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**Lian LIAN**

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**A hierarchical and structured methodology to solve a general delivery problem -  
Resolution of the basic sub-problems in the operational phase**

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|                           |                                                                  |
|---------------------------|------------------------------------------------------------------|
| <b>Président</b>          | <i>Jean-Pierre BOUREY, Professeur, EC-Lille</i>                  |
| <b>Rapporteur</b>         | <i>Besoa, RABENASOLO, Professeur, ENSAIT Roubaix</i>             |
| <b>Rapporteur</b>         | <i>Gilles, GONCALVES, Professeur, FSA Béthune</i>                |
| <b>Membre</b>             | <i>Saïd HANAFI, Professeur, Université de Valenciennes</i>       |
| <b>Membre</b>             | <i>Frédéric SEMET, Professeur, EC-Lille</i>                      |
| <b>Directeur de thèse</b> | <i>Emmanuel, CASTELAIN, Maître de Conférences HDR , EC-Lille</i> |

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# CHAPTER 1: INTRODUCTION

## 1.1 Background

A supply chain is a network of facilities and distribution options that performs the functions of procurement of materials, the transformation of these materials into intermediate and finished products and the distribution of these finished products to customers. There are four major decision areas in supply chain management: location, production, inventory, and transportation. In the last 30 years, decision makers in companies and researchers have a broad consensus that transportation problems in supply chain have a considerable economical impact on distribution center.

In practice, this transportation problem contributes directly to reduce the costs of all logistic systems (Alvarenga et al. 2007). It is in fact the goal of the all private companies. Generally, the transport system can be described as follows: given a node set  $V$  and a set  $E$  of candidate edges, the transport network is simultaneously defined as the underlying network  $G = (V, E)$ . In a broad sense, the edges represent the links between nodes on which the goods are delivered, while the nodes include one or several of the four following types of the operators. Furthermore, one node in the network can simultaneously take the functions of several operators.

- Origins: they are generally suppliers of products who require transport services to move raw materials and intermediate products, and to distribute final goods in order to meet customer demands. Here, the goods being transported can be persons and load goods as well.
- Shippers: they may be the producers of products or some intermediary company (e.g., brokers), thus generate the demand for transportation.
- Carriers: they usually answer this demand by supplying transportation services. In the real world, railways, ocean shipping lines, trucking companies, and postal services are also carriers. They provide seaports, platforms, agencies and other intermediate facilities which could be described as carriers as well.
- Destinations (consumers): they are normally the end of the transportation chain, and serviced by the carriers.

Hierarchical approach and structure are two basic characteristics considered by the transport researches.

The hierarchical approach can be defined as the different time horizons faced by the operators in the network. It involves strategic, tactical and operational problems. In the strategic phase, decisions are found on a long-term horizon (5-10 years). The tactical phase involves a median-term horizon (generally several months). Finally, on operational phase, day-to-day or even real-time decisions are made.

The other characteristic, the structure, can be defined as the layout of the nodes.

Successfully selecting of network structure can not only bring high cost benefits, but also provide compelling evidence of efficiency and practicality. It makes the decision makers select the appropriate network structure that well performs for their distribution system.

Up to now, the research on transport problems generally focuses on a part of transport systems with easy structures in a particular time horizon but the more general transport problems with large scale, complex structures and in several time horizons have not attracted a great deal of attention. For example, Vehicle Routing Problem (VRP) is a transport problem widely studied in the literature. It can be defined as the problem of designing the optimal delivery or collection routes from one or several origins to a number of geographically scattered cities or customers (destinations), subject to side constraints (Laporte 1992). However, VRP is just a particular problem for a part of the transportation chain. Moreover, it is indeed a transport problem in operational phase between origins and destinations.

## 1.2 Problem Statement

This thesis focuses on a *General Delivery Problem (GDP)*, which is more general than the well-known VRP and Traveling Salesman Problem (TSP). In GDP, there is no hypothesis on the way that origins and destinations will be linked to organize and to realize the whole set of deliveries. That is to say, when a company confronts a transportation problem and it just knows the locations and the quantities of the customers, the company needs to decide the strategies and the tactical decisions to manage the transportation process and to organize the routing sequence in operational phase. Generally speaking, TSP or VRP is a kind of the routing sequence problem. TSP and VRP are NP-hard combinatorial optimization problems (Savelsbergh 1985). Therefore, GDP is difficult to solve as TSP and VRP are sub-problems of GDP.

The concept is very general and thus, it means many things to many people:

- It could be huge like intermodal transportation, while it could be small like a Travelling Salesman Problem (TSP). European Conference of Ministers of Transport (2001) gives the definition for intermodal transportation: “movement of goods in one and the same loading unit or vehicle, which uses successively two or more modes of transport without handling the goods themselves in changing modes.” That is to say, the transportation in the intermodal transportation is a combination of truck, rail, and ocean shipping, dedicated rail services to move massive quantities of products (persons or load from the origins) over long distance. While TSP is to find a shortest possible tour for just one vehicle, in which each node is visited exactly once.
- The network structure can be either complex or simple. Furthermore, mostly all types of operators mentioned earlier may, thus, be involved in GDP, either by providing service for part of the transportation chain or by operating a distribution system.
- It takes into consideration of the transport in each time horizon of the whole process, including strategic phase, tactical phase and operational phase. Nevertheless, as mentioned above, the researches mainly concern the transportation problems in a particular time horizon. For instance, VRP in operational phase, Facility Location Problem (FLP) and Hub Location Problem (HLP) in strategic phase and the Facility Allocation Problem (FAP) and Hub Allocation Problem (HAP) in tactical phase.

This thesis proposes a **hierarchical and structured methodology to solve GDP on the whole**. Furthermore, it is mainly concerned with three themes in the thesis: methodology, optimization theory and applications rather than algorithms. They are introduced in detail as follows:

In terms of methodology, it is a hierarchical (strategic, tactical and operational) and structured approach that design and decompose the GDP into basic distribution problems as independent as possible. Here the basic distribution problems are the transport problem with basic structures, for example, traditional transportation problems (TP), VRP, TSP, FLP, HLP, etc , which have already been identified and solved with the existing methods/tools.

For the optimization transport theory, several mixed integer linear programming formulations are proposed for the basic networks which are somewhat alternative to the classical FLP and HLP formulations for the application purposes and the set of possible applications confronted by the private company. They are respectively the new facility single/multi-allocation problem and new single/multi-allocation p-hub median problems in operational phase. Furthermore, the new cost objective and the constraints of the edge capacities are provided according to the distribution characteristics of the private company in operational phase.

At the application level, two real examples are discussed. The first is the delivery system of a company we will call GEDECO, based on the data of a real delivery company. The other one is the delivery system of the Regional Fire and Emergency Center (Le Service Départemental d'Incendie et de Secours du Nord, SDIS59) in the north of France.

## 1.3 Organization of the Thesis

The main topic of the thesis consists of the study of a solution methodology for GDP, the solution of the basic network structures and the two real applications. They are respectively discussed in Part II, III and IV in the thesis. Let us see how the above themes have been developed in the chapters of the thesis.

In Part I, the main issues mentioned in our thesis are overviewed.

In part II, the chapters 3 and 4 introduce our methodology for the solution process of GDP. Chapter 3 is devoted to synthesize the issues in the solution process for GDP following the three somewhat classical phases (strategic, tactical and operational) in operational process. Since GDP covers a very wide research area, we introduce the principle issues in two aspects: transportation system design and terminal operations and discuss the interdependences among the issues and analyze the integrations of the issues in the transportation process.

In chapter 4, we provide a 3-phase decomposition-based heuristic framework to solve GDP. Furthermore, the decomposition methods are in detail introduced. The inspiration source of heuristic framework is the two-phase heuristics to solve VRP. As stated in Cordeau et al. (2007), the idea is based on the decomposition of the VRP solution process into two separate sub-problems:

- (1) Clustering: to determine a partition of the customers into subsets, each corresponding to a route;
- (2) Routing: to determine the sequence of customers on each route.

That is to say, customers are first grouped into clusters, and then the routes are determined by suitably sequencing the customers within each cluster. Different techniques have been proposed for the clustering phase, while the routing phase amounts to solve a TSP. As we stated above, TSP is a typical transportation problem widely researched and up to now there exist solvers like Concorde for instance, which can optimally solve large-scale TSP with instance up to 15,112 cities. Thus, through two-phase heuristic, the VRP which is difficult to solve is divided into some smaller and easier TSP. Improving the two-phase heuristic, we propose a three-phase methodology, including decomposition, routing and improvement phases, to solve our GDP. It can efficiently overcome the large size and the complex structure, which are the two main reasons leading GDP to be difficult to solve. Then, the decomposition methods in each operation phase are discussed in the rest of the chapter. It is important because the different decomposition methods may lead to different resolutions. In order to clearly present our decomposition methods, we firstly classify the attributes of the network. Then four types of basic distribution network structures are introduced, which are widely studied and can be successfully solved. After that, the decomposition methods in strategic view, tactical view and operational view are separately provided. Thus, we can decompose a large-scale GDP with a complex structure into sub-networks with basic distribution network structures as independent as possible. In the same time, we propose some ideas based on the facility multi-allocation to handle the related sub-networks.

Chapter 5, 6 and 7 compose the third part of the thesis. After the decomposition phase, the original networks are divided into several independent sub-networks. Then the transportation problem for each sub-network is regarded as sub-problems of the original GDP, some of which, such as FLP, HLP, FAP, TP, TSP, VRP, etc., could be solved by the existing tools and/or heuristics algorithms. However, some others are not yet studied, especially when we take into account the capacity of the used vehicles. From chapter 5, we solve the sub-problems of these basic network structures in operational phase.

In chapter 5, we apply our proposed framework to solve a special case of GDP: the Capacitated Vehicle Routing Problem (CVRP). CVRP is a special case of VRP with the additional constraint that every vehicle has a uniform capacity of a single commodity. Furthermore, it is a network distribution problem in operational phase with one basic network structure. We first present a Capacitated Clustering Algorithm (CCA) to decompose the original problem into several independent TSPs. Here, CCA is an advanced algorithm based on k-means algorithm. Then the solutions of the TSPs are produced by Concorde. Finally, we improve the routing sequence between groups to approach the global optimal solution with Simulated Annealing and a 3-opt heuristic improvement algorithm.

In chapters 6 and 7, we study the solution to the other two basic distribution networks with the optimal transport theory. The transport studies of these two basic distribution networks are usually mentioned as facility/hub location problems (FLP/HLP). They are commonly defined as locating a finite set of new facilities/hubs with respect to a finite set of possible places, and determining the best strategy for assignment of the products between nodes. They almost focus on the strategic phase since the locations of facility/ hubs do not change for a long time. However, the transport problems in operational phase do not have a great importance for the researchers even though they are also useful and meaningful for the transport process. In operational phase, the existing resources are usually limited. The decisions in the operational phase are made in a short term horizon, generally one or several days that affect how the products are delivered in the transport system. Additionally, they can be classified into two types: facility/hub single location and facility/hub multi-location. In this

case, we propose some new facility/hub single allocation formulations and facility/hub multi-allocation formulations based on the classical FLP/HLP in strategic phase and tactical phase, and then test them on the benchmark CAB, to show the performances of our formulations.

Part IV contains two real examples of our methodology, one of which is the distribution instance of the Regional Fire and Emergency Center (Le Service Départemental d'Incendie et de Secours du Nord, SDIS59) in the north of France described in Chapter 8, and the other is the real delivery problem faced by a general delivery company presented in Chapter 9.

Chapter 8 introduces three distribution strategies of SDIS59, involving the “Old Delivery Strategy”, an improvement delivery strategy and a 0-level distribution strategy. In the old delivery strategy, the delivery process is divided into two parts with the strategic decomposition method. Then we propose an improvement avoiding the round-trips in the network, and efficiently improving the old delivery solution. The third strategy simplifies the delivery process by centralizing the good transport from origins directly to the destinations and deleting the intermediate facilities. In this case, the delivery problem can be defined as a CVRP. And our proposed solution approach developed in Chapter 5 is employed to generate the solutions. Moreover, the solutions are illustrated and show that different strategies can lead to different results. It somewhat provides the reason why SDIS59 has changed the old delivery strategy.

In Chapter 9, we study another real instance, a distribution problem for a 2-level distribution network of a general delivery company in France. In this delivery problem, the network structure has already been determined as a 2-level distribution network, which is composed by suppliers, platforms, agencies and customers. The main activities are to collect the goods from suppliers, then to deliver them throughout France through its parcel distribution network and finally to its customers. We do not take into account the time windows as the goods delivered are bulky and not urgent. In order to solve this distribution network, the strategic decomposition method and the operational decomposition method are applied to divide the original network into a combination of a hub 1-level distribution network and several CVRPs in the administrative regions. Then the formulations proposed in Chapter 7 for hub single/multi-allocation p-hub median problems are used to solve the hub 1-level distribution network, and the approach to CVRP are used to solve the CVRP in each administrative region. The computational results are demonstrated at the end of the chapter. In addition, we compare the solutions between the single-allocation p-hub median problem and multi-allocation p-hub median problem.

As a whole, this thesis presents, in a quite unified setting of GDP involving the solution methodology, optimal theory and the applications, all the researches that we carried out during these PhD studies. We think the most interesting feature of the thesis is that, we propose a new transportation problem, GDP and provide the methodology to solve it. Most of the encountered sub-problems are new. Only some subjects, mainly related to the solution approach of CVRP are not new. But we propose some new clustering approaches. On the other hand, the main issues of the solution process in Chapter 3 and the proposed methodology in Chapter 4 seem to be quite original. Moreover, we provide new extended formulations of classical formulations, in operational phase, which are tested on benchmarks. At least, our thesis does not only develop theory and methodology, but applies them to real problems of the real world (in Part IV).



## Part I

# AN OVERVIEW OF GENERAL DELIVERY PROBLEM



## CHAPTER 2: Bibliography

As mentioned above, GDP is a complex and general domain, with a great number of operators and three phases in operational planning. Furthermore, the decisions to operate the transportation system of GDP have to adapt rapidly to changing political, economic and social conditions and trends. Thus, the efficient solution is required to assist and enhance the analysis of planning and decision-making process in the whole system.

There are a limited number of the previous researches that integrate the whole transportation system in a various number of phases. Crainic and Laporte (1997) identify some of the main issues in freight transportation planning and operations according to the three classical decision-making levels: strategic, tactical and operational phases. Three years later, Crainic (1999) notes that the service network design problem is increasingly used to designate the main issues for freight transportation in tactical planning and present a state-of-the-art of service network design problems. Crainic and Kim (2007) review a part of the main issues in the three operation levels for intermodal transportation. Desaulniers and Hickman (2007) state that the public transit problem could be divided into a set of sub-problems that are usually solved sequentially at various stages of planning process (strategic, tactical and operational phases). Furthermore, they review the state-of-the-art of models and approaches for solving these public transit problems.

However, most researches focus on one part of the transportation systems. The goal of this chapter is to review several main issues in three phases of operation planning in GDP and to present appropriate Operations Research models, methods, and computer-based planning tools. We do not wonder to exhaustively cover all of issues, but just focus on the recent contributions that have been applied or have the potential to be applied into GDP from our point. Especially, we present in detail several issues which we focus on in the remaining parts of the thesis.

## 2.1 Strategic Phase

Strategic phase concerns long-term decisions for company management and capital investments. The strategic decisions determine general development policies and broadly shape the operating strategies of the system. Such decisions involve a set of logistic system design problems, such as physical network design and the structure selection, facility locations, hub locations and transportation mode selection, etc. In this section, we introduce in detail the network design problem, particularly, facility location problem and hub location problems, which are the main three issues widely researched in the literatures. Furthermore, we propose new formulations for facility location problem in Chapter 6 and p-hub location problem in Chapter 7.

### 2.1.1 Network design problem

The network design problem (NDP) is one of the most difficult and challenging problems in transportation. It aims to optimize a transportation network with respect to a system-wide objective while considering the route choice behaviour of network users (Bell et al. 1997).

NDP is normally defined on a graphs with nodes (vertices) and arcs (links). On the graphs, there are certain nodes representing origins of some transportation demand for one or several commodities or products while others (may be the same nodes as the origins) are destinations that demand the commodities or products. The arcs are represented by edges when it is not necessary to specify a direction. They may have various characteristics, such as length, capacity and cost.

Thereby, the aim of NDP is to choose the arcs in order to enable products to flow between the origin-destination pairs (OD pairs) at the lowest global system costs with the capacity constraints. The global system cost is generally defined as the combination of the total fixed cost of selecting the arcs plus the total variable cost of using the network. Here, fixed costs may be associated to some or all arcs, signalling that the fixed cost is occurred as soon as the corresponding arc is chosen to use. And the variable cost is in most cases related to the volume of traffic on the arc.

#### 2.1.1.1 Classical formulation

There are lots of general formulations for NDP, for example, Magnanti and Wong (1984), Minoux (1989), Magnanti and Wolsey (1995), and the book by Ahuja et al. (1993). We present here the classical formulation provided by Ahuja et al. (1995).

Given a node set  $N$  and a set  $A'$  of candidate arcs, and NDP is aimed to simultaneously define the underlying network  $G = (N, A)$  in which  $A \subseteq A'$ , and to determine an optimal flow on it. In the standard version of the problem, Ahuja et al. (1995) define the fixed cost  $F_{ij}$  for including any arc  $(i, j)$  from  $A'$  in the network. To formulate the NDP, the following notations are firstly defined.

$c_{ij}^k$ : the cost for routing all of the flow requirement of commodity  $k$  on arc  $(i, j)$ ;

$r_k$  : a flow requirement;

$f_{ij}^k$  : the fraction of flow requirement of commodity  $k$  that flows on arc  $(i, j)$ ;

$y_{ij}$  : an integer variable indicating how many copies of arc  $(i, j)$  we install on the network;

$u_{ij}$  : units of capacity provided by each copy.

In the standard version of NDP, a fixed cost  $F_{ij}$  is occurred for including any arc  $(i, j)$  from  $A$  in the network. Then, an optimization formulation of NDP is defined as follows:

$$\min \sum_{k=1}^K \sum_{(i,j) \in A'} c_{ij}^k f_{ij}^k + \sum_{(i,j) \in A'} F_{ij} y_{ij} \quad (2.1.1)$$

$$\text{s.t.} \quad \sum_{j \in N} f_{ij}^k - \sum_{j \in N} f_{ji}^k = d_i^k, \quad \text{for all } i \in N, 1 \leq k \leq K, \quad (2.1.2)$$

$$\sum_{k=1}^K r_k f_{ij}^k \leq u_{ij} y_{ij}, \quad \text{for all } (i, j) \in A', \quad (2.1.3)$$

$$f_{ij}^k \leq y_{ij}, \quad \text{for all } (i, j) \in A' \text{ and all } 1 \leq k \leq K, \quad (2.1.4)$$

$$f_{ij}^k \geq 0, \quad \text{for all } (i, j) \in A' \text{ and all } 1 \leq k \leq K, \quad (2.1.5)$$

$$\text{Specific additional constraints on } y_{ij}, \text{ for all } (i, j) \in A' \quad (2.1.6)$$

$$y_{ij} \geq 0 \text{ and integer, for all arcs } (i, j). \quad (2.1.7)$$

where

$$d_i^k = \begin{cases} -1 & \text{if node } i \text{ is the origin of products } k, \\ 1 & \text{if node } i \text{ is the destination of products } k, \\ 0 & \text{otherwise;} \end{cases} \quad (2.1.8)$$

In this multi-commodity flow version of the problem, the objective function (2.1.1) measures the total cost of the system. This is a linear cost. There exist several researches with the nonlinear formulations to solve some important applications, but we focus just on the presentation of the linear cost formulation.

Constraints (2.1.2) and (2.1.7) are mass balance constraints. They express the usual flow conservation and demand satisfaction restrictions.

Constraints (2.1.3), often identified as bundle or forcing constraints, state that the total flow on arc  $(i, j)$  cannot exceed its installed capacity;

Constraints (2.1.4) are the forcing constraints stating that there is not flow on arc  $(i, j)$  if we do not install it. e.g., if  $y_{ij} = 0$ ; these constraints is inefficient in the integer programming version of this model, but not in its linear programming relaxation. Thus, it is useful to include it in the model;

Constraints (2.1.5) permit us to capture additional constraints related to the design of the network or relationships among the flow variables. Together, they may be applied to a wide variety of practical situations, and this leads the NDP more interesting.

Constraints (2.1.6) are additional constraints that are added into the formulation to impose restrictions on the design variables  $y_{ij}$ , for example, degree constraints on the nodes.

An important type of the additional constraints reflects the usually limited nature of the available resources, e.g. budget constraints:

$$\sum_{(i,j) \in S} F_{ij} y_{ij} \leq B, \quad (2.1.9)$$

Where  $S$  is a subset of  $A'$  and constraint (2.1.9) is the budget constraint proposed by Gendron et al. (1997) to illustrate a relatively general class of restrictions imposed upon resources shared by the arc set  $S$ .

Another type of the additional constraints (2.1.5) is partial capacity constraints:

$$f_{ij}^k \leq b_{ij}^k, \quad \text{for all arcs } (i, j) \text{ and all products } p, \quad (2.1.10)$$

The case with capacity constraints on the arcs is known as capacitated network design problem (CNDP) which is the major type of NDP. We will present it in detail in the next subsection.

### 2.1.1.2 Capacitated/Uncapacitated network design problems

In real applications, it is required to send the products to satisfy demands by means of given arcs with existing capacities. Thus, the addition of constraints (2.1.10) leads to the general formulation for an NDP. This type of NDP, which takes the partial capacity constraints on the arcs, is known as capacitated network design problems (CNDP) while uncapacitated network design problem (UNDP) does not restrict the capacity on the arcs.

A lot of studies have been dedicated to UNDP and significant results have been obtained. Very efficient algorithms have been devised for the real-life applications. In particular, the dual-descent method, presented in Balakrishnan et al. (1989), is capable to efficiently solve large size instances of the less-than-truckload consolidation problems.

However, CNDP is much more difficult to solve than UNDP. Hall (1996) states that capacitated minimum spanning tree problem (a special case of NDP) is NP-hard and very difficult to solve in practice. The review of research in CNDP could be found in Gendron and Crainic (1994, 1996). Gendron et al. (1997) note that to solve NDP, researchers have focused on three different approaches: simplex-based cutting plane methods, Lagrangian relaxation and heuristics. Chouman et al. (2003) present a branch-and-cut algorithm following Holberg and Yuan (2000). Later, Kliwer and Timajev (2005), and Sellmann et al. (2002) follow up the algorithm in Chouman et al. (2008).

### 2.1.2 Facility location problem

Simchi-Levi et al. (2004) state that the strategic phase deals with the decisions that have a long-term effect on the company. They involve the decisions of the number, locations and capacities of warehouses and manufacturing plants. Facility location problem (FLP) investigates locating physically a set of facilities (resources) in order to minimize the cost of satisfying some set of demands (customers) subject to some set of constraints (see in Hale and Moberg 2003). It is a critical element in strategic planning for a wide range of private and public firms. It is also an issue which catches more and more attentions of the researches. The number of papers and books has increased tremendously in the last few years and even the American Mathematical Society (AMS) creates specific codes for location problems (90B80 for discrete location and management, and 90B85 for continuous location).

Indeed, there exist two types of facilities: one is the so-called hub between which the goods can be exchanged; the other is named by us as simple facility, between which the good in contrast can not be exchanged. Here, the facility mentioned in the subsection 2.1.2 means simple facility if we do not specially point out. FLP is a particular case of network design

problem (Ahuja et al. 1995). The authors propose to use the familiar node splitting device of network flows to convert the decision as to whether a facility is placed on a node into a decision about placing a facility on an arc.

Various classification schemes are available in the literature to categorize the FLP. For recent reviews on facility location, we refer to Owen and Daskin (1998), Hale and Moberg (2003), Klose and Drexl (2005) and Melo et al. (2009).

The features influencing the classification of FLP are analogous to hub location problem. Like in the next presentation of the bibliography for HLP, facility location models can similarly differ in the distance metric applied, their objective functions, the number and size of the facilities to locate, and several other decision indices. Depending on the specific application, inclusion and consideration of these various indices in the problem formulation will lead to very different location models.

FLPs are generally solved on one of three basic spaces: continuous spaces (spatial), discrete spaces, and network spaces. In the first case of FLPs on a continuous space, any location within the area is a feasible location for a new facility. The second refers to the problems where the facility locations must be chosen from a pre-defined set while the third is mentioned that FLPs are confined to the arcs and nodes of an underlying network. In this section, we will only consider discrete models.

In the discrete facility location problem, there exist three main objectives: minisum, minimax and recently the set covering and maximal covering objective functions. Using the different objectives, the facility location problem is generally divided into p-median problems, p-center and coverage problems, and other research. In the following section, we will introduce the simplest setting for such a problem in which p facilities have to be selected to minimize the total costs for the transportation system.

### **2.1.2.1 Covering problems**

There are covering objective functions studied within the location science community, especially recently. The key issue to locate the facilities in such problem is coverage i.e. selecting locations which minimize the travel distance or time within a specified constraint. Covering problem is typically relevant to public facilities such as health clinics, post office, libraries, school, etc. In literature, the covering problem is divided into two major segments: the set covering problem and the maximal covering problem. The former one attempts to locate the minimum number of new facilities satisfying a prescribed constraint to existing facilities. In contrast, the latter is to locate a given number of facilities to the best response to the demands of the existing facilities within the acceptable service constraint. As explained above, the set covering problem allows the decision makers to examine the number of facilities which are needed to guarantee a certain level of coverage to all customers. In practice, it is sometimes found that the resources are not sufficient to build the facilities in the desired level of coverage. The location goals of coverage subject to the service constraint may be infeasible with respect to the limitation of resources. The potential infeasibility of the set covering problem leads us to shift our location goal to locate the given number of facilities to serve as many customers as possible. The latter location covering problem is that of the maximal covering problem.

### 2.1.2.2 Center problems

Another problem class is p-center problem which is also known as the minimax problem. As stated above, the maximal covering problem is applied to avoid the potential infeasibility of the set covering problem, while p-center problem is another way to avoid the potential infeasibility of the set covering problem. In such cases, instead of taking a constraint of the acceptable coverage distance, the p-center problem locates a given number of facilities so as to determine the minimal coverage distance. Center problems often arise in the location of emergency facilities such as fire or ambulance stations. Suppose, for example, locating a coastal search and rescue station to minimize the maximum response time to maritime accidents.

### 2.1.2.3 Median problems

The p-median problem is stated as follows in Hakimi (1964): find the location of p facilities so as to minimize the total demand-weighted travel distance between demands and facilities. The facility location problem (FLA) is first mentioned by Miehe (1958). Cooper (1963) is the first to state the formulation of p-median facility location problem. Furthermore, they also show that it is neither convex nor concave. As described by Church and ReVelle (1974), the average distance travelled by those who visit it is an important way to measure the effectiveness of a facility location. Facility accessibility decreases and the location effectiveness decreases simultaneously while the average distance increases. P-median location problems are used in a wide variety of applications. These include, but are not limited to, locating warehouses within a transportation system to minimize the average time to customers and other freight distribution systems. On the other hand, locating “undesirable” facilities for instance, landfills, nuclear plants or other hazardous material sites in order to maximize the average travel distance are regarded as the other application domain.

In p-median problem, when p is a variable and the objective function is extended with a term for fixed facility location costs, the new problem defines the Uncapacitated Facility Location Problem (UFLP). The related researches can be found in ReVelle (2008) and Mirchandani and Francis (1990). One of the most important extensions of the UFLP is the capacitated facility location problem (CFLP), in which the facilities are capacitated. For CFLP we refer to Sridharan (1995). Now, we state the formulation for CFLP.

The notations are defined as follows:

$f_j$  : the cost of locating a facility at node j;

$d_i$  : the demands at node i;

$c_{ij}$  : the travel cost per unit of demand between nodes i and j;

$u_j$  : the capacity of a facility located at node j;

$y_j$  : a binary variable. If and only if a facility is located at node j,  $y_j = 1$ ;

$x_{ij}$  : the fraction of the demand of node i served by a facility located at a node j.

Then, we state the following formulation for CFLP.

$$\min \quad \sum_j f_j y_j + \sum_i \sum_j d_i c_{ij} x_{ij} \quad (2.1. 11)$$

$$\text{s.t.} \quad x_{ij} \leq y_j \quad \text{for all } i \text{ and } j, \quad (2.1.12)$$

$$\sum_j x_{ij} = 1 \quad \text{for all } i, \quad (2.1. 13)$$

$$\sum_i d_i x_{ij} \leq u_j y_j \quad \text{for all } j, \quad (2.1.14)$$

$$y_j = 0 \quad \text{or} \quad 1 \quad \text{for all } j, \quad (2.1.15)$$

$$x_{ij} \geq 0 \quad \text{for all } i \text{ and } j. \quad (2.1.16)$$

In this formulation, the objective is the total cost which consists in the fixed facility cost and the transportation cost. Constraints (2.1.12) restrict node  $i$  can be served by node  $j$  only if a facility is located at node  $j$ . Constraints (2.1.13) state the total demand of each node should be satisfied. Constraints (2.1.14) are the facility capacity constraints which ensure the capacity of a facility is not exceeded by its assigned demand.

It is undoubted that this formulation is difficult to provide an exact solution for instances of realistic size. Klose and Drexl (2005) summarize that lagrangian relaxation (dual decomposition and primal-dual decomposition) algorithms are typical and efficient methodology for this formulation. For a detailed description of the algorithms for CFLP, we refer to Cornuejols et al.(1991) and Daskin (1995).

## 2.1.3 Hub location problem (HLP)

### 2.1.3.1 Introduction

Hubs are special facilities that serve as switching, transshipment and sorting points in many distribution systems. The research on hub location problem (HLP) differs from the facility location problems mentioned in the previous subsection in that the demand is specified in terms of flows between origins and destinations. Flows from the same origin with different destinations are concentrated or consolidated on their route to the hub and are combined with flows that have different origins but the same destinations. In fact, the concentration or consolidation is on the route from the origin to the hub and from the hub to the destination as well as between hubs. It can also affect service levels, for example, by allowing more frequent services in a transportation network or by avoiding delays due to congestion.

HLP involves locating hub facilities and allocating demand nodes to hubs in order to route the traffic between origin and destination (OD pairs). It is firstly pioneered by O'Kelly (1987). He presented the first mathematical formulation for HLP by studying airline passenger network. Over the past two decades, HLP forms an important and growing subset of problems in location science. Alumur and Kara (2008) cite more than 100 papers related to the HLP and note that there is a steep increase in the number of publications after the year 2000. Undoubtedly, interest in the hub location area is still strong now. They also conclude that before the year 2000, hub location research is more focused on defining and formulating new problems. These new problems were mainly p-hub median variants due to the first mathematical formulation. After the year 2000, the focus is twisted toward investigating different solution methodologies for these problems.

### 2.1.3.2 Application

Motivated by transportation (air, ground and water) and telecommunication network design, hub location researchers have addressed a wide range of problems over the past two

decades. Here, we just enumerate several instances in four aspects: air passenger carriers, less-than-truckload motor carriers, the express package delivery industry, and a variety of computer communication and telecommunication systems.

For the domain of air passenger carry problem, Hall (1989) analyzes and examined the impact of overnight restrictions and time zones on the configuration of an air freight network. Additionally, the impact of overnight restrictions on multiple terminal networks is also examined. Later, O’Kelly and Lao (1991) propose a 0/1 linear programming model to choose the mode in the hub network discussed by Hall (1989). In the model, they assume that the locations of a master and a mini hub are known and attempt to use the model to determine which may be served by truck rather than air. Iyer and Ratliff (1990) try to locate hubs, called as accumulation points in the paper, to service the OD pairs within a definite time. Jaillet et al. (1996) present models for designing capacitated airline networks on the assumption of non-priori hub-network structure, and propose several different integer programming formulations and a heuristic algorithm for their problem. The resulting network may suggest the presence of hubs, if there is efficiency in cost.

Less-than-truckload (LTL) services serve those customers whose shipments between OD pairs would not fill the truck capacity by weight or volume. In the literature, there are various studies considering less than-truckload (LTL) motor carriers structures. To our best knowledge, Powell and Sheffi (1983) are the first to study the load planning problem for less than-truckload (LTL) motor carriers. Another one may refer to Campbell (2005) for a survey on strategic network design for motor carriers. Cunha and Silva (2007) research the configuring hub-and-spoke networks for trucking companies that operate less-than-truckload (LTL) services in Brazil. A heuristic genetic algorithm is provided, which incorporates an efficient local improvement procedure for each generated individual of the population. They propose a formulation which allows variable scale-reduction factors for the transportation costs according to the total amount of freight between hub terminals, as occurs to less-than-truckload (LTL) flight carries in Brazil. To show the efficiencies of their approach, computational results for benchmark problems and a real world problem are presented.

Campbell (1996) states that the term “hub” is generally used in transportation networks while the corresponding facility in telecommunication network can be called as backbone node, concentrator, gate, switch, access point, etc. These facilities perform similar functions although there exist great differences between transportation and telecommunication hubs. The author presents the main differences between HLP in transportation and in telecommunication network are the cost structures, which could lead to significant differences in problem formulations, solution approaches and problem size. In hub networks, the typical cost is the flow cost for transportation, i.e., moving the freight or people between origins and destinations. In contrast, the most important costs in telecommunications networks may be fixed costs to establish and ensure good performance of the network. Consequently, transportation-oriented models may include fixed costs for hub facilities, but they rarely include delay costs. Some telecommunications hub models focus only on fixed costs to establish a network, while others include delay costs, which are often non-linear, and flow costs. Furthermore, efficient optimal solution procedures have been developed only for the transportation networks with the small size of just 50 OD pairs; in contrast, telecommunications-oriented research typically addresses problems involving hundreds of origins and destinations with heuristic algorithms.

Klincewicz (1998) provides an extensive review of earlier algorithmic work on the integrated problem about HLP, drawing from the literature on facility location, network design, telecommunications, computer systems and transportation. The author discussed certain key issues in modelling HLPs in the particular context of communication networks and proposed possible prospects for future works. Carello et al. (2004) deal with a HLP arising in telecommunication network design in which there are two types of nodes, access nodes and transit nodes (hubs). The goal is to minimize the total cost of the network consisting of connection costs and node fixed costs. In order to solve the problem, the author proposes a local approach and different metaheuristic algorithms, such as tabu search, iterated local search and random multistart. Yaman (2005) studies the uncapacitated hub location problem which is similar to the problem in Carello et al. (2004). The problem named the uncapacitated hub location problem aims to minimize the cost of installing hubs and the integer amounts of capacity units installed on the arcs. Yaman and Carello (2005) deal with a new version of the problem, in which the cost of using an edge is not linear but stepwise, and the capacity of a hub restricts the amount of traffic transiting through the hub rather than the incoming traffic. In the paper, they present an exact and a heuristic method. In the heuristic procedure, a greedy algorithm is applied to find an initial feasible solution, a tabu search step is used to the location sub-problem and local search procedure is provided for the assignment problem.

### 2.1.3.3 Classification

Indeed, various classification schemes are available in the literature to categorize the HLP. The corresponding factors to influence the classifications of the problems include: objective function, basic spaces, the decisions of allocation (single-allocation/multi-allocation), the combination mood between hubs and simple facilities, and several other decision indices. Depending on the specific application, inclusion and consideration of these various indices in the problem formulation will lead to very different hub location problems.

The same to facility location problem (FLP), HLP are also generally solved on one of three basic spaces: continuous spaces, discrete spaces, and network spaces. Moreover, much research has focused on discrete hub location problem.

The HLP is concerned with locating hub facilities and allocating demand nodes to hubs in order to route the traffic between OD pairs. There are two basic types of hub networks: single allocation networks or multiple allocation networks. They differ in how non-hub nodes are allocated to hubs. In single allocation hub network, all the incoming and outgoing traffic of every demand center is routed through a single hub. In multiple allocation hub networks, each demand center can receive and send flow through more than one hub. Some papers are concerned only with the allocation aspect of the problem since optimal hub locations are affected by allocation decisions.

Furthermore, according to the stop number in a trip between each OD pair, the hub location models which are mainly studied could be divided in two types: one-stop model and two-stop model. A two-stop type is in the sense that a trip between each OD pair uses at most two hubs while there is at most one hub in a trip between each OD pair in a one-stop model. (Sasaki and Fukushima (2003))

Besides the basic single allocation and multiple allocation models, Campbell (1994) has classified the p-HLPs into four classes and presented basic formulations with flow thresholds for spokes (non-hub nodes) for each of them. The four classes are the p-hub center

problems, the p-hub median problems, the uncapacitated hub location problems and the hub covering problems.

There exist two predominant objective levels in HLP: service level and cost level. Hub center problems seek to locate hubs to minimize maximum measure related to transportation distance or cost. While hub median-type models focus on economic objectives and hub center and covering models focus more on service level objectives, which combine both dimensions and could provide valuable insights for designing transportation hub networks. Campbell (2009) notes that hub center and hub covering models have generally focused on worst-case service (based on the maximum OD distance or travel time), while ignoring the total cost for transportation. These models often use a discounted inter-hub travel time (analogous to the discounted travel cost in hub median models) to reflect the use of faster vehicles between hubs. However, the underlying practical motivation for many-to-many hub center and hub covering models seems less compelling than that for the one-to-many (non-hub) center and covering models, which have strong motivations from location public sector emergency service facilities. Here, many-to-many network means that there are several original nodes as well as several destination nodes in the network, while one-to-many network has only one original node. On the other hand, many-to-many transportation carriers (airlines, trucking companies, etc.) are generally private sector firms for whom the cost of transport cannot be easily ignored. Thus, models that integrate both cost and service may provide better insights into practical transportation networks.

In the next three sections, we introduce first to the p-hub center problem, then the p-hub covering problem, followed by the p-hub median problem. Especially, we present in detail the p-hub median problem because our GDP mainly handle with the delivery problem of a private company. Additionally, we divide our presentation of p-hub median problem and the related algorithms into two different subsections. They are single-allocation problem and multiple-allocation problem.

### **2.1.3.4 p-hub center problem**

The p-hub center problem is a minimax type problem. Campbell (1994) is the first to formulate and discuss the p-hub center problem in the hub literature. He defines three different types of p-hub center problems: (1) the maximum cost for any origin-destination pair is minimized. (2) The maximum cost for movement on any single link (origin-to-hub, hub-to-hub and hub-to-destination) is minimized. (3) The maximum cost of movement between a hub and an origin/destination is minimized (vertex center). According to Campbell (1994), the first type of hub center problem is important for a hub system involving perishable or time sensitive items in which cost refers to time. As an example of the second type of p-hub center problem, there is the case where the vehicle drivers are subject to a time limit on continuous service. For the third type, similar examples to the second type can be given considering that hub-to-hub links may have some special attributes.

Campbell (1994) presents formulations for both single and multiple allocation versions for all three types of p-hub center problem. Kara and Tansel (2000) provide various linear formulations for the single allocation p-hub center problem. They provide three different linearizations of the first type model of the Campbell (1994) and a new formulation that they proposed. Through computational analysis using CPLEX, their new formulation is better than all of the three linearizations. Their new formulation has  $(n^2+1)$  variables of which one is real variable,  $n^2$  are binary and it has  $(n^3+n^2+n+1)$  linear constraints. Especially, the linearization of their proposed formulation requires  $n^2$  binary variables and  $(n^3+n^2+n+1)$  linear constraints,

while the other three involve respectively  $n^4 + n^2$  for LIN1 and LIN2,  $n^2$  binary and  $n^4$  real variables for LIN3. Kara and Tansel (2000) also provide a combinatorial formulation of the single allocation p-hub center problem and proved that it is NP-complete by a reduction from the dominating set problem to the p-hub center problem. The first heuristic for the single allocation p-hub center problem is presented in Pamuk and Sepil (2001). They propose a single-relocation heuristic for generating location-allocation decisions in a reasonable time and they superpose a tabu search on this underlying algorithm so as to decrease the possibility of being trapped by local optima.

The related research about p-hub center problem could also be found in Ernst et al. (2002 a, b), Gavrilouk and Hamacher (2009), Campbell et al. (2007), etc.

### 2.1.3.5 p-hub covering problem

In a facility covering problem, all the demand nodes which are within a specified distance of a facility that can serve their demand, are considered to be covered. Campbell (1994) defines three coverage criteria for hubs. The OD pair  $(i, j)$  is covered by hubs  $k$  and  $m$  if:

- (1) the cost from  $i$  to  $j$  via  $k$  and  $m$  does not exceed a specified value,
- (2) the cost for each in the path from  $i$  to  $j$  via  $k$  and  $m$  does not exceed a specified value, which means the cost in the path  $i \rightarrow k \rightarrow m \rightarrow j$  has an upper limit,
- (3) each of the origin-hub and hub-destination links meets separate specified values, which is generally found by incorporating a discount factor  $\alpha$  to express the hub arc cost in the research.

The hub set-covering problem is to locate hubs both to cover all demand and to minimize the cost of opening hub facilities. The maximal hub-covering problem is to maximize the demand covered with a given number of hubs to locate.

Campbell (1994) is the first paper presenting mixed integer formulations for both of these two problems. Kara and Tansel (2003) study the single allocation hub set-covering problem and proved that it is NP-hard. The authors present and compare three different linearizations of the original quadratic model. They present also a new model whose performance turns out to be better than all of the other presented linear models. Ernst et al. (2005) present a new formulation for the single allocation hub set covering problem similar to the one proposed in Ernst et al. (2009) for the p-hub center problem and two new formulations for the multiple allocation hub set-covering problem. In the formulation of the single allocation hub set covering problem, they replace the constraint of Kara and Tansel (2003) with its aggregate form and compare their new model with the strengthened formulation of Kara and Tansel (2003). The computational results show their new formulation performed better in terms of CPU time requirement than the strengthened formulation of Kara and Tansel (2003). For the multiple allocation hub set-covering problem, they propose an implicit enumerative method.

### 2.1.3.6 p-hub median problem

The p-hub median problem is to serve a given set of  $n$  demand nodes, with the given flow between OD pairs and the number of hubs to locate ( $p$ ) so that the total transportation cost is minimized.

Campbell (1994) presents models involving flow thresholds for spokes (non-hub nodes), and points out that the p-hub median problem and the uncapacitated hub location

problem have mainly been studied and other classes of problems have not yet been considered seriously. In this section, the studies in p-hub median problem are presented and analyzed in two parts: single allocation-network and multiple-allocation network.

#### 2.1.3.6.1 Single allocation network

In single allocation network, all the incoming and outgoing flow of every demand center is routed through a single hub. In other words, each of the non-hub center can be allocated to only one hub.

The first linear integer programming formulation for the single allocation p-hub median problem is presented in Campbell (1994). In the formulation, there are  $(n^4+n^2+n)$  variables of which  $(n^2+n)$  are binary and  $(n^4+2n^2+n+1)$  linear constraints.

Skorin-Kapov et al. (1996) propose a new mixed integer formulation for the single allocation p-hub median problem due to the highly fractional solutions of the LP relaxation of Campbell (1994) formulation. Their new formulation includes  $(n^4+n^2)$  variables of which  $n^2$  are binary and  $(2n^3+n^2+n+1)$  linear constraints. To the best of our knowledge, the authors are the first to attempt to optimally solve the single allocation p-hub median problem. They show that the linear relaxation of this formulation is tight as it yields most of the time integral solution to a data set of the Civil Aeronautics Board (CAB) which is the data benchmark used by almost all of the hub location researchers. The LP relaxations or the instances with non-integral LP solution result in an objective function value less than 1% below the optimal objective function value.

O'Kelly et al. (1995) provide a reduced size formulation with the assumption of a symmetric flow data. The reduced formulation almost always finds integer solution to LP relaxation. Moreover, O'Kelly et al. (1995) is to discuss the sensitivity of the solutions to the inter-hub discount factor  $\alpha$  which is one important attribute of the paper. Sohn and Park (1998) present a formulation to further reduce in the size of the problem in case of a symmetric flow cost and a proportional distance.

Ernst and Krishnamoorthy (1996) propose a different linear integer programming formulation. They regard the inter-hub (hub-to-hub) transfers as a multi-commodity flow problem in which each commodity represents the traffic flow from a particular original node. Their new formulation has  $(n^3+n^2)$  variables of which  $n^2$  are binary and it requires  $(2n^2+n+1)$  linear constraints. Obviously, the problem size is further reduced from the previous formulation (Skorin-Kapov et al., 1996) in terms of variables and constraints, by a factor around  $n$ . It is the best formulation in terms of computation time requirement in the literature.

Ebery (2001) presents another further reduced size formulation for the single allocation p-hub median problem that just requires  $O(n^2)$  variables and  $O(n^2)$  constraints. This formulation uses fewest variables and constraints in all of the models introduced in the literature. However, to solve the new formulation requires more computation time than the time required solving the Ernst and Krishnamoorthy (1996) formulation.

It is difficult to solve the p-hub median problem because it is NP-hard. Moreover, even if the locations of the hubs are given, the allocation of the problem remains NP-hard (Kara, 1999). Sohn and Park (1998) consider the discrete two-hub location problem in which they

need to choose two hubs from a set of nodes. They show that the problem can be solved in a polynomial time when the hub locations are fixed. In a subsequent study, Sohn and Park (2000) show that the single allocation problem is NP-hard as soon as the number of hubs is three, although the problem in a two-hub system has polynomial time algorithms.

There are many efficient exact solution procedures for p-hub median problem. Ernst and Krishnamoorthy (1996) propose a linear programming based branch-and-bound algorithm. And later, Ernst and Krishnamoorthy (1998b) develop another branch-and-bound algorithm which obtains lower bounds though solving shortest-path problems. Up to now it is the most effective exact algorithm for p-hub median problem. Unlike the traditional branch- and-bound algorithms, their algorithm does not start with a single root node, but with a set of root nodes. They compare its performance with the results provided in Ernst and Krishnamoorthy (1996) on the CAB data set and another data set, the Australia Post (AP) which is based on a postal delivery in the town of Sydney in Australia. The algorithm could optimally solve the largest single allocation problems to date. It solves successfully the problems with 100 nodes and with  $p=2$  and 3 in approximately 228 and 2629 seconds respectively. However, the problem with 100 nodes when  $p>3$  is still unsolved by the algorithm in a reasonable amount of computational time. Ebery (2001) present a formulation for the single allocation p-hub median problem with two or three hubs. The results show that this formulation is better to solve the large problems with  $p=2$  or  $p=3$  than the shortest-path based branch-and-bound approach developed in Ernst and Krishnamoorthy(1998b).

O'Kelly (1987) is the first to propose heuristics for the single allocation p-hub problem. Then, an exchange heuristics for single allocation p-hub median problem is developed by Klinecicz (1991), which is based on a local improvement considering both the single and double exchange procedures which are two typical theories to predict how a node is exchanged between two species. The computational results show that these heuristics are superior to the clustering heuristics and the heuristics proposed in O'Kelly (1987). O'Kelly et al. (1995) provide a lower bounding technique for the single allocation p-hub median problem. The authors linearize the quadratic objective function where the distances are assumed to satisfy the triangle inequality. And then they show that the tabu search method in Skorin-Kapov and Skorin-Kapov (1994) is within an average gap of 3.3% for smaller problems (10-15 nodes) and an average gap of 5.9% for the 20 and 25 node problem. Campbell (1996) proposes two new heuristics for single allocation p-hub median problem with the idea that the multiple allocation p-hub median solution provided a lower bound on the optimal solution of the single allocation p-hub median problem. In these heuristics, the allocations are done by different rules but the location decisions are the same. Pirkul and Schilling (1998) develop an efficient Lagrangian relaxation method which finds tight upper and lower bounds in a reasonable amount of CPU time, which is the most effective heuristic up to that date. They use sub-gradient optimization on the Lagrangian relaxation of the model and provide a cut constraint for one of the sub-problems. They show that the average gaps of this heuristic are 0.048% and even the maximal gaps are under 1%.

Some researchers have introduced meta-heuristics to solve p-hub median problem. Klinecicz (1992) presents a tabu search and a greedy randomized search procedure (GRASP), in both of which the demand nodes are allocated to their nearest hubs. Skorin-Kapov and Skorin-Kapov (1994) develop another tabu search heuristics for the single allocation p-hub median problem. They use the CAB data set to compare their results with the heuristics of O'Kelly (1987) and the tabu search of Klinecicz (1992). Their results are indicated that the performance of their algorithm is superior but CPU time requirement was

greater because they emphasize the allocation phase of the problem. Ernst and Krishnamoorthy (1996) use simulated annealing heuristics to obtain the upper bound for an LP-based branch-and-bound solution method. They present that their procedure is comparable in both solution quality and computational time with the tabu search heuristics of Skorin-Kapov and Skorin-Kapov (1994), but it could not solve any problem greater than  $n=50$  on the CAB and AP data sets. Another simulated annealing heuristics for the single allocation p-hub median problem is developed by Abdinnour-Helm (2001). However, Abdinnour-Helm obtains poorer results than Ernst and Krishnamoorthy (1996). Furthermore, the single allocation p-hub median problem is mapped onto a modified Hopfield neural network by Smith et al. (1996). The authors find that the Hopfield neural network approach performs also effectively as simulated annealing in Ernst and Krishnamoorthy (1996) on the CAB data set. Additionally, they show that the results for the case of the commercial package GAMS with the solve MINOS-5 perform poor than their proposed method and the simulated annealing in Ernst and Krishnamoorthy (1996).

The p-hub location problem is uncapacitated single p-hub location problem if the hubs have not limited capacity for channelling flows between the nodes served by the system. Campbell (1994) introduces that uncapacitated p-hub location problem is one of the four major classes in HLP. In addition, HLP with a capacity in the hub as a constraint is more and more researched. So in the following, we will introduce the studies about the uncapacitated p-hub location problem, followed by the capacitated p-hub location problem.

O'Kelly (1992a) presents a quadratic program for the single allocation hub location problem with fixed cost. Campbell (1994) is the first to present the linear programming formulation for the single/multiple allocation capacitated/uncapacitated p-hub median problems with fixed costs. As the statement in Campbell (1994), like the p-hub median problem, in the absence of capacity constraints on the links, the optimal solution to uncapacity p-hub median problem is always found when all  $X_{ijkm}$  which is the fractional flow from node  $i$  (origin) to node  $j$  (destination), routed via hubs at nodes  $k$  and  $m$  in that order are set to be zero or one.

There are several studies related to the single allocation uncapacitated hub location problem. Abdinnour-Helm and Venkataramanan (1998) present a new quadratic integer formulation for the uncapacitated hub location problem, which is based on the idea of multi-commodity flows in networks. The authors state that their new formulation itself is well used for a branch-and-bound procedure to find optimal solution. Labbé and Yaman (2004) consider two formulations for the uncapacitated hub location problem with single assignment which use multi-commodity flow variables. Furthermore, a family of valid inequalities is derived, which could generalize the facet defining inequalities and which is able to be separated in polynomial time.

In order to solve the single allocation uncapacitated hub location problem with fixed costs, the researchers have provided several algorithms, including exact algorithms and some approximate algorithms. A new branch and bound procedure is presented in Abdinnour-Helm and Venkataramanan (1998) for single allocation uncapacitated hub location problem. Instead of implementing in a traditional fashion, where bounds are obtained by liberalizing the objective function and relaxing the integrity constraints, the bounds are obtained by employing the underlying network structure of the problem in their new procedure. In addition, the authors also propose a genetic algorithm to find solutions quickly and efficiently.

Abdinnour-Helm (1998) proposes a new heuristic method based on a hybrid of genetic algorithm and tabu search. He shows that the results of the new heuristic are very improved comparing to applying the genetic algorithms alone and matched the best solutions found in the literature in all cases but one tested. Then Topcuoglu et al. (2005) present a new and robust solution based on genetic search framework for the uncapacitated single allocation hub location median problem. The authors compare the solutions of their GA-based method with the best solution in the literature over the CAB and AP data sets. The computational results demonstrate that even for larger problems, their method significantly outperform the hybrid heuristic proposed by Abdinnour-Helm (1998) in both solution qualities and CUP time. Chen (2007) provide another hybrid heuristic based on simulated annealing, tabu list and an improvement procedure to determine the upper bound for this problem. Computational results demonstrated that the proposed hybrid heuristic outperforms the genetic algorithm in Topcuoglu et al. (2005). Thus, we conclude that the heuristic in Chen (2007) is the best algorithm proposed for the single allocation uncapacitated hub location median problem up to now in the literature.

In capacitated single p-hub location problem, the hubs have limited capacity for channelling flows between the nodes served by the system. Aykin (1994) formulates for the capacitated hub-and-spoke network design problem under a networking policy allowing both direct (nonstop) and hub connected (one-hub-stop and two-hub-stop) services between the nodes. For the capacitated single allocation hub location problem, Ernst and Krishnamoorthy (1999) present a new mixed integer linear program formulation with fewer variables and constraints than the literature find previously. They used an LP-based branch-and-bound method with the initial upper bound method to obtain the optimal solution of a problem with up to 50 nodes. Notably, the capacity restrictions are only applied to the flows from the non-hub nodes to the hub. Another mixed integer programming formulation is proposed in Labbé et al. (2005) for quadratic capacitated hub location problem. Some polyhedral properties of this problem are investigated, and then, with these properties, a branch-and-bound algorithm is developed. The authors state that for the problems with small values which are the service levels obtained from the formulation, it could be solved by the CPLEX MIP solver for reasonable size, while their proposed algorithm is able to solve the problem with large service level for reasonable size. Costa et al. (2007) present a different method for the capacitated single allocation hub location problem. Instead of using capacity constraints to limit the amount of flow that could be received by the hubs, the authors present a bi-criteria approach. They study two alternative bi-criteria models, in which the first one is to minimize the total service time, and the second is to minimize the maximum service time on the hubs. As stated by the authors, the new models interact with the decision makers and help the decision maker suite better in the final decision.

Aykin (1994) presents Lagrange relaxation based approaches to capacitated hub-and-spoke network design problem. They propose a brand-and-bound algorithm and a heuristic procedure to partition the set of solutions into smaller routing problem on the basis of hub locations, and then apply a Lagrange relaxation with a sub-gradient optimization for the reduced problems. They test their formulation and the proposed algorithms in the CAB data set. Aykin (1995a) introduce a framework for the design of a similar problem with fixed costs and a given number of hubs to locate. The author propose an enumeration algorithm as exact solution procedure and an heuristic which applies greedy interchange based on simulated annealing. Ernst and Krishnamoorthy (1999) develop two heuristic methods based on simulated annealing and random descent that provide good upper bounds. The tested their algorithms on the AP data set. The results show that their heuristics are quite efficient for

most of the test problems. They conclude that their algorithm is preferable on small to medium sized problems than simulated annealing. In Costa et al. (2005), an iterative approach is proposed to solve their different bi-criteria models for the capacitated single allocation hub location problem. Their approach is used to calculate a non-dominated solution. After computational tests, the author presented that, sometimes, the solutions of bi-criteria model has major excess of flow on the hubs or big increases in the value of the costs to reduce just slightly time values. But the bi-criteria model reveals of great utility because a new interesting and viable solution are calculated.

### 2.1.3.6.2 Multiple allocation network

In multiple allocation p-hub median problems, each non-hub center can be connected to some of the p hubs. In other words, each of the non-hub centers can be allocated to more than one hub.

Campbell (1992) is the first to present the linear integer program formulation for the multiple allocation p-hub median problems. Two years later, in Campbell (1994), the same author presents a mixed 0/1 integer formulation with  $n+n^4$  variables and  $1+n+2n^4$  constraints when the number of nodes is  $n$ . As stated in Campbell (1994), in the absence of capacity constraints on the links, all  $X_{ijkm}$  in an optimal solution should be set to zero or one since the total flow for each OD pair should be routed via the least-cost path. So, the  $X_{ijkm}$  are not needed to be restricted to be integer. Moreover, if the hub locations are fixed, the remaining problem is to find a shortest path between each pair of nodes via the given hubs. Skorin-Kapov et al. (1996) provide a modified formulation with  $(2n^3+n^2+1)$  linear constraints and  $(n^4+n)$  variables of which  $n$  are binary. And then, the linear programming (LP) relaxation and branch-and-bound approach for the proposed formulation is considered. The authors show that the formulation resulted in tighter LP relaxations and could produce most always integer solution for the CAB data set. For the instances providing fractional solutions, an implicit enumeration search tree involving very few tree nodes is employed to obtain optimal solutions. Ernest and Krishnamoorthy (1998a) propose a new formulation for the multiple allocation p-hub median problem, which has  $(4n^2+n+1)$  linear constraints and  $(2n^3+n^2+n)$  variables of which  $n$  are binary. The formulation is based on their idea for the single allocation version proposed in Ernst and Krishnamoorthy (1996). The results show that this formulation is more effective than the formulation in Skorin-Kapov et al. (1996).

To obtain exact solutions, Ernst and Krishnamoorthy (1998a) provide a branch-and-bound method based on LP for the multi-allocation p-hub median problems. They identify the inequalities which are violated the constraints of the original formulation and add them to the LP so as to strengthen the lower bound. The same year, in Ernst and Krishnamoorthy (1998b), they develop another but more effective branch-and-bound algorithm which runs 500 time faster and requires significantly less memory than the LP-based branch-and-bound algorithm in Ernst and Krishnamoorthy (1998a). They solve the shortest path problems rather than solving the LP relaxation to obtain lower bounds. The computational results show that this new algorithm is able to produce the exact solution of the largest problem in the literature. They could solve exactly even problems with 200 nodes and  $p=3$  hubs in approximately 632 second. However, they are unable to obtain exact solution to the AP data set problems with 100 nodes,  $p>5$ , and 200 nodes,  $p>3$  in a reasonable amount of computational time.

In order to solve larger multiple allocation p-hub problems, particularly for large  $p$ , other approximate heuristic algorithms are proposed. Campbell (1996) develops a greedy-interchange heuristic for the multiple allocation p-hub median problems. Boland et al. (2004)

develop pre-processing procedures and employed flow cover constraints for existing mixed integer linear programming formulations. The results of their computational experiences show that all of the proposed steps could effectively reduce the computational effort required to obtain optimal solutions. Sakaki et al. (1999) suggest a special case called 1-stop multiple allocation p-hub median problem, where each route between OD pair is allowed to connect just one hub. They formulate the problem as a p-median problem and then proposed a greedy-type heuristic algorithm. They test their algorithm on the CAB data set and some random data for further investigation, and the obtained results show that the proposed algorithms work better than the other algorithms, particularly for relatively small problem.

The same to the presentation in the section of the single allocation, we will focus our presentation of the multiple allocation p-hub median problems in two parts: capacitated and uncapacitated constraint on the hub. Additionally, some papers mentioned in this section refer to the HLP with the fixed cost to open facilities.

Campbell (1994) provides the first linear programming formulations for multiple-allocation capacitated and uncapacitated hub location problems. Moreover, he presents that like in the p-hub problem without fixed cost, there is always an optimal solution where all  $X_{ijkm}$  variables are zero or one as a result of the absence of the capacity constraints on the links.

For the multiple allocation uncapacitated hub location (UHL) problem, the researches are mainly occurred after year 2000. Hamacher et al. (2004) obtain a better formulation, called FACET-UHL, for UHL after some studies for the UHL polyhedron. The new formulation is equivalent to a modified uncapacitated facility location problem where the number of nodes is two, and its feasibility polyhedron has only integer vertices. The authors determine the dimension and some classes of facets for the UHL polyhedron, and then, they develop a general rule to lift facets from the uncapacitated facility location problem to the multiple allocation UHL. Marín (2005b) takes the formulation in Campbell (1994) as a basic formulation and obtain a strengthened formulation in the fact that the transportation costs between hubs satisfy the triangular inequality. He integrates the analysis of the polyhedron associated with their strengthened formulation and the well-known Lagrangian relaxation technique, and implements an efficient relax-and cut algorithm for the problem. They test their algorithm in the AP data set and compare respectively the computational results with Boland et al. (2004), Ernst and Krishnamoorthy (1998a, b) to show the efficiency of their algorithm in CUP time. Marín et al. (2006) gives a new formulation of uncapacitated multiple allocation hub location problem, which generalizes the classic formulation in the literature and includes a more general cost structure that does not need the triangular inequality. The author checks the strength of the new formulations and compares them with the previous formulations by solving instances of two commonly used data sets: the CAB and AP and also randomly generated instances.

In order to solve multiple allocation UHL problems with fixed cost, Klincewicz (1996) provides a means to explicitly address trade-off between the costs of hubs and the cost of transport. The author uses the dual-ascent and the dual adjustment techniques with a branch-and-bound algorithm to compute solutions for UHL that are optimal or within a specified percentage ( $\beta$ ) of optimality which seems to guarantee a particular value in applications. The proposed algorithm is tested in benchmarking of the CAB data set.

Mayer and Wagner (2002) note that the algorithm in Klincewicz (1996) to reduce the computational effort is still restricted due to the specific structure of the problem. In this

situation, the authors develop a new Branch-and-bound solution method, which is called the HubLocator, for the multiple-location HLP. Based on a disaggregated model formulation, the HubLocator determines lower bounds with a dual ascent technique and took some of the complementary slackness conditions into account to calculate upper bounds. The authors state that the performance of the HubLocator is better than Klincewicz (1996) because the dual solution of the relaxed aggregated formulation is first determined and then improved, the lower bounds obtained from the corresponding objective function value are tighter, and the computational effort required by branch-and-bound process is reduced. They compare their method with the algorithm in Klincewicz (1996) and CPLEX on the CAB and AP data set. The computational experiments demonstrate that their algorithm outperformed the one in Klincewicz (1996) but it is not always superior to CPLEX. Furthermore, the optimal solution for the problems with up to 40 nodes can be found in a reasonable amount of time.

Canovas et al. (2007) consider a four-indexed formulation and design a heuristic method also based on a dual-ascent technique to deal with the uncapacitated multiple allocation HLP. The heuristic is embedded in an exact branch-and-bound framework to provide good lower bounds for the nodes of branching. They test their heuristic method in the classical CAB and AP data sets. The computational results show that the proposed method has great effectiveness to even be able to solve instances up to 120 nodes. These are the better computational results for uncapacitated multiple allocation HLP than any of the results in previous literatures.

The same to uncapacitated multiple-location HLP, Campbell (1994) is also the first to present a linear integer programming formulation for capacitated multiple allocation HLP. Then based on formulations in Campbell (1994), Ebery et al. (2000) present new mixed integer linear programming formulations which use fewer variables and constraints than that reported in Campbell (1994) and are similar to the one proposed in Ernst and Krishnamoorthy (1998a). Ebery et al. (2000) is the first to describe a new heuristic method for capacitated multiple allocation HLP in the literature. They develop a LP-based branch-and-bound solution procedure which obtain the upper bound by the shortest path based heuristic.

Marín (2005a) presents tight integer linear programming formulations for the problem based on the same idea used in Ebery et al. (2000). The author designs a solution procedure including pre-processing and branch-and-bound process in which the lower bounds are given by the linear relaxation while the upper bounds are obtained by means of a proposed heuristic method. The author checks the effectiveness of proposed formulation and the methods by a computational experiment carried out on AP data set. Moreover, the related useful properties of the optimal solution to speed up the resolution are used in Marín et al. (2006) to reduce the problem size.

Boland et al. (2004) observe characteristics of optimal solutions for capacitated multiple allocation hub median problems and then use these characteristics to develop pre-processing techniques and tightening constraints to improve the linear programming relaxations for existing formulations. They improved computation time by using flow-cover constraints. The experience results in AP data and the smaller subsets derive from it show that the linear programming formulations have become tighter and the overall computational time is reduced.

## **2.1.4 Summary**

There are a limited number of studies focusing on research of the whole transport system. In this section, some important issues for part of the GDP in strategic phase, involving network design problem (NDP), facility location problem (FLP) and hub location problem (HLP), have been introduced. Furthermore, the NDP is the basic problem and both FLP and HLP are its special problems which are widely studied.

### **2.1.4.1 Facility location problem**

As we have mentioned above, FLP is to locate a set of facilities in order to minimize the objective (for example, the minimum cost) subject to some constraints. The studies of FLP mentioned in this section have several common characteristics, for example, a single-period planning horizon (strategic phase), a single product, one type/layer facility, and location-allocation decisions. However, these are not sufficient to apply to realistic FLP setting. In order to solve the practical cases of the real world, several crucial aspects including multi-period and different types of facilities should be considered.

In the literatures of FLP, the facilities commonly take the function of the origins. So in the transport system studied by FLP, there are just two operators: facilities (origin) and destinations. However, the FLP has not attached much attention in the transport system already having origins and destinations. In such network, the goods flow between origins and destinations and the facilities are located to centralize the goods from different origins and delivery them to the same destination. This type of FLP is somewhat like the 1-stop hub location problem in which there is just one stop between each OD pair. Nevertheless, the 1-stop hub location problem is not widely studied, either.

In our thesis, different to the traditional researches on FLP which mainly focus on the network just one layer facility on strategic phase, we will study the FLP in such a network with origins and destinations. Moreover we will do our research of FLP on operational phase. Several new formulations for the single/multi-allocation facility location problems are proposed, in which the vehicle cost is considered as objective and the capacity on the arcs between nodes are also restricted by the vehicle number.

### **2.1.4.1 Hub location problem**

After summarizing the literatures for HLP, we can note three interesting points that are somewhat ignored by the researchers. They are respectively introduced in the following three paragraphs.

The literatures of hub location problem (HLP) primarily refer to hub location for the network in which there are already origins and destinations. Furthermore, the HLP are considered only on strategic phase both for the single-allocation HLP and multiple allocation HLP. However, the HLP in tactical phase and operational phase also seems to be meaningful but they are not widely studied.

In addition, the HLP can be divided into capacitated HLP and uncapacitated HLP. Here the capacity means that the hubs have limited capacity for channelling flows between the nodes served by system. It can be explained as the operation ability of the hubs in the real

world. However, the capacity on the arcs between nodes is not much considered in the studies. Here the capacity on the arc can be the transport ability, the transport time limitation between two nodes, etc.

Thirdly, HLP, especially the p-hub median location problem, aims to minimize the transport cost, sometimes adding the fixed cost to open the hubs. This cost is usually the sum of the volumetric cost. However, it is not suitable to the examples in the real world. The private company usually pays for the transport according to the vehicle number.

In our thesis, we propose various new formulations for single/multi-allocation HLP in operational phase which have not been mentioned in the literatures. Furthermore, a new cost depending on the transport vehicles are proposed which is much more suitable in the real world and the capacities on the arcs are also considered.

## **2.2 Tactical Phase**

The main aim of the company in tactical phase is to ensure the optimal allocation and utilization of resources to achieve the economic and customer service goals in median horizon. The tactical phase of operation planning refers to a set of interrelated decisions including service network designs, frequency setting problems, facility allocation problems, etc. In the following of the section, we will respectively introduce in detail the service network design and hub allocation problem as they are widely researched. Moreover, we propose a new formulation for hub allocation problem in our thesis.

### **2.2.1 Service network design**

#### **2.2.1.1 Definition and application**

Service network design (SND) is increasingly used to designate the set of services and routes for a physical network with several demands, which is characterized by an origin, a set of intermediate terminal, and a destination terminal. Moreover, the set of links between these terminals is also considered in the definition of the service. Each service has other characteristics such as the speed and the number of cars that can be moved through this service and other main tactical issues and decisions relevant for the services.

The corresponding formulation of the SND is usually based on the modeling of the physical network. The route in physical network is formed by the terminals and the links between the terminals; nevertheless, the route in the service network is composed of a load (at the original terminal), a certain number of intermediate terminals, transfers, and classifications in the intermediate terminals and the unloading in the destination. To further explain, the route on the service network starts by a freight demand which is specified by commodity class according to its origin and destination as well as physical and service characteristics. The freight is moved by carrier services performed by a large number of vehicles, for example, railcars, trailers, etc. The vehicles move, usually on specified routes and sometimes following a given schedule, either individually or grouped in convoys such as trains or multi-trailer assemblies. Convoys are formed and broke down in terminals, while the freight may also be consolidated, loaded and unloaded from vehicles in the terminals. We can see that the number of the possible routes grows exponentially considering a large number of services for the network. Different proportions of the same demand can be assigned to different routes so as to best use the network capacity. The complexity of the problem is high because the capacity must be also optimized.

SND is widely applied in transportation problem. Crainic and Laporte (1997) state that, the main examples of service network design systems are:

- (1) Railway transportation where various train services (e.g., normal, rapid, direct, unit, etc.) correspond to various “modes”.
- (2) Less-than-truckload trucking, eventually incorporating multi-trailer assemblies and use of rail transportation.
- (3) Intermodal container shipping lines.
- (4) Express package services.

- (5) Freight transportation in some countries where a central authority more or less controls a large part of the transportation systems.

Consequently, terminals come in several types and sizes in different applications. For railways, for example, one identifies large and small classification yards pick-up and delivery stations, junction points, etc. In the yard, the railcars are consolidated into blocks which are the groups of cars and trains are formed. Similarly, an LTL network may encompass end-of-line terminals and break-bulk terminals. In the end-of-line terminal, the local traffic is delivered (by smaller pick-up trucks) and consolidated into larger shipments while loads from other parts of the network are unloaded and moved into smaller delivery trucks, and in the break-bulk terminals, the traffic from many end-of-line terminals is unloaded, sorted and consolidated for the next portion of the journey. The rail applications could be found in Crainic (1984), Crainic and Nicolle (1986), Marín and Salmerón (1996a, b), and Cordeau (1998), while the applications of the LTL trucking are presented in Crainic and Roy (1988), Roy and Crainic (1992), and Roy and Delorme (1989).

The main decisions made in the service network for the tactical phase concern the following issues: Selection of the routes including the origins, destinations, and the intermediate terminals with the functions of load, transfer, and unload and the characteristics of each service, particularly their frequency. For recent reviews on SND, we refer to Crainic and Laporte (1997), Crainic (2000), Macharis and Bontekoning (2004).

### **2.2.1.1 Formulation and algorithm**

The SND typically addresses tactical planning issues. The goal of the SND is to generate global strategies to improve the cost and service performance of the system. Furthermore, the global strategies could be used to determine the day-to-day policies for operational phase and also provide an evaluation tool for “what-if” questions raised during the strategy phase.

Crainic (2000) presents that the main issues addressed in SND concern several questions such as:

- (1) What type of service to offer?
- (2) What traffic itineraries to operate?
- (3) How often over the planning horizon to offer it?
- (4) What are the appropriate terminal workloads and policies?

The author further classifies the SND formulations into two types according to the role service levels play in the formulation: decision or output. The former takes the service frequency as integer decision variables in the first class of models; the latter obtains frequencies from traffic flows which are restricted by the minimum service levels as lower bound. In the following, we present a basic formulation to further explain the formulation issues in the SND.

Let  $G=(V, E)$  represent the “physical network”, in which vertices of  $V$  represent the terminals selected for the particular application, while  $E$  is the set of links representing the connections between terminals.

The notations represent as follows:

$m$ : Traffic class which represents a certain commodity  $c$  to be moved from the origin node  $o \in V$ , to a destination  $d \in V$ . To be clear,  $m = (o, d, c)$ ;

$s$ : Service;

$k$ : Itinerary;

$d^m$  : Transportation demand in term of the volume (e.g., number of the vehicles);

$F_s$  : Frequency level of the service  $s$ ;

$X_k^m$  : Amount of flow of the traffic-class in the itinerary  $k$ .

As we have mentioned before, the SND specifies the transportation service  $s$  that could be offered to satisfy the demand  $d^m$ . The service  $s$  is generally defined by the itinerary  $k$ ; it follows from its origin  $o$  to its destination  $d$ , by its service characteristics: mode, speed, capacity, etc., in which frequencies are the most important characteristics. The main elements of the SND are to determine frequencies  $F_s$  and itinerary  $k$  in order to minimize the total system cost. Then, the formulation of SND can be presented as follows:

$$\text{Min } \psi(X_k^m, F_s) \quad (2.2.1)$$

$$\text{s.t. } \sum_k X_k^m = d^m \quad \text{for all } m, \quad (2.2.2)$$

$$X_k^m \geq 0 \quad \text{for all } k, m, \quad (2.2.3)$$

$$F_s \geq 0 \quad \text{and integer for all } s. \quad (2.2.4)$$

The objective function consists of minimizing the total cost. The constraints (2.2.2) ensure the meeting of the demand. The constraints (2.2.3) and (2.2.4) ensure each decision variable must be non-negative.

Note that the total system cost represents a generalized cost including

- (1) The total cost of operating a given service network at Level  $F_s$  ;
- (2) The total cost of moving freight by using the selected itineraries for each traffic-class  $X_k^m$ .

It includes both operating and service costs at this level that the relationship and trade-offs among the various system and policy components are considered. In Crainic and Laporte (1997), the cost components for a rail application and LTL application are presented in detail. There are several efforts in the formulation of SND.

Due to the non-convex optimization model for SND, it is difficult to solve using only exact approaches. Several heuristic and metaheuristic methods have been proposed, for example, Lagrangian relaxation in Keaton (1989, 1992), Simulated Annealing in Marín (1996 a,b), Tabu Search in Pedersen (2009), other heuristics in Crainic and Rousseau (1986), etc.

## 2.2.2 Hub allocation problem

We have introduced the hub location problem in section 2.1. However, the hub locations are generally fixed for some time interval because of long-term lease contracts, equipment at hubs, cost of moving, etc. In this situation, it is important for the efficient operation of the transportation network to decide the optimal assignment of the non-hub nodes

to hubs in tactical phase. The hub allocation problem (HAP) is just to handle this kind of problem.

Due to the close relationship with hub location problem, the HAP could be similarly classified into p-hub center allocation problem, p-hub median allocation problem, p-hub covering allocation problem. Moreover, each of the allocation problems could be further divided into sub-problem according the allocation modes: single allocation and multi-allocation.

