L_0 = 1$).

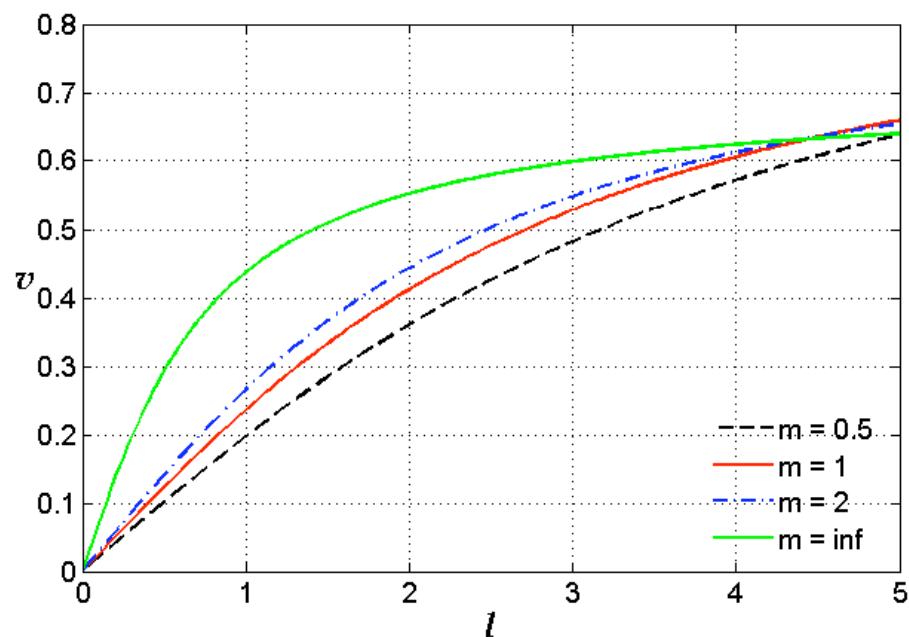


Fig. (7). Temporal distribution of landslide velocity in point $X = L/2$ ($q = 1$, $L_0 = 1$).

Using (19) and (24) we can express velocity distribution in the landslide body through landslide length

$$v = \frac{pX}{L} \sqrt{1 - \left(\frac{L_0}{L}\right)^{\frac{m}{m+1}}}, \quad (33)$$

Velocity increases in both directions from the landslide centre (in modulus) and decreases with time. Fig. (7) demonstrates temporal evolution of the velocity on the landslide for various cross-section shapes.

CONCLUSION

Computing landslide dynamics is an extremely important task in natural hazard assessments, and various physical and

numerical models are developed. Analytical solutions allow studying the physical features of such processes (beginning of movement, direction of the landslide motion, spreading, forming of shocks and so on). Analytical solution for landslide dynamics in underwater canyons approximated by channels of parabolic-like cross-section was obtained here in the framework of the ideal shallow-water flow (Savage-Hutter model). Obtained solutions show that the cross section of the submarine canyon influences significantly on landslide parameters in time. The avalanche edge is quasi beak-shaped in the channel of the cross section ($m = 0.5$) and near lozenge-shaped in the triangular cross section ($m = 1$); it has ellipsoidal shape in the triangular cross section ($m > 1$). Thus, the length of the parabolic cap depends on the cross-section of shape and it

increases faster for concave channels (for the same value of the spreading parameter). Asymptotically, the landslide length varies almost linearly with time. Furthermore, landslide height decreases slowly particularly in channels of parabolic cross section. In whole, for landslides of same thickness nonlinear processes are developed more intensively in narrow canyons.

Analytical solutions given here can be used to test numerical codes of tsunami generation by the submarine landslides.

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REFERENCES

- [1] Heinrich P, Mangeney A, Guibourg S, Roche R. Simulation of water waves generated by a potential debris avalanche in Montserrat, Lesser Antilles. *Geophys Res Lett* 1998; 25: 3697-700.
- [2] Pelinovsky E, Zahibo N, Dunkley P, et al. Tsunami generated by the volcano eruption on July 12-13 2003 at Montserrat, Lesser Antilles. *Sci Tsunami Hazards* 2004; 22: 44-57.
- [3] Zahibo N, Pelinovsky E, Kurkin A, Nikolkina I. In Krishnamurthy RR, Glavovic BC, Kannen A, Green DR, Ramanathan AL, Han Z, Tinti S, Agardy T, Eds. *Tsunami hazard for the French West Indies, Lesser Antilles. Integrated Coastal Zone Management. Research Publ, Singapore* 2008; pp. 517-35.
- [4] Murty TS. Seismic sea waves – tsunamis. *Bull. Fish. Res. Board Canada* 198, Ottawa. 1977; pp. 337.
- [5] Pelinovsky EN. *Tsunami wave hydrodynamics*. Institute Applied Physics Press: Nizhny Novgorod 1996; p 276.
- [6] Levin B, Nosov M. *Physics of Tsunamis*. Springer. 2009; pp. 327.
- [7] Harbitz C. Model simulations of tsunamis generated by the Storegga Slide. *Mar Geol* 1992; 104: 1-21.
- [8] Pelinovsky E, Poplavsky A. Simplified model of tsunami generation by submarine landslides. *Phys Chem Earth* 1996; 21: 13-7.
- [9] Watts P. Tsunami features of solid block underwater landslides. *J Waterway Port Coast Ocean Eng* 2000; 126(3): 144-52.
- [10] Tinti S, Bortolucci E, Chlavitieri C. Tsunami excitation by submarine slides in shallow-water approximation. *Pure Appl Geophys* 2001, 158: 759-97.
- [11] Liu PL-F, Lynett P, Synolakis CE. Analytical solutions for forced long waves on a sloping beach. *J Fluid Mech* 2003; 478: 101-9.
- [12] Pelinovsky E. In Yalciner A, Pelinovsky E, Okal E, Synolakis C, Eds. *Analytical models of tsunami generation by submarine landslides. Submarine landslides and tsunamis. The Netherlands: Kluwer* 2003; Vol. 21: pp. 111-28.
- [13] Sammarco P, Renzi E. Landslide tsunamis propagating along a plane beach. *J Fluid Mech* 2008; 598: 107-119.
- [14] Romstad R, Harbitz CB, Domaas U. A GIS method of rock slide tsunami hazard in all Norwegian lakes and reservoirs. *Nat Hazards Earth Syst Sci* 2009; 9: 353-64.
- [15] Imamura F, Gica EC. Numerical model for tsunami generation due to subaqueous landslide along a coast. *Sci Tsunami Hazards* 1996; 14: 13-28.
- [16] Mangeney A, Heinrich P, Roche R. Analytical solution for testing debris avalanche numerical models. *Pure Appl Geophys* 2000; 157: 1081-96.
- [17] Assier-Rzadkiewicz S, Heinrich P, Sabatier PC, Savoyer B, Bourillet JF. Numerical modeling of a landslide-generated tsunami: the 1979 Nice event. *Pure Appl Geophys* 2000; 157: 1707-27.
- [18] Bouchut F, Mangeney-Castelnau A, Perthame B, Vilotte J-P. A new model of Saint Venant and Savage-Hutter type for gravity driven shallow water flows. *CR Acad Sci Paris Ser I* 2003; 1336: 531-36.
- [19] Fine IV, Rabinovich AB, Thomson RE, Kulikov EA. In: Yalciner A, Pelinovsky E, Okal E, Synolakis C, Eds. *Numerical modeling of tsunami generation by submarine and subaerial landslides. Submarine landslides and tsunamis. The Netherlands: Kluwer* 2003; Vol. 21: pp. 69-88.
- [20] Fine IV, Rabinovich AB, Bornhold BD, Thomson RE, Kulikov EA. The Grand Banks landslide-generated tsunami of November 18, 1929: preliminary analysis and numerical modeling. *Mar Geol* 2005; 215: 45-57.
- [21] Le Friant A, Boudon G, Komorowski J-C, Heinrich P, Semet MP. Potential flank-collapse of Soufriere Volcano, Guadeloupe, Lesser Antilles? Numerical simulation and hazards. *Nat Hazards* 2006; 39: 381-93.
- [22] Fernandez-Feria R. Dam-break flow for arbitrary slopes of the bottom. *J. Eng Math* 2006; 54: 319-31.
- [23] Yalciner AC, Pelinovsky E, Synolakis C, Okal E. *Submarine Landslides and Tsunamis, NATO Science Series. Kluwer Academic Publishers: Netherlands* 2003; p. 240.
- [24] Savage SB, Hutter K. The motion of a finite mass of granular material down a rough incline. *J Fluid Mech* 1989; 199: 177-215.
- [25] Savage SB, Hutter K. Dynamics of avalanches of granular materials from initiation to run-out. Part 1: Analysis. *Acta Mech* 1991; 86: 201-23.
- [26] Pudasaini SP, Hutter K. *Avalanche dynamics: dynamics of rapid flows of dense granular avalanches*. New York: Springer. 2007; p. 602.
- [27] Zahibo N, Pelinovsky E, Golinko V, Osipenko N. Tsunami wave runup on coasts of narrow bays. *Int J Fluid Mech Res* 2006, 33: 106-18.
- [28] Monterey Bay Aquarium Research Institut [homepage on the Internet], updated 2009 April 22, Available from: <http://www.mbari.org/> [cited 2009 May 15].
- [29] Stoker JJ. *Water waves: The mathematical theory with applications*. New York: Interscience 1957; p. 600.
- [30] Carrier GF, Greenspan HP. Water waves of finite amplitude on a sloping beach. *J Fluid Mech* 1958; 4: 97-109.
- [31] Choi BH, Pelinovsky E, Kim DC, Didenkulova I, Woo SB. Two- and three-dimensional computation of solitary wave runup on non-plane beach. *Nonlinear Process Geophys* 2008; 15: 489-502.

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2. Tsunami dans les Petites Antilles: les données et les modélisations numériques

Selon les statistiques, le tsunami se trouve à la 5ème place des catastrophes naturelles (Pelinovsky, 1996). Ainsi, en 2004 lors du tsunami en Indonésie 225 à 300,000 personnes ont été tuées; après avoir traversé l'océan Indien, des vagues de 1.5 m de hauteur ont été enregistrées en Afrique du Sud, qui se trouve environ 8500 km loin du tremblement de terre. Même au Brésil, hauteur des vagues était du même ordre; cet événement est étudié par (Titov et al, 2005; Ioualalen et al, 2007; Candella et al, 2008). Il existe de nombreux cas où les vagues du tsunami avaient une hauteur importante après la traversée de l'océan. Ainsi, le 22 mai 1960, le tsunami survenu sur la côte du Chili, a traversé l'océan Pacifique et il a été enregistré sur la côte Pacifique de l'Extrême-Orient russe 21 à 22 heures après (4,7 m dans les îles Kouriles) (Soloviev et Go, 1974). Le 1 novembre 1755, le tsunami qui s'est produit au Portugal et a détruit la ville de Lisbonne a traversé l'océan Atlantique et a atteint les îles dans mer Caraïbes (Antigua, Dominique et la République dominicaine 3.7 m, Martinique 1.8 m, Saba 6.4 m, Saint-Martin 4.5 m) (NGDC, 2011), ce tsunami a été modélisé par (Baptista et al, 1998, 2003; Mader, 2001; Roger et al, 2010; Yalciner et al., 2010). Plus récemment (le 27 Février 2010) le tsunami, encore une fois généré sur la côte du Chili, a été enregistré par les marégraphes à travers du monde. 22 heures après le tremblement de terre la vague de plus de deux mètres est arrivée sur l'île de Paramushir (nord des îles Kouriles), près d'une heure après l'arrêt d'une alerte au tsunami, ce qui démontre les lacunes des services existants et des systèmes d'alerte aux tsunamis.

Il est important de noter que pour le succès de prévision des tsunamis il est tout d'abord nécessaire d'avoir une bonne connaissance des sources de tsunamis, qui sont étudiés en géophysique. En même temps, l'exemple ci-dessus de vague «en retard» par rapport aux caractéristiques estimées indique la «faiblesse» dans la théorie hydrodynamique du tsunami. Le progrès dans les modèles hydrodynamiques est impossible sans l'utilisation de ces données de tsunamis historiques de vérifier la théorie. Dans ce chapitre, nous aborderons le problème du calcul de la propagation des vagues de tsunami générées par différents sources (séismes, glissements de terrain sous-marine et écoulement volcaniques) sur l'exemple du tsunami dans les Caraïbes. Il faut noter qu'il existe une prévision selon laquelle, dans un avenir proche, il peut arriver un très grand tsunami dans les Caraïbes, et les Caraïbes peuvent être considérées comme un terrain d'essai pour le développement de méthodes de prévision des tsunamis.

Les résultats présentés dans le deuxième chapitre de cette thèse, sont publiés sous:

NIKOLKINA I., ZAHIBO N., PELINOVSKY E. Tsunami in Guadeloupe (Caribbean Sea). *Open Oceanogr. J.*, 2010, vol. 4, p. 44-49.

PELIŇOVSKY E., ZAHİBO N., DUNKLEY P., EDMONDS M., HERD R., TALİPOVA T., KOZELKOV A., NİKOLKİNA I. Tsunami generated by the volcano eruption on July 12-13, 2003 at Montserrat, Lesser Antilles. *Sci Tsunami Hazards*, 2004, vol. 22, n°1, p. 44-57.

ZAHİBO N., PELIŇOVSKY E., KURKİN A., NİKOLKİNA I. Tsunami hazard for the French West Indies, Lesser Antilles. *Integrated Coastal Zone Management (ICZM)* / ed. KRİSHNAMURTHY R. R., GLAVOVİC B. C., KANNEN A., GREEN D.R., RAMANATHAN AI., HAN Z., TİNTİ S., AGARDY T., Research publishing, 2008, p. 515-535.

2.1 Récurrence et statistiques du tsunami dans les Antilles Françaises

Les données statistiques sur la récurrence des tsunamis dans les Antilles Françaises ont été étudiées sur la base des données historiques (Accary et Roger, 2010; O'Loughlin et Lander, 2003; Zahibo et Pelinovsky, 2001; Zahibo et al, 2003a, 2005, 2006). Le nombre exact d'événements valides pour la période 1498-2010 est égal à 18, dont 9 d'origine tectonique, 7 d'origine volcanique et 2 télé-tsunami; la hauteur des vagues sur les Antilles Françaises est représentée dans la Fig. 2.1. Comme dans d'autres zones géographiques, la hauteur des tsunamis d'origine tectonique est mal liée à la magnitude du tremblement de terre, mais la tendance observée est à la corrélation entre ces deux paramètres (Fig. 2.2).

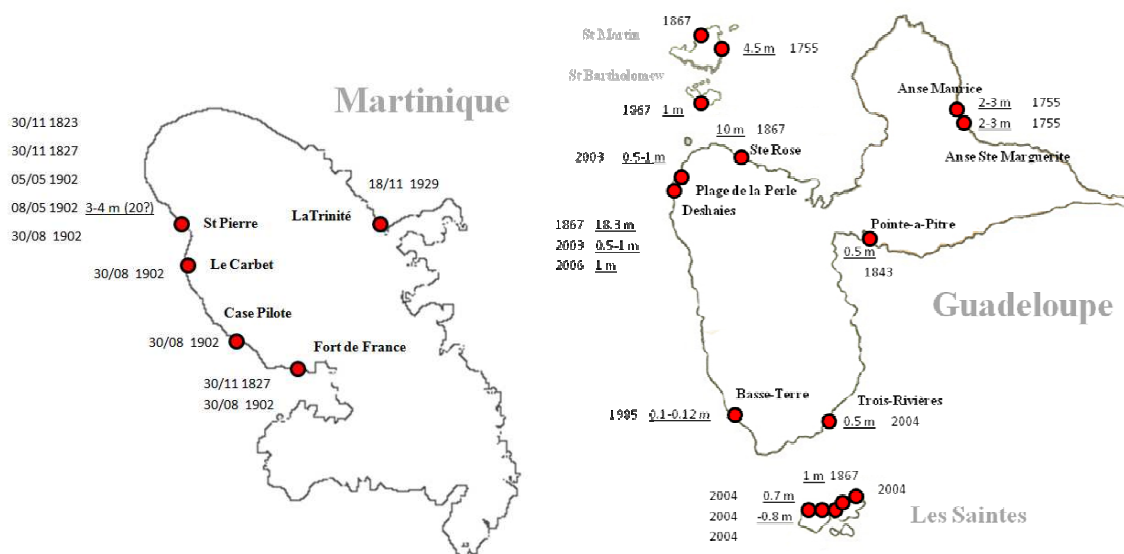


Fig.2.1. Hauteur des vagues de tsunami sur les Antilles Françaises

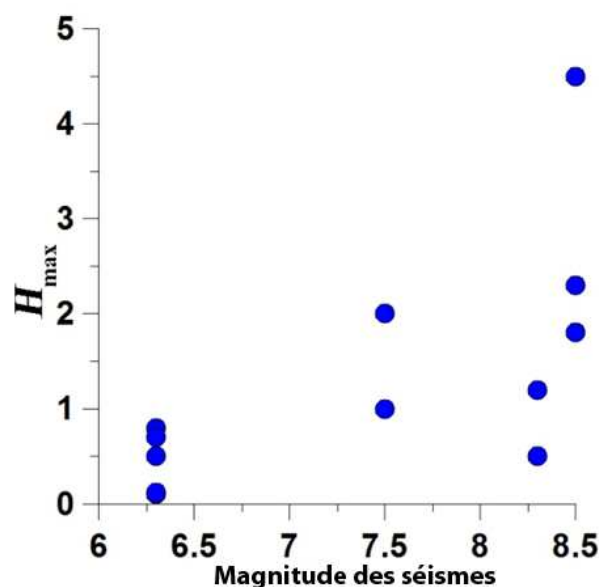


Fig. 2.2. Hauteur des tsunamis en fonction de la magnitude des séismes

2.2 Tsunamis générés par une éruption volcanique de Soufrière Hills (Montserrat) en 2003: enquête de terrain et modélisation

Le 18 Juillet 1995, le volcan de Soufrière Hills (Montserrat, dans la mer des Caraïbes) a repris son activité après une longue pause, et l'éruption continue jusqu'à présent. La moitié de l'île est devenue inhabitable (MVO, 2011). Ainsi, l'effondrement de la coupole, qui se produisit dans la nuit du 12 au 13 Juillet 2003, fut le plus fort dans toute l'histoire des éruptions de l'île. Au total, environ 120 millions de m³ de matériaux et de débris se sont effondrés durant cette nuit. Cette éruption a causé des écoulements pyroclastiques dans la rivière Tar, qui ont changé la côte (Fig. 2.3a) (Herd et al, 2005; Ripepe et al, 2009). Durant l'automne 2003, l'auteur de la thèse a été impliqué dans des études terrain pour détecter les traces du tsunami en Guadeloupe. De telles traces ont été trouvés dans la commune de Deshaies (Fig. 2.3b), où la hauteur des vagues a atteint 1 mètre.



a)



b)

Fig. 2.3. Les écoulements pyroclastiques atteignant la mer dans la vallée de Tar River le 12 Juillet 2003 (a) et une enquête de terrain dans le port de Deshaies en Guadeloupe (b)

Avant de planifier la recherche de traces du tsunami, des simulations numériques de propagation des vagues en eau peu profonde ont été produites. Cette théorie est souvent utilisée pour modéliser les tsunamis dans le territoire français (Assier-Rzadkiewicz et al., 2000; Pelinovsky et al, 2002; Zahibo et al., 2003b; Sladen et al., 2007; Roger et al., 2010; Yalciner et al, 2010).

$$\begin{aligned} \frac{\partial M}{\partial t} + \frac{\partial}{\partial x} \left(\frac{M^2}{D} \right) + \frac{\partial}{\partial y} \left(\frac{MN}{D} \right) + gD \frac{\partial \eta}{\partial x} + \frac{k}{2D^2} M \sqrt{M^2 + N^2} &= 0 \\ \frac{\partial N}{\partial t} + \frac{\partial}{\partial x} \left(\frac{MN}{D} \right) + \frac{\partial}{\partial y} \left(\frac{N^2}{D} \right) + gD \frac{\partial \eta}{\partial y} + \frac{k}{2D^2} N \sqrt{M^2 + N^2} &= 0 \\ \frac{\partial \eta}{\partial t} + \frac{\partial M}{\partial x} + \frac{\partial N}{\partial y} &= 0 \end{aligned} \quad (2.1)$$

où η est la surface de la mer, les composants M et N sont les débits; $D = h(x, y) + \eta$ est la profondeur total du bassin; $h(x, y)$ est la profondeur non perturbée et $k = 0,0025$ est le coefficient de frottement.

Du fait de l'absence d'informations détaillées sur les effets de tsunami, des estimations sur les écoulements pyroclastiques ont été utilisées pour modéliser la source de Soufrière Hills (proposé § 1.1) ainsi que des calculs semblables d'autres auteurs dans des situations similaires: nous avons considéré une zone de 2 km de diamètre avec un déplacement vertical de 10 mètres. Puisque l'objectif principal des calculs était de déterminer l'endroit le plus probable de la manifestation de tsunamis, on a pu appliquer les conditions aux limites telles que les murs verticaux dans les «points» de la mer. Aux frontières ouvertes nous avons utilisé les conditions de radiation de Sommerfeld pour les systèmes hyperboliques de la forme :

$$\frac{\partial \eta}{\partial n} + \sqrt{gh} \frac{\partial \eta}{\partial n} = 0, \quad (2.2)$$

où n est la normale à la frontière du domaine de calcul. Les calculs ont été effectués dans le cadre du code international TUNAMI. Le diagramme de rayonnement des tsunamis et de la distribution des hauteurs de vagues le long de la côte de Guadeloupe sont présentés sur la Fig. 2.4.

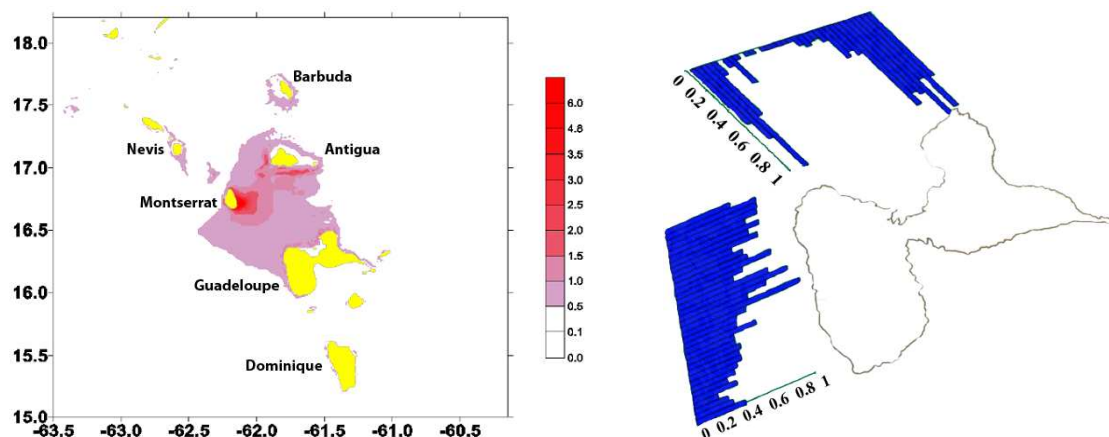


Fig. 2.4. Distribution des amplitudes maximales du tsunami après calcul (en mètres)

Il est intéressant de noter que l'étude de terrain a confirmé les résultats des calculs et des traces des vagues ont été trouvées dans les endroits où l'on avait prédit la grande amplitude.

Nous pouvons tirer de l'analyse du tsunami de 2003 une conclusion importante : un tsunami similaire survenu en 1997 n'est pas officiellement enregistré en Guadeloupe (mais enregistré pour Montserrat) et devrait être inclus dans le catalogue des tsunamis en Guadeloupe en tant que événement probable.

2.3. Simulation des tsunamis d'origine sismique avec des applications pour les événements probables dans la zone Martinique

Dans ce paragraphe, nous présentons les résultats de la modélisation des tsunamis d'origine sismique générés à des profondeurs différentes. Comme modèle de la source du tremblement de terre, nous avons choisi le plus fort séisme de dernières années: celui-ci survint près de la Martinique le 29 Novembre 2007, et a atteint une de magnitude 7.4. Ce tremblement de terre n'a pas provoqué de dégâts importants en raison de la grande profondeur de la rupture (147 km). La simulation numérique a été réalisée dans le cadre de la théorie de l'eau peu profonde, Éqs. (2.1). Le déplacement initial est calculé à l'aide du modèle d'Okada (Okada, 1985). Les résultats des calculs montrent que la configuration de la source dépend fortement de la profondeur du séisme (Fig. 2.5), et que la hauteur décroît exponentiellement avec la profondeur du tremblement de terre (Fig. 2.6). Si le séisme était survenu à une profondeur de 20-40 km, ses conséquences auraient été désastreuses.

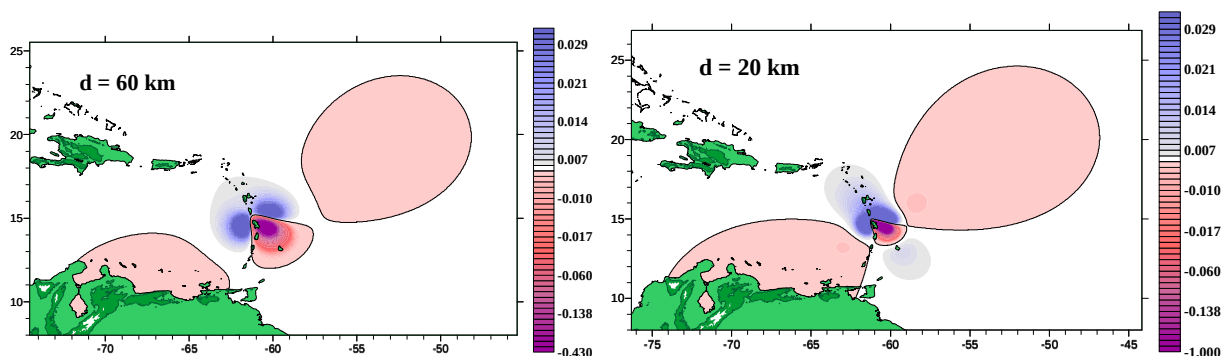


Fig. 2.5. Déplacement d'eau dans la source du tsunami (l'échelle est en mètres), d - la profondeur du séisme

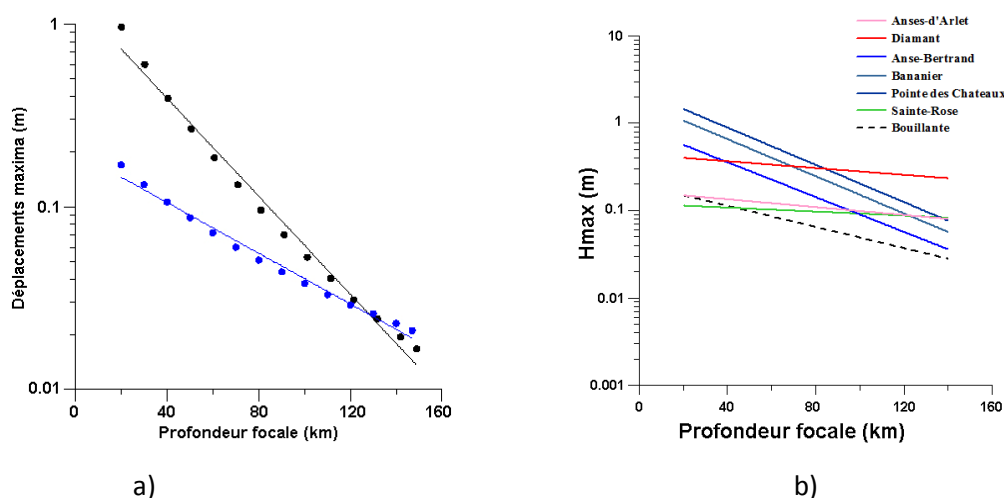


Fig. 2.6. Déplacements maxima positifs (en bleu) et négatifs (noire) en fonction de la profondeur focale du séisme (a); la hauteur maximale des vagues près des côtes en fonction de la profondeur focale (b)

2.4. Modélisation de tsunami probable à partir de «modèles» de sources: cas de la Guadeloupe

Le catalogue de sismicité (ISC-USGS) révèle que la zone de subduction des Petites Antilles est une source potentielle de tremblement de terre. Les séismes intraplaque se produisent le long d'une structure tectonique qui est soit transverse, soit parallèle à l'arc volcanique. Les structures transverses à l'arc se révèlent être dans l'ensemble des failles normales qui présentent une sismicité superficielle (10-30 km). Leur magnitude souvent faible (< 4.5) peut cependant atteindre des valeurs supérieures à 7 (08/10/1974 Ms = 7.4).

Les structures parallèles à l'arc sont caractérisées par des mouvements décrochants. L'activité sismogénique sur ces structures est là encore superficielle et peut atteindre des magnitudes proches de 7 également. Dans la région proche de la Guadeloupe, d'Antigua à la Dominique, ces structures tectoniques ont été cartographiées. La seconde source de séismes provient de l'interface de subduction. Cette interface est supposée avoir généré le séisme de 1843 dont la magnitude maximale estimée serait de 8. Les caractéristiques tectoniques de cette interface sont encore mal connues et font l'objet de nombreuses études auxquelles participe activement le groupe de géologie de l'université des Antilles et de la Guyane (e.g. Mission océanographique INSU-IPGP Sismantilles). Cependant la NOAA (National Oceanic and Atmospheric Administration, Seattle, USA) à la suite d'une campagne géologique mondiale vient de proposer des modèles sources pour toutes les zones de subduction du monde (<http://sift.pmel.noaa.gov>), y compris celles du bassin caribéen. Ces modèles sources (Fig. 2.7) proposent une estimation de plusieurs points de déclenchement des séismes de subduction en fonction de la segmentation de la plaque plongeante et de la plaque chevauchante. Chaque zone de déclenchement de séisme représentée par des rectangles blanc sur la Fig. 2.7 contient de surcroît toutes les fonctions caractéristiques du tremblement de terre potentiel qu'elle pourrait générer. Ce modèle source est de plus associé à un modèle de simulation numérique de propagation et d'inondation des tsunamis (ComMIT/MOST) qui permet donc de générer tous les scénarios possibles de tremblement de terre dans la zone de subduction et d'étudier le tsunami induit.

Il s'agit dans cette action de modéliser un tsunami potentiel généré dans la zone de subduction par un tremblement de terre de très forte magnitude ($M = 8$) et d'estimer son impact sur les côtes de la Guadeloupe. Ce modèle numérique MOST, basé sur les équations de la théorie de l'eau peu profonde, est doté d'une interface ComMIT qui permet de générer la vague initiale en activant un ou plusieurs blocs quelconques dans la zone de subduction et d'étudier ensuite la propagation des vagues en direction des côtes. La précision des hauteurs d'inondation (runup) récoltées sur les côtes dépend de la précision de la résolution bathymétrique/topographique des zones côtières. La méthode utilise 3 grilles imbriquées (A, B et C) avec des résolutions qui s'affinent quand on approche des côtes. La résolution bathymétrique/topographique pour les côtes de la Guadeloupe est de 900 m (GEBCO).

A l'aide du logiciel ComMIT on a fait tourner les simulations pour 78 sources ; les sources de code «atszy39» à «atszy53», de code «atszz39» à «atszz53», de code «atsza30» à «atsza53» et de code «atszb30» à «atszb53» (Fig. 2.9). A chaque source correspond un modèle de tsunami différent.

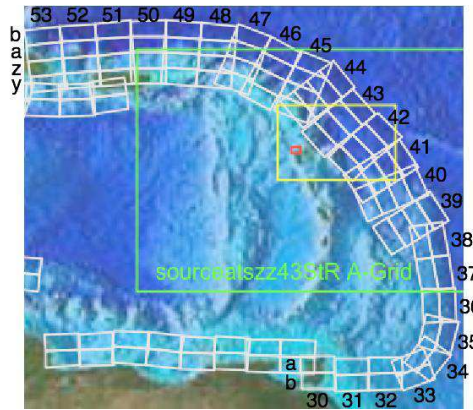


Fig. 2.7. Modèles sources dans les zones de subduction du bassin caribéen utilisées pour l'étude de tsunamis potentiels en Guadeloupe. Les rectangles blancs représentent les zones de déclenchement des séismes

Afin de mesurer la déformation de la surface libre calculée pour chacune des simulations, on a disposé des marégraphes numériques (points de mesure) le long de la côte de la Guadeloupe.

A l'aide du logiciel ComMIT, on a enregistré en chacun de ces marégraphes numériques des séries de points de données mesurés à des temps successifs et espacés d'un intervalle de temps uniforme, représentant l'élévation de la surface libre en ce point de mesure durant le tsunami. On a précédemment estimé la vitesse de propagation d'un tsunami à environ 500 km/h en eau profonde dans la mer des Caraïbes. L'ensemble des sources considérées se situant dans un rayon de 23 km autour de la Guadeloupe, on peut estimer le temps d'arrivée de la première vague du tsunami à environ 30 minutes. Les reliefs des fonds marins et la diminution de la vitesse de propagation à l'approche de la côte peuvent faire varier cette estimation de façon notable. On a décidé, pour chaque simulation, d'une durée d'enregistrement de deux heures après l'évènement déclencheur du séisme. On obtient alors des marégrammes en chaque point de mesure pour chacune des simulations. On a ensuite décidé pour cette étude globale de se focaliser sur les régions de Pointe-à-Pitre et Basse-Terre, présentant les densités de population les plus élevées, et sur la région de Sainte-Rose.

On a déduit des marégrammes obtenus (Fig. 2.8) les caractéristiques (temps d'arrivée de la première vague, hauteur de vague maximale) de chaque tsunami (Tableau 2.1). On a pu observer que les temps d'arrivée des premières vagues diminuent lorsque les sources sont proches de la Guadeloupe. Pour la région de Pointe-à-Pitre ils sont d'environ 20 minutes pour les sources de code «atszy41» et «atszy42» et augmentent jusqu'à atteindre environ 30 minutes pour les sources de code «atszb41» à «atszb46». Pour la région de Basse-Terre les temps d'arrivée de la première vague varient entre 15 minutes pour les sources de code «atszy44» et «atszy45» jusqu'à atteindre 40 minutes pour les sources de code «atszb44» et «atszb45». Pour la région de Sainte-Rose la zone étudiée est si proche de la source que le temps d'arrivée de la première vague est de moins de 5 minutes pour les sources de code «atszy43» et «atszy44» puis augmente jusqu'à être d'environ 45 minutes pour les sources de code «atszb40» et «atszb48».

Ainsi plus les sources de génération de tsunami sont éloignées de la zone étudiée, plus le temps d'arrivée de la première vague est important.

Lorsque la source de génération de tsunami est de code compris entre 39 et 41, la région de Pointe-à-Pitre est celle où les vagues sont les plus rapides à arriver. Lorsque la source de génération de tsunami est de code compris entre 42 et 48, la région de Sainte-Rose est celle où les vagues sont plus rapides à arriver.