We have mentioned the related references for p-hub location problems in section 2.1.3. As stated in Campbell (1994) and Kara and Tansel (2000), many algorithms for the p-hub location problem iteratively select hubs, and then solve the resulting allocation problem. Thus, good approaches to solve the resulting hub allocation problems have already been used in the process of solving the hub location problem, which are presented in section 2.1.3. In the other hand, the researches focusing on allocation problem do not exist so much, except for example Sohn and Park (2000) and Campbell (2005).

### **2.2.3 Summary**

Hub allocation problem can be seen as a part of hub location problem. When the locations of the hubs are fixed, the HLP just needs to allocate the non-hub nodes to the hubs. In this case, the HLP becomes the hub allocation problem. Thereby, studies with respect to the hub allocation problem can be usually found even in the literatures of FLP. Meanwhile, note that neither HLP nor hub allocation problem focus on the long period operation phase (strategic or operational phase). The operational phase is not widely studied.

## 2.3 Operational Phase

Operational phase relate to how the operation should be conducted to offer the proposed service at minimum cost for short term. They include a wide variety of problems. Routing and dispatching of vehicles and crews, scheduling of services, etc. are important operational decisions in this phase. Particularly, real-time control problems are solved in real time during operations and aim at minimizing customer inconvenience. Usually, they consider minor perturbations to the scheduled service. In the section, we focus on introducing vehicle routing problem because we will provide a general framework to solve the vehicle routing problem in operational phase.

### 2.3.1 Vehicle routing problem

#### 2.3.1.1 Introduction

Vehicle routing problem (VRP) is a combinatorial optimization and integer programming problem. Proposed by Dantzig and Ramser in 1959, VRP plays a central role in the fields of transportation, distribution and logistics. It can be defined as the problem of designing optimal delivery or collection routes from one or several depots to a number of geographically scattered cities or customers, subject to side constraints (Gilbert Laporte 1991). There exists a wide variety of VRPs and a broad literature on this class of problems (see, e.g. Cordeau et al. 2007 and references therein). It is mainly concerned with the short distance movement of goods while is faced each day by thousands of companies and organizations in the delivery and collection operations. Obviously, it is included in the operational phase.

#### 2.3.1.2 Classification

Because conditions vary from one setting to another in practice, the objectives and the constraints of the VRPs are highly variable. Various classification elements are available in the literature to categorize the VRPs. We use these as guidelines to show our classifications.

As shown in Table 2.1, there are five elements, each of which has several attributes with corresponding values, influencing the classification of VRPs. The VRP studied in the literature are the combination of one or several attributes. The following Fig. 2.1 shows us the typical VRPs and some particular cases of VRP, TP, TSP, etc., categorizing by the attributes in Table 2.1. Take capacitated vehicle routing problem (CVRP) as an example, it is the problem in which the vehicles have carrying capacity of the goods that must be delivered. Obviously, it is VRP with the “capacitated” value of the attribute “capacitate of vehicle”.

Table 2.1: classification of VRP with regard to its problem perspective

| <b>element</b> | <b>attribute</b>       | <b>value</b>                                                                                                                     |
|----------------|------------------------|----------------------------------------------------------------------------------------------------------------------------------|
| vehicle        | size of vehicle fleets | A: single vehicle<br>B: multiple vehicles                                                                                        |
|                | type of vehicle        | A: single type<br>B: heterogenous fleet                                                                                          |
|                | vehicle capacity       | A: uncapacitated<br>B: capacitated                                                                                               |
|                | speed                  | A: time-dependent<br>B: time-independent                                                                                         |
| assignments    | type of assignment     | A: delivery<br>B: pick up<br>C: mixed                                                                                            |
|                | assignment characters  | A: arcs<br>B: nodes<br>C: mixed                                                                                                  |
| customer       | time windows           | A: unspecified time with no deadlines<br>B: soft time windows with loose deadlines<br>C: hard time windows with strict deadlines |
|                | demand information     | A: static data<br>B: dynamic data                                                                                                |
| depot          | number of depots       | A: single depot<br>B: multiple depots                                                                                            |
|                | type of routes         | A: open route with no need to return the depot<br>B: close route with the need to return the depot                               |
| goods          | type of goods          | A: divisible<br>B: indivisible                                                                                                   |

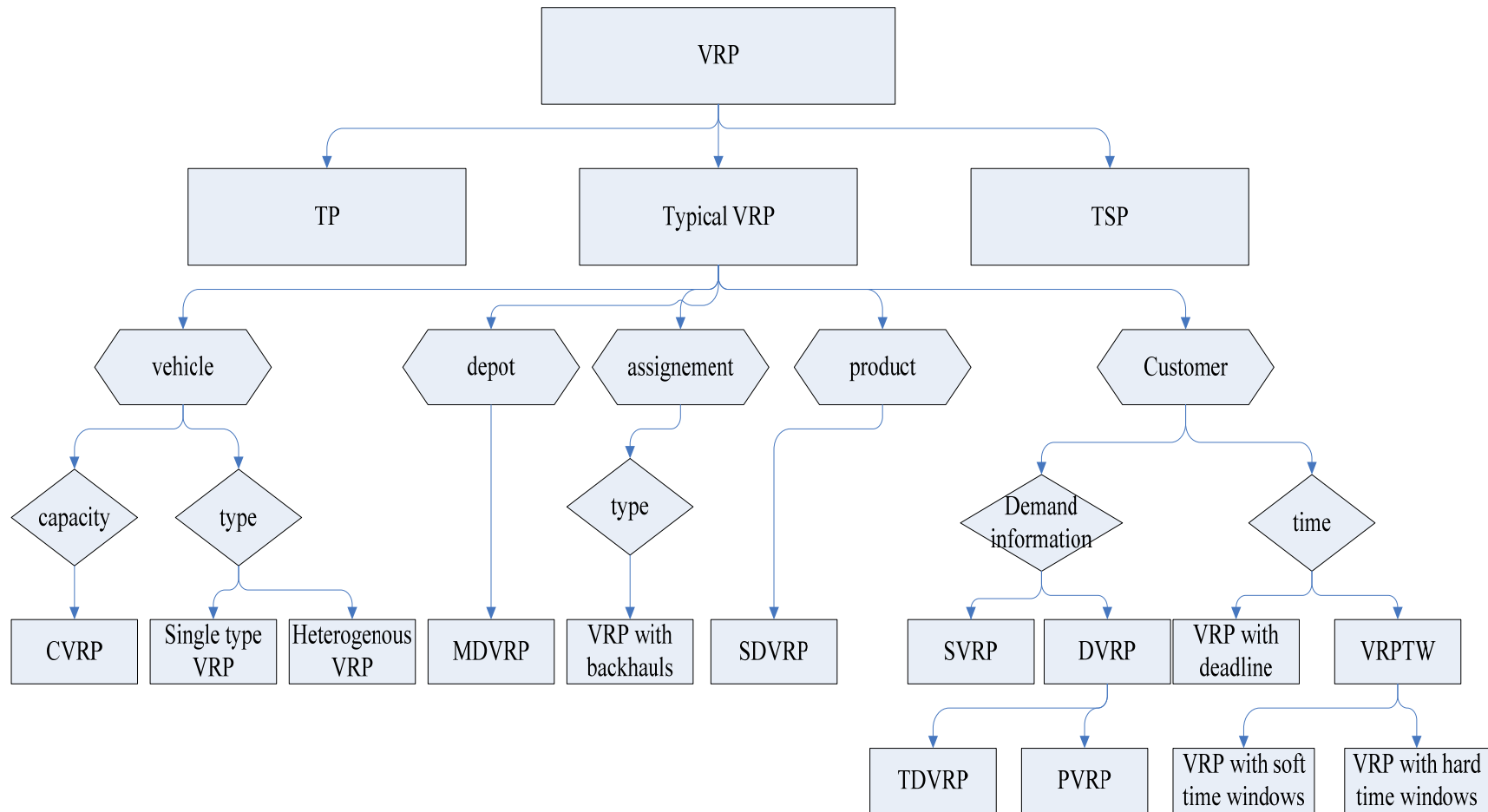


Fig. 2.1: classification of typical VRPs with the attributes

### 2.3.1.3 Formulation for CVRP

The VRP is often defined under capacity and route length restrictions. When only capacity constraints are present, the problem is denoted as CVRP. The CVRP is to determine a set of routes for  $m$  vehicles with the goal of minimizing the total travel cost. It is generally restricted by the following constraints:

- (1) each customer is visited exactly once by one route;
- (2) each route starts and ends at the depot;
- (3) the total demand of the customers served by a route does not exceed the vehicle capacity  $Q$ .

In fact, there are two distinct classes of VRP studies, one of which is symmetric VRP, the other of which is asymmetric VRP. In order to present the formulations for CVRP, the following notations should be firstly defined.

- **Notation**

Let  $G = (V, E)$  be a complete undirected graph;

$V = \{0, \dots, n\}$ : a vertex set in which vertex 0 represents a depot;

$E = \{(i, j) : i, j \in V, i < j\}$ : an edge set;

$i$ : a customer who demands the goods, especially  $i \in V \setminus \{0\}$  ;

$e$ : each edge in the graph;

$c_e$  : a travel cost on the edge  $e$ ;

$m$ : the number of vehicles available at the depot;

$Q$ : the capacity of each vehicle.

Thus, the asymmetric VRP is similarly defined on a direct graph  $G = (V, A)$ , where the edge set is represented as  $A = \{(i, j) : i, j \in V, i \neq j\}$ . As description above, we can see that a solution of symmetric CVRP could be viewed as a set of  $m$  cycles sharing a common vertex at the depot, while the solution of asymmetric CVRP is a set of  $m$  directed cycles associated with the vehicle routes.

- **Variables**

$x_e$  : the integer variable indicating the number of times edge  $e$  is traversed in the solution ;

$r(S)$ : the minimum number of vehicles needed to serve the customers of a subset  $S$  of customers;

- **Formulation**

Finally, for a subset  $S$  of  $V$ , let  $\delta(S) = \{(i, j) : i \in S, j \notin S \text{ or } i \notin S, j \in S\}$ . The formulation proposed in Laporte et al. (1985) is then:

(CVRP)

$$\min \sum_{e \in E} c_e x_e \quad (2.3.1)$$

$$\text{s.t.} \quad \sum_{e \in \delta(i)} x_e = 2, \quad i \in V \setminus \{0\}, \quad (2.3.2)$$

$$\sum_{e \in \delta(0)} x_e = 2m, \quad (2.3.3)$$

$$\sum_{e \in \delta(S)} x_e \geq 2r(S), \quad S \subset V \setminus \{0\}, S \neq \emptyset, \quad (2.3.4)$$

$$x_e \in \{0, 1\}, \quad e \notin \delta(0), \quad (2.3.5)$$

$$x_e \in \{0, 1, 2\}, \quad e \in \delta(0). \quad (2.3.6)$$

The objective is to minimize the total transportation cost.

The constraints (2.3.2) are the degree constraints that state each customer is visited only once.

The constraint (2.3.3) is the depot constraint that means  $m$  routes are created.

The constraints (2.3.4) are the capacitated constraints that impose both the connectivity of the solution and the vehicle capacity requirements by forcing a sufficient number of edges to enter each subset of vertices. Note that  $r(S)$  could be determined by solving an associated Bin Packing Problem (BPP). Since BPP is NP-hard in the strong sense,  $r(S)$  may be approximated from below by any BPP lower bound, such as  $\sum_{i \in S} q_i / Q$ .

Finally, constraints (2.3.5) and (2.3.6) ensure that each edge between two customers is traversed at most once, and each edge associated to the depot is traversed at most twice.

### 2.3.1.4 Algorithm

As we know, TSP and VRP are NP-hard combinatorial optimization problems (Savelsbergh, 1985). In practice, the VRP turns out to be significantly harder to solve than the TSP. The best VRP algorithms can rarely tackle instances involving more than 100 vertices, while TSP instances with even thousands of vertices are now successfully solved to optimality.

Laporte and Nobert (1985) survey that exact algorithms for the VRP can be classified into three broad categories: (1) direct tree search method, (2) dynamic programming, and (3) integer linear programming. Then, Laporte (1992) surveys the main exact and approximate algorithms developed for the VRP, at a level appropriate for a first graduate course in combinatorial optimization. Furthermore, the author divides the algorithm of VRP into two main parts: exact algorithms and heuristic algorithms. Some research efforts were oriented towards the development and analysis of approximate heuristic techniques capable of solving real-size CVRP problems. Bowerman et al. (1994) classify the heuristic approaches to the VRP into five classes: (1) cluster-first/route-second (CFRS), (2) route-first/cluster-second (RFCS), (3) savings/insertion, (4) improvement/exchange and (5) simpler mathematical programming representations through relaxing some constraints. For the two clustering procedures, the cluster-first/route-second looks more effective.

Following Laporte and Nobert (1985), Cordeau et al. (2007) introduce the algorithms in the literatures for CVRP in two types: exact and approximate. Here, we show them in Table 2.2.

Two-phase methods are based on the decomposition of the VRP solution process into the two separate sub-problems. Such decomposition techniques can reduce the problem size and expand the choice of searching strategies. Several authors have previously proposed decomposition technique to solve VRP. Previous works may be classified into three types: (1) using optimization model with Lagrangian relaxation as decomposition technique, i.e., Toth and Vigo (1993), Cheng and Wang (2009) and Sahoo et al. (2005); (2) heuristic approach to construct the groups. The sweep algorithm is often referred to be the first decomposition technique for two-phase methods for VRP. The corresponding literatures could be found in Wren (1971), Wren and Holliday (1972), and Gillett and Miller (1974). Dondo and Cerda (2007) present a novel three-phase heuristic/algorithmic approach which embeds a heuristic-

based clustering algorithm within a VRPTW optimization framework. In other words, they use a preprocessing method to cluster nodes into groups, and then take the group as node to apply the optimization method; (3) cluster analysis is usually proposed. Barreto et al. (2007) integrate several hierarchical and non-hierarchical clustering techniques into a sequential heuristic algorithm for the location-routing problem (LRP) model. Kim et al. (2006) develop a capacitated clustering-based algorithm to deal with the real life waste collection problems. Ganesh and Narendran (2007) provide an initial solution with k-means clustering methods and thereby accelerated convergence of the genetic algorithm to solve the vehicle routing problem with deliveries and pickups.

Table 2.2 Classification of the solution methods for CVRP

| <b>type</b>               | <b>method</b>                                                                                                                                                                                                                                                |
|---------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Exact                     | Branch-and-bound and set partitioning based algorithm<br>Branch-and-cut algorithm                                                                                                                                                                            |
| Approximate<br>Heuristics | route construction methods<br>two-phase methods<br>route improvement methods                                                                                                                                                                                 |
| Meta-heuristics           | local search, including simulated annealing, deterministic annealing, and<br>tabu research<br>population search, including genetic search and adaptive memory<br>procedures<br>learning mechanisms, including neural networks and ant colony<br>optimization |

### 2.3.2 Summary

VRP is a critical issue in the fields of transportation, distributions and logistics. Although there are a great number of literatures on this problem, an efficient algorithm with respect to solve large-scale size VRP seems to be necessary but it does not yet exist. Furthermore, as we have mentioned, the VRP can be categorized into various classes. In this case, a general solution approach to various VRP is needed to be proposed.

## Part II

# SOLUTION PROCESS OF GENERAL DELIVERY PROBLEM



# CHAPTER 3: Solving Procedure

## 3.1 Introduction

As mentioned in the previous chapter, the planning and management of operations in a transportation system can be divided into three phases: the strategic phase, the tactical phase, and the operational phase. In our view, the transportation system management mainly focuses on transportation operations and terminal operations, whose issues have to be confronted according to these three phases of operation planning.

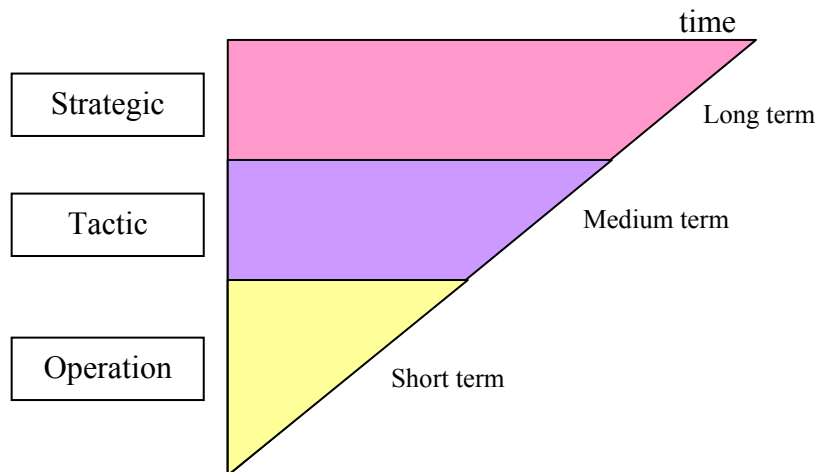
In chapter 2, we have reviewed in detail several main issues of the three phases of operational planning in GDP and we have presented the appropriate Operations Research models, methods, and computer-based planning tools. Addressed as a whole, the issues presented in the previous chapter are a set of sub-problems in the solving process of the GDP that are usually solved sequentially at the three phases (strategic, tactical, and operational or during operations (real-time control)).

The major aim of this chapter is to propose a hierarchical approach including the main issues explained in chapter 2 and other issues in the solving process for GDP to help a decision maker to organize their transportation. We follow the three somewhat classical types of planning phases in transportation management and introduce them in two aspects: transportation system design and terminal operation. Furthermore, the interdependences among the problems in each phase are analyzed and the integrations of the issues in the transportation process are also presented.

The rest of the section is organized as follows: in section 3.2, the three operation phases are introduced and the decisions in each phase are presented in detail. In the following subsections, the major issues and the relationships between each other, which are extremely studied by operational researchers, are respectively integrated in each of the three operation phases.

## 3.2 Solving Process

Generally, transportation systems are rather complex organizations which involve a great number of human and resource elements. To ensure that the transportation process is operating as efficiently as possible, and generate the highest level of customer satisfaction at the lowest cost, companies adopted transportation system in three classical types of planning phases by their time-horizon: strategic, tactical, and operational. The three phases in solving process are shown in Fig.3.1



**Fig. 3.1** The three phases in solving process to GDP

(1) Strategic phase: At this first level, the company management will be looking for long-term decisions (generally several years) to high level strategic decisions concerning the whole organization. Strategic decisions determine general development policies and broadly shape the operating strategies of the system.

The strategic decisions mainly concern the following issues:

- The design of the physical infrastructure network and its evolution (upgrading or resizing);
- The place to locate the facilities (e.g., consolidation terminals, platforms, warehouses, rail yards, and so on);
- The type and quantity of equipment (e.g., cranes) to install at each facility;
- Addition or reduction of the transportation line and the capacities on the line;
- Deciding the direct service zone (the customers are served directly by the company) and indirect service zone (in some cases, the companies demand the carriers to serve the customers);
- The tariff policies...

In strategic phase, the planners are not constrained by existing resources. The data used in this phase are often imprecise. Moreover, an operating plan must be constructed to assess various scenarios depending on the forecasts.

(2) Tactical phase: The activities in tactical phase generate an efficient and rational allocation of the limit resources for a medium horizon, generally from one to several months, that will be executed in the next phase, the operational phase. The goal of this phase is to improve the performance of the whole system.

Tactical decisions mainly focus on adopting measures:

- Developing warehouse strategies to reduce the cost of storing inventory;
- Choosing the routes to provide service;
- Deciding the service type to use, such as door-to-door delivery and the frequency of certain services;
- General operating rules for each terminal and resource;
- Reposition of resources, e.g., empty vehicles;
- And other activities that take functions to the transportation system, and produce cost benefits for the transportation process for the companies.

In the tactical phase, the freedom is very limited to change the existing resources. Data is aggregated and system parameters have no relationship with the day-to-day information. At this phase, the policies and decisions not only aim to an adequate allocation and utilization of existing resources, but also strive to achieve the best trade-off between benefits and service performance. Furthermore, they are commonly used to model and analyze different scenarios, such as determining the incremental operating costs or inventory quantities for a set of volume changes. They are somewhat sensitive only to broad variations in data.

(3) Operational phase: Decisions at this level are made in a short term horizon, generally each or several days that affect how the products are delivered in the transportation system.

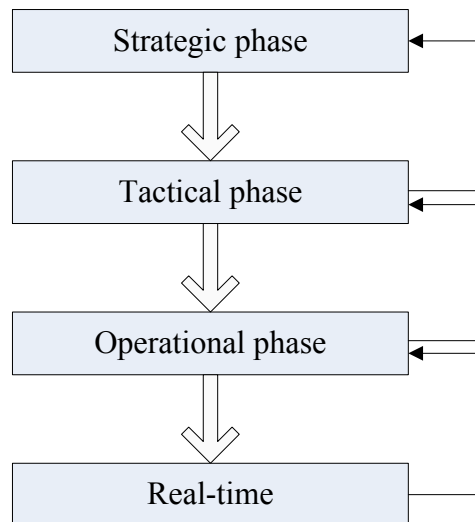
Operational decisions involve several important issues:

- Operational activities of facilities;
- Vehicle routing planning in the whole transportation system;
- Dispatching of vehicles and crews;
- Scheduling of workers and services;
- Especially, some real-time emergency management in the transportation process, which includes preparing the system to adapt to sudden volume changes, recovers from weather disruptions;
- And so on.

In this phase, the time factor plays a highly dynamic role. Notably, sometimes emergency management is regarded as real-time level in the operation process.

These three planning phases highlight how the data circulates among the decision-making level and how to set the policy guidelines. In fact, the strategic phase provides the general policies and guidelines for the decisions made in the tactical phase. Generally, the goals, rules and limits are determined to regulate the transportation system for the operational phase. There exist reverse data flows to supply information for the decision making process to a higher phase. Fig.3.2 illustrates the relationships among the three planning phases and real-time decisions.

The three planning phases occur sequentially and interact with each other. For example, facility location, inventory management, and vehicle routing problem belong respectively to strategic, tactical, and operational phase. When alternatives exist for location of inventory facilities, the vehicle routing sequence between OD pairs through the replacement inventory will also change. Additionally, the routine and the alternative inventory facility necessitate different parameters for inventory management.



**Fig. 3.2** The relationships of the three phases in planning process

Although there are inseparable relationships among the planning phases as we have mentioned above, the formulation of a unique model for the planning transportation systems is prevented because of the complexities of the problem. However, the different models are formulated for specific problems at specific planning level. In the next section, we will summarize the important issues in each of the operation phases.

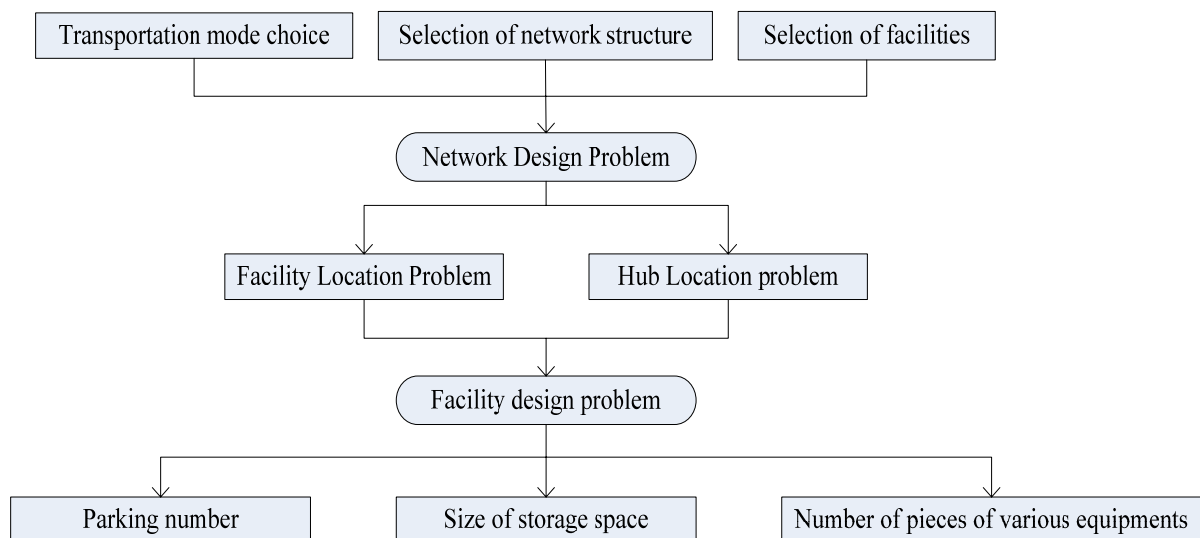
### 3.3 Process Analysis

Now, we describe in detail the solving process in each planning phase for GDP. To solve a GDP, the company is confronted by most transportation problems issues both in transportation system design operation and terminal operation. We summarize the issues in each of the three phases in operation planning and successively present them to help the decision maker to solve their GDP although we do not present a comprehensive and detailed treatment in all subjects. Attentively, the main and basic issues are usually solved sequentially in each of the planning process. These issues may have been widely researched, but they are not yet integrated into a solving process for a global transportation problem.

The remainder of this section is dedicated to present separately important issues for the solving process of GDP in planning and managing transportation systems. The presentation is organized according to the three planning phases.

#### 3.3.1 Strategic phase

For a company, the decisions in the strategic phase determine the general development policies and broadly shape the operating strategies of the system over a long-term horizon. In this phase, prime examples of decisions in the aspect of network system design are the design of the physical network, the location of main facilities and hubs, the transportation mode choice, etc, while the main issues in the aspect of the terminal operation are the design of the terminals. It includes the number of parking, the size of the storage space, the number of pieces of various equipments (e.g. yard cranes), etc. As mentioned in 3.1, these various problems are usually solved sequentially. Fig. 3.3 illustrates the relationship of the issues in strategic phase.

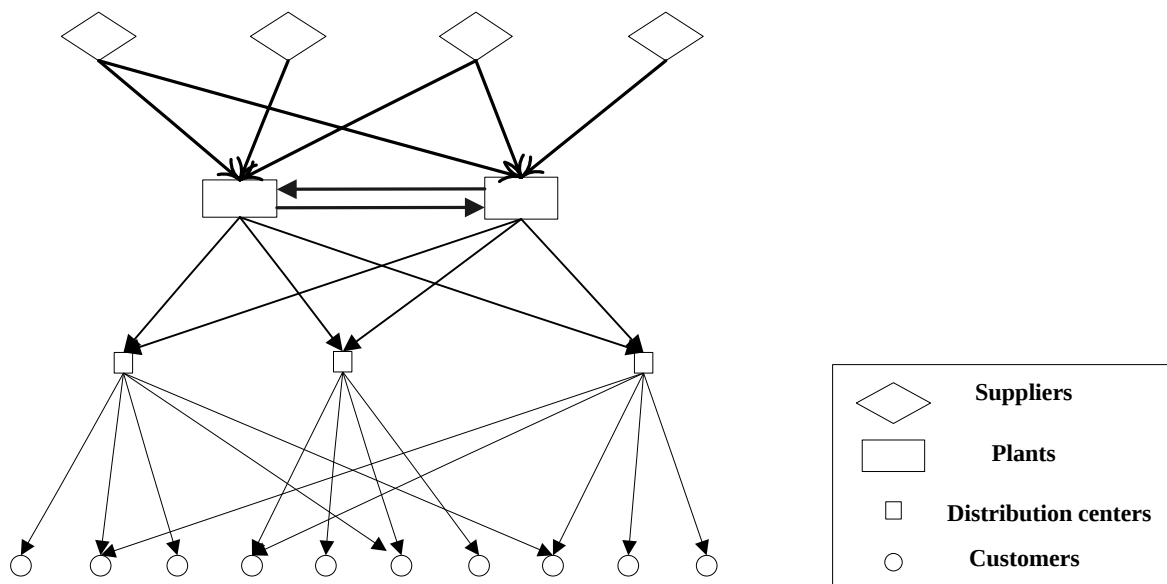


**Fig. 3.3** Relationship between the issues in strategic phase

When a company confronts a GDP in the strategic phase, it firstly decides some basic and general policies. According to the characters of the goods demanded by customers, it should determine the set of transportation modes, for instance, petroleum moves by ship and

pipeline, urgent and long distance movement products by air, even large-scale transportation by combinations of several transportation modes, etc. This issue is usually defined as the *mode choice* model. In the context of economic globalization, different transportation modes are usually a natural consequence of transportation around the world: by air, by sea, or by land.

Then the company selects the transportation *network structures*. There exist different network structures in the transportation system where one vehicle or convey serves different customers with possibly different initial origins and final destinations, and the company provides customized service for each particular customer. We consider a transportation network composed of  $k$  levels, in which the lowest level of facilities is called as level 1 whereas the highest level is called as level  $k$ . The origins and destinations are defined as level 0. Notably, each level of facilities plays a specific role and there is a natural flow between them. Then the transportation network could be classified as 1-level, 2-level, multi-level distribution system according to the number of the facility levels. Fig.3.4 shows us a 2-level transportation network. Furthermore, the possibility of intra-level flows and direct flows from upper levels to lower levels (not immediately lower levels) exist in real cases but they are rarely studied in the literature, because this feature destroys the structure of the constraint matrix and thus prevent to use decomposition methods. After the determination of the network structure, the company must list all of the potentially interesting facilities in each level. The *selection of facilities* often depends on the location of the facilities, the traffic situation, the stock of the facilities, etc.



**Fig.3.4** A 2-level transportation network

After that, the locations and the roles of all possible facilities are known for the company. And then the company needs to determine an optimal route choice behaviour, in the research domain, this kind of problem is usually concerned as *network design problem (NDP)*. In the NDP, the company aims to choose links in the network to delivery goods from origins to destinations with the lowest cost and with the capacity constraints. In fact, NDP is a generalization of *facility location problem (FLP)* and *hub location problem (HLP)*. If the facilities are not yet fixed in a transportation network where there are not intra-flows between the facilities in the same level, we need to decide the number and the locations of the facilities

to organize optimally the delivery process in the lowest cost satisfying the other constraints. Thus, we confront a FLP. If the facilities in the transportation network allow the intra-flows between each other in the same level, then the NDP is defined as a HLP.

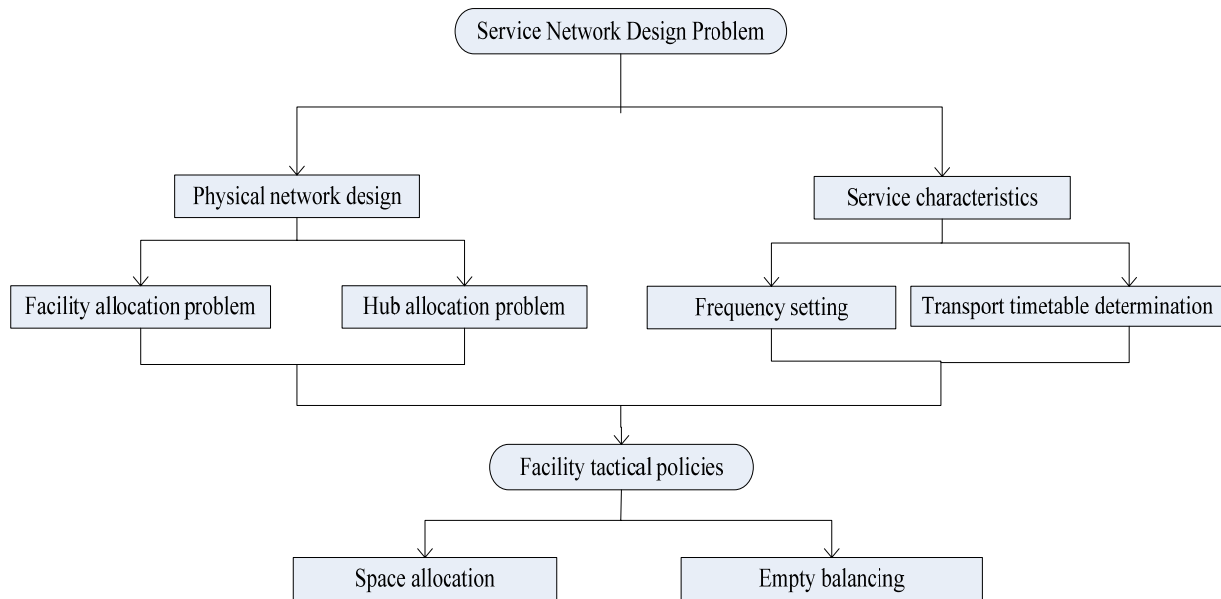
The transfer of goods takes place at the intermediate facilities. After locating the facilities, we consider the physical network has been constructed. Then the *facility design problem* is needed to be solved, which is an important strategic planning issue for transport management. It is related to *parking number*, the *size of storage space*, the *number of pieces of various equipments* to install. To further clarify these notions, consider the case of railway transportation. In a rail terminal, there are three areas: original yard, waiting area and loading area. Goods thus arrive at the rail terminal by truck or other transportation mode and are either directly transferred to a selected rail car or more frequently, are stacked in the waiting area. Then, the goods are brought into the loading area following a pick up operation to rail cars. Once loaded, rail cars are moved to origin yard (may be the same to loading area) where they are sorted, or classified, and grouped into blocks. Here, a block is a set of rail cars with possibly different final destinations. In order to better operate and economize, a block is usually considered as a single from the yard where it is made up, to the destination yard where it is broken down. The block is eventually put on a train and restarts its journey on the next transport leg. On the other hand, when the goods arrive by a train and need to be transferred to trucks for the next journey, the reverse operations take place. In fact, the facility operation issues require a trade-off between the amount of investment and the level of customers. For example, as the number of parking increase, the waiting time and the turnaround time of the trucks decrease. Delay in the transportation process sometimes generates high costs. Thus, availability of empty parting at origin yard is a key issue when operators determine facilities. As some types of equipments (such as yard cranes) are expensive, determining the number of the equipments to be installed is thus another major design issue in facility design problem. Storage space is another critical resource in facility design problem. As the storage space is smaller, the storage stacks become higher. Thus, lower efficiency of the transfer operations is then generated.

### 3.3.2 Tactical phase

In the previous subsection, we have introduced the relationship of the issues in strategic phase. The strategic decisions determine the general policies for the company over long time horizon, generally one or several years. In the real practice, the company needs to adjust these strategic policies for the shorter time term from one to several months, when the operation situations have some changes of existing resources, for example the number of the facilities. The relative issues in this adjust process are concerned as tactical decisions.

The tactical phase aims to ensure an efficient and rational allocation and utilization of existing resources in order to improve the performance of the whole system. In this phase, main decision issues concern *service network design problem* (SNDP) in the aspect of network system design and facility tactical policies in the aspect of the terminal operation. Fig. 3.5 shows the relationship of the main issues and the sequential decision process in tactical phase. Service network design addresses the system-wide planning of operations to decide the physical network design and determination of service characteristics. Particularly, facility allocation and hub allocation are two special issues widely researched to improve the physical network structure. Services offered on the physical routes in the network are including mode, frequency or schedule, etc., in which frequency setting and transport timetable determination

are two special issues indispensable to this service network design process. On the other hand, major decisions performed in consolidation facilities include how to allocate the storage space to prepare for the next planning period. In addition, the policies of repositioning empty vehicles to meet the forecast needs in the next period have also to be determined.



**Fig.3.5** The sequential decision process in tactic phase

In tactical phase, the company firstly confronts the *Service network design problem (SNDP)*. SNDP is to build a transportation plan to ensure the efficient operation of the system and to optimize the profitability of the company. It integrates two types of major decisions : to determine the *physical network design* and to determine the characteristics of each service. The former is to select the routes between each OD pair, to determine the intermediate facilities on each route; the latter is to determine the major characteristics of each service. SNDP is in fact difficult to solve because of the strong interactions among the major decisions and the corresponding tradeoffs between operating costs and service levels. Attentively, although SDN is typically developed to assist the tactical planning of operations, it may be sometimes referred to as strategic/tactical or tactical/operational phase according to the planning traditions and horizons of the companies.

In the process of the physical network design, the company often needs to confront the change of market situation in a medial term period. For example, the customers in one region increase or decrease dramatically because of the season or of the market competition. Another example is when a new market appears in certain region. In these cases, the facilities must be reallocated according to the customers and the origins. This type of problems is defined as *facility allocation problem (FAP)*. More particularly, if special facilities with intra-flows must be allocated, the problem is regarded as *hub allocation problem (HAP)*.

The traditional objective of the transportation is to minimize the total operating cost with both timely and reliable service to satisfy the requirement of customers. Increasingly, the customers not only except low rates, but also require a high-quality service. Thus, to decide what *service characteristics* to include in the transportation plan is another important issue for SNDP. Each service is measured by the speed, flexibility, and reliability of the transportation. Furthermore, the service is characterized by the transportation mode, the route, the capacity on each route, and the service class which indicates characteristics such as preferred traffic,

speed, propriety, etc. Notably, each company needs to determine its transport plan for a median period from one month to several weeks so it can confirm the appointment with the customers for the delivery. In this situation, a transport timetable seems to be necessary. Through it, the customers are divided into several priority-ranked groups according to the requirement deadlines of the customer deliveries. Here, the customers in the same group are served in the same day. Especially, certain service may be operated repeatedly during the planning period, for example, several similar trucks departing during the month and visiting the same plant for the same order. Thus, the design of the transportation must also determine the frequency of each service. The *frequency setting* problem more and more attracts the attention of the researchers.

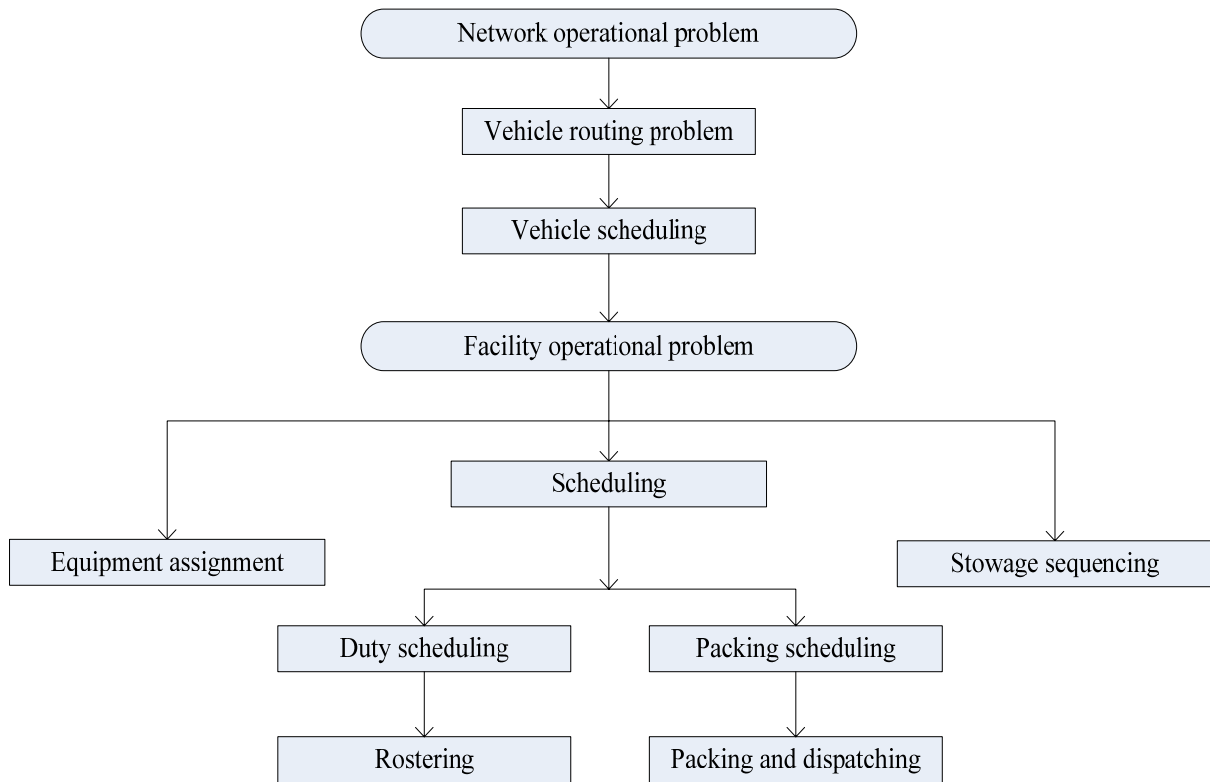
Since the operating rules for the service network design has been determined, the *facility tactic policies* has then to be considered. The operation rules such as how trucks may be sorted, consolidated and grouped also become a part of tactical decisions. In each intermediate facility, the information is interacted between good handlers in the yard and storage operators to decide where the goods are stacked within the yard. The stack and stored mode of the goods in the yard is one of the most important factors that affect the turn-around time of transporters in the yard. Generally, the storage space in the facility is pre-assigned to the goods which will arrive in the near future to maximize the efficiency of the operations. *Space allocation* problem is just concerned with this kind of problem to determine storage locations for the coming goods of the facility. Another important and challenging issue in the tactical phase is to replace empty vehicles between the different facilities to satisfy known and forecast demand in the following periods. Indeed, the transportation of the demands and supplies in different locations results in an accumulation of empty vehicles in regions where they are not needed, or the lack of the vehicles in other regions which require them. Then the vehicles must be moved empty or additional transportation assignment must be found. This operation is known as *empty balancing*. It may decide the vehicle routing dispatch and scheduling of crews and maintenance operations, and it may also determine the procurement of power units. In the most general form, empty balancing problem covers the whole range of planning and management issues from strategy to operational phase. However, it designates a restricted set of activities such as allocation of empty vehicles to meet the needs of customer requests.

So, the tactical phase appears as a critical link in the three phases of the operation planning. Its output is an effective evaluation tool to test the hypotheses provided during the strategic phase, and is also used to determine the day-to-day policies that are regarded as the decisions in operational phase. Taking these main outputs as parameters, the major issues in operational phase will be presented in the next section.

### **3.3.3 Operational phase**

Given a set of inputs obtained in strategic phase and tactical phase such as the frequency of the services, the operational planning phase aims at constructing vehicle routing, schedules of vehicle and duty, and the other assignments of resources to task to minimize total costs while respecting all operational constraints and work regulations. Indeed, the ultimate goal of the company is to minimize the cost and make and even improve its profits so as to maintain its competitive position in the product market. The performance of the company is just determined by its operational capability. Hence, the issues aiming to assisting decision making at an operational level are very important components for the transportation system.

Many different issues in operational phase aim to satisfy the requirements of the customers and to efficiently use the resources within some constraints. Different to strategic and tactical phases, the issues in operational phase are more sensitive in time factor. Consider, for example, a door-to-door service, divers are asked to delivery the goods to customer within the time windows. These main issues and the sequential decision process in tactic phase are summarized into two aspects: network operational problem and facility operational problem, which are illustrated in Fig.3.6.



**Fig.3.6** The sequential decision process in operational phase

In tactical phase, the facilities and hubs in the transportation network are allocated between each OD pair and the routines are sequentially determined. However, these routines are described to apply to long distance transportation over a planning period. The delivery in operational phase takes place in more restricted geographical areas such as the distributions in a local or regional level. The distribution process involves deliveries, collections, or a combination of both. *Vehicle routing problem* (VRP) arises from the distribution management in the operational level and has been extensively researched by operational researchers over the last forty years.

A tactical decision often includes a series of services, each of which is characterized by its own route, intermediate facilities, vehicle and convey type, capacity speed or travel time, and so on. Furthermore, the frequency of the service over a planning period is also identified in tactical phase. However, the service is actually offered by the vehicles in an operational phase according to a schedule which indicates the time of departure at the origin and the time of arrival at the destination, as well as the time and length of stops at the intermediate facilities. The problem to determine the schedule for the transportation assignment is defined as *vehicle scheduling*. The schedule may be partially or totally available

to customers. It means that in some cases, the schedule may be fixed; in other cases, the schedule may indicate a departure interval and the customers and other operational units of the company will be given a cut-off time to finish their services in time. Currently, the company prepares to determine a totally or partially schedule to improve their service and increase its competitiveness.

The facility operational problem may be regarded as allocating resources, such as the empty vehicles (railcars, blocks, etc.), the crews, the storage space, etc., to tasks. It is an extremely rich field both for research and development and for applications including *equipment assignment, scheduling, stowage sequencing*, etc.

Once a vehicle schedule is known, driver *duty scheduling* is performed to ensure a complete coverage of the vehicle schedule at a reasonable cost. Generally, an efficient duty schedule can often be obtained from a near-optimal vehicle schedule. On the other hand, a very efficient schedule may lead to a poor duty schedule or even to an infeasible duty schedule in some rare contexts. Notably, the vehicle schedule must be updated when the duty scheduling is not feasible or not satisfy the requirement of the cost. Once the driver duty schedule is determined, a *rostering problem* is solved to personalize the driver schedules. Given a set of anonymous duties defined for the drivers assigned to a particular facility, crew rostering aims to assign these duties to the available drivers to form their work schedules within the constraints of the safety regulations and collective agreement rules. For example, a driver cannot work more than 8 hours in one day.

*Parking scheduling* is the process of determining the time and position at which arriving vehicle will stop. In congested cities, the parking space is often very restrained and quite crowded. Therefore, when a vehicle of a particular type has to leave the depot according to the parking schedule, several other vehicles might need to be moved to clear the way out, resulting a delay. In this situation, the *vehicle parking and dispatching problem* is to park the vehicles upon their arrival and to dispatch them to the pull-outs in order to minimize the delay time. Additionally, the parking in the background of shipment is known as berth, while parking scheduling is defined as berth scheduling which is largely studied because the berth construction costs are the highest among all relevant cost factors.

When a vehicle arrives at an intermediate facility, some equipment (for example, a crane) is assigned to the vehicle and serves the vehicle in a given service time restricted by the duty schedule. Then, this type of problem is known as *equipment assignment*. During the unloading operation, the equipment picks up the goods and stacks them into a given position in the yard. *Stowage sequencing* next determines the sequence of unloading and loading goods, as well as the precise position the goods being loaded into the next vehicle.

### **3.4 Conclusion**

In this chapter, we have summarized the major issues in GDP depending on the general three phases of operation planning and the relationships and interactions between the issues are also presented sequentially. The goal of this chapter is to show a general solving process of GDP and to provide a general solving framework for decision makers when they confront a GDP. The set of all the decisions are affected to one of the three phases and then sequentially organized in a logical way.

In the following chapters, several particular issues in each operation phase are studied, formulated and finally solved separately since we have mentioned that there was not a common model for the transportation system because the problem is too complicated.

# CHAPTER 4: A Heuristic Framework to General Delivery Problem

## 4.1 Introduction

In chapter 3, we have discussed the solving procedure by dividing the planning and management of operations in transportation system into three phases: the strategic phase, tactical phase, and operational phase. We have also synthesized the major issues in each phase and introduced them by two aspects: transportation system design operations and terminal operations. In this chapter, we focus on the determination of the routing sequence for GDP. In this case, the issues about terminal operation are ignored.

Generally, the structure of the network is complex and the network has a considerable size. That is why the distribution network is hard to solve. As we have mentioned in the previous chapter, VRP is a special issue of GDP in operation phase or somewhat a sub-problem of GDP. Laporte et al.(1986) provide the optimal solution for randomly generated asymmetrical CVRPs involving up to 260 vertices with a branch-and-bound algorithm. So far, it is one of the best solutions to VRP with the greatest number of nodes. But the GDP with a huge number of nodes is more difficult to solve. In order to overcome the limitation and to help the companies solve their special vehicle dispatching and routing problem, we introduce a heuristic framework. The proposed heuristic framework is a structured approach in which we break down a complex GDP into a combination of sub-problems with basic structures as independent as possible to each other. It includes a three-phase heuristic decomposition procedure which can be used to divide the large-scale GDP into some identified sub-problems (FLP, HLP, FAP, HAP Transportation Problem, TSP, VRP or basic GDP). Each sub-problem is solved and the global solution is the sum of all these solutions. Through adding the Improvement Algorithm into the framework, the delivery routes of the sub-problems generated from the proposed framework can be globally optimized.

The rest of the chapter is organized as follows. In section 4.2, the three-phase heuristic framework based on decomposition is introduced in detail. Then the decomposition phase is discussed, in which the decomposition criterions for each planning phase are provided. In the following section, some methods about the improvement heuristic are presented.

#### 4.2 The decomposition framework

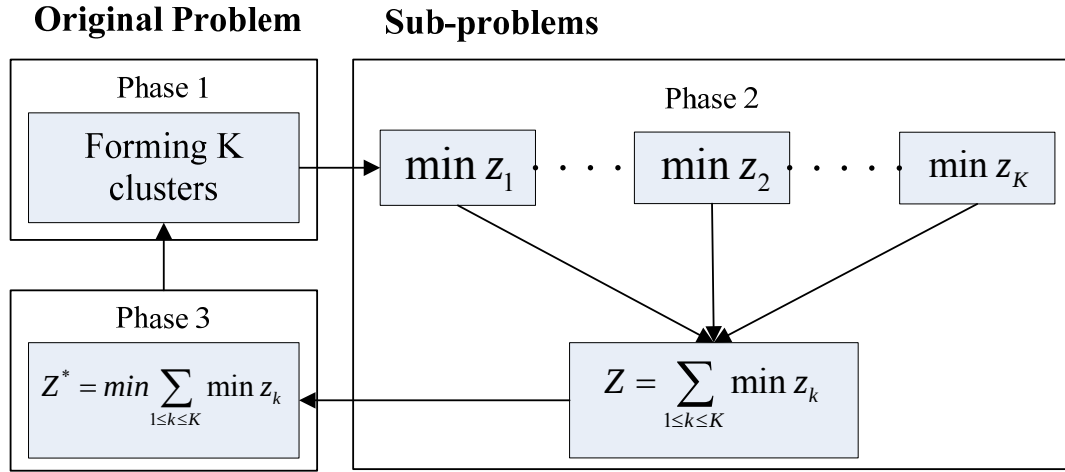
The procedure for the proposed framework consists of three phases as follows:

**Phase 1 (Decomposition phase):** Divide the huge number of nodes (including original nodes and destination nodes) of the GDP into some groups with decomposition techniques.

**Phase 2 (Routing determination phase):** Determine the distribution route for each group with the existing tools and/or heuristics algorithms.

**Phase 3 (Improvement phase):** Improve the routes between groups.

Cheng and Wang (2009) have used an iterative interaction between the original problem and many sub-problems to solve VRPTW. The decomposition procedure presented here is similar to that provided by Cheng and Wang (2009) and it is illustrated in Fig.4.1.



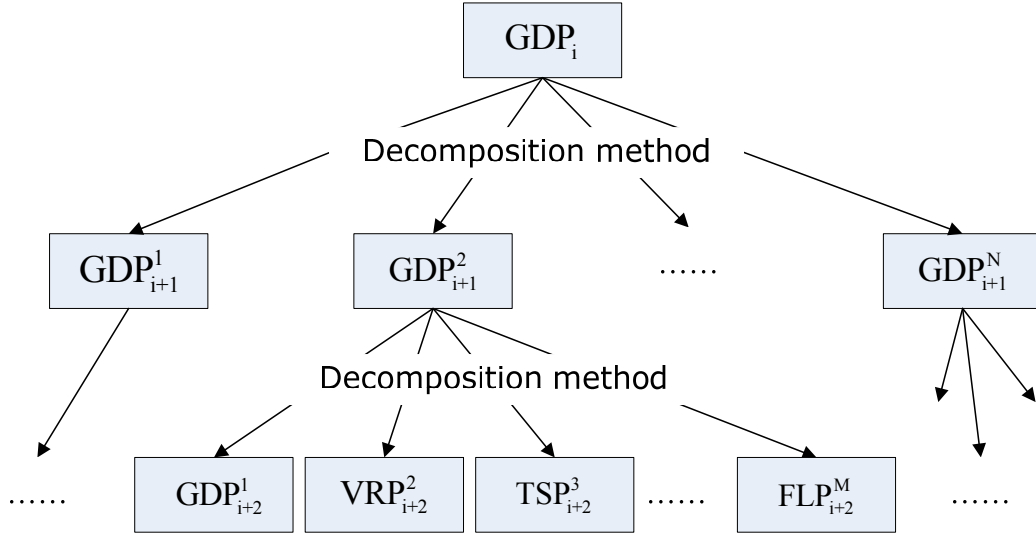
**Fig 4.1** Interaction between original problem and sub-problem

In Phase 1, the decomposition method in strategic view, tactical view and operational view has to be decided at first. Noticeably, this is fundamental. The different decomposition methods may lead to different resolutions. The description of the division methods will be presented in the Section 4.3. The large-scale problem is divided into some sub-problems by a decomposition technique with decomposition methods. The decomposition process continues until the problem is divided into smaller problems that we can solve like TP, TSP, VRP, and FLP and FAP which include general facility and hub location and allocation problems. Here, we name these sub-problems, which are widely studied and can be solved successfully when the scale of the problem is not large, as basic GDP. In fact, the number of the subproblems depends on the complexity of the problem and above all the size of the network. Fig.4.2 shows decomposition process in the Phase 1 of the proposed framework.

In Phase 2, the distribution routes in each group are determined. For TP, it is easy to solve since it is a polynomial problem. We use ILOG CPLEX, high-performance optimization software, to generate the solutions. For TSP, we get the optimal solution by efficient TSP solvers like Concorde, which can solve large-scale TSP instance up to the 15,112 cities in Germany in 2001. For other kinds of VRP, genetic algorithm and other meta-heuristic algorithm can be used. FLP and HLP can also be solved optimally by appropriately formulating.

The decomposition procedure of GDP in Phase 1 and Phase 2 is an interaction between the original problem and its sub-problems. The original problem can be transformed into several sub-problems by dividing all the nodes of the original problem into several groups

according to decomposition methods. And then each sub-problem optimizes its own routing sequence. During the improvement of the routes (Phase 3), improvement heuristics, i.e., 2-opt, 3-opt, Lin-Kernighan, or meta-heuristics like simulated annealing (SA) can be used to reform the current sequence. The sum of the objective values of all new sub-problems is returned to the original problem as a performance indicator for evaluating the current decomposition result.



**Fig. 4.2** Decomposition Process in the Phase 1 of the Proposed Framework

## 4.3 Decomposition Phase

In Phase 1, the origin and destination nodes are sorted and divided into several basic GDP according to decomposition methods. The same to operation planning process, we describe these decomposition methods in three aspects in this section: strategic view, tactical view and operational view.

### 4.3.1 Decomposition in strategic view

#### 4.3.1.1 Basic network structure

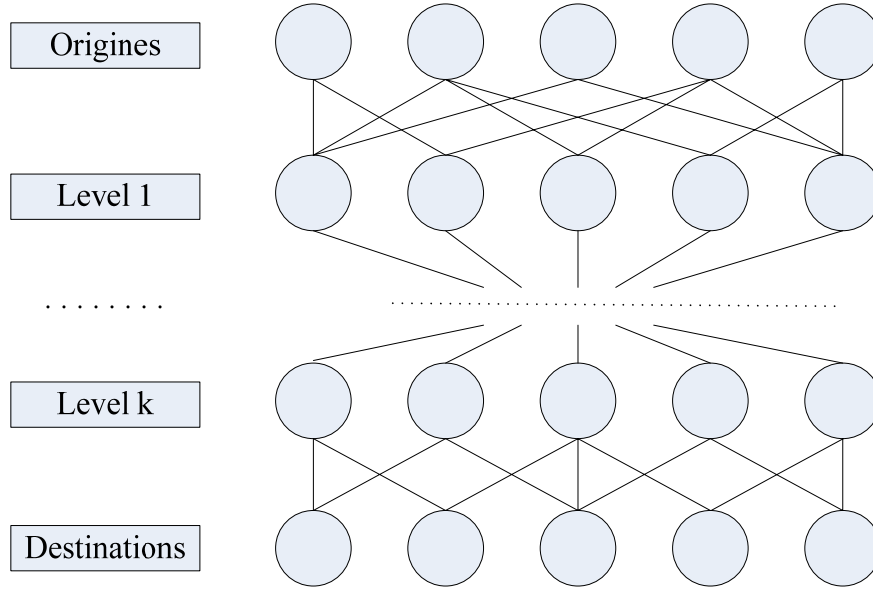
##### 4.3.1.1.1 Definition and Attributes

In chapter 3, we have introduced that the selection of the transportation network structures is a critical issue in the strategic phase in the operation planning of a company. Different network structures exist in the different transportation systems where one or several vehicles serve different customers with possibly different initial origins and final destinations, and the company provides customized service for each particular customer. It is long-lasting, impacting numerous operational and logistical decisions. Successfully selecting network structure can not only bring a high cost benefits, but also provide compelling evidence of the efficacy and practicality. It makes the decision makers select the appropriate network structure that will perform better for their distribution system.

Although there is a vast number of studies that focus on certain transportation networks with special structure and concentrate on the models and reliable algorithms for this network, a comprehensive survey reviewing all the network structures and the related researches has not caught attention of the researchers. Şahina and Süral (2007) state that the distribution systems usually can be defined as a level-by-level structure (i.e. a transportation system with different types of interacting facilities) and review the hierarchical facility location models. The authors are the first to classify the facility problems, according to four attributes. The first attribute, flow pattern, expresses the flow features of services or goods on edges between nodes of the network. The service availability and the spatial configuration of services are two other attributes which describes the vertical interaction between levels of the hierarchy. The objective of the location problem is the last attribute. Here, we use three attributes which are quite similar to the flow pattern and the spatial configuration attributes to investigate the hierarchy structure of the transportation network, and furthermore, to decompose the different structures in transportation networks.

Given a node set  $V$  and a set  $E$  of candidate edges, the transportation network is simultaneously defined as the underlying network  $G = (V, E)$ . In a broad sense, the nodes in transportation network include the origins, destinations and sometimes intermediate facilities, while the edges represent the links between nodes on which the goods are delivered. Normally, a transportation network is composed of  $k+1$  levels, in which the origins and destinations are defined as level 0 while the lowest level of intermediate facilities is named as level 1 whereas the highest level of intermediate facilities is named as level  $k$ . Notably, each level of facilities plays a specific role and a flow of goods is distributed on the edges of the network. Furthermore, we define that the flow in the networks is oriented if the goods are generally

flowed from lower level to higher level facilities. Then the transportation network can be classified as *0-level*, *1-level*, ..., *multi-level distribution network* according to the number of the intermediate facility levels. Fig. 4.3 illustrates a k-level distribution network.



**Fig. 4.3** k-level distribution network

In order to further clarify the description of the transportation network and its attributes, we define the notations as follows:

$O_G$  : Origin set of the transportation network  $G$ , in which  $o \in O_G$  is an origin node;

$D_G$  : Destination set of the transportation network  $G$ , in which  $d \in D_G$  is a destination node;

$F_G$  : The set of intermediate facilities in transportation network  $G$ ;

Especially,  $F_{Gk}$  is the set of the intermediate facilities on level  $k \in \{1, 2, \dots, K\}$ . Thus,

$$V = O_G \cup D_G \cup F_G = O_G \cup D_G \cup \bigcup_{k \in \{1, \dots, K\}} F_{Gk}.$$

$f_n \in F_G$  : An intermediate facility;

$f_{km} \in F_{Gk}$  : An intermediate facility on level  $k$ ;

$E_G$  : Edge set of the transportation network  $G$ , where  $(m, n) \in E_G$  is an edge in  $G$ ;

$g_{mn}$  : The flow on edge  $(m, n)$ ;

Then we note the routing between OD pair as  $r_{ij} = (o, f_{n_1}, f_{n_2}, \dots, f_{n_l}, d)$ . The aim of GDP is to determine a routing set  $R_G = \{r_{od} \mid \forall o \in O_G, \forall d \in D_G\}$  to minimize the total transportation cost.

Now, some network attributes of the network can be introduced.

- *Flow pattern*

The flows in the transportation network can be grouped into two types: *intra-flows* and *out-flows*. The intra-flows are the flows between the nodes of a same level of the intermediate

facilities, while the out-flows are between the intermediate facilities of different levels. Moreover, we define the following two different transportation networks:

1) If  $\exists (m, n) \in E \wedge k \in K, m \in F_{Gk} \wedge n \in F_{Gk},$  we say that there is intra-flow in the transportation network, then the network is defined as an intra-flow network;

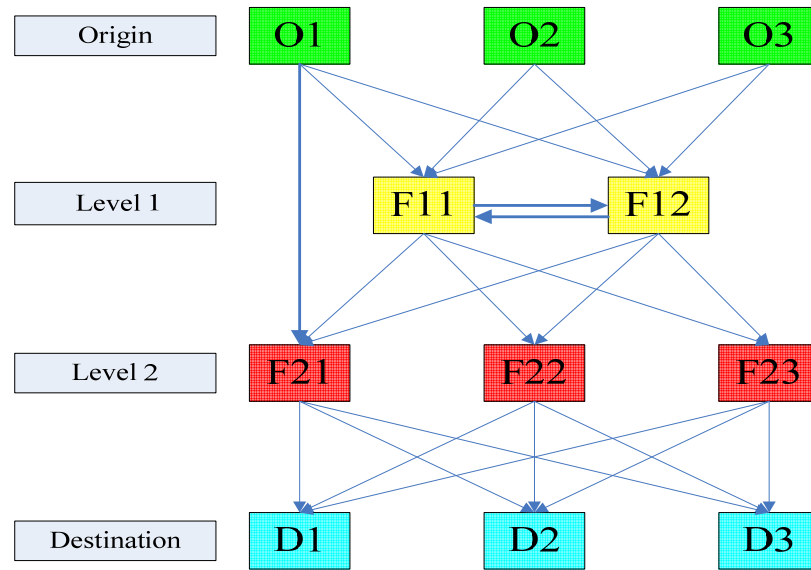
2) If there is not any intra-flow in the transportation network, then we define the network as an out-flow network.

Notably, the intra-flows are only defined for the flows between the intermediate facilities. The intra-flow network is obviously more flexible and more efficient but is more difficult to formulate and to calculate.

The out-flow can be further divided into two types: *layer-by-layer flow* and *cross-layer flow*. The definitions are presented as follows:

- 1) If the flow  $g_{mn}$  on edge  $(m, n) \in E_G, m \in F_{Gk}, n \in F_{Gk+1},$  then the flow  $g_{mn}$  is defined as layer-by-layer flow;
- 2) If the flow  $g_{mn}$  on edge  $(m, n) \in E, m \in F_{Gk}, n \in F_{Gh} \wedge h > k + 1,$  the flow  $g_{mn}$  is defined as cross-layer flow.

In Fig. 4.4, we present a 2-level distribution network, in which O1, O2, O3 are origins, F11 and F12 are the intermediate facilities on level 1, F21, F22 and F23 are the intermediate facilities on level 2, and D1, D2 and D3 are the destinations. Then the flows on the edge (F11, F12) and on the edge (F12, F11) are the intra-flow in the transportation network. The other flows in the network are out-flows. Especially, the flow on the edge (O1, F21) is a cross-layer flow while the others of the out-flows are layer-by-layer flows.



**Fig.4.4** An example for the attribute of flow pattern

Moreover, the following sets are defined for the network:

- 1) The network  $G_c = (V_c, E_c) \subset G = (V, E)$  is the cross-flow subset of  $G$  if  $E_c = \{(m, n) \in E \mid m \in F_{GK}, n \in F_{Gh}, h > k + 1, 0 \leq k < K - 1\}$  is the edge set of the cross-flow in the network, and  $V_c = \{m \in V \mid \exists n \in E_G, (m, n) \in E_c\}$  is the vertex set of the cross-flow.
- 2) The network  $\tilde{G}_c = (\tilde{V}_c, \tilde{E}_c) \subset G = (V, E)$  is the relative complement of the cross-

flow subset  $G_c$  in  $G$ , where  $\tilde{E}_c = \overline{E_c} = E \setminus E_c$  is the absolute complement of  $E_c$  in  $E$ ,  $\tilde{V}_c = \{m \in V \mid \exists n \in V, (m, n) \in \overline{E_c}\}$  is the set of the nodes which are the vertices of  $E_c$ .

The attribute of flow pattern will be used to decompose the distribution network in strategic view which is described in the next section. Especially, the sequence decomposition is proposed according to the attributes of the layer-by-layer flow and the superposition decomposition is introduced to decompose the network with cross-layer flows.

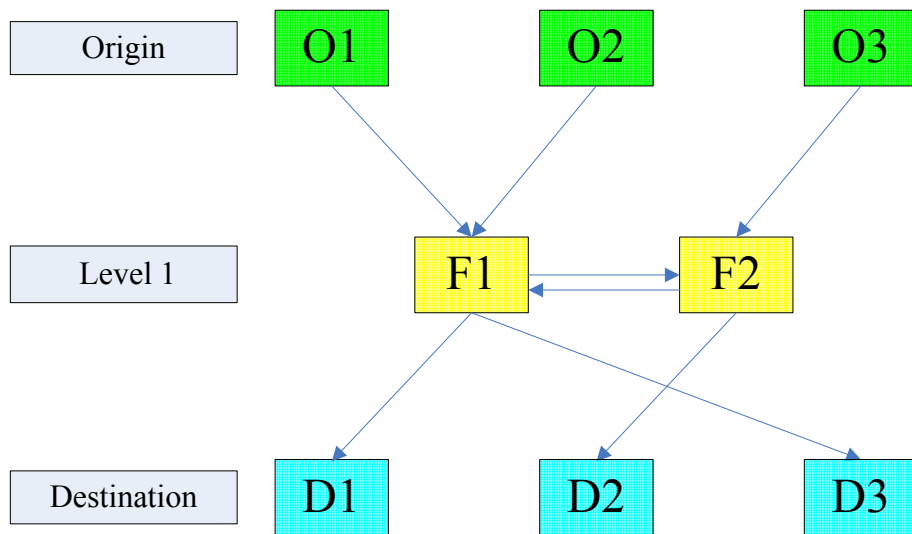
- *Spatial configuration*

Spatial configuration expresses the vertical interaction between levels. It refers to *single-allocation* or *multi-allocation* of lower-level facilities. First of all, we define  $L_n$  the set of nodes of higher-levels which are connected to the node  $n$ .

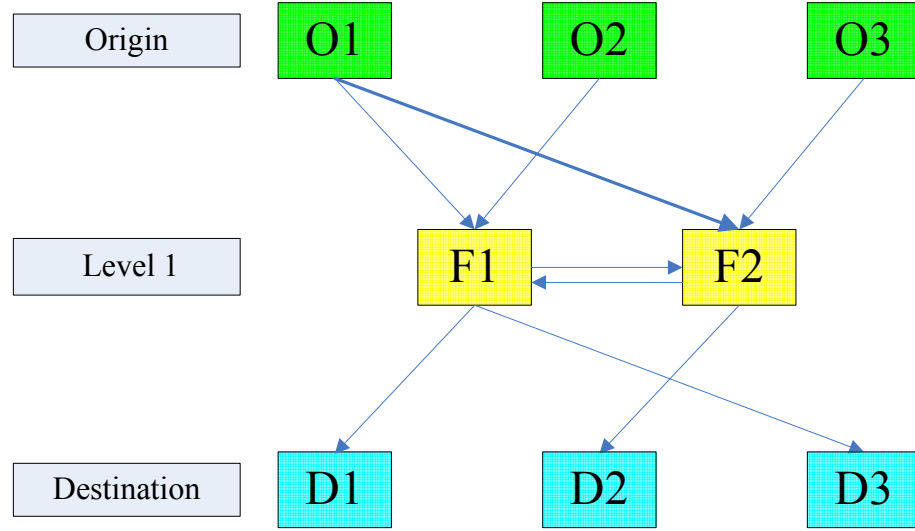
Then we define:

- 1) The network is a single-allocation system, if  $\forall n \in V, \forall m \in V, L_n \cap L_m = \emptyset$ , it is to say that any lower-level node is connected to one and only one higher-level node;
- 2) On the contrary, if  $\exists n \in F_G \wedge m \in F_G, L_n \cap L_m \neq \emptyset$ , it is to say that the lower-level node can be assigned to several higher-level nodes, we define the network as a multi-allocation system.

Fig.4.5 shows us an example of the attribution of spatial configuration in a 1-level transportation network. O1, O2 and O3 are origins, F1 and F2 are intermediate facility on level 1, and D1, D2, and D3 are destinations. The single-allocation system is illustrated in Fig. (a), in which each origin and each destination is only allocated to one intermediate facility. For example, O1 is allocated to F1, D1 is allocated to F1, and so on. On the contrary, O1 is allocated to two facilities, F1 and F2, in Fig. (b), then the transportation network is a multi-allocation system.



(a) Single-allocation system



(b) Multi-allocation system

**Fig. 4.5** An example of the attribute of spatial configuration

According to this attribute of the transportation network, we develop a location and allocation decomposition method to divide the network into smaller sub-networks, which will be introduced in the next section.

In the following, we will respectively introduce the basic 0-level network and the basic 1-level network. These structures are widely studied and can be solved successfully. And then the decomposition methods in strategic view are presented for the complex transportation networks with the combination of attributes we have mentioned above. Finally, we take some examples to show how apply the decomposition methods to divide a distribution network into the basic 0-level network and basic 1-level network in strategic phase.

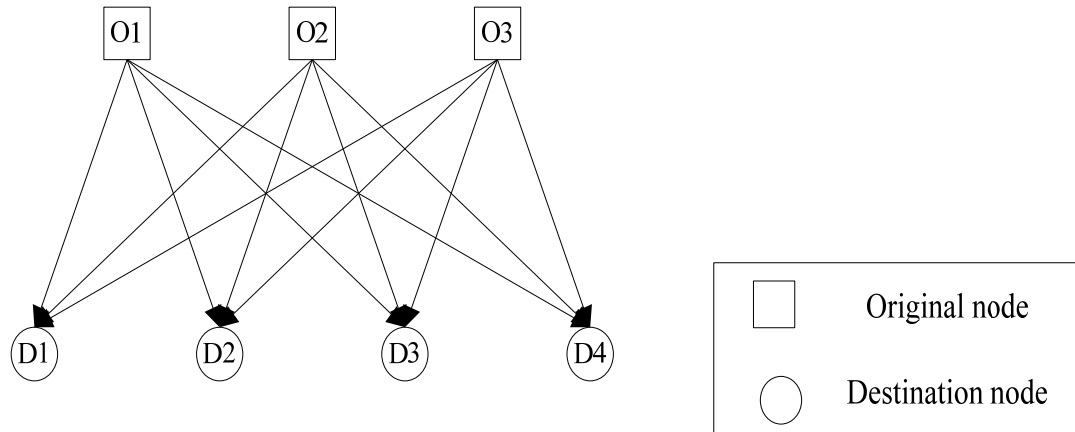
#### 4.3.1.1.2 Basic 0-level network

At first, we define a *0-level network* as a transportation network structure where there are no intermediate facilities between the original nodes and the destination nodes. It is to say that, the network  $G = (V, E)$ , in which  $V = O_G \cup D_G$ .

The 0-level network is frequently used but is not limited to the delivery problem of a local company with a small distribution scale. It is just not necessary to locate intermediate facilities to organize the delivery process. In this type of network, because there is just one level (level 0) in the network, thus the flows can only be from the origins to the destinations in level 0. Then, intra-flows and out-flows do not exist since these attributes are defined for intermediate facilities. Then, single-allocation means  $\forall o \in O_G \wedge d \in D_G, L_n \cap L_m = \phi$ , that is to say that any origin is assigned to one and only one destination. On the other side, multi-allocation implies  $\exists o \in O_G \wedge d \in O_G, L_n \cap L_m \neq \phi$ , that is to say the origins can be assigned to multiple destinations. There are many types of 0-level networks in the real world. Here, we just want to introduce the three basic 0-level distribution networks involving the network structures of TP, TSP and VRP.

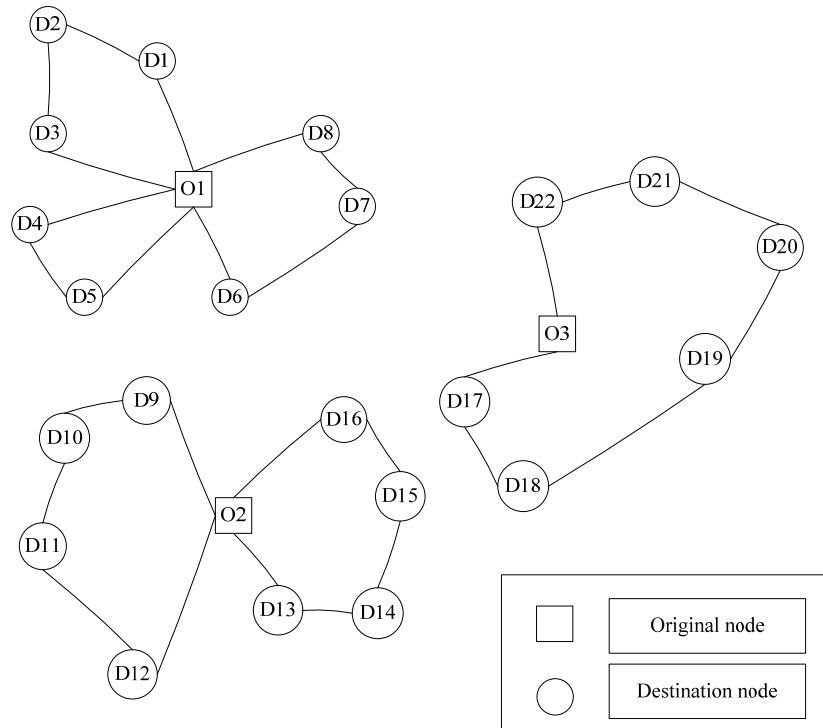
*The traditional transportation problem (TP)* is a typical example of 0-level network, the structure of which is noted as Basic Graph  $BG_{01}$ . It is a programming problem that is concerned with the optimal pattern of the goods distribution from several origins to several

different destinations, with the specified requirements at each destination and it can be solved successfully by simplex method. Fig. 4.6 shows us a TP in which three origins named as O1, O2 and O3 supply the goods to four destinations say D1, D2, D3 and D4. It is multi-allocation system since the destinations are connected to several origins, (for example, the node D1 is connected to O1, O2 and O3).



**Fig. 4.6** Basic Graph  $BG_{01}$

The other basic 0-level networks can be found in *TSP*, *VRP* and generally applied for all the small-scale distribution problems. Here we note the structure of *TSP* and *VRP* as Basic Graph  $BG_{02}$ . As *TSP* is a special case of *VRP*, we just introduce *VRP* here. Often the context is to deliver goods located at one or several central depots (origins) to customers (destinations) who have placed orders for such goods. It is a single-allocation transportation system shown in Fig. 4.7 below. Different to traditional TP, *VRP* is a single-allocation problem since a destination is assigned to only one origin. For instance, the destination D1, D2, and D8 are just assigned to O1.



**Fig. 4.7** Basic Graph  $BG_{02}$

#### 4.3.1.1.3 Basic 1-level network

A *1-level network* is concerned with the distribution network structure with just one level of intermediate facility (level 1) and the origin and destination level (level 0) in the network. It is to say that, the network  $G = (V, E)$ , in which  $K=1, F_G = F_{G1} \wedge V = O_G \cup D_G \cup F_G = O_G \cup D_G \cup F_{G1}$ .

The 0-level distribution network is not always the appropriate answer to customize transportation needs when the transportation distance is long or the scale is large. The relations and trade-offs, on one hand, between volume and frequency of the transportation, and on the other hand, the cost, frequency and delivery time of transportation, often denote the necessity of consolidation transportation services, which means that consolidated facilities will be located in the network.

The 1-level network is usually applied but is not limited to the distribution problem with median and even large transportation scale whose distribution scale and the number of OD pairs is normally larger than 0-level network. In practice, 1-level distribution network is performed by Less-than-truckload (LTL) motor carriers, railways, ocean shipping lines, express postal services, etc. Furthermore, a 1-level network is influenced by the combination of the attributes: flow pattern including the intra-flow and out-flow, and spatial configuration.

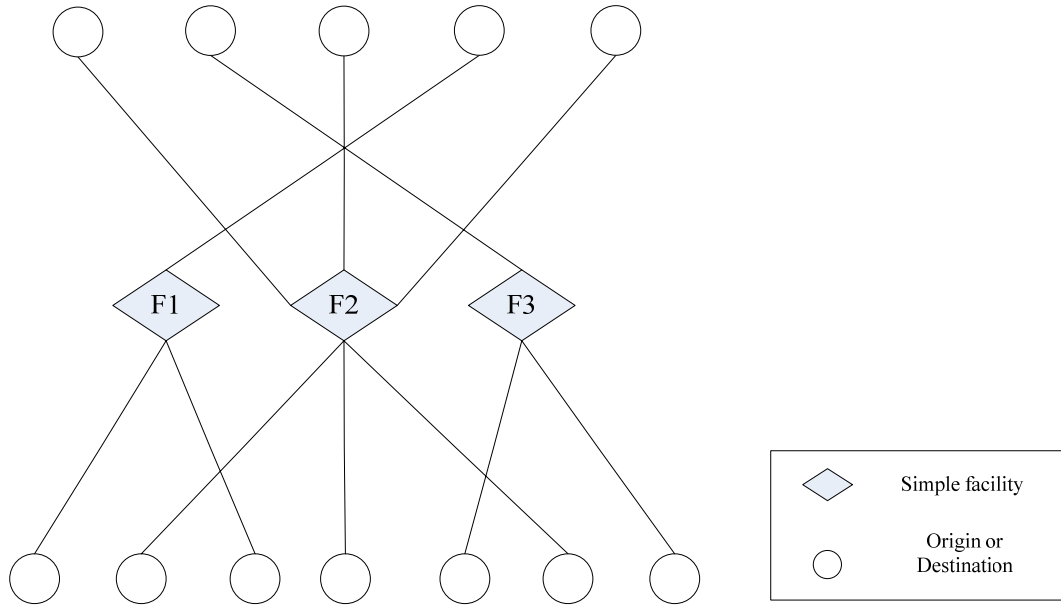
According to attributes of flow pattern, actually intra-flow, there exist two types of intermediate facility: simple facilities and hubs. Here, we define the following notations to represent them.

1)  $s_{km}$ : simple facility on level k if  $s_{km} \in S_{Gk} = \{s_{km} \in F_{Gk} | (s_{km}, s'_{km}) \notin E, \forall s'_{km} \in F_{Gk}\}$ . It is to say that simple facilities do not exchange the goods with the other intermediate facilities in the same level. Especially, we define  $S_{Gk}$  as the simple facility level k at network G;

2)  $h_{km}$ : hubs on level k if  $h_{km} \in H_{Gk} = \{h_{km} \in F_{Gk} | \exists h'_{km} \in F_{Gk}, (h_{km}, h'_{km}) \in E\}$ . Here  $H_{Gk}$  is named as hub level in which the intermediate facility level k including at least one hub  $h_{km}$ . It is to say that hubs are the special facilities exchanging the goods with the other intermediate facilities on the same level. It can also serve as switching, transshipment and consolidating points in many distribution systems.

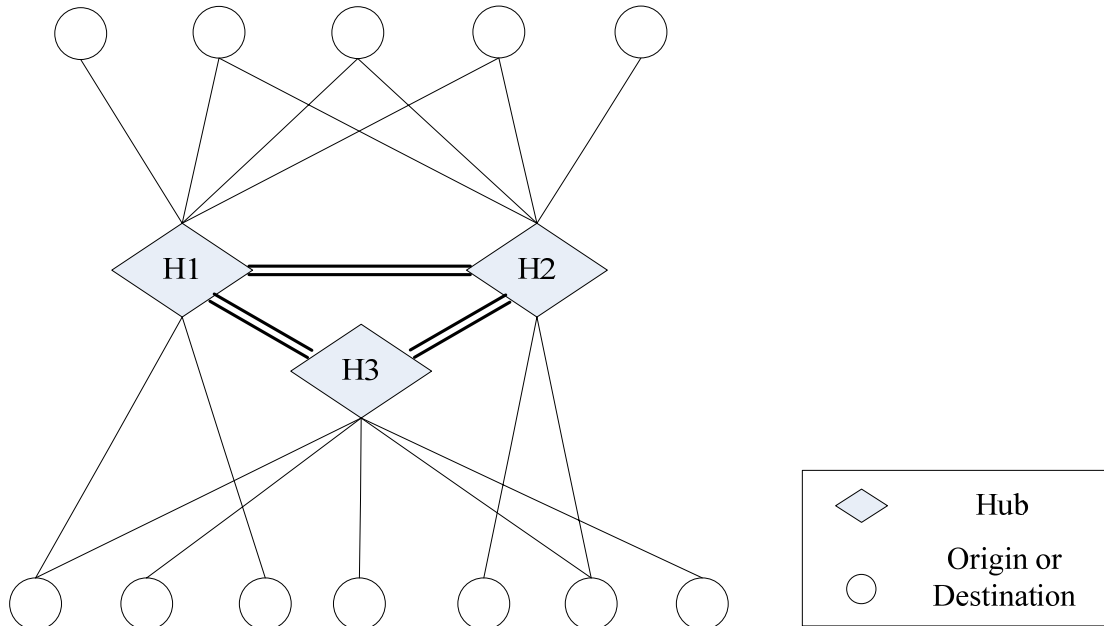
To further clarify, the simple facilities are the intermediate facilities without the function of intra-flow between them, while the goods can pass through the hubs on the same level. In our thesis, the intermediate facility is sometimes called as facility for short if it does not lead to ambiguity. In the following, we will introduce two kinds of basic 1-level network, a *simple 1-level network* (noted with  $BG_{11}$ ) and a *hub 1-level network* (noted with  $BG_{12}$ ).

The *simple 1-level network* is a 1-level network with only simple facilities. That is,  $BG_{11} = (V, E)$  where  $V = O_G \cup D_G \cup F_G = O_G \cup D_G \cup F_{BG1} = O_G \cup D_G \cup S_{BG1}$ . Fig.4.8 illustrates an example of  $BG_{11}$  in which the origins and destinations are marked with white circlets and the lozenges represent the simple facilities. The goods are firstly shipped from origins to facilities. The goods are consolidated and resorted and then delivered to their destinations. In practice, this network structure is used by the company which confronts a mediate or large transport problem. The simple facilities are usually located close either to the origins or to the destinations to better organize the delivery.



**Fig.4.8** an example of simple 1-level network  $BG_{11}$

Another basic 1-level distribution network is the so-called *hub 1-level network* in which the intermediate facilities known as hubs exchange the products with other intermediate facilities in the same level. That is,  $BG_{12} = (V, E)$  where  $V = O_G \cup D_G \cup F_G \wedge F_G = H_{G1}$ . A simple example of  $BG_{12}$  is illustrated in Fig. 4.9. In such a system, low-volume demands are moved first to the hubs (nodes H1, H2, H3 in Fig.4.9) such as airports, seaports, container terminals, rail yards, or platforms. Then the goods are switched, transshipped and consolidated into larger flows that are sometimes routed to other hubs by high-frequency, high-capacity services (thick and parallel lines in Fig. 4.9). Notice that we do not distinguish the destination nodes and origin nodes, both of which are marked with circles and in this case, the distribution between OD pairs performing as either pickup or distribution for customized services.



**Fig.4.9** A simple example of hub 1-level network  $BG_{12}$

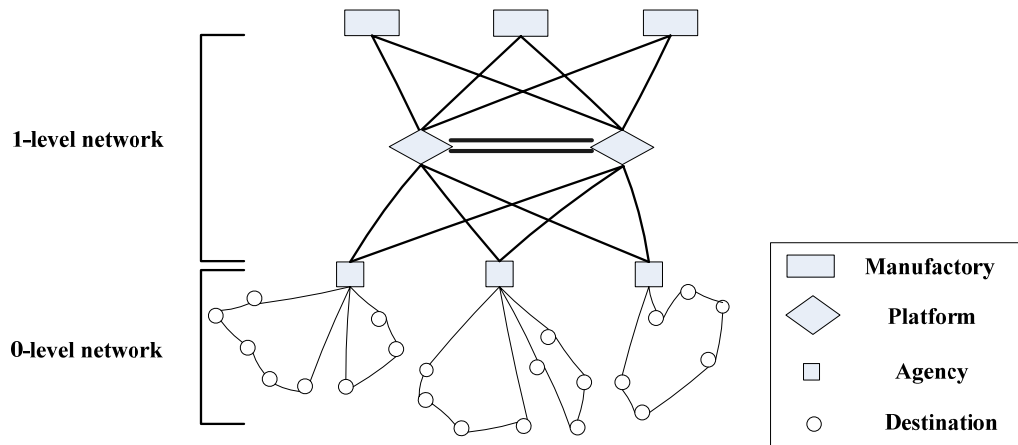
### 4.3.1.2 Decomposition method

#### 4.3.1.2.1 Sequence decomposition

Sequence decomposition is focused on dividing multi-level network into the basic network. Multi-level distribution network occurs when the products are delivered from origins to destinations through two or more than two levels of intermediate facilities, i.e. a multi-level network  $G = (V, E)$ ,  $V = O_G \cup D_G \cup F_G = O_G \cup D_G \cup \bigcup_{k \in \{1, \dots, K\}} F_{Gk} \wedge K \geq 2$ . Moreover,  $F_{Gk}$  can also be either a simple facility level  $S_{Gk}$  or a hub level  $H_{Gk}$ . Normally, the transportation scale of multi-level network is larger than 1-level network and 0-level network.

*Sequence decomposition* is to determine a set of intermediate facility layers  $\{F_{Gk_1}, F_{Gk_2}, \dots, F_{Gk_R}\} \subset N$  to divide sequentially the original network into a set of sub-networks  $\{G_0, G_1, \dots, G_r, \dots, G_R\}$ . Then,  $G = \bigcup_{0 \leq r \leq R} G_r$ , where  $G_0 = (V_0, E_0)$ ,  $V_0 = O_G \cup \bigcup_{0 \leq k \leq k_1} F_{Gk}$ ;  $G_l = (V_l, E_l)$ ,  $V_l = \bigcup_{k_r \leq k \leq k_{r+1}} F_{Gk}$  for  $\forall 1 \leq l \leq L-1$ ;  $G_R = (V_R, E_R)$ ,  $V_R = \bigcup_{k_{R-1} \leq k \leq k_R} F_{Gk} \cup D_G$ . And  $E_r = \{(m, n) \mid m, n \in V_r \wedge (m, n) \in E\}$  for  $\forall 0 \leq r \leq R$ .

We take a 2-level manufacturing transportation network (see in Fig. 4.10) as an example to show how the sequence decomposition divides the network. In the system, the company needs to organize the good (clothing, device, etc.) distribution. Firstly, the trips of transportation is starting from the manufactories (Rectangles in the Fig.4.10) on level 0, the goods are sent to the platforms (intermediate facilities on level 1 marked with diamonds in Fig. 4.10), where the goods are resorted, consolidated and exchanged between the platforms. Then, the goods are transmitted to the agencies (intermediate facilities on level 2 marked with square in Fig. 4.10). The goods are finally distributed to their destinations (marked with circles in the Fig. 4.10) through the agencies in their regions.



**Fig. 4.10** A 2-level manufacturing transportation network

If we try to study a multi-level distribution network as a whole system, we obtain a complex formulation with a great number of variables and constraints even for a 2-level distribution network. The studies on this type of transportation network focus on the heuristic approach to provide the transportation planning. After analyzing the network structure, we can apply a sequence decomposition to divide this 2-level transportation network into a combination of two types of smaller networks, including a hub 1-level network and several 0-

level networks. The combination (see in Fig. 4.10) regards the origins on level 0, the facilities on level 1, the facilities on level 2 and the links between these nodes as a 1-level distribution network, and regards the facilities on level 2, the destinations, and the links between the nodes as several 0-level distribution networks.

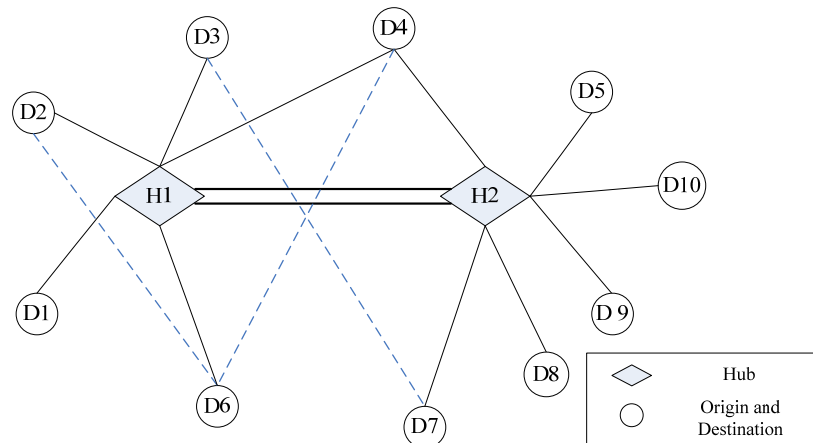
As we can see, the sequence decomposition is an efficient method to decompose the network into smaller ones. In this way, the original problem is simplified into several sub-problems. However, it does not work when there is cross-layer flow in the network. In order to overcome this shortcoming, we introduce superposition decomposition, another decomposition method in the next subsection.

#### 4.3.1.2.2 Superposition decomposition

*Superposition decomposition* primarily deals with the transportation network with cross-layer flow. In the distribution network, the flows generally start from the origins and end at destinations passing through the intermediate facility level by level. However, the cross-layer flow sometimes exists in the network because of the special demands of the nodes (the destination or the intermediate facility), such as the huge demand quantities, the transportation time, the transportation mode, etc. A network with cross-layer flow allows a much higher quantity service in the network and more efficient utilization of the resources. However, it is difficult to solve for the operational research because the cross-flow often destroy the structure of the constraint matrix. In this case, the superposition decomposition heuristic can be applied.

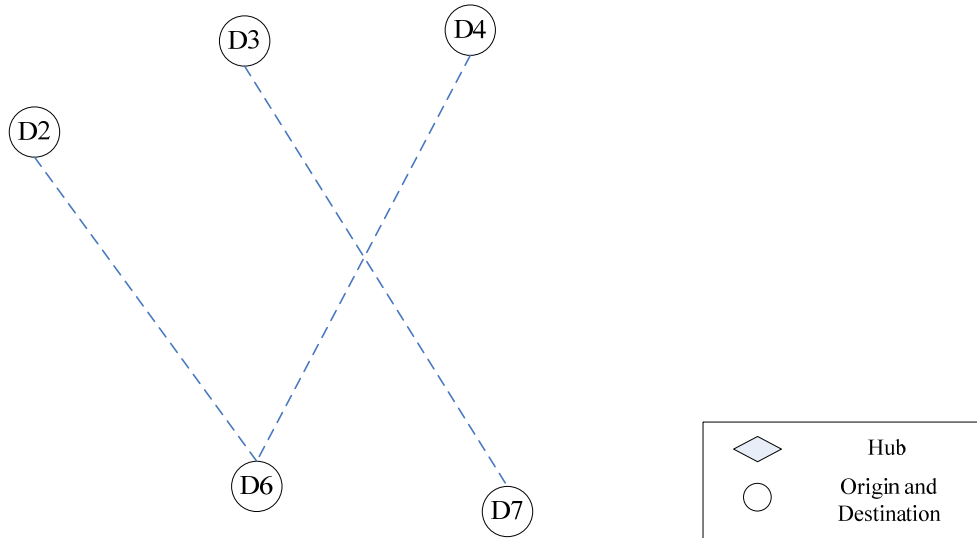
In a network  $G = (V, E)$ , the cross-flow set is  $G_c$ ,  $\tilde{G}_c$  is the relative complement of cross-flow subset  $G_c$  in  $G$ . The *superposition decomposition* is to divide the network  $G$  into two sub-networks  $G_c$  and  $\tilde{G}_c$ . Moreover,  $G = G_c \cup \tilde{G}_c$ .

An example of the distribution network with cross-layer flow is the so-called hub-and-spoke network illustrated in Fig. 4.11, in which the intermediate facilities known as hubs exchange the goods with other intermediate facilities on the same level. In such a system, low-volume demands are moved first from origins to the hubs (nodes H1, H2 in Fig.4.11). Then the goods are switched, transshipped and consolidated into larger flows that are sometimes routed to other hubs by high-frequency, high-capacity services (thick and parallel lines in Fig. 4.11). At last, the goods are shipped to the destinations. Both of the destination nodes and origin nodes are marked with circles. Furthermore, there exist several special links marked with blue lines (D2-D6, D3-D7, and D4-D6) in the Fig. 4.11 which directly connect three OD pairs.

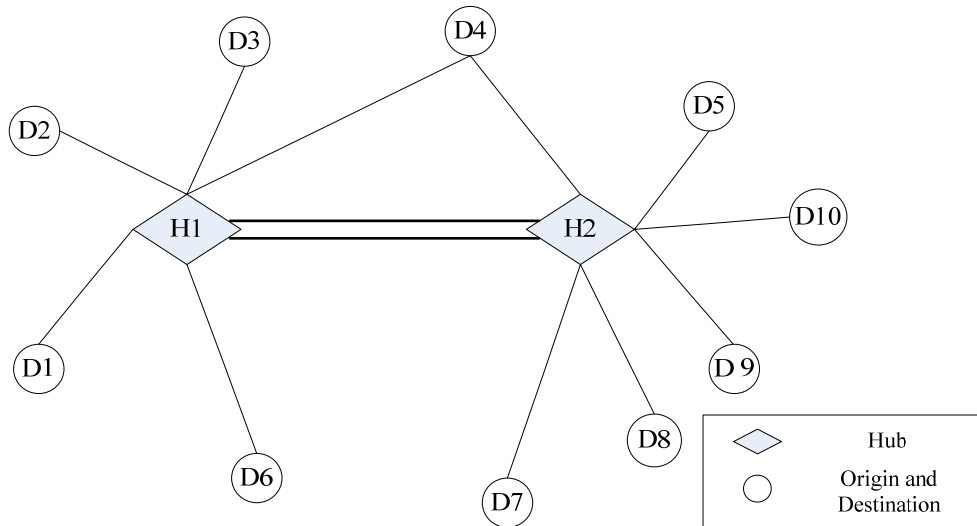


**Fig. 4.11** A hub-and-spoke network

As we have mentioned above, the hub-and-spoke network is difficult to solve with operation research method like all other distribution network with cross-layer flow. Thus, the superposition decomposition method is used to simplify the network structure. Then, the hub-and-spoke transportation network (1-level network) can be decomposed into one basic 0-level distribution network (see in Fig. 4.12 (a)) composed of D2, D3, D4, D6, D7, and one basic 1-level distribution network (see in Fig. 4.12 (b)). After decomposition, the OD pairs directly connected are considered as a new typical TP which can be successfully solved. On the other hand, the transportation problem between the other OD pairs without the direct link can be regarded as a basic 1-level distribution network.



(a) Basic 0-level distribution network



(b) Basic 1-level distribution networks

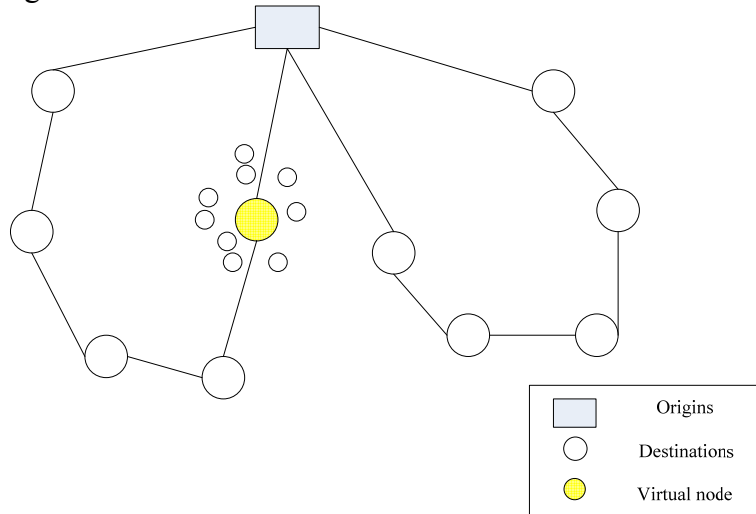
**Fig. 4.12** A superposition decomposition example of hub-and-spoke transportation network

#### 4.3.1.2.3 Aggregation decomposition

There are two aspects that lead the GDP to be difficult to solve: the network structure and the scale of the network. The two decomposition methods mentioned above are mainly focused on handling with the complex network structure, while the aggregation decomposition is concerned with the network with a large number and high density of nodes.

In a network  $G = (V, E)$ , we define that the  $R$  groups of large number and high density of nodes as  $GP_1, GP_2, \dots, GP_R$ . Then the node set  $V = \bigcup_{1 \leq r \leq R} GP_r \cup V_{rest}$ , where the relative complement of  $\bigcup_{1 \leq r \leq R} GP_r$  in node set  $V$ . The *aggregation decomposition* firstly regards a group of large number and high density nodes as a large demand node then solve the new network problem with the new large demand nodes, i.e. it firstly sets  $GP_r \rightarrow LV_r$ , for  $\forall 1 \leq r \leq R$ . Here,  $LV_r$  represents the large demand node. Thus,  $V \rightarrow V' = \bigcup_{1 \leq r \leq R} LV_r \cup V_{rest}$  and then the corresponding edge set  $E'$  are determined. Thereby,  $G \rightarrow G' = (V', E')$ .

The Fig. 4.13 shows an aggregation decomposition example of 0-level distribution network. The origin (marked with square in the Fig. 4.13) distributes the goods to the destinations represented by white circles while the destinations with small demand are represented with smaller white circles. Then, these small demand nodes are served as a huge demand virtual node (the yellow node) with the synthesis restriction of the small demand nodes. Then the routing sequence can be provided for the whole distribution network illustrated in the Fig. 4.13.



**Fig. 4.13** An aggregation decomposition example of 0-level distribution network

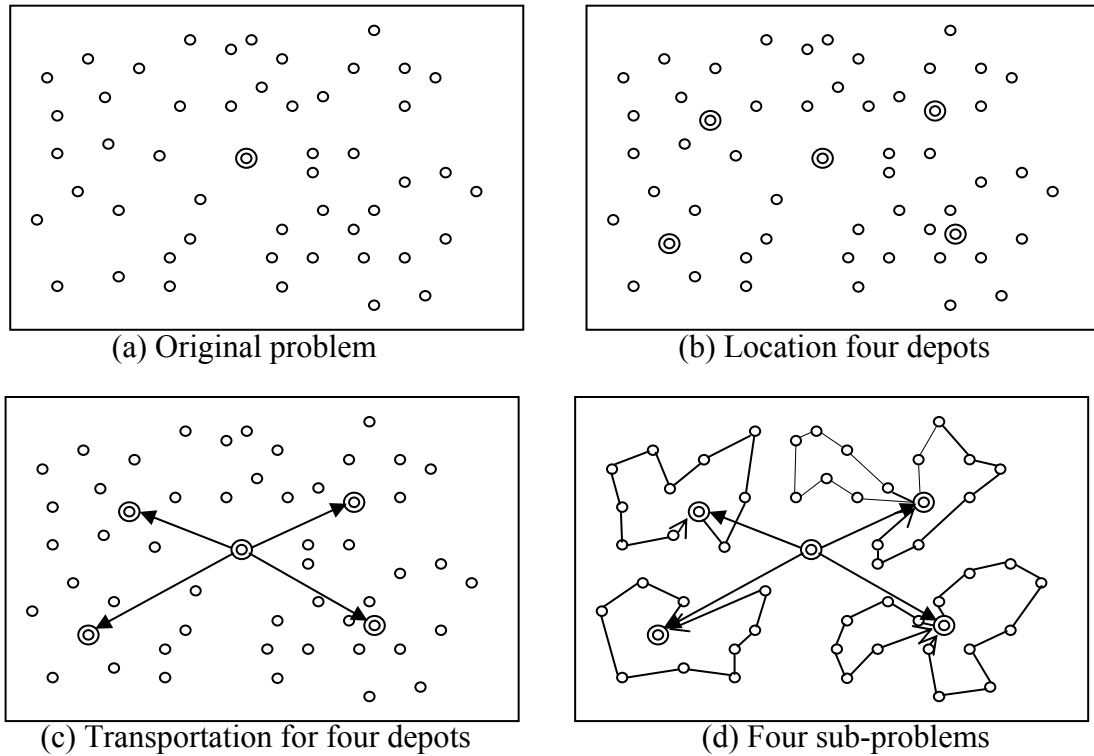
#### 4.3.1.2.4 Facility location and allocation decomposition

Facility location problem is a critical issue in strategic phase, which we have presented in Chapter 3. It is to physically locate a set of intermediate facilities (resources) in order to minimize the cost subject to some set of constraints. After locating the facilities in the distribution network, the origins and the destinations are then needed to be assigned to the one or several facilities. This process is known as facility allocation problem. In fact, the facility location and allocation problem is another method to decompose the distribution network into smaller sub-problems.

We have defined that  $L_n$  was the collection of node  $n$  and lower-level nodes which are

assigned to node  $n$ . The *facility location and allocation decomposition* is firstly to locate several nodes  $\{n_1, n_2, \dots, n_R\}$ , then the corresponding  $L_{n_r}$ , for  $1 \leq r \leq R$  is determined by allocation problem. We define  $G_{n_r} = (L_{n_r}, E_{n_r})$  as new sub-networks of  $G$ , where  $E_{n_r} = \{(e_1, e_2) | e_1 \in L_{n_r} \wedge e_2 \in L_{n_r}\}$ ,  $1 \leq r \leq R$  is the edge set of  $G_{n_r}$ . Let  $E' = \overline{\bigcup_{1 \leq r \leq R} E_{n_r}}$  be the relative complement of  $\bigcup_{1 \leq r \leq R} E_{n_r}$  in  $E$  and let the node set of  $E'$  be  $V' = \{n \in V | \exists m \in V, (n, m) \in E'\}$ . Let  $G' = (V', E')$ , thus  $G = \bigcup_{1 \leq r \leq R} G_{n_r} \cup G'$ .

In order to clearly explain facility location and allocation decomposition method, we take a 0-level distribution network as a simple example showed in Fig. 4.14. In the Fig. 4.14, the ring represents the origin and the other points are destinations. Firstly, the four other depots (simple facility) are located, and then the origin delivers goods to the four distribution centers. At last, the facility allocation has divided the original problem into four sub-problems (TSP or VRP) for each of the 4 depots.

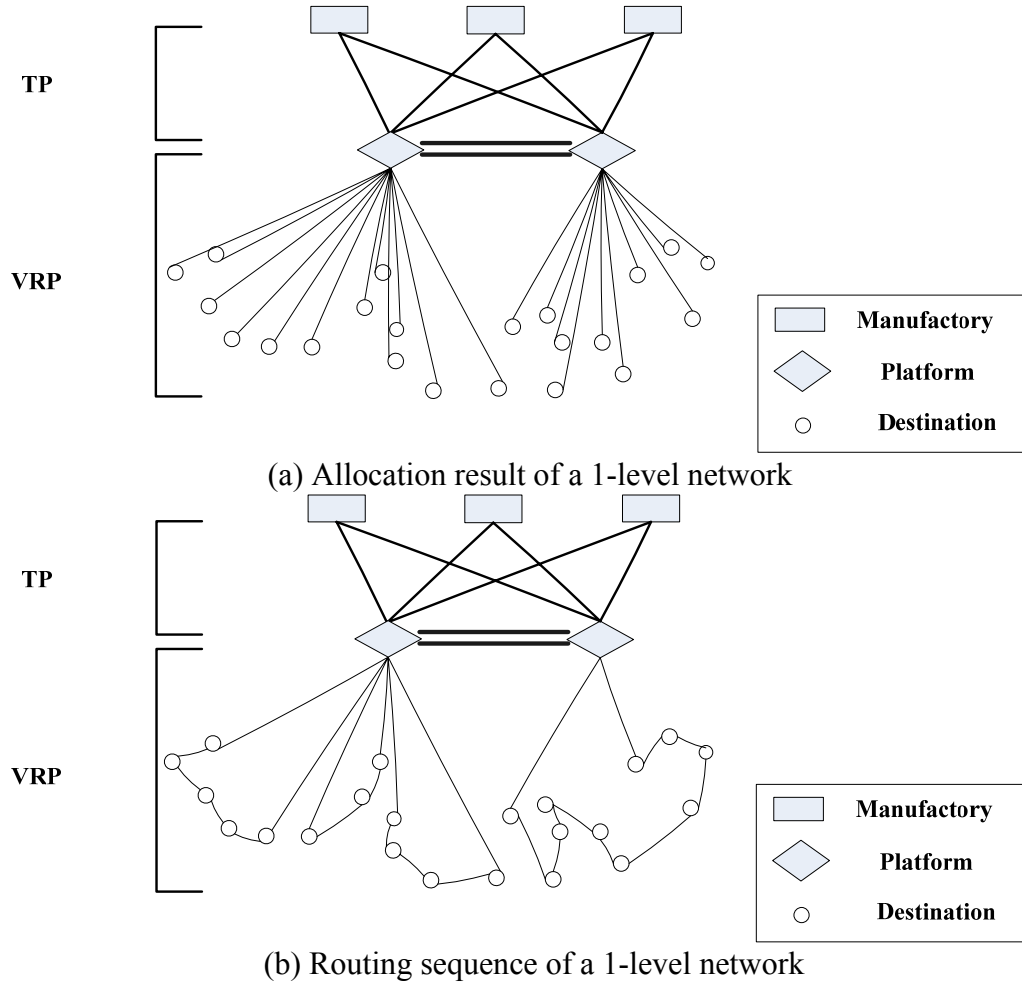


**Fig. 4.14** An example of the facility location and allocation problem in Strategic view

In the real world, there is a type of 1-level network in which the transportation between origins and intermediate facilities is a traditional TP, in the other side, the transportation between intermediate facilities and destinations is somewhat like VRP, and sometimes the intermediate facilities can exchange the goods between.

This is a new structure we have not yet mentioned before. Undoubtedly, it is also difficult to solve with OR because VRP, a sub-problem of the original problem is already hard to solve. Then, the facility location and allocation decomposition is applied to successfully decompose the type of distribution network. Firstly, the routings in VRP are not considered. The distribution network can be regarded as the basic 1-level distribution network  $G_{11}$ . The corresponding hub location and allocation problems are widely studied and there exist a large

number of methods to solve it. After solving the hub location and allocation problem, each destination is assigned to one or several hubs and the transportation quantities between the hub and the destination, between the hubs, and between origins and hubs can also be determined (see in Fig. 4.15 (a)). After that, the transportation between hubs and destinations is solved as a set of smaller VRPs, each of which is composed of a hub as the origin, the destinations assigned to the hub and the links between. The routing sequence (see in Fig. 4.15 (b)) is given out by solving these VRP.



**Fig. 4.15** Facility location and allocation decomposition a special 1-level network in Strategic view

Note that facility allocation can be also applied in tactical phase and in operational phase. In tactical phase, this decomposition method is somewhat similar with the allocation in strategic view. However, the facility allocation as a decomposition method in operational view is different with the facility allocation in the two previous phases in the objective formulation because the quantity of resources is limited. In this case, the number of vehicles, the transportation time and the transportation distance should be restricted when we determine the facility allocations.

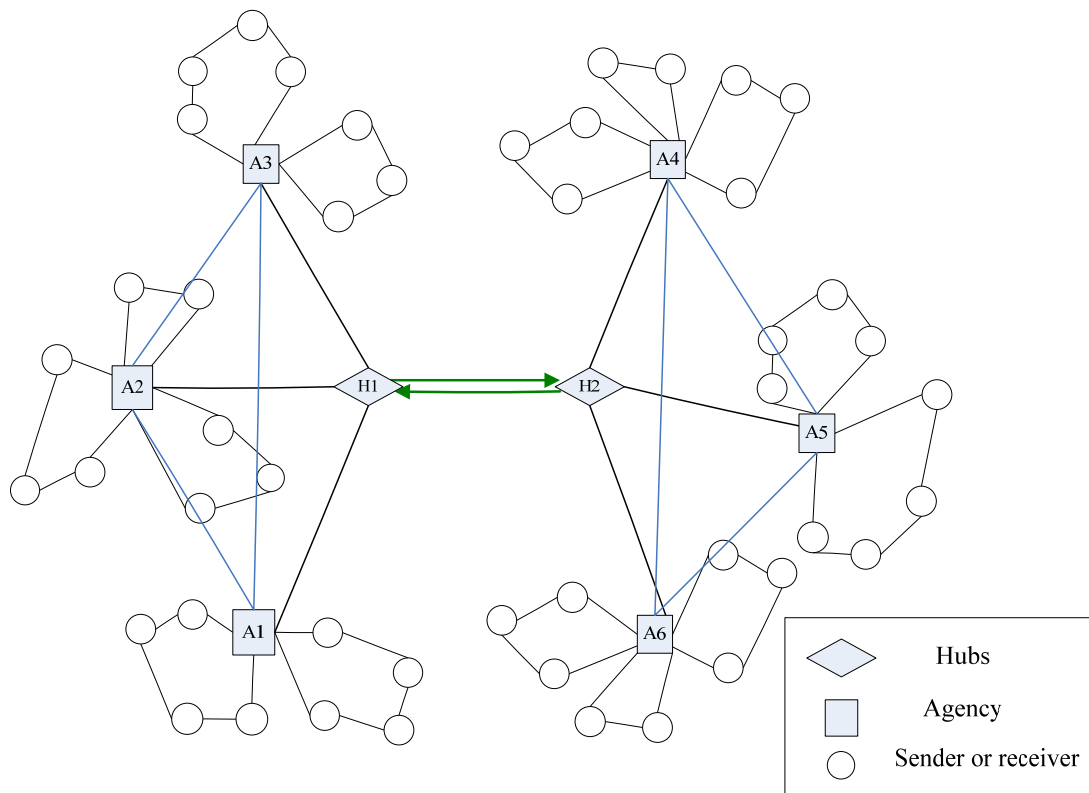
#### 4.3.1.3 A decomposition example in strategic view

In the previous subsection, we have separately introduced four decomposition methods in strategic view to divide the GDP into several smaller problems with basic network structure which we can solve efficiently. To further explain the multi-level distribution network, we

take an international express delivery network (see in Fig. 4.16) as an example to show how to use together these methods to divide the huge GDP. Note that we do not describe the oriented flows in Fig. 4.16.

In the system, the company needs to organize the worldwide and nationwide good (letters and bundles) distribution. In this case, the company usually uses a multi-level distribution network, which is composed of the agencies (simple facilities marked with squares in Fig. 4.16), hubs (hub facilities marked with diamonds in Fig. 4.16) and senders and receivers (origins and destinations on level 0). There exist indeed three different situations of delivery:

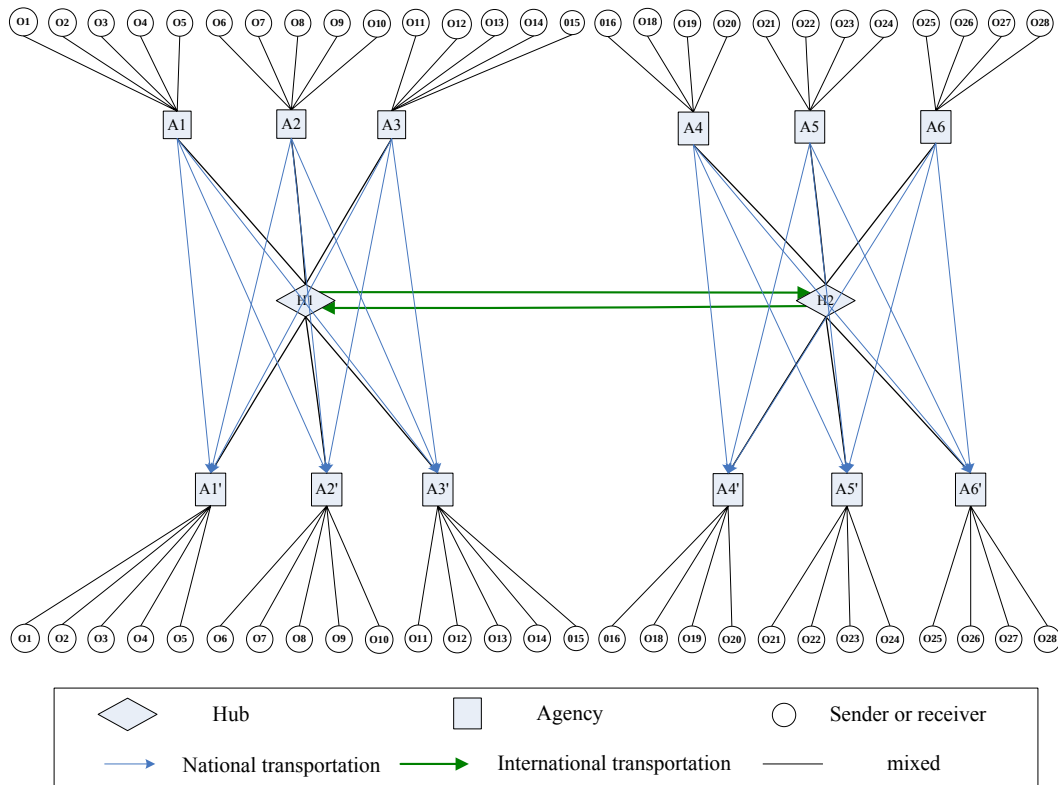
- 1) Firstly, the goods are directly delivered to the customers which are assigned to the same agency with the good sender;
- 2) Secondly, the goods are delivered to the customers belonged to another agency in the same country;
- 3) Thirdly, the goods are exchanged between the hubs and finally transmitted to the hubs in the country of their destination, then shipped to the agencies they belong to.



**Fig. 4.16** A multi-level international express post transportation network

In the first case, the goods to send are consolidated with the other goods, which arrive at the agency and are directly delivered to the customers assigned to the same agency; In the second case, the goods are firstly delivered from senders to their agency, and then they are either directly delivered to the agencies of their destination or to the hub where they are sorted again and consolidated, and then delivered to the agency of their destinations. In the third case, the transportation trip is from the senders on level 0, the goods are sent to their agencies, where the goods are sorted again, consolidated and then delivered to the hubs of its own country and then exchange between the hubs, at last are shipped from the hubs in the destination country to the agencies of the receivers.

It is one of the simplest examples of multi-level distribution networks taking play in many applications such as airlines and other carriers. It is very difficult to formulate for multi-level distribution network, and the studies on this type of transportation network focus on the heuristic approach to give out the transportation planning. Before applying our decomposition methods to divide the network, we need to analyze the network structure. Note that, in this network, the origin nodes are at the same time the destinations. In order to clearly explain, we regard them as two nodes with different performances but with the same locations (see in Fig. 4.17). Furthermore, they are illustrated in the two sides of the distribution network. Similarly, each agency is considered as two different nodes in the network with the different performances: good reception from origins and transportation, good reception and transportation to destinations. Then, we name  $A_r$  the agencies with the performance of good reception from origins and transportation while the agencies with the performance of good reception and transportation to destinations are named  $A'_r$ . Then the flows in the network become 3-level distribution network with orient flows (defined as network G) consist of agency  $A_r$  (Level 1), hubs (Level 2), agency  $A'_r$  (Level 3) and origins and destinations (Level 0). Moreover, the arcs in green bold parallel lines represent the hub arcs used to flow the goods between hubs, the blue arcs represent the national transportation directly from one agency to another agency, while the other black lines are the transportation mixed the national transportation and international transportation together. Particularly, the flow cost (maybe time, distance, transportation cost, etc.) from  $A_r$  to  $A'_r$ ,  $1 \leq r \leq R$  is zero, because they are in fact the same node and we do not consider the terminal cost in our thesis.



**Fig. 4.17** A multi-level international express post transportation network

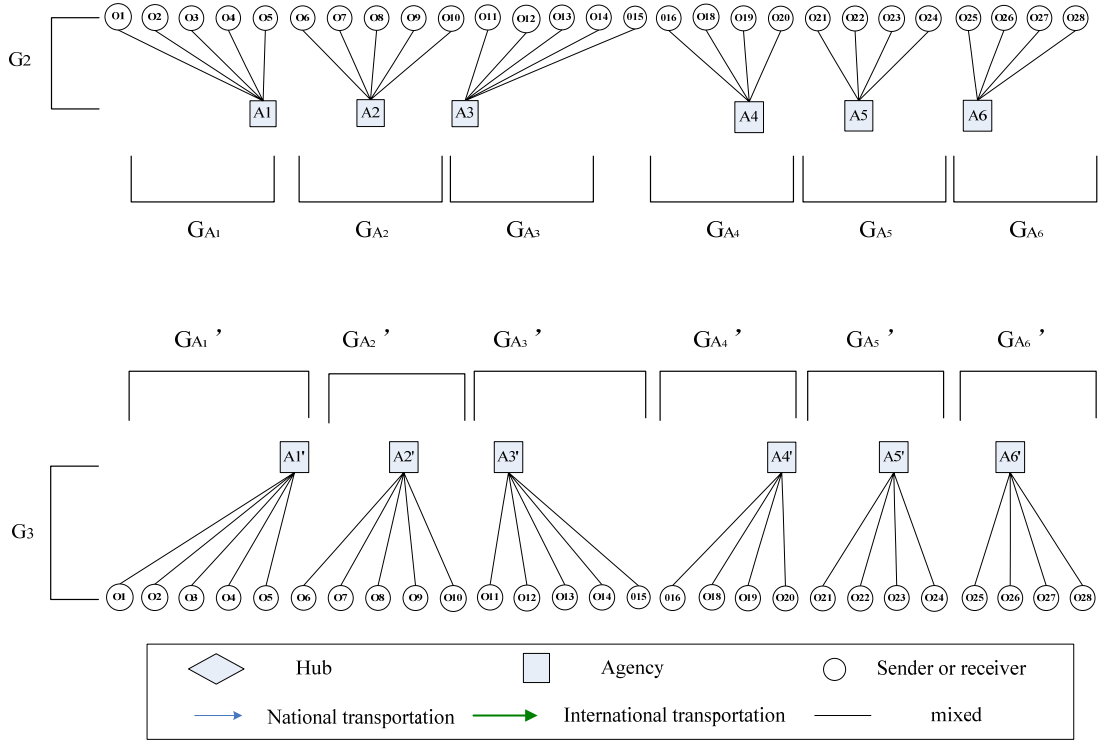
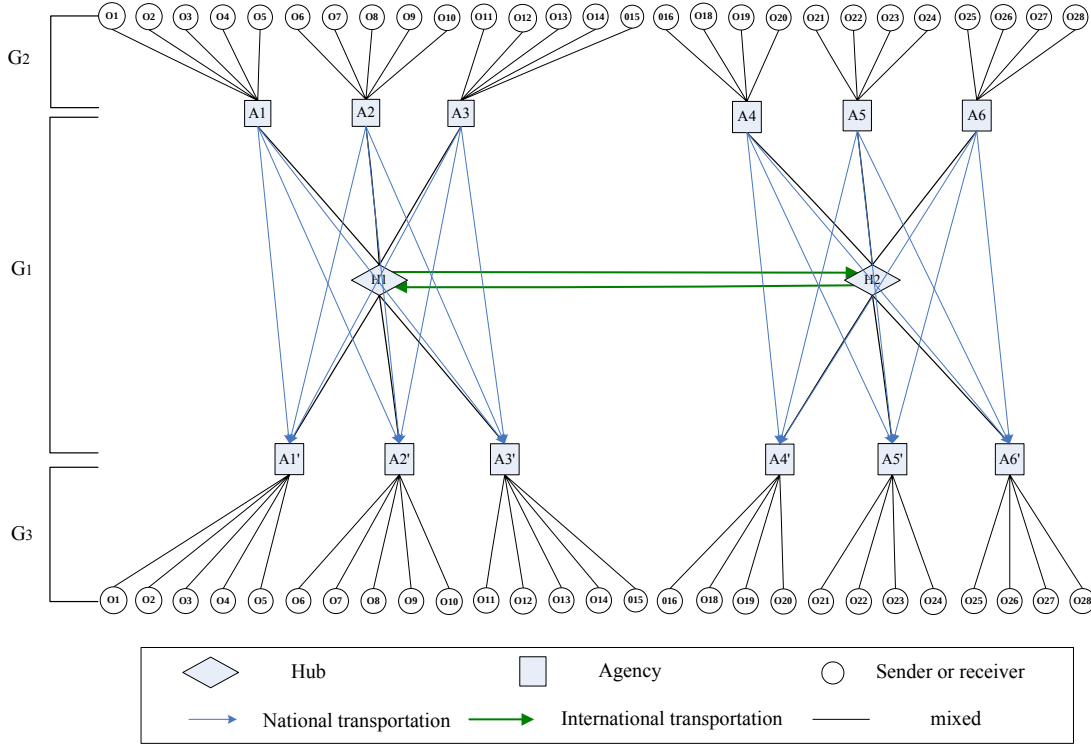
After analyzing the network structure, the decomposition methods in strategic view are applied to divide the origin network G into several smaller sub-problems. Firstly, sequence

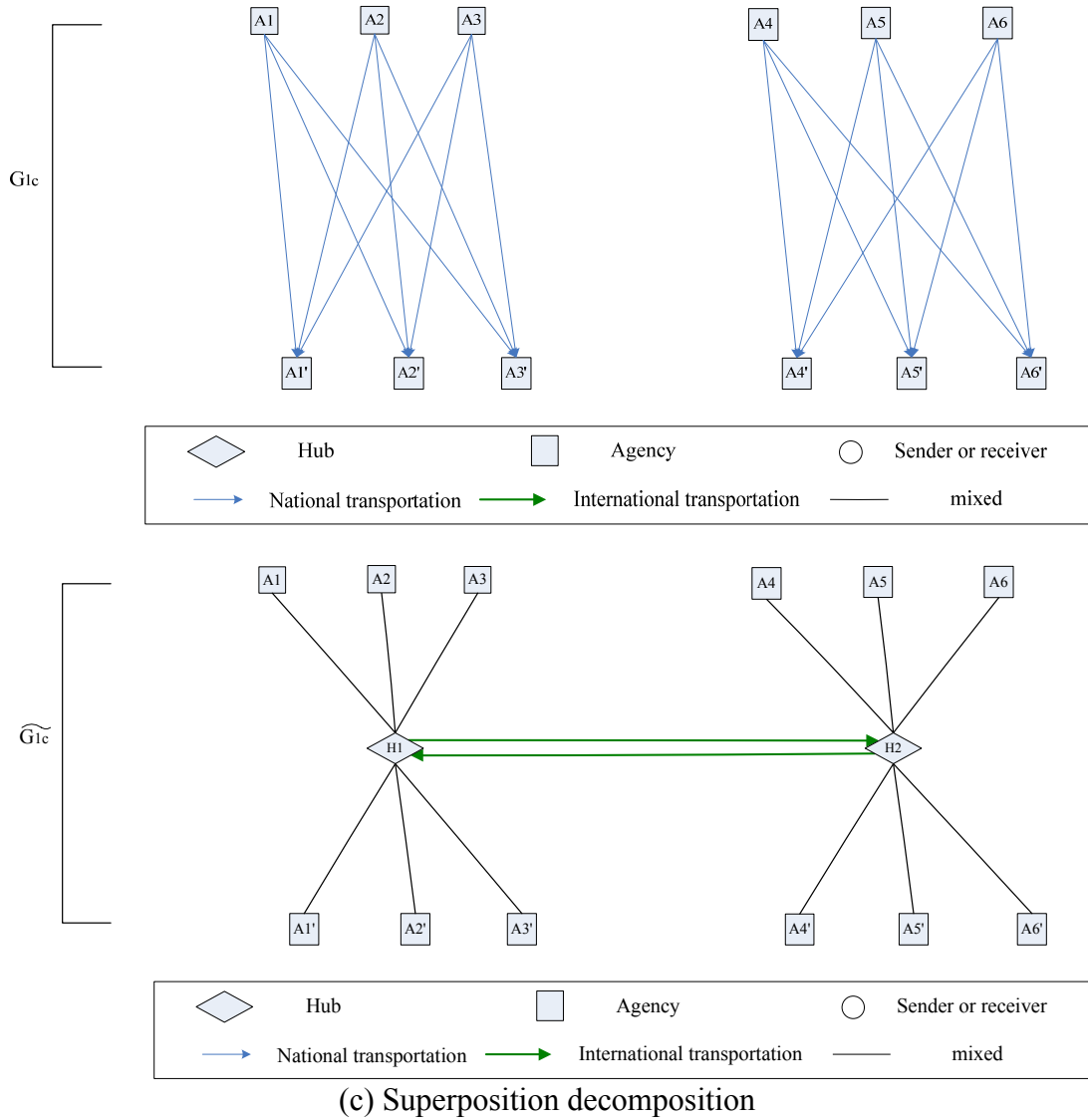
decomposition (see in Fig 4.18 (a)) is used to divide  $G$  into a combination of one 1-level network  $G_1$  and two 0-level distribution networks ( $G_2$  and  $G_3$ ). In Fig.4.18 (a),  $G_2 = (V_2, E_2)$  consists of the origins, agency  $A_r$  (Level 1) and the links between them, in which  $O_{G_2} = O_G$ ,  $D_{G_2} = F_{G_1}$ , so  $V_2 = O_{G_2} \cup D_{G_2}$ . Here,  $F_{G_1}$  is the set of the intermediate facilities of the network  $G_1$ . While  $E_2 = \{(m, n) | m \in V_2, n \in V_2, (m, n) \in E\}$ .  $G_3 = (V_3, E_3)$  is composed of agencies  $A_r'$  (Level 3) and the destinations and the links between them, in which  $D_{G_3} = D_G$ ,  $O_{G_3} = F_{G_3}$ , so  $V_3 = O_{G_3} \cup D_{G_3}$ , while  $E_3 = \{(m, n) | m \in V_3, n \in V_3, (m, n) \in E\}$ . In  $G_1 = (V_1, E_1)$ , the node set is composed of the facilities on level 1, on level 2 and on level 3 and the edge set is the links between the nodes in  $G_1$ . That is to say  $V_1 = F_{G_1} \cup F_{G_2} \cup F_{G_3}$ , where the origin set of  $V_1$  is  $F_{G_1}$ , i.e.  $O_{G_1} = F_{G_1}$ , the destination set of  $V_1$  is  $F_{G_3}$ , i.e.  $D_{G_1} = F_{G_3}$ , the intermediate facility set on level 1 is  $F_{G_2}$ , i.e.  $F_{G_1} = F_{G_2}$  while  $V_1 = \{(m, n) | m \in V_1, n \in V_1, (m, n) \in E\}$ .

The facility location and allocation decomposition (shown in Fig. 4.18 (b)) is applied to divide the 0-level distribution network ( $G_2$  and  $G_3$ ). Firstly a set of agencies  $\{A_1, \dots, A_R\} = \{A_1', \dots, A_R'\}$  is located. Consequentially, the number of agencies and their locations are determined. Here  $R=6$ . As defined before,  $L_n$  is the collection of node  $n$  and the lower-level nodes which are assigned to the node  $n$ . Then the corresponding  $L_{A_r}$ ,  $1 \leq r \leq R$  is determined by allocation problem. We define  $G_{A_r} = (V_{A_r}, E_{A_r})$ ,  $1 \leq r \leq R$  as the new sub-networks of  $G_2$ , where  $E_{A_r} = \{(m, n) | m \in V_{A_r}, n \in V_{A_r}, (m, n) \in E_2\}$ ,  $1 \leq r \leq R$  is the arc set of  $V_{A_r}$ . Then,  $G_2 = \bigcup_{1 \leq r \leq R} G_{A_r}$ . Analogously,  $G_3$  can be divided into six smaller networks denoted as  $G_{A_r'} = (V_{A_r'}, E_{A_r'})$ ,  $1 \leq r \leq R$ , where  $V_{A_r'}$ ,  $1 \leq r \leq R$ , is the set of agency  $A_r'$  and the destinations which are assigned to this agency  $A_r'$ , while  $E_{A_r'} = \{(m, n) | m \in V_{A_r'}, n \in V_{A_r'}, (m, n) \in E_3\}$ ,  $1 \leq r \leq R$  is the edge set of  $V_{A_r'}$ . The networks,  $G_{A_r}$  and  $G_{A_r'}$ ,  $1 \leq r \leq R$  with a few number of nodes, are typically the basic 0-level distribution network  $BG_{02}$  which we can successfully solve. For the network with a large number of nodes in the network  $G_{A_r}$  and  $G_{A_r'}$ ,  $1 \leq r \leq R$ , the aggregation decomposition is applied to divide the problem to several smaller basic 0-level distribution networks ( $BG_{02}$ ). The decomposition process of aggregation decomposition is the same as in the example in the part of aggregation decomposition, so here, we do not repeatedly introduce them and the network after aggregation decomposition are also denoted as the networks  $G_{A_r}$  and  $G_{A_r'}$ ,  $1 \leq r \leq R$ .

The superposition decomposition is used to decompose the 1-level network  $G_1$  (Fig. 4.18 (c)). In  $G_1$ , there exist the cross-flows between level 1 (agencies  $A_r$ ) and level 3 (agencies  $A_r'$ ) marked with blue lines in the Fig. 4.17 because the national transportation needs to directly deliver the goods between agencies in the same country. We define the cross-flow set is  $G_{1c}$ ,  $\tilde{G}_{1c}$  is the relative complement of cross-flow subset  $G_{1c}$  in  $G_1$ . The network  $G_1$  is divided into two sub-networks  $G_{1c}$ ,  $\tilde{G}_{1c}$  after superposition decomposition, in

which  $G_{1c}$  is the basic 0-level distribution network  $BG_{01}$  and  $\tilde{G}_{1c}$  is the basic 1-level distribution network  $BG_{12}$ .





**Fig. 4.18** The four decomposition methods in strategic view

Now, the original network  $G$  is divided into the combination of a set of smaller sub-networks, i.e.  $G = \bigcup_{1 \leq r \leq R} G_{A_r} \cup \bigcup_{1 \leq r \leq R} G_{A'_r} \cup G_{1c} \cup \tilde{G}_{1c}$  after applying the four decomposition methods in strategic view. Moreover, all of the sub-networks are one of the basic distribution networks which can be solved successfully.

### 4.3.2 Decomposition in tactical view

In chapter 3, we have introduced in detail the primary issues in the three phases of the operation planning and management in the transportation system. We focus on designing a framework to determine the routing sequence for GDP in this chapter. In the phase 1 of the framework, the decomposition methods are introduced according to the three phases in operation planning. After the decomposition methods according to the strategic view, the decomposition methods according to the tactical view are now proposed in this subsection.

In practice, the tactical decisions of a company need to adjust the strategic policies for shorter time term from one month to several months so as to meet the changes of the existing resources. Particularly, the facility allocation and frequency setting are two of the most important issues needed to be determined in the aspect of service network design problem. So, in the tactical phase, we provide the decomposition methods depending on these two issues.

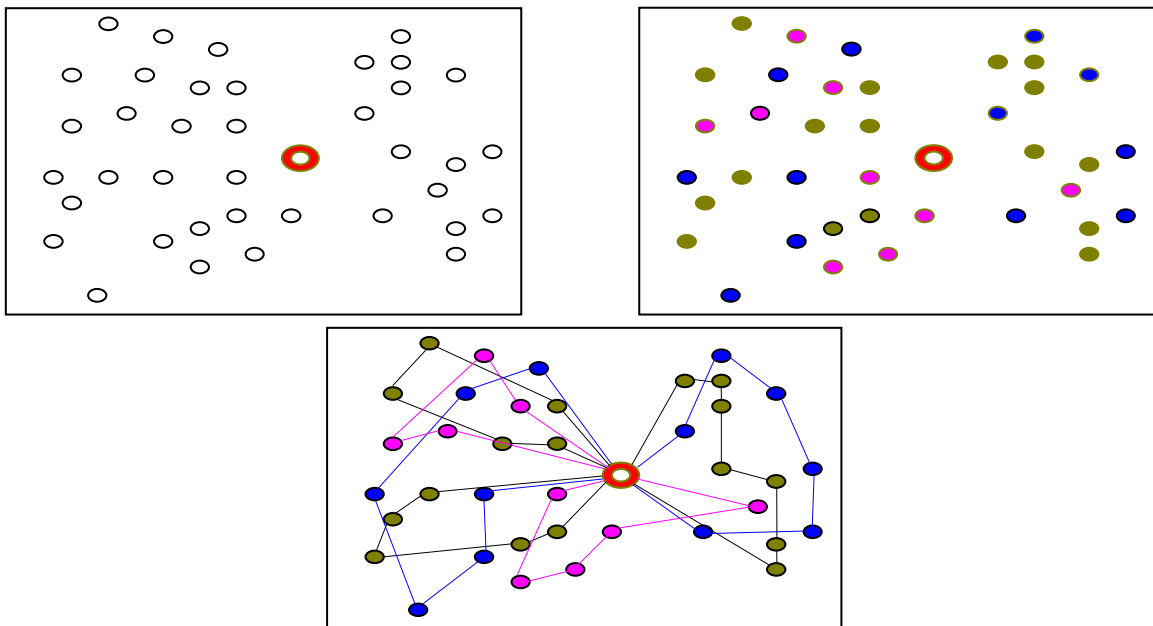
#### 4.3.2.1 Facility allocation decomposition

In the previous subsection, facility location and allocation decomposition method in strategic view has been defined and introduced in detail. Indeed, if the facility allocation is decided in the tactical phase (because of the change of market situation in a medial term period, for example), the decomposition is the same as it was decided in the strategic view.

#### 4.3.2.2 Time decomposition

Suppose that in tactical phase, the company organizes the transportation network  $G = (V, E)$  for a horizon  $[0, T]$ . Then, *time decomposition* is to divide the destination set  $D_G$  into several priority-ranked groups  $\{D_0, D_1, \dots, D_r, \dots, D_R\}$  according to the requirement deadlines of the customer deliveries. So, the original network  $G$  can be divided into  $R$  subsets:  $G_1, G_2, \dots, G_R$ . Here  $D_r$  is the destination subset in which each destination should be served in the time interval  $[t_r, t_{r+1}[$ . Especially, when  $r = 0$ , the time interval is  $[0, t_1[$  and the time interval is  $[t_R, T]$  if  $i = R$ . We define the origin set corresponding to destination set  $D_r$  as  $O_r$ , for  $0 \leq r \leq R$ , and the intermediate facility set of  $G_r$  is  $F_{G_r}$ . Then the node set of  $G_r$  can be denoted as  $V_r = O_r \cup D_r \cup F_{G_r}$ , while the corresponding edge set is then defined as  $E_r = \{(m, n) | m \in V_r, n \in V_r, (m, n) \in E\}$ . Thus,  $G_r = (V_r, E_r)$ , for  $0 \leq r \leq R$ .

In order to clearly explain time decomposition, an example is given in Fig.4.19. According to time decomposition, the delivery destinations are divided into three subsets distinguished by different colors. In other words, the destinations with the same color must be served in the same time interval. The routing sequence for each time interval is finally presented.



**Fig. 4.19** An example of the time decomposition in tactical view

### 4.3.3 Decomposition in operational view

In tactical phase, the physical network is adjusted after facility location decomposition. Furthermore, the transportation network is composed of a set of sub-networks for each time interval. Then, decisions on operational phase have to be made for each short term time interval, generally each day, which affect the way the products are delivered in the transportation system. Particularly, VRP is a primary issue which has to be solved in the operational level. We apply three decomposition methods to decompose the GDP in operational phase: facility allocation decomposition, aggregation decomposition and space decomposition method. We will respectively introduce these three decomposition methods in the following.

#### 4.3.3.1 Facility allocation decomposition and aggregation decomposition

Facility allocation decomposition method has been defined as a decomposition method in strategic view and tactical view. In fact, the facility allocation method can also be applied as a decomposition method in operational view and the definition of facility allocation decomposition method in operational view is similar to the one defined in strategic view and tactical view. However, the decisions in operational phase which are made to affect the goods delivery in the transportation system are restricted by the existing resources during a short term horizon. Therefore, the number of the vehicles, the transportation time, the transportation distance, etc. has to be considered when we determine the facility allocations. Furthermore, the objective formation of the facility allocation decomposition in operational view is sometimes different with the first two facility allocation.

As mentioned in the decomposition methods in strategic view, we know that aggregation decomposition is applied to take in consideration the large scale of the network. It is mainly concerned with the network with a large number and a high density of nodes. Firstly, it regards the group of a large number and high density nodes as a large demand node and then solve the new network problem with the new large demand nodes. In operational phase, due to the limitation of resources, the other attributes such as the distance, the transportation time, etc. have also to be considered when we decide the new large demand nodes.

#### 4.3.3.2 Spatial decomposition

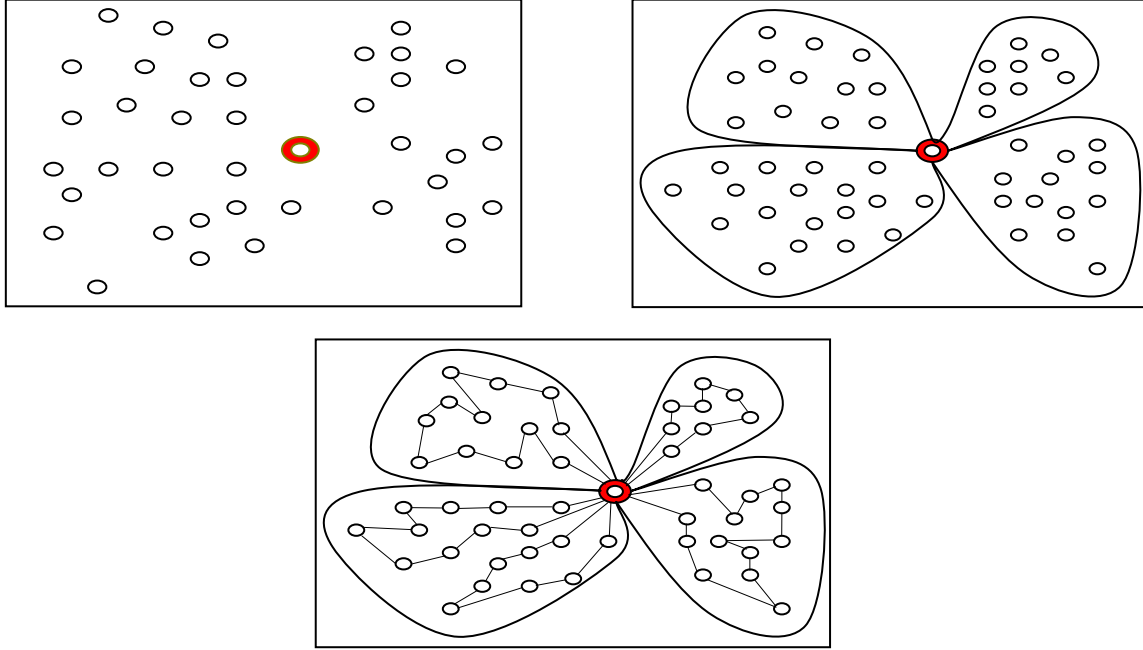
The aggregation decomposition in operational view is one of the methods to deal with the large scale network in operational phase. The other decomposition method in this phase is a spatial decomposition.

There are several researches to divide the large-scale problem with space criterions. Hwang (2002) has developed a sector-clustering algorithm to convert a multi-supply center problem into several single supply center problems. Clarisse (2000) has investigated a spatial decomposition to divide a multi-facility production and distribution problem into some sub-problems and then developed a branch-and-bound algorithm to obtain the exact solutions of the sub-problems.

*The spatial decomposition* is to choose the adjacent nodes on the same level of the distribution network as belonging to the same group. For a network  $G = (V, E)$ , the spatial decomposition firstly divides the nodes into several groups of adjacent nodes. Here we denote these node sets as  $\{V_1, V_2, \dots, V_R\}$ . We define  $E_r$  as the corresponding arc set of  $V_r, 1 \leq r \leq R$ .

Then,  $G = \bigcup_{1 \leq r \leq R} G_r$ , where  $G_r = (V_r, E_r)$ , for  $1 \leq r \leq R$ .

Fig. 4.20 illustrates an example of applying the space decomposition into a simple 0-level distribution network. In the Fig. 4.20, the origin is marked as the red ring, and the other points represent the destinations. First of all, the destinations are divided into 4 groups with space decomposition, and then the four node groups with the origin become four new smaller distribution problems, and then, we decide the itinerary for each of them.



**Fig. 4.20** An example of spatial decomposition

## 4.4 Improvement phase

In our proposed heuristic framework, a three-phase decomposition-based approach is proposed to solve GDP. These three phases involve the decomposition phase, the routing determination phase and, at last, the improvement phase. We have introduced the decomposition phase in the previous section to divide the whole GDP into several smaller groups. The decomposition methods have been presented according to the strategic, tactical and operational views. In Phase 2, the distribution routes in each sub-problem may be determined by the existing methods and tools which we have reviewed in Chapter 2. Moreover, other methods to solve the sub-problems of GDP in operational view will be provided in the next chapter. Now an improvement phase (Phase 3) is explained in this section. It is used to modify the current sequence, and the sum of the objective values of all the new sub-problems is returned to the original problem as a performance indicator to evaluate the current decomposition results.

There exist a large number of improvement methods including heuristics, i.e., 2-opt, 3-opt, Lin-Kernighan, and even the meta-heuristics, i.e. simulated annealing (SA), genetic algorithm, tabu search, etc. In general, improvement heuristics are characterized by a certain type of basic move to alter the current tour. Due to the universality of GDP, we can not appoint a particular method to improve its routing sequence. So we provide a general idea of the decomposition-oriented improvement phase. The idea can be defined as follows: starting from the routing sequence, then choosing one or several nodes from the different sub-networks, and changing their allocation sets, finally combining the new routing sequence in the cheapest way.

In a distribution network  $G = (V, E)$ , the current routing sequence is  $S$ ,  $Z$  is the corresponding objective of  $S$  and  $L_v$  is the allocation set of the node  $v$ .  $S^*$  is the optimal routing sequence and  $Z^*$  is the corresponding objective. The improvement process follows a five-step algorithm:

**Step 0 (Initialization)** Let cycle index  $i = 1$ ; Let the solution  $S$  from Phase 3 as the best solution  $S^*$ , the objective  $Z$  corresponding to  $S$  as the best objective  $Z^*$ .

**Step 1 (Node selection)** Select a set of nodes  $\{v_{i1}, \dots, v_{iJ}\}$ ,  $J \geq 1$  from the several sub-networks and then determine their corresponding allocation set as  $L_{v_{i1}}, L_{v_{i2}}, \dots, L_{v_{iJ}}$ . Notably, the nodes set  $\{v_{i1}, v_{i2}, \dots, v_{iJ}\}$  can be selected randomly or by some special method such as heuristic or metaheuristics.

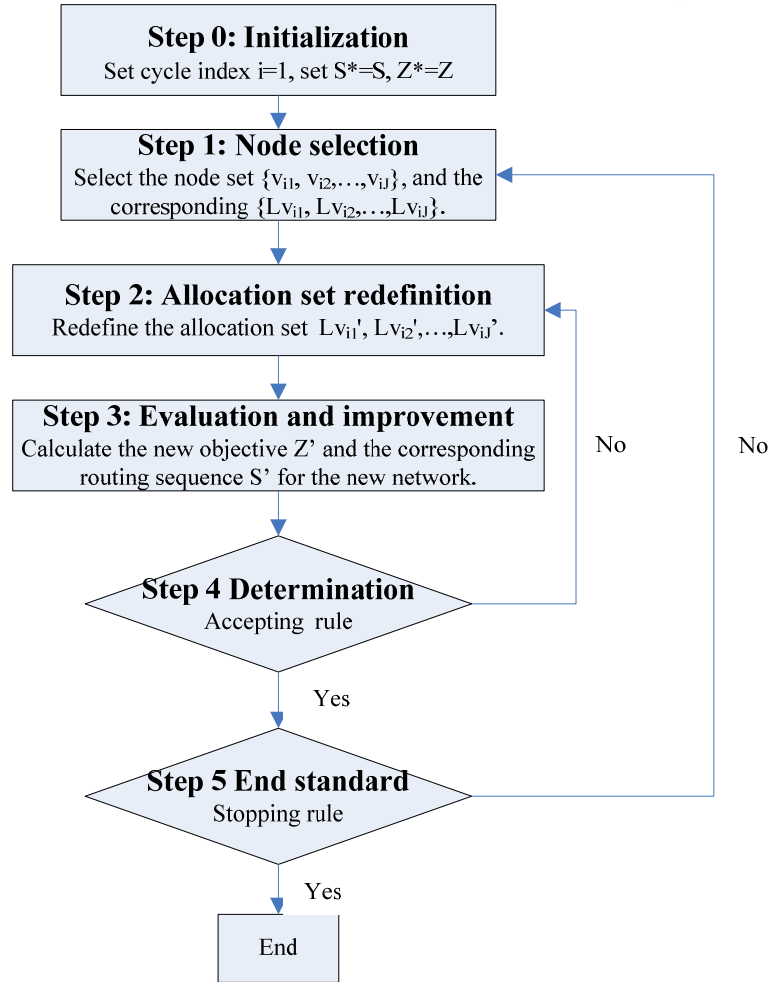
**Step 2 (Allocation set redefinition)** Redefine the allocation set  $L_{v_{i1}}, L_{v_{i2}}, \dots, L_{v_{iJ}}$  for the nodes  $\{v_{i1}, \dots, v_{iJ}\}$  by inserting or removing one or several nodes for each of the allocation set  $L_{v_{ij}}$ , for  $1 \leq j \leq J$  or by exchanging the nodes between the allocation sets  $L_{v_{ij}}$ , for  $1 \leq j \leq J$ .

**Step 3 (Evaluation and improvement)** Calculate the objective  $Z'$  and the solution  $S'$  corresponded to  $Z'$ . If the accepting rule is satisfied, set  $Z^* = Z'$ , and  $S^* = S'$ .

**Step 4 (Determination)** If *stopping rule 1* is satisfied, set  $i = 1$ , go to Step 5; Else, set  $i = i + 1$ , go back to Step 2.

**Step 5 (End standard)** If *Stopping rule 2* is satisfied, end; else, go back to Step 1.

Fig.4.21 shows us the flow chart of this general idea in the improvement phase.



**Fig.4.21** Flow chart of the general idea in the improvement phase

Let us note that the method of the node selection in Step 1 is a crucial factor to directly influence the efficiency and the performance of the improvement algorithm. Up to now, it is still a tough problem in the decomposition-oriented improvement research. A typical node selection method can be found in three aspects: node insertion, local search (k-opt and large-scale neighbourhood search algorithm) and Lin-Kernighan moves.

There are generally two types of accepting rule, one of which is gradient descent and the other of which is heuristic or metaheuristic method. In gradient descent method, if  $Z' < Z$ , then the current solution and the corresponding objective is accepted to the best solution. On the other hand, the heuristic or metaheuristic idea is introduced to accept the current solution. For example, simulated annealing algorithm replaces the current solution by a random "nearby" solution, chosen with a probability that depends on the difference between the corresponding function values and on a global parameter  $T$  (called the temperature), that is gradually decreased during the process.

Moreover, the stopping rule is generally defined either to limit the computational time (see Ferguson (2006)) or to limit the maximum number of iterations to prevent an endless loop in the improvement algorithm.