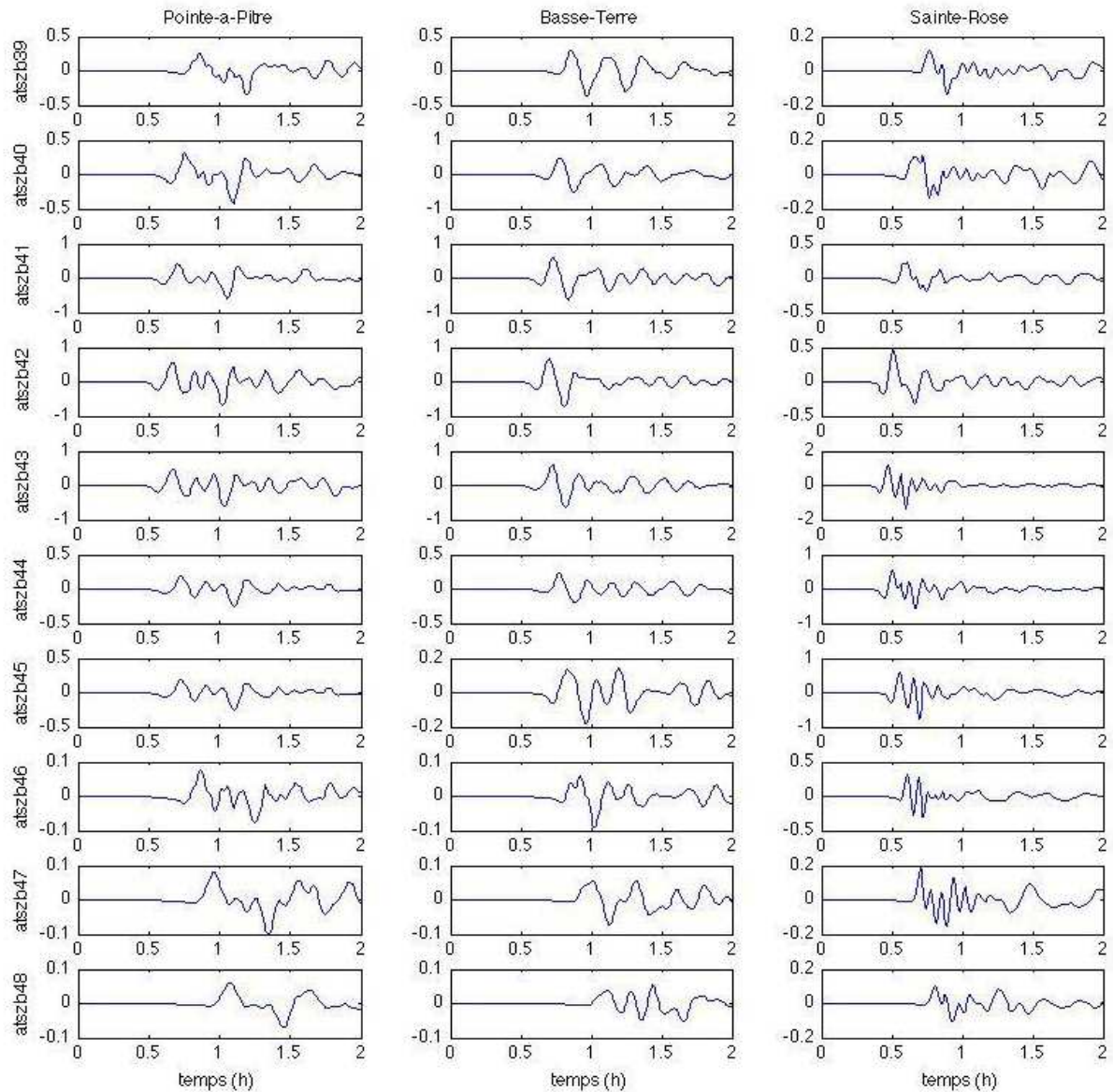


Fig. 2.8. Exemple de marégrammes utilisés lors de l'étude globale (ici pour les sources de code «atszb39» à «atszb48»)

Les hauteurs de vagues observées sont comprises entre 0.1 m dans la région de Sainte-Rose pour la source de code «atszy39» et environ 2 m dans la région de Sainte-Rose pour les sources de code «atszz43» et «atszb43». Lorsque la source de génération de tsunami est de code compris entre 39 et 42, les hauteurs de vagues les plus importantes sont observées dans la région de Pointe-à-Pitre. Lorsque la source de génération de tsunami est de code compris entre 43 et 48, les hauteurs de vagues les plus importantes sont observées dans la région de Sainte-Rose.

Tableau 2.1. Caractéristiques des tsunamis générés

Sources	Position		Temps d'arrivée de la 1ère vague (minutes)	Hauteur maximale (m)
	Lat (deg)	Long (deg)		
atszy39	300.5832	13.9171	28 (Sainte-Rose)	0.5 (Baie-Mahault)
atszy40	299.9869	14.8646	9 (Pointe des Châteaux)	0.8 (Baie-Mahault)
atszy41	299.7771	15.3177	8 (Pointe des Châteaux)	1.3 (Trois-Rivières)
atszy42	299.4355	15.8616	8 (Saint-Francois)	1.8 (Trois-Rivières)
atszy43	298.7182	16.5177	2 (Saint-Francois)	3.2 (Pointe des Châteaux)
atszy44	298.1349	17.3108	2 (Pointe des Châteaux)	0.7 (Pointe des Châteaux)
atszy45	298.0963	17.2569	4 (Deshaies)	0.7 (Pointe des Châteaux)
atszy46	297.1507	17.6477	16 (Vieux-Fort)	0.4 (Sainte-Rose)
atszy47	296.5230	17.9936	16 (Basse-Terre)	0.4 (Baillif)
atszy48	296.0357	18.0465	14 (Sainte-Rose)	0.5 (Saint-Francois)
atszz39	300.9585	14.1543	26 (Vieux-Fort)	0.4 (Baie-Mahault)
atszz40	300.3988	15.0395	14 (Pointe des Châteaux)	1.2 (Trois-Rivières)
atszz41	300.1527	15.5582	11 (Pointe des Châteaux)	1.4 (Sainte-Anne)
atszz42	299.7666	16.1575	11 (Gosier)	2.7 (Sainte-Anne)
atszz43	299.0596	16.8040	3 (Saint-Francois)	3.5 (Deshaies)
atszz44	298.4898	17.5834	3 (Pointe des Châteaux)	1.3 (Deshaies)
atszz45	298.2740	17.6567	11 (Sainte-Rose)	1.4 (Baillif)
atszz46	297.3648	18.0313	20 (Vieux-Fort)	0.5 (Pointe des Châteaux)
atszz47	296.6795	18.3890	13 (Sainte-Anne)	0.8 (Baie-Mahault)
atszz48	296.0784	18.4667	19 (Pointe des Châteaux)	0.2 (Pointe des Châteaux)
atsza39	301.3333	14.3915	25 (Gosier)	0.6 (Pointe des Châteaux)
atsza40	300.8101	15.2143	16 (Pointe des Châteaux)	1.2 (Sainte-Anne)
atsza41	300.5277	15.7987	13 (Pointe des Châteaux)	1.7 (Trois-Rivières)
atsza42	300.0971	16.4535	15 (Sainte-Rose)	2.9 (Sainte-Anne)
atsza43	299.4004	17.0903	6 (Saint-Francois)	4.2 (Deshaies)
atsza44	298.8441	17.8560	7 (Pointe des Châteaux)	2.3 (Deshaies)
atsza45	298.4509	18.0566	20 (Sainte-Rose)	2.3 (Baillif)
atsza46	297.5783	18.4149	27 (Basse-Terre)	1 (Gosier)
atsza47	296.8351	18.7844	28 (Deshaies)	0.3 (Deshaies)
atsza48	296.1200	18.8870	19 (Pointe des Châteaux)	0.3 (Deshaies)
atszb39	301.7087	14.6287	28 (Goyave)	0.6 (Sainte-Anne)
atszb40	301.2219	15.3892	20 (Pointe des Châteaux)	1.2 (Sainte-Anne)
atszb41	300.9034	16.0392	16 (Pointe des Châteaux)	1.6 (Sainte-Anne)
atszb42	300.4284	16.7494	21 (Sainte-Rose)	2 (Sainte-Anne)
atszb43	299.7420	17.3766	9 (Saint-Francois)	3.6 (Deshaies)
atszb44	299.1992	18.1286	11 (Pointe des Châteaux)	1.8 (Deshaies)
atszb45	298.6284	18.4564	26 (Sainte-Rose)	2.3 (Baillif)
atszb46	297.7925	18.7985	20 (Pointe des Châteaux)	0.7 (Deshaies)
atszb47	296.9913	19.1798	29 (Sainte-Rose)	0.4 (Deshaies)
atszb48	298.1618	19.3072	21 (Pointe des Châteaux)	0.2 (Deshaies)

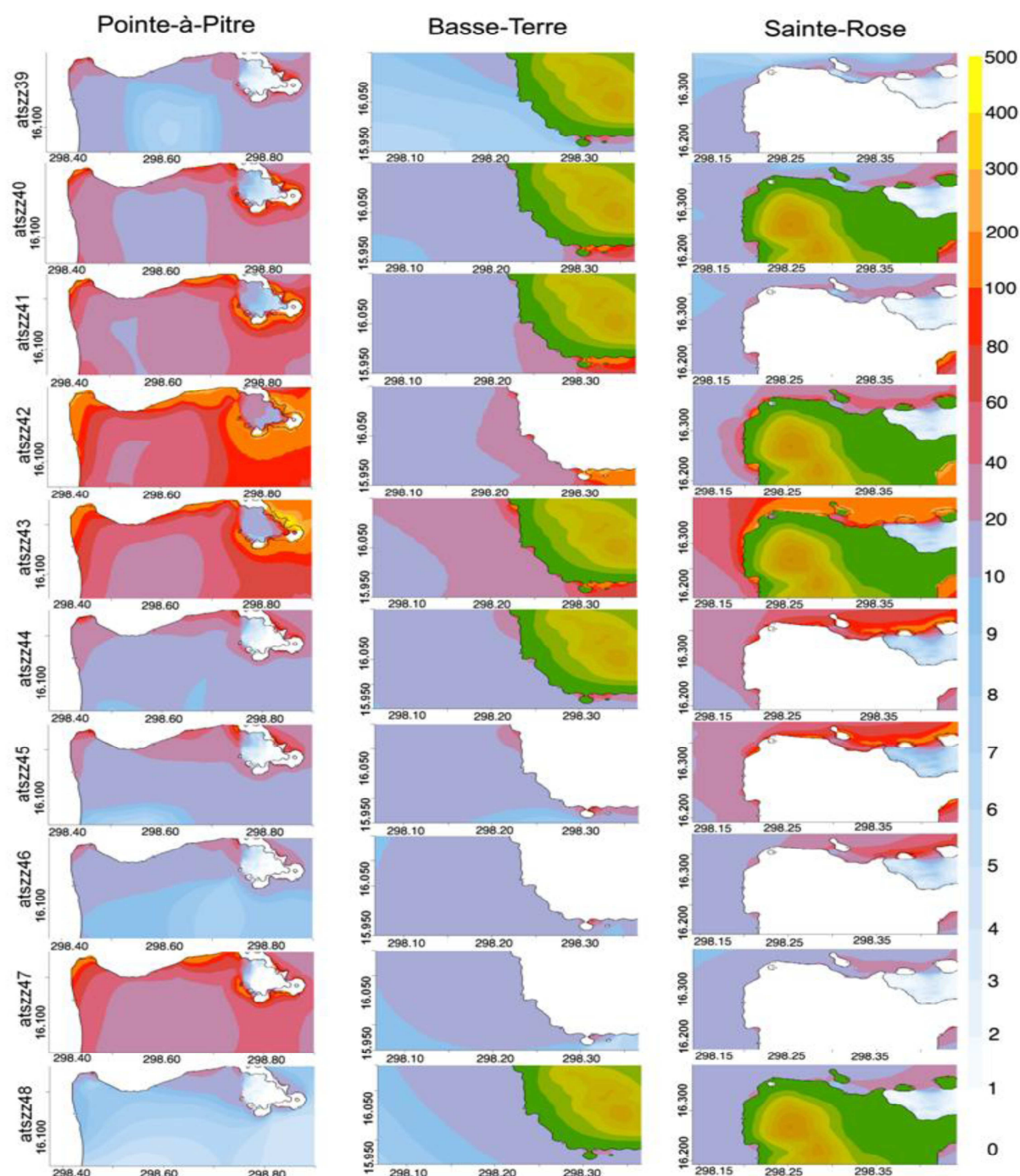


Fig. 2.9. Exemple de cartes d'inondation utilisées lors de l'étude globale (ici pour des tsunamis générés par les sources de code «atszz39» à «atszz48»), les hauteurs de vague sont en centimètres

D'une manière générale, on a constaté que plus la source de génération de tsunami est proche de la zone atteinte, plus la hauteur de vague est importante.

A l'aide du logiciel ComMIT on a ensuite visualisé les zones d'inondation pour les régions de Pointe-à-Pitre, Basse-Terre et Sainte-Rose (Fig. 2.9). On observe que les principales zones inondées se situent à Saint-François (distance de pénétration de la vague : environ 1 km), à la Pointe des Châteaux (distance de pénétration de la vague : environ 1.5 à 2 km), à Pointe-à-Pitre (distance de pénétration

de la vague : environ 1 à 1.5 km), à Vieux-Fort (distance de pénétration de la vague : environ 0.5 à 1 km), à Deshaies (distance de pénétration de la vague : environ 0.5 à 1 km) et à Sainte-Rose (distance de pénétration de la vague : environ 2 à 2.5 km).

Pour l'ensemble des simulations, les hauteurs de vagues atteintes sont comprises entre 0.4 et 5 m dans la région du sud de la Grande-Terre, entre 0.6 et 3 m dans la région de Basse-Terre et entre 0.2 et 4 m dans la région de Sainte-Rose. La hauteur de vague maximale est de 5.2 m. Elle est enregistrée à la Pointe-des-Châteaux pour le tsunami généré par la source de code «atsza43».

Les zones les plus dangereuses car les plus susceptibles d'être inondées sont donc la Pointe des Châteaux, Pointe-à-Pitre, Deshaies et Sainte-Rose.

On a de plus remarqué que les sources provoquant le plus d'inondations sont les sources de code 41 à 44 ; ce sont également les sources les plus proches de la Guadeloupe. Par ailleurs les zones les plus inondées, en particulier la Pointe des Châteaux et Sainte-Rose, sont les plus proches de ces sources. La zone de génération de tsunamis la plus dangereuse est donc la zone comprenant les sources de code 41 à 44.

Le site le plus touché par les inondations est celui de la Pointe des Châteaux. Cependant, bien qu'étant une zone d'inondation, la modélisation de cette zone par le logiciel n'étant pas exactement fidèle à la réalité, l'étendue précise de la zone touchée ne peut pas être exacte.

Tsunami in Guadeloupe (Caribbean Sea)

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Abstract: Detailed analysis of historical data of tsunamis for 1498-2008 period in Guadeloupe (French West Indies, Caribbean sea), collected in different books, papers and sites is presented. Concerning validity scale 10 events are selected as true and almost true, 6 tsunami events were generated by underwater earthquakes; 3 events – by volcano eruptions, and one was a teletsunami. The geographical and temporal distributions of tsunami events are studied.

Keywords: Tsunami, Caribbean Sea, guadeloupe, statistical analysis, natural hazards, ocean waves.

1. INTRODUCTION

Tsunami danger in the Caribbean and especially in Guadeloupe has been widely discussed in the literature. The first attempt to synchronize historical data was produced in [1], results of similar research can be found in [2], databases [3, 4]. Tsunami danger for the Lesser Antilles was studied in [5, 6].

Historical data was collected till 1997 in [1] who suggested that 91 reported events might have been tsunami; he also proposed a validity scale and suggested 27 of 91 reported events to be verified tsunamis; and the rest 9 – to be very likely true tsunamis.

The return period of Caribbean tsunamis is equal to 3 years. Tsunami travel time calculation for the Caribbean Sea was prepared in [7] who estimated time for a complete crossing of the Caribbean as 3.2 hrs laterally and 1.5 hrs meridionally. Two historical events in the Caribbean (the 1918 Puerto Rico tsunami and the 1867 Virgin Island tsunami), induced by earthquakes, were simulated in [8, 9]. According to [10] who modeled the propagation of the Lisbon tsunami (01/11/1755), the wave amplitude east of Saba (Lesser Antilles) reached 5 m, in fact, 7 m wave was observed. Numerical simulation of the 26/12/1997 debris avalanche in Montserrat (Lesser Antilles) that induced tsunami waves up to 3 m height was performed in [11]. The first attempt to evaluate tsunami risk in the Caribbean Sea using the numerical simulation of wave propagation from various prognostic sources was performed in [12]; as a result zones of low tsunami risk were selected.

The goal of the present paper is to revise historical tsunami data for the Guadeloupe Department that includes Guadeloupe and small neighbor islands using all available publications and results of numerical simulations. In this paper only true and almost true events are selected, and this

classification corresponds to the validity 4 (“Definite tsunami. Tsunami did occur. Data corroborated or deemed valid information”) and 3 (“Probable tsunami. Likely or almost true tsunami occurrence. Reliable observations for corroboration are few”) according to definition given in [2]. Totally, ten events are selected as true and almost true, the observed runup locations for all events are given in Fig. (1), and the corresponding runup data are presented in Table 1. Six events were generated by underwater earthquakes; three events – by the volcano eruptions, one is a teletsunami.

The geographical and temporal distributions of tsunami events are studied.

2. TSUNAMI OF THE TECTONIC ORIGIN

Generally, the subduction zone where the French West Indies are situated has the potential to create tsunami genetic earthquakes with magnitude up to $M_s = 8.3$ (1843, Guadeloupe). Totally, 6 tectonic earthquakes are selected in our analysis. The list of the events is given below in the chronicle order.

April 06, 1690

“At St. Thomas, after the earthquake the sea receded so much that one could walk about 9 to 10 fathoms [16.5-18.5 m] into the sea and pick out fishes on dry land. Martinique, Barbados, and St. Lucia were less strongly affected” [2]. “The earthquake caused the collapse of stone buildings at Nevis, where landslides generated on volcanic Nevis Peak caused the sea to withdraw 201 m from Charleston, before returning in two minutes. **Guadeloupe** also incurred much damage”. Also at “a rocky islet called Rockadunda (Redonda Rock Island), a great part of it split and turned into the sea”. Tsunami intensity (Soloviev-Imamura scale) is estimated as 1-2 (0.5 – 3 m) for the Lesser Antilles and US Virgin Islands. The event is estimated in literature as true. The source of this tsunami was an earthquake (magnitude $M \sim 8$, focal depth 33 km). The exact date is unknown. The coordinates of the earthquake epicenter are given in literature: 17.2°N

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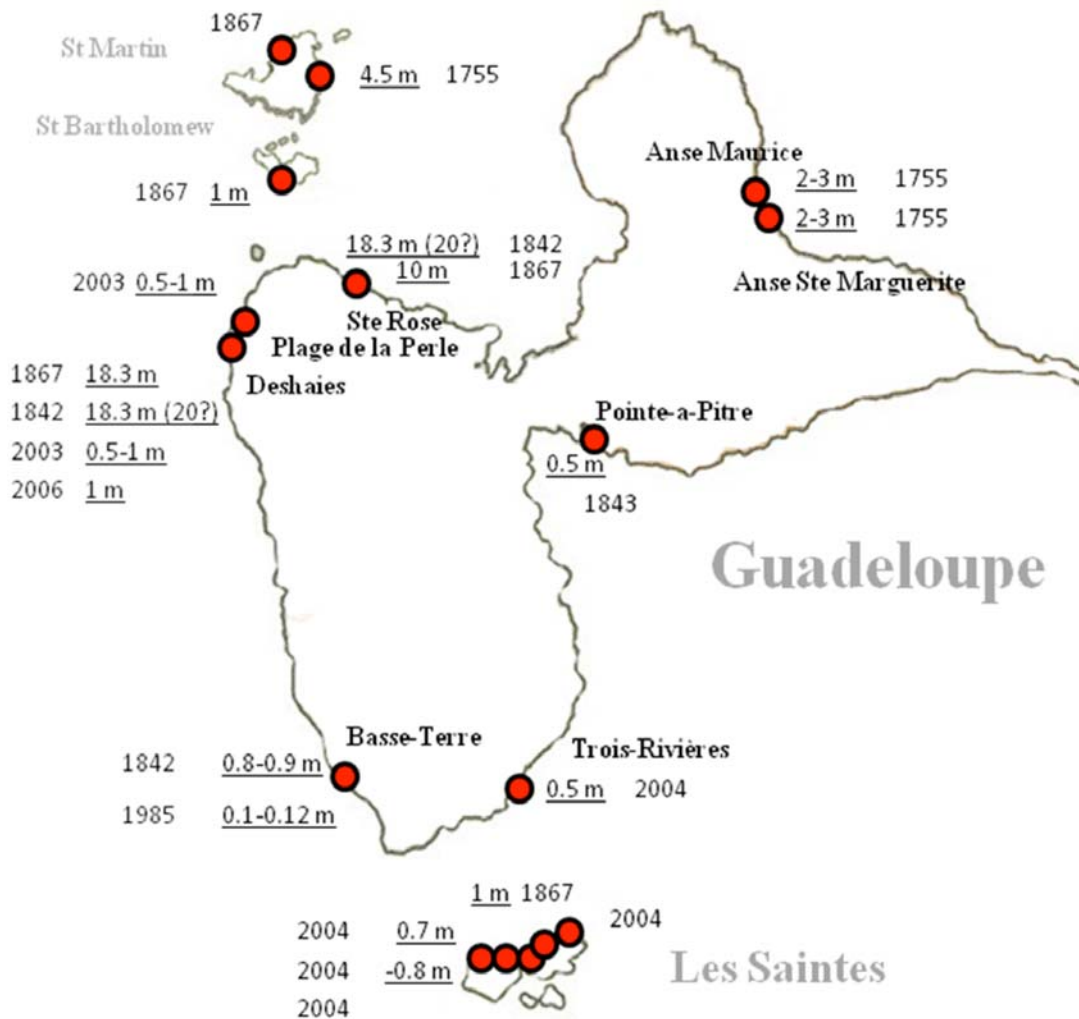


Fig. (1). Runup locations in Guadeloupe Department, St Martin and St Bartholomew.

62.5°W for April 5 [3], 17.5°N 61.5°W for April 6 [4] and April 16 [3].

This tectonic tsunami is considered as true, although it was not recorded in Guadeloupe. Taking into account the large area of the earthquake manifestation, the tsunami in **Guadeloupe** can be estimated as an **almost true (3)** tsunami.

November 30, 1827

"Sequake effects or a tsunami may have disturbed the ships 100 nautical leagues [564 km] west of **Martinique**. All the vessels near Pointe-à-Pitre (**Guadeloupe**) experienced the shock" [2]. Tsunami was caused by an earthquake: magnitude 6.5; epicenter coordinates 14.5°N 61°W [3] or 14.4°N 61°W [4]. Although the magnitude of the earthquake that caused the 1867 tsunami was rather low (6.5), it was higher than the magnitude of the 2004 earthquake in Guadeloupe (6.3) that caused a definite tsunami 50 km from the source. Although tsunami validity given in literature is rather low, we believe that the 1827 tsunami could be estimated as **almost true (3)** for Guadeloupe.

February 08, 1843

According to [2] "columns of water 100 feet high [30.5 m] and several feet thick [1-1.2 m] were observed to be

ejected from fissures in the ground of **Guadeloupe**". The source of this tsunami was the "disastrous earthquake (magnitude 8.3; focal depth 33 km; epicenter coordinates 16.5°N 62.2°W) that struck **Guadeloupe**, leaving Pointe-a-Pitre in ruins" and damaged neighbor islands. Furthermore, in sight of the Redonda Rock Island "a mighty chunk of the rocky mass broke off and fell into the sea, which resulted in a surging rise of the ocean". Tsunami validity is estimated as a **true (4)** event in literature.

November 18, 1867

The huge tsunami of intensity 3 (6 m) crossed all the Lesser Antilles. According to [2] "all of Fond-du-Cure, **Îles des Saintes** was submerged; the water invaded the houses to a height of 1 m. In Basse-Terre, **Guadeloupe** the sea swelled without any outward sign: no waves, no backwash. It rose 1 m in the span of 5 minutes, floating boats tied on the bank. It then dropped in the same span of time, retiring 8 to 10 m. The same phenomenon was produced again. The difference in the extremes of the water levels was not less than 2 m. The wave at Pointe-à-Pitre, **Guadeloupe** was hardly more than a swell. At Sainte-Rose, **Guadeloupe** the ocean suddenly withdrew more than 100 m from the shore. This retreat was preceded by slight oscillations from the earthquake,

Table 1. Runup Records in Guadeloupe and Neighbor Islands

N	Date	Location		Runup, m (or citation)
1	April 06, 1690	Guadeloupe		4.5
2	November 1, 1755	St. Martin		
		Guadeloupe	Anse Maurice	2.3
			Anse St Marguerite	
3	November 30, 1827	Guadeloupe		Seaquake effects or a tsunami disturbed the ships 564 km west of Martinique. All the vessels near Pointe-à-Pitre (Guadeloupe) and in the roadsteads of Fort-de-France and Saint-Pierre (Martinique) also experienced the shock.
4	February 08, 1843	Guadeloupe		Columns of water 100 feet high [30.5 m] and several feet thick [1-1.2 m] were observed to be ejected from fissures in the ground of Guadeloupe.
5	November 18, 1867	Guadeloupe	Ste Rose	10
			Deshaies	18.3
		St Bartholomew		1
		Les Saintes		1
6	March 16, 1985	Guadeloupe	Basse-Terre	0.10-0.12
7	December 26, 1997	Guadeloupe	northern coast	2-3 (modeled)
8	July 12-13, 2003	Guadeloupe	Plage de la Perle	0.5-1
Deshaies				
9	November 21, 2004	Guadeloupe	Trios-Rivières	0.5
			Basse-Terre	weak depression
			Rivière Sens	
		Les Saintes	Village of Grande Anse	sea receded 5 m and dropped 80 cm
			Beach of Grande Anse	0.7
			Village of Petite Anse	water receded a distance of 2-3 m
			Baie du Marigot	an unusual surge
10	May 20, 2006	Guadeloupe	Deshaies	1

which lasted perhaps 5 or 6 seconds. Then felt a first wave, of 60 feet [19.8 m] in elevation, rising to the north, approximately 3 miles [5.6 km] broad, rolled violently towards the land, where it crashed, immersing the whole shore, and swamping the houses that were there. A second and third of these enormous waves rolled in from north to south, one after another, at short intervals, and like the first one, overwhelmed everything along the way. On the northwest shore at Deshaies, **Guadeloupe** a tsunami ravaged the town and smashed nearly all the houses. Everyone has taken refuge in the church. Houses destroyed, sea receded 100 m and returned as an 18.3 m wave". Meanwhile historical material was inspected in [5] and it was concluded that the wave height at Deshaies and Ste Rose could not exceed 10 m, a rather 5 m. This event was simulated in [9] in the framework of the nonlinear shallow-water theory a very good correlation between computed and historic data for all the Lesser

Antilles except Deshaies and Ste Rose (Guadeloupe) was obtained, the authors concluded that observed waves in these locations should be about 2-5 m. The cause of the tsunami was an earthquake occurred in the Virgin Islands (magnitude 7.5; focal depth 33 km [4]; epicenter coordinates 18.1°N 65.1°W [4] or 18.4°N 64.3°W [3]. This event is a **true (4)** tsunami.

March 16, 1985

"A several-centimeter [0.10 - to 0.12-m] of tsunami was recorded at tide-gauge in Basse-Terre, **Guadeloupe**" [2]. It was generated by the earthquake (magnitude 6.3 [2, 3] or 6.4 [4]; focal depth 13 km; epicenter location is 17.013°N 62.448°W [4], or 17°N 62.5°W [5], or 17.1°N 62.4°W [3] which was also felt in Pointe-à-Pitre, **Guadeloupe** and through the other Antillean islands. Furthermore, the land-

slide at Redonda Rock Island was reported in [2]. Tsunami is a **true (4)** event.

November 21, 2004

A tsunami was reported for **Guadeloupe** in [13]: Village of Trois-Rivières (depression 0.5 m), Marina de Rivière Sens, City of Basse-Terre (weak depression), for **Les Saintes, Island of Terre-de-Bas**: Village of Grande Anse (sea receded 5 m and dropped 80 cm), Beach of Grande Anse (70 cm height), Village of Petite Anse (water receded a distance of 2 to 3 m); and for the **Island of Terre-de-Haut**: Baie du Marigot (“*an unusual surge*” reported by a witness). The authors of this paper participated in the field survey in February 2008 that was organized to find tsunami traces on the island on Terre-de-Haut, but no traces were found. The tsunami was caused by an earthquake which occurred in Dominica Passage (between Guadeloupe and Dominica): magnitude 6.3; focal depth of 14 km; coordinates 15.677°N 61.650°W. For the French West Indies this tsunami is a **true (4)** event.

Totally, 6 events can be considered to be true and almost true tsunamis generated by tectonic processes; one of these events (1985) was instrumentally recorded.

It is necessary to mention a strong earthquake that occurred recently in Martinique (November 29, 2007) (19:00 UTC; magnitude 7.4; 14.943°N, 61.244°W, focal depth 146.2 km). Immediately after earthquake the authors inspected the south coast of Guadeloupe but found no tsunami traces. No report has been received from other islands of Lesser Antilles. Based on these data no tectonic tsunami event has occurred on Guadeloupe and Martinique since 2004 meanwhile a strong earthquake took place in this region.

3. TSUNAMI OF THE VOLCANIC ORIGIN

As known, volcanic eruptions can generate numerous destructive tsunamis in the Caribbean region: locally catastrophic, short-period tsunami-like waves can be generated directly by lateral, direct or channelized volcanic blast episodes, or in combination with collateral air pressure perturbations, nuess ardentes, pyroclastic flows, lahars, or cascading debris avalanches. Submarine volcanic caldera collapses can also generate local destructive tsunami waves [14]. Totally, 3 events are selected to be true and almost true, the list of volcanic events is given in the chronicle order.

December 26, 1997

3-meter runup was observed in Montserrat, at Old Road Bay where the wave rolled northwards parallel to the coast, ending up at Old Road Bay, 10 km from the White River Valley. Debris avalanche from the volcano entered in the sea through the River Valley. Formally, the tsunami was not recorded in Guadeloupe. Meanwhile according to the numerical modeling [11], tsunami waves of 2–3 m could reach Guadeloupe and Antigua (50 km from Montserrat) in 10 minutes. Based on similar events of 12/07/2003 and 20/05/2006 (see below) and numerical simulations of Heinrich *et al.* [11] we estimate the 1997 tsunami as **an almost true (3)** tsunami for Guadeloupe.

July 12, 2003

The cause of tsunami was the collapse of a lava dome that occurred at the Soufrière Hills Volcano in Montserrat, the Lesser Antilles. A tsunami was generated when a very large pyroclastic flow impacted the sea. The authors provided a field survey to investigate this event. According to [15] in Deshaies **Guadeloupe** water rose 1.5 m near the entry to the port and surged 25 m inland; on the beach Plage de la Perle the sand level had increased by up to 50 cm. According to [15] who modeled this event, the maximum wave amplitudes were equal to 1.6 m in Guadeloupe. We consider this tsunami as a **true (4)** event.

May 20, 2006

A major lava dome collapse took place at the Soufrière Hills Volcano (Montserrat). According to [16], the effects of the tsunami were felt in the harbor of Deshaies, **Guadeloupe**. The eye-witness said that “*the level of the river roused quickly to a height of approximately 60-70 cm. Two small fishing boats moored in the mouth were slightly damaged.*” This event is estimated as **true (4)** for **Guadeloupe**.

Recently historic and potential flank-collapse of the Soufrière Volcano on Guadeloupe which can be a source of volcanic tsunamis in Guadeloupe has been studied in [17]. Meanwhile, just now there are no historic data of tsunamis from the Guadeloupe volcano; the years of historical eruptions of Soufrière (1797, 1798, and 1836) are not associated in the literature with any unusual sea agitation in the Caribbean.

4. TELE TSUNAMI

November 1, 1755

The event was generated by a well-known Lisbon earthquake (magnitude 8.5-9; focal depth 20-40 km; epicenter coordinates 37°N 11°W) that produced huge tsunami with maximal height of 10 meters, which propagated across the Atlantic Ocean and affected the American Coast, in particular, the Lesser Antilles. “*In Martinique and most of the French Islands, it overflowed the low land, and returned quickly to its former boundaries*” [2]. Recently, the probable geological evidence of the 1755 tsunami on the east coast of Grande-Terre (**Guadeloupe**) at Anse Sainte Marguerite and Anse Maurice on a height of 2-3 m has been found in [18].

4. ANALYSIS OF TSUNAMI OCCURRENCE IN GUADELOUPE

The data of tsunami in Guadeloupe Department and neighbor islands for 1498-2008 (510 years) collected above is analysed. The total number of tsunamis is ten (six of tectonic origin, three of volcanic origin and one teletsunami). The only teletsunami occurred in 1755 in the region (well-known Lisbon event). Volcanic tsunamis were caused by Soufrière Hills (Montserrat) eruptions during the last decade. The variability of the number of tsunamis in 50-yr and 100-yr intervals is studied. The maximum number of events (all of the tectonic origin) occurred in 1800-1849 period. The increase of the volcanic events is vivid; all of them were caused by the activity of volcano in Montserrat in last decade.

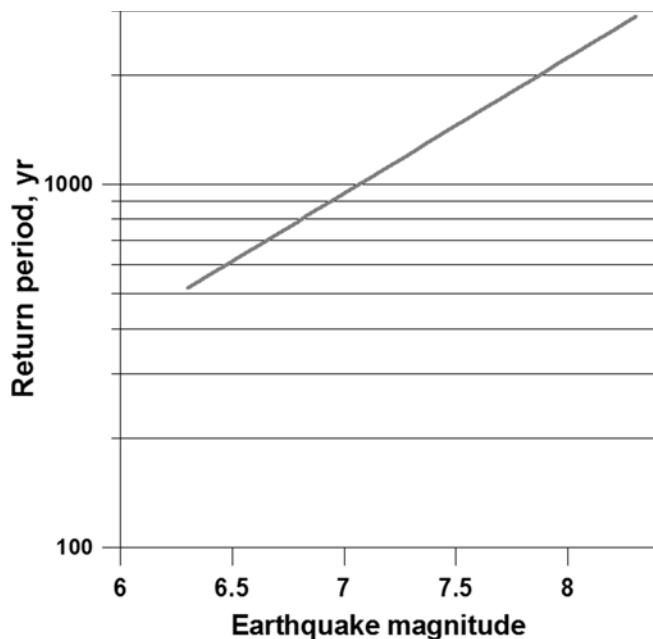


Fig. (2). Return period of tectonic tsunamis in Guadeloupe versus earthquake magnitude.

For all local tectonic events magnitude values are given. The cumulative frequency of tsunamis generated by earthquakes with given parameters is calculated.

These data are fitted very well by the exponential curve with $R = 0.97$ (time is in years):

$$\ln(N/T) = -0.86M + 0.83, \quad (1)$$

As a result, the return period versus magnitude can be determined, see Fig. (2),

$$T = 2.6 \cdot \exp(0.86M) \quad (2)$$

In fact, earthquakes with a magnitude more than 6.3 can generate tsunami; but return period of such tsunamis is more than 200 yrs. As for event with magnitude 8, all strong earthquakes occurred before 1843 and the accuracy of this estimation is very low. Meanwhile, runup values varies from 0.1 m to 1 m irrespective of magnitude, if the values of 10, 18 and 20 m (doubtfulness of these data was discussed previously) are not taken into account.

CONCLUSION

According to historical data tsunami in Guadeloupe is a relatively frequent event; ten tsunamis are selected as true and almost true. Six events were generated by underwater earthquakes; three – by volcano eruptions; and one is a teletsunami. Tsunamis of volcanic origin were induced by Montserrat volcano in last decade. Tsunamis of seismic origin are generated by earthquakes with magnitude more than 6.3. Runup values varies from 0.1 m to 1 m irrespective of magnitude for tectonic events (except abnormal values of the height in 10 or 18 m for Deshaies, Guadeloupe). The possibility of big tsunami which heights exceeds 5 m for Guadeloupe is questionable. Recently Scheffers and Kelletat [19] reported geological traces of tsunami 3 m height that oc-

curred in St. Martin (exactly, on small unnamed island of volcanic rocks at eastern St. Martin) in Holocene (500 BP); this confirm low probability of big tsunami for Guadeloupe. It is necessary to mention that since the coastal utilization is much denser comparing to historical time, it must be taken into account in mitigation strategies that the smaller tsunamis with higher probabilities would also be effective and cause loss of property and life in Guadeloupe.

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REFERENCES

- [1] Lander, JF, Whiteside, LS, Lockridge, PA. A brief history of tsunami in the Caribbean Sea. *Sci Tsunami Hazards* 2002; 20: 57-94.
- [2] O'Loughlin, KF, and Lander, JF. Caribbean Tsunamis: A 500-Year History from 1498-1998, *Adv Nat Technol Hazards Res* 2003; p. 20.
- [3] HTDB/ATL Expert Tsunami Database for the Atlantic. Version 3.6 of March 15, 2002. Tsunami Laboratory, Novosibirsk, Russia, 2003.
- [4] NGDC (National Geophysical Data Center) Tsunami Database, http://www.ngdc.noaa.gov/seg/hazard/tsu_db.shtml, 2006
- [5] Zahibo N, Pelinovsky E. Evaluation of tsunami risk in the Lesser Antilles. *Nat Hazards Earth Sci* 2001; 3: 221-31.
- [6] Zahibo N, Pelinovsky, E. In: Mercado-Irizarry A, Liu P, Eds. Caribbean Tsunami Hazard: Tsunamis in the Lesser Antilles, Caribbean Tsunami Hazard. World Sci Singapore 2006; 244-54.
- [7] Weissert TP. Tsunami travel time charts for the Caribbean. *Sci Tsunami Hazards* 1990; 8: 67-78.
- [8] Mercado, A, McCann, W. Numerical simulation of the 1918 Puerto Rico tsunami. *Nat Hazards* 1998; 18: 57-76.
- [9] Zahibo N, Pelinovsky E, Yalçiner *et al.* The 1867 Virgin Island Tsunami: observations and modeling. *Oceanol Acta* 2003a; 26: 609-21.
- [10] Mader, CL. Modeling the 1755 Lisbon tsunami. *Sci Tsunami Hazards* 2001; 9: 93-98.
- [11] Heinrich, F, Guibourg, S, Mangeney, A, Roche, R. Numerical modelling of a landslide-generated tsunami following a potential explosion of the Montserrat Volcano. *Phys Chem Earth* 1999; A24: 163-168.
- [12] Zahibo N, Pelinovsky E, Kukin A, Kozelkov A. Estimation of far-field tsunami potential for the Caribbean Coast based on numerical simulation. *Sci Tsunami Hazards* 2003b; 21(4): 202-22.
- [13] Zahibo N, Pelinovsky E, Okal E, *et al.* The earthquake and tsunami of November 21, 2004 at Les Saintes, Guadeloupe, Lesser Antilles. *Sci Tsunami Hazards* 2005; 23(1): 25-39.
- [14] Pararas-Carayannis, G. In: Mercado-Irizarry A, Liu P, Eds. Caribbean Tsunami Hazard: Risk assessment of tsunami generation from active volcanic sources in the Eastern Caribbean Region. World Sci, Singapore 2006; 91-137.
- [15] Pelinovsky E, Zahibo N, Dunkley P, *et al.* Tsunami generated by the volcano eruption on July 12-13 2003 at Montserrat, Lesser Antilles. *Sci Tsunami Hazards* 2004; 22(1): 44-57.
- [16] Zahibo N. Deshaies, Guadeloupe tsunami, 20 May 2006, Montserrat, Lesser Antilles, Soufriere volcano tsunami. *Tsunami Newsletter* 2006; XXXVIII, 2: 4-5.
- [17] Le Friant A, Boudon G, Komorowski J-C, Heinrich P, Semet MP. Potential flank-collapse of Soufriere Volcano, Guadeloupe, Lesser Antilles? Numerical simulation and hazards. *Nat Hazards* 2006; 39: 381-393.
- [18] Morton RA, Richmond BM, Jaffe BE, Gelfenbaum G. Reconnaissance investigation of Caribbean extreme wave deposits – prelimi-

- nary observations, interpretations, and research directions, USGS, Open-File Report 2006-1293, 2006; p. 46.
- [19] Scheffers A, Kelletat D. In: Mercado-Irizarry A, Liu P, Eds. Caribbean Tsunami Hazard: new evidence and dating of Holocene paleo-

tsunami events in the Caribbean (Barbados, St. Martin and Anguilla). World Sci Singapore 2006; 178-202.

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TSUNAMI GENERATED BY THE VOLCANO ERUPTION ON JULY 12-13, 2003 AT MONTSERRAT, LESSER ANTILLES

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ABSTRACT

A major collapse of a lava dome occurred at the Soufrière Hills Volcano (Montserrat, Lesser Antilles), culminating late in the evening (11:35 PM local time) on July 12, 2003 (03:35 GMT on 13 July). This generated a tsunami, which was recorded on Montserrat 2-4 km from the generating area and Guadeloupe, 50 km from Montserrat. Results of field surveys are presented. Tsunami wave height on Montserrat may have been about 4 m according to the location of a strandline of charred trees and other floating objects at Spanish Point on the east coast of the island. The wave height on Guadeloupe according to “direct” witnesses was about 0.5-1 m at Deshaies and near Plage de la Perle. The tsunami at Deshaies caused the scattering of boats as confirmed by fishermen and local authorities. Data from the field survey are in agreement with the predicted tsunami scenario obtained by numerical simulation.

1. Introduction

Soufrière Hills Volcano is located on Montserrat, Lesser Antilles (16.72°N , 62.18°W); see Figure 1. The volcano is currently undergoing a prolonged eruption, which began in July 1995 and has caused widespread devastation to the southern part of the island, including the destruction of the airport and the capital of Plymouth. There have also been a number of fatalities. The island's population fell from about 11,000 to 3,500 during the early part of the eruption, but has increased slightly in recent years to about 4,600. Buffonge in series of books (1996 – 1998) provides a chronicle of Montserrat's society experience with the volcano; see also, Pattullo, 2000.



Figure 1. Chart of Montserrat, Lesser Antilles

The first phase of the eruption lasted from July 1995 until March 1998 and is described in a series of short papers published in Geophysical Research Letters (Young et al. 1998) and in a comprehensive collection of papers in a Geological Society of London memoir (Druitt and Kokelaar, 2002). The largest single event of the first phase of the eruption occurred on 26 December 1997, when the southern wall of the old summit crater failed due to loading by the lava dome. This generated a debris avalanche and resulted in the rapid collapse of the active lava dome, unleashing a powerful pyroclastic density current and a volcanic blast, as well as generating a small tsunami.

A period of stagnation in the eruption began in March 1998 and continued until November 1999 when extrusion of lava recommenced and marked the onset of the second phase of dome growth. During this second phase the dome grew and collapsed several times.

By mid-2003 the lava dome had grown to an unprecedented size, having a volume of about 200 million m^3 and a general summit elevation of around 1100 m. On 12-13 July the dome

underwent the largest collapse of the entire eruption to date. This was a prolonged event. Continuous pyroclastic flows (composed of hot rocks, ash and gas) began in the Tar River Valley on the eastern side of the volcano at 09:30 local time (13:30 GMT) on 12 July (Herd et al., 2004). From 10:45 onwards these reached the sea and at 18:30 the flows became larger and more energetic as the collapse progressively cut back into the hotter interior of the dome. The collapse reached its most energetic phase between 21:50 12 July and 0:50 13 July when a sequence of very large pyroclastic flows entered the sea and pyroclastic surges traveled up to 3 km across the surface of the sea. The climax of the collapse occurred at 23:35 when a very large pyroclastic flow impacted the sea and pyroclastic surges devastated 7 km² on the NE flank of the volcano (Edmonds et al., 2004). A tsunami appears to have been generated at this stage. A number of explosive events also took place, with the largest occurring during the climax of the collapse, producing ash clouds to an altitude of 15 km. In total over 120 million m³ of material was removed during the collapse, leaving a large amphitheatre-shaped scar in the dome, which is open to the east above the Tar River Valley. The valley itself was also extensively modified with a deep canyon eroded by the pyroclastic flows. Various photos of this event can be found in the website of the Montserrat Volcano Observatory (<http://www.mvo.ms>). One of the photos of the pyroclastic flow entering sea is shown in Figure 2.



Figure 2. Pyroclastic flow down the Tar River valley on 12th July 2003 Photograph M Edmonds © NERC

Large dome collapses on Montserrat have produced tsunami waves on two occasions during the current eruption. The earlier dome collapse of December 26, 1997 (Boxing Day Collapse) produced an avalanche in the White River Valley (for location, see Figure 1). This resulted in a large amount of debris entering the sea and the generation of a tsunami. Heinrich et al (2001) presented numerical simulations of this debris avalanche. The wave rolled northwards parallel to the coast, ending up at Old Road Bay, 10 km from the landslide site (see Figure 1), where it pushed boats and trailers some distance up on to the shore (Buffonge, 1998). Lander et al (2002) gives the following description of the tsunami: “The wave was estimated to have been about 1 m higher than the road which lies 2-m above water level, and to have moved inland a maximum distance of 80 m. A variety of objects, including a small wooden boat, a roof to a shelter, and a stone table were displaced several meters inland and a large log was carried even farther by the wave. Impact marks up to 1 m were also reported on the side of palm trees facing the sea. The grass was oriented in such a way as to indicate the retreat of the wave. An observer reported seeing the sea move out and then back in, which is typical of a landslide-generated tsunami. The focusing of the wave at Old Road Bay may be attributed to the peculiarities of wave behavior along a coastline and the abrupt change of coast direction at Old Road Bay. The wave moved inland here, because the coast abruptly changes its direction, and the wave moving parallel to the coast would have met the shore head-on. Also, the shallow offshore bathymetry and onshore topography in the area aided extended wave runup.” There is no record of this tsunami on the neighboring islands of Antigua, Guadeloupe, and St Kitts.

The main goal of this paper is to present the data of a field survey conducted on the islands of Montserrat, Guadeloupe and Antigua following the very large collapse of the dome of 12-13 July 2003. The results of numerical simulations of tsunami propagation related to this event are also discussed.

2. Numerical simulation of tsunami waves generated at volcano eruption in Montserrat

A hypothetical tsunami event due to collapse of the lava dome in the Tar River Valley and to a sudden entry of a debris avalanche into sea was in fact modelled numerically by Heinrich et al (1998, 1999). Considering a debris avalanche volume of $40 \cdot 10^6 \text{ m}^3$ (the same volume is estimated for the debris avalanche of 26 December 1997) entering the sea at speed of 40 m/s, their modelling suggested that most of the wave energy propagates in the open sea in the slide

direction, i.e. towards Guadeloupe and Antigua. The distribution of the maximum water surface elevation is presented in Figure 3 taken from Heinrich et al (1998). Along the coast of Montserrat (in fact, near the coast at a depth of 10 m), the computed wave heights are maximal in the epicentral zone (6-10 meters), and 1-2 m at distances of about 10 km from the generation area (Heinrich et al, 1998). According to the modelling, tsunami waves of 2–3 m could reach Guadeloupe and Antigua (50 km from Montserrat) in 10 minutes; see Figure 4 (Heinrich et al, 1999).

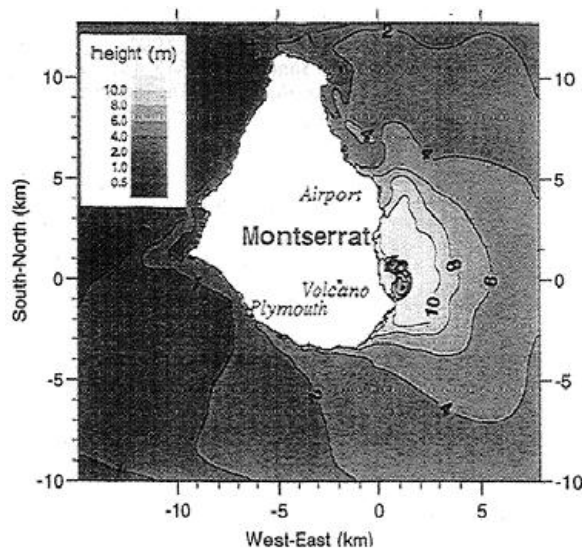


Figure 3. Maximum water surface elevation on Montserrat coast during the tsunami propagation (Heinrich et al, 1998)

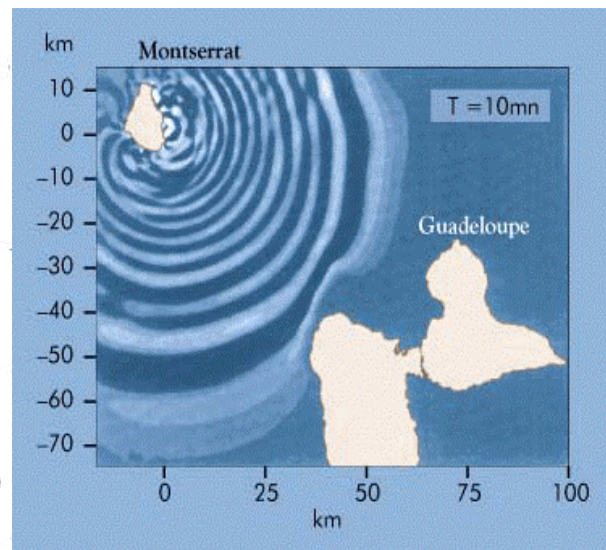


Figure 4. Tsunami wave propagation from Montserrat (Heinrich et al, 1999)

We estimate roughly a comparable distribution of the tsunami wave amplitude along the coasts of Guadeloupe, Antigua and St Kitts. In our simulations the nonlinear shallow water equations are applied based on the numerical code TUNAMI, recently used to study the 1867 Virgin Island tsunami in the Lesser Antilles (see, Zahibo et al., 2003a). The bathymetry of the Eastern Caribbean is taken from the GEBCO Digital Atlas (British Oceanographic Data Centre) with mesh size 1.5 km. As in the modelling of Heinrich et al (1998), the runup stage is not simulated and complete reflection is calculated for the last sea points. The tsunami source is modeled by the initial upward water displacement (up to 10 m) in the “sea” part of the circle of diameter 2 km located at the mouth of the Tar River (16.71N, 62.14W). Such a source was chosen to demonstrate wave propagation along the coasts of neighboring islands far from the source. Figure 5 shows the directivity of the tsunami propagation. It is clearly seen that the tsunami waves mainly propagate towards Antigua and Guadeloupe and relative

weakly towards Nevis and Barbuda, as was predicted in the more exact computing by Heinrich et al (1998, 1999). According to our calculations, the maximum wave amplitudes are 1.6 m at Guadeloupe, 1.9 m at Antigua (1.9 m), 1.2 m at Nevis and 0.9 m at Barbuda. We should stress that these results are very sensitive to the properties of the tsunami source (water displacement and directivity). Although the tsunami source model is only approximate, the comparable wave amplitudes in various coastal locations can be used to select zones where the tsunami effect may be expected to be large. Therefore, from our computing we may conclude that wave heights of the 12-13 July 2003 Montserrat tsunami at Guadeloupe and Antigua were approximately the same, and by comparison were at least twice as small at Nevis and Barbuda. We have calculated the relative wave height distributions along the coast of neighboring islands (Figure 6), normalized on its maximum value for each island. This demonstrates that the 12 July 2003 tsunami should have been manifested on the northern capes and western coast of Guadeloupe, western coast of Antigua, and, perhaps, south-eastern coast of Nevis and west-southern coast of Barbuda.

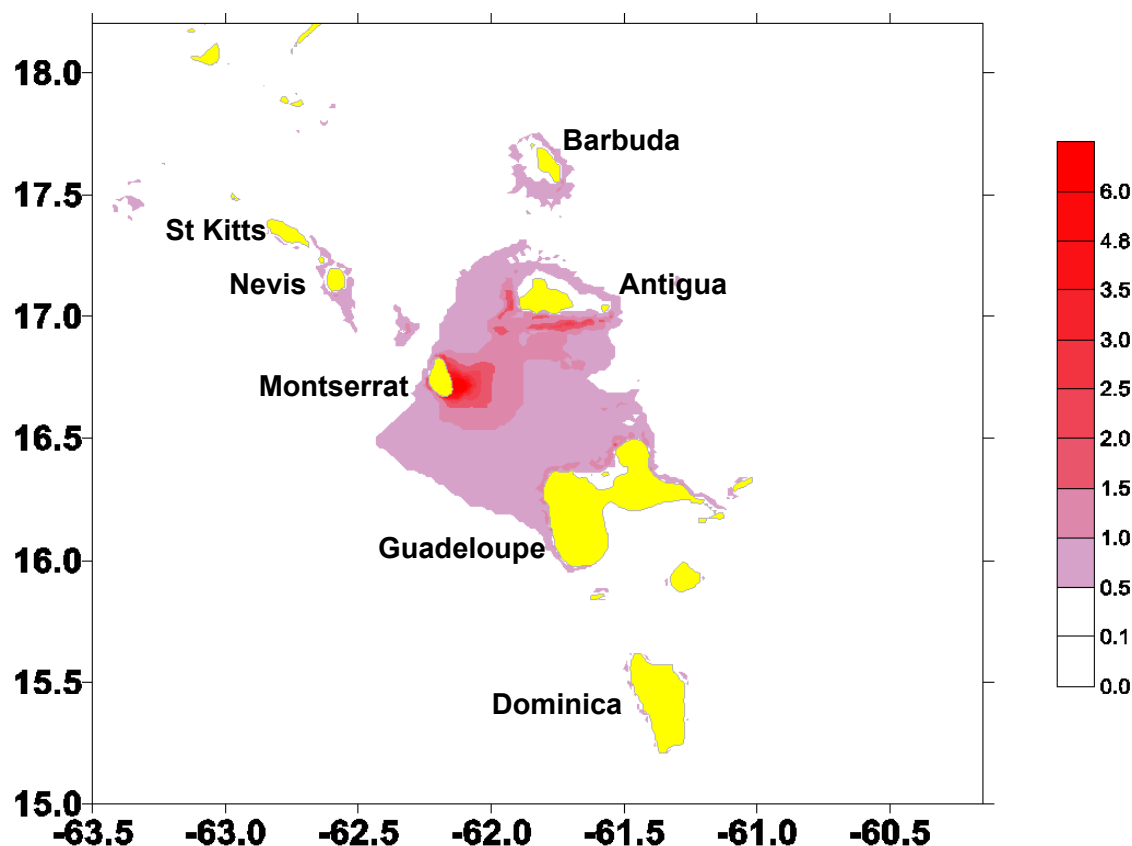


Figure 5. Directivity diagram at the isotropic source

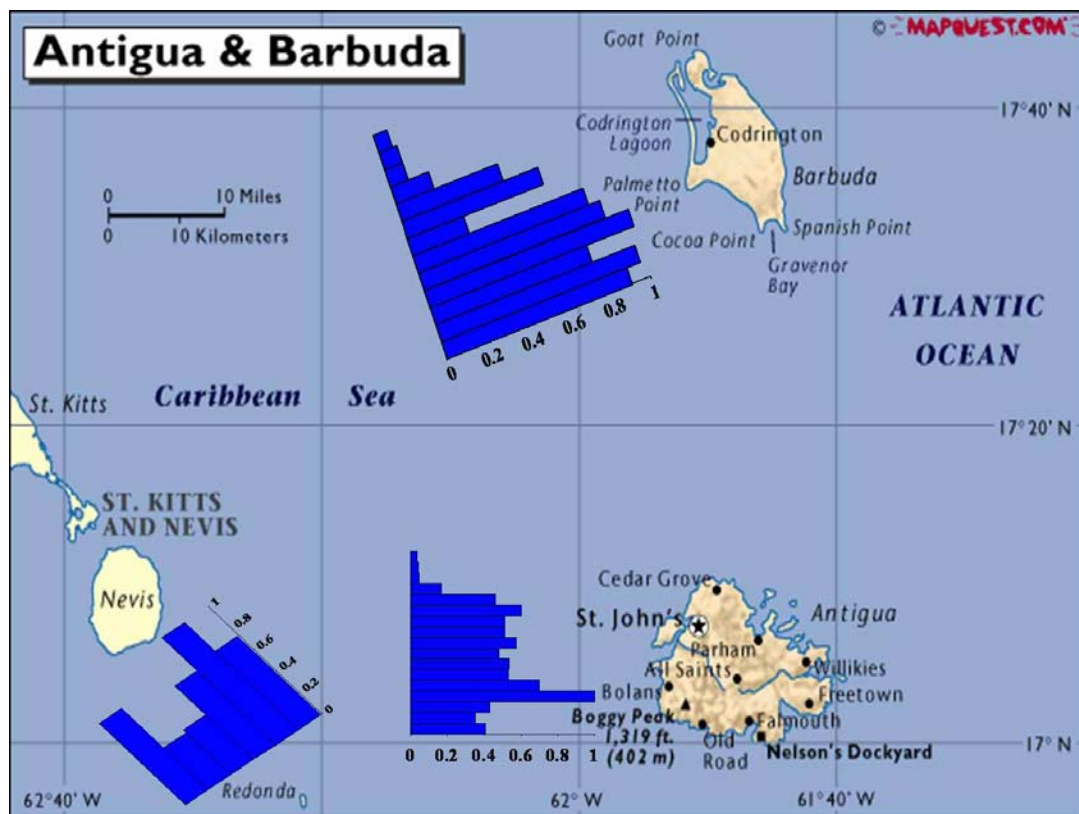


Figure 6. Distribution of the relative wave height along the coasts of Guadeloupe, Antigua, Barbuda and Nevis

3. Field survey data

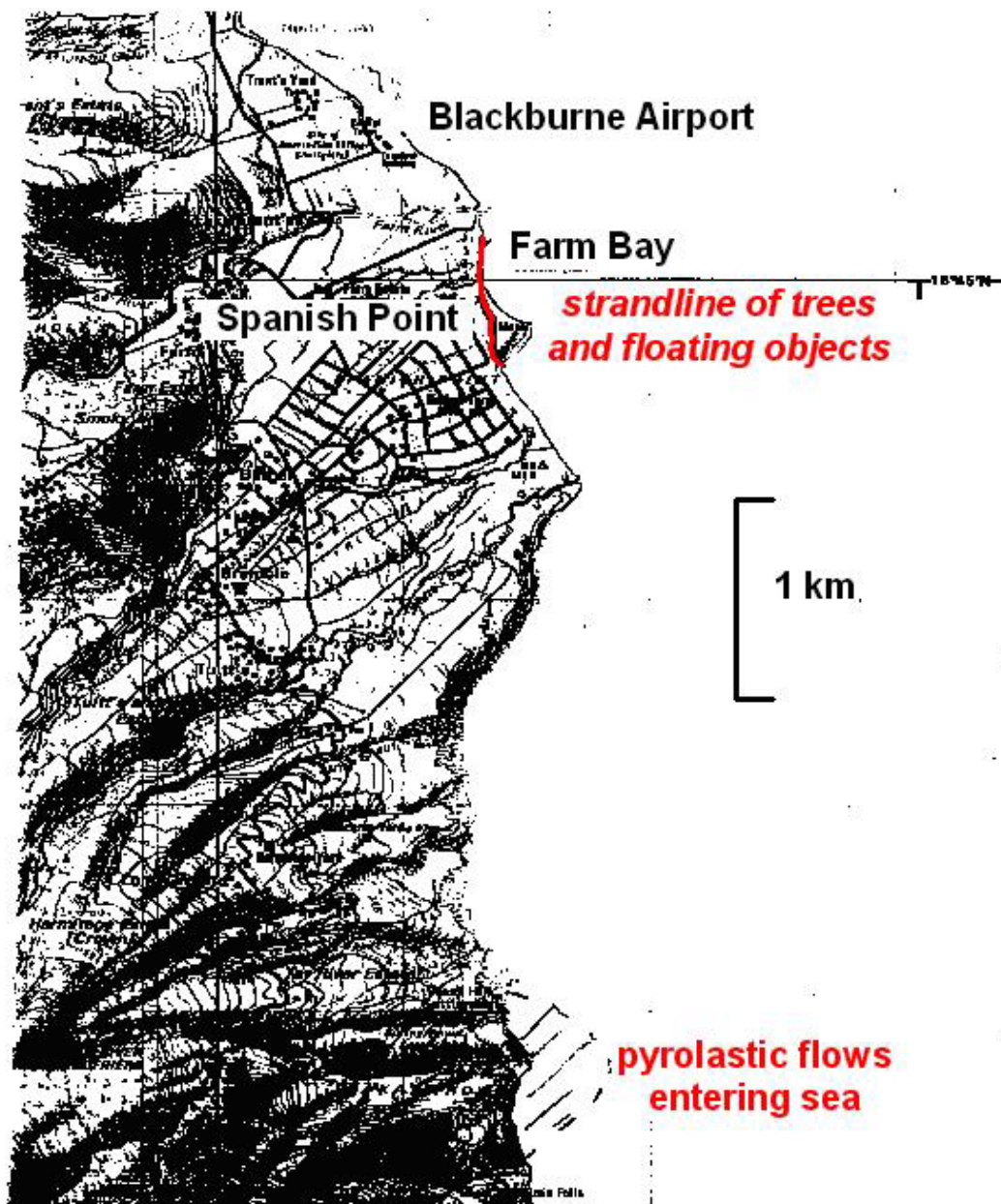


Figure 7. Chart of the area where strandline of trees and floating objects have been found

The first field survey of the coastal area at *Montserrat* was conducted in the middle of September 2003 by staff of the Montserrat Volcano Observatory. A strandline of charred trees and other floating objects was found at Spanish Point on the coast at Farm Bay (16.737N, 62.153W) approximately 3-4 km north of the mouth of the Tar River Valley where the pyroclastic flows impacted the sea on 12-13 July; see Figure 7 for the detailed chart of this area. The strandline is located between about 100-150 m from the shoreline at a height of 4 m above sea level (eye estimates). Cyclone activity was high in September 2003 when two hurricanes passed near Montserrat to the north. These were: "Fabian", category 4, September

3; and "Isabel", category 4-5, September 13. Their tracks taken from the *NOAA Tropical Prediction Center* (UNISYS), are shown in Figure 8. It is possible therefore that the strandline of charred trees may have resulted from a tsunami related to the dome-collapse, as well as from a storm surge. Taking into account the local character of this phenomenon near Tar River Valley, the tsunami origin is preferable. A second field survey around Spanish Point was conducted by the writers in January 2004, at which time the strandline was still apparent (Figure 9).

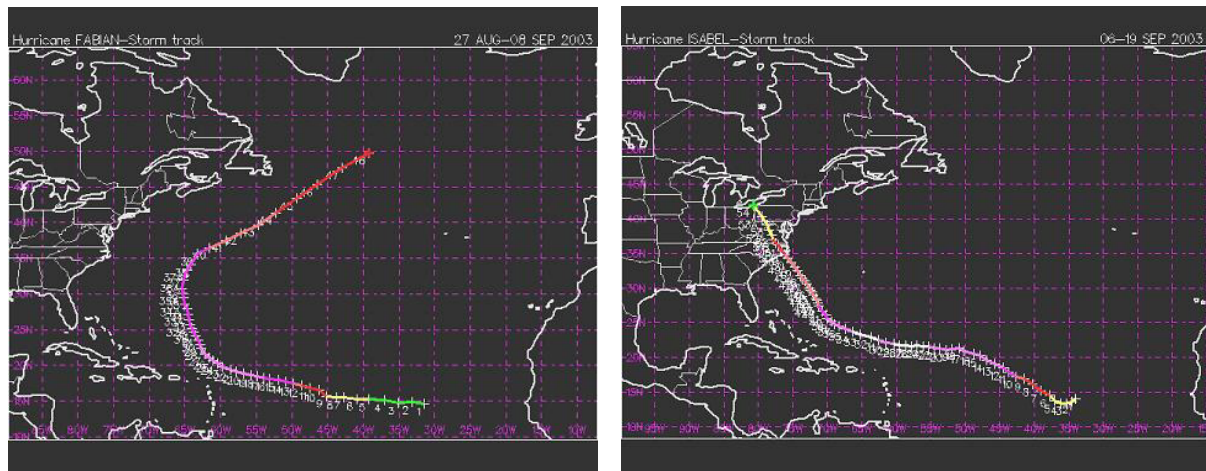


Figure 8. Tracks of hurricanes: “Fabian” (left) and “Isabel” (right)





Figure 9. Strandline of charred trees and other floating objects on the coast at Farm Bay (Spanish Point)

The field survey in *Guadeloupe* was conducted in November 2003. First of all, we should mention that there is no tide gauge in Guadeloupe now to provide quantitative information. Also there was no information on the tsunami in the local newspaper “France - Antilles”, except for a report from Montserrat concerning the eruption on the night of July 12-13, 2003. We could not find direct traces of tsunami manifestation, but obtained witness reports. These were obtained from three locations on the western coast of Guadeloupe at Deshaies, Malendure, and Vieux Habitans (for locations see Figure 6). The distance between Deshaies and Vieux Habitans is about 27 km. At the port of *Deshaies* (Figure 10a), and in the mouth of the river Deshaies (Figure 10b) fishermen on the next morning (13 July) found that approximately 10 boats were scattered in the port and river and had been moved onto the land (up to 15 m). Some of these were slightly damaged. The height of the wall in the place of the boat mooring in the port is 40 cm. One small boat, moored in the mouth of the river Deshaies, was carried for more than 60 meters up the river to the bridge (Figure 10c). The customers of the restaurant “Note Bleue”, who were celebrating a wedding, were surprised by an abrupt rise of water in the mouth of the river of Deshaies during the night of the 12-13 July, which occurred sometime between 23.00 and 01.00 (local time). The appearance of the tsunami

corresponds approximately to the climax of the dome collapse on Montserrat between 11-12 PM. Water overflowed the coast of the river mouth (60 cm); see Figure 10d. No noise of a wave or wind accompanied this phenomenon. The Police investigated the area at 02.00 (July 13). Some people, referred to as “direct” witnesses, reported that the water rose 1.5 m near the entry to the port and surged 25 m inland. The duration of the wave, which broke on the beach, was about 1 min, and is reported to have arrived at 23:30, although this information is not from first hand reports.



Figure 10. Deshaies, a) port, b) the river mouth, c) 60 m upstream, where the boat was found near the bridge, d) restaurant in the river mouth, where the beach was overflowed

On the *Plage de la Perle*, a few kilometers to the north of Deshaies, a shopkeeper told us that on the next morning he found that the sand level on the beach had increased by up to 50 cm. We should add that this beach is protected by a coral reef near the shoreline. At *Malendure* (14.5 km to the south of Deshaies), where according to our simulations wave height should have been at a maximum, the tide that night rose several meters at certain places along the coast where it penetrated for 20 m inland. The sea rose 46 centimeters on the foot of the pontoon of Malendure (Figure 11). At *Vieux Habitans* (13 km to the south of Malendure and 27 km from Deshaies), the water rose up to 60 cm. Unfortunately, we were not able to find “direct” witnesses of the tsunami at Malendure and Vieux Habitans. According to the

information collected at these localities on Guadeloupe, we may conclude that the water rise at Deshaies was due to a real tsunami with a confirmed wave height of about 0.5-1 m. The reliability of the information on the tsunami in other places (Malendure and Vieux Habitants) is not quite clear. The water rise in these places was probably not recorded due to it being nighttime and also because of the low amplitude of the wave.



Figure 11. Pontoon of Malendure (46 cm above sea level)

We visited *Antigua* (St John's and Jolly Harbour (near Bolans) in January 2004 (see Figure 6 for locations, but were not able to find any reports of the tsunami in these places.

4. Conclusion

The climax of the large dome collapse at the Soufrière Hills Volcano, Montserrat occurred late in the evening (23:35 local time) on 12 July, 2003 (03:35 GMT on 13 July). The impact of a large pyroclastic flow on the sea generated a tsunami, which was recorded on **Montserrat**, 2-4 km from generating area and **Guadeloupe**, 50 km from Montserrat. The results of the field surveys are presented. The wave height on Montserrat may have been about 4 m according to the location of the strandline of charred trees and other floating objects at Spanish Point. The wave height on Guadeloupe according to “direct” witnesses was about 0.5-1 m at Deshaies and near Plage de la Perle. The tsunami at Deshaies scattered boats, as confirmed by fishermen and local authorities. Data from the field survey are in agreement with the predicted tsunami scenario of Heinrich et al (1998, 1999) and our simulations.

It is important to point out that the 2003 tsunami is the third event recorded on Guadeloupe during the last 150 years. The two other events include: the catastrophic 1867 tsunami, when waves generated on the Virgin Islands approached Guadeloupe with heights of about 10 m at Deshaies and St Rose; and the weak 1985 tsunami, when a wave several centimeters high was recorded by the tide gauge at Basse-Terra (Zahibo et al., 2001, 2003; Lander et al., 2002). The tsunami risk for the Lesser Antilles should therefore not be ignored and should be evaluated. Such work is now in progress (Zahibo & Pelinovsky, 2001; Zahibo et al, 2003b)

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References

Buffonge, C. *Volcano!* Books 1-4. Ideal Printers, Montserrat, 1996-1998.

Druitt, H.T.H., and Kokelaar, B.P. The eruption of Soufrière Hills Volcano, Montserrat, from 1995 to 1999. *Geological Society Memories*, No. 21. Geological Society of London, 2002.

Edmonds, M., Herd, R. A. and Strutt, M. H. Tephra deposits arising from the lava dome collapse and explosions 12-15 July 2003 at Soufrière Hills Volcano, Montserrat, West Indies. *Journal of Volcanology and Geothermal Research*, 2004 (submitted).

Heinrich, Ph., Mangeney, A., Guibourg, S., and Roche, R. Simulation of water waves generated by a potential debris avalanche in Montserrat, Lesser Antilles. *Geophysical Research Letters*, 1998, vol. 28, 3697-3700.

Heinrich, Ph., Roche, R., Mangeney, A., and Boudon, G. Modéliser un raz de marée créé par un volcan. *La Recherche*, 1999, vol. 318, 67-71.

Heinrich, Ph., Boudon, G., Komorowski, J.C., Sparks, R.S.J., Herd, R., and Voight, B. Numerical simulation of the December 1997 debris avalanche in Montserrat, Lesser Antilles. *Geoph. Research Letters*, 2001, vol. 28, No. 13, 2529-2532.

Herd, R. A., Edmonds, M., Sanchez, J. and Bass, V. A large dome collapse and explosive activity 12-15 July 2003 at Soufrière Hills Volcano, Montserrat, West Indies. *Journal of Volcanology and Geothermal Research*, 2004 (submitted).

Lander, J.F., Whiteside, L.S., and Lockridge, P.A. A brief history of tsunami in the Caribbean Sea. *Science of Tsunami Hazards*, 2002, vol. 20, 57-94.

Pattullo, P. *Fire from the Mountain. The tragedy of Montserrat and the betrayal of its people.* Constable, London. 2000.

UNISYS. *Atlantic Tropical Storm Tracking by Year.* Based on data from the Tropical Prediction Center, NOAA. <http://www.weather.unisys.com/hurricane/atlantic/index.html>

Zahibo, N., Pelinovsky, E. Evaluation of tsunami risk in the Lesser Antilles. *Natural Hazards and Earth System Sciences*, 2001, v. 1, No. 4, 221-231.

Zahibo N., Pelinovsky E., Yalciner A., Kurkin A., Koselkov A., and Zaitsev A. The 1867 Virgin Island Tsunami: observations and modelling. *Oceanologica Acta*, 2003a, vol. 26, No. 5-6, 609 – 621.

Zahibo, N., Pelinovsky, E., Kurkin, A., and Kozelkov, A. Estimation of far-field tsunami potential for the Caribbean Coast based on numerical simulation. *Science Tsunami Hazards*. 2003b, vol. 21, No. 4, 202 – 222.

Young, S.R., Voight, B., Sparks, R.S.J., Rowley, K., Robertson, R.E.A., Lynch, L.L., and Aspinall, W.P. Selected papers on Eruption of the Soufrière Hills Volcano Montserrat. *Geophysical Research Letters*, 1998, vol. 25, No. 18 & 19.

Tsunami hazard for the French West Indies, Lesser Antilles

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Detailed analysis of historical data of tsunamis in the French West Indies collected in different books, papers and sites is given. Totally, 18 events are selected as true and almost true. 10 tsunami events have been generated by underwater earthquakes; 7 events – by the volcano eruptions, and one is a teletsunami. The geographical and temporal distributions of tsunami events are studied.

1. Introduction

The tsunami catalogue has been recently created for the Caribbean Sea (Lander *et al.*, 2002; HTDB/ATL, 2002; O'Loughlin and Lander, 2003; NGDC, 2006) and in particular, for the Lesser Antilles (Zahibo & Pelinovsky, 2001, 2006). In the past 500 years this region has had devastating tsunamis that caused damage in many states of the Caribbean Basin. According to Lander *et al.* (2002), a total of 91 waves reported might have been tsunami. Of these, 27 are judged by the authors to be true, verified tsunamis; and the additional nine are considered to be very likely true tsunamis. The tsunami list for the 20th century contains 33 events, thus one in every three years. Only in the last 40 years there were 8 true and almost true tsunamis: 1969, December 25 (earthquake with magnitude 7.6 in Lesser Antilles, maximal tsunami amplitude of 46 cm at Barbados); 1985, March 16 (moderate earthquake with magnitude 6.3 in Guadeloupe, several-centimeter tsunami was recorded at Basse Terre, Guadeloupe); 1989, November 1 (weak earthquake with magnitude 4.4 off the north coast of Puerto Rico generating a small wave in Cabo Rojo); 1991, April 22 (the earthquake with magnitude 7.6 created the tsunami that affected the coast of Central America from Costa Rica to Panama; wave height is 2 m in Cahuito Perto Viejo, Costa Rica); 1997, July 9 (the earthquake of magnitude 6.8 occurred off the coast of Venezuela and induced a weak tsunami on Tobago); 1997, December 26 (volcanic eruption in Montserrat generated the wave with height 3 m at Old Road Bay); 2003, July 12 (volcanic eruption in Montserrat induced tsunami in Montserrat and Guadeloupe; this and next events will be discussed below); 2004, November 21 (underwater earthquake with magnitude 6.3 generated

tsunami in Guadeloupe and Dominica); 2006, May 20 (volcanic eruption in Montserrat induced tsunami in Montserrat, Antigua and Guadeloupe).

Tsunami phenomenon in the Caribbean Sea has been the subject of a special study in recent years. First of all, we would like to mention the calculation of the tsunami travel time charts for the Caribbean (Weissert, 1990). The estimated time for a complete crossing of the Caribbean is 3.2 hrs laterally and 1.5 hrs meridionally. Two historical events in the Caribbean: the 1918 Puerto Rico tsunami and the 1867 Virgin Island tsunami, induced by the devastating earthquakes, have been simulated by Mercado and McCann (1998) and Zahibo *et al.* (2003a). Computed values of the tsunami wave heights are in a satisfactory agreement with the observed data. The propagation of the trans-atlantic tsunami after the catastrophic Lisbon earthquake (01/11/1755) has been modelled by Mader (2001). According to his calculations, the wave amplitude east of Saba (Lesser Antilles) is 5 m close to the observed value (7 m). Heinrich *et al.* (2001) performed the numerical simulation of the 26/12/1997 debris avalanche in Montserrat (Lesser Antilles) that induced tsunami waves up to 3 m. The first attempt to evaluate the tsunami risk for the Caribbean Sea using the numerical simulation of wave propagation from various prognostic sources has been done by Zahibo *et al.* (2003b); as a results a few zones with low tsunami risk are selected.