## 4.5 Conclusion

In this chapter, we have proposed a three-phase heuristic decomposition framework to solve a GDP according to the characters of the hierarchical distribution network. The procedure for the proposed framework consists of three phases which we can generally define as decomposition phase, routing determination phase and improvement phase.

In the decomposition phase, decomposition methods have been introduced in order to overcome the complexity and the large scale of the delivery network, according to the three aspects in operational planning, the strategic, tactical and operational views. An example of multi-level distribution network has been illustrated to show how the four decomposition methods in strategic view can successfully divide the original hierarchical network into several basic networks, which we can successfully solve as FLP, HLP, FAP, HAP, Transportation Problem, TSP, and VRP.

In improvement phase, a general idea was provided as a five-step improvement algorithm. Furthermore, the basic idea was starting from the current routing sequence, then choosing one or several nodes from the network to change their allocation sets, finally combining the newer routing sequence in the cheapest way.

In routing determination phase, there already exist a great number of studies in the operation planning phase focusing on the basic structure of GDP, such as FLP, HLP, FAP, HAP, TP, TSP, VRP, etc. Some of these studies have been introduced in the bibliography review of the thesis. Some other solutions for the basic networks of GDP in operational phase will be presented in the next chapter.

## Part III

# THE SOLUTION OF BASIC NETWORKS IN OPERATIONAL PHASE



# CHAPTER 5: Heuristic Approach to a Basic 0-level

## Network

### 5.1 Introduction

The aim of this thesis is to design a framework to assist the company to solve their GDPs. The three-phase framework to decompose the large-scale problem and to improve the solution has been designed in Part II. In this framework, the original distribution network is firstly divided into several smaller distribution networks with basic network structures according to the three phases of the operation planning. Then the distribution routing problem for each sub-networks are regarded as the sub-problems of the original GDP such as FLP, HLP, FAP, TP, TSP, VRP, etc., some which could be solved by the existing tools and/or heuristic algorithms. However, some of the sub-problems are not yet studied. So in this part we will provide some solutions to the sub-problems with the basic network structure in operational phase. Above of all, a special case, of GDP: the Capacitated Vehicle Routing Problem (CVRP) is solved by applying a spatial decomposition, our decomposition method in operational phase.

CVRP is a special case of VRP with the additional constraint that every vehicle must have uniform capacity of a single commodity. Furthermore, it is a network distribution problem in operational phase with the basic 0-level network structure  $BG_{02}$  of VRP.

In the last two decades branch-and-bound, including those that make use of the set partitioning formulation and column generation schemes, and branch-and-cut are two main exact approaches to the CVRP. Several branch-and-bound algorithms are available for the solution of CVRP. Toth and Vigo (1998, 2002) reviewed the structure of the branch-and-bound algorithm strategies and dominance rules. Branch-and-cut algorithms currently constitute the best available exact approach for solution of the CVRP. Research in this area has been strongly motivated but the approach is still quite limited by the size of the problem. Naddef and Rinaldi (2002) have given a detailed presentation in this approach. Lian and Castelain (2009) provide a decomposition-based heuristic framework to solve GDP. Furthermore, the framework is employed to solve CVRP. In practice, the best CVRP exact algorithms can rarely tackle instances involving more than 100 vertices. In order to overcome the limitation of the vertices number and to solve larger CVRP, we apply our proposed decomposition method in operational phase to CVRP.

In this case, we take the Capacitated Clustering Algorithm (CCA) as space decomposition technique, apply Concorde as the solver, and then improve the solution with Simulated Annealing (SA). In the remaining parts of this chapter, the heuristic approach based on CCA and SA is firstly provided in section 5.2, then our approach with the decomposition technique is applied into a real instance of the Regional Fire and Emergency

Center in the north of France. The result of our proposed approach is compared with another approach to evaluate its performance.

## 5.2 Heuristic approach based on CCA and SA

### 5.2.1 Algorithm Description

In this section, our proposed approach based on CCA and SA is in detail described. The heuristic approach is in fact based on the three-phase decomposition framework proposed in the previous chapter. Furthermore, we use CCA as a decomposition technique to achieve the spatial decomposition, and employ SA to realize the improvement phase. After embedding the CCA and SA into our proposed three-phase decomposition framework, the heuristic approach to CVRP can be seen as a routing first routing second method which is a method generally used to solve VRP. The algorithm could be defined as follows:

Phase 0: Estimate the number of vehicles,  $N$ , based on the total workload. Construct the distance matrix.

Phase 1: Decompose the large-scale problem into several TSPs (generally more than  $N$ ) with CCA.

Phase 2: Use Concorde to determine the routing sequence for each TSP.

Phase 3: Improve the routes between each TSP with SA.

Phase 4: Finish the heuristic approach.

The number of required vehicles (phase 0) is estimated by the total distribution workload divided by the daily workload capacity of each vehicle. Note that we assume that the vehicles are all of the same type. The explanations of the other phases will be done in the next two subsections.

### 5.2.2 Capacitated clustering algorithm

#### 5.2.2.1 K-means algorithm

In phase 1, we decompose the original problem into several TSPs by CCA. CCA is an advanced algorithm based on k-means algorithm. K-means clustering algorithm has been developed by MacQueen (1967) and then by Hartigan (1975) and Hartigan and Wong (1979). Simply speaking, k-means clustering is one of the most traditional unsupervised learning algorithms that classify or group the objects based on attributes/features into  $k$  number of groups.  $K$  is a fixed positive integer number. The clustering is done by minimizing the sum of squares of distances between data and the corresponding cluster centroid.

The algorithm is generally composed of the following steps:

**Step 1:** Place  $k$  points representing initial cluster centroids into the space. This space consists of the objects that are being clustered.

**Step 2:** Assign each object to the cluster whose centroid is closest to the object.

**Step 3:** Recalculate the positions of the  $k$  centroids when all the objects have been assigned.

**Step 4:** Repeat Steps 2 and 3 until the centroids do not move any more. This produces a separation of the objects into clusters from which the minimal metric can be calculated.

In the Step 1 of the k-means clustering algorithm, the initial centroids should be placed

in a clever way because different locations cause different result. A good choice is to place them as much as possible far away from each other. The following step is to associate each object to the nearest centroids. When all of the points are assigned, decomposition is done. The next step is to recalculate  $k$  new centroids for the clusters resulting from the Step 2. When the new centroids are determined, a new cluster has to be done for the new object set. At this point, a loop has been generated. As a result, we may notice that the  $k$  centroids change their locations loop by loop until no more changes are done. That is to say centroids do not move any more.

The  $k$ -means algorithm is not necessary to find the optimal configuration for the object set and the algorithm is significantly sensitive to initial randomly selected cluster centroids. The algorithm could be run multiple times to reduce the deviation of the different initial centroid selections. Moreover, there is no general theoretical solution to find the optimal number of clusters  $k$  for a given object set. A simple approach is to compare the results of multiple runs with different  $k$  and choose the best one according to a given criterion. Note that we need to be careful to increase  $k$ , because it may sometimes lead to smaller error function values, but also an increasing risk of over-fitting.

### 5.2.2.2 Capacitated clustering algorithm

In standard  $k$ -means algorithm, the objects are clustered according to the distances between the objects and the centroids, i.e. an object is assigned to the cluster whose centroid is closest to the object. CVRP is the vehicle routing problem with the restriction of the vehicle capacity. We need to consider the vehicle workload when we do the clustering, in the same time to minimize the vehicle number as less as possible. In this situation, we provide CCA, an advanced algorithm of  $k$ -means to solve CVRP. In the CCA, the centroid of the centroids is also considered, which would be called “the grand centroid”. Additionally, in order to minimize the number of vehicles, we begin the algorithm with the number obtained in Phase 0 of our proposed heuristic method in Section 5.2.1.

**Step 0:** Set  $N$  be the vehicle number estimated by phase 0, the maximal cycle index be  $m_{\max}$ . The cycle index  $m=0$ .

**Step 1:**  $N$  initial centroid seed nodes are selected and the corresponding grand centroid is calculated.

**Step 2:** The nodes are sorted in decreasing order based on the distances up to the grand centroid. Repeat the assignation until there is no more nodes to assign.

**Step 3:** If all the nodes are assigned to clusters, go to Step 4; else, there are still some nodes which are not assigned to any cluster, the cycle index  $m = m + 1$ . If  $m > m_{\max}$ ,  $N=N+1$ , return to step 1; else, go to Step 4.

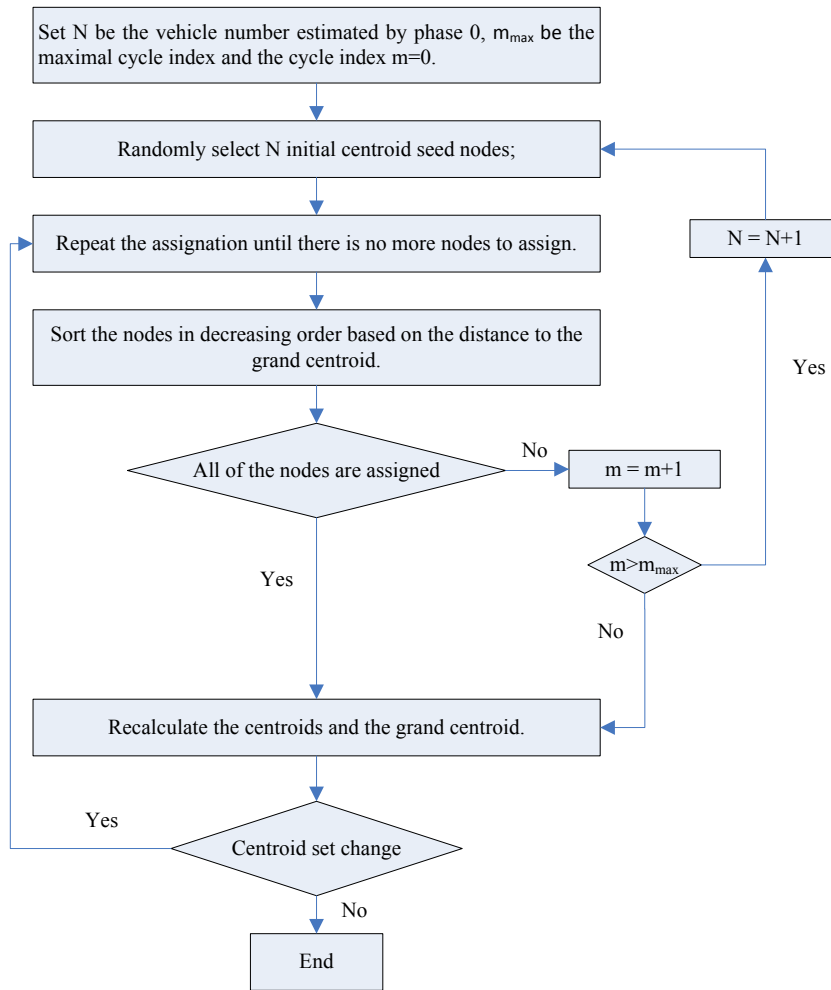
**Step 4:** A new set of centroids for each existing cluster and the grand centroid are calculated.

**Step 5:** If the centroids do not change, go to Step 6; else return to Step 2.

**Step 6:** Finish the algorithm when the groups of nodes are found.

To further clarify the algorithm, we explain in detail Step 4 of CCA. In Step 4, the farthest node from the grand centroid is assigned first to its nearest centroid, then the next farthest node and so on. The sum of the node demand in the cluster defined as the capacity of the cluster is considered when a new node needs to be assigned to the cluster. The node would be assigned to another cluster when its closest cluster has already reached its capacity.

This algorithm can also be described as the following flow chart in Fig.4.1.



**Fig. 4.1** Flow chart of Capacitated Clustering Algorithm

### 5.2.3 Simulated annealing improvement

The optimal solution to each group is produced by Concorde in Phase 2. In Phase 3, we modify the routing sequence between the routings to approach the global optimal solution. SA with a 3-opt improvement heuristic algorithm has been developed for this problem.

Simulated annealing (SA) is a generic probabilistic metaheuristic proposed in Kirkpatrick et al.(1983), aiming merely to find an acceptably good solution in a fixed amount of time. The inspiration derives from the physical cooling of solids to eventually obtain a strong crystalline structure, where a minimum energy configuration happens. The SA algorithm is an iterative approach, each step of which is analogous with this physical process. Moreover, the current solution is replaced by a random “nearby” solution in each step, chosen with a probability so as to obtain other “good” solution in further stages. The local optima could be effectively avoided by the introduction of this probability.

### 5.2.3.1 Basic element

As statement in Bertsimas and Tsitsiklis (1993), the basic elements of SA are the following:

1. **State space:** A finite set  $S$ .
2. **Cost function:** A real-valued cost function  $J$  defined on  $S$ . Let  $S^* \subset S$  be the set of global minima of the function  $J$ , assumed to be a proper subset of  $S$ .
3. **Neighbor set:** For each  $i \in S$ , a set  $S(i) \subset S - \{i\}$ , called the set of neighbors of  $i$ .
4. **Acceptance probabilities:** For every  $i$ , a collection of positive coefficients  $q_{ij}$ ,  $j \in S(i)$ , such that  $\sum_{j \in S(i)} q_{ij} = 1$ . It is assumed that  $j \in S(i)$  if and only if  $i \in S(j)$ .
5. **Annealing schedule:** A nonincreasing function  $T: N \rightarrow (0, \infty)$ , called the annealing schedule. Here  $N$  is the set of positive integers, and  $T(t)$  is called the temperature at time  $t$ .
6. **Initial state:** An initial "state"  $x(0) \in S$ .

### 5.2.3.2 Parameter selection

In order to apply the SA to our specific CVRP, some elements and their parameters have to be adjusted. These adjustments can have a significant impact on the effectiveness of the SA. Unfortunately, there are no choices of these parameters that will be good, and there is no general way to find the best choices for our CVRP, even for any given problem. We point out our choice for our CVRP as follows:

1. **State space  $S$ :** the set of routing sequence.
2. **Initial state  $R_0 = \{r_{01}, r_{02}, \dots, r_{0N}\} \in S$ ,** where  $R_0$  represents the routing sequence provided by Phase 2, and it consists of the routings  $r_{0n}, 1 \leq n \leq N$ .
3. **Objective function  $c_k$ :** the sum of the transportation cost in each route of  $R_k$ .
4. **Acceptance probabilities and candidate generation selection**

Now, we explain the way we follow for accepting or refusing the new routing sequence with worse than the current one.

Given the above elements, we now describe how a state evolves to another by SA algorithm. If the current state is  $R_k$ , choose a neighbor  $R_{k'}$  of  $R_k$ ; the acceptance probability that any particular  $R_{k'}$  is selected is equal  $p = \exp[-(c_{k'} - c_k / T(k))]$ .

Once the neighbor  $R_{k'}$  of  $R_k$  is chosen, the next state  $R_{k+1}$  is determined as follows:

If  $c_{k'} \leq c_k$ , then  $R_{k+1} = R_{k'}$ .

If  $c_{k'} > c_k$ , then  $p = \exp[-(c_{k'} - c_k / T(k))]$ .

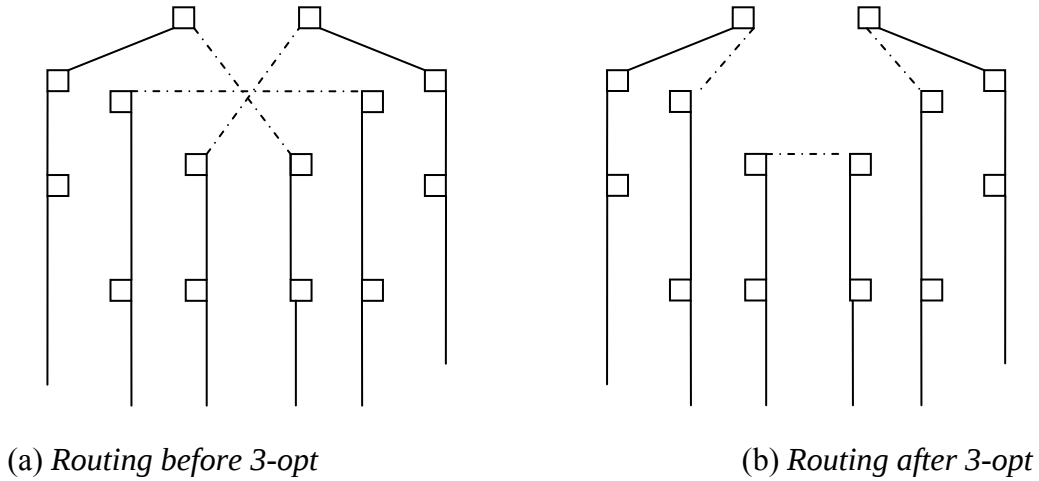
Then,  $R_{k+1} = \begin{cases} R_{k'}, & \text{c= random } [0, 1] < p; \\ R_k, & \text{otherwise.} \end{cases}$

### 5. Annealing schedule

We set the initial value for the temperature  $T_0 = 1$  after several experiments. And set the temperature  $T_{k+1} = \alpha T_k$ , where  $\alpha \in (0, 1) \wedge \alpha \rightarrow 1$ . The Lower limit of the temperature  $T_{\min}$  is defined as a given constant. The SA finishes when the temperature is smaller than  $T_{\min}$ .

## 6. Candidate neighbor generation

When choosing the candidate generator neighbor, one must consider that after a few iterations of the SA, the current state is expected to have much lower energy than a random state. In the basic iteration of SA, we obtain the neighbors of each state by 3-opt improvement heuristics. First, 3-opt improvement heuristics is employed. The basic idea of 3-opt local search algorithm (see in Fig. 5.2) is to start from the current routing sequence, to randomly choose three edges from the routing sequence, to remove them, and to combine the six parts to a new routing sequence. After that, the capacity in each route is examined for each of the 14 new routing sequences. Here, the number of the routing sequences  $= C_6^2 - 1$ . The routing sequence satisfying the vehicle's workload is improved by SA and then is regarded as a candidate neighbor of the current state.



**Fig. 5.2** Idea of 3-opt Algorithm

Let  $R_k = \{r_{k1}, r_{k2}, \dots, r_{kN}\} \in S$  be the current routing sequence, and  $r_{kn}$ ,  $0 \leq n \leq N$  is the route in the routing sequence. And  $c_k$  is the transportation cost of  $R_k$ . Then the procedure to obtain the neighbor set  $N_T(R_k)$  of  $R_k$  is defined as follows:

**Step 0 (Initialization)** Let cycle index  $t = 1$  and the maximal cycle index is  $t_{\max}$ ; Let the solution  $R_k^*$  be the best solution  $R_k$ , and the objective  $c_k^*$  corresponding to  $R_k^*$  be the best objective  $c_k$ .

**Step 1 (Edges choosing)** Apply 3-opt algorithm to randomly choose three edges noted as  $r_1, r_2, r_3$  from the edge set of the three different routings. Then, at most, 15 new solutions  $R_{k1}, R_{k2}, \dots, R_{k15}$  can be produced.

**Step 2 (Capacity examination)** The capacity of each route in each of the 15 new routing sequences is calculated. If the capacity is surpassed in any route, remove the corresponding routing sequence; otherwise, the routing sequence, all routes of which satisfy the vehicle's

workload, is an element of the candidate routing sequence set. Suppose the candidate routing sequence set be  $\{R_{k1}, R_{k2}, \dots, R_{kM}\}$ .

**Step 3 (Routing improvement)** If  $M = 0$ , set  $t=t+1$ , Go to Step 5; else, SA is embedded into this step to improve all of the routes in the routing sequence  $R_{km}$ ,  $1 \leq m \leq M$  as TSP. Then a new routing sequence set  $\{R_{k1}', R_{k2}', \dots, R_{kM}'\}$  are generated.

**Step 4 (Exchange evaluation)** Calculate the objective (the total distance)  $c_{k1}', c_{k2}' \dots, c_{kM}'$ . Index  $c' = \min\{c_{k1}', c_{k2}' \dots, c_{kM}'\}$ , and  $R'$  corresponding to  $c'$  are added into the candidate neighbor set  $N_T(R_k)$ .

**Step 5 (End standard)** If  $t \leq t_{\max}$ , go back to Step 1; else, end.

### 5.2.3.3 Algorithm description

After introducing the basic elements and all the parameter selections for SA, we could provide our SA improvement algorithm as follows:

**Step 0:** Define initial temperature  $T_0$ , the lower limit of the temperature  $T_{\min}$ , the descend index  $\alpha$ , the maximal iterations number of  $K_{\max}$ . Set the loop index  $k = 0$ ,  $l = 0$ ; given the initial state (routing sequence)  $R_0$ , set the current best state  $R^* = R_0$ , the corresponding best cost  $c^* = c_0$ .

**Step 1:** Generate the neighbor set  $N_T(R_{new}')$  according to the method mentioned above, and then randomly choose one neighbor  $R_{new}' \in N_T(R_{new}')$ ,  $c_{new}'$  is the corresponding cost of the routing sequence  $R_{new}'$ .

**Step 2:** Determine the next state  $R_{new}$  according to the method of **candidate generation selection** which has been introduced in the previous subsection. That is to say: the new state  $R_{new}$  is determined as follows:

If  $c_{k'} \leq c^*$ , then  $R_{new} = R_{new}'$ .

If  $c_{k'} > c^*$ , then  $p = \exp[-(c_{new} - c^* / T(l))]$ .

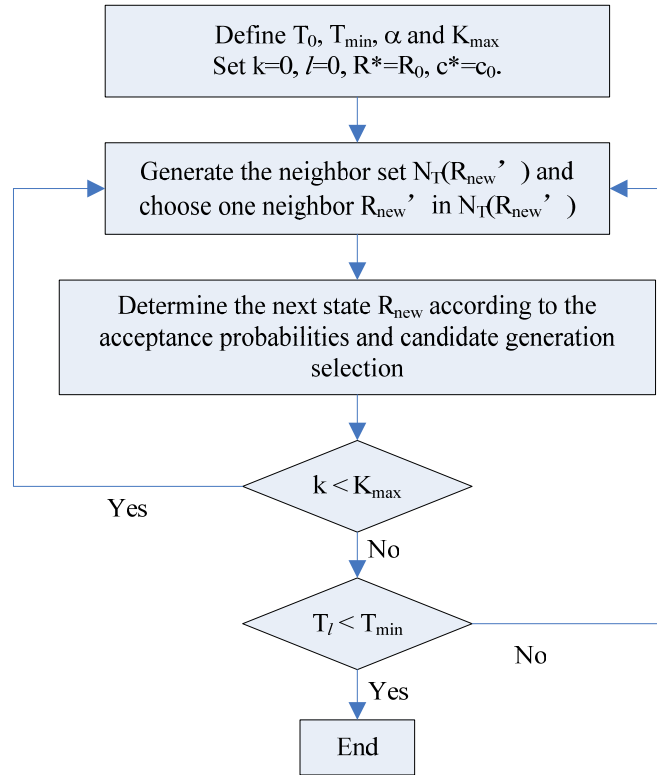
Then,  $R_{new} = \begin{cases} R_{new}', & c = \text{random}[0, 1] < p; \\ R^*, & \text{otherwise.} \end{cases}$

And then, set  $R^* = R_{new}$  and  $c^* = c_{new}$ , moreover  $k = k + 1$ .

**Step 3:** If  $k \leq K_{\max}$ , go back to Step 1; else go to Step 4.

**Step 4:** Set  $l = l + 1$ . If  $T_l > T_{\min}$ , go to Step 1; else end.

The flow chart of our proposed simulated annealing algorithm is shown in the Fig. 5.3.



**Fig. 5.3** The flow chart of simulated annealing algorithm

## 5.3 Computational results

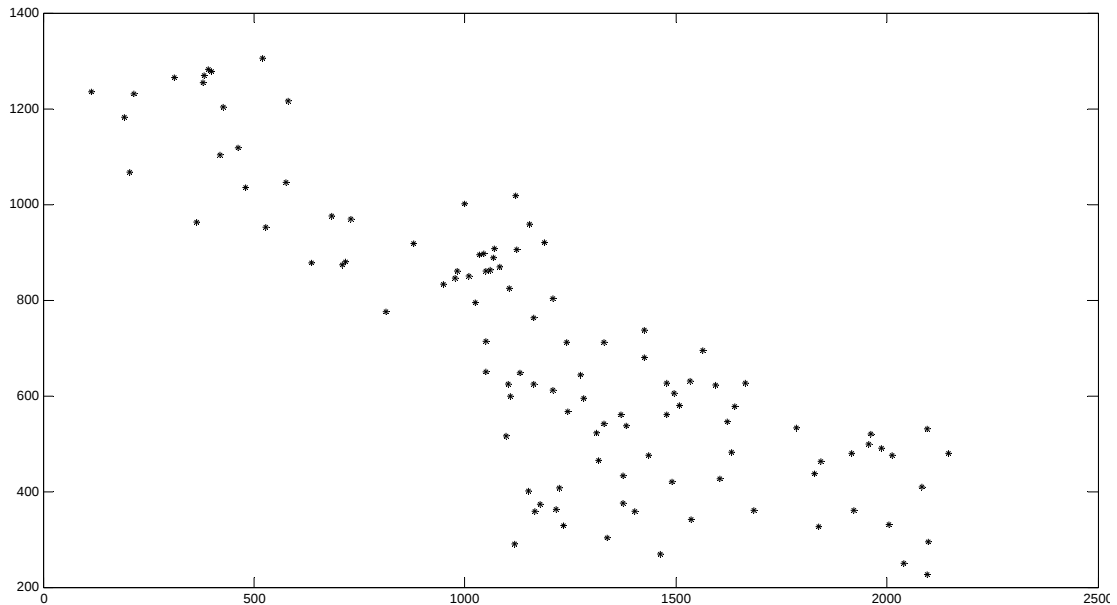
To evaluate the performance of the proposed approach, our approach with the decomposition technique is compared with the other five decomposition approaches, the old distribution strategy used by a distribution instance of the Regional Fire and Emergency Center (SDIS59) in the north of France.

The regional service centre needs to delivery the medicine to its firefighter centers in five regions each week. In the case of SDIS59, the distribution process is not associated with the time windows. From the performance point of view, the center just aims to minimize the number of the vehicles and the traveling distance. The centre has 10 vehicles with the same capacity. There are 110 firefighter centers in five different regions.

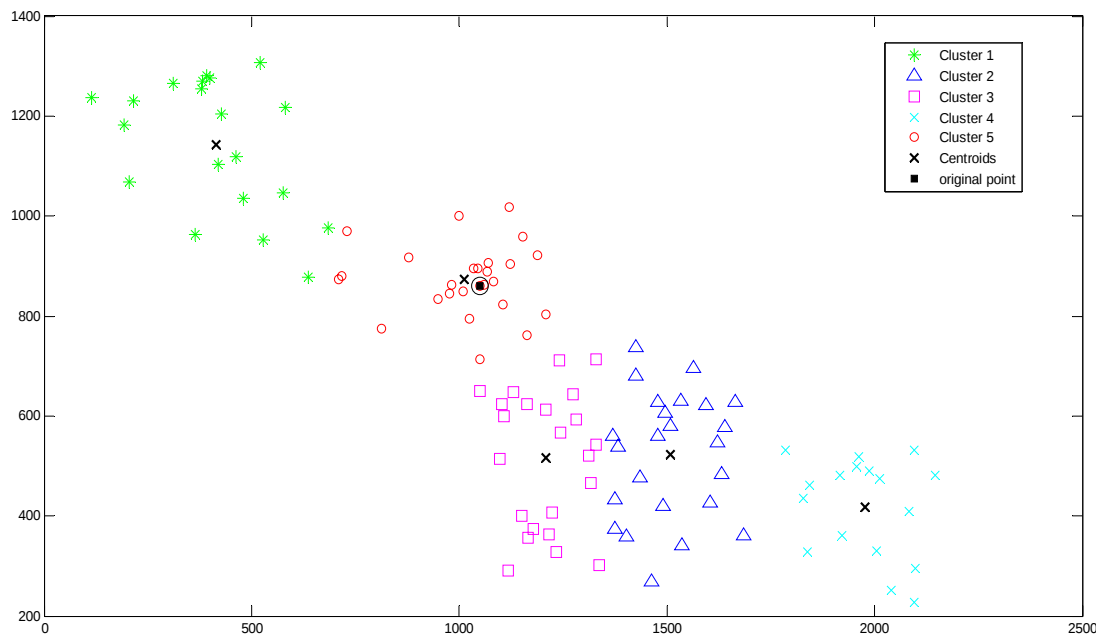
We set  $N=5$  to begin the algorithm from Phase 0. After several experience, we get our best results when we set  $m_{\max}=50$  for CCA in Phase 1 and set  $T_0=1$ ,  $T_{\min}=0.05$ ,  $\alpha=0.95$  and  $K_{\max}=50$  in the SA improvement of Phase 4. The routing sequence provided by our proposed approach is illustrated in the following subsection.

### 5.3.1 Performance of the proposed heuristic approach

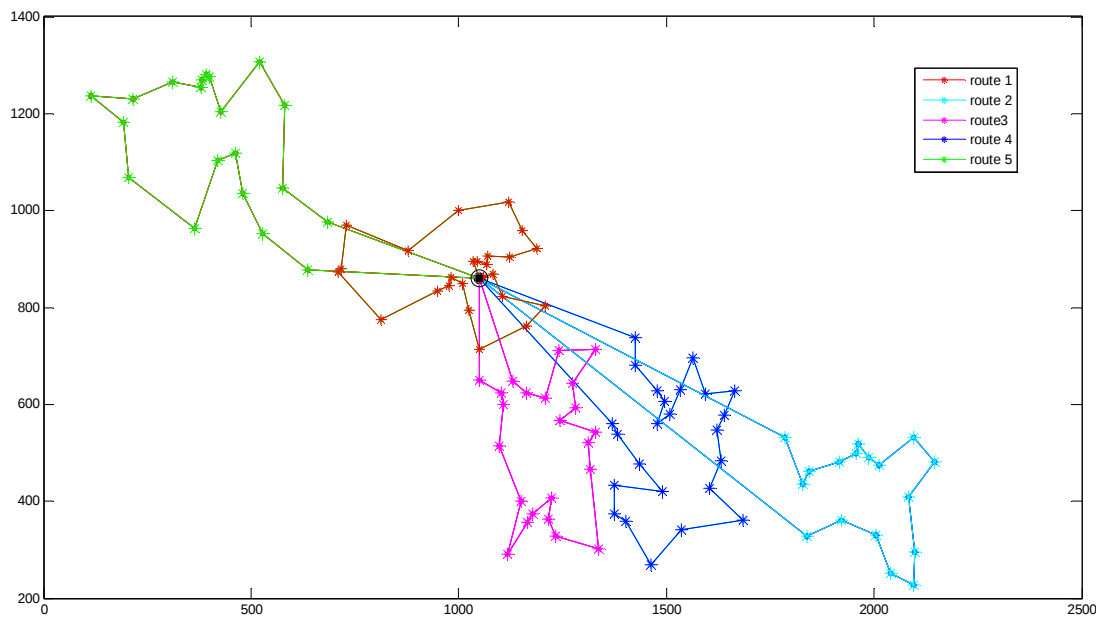
Fig. 5.4 shows the results of each phase in the heuristic approach based on CCA and SA when the approach is used to solve the problem instance. Fig. 5.4(a) displays the distribution of the 110 firefighters; Fig. 5.4(b) indicates the groups of firefighter centers produced by Phase 1. After Phase 2, routing sequence for each group of the firefighter centers is shown in Fig. 5.4(c). At last, Fig. 5.4(d) shows the routing sequence improved by SA in Phase 3.



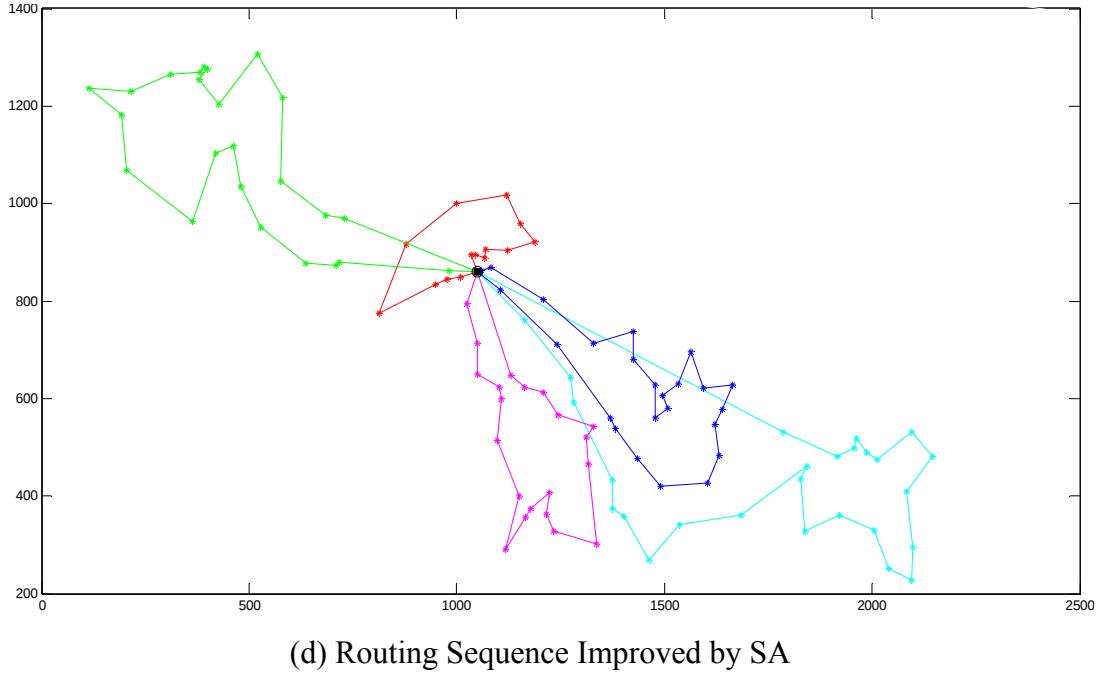
(a) The distribution of the 110 firefighters



(b) Groups Produced by Capacitated Clustering



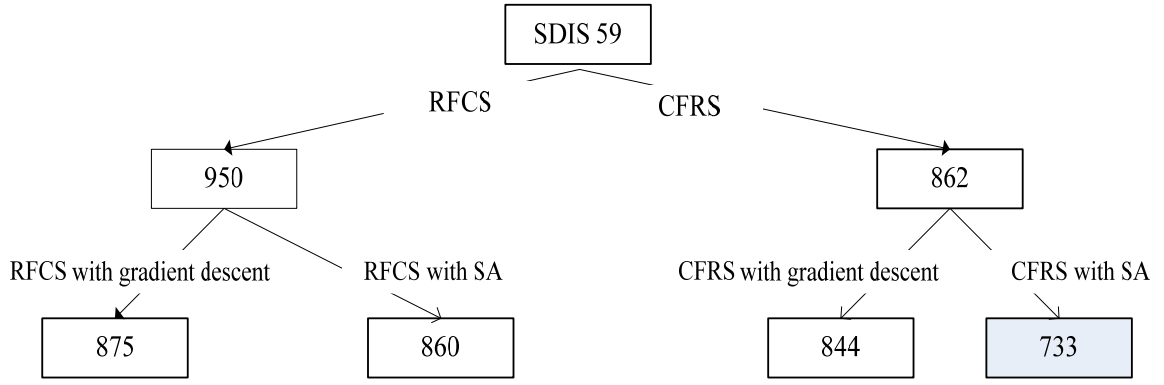
(c) Routing Sequence after Clustering the Destination Nodes



**Fig. 5.4** Performance of the proposed heuristic approach for SDIS59

### 5.3.2 Result comparison

The performance of our proposed heuristic is compared with five other approaches: route-first-cluster-second approach (RFCS) without the improvement heuristic, RFCS with 3-opt improvement technique, RFCS with SA, and the other approaches related to our proposed approach, which are respectively the clustering and routing approach without any improvement technique (Noted as CFRS), the combination approach of CCA and 3-opt (Noted as CFRS with 3-opt) and our proposed results (Noted as CFRS with SA) to the distribution instance. In a route-first-cluster-second method, the route is first determined by suitably sequencing the all of the customers, and then the customers are then grouped into clusters. Different techniques have been proposed for the clustering phase, while the routing phase amounts to solving a TSP. Here, we take the results in the Castillo's practice report. The 3-opt is a kind of local search metaheuristic algorithm for solving the traveling salesman problem and related network optimization problems. In our proposed approach, the 3-opt is embedded into the improvement phase of simulated annealing algorithm as the neighbor selection method. In order to compare the performance of the proposed approach, 3-opt as a stochastic descent improvement method, is separately applied to improve the solution provided by RFCS and CFRS. And the 3-opt improvement stops when the results do not change within 50 iterations. Computational results by different approaches in term of distance (km) are shown in Figure 5.5.



**Fig. 5.5** Computational Result in Different Approaches

Our proposed approach outperforms the other five approaches in terms of the total distance. It can be proved that CCA is an effective approach by comparing the result of CFRS with the result of RFCS, i.e. 862 is much better than 950. Furthermore, by comparison the result of RFCS with 3-opt (875) and RFCS with SA (860) and the result of CFRS with 3-opt (844) and CFRS with SA (733), our proposed SA improvement approach is also a good improvement approach, because it is better than the 3-opt improvement approach and it can improve the results of the routing problem.

## 5.4 Conclusion

CVRP as a special case of VRP, is an important transportation routing problem in operational phase with the basic 0-level distribution network structure  $BG_{02}$ . In this chapter, we have proposed a heuristic approach to solve CVRP, which was based on the three-phase decomposition framework and took CCA as decomposition technique and SA as improvement approach. And then our proposed heuristic approach with CCA and SA was applied to solve the delivery problem instance of SDIS59 in the north of France.

The performance of the proposed approach was shown and its result was evaluated by comparing with the five other approaches, including RFCS without the improvement heuristic, RFCS with 3-opt improvement technique, RFCS with SA, CFRS without the improvement heuristic, CFRS with 3-opt improvement technique. Computational results showed that our approach outperformed these six approaches.

In the next chapter, the distribution problem in operational phase with another basic distribution network structure, 1-level facility distribution network structure  $BG_{11}$ , will be studied and its formulations and the related experience results will also be provided.

# CHAPTER 6: Facility Allocation Problem for A

## Simple 1-level Network

### 6.1 Introduction

As stated in the previous chapter, this part of the thesis is aimed to resolve the distribution problems with basic network structure in the operational view. The basic network structure presented in chapter 4 in Phase 1 of our proposed framework consists of two types: 0-level basic network structure and 1-level basic network structure, which involves *simple 1-level network*  $BG_{11}$  and *hub 1-level network*  $BG_{12}$ . The solution of CVRP with the basic 0-level basic structure has been introduced in Chapter 5. Now we focus on solving the distribution problem with the simple 1-level network  $BG_{11}$ .

The *simple 1-level network* is a 1-level network with only simple facilities as intermediate facilities. In the research area, the distribution problem with the simple 1-level network structure is usually studied as the facility location-allocation (FLA) problem which can be used to determine the mode, the structure and the form of the whole distribution system. It is firstly mentioned by Miehe (1958) and firstly formulated in Cooper (1963). It commonly involves locating a finite set of new facilities with respect to a finite set of existing facilities, and determining the best strategy for allocation of the products between the lower level facilities and the higher level facilities. In this case, the studies with respect to FLA are focused on the strategic phase and tactical phase. There are a large number of relative publications of FLA in strategic phase and tactical phase which have been reviewed in Part II. However, the distribution problem with a simple 1-level network structure in operational phase has not a great importance for the researchers even though it is also a useful and meaning problem. As we have mentioned before, since the facility location problem is a critical issue in strategic phase, the distribution problem with a simple 1-level network structure does not need to study how to locate the facilities, but just needs to focus on the facility allocation. Furthermore, different to the strategic phase and tactical phase, the limitation of the existing resources are usually considered because the decisions on operational phase are made in a short term horizon, generally each or several days, that affect how the products are delivered in the transportation system.

In this chapter, we discuss four variants of facility allocation problems, including *Facility Multi-allocation* (FMA) problem and *Facility Single-allocation* (FSA) problem with the constraints of the vehicle number and a new cost function which are respectively named as *Facility Multi-allocation Problem with Vehicle Number* (FMA-VN) and *Facility Single-allocation Problem with Vehicle Number* (FSA-VN). The traditional FMA and FSA with the considerations of the product volume are respectively named as *Traditional Facility Multi-allocation* (T-FMA) Problem and *Traditional Facility Single-allocation* (T-FSA) Problem.

The remainder of this chapter is organized as follows. In the next section, the formulations of T-FMA and FMA-VN are proposed. In Section 6.3 and 6.4, we introduce the formulations for FMA-VN and FSA-VN. In Section 6.5, the FMA-VN and FSA-VN are reformulated to add cuts to tighten the formulation. And then, the formulations are tested on the standard CAB data set with the addition of the vehicle capacity and the cost for each vehicle. The computational results are shown and analyzed in Section 6.6. The comparison of Single to Multiple Facility allocation problems is illustrated in Section 6.7.

## 6.2 Previous model

FLA science investigates locating physically a set of facilities (resources) in order to minimize the cost of satisfying a set of demands (customers) subject to a set of constraints (see in Hale and Moberg 2003). Using the different objectives, the facility location problem is generally divided into p-median problems, p-center and coverage problems, and other research. The facility multi-allocation problem we focus here is indeed a p-median problem. As stated in Hakimi (1964), it is to find the location of p facilities so as to minimize the total demand-weighted travel distance between destinations and facilities.

P-median problem is also referred to as uncapacitated multi-facility location-allocation problem. For p-median problem, the problem is to locate a set of K new facilities  $f_1, f_2, \dots, f_K$  with respect to a given set of N existing nodes  $V = \{v_1, v_2, \dots, v_N\}$  in order to minimize the total, positively weighted travel distance. The formulation, marked as D-FLA, can be defined as follows:

(D-FLA)

$$\text{Min} \quad \sum_{k=1}^K \sum_{n=1}^N y_{nk} w_n d(f_k, v_n) \quad (6.1)$$

$$\text{s.t.} \quad \sum_{k=1}^K y_{nk} = 1, \quad k = 1, \dots, K \quad (6.2)$$

$$y_{nk} \in \{0, 1\}, \quad n = 1, \dots, N, k = 1, \dots, K \quad (6.3)$$

$$f_k \in \mathbb{R}^2, \quad k = 1, \dots, K.$$

(6.4)

Where, the binary variables  $y_{nk}$  contain the information on the assignment of the existing nodes to new facilities, i.e.  $y_{nk} = 1$  if  $v_n$  is assigned to  $f_k$ , for  $n = 1, \dots, N, k = 1, \dots, K$ ;  $y_{nk} = 0$ , otherwise. The positive weights  $w_n$ ,  $n = 1, \dots, N$  may represent, for example, the demands of the existing node  $v_n$ ,  $n = 1, \dots, N$ . The function  $d$  is assumed to be an arbitrary distance function that is induced by a norm  $\|\bullet\|_d$ . Basically,  $d$  is the Euclidean distance.

However, the objective of the distribution problem is not always to minimize the total weighted distance in practice. The FLA for a private company usually goals to minimize the total transportation cost, which is the sum of the unit volume transportation cost in strategic phase and tactical phase and the sum of the vehicle cost in operational phase. The allocation can be limited to multi-allocation, or to single allocation. As we have mentioned in Chapter 4, single allocation and multi-allocation are two basic attributes of the distribution network. In a single allocation simple 1-level network, each node is assigned to one and only one facility. The formulation D-FLA is actually for this kind of network, in which the constraints (6.4) ensure the nodes are single allocated to one facility. If there is no restriction on the number of facilities to which a non-facility node is allocated, the location problem is the version of the multi-allocation problem. In the next two subsections, we introduce the formulations to each formulation of multiple/single allocation p-hub median problems in strategic phase, in tactical phase and in operational phase.

## 6.3 Facility Multi-allocation Problem

### 6.3.1 Traditional Facility Multi-allocation Problem

In this section, we focus on providing a path-based formulation for the T-FMA confronted by a private company. As we have mentioned earlier, the T-FMA focuses on allocating the non-facilities to one or several facilities in strategic phase and tactical phase. The resources are generally supposed to be unlimited in strategic phase and tactical phase since the strategic and tactical determines mainly refer to the long/mediate time horizon. In this case, the private company goals to minimize the total transport cost which is defined as the sum of the unit volume transportation cost.

We are given a complete graph  $G = (V, E)$ , where  $V = N_{od} \cup F$ , in which  $N_{od} = \{1, 2, \dots, N\}$  is the set of nodes on level 0 and  $F = \{1, \dots, K\}$  is the facility set, and  $E$  is the edge set. For each OD pair  $(i, j)$ , we are given a non-negative flow demand  $W_{ij}$  which characterizes the transport volume between node pair  $(i, j)$ .  $c_{v_1 v_2}$  is the transport cost per unit flow from node  $v_1$  to node  $v_2$ , where  $v_1, v_2 \in V$ .

To define the path-based model, we introduce the following set of variables:

$x_{ijk}$  = the fractional flow from node  $i$  (origin) to node  $j$  (destination), routed via facility  $k$  following the path  $i \rightarrow k \rightarrow j$ .

Then, the cost of the path  $i \rightarrow k \rightarrow j$  is defined as:  $c_{ijk} = c_{ik} + c_{kj}$ .

The path-based formulation of the T-FMA can then be written as follows:

(T-FMA)

$$\text{Min} \quad \sum_{k=1}^K \sum_{j=1}^N \sum_{i=1}^N W_{ij} \cdot x_{ijk} \cdot c_{ijk} \quad (6.5)$$

$$\text{s.t.} \quad \sum_{k=1}^K x_{ijk} = 1, \quad i, j = 1, \dots, N \quad (6.6)$$

$$x_{ijk} \geq 0, \quad i, j = 1, \dots, N, k = 1, \dots, K. \quad (6.7)$$

The objective (6.5) is to minimize the overall transportation cost. Constraints (6.6) ensure that the flow between every OD pair  $(i, j)$  should be routed via one or several facilities.

The above formulation T-FMA is a problem involving  $KN^2$  variables and  $N^2$  linear constraints. Note that the  $x_{ijk}$  may be fractional for multiple allocation problems. However, it is easy to prove the following lemma.

**Lemma 1:** T-FMA has an optimal solution in which all of the  $x_{ijk}$  are either zero or one. In other words, there exists a solution in which all flow between any given pair of nodes is transferred along only one path.

**Proof:** Since  $W_{ij} \geq 0$ ,  $G$  is a complete graph and  $p \geq 1$ , there will always exist a feasible solution to T-FMA.

If  $x_{ijk}$  is fractional for some  $i, j$  and  $k$ , then there exist at least one other fractional  $x_{ijk'}$ , for some  $k' \in F$ , since the constraints (6.6) hold.

Now, if  $c_{ijk} > c_{ijk'}$ , then the solution is not optimal since a cheaper solution could be found by setting  $x_{ijk} + x_{ijk'}$  be  $x_{ijk'}$  and  $x_{ijk} = 0$ . A similar argument holds if  $c_{ijk'} > c_{ijk}$ .

If  $c_{ijk} = c_{ijk'}$ , the flow can be routed via either the route  $i \rightarrow k \rightarrow j$  or the route  $i \rightarrow k' \rightarrow j$ .

Then, we can say that the optimal solution can be obtained by selecting some  $k \in F$  for each  $i, j \in N$  so that  $c_{ijk} = \min_{k' \in K} c_{ijk'}$  and setting  $x_{ijk} = 1$ . (End)

### 6.3.2 Facility Multi-allocation Problem with Vehicle Number

In operational decisions, the private company can own or rent the vehicles. Then the objective is now to minimize the transportation cost which is the sum of the vehicle transport cost.

To define the formulation of *Facility Multi-allocation Problem with Vehicle Number* (FMA-VN), we add  $P$  as the data related to the vehicle capacities and  $C_{v_1 v_2}$  as the data related to the vehicle transport cost, in which:

$Q$  = volumetric capacity of a vehicle transporting products units between node on level 0 and any facility on level 1;

$C_{v_1 v_2}$  = the transportation cost for each vehicle from node  $v_1$  to node  $v_2$ , where  $v_1, v_2 \in V$ .

Then,  $X_{ik}, Y_{kj}$  as general variables are defined as follows:

$X_{ik}$  = the number of vehicles transferred from Node  $i$  to Facility  $k$ ;

$Y_{kj}$  = the number of vehicles transferred from Facility  $k$  to Node  $j$ .

Then, the objective formula of FMA-VN is changed to:

$$\min \sum_{i=1}^N \sum_{k=1}^K C_{ik} X_{ik} + \sum_{k=1}^K \sum_{j=1}^N C_{kj} Y_{kj} \quad (6.8)$$

Thus, the capacity constraints (represented by the number of vehicles used on the edges) are as follows:

$$\sum_{j=1}^N W_{ij} x_{ijk} - Q X_{ik} \leq 0, \quad i = 1, \dots, N, k = 1, \dots, K \quad (6.9)$$

$$\sum_{i=1}^N W_{ij} x_{ijk} - Q Y_{kj} \leq 0, \quad j = 1, \dots, N, k = 1, \dots, K \quad (6.10)$$

The constraint set (6.9)-(6.10) enforce respectively that the volume transported between the facility node and non-facility node are less than the transportation ability of the needed vehicles on the edges.

Now we present the 0/1 Mixed Integer linear programming formulations for the FMA-VN as follows. It is based on the formulation of T-FMA.

(FMA-VN)

$$\text{Min } \sum_{i=1}^N \sum_{k=1}^K C_{ik} X_{ik} + \sum_{k=1}^K \sum_{j=1}^N C_{kj} Y_{kj} \quad (6.8)$$

$$\text{s.t. } \sum_{k=1}^K x_{ijk} = 1, \quad i, j = 1, \dots, N \quad (6.6)$$

$$\sum_{j=1}^N w_{ij} x_{ijk} - QX_{ik} \leq 0, \quad i = 1, \dots, N, \quad k = 1, \dots, K, \quad (6.9)$$

$$\sum_{i=1}^N w_{ij} x_{ijk} - QY_{kj} \leq 0, \quad j = 1, \dots, N, \quad k = 1, \dots, K, \quad (6.10)$$

$$x_{ijk} \geq 0, \quad i, j = 1, \dots, N, \quad k = 1, \dots, K \quad (6.7)$$

$$X_{ik}, Y_{kj} \geq 0 \text{ and integer}, \quad i, j = 1, \dots, N, \quad k = 1, \dots, K. \quad (6.11)$$

Note that FMA-VN is much more complex than T-FMA after considering the vehicle number in the cost function and also the capacity constriction on the edges. The resulting formulation proposed for FMA-VN has  $N^2 + 2KN$  constraints and  $KN^2 + 2KN$  variables of which  $2KN$  are integer. Note that for T-FMA, we present a Lemma to prove that there exists an optimal solution in which all of the  $x_{ijk}$  are either zero or one. Here, we can not provide the similar conclusion because of the capacity constraints (6.9)-(6.10).

## 6.4 Facility Single-allocation Problem

### 6.4.1 Traditional Facility Single-allocation Problem

In this section, we introduce the path-based formulations for Facility single-allocation problem confronted by a private company in strategic phase and tactical phase. As mentioned before, each non-facility node is assigned to one and only one facility in facility single-allocation problem.

This is the major different point between the T-FMA problem and T-FSA problem. The formulation T-FMA can be modified by making the allocation choice of the non-facility node  $i$  independent of its destination  $j$ . To do that, we introduce the allocation variables  $y_{ik}$ . Then,  $y_{ik} = 1$ , if the node  $i$  is allocated to facility  $f_k$ ;  $y_{ik} = 0$ , otherwise.

Based on T-FMA, the following mixed 0/1 formulation T-FSA can then be presented:

(T-FSA)

$$\text{Min} \quad \sum_{k=1}^K \sum_{j=1}^N \sum_{i=1}^N W_{ij} \cdot x_{ijk} \cdot c_{ijk} \quad (6.5)$$

$$\text{s.t.} \quad \sum_{k=1}^K x_{ijk} = 1 \quad i, j = 1, \dots, N, \quad (6.6)$$

$$\sum_{k=1}^K y_{ik} = 1 \quad i = 1, \dots, N, \quad (6.12)$$

$$x_{ijk} - y_{ik} \leq 0 \quad i = 1, \dots, N, k = 1, \dots, K, \quad (6.13)$$

$$y_{ik} \in \{0, 1\} \quad i = 1, \dots, N, k = 1, \dots, K, \quad (6.14)$$

$$x_{ijk} \geq 0 \quad i, j = 1, \dots, N, k = 1, \dots, K. \quad (6.7)$$

The changes from the T-FMA occur in constraints (6.12) and (6.13).

Constraints (6.12) ensure that each node must be allocated to exactly one facility;

Constraints (6.13) state that for every origin and every facility  $f_k$ , a flow through the path  $i \rightarrow k \rightarrow j$  is feasible only when the node  $i$  is allocated to the facility  $f_k$ , and vice versa.

By using the definition of allocation variables  $y_{ik}$ , the constraints (6.12) and (6.13) are added to the formulation of the T-FMA. The resulting formulation T-FSA has  $KN^2 + KN$  variables, of which  $KN$  are binary. It requires  $N^2 + (K + 1)N$  linear constraints.

## 6.4.2 Facility Single-allocation Problem with Vehicle Number

In the previous sections, we have formulated the Traditional Facility Single-allocation Problem (T-FSA) which is the facility single-allocation problem in strategic phase and tactical phase for the simple 1-level distribution network. Now we introduce the facility single-allocation problem in operational phase, marked as FSA-VN, in which the objective cost is sum of the vehicle transport cost.

By introducing the allocation variables  $y_{ik}$ , the FMA-VN can be similarly modified to describe FSA-VN. The new formulation for FSA-VN can be defined as follows:

(FSA-VN)

$$\mathbf{Min} \quad \sum_{i=1}^N \sum_{k=1}^K C_{ik} X_{ik} + \sum_{k=1}^K \sum_{j=1}^N C_{kj} Y_{kj} \quad (6.8)$$

$$\text{s.t.} \quad \sum_{k=1}^K x_{ijk} = 1, \quad i, j = 1, \dots, N \quad (6.6)$$

$$\sum_{j=1}^N W_{ij} x_{ijk} - QX_{ik} \leq 0, \quad i = 1, \dots, N, \quad k = 1, \dots, K \quad (6.9)$$

$$\sum_{i=1}^N W_{ij} x_{ijk} - QY_{kj} \leq 0, \quad j = 1, \dots, N, \quad k = 1, \dots, K \quad (6.10)$$

$$\sum_{k=1}^K y_{ik} = 1, \quad i = 1, \dots, N, \quad (6.12)$$

$$x_{ijk} - y_{ik} \leq 0, \quad i = 1, \dots, N, \quad k = 1, \dots, K \quad (6.13)$$

$$y_{ik} \in \{0, 1\}, \quad i = 1, \dots, N, \quad k = 1, \dots, K \quad (6.14)$$

$$x_{ijk} \geq 0, \quad i, j = 1, \dots, N, \quad k = 1, \dots, K \quad (6.7)$$

$$X_{ik}, Y_{kj} \geq 0 \text{ and integer}, \quad i, j = 1, \dots, N, \quad k = 1, \dots, K \quad (6.11)$$

Comparing with the T-FSA, the constraints (6.9) and (6.10) are added to FSA-VN. The corresponding explanations of these constraints have been already presented in the previous section. The resulting formulation for FSA-VN involves  $N^2 + (3K + 1)N$  constraints and  $KN^2 + 3KN$  variables of which  $KN$  are binary and  $2KN$  are integer.

## 6.5 Reformulations

In previous sections, we formulated respectively the traditional facility multi-allocation problem (T-FMA), the facility multi-allocation problem with vehicle number (FMA-VN), the traditional facility single-allocation problem (T-FSA) and the facility single-allocation problem with vehicle number (FSA-NV). For the new formulations of FMA-VN in Section 6.3, although the LP relaxation is, in general, tight, it sometimes leads the formulation to be unable to solve due to memory limitation. It is supported by our computational results, for example, the computational time to solve the instance with  $N=15$ ,  $K=3$  in FMA-VN of this linear program is more than 20 hours of CPU time.

To deal with this, we add cuts to tighten the formulation of FMA-VN. These are:

$$X_{ik} \leq \left\lceil \sum_{j=1}^N W_{ij} / Q \right\rceil, \quad i = 1, \dots, N, k = 1, \dots, K \quad (6.15)$$

$$Y_{kj} \leq \left\lceil \sum_{i=1}^N W_{ij} / Q \right\rceil, \quad j = 1, \dots, N, k = 1, \dots, K \quad (6.16)$$

The constraints (6.15) enforce that the vehicle number on edge  $(i, k)$  can not be greater than the vehicle number needed to transport all the goods offered by origin  $i$ ;

The constraints (6.16) ensure that the vehicle number on edge  $(k, j)$  can not be greater than the vehicle number needed to transport all the goods demanded by destination  $j$ .

The constraints (6.15) and (6.16) can be further transformed into the constraints (6.17) and constraints (6.18) which are chosen to maximize the constraint violation of the current solution.

$$X_{ik} \leq \sum_{j=1}^N W_{ij} / Q + 1, \quad i = 1, \dots, N, k = 1, \dots, K, \quad (6.17)$$

$$Y_{kj} \leq \sum_{i=1}^N W_{ij} / Q + 1, \quad j = 1, \dots, N, k = 1, \dots, K, \quad (6.18)$$

Then, if we add these valid inequalities to the problem FMA-VN, the following problem, denoted as FMA-VN', is obtained:

(FMA-VN')

$$\text{Min} \quad \sum_{i=1}^N \sum_{k=1}^K C_{ik} X_{ik} + \sum_{k=1}^K \sum_{j=1}^N C_{kj} Y_{kj} \quad (6.8)$$

$$\text{s.t.} \quad \sum_{k=1}^K x_{ijk} = 1, \quad i, j = 1, \dots, N \quad (6.6)$$

$$\sum_{j=1}^N W_{ij} x_{ijk} - QX_{ik} \leq 0, \quad i = 1, \dots, N, k = 1, \dots, K, \quad (6.9)$$

$$\sum_{i=1}^N W_{ij} x_{ijk} - QY_{kj} \leq 0, \quad j = 1, \dots, N, k = 1, \dots, K, \quad (6.10)$$

$$X_{ik} \leq \sum_{j=1}^N W_{ij} / Q + 1, \quad i = 1, \dots, N, k = 1, \dots, K, \quad (6.17)$$

$$Y_{kj} \leq \sum_{i=1}^N W_{ij} / Q + 1, \quad j = 1, \dots, N, k = 1, \dots, K, \quad (6.18)$$

$$x_{ijk} \geq 0, \quad i, j = 1, \dots, N, k = 1, \dots, K, \quad (6.7)$$

$$X_{ik}, Y_{kj} \geq 0 \text{ and integer}, \quad i, j = 1, \dots, N, k = 1, \dots, K. \quad (6.11)$$

The various experiments for FMA-VN' indicate the additional constraints (6.17) and (6.18) are very useful in practice which will be shown in the next section. In addition, we suppose that the constraints (6.17) and (6.18) can also be useful to strengthen the formulation of FSA-VN. We do not mention it here because our computational results show us the FSA-VN is already tightening enough for the experience data. In the next section, we present numerical results comparing the performance of the different formulations in practice.

## 6.6 Computational results

In this section we present the results of computational experiments to evaluate the effectiveness of the formulations discussed in Section 6.4 and Section 6.5. All numerical tests are carried out on an Intel(R) Core(TM)2 CPU operating at 1.66GHz and equipped with 1 Gig RAM. The algorithms were coded in C. We use branch-and-bound algorithm from Cplex version 9.0 for solving the linear programs. It is well suited for LP problems. The computational comparisons are based on the CAB data set published previously.

- CAB: The Civil Aeronautics Board (CAB) data set is a benchmark data set frequently used in the literature to test p-hub problems. A detailed description can be found in Fotheringham(1983) or O'Kelly(1987). It can be obtained from the site of OR-Library (<http://people.brunel.ac.uk/~mastijb/jeb/orlib/phubinfo.html>). The data set consists of 25 interacting nodes along with their flow volumes and transportation costs (see in Annex I). The 10, 15 and 20 node subsets are found by taking the top 10x10 15x15 and 20x20 sub-matrices. However, the dataset does not contain the vehicle cost on each edge. We name vehicle A be the type of vehicles between non-facility nodes and facilities. We define the capacity of this type of vehicle is  $Q$ , then the vehicle transportation cost on the edge arcs is generated as follows: Vehicle Transportation Cost = unit volume cost \* (capacity of vehicle A). In the test, we determine the vehicle capacity  $Q = 80m^3$  which is the capacity of a standard truck. Then, we can generate the data for our FMA-VN and FSA-VN. For example, the volumetric transportation cost between Node 1 and Node 3 is 0.9464954 per  $m^3$ . The vehicle transportation cost on edge (1, 3) is then 75.71963280 which is 80 times of 0.9464954. Additionally, the selection of the facility set is determined according to the data in Skorin-Kapov et al. (1996). Annex II and III show respectively the samples of FMA and FSA.

In the following subsections, we first present the detail results for the T-FMA and FMA-VN, followed by the results for the T-FSA and FSA-VN cases.

### 6.6.1 Facility multi-allocation problem

Our computational results for facility multi-allocation problem involving T-FMA, FMA-VN and FMA-VN' are respectively displayed in Table 6.1-6.2. The column "Sample Number" provides the sample we select, "n" is the total number of nodes in the network including facility nodes and non-facility nodes, and "p" is the number of facility nodes. The selected facilities for the corresponding sample are shown in the column "Facility". Both of tables show the transportation cost ("objective" in the table) and the computational time (in second) to solve the formulation. Note that we display just 11 samples (10 Node Samples and 15 Node Samples) in Table 6.2 due to the computational complexity of the FMA-VN and FMA-VN'. The average vehicle load rate (AVLR) of FMA-VN' is presented in Table 6.2. The average VLR in Sample 3 and Sample 8 are less than 0.5. In this case, we propose to change to smaller vehicles.

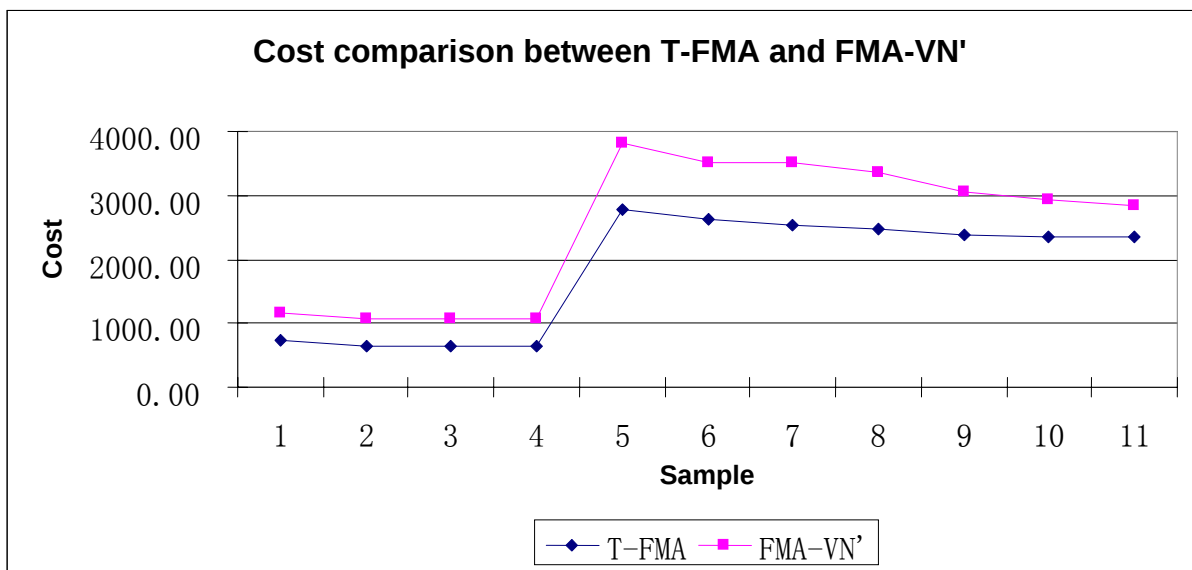
**Table 6.1:** Results for T-FMA

| <b>Sample Number</b> | <b>n</b> | <b>p</b> | <b>Facility</b> | <b>Objective</b> | <b>Computational<br/>time (sec.)</b> |
|----------------------|----------|----------|-----------------|------------------|--------------------------------------|
| 1                    | 10       | 2        | 7,9             | 720.50           | 0.00                                 |
| 2                    |          | 3        | 4,6,7           | 653.71           | 0.00                                 |
| 3                    |          | 4        | 3,4,6,7         | 636.30           | 0.00                                 |
| 4                    |          | 4        | 2,4,6,7         | 631.57           | 0.02                                 |
| 5                    | 15       | 2        | 4,12            | 2787.51          | 0.02                                 |
| 6                    |          | 2        | 4,7             | 2635.83          | 0.00                                 |
| 7                    |          | 3        | 4,7,12          | 2533.94          | 0.00                                 |
| 8                    |          | 3        | 1,4,7,          | 2458.10          | 0.00                                 |
| 9                    |          | 4        | 4,7,12,14       | 2383.04          | 0.02                                 |
| 10                   |          | 4        | 1,4,7,12        | 2356.62          | 0.00                                 |
| 11                   |          | 4        | 1,4,7,8         | 2339.50          | 0.00                                 |

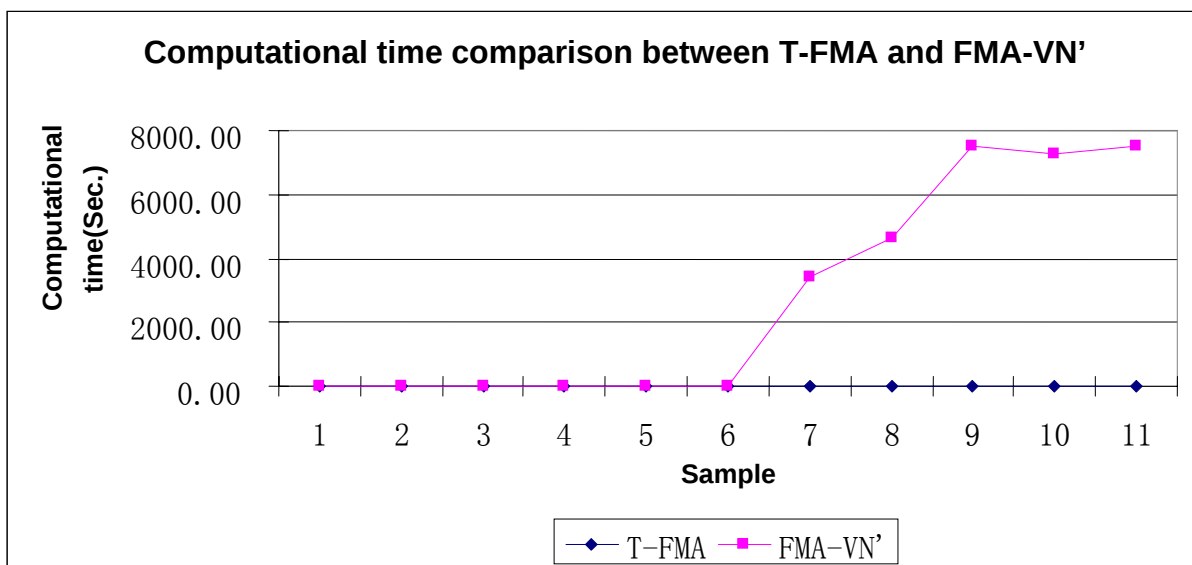
**Table 6.2:** Results for FMA-VN and FMA-VN'

| SampleNumber | n  | p | Facility  | Objective<br>FMA-VN' | Computational time (sec.) |          | AVLR   |
|--------------|----|---|-----------|----------------------|---------------------------|----------|--------|
|              |    |   |           |                      | FMA-VN                    | FMA-VN'  |        |
| 1            | 10 | 2 | 7,9       | 1152.36              | 0.08                      | 0.05     | 0.7393 |
| 2            |    | 3 | 4,6,7     | 1056.15              | 1.03                      | 0.78     | 0.4679 |
| 3            |    | 4 | 3,4,6,7   | 1056.15              | 1.88                      | 3.59     | 0.3217 |
| 4            |    |   | 2,4,6,7   | 1056.15              | 6.83                      | 4.28     | 0.6729 |
| 5            | 15 | 2 | 4,12      | 3808.77              | 0.70                      | 0.99     | 0.6679 |
| 6            |    |   | 4,7       | 3525.90              | 0.30                      | 1.11     | 0.7200 |
| 7            |    | 3 | 4,7,12    | 3504.46              | 5407.46                   | 3396.64  | 0.6830 |
| 8            |    |   | 1,4,7,    | 3477.53              | 6434.79                   | 4615.23  | 0.2319 |
| 9            |    | 4 | 4,7,12,14 | 3052.26              | 11693.02                  | 9458.38  | 0.6962 |
| 10           |    |   | 1,4,7,12  | 2944.53              | 11879.11                  | 9867.59  | 0.7872 |
| 11           |    |   | 1,4,7,8   | 2840.82              | 12958.69                  | 10286.45 | 0.7239 |

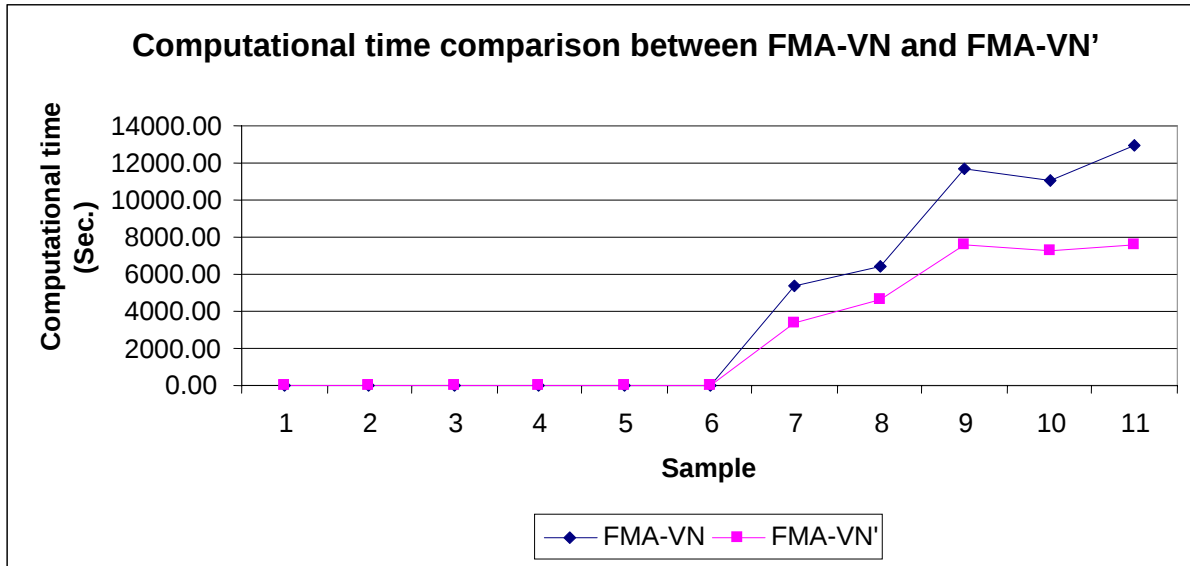
The objective comparison between T-FMA and FMA-VN is shown in Fig.6.1. As we can see, the cost of FMA-VN (Rose line) is always greater than the cost of T-FMA (Blue line). It is because of the existence of the non-full load of the vehicles. Fig. 6.2 gives the computational time comparison between T-FMA and FMA-VN. We can see the computational time required by FMA-VN is much more than the T-FMA. It is because the addition of the variables of the vehicle number that leads the formulation to become larger and the Lemma 1 to be invalid. Meanwhile, the edge capacity is restricted by the transportation ability of the vehicles on the edge. From Fig. 6.2, we can see that the computational time required by FMA-VN' increases rapidly when the number of nodes and facilities increases. Fig. 6.3 illustrates the comparison of the CPU time between FMA-VN and FMA-VN'. As we can see, the FMA-VN' appears to be a more efficient formulation than FMA-VN in terms of computational time. Particularly for large samples, FMA-VN' requires less computational time than FMA-VN.