The present chapter has a goal to revise historical data of tsunamis for the French West Indies (Guadeloupe, Martinique, St Martin and several small islands), Lesser Antilles using all available publications and results of numerical simulations. In this paper only true and almost true events are selected, and this classification corresponds to the validity 4 (“**Definite tsunami**. Tsunami did occur. Data corroborated or deemed valid information”) and 3 (“**Probable tsunami**. Likely or probable tsunami occurrence. Reliable observations for corroboration are few”) according to the definition by O’Loughlin and Lander (2003). Totally, eighteen events are selected as true and almost true. Ten events have been generated by underwater earthquakes; seven events – by the volcano eruptions, and one is a teletsunami. The geographical and temporal distributions of tsunami events are studied.

2. Tele tsunamis in French West Indies

According to the all the catalogues the French West Indies were affected by two teletsunamis that crossed the Atlantics between 1498 and 2006. Both were produced by Lisbon earthquakes in the XVIII century. They are determined in literature as true events.

The first **true** event occurred on **November 1, 1755** was related with a well-known Lisbon earthquake (magnitude 8.5–9; focal depth 20–40 km; epicenter coordinates 37°N 11°W). This earthquake produced huge tsunami with maximal height of more than 10 meters (intensity 4), which propagated across the Atlantic Ocean and affected the American Coast, in particular, the French West Indies (Fig. 1). “*The sea waves swept across the Atlantic and were observed in several of the West Indian Islands, where the usual rise of the tide is little more than 2 feet [60 cm]. In St. Martin, the sea retired so far that a sloop, riding at anchor in 15 feet [4.6 m] of water, was laid dry on her broadside. In Martinique and most of the French Islands, it overflowed the low land, and returned quickly to its former boundaries. In the West Indies, this extraordinary motion of the waters was observed 6 hours after the first shock was felt at Lisbon. In Martinique, in that remarkable flux and reflux of the sea, some places were left dry on about a mile [1.5 km]*” (O’Loughlin & Lander, 2003). Near Fort-de-France, Martinique “*Sea withdrew 1.6 km and returned to inundate the upper floors of houses*” (NGDC, 2006).

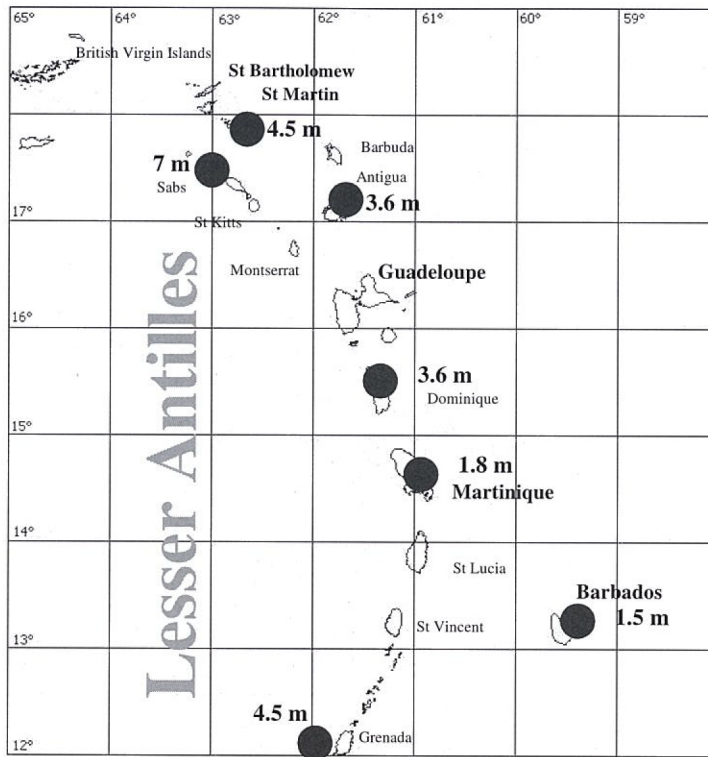


Figure 1 The map of the 1755 event for Lesser Antilles (hereafter circles correspond to runup location).

The second teletsunami (1761, 31 March) was caused by a great earthquake in Lisbon, Portugal (magnitude value and focal depth are unknown; epicenter coordinates 37°N 10°W [2]). "The waves were noted throughout the British Isles and the Low Countries as well as on Barbados (1.2–2.4 m)" (O'Loughlin & Lander, 2003). The tsunami validity for Barbados is estimated as a definite event. Taking into account that this event was not recorded on the French West Indies and the earthquake magnitude is unknown, we do not include this event in list of tsunamis for the French West Indies.

So, only one teletsunami (1755 event) is a **true tsunami** in the French West Indies.

3. Volcanic tsunamis in French West Indies

A total of nine tsunamis of volcanic origin were reported in the literature for the French West Indies between 1498 and 2006. They are related with a series of the eruptions of the Mt Pelée Volcano (14.82°W 61.17°N) in Martinique in 1902, and the Soufrière Hills Volcano (16.72°W 62.18°N) in Montserrat in 1995–2006. Below the list of true and almost true tsunamis of volcanic origin is given.

Mt Pelée Volcano, Martinique

- Four tsunami events (Fig. 2)

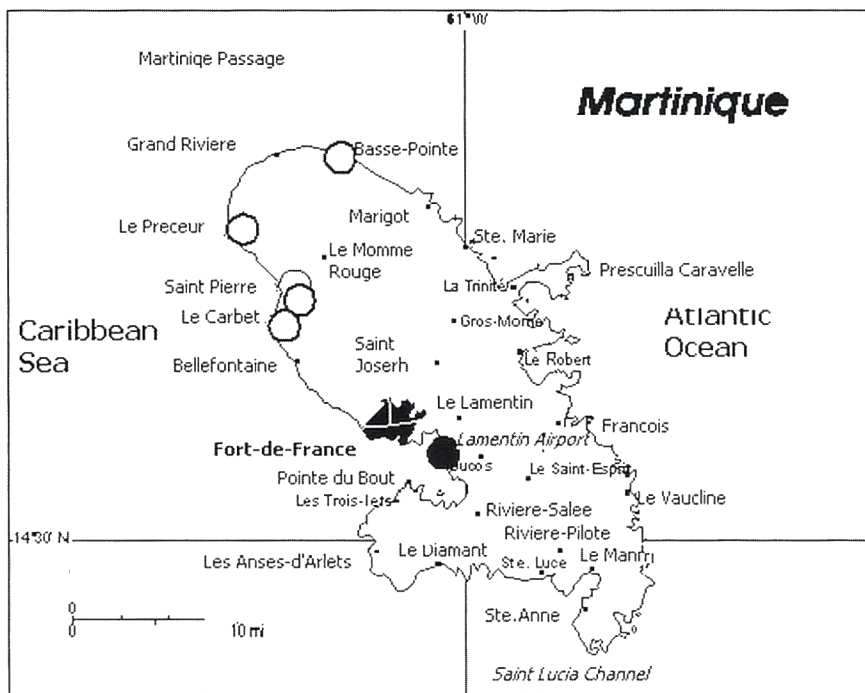


Figure 2 Map for 1902 events (white color – 5 and 8 May, grey color – 20 May, dark color – 30 August).

1902, May 5

“On May 5, the crater rim gave way, sending a torrent of scalding water cascading down the River Blanche. The hot water mixed with loose pyroclastic debris with a downslope speed of nearly 100 kilometers per hour. Near the mouth of the river, north of St. Pierre, it overran a rum distillery, killing 23 workmen. The lahar generated a three-meter-high tsunami which flooded the low-lying areas along the waterfront of St. Pierre” (Contour, 1989).

1902, May 8

A large black cloud composed of superheated gas, ash and rock rolled headlong down the south flank of Mt. Pelée at more than 100 miles per hour (about 45 meters per second) in less than one minute, which struck St. Pierre with hurricane force. Of the 28,000 people in St. Pierre, there were only two known survivors. *“Every craft heeled over to the great tidal wave and seemed to careen and sink. The sea was rolling like the heaviest kind of ground swell. Following the wave, the sea was a confused mass of boiling mud”* (O’Loughlin & Lander, 2003). *“The fall of lava into the sea had pushed all the water out to the open ocean, as if trying to topple the harbor into the Atlantic a league away”* (NGDC, 2006). According to (Lander et al., 2002) *“nuee ardente (blazing cloud of pyroclastic flow) entered the harbor”* in Saint-Pierre. At least twenty ships were destroyed.

1902, May 18–20

"20 May the sea itself seems troubled. It has invaded Le Prêheur, 14.8°N 61.22°W (NGDC, 2006), undermining several houses, and adding the ravages of inundation to those of fire. A severe inundation at Basse Pointe, 14.87°N 61.12°W (NGDC, 2006), on the northeast coast of this island, swept away twenty houses. A tidal wave had destroyed a portion of the village of Le Carbet, 14.7°N 61.18°W (NGDC, 2006), and added to the terror of the population in Fort-de-France". Meanwhile according to (HTDB/ATL, 2002) a wave was recorded in Saint-Pierre.

1902, August 30

"The sea retreated from Fort-de-France, followed by a more rapid rise of 1 m, which covered the quays and inundated to the border of the savanna" (O'Loughlin & Lander, 2003).

Soufrière Hills Volcano, Montserrat, 1995–2006

Three tsunami events.

1997, December 26

3-meter runup was observed in Montserrat, at Old Road Bay where the wave rolled northwards parallel to the coast, ending up at Old Road Bay, 10 km from the White River Valley. Debris avalanche from the volcano entered in the sea through the River Valley (Fig. 3). "The wave was estimated to have been about 1 m higher than the road which lies 2-m above water level, and to have moved inland a maximum distance of 80 m. A variety of objects, including a small wooden boat, a roof to a shelter, and a stone table were displaced several meters inland and a large log was carried even farther by the wave.

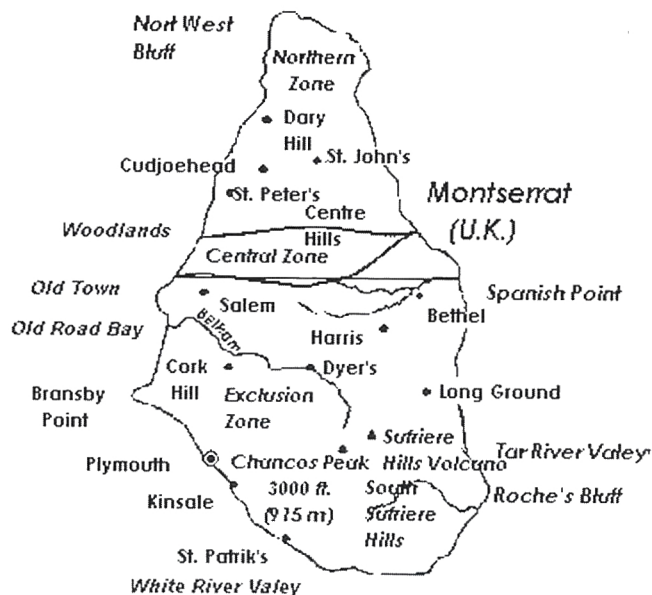


Figure 3 Chart of Montserrat, Lesser Antilles.

Impact marks up to 1 m were also reported on the side of palm trees facing the sea". This event was modeled by Heinrich *et al.* (1998). Formally, the tsunami was not recorded in the French West Indies. Meanwhile according to the numerical modeling (Heinrich *et al.*, 1999), tsunami waves of 2–3 m could reach Guadeloupe and Antigua (50 km from Montserrat) in 10 minutes. Based on the two similar events of 12.07.2003 and 20.05.2006 (see below) and numerical simulations of Heinrich *et al.* (1999) we estimate the 1997 tsunami as **an almost tsunami** for Guadeloupe.

2003, July 12

The cause of tsunami was the collapse of a lava dome that occurred at the Soufrière Hills Volcano in Montserrat, the Lesser Antilles. A tsunami was generated when a very large pyroclastic flow impacted the sea and pyroclastic surges devastated 7 km² on the NE flank of the volcano. According to Pelinovsky *et al.* (2004) in Montserrat a strandline of charred trees and other floating objects was found at Spanish Point (Fig. 3) about 3–4 km north of the mouth of the Tar River Valley where the pyroclastic flows impacted the sea on 12–13 July. The strand-line is located between 100 and 150 m from the shoreline at a height of 4 m above sea level (eye estimates). For *Guadeloupe* (Fig. 4), in Deshaies water rose 1.5 m near the entry to the port and surged 25 m inland; on the nearest beach (Plage de la Perle) the sand level had increased by up to 50 cm. Data of this tsunami is given in the field report (Pelinovsky *et al.*, 2004). The 2003 tsunami was modeled by Pelinovsky *et al.* (2004). According to these calculations, the maximum wave amplitudes were equal to 1.6 m at Guadeloupe.

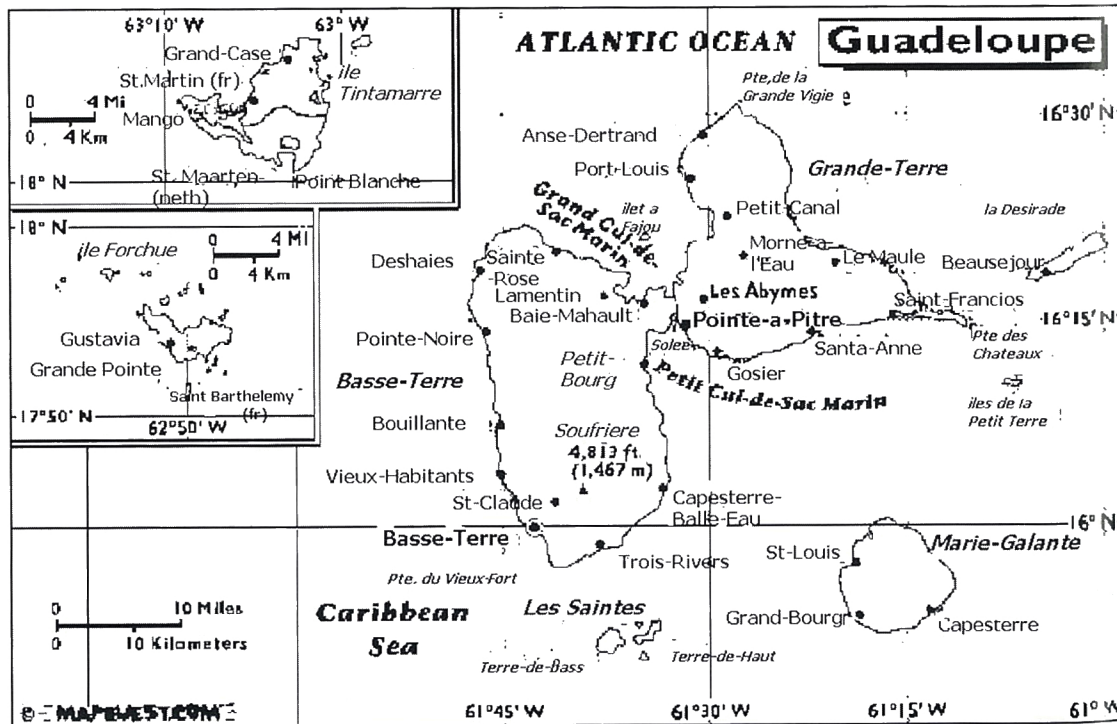


Figure 4 Chart of Guadeloupe, Lesser Antilles.

2006, May 20

A major lava dome collapse took place at the Soufriere Hills Volcano and a volume of the lava dome was believed to be about 90 million cubic meters and most of this collapsed over a period of less than three hours. Regular pulses of pyroclastic flows were reaching the sea down the Tar River valley with major pulses recorded in seismic amplitude at 07:36, 07:43 and between 08:01 and 08:04. At 07:43 pyroclastic surges were observed spreading across the northeastern flanks of the volcano reaching the Spanish Point area (MVO, 2006). According to the Zahibo (2006), the effects of the tsunami were felt in the harbour of *Deshaies* (Guadeloupe). The fisherman said "the water was at the level of its calf near his boat and he felt the water rising gradually to its thigh. He saw great water swirls at the entry of the mouth of the river and the marina. The level of the river roused quickly to a height of approximately 60–70 cm. A great current started to be established between the river and the marina which are actually connected by large tubes. These exchanges between the marina and the river sometimes caused drawdown in the marina from approximately 1 meter (the depth of the marina is about 4.5–5 m). This phenomenon lasted nearly one hour. Two small fishing boats moored in the mouth were slightly damaged. This one sank at the riverbed. The height of this tsunami in this harbour is less than one meter." Also, swells were detected in Little Bay, Montserrat and at Jolly Harbour and English Harbour, Antigua (MVO, 2006).

Events (March 6, 1718 and February 17, 1843) are estimated in literature as questionable tsunamis and we have no data to believe that these events really occurred.

Thus the total number of true and almost true volcanic tsunamis is equal to seven; these volcanic tsunamis of the French West Indies are presented in Table 1.

4. Tectonic tsunamis in French West Indies

In total, seventeen tsunamis of tectonic origin were reported in literature for the French West Indies between 1498 and 2005. The list of tectonic tsunamis estimated as true or almost true events is given below in the chronological order.

1690, April 6

"At St. Thomas, after the earthquake the sea receded so much that one could walk about 9 to 10 fathoms into the sea and pick out fishes on dry land, "retreated vertically" 16.5–18.5 m. **Martinique**, Barbados, and St. Lucia were less strongly affected" (O'Loughlin and Lander, 2003). "The earthquake caused the collapse of stone buildings at Nevis, where landslides generated on volcanic Nevis Peak caused the sea

Table 1 True and almost true volcanic tsunamis in French West Indies.

	Date	Volcano	Runup Location	Runup height, m
1	1902, May 5	Mt. Pelée	St Pierre, the bay in the Blanche River, Martinique	3
2	1902, May 8	Mt. Pelée	St Pierre, Martinique	3
3	1902, May 18–20	Mt. Pelée	Le Prêheur, Basse Pointe, Le Carbet, Saint-Pierre, Martinique	
4	1902, August 30	Mt. Pelée	Fort-de-France, Martinique	1
5	1997, December 26	Soufrière Hills	Northern Guadeloupe	2–3 (modeled)
6	2003, 12–13 July	Soufrière Hills	Plage de la Perle and Deshaies, Guadeloupe	0.5–1
7	2006 20 May	Soufrière Hills	Deshaies, Guadeloupe	1

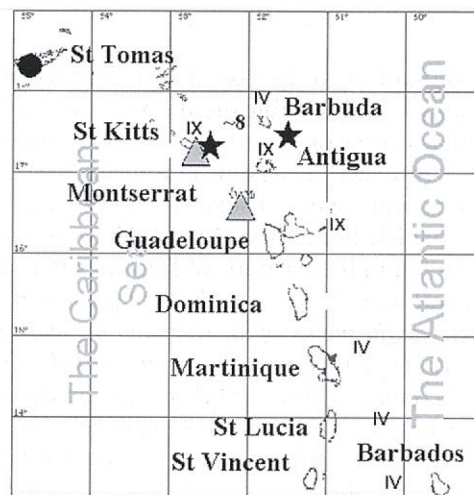


Figure 5 Map of the 1690 event (hereafter earthquake epicenters are shown by stars, intensity – by Roman numbers, landslides – by triangles).

to withdraw 201 m from Charleston, before returning in two minutes. *Guadeloupe* also incurred much damage". Also at "a rocky islet called Rockadunda (Redonda Rock Island), a great part of it split and turned into the sea". Tsunami intensity is estimated as 1–2 for the Lesser Antilles and US Virgin Islands. The validity of tsunami is estimated in literature as *definite event*. The source of this tsunami was a terrible earthquake (magnitude $M \sim 8$, focal depth 33 km); its intensity in Antigua was 9, St Kitts – 9, and particularly 4 in *Martinique* (Fig. 5). The exact date is unknown; perhaps it was 26 February, 5 April, 10 April, or 16 April. The coordinates of the earthquake epicenter are given in literature: $17.2^\circ\text{N } 62.5^\circ\text{W}$ for 5 April (HTDB/ANL, 2002) and $17.5^\circ\text{N } 61.5^\circ\text{W}$ for 16 April (HTDB/ANL, 2002).

We may conclude that this earthquake tsunami was true, but not recorded in the French West Indies. Taking into account the large area of the earthquake manifestation, the tsunami in the French West Indies can be estimated as an **almost true tsunami**.

1767, April 24

At *Martinique* and Barbados (Fig. 6), "the sea ebbed and flowed in an unusual way. At the same time, at *Martinique*, the waters of the sea were very agitated". Tsunami had intensity 1 on Soloviev-Imamura Scale (0.5 m). The validity of tsunami is estimated in literature from questionable to probable. We believe that this tsunami can be estimated as an **almost true** event in the French West Indies. Tsunami cause is considered to be an earthquake (magnitude 7) with epicenter coordinates $14.5^\circ\text{N } 61^\circ\text{W}$ (HTDB/ANL, 2002), or $14.4^\circ\text{N } 61^\circ\text{W}$ (NGDC, 2005) in the southern coast of *Martinique*.

1823, November 30

"A tidal wave" (tsunami intensity 2) occurred in Saint-Pierre, *Martinique*, "the sea rose after the shock, and occasioned some damage in the harbors. It was preceded by a very intense noise" (O'Loughlin

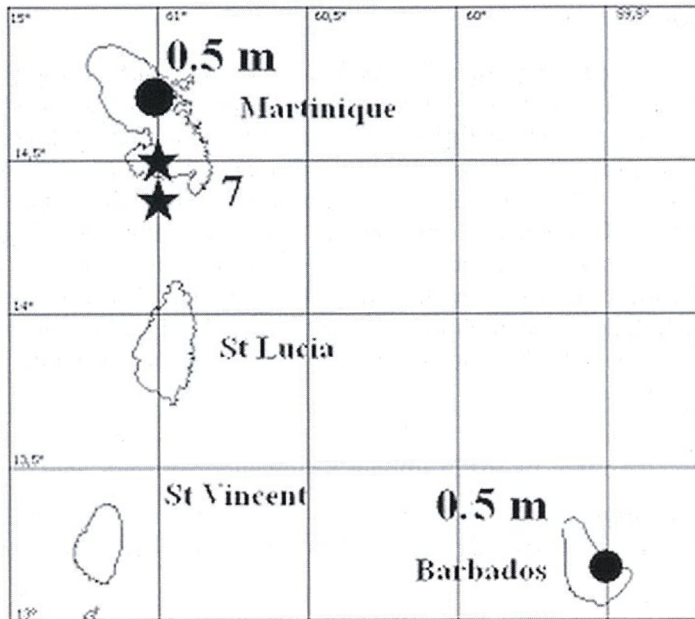


Figure 6 Map of 1767 event (hereafter tsunami runup is given by numbers).

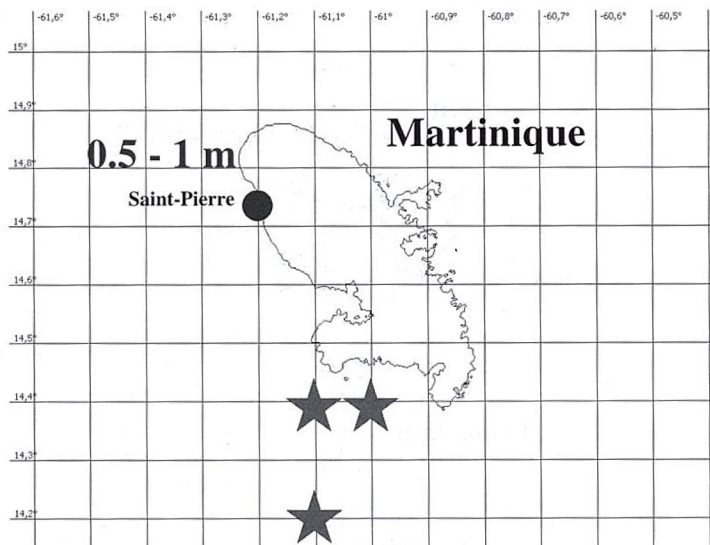


Figure 7 Map of 1823 event.

and Lander, 2003). The tsunami validity is estimated as a **true event** in literature. The source of the tsunami was the earthquake with the epicenter coordinates 14.2°N 61.1°W (HTDB/ATL, 2002), or 14.4°N 61.1°W (NGDC, 2005), or 14.4°N 61°W (NGDC, 2005). The locations of the earthquake epicenter and tsunami observations are presented in Fig. 7.

1827, November 30

"Seauquake effects or a tsunami may have disturbed the ships 100 nautical leagues [564 km] west of **Martinique**. All the vessels near **Pointe-à-Pitre (Guadeloupe)** and in the roadsteads of **Port Royal (Fort-de-France?)** and **Saint-Pierre (Martinique)** also experienced the shock" [3]. Tsunami was caused by an earthquake: magnitude 6.5; epicenter coordinates 14.5°N 61°W (HNDB/ATL, 2002) or 14.4°N 61°W (NGDC, 2005), the shocks are reported in Fort-de-France and Saint-Pierre, **Martinique** (intensity 6); **Guadeloupe** (intensity 4). The map of this event is given in the Fig. 8. The tsunami validity is estimated from doubtful to questionable. Although the magnitude of the earthquake that caused the 1867 tsunami was rather low (6.5), it was higher than the magnitude of the 2004 earthquake in Guadeloupe (6.3) that caused a definite tsunami 50 km from the source (see below). We believe that the tsunami in 1827 could be estimated as an **almost true** tsunami for the French West Indies.

1839, January 11

"In the harbor of **Martinique**, all the ships were violently shaken during the shock that lasted almost 40 seconds. This may have been seaquake effects, although the possibility cannot be ruled out that a

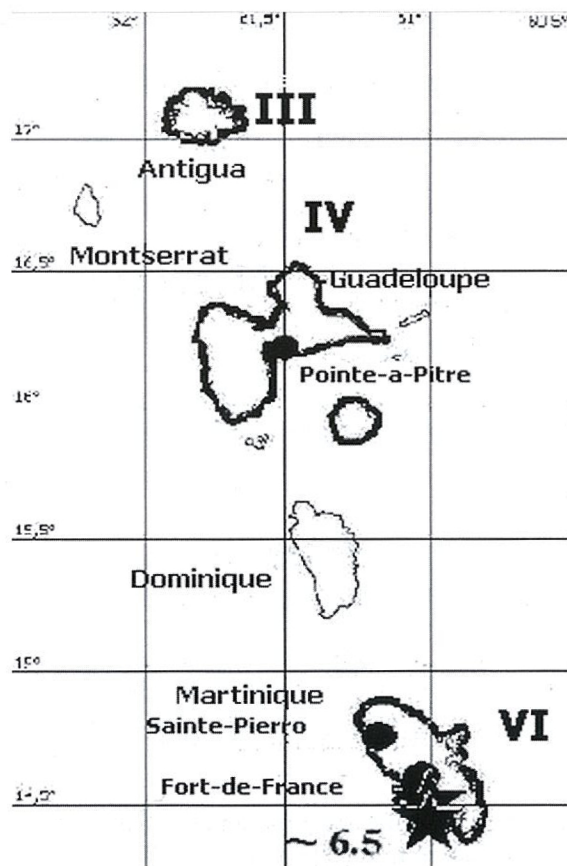


Figure 8 Map of 1827 event.

tsunami added to the tumult". The cause of possible tsunami was the terrible earthquake (magnitude 7.8; focal depth 33 km; epicenter coordinates 14.4°N 61°W) that occurred at Fort-de-France and St Pierre, *Martinique* (intensity 9), and was felt in *Guadeloupe* and other the Antilles. The tsunami validity is estimated as *doubtful tsunami*, but we believe that the tsunami generated by this strong earthquake could have occurred in Martinique and we estimated this event as **almost true**.

1842, May 7

A huge tsunami with intensity of 3 on Soloviev-Imamura Scale (1 m) occurred on the Island of Haiti. Tsunamis also were reported in Grenada, St Vincent and the Grenadines and St John, but the maximum runup reported was 18.3 m in Guadeloupe. According to (NGDC, 2005) "*20 m wave carried off floatable objects*" in Sainte-Rose and Deshaies, *Guadeloupe* and 0.8–0.9 m above sea level in Basse-Terre, *Guadeloupe*. The map of the event is presented in the Fig. 9. The cause of the tsunami was an earthquake with a magnitude of 8.1 that occurred in Haiti (19.75°N 72.2°W (NGDC, 2005) or 18.5°N 72.5°W (HTDB/ATL, 2002)). Tsunami validity is estimated as a *true* event. Concerning the French Indies perhaps this information was mixed with the data of 1867 (see below). That is why we estimate the 1842 event as an **almost true** tsunami.

1843, February 8

According to (NGDC, 2005) "*sea rose 1.2 m then fell*" in *Guadeloupe*. The tsunami intensity was estimated as 1 on Soloviev-Imamura scale (0.5 m). The source of this tsunami was the "*disastrous earthquake* (magnitude 8.3; focal depth 33 km; epicenter coordinates 16.5°N 62.2°W)

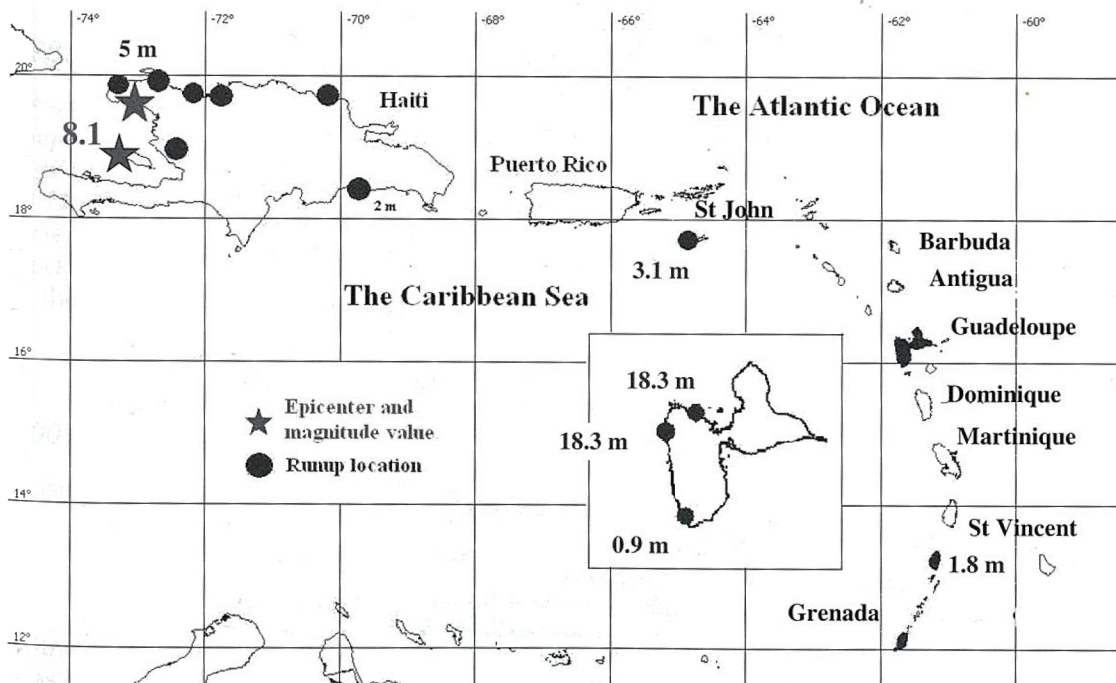


Figure 9 Map of 1842 event.

that struck *Guadeloupe*, leaving *Pointe-a-Pitre* in ruins" (intensity 9). The earthquake ravaged *Marie-Galante*, *St. Desirade* (*La Desirade*), *Martinique* (intensity 7); *St. Bartholomew*, *St. Martin*. Almost all the Antilles and Guyana felt the effects. Meanwhile (HNDB/ATL. 2002) gives Antigua and *Guadeloupe* as a tsunami source area. Furthermore, in sight of the Redonda Rock Island "a mighty chunk of the rocky mass broke off and fell into the sea, which resulted in a surging rise of the ocean". Tsunami validity is estimated as a **true** event in literature.

1867, November 18

The huge tsunami (intensity is estimated as 3 on Soloviev-Imamura scale) crossed all the Lesser Antilles; the map of event is presented in the Fig. 10. "Damaging waves inundated *St. Martin*, and *St.-Barthélemy*. At 2:50 p.m., the 2.1-m-deep *St.-Barthélemy*, Carenage [Gustavia – our mark] was left dry. Vessels were abruptly deposited on the ground and several were set adrift by the return wave. A schooner rushed about with the currents, at a rate of 10 knots, and was not able to be secured until 8 p.m. The wharves were submerged by 1 m of water." "Waves were noted in *Martinique*". All of Fond-du-Cure, *Îles des Saintes* (15.8°N 61.5°W) was submerged; the water invaded the houses to a height of 1 m. The inhabitants ran away before the looming sea. In Basse-Terre, *Guadeloupe* an extraordinary



Figure 10 Observed runup amplitudes (m) during the 1867 tsunami (Zahibo et al., 2003).

phenomenon was produced in the roadstead. The sea swelled without any outward sign: no waves, no backwash. It rose 1 m in the span of 5 minutes, floating boats tied on the bank. It then dropped in the same span of time, retiring 8 to 10 m, leaving places that were always submerged, revealed. The same phenomenon was produced again, and then the sea went back to normal. The difference in the extremes of the water levels was not less than 2 m. The wave at Pointe-à-Pitre, **Guadeloupe** was hardly more than a swell. At Sainte-Rose, **Guadeloupe** the ocean suddenly withdrew more than 100 m from the shore. This retreat was preceded by slight oscillations from the earthquake, which lasted perhaps an estimated 5 or 6 seconds. Then all a first wave, of 60 feet [19.8 m] in elevation, rising to the north, approximately 3 milles [5.6 km] broad, rolled violently towards the land, where it crashed, immersing the whole shore, and swamping the houses that were there. Boats drawn up on shore, fishing nets, wood, and other materials—everything was left adrift. A second and third of these enormous waves rolled in from north to south, one after another, at short intervals, and like the first one, overwhelmed everything along the way". "At Sainte-Rose there was a 10 m wave." On the northwest shore at Deshaies, **Guadeloupe** a tsunami ravaged the town and smashed nearly all the houses. Everyone has taken refuge in the church". "Houses destroyed, sea receded 100 m and returned as an 18.3 m wave". Meanwhile Zahibo and Pelinovsky (2001) inspecting historical material conclude that the wave height at Deshaies and St Rose could not exceed 10 m, a rather 5 m. Then they simulated this event in the framework of the nonlinear shallow-water theory and obtained a very good correlation between computed and historic data for all the Lesser Antilles except Deshaies and St Rose, Guadeloupe, and concluded from this analysis that observed waves in these locations should be about 2–5 m (Zahibo *et al.*, 2003a). The cause of the tsunami was an earthquake occurred that in the Virgin Islands, 5–20 km southwest of St. Thomas (magnitude 7.5; epicenter coordinates 18.4°N 64.3°W). This event is a **true tsunami**.

1985, March 16

"A several-centimeter [0.10 – to 0.12-m] of tsunami was recorded at tide-gauge in Basse-Terre, **Guadeloupe**". It generated by the earthquake (magnitude 6.3; focal depth 13 km; epicenter location is 17.013°N 62.448°W (NGDC, 2005), or 17°N 62.5°W (Zahibo and Pelinovsky, 2001), or 16°N 61.72°W (HTDB/ATL, 2002)) which was also felt in **Martinique**, in Point-à-Pitre, **Guadeloupe** (intensity 6) and through the other Antillean islands. Furthermore, the landslide at Redonda Rock Island was reported by O'Loughlin and Lander (2000). The map of the event is presented in the Fig. 11. Tsunami is a **true event**.

2004, November 21

A tsunami was reported for **Guadeloupe** (Zahibo *et al.*, 2005): Village of Trois-Rivières (depression was about 50 cm), Marina de Rivière Sens, city of Basse-Terre (weak depression), for **Les Saintes**, **Island of Terre-de-Bas**: Village of Grande Anse (sea receded 5 m and dropped 80 cm), Beach of Grande Anse (70 cm height), Village of Petite Anse (water receded a distance of 2 to 3 m); and for the **Island of Terre-de-Haut**: Baie du Marigot ("an unusual surge" reported by a witness); see Fig. 12. For the French West Indies this tsunami is a **true tsunami**. The tsunami was caused by an earthquake which occurred in Dominica Passage (between Guadeloupe and Dominica): magnitude 6.3; focal depth of 14 km; coordinates 15.677°N 61.650°W.

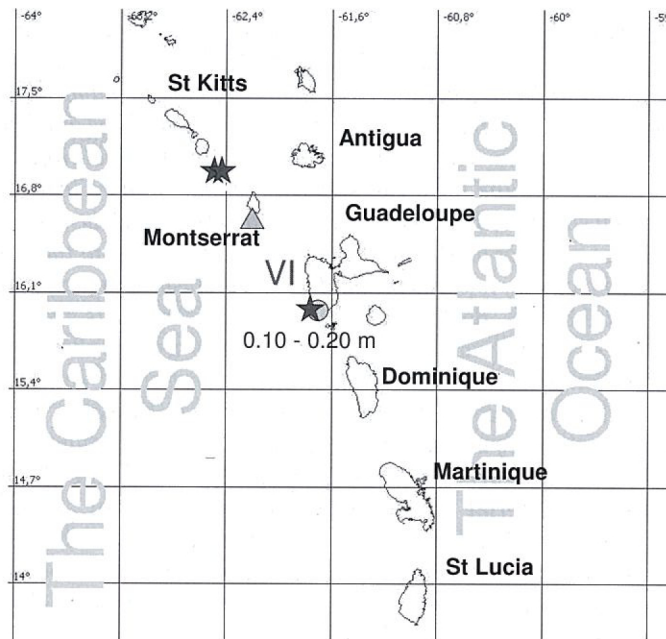


Figure 11 Map for 1985 events.

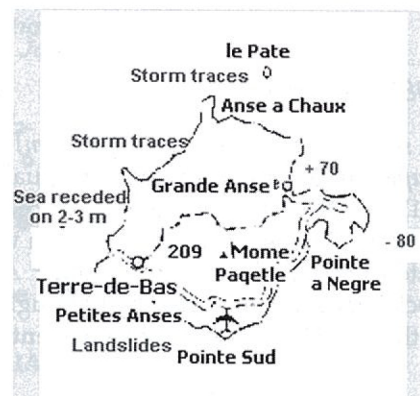
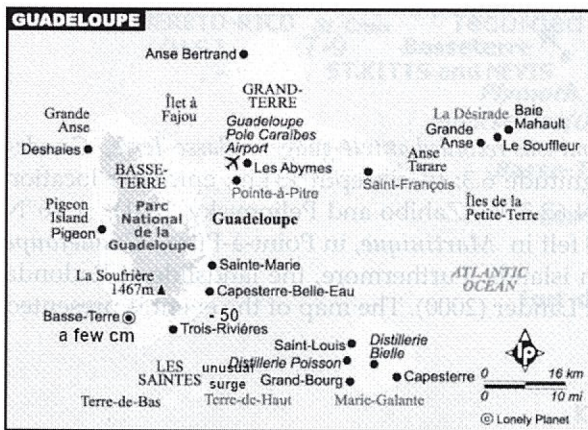


Figure 12 Map of the 2004 event with location of tsunami height (Zahibo et al., 2005).

Thus, we the number of true and almost true tsunamis caused by tectonic events is equal to ten, their characteristics are given in a Table 2.

The following events of September 1702, 6 March 1718, 21 November 1751, 9–13 September 1824 and 30 November 1824) are not listed here because they are not estimated in literature as real events.

Table 2 Tectonic Tsunamis in French West Indies.

Date	Epicenter characteristics		Runup characteristics			
	Location, Coordinates	M	Maximum runup, m	height m	Island	Runup location Coordinates
1 1690 04 06	17.5°N 61.5°W	~ 8			Guadeloupe, Martinique	16.25°N 61.583°W
2 1767 04 24	14.4°N 61°W	7	0.5		Martinique	14.5°N 61°W
3 1823 11 30	14.4°N 61.1°W		1		Martinique	14.73°N 61.18°W
4 1827 11 30	14.5°N 61°W	~ 6.5			Martinique	
5 1839 01 11	14.4°N 61°W	7.8			Guadeloupe	
6 1842 05 07	19.75°N 72.2°W	8.1	18.3 (20?)		Martinique	16.33°N 61.7°W
			0.8–0.9		Guadeloupe	16.3°N 61.58°W
7 1843 02 08	16.5°N 62.2°W	8.3	0.5	1.2	Guadeloupe	16°N 61.73°W
8 1867 11 18	18.4°N 64.3°W	7.5			Guadeloupe	16.25°N 61.58°W
			1		St. Martin	18.08°N 63.08°W
			1		St. Barthelemy	17.9°N 62.83°W
				2	Les Saintes	15.8°N 61.5°W
					Guadeloupe	16°N 61.72°W
			10			17.9°N 62.83°W
			18.3 (10?)			16.33°N 61.7°W
9 1985 03 16	17.013°N 62.448°W	6.3			Martinique	16.3°N 61.78°W
10 2004 11 21	15.677°N 61.65°W	6.3		0.10–0.12	Guadeloupe	14.67°N 61°W
			0.5		Guadeloupe	16°N 61.72°W
					Guadeloupe	15.58084°N
					Les Saintes, Island of Terre-de-Bas	61.38695°W
			0.8			15.51260°N
			0.7			61.36968°W
						15.51547°N
			0.5		Les Saintes, Island of Terre-de-Haut	61.37/47°W

5. Analysis of tsunami occurrence for French West Indies

The revised data of tsunami in the French West Indies for 1498–2006 is analysed. Geographically, the French West Indies could be divided into three groups: St Barthelemy and St Martin, Guadeloupe and neighbor islands (La Desirade, Les Saintes and Marie-Galante), and Martinique. The majority of tsunamis occurred in Martinique. Figure 13 below compare the number of events occurred in Martinique and in Guadeloupe.

The total number of tsunamis is eighteen (ten of tectonic origin, seven of volcanic origin and one teletsunami); see Fig. 14.

The 508 years (1498–2006) period is analyzed. The variability of the number of tsunamis in 50 years intervals is found (Fig. 15). On average, two events occurred each 50 years. Anomalous time for tectonic tsunamis it was between 1800 and 1850, and for volcanic tsunamis it was between 1900 and 1950.

The total amount of tectonic tsunamis is ten, for nine of them magnitude values are given. The cumulative frequency of tsunamis generated by earthquakes with given parameters is calculated (Fig. 16).

These data are fitted very well by the exponential curve with $R = 0.94$ (time is in years):

$$\ln(N/T) = -0.5M - 0.95, \quad (1)$$

As a result, the return period versus magnitude can be determined,

$$T = 2.58 \exp(0.5M). \quad (2)$$

So, the tsunamigeneric earthquakes expected for 100 years has a magnitude 7.3, and this value corresponds to other active tsunamigeneric areas in Pacific. It demonstrates the high tsunami

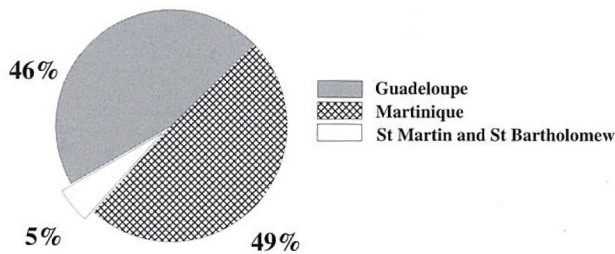


Figure 13 Geographic distribution of tsunamis in French West Indies.

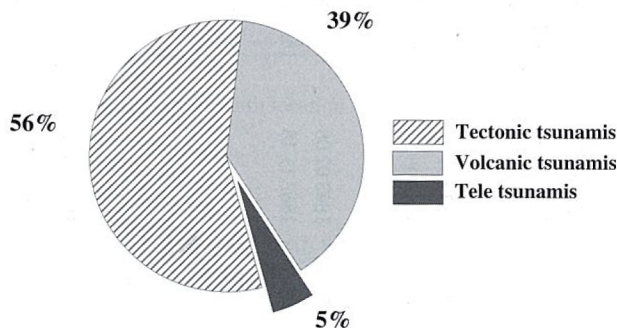


Figure 14 Tsunamis of different origin in French West Indies.

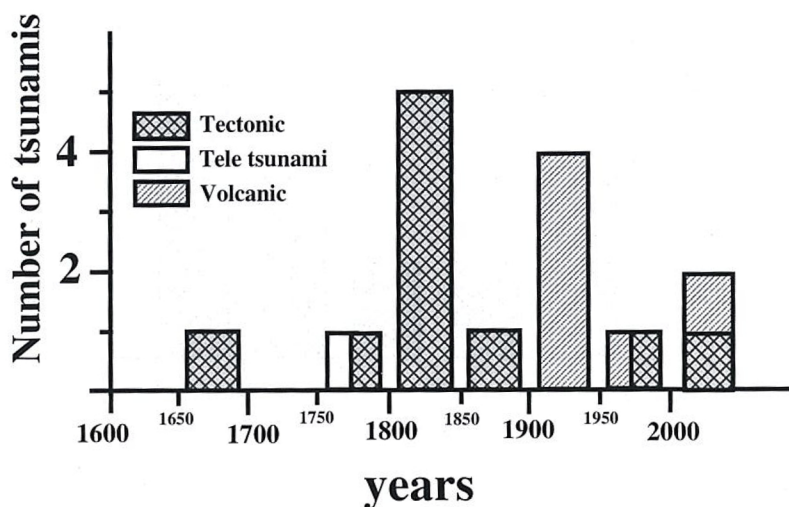


Figure 15 The variability of the number of tsunamis in 50-yr periods.

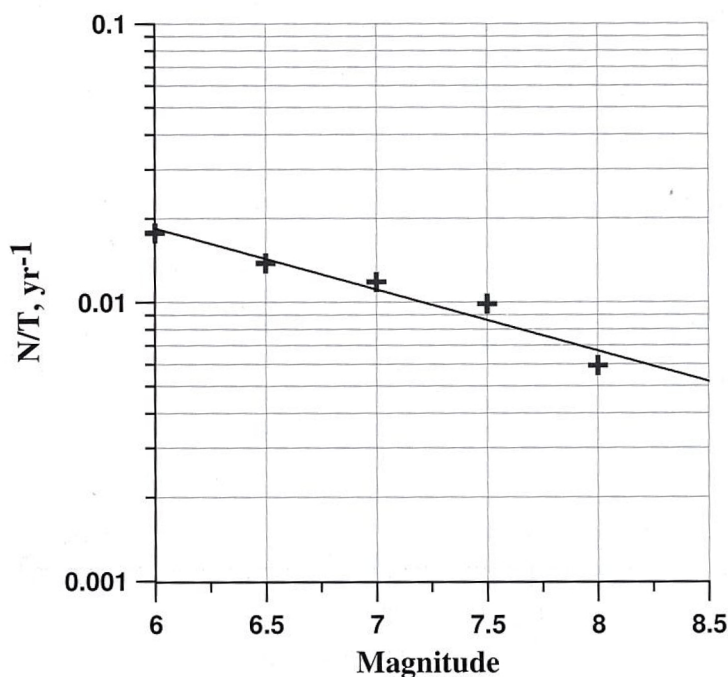


Figure 16 Cumulative frequency of tectonic tsunamis versus magnitude.

potential for the Lesser Antilles. In fact, the earthquakes with a magnitude of more than 6.3 can generate tsunami; see Fig. 17. Meanwhile, the runup values are over the range 0.1 m to 1 m irrespective of magnitude (Fig. 18), if the values of 10, 18 and 20 m (doubtfulness of these data was discussed previously) are not taken into account.

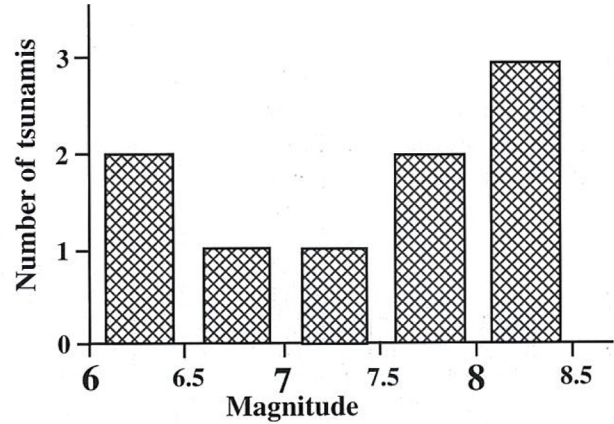


Figure 17 The distribution of tectonic tsunamis on magnitude.

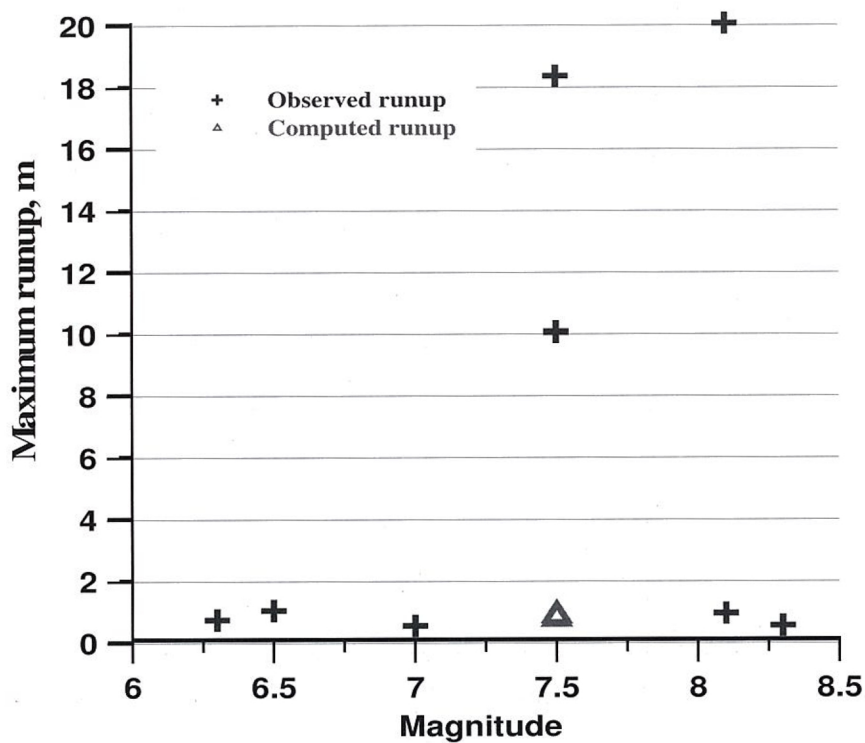


Figure 18 Runup distribution versus magnitude.

6. Conclusion

Tsunami in the French West Indies is a frequent event; and eighteen tsunamis are selected as true and almost true. Ten events have been generated by underwater earthquakes; seven – by the volcano eruptions, and one is a teletsunami. Tsunamis of volcanic origin are induced by

volcano in Martinique and Montserrat. Tsunamis of seismic origin are generated by earthquakes with magnitudes of more than 6.3. Meanwhile, the runup values are over the range 0.1 m to 1 m irrespective of magnitude for tectonic events (expect abnormal values of the height in 10 or 18 m for Deshaies, Guadeloupe). The number of tsunamis in Martinique and Guadeloupe is almost the same.

Acknowledgement

The study is supported particularly by the University of Antilles Guyane and RFBR (05-05-64265).

References

- Contour S. (1989). *Sainte-Pierre (Martinique). La catastrophe et les suites*, Editions Caribéennes.
- Heinrich F., Guibourg S., Mangeney A. and Roche R. (1999). Numerical modelling of a landslide-generated tsunami following a potential explosion of the Montserrat Volcano. *Phys. Chem. Earth*, **A24**:163–168.
- Heinrich F., Mangeney A., Guibourg S. and Roche R. (1998). Simulation of water waves generated by a potential debris avalanche in Montserrat, Lesser Antilles. *Geophys. Research Letters*, **25**:3697–3700.
- Heinrich F., Boudon G., Komorowski J. C., Sparks R. S. J., Herd R. and Voight B. (2001). Numerical simulation of the December 1997 debris avalanche in Montserrat. *Geophys. Research Letters*, **28**:2529–2532.
- HTDB/ATL. (2002). *Expert Tsunami Database for the Atlantic*. Version 3.6 of March 15, 2002. Tsunami Laboratory, Novosibirsk, Russia.
- Lander J. F., Whiteside L. S. and Lockridge P. A. (2002). A brief history of tsunami in the Caribbean Sea. *Science of Tsunami Hazards*, **20**:57–94.
- Mader C. L. (2001). Modeling the 1755 Lisbon tsunami. *Science of Tsunami Hazards*, **9**:93–98.
- Mercado A. and McCann W. (1998). Numerical simulation of the 1918 Puerto Rico tsunami. *Natural Hazards*, **18**:57–76.
- MVO. (2006). *Dome collapse report*. www.mvo.ms/Dome_collapse_report.html
- NGDC (National Geophysical Data Center) Tsunami Database. (2006). <http://www.ngdc.noaa.gov/seg/hazard/tsu.db.shtml>
- O'Loughlin K. F. and Lander J. F. (2003). *Caribbean Tsunamis: A 500-Year History from 1498–1998*. Advances in Natural and Technological Hazards Research, V. 20, Kluwer.
- Pelinovsky E., Zahibo N., Dunkley P., Edmonds M., Herd R., Talipova T., Kozelkov A. and Nikolkina I. (2004). Tsunami generated by the volcano eruption on July 12–13, 2003 at Montserrat, Lesser Antilles. *Science of Tsunami Hazards*, **22**(1):44–57.
- Weissert T. P. (1990). Tsunami travel time charts for the Caribbean. *Science of Tsunami Hazards*, **8**:67–78.
- Zahibo N. and Pelinovsky E. (2001). Evaluation of tsunami risk in the Lesser Antilles. *Natural Hazard and Earth Sciences*, **3**:221–231.
- Zahibo N., Pelinovsky E., Yalciner A., Kurkin A., Kozelkov A. and Zaitsev A. (2003a). The 1867 Virgin Island Tsunami: observations and modelling. *Oceanologica Acta*, **26**:609–621.
- Zahibo N., Pelinovsky E., Kurkin A. and Kozelkov A. (2003b). Estimation of far-field tsunami potential for the Caribbean Coast based on numerical simulation. *Science Tsunami Hazards*, **21**(4):202–222.
- Zahibo N., Pelinovsky E., Okal E., Yalciner A., Kharif C., Talipova T. and Kozelkov A. (2005). The earthquake and tsunami of November 21, 2004 at Les Saintes, Guadeloupe, Lesser Antilles. *Science of Tsunami Hazards*, **23**(1):25–39.
- Zahibo N. and Pelinovsky E. (2006). Tsunamis in the Lesser Antilles. *Caribbean Tsunami Hazard* (Eds. A. Mercado-Uriazary and P. Liu). World Sci., Singapore, 244–254.
- Zahibo N. *Tsunami Bulletin Board*, 30 May 2006.

3. Les houles cycloniques: cas de la Guadeloupe

Selon les statistiques, les inondations occupent la première place mondiale (nombre de personne tué) des catastrophes naturelles. Parmi les sources d'inondation on distingue les ouragans et les cyclones dans l'océan, qui «pousse» l'eau vers la côte; les ruptures des barrages sur les rivières et les réservoirs d'eau, de fortes pluies et des rivières en crue. À la suite des fortes pluies une inondation s'est produite en Guadeloupe en janvier 2011 et a conduit à la mort de 5 personnes.

Les inondations sont les catastrophes naturelles qui se produisent souvent dans les Caraïbes, y compris en Guadeloupe. Comme il est montré plus loin dans la thèse, cet événement est plus fréquent que les tsunamis. On parlera spécialement de houles cycloniques qui brouillent la côte de l'île et endommagent l'infrastructure touristique. Une protection anthropologique a été effectuée sur la côte ouest de l'île: la couche de roche banque jusqu'à 2 mètres (Fig. 3.1). Les ondes de tempête sont des vagues longues avec des paramètres similaires au cas du tsunami. Cela nous permet d'utiliser les mêmes modèles mathématiques et numériques qui sont basés sur la théorie des vagues longues.



Fig. 3.1. Enrochement sur les berges près de Basse-Terre, la préfecture du Département de la Guadeloupe

Les principaux résultats du troisième chapitre ont été publiés:

ZAHIBO N., PELINOVSKY E., TALIPOVA T., RABINOVICH A., KURKIN A., NIKOLKINA I. Statistical analysis of cyclone hazard for Guadeloupe, Lesser Antilles. *Atmos. Res.*, 2007, vol. 84, n°1, p. 13-29.

ZAHIBO N., NIKOLKINA I., DIDENKULOVA I. Extreme waves generated by cyclones in Guadeloupe. *Extreme Ocean Waves* / ed. PELINOVSKY E., KHARIF C., Springer, 2008, p. 161-179.

3.1 Récurrence et statistiques des houles cycloniques dans les Antilles Françaises

La statistique des cyclones sur l'île de Guadeloupe pour les 500 dernières années est discutée sur la base des données historiques (Dorville et Zahibo, 2010; Saffache et al, 2002, 2003; Neumann et al, 1978). Le délai moyen d'apparition des cyclones est de 3-4 ans; et environ 75 ans pour un cyclone d'intensité IV (sur cinq dans l'échelle de Saffir-Simpson). Les données de la fréquence cumulative en fonction de la vitesse du vent et le gradient de pression atmosphérique sont approximées par les courbes exponentielles qui sont caractéristiques des statistiques extrêmes (Fig. 3.2). L'analyse a montré que les houles cycloniques sont mal corrélées avec la baisse de la pression atmosphérique mais en même temps les houles sont plus fortes pour le vent le plus fort. (Fig. 3.3).

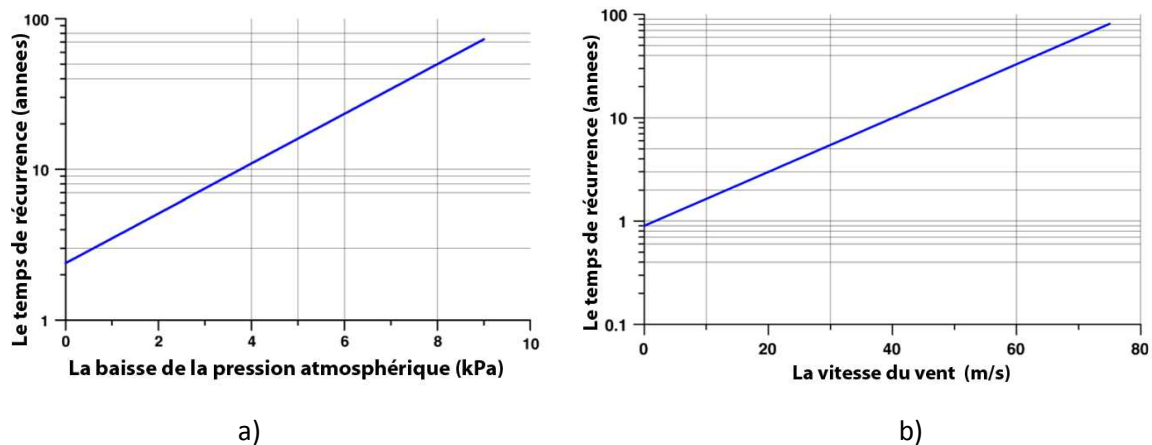


Fig. 3.2. Temps de récurrence des cyclones en fonction de la pression atmosphérique (a) et de la vitesse du vent (b)

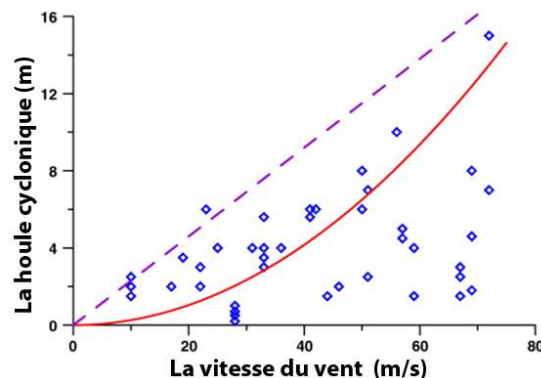


Fig. 3.3. Corrélation entre hauteurs des houles cycloniques et vitesse du vent

3.2 Etude de terrain et analyse de l'impact du cyclone Dean

La nuit du 16 au 17 août 2007 l'ouragan Dean a traversé l'arc des Antilles. Quand il est passé par le canal de Sainte-Lucie (Sud de la Martinique); il est passé en eu catégorie III sur l'échelle de Saffir-Simpson avec une vitesse moyenne de vent 160-180 km/h et des rafales de 200 km/h. Le centre de l'ouragan est passé à quelques kilomètres de la Martinique (15 km de la ville de Sainte-Anne, à 30 km de l'aéroport du Lamentin). Le cyclone Dean a pris la vies de deux personnes. Des dommages importants ont été causés en Guadeloupe et Martinique: les arbres déracinés (en particulier dans les plantations de cocotiers Faula au Vauclin au sud-est de la Martinique), les routes ont été coupées. En Martinique certaines plages (Sainte-Anne et Le Diamant) ont été immergées dans l'eau. Les bananes (100% des cultures en Martinique et 80% en Guadeloupe), les fruits tropicaux et la canne à sucre (70% de récolte en Martinique) ont été le plus touchées.

Dans les territoires français la vitesse du vent maximale a atteint 126 km/h (Les Saintes) et 119 km/h (Marie-Galante). La vitesse du vent en Guadeloupe a atteint 100 km/h sur les plaines et les 175 km/h à proximité du volcan de la Soufrière. Selon l'échelle de Saffir-Simpson, ceci correspond à des houles cycloniques de 0.09 à 3.3 m dans Grande-Terre et Basse-Terre, respectivement.

Dans ce paragraphe, nous présentons les résultats d'une enquête de terrain de l'ouragan Dean, qui a balayé les Caraïbes en août 2007, l'auteur de la thèse a participé à cette étude terrain qui a été organisé immédiatement après le cyclone pour obtenir les données des houles cycloniques. Le caractère typique d'inondations côtières est représenté sur la Fig. 3.4.



a)



b)

Fig. 3.4. La plage (a) et la place centrale (b) au Gosier

3.3 Simulation numérique des houles cycloniques causées par l'ouragan Lili en 2002

Pour produire les simulations numériques on a choisi le cyclone Lili, qui a traversé les Petites Antilles le 23-25 Septembre 2002. Les trajectoires des cyclones tropicaux dans l'océan Atlantique en 2002 sont présentées dans la Fig. 3.5 (UNISYS, 2011). Le cyclone tropical Lili est passé près de la Guadeloupe la nuit de 23 au 24 Septembre à une distance de 414 km, avec des vents violents jusqu'à 93 km/h. Le régime de la surveillance (tropical storm watch) a été déclenché immédiatement sur la Guadeloupe et les îles voisines. Plus tard, en Louisiane (USA) l'ouragan a provoqué un glissement terrain sur la côte et des inondations importantes. Au total, l'ouragan Lili a tué treize personnes, laissant 400 personnes sans-abri à la Barbade, et «ruiné» la moitié de la récolte de bananes sur Sainte-Lucie.

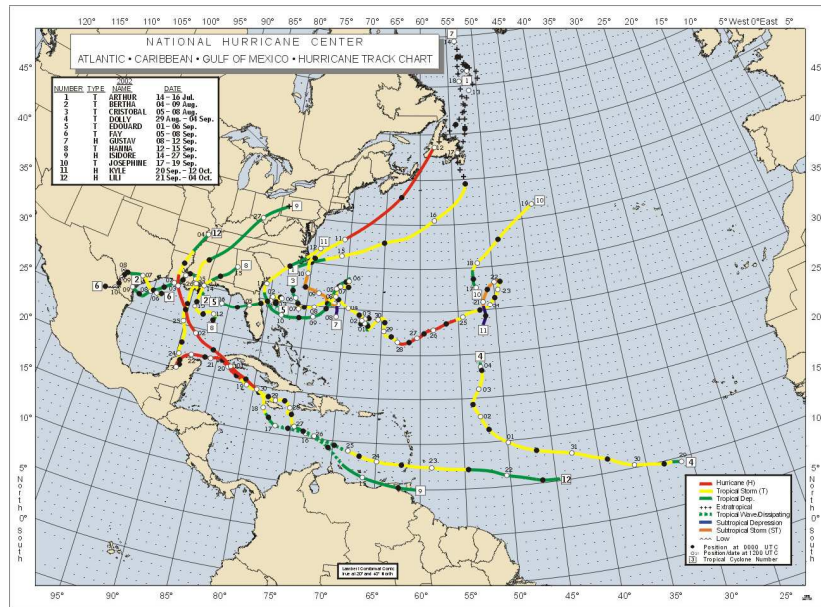


Fig. 3.5. Les trajectoires des cyclones tropicaux dans l'océan Atlantique Nord en 2002 (l'ouragan Lili est marqué par le chiffre 12)

Dans ce paragraphe on présente les résultats de simulations numériques des houles de tempête causée par le cyclone Lili en 2002.

La simulation numérique est faite dans le cadre de la théorie non linéaire de l'eau peu profonde

$$\frac{\partial M}{\partial t} + \frac{\partial}{\partial x} \left(\frac{M^2}{D} \right) + \frac{\partial}{\partial y} \left(\frac{MN}{D} \right) + gD \frac{\partial \eta}{\partial x} - fN = - \frac{D}{\rho_a} \frac{\partial P_a}{\partial x} + \frac{\tau_{sx} - \tau_{bx}}{\rho_a}, \quad (3.1)$$

$$\frac{\partial N}{\partial t} + \frac{\partial}{\partial x} \left(\frac{MN}{D} \right) + \frac{\partial}{\partial y} \left(\frac{N^2}{D} \right) + gD \frac{\partial \eta}{\partial x} + fM = - \frac{D}{\rho_a} \frac{\partial P_a}{\partial y} + \frac{\tau_{sy} - \tau_{by}}{\rho_a}, \quad (3.2)$$

$$\frac{\partial \eta}{\partial t} + \frac{\partial M}{\partial x} + \frac{\partial N}{\partial y} = 0, \quad (3.3)$$

où les différences principales des équations utilisées précédemment dans le chapitre 2 sont liées à des forces extérieures; en particulier avec le gradient de pression atmosphérique $Pa(x, y, t)$ et la tension du vent τ_s . La tension superficielle est généralement paramétrée par

$$\frac{\tau_s}{\rho} = c_* W |W|, \quad (3.4)$$

ou W est la vitesse du vent, c_* est le coefficient constant de la proportionnalité. La tension du fond τ_b est «responsable» de la dissipation de l'énergie; généralement elle est paramétrée par le terme $u|u|$, comme dans l'étude de tsunami. On utilise le mur «verticale» près de la côte et la non-réflexion pour les conditions limites comme dans le paragraphe 2.2.

On a calculé les champs de vitesse lors du passage de cyclone pour des temps différents (Fig. 3.6). Selon les données numériques de changement de niveau de la mer pour deux points en Guadeloupe (Pointe-à-Pitre, Deshaies), la hauteur des vagues ne dépasse pas 10 cm (Fig. 3.7). Étant donné que la vague augmente près de la côte (Pelinovsky, 1996), la hauteur de la vague sera de 1-2 mètres, ce qu'est habituellement observé en Guadeloupe.

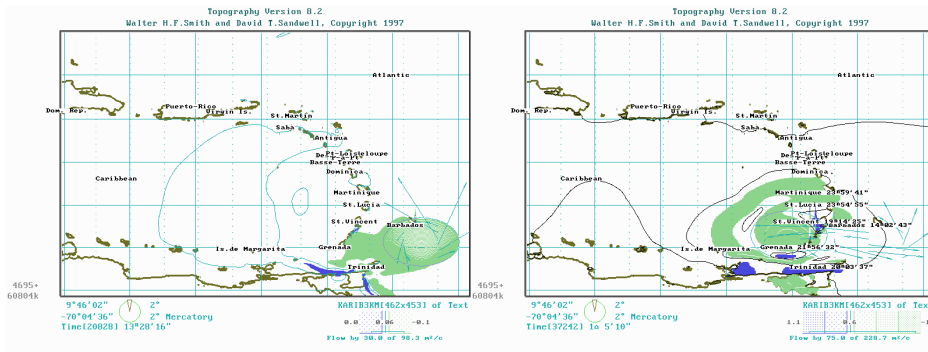


Fig. 3.6. Les iso lignes du déplacement de la surface de l'eau, causé par le cyclone Lili pour des moments différents

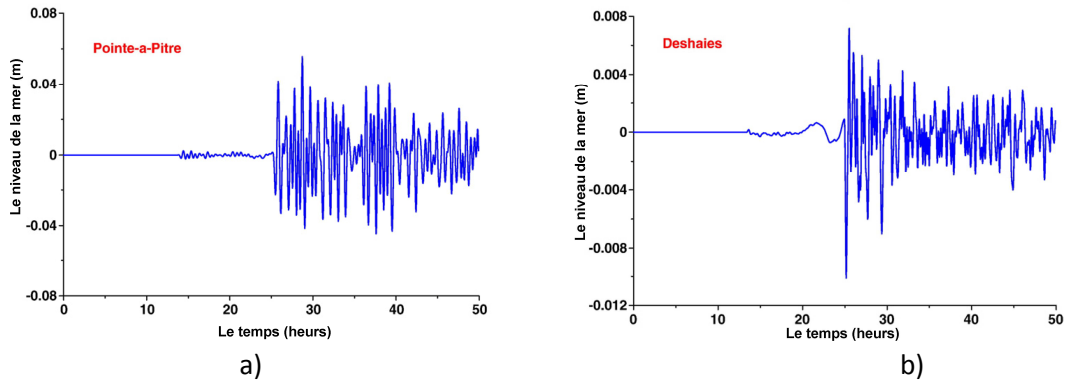


Fig. 3.7. Le niveau de la mer calculée à Deshaies (a) et Pointe-à-Pitre (b)

Statistical analysis of cyclone hazard for Guadeloupe, Lesser Antilles

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Abstract

The statistical analysis of the tropical cyclone activity for Guadeloupe (Lesser Antilles, Caribbean Sea) is given. In total, 162 events are selected for period 1635–2000. The number of cyclones in different periods has a large variability. In average, the time interval about 100 years can be considered as stationary one to have the reliability assessment of cyclone hazard. The distribution functions and cumulative frequencies of the cyclones depending on the atmospheric pressure drop and wind speed are calculated. They are used to determine the return period for cyclones with given characteristics (category, atmospheric pressure, and wind speed). The data of the storm-induced surges in the coastal locations of Guadeloupe is collected. It is shown that the surge height increases with increase of the wind speed, but observed data have great dispersion. Numerical simulation of the long water waves induced by the hurricane “Lili” in 2002 is performed; surge heights in Guadeloupe exceed 10 cm.

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Keywords: Cyclones; Hazards; Storm surges; Statistics; Numerical modelling; Lesser Antilles

1. Introduction

Tropical cyclones are formed over the warm waters of the tropical oceans, mainly in the belt between 20° N and 20° S. In comparison with extra-tropical (continental) cyclones, tropical cyclones are characterized by much smaller size and much higher intensity, i.e. by much higher atmospheric pressure gradients and respective wind speeds (Murty, 1984). Tropical

cyclones which sustain maximum wind speeds greater than 33 m/s are known as *typhoons* in the Pacific Ocean and as *hurricanes* in the Atlantic Ocean. Severe hurricanes have maximum wind speeds more than 100 m/s. In an average hurricane, the release of latent heat from the condensation of water vapor provides as much energy as the detonation of 400 20-megaton hydrogen bombs (Davis, 1993). The cyclones induce the storm surges and the evaluation of storm surge hazard for Canada and Bangladesh is actively discussed (Sharma and Murty, 1988; Danard et al., 2003; Rao et al., 2004).

Neumann and Hill (1976) and Neumann et al. (1978) gave detailed tracks and statistics of the tropical cyclones of the North Atlantic Ocean for the period

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1871–1977. During this 107-year period, at least 850 tropical cyclones occurred in this region, mainly in the period from June to November (maximum number of these cyclones is related to the end of August–beginning of September). Each year on average, 10 tropical storms (on which 6 become hurricanes) develop over the Atlantic Ocean, Caribbean Sea, or Gulf of Mexico. Many of these remain in the ocean. The deadliest Atlantic tropical cyclones during 1492–1996 are described by Rappaport and Fernandez-Partagas (1997). Various data of tropical storms and hurricanes in the Atlantic can be found in sites (NTC; UNISYS; Hurricanecity) and in papers (Gray and Landsea, 1992; Gray et al., 1993; Gray et al., 1992; Landsea et al., 1992). Recently, Goldenberg et al. (2001) studied the major North Atlantic hurricanes for high and low activity periods, and found the average numbers of major hurricane per year 2.7, 1.5 and 3.8 for the 1944–1970, 1971–1994, 1995–2000 respectively.

Recently Pielke et al. (2003) discussed the hurricane vulnerability in Latin America and the Caribbean after catastrophic Hurricane Mitch killed more than 10,000 people and caused as much as \$8.5 billion in damage in Honduras and Nicaragua. The fatalities and destruction caused by Hurricane Mitch were the greatest in at least several decades. The Caribbean Sea basin has a great history of natural disasters; for instance, the Great Hurricane in October 1780 killed about 22,000 people in the Lesser Antilles (Martinique, Barbados). Authors

analyze the regional climatology of hurricanes in the Caribbean Sea and point out that hurricane frequency is not constant: in 1968–1991 it was less than in the previous period 1944–1967. Webster et al. (2005) examined the characteristics of tropical cyclones over the past 35 years and concluded that a large increase was seen in the number of hurricanes reaching categories 4 and 5 which has not been accompanied by an increase in the actual intensity of the most intense hurricanes: the maximum intensity remained remarkably static over the past 35 years; and claimed that the smallest percentage increase occurred in the North Atlantic Ocean.

The French West India situated in the Lesser Antilles has a huge experience with tropical cyclones. Tropical cyclones are a very important part of natural disasters and influence all components of life in the French West Indies (Yacou, 1999; Martin, 2002). Saffache et al. (2002, 2003) collected the descriptions of cyclone activity in Martinique and Guadeloupe. In particular, about 160 events occurred in Martinique and Guadeloupe during 1635–1999, and therefore, the mean return period for the cyclones in the French West India is 3–4 years. The very catastrophic cyclone of September 6, 1776 (its category is highest, 5 according to the Saffir–Simpson scale (Simpson, 1974)) killed more than 6000 people in Pointe-à-Pitre Bay (Guadeloupe). In September 17, 1989 the Hugo Hurricane slammed Guadeloupe (the wind speed reached 60 m/s), killing 5, injuring 80 and leaving 11,000 homeless (Duteil, 1999).