**Fig. 6.1** Cost comparison between T-FMA and FMA-VN'



**Fig. 6.2** Computational time comparison between T-FMA and FMA-VN'



**Fig. 6.3** Computational time comparison between FMA-VN and FMA-VN'

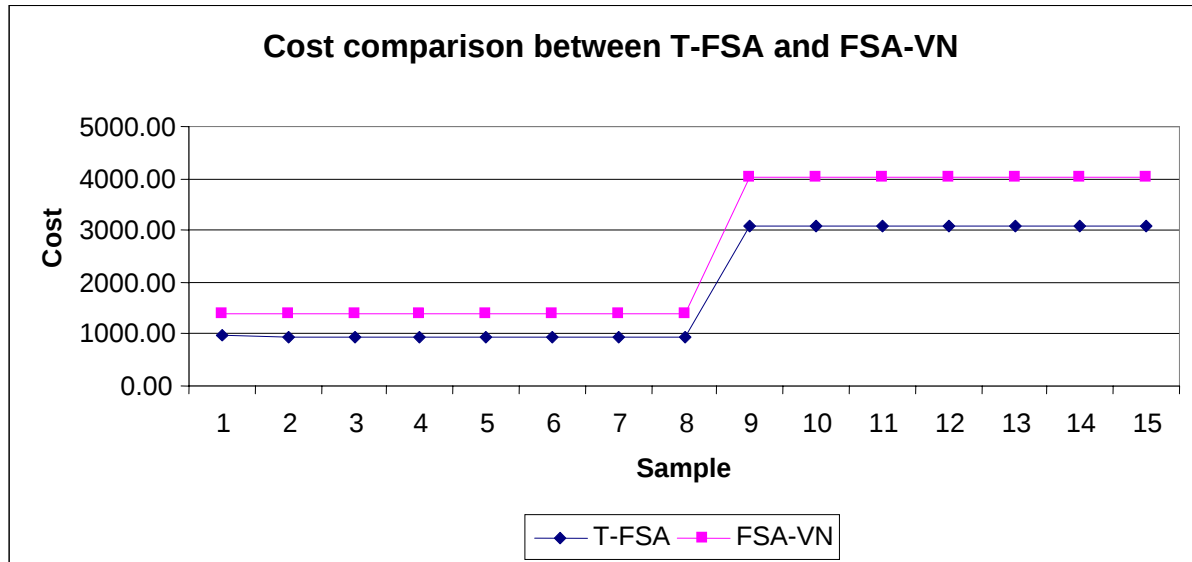
### 6.6.2 Facility single-allocation problem

Table 6.3 shows the computational results of the samples 1-15 for the single-allocation problems including T-FSA and FSA-VN. The transportation costs (“objective” column in Table 6.3) and the computational time (in second) of T-FSA and FSA-VN are also presented. Furthermore, the average vehicle load rate (AVLR) is additionally provided. We can see that the AVLR of Sample 2 ( $n=10$ ,  $p=2$  and facility=4, 7) does not exceed 0.5, which appears the same result with the FMA-VN, it means that we need to select a smaller type of vehicles.

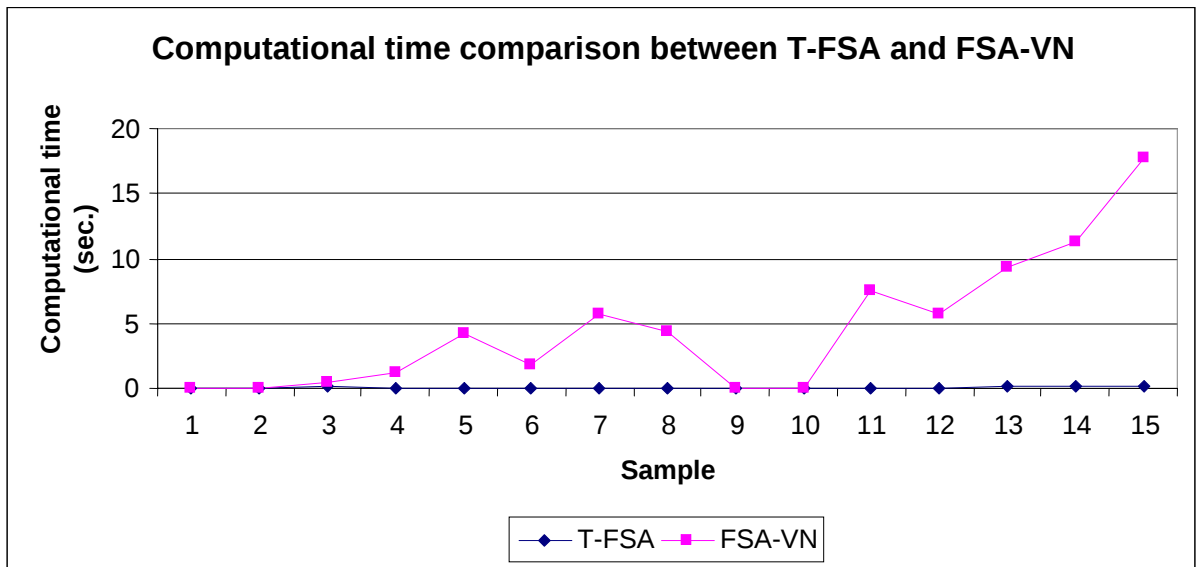
**Table 6.3:** Results for T-FSA and FSA-VN

| Sample Number | n  | p | Facility  | Objective |         | Computational time (sec.) |        | AVLR   |
|---------------|----|---|-----------|-----------|---------|---------------------------|--------|--------|
|               |    |   |           | T-FSA     | FSA-VN  | T-FSA                     | FSA-VN |        |
| 1             | 10 | 2 | 7,9       | 970.06    | 1390.39 | 0                         | 0.03   | 0.7393 |
| 2             |    | 2 | 4,7       | 930.15    | 1380.99 | 0                         | 0.03   | 0.4863 |
| 3             |    | 3 | 4,6,7     | 930.15    | 1380.99 | 0.20                      | 0.45   | 0.3217 |
| 4             |    | 3 | 4,7,9     | 930.15    | 1380.99 | 0.03                      | 1.20   | 0.6729 |
| 5             |    | 4 | 3,4,6,7   | 930.15    | 1380.99 | 0.05                      | 4.23   | 0.7017 |
| 6             |    | 4 | 4,6,7,8   | 930.15    | 1380.99 | 0.05                      | 1.77   | 0.7200 |
| 7             |    | 4 | 4,6,7,9   | 930.15    | 1380.99 | 0.05                      | 5.64   | 0.6174 |
| 8             |    | 4 | 1,4,7,9   | 930.15    | 1380.99 | 0.05                      | 4.34   | 0.6962 |
| 9             | 15 | 2 | 4,12      | 3090.18   | 4024.45 | 0.02                      | 0.02   | 0.5926 |
| 10            |    | 5 | 4,11      | 3090.18   | 4024.45 | 0.02                      | 0.03   | 0.6280 |
| 11            |    | 3 | 4,7,12    | 3090.18   | 4024.45 | 0.06                      | 7.45   | 0.7552 |
| 12            |    | 3 | 4,7,8     | 3090.18   | 4024.45 | 0.06                      | 5.74   | 0.8083 |
| 13            |    | 4 | 4,7,12,14 | 3090.18   | 4024.45 | 0.09                      | 9.34   | 0.7994 |
| 14            |    | 4 | 1,4,7,12  | 3090.18   | 4024.45 | 0.14                      | 11.28  | 0.7590 |
| 15            |    | 4 | 1,4,7,8   | 3090.18   | 4024.45 | 0.14                      | 17.67  | 0.7452 |

The cost comparison between T-FSA and FSA-VN is shown in Fig. 6.4. The cost of FSA-VN (Rose line) is always higher than T-FSA (Blue line) for each sample due to the non-full load of the vehicles. As we can see in Fig. 6.4, the samples 2-8 (10 node samples) have all the same cost, and the costs of Sample 9-15 are the same. It is because the allocation does not change when a new facility is added into the network. It is a characteristic of the single-allocation distribution network.

**Fig. 6.4** Cost comparison between T-FSA and FSA-VN

The computational time comparison between T-FSA and FSA-VN is shown in Fig. 6.5. We can see the computational time required by FSA-VN is greater than the T-FSA for every sample. Then, a similar conclusion with multi-allocation problem can be presented. Because of the addition of the variables of the vehicle numbers, the formulation becomes larger.



**Fig. 6.5** Computational time comparison between T-FSA and FSA-VN

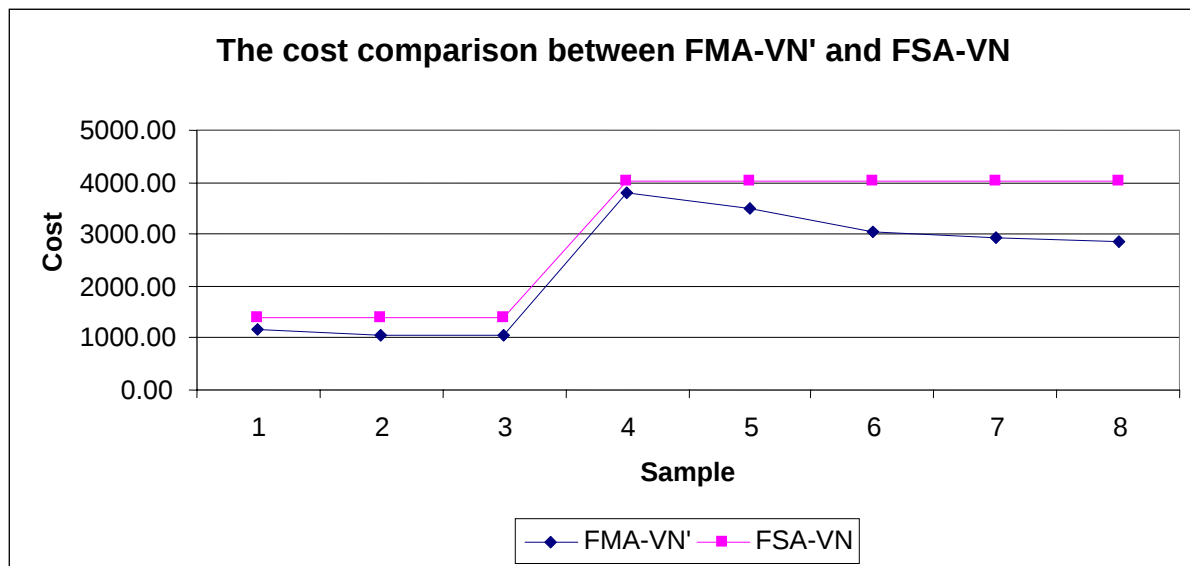
## 6.7 Comparison of Single to Multiple Facility allocation problems

Now, we compare, in Table 6.4, the performance of our FMA-VN' with FSA-VN for the samples with the same n, p and facilities.

**Table 6.4** Comparison between FMA-VN' and FSA-VN

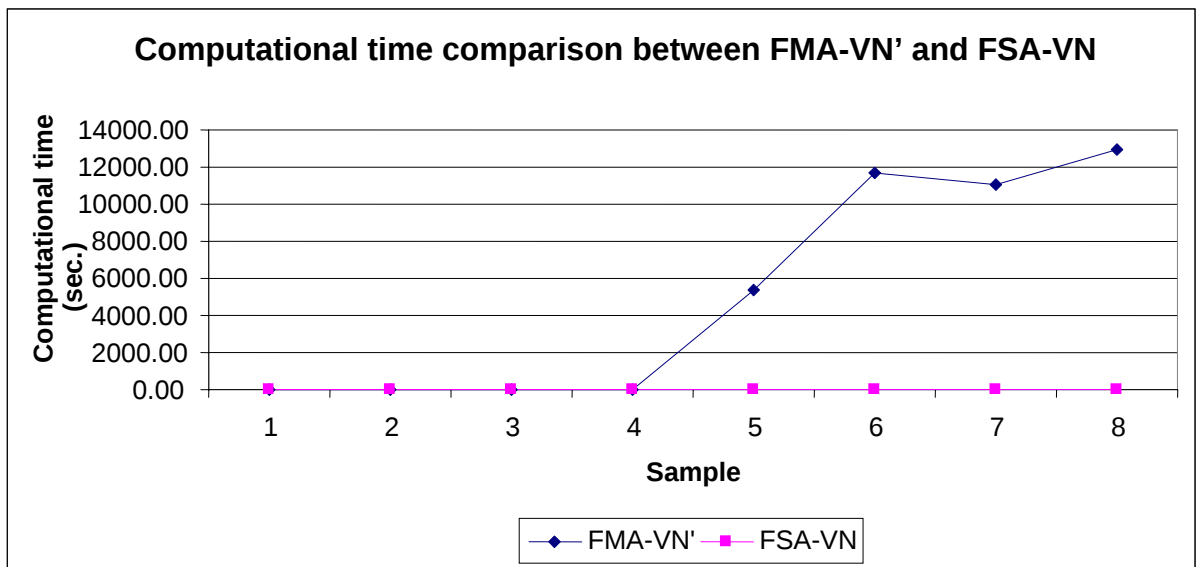
| Sample Number | n  | p | Facility  | Objective |         | Computational time (sec.) |        |
|---------------|----|---|-----------|-----------|---------|---------------------------|--------|
|               |    |   |           | FMA-VN'   | FSA-VN  | FMA-VN'                   | FSA-VN |
| 1             | 10 | 2 | 7,9       | 1152.36   | 1390.39 | 0.08                      | 0.03   |
| 2             | 10 | 3 | 4,6,7     | 1056.15   | 1380.99 | 1.03                      | 0.45   |
| 3             | 10 | 4 | 3,4,6,7   | 1056.15   | 1380.99 | 1.88                      | 4.23   |
| 4             | 15 | 2 | 4,12      | 3808.77   | 4024.45 | 0.70                      | 0.02   |
| 5             | 15 | 3 | 4,7,12    | 3504.46   | 4024.45 | 5407.46                   | 7.45   |
| 6             | 15 | 4 | 4,7,12,14 | 3052.26   | 4024.45 | 11693.02                  | 9.34   |
| 7             | 15 | 4 | 1,4,7,12  | 2944.53   | 4024.45 | 11089.11                  | 11.28  |
| 8             | 15 | 4 | 1,4,7,8   | 2840.82   | 4024.45 | 12958.69                  | 17.67  |

The cost of FSA-VN is always greater than FMA-VN' (see in Fig.6.6) since the single allocation hypothesis is a special case of multiple allocation. In the Fig. 6.6, the pink line (the cost of FSA-VN) is always above the blue line (the cost of FMA-VN').



**Fig.6.6** The cost comparison between FMA-VN' and FSA-VN

Moreover, the computational time for FMA-VN' (blue line in Fig.6.7) is longer than for FSA-VN (pink line) since there are more constraints and variables in FMA-VN'.



**Fig. 6.7** Computational time comparison between FMA-VN' and FSA-VN

## 6.8 Conclusion

The *simple 1-level network* is one of the basic distribution networks. It is a 1-level network with only simple facilities as intermediate facilities. In the research area, the distribution problem of the simple 1-level network structure is usually studied as a facility location-allocation (FLA) problem in which the p-median facility location problem is widely researched for private companies in the strategic and tactical phases of their planning management.

In this chapter, we have provided a new path-based formulation for the facility single/multi-allocation problems in strategic and tactical phase, denoted as T-FSA and T-FMA. Two other new facility allocation problems, the facility multi/single-allocation problems in the operational phase, have been introduced, in which the limitation of the resources are considered and the corresponding formulations with the limited vehicle number are presented. We have defined respectively these two new formulations as FMA-VN and FSA-VN. Additionally, a reformulation FMA-VN' has been provided to tighten the formulation of FMA-VN, and to improve the solution process. All these formulations have been tested on the benchmark set, CAB. The computational results have been presented and compared in the end of this chapter.

In all cases, the objective with vehicle costs is bigger than the objective with volumetric costs because of the non-full vehicle load. On the other hand, the objective of the facility multi-allocation is better than the objective of the facility single-allocation but more difficult to solve. That is to say facility the multi-allocation problem requires more computational time than the facility single-allocation problem. It is because that the facility single-allocation problem is tighter than facility multi-allocation problem.

In the next chapter, the distribution problem for another basic distribution network, the hub 1-level distribution network will be studied and new formulations will be given, the computational results on the CAB will be illustrated and compared.

# CHAPTER 7: Hub Allocation Problem for Basic 1-level Network

## 7.1 Introduction

The hub 1-level network  $BG_{12}$  is one of the two types of the 1-level basic network structures. The other one, the simple 1-level network  $BG_{11}$  has been introduced in the previous chapter. Now, we focus on solving the distribution problem with the hub 1-level network  $BG_{12}$  in this chapter.

In the hub 1-level distribution network, there are origins, destinations, and just one level of intermediate facilities known as hubs. The trips of the transportation start from the origins. Then, the goods are transferred, resorted and consolidated into larger flows at hubs, and finally exchanged between hubs on the same level by high-frequency, high-capacity services. That is to say that the good flows from the same origin with different destinations are concentrated or consolidated on their route to the hub and are combined with flows that have different origins but the same destinations. Here, the hubs are central facilities designed to act as switching nodes for inter-nodal flows and are interconnected. The hub 1-level distribution network is highly applied in the real world. The less-than-truckload (LTL) company is one of the typical applications, which serves those customers whose shipment, between one OD pair, would not fill the truck capacity by weight or volume. Then many LTL trucking companies operate their network as hub 1-level distribution network to achieve high level of efficiency as well as to improve customer service and short transit times.

In the research area, the distribution problem with the simple 1-level network structure is usually studied as Hub Location Problem (HLP) which has been precisely presented in Part II. HLP generally involves locating hub facilities and allocating demand nodes to hubs in order to route the traffic between OD pairs. It is firstly pioneered by O'Kelly (1987). Recently, the number of the researches related to HLP has steeply increased since the year 2000. These researches can be mainly summarized into three aspects: p-hub center problem, p-hub covering problem and p-hub median problem. Recall that, p-hub median location problem aims to minimize the total transportation cost to node interactions between nodes of the network via the set of hubs. Therefore, the real transportation problems for the private companies are generally defined as p-hub median problem. Furthermore, because of the complexity of the HLP, the relative researches primarily focus on the 2-stop. 2-stop means there are at most two stops (indeed hubs) on the trip between OD pair. Fig.7.1 shows a typical 2-stop location-distribution network. If node  $i$  and node  $j$  are assigned to the same hub, the flow from  $i$  is first sent to hub and then to node  $j$ . If node  $i$  and node  $j$  are assigned to the different hubs, the flow from node  $i$  to node  $j$  is first sent to the hub  $k$  to which node  $i$  is

assigned, and to the other hub  $m$ , and then to node  $j$ . This situation is marked by black thick lines in Fig.7.1.

In this chapter, we focus on the 2-stop single/multi-allocation p-hub median problems with a new transportation cost in operation phase. Notably, we do not mention “2-stop” afterwards because it does not lead to any misunderstanding. Then, the 2-stop single/multi-allocation p-hub median problems are respectively named as the single-allocation p-hub median problem (SAPhM) and the multi-allocation p-hub median problem (MApHM). Similar with facility location-allocation problem, the studies on HLP are mainly referred to the transportation problem in strategic phase or in tactical phase, and rarely in operational phase. The hub locations are, in fact, fixed in the operational phase, and then the HLP involves only the Hub Allocation Problem (HAP). Furthermore, the companies more and more prefer to rent vehicles from third party logistic companies to organize their distribution process. Then the transportation cost is usually calculated by vehicle numbers. This type of transportation cost can be also reasonably employed when the companies use their own vehicles because the drivers are paid day by day. In this chapter, we consider a new formulation with the cost arising from this context.

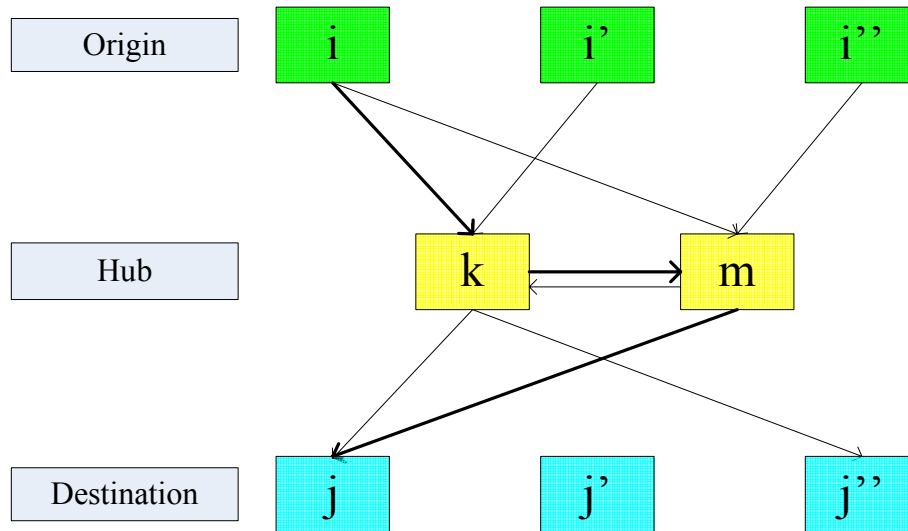


Fig.7.1 Typical 2-stop hub location-distribution network

As stated in Alumer S and Kara B.Y. (2008), studies on the hub location problem often have three assumptions:

1. the  $p$  hubs are fully connected by  $p(p-1)/2$  hub arcs, but the other non-hub nodes interact only via hub and each origin and destination node must be connected to one or several hubs;
2. there is a discounted factor  $\alpha$  with  $0 \leq \alpha \leq 1$  for the transportation cost between 2 hubs, presumably to capture the economies of scale from consolidated transportation;
3. no direct service between two non-hub nodes is allowed.

In this section, we assume that the problems we want to solve satisfy the three hypotheses mentioned above.

The remainder of this chapter is organized as follows. In the next section, previous model for the multi-allocation p-hub median problem is firstly presented, and then our formulation for MAH is proposed. In Section 7.3, we introduce the previous formulation and our proposed formulation for SAH. In Section 7.4, the formulations are tested on the standard CAB data set with the addition of the vehicle capacity and the cost for each vehicle.

## 7.2 2-stop multiple allocation p-hub median problem

Single allocation and multi-allocation are two basic types of hub networks. In single allocation problem, each non-hub node is assigned to only one hub. If there is no restriction on the number of hubs to which a non-hub node may be allocated, the location problem is the version of the multi-allocation problems. In this section, we will introduce the formulations and the corresponding simplification to the formulation of MAH for our problem.

### 7.2.1 Previous model

Campbell (1994) is the first to present a mixed 0/1 integer linear programming formulation. Using the notation from the Campbell (1994), let us define the following variables:

$x_{ijkm}$  = the fractional flow from node  $i$  (origin) to node  $j$  (destination), routed via hubs at nodes  $k$  and  $m$  in that order;

$y_k = 1$  if node  $k$  is a hub and 0 otherwise.

The input data are given as follows:

$n$  = the number of nodes;

$p$  = the required number of hubs to open;

$W_{ij}$  = the flow from node  $i$  to node  $j$ ;

$c_{ij}$  = the cost per unit flow from node  $i$  to node  $j$ ;

$\alpha \leq 1$  is the discount factor on the unit cost of flow between hubs. So, the cost per unit of flow from node  $i$  (origin) to node  $j$  (destination), routed via hubs  $k$  and  $m$  in that order, is given by  $c_{ijkm} = c_{ik} + \alpha c_{km} + c_{mj}$ . It is assumed that  $c_{ii} = 0$ ,  $i = 1, \dots, n$ , so the formula for  $c_{ijkm}$  remains valid when  $i$  and/or  $j$  is a hub.

Then, Campbell (1994) formulated the multi-allocation  $p$ -hub median location (MApHML) problem as follows:

(MApHML)

$$\min \sum_{i=1}^n \sum_{j=1}^n \sum_{k=1}^n \sum_{m=1}^n W_{ij} c_{ijkm} x_{ijkm} \quad (7.2.0)$$

$$\text{s.t.} \quad \sum_{k=1}^n y_k = p \quad (7.2.1)$$

$$\sum_{k=1}^n \sum_{m=1}^n x_{ijkm} = 1 \quad i, j = 1, \dots, n \quad (7.2.2)$$

$$x_{ijkm} \leq y_k \quad i, j, k, m = 1, \dots, n \quad (7.2.3)$$

$$x_{ijkm} \leq y_m \quad i, j, k, m = 1, \dots, n \quad (7.2.4)$$

$$y_k \in \{0, 1\} \quad k = 1, \dots, n \quad (7.2.5)$$

$$x_{ijkm} \geq 0 \quad i, j, k, m = 1, \dots, n \quad (7.2.6)$$

The objective is to minimize the overall transportation cost, which is the transportation cost sum of the volumetric unit subject to:

Constraints (7.2.1) ensure that there are exactly  $p$  hubs, constraints (7.2.2) state that the flow between every OD pair  $(i, j)$  should be routed via some hub pair. And constraints (7.2.3) and (7.2.4) assure that the flows can be routed only via the hubs. Variables  $y_k$ , serving as hub indicators, are restricted to be 0 or 1, and flow variables  $x_{ijkm}$  cannot have values bigger than 1. Due to constraints (7.2.2), it is clear that variables  $x_{ijkm}$  can not be bigger than 1.

This problem MAPHML is a very large mixed 0/1 linear problem (with  $n + n^4$  variables,  $1 + n^2 + 2 \cdot n^4$  constraints). Campbell points out that the optimal solution is obtained when all  $x_{ijkm}$  equal to zero or one. It is because that, since there are no capacity constraints on the arcs, the total flow for each OD pair should be routed via the least costly path. Two years later, Ernst and Krishnamoorthy (1996) provide the similar conclusion and state the corresponding mathematic proof.

As stated in Skorin-Kapov et al. (1996), the constraints (7.2.3) and (7.2.4) are not “strong enough” with respect to hub locations because the fractional solutions are obtained when relaxing the integrality of the variables  $y_k$ . Relaxing integrality gives lots of “partial” hubs depending on the cheapest routes indicated via  $x_{ijkm}$  variables, since there are no fixed costs for opening the hubs. Then, Skorin-Kapov et al. (1996) provide a modified formulation for MAPHML in Campbell (1994). They replace constraints (7.2.3) and (7.2.4) by:

$$\sum_{m=1}^n x_{ijkm} \leq y_k \quad i, j, k=1, \dots, n \quad (7.2.7)$$

$$\sum_{k=1}^n x_{ijkm} \leq y_m \quad i, j, m=1, \dots, n \quad (7.2.8)$$

Then, the formulation of Skorin-Kapov et al. (1996) for MAPHMP is as follows:

(MAPHM')

$$\min \sum_{i=1}^n \sum_{j=1}^n \sum_{k=1}^n \sum_{m=1}^n W_{ij} C_{ijkm} x_{ijkm} \quad (7.2.0)$$

$$\text{s.t.} \quad \sum_{k=1}^n y_k = p \quad (7.2.1)$$

$$\sum_{k=1}^n \sum_{m=1}^n x_{ijkm} = 1 \quad i, j = 1, \dots, n \quad (7.2.2)$$

$$\sum_{m=1}^n x_{ijkm} \leq y_k \quad i, j, k=1, \dots, n \quad (7.2.7)$$

$$\sum_{k=1}^n x_{ijkm} \leq y_m \quad i, j, m=1, \dots, n \quad (7.2.8)$$

$$y_k \in \{0, 1\} \quad k=1, \dots, n \quad (7.2.5)$$

$$x_{ijkm} \geq 0 \quad i, j, k, m=1, \dots, n \quad (7.2.6)$$

It is a formulation with  $n + n^4$  variables and  $1 + n^2 + 2 \cdot n^3$  constraints. Compared to MApHML in Campbell (1994), the constraint set has been reduced by  $2 \cdot n^3(n-1)$  constraints. The relaxation of MApHM' is tighter than the linear relaxation of MApHML, because every nonnegative solution satisfying the constraint sets of MApHM' satisfies (7.2.1)-(7.2.4), but not vice versa.

## 7.2.2 Proposed formulation

Now, we will provide a new formulation for the transportation in operational phase, in which a new cost function and the capacity constraints of the vehicles on each edge are considered. Moreover, the transportation cost is calculated as the total cost of vehicles, but not by the total transportation cost of the volumetric units.

Gendron and Semet (2009) present two mixed-integer programming (MIP) formulation, an arc-based formulation and a path-based formulation, for a multi-echelon capacitated location-distribution problem. The authors show that the linear programming relaxation (LP) of the path-based model provides a better bound than the LP relaxation of the arc-based model. We just list here the classical path formulations for HLP.

In their formulation, they take into account both the transportation costs and the location costs. Notably, they formulate the transportation costs as the vehicle costs between origins and hubs, between hubs and hubs, and between hubs and destinations. It is different from the traditional cost structure used in most location problems discussed in the literature, which typically exhibit transportation costs that are linear in the number of product units. Moreover, the vehicles on the edge restrict the volumetric capacity of each edge.

Using a similar idea, we provide the formulations for our problem. We suppose that there exist three types of vehicles in the network: Vehicle A from non-hub node to hub node, Vehicle B from hub node to non-hub node, and Vehicle C between two hub nodes. Furthermore, we first define the following variables:

$X_{ik}$  = the number of Vehicles A from node  $i$  to hub  $k$ ;

$Y_{mj}$  = the number of Vehicles B from hub  $m$  to node  $j$ ;

$Z_{km}$  = the number of Vehicles C from hub  $k$  to hub  $m$ .

The given input data are as follows:

$C_{ik}$  = the cost of Vehicle A between node  $i$  and hub  $k$ ;

$C_{mj}$  = the cost of Vehicles B from hub  $m$  to node  $j$ ;

$C_{km}$  = the cost of Vehicle C between hub  $k$  and hub  $m$ ;

$Q$  = the capacity of the vehicle A;

$T$  = the capacity of the vehicle B;

$R$  = the capacity of the vehicle C.

Obviously,  $C_{ik} = C_{ki}$  for  $i, j, k = 1, \dots, n$ . Then, the total transportation cost can be stated as follows:

$$\sum_{i=1}^n \sum_{k=1}^n C_{ik} \cdot X_{ik} + \sum_{k=1}^n \sum_{m=1}^n C_{km} \cdot Z_{km} + \sum_{m=1}^n \sum_{j=1}^n C_{jm} \cdot Y_{mj} \quad (7.2.9)$$

Thus, the capacity constraints (represented by the number of vehicles used on the edges) are defined as follows:

$$\sum_{j=1}^n \sum_{m=1}^n W_{ij} x_{ijk} - QX_{ik} \leq 0, \quad i, k = 1, \dots, n, i \neq k, \quad (7.2.10)$$

$$\sum_{i=1}^n \sum_{j=1}^n W_{ij} x_{ijk} + \sum_{j=1}^n \sum_{t=1}^n W_{kj} x_{kjt} + \sum_{i=1}^n \sum_{t=1}^n W_{im} x_{imtk} - RZ_{km} \leq 0, \quad k, m = 1, \dots, n, k \neq m, \quad (7.2.11)$$

$$\sum_{i=1}^n \sum_{k=1}^n W_{ij} x_{ijk} - TY_{mj} \leq 0, \quad j, k = 1, \dots, n, m \neq j. \quad (7.2.12)$$

The constraint sets (7.2.10)-(7.2.12) enforce respectively that the volume transported on the edge, from non-hub nodes to hubs, from hubs to hubs, and from hubs to non-hub nodes, are less than the transportation ability of the needed vehicles on the edges. Then, our proposed formulation for MApHM with the vehicle number variables, denoted as MApHM-VN1, can be presented as follows:

(MApHM-VN1)

$$\min \quad \sum_{i=1}^n \sum_{k=1}^n C_{ik} \cdot X_{ik} + \sum_{k=1}^n \sum_{m=1}^n C_{km} \cdot Z_{km} + \sum_{m=1}^n \sum_{j=1}^n C_{mj} \cdot Y_{mj} \quad (7.2.9)$$

$$\text{s.t} \quad \sum_{k=1}^n y_k = p \quad (7.2.1)$$

$$\sum_{k=1}^n \sum_{m=1}^n x_{ijk} = 1 \quad i, j = 1, \dots, n, \quad (7.2.2)$$

$$\sum_{m=1}^n x_{ijk} \leq y_k \quad i, j, k = 1, \dots, n, \quad (7.2.7)$$

$$\sum_{k=1}^n x_{ijk} \leq y_m \quad i, j, m = 1, \dots, n, \quad (7.2.8)$$

$$\sum_{j=1}^n \sum_{m=1}^n W_{ij} x_{ijk} - QX_{ik} \leq 0, \quad i, k = 1, \dots, n, i \neq k \quad (7.2.10)$$

$$\sum_{i=1}^n \sum_{j=1}^n W_{ij} x_{ijk} + \sum_{j=1}^n \sum_{t=1}^n W_{kj} x_{kjt} + \sum_{i=1}^n \sum_{t=1}^n W_{im} x_{imtk} - RZ_{km} \leq 0, \quad k, m = 1, \dots, n, k \neq m, \quad (7.2.11)$$

$$\sum_{i=1}^n \sum_{k=1}^n W_{ij} x_{ijk} - TY_{mj} \leq 0, \quad j, m = 1, \dots, n, m \neq j, \quad (7.2.12)$$

$$X_{ik} \geq 0 \text{ and integer}, \quad i, k = 1, \dots, n, \quad (7.2.13)$$

$$Y_{km} \geq 0 \text{ and integer}, \quad k, m = 1, \dots, n, \quad (7.2.14)$$

$$Z_{mj} \geq 0 \text{ and integer}, \quad j, m = 1, \dots, n, \quad (7.2.15)$$

$$y_k \in \{0, 1\} \quad k = 1, \dots, n, \quad (7.2.5)$$

$$x_{ijk} \geq 0 \quad i, j, k, m = 1, \dots, n. \quad (7.2.6)$$

As stated in Campbell (1994), if the hub locations are fixed, the remaining problem for hub location problem, which is indeed our problem in operational phase, is to find cheapest path set between each OD pairs via the given hubs. So the HLP can be divided into two

phases, fixing the hub locations first, and then finding the optimal allocation for non-hub nodes. Normally, if the hub locations are fixed, the formulations for MApHMP-VN1 can be simplified. Now we present our formulations for the fixed hubs MApHM. Let  $H$  be the set of fixed hubs with  $|H| = p$ . Then our formulation MApHMP-VN1 can be modified as follows:

(MApHM-VN2)

$$\min \sum_{i=1, i \notin H}^n \sum_{k=1}^p C_{ik} \cdot X_{ik} + \sum_{k=1}^p \sum_{m=1, m \neq k}^p C_{km} \cdot Z_{km} + \sum_{m=1}^p \sum_{j=1, j \notin H}^n C_{mj} \cdot Y_{mj} \quad (7.2.16)$$

$$\text{s.t.} \quad \sum_{k=1}^p \sum_{m=1}^p x_{ijkm} = 1, \quad i, j = 1, \dots, n, \quad (7.2.17)$$

$$\sum_{j=1}^n \sum_{m=1}^p W_{ij} x_{ijkm} - QX_{ik} \leq 0, \quad i = 1, \dots, n, k \in H, i \notin H, \quad (7.2.18)$$

$$\sum_{i=1}^n \sum_{j=1}^p W_{ij} x_{ijkm} + \sum_{j=1}^n \sum_{t=1}^p W_{kj} x_{kjmt} + \sum_{i=1}^n \sum_{t=1}^p W_{im} x_{imtk} - RZ_{km} \leq 0, k, m \in H, k \neq m, \quad (7.2.19)$$

$$\sum_{i=1}^n \sum_{k=1}^p W_{ij} x_{ijkm} - TZ_{mj} \leq 0, \quad j = 1, \dots, n, m \in H, j \notin H, \quad (7.2.20)$$

$$X_{ik} \geq 0 \text{ and integer}, \quad i = 1, \dots, n, k \in H, \quad (7.2.21)$$

$$Y_{km} \geq 0 \text{ and integer}, \quad k, m \in H, \quad (7.2.22)$$

$$Z_{mj} \geq 0 \text{ and integer}, \quad j = 1, \dots, n, m \in H \quad (7.2.23)$$

$$x_{ijkm} \geq 0 \quad i, j = 1, \dots, n, k, m \in H. \quad (7.2.24)$$

Note that MApHM-VN2 is much simpler than MApHM-VN1 after fixing the hub set. The constraints (7.2.1), (7.2.5), (7.2.7), (7.2.8) become invalid when the hub locations are given. The resulting formulation proposed in MApHM-VN2 has  $n^2 + 2pn - p^2 - p$  constraints and  $p^2 n^2 + 2pn + p^2$  variables,  $2pn + p^2$  of which are integer. Moreover, compared to MApHM-VN1, the constraint set has been reduced by  $2n^3 + 3n^2 - (2p + 3)n + (1 + p)p$  for the constraints and by  $(n + p)(n - p)n^2 + (3n - 2p)n + (n - p^2)$  for the variables. The computational results and the corresponding analysis are presented in Section 7.4.

## 7.3 2-stop single allocation p-hub median problem

### 7.3.1 Previous formulation

Single-allocation p-hub median problem (SApHM) is to restrict non-hub nodes to be connected exactly to one hub. It could be found in several applications with the advantages in terms of the amalgamation of flows into efficient consolidations.

O’Kelly (1987) presented the first recognized mathematical formulation for a SApHM by studying airline passenger networks. After introducing the location variable  $z_{ik}$ , his formulation marked as SApHMP-Q is referred as the following quadratic 0/1 formulation.

(SApHMP-Q)

$$\min \sum_{i=1}^n \sum_{j=1}^n \sum_{k=1}^n \sum_{m=1}^n W_{ij} (c_{ik} z_{ik} + \alpha c_{km} z_{ik} z_{jm} + c_{jm} z_{jm}) \quad (7.3.1)$$

$$\text{s.t.} \quad \sum_{k=1}^n z_{kk} = p, \quad (7.3.2)$$

$$\sum_{k=1}^n z_{ik} = 1, \quad i = 1, \dots, n, \quad (7.3.3)$$

$$z_{ik} \leq z_{kk}, \quad i, k = 1, \dots, n, \quad (7.3.4)$$

$$z_{ik} \in \{0, 1\}, \quad i, k = 1, \dots, n. \quad (7.3.5)$$

Where variable  $z_{ik} = 1$  if node  $i$  is allocated to hub  $k$ , and 0 otherwise. Especially, for  $z_{kk}$ , when node  $k$  is a hub, node  $k$  is surely allocated to hub  $k$ , then  $z_{kk} = 1$ ; while node  $k$  is not a hub,  $z_{kk} = 0$ . The above problem can be linearized by introducing the variables  $x_{ijkm} = z_{ik} z_{jm}$ . Constraints (7.3.3) state that each node should be allocated to exactly one hub. Campbell (1994) is also the first to propose the mixed 0/1 linear formulation for the SApHM, defined as SApHM-L. It is stated as follows:

(SApHM-L)

$$\min \sum_{i=1}^n \sum_{j=1}^n \sum_{k=1}^n \sum_{m=1}^n W_{ij} c_{ijkm} x_{ijkm} \quad (7.3.6)$$

$$\text{s.t.} \quad (7.3.2)-(7.3.5),$$

$$\sum_{k=1}^n \sum_{m=1}^n x_{ijkm} = 1, \quad i, j = 1, \dots, n, \quad (7.3.7)$$

$$z_{ik} \leq y_k, \quad i, k = 1, \dots, n, \quad (7.3.8)$$

$$\sum_{j=1}^n \sum_{m=1}^n (W_{ij} x_{ijkm} + W_{ji} x_{jikm}) = \sum_{j=1}^n (W_{ij} + W_{ji}) z_{ik}, \quad i, k = 1, \dots, n, \quad (7.3.9)$$

$$y_k \in \{0, 1\}, \quad k = 1, \dots, n, \quad (7.3.10)$$

$$x_{ijkm} \geq 0, \quad i, j, k, m = 1, \dots, n. \quad (7.3.11)$$

Where  $y_k = 1$  if  $k$  is a hub, and 0 otherwise. And when a hub is allocated to itself, it was denoted by  $z_{kk} = y_k$ ,  $k = 1, \dots, n$ . The constraint set (7.3.9) enforces a single allocation requirement: for a given allocation of node  $i$  to hub  $k$ , the total flow between  $i$  and all the other nodes  $j$  passes via the link  $(i, k)$ . Therefore,  $i$  cannot be allocated to another hub, if it is already allocated to hub  $k$ .

To simplify this formulation, Campbell (1994) also proposed another slightly different formulation by replacing constraints (7.3.9) with  $z_{ik} + z_{jm} - 2x_{ijkm} \geq 0$ ,  $i, j, k, m = 1, \dots, n$ . However, the linear relaxations of Campbell's formulation SApHM-L and the modified formulation are not tight. They lead to fractional solutions with objective function values significantly lower to the optimal objective functions values.

In consequence, Skorin-Kapov et al. (1996) propose a reduced mixed 0/1 linear program formulation based on SApHMP-Q and SApHM-L. The modification consists in making the allocation choice of an origin node  $i$  independent of a destination node, and vice versa. Consequently, they firstly used the "allocation" variables  $z_{ik}$  to improve SApHMP-L. Then, they simplify further their formulation by using a quadratic programming formulation to redefine the meaning of  $x$  variables. The final mixed 0/1 linear program in Skorin-Kapov et al. (1996) for the SApHM, marked as SApHM-FL, can be defined as follows.

(SApHM-FL)

$$\min \sum_{i=1}^n \sum_{j=1}^n \sum_{k=1}^n \sum_{m=1}^n W_{ij} c_{ijkm} x_{ijkm} \quad (7.3.6)$$

$$\text{s.t.} \quad \sum_{k=1}^n z_{kk} = p, \quad (7.3.2)$$

$$\sum_{k=1}^n z_{ik} = 1, \quad i = 1, \dots, n, \quad (7.3.3)$$

$$z_{ik} \leq z_{kk}, \quad i, k = 1, \dots, n, \quad (7.3.4)$$

$$\sum_{j=1}^n \sum_{m=1}^n (W_{ij} x_{ijkm} + W_{ji} x_{jikm}) = \sum_{j=1}^n (W_{ij} + W_{ji}) z_{ik}, \quad i, k = 1, \dots, n, \quad (7.3.9)$$

$$\sum_{m=1}^n x_{ijkm} = z_{ik}, \quad i, j, k = 1, \dots, n, \quad (7.3.12)$$

$$\sum_{k=1}^n x_{ijkm} = z_{jm}, \quad i, j, m = 1, \dots, n, \quad (7.3.13)$$

$$x_{ijkm} \geq 0, \quad i, j, k, m = 1, \dots, n, \quad (7.3.11)$$

$$z_{ik} \in \{0, 1\}, \quad i, k = 1, \dots, n. \quad (7.3.5)$$

Where constraints (7.3.12) assure that, for every destination  $j$ , the sum  $\sum_{m=1}^n x_{ijkm}$  (i.e. the total flow from origin  $i$  to destination  $j$  routed via all paths using link  $i-k$  will be nonzero only if location  $i$  is allocated to hub  $k$ , which is independent to the destination. Similarly, constraints (7.3.13) assure that for every origin  $i$  and every hub  $k$ , a flow through the path  $i \rightarrow k \rightarrow m \rightarrow j$  is feasible only if  $j$  is allocated to hub  $m$ . The formulation SApHM-FL has  $n^2$  0/1 variables,  $n^4$  continuous variables, and  $1 + n + n^2 + 2n^3$  linear constraints. Skorin-

Kapov et al. (1996) proved by the computational results that the linear relaxation of SApHMP-FL is tighter than the other formulations mentioned above.

### 7.3.2 Proposed formulation

Now, we are going to formulate a mixed integer linear program for our problem, SApHM taking into account the number of vehicles. We use the same notations as well as in the MApHM with the vehicle number variables. Then, the total transportation can be defined as follows:

$$\sum_{i=1}^n \sum_{k=1}^n C_{ik} \cdot X_{ik} + \sum_{k=1}^n \sum_{m=1}^n C_{km} \cdot Z_{km} + \sum_{m=1}^n \sum_{j=1}^n C_{mj} \cdot Y_{mj} \quad (7.3.14)$$

Thus, the capacity constraints (represented by the number of vehicles used on the arcs) are as follows:

$$\sum_{m=1}^n W_{ij} x_{ijkm} - QX_{ik} \leq 0, \quad i, k = 1, \dots, n, i \neq k, \quad (7.3.15)$$

$$\sum_{i=1}^n \sum_{j=1}^n W_{ij} x_{ijkm} + \sum_{j=1}^n \sum_{t=1}^n W_{kj} x_{kjmt} + \sum_{i=1}^n \sum_{t=1}^n W_{im} x_{imtk} - RZ_{km} \leq 0, \quad k, m = 1, \dots, n, k \neq m, \quad (7.3.16)$$

$$\sum_{k=1}^n W_{ij} x_{ijkm} - TY_{mj} \leq 0, \quad j, m = 1, \dots, n, m \neq j. \quad (7.3.17)$$

The constraint set (7.3.15)-(7.3.17) enforces respectively that the volume transported on the arc, from origins to hubs, from hubs to hubs, and from hubs to destinations, are less than the transportation ability of the needed vehicles on the arcs. Note that, unlike multiple allocation problems, there is no need to do the summation for  $j$  in constraints (7.3.15) because of the definition of the single allocation. Similarly, constraints (7.3.17) do not take summation for  $i$ .

Then, our proposed formulation (SApHM -VN1) for SApHM with the vehicle number variables can be presented as follows:

(SApHM -VN1)

$$\min \quad \sum_{i=1}^n \sum_{k=1}^n C_{ik} \cdot X_{ik} + \sum_{k=1}^n \sum_{m=1}^n C_{km} \cdot Z_{km} + \sum_{m=1}^n \sum_{j=1}^n C_{mj} \cdot Y_{mj} \quad (7.3.14)$$

$$\text{s.t.} \quad \sum_{k=1}^n z_{kk} = p, \quad (7.3.2)$$

$$\sum_{k=1}^n z_{ik} = 1, \quad i = 1, \dots, n, \quad (7.3.3)$$

$$z_{ik} \leq z_{kk}, \quad i, k = 1, \dots, n, \quad (7.3.4)$$

$$\sum_{m=1}^n x_{ijkm} = z_{ik}, \quad i, j, k = 1, \dots, n, \quad (7.3.12)$$

$$\sum_{k=1}^n x_{ijkm} = z_{jm}, \quad i, j, m = 1, \dots, n, \quad (7.3.13)$$

$$\sum_{m=1}^n W_{ij} x_{ijkm} - QX_{ik} \leq 0, \quad i, k = 1, \dots, n, i \neq k, \quad (7.3.15)$$

$$\sum_{i=1}^n \sum_{j=1}^n W_{ij} x_{ijk} + \sum_{j=1}^n \sum_{t=1}^n W_{kj} x_{kjmt} + \sum_{i=1}^n \sum_{t=1}^n W_{im} x_{imtk} - RZ_{km} \leq 0, \quad k, m = 1, \dots, n, k \neq m, \quad (7.3.16)$$

$$\sum_{k=1}^n W_{ij} x_{ijk} - TY_{mj} \leq 0, \quad j, m = 1, \dots, n, m \neq j, \quad (7.3.17)$$

$$X_{ik} \geq 0 \text{ and integer}, \quad i = 1, \dots, n, \quad (7.3.18)$$

$$Y_{km} \geq 0 \text{ and integer}, \quad k, m = 1, \dots, n, \quad (7.3.19)$$

$$Z_{mj} \geq 0 \text{ and integer}, \quad j, m = 1, \dots, n, \quad (7.3.20)$$

$$z_{ik} \in \{0, 1\}, \quad i, k = 1, \dots, n, \quad (7.3.5)$$

$$x_{ijk} \geq 0, \quad i, j, k, m = 1, \dots, n. \quad (7.3.11)$$

Let  $H$  be the set of fixed hubs with  $|H| = p$ . Then our formulation can be modified as follows:

(SApHMP-VN2)

$$\min \quad \sum_{i=1, i \notin H}^n \sum_{k=1}^p C_{ik} \cdot X_{ik} + \sum_{k=1}^p \sum_{m=1}^p C_{km} \cdot Z_{km} + \sum_{m=1}^p \sum_{j=1, j \notin H}^n C_{mj} \cdot Y_{mj} \quad (7.3.21)$$

$$\text{s.t.} \quad z_{kk} = 1, \quad k \in H, \quad (7.3.22)$$

$$\sum_{k=1}^p z_{ik} = 1, \quad i = 1, \dots, n, k \in H, \quad (7.3.23)$$

$$\sum_{m=1}^p x_{ijk} = z_{ik}, \quad i, j = 1, \dots, n, k \in H, \quad (7.3.24)$$

$$\sum_{k=1}^p x_{ijk} = z_{jm}, \quad i, j = 1, \dots, n, m \in H, \quad (7.3.25)$$

$$\sum_{m=1}^p W_{ij} x_{ijk} - QX_{ik} \leq 0, \quad i = 1, \dots, n, i \notin H, k \in H, \quad (7.3.26)$$

$$\sum_{i=1}^n \sum_{j=1}^p W_{ij} x_{ijk} + \sum_{j=1}^n \sum_{t=1}^p W_{kj} x_{kjmt} + \sum_{i=1}^n \sum_{t=1}^p W_{im} x_{imtk} - RZ_{km} \leq 0, k, m \in H, k \neq m, \quad (7.3.27)$$

$$\sum_{k=1}^p W_{ij} x_{ijk} - QY_{mj} \leq 0, \quad j = 1, \dots, n, j \notin H, m \in H, \quad (7.3.28)$$

$$X_{ik} \geq 0 \text{ and integer}, \quad i = 1, \dots, n, k \in H, \quad (7.3.29)$$

$$Z_{km} \geq 0 \text{ and integer}, \quad k, m \in H, \quad (7.3.30)$$

$$Y_{mj} \geq 0 \text{ and integer}, \quad j = 1, \dots, n, m \in H, \quad (7.3.31)$$

$$z_{ik} \in \{0, 1\}, \quad i = 1, \dots, n, k \in H, \quad (7.3.5)$$

$$x_{ijk} \geq 0, \quad i, j = 1, \dots, n, k, m \in H. \quad (7.3.32)$$

Note that SApHM-VN2 is much simpler than SApHMP-VN1 after fixing the hub set. The constraints (7.3.2), (7.3.4) become invalid when the hub locations are given. The resulting formulation proposed in SApHMP-VN2 has  $2np(2n-p) + p(n+p)$  constraints and  $n^2 p^2 + p(3n+p)$  variables in which  $p(2n+p)$  are integer,  $p^n$  are binary. Moreover, compared to SApHMP-VN1, the formulation size is greatly reduced.

Depending on the characterization of the single allocation problem, the volumetric quantity on the edges between hub nodes and non-hub nodes can be also defined as follows:

$$Q_{ik} = \sum_{j=1}^n W_{ij} \cdot z_{ik}, \quad i, j = 1, \dots, n, \quad i \notin H, \quad k \in H,$$

$$Q_{mj} = \sum_{i=1}^n W_{ij} \cdot z_{jm}, \quad i, j = 1, \dots, n, \quad j \notin H, \quad m \in H,$$

Where  $Q_{ik}$  represents the volumetric quantity transported from non-hub node  $i$  to hub node  $k$ ,  $Q_{mj}$  represents the volumetric quantity between hub node  $m$  and non-hub node  $j$ .

Without loss of generality, SApHM-VN2 can be further simplified to the following mixed integer linear program SApHM-VN3 by using constraints set (7.3.33) and (7.3.34) to replace constraints set (7.3.26) and (7.3.28).

(SApHM-VN3)

$$\min \quad \sum_{i=1, i \notin H}^n \sum_{k=1}^P C_{ik} \cdot X_{ik} + \sum_{k=1}^P \sum_{m=1}^P C_{km} \cdot Z_{km} + \sum_{m=1}^P \sum_{j=1, j \notin H}^n C_{mj} \cdot Y_{mj} \quad (7.3.21)$$

$$\text{s.t.} \quad z_{kk} = 1, \quad k \in H, \quad (7.3.22)$$

$$\sum_{k=1}^P z_{ik} = 1, \quad i = 1, \dots, n, \quad k \in H, \quad (7.3.23)$$

$$\sum_{m=1}^P x_{ikm} = z_{ik}, \quad i, j = 1, \dots, n, \quad k \in H, \quad (7.3.24)$$

$$\sum_{k=1}^P x_{ijkm} = z_{jm}, \quad i, j = 1, \dots, n, \quad m \in H, \quad (7.3.25)$$

$$\sum_{j=1}^n W_{ij} \cdot z_{ik} - QX_{ik} \leq 0, \quad i = 1, \dots, n, \quad i \notin H, \quad k \in H, \quad (7.3.33)$$

$$\sum_{i=1}^n \sum_{j=1}^P W_{ij} x_{ijkm} + \sum_{j=1}^n \sum_{t=1}^P W_{kj} x_{kjmt} + \sum_{i=1}^n \sum_{t=1}^P W_{im} x_{imtk} - RZ_{km} \leq 0, \quad k, m \in H, \quad k \neq m \quad (7.3.27)$$

$$\sum_{i=1}^n W_{ij} \cdot z_{jm} - TY_{mj} \leq 0, \quad j = 1, \dots, n, \quad j \notin H, \quad m \in H, \quad (7.3.34)$$

$$X_{ik} \geq 0 \text{ and integer}, \quad i = 1, \dots, n, \quad i \notin H, \quad (7.3.29)$$

$$Y_{km} \geq 0 \text{ and integer}, \quad k, m \in H, \quad k \neq m, \quad (7.3.30)$$

$$Z_{mj} \geq 0 \text{ and integer}, \quad j = 1, \dots, n, \quad j \notin H, \quad (7.3.31)$$

$$z_{ik} \in \{0, 1\}, \quad i, k = 1, \dots, n, \quad (7.3.5)$$

$$x_{ijkm} \geq 0, \quad i, j = 1, \dots, n, \quad k, m \in H. \quad (7.3.32)$$

Actually, although there are not either redundant variables or constraints in SApHM-VN3 compared to p-MApHM-VN2, the form has been much simplified. It is tested on the benchmark CAB, and the computational results are shown in the next Section.

## 7.4 CAB computational results

In this section, we present the computational results to evaluate the effectiveness of the various formulations discussed in this chapter. The same to the previous chapter, all the numerical tests were carried out with LP/MIP solver CPLEX (version 9.0) on the same computer used in Chapter 6.

The computational comparisons are based on the benchmark data set CAB which has been introduced in the previous chapter and we add the parameter  $\alpha \in \{0.2, 0.4, 0.6, 0.8, 1.0\}$ . The factor  $\alpha$  is used to express the cost discount on the arcs between hubs. Annexes III and IV show respectively the samples for our MApHM-VN2 and SApHM-VN3 in the data set. And we select respectively the samples 1-30 with 10 nodes and 15 nodes from the Annex III and Annex IV. Furthermore, we set the capacity of vehicle A (Vehicle from non-hub node to hub node) and the capacity of vehicle B (Vehicle from hub node to non-hub node) be  $80 \text{ m}^3$  and set the capacity of the vehicle C (the vehicle between the hubs) be  $110 \text{ m}^3$ .

In the next subsection, we first present the detail results for MApHM-VN2, followed by the results for SApHM-VN3.

### 7.4.1 Multiple allocation p-hub median problem

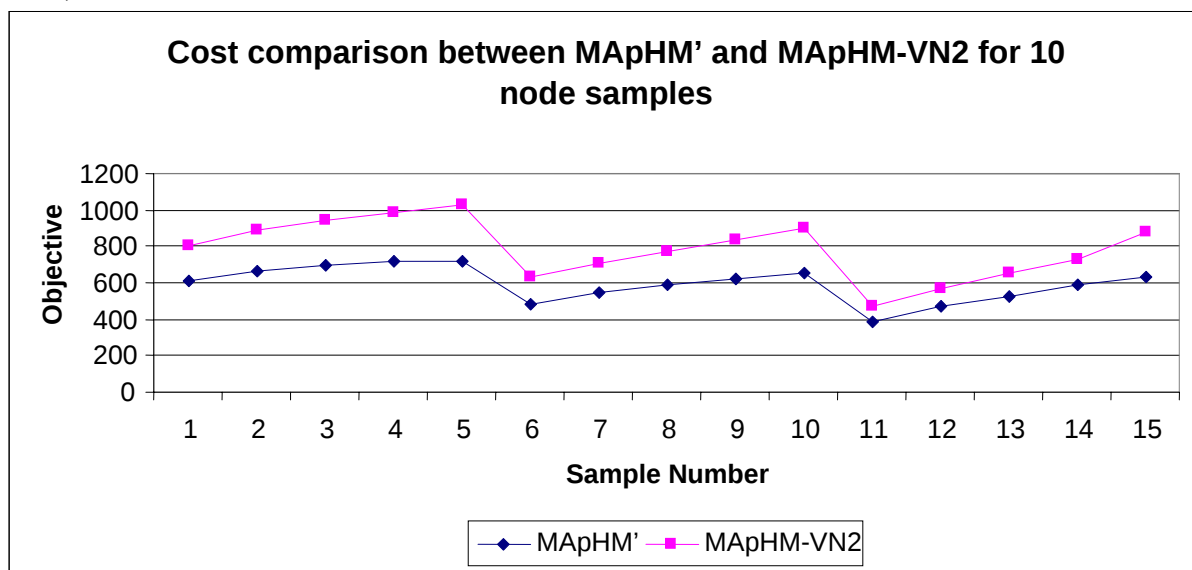
Our computational results are different to the results of MApHM' in Skorin-Kapov et al. (1996), since we use the new cost function as the objective. Moreover, we add the arc capacity constraints on each arc. Table 7.1 shows us the computational results of our MApHM-VN2 for Sample 1–30 in Annex III, including the objective of MApHM-VN2, the computational time (in second) and multi-allocation. It also illustrates the objective of MApHM' to compare with the results.

**Table 7.1** Computational results of the Sample 1-30 for MApHM-VN2

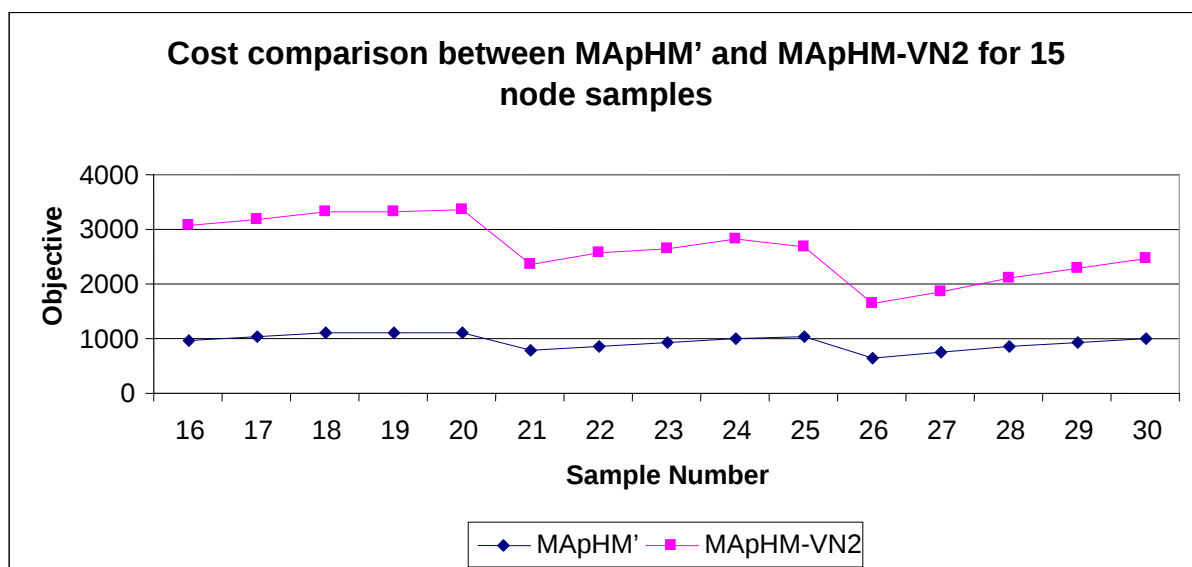
| Sample Number | n  | p | $\alpha$ | Hub locations | Objective |           | Computational time (sec.) | Multi-allocation |
|---------------|----|---|----------|---------------|-----------|-----------|---------------------------|------------------|
|               |    |   |          |               | MApHM'    | MApHM-VN2 |                           |                  |
| 1             | 10 | 2 | 0.2      | 7,9           | 612.12    | 806.10    | 0.03                      | No               |
| 2             |    | 2 | 0.4      | 7,9           | 662.11    | 887.73    | 0.06                      | No               |
| 3             |    | 2 | 0.6      | 7,9           | 698.69    | 937.83    | 0.09                      | Yes              |
| 4             |    | 2 | 0.8      | 7,9           | 713.55    | 981.07    | 0.09                      | Yes              |
| 5             |    | 2 | 1        | 7,9           | 721.2     | 1024.31   | 0.53                      | Yes              |
| 6             |    | 3 | 0.2      | 4,6,7         | 487.26    | 636.51    | 0.30                      | No               |
| 7             |    | 3 | 0.4      | 4,6,7         | 543.73    | 709.05    | 1.25                      | No               |
| 8             |    | 3 | 0.6      | 4,6,7         | 586.47    | 775.06    | 2.53                      | No               |
| 9             |    | 3 | 0.8      | 4,6,7         | 625.48    | 839.94    | 5.03                      | No               |
| 10            |    | 3 | 1        | 4,6,7         | 654.35    | 904.81    | 16.86                     | Yes              |
| 11            | 15 | 4 | 0.2      | 3,4,6,7       | 389.04    | 476.36    | 1.86                      | Yes              |
| 12            |    | 4 | 0.4      | 3,4,6,7       | 466.53    | 566.69    | 3.70                      | No               |
| 13            |    | 4 | 0.6      | 3,4,6,7       | 530.26    | 650.50    | 5.14                      | No               |
| 14            |    | 4 | 0.8      | 3,4,6,7       | 589.36    | 733.17    | 8.39                      | No               |
| 15            |    | 4 | 1        | 2,4,6,7       | 632.18    | 874.44    | 182.05                    | Yes              |
| 16            |    | 2 | 0.2      | 4,12          | 970.65    | 3066.34   | 0.13                      | No               |
| 17            |    | 2 | 0.4      | 4,12          | 1033.86   | 3195.63   | 4.74                      | Yes              |
| 18            |    | 2 | 0.6      | 4,12          | 1091.91   | 3307.11   | 7.31                      | Yes              |
| 19            |    | 2 | 0.8      | 4,7           | 1108.79   | 3326.53   | 810.33                    | Yes              |
| 20            |    | 2 | 1        | 4,7           | 1114.54   | 3357.90   | 690.19                    | Yes              |
| 21            |    | 3 | 0.2      | 4,7,12        | 783.8     | 2360.81   | 1574.97                   | Yes              |
| 22            |    | 3 | 0.4      | 4,7,12        | 864.58    | 2555.03   | 3226.49                   | Yes              |
| 23            |    | 3 | 0.6      | 4,7,12        | 941.21    | 2642.88   | 2184.69                   | Yes              |
| 24            |    | 3 | 0.8      | 4,7,12        | 1012.04   | 2815.20   | 8810.00                   | Yes              |
| 25            |    | 3 | 1        | 1,4,7,        | 1039.39   | 2671.58   | 2777.20                   | Yes              |
| 26            |    | 4 | 0.2      | 4,7,12,14     | 626.33    | 1654.86   | 4833.44                   | Yes              |
| 27            |    | 4 | 0.4      | 4,7,12,14     | 738.52    | 1846.76   | 9551.89                   | Yes              |
| 28            |    | 4 | 0.6      | 4,7,12,14     | 841.78    | 2096.01   | 13348.89                  | Yes              |
| 29            |    | 4 | 0.8      | 1,4,7,12      | 927.42    | 2283.42   | 18763.61                  | Yes              |
| 30            |    | 4 | 1        | 1,4,7,8       | 989.24    | 2461.49   | 19478.80                  | Yes              |

The column Objective shows us the cost of the MApHM' and MApHM-VN2. Fig. 7.2 shows the objective comparison between MApHM' and our MApHM-VN2. As we can see, the objective of MApHM' is, not surprisingly, lower than MApHM-VN2. It can be explained because of the non-full vehicle load. Furthermore, the objective decreases when the hub number increases. The use of hubs is then a good way to decrease the transport costs.

Fig. 7.3 shows the cost-hub number relationship for the samples 1-15 and the samples 16-30. Take Figure (a) as an example to further explain, the three lines (blue, pink and yellow) respectively represent the  $\alpha$ -objective for the 10 node samples with 2 hubs, 3 hubs and 4 hubs. Then the objective of the sample with  $n=10$  and  $p=2$  (blue line) is always higher than the corresponding objective of the samples with  $n=10$  and  $p=3$  while the objective of the sample with  $n=10$  and  $p=4$  (yellow line) is the lowest in the three lines. In addition, Fig. 7.3 shows the objective generally increases when  $\alpha$  increases except for the Sample 30 with  $n=15$ ,  $p=3$  and  $\alpha=1$ ).

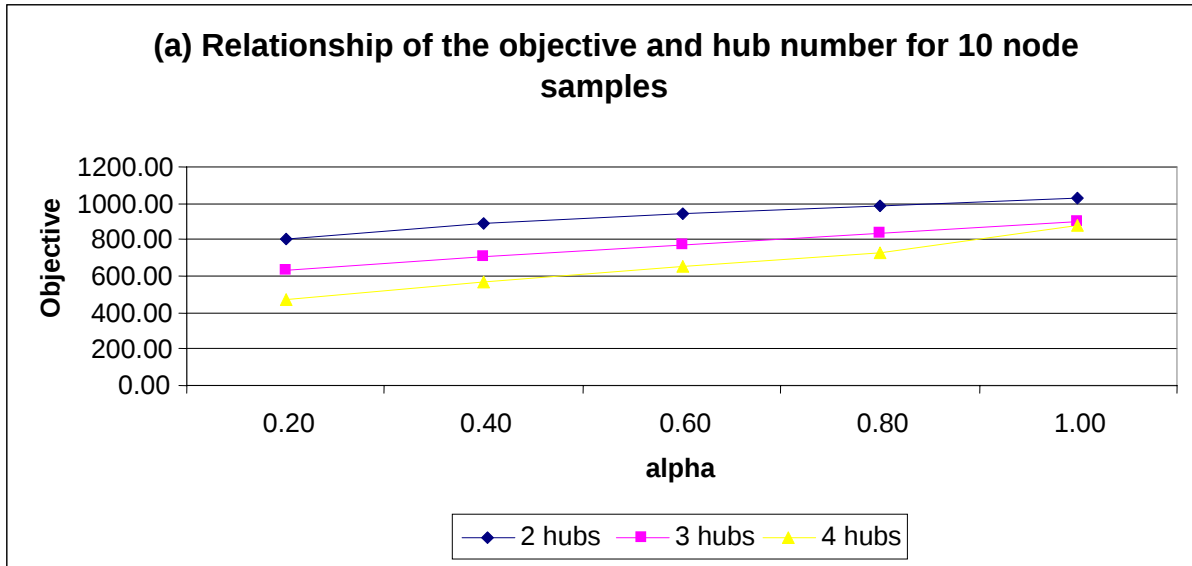


(a) Cost comparison between MApHM' and MApHM-VN2for 10 node samples

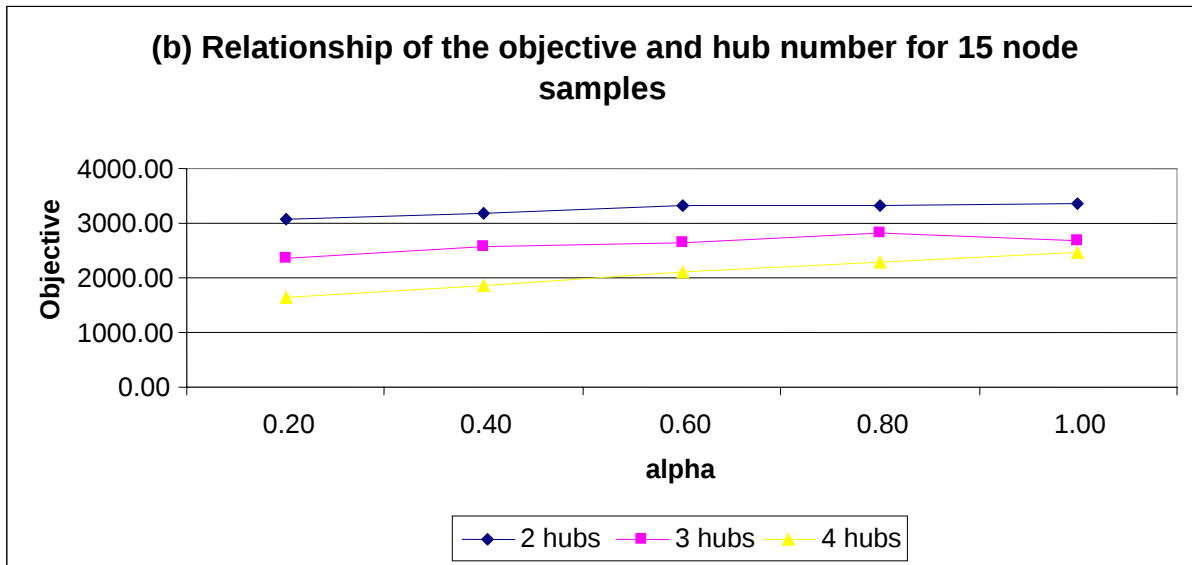


(b) Cost comparison between MApHM' and MApHM-VN2for 15 node samples

**Fig. 7.2** Cost comparison between MApHM' and MApHM-VN2



(a) Relationship between the objective to the value of  $\alpha$  for 10 node samples



(b) Relationship between the objective to the value of  $\alpha$  for 15 node samples

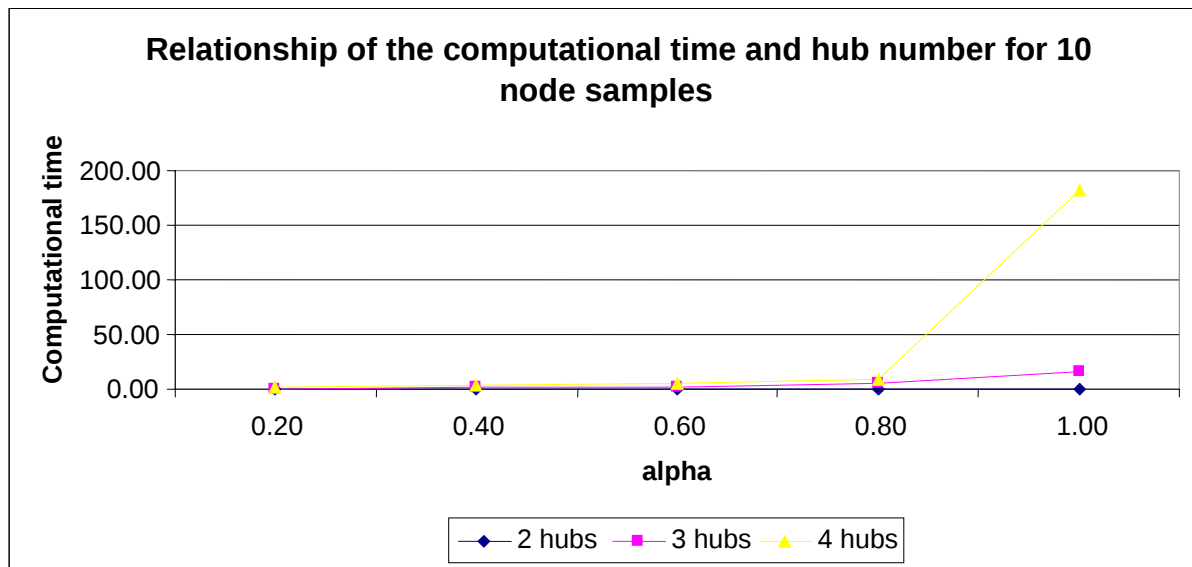
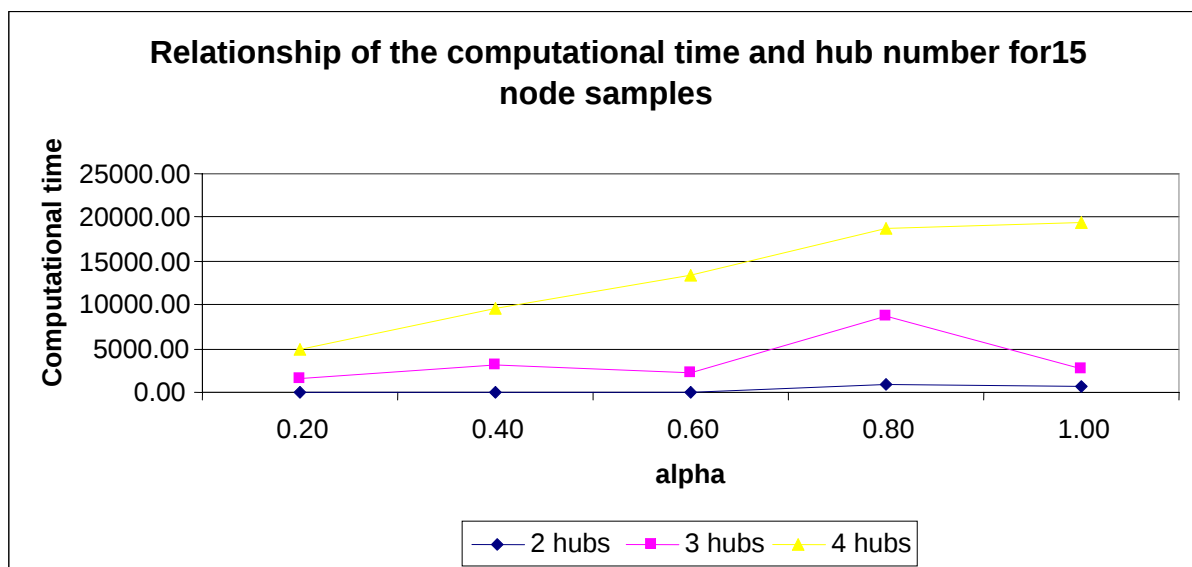
**Fig. 7.3** Relationship between the objectives to the value of  $\alpha$  for several values of  $p$

The allocation results may change between MApHM' and MApHM-VN2 (Column Multi-allocation in Table 7.1). There exists multi-allocation for some samples. It means non-integer solutions of MApHM-VN2 are generated. It can be explained by the restriction of the vehicle transportation capacity on the arcs. A node may divide its goods into several parts to distribute them to their destinations in order to fill the vehicles and thereby to save cost. Table 7.2 illustrates the multi-allocation solution of MApHM-VN2 for the Sample 25 with  $n=15$ ,  $p=3$ ,  $\alpha=1.0$ . The Node 2, 3, 8, 10, 11 (distinguished in red) are allocated to several hubs in which Node 2 and Node 3 are allocated to Hub 1 and Hub 4, Node 10 and Node 11 are assigned to Hub 1 and Hub 7, meanwhile, Node 8 is allocated to Hub 4 and Hub 7. Nevertheless, this result is totally different since the integer solution of MApHM' can always be found as stated in Skorin-Kapov et al. (1996).

**Table 7.2** A multi-allocation solution of Sample 25

| Hub | Non-hub node         |
|-----|----------------------|
| 1   | 2, 3, 10, 11, 13, 14 |
| 4   | 2, 3, 5, 6, 8, 9, 15 |
| 7   | 8, 10, 11, 12        |

The computational times of MApHM-VN2 (Table 7.1) are dimension dependent: around few seconds for  $n=10$ , few minutes for  $n=15$ , and 2-3 hours for  $n=25$ . Computational time required by our MApHM-VN2 is around one minute for  $n=10$ , few minutes for  $n=15$ ,  $p=2$ , around half an hour for  $n=15$ ,  $p=3$ , and from 1.5 hours to 3 hours for  $n=25$ . Moreover, the computational time increases as the hub number, in the same time, it also seems to increase with  $\alpha$ . Fig. 7.4 separately shows the relationship between the CPU time to the Hub number for  $n=10$  and  $n=15$ .

(a) The relationship between the Computational time to the hub number for  $n=10$ (b) The relationship between the Computational time to the hub number for  $n=15$ **Fig. 7.4** The relationship between the Computational time to the hub number

Let us take Fig. 7.4 (b) as an example. In this figure, the blue line, pink line and yellow line represent the  $\alpha$ -Computational time. As we can see, the yellow line is always higher than the pink line and yellow line. Meanwhile, the blue line is the lowest line in the three lines. So, for the samples with the same  $n$  and  $\alpha$ , the computational time increases as the hub number. Moreover, we can also find in Fig.7.4 that the computational time generally increases as  $\alpha$ .

The average load rates for vehicle A, B and C in each sample are displayed in Table 7.3. We can see that the average load rates for all of the samples 1-30 are greater than 0.7. It means our computational results are good. Additionally, when the average load rate is lesser than 0.5, smaller vehicles can be the used.

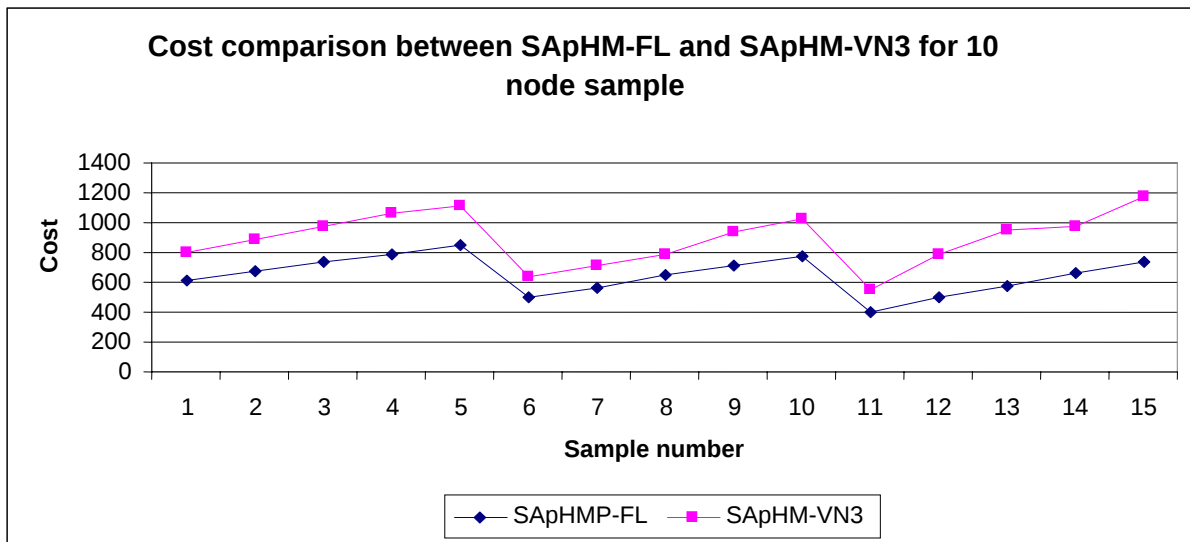
**Table 7.3:** The load rates for vehicle A, B and C in Sample 1-30 for MApHM-VN2

| Sample Number | n  | p | $\alpha$ | Hub locations | Vehicle average load rate(%) |           |           |
|---------------|----|---|----------|---------------|------------------------------|-----------|-----------|
|               |    |   |          |               | Vehicle A                    | Vehicle B | Vehicle C |
| 1             | 10 | 2 | 0.2      | 7,9           | 74.61                        | 74.61     | 73.85     |
| 2             |    | 2 | 0.4      | 7,9           | 74.61                        | 74.61     | 86.67     |
| 3             |    | 2 | 0.6      | 7,9           | 77.40                        | 77.40     | 94.19     |
| 4             |    | 2 | 0.8      | 7,9           | 77.40                        | 77.40     | 89.95     |
| 5             |    | 2 | 1        | 7,9           | 77.33                        | 77.40     | 89.49     |
| 6             |    | 3 | 0.2      | 4,6,7         | 73.43                        | 73.43     | 95.46     |
| 7             |    | 3 | 0.4      | 4,6,7         | 73.43                        | 73.43     | 90.75     |
| 8             |    | 3 | 0.6      | 4,6,7         | 73.43                        | 73.43     | 86.70     |
| 9             |    | 3 | 0.8      | 4,6,7         | 73.43                        | 73.43     | 85.66     |
| 10            |    | 3 | 1        | 4,6,7         | 73.43                        | 73.43     | 89.07     |
| 11            | 15 | 4 | 0.2      | 3,4,6,7       | 74.39                        | 74.39     | 96.92     |
| 12            |    | 4 | 0.4      | 3,4,6,7       | 74.39                        | 74.39     | 92.88     |
| 13            |    | 4 | 0.6      | 3,4,6,7       | 74.39                        | 74.39     | 89.12     |
| 14            |    | 4 | 0.8      | 3,4,6,7       | 74.39                        | 74.39     | 92.58     |
| 15            |    | 4 | 1        | 2,4,6,7       | 72.40                        | 72.40     | 83.48     |
| 16            |    | 2 | 0.2      | 4,12          | 74.48                        | 74.48     | 92.87     |
| 17            |    | 2 | 0.4      | 4,12          | 73.37                        | 73.37     | 99.75     |
| 18            |    | 2 | 0.6      | 4,12          | 73.37                        | 73.37     | 99.77     |
| 19            |    | 2 | 0.8      | 4,7           | 74.49                        | 75.37     | 99.02     |
| 20            |    | 2 | 1        | 4,7           | 75.97                        | 75.97     | 90.49     |
| 21            |    | 3 | 0.2      | 4,7,12        | 75.29                        | 75.28     | 99.59     |
| 22            |    | 3 | 0.4      | 4,7,12        | 73.51                        | 73.40     | 99.09     |
| 23            |    | 3 | 0.6      | 4,7,12        | 74.79                        | 74.79     | 97.23     |
| 24            |    | 3 | 0.8      | 4,7,12        | 74.47                        | 74.19     | 98.93     |
| 25            |    | 3 | 1        | 1,4,7,        | 73.95                        | 74.83     | 92.31     |
| 26            |    | 4 | 0.2      | 4,7,12,14     | 73.90                        | 73.90     | 96.05     |
| 27            |    | 4 | 0.4      | 4,7,12,14     | 73.90                        | 74.11     | 98.23     |
| 28            |    | 4 | 0.6      | 4,7,12,14     | 73.90                        | 74.11     | 97.41     |
| 29            |    | 4 | 0.8      | 1,4,7,12      | 72.51                        | 74.08     | 96.24     |
| 30            |    | 4 | 1        | 1,4,7,8       | 76.17                        | 71.97     | 89.90     |

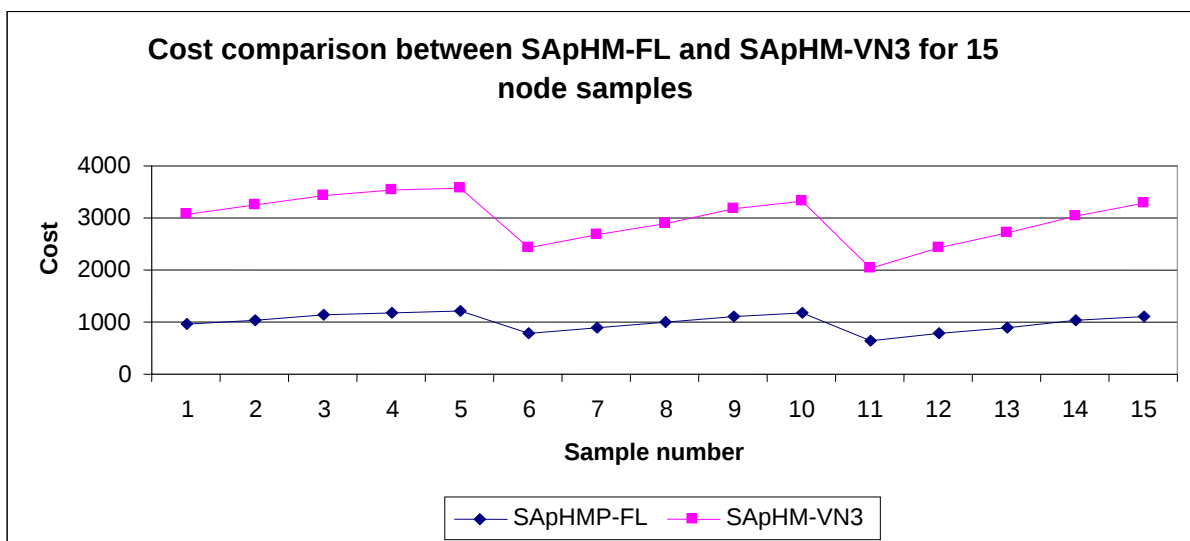
## 7.4.2 Single allocation p-hub median problem

In this section, we detail the computational results of our SApHM-VN3 for Sample 1–30 shown in Annex IV. The results are analyzed and compared with SApHM-FL, the final mixed 0/1 linear program of Skorin-Kapov et al. (1996). The comparison with our proposed formulation for multi-allocation p-hub median problem, MApHM-VN3 will be made in the next part. Table 7.4 shows the computational results of the Sample 1-30 for SApHMP-VN3 involving the objectives of SApHMP-FL and SApHM-VN3 and the computational time (in second). The load rates for vehicle A, B and C in each sample are displayed in Table 7.5.

As we can see in Fig.7.5, the cost of SApHM-FL (Blue line) is always lower than the cost of SApHM-VN3 (Rose line). Then a similar conclusion with objective comparison between MApHM' and our MApHM-VN2 can be presented. It is because of the non-full vehicle load.



(a) Cost comparison between SApHM-FL and SApHM-VN3 for 10 node samples



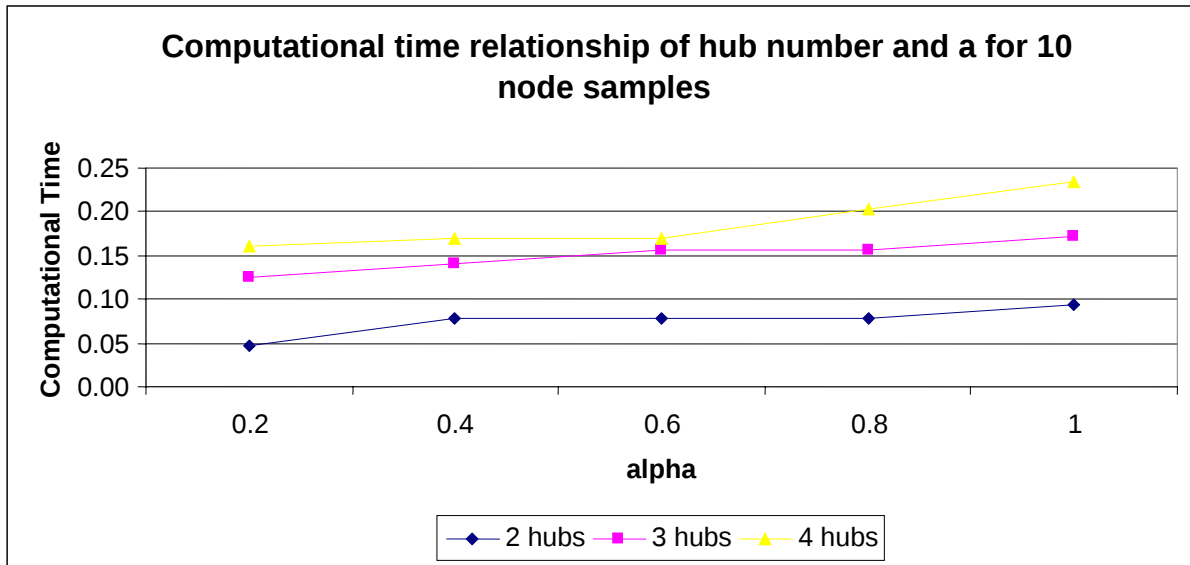
(b) Cost comparison between SApHM-FL and SApHM-VN3 for 15 node samples

**Fig.7.5** Cost comparison between SApHM-FL and SApHM-VN3

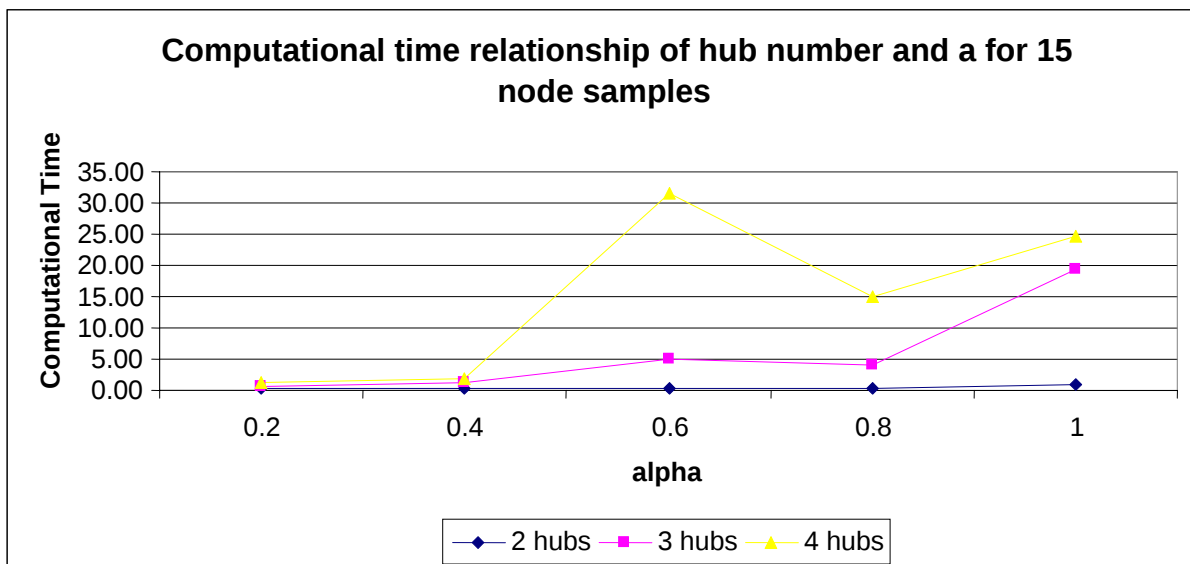
**Table 7.4** Computational results of the Sample 1-30 for SApHM-VN3

| Sample Number | n  | p | $\alpha$ | Hub locations | Objective |           | Computational time (sec.) |
|---------------|----|---|----------|---------------|-----------|-----------|---------------------------|
|               |    |   |          |               | SApHM-FL  | SApHM-VN3 |                           |
| 1             | 10 | 2 | 0.2      | 7,9           | 615.99    | 806.10    | 0.05                      |
| 2             |    |   | 0.4      | 7,9           | 674.31    | 892.58    | 0.08                      |
| 3             |    |   | 0.6      | 7,9           | 732.63    | 979.06    | 0.08                      |
| 4             |    |   | 0.8      | 7,9           | 790.94    | 1065.54   | 0.08                      |
| 5             |    |   | 1        | 4,7           | 853.81    | 1114.57   | 0.09                      |
| 6             |    | 3 | 0.2      | 4,6,7         | 497.93    | 638.47    | 0.13                      |
| 7             |    |   | 0.4      | 4,6,7         | 567.91    | 715.98    | 0.14                      |
| 8             |    |   | 0.6      | 4,6,7         | 643.89    | 793.50    | 0.16                      |
| 9             |    |   | 0.8      | 4,7,9         | 716.98    | 939.64    | 0.16                      |
| 10            |    |   | 1        | 4,7,9         | 776.68    | 1023.34   | 0.17                      |
| 11            | 15 | 4 | 0.2      | 3,4,6,7       | 395.13    | 555.10    | 0.17                      |
| 12            |    |   | 0.4      | 4,6,7,8       | 493.79    | 788.21    | 0.13                      |
| 13            |    |   | 0.6      | 4,6,7,8       | 577.83    | 954.95    | 0.16                      |
| 14            |    |   | 0.8      | 4,6,7,9       | 661.41    | 971.30    | 0.20                      |
| 15            |    |   | 1        | 1,4,7,9       | 736.26    | 1174.40   | 0.23                      |
| 16            |    | 2 | 0.2      | 4,12          | 981.28    | 3076.87   | 0.22                      |
| 17            |    |   | 0.4      | 4,12          | 1026.63   | 3244.09   | 0.22                      |
| 18            |    |   | 0.6      | 4,12          | 1134.97   | 3411.31   | 0.20                      |
| 19            |    |   | 0.8      | 4,11          | 1190.77   | 3520.85   | 0.42                      |
| 20            |    |   | 1        | 4,11          | 1221.92   | 3585.87   | 1.06                      |
| 21            |    | 3 | 0.2      | 4,7,12        | 799.97    | 2422.86   | 0.63                      |
| 22            |    |   | 0.4      | 4,7,12        | 905.1     | 2667.74   | 1.34                      |
| 23            |    |   | 0.6      | 4,7,12        | 1009.93   | 2895.06   | 5.13                      |
| 24            |    |   | 0.8      | 4,7,8         | 1099.51   | 3174.99   | 4.14                      |
| 25            |    |   | 1        | 4,7,8         | 1168.68   | 3333.92   | 19.34                     |
| 26            |    | 4 | 0.2      | 4,7,12,14     | 639.77    | 2029.66   | 1.23                      |
| 27            |    |   | 0.4      | 4,7,12,14     | 779.71    | 2417.53   | 1.72                      |
| 28            |    |   | 0.6      | 1,4,7,12      | 910.21    | 2711.44   | 31.59                     |
| 29            |    |   | 0.8      | 1,4,7,8       | 1026.52   | 3045.38   | 15.00                     |
| 30            |    |   | 1        | 1,4,7,8       | 1118.23   | 3292.30   | 24.74                     |

The computational time is also displayed in Table 7.4. The time increases when  $\alpha$  increases and when the hub number increases. Fig. 7.6 shows the relationship between the computational time to  $\alpha$  for  $p=2$ ,  $p=3$  and  $p=4$ . In Fig. 7.6 (a) for 10 node samples, the computational time increases as  $\alpha$ , but it is disordered in Fig. 7.6 (b), because the computational time with  $\alpha=0.6$ (Yellow line) is longer than with  $\alpha=0.8$ (Blue line). Fig. 7.6 also illustrates the relationship of computational time and hub number for  $\alpha=0.2, 0.4, 0.6$  and  $1.0$ . We can see that the computational time for Sample 1-15 and Sample 16-30 increases respectively in the order of  $p=2$ ,  $p=3$  and  $p=4$  since the yellow line (the samples with 4 hubs) is always the highest in Fig. 7.6 (a) and (b), meanwhile the pink one (the samples with 3 hubs) is higher than the blue one (the samples with 2 hubs).



(a) Relationship between the computational time to  $\alpha$  for 10 node samples



(b) Relationship between the computational time to  $\alpha$  for 15 node samples

**Fig. 7.6** Relationship between the computational time to  $\alpha$  for  $p=2$ ,  $p=3$  and  $p=4$

The vehicle (A, B and C) average load rates are displayed in Table 7.5. All the average load rates for vehicle A and B are bigger than 0.65. It suggests that Vehicle A and B are the appropriate vehicle type for these samples. For Vehicle C, almost the entire average load rates for all 30 samples are bigger than 0.55 except for the Sample 11-13 and Sample 15 which are distinguished by pink in the table. In these cases, a smaller type of Vehicle is advised.

**Table 7.5:** The load rates for vehicle A, B and C in Sample 1-30

| Sample Number | n  | p | $\alpha$ | Hub locations | Average load rate(%) |           |           |
|---------------|----|---|----------|---------------|----------------------|-----------|-----------|
|               |    |   |          |               | Vehicle A            | Vehicle B | Vehicle C |
| 1             | 10 | 2 | 0.2      | 7,9           | 74.65                | 74.65     | 67.37     |
| 2             |    |   | 0.4      | 7,9           | 74.65                | 74.65     | 67.37     |
| 3             |    |   | 0.6      | 7,9           | 74.65                | 74.65     | 67.37     |
| 4             |    |   | 0.8      | 7,9           | 74.65                | 74.65     | 67.37     |
| 5             |    |   | 1        | 4,7           | 71.39                | 71.39     | 67.37     |
| 6             |    | 3 | 0.2      | 4,6,7         | 73.48                | 73.48     | 71.34     |
| 7             |    |   | 0.4      | 4,6,7         | 73.48                | 73.48     | 71.34     |
| 8             |    |   | 0.6      | 4,6,7         | 73.48                | 73.48     | 71.34     |
| 9             |    |   | 0.8      | 4,7,9         | 71.08                | 71.08     | 73.43     |
| 10            |    |   | 1        | 4,7,9         | 71.08                | 71.08     | 73.43     |
| 11            | 15 | 4 | 0.2      | 3,4,6,7       | 74.44                | 74.44     | 47.01     |
| 12            |    |   | 0.4      | 4,6,7,8       | 71.58                | 71.58     | 38.50     |
| 13            |    |   | 0.6      | 4,6,7,8       | 71.58                | 71.58     | 38.50     |
| 14            |    |   | 0.8      | 4,6,7,9       | 73.47                | 73.47     | 58.55     |
| 15            |    |   | 1        | 1,4,7,9       | 67.29                | 67.29     | 42.31     |
| 16            |    | 2 | 0.2      | 4,12          | 74.34                | 74.34     | 83.67     |
| 17            |    |   | 0.4      | 4,12          | 74.34                | 74.34     | 83.67     |
| 18            |    |   | 0.6      | 4,12          | 74.34                | 74.34     | 83.67     |
| 19            |    |   | 0.8      | 4,11          | 76.00                | 76.00     | 84.35     |
| 20            |    |   | 1        | 4,11          | 76.00                | 76.00     | 84.35     |
| 21            |    | 3 | 0.2      | 4,7,12        | 74.96                | 74.96     | 85.38     |
| 22            |    |   | 0.4      | 4,7,12        | 74.96                | 74.96     | 79.48     |
| 23            |    |   | 0.6      | 4,7,12        | 74.96                | 74.96     | 79.48     |
| 24            |    |   | 0.8      | 4,7,8         | 75.74                | 75.74     | 75.67     |
| 25            |    | 4 | 1        | 4,7,8         | 75.74                | 75.74     | 75.67     |
| 26            |    |   | 0.2      | 4,7,12,14     | 74.45                | 74.45     | 66.70     |
| 27            |    |   | 0.4      | 4,7,12,14     | 74.45                | 74.45     | 66.70     |
| 28            |    |   | 0.6      | 1,4,7,12      | 74.72                | 74.72     | 66.70     |
| 29            |    |   | 0.8      | 1,4,7,8       | 75.57                | 75.57     | 71.26     |
| 30            |    |   | 1        | 1,4,7,8       | 75.57                | 75.57     | 71.26     |

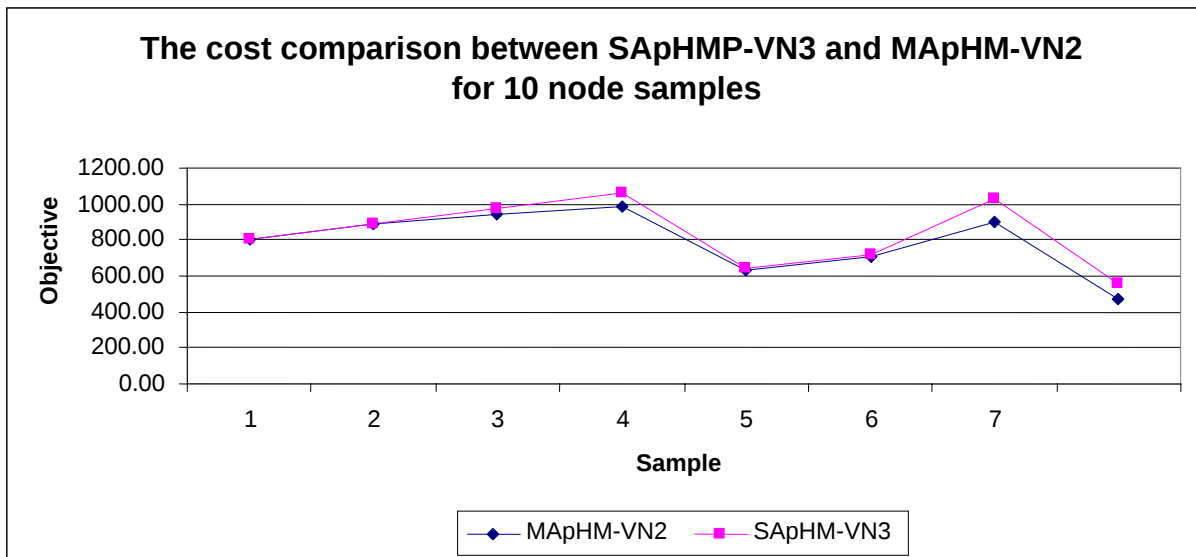
Indeed, the allocation situation changes according to the value of  $\alpha$  for the samples with the same node number n and same hub number p. Take the Sample 21-23 as an example, the allocation situations are shown in Table 7.6. It is because that the volumes transported between the hubs increase when  $\alpha$  decreases.

**Table 7.6:** Allocation results for Sample 21-23

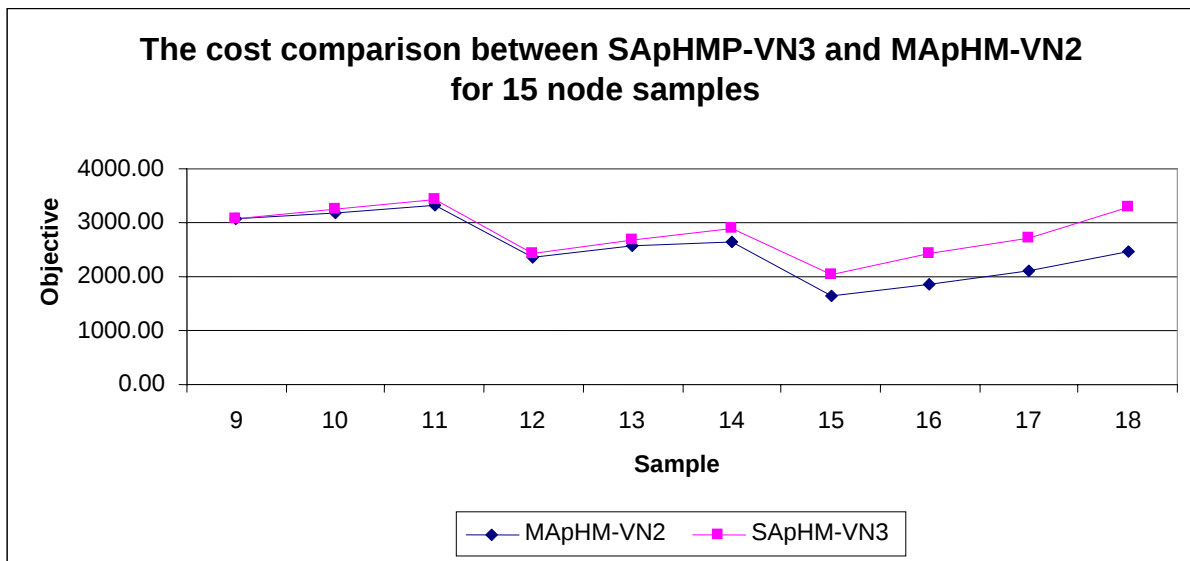
| Sample Number | n  | p | allocation |
|---------------|----|---|------------|
| 21            | 15 | 3 | 0.2        |
| 22            |    |   | 0.4        |
| 23            |    |   | 0.6        |

### 7.4.3 Comparison of Single to Multiple allocation p-hub median problems

Now, we compare the performance of SApHM-VN3 with MApHM-VN2 for the samples with the same  $n$ ,  $p$ ,  $\alpha$  and facilities shown in Table 7.7. The cost of SApHM-VN3 is always higher than or equal to MApHM-VN2 (see in Fig. 7.7). SApHM-VN3 is stronger than MApHM-VN2 since the solution to SApHM-VN3 must be a feasible allocation of MApHM-VN2, but not necessarily the best. As we can see in Fig. 7.7, the cost line of MApHM-VN2 (blue line) meets the cost line of SApHM-VN3 (pink line) at several points, for example, Sample 1 ( $n=10$ ,  $p=2$ ,  $\alpha = 0.2$ ). It is because the results of MApHM-VN2 are integer solution. In this case, the solution of MApHM-VN2 is the same to SApHM-VN3.



(a) The cost comparison between SApHM-VN3 and MApHM-VN2 for 10 node samples



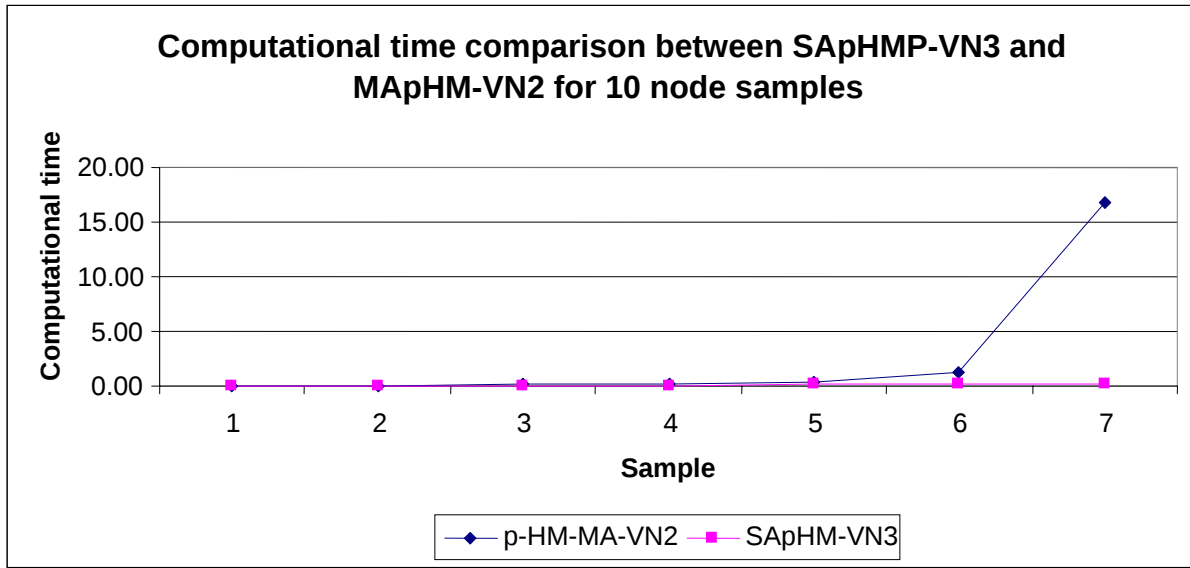
(b) The cost comparison between SApHM-VN3 and MApHM-VN2 for 10 node samples  
(c)

Fig.7.7 The cost comparison between SApHM-VN3 and MApHM-VN2

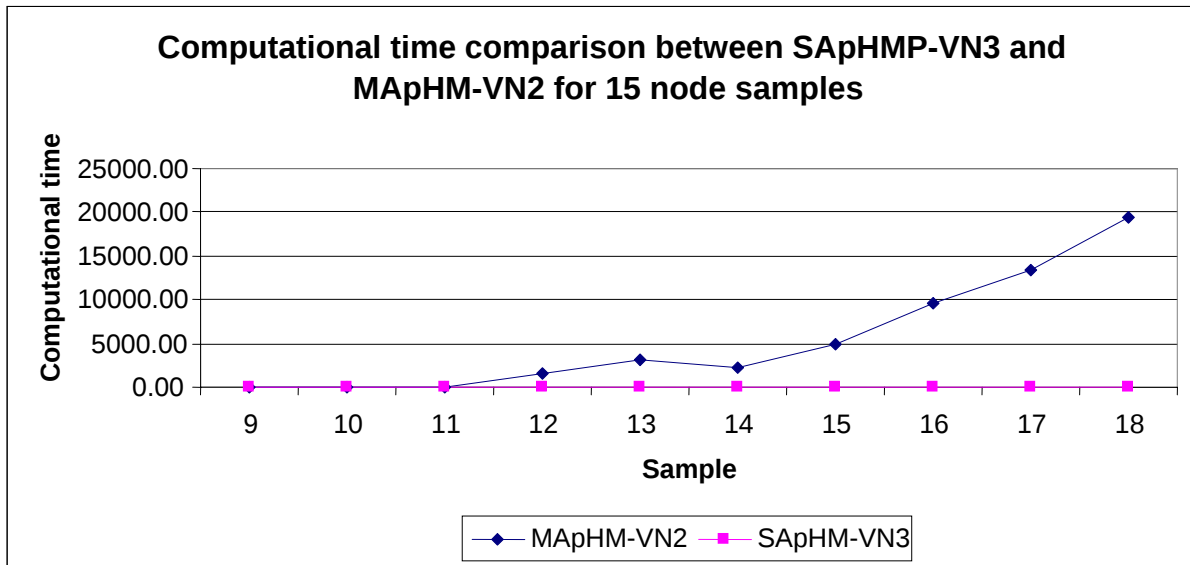
Table 7.7 Comparison between SApHM-VN3 and MApHM-VN2

| Sample Number | n  | p | $\alpha$ | Hub locations | Objective |           | Computational time(sec.) |           |
|---------------|----|---|----------|---------------|-----------|-----------|--------------------------|-----------|
|               |    |   |          |               | MApHM-VN2 | SApHM-VN3 | MApHM-VN2                | SApHM-VN3 |
| 1             | 10 | 2 | 0.2      | 7,9           | 806.10    | 806.10    | 0.03                     | 0.05      |
| 2             |    | 2 | 0.4      | 7,9           | 887.73    | 892.58    | 0.06                     | 0.08      |
| 3             |    | 2 | 0.6      | 7,9           | 937.83    | 979.06    | 0.09                     | 0.08      |
| 4             |    | 2 | 0.8      | 7,9           | 981.07    | 1065.54   | 0.09                     | 0.08      |
| 5             |    | 3 | 0.2      | 4,6,7         | 636.51    | 638.47    | 0.30                     | 0.13      |
| 6             |    | 3 | 0.4      | 4,6,7         | 709.05    | 715.98    | 1.25                     | 0.14      |
| 7             |    | 3 | 1        | 4,6,7         | 904.81    | 1023.34   | 16.86                    | 0.17      |
| 8             |    | 4 | 0.2      | 3,4,6,7       | 476.36    | 555.10    | 1.86                     | 0.16      |
| 9             | 15 | 2 | 0.2      | 4,12          | 3066.34   | 3076.87   | 0.13                     | 0.22      |
| 10            |    | 2 | 0.4      | 4,12          | 3195.63   | 3244.09   | 4.74                     | 0.22      |
| 11            |    | 2 | 0.6      | 4,12          | 3307.11   | 3411.31   | 7.31                     | 0.20      |
| 12            |    | 3 | 0.2      | 4,7,12        | 2360.81   | 2422.86   | 1574.97                  | 0.63      |
| 13            |    | 3 | 0.4      | 4,7,12        | 2555.03   | 2667.74   | 3226.49                  | 1.34      |
| 14            |    | 3 | 0.6      | 4,7,12        | 2642.88   | 2895.06   | 2184.69                  | 5.13      |
| 15            |    | 4 | 0.2      | 4,7,12,14     | 1654.86   | 2029.66   | 4833.44                  | 1.23      |
| 16            |    | 4 | 0.4      | 4,7,12,14     | 1846.76   | 2417.53   | 9551.89                  | 1.72      |
| 17            |    | 4 | 0.6      | 4,7,12,14     | 2096.01   | 2711.44   | 13348.89                 | 31.59     |
| 18            |    | 4 | 1        | 1,4,7,8       | 2461.49   | 3292.30   | 19478.80                 | 24.74     |

Moreover, the computational time of MApHM-VN2 is longer than SApHMP-VN3 (see in Fig.7.8) although more constraints and variables are needed in SApHM-VN3.



(a)Computational time comparison between SApHM-VN3 and MApHM-VN2



(b) Computational time comparison between SApHM-VN3 and MApHM-VN2 for 15 node samples

Fig. 7.8 Computational time comparison between SApHM-VN3 and MApHM-VN2

#### 7.4.4 Comparison between FMA-VN', FSA-VN, MApHM-VN2 and SApHMP-VN3

In Chapter 6 and Chapter 7, we have provided four allocation problems, including FMA-VN, FSA-VN, MApHM and SApHM in operational phase and proposed their respective formulations. Their costs are compared in Table 7.8 for the samples with the same

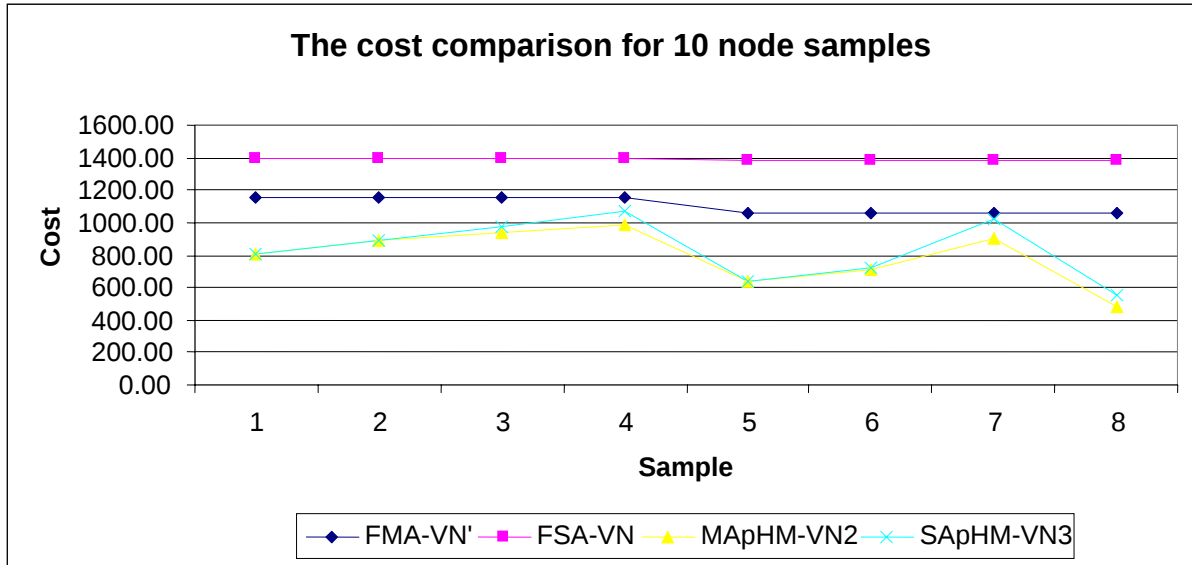
$n$ ,  $p$ ,  $\alpha$  and intermediate facilities. However, the index  $\alpha$  does not exist in the samples for FMA-VN and FSA-VN. In this case, we define the sample with  $n$ ,  $p$  and intermediate facilities in facility allocation problem as the same sample with the same  $n$ ,  $p$  and hubs in the hub allocation problem, without consideration of  $\alpha$ . For example, the sample of FMA-VN' and FSA-VN3 with  $n=10$ ,  $p=2$  and intermediate facilities=7, 9 is compared with the samples of MApHM-VN2 and SApHM-VN3 with  $n=10$ ,  $p=2$ , intermediate facilities = 7, 9 and  $\alpha = \{0.2, 0.4, 0.6, 0.8\}$ .

**Table 7.8** Cost comparison between FMA-VN', FSA-VN, SApHM-VN3 and MApHM-VN2

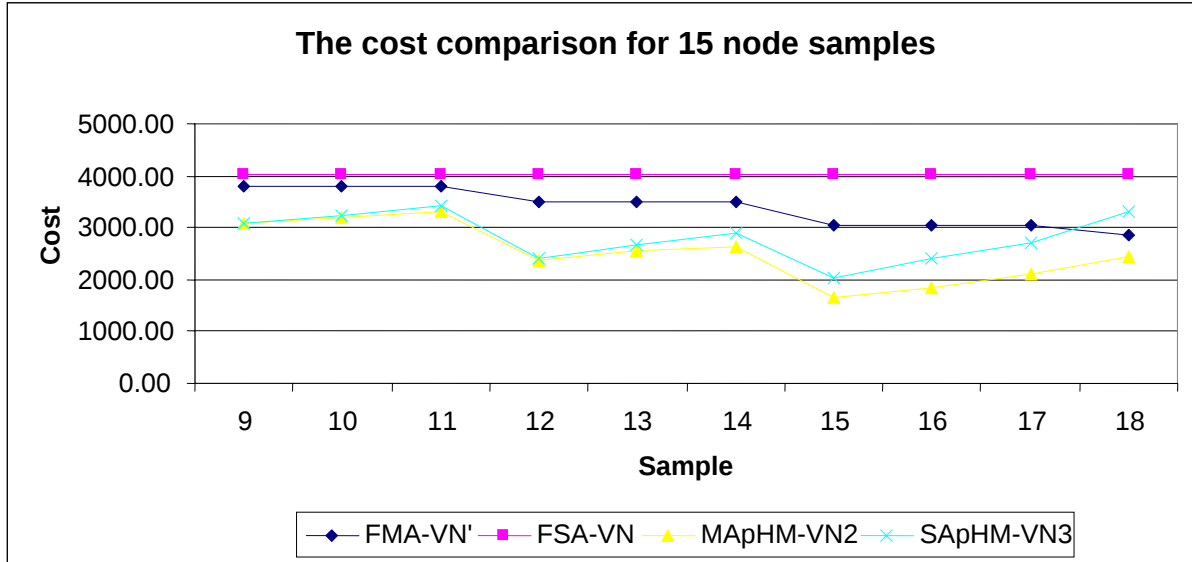
| Sample Number | n  | p | $\alpha$ | Intermediate | Objective |         |           |           |
|---------------|----|---|----------|--------------|-----------|---------|-----------|-----------|
|               |    |   |          | facility     | FMA-VN'   | FSA-VN  | MApHM-VN2 | SApHM-VN3 |
| 1             | 10 | 2 | 0.2      | 7,9          | 1152.36   | 1390.39 | 806.1     | 806.1     |
| 2             |    | 2 | 0.4      | 7,9          | 1152.36   | 1390.39 | 887.73    | 892.58    |
| 3             |    | 2 | 0.6      | 7,9          | 1152.36   | 1390.39 | 937.83    | 979.06    |
| 4             |    | 2 | 0.8      | 7,9          | 1152.36   | 1390.39 | 981.07    | 1065.54   |
| 5             |    | 3 | 0.2      | 4,6,7        | 1056.15   | 1380.99 | 636.51    | 638.47    |
| 6             |    | 3 | 0.4      | 4,6,7        | 1056.15   | 1380.99 | 709.05    | 715.98    |
| 7             |    | 3 | 1        | 4,6,7        | 1056.15   | 1380.99 | 904.81    | 1023.34   |
| 8             |    | 4 | 0.2      | 3,4,6,7      | 1056.15   | 1380.99 | 476.36    | 555.1     |
| 9             | 15 | 2 | 0.2      | 4,12         | 3808.77   | 4024.45 | 3066.34   | 3076.87   |
| 10            |    | 2 | 0.4      | 4,12         | 3808.77   | 4024.45 | 3195.63   | 3244.09   |
| 11            |    | 2 | 0.6      | 4,12         | 3808.77   | 4024.45 | 3307.11   | 3411.31   |
| 12            |    | 3 | 0.2      | 4,7,12       | 3504.46   | 4024.45 | 2360.81   | 2422.86   |
| 13            |    | 3 | 0.4      | 4,7,12       | 3504.46   | 4024.45 | 2555.03   | 2667.74   |
| 14            |    | 3 | 0.6      | 4,7,12       | 3504.46   | 4024.45 | 2642.88   | 2895.06   |
| 15            |    | 4 | 0.2      | 4,7,12,14    | 3052.26   | 4024.45 | 1654.86   | 2029.66   |
| 16            |    | 4 | 0.4      | 4,7,12,14    | 3052.26   | 4024.45 | 1846.76   | 2417.53   |
| 17            |    | 4 | 0.6      | 4,7,12,14    | 3052.26   | 4024.45 | 2096.01   | 2711.44   |
| 18            |    | 4 | 1        | 1,4,7,8      | 2840.82   | 4024.45 | 2461.49   | 3292.3    |

Fig. 7.8 shows us the cost comparison between FMA-VN', FSA-VN, MApHM-VN2 and SApHM-VN3. It is not difficult to find that the cost of FSA-VN (pink line) is always the highest case followed by FMA-VN' (blue line), and then is the cost of SApHM-VN2 (light blue line). At last, MApHM-VN2 (yellow line) is the case with the lowest cost. In a word, the cost of these four cases can be sorted as follows:

$$Cost_{FMA-VN'} \geq Cost_{FSA-VN} \geq Cost_{SApHM-VN3} \geq Cost_{MApHM-VN2}.$$



(a) The cost comparison for 10 node samples



(b) The cost comparison for 15 node samples

**Fig.7.8** Cost comparison between FMA-VN', FSA-VN, MApHM-VN2 and SApHM-VN3

So, it seems that the hub structure is better than the facility structure both for single-allocation and for multi-allocation problems. It is because that in facility location problems, there is no exchange between the facilities. Additionally, the multi-allocation seems better than the single-allocation both for the hub structure and for the facility structure. It can be explained because of the tighten constraints of single-allocation.

## 7.5 Conclusion

Hub 1-level network  $BG_{12}$  is one of the two types of the 1-level basic network structure, in which the intermediate facilities are known as hubs. Different to simple 1-level network, the goods can be exchanged between hubs on the same level by high-frequency, high-capacity services. The distribution problem with the hub 1-level network structure is generally researched as Hub Location Problem (HLP) in the research area. Furthermore, the transportation researches on the hub 1-level network structure mainly focus on the strategic phase and tactical phase.

In this chapter, the previous formulations for the 2-stop single/multi-allocation p-hub median problems in strategic phase and tactical phase are presented, and then a new cost considering the vehicle costs instead of the volumetric costs of the goods is considered. The formulations MApHM-VN2 and SApHMP-VN3 are provided for SApHM and MApHM problem in operational phase. In the last section of the chapter, the formulations are tested on the benchmark CAB, and the computational results are presented, analyzed and compared with the formulations proposed in Skorin-Kapov et al. (1996) for the volumetric costs.

In all cases, the objective with vehicle costs is greater than the objective with volumetric costs because of the non-full vehicle load. We have already got this same result for the facility allocation problem in chapter 6. In fact, the objective with multiple allocations is lower than the objective with only single allocation but more difficult to solve (i.e. a longer Computational time). We have already got this same result for the facility allocation problem in chapter 6.

When we compare the results obtained at chapter 6 for facilities and the results obtained in this chapter for hubs, the objective for hub-allocation problem is lower than the objective of the facility-allocation. So the hub structure is profitable. It is true for volumetric costs as well as for vehicle costs.

In the next part, two real distribution problems encountered by two French companies are introduced. The whole resolution process is divided into some smaller sub-networks by applying our hierarchical and structured methodology presented in Part II and then the resulting basic distribution problems in operational phase are solved. The results are provided to show the efficiency of our proposed methodology and the formulations provided in Part III.



## Part IV

# APPLICATIONS OF THE PROPOSED APPROACH



# CHAPTER 8: Example SDIS59

## 8.1 Introduction

In the two previous parts, Part II and Part III, we have respectively introduced in detail the decomposition process to divide the original transport system into several smaller basic distribution networks in strategic phase, tactical phase and operational phase with our proposed hierarchical and structure methodology, and the solution methods in operational phase to the basic distribution networks, involving basic 0-level network  $BG_{02}$ , basic simple 1-level network  $BG_{11}$  and basic hub 1-level network  $BG_{12}$ . In this chapter, our proposed methodology is applied to a real example, the distribution instance of the Regional Fire and Emergency Center (Le Service Départemental d'Incendie et de Secours du Nord, SDIS59) in the north of France, which we have briefly mentioned in Chapter 5.

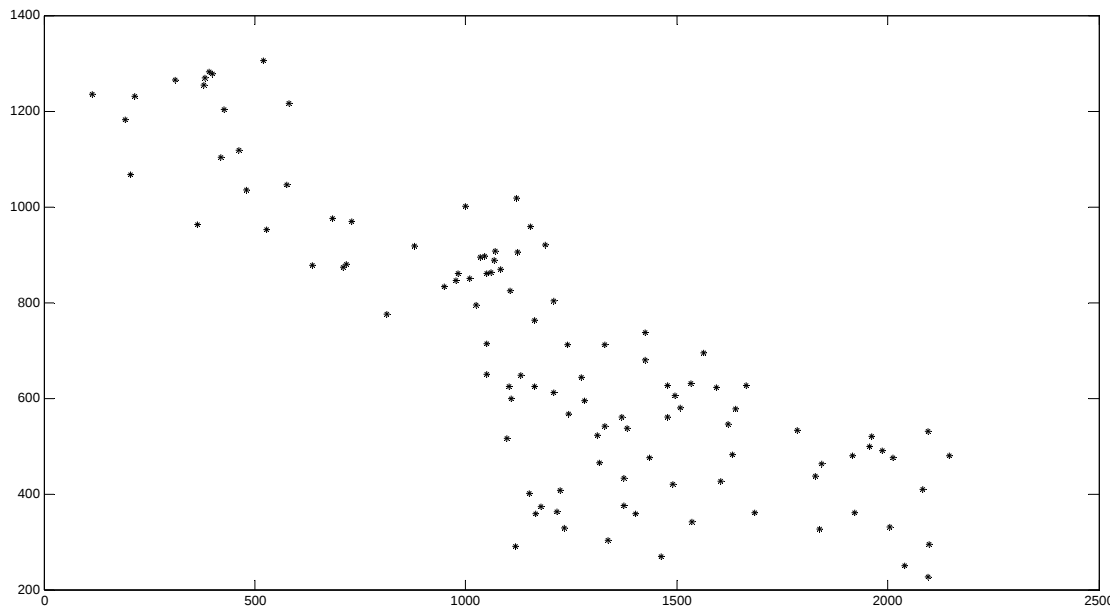
This chapter gives the results and analysis of the various distribution strategies that have been designed using different network structures described in Chapter 4. The remaining sections of this chapter are organized as follows: in Section 8.2, the problem background of the SDIS59 is stated; then in Section 8.3, three different network structures are compared to realize the transport system of SDIS59, one of them is the basic 0-level network  $BG_{02}$  and the two others are based on the basic simple 1-level distribution network  $BG_{12}$ . Especially, two different transport strategies are provided on the basis of the network  $BG_{02}$ . The performances in term of the transport distance are finally compared to provide certain support for the company to select the appropriate transport strategy.

## 8.2 Problem statement

SDIS59 is the Regional Fire and Emergency Center which is in charge of the rescue service for firefighter department of northern France. In 2007, there were nearly 132,000 operations within which 70% were for the incidences of victim rescue. The activities of the regional service center (Pharmacie à usage intérieur ou Pharmacie départementale, PUI which is a department of the SDIS59, are to supply the medicines to the 110 firefighter centers (Centre d'incendie et de secours, CIS) in the northern region and to maintain their equipments.

The distribution of the 110 firefighters is shown in the Fig.8.1 below, in which the firefighters are located throughout the whole northern region and the regional service center is located in the center of Lille. The coordinates of the firefighters and of the regional service center are displayed in Annex VI.

Our problem in this chapter focuses on the medicine distribution between the regional service center and the 110 firefighter centers. In this case, some strategic decisions and long-term policies should be firstly determined. Thus, three different strategies with respect to the distribution network structure are respectively proposed. The comparisons of the corresponding distribution sequences in operational phase are fed back to offer some basis to help the decision maker to make decisions.



**Fig.8.1** The distribution of the 110 firefighters

## 8.3 Performance of the different strategies

### 8.3.1 Performance of the old strategy

First of all, the old strategy employed by SDIS59 is presented. Before, the firefighter centers were decomposed into five groups according to their post code, i.e. the firefighters in the same administrative region were severed as the same group. These five groups are displayed in Fig.8.2, in which the red points represent the 110 firefight centers, the red star is the regional service center, and the five groups are distinguished by different colors.

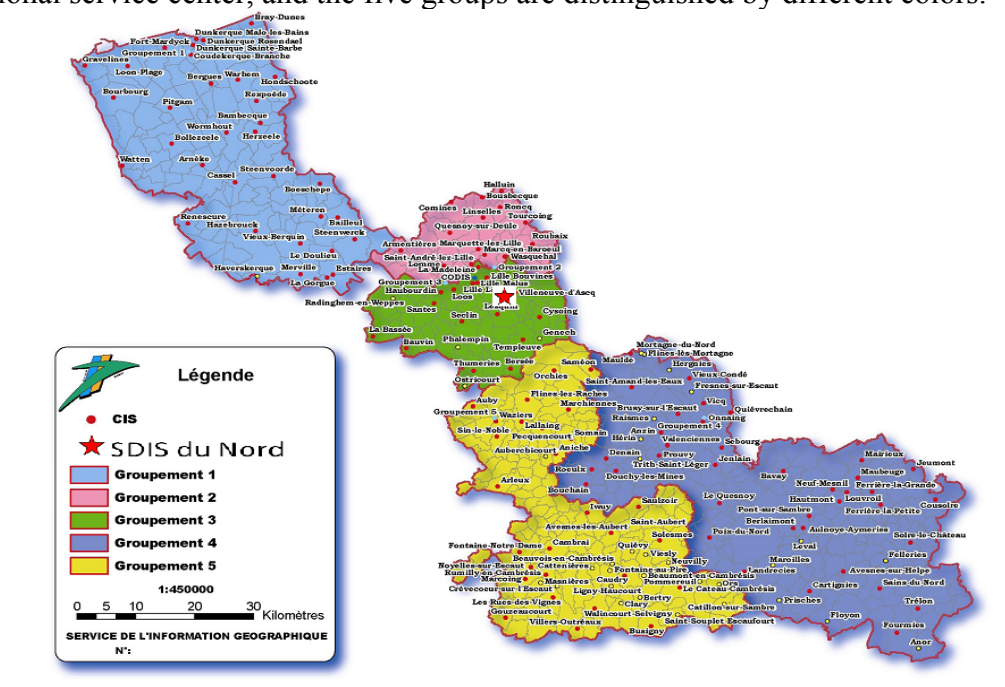
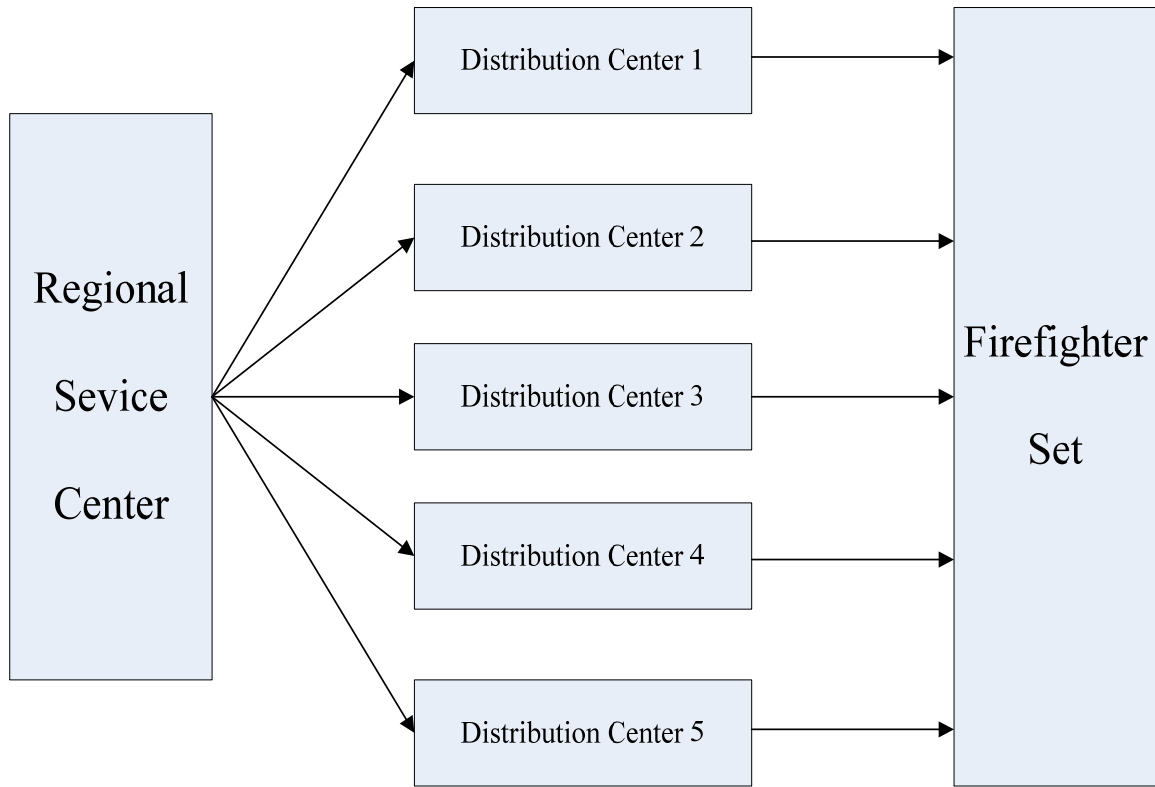


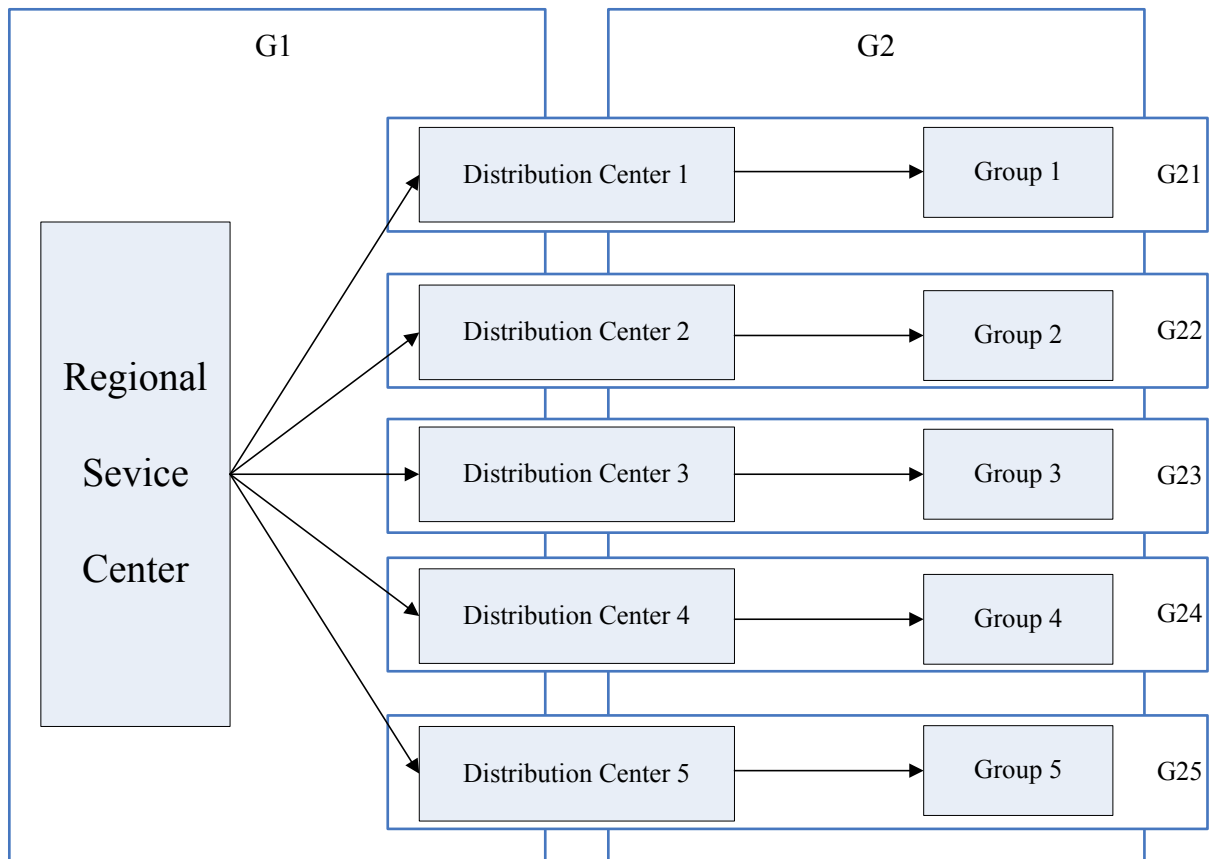
Fig. 8.2 Five groups of the SDIS59

The distribution process is a basic facility 1-level distribution network, shown in Fig.8.3, composed by three elements: the one regional service center, the distribution centers in five regions and the 110 firefighter centers. The regional service center firstly distributed the medicine to five distribution centers in each administrative region. Each distribution center organized the transport in the form of round-trip vehicles between the distribution center and each firefighter centers in its region. Here, we name this transportation strategy as Old Strategy.

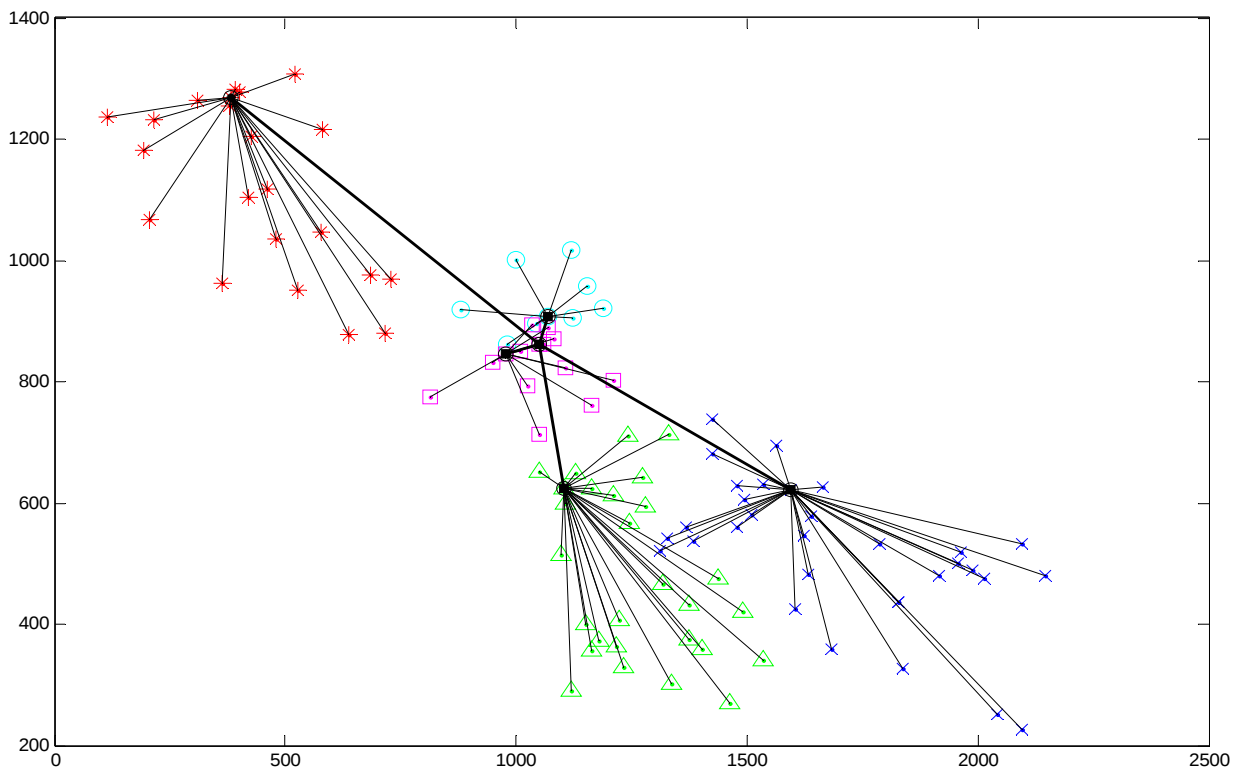


**Fig. 8.3** Distribution process of Old Strategy

Obviously, the distribution network structure of the Old Strategy is a kind of 1-level distribution network in which the regional service center is the origin, all of the firefighters are the destinations and the distribution centers in the five regions are the simple facilities. Then the sequence decomposition method is employed to divide the network into one basic 0-level distribution network  $G_1$  with the structure  $BG_{01}$  and the other basic 0-level distribution network  $G_2$  with the structure  $BG_{02}$ . Notably, the  $G_2$  can be further divided into five sub-networks  $G_{21}$ ,  $G_{22}$ ,  $G_{23}$ ,  $G_{24}$ ,  $G_{25}$ , taking the distribution center as origin and the firefighter centers in its administrative region as destinations. The decomposition process is displayed in the Fig.8.4 and the routing sequence is shown in Fig.8.5.



**Fig.8.4** The decomposition process of the 1-level distribution network



**Fig. 8.5** Distribution sequence of Old Strategy

### 8.3.2 Performance of Improvement Old Strategy

In the Old Strategy, the routing sequence is not good because there are too many round-trips. Then an improvement distribution strategy appears to be necessary. It is named as Improvement Old Strategy in this thesis. In this strategy, the distribution process is starting from the regional service center to the five distribution centers in each administrative region where the medicines are sorted and distributed to the firefighter centers in its region. The distribution routing problem from the regional service center to the firefighter centers in each region is regarded as a TSP.

In this case, the transport system can be defined as a 1-level distribution network with the same structure  $BG_{11}$  as in the Old strategy. Fig. 8.6 illustrates the routing of this strategy. In the Fig. 8.6, the firefighters in five regions are distinguished by five different colors, the corresponding routings are marked with the same color as the firefighters on the routing, the regional service centers are represented by black points and the routing between the regional service center and the distribution centers are black dotted lines.

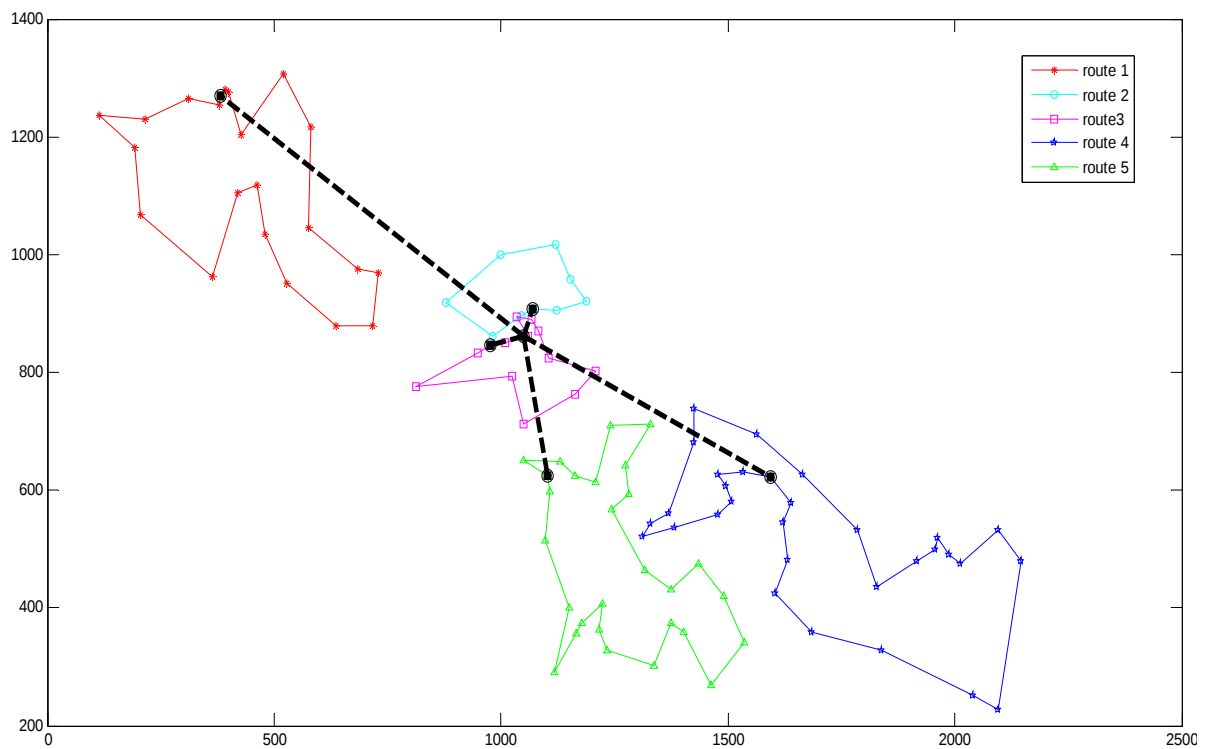


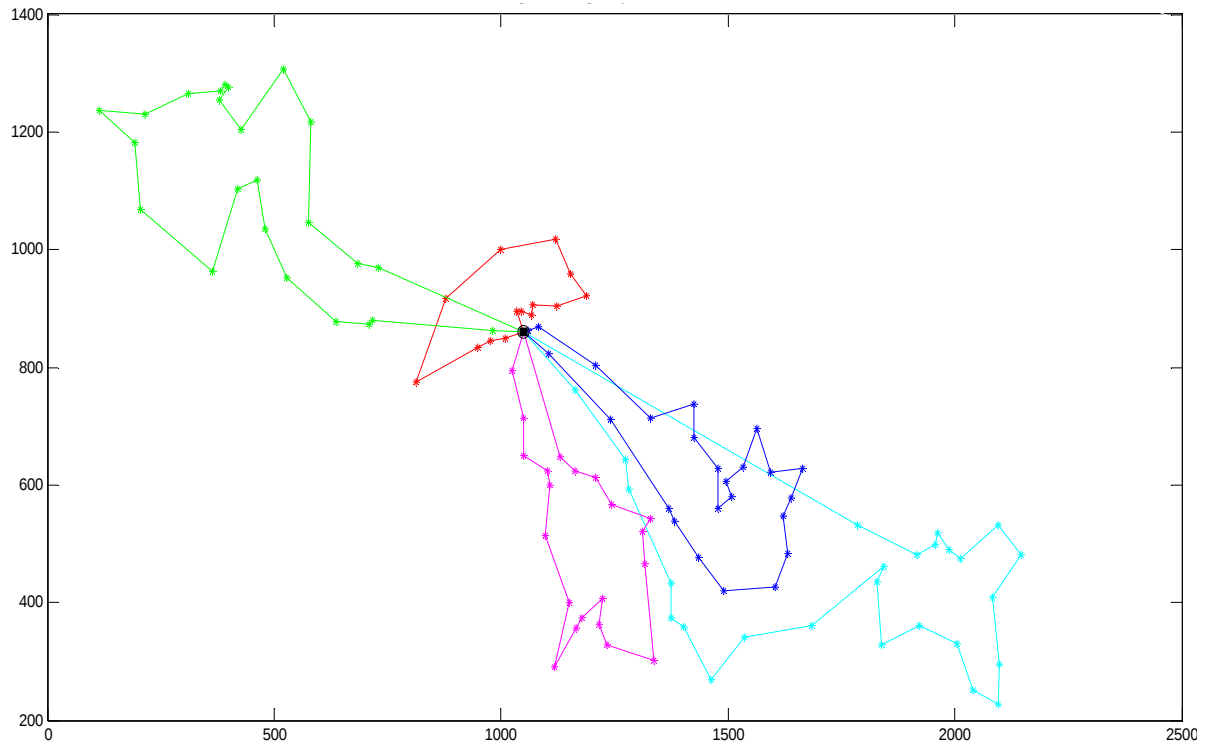
Fig. 8.6 Distribution sequence of Improvement Old Strategy

### 8.3.3 Performance of 0-level distribution strategy

In practice, the regional service center cannot handle all distributions by itself because there is not enough space for stocking the medicines. It is the reason why the five distribution centers are located and the distribution network is selected as a 1-level distribution network. In this situation, it is feasible to centralize the transport system when a bigger stockiest for all

of the medicines is located nearby the regional service center, several new vehicle types are selected, some divers are employed to the distribution.