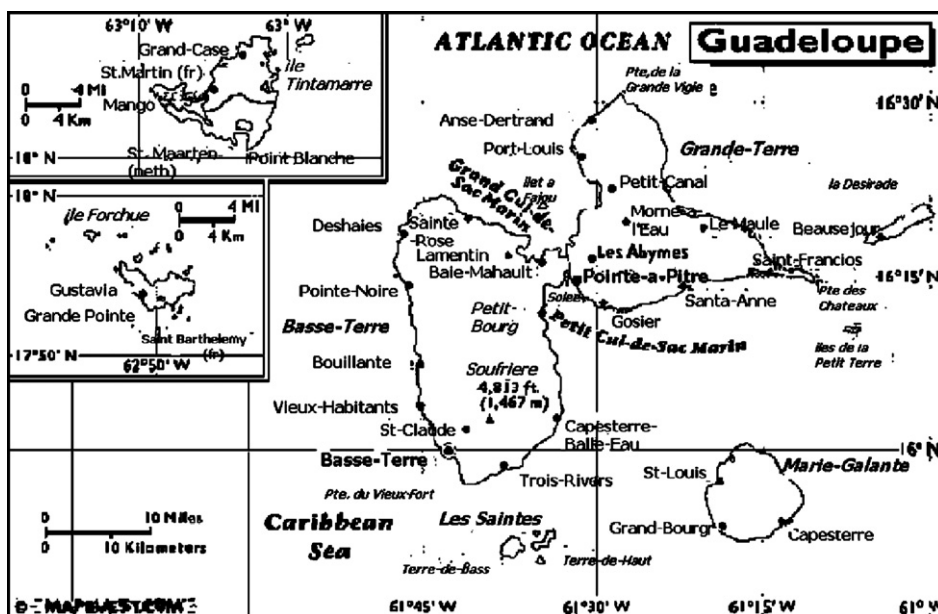


Fig. 1. Chart of the Guadeloupe Department.

The main goal of this paper is to give a quantitative analysis of the tropical cyclone activity for Guadeloupe Department included islands: Guadeloupe, Marie-Galante, La Desirade, Les Saintes, St Martin and St Barthelemy (Fig. 1). The paper is organized as follows. Data of cyclone observations in Guadeloupe and corresponding literature sources for data are discussed in Section 2. The analysis of the variability of the cyclone activity over time is given in Section 3. In average, the time interval of about 100 years can be considered as a stationary one to have the reliability assessment of cyclone characteristics. The distribution functions and cumulative frequencies of the cyclones depending on the atmospheric pressure drop and wind speed are studied in Section 4. The data of the storm-induced surges in the coastal locations of Guadeloupe are given in Section 5. It is shown that the surge height increases with increase of the wind speed. Numerical simulation of water waves induced by the hurricane “Lili” in 2002 is performed in the framework of the shallow water theory. The surge heights in Guadeloupe exceed 10 cm. Obtained results are summarized in conclusion.

2. Data

Recently published book by Saffache et al. (2003) collects the various descriptions of the cyclones on Guadeloupe Department for period 1635–2000 (total number of cyclones is 76). Quantitative information of the cyclones tracks, wind speed, atmospheric pressure in Guadeloupe for the period 1851–2002 can be extracted from site (UNISYS) based on the data of the Tropical Prediction Center, NOAA. Also, some data of deadliest

cyclones in Guadeloupe (for instance for 1656, 1666 and 1821) can be found for example in Rappaport and Fernandez-Partagas (1997). Rough estimates of the return period have been done in (HurricaneCity); in particular, cyclones affect Guadeloupe each 3.4 years, and St Martin — 2.8 years; hurricane hits Guadeloupe each 14.8 years and St Martin — 8.3 year.

The quantitative information of cyclones in the Guadeloupe Department contained in (UNISYS; Saffache et al., 2003; Rappaport and Fernandez-Partagas, 1997) for the period 1635–2000 (totally 162 events) is summarized, and Table 1 gives the information of cyclones with category exceeded 4. The cyclone category is determined according to the Saffir–Simpson scale (Simpson, 1974). The number of cyclones contained in these literature sources is shown in Fig. 2. These characteristics of the cyclones in the Guadeloupe Department will be used for the statistical analysis of the cyclone activity in Guadeloupe.

Of course, some uncertainty in the tropic cyclones characteristics determination occur in the used datasets, this could probably influence on the statistical analysis. Recently, a good documentation of the inherent potential for possible errors was published in (Buckley et al., 2003) for the Australian Basin, the same approach is important for the Caribbean basin. But we believe that main statistical conclusions given below are not “sensitive” for possible errors in tropical cyclone parameters due to large amount of data. In fact, as it is shown further, the statistical characteristics are the same for 100-yr and 150-yr datasets.

Geographically, the Guadeloupe Department contains the two groups of the nearest islands; the one group contains Guadeloupe, Marie-Galante, La Desirade and

Table 1
Data of catastrophic cyclones (categories 4 and 5) on the Guadeloupe for 1635–2000

No.	Date (yr, day/m)	Name	Place	Category	Wind m/s	Pressure, kPa	Reference
1	1642		Guadeloupe	4			[2,p.29]
2	1642		Guadeloupe	4			[2,p.29]
3	1666, 23–24/10		Guadeloupe, Les Saintes	4			[2,p.34]
4	1880, 04–14/08	Hurricane #2	Guadeloupe	4		93	[1]
5	1900, 27/08–15/09	Hurricane #1	Guadeloupe, St Martin	4	23	96	[1]
6	1906, 23/09	Hurricane #3	Guadeloupe	4			[1],[2,p.55]
7	1928, 12/09	Hurricane #4	Guadeloupe	4	33	94	[1],[2,p.72],[3]
8	1960, 29/08–14/09	Hurricane Donna	Les Antilles	4	21		[1],[2,p.76]
9	1964, 20/08–05/09	Hurricane Cleo	Guadeloupe Marie-Galante, Les Saintes	4	59	96	[1],[2,p.80],[3]
10	1966,21/09–11/10	Hurricane Inez	Guadeloupe	4	15	96	[1],[2,p.89]
11	1979, 29/08	Hurricane David	Guadeloupe Desirade	5	75	92	[1],[2,p.98]
12	1989, 13/09	Hurricane Hugo	Guadeloupe, St Martin, Desirade	4	21	94	[1],[2,p.124],[3]
13	1998, 20–21/09	Hurricane Georges	Guadeloupe, St Martin	5	42	97	[1]
14	1999, 17/11	Hurricane Lenny	Guadeloupe, St Martin, St Barthelemy	4	39	97	[1],[2,p.232]

References: [1]—UNISYS, [2] — Saffache et al., 2003, [3] — Rappaport and Fernandez-Partagas, 1997.

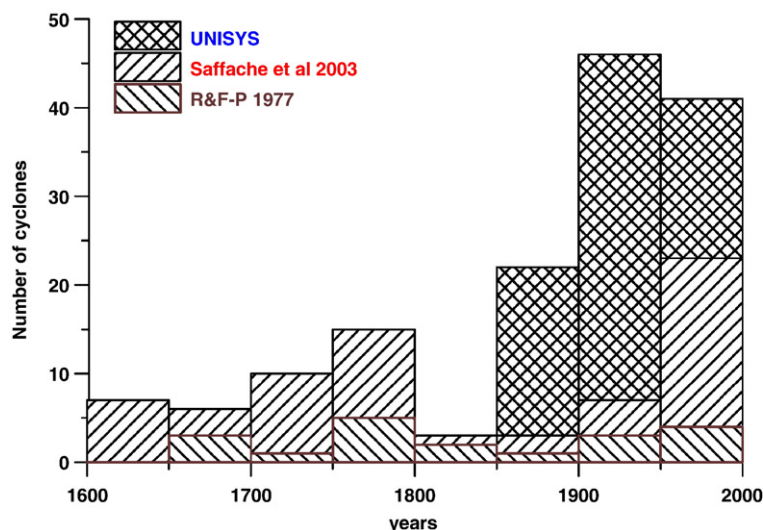


Fig. 2. Number of cyclones presented in various literature sources (R and F-P is Rappaport and Fernandez-Partagas).

Les Saintes, and the second one — St Martin and St Barthelemy; the distance between these groups is about 180 km (Fig. 1). The distribution of the recorded cyclones for 1635–2000 in Guadeloupe is presented in Fig. 3. Cyclones were recorded mainly on Guadeloupe (59%), and on St Martin (26%), and significantly less on Marie-Galante (4%), La Desirade (3%), Les Saintes (3%), and St Barthelemy (4%). The largely different distributions observed in the six different islands of the study area is connected with the relative space scales of the cyclones and the islands, also the different distributions occurred due to a density population bias; it is a possible subject of further study.

3. Analysis of the tropical cyclone occurrence for Guadeloupe

According to the data discussed in the Section 2 and particularly presented in Table 1 the mean value of the frequency of the cyclone appearance for Guadeloupe is 0.44 1/yr. It corresponds to the return period 2.25 years confirming high frequency of the tropical cyclones.

Pielke et al. (2003) analyzed the cyclone activity for the whole Caribbean for the period 1944–1999 and demonstrated the annual and decade variability of the number of cyclones; see also Landsea (1993); Landsea et al. (1996); Gray et al. (1997). They concluded that the

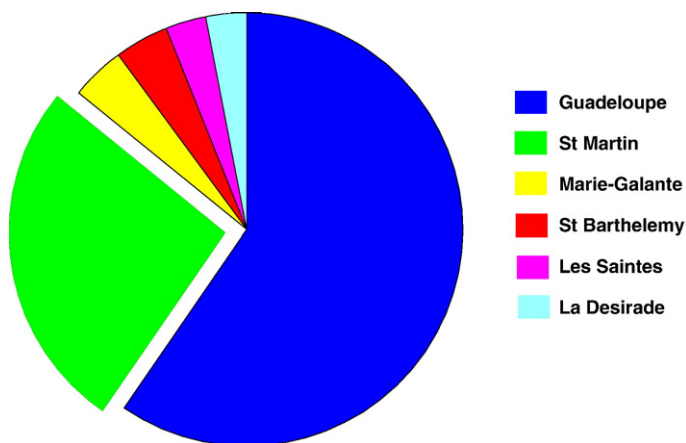


Fig. 3. Geographic distribution of cyclones in the Guadeloupe Department.

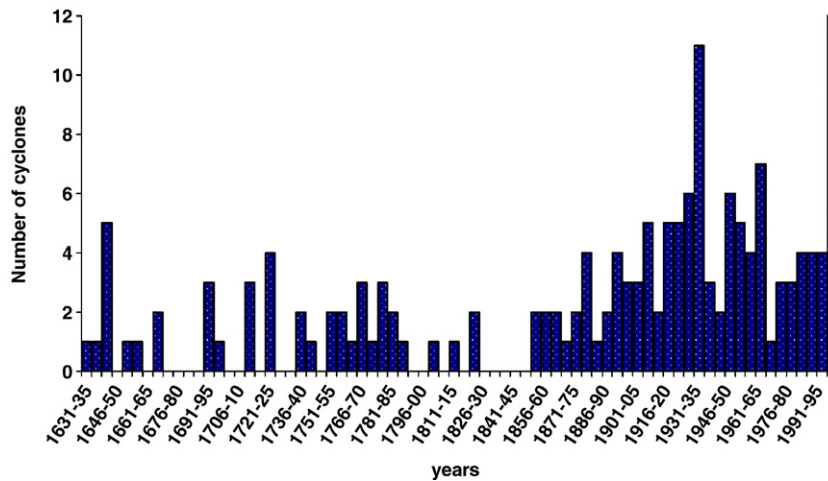


Fig. 4. The variability of the number of tropical cyclones in 5-yr intervals.

empirical probabilities for Caribbean hurricanes can vary dramatically from one decade to another. The same analysis for the Guadeloupe department shows the large variability of cyclone activity in short scale intervals. Fig. 4 presents the number of cyclones in a histogram with 5-year time intervals. Temporal distribution of the cyclones is strongly non-uniform. Maximum number of cyclones (12) occurred in Guadeloupe in 1996–2000. The period 1931–1935 was also productive for cyclones (11 events). Minimum number of cyclones during the 20th century was recorded for 1966–1970 (one event only). The number of cyclones was constant for period 1981–1995, approximately 0.9 cyclones per year. The

ten-year average smoothes the cyclone distribution (Fig. 5). The cyclone activity after 1931–1940 reduces to 1971–1980, and then again increases. The 25-year average smoothes demonstrate the highest cyclone activity (28) on Guadeloupe in 1926–1950 (Fig. 6). Last 25 years of the 20th century are characterized by large number of cyclones; totally 27 cyclones occurred. The 50-year temporal period lead to the smooth distribution (Fig. 7). The amount of cyclones for the last 150 years is relatively huge due to modern organization of the documental data of the cyclone observations. It is important to mention that the number of cyclones in the first half and in the second half of the

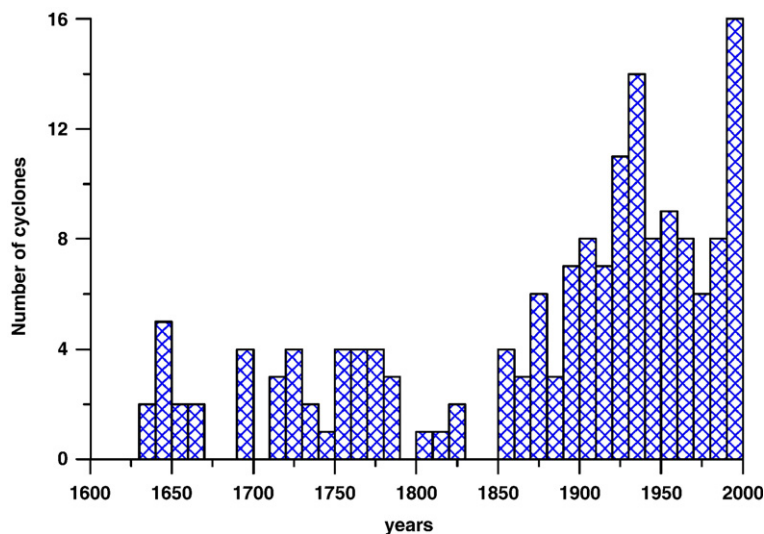


Fig. 5. The variability of the number of tropical cyclones in 10-yr intervals.

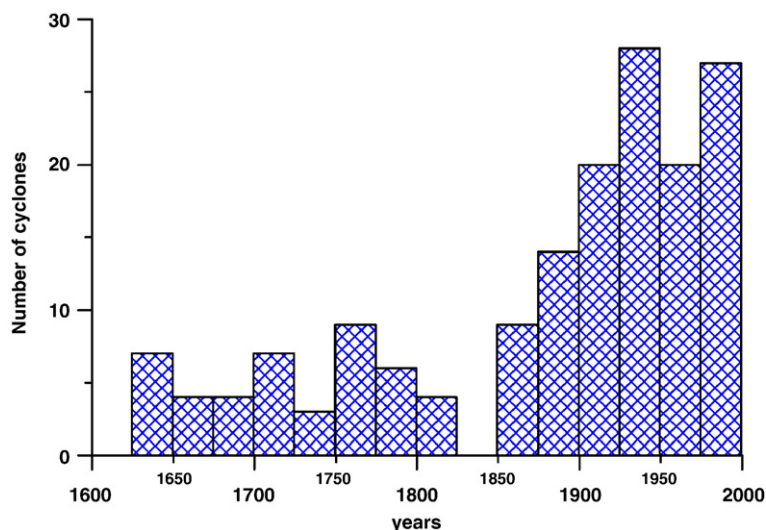


Fig. 6. The variability of the number of tropical cyclones in 25-yr intervals.

20th century is almost the same (48 and 49), and therefore, roughly, this process can be considered as the stationary one in scales 50–100 years.

Fig. 8 characterizes temporal distribution of the cyclones according to the last 250 years (1635–2000). The maximum number of cyclones is related to August–September.

4. Statistical characteristics of the cyclone hazard for Guadeloupe

Quantitative information of the cyclone parameters (category, wind speed and atmospheric pressure) is

known mostly for the last 200 years. The statistical characteristics for 100–200 years can be considered roughly as the stationary estimates, and they may be used to characterize the statistical series for the next 100–200 years. First of all, the distribution of the number of cyclones on categories for Guadeloupe can be analyzed (Fig. 9) for period 1851–2000 (total number of cyclones, $n=109$) and for period 1901–2000 (total number of cyclones, $n=90$). Hereafter, tropical storm is abbreviated “TS” and tropical depression “TD”. Both distributions are almost the same and this demonstrates that the considered series are stationary. The number of cyclones of the highest category is equal to two. The

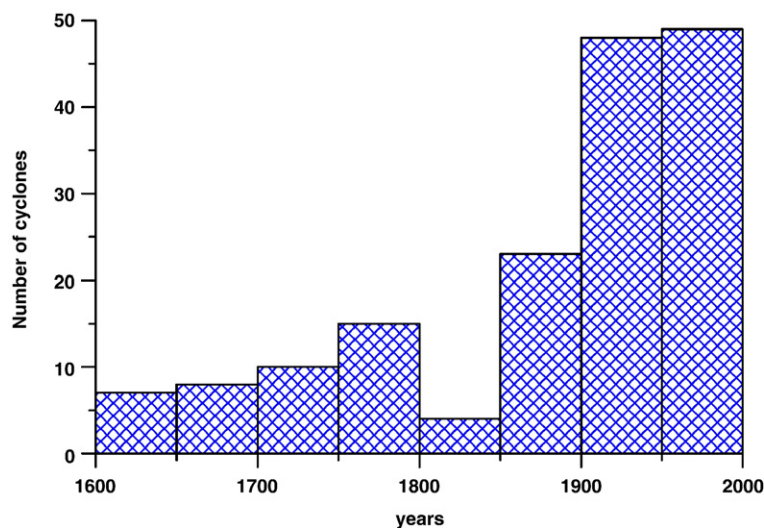


Fig. 7. The variability of the number of tropical cyclones in 50-yr intervals.

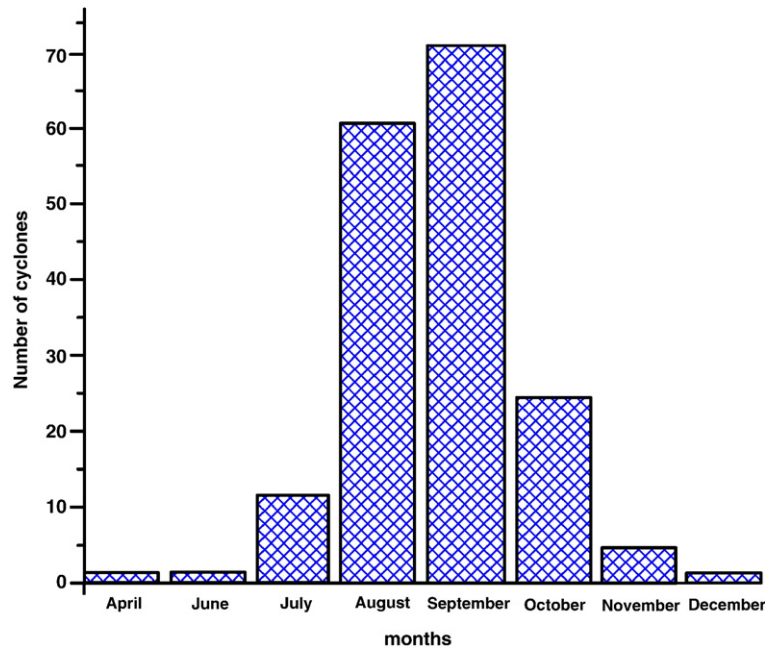


Fig. 8. The variability of the number of tropical cyclones for months in 250 years.

number of tropical depressions is also relatively small, and it is related with weak identification of the weak cyclones. As expected, the number of cyclones decreases with increase of category, but this dependence is not monotonic. For instance, the number of Category 3 hurricanes is less than the number Category 4 hurricanes. Possible explanations of this phenomenon include a shift in operation practices depressions or changes in observed technology that changed whether some disturbances were classified as depressions or not.

Furthermore, Landsea (1993) documented a “high bias” in intensity classification from the 1940s to the 1960s of approximately 2.5 m/s. As it is mentioned in the Section 2, we believe that a large amount of data leads to the fact that the main statistical conclusions are not “sensitive” for possible errors in tropical cyclone parameters.

The exceedance characteristics are used for applications. Fig. 10 shows the cumulative frequency of the cyclones exceeded the given category. And here the estimates based on 100 and 150-year intervals are

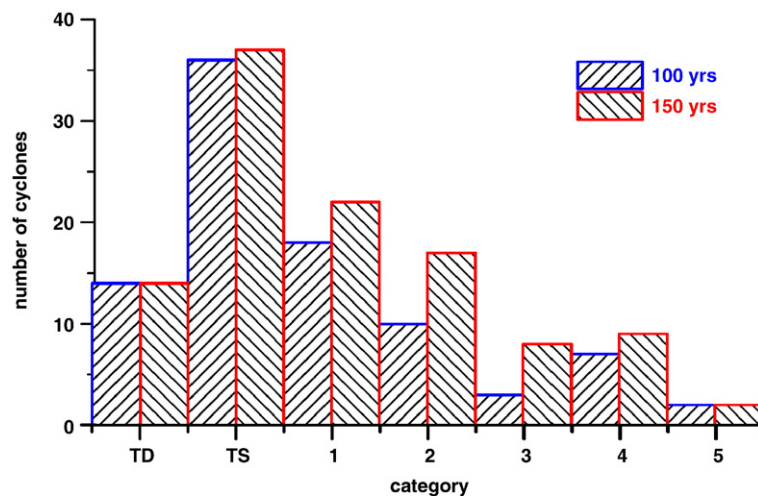


Fig. 9. The distribution of cyclones on categories.

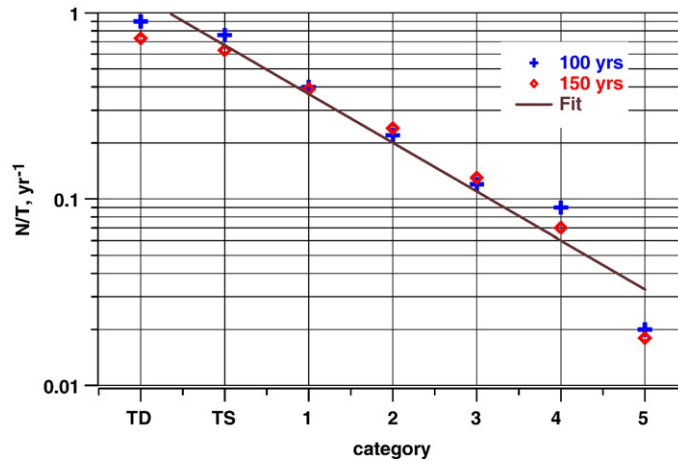


Fig. 10. Cumulative frequency of cyclones versus categories.

almost the same. The best fit for data is an exponential curve with coefficient of determinations, $R=0.95$

$$\log(N/T) = -0.4 - 0.6C, \quad (1)$$

where C is category, and T is measured in years. They can be used to evaluate the return period required for the “design” cyclone definition and corresponding formula is (Fig. 11)

$$T = 0.67 \exp(0.60C). \quad (2)$$

In average, the cyclone, which can be categorized, occurred almost each year. Cyclone with category more than 1 occurs each 2.3 year and this estimation is close to the estimation given by (HurricaneCity). The strongest cyclones with category more 2, 3 and 4 occur each 4, 7.6 and 13.4 years, respectively. The cyclone with highest category 5 occurs once in 75 years, but this is not a

reliable estimate taking into account that there were only two such cyclones in Guadeloupe.

The integral distribution of the cyclone occurrence (cumulative frequency) versus the atmospheric pressure drop is shown in Fig. 12 for the period 150 years (1851–2000, total number of cyclones, n is 41) and for period 100 years (1901–2000, $n=34$). As accepted in Sharma and Murty (1988), for the further discussion in order to have a precise view, the difference between the normal value ($P_0=101.325$ kPa) and real value, p is taken into consideration. Atmospheric pressure drop ($\Delta P = P_0 - P$) is measured in kPa as the difference between the normal value P_0 and real value. The difference between two curves is weak confirming that 100–150 years can be considered as the stationary time for the process of the cyclone occurrence. These data are fitted very well by the exponential curve with $R=0.91$

$$\ln(N/T) = -0.38\Delta P - 0.87. \quad (3)$$

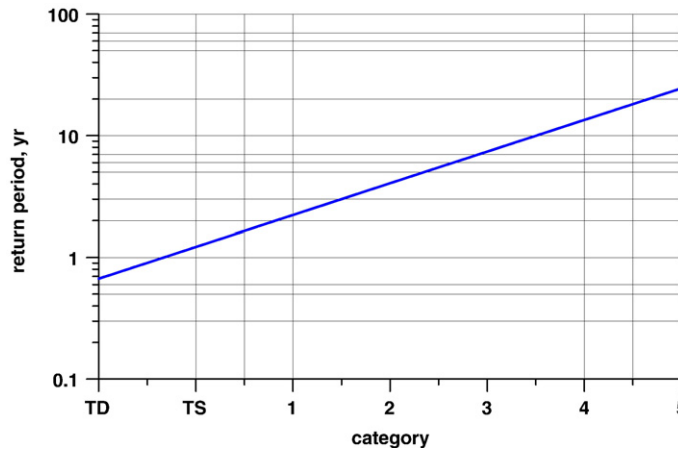


Fig. 11. Return period versus categories.

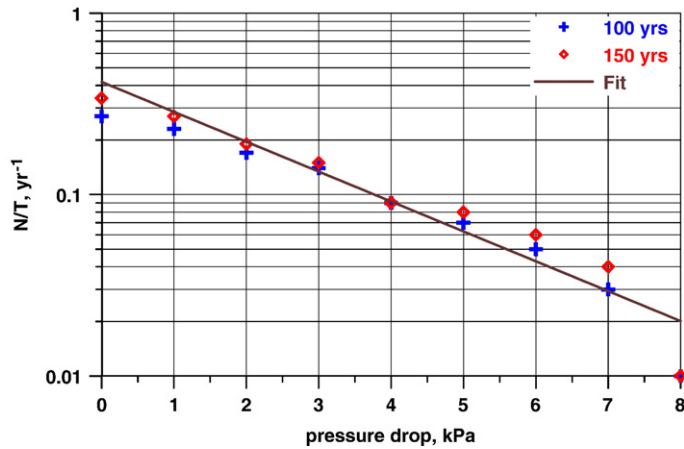


Fig. 12. Cumulative frequency versus the atmospheric pressure.

As a result, the return period versus the atmospheric pressure can be determined in (Fig. 13)

$$T = 2.39 \exp(0.38 \Delta P). \quad (4)$$

Cyclones with huge fall of the atmospheric pressure ($\Delta P = 5$ kPa) may occur in Guadeloupe each 15 years.

The integral distribution of the cyclone occurrence versus the wind speed is presented in Fig. 14 for period 150 years (1851–2000, $n=108$), and for period 100 years (1901–2000, $n=88$). And here the difference between the curves is relatively small. Presented data are fitted by the exponential curve with $R=0.89$

$$\ln(N/T) = 1.07 - 0.06W, \quad (5)$$

where W is the wind speed in m/s. Measured data have some oscillations around the exponential curve, but its

nature is not clear. As a result, the return period versus the wind speed can be determined (Fig. 15)

$$T = 0.9 \exp(0.06W). \quad (6)$$

The cyclones with wind speed exceeding 10 m/s occurred each year, catastrophic cyclones with speed about 50 m/s occurred approximately each 11 years.

5. Storm surges induced by cyclones in Guadeloupe

Cyclones passed through Guadeloupe induce flooding in many coastal locations. Atmospherically generated sea level disturbances, S in a general case may be expressed as the superposition of three major components (Murty, 1984)

$$S = S_1 + S_2 + S_3, \quad (7)$$

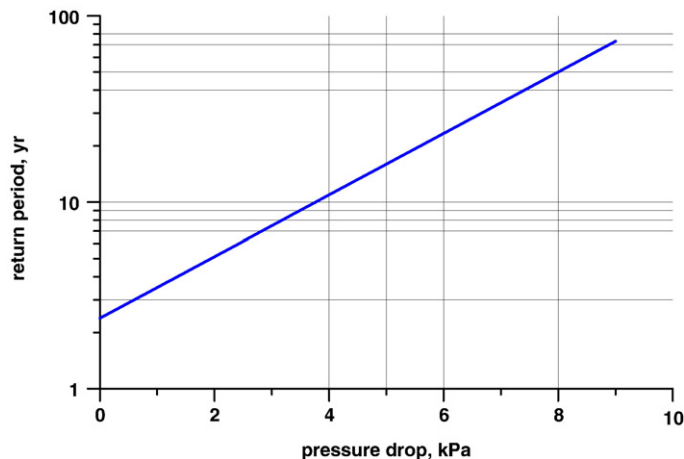


Fig. 13. Return period of cyclones versus atmospheric pressure.

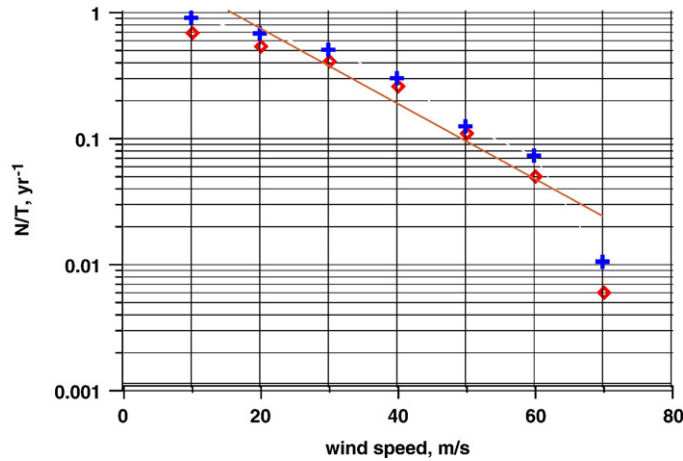


Fig. 14. Cumulative frequency versus the wind speed.

where

$$S_1 = \alpha \Delta P \quad (8)$$

is the dynamic sea level rise associated with the atmospheric pressure drop, ΔP and α is the barometric factor,

$$S_2 = \beta |W| W \cos \theta \quad (9)$$

is the sea level rise caused by onshore component of the wind stress, W is the wind speed, and θ is the angle between the wind vector and orientation of the coastal line, and β is the respective empiric coefficient depending on shelf parameters

$$S_3 = \gamma f v \quad (10)$$

is the bathystrophic storm surge, f is the Coriolis parameter, v is the alongshore drift flow, initiated mainly by the alongshore component of wind stress,

and γ is the coefficient depending on water depth and bottom friction (see Pararas-Carayannis (1975) for details).

The source of the coastal flooding in many historic materials is not clarified; sometimes it can be related with tsunamis (Zahibo and Pelinovsky, 2001; Lander et al., 2002). The quantitative information of the recorded surges and parameters of cyclones is summarized in Table 2. In fact, the surge heights are known for 17 cyclones during 1928–2000 only. Taking into account various points of observations, the total number of data is 46, and they can be used to analyze the relation between the surge heights in the coastal locations and cyclone characteristics (wind speed and atmospheric pressure) and clarify the major factors of storm surges in Guadeloupe.

Fig. 16 shows the surge height versus the atmospheric pressure drop, ΔP for Guadeloupe. The

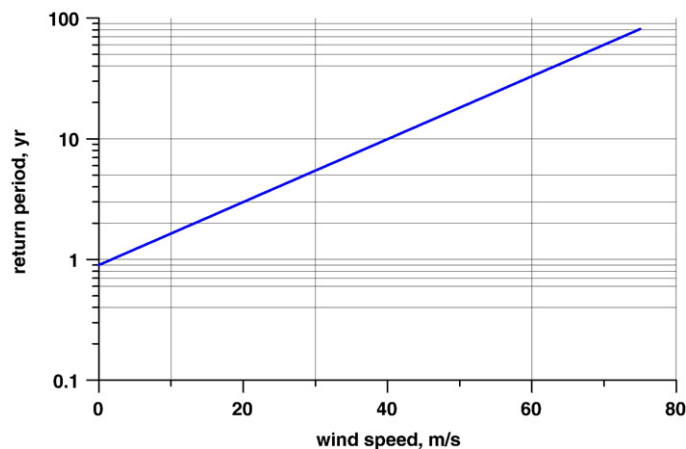


Fig. 15. Return period of cyclones versus wind speed.

Table 2
Data of storm surges for Guadeloupe (references as in Table 1)

No.	Cyclone name	Date	Point	Wind, m/s	Pressure, kPa	Height, m	Reference
1		1928/09/12	Petit Cul-de-Sac-Marin, Pointe-a-Pitre	57	94	5 4.5	[2, p.75] [2, p.67]
2		1949/09	Basse-Terre	23		6	[2, p.75]
3	Inez	1966/09/27	North Grande-Terre	46	96.1	2	[2, p.84]
4	David	1979/08/29–30	Basse-Terre, sw Basse-Terre, ssw Gosier, St Francois, La Desirade Basse-Terre, wsw	50 33 31 25	93.3 93.3 93.3 92.4	6 3.5 4 4	[2, p.99–100]
5	Frederic	1979/09/03	Boucan St Claude Gosier	28	99.9	0.7 0.5 0.2	[2, p.118–120]
6	Hugo	1989/09/17–19	La Pointe des Chateaux, Moule, Gosier, Goyave, Pointe-a-Pitre, Abymes Canal de la Dominique St Rose Grand Cul-de-Sac Marin Grande-Terre Petit Cul-de-Sac Marin	72 72 67 67 67 67	95.9 95.9 95 93.4 95 94.1	15 7 3 3 2.5 1.5	[2, p.125–134]
7	Luis	1995/09/05	Illes de Nord St Martin Pointe-a-Pitre	56 69 69		10 8 8	[2, p.174–182]
8.	Iris	1995/08/25	Marie-Galante, les Saintes, La Desirade, le Petit Cul-de-Sac Marin, Gourbeyre, Petit-Bourg	22	99.5	3	[2, p.162]
9	Marilyn	1995/09/14–18	Marie-Galante, les Saintes, Capesterre, Petit-Bourg Basse-Terre Boucan Deshaies	42 36 41 51 44	96.9 95 96.3	6 4 6 2.5 1.5	[2, p.184–200]
10	Erica	1997/09/06	Illes de Nord	33 19		4 3.5	[2, p.224–225]
11	George	1998/09/20–21	Port Luis	51 69	96.3 93.7	7 4.6	[1]
12	Bonnie	1998/08/20	Guadeloupe	22		2	[2, p.227]
13	Danielle	1998/08/26	Illes de Nord	10		2.5 2 1.5	[2, p.229]
14	Jose	1999/10/19–21	La Desirade	50 41 33 44 33	99 97.9 99 97.9 99.2	8 5.6 5.6 3 2.5	[2, p.230] [1] [2, p.229] [1] [1]
15	Lenny	1999/11/17–20	Guadeloupe, sw Petit-Perou Pointe-a-Pitre Point-Moir	59 69 59 28	93.3 93.3 93.3 99.4	4 1.8 1.5 1	[1] [2, p.355] [2, p.247] [2, p.242]
16.	Debby	2000/08/21–22	La Desirade	36 33	99.3 99.3	4 3	[1]
17	Joice	2000/09/29	L'est-sud-est de la Guadeloupe	17		2	[2, p.258]

variations in the surge height are rather significant, but in average our data is in a good agreement with the data from other regions. For instance, the surge height along the east coast of India is varied twice, having the order 1–2 m for pressure drop 5 kPa (Sharma and Murty, 1988). The same numbers of storm surges are

characterized for storm surges in the Sea of Azov (Eremeev et al., 2000). Formally we may conclude that the correlation between storm surges and atmospheric pressure is very poor, and therefore, the contribution of the atmospheric pressure mechanism in the dynamic sea level rise cannot be clarified based on observed data.

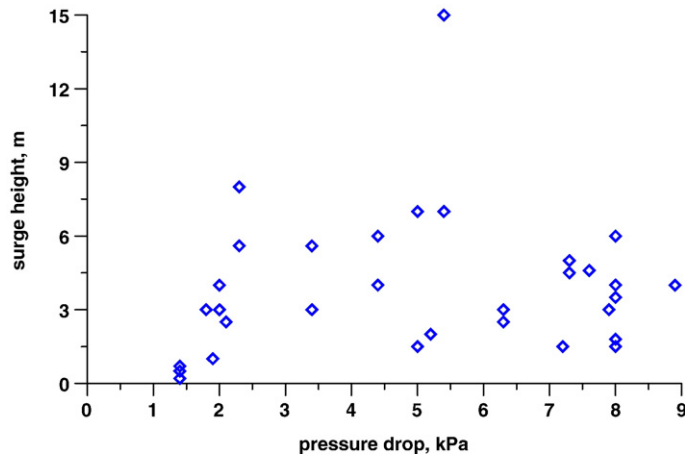


Fig. 16. Surge height versus atmospheric pressure (36 events for 1928–2000).