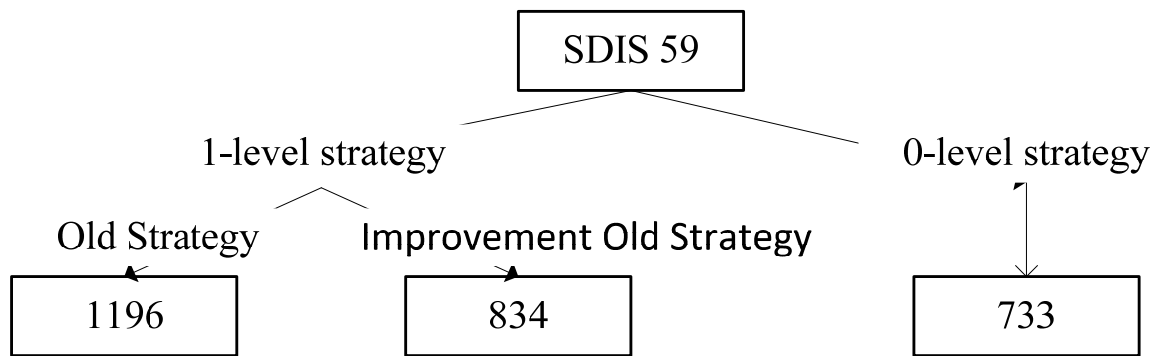
Then the transport system becomes a 0-level distribution network with just the regional distribution center as the origin and the 110 firefighter centers as destinations. We have proposed a decomposition-based heuristic approach to solve the 0-level distribution network in operational phase in Chapter 5 and applied it to solve the transport problem of the 0-level distribution network. The routing sequence for SDIS59 with the 0-level distribution network structure is shown again in Fig.8.7 below.



**Fig. 8.7** Distribution sequence of 0-level distribution strategy

### 8.3.4 Performance comparison

The distribution routing sequences for the three distribution strategies have been displayed in the three previous subsections. As we can see, different strategies can lead to totally different routing sequence in operation. Furthermore, it generates the different transport costs shown in Fig.8.8.



**Fig.8.8** Cost (in kilometers) of the three transport strategies

As we can see in Fig.8.8, the 0-level strategy is the cheapest transport strategy. Furthermore, the management costs of the five distribution centers are not needed. It is also practical in the real world since the intermediate facilities are deleted and the transport process is simplified.

## 8.4 Conclusion

This chapter presents a real example, SDIS59 in the north of France, solved by our hierarchical and structure methodology. Three different transport strategies are provided. For each distribution strategy, the network structure is firstly analyzed and the decomposition-based methodology is employed to generate the distribution sequence. The computational results are displayed to show that different strategies can lead to different routing sequences.

In the next chapter, another real example will be presented and the resolution process provided by our methodology will be introduced. Furthermore, our formulations presented in Chapter 7 for MApHM and SApHM with consideration of the vehicle number are applied to solve the transport problem in operational phase for the sub-network generated after the division of the network.



# CHAPTER 9: An Example of Express Company

## 9.1 Problem statement

In this chapter, our hierarchical and structured methodology is applied to another real delivery problem faced by a general delivery company, denoted as GEDECO here.

GEDECO is a subsidiary of a company group and deals with some of the movements of parcels for this group. It is structured in a conventional manner (with respect to different services) as the other companies of this type and size with human resource branch, operation branch, administrative branch and also transportation service department. Our study here is in collaboration with the last one, which is responsible of international transport, all urgent matters, warehousing and national logistics. In this circumstance, GEDECO takes a role of freight carriers with more than 600 vehicles. Its main activities are to collect the goods from suppliers, to deliver them throughout France by its parcel distribution network and finally among its customers. Furthermore, the group can offer its service in several forms (24-hour package, door-to-door delivery, parcel delivery relay...). We focus here on the bulky goods (furniture, household appliances...) transports on behalf of GEDECO. In this case, there is no 24-hour constraint because they are not urgent. In addition, we neglect the backflow of the packages that requires another level of complexity.

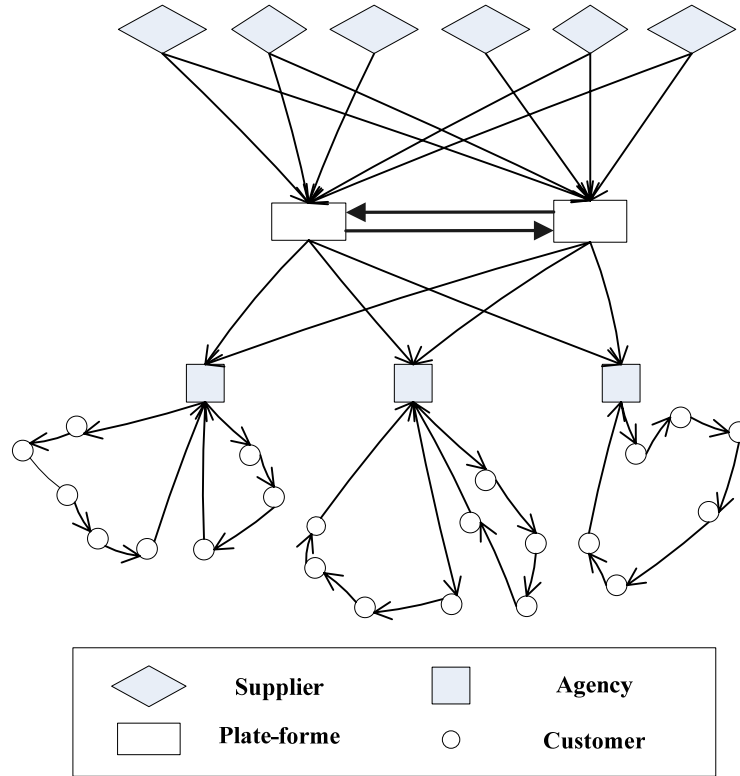
In order to achieve a higher service level and to provide a good transport process, GEDECO determines a 2-level distribution network structure in strategic phase to organize the national big parcels delivery. The 2-level transport system consists of 49 suppliers and thousands of customers on Level 0, two plate-forms on Level 1, and 26 agencies on Level 2 to directly serve the customers in their region. Fig. 9.1 illustrates the 2-level distribution network of GEDECO.

The starting point of the goods is generally from suppliers. Either these suppliers are based in France (or Area: Belgium, Netherlands ...), or they are distant suppliers, in which case they transport large quantities of goods in containers and store them in the stockists. The starting points of parcels in the national transport trip are either suppliers or stockists. Geographically, the stockists and the suppliers are spread throughout France. Thus, GEDECO takes care in negotiation to the carriers on a daily (or a few days in advance) to remove the goods on these starting points and to deliver them to one of the two platforms.

GEDECO has three platforms: P1, P2 and P3. P1 only handles the 24 hours packages (urgent parcels). Moreover, it dispatches the textile articles (urgent or not urgent) and the small parcels. We are not interested in this platform since our problem is with regard to bulky goods. The P2 mainly receives large number of imports because it is located close to most stockists. The P3 specifically receives the goods from suppliers. It is more important in terms of space and the number of docks than P2. Since neither P2 nor P3 does serve all agencies, shuttle vehicles make daily round trips between the two platforms. In this case, the platforms play the role of hubs, in which the goods are sorted again, grouped and consolidated into large

volume. The goods are shipped to the agencies after they are exchanged between the platforms.

The 26 agencies are the last elements of the delivery trip before the clients. They are distributed throughout France and Corsica and take responsibility for door-to-door delivery after receipt of the package from the platforms. The agencies have numerous drivers and vehicle types.



**Fig. 9.1** 2-level distribution network of GEDECO

According to the statement above, it is not difficult to conclude that the national transportation process of GEDECO for the bulky goods can be concerned with multi-allocation 2-level oriented distributional network, not involving any cross-layer flow.

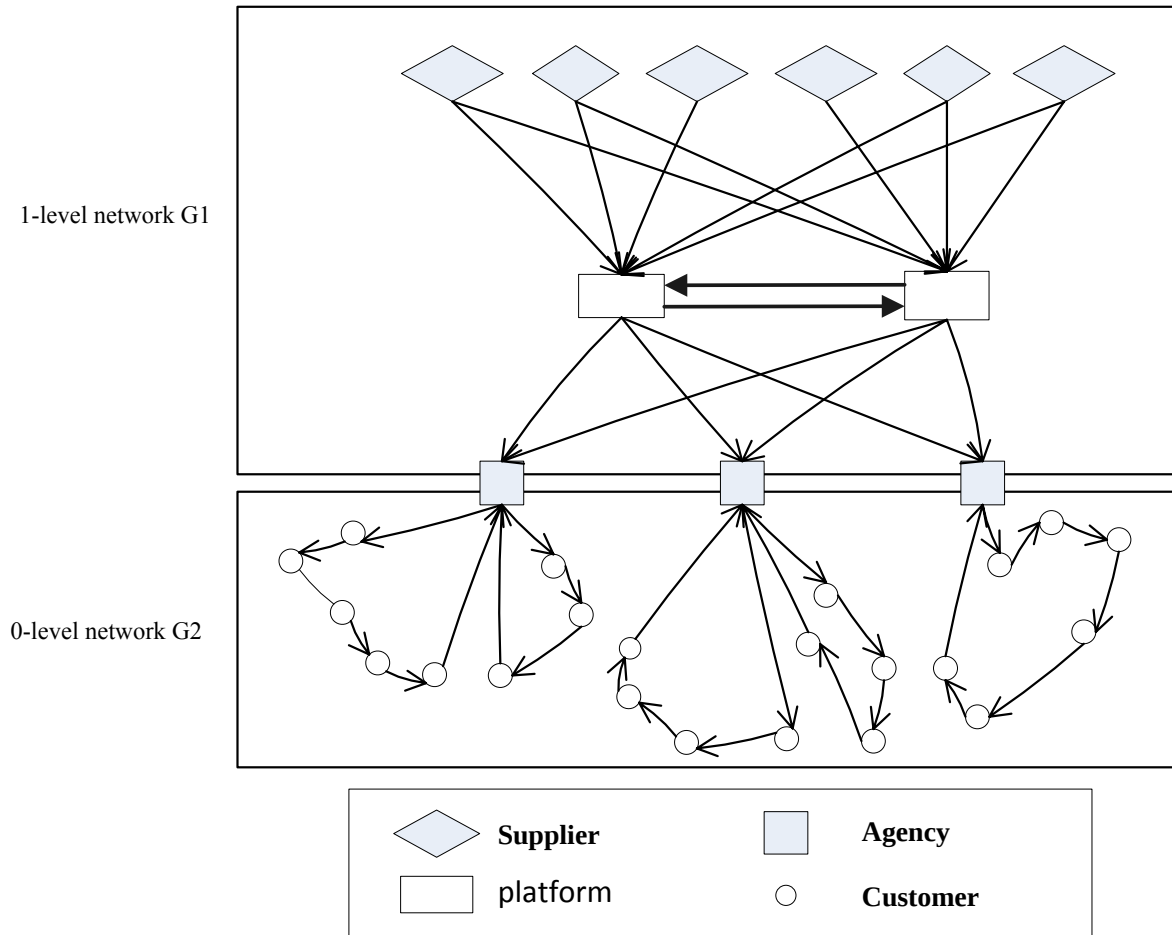
In the remaining sections, the resolution process of the transport system is respectively introduced in strategic phase, tactical phase and operational phase. In Section 9.2, our decomposition framework is employed to decompose the 2-level distribution network into the combination of several basic 0-level networks and a basic hub 1-level network. Then, the spatial decomposition in operational phase is presented to divide the large transport problem into smaller ones. In Section 9.3, our proposed approach for multi-allocation p-hub median is applied to solve the hub 1-level network, and the computational results are compared with the actual transport cost to show the performance of our approach.

## 9.2 Decomposition process

As we have mentioned in Chapter 4, the decomposition process is introduced in three phases in operation planning of the company. In this section, the transport problem of GEDECO is decomposed into smaller sub-problems by our proposed decomposition-based framework in three phases, involving strategic phase, tactical phase and operational phase.

### 9.2.1 Strategic phase

Recall that, there exist four decomposition methods in strategic phase. They are sequence decomposition, superposition decomposition, aggregation decomposition and facility location and allocation decomposition. Sequence decomposition focuses on handling the oriented distribution network with just layer-by-layer flow, while the superposition decomposition targets the cross-layer flow in the network. Obviously, the sequence decomposition needs to be applied since there is no cross-layer flow in the network. The distribution network is divided into two smaller sub-networks, one of which is a 1-level distribution network (G1) and the other of which is a 0-level distribution network (G2). Fig.9.2 shows us the results after the sequence decomposition. The 1-level network G1 is composed by the 49 suppliers, the two platforms and the 26 agencies. As we can see in Fig.9.2, G1 takes the basic hub 1-level distribution network, in which the suppliers are the origins, the agencies are the destinations and the 2 platforms play a role of hubs. On the other hand, the 0-level distribution network G2 is the basic 0-level distribution network, in which there is not any intermediate facility. G2 consists of 26 agencies and the thousands of customers throughout the country. Moreover, the agencies are the origins of G2 while the customers are the destinations in the network.

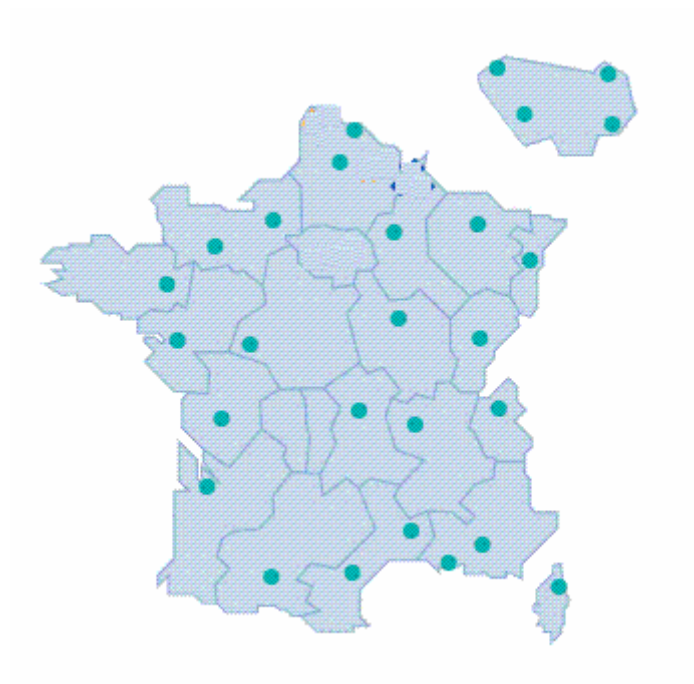


**Fig. 9.2** The results after sequence decomposition

### 9.2.2 Operational phase

In the daily transport management, the sub-network G1 is divided again into several smaller 0-level distribution networks according to the spatial decomposition method. Recall that, spatial decomposition is a method to decompose the distribution network in operational phase. It is to choose the adjacent nodes on the same level of the distribution network to belong to the same group. GEDECO applies a spatial decomposition in the manner to assign a customer to the agency in the corresponding administrative region. In other words, it decides each agency serves the customers with the same post code. Fig.9.3 shows us the spatial decomposition for the agencies, in which the green nodes represent the 26 agencies throughout the country, and the sub-regions for the agencies are delimited with blue lines.

Obviously, this manner to use spatial decomposition is not optimal and there surely exists better methods. For example, we could employ the cluster method used in chapter 5 to decompose it and then to generate the routing sequences.

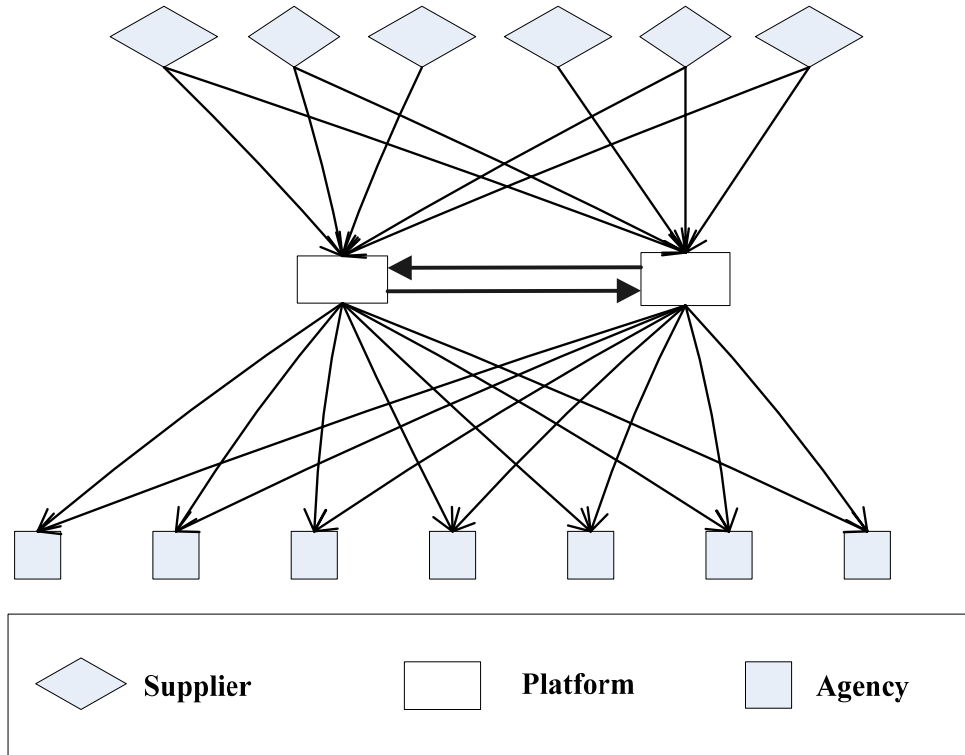


**Fig. 9.3** Territorial distribution of agencies

### 9.3 Performance of the 1-level distribution network

After the decomposition process in strategic phase and tactical phase, the whole national distribution network of GEDECO is decomposed into a set of one basic hub 1-level distribution network G1 and G2, which can be further divided into several smaller basic 0-level distribution networks. As we have mentioned above, we can use the same approach, RFCS, to divide G2 and to generate the routing sequence for G2. But we do not do it for two reasons: the solution process has been tested and verified in the previous chapter; we do not have enough real data. In this section, the solution process of G1 is provided according to our formulations (M<sub>Ap</sub>HM-VN2 and S<sub>Ap</sub>HM-VN3) in Chapter 7. The computational results are compared with respect to the costs provided by multi-allocation and single-allocation.

The sub-network, G1, of the national transport system can be regarded as a multi-allocation basic 1-level distribution network, having 49 suppliers, 26 agencies and 2 platforms. Fig. 9.4 is presented G1 so as to further clearly describe.



**Fig. 9.4** The sub-network G1

The computational experience is tested with always the same computer: All numerical tests are carried out on an Intel(R) Core(TM)2 CPU operating at T5500 @ 1.66GHz and the memory is 1 Gig RAM. The relative data, which are provided by the company, are given in Appendix VII-VIII: the volume required by each agency to suppliers and platforms and the cost from suppliers and agencies to platforms. There exist three vehicles types in the network. We take, here, the same notations as Vehicle A, Vehicle B and Vehicle C denoted in Chapter 7. Vehicle A is the tractor-trailer from suppliers to platforms, Vehicle B is the tractor-trailer from platforms to agencies, both of which have the capacity  $Q=T= 80\text{m}^3$  while vehicle C is the bigger truck-trailer shuttled between the two platforms with the capacity  $R=110\text{ m}^3$ .

The proposed formulations in Chapter 7 are difficult to solve for this real example, because the proposed formulation MApHM-VN2 leads to a mixed integer linear programming with 25284 variables and 6549 constraints. On the other hand, the formulation SApHM-VN3 leads to a mixed integer linear programming with 25442 variables and 49458 constraints. The numerical tests are carried out with LP/MIP solver CPLEX (version 9.0). Nevertheless, we do not use the default CPLEX to solve the formulations since these problems are much bigger than the samples in the Chapter 7.

The technical report of Baz et al. (2007) states a method called Selection Tool for Optimization Parameters (STOP), to tune the software parameters. The method attempts to find the good parameter values using a relatively small number of optimization trials. They select six parameters in CPLEX 9.0, shown in Column “Parameter” in Table. 9.1. After testing 1296 settings for these six parameters, they find the best configurations, shown in Column “Effect” in Table 9.1, to find the true best solution time.

**Table 9.1:** CPLEX default settings and the suggested settings of STOP

| Parameter           | Effect           |
|---------------------|------------------|
| MIP emphasis        | feasibility      |
| Node selection      | best-bound       |
| Branching var. sel. | automatic        |
| Dive type           | traditional dive |
| Fractional          | off              |
| MIR cuts            | off              |

Here, according to this report, we set the parameters shown in Table 9.2 to calculate MApHM-VN2.

**Table 9.2:** Parameters configuration to calculate MApHM-VN2

| Parameter           | Name                  | Value | Effect                                 |
|---------------------|-----------------------|-------|----------------------------------------|
| MIP emphasis        | CPX_PARAM_MIPEMPHASIS | 1     | Emphasize feasibility over optimality  |
| Node selection      | CPX_PARAM_NODESEL     | 1     | best-bound                             |
| Branching var. sel. | CPX_PARAM_VARSEL      | 0     | Let CPLEX choose variable to branch on |
| Dive type           | CPX_PARAM_DIVETYPE    | 1     | Traditional dive                       |
| Fractional          | CPX_PARAM_FRACCUTS    | -1    | Do not generate Gomory fractional cuts |
| MIR cuts            | CPX_PARAM_MIRCUTS     | -1    | Do not generate MIR cuts               |

Indeed, we have tried other configurations for the parameters of CPLEX 9.0, the configuration in Baz et al. (2007) performs better than the others. Furthermore, we set CPX\_PARAM\_TRELIM=500, i.e. the upper limit on the size of the branch and cut tree be

1000, to control the computational time. The computational results of MApHM-VN2 for GEDECO are displayed in Table 9.3. As we can see, the cost, computational time and the vehicle average load rate are separately shown in the table. Since we have limited the node file size, the solutions we have provided are not optimal, nevertheless, they are feasible. Furthermore, the solution accuracy shown in the table is below 0.05. Particularly, the solution accuracy of Sample 1 is below 0.0003. The computation time is around 3 hours since the limit size of node file is 500M and all the vehicle average load rates of the three type vehicles are more than 0.6. It means the vehicles we use now are of the appropriate types.

**Table 9.3** Results of MApHM-VN2 for GEDECO

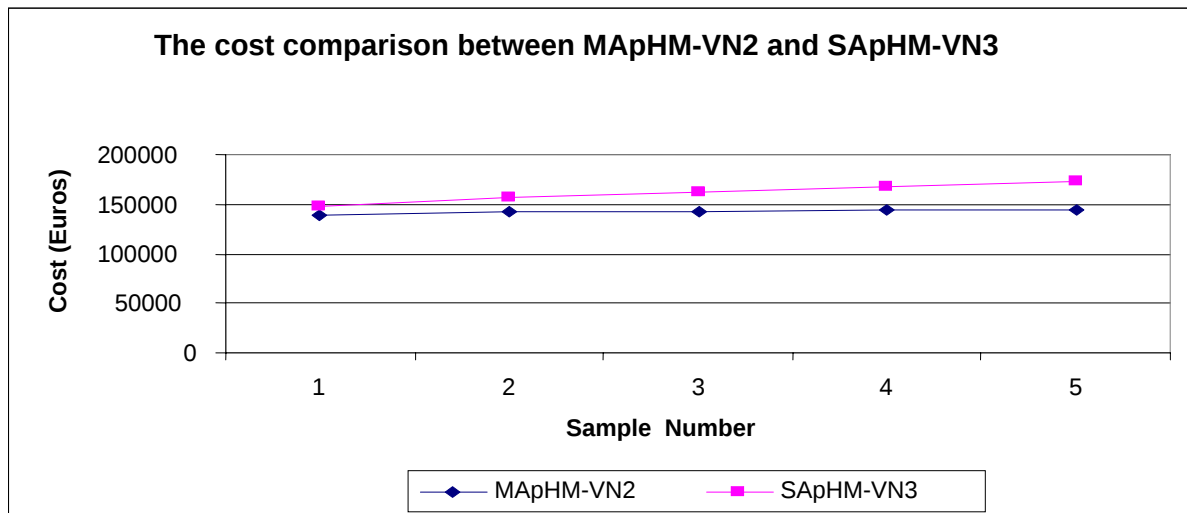
| Sample | alpha | Cost      | Computational | Vehicle average load rate |           |           |
|--------|-------|-----------|---------------|---------------------------|-----------|-----------|
| Number |       | (Euros)   | time (sec.)   | Vehicle A                 | Vehicle B | Vehicle C |
| 1      | 0,2   | 138261,65 | 11123,58      | 0,6564                    | 0,9363    | 0,998     |
| 2      | 0,4   | 142722,01 | 12499,94      | 0,6801                    | 0,9392    | 0,9227    |
| 3      | 0,6   | 143110    | 11183,66      | 0,6776                    | 0,9392    | 0,9807    |
| 4      | 0,8   | 143610,57 | 12222,92      | 0,6902                    | 0,9392    | 0,9989    |
| 5      | 1     | 143875,27 | 10905,53      | 0,6832                    | 0,9416    | 0,9944    |

We also test SApHM-VN3 with the parameter configuration in Table 9.2 and the results are shown in Table 9.4. As we have mentioned earlier, the SApHM-VN3 is much easier to solve than MApHM-VN2, so we do in addition set CPX\_PARAM\_EPGAP=0.00001, i.e. the relative MIP gap tolerance be 0.00001, to control the solution accuracy.

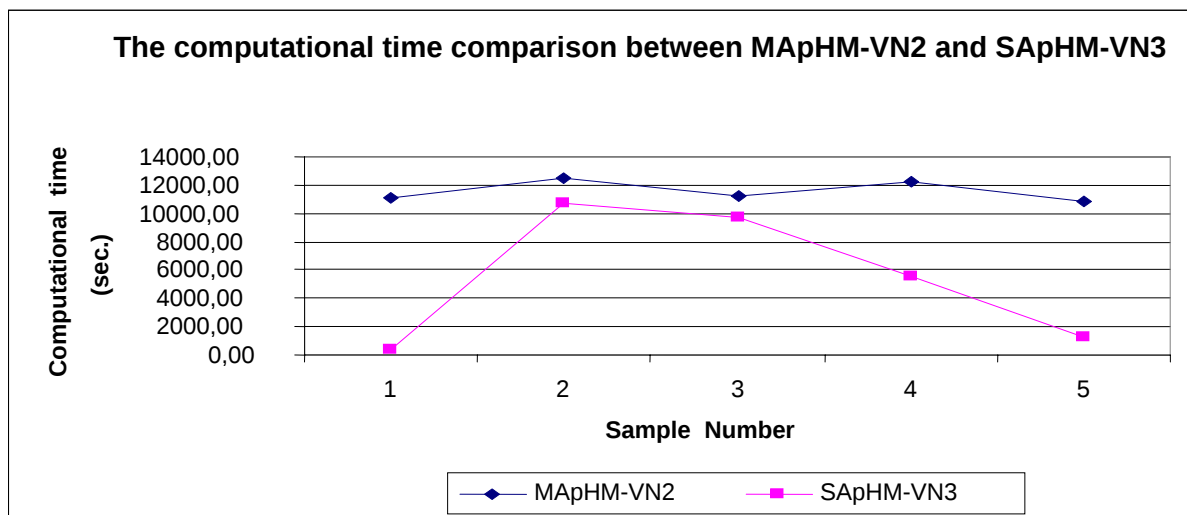
**Table 9.4** Results of SApHM-VN3 for GEDECO

| Sample | alpha | Cost      | Computational | Vehicle average load rate |           |           |
|--------|-------|-----------|---------------|---------------------------|-----------|-----------|
| Number |       | (Euros)   | time (sec.)   | Vehicle A                 | Vehicle B | Vehicle C |
| 1      | 0,2   | 148269,6  | 426,47        | 0,6453                    | 0,9296    | 0,9864    |
| 2      | 0,4   | 156113,29 | 10766,31      | 0,6453                    | 0,9296    | 0,9986    |
| 3      | 0,6   | 162830,45 | 9679,44       | 0,6453                    | 0,9296    | 0,9909    |
| 4      | 0,8   | 168389,26 | 5543,28       | 0,6453                    | 0,9296    | 0,9973    |
| 5      | 1     | 172990    | 1282,19       | 0,6453                    | 0,9296    | 0,9962    |

The cost comparison between MApHM-VN2 and SApHM-VN3 is shown in Fig.9.5. The cost of MApHM-VN2 (blue line) is much lower than SApHM-VN3 (pink line) even when it is just the feasible solution. And the computational time comparison is shown in Fig.9.6. Undoubtedly, the computational time of MApHM-VN2 (blue line) with the limitation of the node file size equal to 500M is already longer than SApHM-VN3 (pink line).



**Fig. 9.5** The cost comparison between MApHM-VN2 and SApHM-VN3



**Fig. 9.6** The computational time comparison between MApHM-VN2 and SApHM-VN3

## 9.4 Conclusion

In this chapter, another real example is solved according to our hierarchical and structured methodology. This example is mentioned about the national bulky goods transport system of a delivery company in France. It is, in fact, a multi-allocation 2-level distribution network, in which the trip starts from 49 suppliers/stockiest throughout the France, then the goods are transferred, grouped and consolidated in the 2 platforms, after that the goods are exchanged between the platforms and then delivered to the 26 agencies, at last the agencies delivered the goods to the customers by door-to-door delivery of 24-hours delivery service.

According to our decomposition-based methodology, we successively present the solution process of the 2-level distribution network in strategic phase and operational phase. Here, the decomposition in tactical phase is not mentioned and we consider it is an interesting subject for further research. The sequential decomposition method is used to divide the original network into the combination of one basic hub 1-level distribution network  $G1$  and one 0-level distribution network  $G2$  in strategic phase. Then  $G2$  is divided into several smaller sub-networks with the same basic 0-level distribution network structure. After that, the formulations provided for multi-allocation  $p$ -hub median problem (MApHM) of chapter 7 are employed, the computational test is carried out and the results are compared with the results generated by the single-allocation  $p$ -hub median problem (SApHM) to verify the efficiency of our formulations.

# CHAPTER 10: CONCLUSION AND PROSPECTIVES

## 10.1 Conclusions

In this research, a new transport problem, the General Delivery Problem, is proposed. It is a transport problem of the private companies throughout the whole operational process (strategic, tactical and operational phases). GDP is more general than VRP and TSP and this research centers to investigate a **hierarchical and structured methodology to globally solve the GDP**.

In fact, the main idea is to divide the original problem into some smaller basic sub-problems as independent as possible. In this way, the problem size is reduced because it is much simpler to solve small-scale sub-problems than to solve the original problem. Due to its advantages, our methodology is a promising way to find good solutions for large problems. The results of the research will contribute for the decision makers of the companies to select their transport strategy and to organize their transport.

This research consists of three major components: methodology, optimal transport theory and applications. According to these three themes, the major contributions that are derived for this study are summarized below.

### 10.1.1 Methodology

As we have mentioned above, the research on the transport problem is generally concentrated on a part of the supply chain and the distribution network with a simple structure and a special temporal horizon. However, the transport problem with complex network structure and confronted by the enterprise in several time horizons has been rarely considered by the researchers. We have summarized the major issues in the general three phases of operation planning and sequentially presented the relationships and interactions between the issues. In this way, the general solving process of GDP has been shown to provide a general solving methodology for decision makers. To be really operational, this methodology needs an instruction manual to guide the decomposition approach. This manual is not yet defined (it is one of the prospectives we plan to do) although we have already recommended along this thesis some principles to help the decision makers.

Since there is not a common model for the transportation system due to the complexity of the problem, a three-phase heuristic decomposition framework to solve GDP is proposed. The general approach gives a useful tool to support network planners. The procedure for the proposed framework consists of three phases involving decomposition phase, routing determination phase and improvement phase. In decomposition phase, four decomposition methods including sequence decomposition, superposition decomposition, aggregation decomposition and facility location and allocation decomposition are provided to simplify the complex network structure. And then, time decomposition and spatial decomposition are introduced to reduce the size of the sub-networks. A general idea of the improvement phase

has been presented because of the wide variety of GDP. The basic idea is to start from the routing sequence, then to choose one or several nodes from the network, and to change their allocation sets, finally to combine the new routing sequence in the cheapest way.

### 10.1.2 Optimal theory

In this decomposition framework, the original distribution network is firstly divided into several smaller distribution networks with basic networks. Then the distribution routing problem for each sub-networks are regarded as the sub-problems of the original GDP such as FLP, HLP, FAP, TP, TSP, VRP, etc, some which could be solved by the existing tools and/or heuristics algorithms. However, there were still some sub-problems that had not yet been studied. We have studied the sub-problems corresponding to the basic network structures in operational phase. The solutions of three sub-problems are proposed:

- Firstly, Capacitated Vehicle Routing Problem (CVRP), as a basic 0-level network is solved. This way of solving the CVRP problem is a direct application of our three-phase heuristic decomposition framework. We have designed a heuristic approach taking the Capacitated Clustering Algorithm (CCA) as spatial decomposition technique, applying Concorde as solver of TSPs, and then improving the solution with Simulated Annealing (SA).
- Then, we have studied the transport problem in operational phase for a 1-level network composed of simple facilities. We provide two new path-based mixed integer linear programming formulations, FSA-VN and FMA-VN for the facilities with respectively single/multi-allocation, in which a new cost is provided where the limitation of the resources is considered. The corresponding formulations taking into account the number and the limited capacity of the vehicles are presented. Additionally, the reformulations are provided to tighten the formulations and to improve the solution process. The formulations are tested on the benchmark set CAB. The Civil Aeronautics Board data set is a benchmark data set frequently used in the literature to test p-hub problems. The data set consists of 25 interacting nodes along with their flow volumes and transportation costs. The computational results show us that our new formulations are difficult to calculate. It is because the introduction of the vehicle number and the edge capacity constraints leads the formulation to become larger. Additionally, the costs of our new formulations are higher than the corresponding formulations in strategic phase and operational phase due to the existence of non-full load rate.
- Thirdly, we have studied the distribution problem in operational phase with the hub 1-level network  $BG_{12}$ . A new cost with consideration of the vehicle cost instead of the volumetric cost of the goods is proposed and the edge capacity is restricted by the vehicle numbers. The new formulations, MApHM-VN2 and SApHM-VN3, are mixed integer linear programming based on the classical formulations proposed by Skorin-Kapov et al. (1996). Then, these new formulations are tested on the standard CAB data set with the addition of the vehicle capacity and the cost for each vehicle. We compare the computational results of our new formulations for the single-allocation p-hub median problem (SApHMP) and the multi-allocation p-hub median problem (MApHMP) and the results in Skorin-Kapov et al. (1996). Similar conclusions to the computation result of the FMA-VN and FSA-VN are obtained.

### 10.1.3 Real application

In this research, two real examples in France are solved with our proposed methodology: SDIS591 and a general delivery company which is called GEDECO.

In SDIS59, three transport strategies, Old Strategy, Improvement Old Strategy and 0-level Distribution Strategy, are discussed. For each distribution strategy, the network structure is firstly analyzed and then our decomposition-based methodology is employed to generate the routing sequence. Particularly, for the 0-level Distribution Strategy, the routing sequence is provided by our heuristic approach for CVRP. The computational result of this approach is compared with five other methods to show that it outperforms these six approaches. Furthermore, the result comparison of the different distribution strategies offers the decision maker a tool to make the strategy and policies.

In the case of the general delivery company GEDECO, the distribution network structure and the distribution strategy were already fixed. To solve the transport problem, the sequential decomposition in strategic view and the spatial decomposition in operational view are successfully employed to divide the original network into the combination of one basic hub 1-level distribution network and several basic 0-level distribution networks. After that, our formulation MApHM-VN2 and SApHMP-VN3 are employed for the basic hub 1-level distribution network. The computational results show us that the cost of the SApHMP-VN3 is higher than MApHM-VN2 and it requires less computational time. As we have mentioned earlier, the objective of the private company is to minimize the cost. Then it is reasonable to propose the company GEDECO to select a multi-allocation strategy to organize its delivery process.

## 10.2 Prospective

Despite these encouraging results, there are yet many opportunities ahead. Further researches include the following aspects.

- We have provided a hierarchical and structured methodology to solve a general delivery problem that is usually large-scale and generally difficult to solve. However, the methodology is not yet completely operational. We propose two aspects to systematise this approach: (1) to define an instruction manual to guide the decomposition approach; (2) to design a management software to imbed it.
- Some new tightened formulations and some concrete improvement algorithms in our proposed 3-phase decomposition framework need to be proposed, which are more general and can be applied into more instances. In this case, some of the research can be investigated in three aspects including heuristics, meta-heuristics (tabu-search, genetic algorithm) and exact algorithms.
- The optimal theory can be introduced to the time decomposition in the tactical phase. In our research, the time decomposition method has been presented but must be developed. The optimal theory has to be employed to determine the exact time to serve the customers.
- In term of solution for the proposed formulations in our study involving FMA-VN, FSA-VN SApHMP-VN3 and MApHM-VN2, there are three points that can be suggested to obtain better performances. They are presented as follows: (1) the formulations are proposed to reformulate in order to tighten the constraints; (2) since we use Cplex as solver, the parameter setting should be further tested to find the accurate values of the solving parameters; (3) some other algorithms, for instance, the branch-and-cut or branch-and-bound, can be provided.
- Our proposed methodology needs to be tested on more instances to demonstrate the performance of the proposed approaches. Furthermore, the solutions for the two real examples just focus some sub-problems provided by the decomposition phase. Thus, the instances to solve all the sub-problems and then to integrate the solutions together seem to be necessary.

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# APPENDICES

## Appendix I

### The transport volume and cost between the nodes in CAB

- In the table, the 1000 times of the transportation volume and transportation cost are presented.

| Node $i$ | Node $j$ | Transportation Volume $W_{ij}$ | Transportation Cost $c_{ij}$ |
|----------|----------|--------------------------------|------------------------------|
| 1        | 1        | 0                              | 0                            |
| 1        | 2        | 6469                           | 576.9631                     |
| 1        | 3        | 7629                           | 946.4954                     |
| 1        | 4        | 20036                          | 597.5972                     |
| 1        | 5        | 4690                           | 373.8127                     |
| 1        | 6        | 6194                           | 559.7673                     |
| 1        | 7        | 11688                          | 709.0215                     |
| 1        | 8        | 2243                           | 1208.328                     |
| 1        | 9        | 8857                           | 603.6477                     |
| 1        | 10       | 7248                           | 695.208                      |
| 1        | 11       | 3559                           | 680.709                      |
| 1        | 12       | 9221                           | 1936.572                     |
| 1        | 13       | 10099                          | 332.4644                     |
| 1        | 14       | 22866                          | 592.5679                     |
| 1        | 15       | 3388                           | 908.7715                     |
| 1        | 16       | 9986                           | 426.1877                     |
| 1        | 17       | 46618                          | 756.1987                     |
| 1        | 18       | 11639                          | 672.5906                     |
| 1        | 19       | 1380                           | 1590.224                     |
| 1        | 20       | 5261                           | 527.3008                     |
| 1        | 21       | 5985                           | 483.4673                     |
| 1        | 22       | 6731                           | 2140.978                     |
| 1        | 23       | 2704                           | 2184.402                     |
| 1        | 24       | 12250                          | 408.1648                     |
| 1        | 25       | 16132                          | 540.7388                     |
| 2        | 1        | 6469                           | 576.9631                     |
| 2        | 2        | 0                              | 0                            |
| 2        | 3        | 12999                          | 369.5327                     |
| 2        | 4        | 13692                          | 613.0386                     |
| 2        | 5        | 3322                           | 429.1079                     |
| 2        | 6        | 5576                           | 312.8831                     |
| 2        | 7        | 3878                           | 1196.489                     |

| Node $i$ | Node $j$ | Transportation Volume $W_{ij}$ | Transportation Cost $c_{ij}$ |
|----------|----------|--------------------------------|------------------------------|
| 2        | 8        | 3202                           | 1502.14                      |
| 2        | 9        | 6699                           | 405.8975                     |
| 2        | 10       | 4198                           | 1241.961                     |
| 2        | 11       | 2454                           | 960.3459                     |
| 2        | 12       | 7975                           | 2318.076                     |
| 2        | 13       | 1186                           | 786.5959                     |
| 2        | 14       | 7443                           | 949.5669                     |
| 2        | 15       | 1162                           | 938.7461                     |
| 2        | 16       | 5105                           | 999.5005                     |
| 2        | 17       | 24817                          | 179.2426                     |
| 2        | 18       | 6532                           | 96.2744                      |
| 2        | 19       | 806                            | 1999.584                     |
| 2        | 20       | 8184                           | 210.7656                     |
| 2        | 21       | 3896                           | 736.3755                     |
| 2        | 22       | 7333                           | 2456.263                     |
| 2        | 23       | 3719                           | 2339.509                     |
| 2        | 24       | 2015                           | 844.1663                     |
| 2        | 25       | 565                            | 36.4947                      |
| 3        | 1        | 7629                           | 946.4954                     |
| 3        | 2        | 12999                          | 369.5327                     |
| 3        | 3        | 0                              | 0                            |
| 3        | 4        | 35135                          | 858.3308                     |
| 3        | 5        | 5956                           | 749.6018                     |
| 3        | 6        | 14121                          | 556.0706                     |
| 3        | 7        | 5951                           | 1541.273                     |
| 3        | 8        | 5768                           | 1764.791                     |
| 3        | 9        | 16578                          | 621.3306                     |
| 3        | 10       | 4242                           | 1603.165                     |
| 3        | 11       | 3365                           | 1250.962                     |
| 3        | 12       | 22254                          | 2600.078                     |
| 3        | 13       | 1841                           | 1137.335                     |
| 3        | 14       | 23665                          | 1266.851                     |
| 3        | 15       | 6517                           | 1124.778                     |
| 3        | 16       | 3541                           | 1368.267                     |
| 3        | 17       | 205088                         | 190.3157                     |
| 3        | 18       | 37669                          | 274.3105                     |
| 3        | 19       | 2885                           | 2299.429                     |
| 3        | 20       | 13200                          | 494.2224                     |
| 3        | 21       | 7116                           | 1043.484                     |
| 3        | 22       | 17165                          | 2703.402                     |
| 3        | 23       | 4284                           | 2503.828                     |
| 3        | 24       | 8085                           | 1188.549                     |
| 3        | 25       | 51895                          | 405.7886                     |
| 4        | 1        | 20036                          | 597.5972                     |
| 4        | 2        | 13692                          | 613.0386                     |
| 4        | 3        | 35135                          | 858.3308                     |
| 4        | 4        | 0                              | 0                            |

| Node $i$ | Node $j$ | Transportation Volume $W_{ij}$ | Transportation Cost $c_{ij}$ |
|----------|----------|--------------------------------|------------------------------|
| 4        | 5        | 19094                          | 255.0303                     |
| 4        | 6        | 35119                          | 311.3071                     |
| 4        | 7        | 21423                          | 790.1213                     |
| 4        | 8        | 27342                          | 907.4331                     |
| 4        | 9        | 51341                          | 237.0703                     |
| 4        | 10       | 15826                          | 932.2173                     |
| 4        | 11       | 28537                          | 406.3386                     |
| 4        | 12       | 65387                          | 1741.873                     |
| 4        | 13       | 12980                          | 485.5564                     |
| 4        | 14       | 44097                          | 1186.858                     |
| 4        | 15       | 51525                          | 345.8738                     |
| 4        | 16       | 14354                          | 830.3635                     |
| 4        | 17       | 172895                         | 720.4687                     |
| 4        | 18       | 37305                          | 675.3437                     |
| 4        | 19       | 15418                          | 1447.104                     |
| 4        | 20       | 26221                          | 403.8657                     |
| 4        | 21       | 42303                          | 255.8823                     |
| 4        | 22       | 35303                          | 1853.617                     |
| 4        | 23       | 13618                          | 1733.132                     |
| 4        | 24       | 17580                          | 1005.761                     |
| 4        | 25       | 40708                          | 592.0278                     |
| 5        | 1        | 4690                           | 373.8127                     |
| 5        | 2        | 3322                           | 429.1079                     |
| 5        | 3        | 5956                           | 749.6018                     |
| 5        | 4        | 19094                          | 255.0303                     |
| 5        | 5        | 0                              | 0                            |
| 5        | 6        | 7284                           | 225.8954                     |
| 5        | 7        | 3102                           | 794.1726                     |
| 5        | 8        | 1562                           | 1080.374                     |
| 5        | 9        | 7180                           | 238.944                      |
| 5        | 10       | 1917                           | 879.5647                     |
| 5        | 11       | 2253                           | 533.156                      |
| 5        | 12       | 5951                           | 1889.528                     |
| 5        | 13       | 1890                           | 402.3291                     |
| 5        | 14       | 7097                           | 947.3188                     |
| 5        | 15       | 2009                           | 598.541                      |
| 5        | 16       | 1340                           | 700.4368                     |
| 5        | 17       | 25303                          | 578.3286                     |
| 5        | 18       | 6031                           | 512.3965                     |
| 5        | 19       | 1041                           | 1570.725                     |
| 5        | 20       | 4128                           | 255.6551                     |
| 5        | 21       | 5452                           | 307.3289                     |
| 5        | 22       | 3344                           | 2036.128                     |
| 5        | 23       | 1067                           | 1967.256                     |
| 5        | 24       | 4608                           | 775.239                      |
| 5        | 25       | 7050                           | 399.2253                     |
| 6        | 1        | 6194                           | 559.7673                     |

| Node $i$ | Node $j$ | Transportation Volume $W_{ij}$ | Transportation Cost $c_{ij}$ |
|----------|----------|--------------------------------|------------------------------|
| 6        | 2        | 5576                           | 312.8831                     |
| 6        | 3        | 14121                          | 556.0706                     |
| 6        | 4        | 35119                          | 311.3071                     |
| 6        | 5        | 7284                           | 225.8954                     |
| 6        | 6        | 0                              | 0                            |
| 6        | 7        | 5023                           | 1009.689                     |
| 6        | 8        | 3512                           | 1216.868                     |
| 6        | 9        | 10419                          | 94.2588                      |
| 6        | 10       | 3543                           | 1104.574                     |
| 6        | 11       | 2752                           | 694.9153                     |
| 6        | 12       | 14412                          | 2047.122                     |
| 6        | 13       | 2043                           | 627.115                      |
| 6        | 14       | 15642                          | 1084.5                       |
| 6        | 15       | 5014                           | 626.1548                     |
| 6        | 16       | 2016                           | 922.3181                     |
| 6        | 17       | 62034                          | 409.3542                     |
| 6        | 18       | 15385                          | 365.6853                     |
| 6        | 19       | 2957                           | 1743.432                     |
| 6        | 20       | 5035                           | 104.6478                     |
| 6        | 21       | 7482                           | 491.1125                     |
| 6        | 22       | 6758                           | 2164.855                     |
| 6        | 23       | 2191                           | 2027.319                     |
| 6        | 24       | 6599                           | 933.196                      |
| 6        | 25       | 14181                          | 298.8486                     |
| 7        | 1        | 11688                          | 709.0215                     |
| 7        | 2        | 3878                           | 1196.489                     |
| 7        | 3        | 5951                           | 1541.273                     |
| 7        | 4        | 21423                          | 790.1213                     |
| 7        | 5        | 3102                           | 794.1726                     |
| 7        | 6        | 5023                           | 1009.689                     |
| 7        | 7        | 0                              | 0                            |
| 7        | 8        | 11557                          | 663.8762                     |
| 7        | 9        | 6479                           | 982.7378                     |
| 7        | 10       | 34261                          | 221.422                      |
| 7        | 11       | 10134                          | 447.8044                     |
| 7        | 12       | 27350                          | 1249.763                     |
| 7        | 13       | 6929                           | 411.1133                     |
| 7        | 14       | 7961                           | 1097.608                     |
| 7        | 15       | 4678                           | 851.8228                     |
| 7        | 16       | 13511                          | 423.7053                     |
| 7        | 17       | 29801                          | 1362.874                     |
| 7        | 18       | 7549                           | 1288.966                     |
| 7        | 19       | 5550                           | 895.0908                     |
| 7        | 20       | 3089                           | 1049.266                     |
| 7        | 21       | 9958                           | 537.6206                     |
| 7        | 22       | 14110                          | 1493.843                     |
| 7        | 23       | 4911                           | 1686.675                     |

| Node $i$ | Node $j$ | Transportation Volume $W_{ij}$ | Transportation Cost $c_{ij}$ |
|----------|----------|--------------------------------|------------------------------|
| 7        | 24       | 2722                           | 912.2104                     |
| 7        | 25       | 10802                          | 1161.676                     |
| 8        | 1        | 2243                           | 1208.328                     |
| 8        | 2        | 3202                           | 1502.14                      |
| 8        | 3        | 5768                           | 1764.791                     |
| 8        | 4        | 27342                          | 907.4331                     |
| 8        | 5        | 1562                           | 1080.374                     |
| 8        | 6        | 3512                           | 1216.868                     |
| 8        | 7        | 11557                          | 663.8762                     |
| 8        | 8        | 0                              | 0                            |
| 8        | 9        | 5615                           | 1143.791                     |
| 8        | 10       | 7095                           | 874.5181                     |
| 8        | 11       | 10753                          | 551.6299                     |
| 8        | 12       | 30362                          | 841.624                      |
| 8        | 13       | 1783                           | 880.0728                     |
| 8        | 14       | 3437                           | 1714.651                     |
| 8        | 15       | 8897                           | 694.0088                     |
| 8        | 16       | 2509                           | 1066.563                     |
| 8        | 17       | 23273                          | 1625.87                      |
| 8        | 18       | 5160                           | 1574.822                     |
| 8        | 19       | 8750                           | 593.4216                     |
| 8        | 20       | 2583                           | 1301.511                     |
| 8        | 21       | 7288                           | 780.9512                     |
| 8        | 22       | 17481                          | 955.802                      |
| 8        | 23       | 7930                           | 1024.566                     |
| 8        | 24       | 1278                           | 1519.174                     |
| 8        | 25       | 8447                           | 1475.479                     |
| 9        | 1        | 8857                           | 603.6477                     |
| 9        | 2        | 6699                           | 405.8975                     |
| 9        | 3        | 16578                          | 621.3306                     |
| 9        | 4        | 51341                          | 237.0703                     |
| 9        | 5        | 7180                           | 238.944                      |
| 9        | 6        | 10419                          | 94.2588                      |
| 9        | 7        | 6479                           | 982.7378                     |
| 9        | 8        | 5615                           | 1143.791                     |
| 9        | 9        | 0                              | 0                            |
| 9        | 10       | 4448                           | 1094.906                     |
| 9        | 11       | 5076                           | 636.9045                     |
| 9        | 12       | 22463                          | 1978.943                     |
| 9        | 13       | 4783                           | 620.488                      |
| 9        | 14       | 24609                          | 1151.868                     |
| 9        | 15       | 9969                           | 535.0244                     |
| 9        | 16       | 4224                           | 936.2502                     |
| 9        | 17       | 79945                          | 489.5645                     |
| 9        | 18       | 20001                          | 453.2583                     |
| 9        | 19       | 4291                           | 1682.489                     |
| 9        | 20       | 10604                          | 198.9058                     |

| Node $i$ | Node $j$ | Transportation Volume $W_{ij}$ | Transportation Cost $c_{ij}$ |
|----------|----------|--------------------------------|------------------------------|
| 9        | 21       | 11925                          | 450.2585                     |
| 9        | 22       | 13091                          | 2086.845                     |
| 9        | 23       | 4172                           | 1936.304                     |
| 9        | 24       | 12891                          | 992.3379                     |
| 9        | 25       | 19500                          | 392.9045                     |
| 10       | 1        | 7248                           | 695.208                      |
| 10       | 2        | 4198                           | 1241.961                     |
| 10       | 3        | 4242                           | 1603.165                     |
| 10       | 4        | 15826                          | 932.2173                     |
| 10       | 5        | 1917                           | 879.5647                     |
| 10       | 6        | 3543                           | 1104.574                     |
| 10       | 7        | 34261                          | 221.422                      |
| 10       | 8        | 7095                           | 874.5181                     |
| 10       | 9        | 4448                           | 1094.906                     |
| 10       | 10       | 0                              | 0                            |
| 10       | 11       | 4370                           | 642.2092                     |
| 10       | 12       | 17267                          | 1375.635                     |
| 10       | 13       | 3929                           | 477.459                      |
| 10       | 14       | 8602                           | 963.7202                     |
| 10       | 15       | 2753                           | 1046.119                     |
| 10       | 16       | 20013                          | 305.3132                     |
| 10       | 17       | 28080                          | 1417.072                     |
| 10       | 18       | 5971                           | 1337.648                     |
| 10       | 19       | 2131                           | 1017.332                     |
| 10       | 20       | 3579                           | 1125.041                     |
| 10       | 21       | 6809                           | 677.0608                     |
| 10       | 22       | 8455                           | 1649.619                     |
| 10       | 23       | 2868                           | 1891.166                     |
| 10       | 24       | 2336                           | 795.2136                     |
| 10       | 25       | 5616                           | 1205.747                     |
| 11       | 1        | 3559                           | 680.709                      |
| 11       | 2        | 2454                           | 960.3459                     |
| 11       | 3        | 3365                           | 1250.962                     |
| 11       | 4        | 28537                          | 406.3386                     |
| 11       | 5        | 2253                           | 533.156                      |
| 11       | 6        | 2752                           | 694.9153                     |
| 11       | 7        | 10134                          | 447.8044                     |
| 11       | 8        | 10753                          | 551.6299                     |
| 11       | 9        | 5076                           | 636.9045                     |
| 11       | 10       | 4370                           | 642.2092                     |
| 11       | 11       | 0                              | 0                            |
| 11       | 12       | 15287                          | 1358.213                     |
| 11       | 13       | 3083                           | 378.5906                     |
| 11       | 14       | 4092                           | 1236.192                     |
| 11       | 15       | 7701                           | 405.0906                     |
| 11       | 16       | 2809                           | 674.479                      |
| 11       | 17       | 17291                          | 1096.712                     |

| Node $i$ | Node $j$ | Transportation Volume $W_{ij}$ | Transportation Cost $c_{ij}$ |
|----------|----------|--------------------------------|------------------------------|
| 11       | 18       | 4462                           | 1038.645                     |
| 11       | 19       | 3239                           | 1048.539                     |
| 11       | 20       | 2309                           | 768.1641                     |
| 11       | 21       | 16003                          | 229.4867                     |
| 11       | 22       | 8381                           | 1506.451                     |
| 11       | 23       | 3033                           | 1503.794                     |
| 11       | 24       | 1755                           | 1038.624                     |
| 11       | 25       | 7266                           | 931.7148                     |
| 12       | 1        | 9221                           | 1936.572                     |
| 12       | 2        | 7975                           | 2318.076                     |
| 12       | 3        | 22254                          | 2600.078                     |
| 12       | 4        | 65387                          | 1741.873                     |
| 12       | 5        | 5951                           | 1889.528                     |
| 12       | 6        | 14412                          | 2047.122                     |
| 12       | 7        | 27350                          | 1249.763                     |
| 12       | 8        | 30362                          | 841.624                      |
| 12       | 9        | 22463                          | 1978.943                     |
| 12       | 10       | 17267                          | 1375.635                     |
| 12       | 11       | 15287                          | 1358.213                     |
| 12       | 12       | 0                              | 0                            |
| 12       | 13       | 5454                           | 1608.082                     |
| 12       | 14       | 15011                          | 2335.816                     |
| 12       | 15       | 17714                          | 1530.57                      |
| 12       | 16       | 10037                          | 1661.778                     |
| 12       | 17       | 105507                         | 2453.352                     |
| 12       | 18       | 20040                          | 2396.794                     |
| 12       | 19       | 31780                          | 358.3762                     |
| 12       | 20       | 10822                          | 2125.512                     |
| 12       | 21       | 16450                          | 1582.369                     |
| 12       | 22       | 92083                          | 361.5388                     |
| 12       | 23       | 32908                          | 986.8149                     |
| 12       | 24       | 3865                           | 2157.517                     |
| 12       | 25       | 24583                          | 2288.748                     |
| 13       | 1        | 10099                          | 332.4644                     |
| 13       | 2        | 1186                           | 786.5959                     |
| 13       | 3        | 1841                           | 1137.335                     |
| 13       | 4        | 12980                          | 485.5564                     |
| 13       | 5        | 1890                           | 402.3291                     |
| 13       | 6        | 2043                           | 627.115                      |
| 13       | 7        | 6929                           | 411.1133                     |
| 13       | 8        | 1783                           | 880.0728                     |
| 13       | 9        | 4783                           | 620.488                      |
| 13       | 10       | 3929                           | 477.459                      |
| 13       | 11       | 3083                           | 378.5906                     |
| 13       | 12       | 5454                           | 1608.082                     |
| 13       | 13       | 0                              | 0                            |
| 13       | 14       | 3251                           | 858.251                      |

| Node $i$ | Node $j$ | Transportation Volume $W_{ij}$ | Transportation Cost $c_{ij}$ |
|----------|----------|--------------------------------|------------------------------|
| 13       | 15       | 1126                           | 700.8213                     |
| 13       | 16       | 5926                           | 348.2725                     |
| 13       | 17       | 10653                          | 955.6191                     |
| 13       | 18       | 3062                           | 879.9795                     |
| 13       | 19       | 759                            | 1265.573                     |
| 13       | 20       | 1255                           | 651.1179                     |
| 13       | 21       | 6173                           | 254.9977                     |
| 13       | 22       | 2974                           | 1808.52                      |
| 13       | 23       | 1056                           | 1872.696                     |
| 13       | 24       | 1504                           | 660.5173                     |
| 13       | 25       | 4588                           | 751.4614                     |
| 14       | 1        | 22866                          | 592.5679                     |
| 14       | 2        | 7443                           | 949.5669                     |
| 14       | 3        | 23665                          | 1266.851                     |
| 14       | 4        | 44097                          | 1186.858                     |
| 14       | 5        | 7097                           | 947.3188                     |
| 14       | 6        | 15642                          | 1084.5                       |
| 14       | 7        | 7961                           | 1097.608                     |
| 14       | 8        | 3437                           | 1714.651                     |
| 14       | 9        | 24609                          | 1151.868                     |
| 14       | 10       | 8602                           | 963.7202                     |
| 14       | 11       | 4092                           | 1236.192                     |
| 14       | 12       | 15011                          | 2335.816                     |
| 14       | 13       | 3251                           | 858.251                      |
| 14       | 14       | 0                              | 0                            |
| 14       | 15       | 5550                           | 1500.774                     |
| 14       | 16       | 9473                           | 675.7505                     |
| 14       | 17       | 169397                         | 1098.282                     |
| 14       | 18       | 25073                          | 1021.611                     |
| 14       | 19       | 1170                           | 1977.613                     |
| 14       | 20       | 14272                          | 1015.165                     |
| 14       | 21       | 8543                           | 1065.599                     |
| 14       | 22       | 8064                           | 2591.447                     |
| 14       | 23       | 1840                           | 2725.79                      |
| 14       | 24       | 20618                          | 197.8015                     |
| 14       | 25       | 20937                          | 923.2229                     |
| 15       | 1        | 3388                           | 908.7715                     |
| 15       | 2        | 1162                           | 938.7461                     |
| 15       | 3        | 6517                           | 1124.778                     |
| 15       | 4        | 51525                          | 345.8738                     |
| 15       | 5        | 2009                           | 598.541                      |
| 15       | 6        | 5014                           | 626.1548                     |
| 15       | 7        | 4678                           | 851.8228                     |
| 15       | 8        | 8897                           | 694.0088                     |
| 15       | 9        | 9969                           | 535.0244                     |
| 15       | 10       | 2753                           | 1046.119                     |
| 15       | 11       | 7701                           | 405.0906                     |

| Node $i$ | Node $j$ | Transportation Volume $W_{ij}$ | Transportation Cost $c_{ij}$ |
|----------|----------|--------------------------------|------------------------------|
| 15       | 12       | 17714                          | 1530.57                      |
| 15       | 13       | 1126                           | 700.8213                     |
| 15       | 14       | 5550                           | 1500.774                     |
| 15       | 15       | 0                              | 0                            |
| 15       | 16       | 2152                           | 1039.77                      |
| 15       | 17       | 26816                          | 1018.399                     |
| 15       | 18       | 6931                           | 987.8645                     |
| 15       | 19       | 4947                           | 1280.737                     |
| 15       | 20       | 2676                           | 728.3743                     |
| 15       | 21       | 8033                           | 450.3982                     |
| 15       | 22       | 12692                          | 1589.835                     |
| 15       | 23       | 6157                           | 1401.321                     |
| 15       | 24       | 3065                           | 1311.21                      |
| 15       | 25       | 12044                          | 922.3145                     |
| 16       | 1        | 9986                           | 426.1877                     |
| 16       | 2        | 5105                           | 999.5005                     |
| 16       | 3        | 3541                           | 1368.267                     |
| 16       | 4        | 14354                          | 830.3635                     |
| 16       | 5        | 1340                           | 700.4368                     |
| 16       | 6        | 2016                           | 922.3181                     |
| 16       | 7        | 13511                          | 423.7053                     |
| 16       | 8        | 2509                           | 1066.563                     |
| 16       | 9        | 4224                           | 936.2502                     |
| 16       | 10       | 20013                          | 305.3132                     |
| 16       | 11       | 2809                           | 674.479                      |
| 16       | 12       | 10037                          | 1661.778                     |
| 16       | 13       | 5926                           | 348.2725                     |
| 16       | 14       | 9473                           | 675.7505                     |
| 16       | 15       | 2152                           | 1039.77                      |
| 16       | 16       | 0                              | 0                            |
| 16       | 17       | 21806                          | 1178.439                     |
| 16       | 18       | 4519                           | 1095.657                     |
| 16       | 19       | 886                            | 1304.043                     |
| 16       | 20       | 1742                           | 918.5615                     |
| 16       | 21       | 4782                           | 601.9917                     |
| 16       | 22       | 6453                           | 1916.578                     |
| 16       | 23       | 2022                           | 2090.089                     |
| 16       | 24       | 3546                           | 496.4224                     |
| 16       | 25       | 5065                           | 963.0435                     |
| 17       | 1        | 46618                          | 756.1987                     |
| 17       | 2        | 24817                          | 179.2426                     |
| 17       | 3        | 205088                         | 190.3157                     |
| 17       | 4        | 172895                         | 720.4687                     |
| 17       | 5        | 25303                          | 578.3286                     |
| 17       | 6        | 62034                          | 409.3542                     |
| 17       | 7        | 29801                          | 1362.874                     |
| 17       | 8        | 23273                          | 1625.87                      |

| Node $i$ | Node $j$ | Transportation Volume $W_{ij}$ | Transportation Cost $c_{ij}$ |
|----------|----------|--------------------------------|------------------------------|
| 17       | 9        | 79945                          | 489.5645                     |
| 17       | 10       | 28080                          | 1417.072                     |
| 17       | 11       | 17291                          | 1096.712                     |
| 17       | 12       | 105507                         | 2453.352                     |
| 17       | 13       | 10653                          | 955.6191                     |
| 17       | 14       | 169397                         | 1098.282                     |
| 17       | 15       | 26816                          | 1018.399                     |
| 17       | 16       | 21806                          | 1178.439                     |
| 17       | 17       | 0                              | 0                            |
| 17       | 18       | 9040                           | 84.3365                      |
| 17       | 19       | 11139                          | 2143.565                     |
| 17       | 20       | 63153                          | 328.7515                     |
| 17       | 21       | 34092                          | 880.5469                     |
| 17       | 22       | 70935                          | 2574.082                     |
| 17       | 23       | 14957                          | 2415.489                     |
| 17       | 24       | 28398                          | 1008.2                       |
| 17       | 25       | 166694                         | 215.561                      |
| 18       | 1        | 11639                          | 672.5906                     |
| 18       | 2        | 6532                           | 96.2744                      |
| 18       | 3        | 37669                          | 274.3105                     |
| 18       | 4        | 37305                          | 675.3437                     |
| 18       | 5        | 6031                           | 512.3965                     |
| 18       | 6        | 15385                          | 365.6853                     |
| 18       | 7        | 7549                           | 1288.966                     |
| 18       | 8        | 5160                           | 1574.822                     |
| 18       | 9        | 20001                          | 453.2583                     |
| 18       | 10       | 5971                           | 1337.648                     |
| 18       | 11       | 4462                           | 1038.645                     |
| 18       | 12       | 20040                          | 2396.794                     |
| 18       | 13       | 3062                           | 879.9795                     |
| 18       | 14       | 25073                          | 1021.611                     |
| 18       | 15       | 6931                           | 987.8645                     |
| 18       | 16       | 4519                           | 1095.657                     |
| 18       | 17       | 9040                           | 84.3365                      |
| 18       | 18       | 0                              | 0                            |
| 18       | 19       | 2802                           | 2082.316                     |
| 18       | 20       | 30224                          | 273.4106                     |
| 18       | 21       | 7982                           | 818.1228                     |
| 18       | 22       | 14964                          | 2526.562                     |
| 18       | 23       | 4589                           | 2388.689                     |
| 18       | 24       | 6227                           | 926.6267                     |
| 18       | 25       | 12359                          | 132.7684                     |
| 19       | 1        | 1380                           | 1590.224                     |
| 19       | 2        | 806                            | 1999.584                     |
| 19       | 3        | 2885                           | 2299.429                     |
| 19       | 4        | 15418                          | 1447.104                     |
| 19       | 5        | 1041                           | 1570.725                     |

| Node $i$ | Node $j$ | Transportation Volume $W_{ij}$ | Transportation Cost $c_{ij}$ |
|----------|----------|--------------------------------|------------------------------|
| 19       | 6        | 2957                           | 1743.432                     |
| 19       | 7        | 5550                           | 895.0908                     |
| 19       | 8        | 8750                           | 593.4216                     |
| 19       | 9        | 4291                           | 1682.489                     |
| 19       | 10       | 2131                           | 1017.332                     |
| 19       | 11       | 3239                           | 1048.539                     |
| 19       | 12       | 31780                          | 358.3762                     |
| 19       | 13       | 759                            | 1265.573                     |
| 19       | 14       | 1170                           | 1977.613                     |
| 19       | 15       | 4947                           | 1280.737                     |
| 19       | 16       | 886                            | 1304.043                     |
| 19       | 17       | 11139                          | 2143.565                     |
| 19       | 18       | 2802                           | 2082.316                     |
| 19       | 19       | 0                              | 0                            |
| 19       | 20       | 1869                           | 1814.83                      |
| 19       | 21       | 3716                           | 1264.193                     |
| 19       | 22       | 11510                          | 661.6543                     |
| 19       | 23       | 3519                           | 1129.327                     |
| 19       | 24       | 569                            | 1800.098                     |
| 19       | 25       | 3520                           | 1968.689                     |
| 20       | 1        | 5261                           | 527.3008                     |
| 20       | 2        | 8184                           | 210.7656                     |
| 20       | 3        | 13200                          | 494.2224                     |
| 20       | 4        | 26221                          | 403.8657                     |
| 20       | 5        | 4128                           | 255.6551                     |
| 20       | 6        | 5035                           | 104.6478                     |
| 20       | 7        | 3089                           | 1049.266                     |
| 20       | 8        | 2583                           | 1301.511                     |
| 20       | 9        | 10604                          | 198.9058                     |
| 20       | 10       | 3579                           | 1125.041                     |
| 20       | 11       | 2309                           | 768.1641                     |
| 20       | 12       | 10822                          | 2125.512                     |
| 20       | 13       | 1255                           | 651.1179                     |
| 20       | 14       | 14272                          | 1015.165                     |
| 20       | 15       | 2676                           | 728.3743                     |
| 20       | 16       | 1742                           | 918.5615                     |
| 20       | 17       | 63153                          | 328.7515                     |
| 20       | 18       | 30224                          | 273.4106                     |
| 20       | 19       | 1869                           | 1814.83                      |
| 20       | 20       | 0                              | 0                            |
| 20       | 21       | 5020                           | 552.4229                     |
| 20       | 22       | 6610                           | 2253.211                     |
| 20       | 23       | 2139                           | 2128.828                     |
| 20       | 24       | 5431                           | 875.2542                     |
| 20       | 25       | 13541                          | 194.5945                     |
| 21       | 1        | 5985                           | 483.4673                     |
| 21       | 2        | 3896                           | 736.3755                     |

| Node $i$ | Node $j$ | Transportation Volume $W_{ij}$ | Transportation Cost $c_{ij}$ |
|----------|----------|--------------------------------|------------------------------|
| 21       | 3        | 7116                           | 1043.484                     |
| 21       | 4        | 42303                          | 255.8823                     |
| 21       | 5        | 5452                           | 307.3289                     |
| 21       | 6        | 7482                           | 491.1125                     |
| 21       | 7        | 9958                           | 537.6206                     |
| 21       | 8        | 7288                           | 780.9512                     |
| 21       | 9        | 11925                          | 450.2585                     |
| 21       | 10       | 6809                           | 677.0608                     |
| 21       | 11       | 16003                          | 229.4867                     |
| 21       | 12       | 16450                          | 1582.369                     |
| 21       | 13       | 6173                           | 254.9977                     |
| 21       | 14       | 8543                           | 1065.599                     |
| 21       | 15       | 8033                           | 450.3982                     |
| 21       | 16       | 4782                           | 601.9917                     |
| 21       | 17       | 34092                          | 880.5469                     |
| 21       | 18       | 7982                           | 818.1228                     |
| 21       | 19       | 3716                           | 1264.193                     |
| 21       | 20       | 5020                           | 552.4229                     |
| 21       | 21       | 0                              | 0                            |
| 21       | 22       | 9942                           | 1735.937                     |
| 21       | 23       | 3276                           | 1712.136                     |
| 21       | 24       | 3820                           | 871.6396                     |
| 21       | 25       | 11799                          | 706.5024                     |
| 22       | 1        | 6731                           | 2140.978                     |
| 22       | 2        | 7333                           | 2456.263                     |
| 22       | 3        | 17165                          | 2703.402                     |
| 22       | 4        | 35303                          | 1853.617                     |
| 22       | 5        | 3344                           | 2036.128                     |
| 22       | 6        | 6758                           | 2164.855                     |
| 22       | 7        | 14110                          | 1493.843                     |
| 22       | 8        | 17481                          | 955.802                      |
| 22       | 9        | 13091                          | 2086.845                     |
| 22       | 10       | 8455                           | 1649.619                     |
| 22       | 11       | 8381                           | 1506.451                     |
| 22       | 12       | 92083                          | 361.5388                     |
| 22       | 13       | 2974                           | 1808.52                      |
| 22       | 14       | 8064                           | 2591.447                     |
| 22       | 15       | 12692                          | 1589.835                     |
| 22       | 16       | 6453                           | 1916.578                     |
| 22       | 17       | 70935                          | 2574.082                     |
| 22       | 18       | 14964                          | 2526.562                     |
| 22       | 19       | 11510                          | 661.6543                     |
| 22       | 20       | 6610                           | 2253.211                     |
| 22       | 21       | 9942                           | 1735.937                     |
| 22       | 22       | 0                              | 0                            |
| 22       | 23       | 35285                          | 694.9363                     |
| 22       | 24       | 2566                           | 2404.839                     |

| Node $i$ | Node $j$ | Transportation Volume $W_{ij}$ | Transportation Cost $c_{ij}$ |
|----------|----------|--------------------------------|------------------------------|
| 22       | 25       | 19926                          | 2430.269                     |
| 23       | 1        | 2704                           | 2184.402                     |
| 23       | 2        | 3719                           | 2339.509                     |
| 23       | 3        | 4284                           | 2503.828                     |
| 23       | 4        | 13618                          | 1733.132                     |
| 23       | 5        | 1067                           | 1967.256                     |
| 23       | 6        | 2191                           | 2027.319                     |
| 23       | 7        | 4911                           | 1686.675                     |
| 23       | 8        | 7930                           | 1024.566                     |
| 23       | 9        | 4172                           | 1936.304                     |
| 23       | 10       | 2868                           | 1891.166                     |
| 23       | 11       | 3033                           | 1503.794                     |
| 23       | 12       | 32908                          | 986.8149                     |
| 23       | 13       | 1056                           | 1872.696                     |
| 23       | 14       | 1840                           | 2725.79                      |
| 23       | 15       | 6157                           | 1401.321                     |
| 23       | 16       | 2022                           | 2090.089                     |
| 23       | 17       | 14957                          | 2415.489                     |
| 23       | 18       | 4589                           | 2388.689                     |
| 23       | 19       | 3519                           | 1129.327                     |
| 23       | 20       | 2139                           | 2128.828                     |
| 23       | 21       | 3276                           | 1712.136                     |
| 23       | 22       | 35285                          | 694.9363                     |
| 23       | 23       | 0                              | 0                            |
| 23       | 24       | 940                            | 2528.479                     |
| 23       | 25       | 4951                           | 2321.873                     |
| 24       | 1        | 12250                          | 408.1648                     |
| 24       | 2        | 2015                           | 844.1663                     |
| 24       | 3        | 8085                           | 1188.549                     |
| 24       | 4        | 17580                          | 1005.761                     |
| 24       | 5        | 4608                           | 775.239                      |
| 24       | 6        | 6599                           | 933.196                      |
| 24       | 7        | 2722                           | 912.2104                     |
| 24       | 8        | 1278                           | 1519.174                     |
| 24       | 9        | 12891                          | 992.3379                     |
| 24       | 10       | 2336                           | 795.2136                     |
| 24       | 11       | 1755                           | 1038.624                     |
| 24       | 12       | 3865                           | 2157.517                     |
| 24       | 13       | 1504                           | 660.5173                     |
| 24       | 14       | 20618                          | 197.8015                     |
| 24       | 15       | 3065                           | 1311.21                      |
| 24       | 16       | 3546                           | 496.4224                     |
| 24       | 17       | 28398                          | 1008.2                       |
| 24       | 18       | 6227                           | 926.6267                     |
| 24       | 19       | 569                            | 1800.098                     |
| 24       | 20       | 5431                           | 875.2542                     |
| 24       | 21       | 3820                           | 871.6396                     |

| <b>Node <math>i</math></b> | <b>Node <math>j</math></b> | <b>Transportation Volume <math>W_{ij}</math></b> | <b>Transportation Cost <math>c_{ij}</math></b> |
|----------------------------|----------------------------|--------------------------------------------------|------------------------------------------------|
| 24                         | 22                         | 2566                                             | 2404.839                                       |
| 24                         | 23                         | 940                                              | 2528.479                                       |
| 24                         | 24                         | 0                                                | 0                                              |
| 24                         | 25                         | 6237                                             | 813.5513                                       |
| 25                         | 1                          | 16132                                            | 540.7388                                       |
| 25                         | 2                          | 565                                              | 36.4947                                        |
| 25                         | 3                          | 51895                                            | 405.7886                                       |
| 25                         | 4                          | 40708                                            | 592.0278                                       |
| 25                         | 5                          | 7050                                             | 399.2253                                       |
| 25                         | 6                          | 14181                                            | 298.8486                                       |
| 25                         | 7                          | 10802                                            | 1161.676                                       |
| 25                         | 8                          | 8447                                             | 1475.479                                       |
| 25                         | 9                          | 19500                                            | 392.9045                                       |
| 25                         | 10                         | 5616                                             | 1205.747                                       |
| 25                         | 11                         | 7266                                             | 931.7148                                       |
| 25                         | 12                         | 24583                                            | 2288.748                                       |
| 25                         | 13                         | 4588                                             | 751.4614                                       |
| 25                         | 14                         | 20937                                            | 923.2229                                       |
| 25                         | 15                         | 12044                                            | 922.3145                                       |
| 25                         | 16                         | 5065                                             | 963.0435                                       |
| 25                         | 17                         | 166694                                           | 215.561                                        |
| 25                         | 18                         | 12359                                            | 132.7684                                       |
| 25                         | 19                         | 3520                                             | 1968.689                                       |
| 25                         | 20                         | 13541                                            | 194.5945                                       |
| 25                         | 21                         | 11799                                            | 706.5024                                       |
| 25                         | 22                         | 19926                                            | 2430.269                                       |
| 25                         | 23                         | 4951                                             | 2321.873                                       |
| 25                         | 24                         | 6237                                             | 813.5513                                       |
| 25                         | 25                         | 0                                                | 0                                              |

## Appendix II

### The multi-allocation samples in CAB

| Sample Number | n  | p | Facility   |
|---------------|----|---|------------|
| 1             | 10 | 2 | 7,9        |
| 2             |    | 3 | 4,6,7      |
| 3             |    | 4 | 3,4,6,7    |
| 4             |    | 4 | 2,4,6,7    |
| 5             |    | 2 | 4,12       |
| 6             | 15 | 2 | 4,7        |
| 7             |    | 3 | 4,7,12     |
| 8             |    | 3 | 1,4,7,     |
| 9             |    | 4 | 4,7,12,14  |
| 10            |    | 4 | 1,4,7,12   |
| 11            | 20 | 4 | 1,4,7,8    |
| 12            |    | 2 | 4,17       |
| 13            |    | 2 | 11,17      |
| 14            |    | 2 | 11,18      |
| 15            |    | 3 | 4,12,17    |
| 16            |    | 3 | 4,7,17     |
| 17            |    | 4 | 4,12,16,17 |
| 18            |    | 4 | 1,4,12,17  |
| 19            |    | 4 | 4,7,12,17  |
| 20            |    | 4 | 4,7,12,18  |
| 21            | 25 | 4 | 4,16,17,19 |
| 22            |    | 2 | 12,20      |
| 23            |    | 3 | 12,17,21   |
| 24            |    | 3 | 12,18,21   |
| 25            |    | 4 | 4,12,17,24 |
| 26            |    | 4 | 1,4,12,17  |

## Appendix III

### The single-allocation samples in CAB

| Sample Number | n  | p | Facility   |
|---------------|----|---|------------|
| 1             | 10 | 2 | 7,9        |
| 2             |    | 2 | 4,7        |
| 3             |    | 3 | 4,6,7      |
| 4             |    | 3 | 4,7,9      |
| 5             |    | 4 | 3,4,6,7    |
| 6             |    | 4 | 4,6,7,8    |
| 7             |    | 4 | 4,6,7,9    |
| 8             |    | 4 | 1,4,7,9    |
| 9             | 15 | 2 | 4,12       |
| 10            |    | 2 | 4,11       |
| 11            |    | 3 | 4,7,12     |
| 12            |    | 3 | 4,7,12     |
| 13            |    | 3 | 4,7,8      |
| 14            |    | 4 | 4,7,12,14  |
| 15            |    | 4 | 1,4,7,12   |
| 16            |    | 4 | 1,4,7,8    |
| 17            | 20 | 2 | 4,17       |
| 18            |    | 2 | 4,2        |
| 19            |    | 3 | 4,12,17    |
| 20            |    | 3 | 4,8,17     |
| 21            |    | 3 | 4,11,20    |
| 22            |    | 4 | 4,12,16,17 |
| 23            |    | 4 | 1,4,12,17  |
| 24            |    | 4 | 1,4,8,17   |
| 25            | 25 | 4 | 4,8,13,20  |
| 26            |    | 2 | 12,20      |
| 27            |    | 3 | 12,17,21   |
| 28            |    | 3 | 4,12,17    |
| 29            |    | 3 | 12,18,21   |
| 30            |    | 4 | 4,12,17,24 |
| 31            |    | 4 | 1,4,12,17  |

## Appendix IV

### The samples for MApHM-VN2

| Sample Number | n  | p |     | Facility   |
|---------------|----|---|-----|------------|
| 1             | 10 | 2 | 0,2 | 7,9        |
| 2             |    | 2 | 0,4 | 7,9        |
| 3             |    | 2 | 0,6 | 7,9        |
| 4             |    | 2 | 0,8 | 7,9        |
| 5             |    | 2 | 1   | 7,9        |
| 6             |    | 3 | 0,2 | 4,6,7      |
| 7             |    | 3 | 0,4 | 4,6,7      |
| 8             |    | 3 | 0,6 | 4,6,7      |
| 9             |    | 3 | 0,8 | 4,6,7      |
| 10            |    | 3 | 1   | 4,6,7      |
| 11            | 15 | 4 | 0,2 | 3,4,6,7    |
| 12            |    | 4 | 0,4 | 3,4,6,7    |
| 13            |    | 4 | 0,6 | 3,4,6,7    |
| 14            |    | 4 | 0,8 | 3,4,6,7    |
| 15            |    | 4 | 1   | 2,4,6,7    |
| 16            |    | 2 | 0,2 | 4,12       |
| 17            |    | 2 | 0,4 | 4,12       |
| 18            |    | 2 | 0,6 | 4,12       |
| 19            |    | 2 | 0,8 | 4,7        |
| 20            |    | 2 | 1   | 4,7        |
| 21            | 20 | 3 | 0,2 | 4,7,12     |
| 22            |    | 3 | 0,4 | 4,7,12     |
| 23            |    | 3 | 0,6 | 4,7,12     |
| 24            |    | 3 | 0,8 | 4,7,12     |
| 25            |    | 3 | 1   | 1,4,7,     |
| 26            |    | 4 | 0,2 | 4,7,12,14  |
| 27            |    | 4 | 0,4 | 4,7,12,14  |
| 28            |    | 4 | 0,6 | 4,7,12,14  |
| 29            |    | 4 | 0,8 | 1,4,7,12   |
| 30            |    | 4 | 1   | 1,4,7,8    |
| 31            | 20 | 2 | 0,2 | 4,17       |
| 32            |    | 2 | 0,4 | 4,17       |
| 33            |    | 2 | 0,6 | 4,17       |
| 34            |    | 2 | 0,8 | 11,17      |
| 35            |    | 2 | 1   | 11,18      |
| 36            |    | 3 | 0,2 | 4,12,17    |
| 37            |    | 3 | 0,4 | 4,12,17    |
| 38            |    | 3 | 0,6 | 4,12,17    |
| 39            |    | 3 | 0,8 | 4,7,17     |
| 40            |    | 3 | 1   | 4,7,17     |
| 41            |    | 4 | 0,2 | 4,12,16,17 |
| 42            |    | 4 | 0,4 | 1,4,12,17  |

| Sample Number | n  | p |     | Facility   |
|---------------|----|---|-----|------------|
| 43            | 25 | 4 | 0,6 | 4,7,12,17  |
| 44            |    | 4 | 0,8 | 4,7,12,18  |
| 45            |    | 4 | 1   | 4,16,17,19 |
| 46            |    | 2 | 0,2 | 12,20      |
| 47            |    | 2 | 0,4 | 12,20      |
| 48            |    | 2 | 0,6 | 12,20      |
| 49            |    | 2 | 0,8 | 12,20      |
| 50            |    | 2 | 1   | 12,20      |
| 51            |    | 3 | 0,2 | 12,17,21   |
| 52            |    | 3 | 0,4 | 4,12,17    |
| 53            |    | 3 | 0,6 | 4,12,17    |
| 54            |    | 3 | 0,8 | 4,12,17    |
| 55            |    | 3 | 1   | 12,18,21   |
| 56            |    | 4 | 0,2 | 4,12,17,24 |
| 57            |    | 4 | 0,4 | 4,12,17,24 |
| 58            |    | 4 | 0,6 | 1,4,12,17  |
| 59            |    | 4 | 0,8 | 1,4,12,17  |
| 60            |    | 4 | 1   | 1,4,12,17  |

## Appendix V

### The samples for SApHM-VN3

| Sample Number | n  | p | $\alpha$ | Facility   |
|---------------|----|---|----------|------------|
| 1             | 10 | 2 | 0,2      | 7,9        |
| 2             |    | 2 | 0,4      | 7,9        |
| 3             |    | 2 | 0,6      | 7,9        |
| 4             |    | 2 | 0,8      | 7,9        |
| 5             |    | 2 | 1        | 4,7        |
| 6             |    | 3 | 0,2      | 4,6,7      |
| 7             |    | 3 | 0,4      | 4,6,7      |
| 8             |    | 3 | 0,6      | 4,6,7      |
| 9             |    | 3 | 0,8      | 4,7,9      |
| 10            |    | 3 | 1        | 4,7,9      |
| 11            | 15 | 4 | 0,2      | 3,4,6,7    |
| 12            |    | 4 | 0,4      | 4,6,7,8    |
| 13            |    | 4 | 0,6      | 4,6,7,8    |
| 14            |    | 4 | 0,8      | 4,6,7,9    |
| 15            |    | 4 | 1        | 1,4,7,9    |
| 16            |    | 2 | 0,2      | 4,12       |
| 17            |    | 2 | 0,4      | 4,12       |
| 18            |    | 2 | 0,6      | 4,12       |
| 19            |    | 2 | 0,8      | 4,11       |
| 20            |    | 2 | 1        | 4,11       |
| 21            | 20 | 3 | 0,2      | 4,7,12     |
| 22            |    | 3 | 0,4      | 4,7,12     |
| 23            |    | 3 | 0,6      | 4,7,12     |
| 24            |    | 3 | 0,8      | 4,7,8      |
| 25            |    | 3 | 1        | 4,7,8      |
| 26            |    | 4 | 0,2      | 4,7,12,14  |
| 27            |    | 4 | 0,4      | 4,7,12,14  |
| 28            |    | 4 | 0,6      | 1,4,7,12   |
| 29            |    | 4 | 0,8      | 1,4,7,8    |
| 30            |    | 4 | 1        | 1,4,7,8    |
| 31            | 20 | 2 | 0,2      | 4,17       |
| 32            |    | 2 | 0,4      | 4,17       |
| 33            |    | 2 | 0,6      | 4,17       |
| 34            |    | 2 | 0,8      | 4,17       |
| 35            |    | 2 | 1        | 4,2        |
| 36            |    | 3 | 0,2      | 4,12,17    |
| 37            |    | 3 | 0,4      | 4,12,17    |
| 38            |    | 3 | 0,6      | 4,12,17    |
| 39            |    | 3 | 0,8      | 4,8,17     |
| 40            |    | 3 | 1        | 4,11,20    |
| 41            |    | 4 | 0,2      | 4,12,16,17 |
| 42            |    | 4 | 0,4      | 1,4,12,17  |

| Sample Number | n  | p | $\alpha$ | Facility   |
|---------------|----|---|----------|------------|
| 43            | 25 | 4 | 0,6      | 1,4,12,17  |
| 44            |    | 4 | 0,8      | 1,4,8,17   |
| 45            |    | 4 | 1        | 4,8,13,20  |
| 46            |    | 2 | 0,2      | 12,20      |
| 47            |    | 2 | 0,4      | 12,20      |
| 48            |    | 2 | 0,6      | 12,20      |
| 49            |    | 2 | 0,8      | 12,20      |
| 50            |    | 2 | 1        | 12,20      |
| 51            |    | 3 | 0,2      | 12,17,21   |
| 52            |    | 3 | 0,4      | 4,12,17    |
| 53            |    | 3 | 0,6      | 4,12,17    |
| 54            |    | 3 | 0,8      | 4,12,17    |
| 55            |    | 3 | 1        | 12,18,21   |
| 56            |    | 4 | 0,2      | 4,12,17,24 |
| 57            |    | 4 | 0,4      | 4,12,17,24 |
| 58            |    | 4 | 0,6      | 1,4,12,17  |
| 59            |    | 4 | 0,8      | 1,4,12,17  |
| 60            |    | 4 | 1        | 1,4,12,17  |

## Appendix VI

### The coordinates of the firefighters and the regional service center in SDIS59

The coordinates of the regional service centers: x=1048.5; y=861.1.

The coordinates of the firefighters

| No. | City                 | x      | y      |
|-----|----------------------|--------|--------|
| 1   | DUNKERQUE CIS        | 382,5  | 1269,6 |
| 2   | FORT MARDYCK         | 310,5  | 1265   |
| 3   | GRAVELINES CIS       | 114,9  | 1235,9 |
| 4   | HAZEBROUCK CIS       | 528,8  | 951,9  |
| 5   | ARMENTIERES          | 878,6  | 917,9  |
| 6   | LOMME                | 980,5  | 861,4  |
| 7   | MARCQ EN BAROEUL     | 1069,1 | 906,4  |
| 8   | COMINES              | 1000   | 1000,1 |
| 9   | ROUBAIX              | 1189,4 | 921,1  |
| 10  | TOURCOING            | 1153,8 | 957,7  |
| 11  | LA BASSEE            | 812,7  | 775    |
| 12  | LESQUIN              | 1105,8 | 823,6  |
| 13  | SECLIN               | 1025,4 | 793,8  |
| 14  | HAUBOURDIN           | 977,5  | 845,7  |
| 15  | LILLE BOUVINES       | 1035,6 | 894,2  |
| 16  | LILLE LITRE          | 1048,5 | 861,1  |
| 17  | LILLE MALUS          | 1060,7 | 862,2  |
| 18  | VILLENEUVE D'ASCQ    | 1082,9 | 869,9  |
| 19  | ANZIN                | 1495,2 | 606,2  |
| 20  | DENAIN               | 1369,5 | 560    |
| 21  | DOUCHY LES MINES     | 1382,4 | 537,8  |
| 22  | LE QUESNOY           | 1632,8 | 482,8  |
| 23  | SAINT AMAND LES EAUX | 1426,3 | 680,8  |
| 24  | VALENCIENNES         | 1508,6 | 580,6  |
| 25  | VIEUX CONDE          | 1564,1 | 694,9  |
| 26  | AULNOYE AYMERIES     | 1828,6 | 436,4  |
| 27  | AVESNES SUR HELPE    | 1922,1 | 361,5  |
| 28  | FOURMIES             | 2040,8 | 250,7  |
| 29  | HAUTMONT             | 1915,5 | 480,7  |
| 30  | JEUMONT              | 2096,2 | 531,8  |
| 31  | MAUBEUGE             | 1961,6 | 519,5  |
| 32  | CAMBRAI              | 1224,4 | 407,6  |

| <b>No.</b> | <b>City</b>         | <b>x</b> | <b>y</b> |
|------------|---------------------|----------|----------|
| 33         | CAUDRY              | 1403,2   | 359,2    |
| 34         | WAZIERS             | 1103,3   | 624,1    |
| 35         | ORCHIES             | 1241,5   | 710,8    |
| 36         | SOMAIN              | 1280,4   | 594      |
| 37         | BAILLEUL            | 728,8    | 970      |
| 38         | CASSEL              | 481,2    | 1034,9   |
| 39         | ESTAIRES            | 715,9    | 879,9    |
| 40         | LA GORGUE           | 710,3    | 873,6    |
| 41         | MERVILLE            | 636,5    | 878,3    |
| 42         | METEREN             | 684,7    | 975,8    |
| 43         | RENESECURE          | 364,8    | 963      |
| 44         | STEENVOORDE         | 576,6    | 1046     |
| 45         | BERGUES             | 426,8    | 1203,7   |
| 46         | BOURBOURG           | 193,3    | 1182,3   |
| 47         | BRAY DUNES          | 520,8    | 1306,3   |
| 48         | BOLLEZEELE YSER     | 420      | 1103,9   |
| 49         | COUDEKERQUE BRANCHE | 379,7    | 1254,4   |
| 50         | HONDSCHOOTE         | 579,9    | 1216,5   |
| 51         | LOON PLAGE          | 214,9    | 1231     |
| 52         | DUNKERQUE MALO      | 391,8    | 1281,3   |
| 53         | ROSENDAEL           | 400,1    | 1277,1   |
| 54         | WATTEN              | 205,5    | 1067,2   |
| 55         | WORMHOUT            | 462,4    | 1118,5   |
| 56         | SAINT ANDRE         | 1044,5   | 896,2    |
| 57         | HALLUIN             | 1120,4   | 1017,9   |
| 58         | WASQUEHAL           | 1123,6   | 904,5    |
| 59         | LA MADELEINE        | 1067,5   | 889      |
| 60         | SANTES              | 949,7    | 833      |
| 61         | CYSOING             | 1209,5   | 803,2    |
| 62         | TEMPLEUVE           | 1163,3   | 761,9    |
| 63         | THUMERIES           | 1049,7   | 712,9    |
| 64         | LOOS                | 1008,7   | 850      |
| 65         | FERRIERE LA PETITE  | 2013,6   | 474,8    |
| 66         | JENLAIN             | 1622,7   | 546,4    |
| 67         | MAULDE              | 1426,2   | 738,1    |
| 68         | POIX DU NORD        | 1603,8   | 425,6    |
| 69         | PONT SUR SAMBRE     | 1842,7   | 462,2    |
| 70         | ROEULX              | 1328,5   | 542,3    |
| 71         | SAINS DU NORD       | 2005,4   | 330,4    |
| 72         | SEBOURG             | 1638,5   | 578,4    |
| 73         | ANOR                | 2094,5   | 227,2    |
| 74         | BAVAY               | 1786     | 532,3    |
| 75         | CARTIGNIES          | 1838,5   | 327,6    |
| 76         | FERRIERE LA GRANDE  | 1987     | 489,8    |
| 77         | QUIEVRECHAIN        | 1664,5   | 627,4    |

| No. | City                  | x      | y     |
|-----|-----------------------|--------|-------|
| 78  | SOLRE LE CHATEAU      | 2083,3 | 409,5 |
| 79  | TRELON                | 2097,8 | 294,2 |
| 80  | TRITH SAINT LEGER     | 1479,2 | 559,7 |
| 81  | BOUCHAIN              | 1310,7 | 521,5 |
| 82  | COUSOLRE              | 2145,6 | 480,7 |
| 83  | LANDRECIES            | 1683,8 | 360   |
| 84  | LOUVROIL              | 1956,7 | 499,5 |
| 85  | RAISMES               | 1477,5 | 627,5 |
| 86  | BRUAY SUR L'ESCAUT    | 1534   | 631,2 |
| 87  | ONNAING               | 1593,6 | 621,5 |
| 88  | AVESNES LES AUBERT    | 1374,3 | 432,5 |
| 89  | BEAUVOIS EN CAMBRESIS | 1375,1 | 374,5 |
| 90  | BUSIGNY               | 1463,3 | 269,7 |
| 91  | FLINES LES RACHES     | 1130   | 648,6 |
| 92  | FONTAINE NOTRE DAME   | 1150,5 | 401,1 |
| 93  | GOUZEAUCOURT          | 1118,1 | 290,8 |
| 94  | IWUY                  | 1316,6 | 465,3 |
| 95  | LES RUES LES VIGNES   | 1233   | 328,7 |
| 96  | MARCHIENNE            | 1272,7 | 642,7 |
| 97  | NOYELLE SUR ESCAULT   | 1178,6 | 373,5 |
| 98  | RUMILLY EN CAMBRAISIS | 1215,9 | 363,1 |
| 99  | SAMEON                | 1328,8 | 712,5 |
| 100 | SAULZOIR              | 1436,3 | 476,4 |
| 101 | WALLINCOURT SELVIGNY  | 1336   | 302,5 |
| 102 | LALLAING              | 1163,3 | 624   |
| 103 | PECQUENCOURT          | 1209,4 | 612,7 |
| 104 | ANICHE                | 1244,5 | 566,4 |
| 105 | MARCOING              | 1164,8 | 357,5 |
| 106 | AUBY                  | 1048,7 | 650,6 |
| 107 | LE CATEAU             | 1536,5 | 341,1 |
| 108 | SOLESMES              | 1491,2 | 421   |
| 109 | ARLEUX                | 1097,8 | 515,2 |
| 110 | SIN LE NOBLE          | 1107,3 | 598,9 |

## Appendix VII

### The transport volume between the nodes in GEDECO

(a) The transport volume between suppliers/platforms and agency 1-9

| Agency<br>Supplier | A1    | A2    | A3    | A4    | A5    | A6    | A7    | A8    | A9    |
|--------------------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| S1                 | 4,14  | 1,27  | 3,25  | 2,76  | 2,72  | 2,33  | 1,98  | 1,7   | 1,56  |
| S2                 | 6,62  | 2,04  | 5,21  | 4,41  | 4,36  | 3,73  | 3,17  | 2,72  | 2,49  |
| S3                 | 0,83  | 0,25  | 0,65  | 0,55  | 0,54  | 0,47  | 0,4   | 0,34  | 0,31  |
| S4                 | 13,24 | 4,07  | 10,41 | 8,83  | 8,71  | 7,47  | 6,34  | 5,43  | 4,98  |
| S5                 | 46,34 | 14,26 | 36,44 | 30,89 | 30,5  | 26,14 | 22,18 | 19,01 | 17,43 |
| S6                 | 6,62  | 2,04  | 5,21  | 4,41  | 4,36  | 3,73  | 3,17  | 2,72  | 2,49  |
| S7                 | 19,86 | 6,11  | 15,62 | 13,24 | 13,07 | 11,2  | 9,5   | 8,15  | 7,47  |
| S8                 | 10,76 | 3,31  | 8,46  | 7,17  | 7,08  | 6,07  | 5,15  | 4,41  | 4,05  |
| S9                 | 3,31  | 1,02  | 2,6   | 2,21  | 2,18  | 1,87  | 1,58  | 1,36  | 1,24  |
| S10                | 0,17  | 0,05  | 0,13  | 0,11  | 0,11  | 0,09  | 0,08  | 0,07  | 0,06  |
| S11                | 33,1  | 10,18 | 26,03 | 22,07 | 21,78 | 18,67 | 15,84 | 13,58 | 12,45 |
| S12                | 0,83  | 0,25  | 0,65  | 0,55  | 0,54  | 0,47  | 0,4   | 0,34  | 0,31  |
| S13                | 26,48 | 8,15  | 20,82 | 17,65 | 17,43 | 14,94 | 12,67 | 10,86 | 9,96  |
| S14                | 19,86 | 6,11  | 15,62 | 13,24 | 13,07 | 11,2  | 9,5   | 8,15  | 7,47  |
| S15                | 6,62  | 2,04  | 5,21  | 4,41  | 4,36  | 3,73  | 3,17  | 2,72  | 2,49  |
| S16                | 13,24 | 4,07  | 10,41 | 8,83  | 8,71  | 7,47  | 6,34  | 5,43  | 4,98  |
| S17                | 6,62  | 2,04  | 5,21  | 4,41  | 4,36  | 3,73  | 3,17  | 2,72  | 2,49  |
| S18                | 13,24 | 4,07  | 10,41 | 8,83  | 8,71  | 7,47  | 6,34  | 5,43  | 4,98  |
| S19                | 6,62  | 2,04  | 5,21  | 4,41  | 4,36  | 3,73  | 3,17  | 2,72  | 2,49  |
| S20                | 53,2  | 16,37 | 41,84 | 35,47 | 35,01 | 30,01 | 25,47 | 21,83 | 20,01 |
| S21                | 99,29 | 30,55 | 78,08 | 66,2  | 65,35 | 56,01 | 47,52 | 40,74 | 37,34 |
| S22                | 57,92 | 17,82 | 45,54 | 38,61 | 38,12 | 32,67 | 27,72 | 23,76 | 21,78 |
| S23                | 6,62  | 2,04  | 5,21  | 4,41  | 4,36  | 3,73  | 3,17  | 2,72  | 2,49  |
| S24                | 52,96 | 16,29 | 41,64 | 35,3  | 34,85 | 29,87 | 25,35 | 21,73 | 19,92 |
| S25                | 13,24 | 4,07  | 10,41 | 8,83  | 8,71  | 7,47  | 6,34  | 5,43  | 4,98  |
| S26                | 3,31  | 1,02  | 2,6   | 2,21  | 2,18  | 1,87  | 1,58  | 1,36  | 1,24  |
| S27                | 33,1  | 10,18 | 26,03 | 22,07 | 21,78 | 18,67 | 15,84 | 13,58 | 12,45 |
| S28                | 9,1   | 2,8   | 7,16  | 6,07  | 5,99  | 5,13  | 4,36  | 3,73  | 3,42  |
| S29                | 1,65  | 0,51  | 1,3   | 1,1   | 1,09  | 0,93  | 0,79  | 0,68  | 0,62  |
| S30                | 7,45  | 2,29  | 5,86  | 4,96  | 4,9   | 4,2   | 3,56  | 3,06  | 2,8   |
| S31                | 0,83  | 0,25  | 0,65  | 0,55  | 0,54  | 0,47  | 0,4   | 0,34  | 0,31  |
| S32                | 13,24 | 4,07  | 10,41 | 8,83  | 8,71  | 7,47  | 6,34  | 5,43  | 4,98  |
| S33                | 0,83  | 0,25  | 0,65  | 0,55  | 0,54  | 0,47  | 0,4   | 0,34  | 0,31  |
| S34                | 11,58 | 3,56  | 9,11  | 7,72  | 7,62  | 6,53  | 5,54  | 4,75  | 4,36  |

| Agency<br>Supplier | A1     | A2     | A3     | A4     | A5     | A6     | A7     | A8     | A9     |
|--------------------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| S35                | 2,65   | 0,81   | 2,08   | 1,77   | 1,74   | 1,49   | 1,27   | 1,09   | 1      |
| S36                | 1,65   | 0,51   | 1,3    | 1,1    | 1,09   | 0,93   | 0,79   | 0,68   | 0,62   |
| S37                | 1,65   | 0,51   | 1,3    | 1,1    | 1,09   | 0,93   | 0,79   | 0,68   | 0,62   |
| S38                | 5,79   | 1,78   | 4,55   | 3,86   | 3,81   | 3,27   | 2,77   | 2,38   | 2,18   |
| S39                | 0,33   | 0,1    | 0,26   | 0,22   | 0,22   | 0,19   | 0,16   | 0,14   | 0,12   |
| S40                | 0,74   | 0,23   | 0,59   | 0,5    | 0,49   | 0,42   | 0,36   | 0,31   | 0,28   |
| S41                | 6,62   | 2,04   | 5,21   | 4,41   | 4,36   | 3,73   | 3,17   | 2,72   | 2,49   |
| S42                | 2,07   | 0,64   | 1,63   | 1,38   | 1,36   | 1,17   | 0,99   | 0,85   | 0,78   |
| S43                | 3,72   | 1,15   | 2,93   | 2,48   | 2,45   | 2,1    | 1,78   | 1,53   | 1,4    |
| S44                | 3,72   | 1,15   | 2,93   | 2,48   | 2,45   | 2,1    | 1,78   | 1,53   | 1,4    |
| S45                | 0,83   | 0,25   | 0,65   | 0,55   | 0,54   | 0,47   | 0,4    | 0,34   | 0,31   |
| S46                | 48,82  | 15,02  | 38,39  | 32,55  | 32,13  | 27,54  | 23,37  | 20,03  | 18,36  |
| S47                | 8,27   | 2,55   | 6,51   | 5,52   | 5,45   | 4,67   | 3,96   | 3,39   | 3,11   |
| S48                | 3,31   | 1,02   | 2,6    | 2,21   | 2,18   | 1,87   | 1,58   | 1,36   | 1,24   |
| S49                | 5,79   | 1,78   | 4,55   | 3,86   | 3,81   | 3,27   | 2,77   | 2,38   | 2,18   |
| P1                 | 343,72 | 105,76 | 270,27 | 229,15 | 226,21 | 193,89 | 164,51 | 141,01 | 129,26 |
| P2                 | 293,24 | 90,23  | 230,59 | 195,5  | 192,99 | 165,42 | 140,36 | 120,31 | 110,28 |

(b) The transport volume between suppliers/platforms and agency 10-18

| Agency<br>Supplier | A10   | A11   | A12   | A13   | A14   | A15   | A16   | A17   | A18   |
|--------------------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| S1                 | 2,26  | 2,12  | 1,94  | 0,64  | 1,17  | 1,49  | 2,12  | 0,88  | 1,27  |
| S2                 | 3,62  | 3,39  | 3,11  | 1,02  | 1,87  | 2,38  | 3,39  | 1,41  | 2,04  |
| S3                 | 0,45  | 0,42  | 0,39  | 0,13  | 0,23  | 0,3   | 0,42  | 0,18  | 0,25  |
| S4                 | 7,24  | 6,79  | 6,22  | 2,04  | 3,73  | 4,75  | 6,79  | 2,83  | 4,07  |
| S5                 | 25,35 | 23,76 | 21,78 | 7,13  | 13,07 | 16,63 | 23,76 | 9,9   | 14,26 |
| S6                 | 3,62  | 3,39  | 3,11  | 1,02  | 1,87  | 2,38  | 3,39  | 1,41  | 2,04  |
| S7                 | 10,86 | 10,18 | 9,34  | 3,06  | 5,6   | 7,13  | 10,18 | 4,24  | 6,11  |
| S8                 | 5,88  | 5,52  | 5,06  | 1,65  | 3,03  | 3,86  | 5,52  | 2,3   | 3,31  |
| S9                 | 1,81  | 1,7   | 1,56  | 0,51  | 0,93  | 1,19  | 1,7   | 0,71  | 1,02  |
| S10                | 0,09  | 0,08  | 0,08  | 0,03  | 0,05  | 0,06  | 0,08  | 0,04  | 0,05  |
| S11                | 18,1  | 16,97 | 15,56 | 5,09  | 9,34  | 11,88 | 16,97 | 7,07  | 10,18 |
| S12                | 0,45  | 0,42  | 0,39  | 0,13  | 0,23  | 0,3   | 0,42  | 0,18  | 0,25  |
| S13                | 14,48 | 13,58 | 12,45 | 4,07  | 7,47  | 9,5   | 13,58 | 5,66  | 8,15  |
| S14                | 10,86 | 10,18 | 9,34  | 3,06  | 5,6   | 7,13  | 10,18 | 4,24  | 6,11  |
| S15                | 3,62  | 3,39  | 3,11  | 1,02  | 1,87  | 2,38  | 3,39  | 1,41  | 2,04  |
| S16                | 7,24  | 6,79  | 6,22  | 2,04  | 3,73  | 4,75  | 6,79  | 2,83  | 4,07  |
| S17                | 3,62  | 3,39  | 3,11  | 1,02  | 1,87  | 2,38  | 3,39  | 1,41  | 2,04  |
| S18                | 7,24  | 6,79  | 6,22  | 2,04  | 3,73  | 4,75  | 6,79  | 2,83  | 4,07  |
| S19                | 3,62  | 3,39  | 3,11  | 1,02  | 1,87  | 2,38  | 3,39  | 1,41  | 2,04  |
| S20                | 29,1  | 27,28 | 25,01 | 8,19  | 15,01 | 19,1  | 27,28 | 11,37 | 16,37 |
| S21                | 54,31 | 50,92 | 46,68 | 15,28 | 28,01 | 35,64 | 50,92 | 21,22 | 30,55 |
| S22                | 31,68 | 29,7  | 27,23 | 8,91  | 16,34 | 20,79 | 29,7  | 12,38 | 17,82 |
| S23                | 3,62  | 3,39  | 3,11  | 1,02  | 1,87  | 2,38  | 3,39  | 1,41  | 2,04  |
| S24                | 28,97 | 27,16 | 24,89 | 8,15  | 14,94 | 19,01 | 27,16 | 11,32 | 16,29 |
| S25                | 7,24  | 6,79  | 6,22  | 2,04  | 3,73  | 4,75  | 6,79  | 2,83  | 4,07  |
| S26                | 1,81  | 1,7   | 1,56  | 0,51  | 0,93  | 1,19  | 1,7   | 0,71  | 1,02  |
| S27                | 18,1  | 16,97 | 15,56 | 5,09  | 9,34  | 11,88 | 16,97 | 7,07  | 10,18 |
| S28                | 4,98  | 4,67  | 4,28  | 1,4   | 2,57  | 3,27  | 4,67  | 1,94  | 2,8   |
| S29                | 0,91  | 0,85  | 0,78  | 0,25  | 0,47  | 0,59  | 0,85  | 0,35  | 0,51  |
| S30                | 4,07  | 3,82  | 3,5   | 1,15  | 2,1   | 2,67  | 3,82  | 1,59  | 2,29  |
| S31                | 0,45  | 0,42  | 0,39  | 0,13  | 0,23  | 0,3   | 0,42  | 0,18  | 0,25  |
| S32                | 7,24  | 6,79  | 6,22  | 2,04  | 3,73  | 4,75  | 6,79  | 2,83  | 4,07  |
| S33                | 0,45  | 0,42  | 0,39  | 0,13  | 0,23  | 0,3   | 0,42  | 0,18  | 0,25  |
| S34                | 6,34  | 5,94  | 5,45  | 1,78  | 3,27  | 4,16  | 5,94  | 2,48  | 3,56  |
| S35                | 1,45  | 1,36  | 1,24  | 0,41  | 0,75  | 0,95  | 1,36  | 0,57  | 0,81  |
| S36                | 0,91  | 0,85  | 0,78  | 0,25  | 0,47  | 0,59  | 0,85  | 0,35  | 0,51  |
| S37                | 0,91  | 0,85  | 0,78  | 0,25  | 0,47  | 0,59  | 0,85  | 0,35  | 0,51  |
| S38                | 3,17  | 2,97  | 2,72  | 0,89  | 1,63  | 2,08  | 2,97  | 1,24  | 1,78  |
| S39                | 0,18  | 0,17  | 0,16  | 0,05  | 0,09  | 0,12  | 0,17  | 0,07  | 0,1   |
| S40                | 0,41  | 0,38  | 0,35  | 0,11  | 0,21  | 0,27  | 0,38  | 0,16  | 0,23  |
| S41                | 3,62  | 3,39  | 3,11  | 1,02  | 1,87  | 2,38  | 3,39  | 1,41  | 2,04  |
| S42                | 2,04  | 1,91  | 1,75  | 0,57  | 1,05  | 1,34  | 1,91  | 0,8   | 1,15  |

| Agency<br>Supplier | A10    | A11    | A12    | A13   | A14   | A15    | A16    | A17   | A18    |
|--------------------|--------|--------|--------|-------|-------|--------|--------|-------|--------|
| S44                | 2,04   | 1,91   | 1,75   | 0,57  | 1,05  | 1,34   | 1,91   | 0,8   | 1,15   |
| S45                | 0,45   | 0,42   | 0,39   | 0,13  | 0,23  | 0,3    | 0,42   | 0,18  | 0,25   |
| S46                | 26,7   | 25,04  | 22,95  | 7,51  | 13,77 | 17,52  | 25,04  | 10,43 | 15,02  |
| S47                | 4,53   | 4,24   | 3,89   | 1,27  | 2,33  | 2,97   | 4,24   | 1,77  | 2,55   |
| S48                | 1,81   | 1,7    | 1,56   | 0,51  | 0,93  | 1,19   | 1,7    | 0,71  | 1,02   |
| S49                | 3,17   | 2,97   | 2,72   | 0,89  | 1,63  | 2,08   | 2,97   | 1,24  | 1,78   |
| P1                 | 188,02 | 176,27 | 161,58 | 52,88 | 96,95 | 123,39 | 176,27 | 73,44 | 105,76 |
| P2                 | 160,41 | 150,38 | 137,85 | 45,11 | 82,71 | 105,27 | 150,38 | 62,66 | 90,23  |

(c) The transport volume between suppliers/platforms and agency 19-26

| Agency<br>Supplier | A19   | A20   | A21   | A22   | A23   | A24   | A25  | A26   |
|--------------------|-------|-------|-------|-------|-------|-------|------|-------|
| S1                 | 1,73  | 2,02  | 1,87  | 2,09  | 2,09  | 1,41  | 0,32 | 2,86  |
| S2                 | 2,77  | 3,22  | 3     | 3,34  | 3,34  | 2,26  | 0,51 | 4,58  |
| S3                 | 0,35  | 0,4   | 0,37  | 0,42  | 0,42  | 0,28  | 0,06 | 0,57  |
| S4                 | 5,54  | 6,45  | 6     | 6,68  | 6,68  | 4,53  | 1,02 | 9,17  |
| S5                 | 19,41 | 22,57 | 20,99 | 23,37 | 23,37 | 15,84 | 3,56 | 32,08 |
| S6                 | 2,77  | 3,22  | 3     | 3,34  | 3,34  | 2,26  | 0,51 | 4,58  |
| S7                 | 8,32  | 9,67  | 9     | 10,01 | 10,01 | 6,79  | 1,53 | 13,75 |
| S8                 | 4,5   | 5,24  | 4,87  | 5,42  | 5,42  | 3,68  | 0,83 | 7,45  |
| S9                 | 1,39  | 1,61  | 1,5   | 1,67  | 1,67  | 1,13  | 0,25 | 2,29  |
| S10                | 0,07  | 0,08  | 0,07  | 0,08  | 0,08  | 0,06  | 0,01 | 0,11  |
| S11                | 13,86 | 16,12 | 14,99 | 16,69 | 16,69 | 11,32 | 2,55 | 22,91 |
| S12                | 0,35  | 0,4   | 0,37  | 0,42  | 0,42  | 0,28  | 0,06 | 0,57  |
| S13                | 11,09 | 12,9  | 11,99 | 13,35 | 13,35 | 9,05  | 4,07 | 36,66 |
| S14                | 8,32  | 9,67  | 9     | 10,01 | 10,01 | 6,79  | 1,53 | 13,75 |
| S15                | 2,77  | 3,22  | 3     | 3,34  | 3,34  | 2,26  | 0,51 | 4,58  |
| S16                | 5,54  | 6,45  | 6     | 6,68  | 6,68  | 4,53  | 1,02 | 9,17  |
| S17                | 2,77  | 3,22  | 3     | 3,34  | 3,34  | 2,26  | 0,51 | 4,58  |
| S18                | 5,54  | 6,45  | 6     | 6,68  | 6,68  | 4,53  | 1,02 | 9,17  |
| S19                | 2,77  | 3,22  | 3     | 3,34  | 3,34  | 2,26  | 0,51 | 4,58  |
| S20                | 22,28 | 25,92 | 24,1  | 26,83 | 26,83 | 18,19 | 4,09 | 36,83 |
| S21                | 41,58 | 48,37 | 44,98 | 50,07 | 50,07 | 33,95 | 7,64 | 68,74 |
| S22                | 24,26 | 28,22 | 26,24 | 29,21 | 29,21 | 19,8  | 4,46 | 40,1  |
| S23                | 2,77  | 3,22  | 3     | 3,34  | 3,34  | 2,26  | 0,51 | 4,58  |
| S24                | 22,18 | 25,8  | 23,99 | 26,7  | 26,7  | 18,1  | 4,07 | 36,66 |
| S25                | 5,54  | 6,45  | 6     | 6,68  | 6,68  | 4,53  | 1,02 | 9,17  |
| S26                | 1,39  | 1,61  | 1,5   | 1,67  | 1,67  | 1,13  | 0,25 | 2,29  |
| S27                | 13,86 | 16,12 | 14,99 | 16,69 | 16,69 | 11,32 | 2,55 | 22,91 |
| S28                | 3,81  | 4,43  | 4,12  | 4,59  | 4,59  | 3,11  | 0,7  | 6,3   |
| S29                | 0,69  | 0,81  | 0,75  | 0,83  | 0,83  | 0,57  | 0,13 | 1,15  |
| S30                | 3,12  | 3,63  | 3,37  | 3,76  | 3,76  | 2,55  | 0,57 | 5,16  |
| S31                | 0,35  | 0,4   | 0,37  | 0,42  | 0,42  | 0,28  | 0,06 | 0,57  |
| S32                | 5,54  | 6,45  | 6     | 6,68  | 6,68  | 4,53  | 1,02 | 9,17  |
| S33                | 0,35  | 0,4   | 0,37  | 0,42  | 0,42  | 0,28  | 0,06 | 0,57  |
| S34                | 4,85  | 5,64  | 5,25  | 5,84  | 5,84  | 3,96  | 0,89 | 8,02  |
| S35                | 1,11  | 1,29  | 1,2   | 1,34  | 1,34  | 0,91  | 0,2  | 1,83  |
| S36                | 0,69  | 0,81  | 0,75  | 0,83  | 0,83  | 0,57  | 0,13 | 1,15  |
| S37                | 0,69  | 0,81  | 0,75  | 0,83  | 0,83  | 0,57  | 0,13 | 1,15  |
| S38                | 2,43  | 2,82  | 2,62  | 2,92  | 2,92  | 1,98  | 0,45 | 4,01  |
| S39                | 0,14  | 0,16  | 0,15  | 0,17  | 0,17  | 0,11  | 0,03 | 0,23  |
| S40                | 0,31  | 0,36  | 0,34  | 0,38  | 0,38  | 0,25  | 0,06 | 0,52  |
| S41                | 2,77  | 3,22  | 3     | 3,34  | 3,34  | 2,26  | 0,51 | 4,58  |
| S42                | 1,56  | 1,81  | 1,69  | 1,88  | 1,88  | 1,27  | 0,29 | 2,58  |

| Agency<br>Supplier | A19    | A20    | A21    | A22    | A23    | A24    | A25   | A26    |
|--------------------|--------|--------|--------|--------|--------|--------|-------|--------|
| S44                | 1,56   | 1,81   | 1,69   | 1,88   | 1,88   | 1,27   | 0,29  | 2,58   |
| S45                | 0,35   | 0,4    | 0,37   | 0,42   | 0,42   | 0,28   | 0,06  | 0,57   |
| S46                | 20,45  | 23,78  | 22,11  | 24,62  | 24,62  | 16,69  | 3,76  | 33,8   |
| S47                | 3,47   | 4,03   | 3,75   | 4,17   | 4,17   | 2,83   | 0,64  | 5,73   |
| S48                | 1,39   | 1,61   | 1,5    | 1,67   | 1,67   | 1,13   | 0,25  | 2,29   |
| S49                | 2,43   | 2,82   | 2,62   | 2,92   | 2,92   | 1,98   | 0,45  | 4,01   |
| P1                 | 143,95 | 167,45 | 155,7  | 173,33 | 173,33 | 117,51 | 26,44 | 237,96 |
| P2                 | 122,81 | 142,86 | 132,84 | 147,88 | 147,88 | 100,25 | 22,56 | 203,02 |

## Appendix VIII

### The transport cost between the nodes in GEDECO

(a) The transportation cost between supplier and platform

| <b>Platform</b> |           |           |
|-----------------|-----------|-----------|
| <b>Supplier</b> | <b>P1</b> | <b>P2</b> |
| S1              | 442       | 309       |
| S2              | 503       | 382       |
| S3              | 198       | 370       |
| S4              | 350       | 220       |
| S5              | 492       | 350       |
| S6              | 198       | 360       |
| S7              | 380       | 350       |
| S8              | 550       | 382       |
| S9              | 499       | 370       |
| S10             | 350       | 198       |
| S11             | 201       | 350       |
| S12             | 380       | 198       |
| S13             | 420       | 357       |
| S14             | 201       | 360       |
| S15             | 201       | 360       |
| S16             | 506       | 380       |
| S17             | 506       | 380       |
| S18             | 201       | 360       |
| S19             | 201       | 360       |
| S20             | 590       | 405       |
| S21             | 494       | 320       |
| S22             | 591       | 380       |
| S23             | 697       | 625       |
| S24             | 535       | 406       |
| S25             | 567       | 198       |
| S26             | 645       | 439       |
| S27             | 201       | 360       |
| S28             | 450       | 364       |
| S29             | 380       | 346       |
| S30             | 198       | 370       |
| S31             | 320       | 300       |
| S32             | 488       | 512       |
| S33             | 526       | 380       |
| S34             | 551       | 420       |
| S35             | 370       | 198       |
| S36             | 562       | 507       |
| S37             | 507       | 380       |
| S38             | 561       | 502       |
| S39             | 885       | 617       |
| <b>Platform</b> | <b>P1</b> | <b>P2</b> |

| <b>Supplier</b> |     |     |
|-----------------|-----|-----|
| S40             | 744 | 459 |
| S41             | 420 | 354 |
| S42             | 494 | 354 |
| S43             | 350 | 198 |
| S44             | 350 | 300 |
| S45             | 459 | 380 |
| S46             | 198 | 320 |
| S47             | 198 | 320 |
| S48             | 198 | 198 |
| S49             | 198 | 320 |

(b) The transportation cost between agency 1-26 and platform

| <b>Agency</b> | <b>Platform</b> | <b>P1</b> | <b>P2</b> |
|---------------|-----------------|-----------|-----------|
| A1            |                 | 526       | 380       |
| A2            |                 | 442       | 309       |
| A3            |                 | 198       | 320       |
| A4            |                 | 350       | 198       |
| A5            |                 | 562       | 507       |
| A6            |                 | 503       | 382       |
| A7            |                 | 198       | 370       |
| A8            |                 | 503       | 435       |
| A9            |                 | 1255      | 866       |
| A10           |                 | 1380      | 850       |
| A11           |                 | 1013      | 821       |
| A12           |                 | 335       | 381       |
| A13           |                 | 473       | 487       |
| A14           |                 | 571       | 469       |
| A15           |                 | 433       | 300       |
| A16           |                 | 1400      | 975       |
| A17           |                 | 1300      | 839       |
| A18           |                 | 1000      | 724       |
| A19           |                 | 731       | 598       |
| A20           |                 | 640       | 460       |
| A21           |                 | 700       | 477       |
| A22           |                 | 600       | 380       |
| A23           |                 | 385       | 368       |
| A24           |                 | 500       | 415       |
| A25           |                 | 183       | 307       |
| A26           |                 | 183       | 307       |

(c) The transportation cost between platforms

| <b>Platform</b><br><b>Platform</b> | <b>Platform</b> |           |
|------------------------------------|-----------------|-----------|
|                                    | <b>P1</b>       | <b>P2</b> |
| P1                                 | 0               | 598       |
| P2                                 | 598             | 0         |

# RESUME ETENDU EN FRANCAIS

## CHAPITRE 1 : INTRODUCTION

Les entreprises de transport et de distribution sont confrontées à des difficultés d'exploitation liées à la taille et à la complexité de leur processus de livraison. Il s'agit globalement de minimiser les coûts logistiques tout en assurant une prestation de qualité pour les clients. Le modèle sous-jacent est un graphe  $G = (V, E)$ . Les nœuds de  $V$  représentent les points origines et destinations, les entrepôts, agences et hubs intermédiaires. Les arcs de  $E$  représentent les transports possibles entre ces nœuds pondérés par le coût de transport et, en phase opérationnelle, la capacité des moyens de transport mis en œuvre. Dans cette problématique, nous proposons une approche globale du Problème Général de Livraison (appelé ici GDP pour General Delivery Problem).

Au niveau méthodologique, c'est une approche à la fois **hiérarchique** et **structurée**. L'aspect hiérarchique (chapitre 3) consiste à hiérarchiser les prises de décision en trois niveaux (stratégique, tactique et opérationnel). L'aspect structuré (chapitre 4) consiste à concevoir et à exploiter un GDP en le décomposant en problèmes de livraisons élémentaires identifiés et le plus possible indépendants les uns des autres (problèmes de transport, de hubs, d'agences, de tournées...).

Au niveau algorithmique, les modèles et algorithmes de résolution ont été proposés pour résoudre ces problèmes élémentaires de livraison dans la phase opérationnelle en tenant compte, en particulier, du nombre et de la capacité limités des moyens de transport (chapitres 5, 6 et 7).

Au niveau applicatif, deux exemples réels sont traités : le système de livraison des casernes de pompiers du Nord de la France à partir de la pharmacie centrale de Lille (chapitre 8) et le système de livraison d'une entreprise de Vente à Distance (chapitre 9).

## CHAPITRE 2 : BIBLIOGRAPHIE

Peu de travaux appréhendent les GDP dans leur globalité. En général, les études privilégient un niveau de prise de décision et/ou une partie du processus de livraison.

Au niveau stratégique, on peut citer les problèmes de conception de réseaux (Network Design Problem), de localisation d'entrepôts ou de hubs (Facility Location Problem et Hub Location Problem).

Le niveau tactique concerne le « Service Network Design » et le problème d'allocation de hubs (Hub Allocation Problem).

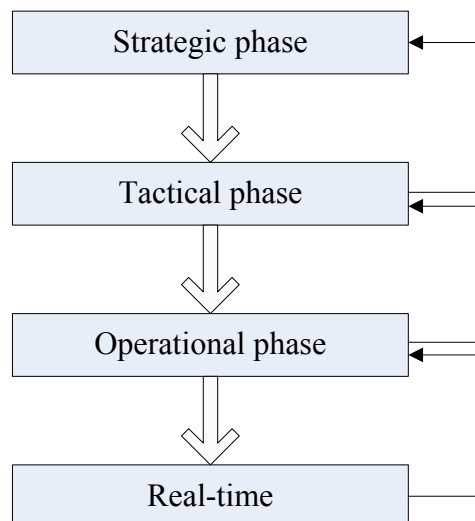
De nombreux travaux s'inscrivent au niveau opérationnel : résolution de problèmes de transport, de voyageur de commerce, et en particulier les problèmes de tournées où l'on tient

compte du nombre et de la capacité limités des véhicules utilisés (Capacited Vehicle Routing Problem).

La formulation de ces problèmes aboutit en général à un système de contraintes linéaires avec des variables continues (les volumes transportés) et des variables discrètes (booléennes ou entières) qui mènent à des problèmes NP-difficiles. De plus, la taille du système de livraison induit un nombre très important de contraintes et de variables. En pratique, des méthodes de résolution exactes peuvent être envisagées pour des problèmes de petite taille. Par contre, les problèmes réels de grande taille nécessitent l'usage d'heuristiques ou de méta-heuristiques.

### CHAPITRE 3 : PROCEDURE DE RESOLUTION

La procédure de résolution (figure 3.2) consiste à hiérarchiser l'ensemble des décisions en trois niveaux (stratégique, tactique et opérationnel) et à résoudre séquentiellement ces niveaux. Les résultats des décisions prises à un niveau deviennent des données pour le niveau inférieur. Cette approche traditionnelle en gestion de production s'étend désormais à l'ensemble de la chaîne logistique et peut, en particulier, s'appliquer au GDP.



**Fig. 3.2** Les différentes phases du processus décisionnel

Le niveau stratégique concerne les décisions à long terme (plusieurs années) : structure et dimensionnement du réseau de transport, localisation et dimensionnement des hubs, entrepôts et agences.

Le niveau tactique concerne les décisions à moyen terme (un à plusieurs mois) : allocation des flux origine-destination à des entrepôts, agences ou hubs particuliers, détermination des fréquences de transport...

La phase opérationnelle concerne les décisions à court terme (au jour le jour) et en particulier le nombre, le trajet et le chargement des moyens de transport mis en œuvre.

### CHAPITRE 4 : UNE APPROCHE HEURISTIQUE DE RESOLUTION D'UN GDP

#### 4.1 Introduction

Nous proposons de décomposer un GDP en problèmes de livraisons élémentaires identifiés et le plus possible indépendants les uns des autres. Cette méthode structurée peut être utilisée en cas de conception d'un nouveau système de transport ou pour exploiter un système déjà

existant. Cette approche structurée s'applique conjointement à l'approche hiérarchique présentée au chapitre 3. On utilisera donc notre méthode de décomposition au niveau stratégique puis tactique et enfin opérationnel.

## 4.2 L'approche de décomposition

Notre approche heuristique structurée (figure 4.1) s'opère en trois phases :

**Phase 1 : décomposition.** On divise l'ensemble des nœuds du GDP initial en K sous-ensembles en utilisant les techniques de décomposition spécifiques au niveau concerné. Chaque sous-ensemble doit correspondre à un problème élémentaire identifié qu'on sait résoudre. Sinon, le sous-problème devient un nouveau problème original auquel on applique récursivement la méthode structurée (figure 4.2).

**Phase 2 : résolution.** Chaque sous-problème est résolu avec les outils, algorithmes ou heuristiques existants. La solution du GDP initial est la concaténation (ou la somme) des solutions des sous-problèmes.

**Phase 3 : amélioration.** Si les sous-problèmes identifiés dans la phase 1 ne sont pas indépendants les uns des autres, une heuristique spécifique (2-opt, 3-opt, Lin-Kernighan...) ou une méta-heuristique (recuit simulé ou autre) peut être utilisée pour améliorer la solution globale.

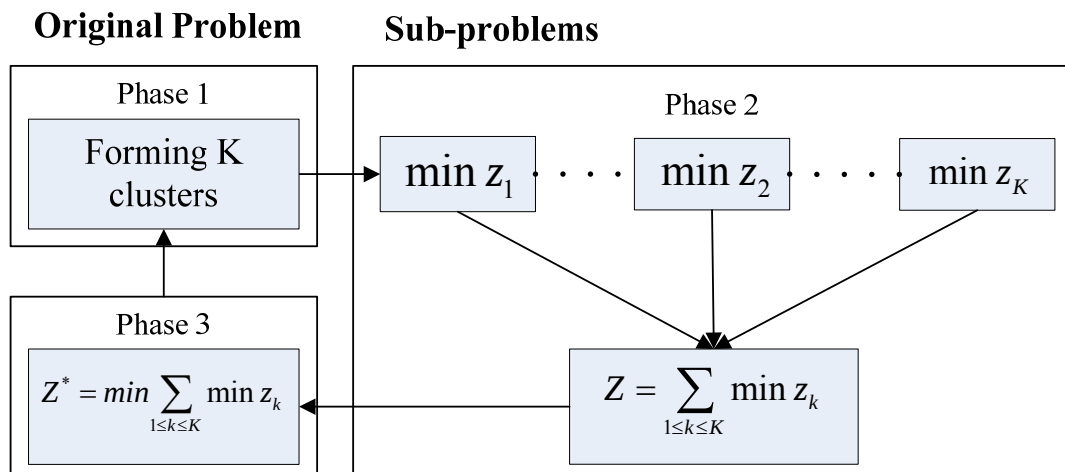


Fig 4.1 L'approche structurée de résolution

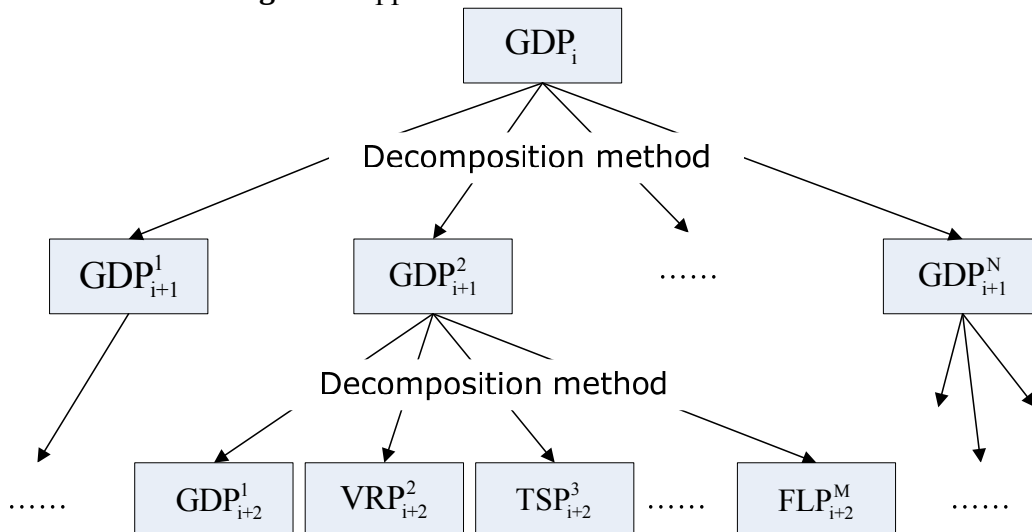


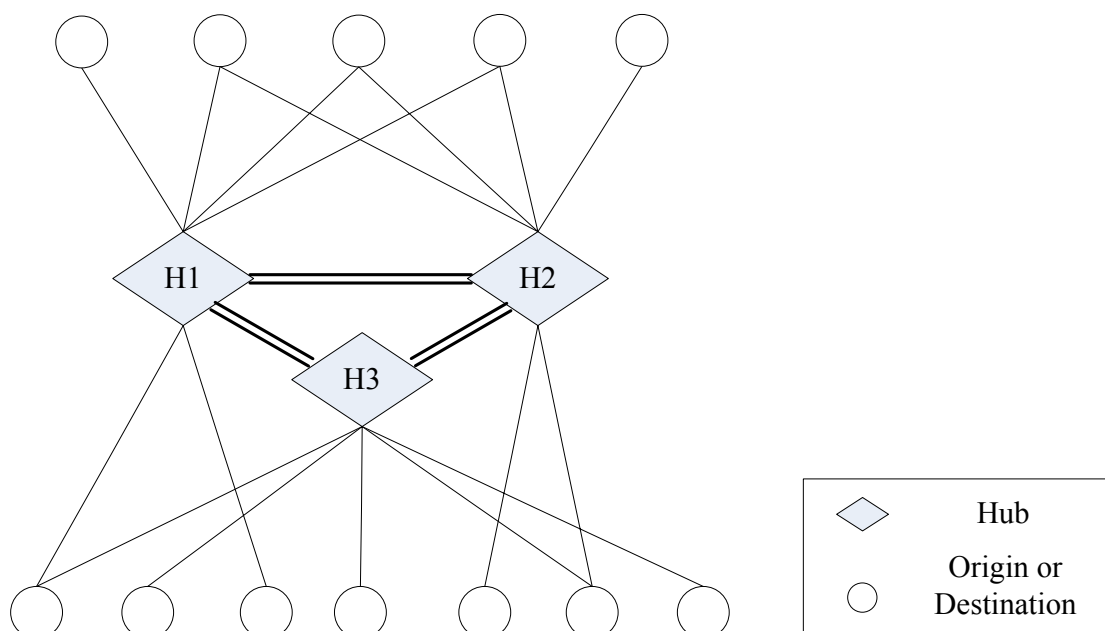
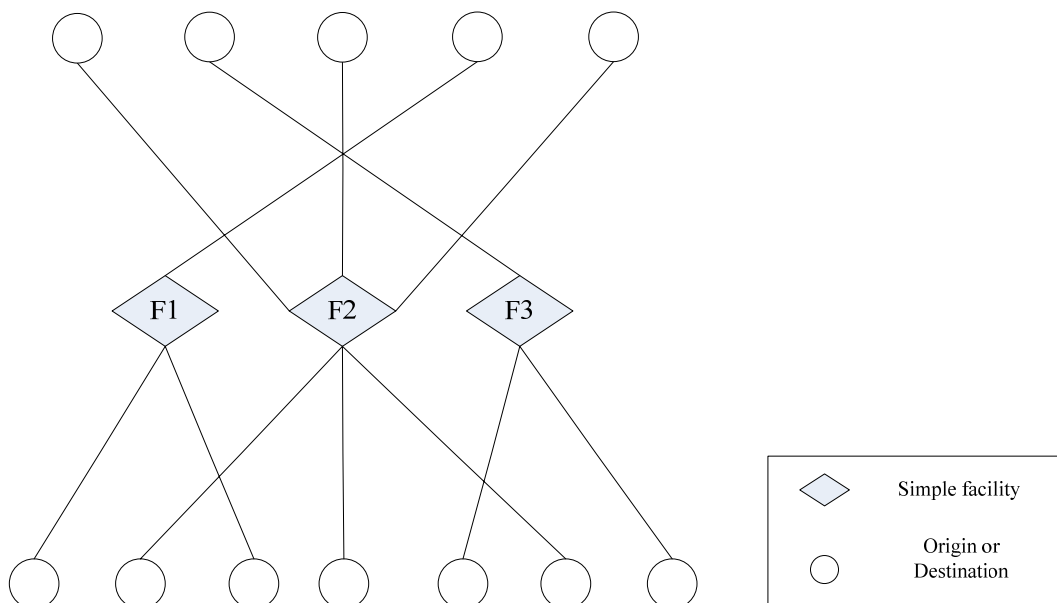
Fig. 4.2 Processus de décomposition d'un GDP en GDP élémentaires

### 4.3 La phase de décomposition

Le réseau de transport est qualifié de 0-niveau, 1 niveau, multi-niveaux, selon le nombre de nœuds d'entreposage intermédiaires sur les chemins reliant les nœuds origines et destinations.

Les réseaux 0-niveau regroupent les problèmes de transport (TP), de voyageur de commerce (TSP) et de tournées de livraison (VRP).

Nous distinguons deux types de nœuds d'entreposage intermédiaires : les nœuds simples pour lesquels il n'y a pas d'intra-flots entre nœuds d'un même niveau et les nœuds de type hub permettant les intra-flots. Un réseau 1-niveau simple (simple 1-level network) est composé de nœuds intermédiaires simples (figure 4.8). Un réseau 1-niveau de type hub (hub 1-level network) est composé de nœuds intermédiaires de type hub (figure 4.9).



Au niveau stratégique, les techniques de décomposition (séquence, superposition, agrégation, localisation et allocation) permettent de décomposer un réseau multi-niveaux en réseaux élémentaires.

Au niveau tactique, il s'agit essentiellement de décomposer temporellement le GDP à moyen terme en une succession de GDP à court terme.

Enfin, au niveau opérationnel, les techniques de décomposition (agrégation, allocation et spatiale) permettent encore de réduire la taille des problèmes élémentaires à résoudre.

## **CHAPITRE 5 : RESOLUTION DU CVRP AU NIVEAU OPERATIONNEL**

Les meilleurs algorithmes exacts de résolution du CVRP permettent d'en résoudre des instances d'une centaine de nœuds. Au-delà, nous proposons d'utiliser l'approche heuristique de décomposition présentée au chapitre 4. Nous utilisons le CCA (Capacitated Clustering Algorithm) comme technique de décomposition spatiale afin de décomposer le VRP initial en K problèmes de voyageur de commerce (TSP). Chacun des TSP est ensuite résolu de façon indépendante, en l'occurrence avec le solveur Concorde. La solution globale est ensuite améliorée par Recuit Simulé, la génération d'un voisin se faisant par un opérateur 3-opt.

Afin d'évaluer la pertinence de notre approche, nous traitons l'exemple réel de la livraison des 110 casernes de pompiers du Nord de la France à partir de la pharmacie centrale de Lille (SDIS59).

## **CHAPITRE 6 : RESOLUTION DU PROBLEME D'ALLOCATION POUR UN RESEAU 1-NIVEAU SIMPLE**

Il s'agit d'un réseau à un seul niveau intermédiaire où les nœuds de stockages intermédiaires sont simples, ie. sans intra-flots (figure 4.8).

Nous proposons une nouvelle formulation pour résoudre les problèmes d'allocation unique (T-FSA) ou multiple (T-FMA) aux niveaux stratégiques et tactiques, en tenant compte uniquement des coûts volumiques de transport. Nous proposons les formulations analogues au niveau opérationnel en tenant compte du nombre et de la capacité limités des véhicules utilisés : FSA-VN' et FMA-VN.

Toutes ces formulations sont testées sur le benchmark CAB (pour « Civil Aeronautics Board ») couramment utilisés dans la littérature pour tester les problèmes de hubs et résolues avec le solveur LP/MIP CPLEX 9.0. Dans tous les cas, le coût global est supérieur quand on tient compte du coût des véhicules plutôt que des coûts volumiques. Ceci est dû au fait que les véhicules ne sont pas complètement remplis. D'autre part, la multi-allocation donne de meilleurs résultats que l'allocation simple, mais elle est plus complexe à résoudre.

## **CHAPITRE 7 : RESOLUTION DU PROBLEME D'ALLOCATION POUR UN RESEAU 1-NIVEAU DE TYPE HUB**

Il s'agit d'un réseau à un seul niveau intermédiaire où les nœuds de stockages intermédiaires sont des hubs (figure 4.9).

Nous proposons, comme au chapitre 6, une nouvelle formulation pour résoudre les problèmes d'allocation unique (SApHM-VN3) ou multiple (MApHM-VN2) au niveau opérationnel en tenant compte du nombre et de la capacité limités des véhicules utilisés. Ces formulations sont testées également sur le benchmark CAB et comparées aux formulations de Shorin-Kapov et al. (1996), SApHM-FL et MApHM'. La comparaison de ces diverses formulations donne les mêmes résultats qu'au chapitre 6.

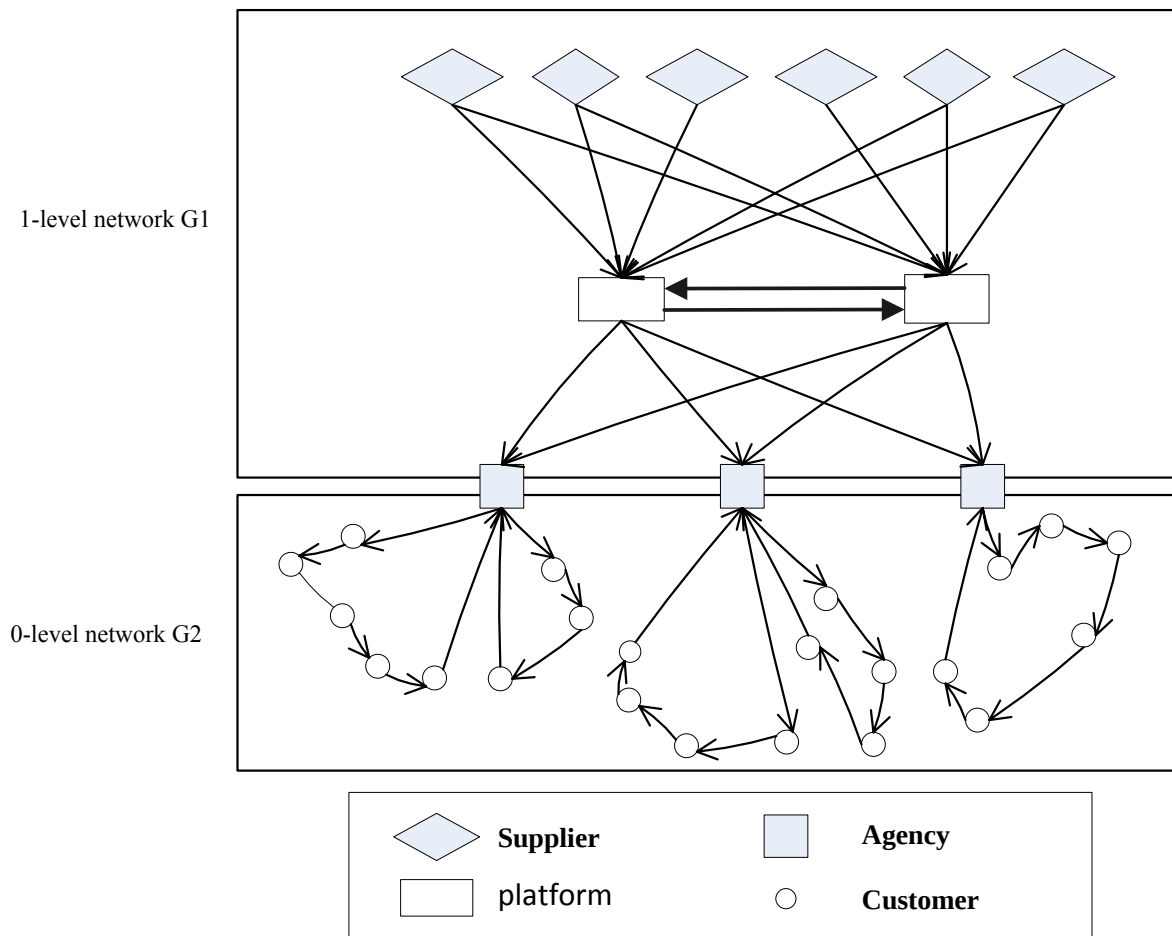
Enfin, si l'on compare les résultats trouvés aux chapitres 6 et 7, il apparaît que les hubs donnent des solutions meilleures que les nœuds simples sur le benchmark CAB.

## CHAPITRE 8 : APPLICATION AU SDIS59

Dans ce chapitre, nous revenons sur le réseau de livraison des 110 casernes de pompiers du Nord de la France à partir de la pharmacie centrale de Lille, exemple déjà présenté et traité au chapitre 5. Notre but est de montrer l'impact des décisions stratégiques sur les solutions mises en œuvre au niveau opérationnel. Nous comparons l'ancienne stratégie 1-niveau qui était en application, une variante, toujours 1-niveau, de cette stratégie et enfin la stratégie 0-niveau présentée au chapitre 5. C'est cette dernière qui donne le moindre coût.

## CHAPITRE 9 : APPLICATION AU CAS GEDECO

GEDECO est la filiale transport d'une société de Vente à Distance, dont les données sont inspirées d'une entreprise réelle de VAD du Nord de la France.



**Fig. 9.2** Le réseau 2-niveaux de GEDECO

Il s'agit d'un réseau 2-niveaux (figure 9.2) composé de 49 fournisseurs, 2 hubs, 26 agences et des milliers de clients. Pour le réseau 1-niveau de type hub, nous avons comparé deux stratégies d'exploitation, d'une part l'allocation unique, plus facile à mettre en œuvre, où chaque fournisseur est relié à un seul hub et où chaque agence n'est livrée que par un seul hub, et d'autre part l'allocation multiple.

Les résultats montrent que la multi-allocation est préférable. Elle permet une meilleure massification des transports (le taux de chargement des camions est meilleur) à moindre coût.

## **CHAPITRE 10 : CONCLUSION**

Notre travail de recherche constitue une contribution pour appréhender les réseaux de livraison de grande taille. La méthodologie hiérarchique et structurée que nous proposons permet de hiérarchiser et de mieux cerner les décisions à prendre, que ce soit en cas de conception (ou de re-conception comme dans l'exemple de SDIS59) d'un nouveau système de transport ou dans le cas de l'exploitation d'un réseau déjà existant (exemple GEDECO). Dans cette approche, un GDP apparaît comme l'assemblage de réseaux élémentaires dont nous proposons la formulation et la résolution.

De nombreuses perspectives s'inscrivent dans la continuité de ce travail :

- une étude plus approfondie des méthodes d'amélioration de la solution globale (phase 3 du chapitre 4)
- la confrontation de notre approche à d'autres cas réels
- la définition d'un véritable guide opérationnel pour assister le décideur lors de la décomposition du système de transport. Un certain nombre de principes de décomposition ont été présentés tout au long de ce mémoire. Leur usage doit être systématisé. Une solution de type atelier de Génie Logiciel peut être envisagée en ce sens.

## ABSTRACT

Transport and delivery companies are confronted by difficulties in their transportation process due to the scale and the complexity of their distribution process. In this context, we propose a comprehensive approach to General Delivery Problem (GDP).

In terms of methodology, it is a **hierarchical** (strategic, tactical and operational) and **structured** approach. It consists of designing and decomposing the GDP into well identified basic delivery problems as independent as possible. These basic transport problems involve the problems about transportation, intermediate facility, agencies, routings, etc.

At the algorithm level, models and solution algorithms have been proposed to solve these basic delivery problems in the operational phase, taking account in particular transportation restriction about the number and capacity of vehicles.

At the application level, two real examples are discussed: one is the delivery system of a delivery company; the other one is the delivery system of the Regional Fire and Emergency Center in the north of France.

## RESUME

Les entreprises de transport et de distribution sont confrontées à des difficultés d'exploitation liée à la taille et à la complexité de leur processus de livraison. Dans cette problématique, nous proposons une approche globale du Problème Général de Livraison (PGL).

Au niveau méthodologique, c'est une approche **hiérarchique** (stratégique, tactique, opérationnelle) et **structurée**. Il s'agit de concevoir et d'exploiter un PGL en le décomposant en problèmes de livraisons élémentaires identifiés et le plus possible indépendants les uns des autres (problèmes de transport, de hubs, d'agences, de tournées...).

Au niveau algorithmique, des modèles et algorithmes de résolution ont été proposés pour résoudre ces problèmes élémentaires de livraison dans la phase opérationnelle en tenant compte, en particulier, du nombre et de la capacité limités des moyens de transport.

Au niveau applicatif, deux exemples réels sont traités : le système de livraison d'une entreprise de Vente à Distance et le système de livraison des casernes de pompiers du Nord de la France à partir de la pharmacie centrale de Lille.