The relation between the surge height and atmospheric pressure cannot be used to forecast the storm surges in Guadeloupe.

Fig. 17 shows the surge height versus the wind speed during the cyclone. In general, the surge height increases for strong cyclones. The solid line corresponds to the formula (9) with coefficient $\beta=0.0026 \text{ s/m}^2$ and $\theta=0$. Observed data have a huge dispersion, and expression (9) leads to a decrease of the surge heights at light winds, and to an increase of the surge heights at strong winds. For prognostic purposes the linear function

$$S(\text{m}) = 0.23W(\text{m/s}) \quad (11)$$

can be more convenient because it gives the “upper” estimation of the storm surges in Guadeloupe. This

prognostic curve is shown in Fig. 17 by the dashed line.

Of course, the use of observed data to forecast the storm surges only is the first, very rough step. Here the mathematical modeling is the effective tool to study the storm-induced coastal flooding, and main factors influenced on regional characteristics of the storm surges.

6. Numerical simulation of long wave generation by hurricane “Lili”

Tracks of the 2002 tropical cyclones in the North Atlantic taken from (UNISYS) are shown in Fig. 18. During that year were no catastrophic strong hurricanes

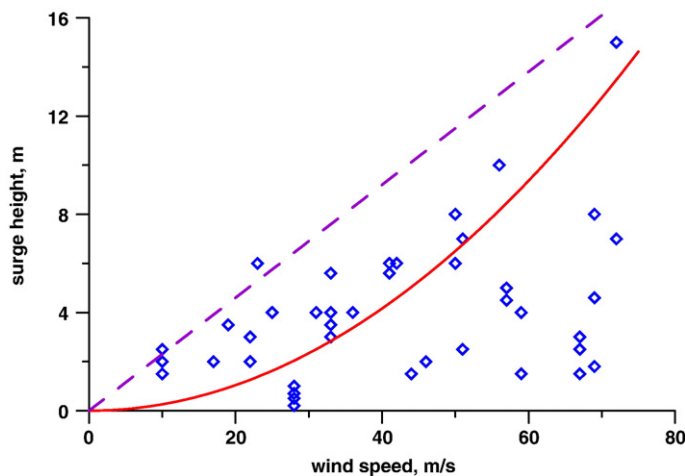
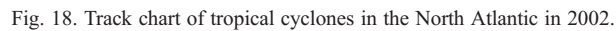


Fig. 17. Surge height versus the wind speed (46 events for 1928–2000). Solid line (9) and dashed line (11).



over the Gulf of Mexico. Landslides initiated by this hurricane killed four people in St Vincent (Caribbean Sea), including one child. About 400 homes in Barbados were damaged and half the banana crop of St Lucia was totally destroyed. Time variations of the characteristics of hurricane Lili are presented in [Table 3](#).

The numerical simulation of the water waves generated by the hurricane “Lili” is done in the framework of the shallow-water theory widely used for simulation of the storm-induced flooding (Murty,

Date/time (UTC)	Position		Pressure (kPa)	Wind speed (m/s)	Stage
	Lat. (° N)	Lon. (° W)			
21 / 1800	10.2	44.6	109	13	Tropical depression
22 / 0000	10.3	46.5	107	15	“
22 / 0600	10.8	48.5	106	15	“
23 / 0000	12.1	54.6	105	18	Tropical storm
23 / 0600	12.2	56.8	105	21	“
23 / 1200	12.4	58.7	104	23	“
23 / 1800	12.5	60.4	105	26	“
24 / 0600	12.8	63.7	106	26	“
24 / 1200	13.0	64.9	104	31	“
24 / 1800	13.2	66.0	107	26	“
25 / 0000	13.5	66.9	108	18	“
25 / 1200	14.0	68.2	108	21	Tropical wave
25 / 1800	14.2	68.9	107	21	

1984; Poplavsky et al., 1997; Ereemeev et al., 2000; Cheung et al., 2003)

$$\begin{aligned} \frac{\partial M}{\partial t} + \frac{\partial}{\partial x} \left(\frac{M^2}{D} \right) + \frac{\partial}{\partial y} \left(\frac{MN}{D} \right) + gD \frac{\partial \eta}{\partial x} - fN \\ = - \frac{D}{\rho_a} \frac{\partial P_a}{\partial x} + \frac{\tau_{sx} - \tau_{bx}}{\rho_0}, \end{aligned} \quad (12)$$

$$\begin{aligned} \frac{\partial N}{\partial t} + \frac{\partial}{\partial x} \left(\frac{MN}{D} \right) + \frac{\partial}{\partial y} \left(\frac{N^2}{D} \right) + gD \frac{\partial \eta}{\partial y} + fM \\ = - \frac{D}{\rho_a} \frac{\partial P_a}{\partial y} + \frac{\tau_{sy} - \tau_{by}}{\rho_a}, \end{aligned} \quad (13)$$

$$\frac{\partial \eta}{\partial t} + \frac{\partial M}{\partial x} + \frac{\partial N}{\partial y} = 0, \quad (14)$$

where η is the water surface elevation, t is time, x and y are horizontal coordinates in zonal and meridian directions, M and N are discharge fluxes in horizontal plane along x and y coordinates, $D = h(x, y) + \eta$ is the total water depth, $h(x, y)$ is the unperturbed basin depth, g is the gravity acceleration, f is the Coriolis parameter, $P_a(x, y, t)$ is an atmospheric pressure, τ_s and τ_b are the

wind and bottom stresses expressed as quadratic functions of wind and fluid velocities, respectively. The shallow-water system is realized numerically by Vasily Khramushin (Poplavsky et al., 1997), as the special package. This package was applied early to describe the tsunami waves in the Lesser Antilles (Zahibo and Pelinovsky, 2001) and obtained results are in a good agreement with the simulation for the Caribbean Sea within the international code TUNAMI (Zahibo et al., 2003; Pelinovsky et al., 2004). The bathymetry used is ETOPE3 (Smith and Sandwell, 1997) with grid resolution 3 min. Near the coast the reflected boundary conditions are used (approximation of the vertical wall in last sea points on depth 10–20 m). The open boundaries of the computed domain are simulated by absorbing conditions.

We chose the storm positions from 23 September (12:00) to 25 September (12:00) with observed parameters (Table 3) as input for numerical modeling. The storm initial position and the hurricane track used for modeling are shown in Fig. 19.

The snapshot of hurricane motion and induced waves is shown in Fig. 20. Results of the calculations of the sea level variations near several coastal locations at Guadeloupe are presented in Fig. 21 starting from 12:00 on 23 September. Wave heights in both locations (Pointe-a-Pitre and Deshaies, see for locations Fig. 1)

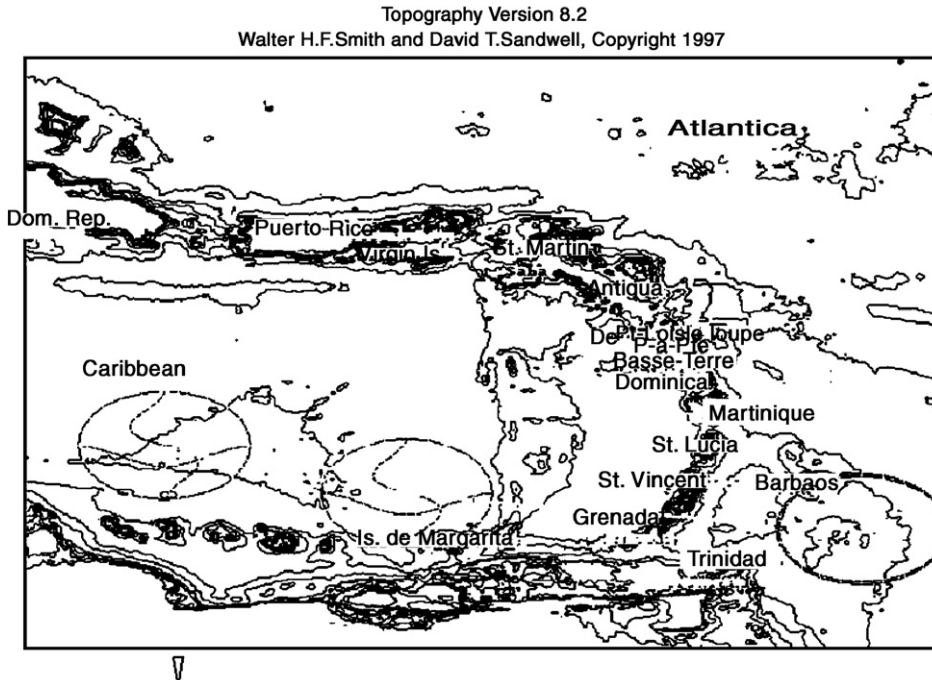


Fig. 19. Storm initial position (black cycle) and the following positions (grey cycles).

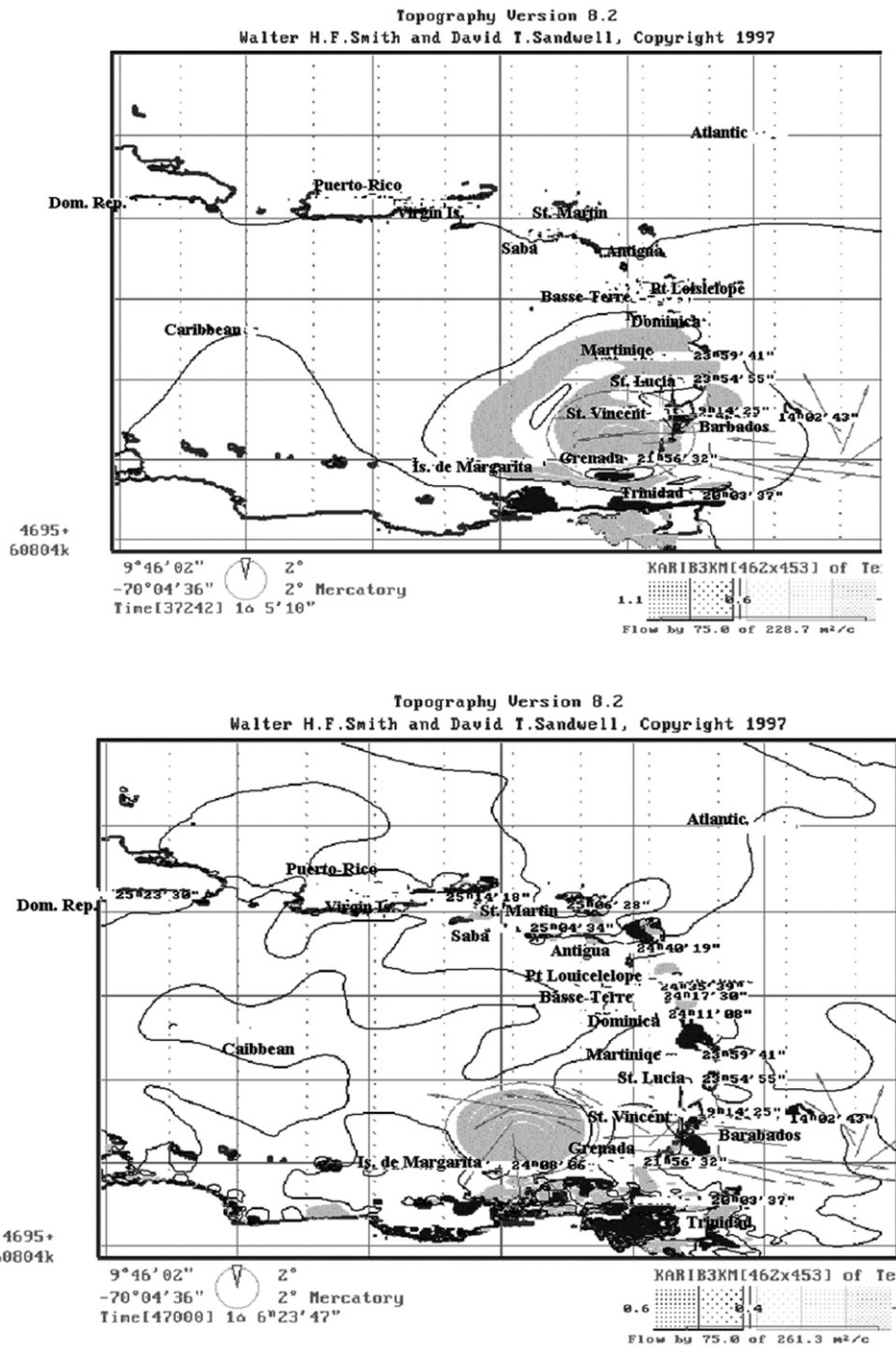


Fig. 20. Snapshots of hurricane motion and water wave generation.

exceed 10 cm. In fact, the model does not take into account the runup due to not high resolution of the coastal zone, so the storm surge heights can be higher in several times in agreement with observed data. We plan to model the storm surges using more detail bathymetry of Guadeloupe.

7. Conclusion

The cyclone activity for Guadeloupe is analyzed. In total, 162 events are selected for period 1635–2000 from various literature sources. Cyclones were recorded mainly on Guadeloupe (59%), and on St Martin (26%),

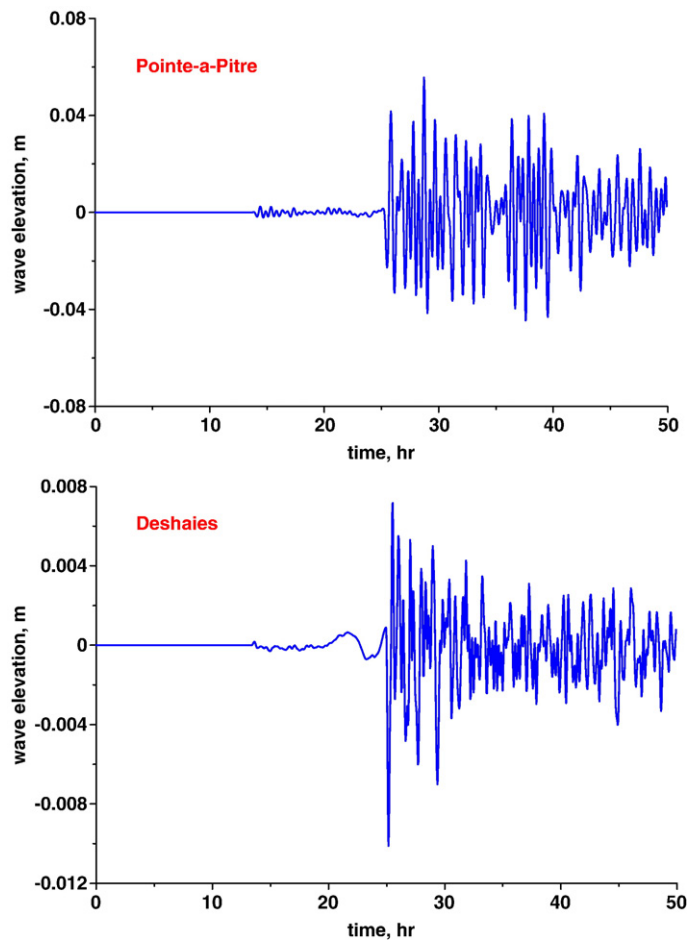


Fig. 21. Computed sea level oscillations in Guadeloupe during the Lili passing.

and significantly less on Marie-Galante (4%), La Desirade (3%), St Barthelemy (4%) and Les Saintes (3%). The number of cyclones in different periods has a large variability. In 20th century, the maximum number of cyclones (12) occurred in Guadeloupe in 1996–2000. Productive for cyclones was also the period 1931–1935, when 11 cyclones occurred. Minimum number of cyclones during 20th century recorded for 1966–1970 (one event only). On average, the time interval of about 100 years can be considered a stationary one to have the reliability assessment of cyclone characteristics. The distribution functions and cumulative frequencies of the cyclones depending on the atmospheric pressure and wind speed are calculated. They are used to determine the return period for cyclones with given characteristics (category, atmospheric pressure, and wind speed). In average, the cyclone, which can be categorized, occurs in Guadeloupe almost each year. Cyclone with category more than 1 occurs each 2.6 year, and the cyclone with

highest category (5) occurs once in 75 years. The data of the storm-induced surges in the coastal locations of Guadeloupe is collected and analyzed. It is shown that the surge height increases with increase of the wind speed, but observed data has great dispersion. The regression between the surge height and atmospheric pressure is not followed from observed data. The modeling of the wave generation by the hurricane “Lili” in 2002 passed through the Lesser Antilles is performed in the framework of the shallow-water theory. Surge heights computed for several locations in Guadeloupe exceed 10 cm. Obtained results may be applied for the risk assessment for Guadeloupe.

Acknowledgements

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References

- Buckley, B.W., Leslie, L.M., Speer, M.S., 2003. The impact of observational technology on climate database quality: tropical cyclones in the Tasman Sea. *Journal of Climate* 16 (15), 2640–2645.
- Cheung, K.F., Phadke, A.C., Wei, Y., Rojas, R., Douyere, Y.J.-M., Martino, C.D., Houston, S.H., Liu, P.L.-F., Lynett, P.J., Dodd, N., Liao, S., Nakazaki, E., 2003. Modeling of storm-induced coastal flooding for emergency management. *Ocean Engineering* 30, 1353–1386.
- Danard, M., Munro, A., Murty, T., 2003. Storm surge hazards in Canada. *Natural Hazards* 28, 407–431.
- Davis, L., 1993. *Encyclopedia of Natural Hazards*. Headline Book Publishing, p. 433.
- Duteil, A., 1999. *Hugo ou l'hiver en Guadeloupe*. Centre d'Édition et de Diffusion International du Livre, Paris, p. 143.
- Eremeev, V.N., Konovalov, A.V., Manilyuk, Yu.V., Cherkesov, L.V., 2000. Modeling of long waves in the Sea of Azov generated by cyclone propagation. *Oceanology* 40 (5), 616–623.
- Goldenberg, S.B., Landsea, C.W., Mestas-Nunez, A.M., Gray, W.M., 2001. The recent increase in the Atlantic hurricane activity: causes and implications. *Science* 309, 1844–1846.
- Gray, W.M., Landsea, C.W., 1992. Examples of the large modification in US east coast hurricane spawned destruction by prior occurring west African rainfall conditions. ICSU/WIVIO International Symposium on Tropical Cyclone Disasters October 12–16, Beijing.
- Gray, W.M., Landsea, C.W., Mielke Jr., P.W., Berry, K.J., 1992. Predicting Atlantic seasonal hurricane activity 6–11 months in advance. *Weather Forecasting* 7, 440–455.
- Gray, W.M., Landsea, C.W., Mielke Jr., P.W., Berry, K.J., 1993. Predicting Atlantic basin seasonal tropical cyclone activity by 1 August. *Weather Forecasting* 8, 73–86.
- Gray, W.M., Sheaffer, J.D., Landsea, C.W., 1997. Climate trends associated with multi-decadal variability of Atlantic hurricane activity. In: Diaz, H.F., Pulwarty, R.S. (Eds.), *Hurricanes: Climate and Socioeconomic Impacts*. Springer-Verlag, New York, pp. 15–53.
- Hurricanecity, <<http://www.hurricanecity.com/city/Rank.htm>>.
- Lander, J.F., Whiteside, L.S., Lockridge, P.A., 2002. A brief history of tsunami in the Caribbean Sea. *Science of Tsunami Hazards* 20, 57–94.
- Landsea, C.W., 1993. A climatology of intense (or major) Atlantic hurricanes. *Monthly Weather Review* 121, 1703–1713.
- Landsea, C.W., Gray, W.M., Mielke, P.W., Berry, K.J., 1992. Long-term variations of western Sahelian monsoon rainfall and intense U.S. landfalling hurricanes. *Journal of Climate* 5, 1528–1534.
- Landsea, C.W., Nicholls, N., Gray, W.M., Avila, L.A., 1996. Downward trends in the frequency of intense Atlantic hurricanes during the past five decades. *Geophysical Research Letters* 23, 1697–1700.
- Martin, J.-L., 2002. *Le Grand livre des cyclones et tempêtes tropicales*. ORPHIE, p. 157 (in French).
- Murty, T.S., 1984. Storm surges: meteorological ocean tides. *Bull.*, 212. Dep. Fish. Oceans, Ottawa, p. 897.
- Neumann, C.J., Hill, D.A., 1976. Computerized tropical cyclone climatology. *Monthly Weather Log* 20 (5), 257–262.
- Neumann, C.J., Cry, G.W., Caso, E.L., Jarvinen, B.R., 1978. Tropical cyclones of the North Atlantic Ocean, 1871–1977. National Climatic Center, Asheville, NC, p. 178.
- NTC. National Hurricane Center, Tropical Prediction Center, National Weather Service, NOAA, <<http://www.tpc.ncep.noaa.gov>>.
- Pararas-Carayannis, G., 1975. Verification study of bathystrophic storm surge model. *Techn. Mem.*, vol. 50. Coast. Eng. Res. Center, p. 248.
- Pelinovsky, E., Zahibo, N., Dunkley, P., Edmonds, M., Herd, R., Talipova, T., Kozelkov, A., Nikolkina, I., 2004. Tsunami generated by the volcano eruption on July 12–13 2003 at Montserrat, Lesser Antilles. *Science of Tsunami Hazards* 22 (1), 44–57.
- Pielke Jr., R.A., Rubiera, J., Landsea, C., Fernandez, C.M.L., Klein, R., 2003. Hurricane vulnerability in Latin America and the Caribbean: normalized damage and loss potentials. *Natural Hazards Review* 4 (3), 101–113.
- Poplavsky, A., Khramushin, V., Nepon, K., Korolev, Yu., 1997. Tsunami prediction for the sea coasts of the far east of Russia. Institute of Marine Geology and Geophysics, Yuzhno – Sakhalinsk.
- Rao, A.D., Babu, S.V., Dube, S.K., 2004. Impact of tropical cyclones on coastal upwelling processes. *Natural Hazards* 31, 415–435.
- Rappaport, E.N., Fernandez-Partagas, J., 1997. The deadliest Atlantic Tropical Cyclones, 1492–1996. NOAA Technical Memorandum NWS NHC 47. <<http://www.nhc.noaa.gov/pastdeadly.shtml>>.
- Saffache, P., Marc, J.-V., Cospar, O., 2002. Les cyclones en Martinique: quatre siècles cataclysmiques. IBIS Rouge Editions, Martinique, p. 197.
- Saffache, P., Marc, J.-V., Huyghes-Belrose, V., 2003. Les cyclones en Guadeloupe: quatre siècles cataclysmiques. IBIS Rouge Editions, Martinique, p. 276.
- Simpson, R.H., 1974. The hurricane disaster potential scale. *Weatherwise* 27 (4), 169–186.
- Sharma, G.S., Murty, A.S., 1988. Storm surges along the East Coast of India. In: El-Sabh, M., Murty, T. (Eds.), *Natural and Made-Man hazards*. Reidel Publ. Company, pp. 255–277.
- Smith, W.H.F., Sandwell, D.T., 1997. Global sea floor topography from satellite altimetry and ship depth soundings. *Science* 277, 1956–1962.
- Webster, P.J., Holland, G.J., Curry, J.A., Chang, H.-R., 2005. Changes in tropical cyclone number, duration, and intensity in a warming environment. *Science* 293, 474–479.
- UNISYS, Atlantic Tropical Storm Tracking by Year (1851–2002), Based on data from the Tropical Prediction Center, NOAA, <<http://www.weather.unisys.com/hurricane/atlantic/index.html>>.
- Yacou, A., 1999. *Les catastrophes naturelles aux Antilles*. Editions Karthala, Paris, p. 334.
- Zahibo, N., Pelinovsky, E., 2001. Evaluation of tsunami risk in the Lesser Antilles. *Natural Hazard and Earth Sciences* 3, 221–231.
- Zahibo, N., Pelinovsky, E., Yalciner, A., Kurkin, A., Koselkov, A., Zaitsev, A., 2003. The 1867 Virgin Island Tsunami: observations and modelling. *Oceanologica Acta* 26 (5–6), 609–621.

Extreme Waves Generated by Cyclones in Guadeloupe

Narcisse Zahibo, Irina Nikolkina, and Ira Didenkulova

Abstract Cyclones produce some harm by terrific wind speed, abnormal precipitations, and sea motion. Historically, storm surge inundation has been the most destructive and murderous. The extreme waves caused by cyclones on the coast of Guadeloupe for a whole historic period are discussed. The first documented sea damage during hurricane passage in Guadeloupe occurred in 1642. Data of extreme waves induced by hurricanes from 1928 to 2007 in Guadeloupe are collected and discussed. The heights of extreme waves for Caribbean and Atlantic coasts are compared; the most dangerous regions are also evaluated.

1 Introduction

Cyclones known as hurricanes in North Atlantic and typhoons in North Pacific are categorized according to their maximum wind speed and lowest central pressure. In the Atlantic, the Saffir–Simpson Scale (SSS) is used to classify most of the tropical cyclones that exceed the intensities of “tropical depressions” and “tropical storms.” It is a scale with the range of 1–5 based on wind speed that shows expected damage to structures, the effects of storm surge, and flooding (Table 1).

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Table 1 Saffir–Simpson hurricane scale

Category	Wind speed (km h ⁻¹)	Storm surge (m)
Tropical depression	0–62	0
Tropical storm	63–117	0–0.9
1	119–153	1.2–1.5
2	154–177	1.8–2.4
3	178–209	2.7–3.7
4	210–249	4–5.5
5	>250	>5.5

Various data of tropical storms and hurricanes in the Atlantic can be found now in sites (NTC; UNISYS; Hurricanecity). The Atlantic tropical cyclones that resulted in great loss of life during 1492–1996 are introduced and discussed in Rappaport and Fernandez-Partagas (1997). Recently, Pielke et al. (2003) discussed the hurricane vulnerability in Latin America and the Caribbean after catastrophic Hurricane Mitch, which caused deaths of more than 10,000 people and resulted in property damage at a valuation of more than \$8.5 billion in Honduras and Nicaragua.

The French West Indies located in the Lesser Antilles, North Atlantic has a huge experience of tropical cyclones. The most detailed catalogue of cyclone activity in Martinique and Guadeloupe was compiled by Saffache et al. (2002, 2003) based on historical documents. This data was used by Zahibo et al. (2007) to obtain the statistical analysis of cyclone characteristics in Guadeloupe.

Cyclones produce some harm by terrific wind speed, abnormal precipitations, and sea action. Historically, storm surge inundation has been the most destructive and murderous. One of the most catastrophic recent natural disasters occurred in Bangladesh in 1970; where more than 300,000 people were lost in storm surge flooding during a powerful cyclone (Wolshon et al. 2005). Storm surges are created by large wind speed (in cyclones category 1–5 SSS wind speed is less than 33 m s⁻¹) and high atmospheric pressure fall (the maximum value of 5 h Pa in the Atlantic was recorded in September 1966 when Hurricane Inez went over Guadeloupe).

A storm surge history is rather extensive; thus on 10 March 1899 during cyclone Mahina (the most fatal natural disaster in Australian history) an extreme wave occurred. Just before the eye of the cyclone passed overland to the north, a tidal wave (caused by storm surge), variously reported as either 13 m or 48 ft (14.6 m) high, swept inland for about 5 km, destroying anything that was left of the Bathurst Bay pearling fleet along with the settlement. The analysis of this event made by Nott and Hayne (2000) suggested that marine inundations have not exceeded 3–5 m elevation.

The analysis of the storm surge (up to 28 ft height) on American coast during Katrina passage has been produced under the direction of the Federal Emergency Management Agency (Knabb et al. 2005). Afterwards, numerical simulation was performed for forecasting storm surges propagating up the Mississippi River (Reed and Stucky 2005). Nowadays, the Atlantic Storm Surge and Tsunami System is a work in progress, NOAA (O'Reilly et al. 2006).

Recently, Atlantic has been struck by a destructive event of category 5. The Hurricane Dean was formed on 13 August 2007 over west coast of Africa. This powerful

tropical cyclone is considered as the most intense event in the Atlantic basin since Hurricane Wilma that occurred in 2005. Dean fell on the Lesser Antilles on the night of 16th to 17th of August, 2007. The field survey has been performed by authors in August 2007 in Guadeloupe after the attack of the Hurricane Dean.

The extreme waves caused by cyclones on the coast of Guadeloupe for a whole historic period are discussed here. The paper is organized as follows. Data of extreme waves induced by the last Hurricane Dean in Guadeloupe is given in Sect. 2. Analysis of available historical data of storm surges is made in Sect. 3. The main conclusions are summarized in Sect. 4.

2 Extreme Waves Induced by Hurricane Dean

2.1 Cyclone General Information

The Hurricane Dean crossed the center of an Antillean arc on 16–17 August 2007. After traversing the channel Sainte-Lucie (south of Martinique), Dean reached to the third stage of SSS; its average maximal wind speed was in the order of $160\text{--}180\text{ km h}^{-1}$ with blasts of 200 km h^{-1} . Between 4 a.m. and 7 a.m. the center of the hurricane passed several kilometers to south of Martinique (15 km to Sainte-Anne, 30 km to Airport Lamentin) (Communiqué météorologique de presse, 2007), see satellite image and Guadeloupe map in Figs. 1 and 2.

Two deaths were caused by Dean in Martinique: an old man died because of cardiac seizure and another man died because of a very strong wind. Great damages occurred in French territories, trees were overthrown and root out (particularly cocowood of Faula au Vauclin, South-east of Martinique; and Sainte-Anne, South of Martinique), roads were destroyed, some beaches of white sand in Sainte-Anne and Diamant disappeared under the water. But the most considerable damage occurred to bananas (100% of production in Martinique and 80% in Guadeloupe), tropical fruits, and sugar-cane (70% of production in Martinique) (Sergent, 2007), see Fig. 3.

The Guadeloupe archipelago was 200 km far from the center of the hurricane; since hurricane winds reached its maximum strength at 10–100 km from the center, wind speed was not so violent there. The highest wind speed was observed in the South: on the island Marie-Galante (119 km h^{-1}) and Les Saintes (126 km h^{-1}) (Damase, 2007). In general, maximum wind speed in Guadeloupe was less than 100 km h^{-1} on the plain and about 175 km h^{-1} on the mountain Soufriere (Communiqué météorologique de presse, 2007). According to SSS these wind speed values correspond to storm surges of 0.09–2.4 m height in Grande-Terre and Basse-Terre, respectively.

The wave height was about 6–10 m; surge waves were measured in Basse Poiné, North Atlantic of Martinique (10.4 m), and in the channel Sainte-Lucie (9 m). In the vicinity of Guadeloupe surge waves were estimated to be 6–8 m, in the channel of Les Saintes the heights exceeded 7 m (Communiqué météorologique de presse, 2007).

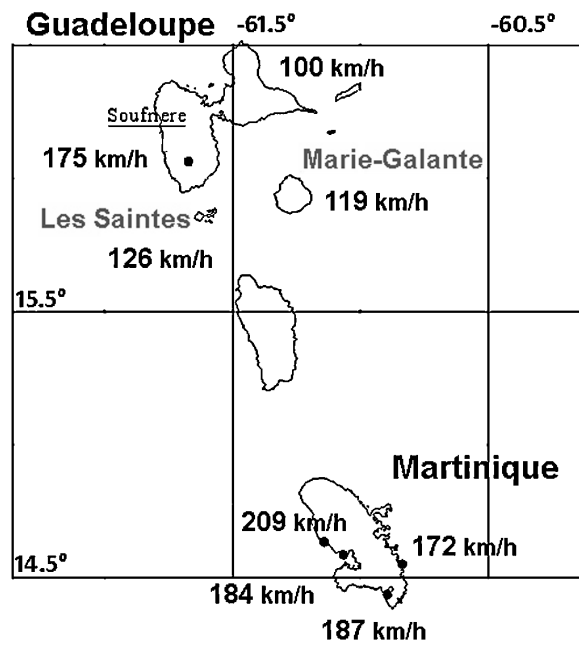


Fig. 1 Observed wind speed during hurricane passage

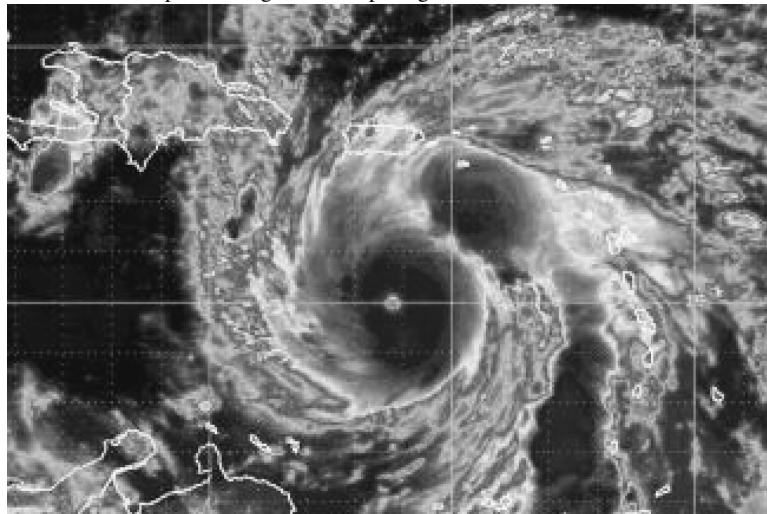


Fig. 2 Hurricane Dean category 4 in the Caribbean, satellite image 17 August 2007, 5.15 a.m. GMT (Les cyclones, typhons et autres ouragans, 2007)



Fig. 3 About 80% of bananas were damaged in Guadeloupe

Field survey of the Guadeloupe Island was carried out 2 days after hurricane passage by a survey team from the University of Antilles Guyane: Prof. Narcisse Zahibo, MSc student Irina Nikolkina and Dr. Ira Didenkulova, which lasted for four days (19–22 August 2007).

As hurricane passed from 200 km to the south from Guadeloupe (channel Sainte-Lucie, south of Martinique), particular damage was caused on the southern shore of Guadeloupe. The most damaged areas were observed from North West to South of Basse-Terre and from South to South East of Grande-Terre. The expedition started in Sainte-Rose (north-west of Basse-Terre) and finished in Sainte-Anne (south-east of Grande-Terre).

2.2 Severely Damaged Territories

Authors performed the field survey and suggested that Grande-Terre was especially damaged by storm surges. This amplification could be caused by bottom topography as the beaches in Grand-Terre are the flattest. The most damaged territories were located in the northern coast of the Caribbean shore, St Anne, St Félix, and Petit-Havre. The main harm observed in Grande-Terre during the field survey included soil degradation, inundation (Sainte-Anne), destruction of littoral buildings (Petit Havre), and debris of marine origin up to 50–60 m inshore (Saint-Felix). North-western part of Basse-Terre was damaged mainly by wind, as it reached its maximum speed near the volcano Soufriere (Communiqué météorologique de presse, 2007). The field survey results showed that the most significant damage was caused

in north and north-western parts of Basse-Terre (Vieux-Habitants, Sainte-Rose, Malendure, Vieux-Habitants). Here is the short description of the observed damage.

2.3 Gosier

In Center of Gosier the main beach was inundated with the marine (sea shells, algae, grass-wrack) and anthropogenic (plastic bottles, bauchles) origin materials (Fig. 4). The small beach near the central park was covered with sea-grass on a horizontal distance of more than 5 m (Fig. 5). The central square of Gosier, which is situated on the sea shore, was cluttered up with the rubbish of all sorts (Fig. 6). The horizontal distance covered with the sea garbage exceeded 10 m. A berth to get to an island Ile de Gosier was totally destroyed by the hurricane, and stone plates near the berth were broken (Fig. 7).

2.4 Saint Félix

In Anse du Mont, Saint-Félix, garbage of marine origin was drifted in the water 100 m offshore (Fig. 8). Anse Canot was over covered by sea-grass along a distance more than 40 m inland (Fig. 9), and trees were uprooted. This beach was washed away, and thickness of a depredated layer exceeded 20 cm. The beach degradation can be seen in Fig. 9.



Fig. 4 Local people gathering cockle-shell and sea plants, Gosier



Fig. 5 A small beach covered with sea-grass, Gosier



Fig. 6 Garbage on the central square, Gosier



Fig. 7 Destroyed berth, Gosier



Fig. 8 Yachts and sea garbage in Marina Saint-Felix (Anse du Mont)



Fig. 9 Beach of Saint-Felix (Anse Canot): before the hurricane, February 2007 (*left*); after the Hurricane Dean, August 2007 (*right*)

2.5 *Petit Havre*

According to an eye-witness, in the bay Petit Havre the wave of 6 m height (Lapinard, 2007) destroyed a berth (Fig. 10). The small beach is totally ruined (Fig. 11), sea garbage and rubbish of anthropogenic origin materials covered the shore. The hurricane uprooted the trees and bent a stone more than 1 m height. Along the beach of Petit Havre the soil was washed away for a distance about 45 m (Fig. 12). A summerhouse and a table were broken and the trees were root out because of beach degradation (Fig. 13). A metallic frame (probably a part of local berth) was dragged by the waves.

2.6 *Sainte-Anne*

The local newspaper declaimed on the first day after the hurricane passage “The Sainte-Anne beach doesn’t exist any more; the road, parking – everything is covered



Fig. 10 Destroyed berth in Petit Havre



Fig. 11 The beach of Petit Havre: January, 2006 (*left*) and August, 2007 (*right*)



Fig. 12 Beach degradation



Fig. 13 Consequences of the hurricane: broken table (*left*), inclined summerhouse (*right*)

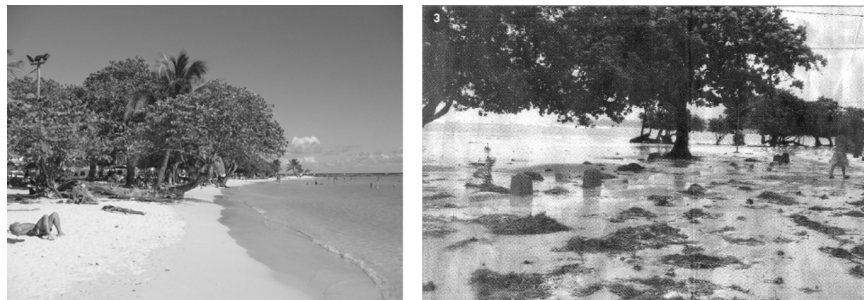


Fig. 14 Sainte-Anne beach: before the hurricane, December 2005 (*left*); after the Hurricane Dean, August 2007 (*right*)

with sand up to apartment houses,” see Fig. 14 (Lapinard, 2007). The destroyed berth (Fig. 15) and several tired up palms (Fig. 16) were observed. The diameter of a rhizome is comparable with human height, mounts to 1.8 m with 30–40 cm thickness. A layer of degraded soil was in 40 cm thickness (Fig. 17). The small hidden beach Village Artisanal de Sainte-Anne near the main road was covered by sea-grass (Fig. 18). The grass was spread out to a distance of 50 m.



Fig. 15 Destroyed berth, Sainte-Anne



Fig. 16 Fallen palms, Sainte-Anne



Fig. 17 Soil degradation, Sainte-Anne



Fig. 18 Sea grass on a small beach, Sainte-Anne



Fig. 19 Community of Marine Capture Fisheries was heavily damaged (Vieux-Habitants)

2.7 Basse-Terre Region

In Vieux-Habitants, run-up of severe storm surges of 3–4 m and heavy winds were observed (Figs. 19 and 20). The beach in Malendure was covered by sea-grass, placed out to several meters (Fig. 21). According to a local paper, some damage was caused in the beach Sainte-Claire, Goyave (Fig. 22), sea damage was drifted 30–40 m offshore, and terrific storm surges were recorded in Vieux-Habitants (Fig. 23) (Lapinard, 2007).



Fig. 20 Storm surges in Vieux-Habitants (France-Antilles Guadeloupe, samedi 18 et dimanche 19 août 2007)



Fig. 21 Extreme left sea-grass several meters inshore in Malendure (on the left photo: a member of expedition Ira Didenkulova)

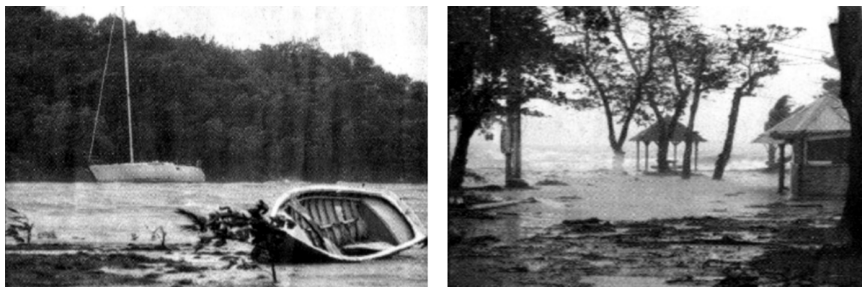


Fig. 22 Data of hurricane destruction in Sainte-Claire, Goyave (destroyed beach)

2.8 Tide-Gauge Data

Since 1984, the network of tide-gauges exists in Guadeloupe. It consists of six mareographs nowadays (two of them are nonworking), Fig. 23 (Etude du risque tsunami en Guadeloupe, 2006). According to the report of the Volcanic and Seismic

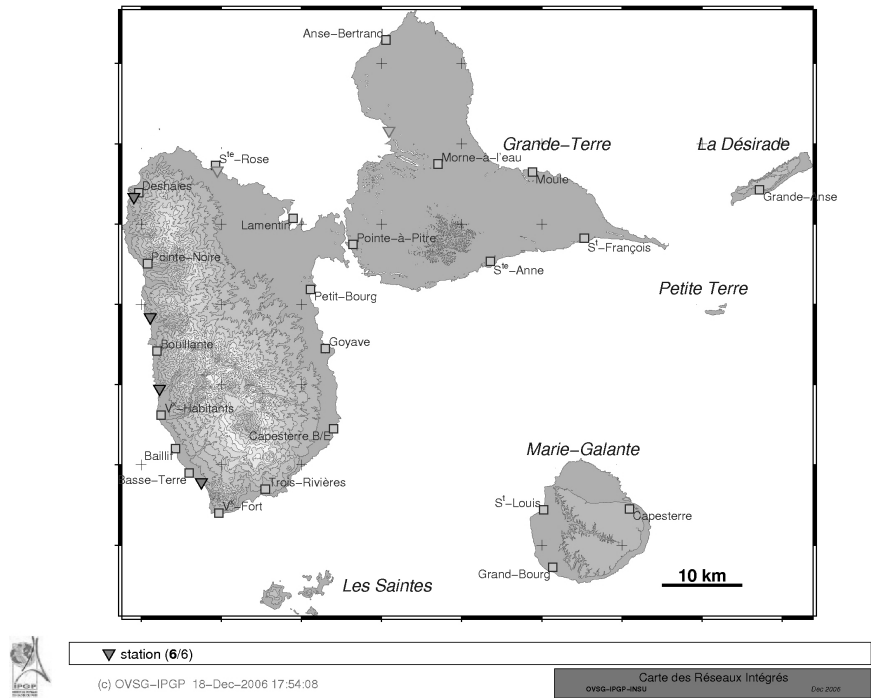


Fig. 23 The location of the tide-gauges in Guadeloupe (triangles and squares indicate actual tide-gauges and planned stations, respectively)

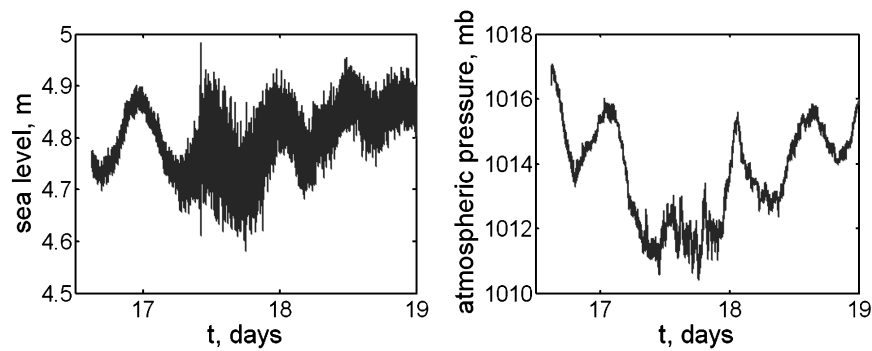


Fig. 24 Records of sea level and atmospheric pressure taken in August, 2007 in Deshaies

Observatory of Guadeloupe, the tide-gauges, which are available now, are located in average depth of about 2 m in Deshaies, Bouillante, Vieux-Habitants and Gourbeyre. The port in Deshaies was the only place where the tide-gauge record was available (Fig. 24). According to the record, storm surges in Deshaies reached 3 m height.

The field survey results mark that, being nowadays well-protected by stone constructions, Deshaies was slightly damaged by wind and not by sea waves. However,

there was some water inside the mareograph (about 2 m above mean water level). This statement is in a good agreement with data from the western part of Basse-Terre (Vieux-Habitants), where storm surges of 3–4 m height were observed. Apparently, the amplitude of storm surges might have had similar order in all Basse-Terre.

The atmospheric pressure is in an obvious correlation with sea waves; here we observe so-called rule of reverse barometer (Lamb, 1932). The experience of the Volcanic and Seismic Observatory of Guadeloupe in 2006 showed that normally the short-term atmospheric pressure variations have an order of 5 mb that causes sea level variations of 5 cm (Etude du risque tsunami en Guadeloupe, 2006). During the cyclone passage the short-term atmospheric pressure variations of 7 mb were accompanied by sea level variations of about 40 cm.

Because of the large distance between the hurricane eye and Guadeloupe (200 km), all mentioned phenomena were recorded by tide-gauge several hours after hurricane passage.

The locations and amplitudes of the storm surges are given in Fig. 25. According to Fig. 25, the Caribbean coast, especially south of Grande-Terre and west of Basse-Terre, was damaged significantly.

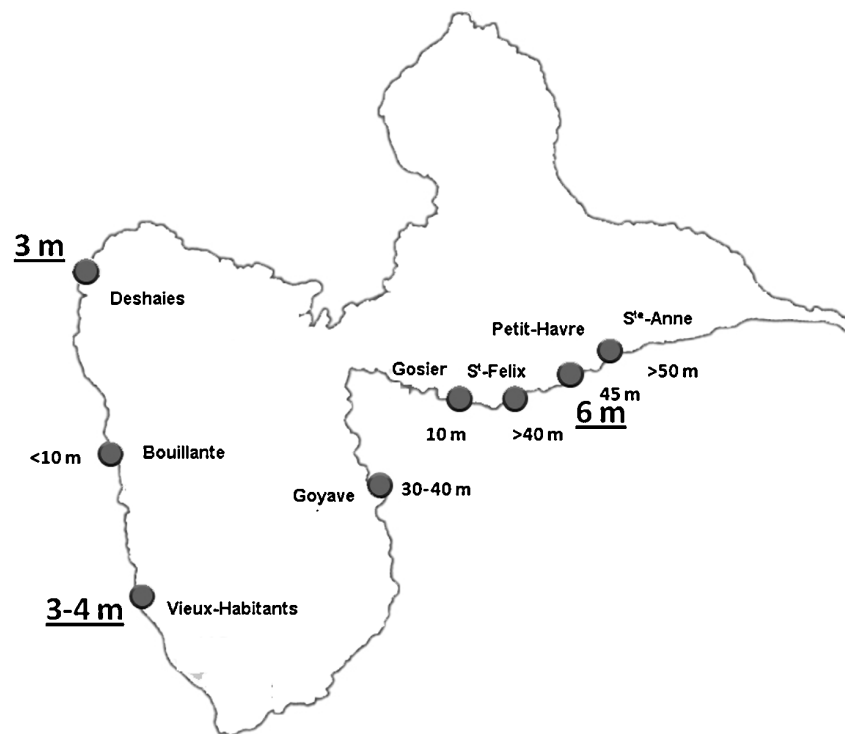


Fig. 25 Locations and amplitude of storm surges produced by Hurricane Dean (run-up height, big underlined letters; run-up length, small letters)

3 Historical Data of Storm Surges

The first mentioned sea-damage during hurricane passage in Guadeloupe occurred in 1642. “Dead fish was partly dug in sand along the whole shore of Basse-Terre; here a refugee was found from its natural world <the sea>.” In Table 2, events are presented in the chronicle order. Here we give some description found in Saffache (2003). Figure 26 presents data of extreme waves in Guadeloupe from 1928 to 2007. Here we use only presided data for storm surge locations for several cyclones; the description does not include the name of district, but a storm surge height for the whole island is given. Thus, the terrible cyclone of 1928 (category 4) produced extreme waves of 15 m height; the famous Hurricane Hugo (category 4) that traversed the island from east to west causing huge damage by wind caused “small storm surges” (“modest houle” in French). This phenomenon is discussed in Pagney and Dalby (1991), where the authors suggested that the coast morphology could have reduced storm surge. During 2000–2006, cyclones produced extreme waves less than 2.5 m, except Hurricane Fabian (category 3) when 4 m storm surges were reported.

Table 2 Damaging cyclones in Lesser Antilles

Date	Cyclone	Category	Storm surge (m)	Location
1928		4	15	Guadeloupe coast
1966	Inez	3	“Probable storm surge”	Saint-François, Pointe-a-Pitre and Sainte-Rose
1979	David	4	4–5	Saint-François
1989	Hugo	4	“Minor storm surge”	Guadeloupe coast
1995	Luis	4	“Strong storm surge”	Moule, Port-Louis, Malendure, Deshaies and Bouillante
1995	Marilyn	1	4–6	Caribbean coast of Grande-Terre and on the shore of Basse-Terre, between Capesterre and Petit-Bourg
1998	Bonnie	TS	1.5–2	South-east of Guadeloupe
1998	Danielle	2	2–2.5	Atlantic coast (east-north-east and north-east)
1999	Jose	2	4–5	South-east of Guadeloupe
1999	Lenny	TS	4–5	Caribbean coast
			3	Deshaies
2000	Joyce	TS	2	Guadeloupe coast
2001	Chantal	TS	“Storm surge”	Guadeloupe coast
2003	Fabian	4	4	Guadeloupe coast
2003	Isabel	4	2.5	Guadeloupe coast
2004	Ivan	3	“Minor storm surge”	Guadeloupe coast
2007	Dean	4	6–8	Petit Havre (Gosier)
			3	Deshaies
			3–4	Vieux-Habitants

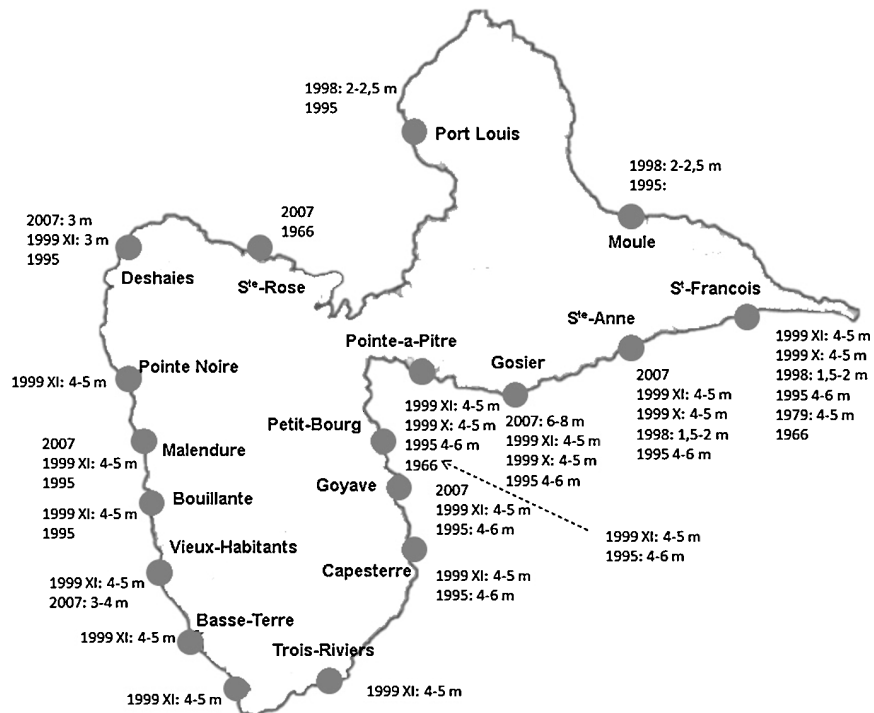


Fig. 26 Locations and dates of extreme waves in Guadeloupe in 1928–2007

The most important storm surges were recorded in 2007 (6–8 m, Petit-Havre), in 1995 (4–6 m, Caribbean coast of Grande-Terre and south-eastern coast of Basse-Terre), and in 1999 (4–5 m, Caribbean coast).

Both Atlantic and Caribbean coasts of the island are exposed to extreme waves; the mean value of storm surge height for the Atlantic coast (2.2 m) is twice lower than that for the Caribbean one (4.4 m). The most dangerous regions are observed in the southern shore of Grande-Terre: Gosier, Sainte-Anne, and Saint-François.

The maximum runup was recorded in 1999 during Hurricane Lenny (category 4), when waves of 3 m height in Deshaies flooded a local boulevard up to 200 m inland. In 2007, after Dean passage, runup length in Anse du Mont, Gosier, reached 100 m.

4 Conclusion

Since 1928, several terrific hurricanes have occurred in the Caribbean, and extreme waves produced by these cyclones damaged Guadeloupe. The tide-gauge record of the first storm surge (hurricane Dean, August 2007) was presented and analyzed.

According to it, the atmospheric pressure is in an obvious correlation with storm surges; here we observe so-called rule of reverse barometer.

The maximal storm surges in Guadeloupe reached 15 m and were produced by hurricane in 1928. The most important storm surges were recorded in 2007 (6–8 m, Petit-Havre), in 1995 (4–6 m, Caribbean coast of Grande-Terre and south-eastern coast of Basse-Terre), and in 1999 (4–5 m, Caribbean coast). Both Atlantic and Caribbean coasts of the island felt extreme waves, and the mean value of storm surge height for the Atlantic coast (2.2 m) is twice lower than that for the Caribbean one (4.4 m). The most dangerous regions are located in the southern shore of Grande-Terre: Gosier, Sainte-Anne, and Saint François.

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References

- Communiqué météorologique de presse, le 21 août 2007 à 12h00 locales, Direction Interrégionale Antilles-Guyane, Météo France
- Damase M.A (2007) Catastrophe naturelle ou pas? Damase M. France-Antilles: le quotidien d'information de la Guadeloupe, N 11251, p 2
- Etude du risque tsunami en Guadeloupe (2006) Installation de marégraphes permanents pour l'analyse des effets liés à l'activité volcanique, Institut de physique du globe de Paris, Observatoire Volcanologique et sismologique de Guadeloupe. Rapport final-décembre 2006, <http://tempetropicale.blogspot.com/2007/08/etude-du-risque-tsunami-en-guadeloupe.html>
- Knabb RD, Rheme JR, Brown DP (2005) Tropical cyclone report Hurricane Katrina 23–30 August 2005, National Hurricane Center, 20 December 2005 http://www.nhc.noaa.gov/pdf/TCR-AL122005_Katrina.pdf
- Lamb H (1932) Hydrodynamics, 6th edn. Cambridge University Press, Cambridge, London
- Lapinard YJ (2007) Cyclone Dean en images / Lapinard YJ, Mercader H (2007) France-Antilles: le quotidien d'information de la Guadeloupe. N 11244. P 4–5
- Les cyclones typhons et autres ouragans (2007). <http://youpifrance.blogspot.com/2007/08/les-cyclones-typhons-et-autres-ouragans.html>
- Nott J, Hayne M (2000) How high was the storm surge from tropical cyclone Mahina? Aust J Emerg Manage (Autumn):11–13
- O'Reilly C, Macaulay P, Parkes G (2007) Atlantic Storm Surge and Tsunami Warning System. http://www.quebec2007.ca/pdf/salle205c/seance30/articles/c_oreilly.pdf
- Pagney P, Dalby M A (1991) propos de l'ouragan Hugo. Les phénomènes cycloniques et les Petites Antilles françaises. Les risques cycloniques et les Petites Antilles françaises. Les cyclones et l'archipel guadeloupéen. S.l., S.N. p 53
- Pielke RA Jr, Rubiera J, Landsea C, Fernandez CML, Klein R (2003) Hurricane vulnerability in Latin America and the Caribbean: Normalized damage and loss potentials. Nat Hazards Rev 4(3):101–113
- Rappaport EN, Fernandez-Partagas J (1997) The deadliest Atlantic tropical cyclones, NOAA Technical Memorandum NWS NHC 47:1492–1996. <http://www.nhc.noaa.gov/pastdeadly.shtml>

- Reed DB, Stucky BE (2005) Forecasting Hurricane storm surge on the Mississippi river. Solutions to coastal disasters :52–60
- Saffache P, Marc J-V, Cospar O (2002) Les cyclones en Martinique: Quatre siècles cataclysmiques. IBIS Rouge Editions, Martinique
- Saffache P, Marc J-V, Huyghes-Belrose V (2003) Les cyclones en Guadeloupe: Quatre siècles cataclysmiques. IBIS Rouge Editions, Martinique, p 276
- Sergent D (2007) Aux Antilles, le cyclone Dean a ravage les bananeraies / Sergent D. La Croix, N 37829, p 7
- Wolshon B, Urbina E, Wilmot C, Levitan M (2005) Review of policies and practices for hurricane evacuation. I. Transportation planning, preparedness, and response. Nat Hazards Rev, ASCE/August:129–142
- Zahibo N, Pelinovsky E, Talipova T, Rabinovich A, Kurkin A, Nikolkina I (2007) Statistical analysis of cyclone hazard for Guadeloupe, Lesser Antilles. Atmos Res 84:13–29

Conclusions et perspectives

1. On a trouvé des nouvelles solutions analytiques dans le cadre du modèle de Savage-Hutter qui décrit la dynamique non linéaire d'écoulement gravitationnel dans des canaux inclinés. En particulier, on décrit la déformation non linéaire du corps des glissements de terrain (un analogue de vague de Riemann dans hydrodynamique); on définit le temps d'effondrement du front d'avalanche. On a trouvés des solutions auto-similaires (de rupture de barrage, le M-vague, le chapeau parabolique), connue auparavant uniquement pour l'écoulement à deux dimensions sur le plan (pas le canal). On a montré que la section de l'inclinaison du canal affecte la vitesse de déformation non linéaire de l'écoulement gravitationnel.
2. Le mouvement du front d'écoulement gravitationnel est étudié dans le modèle de Savage-Hutter pour le canal de section parabolique. Les équations initiales non linéaires sont résolues exactement en utilisant la transformation d'hodographe (Legendre). La dynamique du front d'avalanche dépend de la configuration initiale du front d'écoulement: l'effondrement et la diffusion sont possibles. Le critère de changement de régime est étudié.
3. Le tsunami d'origine volcanique généré par l'éruption volcanique de Montserrat en 2003 a été étudié. Les traces du tsunami sur l'île de la Guadeloupe ont été trouvées pendant l'étude terrain à laquelle l'auteur a participé. La simulation numérique a permis de déterminer les zones de la hauteur maximale de tsunami ce qui a été confirmés par les résultats d'étude terrain.
4. Avec l'aide de la simulation numérique on a étudié la hauteur des vagues du tsunami en fonction de la profondeur du tremblement de terre sous-marin (sur l'exemple du tsunami possible près de la Martinique). Les conclusions théoriques et empiriques sur la forte baisse de la hauteur des vagues avec la profondeur sont confirmés; les caractéristiques régionales de cette dépendance ont été définies.
5. Nous avons trouvé de nouvelles solutions analytiques qui décrivent la génération des ondes longues linéaires associé à les glissements de terrain de la masse d'eau variable, se déplaçant à une vitesse variable dans un bassin de profondeur variable. Ils ont été obtenus pour des conditions spécifiques du profil marin et les caractéristiques spéciaux des glissements de terrain. Le cas de la résonance lors du déplacement des ondes avec la même vitesse que le glissement de terrain a été étudié en détail. Il est montré que dans le cas de résonance l'amplitude des vagues dans le bassin d'une profondeur variable peut être réduite, et les conditions correspondantes sont définies.
6. L'étude de terrain pour examiner des traces de destruction côtière en Guadeloupe causées par le cyclone Dean en 2007 a été effectuée. Grâce à cette étude terrain le matériau unique est récupéré, et la comparaison des observations des cyclones avec des données historiques pour les 500 ans a été faite. Cela nous a permis d'estimer la fréquence d'occurrence des cyclones, selon la vitesse du vent et la pression atmosphérique.
7. La simulation numérique des houles cycloniques en Guadeloupe causés par le cyclone Lili en 2002, a été faite dans le cadre de la théorie de l'eau peu profonde sur la grille de base résolution. La hauteur des houles prévues peut être une base pour le calcul des zones d'inondations et pour fournir les cartes des risques côtiers dans cette région.

Bibliographie

- ACCARY F., ROGER J. Tsunami catalog and vulnerability of Martinique (Lesser Antilles, France). *Sci Tsunami Hazards*, 2010, vol.29, n°3, p. 148-174.
- ARANSON I. S., TSIMRING L.S., MALLOGGI F., CLÉMENT E. Nonlocal rheological properties of granular flows near a jamming limit. *Phys. Rev. E*, 2008, vol. 78, p. 1-14.
- ASSIER-RZADKIEWICZ S., HEINRICH P., SABATIER P.C., SAVOYER B., BOURILLET J.F. Numerical modeling of a landslide-generated tsunami: the 1979 Nice event. *Pure Appl. Geophys.*, 2000, vol. 157, p. 1707-1727.
- BAPTISTA M. A., MIRANDA P. M. A., MIRANDA J. M., MENDES-Victor L. Constraints on the source of the 1755 Lisbon tsunami inferred from numerical modeling of historical data on the source of the 1755 Lisbon tsunami. *J. Geodyn.*, 1998, vol. 25, p. 159-174.
- BAPTISTA M. A., MIRANDA J. M., CHIERICI F., ZITELLINI N. New study of the 1755 earthquake source based on multi-channel seismic survey data and tsunami modeling. *Nat. Hazards Earth Sci. Syst.*, 2003, vol. 3, p. 333-340.
- BOUCHUT F., WESTDICKENBERG, M. Gravity driven shallow water models for arbitrary topography. *Commun. Math. Sci.*, 2004, vol. 2, p. 359–389.
- BOUCHUT F., MANGENEY-CASTELNAU A., PERTHAME B., VILOTTE J.-P. A new model of Saint Venant and Savage-Hutter type for gravity driven shallow water flows. *C. R. Acad. Sci.*, 2003, Ser. I, vol. 336, p. 531–536.
- CANDELLA R. N., RABINOVICH A. B., THOMSON R. E. The 2004 Sumatra tsunami as recorded on the Atlantic coast of South America. *Adv. Geosci.*, 2008, vol. 14, p. 117-128.
- DEPLUS C., LE FRIANT A., BOUDON G., KOMOROWSKI J.C., VILLEMANT B., HARFORD C., SÉGOUFIN J., CHEMINÉE J.-L. Submarine evidence for large-scale debris avalanches in the Lesser Antilles arc. *Earth Planet. Sci. Lett.*, 2001, vol. 192, p. 145–157.
- DIDENKULOVA I. I., PELINOVSKY E. N. Phenomena similar to tsunami in Russian internal basins. *Russ. J. Earth Sci.*, 2006, vol. 8, n°6, ES6002, doi: 10.2205/2006ES000211.
- DIDENKULOVA I., PELINOVSKY E., SOOMERE T. Long surface wave dynamics along a convex bottom. *J. Geophys. Res. - Oceans*. 2009. Vol. 114, C07006. doi:10.1029/2008JC005027.
- DIDENKULOVA I., PELINOVSKY E. Traveling water waves along quartic bottom profile. *Proc. Estonian Acad. Sci.*, 2010, vol. 59, n°2, 166-171.
- DORVILLE J.-F., ZAHIBO N. Hurricane Omar Waves Impact on the West Coast of the Guadeloupe Island, October 2008. *Open Oceanogr. J.*, 2010, vol. 4, p.83-91.
- FERNANDEZ-FERIA R. Dam-break flow for arbitrary slopes of the bottom. *J. Eng. Math.*, 2006, vol. 54, p. 319-331.
- FRITZ H. M., HAGER W. H., MINOR H. E. Lituya Bay Case: Rockslide Impact and Wave Runup. *Sci Tsunami Hazards*, 2001, vol. 19, n°1, p. 3-22.
- GRINDLAY N. R., HEARNE M., MANN P. High Risk of Tsunami in the Northern Caribbean. *Eos*, 2005. vol. 86, n 12, p. 121-132.
- HARBITZ, C. B. Model simulations of tsunamis generated by the Storegga Slides. *Mar. Geol.*, 1992. vol. 105, p. 1-21.

- HEINRICH P., MANGENEY A., GUIBOURG S., ROCHE R. Simulation of water waves generated by potential debris avalanche in Montserrat, Lesser Antilles. *Geophys. Res. Lett.*, 1998. vol. 25, n°18, p. 36971-3700.
- HEINRICH P., BOUDON G., KOMOROWSKI J. C., SPARKS R. S. J., HERD R., VOIGHT B. Numerical simulation of the December 1997 debris avalanche in Montserrat, Lesser Antilles. *Geophys. Res. Lett.*, 2001, vol. 28, n°13, p. 2529-2532.
- HERD R. A., EDMONDS M., BASS V. A. Catastrophic lava dome failure at Soufrière Hills Volcano, Montserrat, 12–13 July 2003. *J. Volcanol. and Geoth. Res.*, 2005, vol. 148, p. 234-252.
- HTDB/WLD, Historical tsunami database for the World ocean, 2011. Disponible sur: <http://tsun.sccc.ru/nh/tsunami.php>.
- IOUALALEN M., ASAVANANT J., KAEWBANJAK N., GRILLI S., KIRBY J., WATTS, P. Modeling the 26th December 2004 Indian Ocean tsunami: Case study of impact in Thailand. *J. Geophys. Res. – Oceans*, 2007, vol. 112, C07024, doi:10.1029/2006JC003850.
- LE FRIANT A, HEINRICH P., DEPLUS C., BOUDON G. Numerical simulation of the last flank-collapse event of Montagne Pelee, Martinique, Lesser Antilles. *Geophys. Res. Lett.*, 2003, vol. 30, n°2, p. 1034-1037.
- LE FRIANT A, LOCK E. J., HART M. B., BOUDON G., SPARKS R. S. J., LENG M. J., SMART C. W., KOMOROWSKI J. C., DEPLUS C., FISHER J. K. Late Pleistocene tephrochronology of marine sediments adjacent to Montserrat, Lesser Antilles volcanic arc. *J. Geol. Soc. London*, 2008, vol. 165, p. 279-289.
- LIU P. L.-F., LYNNETT P., SYNOLAKIS C. E. Analytical solutions for forced long waves on a sloping beach. *J. Fluid Mech.*, 2003, vol. 478, p. 101-109.
- MANGENEY A., HEINRICH P., ROCHE R. Analytical solution for testing debris avalanche numerical models. *Pure Appl. Geophys.*, 2000, vol. 157, p. 1081-1096.
- MADER C. L. Modeling the 1755 Lisbon tsunami. *Sci. Tsunami Hazards*, 2001, vol. 19, p. 93-116.
- MVO, The Montserrat Volcano Observatory, 2011. Disponible sur: <http://www.mvo.ms>.
- NEUMANN C. J., CRY G. W., CASO E. L., JARVINEN B. R. Tropical cyclones of the North Atlantic Ocean, 1871–1977. National Climatic Center, Asheville, NC, 1978, 178 p.
- NGDC, The National Geophysical Data Center, 2011. Disponible sur: <http://www.ngdc.noaa.gov>.
- NOAA, The *National Oceanic and Atmospheric Administration*, [<http://www.noaa.gov>], 2011.
- OKADA Y. Surface deformation due to shear and tensile faults in a half space. *Bull. Seismol. Soc. Am.*, 1985, vol. 75, p. 1135-1154.
- O'LOUGHLIN K. F., LANDER J. F. *Caribbean Tsunamis: A 500-Year History from 1498–1998*. Adv. Nat. Technol. Hazards Rese, 2003, 280 p.
- PARARAS-CARAYANNIS G. Volcanic tsunami generating source mechanisms in the Eastern Caribbean region. *Sci. Tsunami Hazards*, 2004, vol. 22, n°2, p. 74-114.
- PELINOVSKY E. N. Analytical models of tsunami generation by submarine landslides. *Submarine landslides and tsunamis*, NATO Science Series: IV. Earth Env. Sci. / ed. par Yalçiner A., Pelinovsky E., Okal E., Synolakis C. E., Kluwer, 2003, p. 111-128.
- PELINOVSKY E., KHARIF C. RIABOV I., FRANCIUS M. Modelling of tsunami propagation in the vicinity of the French coast of the Mediterranean, *Nat. hazards*, 2002, vol. 25, n°2, p. 135-159.
- PELINOVSKY E. N. *Hydrodynamics of tsunami waves*, Institute of Applied Physics, Nizhny Novgorod, 1996, 276 p.

- PELINOVSKY E., POPLAVSKY A. Simplified Model of Tsunami Generation by Submarine Landslides, *Phys. Chem. Earth*, 1996, vol. 21, n°12, p. 13–17.
- PIRULLI M., MANGENEY A. Results of back-analysis of the propagation of rock avalanches as a function of the assumed rheology. *Rock Mech. Rock Eng.*, 2008. vol. 41, n°1, p. 59–84.
- PUDASAINI S. P., HUTTER K. *Avalanche Dynamics: Dynamics of Rapid Flows of Dense Granular Avalanches*, New York: Springer, 2007, 602 p.
- RIPEPE M., DE ANGELIS S., LACANNA G., POGGI P., WILLIAMS C., MARCHETTI E., DELLE DONNE D., ULIVIERI G. Tracking Pyroclastic Flows at Soufrière Hills Volcano. *Eos*, 2009, vol. 90, n°27, p. 229.
- ROGER J., ALLGEYER S., HÉBERT H., BAPTISTA M. A., LOEVENBRUCK A., SCHINDELÉ F. The 1755 Lisbon tsunami in Guadeloupe Archipelago: source sensitivity and investigation of resonance effects. *Open Oceanogr. J.*, 2010, n°4, p. 58-70.
- SAFFACHE P., MARC J.-V., COSPAR O. Les cyclones en Martinique: quatre siècles cataclysmiques. Martinique: IBIS Rouge Editions, 2002, 197 p.
- SAFFACHE P., MARC J.-V., HUYGHES-BELROSE V. Les cyclones en Guadeloupe: quatre siècles cataclysmiques. Martinique: IBIS Rouge Editions, 2003, 276 p.
- SAMMARCO P., RENZI E. Landslide tsunamis propagating along a plane beach. *J. Fluid Mech.*, 2008. vol. 598, p. 107-119.
- SAVAGE S. B., HUTTER K. The motion of a finite mass of granular material down a rough incline. *J. Fluid Mech.*, 1989, vol. 199, p. 177-215.
- SAVAGE S. B., HUTTER K. Dynamics of avalanches of granular materials from initiation to run-out. Part 1: Analysis. *Acta Mech.*, 1991, vol. 86, p. 201-223.
- SLADEN A., HEBERT H., SCHINDELE F., REYMOND D. L'aléa tsunami en Polynésie française: apports de la simulation numérique. *CR. Geosci.*, 2007, vol. 339, n°5, p. 303-316.
- SOLOVIEV S.L., GO C. N. *A catalogue of tsunamis on the western shore of the Pacific Ocean*. Academy of Sciences of the USSR, Nauka Publishing House, Moscow, 1974, 310 p.
- TINTI S., BORTOLUCCI E., CHIAVETTIERI C. Tsunami excitation by submarine slides in shallow-water approximation. *Pure Applied Geophys.*, 2001, vol. 158, p. 759-797.
- TITOV V., RABINOVICH A. B., MOFJELD H. O., THOMSON R. E., GONZÁLEZ F. I. The Global Reach of the 26 December 2004 Sumatra Tsunami, *Science*, 2005, vol. 309, n°5743, p. 2045-2048.
- UNISYS, Atlantic Tropical Storm Tracking by Year (1851–2002), based on data from the Tropical Prediction Center, NOAA, 2011. Disponible sur: <http://weather.unisys.com/hurricane/atlantic/>.
- WATTS P. Tsunami features of solid block underwater landslides. *J. Waterw. Port C. Div.*, 2000, vol. 126, n°3, p. 144-152.
- YALCINER A. C., PELINOVSKY E. N. OKAL E, SYNOLAKIS C. E. *Submarine landslides and tsunamis*. NATO Science Series: IV. Earth and Environmental Sciences, Kluwer, 2003. vol. 21, 352 p.
- YALCINER A., ZAHIBO N., PELINOVSKY E., INSEL I., DILMEN D. I., ZAYTSEV A., CHERNOV A., OZER C. Understanding the possible effects of near and far field tsunamis on Lesser Antilles by numerical modelling. *Open Oceanogr. J.*, 2010, n°4, p. 50-57.
- ZAHIBO N. Deshaies, Guadeloupe tsunami, 20 May 2006, Montserrat, Lesser Antilles, Soufriere volcano tsunami. Tsunami Newsletter, 2006, vol. 38, n°2, May–June 2006, p. 4–5. Disponible sur: www.ioc3.unesco.org/itic/files/2006_May-June_screen.pdf.

ZAHIBO N., PELINOVSKY E., OKAL E., YALCINER A., KHARIF C., TALIPOVA T., KOZELKOV A. The earthquake and tsunami of November 21, 2004 at Les Saintes, Guadeloupe, Lesser Antilles. *Sci. Tsunami Hazards*, 2005, vol. 23, n°1, p. 25-39.

ZAHIBO N., PELINOVSKY E., YALCINER A., KURKIN A., KOZELKOV A., ZAITSEV A. The 1867 Virgin Island Tsunami: observations and modelling. *Oceanol. Acta*, 2003a, vol. 26, n°5-6, p. 609-621.

ZAHIBO N., PELINOVSKY E., KURKIN A., KOZELKOV A. Estimation of far-field tsunami potential for the Caribbean Coast based on numerical simulation. *Sci. Tsunami Hazards*, 2003b, vol. 21, n°4, p. 202-222.

ZAHIBO N., PELINOVSKY, E. Evaluation of tsunami risk in the Lesser Antilles. *Nat. Hazards Earth Sci. Syst.*, 2001, vol. 3, p. 221-231.